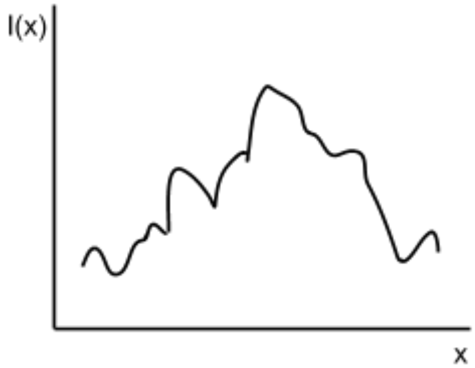
Edge Detection

- What can cause change in intensity of light?
  - Reflectance change
  - Shadows: relatively abrupt change in illumination
  - Occlusions
  - Convex + Concave surface edges
  - Specular highlights (shiny spots)



- How to find changes in intensity?
  - Find changes in intensity by measuring slopes, which is the derivative of the image.
    - f'(x) = $lim_{h\to 1}\frac{f(x+h)-f(x)}{h}$ => $\frac{f(x+1)-f(x)}{1}$
    - Convolution mask [1 | -1] OR [-1 | 1] can be used to find f'(x) horizontally
  - In two dimensional image f(x, y), use the mask both horizontally and vertically
    - $\frac{\partial f(x,y)}{\partial x} = lim_{h\to 1}\frac{f(x+h,y)-f(x,y)}{h}$
    - $\frac{\partial f(x,y)}{\partial y} = lim_{h\to 1}\frac{f(x,y+h)-f(x,y)}{h}$
    - Combining the above two derivatives together, the Gradient is given by
      ∇f = $(\frac{\partial f(x,y)}{\partial x}, \frac{\partial f(x,y)}{\partial y})$
    - Magnitude of the gradient is the 'slope' we need. To find edges we find where the magnitude of the gradient is big.
  - Problem
    - Derivative is very 'local' because we take only two neighboring pixels and calculate the gradient, we may end up finding many wiggles with big gradient too.
  - Consider theoretical case of an image with just two intensities
    - Define step function

$$u(x)=\begin{cases}1 & x>0\\ \frac{1}{2} & x=0\\ 0 & x<0\end{cases}$$

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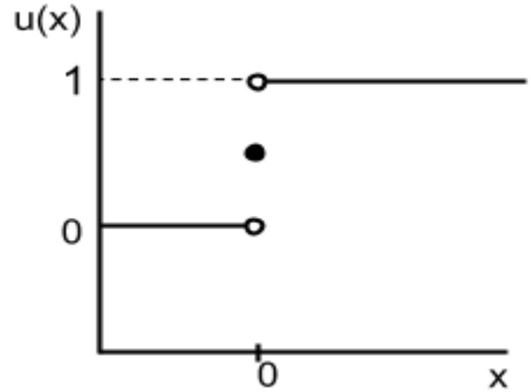
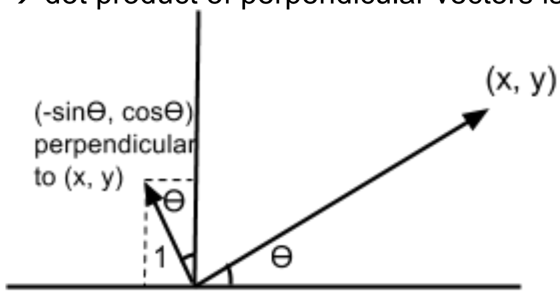
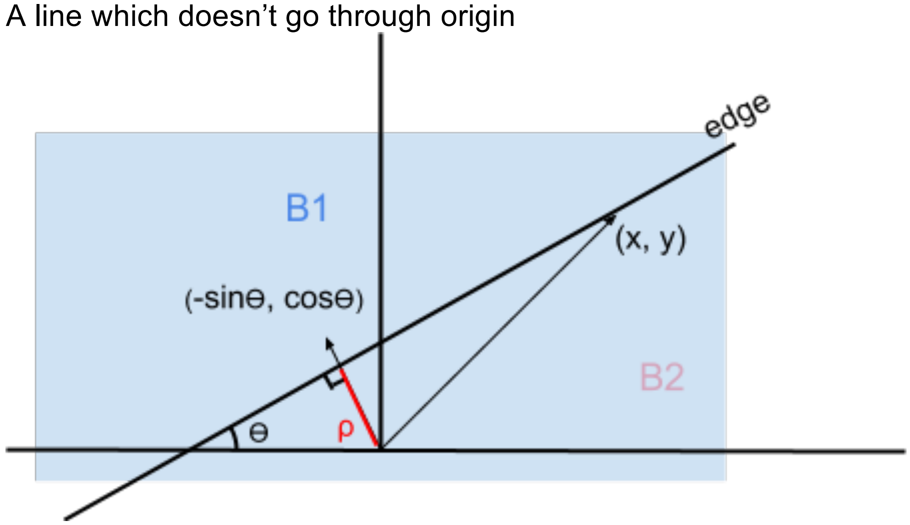


- A line through origin: $(x, y) * (-\sin\theta, \cos\theta) = 0$
  → dot product of perpendicular vectors is zero



- A line which doesn't go through origin



  - $E(x, y) = B_1 + [B_2 - B_1]u(x * sin\theta - y * cos\theta + \rho)$
    $u = 0 => E(x, y) = B_1$
    $u = 1 => E(x, y) = B_2$
    $\frac{\partial E}{\partial x} = (B_2 - B_1)sin\theta * u'(xsin\theta - ycos\theta + \rho)$
    $\frac{\partial E}{\partial y} = (B_2 - B_1)(-cos\theta) * u'(xsin\theta - ycos\theta + \rho)$

- Consider the step function as the limit of the sequence $\{u_\epsilon(x)\}$ where

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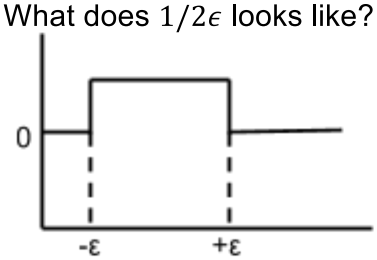
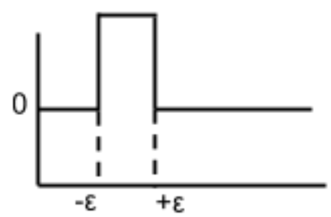
$$u_\epsilon(x)\begin{cases}1 & x>\epsilon\\ \frac{1}{2}(1+{}^{x}/_{\epsilon}) & |x|<\epsilon\\ 1 & x<-\epsilon\end{cases}$$

  - Taking the derivative of each $\{u_\epsilon(x)\}$ gives a sequence the limit of which is the Dirac Delta. We use it as a point source, written as $\boldsymbol{\delta}$
  - As $\epsilon \to 0$

$$u'_\epsilon(x) = \frac{du_\epsilon(x)}{dx} = \begin{cases}\frac{1}{2\epsilon} & |x| \le \epsilon\\ 0 & |x| > \epsilon\end{cases}$$

  - What does $1/2\epsilon$ looks like?





    - When ϵ gets smaller, the central part gets taller
    - Area under the curve = $2\epsilon * (1/2\epsilon)$ = 1
  - $\frac{\partial E}{\partial x} = sin\theta(B_2 - B_1)\delta(xsin\theta - ycos\theta + \rho)$
  - $\frac{\partial E}{\partial y} = -cos\theta(B_2 - B_1)\delta(xsin\theta - ycos\theta + \rho)$
  - δ has a 'sifting' property: $\int_{-\infty}^{+\infty} \delta(x)h(x)dx = h(0)$
  - ∇E = $(\frac{\partial E}{\partial x}, \frac{\partial E}{\partial y})$ = $(B_2 - B_1)(sin\theta - cos\theta)$ → perpendicular to the edge
- Problem
  - The same scene with difference in brightness may result in different gradients. This can be eliminated by threshold, but using a threshold can neglect some local maximums of gradient.
- Find local maxima or minima in I'(x) where I''(x) = 0
  - $\frac{\partial^2 E}{\partial x^2} = sin^2\theta(B_2 - B_1)\delta'(xsin\theta - ycos\theta + \rho)$
  - $\frac{\partial^2 E}{\partial y^2} = cos^2\theta(B_2 - B_1)\delta'(xsin\theta - ycos\theta + \rho)$
  - Laplacian: $\nabla^2 E = \frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} = \delta'(B_2 - B_1)(xsin\theta - ycos\theta + \rho)$
  - To find local maxima/minima we need to locate where $\nabla^2 E$ is 0 by finding where δ' is 0

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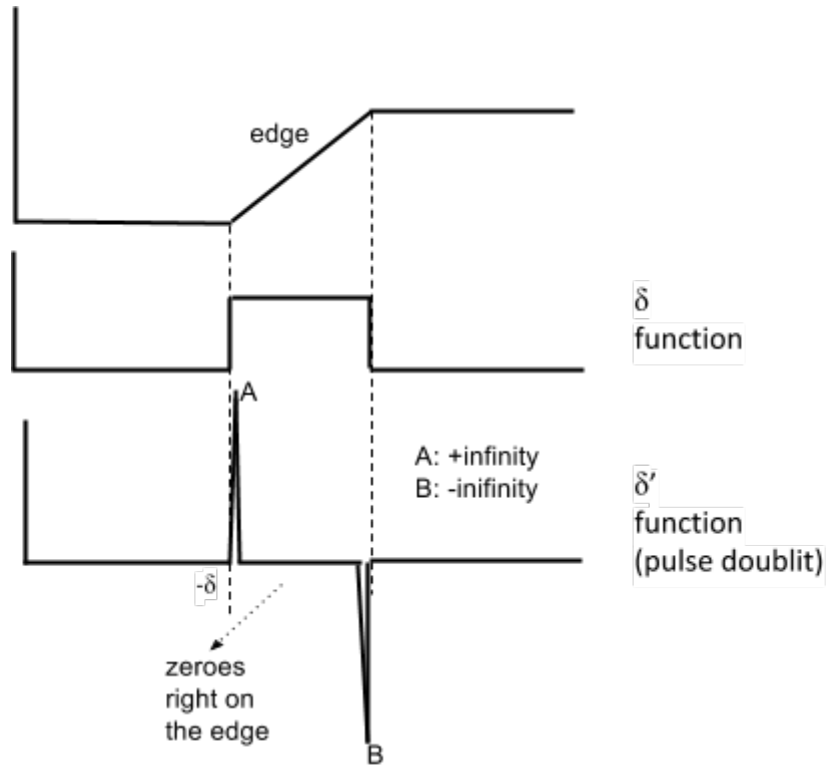

- We look for the zero crossings: the segment between A and B in the above graph is the edge.

- How to find Laplacian in discrete case
  - $\frac{\partial E}{\partial x} = lim_{h \to 1} \frac{E(x+h) - E(x)}{h} = E(x+1) - E(x)$ → use [-1|+1] mask
  - $\frac{\partial^2 E}{\partial x^2} = lim_{h \to 1} \frac{\frac{dE(x+h)}{dx} - \frac{dE(x)}{dx}}{h} = E(x+2) - 2E(x+1) + E(x)$
    → use [1|-2|1] mask
  - Combining the mask horizontally and vertically we get

0	1	0
1	-4	1
0	1	0

❖ Strategy in edge detection: 1. Smooth the image 2. Take the Laplacian
- Smoothing (blurring) the image using weighted average
  - Gaussian in 2D

    $G(x, y) = -\frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$

    $G(r) = -\frac{1}{2\pi\sigma^2} e^{-\frac{r^2}{2\sigma^2}}$ where $r = x^2 + y^2$
  - Input image → Output weighted average of each pixel
  - σ controls the output → the bigger σ is, the bigger the mask is, the more blurring the output is

❖ ∇*(G*E) = (∇*G)*E → Apply the Laplacian of Gaussian to different images

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