Expert Systems

Lecture 5: Fuzzy expert systems

Dr. Mohammed A. Altahrawi

University College of Applied Sciences

December 29, 2024

Outlines

الكلية الجامعية للعلوم التطبيقية
University College of Applied Sciences

Outlines

1. Membership Functions
2. Scikit-Fuzzy Library
3. Example: Managing Project Success

Membership Functions

Fuzzy membership functions represent the degree of membership of an element in a fuzzy set. They map elements in the universe of discourse to values between 0 and 1.

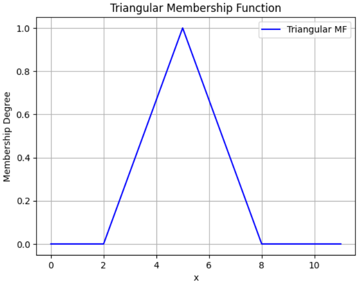
Membership functions are fundamental in fuzzy logic systems and come in various shapes to model different types of relationships.

Triangular Membership Function

Shape: Triangle

Mathematical Equation:

$$\mu(x) = \begin{cases} 0 & x \leq a \text{ or } x \geq c, \\ \frac{x-a}{b-a} & a < x \leq b, \\ \frac{c-x}{c-b} & b < x < c \end{cases}$$



Parameters:

- a: Start of the triangle
- b: Peak of the triangle
- c: End of the triangle

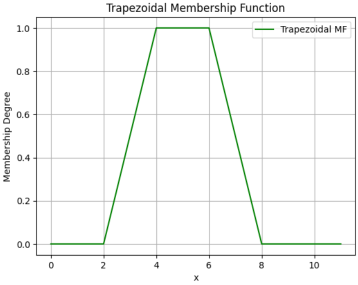
Application: Simple and widely used for gradual transitions.

Trapezoidal Membership Function

Shape: Trapezoid

Mathematical Equation:

$$\mu(x) = \begin{cases} 0 & x \leq a \text{ or } x \geq d, \\ \frac{x-a}{b-a} & a < x \leq b, \\ 1 & b < x \leq c, \\ \frac{d-x}{d-c} & c < x < d \end{cases}$$



Parameters:

- a, b: Left base
- c, d: Right base

Application: Represents situations with a plateau.

Gaussian Membership Function

Shape: Bell Curve

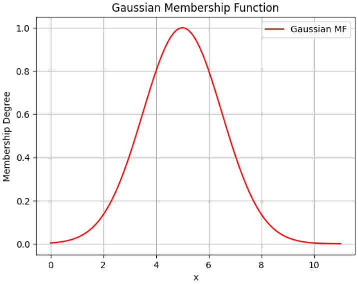
Mathematical Equation:

$$\mu(x) = e^{-\frac{(x-c)^2}{2\sigma^2}}$$

Parameters:

- c: Mean (center)
- σ: Standard deviation (width)

Application: Ideal for smooth transitions and normal distributions.



University College of Applied Sciences

Generalized Bell Membership Function

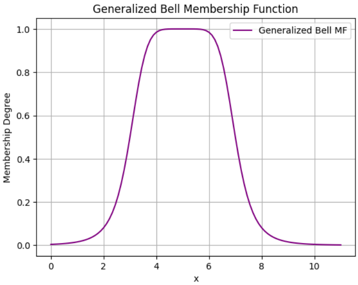
Shape: Generalized Bell Curve

Mathematical Equation:

$$\mu(x) = \frac{1}{1 + \left|\frac{x-c}{a}\right|^{2b}}$$

Parameters:

- a: Width
- b: Slope
- c: Center



Application: Flexible control over width and slope.

Sigmoidal Membership Function

Shape: S-Shaped Curve

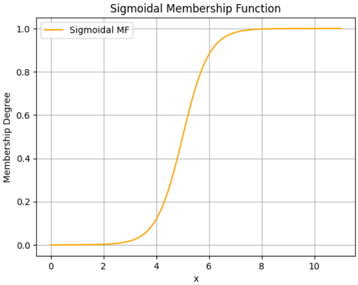
Mathematical Equation:

$$\mu(x) = \frac{1}{1 + e^{-a(x-c)}}$$

Parameters:

- a: Slope
- c: Center

Application: Smooth transitions between states.

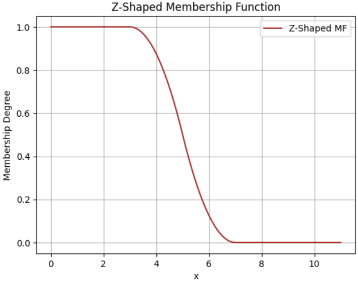


Z-Shaped Membership Function

Shape: Decreasing S-Shaped Curve

Mathematical Equation:

$$\mu(x) = \begin{cases} 1 & x \leq a, \\ 1 - 2\left(\frac{x-a}{b-a}\right)^2 & a < x \leq \frac{a+b}{2}, \\ 2\left(\frac{b-x}{b-a}\right)^2 & \frac{a+b}{2} < x \leq b, \\ 0 & x > b \end{cases}$$



Parameters:

- a: Start point
- b: End point

Application: Smooth decrease.

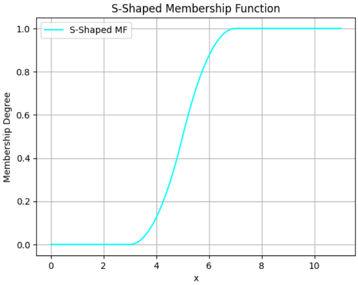
الكلية الجامعية للعلوم التطبيقية
University College of Applied Sciences

S-Shaped Membership Function

Shape: Increasing S-Shaped Curve

Mathematical Equation:

$$\mu(x) = \begin{cases} 0 & x \leq a, \\ 2\left(\frac{x-a}{b-a}\right)^2 & a < x \leq \frac{a+b}{2}, \\ 1 - 2\left(\frac{b-x}{b-a}\right)^2 & \frac{a+b}{2} < x \leq b, \\ 1 & x > b \end{cases}$$



figureS-Shaped Membership Function

Parameters:

- a: Start point
- b: End point

Application: Smooth increase.

Singleton Membership Function

Shape: Vertical Spike

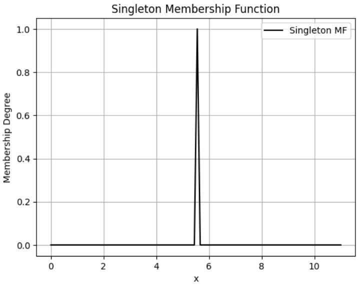
Mathematical Equation:

$$\mu(x) = \begin{cases} 1 & x = c, \\ 0 & x \neq c \end{cases}$$

Parameters:

- c: Singleton point

Application: Represents crisp values.



Outlines

Scikit-Fuzzy Library

What is Scikit-Fuzzy?

- An open-source Python library for implementing fuzzy logic systems.
- Built on top of NumPy and SciPy for fast numerical computations.
- Provides tools for creating and simulating fuzzy inference systems.

Key Features:

- Supports Mamdani and Sugeno fuzzy inference systems.
- Provides membership function generators (e.g., triangular, trapezoidal, Gaussian).
- Includes defuzzification methods (e.g., centroid).
- Offers fuzzy clustering algorithms like c-means.

Documentation:

`https://scikit-fuzzy.github.io/scikit-fuzzy/index.html`

Outlines

Balancing Staffing, Risk, and Funding

- This example illustrates the use of fuzzy logic to define and evaluate key variables in project management: Project Funding, Project Staffing, and Project Risk.
- **Handles Uncertainty:** Allows partial memberships in multiple categories (e.g., normal and high risk).
- **Improves Decision-Making:** Models relationships between inputs (funding, staffing) and output (risk) using fuzzy rules.
- **Enhanced Clarity:** Converts complex scenarios into understandable terms for stakeholders.
- In the following slides, the procedures to build a fuzzy system will be presented in detail.

الكلية الجامعية للعلوم التطبيقية
University College of Applied Sciences

Recap

Steps to build fuzzy system:

1. Define linguistic variables and terms.
2. Construct membership functions.
3. Construct Fuzzy rules.
4. Fuzzification.
5. Rule Evaluation.
6. Aggregation of Rule Consequent.
7. Defuzzification.

الكلية الجامعية للعلوم التطبيقية
University College of Applied Sciences

Step 1: Define linguistic variables and terms

- **For Input:**
  - **Project funding:** Inadequate, Marginal, Adequate
  - **Project staffing:** Small, Large
- **For Output:**
  - **Project risk:** Low, Normal, High

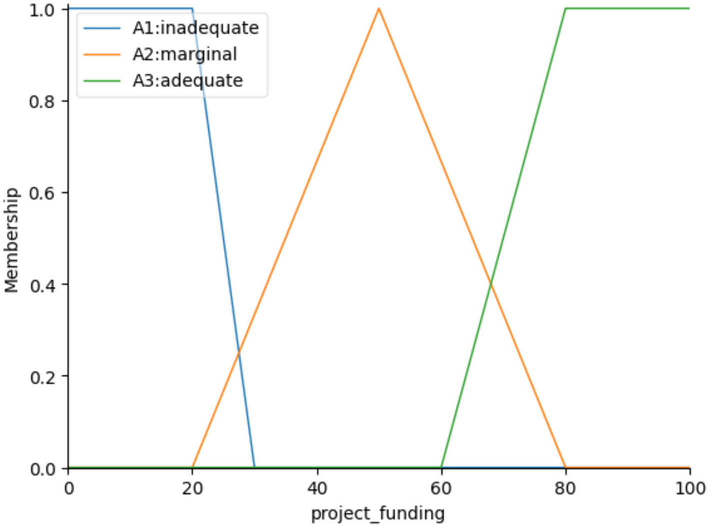
Expert Domain

The linguistic variables and terms are defined and determined by **expert domain**.

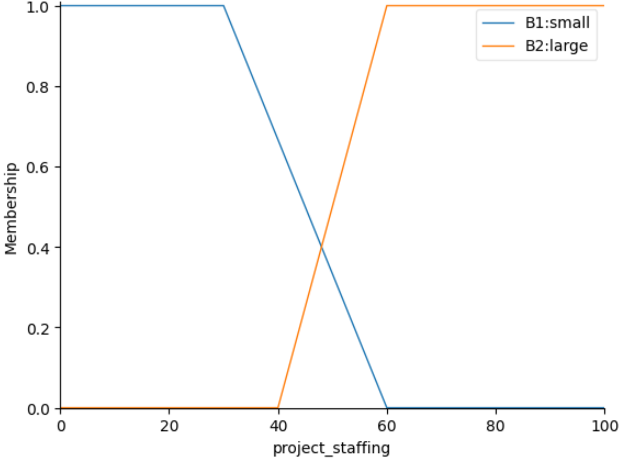
الكلية الجامعية للعلوم التطبيقية
University College of Applied Sciences

Step 2: Construct membership function

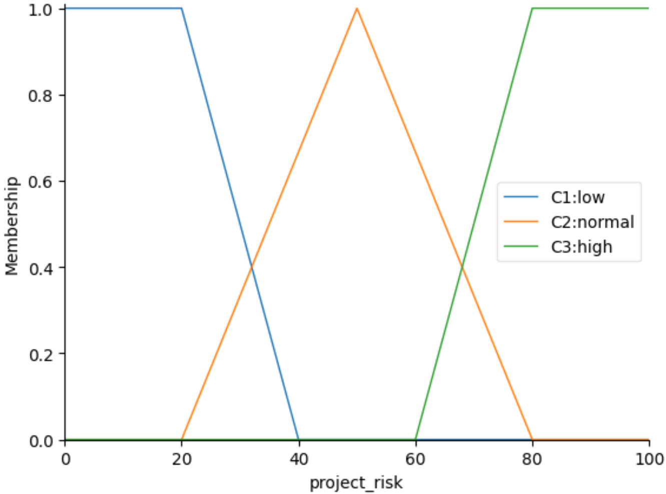
1: Project Funding Membership (A)



2: Project Staffing Membership (B)



3: Project Risk Membership (C)



Step 3: Construct Rules base

Rules:

1. **IF** Project Funding is Adequate **OR** Project Staffing is Small **THEN** Risk is Low.
2. **IF** Project Funding is Marginal **AND** Project Staffing is Large **THEN** Risk is Normal.
3. **IF** Project Funding is Inadequate **THEN** Risk is High.

Step 4: Fuzzification

Definition

Fuzzification is the process of mapping crisp inputs into appropriate fuzzy sets based on their degrees of membership.

Example

- **Crisp Input:** Project funding rated as 26%.
- **Mapped Fuzzy Sets:**
  - $\mu(A1) = 0.4$ (inadequate)
  - $\mu(A2) = 0.2$ (marginal)
- **Crisp Input:** Project staffing rated as 54%.
- **Mapped Fuzzy Sets:**
  - $\mu(B1) = 0.2$ (small)
  - $\mu(B2) = 0.7$ (large)

Step 5: Rule Evaluation

Definition

Rule evaluation determines the result of the antecedent evaluation and applies it to the consequent membership function.

Process

- Fuzzified inputs: $\mu(A1) = 0.4$, $\mu(A2) = 0.2$, $\mu(B1) = 0.2$, $\mu(B2) = 0.7$.
- Logical operation for antecedents:
  - Disjunction (OR): $\mu_{A \cup B}(x) = \max[\mu_A(x), \mu_B(x)]$
  - Conjunction (AND): $\mu_{A \cap B}(x) = \min[\mu_A(x), \mu_B(x)]$

Cont.,

Example Rules

- **Rule 1: IF** Project Funding is Adequate **OR** Project Staffing is Small **THEN** Risk is Low.

$$\text{IF } A \text{ is } A3(0.0) \text{ OR } B \text{ is } B1(0.2) \text{ THEN } C \text{ is } C1(0.2)$$

$$\mu_C(C1) = \max[\mu_A(A3), \mu_B(B1)] = 0.2$$

- **Rule 2: IF** Project Funding is Marginal **AND** Project Staffing is Large **THEN** Risk is Normal.

$$\text{IF } A \text{ is } A2(0.2) \text{ AND } B \text{ is } B2(0.7) \text{ THEN } C \text{ is } C2(0.2)$$

$$\mu_C(C2) = \min[\mu_A(A2), \mu_B(B2)] = 0.2$$

- **Rule 3: IF** Project Funding is Inadequate **THEN** Risk is High.

$$\text{IF } A \text{ is } A1(0.4) \text{ THEN } C \text{ is } C3(0.4)$$

Clipping:

- The membership function is "sliced" at the firing strength (truth value of the rule antecedent).
- Values above the firing strength are set to that level.
- **Advantage:** Simple and computationally efficient.
- **Disadvantage:** Loses the original shape of the membership function.

Scaling:

- The membership function is scaled proportionally by multiplying all values by the firing strength.
- Preserves the shape of the original membership function.
- **Advantage:** Retains more information.
- **Disadvantage:** Computationally more intensive.

الكلية الجامعية للعلوم التطبيقية
University College of Applied Sciences

Numerical Example

Consider a triangular membership function with:

- Base: [0, 5, 10]
- Peak: $\mu(5) = 1.0$

The rule firing strength is 0.7.

Original Membership Values:

x	0	1	2	3	4	5	6	7	8	9	10
$\mu(x)$	0.0	0.2	0.4	0.6	0.8	1.0	0.8	0.6	0.4	0.2	0.0

Clipped Membership Function

Clipping: The membership values above 0.7 are set to 0.7.

Resulting Membership Values:

x	0	1	2	3	4	5	6	7	8	9	10
$\mu(x)$	0.0	0.2	0.4	0.6	0.7	0.7	0.7	0.6	0.4	0.2	0.0

Explanation: The peak is flattened, and information is lost.

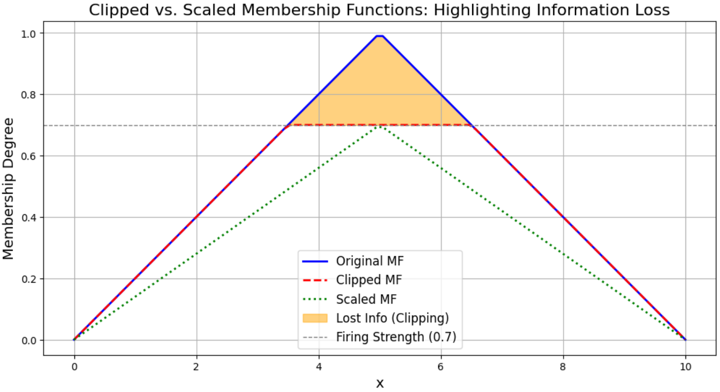
Scaling: Each membership value is multiplied by the firing strength 0.7.

Resulting Membership Values:

x	0	1	2	3	4	5	6	7	8	9	10
$\mu(x)$	0.0	0.14	0.28	0.42	0.56	0.70	0.56	0.42	0.28	0.14	0.0

Explanation: The original shape is preserved but proportionally reduced.

Visualization of Clipping vs Scaling



Comparison of Clipping and Scaling

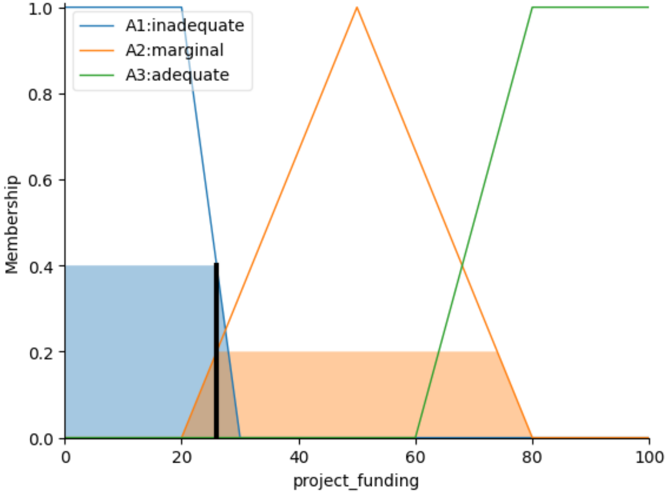
Clipping:

- Simple and efficient.
- Information loss at the peak of the membership function.

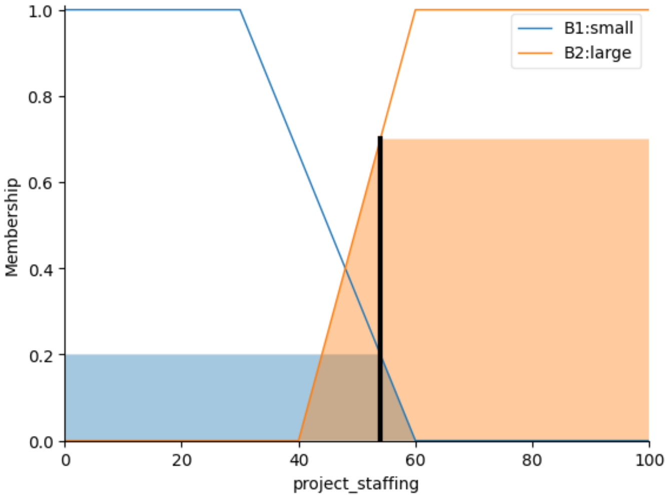
Scaling:

- Retains the original shape.
- Computationally more complex.

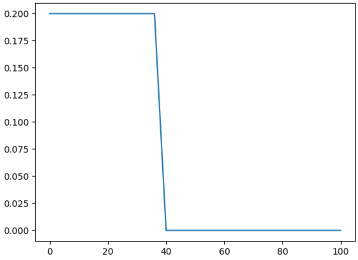
clipping of membership A

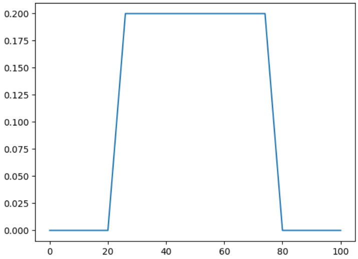


clipping of membership B

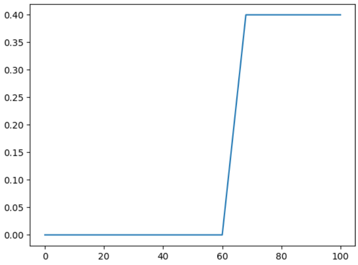


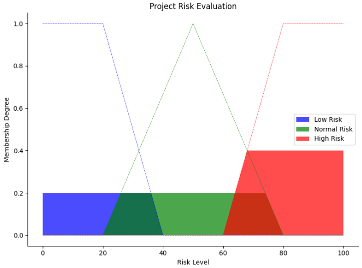
Step 6: Aggregation of Rule Consequent (Part 1)





Step 6: Aggregation of Rule Consequent (Part 2)





Step 7: Defuzzification

Defuzzification

- The final step in the fuzzy inference process.
- Converts the aggregate fuzzy set into a crisp number.
- Popular method: **Centroid Technique**.

Centroid Technique

- Finds the **centre of gravity** (COG) of the fuzzy set.
- Mathematically expressed as:

$$COG = \frac{\int_a^b \mu_A(x) x \, dx}{\int_a^b \mu_A(x) \, dx}$$

- For a discrete set:

$$COG = \frac{\sum_{x=a}^{b} \mu_A(x) x}{\sum_{x=a}^{b} \mu_A(x)}$$

Cont.,

Example: Centroid Method Calculation

- Calculation:

$$COG = \frac{60 \times 0.2 \times 30 + 40 \times 0.4 \times 80}{60 \times 0.2 + 40 \times 0.4} = \frac{1640}{28} = 58.36$$

- Result: Risk involved in the fuzzy project is 58.3%.

Cont.,

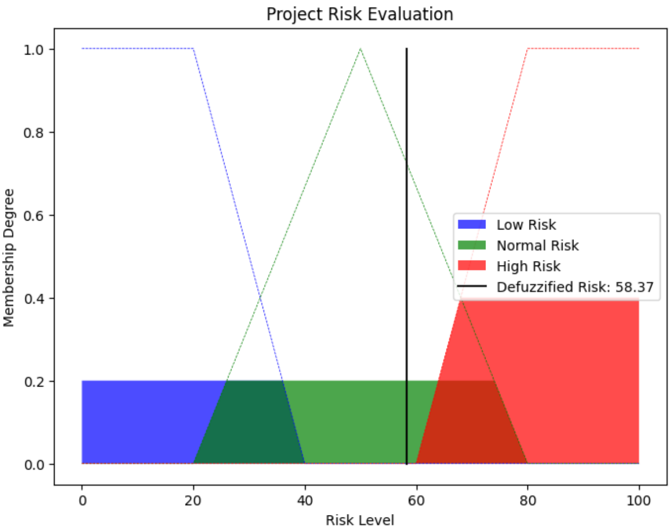


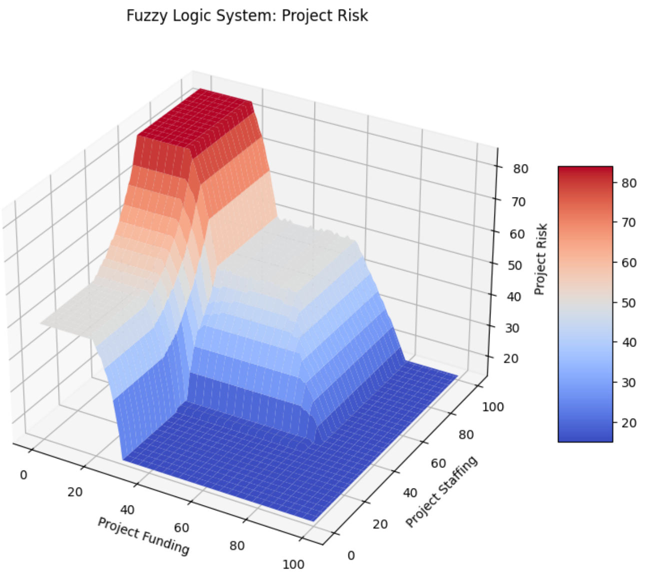
Figure: Defuzzification

Weighted Average (WA) Calculation

$$WA = \frac{\mu(k_1) \times k_1 + \mu(k_2) \times k_2 + \mu(k_3) \times k_3}{\mu(k_1) + \mu(k_2) + \mu(k_3)}$$

$$WA = \frac{0.2 \times 10 + 0.2 \times 50 + 0.4 \times 80}{0.2 + 0.2 + 0.4} = 55$$

3D plotting



Fuzzy Logic System: Project Risk
Project Risk
Project Funding
Project Staffing
0
20
40
60
80
100
20
30
40
50
60
70
80

Explore the Code

Interested in Learning More?

- Visit our GitHub repository for detailed code and explanations.
- Explore how fuzzy logic is implemented step-by-step using Python and the `scikit-fuzzy` library.

GitHub Repository:

`https://github.com/mtahrawi/fuzzy-example.git`

Enhance Your Understanding of Fuzzy Systems!

THANKS