Expert Systems

Lecture 5: Fuzzy Expert Systems

Dr. Mohammed A. Altahrawi

University College of Applied Sciences

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Outlines

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What is Fuzzy Thinking?

Definition

Fuzzy thinking refers to reasoning and decision-making based on vague and ambiguous terms that mimic human common sense.

Challenge

Computers and traditional logic struggle to process vague terms like *"slightly overloaded"* or *"a while"*. How can these terms be represented in a meaningful way?

Real-Life Example

- An expert might say, *"Though the power transformer is slightly overloaded, I can keep this load for a while."*
- Humans understand this intuitively, but computers require structured representation.

Fuzzy Logic vs. Boolean Logic

Boolean Logic

Traditional Boolean logic uses binary values:

- **True (1):** If the condition is fully satisfied.
- **False (0):** If the condition is not satisfied.

Example: An electric car is classified as "short-range" if it can travel less than 300 km.

Fuzzy Logic

Fuzzy logic adds a spectrum of truth values between 0 and 1:

- Degrees of membership allow a car with a 290 km range to be "*mostly short-range*" and a car with a 310 km range to be "*slightly long-range*".

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Key Features of Fuzzy Thinking

Why Fuzzy Logic?

Fuzzy logic enables smooth transitions and more realistic modeling of real-world phenomena.

Core Principles

- Everything operates on a *sliding scale*.
- Membership is not binary but exists in degrees.

Real-Life Analogies

- **Height:** Someone can be "*very tall*" or "*slightly tall*".
- **Speed:** A car can be "*moderately fast*" or "*extremely fast*".

Cont.,

Advantages

- Models uncertainty and vagueness effectively.
- Mimics human reasoning for complex decision-making.
- Provides a bridge between linguistic terms and numerical data.

Application

Example: A motor is *"slightly overloaded"* or *"very hot."*

- Fuzzy logic can quantify these terms and integrate them into control systems.

Distribution Over Range domain

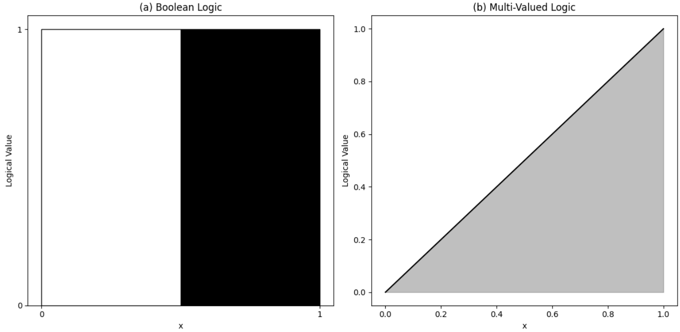

Figure: Range of logical values in Boolean and fuzzy logic

Outlines

Classical Sets vs. Fuzzy Sets

Classical (Crisp) Sets

- An element either belongs to a set ($x \in X$) or does not ($x \notin X$).

Fuzzy Sets

- Membership function defines the degree to which an element belongs to a set.
- Fuzzy sets allow for gradual transitions between membership and non-membership.

Fuzzy Sets and Degrees of Membership

Fuzzy Set Theory

- Membership is not binary.
- Allows intermediate degrees (e.g., 0.5 membership in a set).
- Example: Tall men set has varying degrees based on height.

Crisp vs. Fuzzy Sets

- Crisp Set: "Is the man tall?" (Yes or No).
- Fuzzy Set: "How tall is the man?" (Membership values vary).

Degree of Membership in Fuzzy Sets

Example: Tall Men

The table below represents the degrees of membership in the fuzzy set of tall men:

Name	Height (cm)	Membership Degree
Chris	208	1.00
Mark	205	1.00
John	198	0.98
Tom	181	0.82
David	179	0.78
Mike	172	0.24
Bob	167	0.15
Steven	158	0.06
Bill	155	0.01
Peter	152	0.00

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Crisp vs. Fuzzy Membership

Crisp Membership Function

In a crisp set, membership is binary:

$$f_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases}$$

Fuzzy Membership Function

In a fuzzy set, membership is gradual. For example:

- David (179 cm): Membership = 0.78.
- Tom (181 cm): Membership = 0.82.

Comparison of Crisp and Fuzzy Memberships

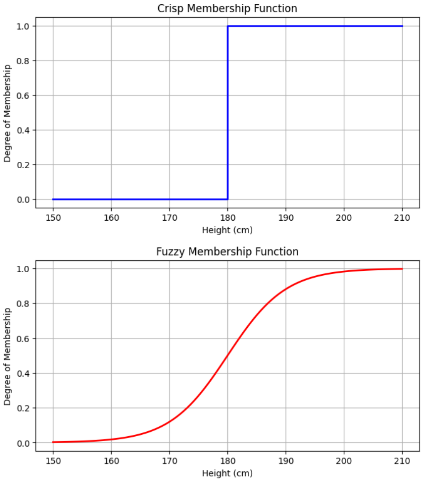

Figure: Comparison of Crisp (a) and Fuzzy (b) Memberships

What is a Fuzzy Set?

Definition

A fuzzy set can be defined as a set with fuzzy boundaries. Each element x in the set has a degree of membership $\mu_A(x)$ where:

$$\mu_A(x) \in [0, 1]$$

Comparison to Crisp Sets

- Crisp Set: Sharp boundaries, $\mu_A(x) = 0$ or 1.
- Fuzzy Set: Smooth transitions, $\mu_A(x)$ ranges from 0 to 1.

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Membership Function of a Fuzzy Set

Definition of Membership Function

For a universe X, the fuzzy set A is defined by a membership function $\mu_A(x)$:

$$\mu_A(x) \in [0, 1]$$

- $\mu_A(x) = 1$ if x is completely in A.
- $\mu_A(x) = 0$ if x is not in A.
- $0 < \mu_A(x) < 1$ if x is partially in A.

Significance of Membership Function

The membership function provides a continuum of choices, representing the degree to which an element belongs to the set A.

How to Represent a Fuzzy Set in a Computer?

Determining the Membership Function

- Use expert opinions to determine the degree of membership.
- Employ artificial neural networks to learn fuzzy sets automatically from data.

Example: 'Tall Men'

Consider a man with a height of 184 cm:

- Membership in the "Average Men" set: $\mu_A = 0.1$.
- Membership in the "Tall Men" set: $\mu_T = 0.4$.

A man can belong to multiple fuzzy sets with varying degrees of membership.

Visualizing the Membership Function

Membership Representation

Fuzzy sets can be represented as a set of pairs:

$$A = \{(x_1, \mu_A(x_1)), (x_2, \mu_A(x_2)), \ldots, (x_n, \mu_A(x_n))\}$$

Cont.,

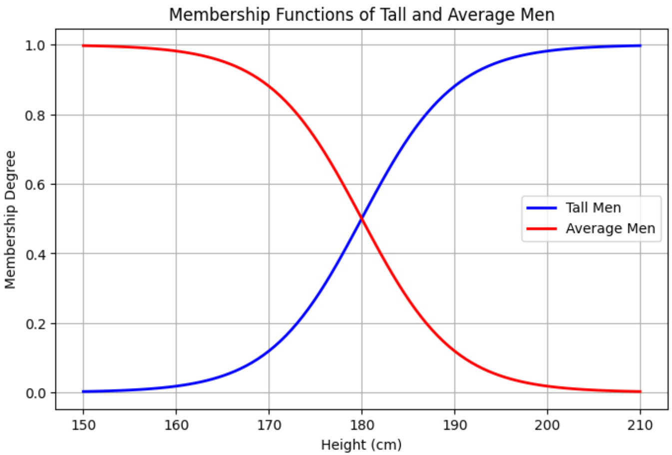

Figure: Graphical Representation of Membership Functions

Representation of Fuzziness

Special Case: Crisp Subset

When $\mu_A(x)$ can only take values 0 or 1, the fuzzy subset A becomes a crisp subset.

Graphical Representation

- UP: Crisp subset with binary membership.
- Down: Fuzzy subset showing degrees of membership.

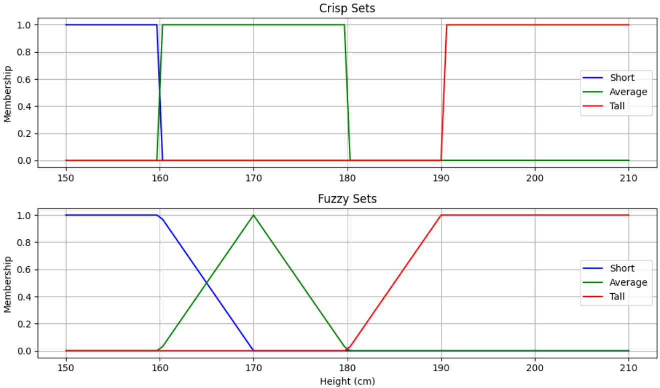

Figure: Crisp vs. Fuzzy Subsets

Representing Fuzzy Sets

Fuzzy Subset Representation

A fuzzy subset A of the finite reference super set X can be expressed as:

$$A = \{(x_1, \mu_A(x_1)), (x_2, \mu_A(x_2)), \ldots, (x_n, \mu_A(x_n))\}$$

Alternatively, it can be represented in a compact form:

$$A = \{\mu_A(x_1)/x_1, \mu_A(x_2)/x_2, \ldots, \mu_A(x_n)/x_n\}$$

where $\mu_A(x)$ is the membership value.

Frame Title

Fit-Vector Example

For tall men, the fuzzy set can be represented as:

$$\text{tall men} = (0/180, 0.5/185, 1/190)$$

Similarly:

$$\text{short men} = (1/160, 0.5/165, 0/170)$$

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Linguistic Variables and Hedges

Linguistic Variables

A linguistic variable is characterized by:

- A name (e.g., **John**).
- A linguistic value (e.g., **tall**).

Linguistic variables are commonly used in fuzzy rules. For example:

- IF **wind** is **strong**, THEN **sailing** is **good**.
- IF **speed** is **slow**, THEN **stopping distance** is **short**.

Universe of Discourse

The universe of discourse of a linguistic variable defines its range of possible values. For example:

- The variable **speed** might range from 0 to 220 km/h, with fuzzy subsets like **very slow**, **medium**, and **very fast**.

Cont.,

Hedges and Their Applications

A hedge modifies the shape of fuzzy sets and can be used in:

- **All-purpose modifiers:** very, quite, or extremely.
- **Truth values:** quite true or mostly false.
- **Probabilities:** likely or not very likely.
- **Quantifiers:** most, several, or few.
- **Possibilities:** almost impossible or quite possible.

Example: The Hedge **Very**

The hedge **very** performs concentration, narrowing a fuzzy set:

- **Tall Men:** From the original set, derive the subset of **very tall men**.
- **Extreme Case:** The hedge **extremely** performs this to an even greater extent.

Graphical Representation of Hedges

Hedges in Fuzzy Sets

- **Concentration:** Narrows the fuzzy set (**very**).
- **Dilation:** Expands the fuzzy set (**more or less**).

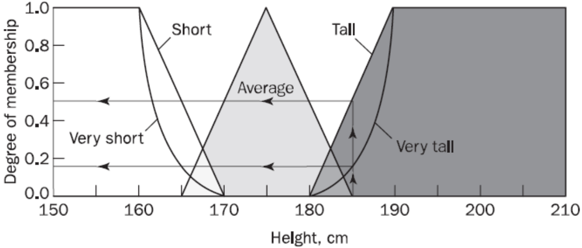

Figure: Fuzzy sets with very hedge

Practical Applications of Hedges

Applications

Hedges help to model human linguistic descriptors such as temperature:

- **Examples:** very cold, moderately cold, neutral, slightly hot, moderately hot, very hot.

These overlaps enable better human-computer interaction and decision-making.

Key Insight

Humans cannot easily distinguish between **slightly hot** and **moderately hot**, making hedges a powerful tool for bridging linguistic gaps.

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Operations with Hedges in Fuzzy Sets

Hedge: Very

The operation of concentration:

$$\mu_A^{\text{very}}(x) = [\mu_A(x)]^2$$

Example: If Tom has a 0.86 membership in the *tall men* set:

$$\mu_A^{\text{very}}(x) = 0.7396 \text{ (membership in } \textit{very tall men}\text{)}$$

Hedge: Extremely

Performs concentration to a greater extent:

$$\mu_A^{\text{extremely}}(x) = [\mu_A(x)]^3$$

Example: If Tom has a 0.86 membership in the *tall men* set:

$$\mu_A^{\text{extremely}}(x) = 0.6361 \text{ (membership in } \textit{extremely tall men}\text{)}$$

Extensions and Other Operations with Hedges

Hedge: Very Very

Extends the concentration operation:

$$\mu_A^{\text{very very}}(x) = (\mu_A^{\text{very}}(x))^2 = [\mu_A(x)]^4$$

Example: Membership in *very very tall men* is 0.547 for Tom.

Hedge: More or Less

Performs dilation (expands the set):

$$\mu_A^{\text{more or less}}(x) = \sqrt{\mu_A(x)}$$

Example: Membership in *more or less tall men* is 0.9274 for Tom.

Cont.,

Hedge: Indeed

Intensifies the meaning:

$$\mu_A^{\text{indeed}}(x) = \begin{cases} 2[\mu_A(x)]^2 & \text{if } 0 \leq \mu_A(x) \leq 0.5 \\ 1 - 2[1 - \mu_A(x)]^2 & \text{if } 0.5 < \mu_A(x) \leq 1 \end{cases}$$

Example: Membership in *indeed tall men* is 0.9608 for Tom.

Hedges and Their Mathematical Expressions

Hedges modify fuzzy sets to represent varying degrees of intensity. Below are some common hedges and their mathematical formulations:

Representation of Hedges in Fuzzy Logic

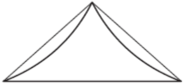
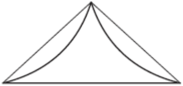
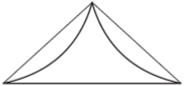
Hedge	Mathematical Expression	Graphical Representation
A Little	$[\mu_A(x)]^{1.3}$	
Slightly	$[\mu_A(x)]^{1.7}$	
Very	$[\mu_A(x)]^2$	

Table: Hedge representation in fuzzy logic (Part 1)

Representation of Hedges in Fuzzy Logic (Cont.)

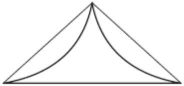
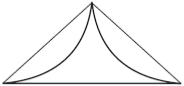
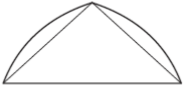
Hedge	**Mathematical Expression**	**Graphical Representation**
Extremely	$[\mu_A(x)]^3$	
Very Very	$[\mu_A(x)]^4$	
More or Less	$\sqrt{\mu_A(x)}$	

Table: Hedge representation in fuzzy logic (Part 2)

Representation of Hedges in Fuzzy Logic (Final Part)

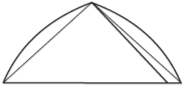
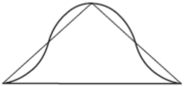
Hedge	Mathematical Expression	Graphical Representa
Somewhat	$\sqrt{\mu_A(x)}$	
Indeed	$2[\mu_A(x)]^2$ if $0 \leq \mu_A(x) \leq 0.5$ $1 - 2[1 - \mu_A(x)]^2$ if $0.5 < \mu_A(x) \leq 1$	

Table: Hedge representation in fuzzy logic (Part 3)

Outlines

1. Introduction to Fuzzy Thinking
2. Fuzzy Sets
3. Linguistic Variables and Hedges
4. **Operations of fuzzy sets**
5. Fuzzy rules
6. Fuzzy inference

- **Complement:** Who does not belong to the set?
- **Containment:** Which sets belong to other sets?
- **Intersection:** Which element belongs to both sets?
- **Union:** Which element belongs to either set?

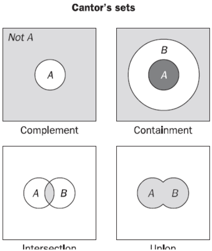


Figure: Cantor's sets representation

Complement in Fuzzy and Crisp Sets

Crisp Sets

- The complement is the set of elements not in the set.

Fuzzy Sets

- How much do elements not belong to the set?
- Formula: $\mu_{\neg A}(x) = 1 - \mu_A(x)$

Example

Tall men set: $(0/180, 0.25/182.5, \ldots, 1/190)$
NOT tall men: $(1/180, 0.75/182.5, \ldots, 0/190)$

Containment in Fuzzy and Crisp Sets

Crisp Sets

- Determines which sets belong to others.

Fuzzy Sets

- Determines which sets partly or fully belong to others.

Subset in Fuzzy and Crisp Sets

Crisp Sets

- A smaller set is entirely contained in a larger set.
- Example: *Very tall men* is a subset of *Tall men*.

Fuzzy Sets

- Elements may belong less to the subset than the larger set.
- **Example:**

$$\text{Tall men: } (0/180, 0.25/182.5, \ldots, 1/190)$$
$$\text{Very tall men: } (0/180, 0.06/182.5, \ldots, 1/190)$$

Intersection in Fuzzy and Crisp Sets

Crisp Sets

- Identifies elements common to both sets.

Fuzzy Sets

- Determines the extent of membership in both sets.
- Formula: $\mu_{A\cap B}(x) = \min[\mu_A(x), \mu_B(x)]$

Example

Tall men: $(0/165, 0/175, 0/180, 0.25/182.5, 0.5/185, 1/190)$
Average men: $(0/165, 1/175, 0.5/180, 0.25/182.5, 0/185, 0/190)$
Intersection: $(0/165, 0.25/182.5, 0/185)$

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Union in Fuzzy and Crisp Sets

Crisp Sets

- Identifies elements belonging to either set.
- Example: All men who are tall OR fat.

Fuzzy Sets

- Calculates the largest membership value for an element in either set.
- Formula: $\mu_{A \cup B}(x) = \max[\mu_A(x), \mu_B(x)]$

Example

Tall men: $(0/165, 0/175, 0/180, 0.25/182.5, 0.5/185, 1/190)$
Average men: $(0/165, 1/175, 0.5/180, 0.25/182.5, 0/185, 0/190)$
Union: $(0/165, 1/175, 0.5/180, 0.25/182.5, 0.5/185, 1/190)$

Properties of Fuzzy Sets

Commutativity

$$A \cup B = B \cup A$$
$$A \cap B = B \cap A$$

Example:

- **Union:** Tall men OR Short men = Short men OR Tall men.
- **Intersection:** Tall men AND Short men = Short men AND Tall men.

Associativity

$$A \cup (B \cup C) = (A \cup B) \cup C$$
$$A \cap (B \cap C) = (A \cap B) \cap C$$

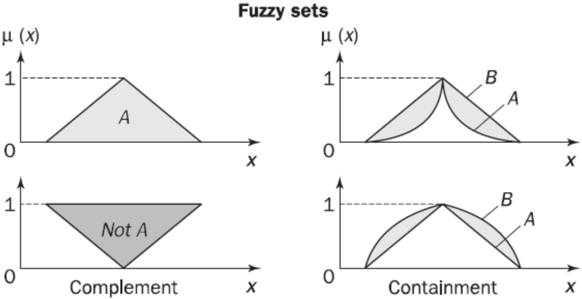

Figure: Complement and Containment

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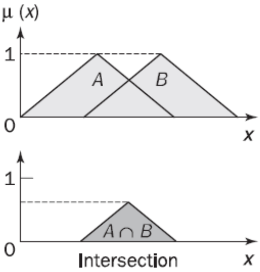

Figure: Intersection and union

Example

Example:

- **Union:** Tall men OR Short men OR Average men
- **Intersection:** Tall men AND Short men AND Average men

Distributivity Property in Fuzzy Sets

Definition:

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Example:

- **Union:**

  (Tall men OR Short men) AND Average men

- **Intersection:**

  (Tall men AND Short men) OR Average men

Idempotency in Fuzzy Sets

Definition:

$$A \cup A = A$$
$$A \cap A = A$$

Example:

- Tall men OR Tall men = Tall men
- Tall men AND Tall men = Tall men

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Identity in Fuzzy Sets

Definition:

$$A \cup \emptyset = A$$
$$A \cap X = A$$
$$A \cap \emptyset = \emptyset$$
$$A \cup X = X$$

Example:

- Tall men OR Undefined = Tall men
- Tall men AND Unknown = Tall men
- Tall men AND Undefined = Undefined
- Tall men OR Unknown = Unknown

Involution in Fuzzy Sets

Definition:

$$\neg(\neg A) = A$$

Example:

- NOT (NOT Tall men) $=$ Tall men

Transitivity in Fuzzy Sets

Definition:

If $A \subset B$ and $B \subset C$, then $A \subset C$.
Every set contains the subsets of its subsets.

Example:

- If Extremely Tall men $\subset$ Very Tall men
- AND Very Tall men $\subset$ Tall men
- THEN Extremely Tall men $\subset$ Tall men

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De Morgan's Laws in Fuzzy Sets

Definition:

$$\neg(A \cap B) = \neg A \cup \neg B$$
$$\neg(A \cup B) = \neg A \cap \neg B$$

Examples:

- NOT (Tall men AND Short men) = NOT Tall men OR NOT Short men
- NOT (Tall men OR Short men) = NOT Tall men AND NOT Short men

Fuzzy Operations:

$$\mu_C(x) = [1 - \mu_A(x)^2] \cap [1 - \mu_B(x)^2]$$
$$\mu_D(x) = [1 - \mu_A(x)^4] \cap [1 - \mu_B(x)^4]$$

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What is a Fuzzy Rule?

Definition:

A fuzzy rule can be defined as a conditional statement:

$$\textbf{IF } x \textbf{ is } A, \ \textbf{THEN } y \textbf{ is } B$$

where:

- x, y are linguistic variables
- A, B are linguistic values determined by fuzzy sets

Classical vs Fuzzy Rules

Classical Rules:

- Rule 1: IF speed $>$ 100, THEN stopping_distance is long
- Rule 2: IF speed $<$ 40, THEN stopping_distance is short

Fuzzy Rules:

- Use linguistic variables (e.g., speed can be slow, medium, fast)
- Include ranges of values for inputs and outputs

Representing Classical Rules in Fuzzy Form

Classical Rules:

- **Rule 1:** IF speed is fast, THEN stopping_distance is long.
- **Rule 2:** IF speed is slow, THEN stopping_distance is short.

Fuzzy Variables:

- **Speed:** slow, medium, fast (range: 0 to 220 km/h)
- **Stopping Distance:** short, medium, long (range: 0 to 300 m)

Advantages of Fuzzy Rules

- Fuzzy rules reduce the number of required rules significantly.
- Linguistic variables simplify the representation of complex systems.
- Fuzzy rules cut the number of rules by at least 90%.

Note:

Fuzzy expert systems merge rules to create more efficient and adaptable systems.

Reasoning with Fuzzy Rules

Steps:

1. **Antecedent Evaluation:** Determine the degree of truth for the IF part.
2. **Implication:** Apply the result to the THEN part of the rule.

Example:

Two fuzzy sets:

- Tall men
- Heavy men

Fuzzy sets of tall and heavy men

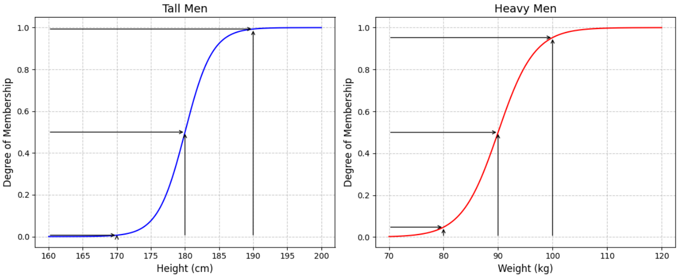

Figure: Fuzzy sets of tall and heavy men

Combining Output Fuzzy Sets

Key Process:

- Aggregate all output fuzzy sets into a single output fuzzy set.
- Defuzzify the resulting fuzzy set into a single crisp number.

Goal:

To obtain a precise solution rather than a fuzzy one.

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Mamdani-Style Fuzzy Inference

Definition:

Fuzzy inference is the process of mapping from a given input to an output using the theory of fuzzy sets.

Mamdani Overview

Developed by Professor Ebrahim Mamdani in 1975 to control steam engines and boilers based on fuzzy logic.

Steps:

- Fuzzification
- Rule Evaluation
- Aggregation of Outputs
- Defuzzification

Sugeno-Style Inference

Motivation

- Mamdani-style inference requires finding the centroid of a 2D shape.
- Computationally expensive for continuously varying functions.
- Sugeno method offers a simpler alternative.

Sugeno-Style Method

- Uses a single spike (singleton) as the membership function of the rule consequent.
- Introduced by Michio Sugeno in 1985.
- Format of Sugeno-style fuzzy rule:

$$\text{IF } x \text{ is } A \text{ AND } y \text{ is } B \text{ THEN } z \text{ is } f(x, y)$$

- $f(x, y)$: Mathematical function of input variables x and y.

Zero-Order Sugeno Fuzzy Model

Sugeno Fuzzy Rule Format

IF x is A AND y is B THEN z is k

- Here, k is a constant.
- Output of each fuzzy rule is represented by singleton spikes.

Choosing Mamdani or Sugeno Method

- **Mamdani Method:** Intuitive, captures expertise, but computationally intensive.
- **Sugeno Method:** Efficient for dynamic systems, supports optimization and adaptive techniques.

What is the next

Next

- We will demonstrate a fuzzy example to clarify the procedures of building a fuzzy system.
- We will illustrate the difference between the Mamdani and Sugneno methods.
- In the end, building a fuzzy expert system will be presented.

THANKS