Now Methods to find Particular Solution of the the NHDE → $y'' + P(x)y' + Q(x)y = r(x)$ — (1)

Method (1) → Method of undetermined Coeffecients (MOUC)

In case the Coeffecients P + Q in NHDE (1) are Constants then we can use MOUC. Here we will Consider main (4) cases of MOUC depending upon what $r(x)$ is given in NHDE

Case (1) → If $r(x)$ is of form ke^{ax} (exponential form)

Case (2) → If $r(x)$ is of trignometric form $k\cos bx$ or $k\sin bx$ or their linear Combination $k_1\cos bx + k_2\sin bx$

Case (3) If $r(x)$ is of polynomial form $a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$, $a_n \neq 0$

Case (4) If $r(x)$ is of form → $ke^{ax}\cos bx$ or $ke^{ax}\sin bx$ or $e^{ax}(k_1\cos bx + k_2\sin bx)$

Case ① When $r(x)$ is of form ke^{ax} (exponential)

That is, $y'' + Py' + Qy = ke^{ax}$ — ①

Let $y = Ae^{ax}$ ② be trial (Particular) soln of ① where A is the undetermined (or unknown) coeffecient that we have to determine so that ② becomes soln of ①.

So $y' = aAe^{ax}$, $y'' = a^2Ae^{ax}$.

Putting values of y, y' & y'' in ① we get

$$A(a^2 + Pa + Q)e^{ax} = e^{ax} \quad \text{— ③}$$

Comparing both sides we get

$$A(a^2 + Pa + Q) = 1 \Rightarrow \boxed{A = \frac{1}{a^2 + aP + Q}} \quad \text{— ④}$$

So ④ gives the value of A for which ② is the P. soln of ①, provided $a^2 + aP + Q \neq 0$ (as then A will be undefined)

(Also HDE is → $y'' + Py' + Qy = 0$
A.E → $m^2 + Pm + Q = 0$) → Now if we put in place of m, value of a we get $a^2 + aP + Q$. So we can also say that ② is P. soln of ① provided a does not satisfy A.E ($m^2 + Pm + Q = 0$)

So trial choice of P.S is taken as $= \boxed{Ae^{ax}}$ when a is not root of A.E or a does not satisfy A.E or $r(x)$ does not satisfy HDE ($y'' + Py' + Qy = 0$)

→ In case (a) is single root of A.E of HDE ($y''+Py'+Qy=0$)

then Trial P. Soln of D.E (1) is taken as $= Axe^{ax}$
(Multiply Ae^{ax} by x)

[otherwise L.H.S becomes zero]

→ In case (a) is double root of A.E of HDE →

Trial P. Soln is $= Ax^2e^{ax}$
(Multiply Ae^{ax} by x^2)

[Otherwise R.H.S becomes zero as (a) is root of A.E] (see this eg)

Q is related to case (1)

eg (1) Find the G.S of D.E → $y''-2y'+y = 6e^x$ — (1)
Use MOUC to find the P. Soln.

Soln
Step (1) (To find G.S of HDE)

HDE associated with (1) is →
$$y''-2y'+y=0 \quad \text{—(2)}$$
A.E → $m^2-2m+1=0$ —(3)
⇒ Roots → $m=1,1$. So $y_g=(c_1+c_2x)e^x$
(Gen. Soln of HDE (2))

Step (2) (To find Particular soln of NHDE (1))

∴ NHDE is $y''-2y'+y = 6e^x$ —(1) where
$r(x)=6e^x = ke^{ax}$, $\boxed{a=1}$

$r(x)=6e^x$ ∴

So trial choice of P. Soln is →
$y_p = Ae^x$. But since
$a=1$ is double root of A.E (3) ($m=1,1$)
So we must multiply Ae^x by x^2 so that our actual trial choice of P. Soln for this Q is
$$\boxed{y_p = Ax^2e^x} \quad \text{—(4)}$$
So that $y_p' = Ax^2e^x + 2Axe^x$
$y_p'' = A(x^2e^x+e^x2x) + A(2xe^x+2e^x)$

Put y_p, y_p', y_p'' in ① we get after solving →

$$2Ae^{x} = 6e^{x}$$

Comparing coeffecients of e^{x} on both sides we get →

$\boxed{A=3}$ So that $\boxed{y_p = 3x^2 e^{x}}$

In same Q if by mistake you consider $y = Ae^{x}$ or $y = Axe^{x}$ then in both cases L.H.S of ① will be become zero on putting values of y, y', y'' in D.E ①. That shows your choice of y_p is wrong

Hence Gen Soln of NHDE ① is →

$$= y_g + y_p$$

$$= (c_1 + c_2 x)e^{x} + 3x^2 e^{x}$$

eg ② → $y'' - 4y = 6e^{2x}$ — ①

Here $r(x) = 6e^{2x} = ke^{ax}$, $a = 2$

Soln

Step ① HDE → $y'' - 4y = 0$ — ②

A.E → $m^2 - 4 = 0 \Rightarrow m = \pm 2$

So $y_g = c_1 e^{2x} + c_2 e^{-2x}$ — ③

Step ② $r(x) = 6e^{2x}$ Here ∵ $a = 2$ is single root of A.E

So y_p is taken as → $\boxed{y_p = Axe^{2x}}$ — ④

So $y_p' = 2Axe^{2x} + Ae^{2x}$

$y_p'' = 2A(xe^{2x}\cdot 2 + e^{2x}) + 2Ae^{2x}$

$= 4Ae^{2x} + 4Axe^{2x}$

In case → D.E is $y'' + 4y = 6e^{2x}$

→ HDE → $m^2 + 4 = 0$, $m = \pm 2i$

$y_g = c_1 \cos 2x + c_2 \sin 2x$

Trial P.S $y_p = Ae^{2x}$

Put y_p, y_p' & y_p'' in ① we get →

$$(4Ae^{2x} + 4Axe^{2x}) - 4Axe^{2x} = 6e^{2x}$$

$$4Ae^{2x} = 6e^{2x}$$

$\boxed{A = 3/2}$ So $\boxed{y_p = 3/2\, xe^{2x}}$ — ⑤

→ G.S of NHDE ① is → $= y_g + y_p = c_1 e^{2x} + c_2 e^{-2x} + 3/2\, xe^{2x}$

eg (3) Solve the D.E →

$$y'' + y' - 2y = e^{x} + e^{+2x} \quad \text{--- (1)}$$

Soln

Step (1)

HDE → $y'' + y' - 2y = 0$

A.E → $m^2 + m - 2 = 0$, $m^2 + 2m - m - 2 = 0$

$m(m+2) - (m+2) = 0$, $\underline{m = 1, -2}$

Roots → $m = 1, -2$

So $\left. y_g = c_1 e^{x} + c_2 e^{-2x} \right] \text{--- (2)}$

Step (2) → Here $r(x) = e^{x} + e^{2x}$

So accordingly → $y_p = (\text{As per } e^{x}) + (\text{As per } e^{2x})$ ← e^{ax}

$= (Axe^{x}) + (Be^{2x})$

[∴ for e^{x} in $r(x)$, $a = 1$ & it is one root of A.E]

[But for e^{2x} in $r(x)$ $a = 2$ is not root of A.E]

$$y_p = Axe^{x} + Be^{2x} \quad \text{--- (3)}$$

$$y_p' = A[xe^{x} + e^{x}] + 2Be^{2x} \quad \text{--- (4)}$$

$$y_p'' = A[xe^{x} + 2e^{x}] + 4Be^{2x} \quad \text{--- (5)}$$

Put y_p'', y_p' & y_p in (1) we get

$$\left(A[xe^{x} + 2e^{x}] + 4Be^{2x}\right) + \left(A(xe^{x} + e^{x}) + 2Be^{2x}\right) - 2\left(Axe^{x} + Be^{2x}\right) = e^{x} + e^{2x}$$

$$e^{x}[Ax + 2A + Ax + A - 2Ax] + e^{2x}[4B + 2B - 2B] = e^{x} + e^{2x}$$

$$e^{x}(3A) + e^{2x}(4B) = e^{x} + e^{2x}$$

Comparing both sides

$3A = 1$, $4B = 1$

$A = 1/3$, $B = 1/4$

So P. Soln

$$y_p = \frac{1}{3} xe^{x} + \frac{1}{4} e^{2x} \quad \text{--- (6)}$$

Hence G.S of (1) is

$= y_g + y_p$

where y_g is given by (2) & y_p is given by (6)

Qs for Practice

→ Method of Undetermined Coefficients (MOUC)

Find the G.S of following D.E. Use MOUC to find the P.Soln.

Q1 $3y'' + 5y' - 2y = 14e^{x/3}$

Q2 $y'' + 3y' - 10y = 6e^{4x}$

Q3 $y'' + 10y' + 25y = 14e^{-5x}$

Soln

1) $Ae^{-2x} + Be^{x/3} + 2x \cdot e^{x/3}$

2) $c_1e^{2x} + c_2e^{-5x} + \frac{1}{3}e^{4x}$

3) $c_1e^{-5x} + c_2xe^{-5x} + 7x^2e^{-5x}$.

→ Try this also →

$$2y'' + 3y' - 2y = 5e^{-2x} + e^{x}$$

Soln → $y = Ae^{-2x} + Be^{x/2} + \frac{1}{3}e^{x} - xe^{-2x}$

Case 2 When $r(x)$ is of form $k\sin bx$ or $k\cos bx$ or $k_1\sin bx + k_2\cos bx$.

→ In such cases we take trial Particular soln as → $\left[y_p = A\cos bx + B\sin bx\right]$ — (1)

However, if $r(x)$ satisfies the HDE associated with NHDE (or $r(x)$ can be generated from y_g of HDE), then ~~P~~ Soln is taken as →

$$\left[y_p = x\,(A\cos bx + B\sin bx)\right] \quad \text{— (2)}$$

eg 1 Solve the D.E → $y'' + y = 2\cos x$ — (1) by using MOUC

Soln → Here HDE → $y'' + y = 0$ — (2)

Step 1 A.E → $m^2 + 1 = 0 \Rightarrow m = \pm i$ → Roots.

$$\text{So } y_g = e^{0x}\left[c_1\cos x + c_2\sin x\right]$$
$$= c_1\cos x + c_2\sin x \quad \text{— (3)}$$

Step 2 Here $r(x) = 2\cos x$

~~As~~ So our choice of P. Soln would be

$$y_p = A\cos x + B\sin x.$$

→ No need to multiply with x then.

(If $r(x) = 2\cos 3x$, we would have taken $y_p = A\cos 3x + B\sin 3x$)

However $r(x) = 2\cos x$ satisfies the HDE (2) $y'' + y = 0$ (We need not to find y' & y'' & substitute it in (2)).

We can check it from y_g. If $r(x)$ can be obtained from the G. Soln (y_g) of HDE by giving values to Constant c_1 & c_2 then it satisfies HDE (or soln of HDE) otherwise not.

→ In this Q, $r(x) = 2\cos x$ can be obtained from $y_g = c_1\cos x + c_2 \sin x$ by putting $\underline{c_1 = 2 \text{ \& } c_2 = 0}$

So $r(x)$ satisfies HDE.

→ Hence actual trial choice of P. Soln will be →

$$\boxed{y_p = x(A\cos x + B\sin x)} \quad \text{—} (4)$$

* (See nxt page bottom)

~~Gen. Soln of NHDE~~ → Find y_p' & y_p'' and ~~$= y_g + y_p$~~ Substitute y_p, y_p', y_p'' ~~$(c_1\cos x + c_2 \sin x)$~~ in eqn (1) we get →

$y_p' = (-Ax + B)\sin x + (Bx + A)\cos x$

$y_p'' = \cos x(-Ax + 2B) + \sin x(-2A - Bx)$

$$-2A\sin x + 2B\cos x = 2\cos x + 0\cdot\sin x$$

Comparing Coefficients of $\cos x$ & $\sin x$ on both sides we get →

$$-2A = 0 \quad | \quad 2B = 2$$

$\boxed{A = 0}$, $\boxed{B = 1}$. So $\boxed{y_p = x\sin x}$ — (5)

So Gen soln of NHDE (1) is

$$= y_g + y_p$$

$$= \underline{(c_1\cos x + c_2 \sin x) + x\sin x} \quad \text{—} (6)$$

eg ② If $y'' + 4y = 2\cos x$ — ① is given D.E

~~Solve~~ it by MOUC.

Soln → HDE → $y'' + 4y = 0$

A.E → $m^2 + 4 = 0 \Rightarrow m = \pm 2i$ → Roots

So $y_g = c_1\cos 2x + c_2\sin 2x$ — ②

Now here $r(x) = 2\cos x$. Clearly $r(x)$ does not Satisfy the HDE as $r(x)$ can not be obtained from Gen. Soln (y_g) of HDE by giving any value to c_1 & c_2.

So y_p is taken as → $\boxed{y_p = A\cos x + B\sin x}$ — ③

(If $r(x) = 2\cos 2x$, then we would have taken $y_p = x(A\cos 2x + B\sin 2x)$)

Find y_p', y_p'' and substitute y_p, y_p', y_p'' in ① we get

$3A\cos x + 3B\sin x = 2\cos x$ — ④

Comparing coefficients of $\cos x$ & $\sin x$ on both sides we get →

$A = 2/3$ & $B = 0$ So that $\boxed{y_p = \frac{2}{3}\cos x}$

Hence G.S of D.E ① is

$= y_g + y_p$

$= (c_1\cos 2x + c_2\sin 2x) + \frac{2}{3}\cos x$

$y_p' = -A\sin x + B\cos x$

$y_p'' = -A\cos x - B\sin x$

(*) If we have taken by mistake $y_p = A\cos x + B\sin x$ in eq ① we would have got in end

$0\cdot\cos x + 0\cdot\sin x = 2\cos x + 0\cdot\sin x$

$\Rightarrow 0 = 2$ (Contradiction). So choice of y_p is wrong.

eg ③ Solve the D.E →

$$y'' - y' = \cos x + e^{2x} \quad \text{— ①}$$

Soln → HDE → $y'' - y' = 0$

A.E → $m^2 - m = 0$, $m(m-1) = 0$

Roots → $m = 0, 1$.

So $y_g = c_1 + c_2 e^{x}$ — ②

→ ∵ $r(x) = \cos x + e^{2x}$.

So. $y_p = (A\cos x + B\sin x) + Ce^{2x}$ — ③

① If in D.E ①, L.H.S is → $y'' + y$

Then our choice will be → $y_p = \underline{\underline{x}}(A\cos x + B\sin x) + Ce^{2x}$

∵ y_g then will be $= c_1\cos x + c_2\sin x$.

& $\underline{\cos x}$ in $r(x)$ can be obtained by y_g by putting $c_1 = 1, c_2 = 0$ here

② If in DE ①, L.H.S is → $y'' + 4y$

Then $y_p = (A\cos x + B\sin x) + \underline{\underline{Cx}}e^{2x}$

∵ y_g then will be $= c_1 e^{2x} + c_2 e^{-2x}$

& e^{2x} in $r(x)$ has $a = 2$ has one root of A.E $m^2 - 4 = 0$, $m = \pm 2$

Now $y_p' = -A\sin x + B\cos x + 2Ce^{2x}$ — ④

$y_p'' = -A\cos x - B\sin x + 4Ce^{2x}$ — ⑤

Put ③, ④ & ⑤ in ① we get

$$-A\cos x - B\sin x + 4Ce^{2x} - (-A\sin x + B\cos x + 2Ce^{2x}) = \cos x + e^{2x}$$

$$\cos x(-A-B) + \sin x(-B+A) + e^{2x}(4C-2C) = \cos x + 0\cdot\sin x + e^{2x}$$

Compare Coeffts of $\cos x$, $\sin x$ & e^{2x} on both sides we get $\longrightarrow$

$$\left.\begin{array}{l} A+B=-1 \\ A-B=0 \\ 2C=1 \end{array}\right\} \longrightarrow \text{Solving we get } \left[\begin{array}{l} A=-\tfrac{1}{2}=B \\ C=\tfrac{1}{2} \end{array}\right]$$

So $y_p = -\frac{1}{2}\cos x + \frac{1}{2}\sin x + \frac{1}{2}e^{2x}$ — (6)

Hence G.S of NHDE (1) is

$= y_g + y_p$

where y_g & y_p are given by (2) & (6).

Qs for Practice → (When $r(x)$ is of form $k\cos bx$, Or $k\sin bx$ or $k_1\cos bx + k_2\sin bx$) 23 C

Q1 $y'' + 3y' + 2y = \cos x + \sin x$. [Use MOVC to find P. Soln]

Q2 $y'' + 25y = 50\cos 5x + 30\sin 5x$

Q3 $y'' + y' - 6y = 39\cos 3x$.

Soln

1 $Ae^{-x} + Be^{-2x} + \frac{1}{5}(2\sin x - \cos x)$

2 $c_1\cos 5x + c_2\sin 5x + 5x\sin 5x - 3x\cos 5x$.

3 $c_1e^{2x} + c_2e^{-3x} + \frac{1}{2}(\sin 3x - 5\cos 3x)$

Case 3 → When $r(x)$ is some polynomial fn of the form →

$$r(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n \quad \text{— (1)}$$

Then Choice of P. Soln is taken as

$$y_p = A_0 + A_1 x + A_2 x^2 + \cdots + A_n x^n \quad \text{— (2)}$$

However if in NHDE → $y'' + Py' + Qy = r(x)$ — (3)

if there is no term containing y (or $Q = 0$) in L.H.S of D.E then choice of P. Soln is taken as →

$$y_p = x(A_0 + A_1 x + \cdots + A_n x^n) \quad \text{— (4)}$$

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eg (1) Solve the D.E → $y'' - 2y' + 5y = 25x^2 + 12$ — (1)

using MOUC

Soln →

HDE → $y'' - 2y' + 5y = 0$

A.E → $m^2 - 2m + 5 = 0$

Roots → $m = 1 \pm 2i$

So $y_g = e^x [C_1 \cos 2x + C_2 \sin 2x]$ — (2)

Now ∵ $r(x) = 25x^2 + 12$ is a quadratic Polynomial.

So trial choice of P. Soln is →

$$\boxed{y_p = Ax^2 + Bx + C} \quad \text{— (3)}$$

If there would have been no term of y on L.H.S of (1), we have to then multiply by x, that is,

$$\boxed{y_p = x(Ax^2 + Bx + C)}$$

Find y_p', y_p'' & substitute y_p, y_p', y_p'' in (1) we get →

* (Even if $r(x) = x^2$, we would have taken $y_p = Ax^2 + Bx + C$.)

$$(5A)x^2 + (-4A+5B)x + (2A-2B+5C) = 25x^2+12 \quad \text{--- (4)}$$

Comparing coefficients of x^2, x & constant term on both sides of (4) we get →

$5A = 25$, $-4A+5B=0$, $2A-2B+5C=12$.

Solving we get $A=5$, $B=4$, $C=2$

So that → $\boxed{y_p = 5x^2+4x+2}$

So G.S of NHDE (1) is → $= y_g + y_p$

$$= e^x[c_1\cos 2x + c_2 \sin 2x] + (5x^2+4x+2)$$

eg (2) Solve D.E → $y'' - 4y' = 25x^2+12$. — (1)
using MOUC

Soln →

HDE → $y'' - 4y' = 0$

A.E → $m^2 - 4m = 0$

Roots → $m = 0, 4$.

$$y_g = c_1 e^{0x} + c_2 e^{4x}$$
$$= c_1 + c_2 e^{4x} \quad \text{--- (2)}$$

Now ∵ $r(x) = 25x^2+12$

So y_p would be $= Ax^2+Bx+C$.

But since there is no term of (y) on the L.H.S of D.E (1) so our actual choice of P.soln will be →

$$\boxed{y_p = x(Ax^2+Bx+C)} \quad \text{--- (3)}$$

Put y_p, y_p', y_p'' in (1) we get →

$$-12Ax^2 + (6A-8B)x + (2B-4C) = 25x^2+12. \quad \text{--- (4)}$$

Comparing Coeffecients of x^2, x & constant terms on both sides we get →

$-12A = 25$, $6A-8B=0$, $2B-4C=12$.

⇒ Solving we get →

$A = -25/12$, $B = -25/16$, $C = \frac{-121}{32}$.

So $y_p = x\left(\frac{-25}{12}x^2 - \frac{25}{16}x - \frac{121}{32}\right)$

Hence the G. Soln of NHDE (1) is

$= y_g + y_p$.

$$= (C_1 + C_2 e^{4x}) + x\left(\frac{-25}{12}x^2 - \frac{25}{16}x - \frac{121}{32}\right)$$

eg (3) Solve the D.E →

$$y'' + y' = e^{-x} + x \quad \text{--- (1)}$$

Soln →

→ HDE → $y'' + y' = 0$

A.E → $m^2 + m = 0$, $m(m+1) = 0$

Roots → $m = 0, -1$

$$\text{So } y_g = c_1 + c_2 e^{-x} \quad \text{--- (2)}$$

→ ∵ $r(x) = e^{-x} + x$

So default choice ~~xx~~ of y_p will be →

$$y_p = Ae^{-x} + Bx + C$$

→ [But since e^{-x} in $r(x)$ has $a = -1$, which is root of A.E so Ae^{-x} be taken as Axe^{-x}.]

→ [Also there is no term of (y) in L.H.S of (1) so the choice $Bx + C$ must be multiplied with (x).]

Hence $\quad y_p = Axe^{-x} + x(Bx + C)$ --- (3)

$$y_p' = A\left[xe^{-x}(-1) + e^{-x}\right] + 2Bx + C$$

$$= A\left[-xe^{-x} + e^{-x}\right] + 2Bx + C \quad \text{--- (4)}$$

$$y_p'' = A\left[+xe^{-x} - e^{-x} - e^{-x}\right] + 2B$$

$$= A\left[xe^{-x} - 2e^{-x}\right] + 2B \quad \text{--- (5)}$$

Put y_p'' & y_p' in (1) we get

$$\left(A\left[xe^{-x} - 2e^{-x}\right] + 2B\right) + A\left(-xe^{-x} + e^{-x}\right) + 2Bx + C = e^{-x} + x$$

$e^{-x}[Ax-2A-Ax+A] + 2Bx + (2B+C) = e^{-x}+x$

$e^{-x}(-A) + (2B)x + (2B+C) = e^{-x} + x + 0$ 26B

Compare both sides we get →

$-A = 1 \mid 2B = 1, \mid 2B + C = 0$

Solving → $\boxed{A = -1}$, $\boxed{B = 1/2}$, $\boxed{C = -1}$

So that $\boxed{y_p = -xe^{-x} + \frac{1}{2}x^2 - x}$ (6)

Hence G.S of NHDE (1) is

$= y_g + y_p$, where y_g is given by (2)

& y_p is given by (6)

When $r(x)$ is of Polynomial form

Q1 $2y'' - y' - 3y = x^3 + x + 1$.

Q2 $y'' + 2y' + y = x - e^{x}$. → | **Q4** $y'' + 2y' = x - e^{x}$.

Soln $\left(= C_1 + C_2 e^{-2x} + \frac{1}{4}x^2 - \frac{1}{4}x - \frac{1}{3}e^{x} \right)$

Q3 $y'' + 3y' + 2y = \cos x + x$.

Soln

1.) $C_1 e^{-x} + C_2 e^{3x/2} + \frac{1}{27}\left[-9x^3 + 9x^2 - 51x + 20\right]$

2) $(C_1 + C_2 x)e^{-x} + x - 2 - \frac{1}{4}e^{x}$

3) $C_1 e^{-x} + C_2 e^{-2x} + \left(\frac{x}{2} - \frac{3}{4}\right) + \frac{(\cos x + 3\sin x)}{10}$

In Q 2, if R.H.S is $= x - e^{-x}$. Then find soln

Hint → $\boxed{y_g \text{ is same}, \ y_p = x - 2 - \frac{1}{2}x^2 e^{-x}}$

Case 4 When $r(x)$ is of form $ke^{ax}\sin bx$ or $ke^{ax}\cos bx$ or $k_1 e^{ax}\cos bx + k_2 e^{ax}\sin bx$

Then the choice of P. Soln is taken as →

$$y_p = e^{ax}(A\cos bx + B\sin bx) \quad \text{—— (1)}$$

However, if $r(x)$ satisfies HDE ~~associa~~ $(y'' + Py' + Qy = 0)$ or can be obtained from G. Soln (y_g) of HDE (similarily as done in Case 2) then choice of P. Soln is →

$$y_p = x e^{ax}\left[A\cos bx + B\sin bx\right] \quad \text{—— (2)}$$

(*) Here we are considering the case of product of Exponential fn and Sine or Cosine fn as $r(x)$

→ Two other Cases are

2) Product of Polyn fn with sine or Cosine fn

eg → $y'' + 9y = x \sin 3x$

3) Product of Polyn fn with exp fn

eg → $y'' + 2y' + y = 2x^2 e^{-2x} + 3e^{2x}$

→ [But we will only consider here case of → product of exp fn with sine or cosine fn

$k_1 e^{ax} \cos bx$ or $k_2 e^{ax} \sin bx$ or there L.C]

eg (1) Find the Gen. Soln of →

$$y'' - 2y' + 2y = e^{x} \sin x \quad \text{—(1)}$$

Use MOUC to find the P. Soln

Soln

Step (1)

HDE → $y'' - 2y' + 2y = 0$ —(2)

A.E → $m^2 - 2m + 2 = 0$

Roots → $m = 1 \pm i$

So $y_g = e^{x}[C_1 \cos x + C_2 \sin x]$ —(3)

Step (2)

Now as $r(x) = e^{x} \sin x$.

So our trial choice of P. Soln is →

$$y_P = e^{x}[A \cos x + B \sin x] \quad \text{—(4)}$$

But since $r(x) = e^{x} \sin x$ satisfies the HDE (2) as

it can be generated from $y_g = e^x(c_1 \cos x + c_2 \sin x)$
by taking $c_1 = 0$ & $c_2 = 1$

→ So actual choice of P. Soln will be →

$$y_p = x e^x (A \cos x + B \sin x) \quad \text{--- (5)}$$

Put y_p, y_p', y_p'' in (1) ~~and get~~ and compare coeffecients of $\cos x$ & $\sin x$ we get →

$$A = -\tfrac{1}{2} \text{ \& } B = 0$$

So that $y_p = x e^x \left(-\tfrac{1}{2} \cos x\right)$

$$= -\tfrac{1}{2} x e^x \cos x$$

Hence the Gen Soln is $= y_g + y_p$

$$= e^x [c_1 \cos x + c_2 \sin x] - \tfrac{1}{2} x e^x \cos x$$

(*) In same question if $r(x) = e^x \sin 2x$

that is, $y'' - 2y' + 2y = e^x \sin 2x$

Then our choice of P. Soln would have been →

as $r(x) = e^x \sin 2x$

& $y_g = e^x [c_1 \cos x + c_2 \sin x]$

So

$$y_p = e^x [A \cos 2x + B \sin 2x]$$

∵ $r(x)$ does not satisfy HDE associated with (1).

Qs for Practice (Try yourself) (MODC) 28A

Q1 $y'' - 2y' = e^{x}\sin x$

Q2 $y'' + 2y' + 10y = e^{-x}\sin 3x$.

Soln

1) $c_1 + c_2 e^{2x} - \frac{1}{2}e^{x}\sin x$

2) $e^{-x}(c_1\cos 3x + c_2\sin 3x) - \frac{1}{6}x e^{-x}\cos 3x$