UPH004: APPLIED PHYSICS
SOLITION OF
Tutorial Sheet # 6
[DIFFRACTION]

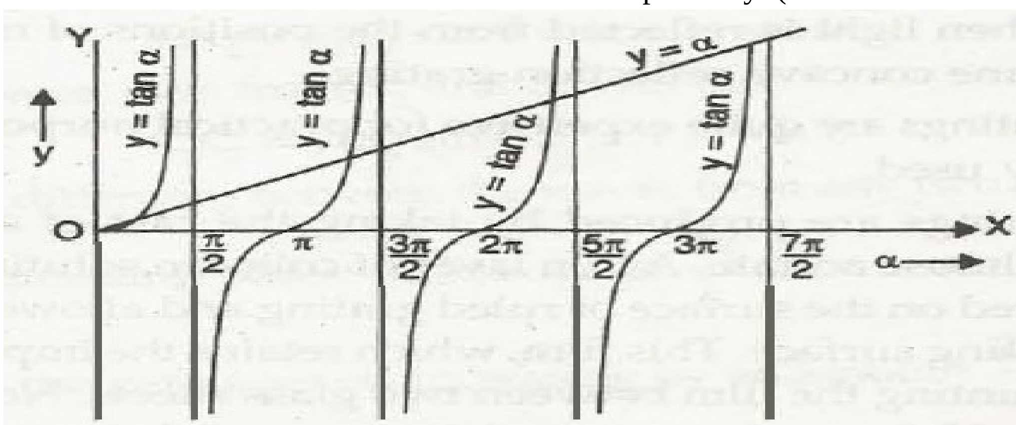
Q1. For a single slit diffraction fringe find the percentage intensities of 1st and 2nd order maxima with respect to that of the central maximum.

Solution:

For a single slit diffraction pattern the intensity is given by

$I=I_0\frac{\sin^2\alpha}{\alpha^2}$; where I_0 is the intensity of the CENTRAL MAXIMUM. (Corresponds to $\alpha=0$).

The 1st and 2nd order maxima occurs at $\alpha=1.43\pi$ and 2.46π respectively. (not at $\alpha=1.5\pi$ and 2.5π)



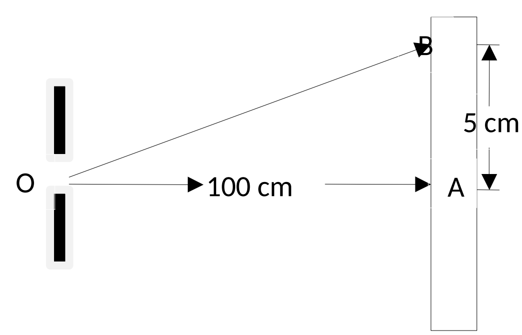
$\therefore$ Intensity of the 1st Maxima: $I_1=I_0\frac{\sin^2(1.43\pi)}{(1.43\pi)^2}=0.04719I_0$ => ~ 4.72% of the central Maximum.

$\therefore$ Intensity of the 2nd Maxima: $I_2=I_0\frac{\sin^2(2.46\pi)}{(2.46\pi)^2}=0.01647\,I_0$ => ~ 1. 65% of the central Maximum.

Answer:	**Respectively, 4.72% and 1. 65% of the central Maximum**

Q2. The eleventh order minima of a single slit diffraction pattern are found at a distance of 5 *cm* on either side of the central maximum. Find the wavelength of the monochromatic radiation used, while the distance between the slit and screen is 1*m* and slit width is 0.1*mm*

Solution:



For the minima of a single slit diffraction pattern

$$a\sin\theta=n\lambda$$

Here, *a*=> slit width =0.1 mm=0.01 cm.

n=> order number =11

θ=> the diffraction angle.

Let, O is the slit position and B is the location of 11th order minima.

$$\sin\theta=\frac{AB}{OB}=\frac{5}{\sqrt{100^2+5^2}}=\frac{5}{100.1249}=0.04994$$

$\therefore$ The wavelength of the monochromatic radiation $\lambda = \frac{a\, sin\theta}{n} = \frac{0.01 \times 0.04994}{11} cm = \frac{49940}{11}$ Å= 4540 Å

Answer:	**The wavelength of the monochromatic radiation $\lambda =$ ¿4540 Å**

Q3. A thin needle is placed at the centre of an aperture (as in figure), having width thrice that of the needle. If a laser beam incidences normally on this arrangement, which order spectrum will be absent from the diffraction pattern?

The needle divides the aperture into two equal slits.

Let, width of the opaque needle is =b,

Width of each slit is =*a*.

Then, according to the condition (and figure) *a*=b

Now, condition for INTERFERENCE MAXIMA:

$$(a+b)\, sin\theta = m\lambda, \text{ where } m = 0,1,2,3...\text{etc.} \qquad (1)$$

Condition for DIFFRACTION MINIMA:

$$a\, sin\theta = n\lambda, \text{ where } n = 1,2,3...\text{m.....2m.....3m....etc.} \qquad (2)$$

When, INTERFERENCE MAXIMA superpose on DIFFRACTION MINIMA that particular INTERFERENCE MAXIMA vanish.

Let the superposition occurs at an angleΦ.

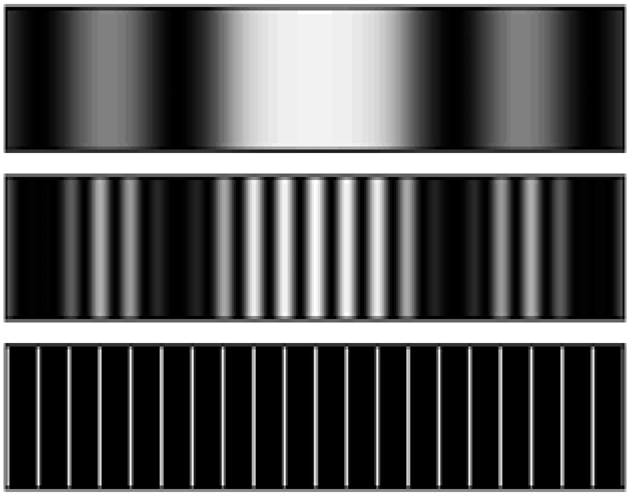
$$\therefore \frac{(a+b)\, sin\Phi}{a\, sin\Phi} = \frac{m\lambda}{n\lambda} \qquad \text{[By dividing eq.(1) by eq.(2)]}$$

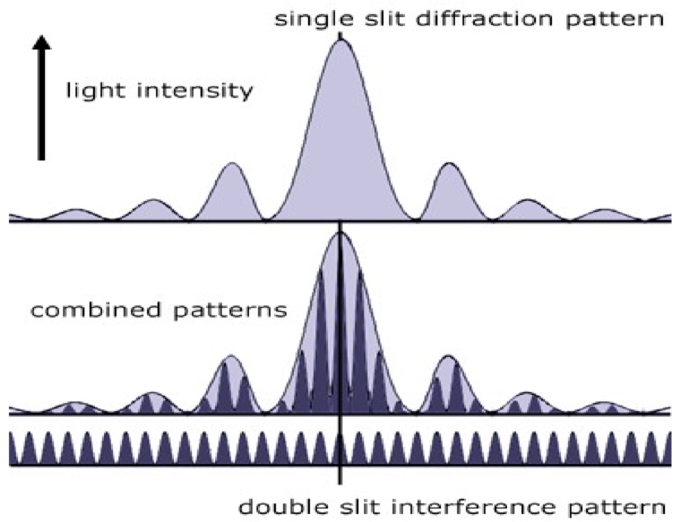
$$\text{Or } \frac{(a+a)}{a} = \frac{m}{n}$$

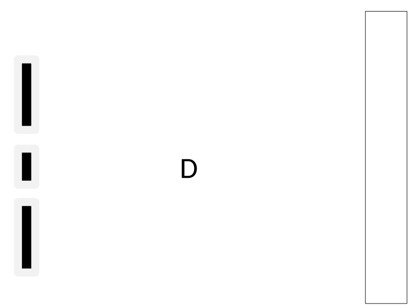
Or $m = 2n$= 2,4,6,8....

Answer:	**2nd , 4th , 6th ...order Interference Maxima will be absent/Vanished**

Q4.A double-slit, each slit having width 0.05 *cm* and a separation of 0.5 *cm* between them, forms diffraction pattern on a screen placed 1.5 *m* away from the slits. If the diffraction fringe width is 0.15*mm* find the wavelength of the monochromatic light used.







The figures shows that **the width of the final diffraction pattern**, (which is combination of the single-slit diffraction pattern and interference pattern) **is limited by the interference fringe**.

If d_n is the distance of the *n*-th order Interference Maxima from center and D is the slit-screen distance, then,

$$\sin\theta \approx \tan\theta = \frac{d_n}{D}$$

Now, $(a+b)\,sin\theta = n\lambda$

Or, $(a+b)\dfrac{d_n}{D} = n\lambda$

Or, $d_n = \dfrac{n\lambda D}{(a+b)}$

$\therefore\ d_{n+1} = \dfrac{(n+1)\lambda D}{(a+b)}$.

Thus, fringe width, say $\delta = d_{n+1} - d_n = \dfrac{\lambda D}{(a+b)}$

$\therefore\quad \lambda = \dfrac{\delta(a+b)}{D} = \dfrac{0.015\times(0.05+0.5)}{150}$ cm [Putting the given values of a, b, D and δ in *cm* unit]

$= 55\times10^{-6}$ cm $=5500$ **Å**

Answer:	The wavelength of the monochromatic light used $\lambda = 5500$ Å

Q5. 15,000 numbers of long chain Iodine molecules (opaque) are arranged parallel on a transparent thin film of length 1 inch. Let, the film is illuminated by a light of wavelength 5600 Å. How many bright spots will be observed on the screen? Label their order.

The arrangement of long chain opaque Iodine molecules on transparent thin film is equivalent to a plane transmission grating.

Using the relation for grating:

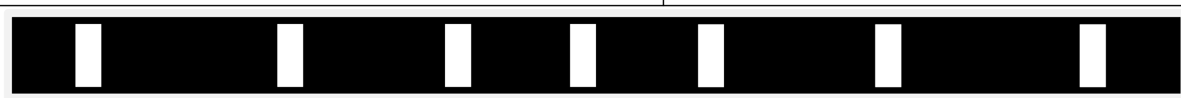
$$(a+b)\,sin\theta = n\lambda$$

Or, $$n = \frac{(a+b)\,sin\theta}{\lambda}$$

Or, $$n_{max} = \left.\frac{(a+b)\,sin\theta}{\lambda}\right|_{max} = \frac{(a+b)\times 1}{\lambda}$$

Or, $$n_{max} = \frac{2.54\,cm}{14999 \times 5600 \times 10^{-8}\,cm} = 3.024$$

$(a+b)$=> Grating element $= \frac{1\,inch}{15000-1} = \frac{2.54\,cm}{14999}$



n=3 n=2 n=1 n=0 n=1 n=2 n=3

Answer:	Total seven bright spot, each corresponds to the principal maxima will be observed

Q6. Prove that for white light (wavelength range 4000 Å to 7000 Å) the second and third order spectrum will partially overlap for any grating.

Condition for grating diffraction maxima:

$$d\,sin\theta = n\lambda, \qquad d = (a+b) \text{=> Grating element}$$

$$\theta = \sin^{-1}\left(\frac{n\lambda}{d}\right)$$

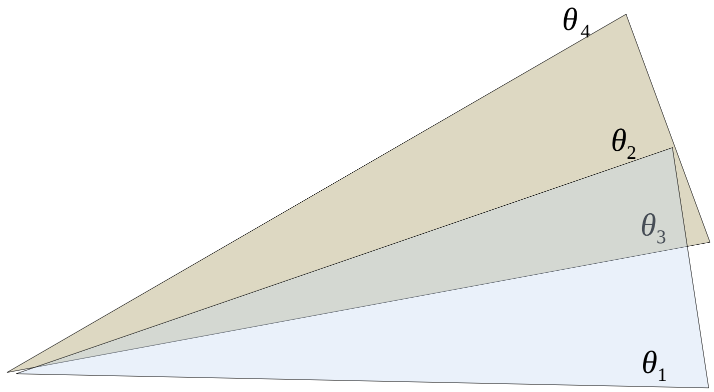
The edge colours of the spectrum	n=2 2nd order spectrum extends from θ_1 to θ_2	n=3 3rd order spectrum extends from θ_3 to θ_4
λ_1=4000 Å	$\theta_1 = \sin^{-1}\left(\frac{2\times 4000}{d}\right) = \sin^{-1}\left(\frac{8000}{d}\right)$	$\theta_3 = \sin^{-1}\left(\frac{3\times 4000}{d}\right)$ ¿ $\sin^{-1}\left(\frac{12000}{d}\right)$
λ_2=7000 Å	$\theta_2 = \sin^{-1}\left(\frac{2\times 7000}{d}\right)$ ¿ $\sin^{-1}\left(\frac{14000}{d}\right)$	$\theta_4 = \sin^{-1}\left(\frac{3\times 7000}{d}\right)$ ¿ $\sin^{-1}\left(\frac{21000}{d}\right)$

2^{nd} order spectrum extends from θ_1 to θ_2, while 3^{rd} order spectrum extends from θ_3 to θ_4.

Conclusions:

1. *Since* $,\theta_1<\theta_3<\theta_2<\theta_4,$ the 2^{nd} and 3^{rd} order spectrum will partially overlap.

2. Since, this is true for any Grating element d, the overlapping will be for any grating.



7. A plane transmission grating has 300 rulings per *mm*. Determine the dispersive power of violet (wavelength 4000 Å) and red (wavelength 6328 Å) light for second order diffraction pattern.

The DISPERSIVE POWER of a plane transmission grating is given by:

$$\frac{d\theta}{d\lambda}=\frac{n}{(a+b)\cos\theta}.$$

Here, n=> order=2

$(a+b)$=> Grating element$=\frac{1mm}{300-1}=\frac{10^{-3}}{299}m=3.344481\times10^{-6}m=33444.81$ Å.

We know, $(a+b)\,sin\theta=n\lambda$

$$sin\theta=\frac{n\lambda}{(a+b)}$$

$$cos\theta=\sqrt{1-\left(\frac{n\lambda}{(a+b)}\right)^2}$$

<u>For **violet (wavelength 4000 Å)**</u>:

$cos\theta=\sqrt{1-\left(\frac{2\times4000}{33444.81}\right)^2}$=0.97097.

Dispersive power for violate= $\frac{n}{(a+b)\cos\theta}=\frac{2}{3.344481\times10^{-4}\times0.97097}$ cm/rad=0.615879 $\times10^4$ rad/cm

=6158.79 rad/cm

<u>For **Red (wavelength 6328 Å)**</u>:

$cos\theta = \sqrt{1 - \left(\frac{2 \times 6328}{33444.81}\right)^2}$=0.92563.

Dispersive power for violate= $\frac{n}{(a+b)\cos\theta} = \frac{2}{3.344481 \times 10^{-4} \times 0.92563}$ cm/rad=0.646046 $\times 10^4$ rad/cm

=6460.46 rad/cm

Answer:	**Dispersive power for (i) violet(wavelength 4000 Å): 6158.79** rad/cm **(ii) Red (wavelength 6328 Å): =6460.46** rad/cm

Q8. A plane transmission grating can just resolve two spectral lines of wavelength 5499.5 Å and 5500.5 Å in the first order diffraction pattern. Determine the minimum order spectrum the same grating can resolve, while using another pair of wavelength 6500 Å and 6500.5 Å.

The resolving power of a plane transmission grating can be given by

$$\frac{\lambda}{d\lambda} = nN$$

‘Required’ RP to separate two spectral lines

‘Available’ RP of the Grating

Here, for the first case:

$\lambda = \frac{5499.5 + 5500.5}{2}$=5500 Å, and $d\lambda = 5499.5 - 5500.5$ Å=1 Å, n=1.

$N = \frac{\lambda}{nd\lambda} = \frac{5500}{1 \times 1}$=5500 => Total number of rulings on the grating.

For the SECOND case:

$\lambda = \frac{6500 + 6500.5}{2}$=6500.25 Å, and $d\lambda = 6500 - 6500.5$ Å=0.5 Å,

n=?

From the formula, $n = \frac{\lambda}{d\lambda N} = \frac{6500.25}{0.5 \times 5500}$=2.36.

Therefore, the same grating can resolve n=3, i.e., 3rd order spectrum.

Answer:	The grating can resolve 3rd order spectrum.