School of Mathematics
Thapar Institute of Engineering and Technology, Patiala,
UMA 004: Tutorial Sheet 02

1. Solve the following differential equations:

   (i) $2x(x+y)\dfrac{dy}{dx} = 3y^2 + 4xy; \quad y(1) = 1$ (ii) $(x^2+y^2)\dfrac{dy}{dx} = xy; \quad y(1) = 2$

   (iii) $x^2\dfrac{dy}{dx} = y^2 + 2xy$ (iv) $(x^3+y^3)dx - xy^2dy = 0$

2. Solve the differential equations:

   (i) $\dfrac{dy}{dx} = \dfrac{x+y+4}{x+y-6}$ (ii) $(2x+3y-1)dx - 4(x+1)dy = 0$

3. Determine which of the following equations are exact or not and if so then find its solution.

   (i) $\cos x \cos^2 y\, dx + 2\sin x \sin y \cos y\, dy = 0$ (ii) $(2xy^4 + \sin y)\, dx + (4x^2y^3 + x\cos y)\, dy = 0$

4. Find the value of n for which the equation $(xy^2 + nx^2y)dx + (x^3 + x^2y)dy = 0$ is exact and hence solve for that value of n.

5. Show that if $\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{(N-M)}$ is a function of $g(z)$ where $z = x+y$, then $\mu = e^{\int g(z)dz}$ is an integrating factor for equation $M(x,y)dx + N(x,y)dy = 0$.

6. Solve the following equations by finding an integrating factor

   (i) $y\ dx - x\ dy = xy^3\ dy$ (ii) $x\ dy - y\ dx = (2x^2-3)dx$
   (iii) $x\ dy = (x^5 + x^3y^2 + y)\ dx$ (iv) $(2xy^2 - y)\ dx + x\ dy = 0$
   (v) $(3x^2 - y^2)\ dy - 2xy\ dx = 0$ (vi) $(x+2)\sin y\ dx + x\cos y\ dy = 0$
   (vii) $(3x^2y^3e^y + y^3 + y^2)\ dx + (x^3y^3e^y - xy)\ dy = 0$ (viii) $(5x^3 + 12x^2 + 6y^2)\ dx + 6xy\ dy = 0$

7. Solve the following linear equations

   (i) $y' + y = \dfrac{1}{1+e^{2x}}$ (ii) $y' + y\cot x = 2x\csc x$

8. The equation $\frac{dy}{dx} + P(x)y = Q(x)y^n$ which is known as Bernoulli's equation, is linear when $n = 0$ or 1. Show that it can be reduced to a linear equation for any other value of n by the change of variable $z = y^{1-n}$ and apply this method to solve the following equations:

   (i) $xy' + y = x^4y^3$ (ii) $x\ dy + y\ dx = xy^2\ dx$

Answers:

1. (i) $y^2 + 2xy = 3x^3$ (ii) $2y^2\ln(cy) = x^2$, (iii) $y = \frac{cx^2}{1-cx}$ (iv) $y^3 = x^3\log(cx^3)$

2. (i) $y - x = 5\log(x+y-1) + c$ (ii) $(2x - y + 3)^4 = c(x+1)^3$

3. (i) Not Exact (ii) $x^2y^4 + x\sin y = c$

4. $n = 3$; $x^2y^2 + 2x^3y = c$

6. (i) $\log\frac{x}{y} = \frac{y^3}{3} + c$ (ii) $y = 2x^2 + 3 + cx$ (iii) $\tan^{-1}\frac{x}{y} = -\frac{x^4}{4} + c$ (iv) $y = \frac{x}{x^2+c}$
   (v) $x^2 - y^2 = cy^3$ (vi) $x^2e^x\sin y = c$ (vii) $x^3e^y + x + \frac{x}{y} = c$ (viii) $x^5 + 3x^4 + 3x^2y^2 = c$

7. (i) $y = e^{-x}\tan^{-1}(e^x) + ce^{-x}$ (ii) $y = x^2\csc x + c\csc x$

8. (i) $\frac{1}{y^2} = -x^4 + cx^2$ (ii) $1 + xy\log x = cxy$