Tutorial Sheet # 4.

HINTS

1. At peak $\vec{\nabla} h(x,y) = 0$

$\Rightarrow 5(2y-6x-18)\hat{i} + 5(2x-8y+28)\hat{j} = 0$

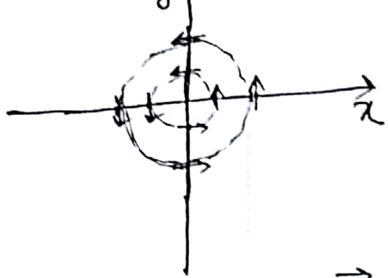
$\Rightarrow$ $\left.\begin{array}{l} y-3x-9=0 \\ x-4y+14=0 \end{array}\right\}$ Solving these, we get $x=-2$, $y=3$

So Location of peak is at $(-2,3)$

Height of peak is $h(-2,3) = 330$ units.

2. $\vec{\nabla}\cdot\vec{f} = 0$ i.e. f is not diverging.

3. $\vec{\nabla}\times\vec{f} = 2\hat{k}$

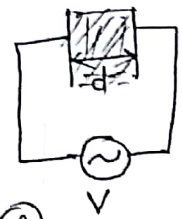


4. Displacement current density $\vec{J}_d = \frac{\partial \vec{D}}{\partial t}$

Displacement vector $\vec{D} = \epsilon\vec{E}$

$|\vec{D}| = \epsilon E = \epsilon\frac{V}{d}$

$= \frac{\epsilon}{d}V_0 \sin 2\pi\nu t$



$\therefore J_d = \frac{\epsilon}{d}V_0 \cdot 2\pi\nu \cos 2\pi\nu t$ — (A)

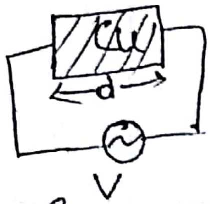
The conduction current density

$J_c = \sigma E = \frac{1}{\rho}\frac{V}{d} = \frac{1}{\rho d}V_0 \sin 2\pi\nu t$ (B)

From (A) & (B)

$\frac{J_c}{J_d} = \frac{1}{2\pi\nu\rho\epsilon} \tan 2\pi\nu t$ — (C)

5. This question is somewhat similar to Q. N. (4)



Put values of ν, ρ & ϵ in Eq. (C).

$\frac{J_c}{J_d} \simeq 3.5\times10^{15} \tan 100\pi t$

$\left\{\begin{array}{l} I = \frac{V_0}{\rho d}\sin 2\pi\nu t \\ J_d = \frac{2\pi\nu\epsilon V_0}{d}\cos 2\pi\nu t \end{array}\right\}$ Metals are opaque for such frequencies.

6. Induced emf $= -\frac{\partial\phi}{\partial t}$

$\phi = \vec{B}\cdot\vec{A} = BA\cos 60^\circ = 0.5\,BA$

emf $= -\frac{\partial}{\partial t}(0.5\,BA) = -0.5A(0-0.006t)$

emf at $t=2s = 0.5\times25\times10^{-4}\times0.024$ T

$= 3.0\times10^{-5}$ T

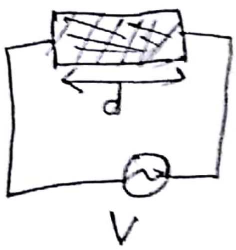
7. $\vec{E}$ and $\vec{B}$ are $\perp^{er}$ to each other.

$\vec{E}$ and $\vec{B}$ are also $\perp^{er}$ to direction of propagation (i.e. $\vec{E}\times\vec{B}$)

8. From Ques. 5

$(J_d)_{max} = (J_c)_{max}$

$\frac{2\pi\nu V_0\epsilon}{\cancel{d}} = \frac{\cancel{V_0}}{\rho d}$



$\nu = \frac{1}{2\pi\epsilon\rho}$

$\nu = \frac{\sigma}{2\pi\epsilon}$

$\simeq 9\times10^{10}$ Hz

$\left\{\begin{array}{l} \epsilon = \epsilon_0 = 8.85\times10^{-12} \\ C^2/N\cdot m^2 \end{array}\right.$

9. $\tan\phi = \frac{k_-}{k_+} = 1$ $\left\{\begin{array}{l} \text{For good conductor} \\ k_- = k_+ \end{array}\right.$

$\Rightarrow \phi = 45^\circ$

10. Skin depth $d = \frac{1}{k_-} = \sqrt{\frac{2}{\omega\sigma\mu}}$

$\omega = 2\pi\nu$.

$d = \sqrt{\frac{2}{2\pi\nu\sigma\mu}} = 2\times10^{-9}$ m