Homework #2

1. Initial Value theorem

$$f(t) \rightleftharpoons F(s)$$

$$f(0^+) = \lim_{s \to \infty} sF(s)$$

proof $L\left(\frac{d^n f(t)}{dt^n}\right) = s^n F(s) - s^{n-1} f(0^-) - s^{n-2} f'(0^-) \ldots\ldots$

for $n=1$

$$L\left(\frac{d f(t)}{dt}\right) = sF(s) - f(0^-)$$

$$\int_0^\infty \frac{df(t)}{dt} e^{-st} dt = sF(s) - f(0)$$

Assuming $\frac{df}{dt}$ is continuous and taking limit $s \to \infty$

$$\lim_{s \to \infty} e^{-st} \to 0$$

$$\therefore \quad 0 = \lim_{s \to \infty} [sF(s)] - f(0)$$

$$f(0) = \lim_{s \to \infty} [sF(s)]$$

1 The Final Value theorem

$$f(t) = F(s)$$

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} s\,F(s)$$

$$L\left(\frac{df(t)}{dt}\right) = sF(s) - f(0)$$

taking limit $s \to 0$, $e^{-st} \to 1$

$$\lim_{s\to 0} \int_0^\infty \frac{df(t)}{dt} e^{-st}\, dt = sF(s) - f(0)$$

$$\lim_{s\to 0} \int_0^\infty \frac{df}{dt}\, dt = sF(s) - f(0)$$

$$f(\infty) - f(0) = \lim_{s\to 0} sF(s) - f(0)$$

$$\cancel{\lim_{s\to 0}}\; f(\infty) = \lim_{s\to 0} sF(s)$$

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$$

Page No.
Date :

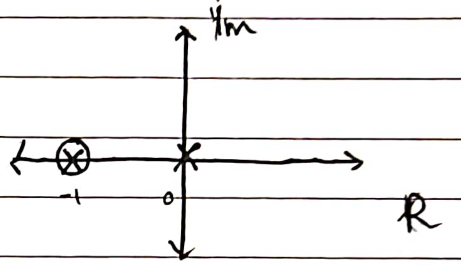
(2) a) Since the input is step function, $U(s) = 1/s$

a) $G(s) = \dfrac{6(s+1)}{s^2(s+1)}$; zeros : $\varnothing$ / -1 ; poles : $0, -1$

$Y(s) = \dfrac{6(s+1)}{s^2(s+1)} \cdot \dfrac{1}{s} \Rightarrow \dfrac{6}{s^3}$; pole on left side ; hence can cancel

$L^{-1}(Y(s)) = L^{-1}(6/s^3) = \dfrac{6t^2}{2} \Rightarrow 3t^2$

it unbounded integrating system.



b) $G(s) = \cancel{6}\ \dfrac{12(s+2)}{s(s^2+9)}$ ~~pot~~ zeros : -2 ; poles : $0, +3i, -3i$

$Y(s) = \dfrac{12(s+2)}{s(s^2+9)} \cdot \dfrac{1}{s}$

$b = 8/3$

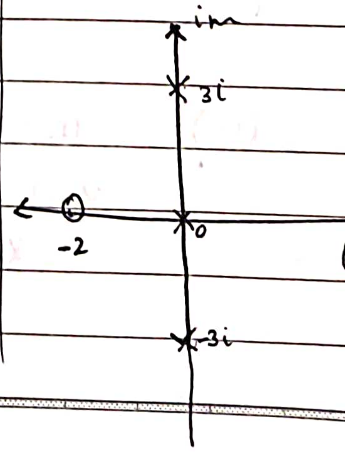
$\Rightarrow \dfrac{a}{s} + \dfrac{b}{s^2} + \dfrac{cs+d}{s^2+9}$

$\Rightarrow \dfrac{4}{3s} + \dfrac{8}{3s^2} - \dfrac{4}{3}\left(\dfrac{s+2}{s^2+9}\right)$

taking Inverse laplace transform

$y(t) = \dfrac{4}{3} s(t) + \dfrac{8}{3} t - \dfrac{4}{3}\left[\cos 3t + \dfrac{2}{3}\sin 3t\right]$

→ unbounded, integrating, no damping



c) $G \cancel{X}(s) = \dfrac{(s+2)(s+3)}{(s+4)(s+5)(s+6)}$

poles : $-4, -5, -6$

zeros : $-2, -3$

$$Y(s) = \frac{(s+2)(s+3)}{(s+4)(s+5)(s+6)} \cdot \frac{1}{s}$$

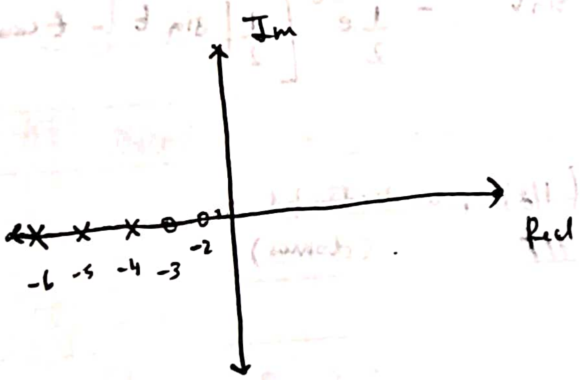
$$= \frac{a}{s+4} + \frac{b}{s+5} + \frac{c}{s+6} + \frac{d}{s}$$

$$= \frac{1}{20s} - \frac{1}{4(s+4)} + \frac{6}{5(s+5)} - \frac{1}{s+6}$$

taking inverse laplace,

$$\frac{1}{20} - \frac{1}{4}e^{-4t} + \frac{6}{5}e^{-5t} - e^{-6t}$$

→ Bounded, having steady state (1/20), ~~[illegible]~~ under-damped (dumnas)



d) $G(s) = \dfrac{1}{[(s+1)^2+1]^2(s+2)}$ zeros

$G(s) \cdot U(s) = Y(s) = \dfrac{1}{[(s+1)^2+1]^2(s+2)} \cdot \dfrac{1}{s}$

$= \dfrac{a}{s} + \dfrac{b}{s+2} + \dfrac{cs+d}{(s+1)^2+1} + \dfrac{es+f}{[(s+1)^2+1]^2}$

$a = 1/8$
$b = -1/8$
$(s+1)^2 = -1$
$s = -1+i$
$(1+i)(1-i)$
2
$c=0 \quad f = -1/2$

$\Rightarrow \dfrac{1}{8s} - \dfrac{1}{8(s+2)} - \dfrac{1}{4[(s+1)^2+1]} - \dfrac{1}{2[(s+1)^2+1]^2}$

$\dfrac{1}{25 \cdot 3} \quad \dfrac{1}{75}$

$\left(\dfrac{1}{8} - \dfrac{1}{24} - \dfrac{1}{2 \times 25}\right)3$

$c=0 \quad d = -1/4$

$-\dfrac{1}{2}e^{-t}\left(L^{-1}\left(\dfrac{1}{(s^2+1)^2}\right)\right)$

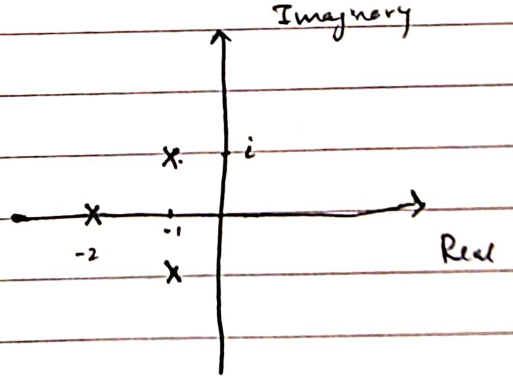
$\hookrightarrow \left(1/(s+1)\right) + \dfrac{bs+1}{(s^2+1)}$

taking inverse laplace transform,

$y(t) = \dfrac{1}{8} - \dfrac{1}{8}e^{-2t} - \dfrac{1}{4}e^{-t}\sin t - \dfrac{1}{2}e^{-t}\left[\dfrac{1}{2}[\sin t - t\cos t]\right]$

Bounded, steady state $(1/8)$, underdamped (dsmos)

poles : -2, $i-1$, $-i-1$



c)

③ Conditions for LT to exist

(i) $f(t)$ is piecewise continuous on $0 \le t \le A$ for every $A > 0$,

the function $1/(t-1)$ has a discontinuity at $t = 1$

$\therefore \frac{1}{t-1}$ does not have Laplace transform.

④ Steady State gain $\frac{y_{ss_2} - y_{ss_1}}{U_2 - U_1}$

a) (i) $U_2 = 2$ $\quad y_{ss_2} = 0$

$U_1 = 1$ $\quad y_{ss_1} = 1$

$\therefore k = \frac{0-1}{2-1} = \boxed{-1}$

(ii) $U_2 = 2$ $\quad y_{ss_2} = 2$

$U_1 = 1$ $\quad y_{ss_1} = 1$

$\therefore k = \frac{2-1}{2-1} \boxed{=1}$

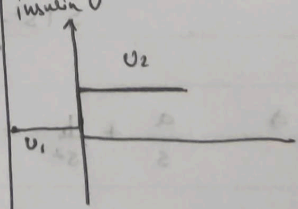
(iii) $U_2 = 2$ $\quad y_{ss_2} = -1$

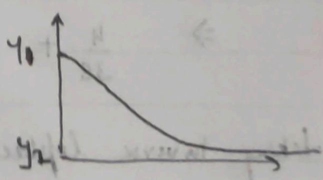
$U_1 = 1$ $\quad y_{ss_1} = 2$

$\therefore K = \cancel{2 + -1}$ $\frac{-1-(2)}{2-1} = \boxed{-3}$

b) body insulin & blood glucose

An increase in body insulin leads to decrease in blood suger





$y_2 - y_1$ is negative

$U_1 - u_1$ is positive

$\frac{y_2 - y_1}{U_2 - U_1} < 0$

c)

(3) Conditions for LT to exist

(i) $f(t)$ is piecewise continuous on $0 \le t \le A$ for every $A > 0$,

the function $1/(t-1)$ has a discontinuity at $t = 1$

$\therefore \frac{1}{t-1}$ does not have Laplace transform.

(4) Steady State gain $\frac{y_{ss_2} - y_{ss_1}}{u_2 - u_1}$

a) (i) $u_2 = 2$ $\quad y_{ss_2} = 0$

$u_1 = 1$ $\quad y_{ss_1} = 1$

$\therefore k = \frac{0 - 1}{2 - 1} = \boxed{-1}$

(ii) $u_2 = 2$ $\quad y_{ss_2} = 2$

$u_1 = 1$ $\quad y_{ss_1} = 1$

$\therefore k = \frac{2-1}{2-1} \boxed{= 1}$

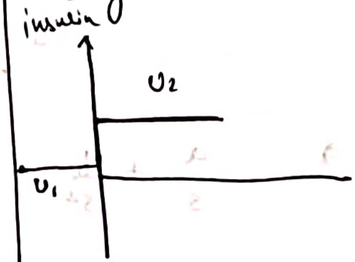
(iii) $u_2 = 2$ $\quad y_{ss_2} = -1$

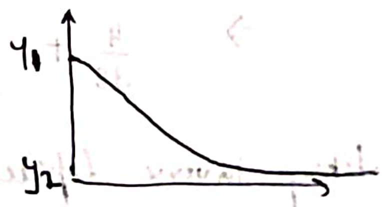
$u_1 = 1$ $\quad y_{ss_1} = 2$

$\therefore k = \cancel{2+-1}$ $\frac{-1-(2)}{2-1} = \boxed{-3}$

b) body insulin & blood glucose

An increase in body insulin leads to decrease in blood suger





$y_2 - y_1$ is negative

$u_2 - u_1$ is positive

$\frac{y_2 - y_1}{u_2 - u_1} < 0$

(3 Q5) 5

$$G(s) = \frac{1}{(1.5s+1)^2}$$

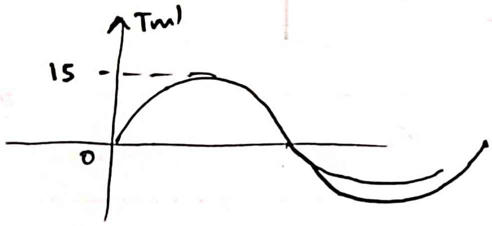
$f = 0.1$ cycle/min $\omega = 2\pi f = 0.2\pi$ rad/min

$T_{meas} = 350°C$ $T = \frac{T_c + 2T_w}{3}$

$$\frac{T_m'(s)}{T'(s)} = \frac{1}{(1.5s+1)^2}$$ ($T_m' = T - \bar{T}$)

$\bar{T}_m(t) = 350°C$

$T_m'(t) = 15\sin(\omega t)$ $T_m'(s) = \frac{15\omega}{\cancel{s^2+\omega^2}}\ s^2+\omega^2$ $\frac{A\omega}{s^2+\omega^2}$ ∴ input

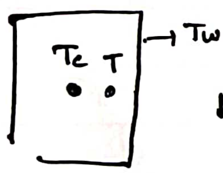


a) At steady state, $T_m' = 0 \Rightarrow \bar{T}_m - T_m = 0$

$T_m = \bar{T}_m = 350°C$

$\bar{T}_w = 200°C$

$T_c = 3T - 2T_w \Rightarrow 650°C$

b) 

Here T is output

Page No.
Date :

5

b) $Y(s) = \frac{1}{(1.5s+1)^2} \cdot \frac{A\omega}{s^2+\omega^2}$ $\Rightarrow \left(\frac{1}{k\tau^2 s^2 + 2\tau\zeta s + 1} \cdot \frac{A\omega}{s^2+\omega^2} \right)$

$y'(t) = \frac{KA}{\sqrt{[1-(\omega\tau)^2]^2 + (2\zeta\omega\tau)^2}} \sin(\omega t + \phi)$

comparing,
$\zeta = 1$
$\tau = 1.5$
$k = 1$

(After transient dies down)

$y'(t) = 15 \sin \omega t$ $\hat{A} = 15$

$\frac{\hat{A}}{KA} = \frac{1}{\sqrt{[1-(\omega\tau)^2]^2 + (2\zeta\omega\tau)^2}}$

$= \frac{1}{1+(\omega\tau)^2}$

$A = K\hat{A}\left(1+(\omega\tau)^2\right) = 28.32°C$

$T = \bar{T} + A\sin(\omega t + \phi)$
$T_{max} = \bar{T} + A = 378.32°C$
$T_{min} = \bar{T} - A = 321.68°C$

c) $T_c = 3T - 2T_w$

$T_{c_{max}} = 3T_{max} - 2T_w = 734.96°C$
$T_{c_{min}} = 3T_{min} - 2T_w = 565.04°C$

d)

(5)

a) for $715^\circ C$ $\qquad T < (715 - 680\,350) = 365^\circ$

$A = \hat{A}\,(1+(\omega^2\tau)^2) < 365 \quad \Rightarrow \quad \omega^2\tau^2 < 23.33$

$\omega \tau < \dfrac{4.83}{1.5} = 3.22$

$2\pi f = \omega \qquad f \leq 0.512$ cycles/min

Page No.
Date :

Q6) Falling of a ball under air resistance,
the air resistance acts as a damper and
hence reduces the ~~damper~~ heigh (Amplitude)
The mechanics almost work similar to a spring,
After each the height with which the ball raises to
gradually decreases and attain steady state at $h=0$

Q6) Oscillation of pendulum with disk

Q7) NO, because the denominator of transfer function for an underdamped system has complex roots.

A pair of first order system TF can be represented as

$$\frac{k_1}{Z_1 s+1} \quad \text{and} \quad \frac{k_2}{Z_2 s+1}$$

A series combination of these two results in a TF

$$\frac{k_1 k_2}{(Z_1 s+1)(Z_2 s+1)}$$

As visible here, ~~these~~ the denominator has real roots of $s = -1/Z_1$ and $s = -1/Z_2$

Hence a two first order in series won't produce an underdamped system.

While two first order in parallel too will not form

TF $G(s) = \frac{k_1}{(Z_1 s+1)} + \frac{k_2}{(Z_2 s+1)}$

Q8

Page No.
Date :

Q8 ~~$\frac{d^2h'}{}$~~ $\frac{d^2h'}{dt^2} + \frac{6\mu}{R^2\rho}\frac{dh'}{dt} + \frac{3}{2}\frac{g}{l}h' = \frac{3}{4\rho L}p'(t)$

taking LT of both sides results in

$$s^2H'(s) + \frac{6\mu}{R^2\rho}sH'(s) + \frac{3g}{2L}H' = \frac{3}{4\rho L}P'(s)$$

$$H'(s) = \frac{3/4\rho L \; P'(s)}{s^2 + \frac{6\mu}{R^2\rho}s + \frac{3g}{2L}}$$

the form ⇒

$$\frac{\frac{\cancel{3}}{2\,4\rho L} \times \frac{2L}{\cancel{3}g} \quad p(s)}{\frac{2L}{3g}s^2 + \frac{4\mu L}{R^2\rho g} + 1}$$

Comparing with $\frac{k}{\tau^2s^2 + 2\xi\tau s + 1}$

$k = 1/2\rho g$

$\tau^2 = \frac{2L}{3g} \quad \therefore \tau = \sqrt{\frac{2L}{3g}}$

$2\xi\tau = \frac{4\mu L}{R^2\rho g} \quad = \frac{2\mu L}{R^2\rho g}\cdot\sqrt{\frac{3g}{2L}}$

$\xi = \sqrt{\frac{6L}{g}}\cdot\frac{\mu}{R^2\rho}$

ω^2 $1/8$ $1/8$ -1 i $1-i)$ $-1/2$ $\frac{1}{2\times 25})^3$ 4 3×1

b) for oscillation, system should be underdamped i.e

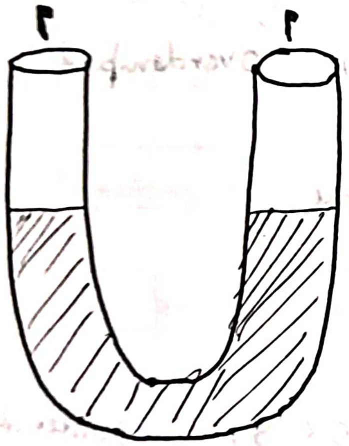
$0 < \zeta < 1$

$$0 < \sqrt{\frac{6L}{g}} \frac{\mu}{R^2 \rho} < 1$$

for greater ρ, ζ decrease and approaches zero, hence more oscillation.

for greater μ, ζ increase hence increasing it would result in decrease oscillations.

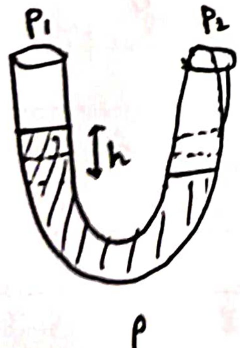
9)



A U-tubed equipment used to measure pressure differences on the free ends of the tube.

A manometer tube with equal pressure on either sides.

Applying different pressures would result in the rise of liquid in one column and fall in another.



$P_2 - P_1 = \rho g h$

ρ is the density of liquid.

(usually one end of manometer is exposed to atmosphere)

Page No.
Date

b) i) $\rho = 2000$, $\mu = 0.645 \times 10^{-1}$, $R = 0.5 \times 10^{-2}$
$L = 1$, $g = 9.8$

$k = \frac{1}{2}\rho g = 2.55 \times 10^{-5}$

$\tau = \sqrt{\frac{2L}{3g}} = 0.26$

$\zeta = \sqrt{\frac{6L}{g}} \frac{\mu}{R^2 \rho} = 1.01 \approx 1$ $\zeta \approx 1$, ~~overdamped~~ critically damped.

ii) $\rho = 2000$, $\mu = 2e-1$, $R = 0.5e^{-2}$ $L = 1$ $g = 9.8$

$k = 2.55 e{-5}$
$\tau = 0.26$ $\zeta > 1$ overdamped
$\zeta = 3.13$

iii) $\mu = 1e-2$, rest alls same,

$K = 2.55\ e{-5}$
$\tau = 0.26$ $0 < \zeta < 1$ underdamped
$\zeta = 0.157$

c) case 1: ~~overdamped~~ critically damped if $\zeta \approx 1$
case 2: Over damped
case 3: underdamped

$\xi\ \xi\ \xi\ \xi$
$\xi\ \xi\ \xi$

8

d) $p'(t) = 1e^4 u$ $\quad \therefore p'(s) = \frac{1e4}{s}$

Case 1 ($\zeta = 1$)

$$\frac{H'(s)}{P'(s)} = \frac{k}{\tau^2 s^2 + 2\zeta\tau s + 1} \cdot \frac{1e^4}{s} \Rightarrow \frac{\cancel{2.55\times10^{-5}}\ k}{(\tau s+1)^2} \cdot \frac{\cancel{1e}10^4}{s}$$

finding inverse LT,

$$LT^{-1}\left\{ \frac{k \cdot 10^4}{(\tau s+1)^2 s} \right\} = 10^4 k\left[1 - \left(1+\frac{t}{\tau}\right)e^{-t/\tau}\right]$$

putting in values, we get

$$\cancel{\frac{H'(s)}{P'(s)}}\ \ \frac{h'(t)}{\cancel{p'(t)}} = 0.255\left[1 - \left(1+\frac{t}{0.26}\right)e^{-t/0.26}\right]$$

Case 2: Overdamped

the solution for step response is given by

$$h'(t) = km\left[1 - \frac{\tau_1 e^{-t/\tau_1} - \tau_2 e^{-t/\tau_2}}{\tau_1 - \tau_2}\right]$$

According to formula,

$$\tau_1 = \frac{\tau}{\zeta - \sqrt{\zeta^2 - 1}} \quad \text{and} \quad \tau_2 = \frac{\tau}{\zeta + \sqrt{\zeta^2 + 1}}$$

$\tau_1 = 1.59 \qquad \tau_2 = 0.043$

$$h'(t) = 0.255\left[1 - \frac{1.59\,e^{-t/1.59} - 0.043\,e^{-t/0.043}}{1.55}\right]$$

Page No.
Date :

case 3 : underdamped.

the solution for step response is given as.

$$h'(t) = km \left\{ 1 - e^{-\zeta t/2} \left[\cos\left(\frac{\sqrt{1-\zeta^2}\, t}{2}\right) + \frac{\zeta}{\sqrt{1-\zeta^2}} \sin\left(\frac{\sqrt{1-\zeta^2}\, t}{2}\right) \right] \right.$$

$$\cancel{\delta l = 1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta \omega_n}}$$

putting in values.

$$= \cancel{2.5}\ 0.255 \left[1 - e^{-0.157\, t/0.26} \left[\cos\left(\frac{\sqrt{1-0.157^2}\, t}{0.26}\right) \right. \right.$$

$$+ \frac{0.157}{\sqrt{1-0.157^2}} \sin\left(\frac{\sqrt{1-0.157^2}\, t}{0.26}\right)$$

$$= 0.255 \left\{ 1 - e^{-0.6t} \left[\cos(3.8t) + 0.16 \sin(3.8t) \right] \right\}$$

d Decay Ratio : $\exp\left(\frac{-2\pi\zeta}{\sqrt{1-\zeta^2}}\right) = 0.36$

Overshoot : $\sqrt{DR} = 0.6$

Period : $\frac{2\pi\tau}{\sqrt{1-\zeta^2}} = 1.65$

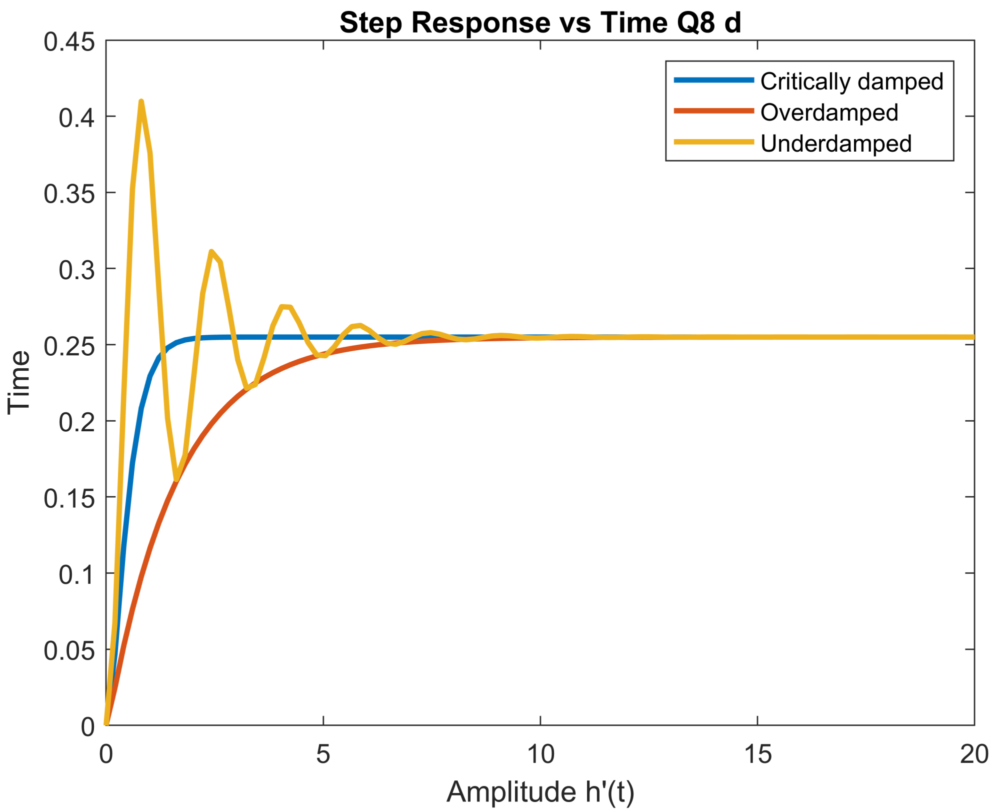
Q9) $G(s) = \frac{1}{(3s+1)^n}$; $n = 1, 2 \ldots n$

b) Yes, partially
Missed features
1) Slope zero initially in original function and increases gradually and then decrease.
while the approximation, slope 0 for tw and then shoots to a large value, then decreases.

c) Yes, it gives a better approximation than 1st order
the slope varies similar.
there is a only sudden increase of slope for 2nd order system
~~sort it~~

1) $\lambda = 20$

```
% Q8 d
t=linspace(0,20);
y=case1(0.26,t);
plot(t,y,LineWidth=2)
hold on
y2=case2(1.59,0.043,t);
plot(t,y2,LineWidth=2)
y3=case3(0.26,0.157,t);
plot(t,y3,LineWidth=2)
hold off
title("Step Response vs Time Q8 d")
xlabel("Amplitude h'(t)")
ylabel("Time")
legend("Critically damped","Overdamped","Underdamped")
```




```
function outp = case1(tau,t)
    outp = 0.255.*(1-(1+t./tau).*exp(-t./tau));
end

function outp = case2(tau1,tau2,t)
    %outp = 0.255.*(1-tau1.*exp(-t./tau1) -tau2.*exp(-t./tau2))./(tau1-tau2)
    outp=0.255.*(1-(1.59.*exp(-t./1.59)-0.043.*exp(-t./0.043))./1.55);
```

```
end

function outp=case3(tau,eta,t)
    %outp=0.255*(1-exp(-eta.*t./tau).*(cos(sqrt(1-eta^2).*t./tau) + (eta/(sqrt(1-eta*eta)))) .*
    outp = 0.255.*(1-exp(-0.6*t).*(cos(3.8*t)+0.16.*sin(3.8.*t)));
end
```

```
% Q8 e
syms s
syms t
rho = 2000;
mu = [0.01];
R=0.005;
L=1;
g=9.8;
M=10e4;
K=1/(2*rho*g);
tau=sqrt(2*L/(3*g));
zeta= (4*L*mu./(g*R*R*rho))/(2*tau)
```

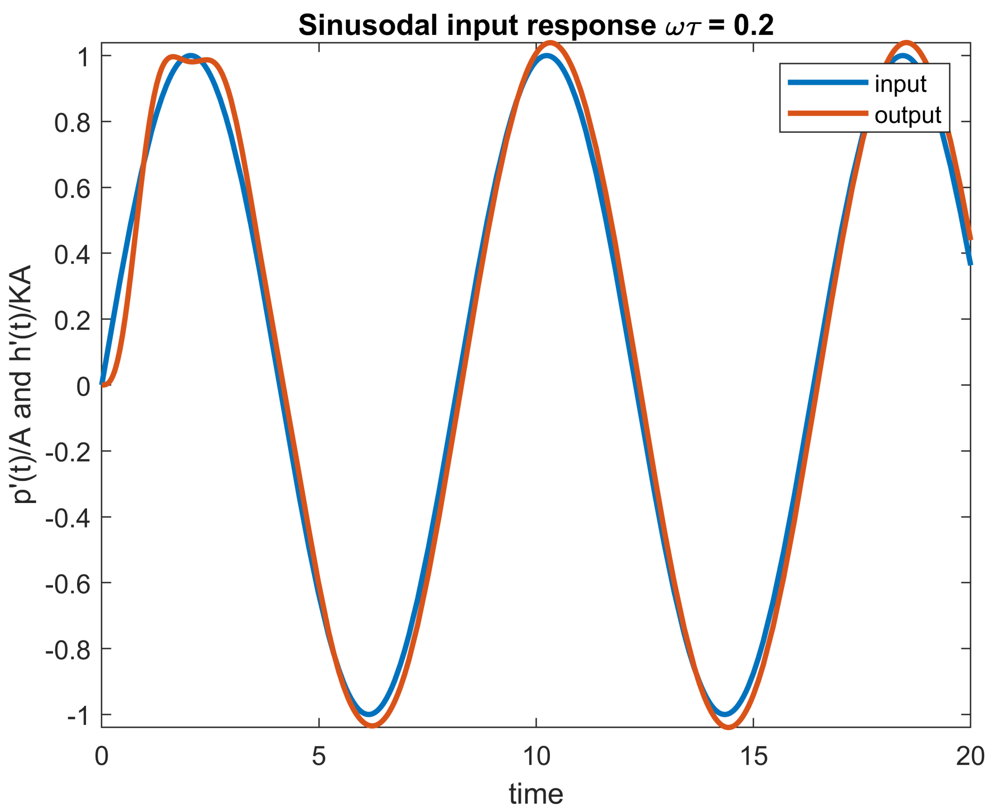
```
zeta = 0.1565
```

```
w=[0.2 1 2.5]/tau;
part1=sin(w(1)*t)
```

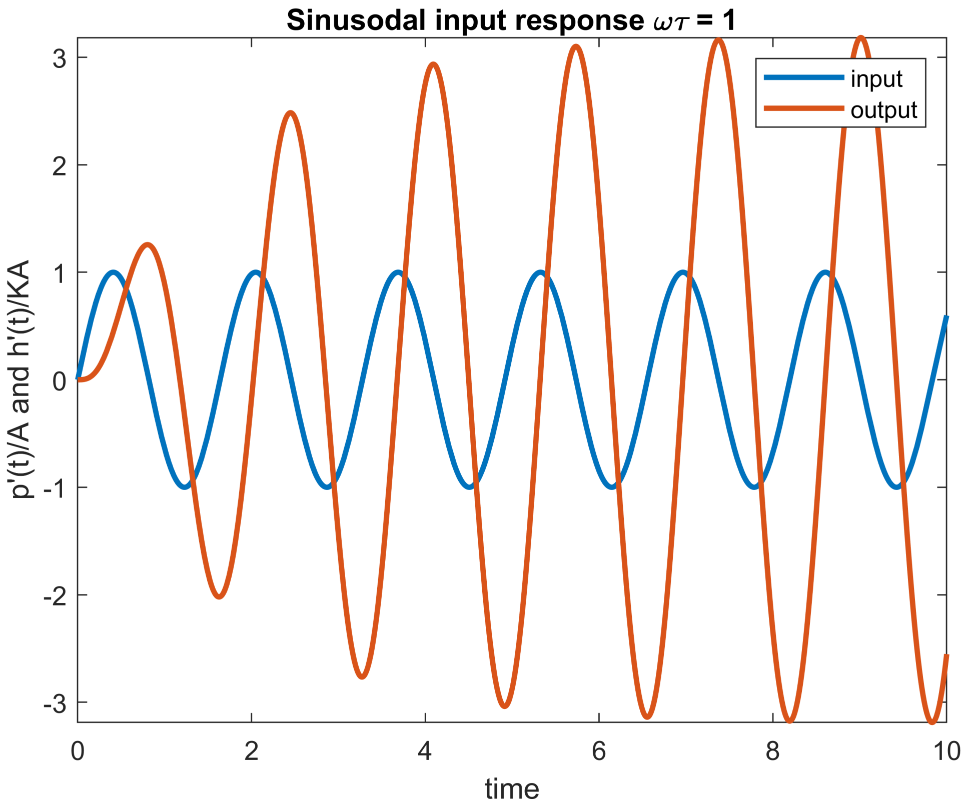
part1 =

$$\sin\left(\frac{7\sqrt{3}\ \sqrt{10}\ t}{50}\right)$$

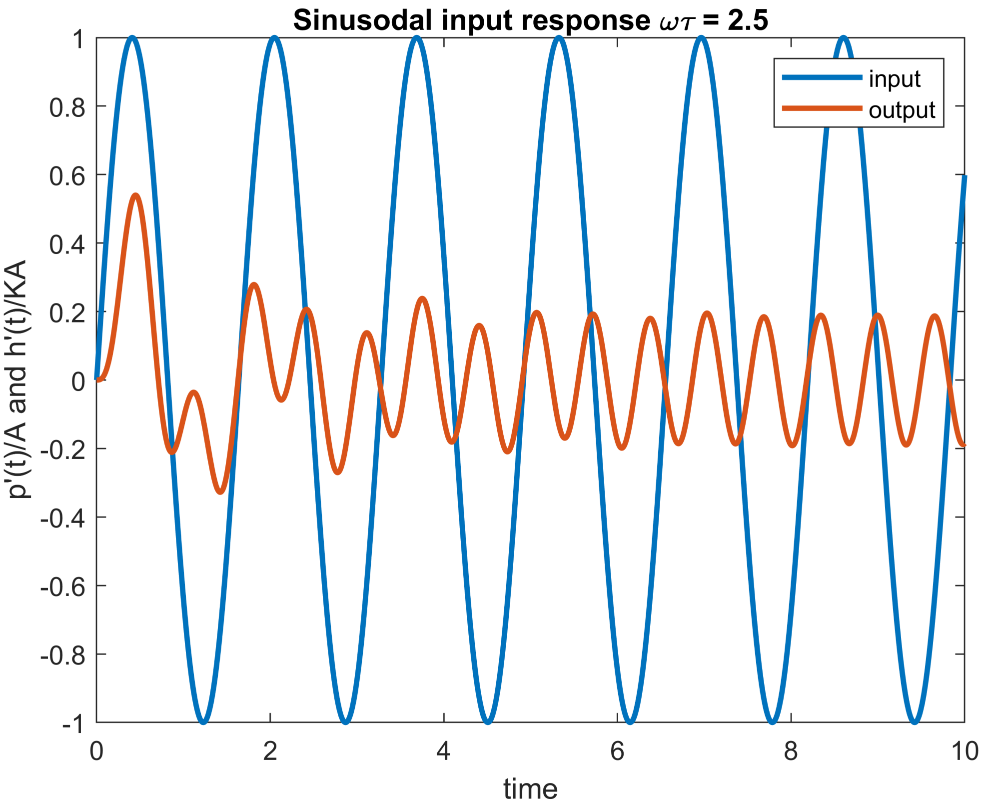
```
fplot(part1,[0 20],linewidth=2)
hold on
y1 = ilaplace(K*M*w(1)/((s^2 + (w(1))^2)*(tau^2*s^2 + 2*zeta*tau*s+1)));
fplot(y1/(K*M),[0 20],LineWidth=2)
legend("input","output")
title("Sinusodal input response \omega\tau = 0.2")
xlabel("time")
ylabel("p'(t)/A and h'(t)/KA")
hold off
```




```
part2=sin(w(2)*t);
fplot(part2,[0 10],Linewidth=2)
hold on
y2 = ilaplace(K*M*w(2)/((s^2 + (w(2))^2)*(tau^2*s^2 + 2*zeta*tau*s+1)));
fplot(y2/(K*M),[0 10],LineWidth=2)
legend("input","output")
title("Sinusodal input response \omega\tau = 1")
xlabel("time")
ylabel("p'(t)/A and h'(t)/KA")
hold off
```

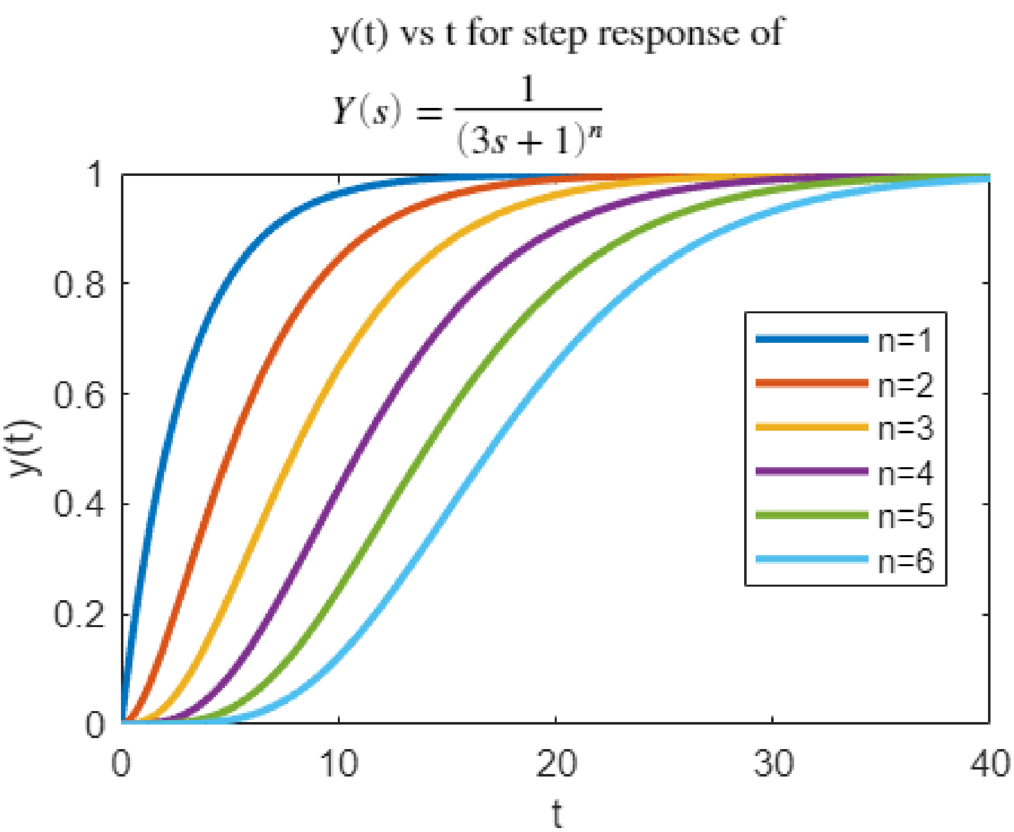



```
part3=sin(w(2)*t);
fplot(part3,[0 10],Linewidth=2)
hold on
y3 = ilaplace(K*M*w(3)/((s^2 + (w(3))^2)*(tau^2*s^2 + 2*zeta*tau*s+1)));
fplot(y3/(K*M),[0 10],LineWidth=2)
legend("input","output")
title("Sinusodal input response \omega\tau = 2.5")
xlabel("time")
ylabel("p'(t)/A and h'(t)/KA")
hold off
```



Sinusodal input response $\omega\tau$ = 2.5
input
output
p'(t)/A and h'(t)/KA
time
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
2
4
6
8
10

```
% For Question 9a
clear
clc
syms s
for i = 1:6
    Y=1/(((3*s+1)^i)*s);
    y=ilaplace(Y);
    fplot(y,[0,40],LineWidth=2)
    hold on;
end
title('y(t) vs t for step response of   $$Y(s) = \frac{1}{(3s +1)^n}$$','interpreter','latex')
xlabel("t")
ylabel("y(t)")
legend("n=1","n=2","n=3","n=4","n=5","n=6",'Location','best')
hold off;
```



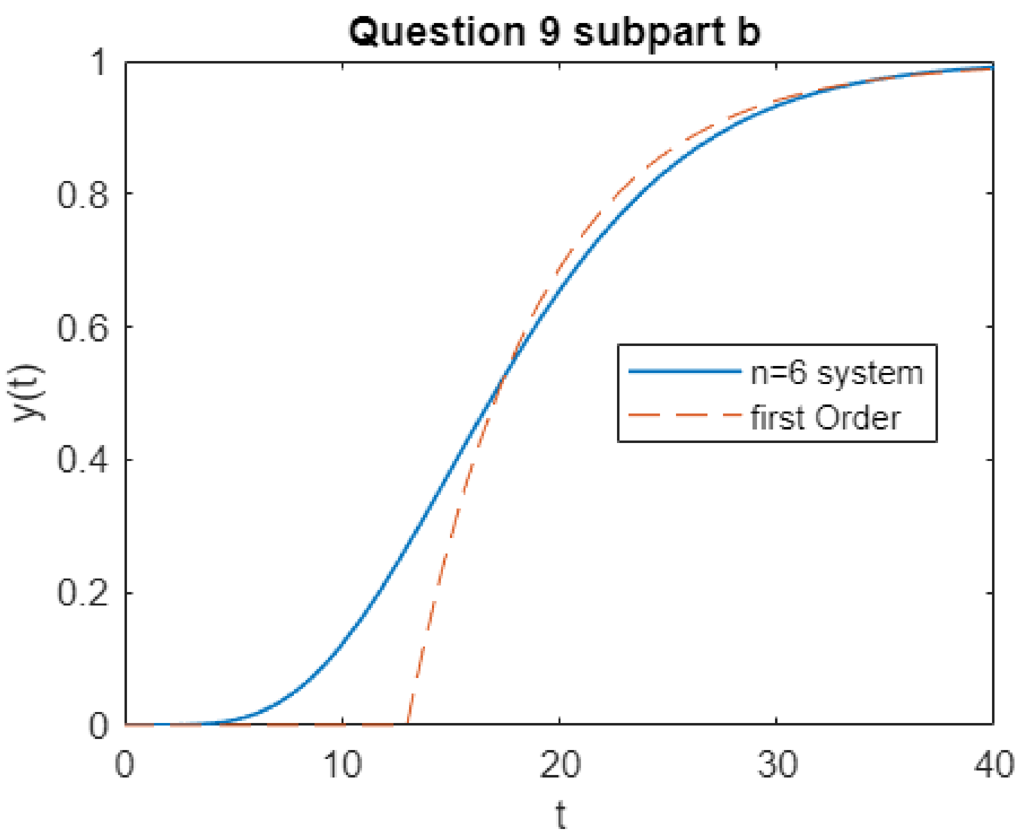

```
% For Question 9b
fplot(y,[0,40],LineWidth=1)
hold on
tw =13;
tau =6     ;
Y_app = exp(-tw*s)/(s*(tau*s+1));
y_app = ilaplace(Y_app)
```

y_app =

$$-\text{heaviside}(t-13)\left(e^{\frac{13}{6}-\frac{t}{6}}-1\right)$$

```
xlabel("t")
ylabel("y(t)")
title("Question 9 subpart b ")

fplot(y_app,[0,40],"--")
legend("n=6 system","first Order",'Location','best')
hold off;
```




```
% For Question 9c
fplot(y,[0,40],LineWidth=1)
hold on
tw =7;
tau =6
```

```
tau = 6
```

```
zeta=0.9;
Y_app = exp(-tw*s)/(s*(tau*tau*s*s+1+2*zeta*tau*s));
y_app = ilaplace(Y_app)
```

y_app =

$$-\mathrm{heaviside}(t-7)\left(e^{\frac{21}{20}-\frac{3t}{20}}\left(\cos\left(\frac{\sqrt{19}\,(t-7)}{60}\right)+\frac{9\sqrt{19}\sin\left(\frac{\sqrt{19}\,(t-7)}{60}\right)}{19}\right)-1\right)$$

```
title("Question 9 subpart c ")
fplot(y_app,[0,40],"--","LineWidth",2)
```

```
xlabel("t")
ylabel("y(t)")
legend("n=6 system","Second Order",'Location','best')
hold off;
```