

# HW5

CL302

Q1

a)  $\check{G}(s) = \frac{k(z_3 s + 1) e^{-\theta s}}{(z_1 s + 1)(z_2 s + 1)}$

$$\tilde{G}(s) = \tilde{G}^+(s) \cdot \tilde{G}^-(s)$$

$$\tilde{G}_c^*(s) = \frac{1}{z_c s + 1} \cdot \tilde{G}_c^-(s)$$

$$G(s) = \frac{\tilde{G}_c^*(s)}{1 - \tilde{G}_c^*(s) \tilde{G}_c(s)} = \frac{(\tilde{G}_c^*(s)) F}{1 - (\tilde{G}_c^*(s)) F \tilde{G}_c^+(s) \cdot \tilde{G}_c^-(s)}$$

$$= \frac{\tilde{G}_c^*(s) F}{(\tilde{G}_c^-(s)) [1 - F \tilde{G}_c^+(s)]}$$

substituting.

$$\begin{aligned} G(s) &= \frac{\tilde{G}_c^+(s) \cdot e^{-\theta s}}{\tilde{G}_c^-(s)} \\ \tilde{G}_c^-(s) &= \frac{k(z_3 s + 1)}{(z_1 s + 1)(z_2 s + 1)} \end{aligned}$$

$$G(s) = \frac{1}{\left( \frac{(z_1 s + 1)(z_2 s + 1)}{k(z_3 s + 1)} \right) \left[ 1 + z_c s - e^{-\theta s} \right]}$$

$e^{-\theta s} \approx 1 - \theta s$  Taylor approximation

$$\begin{aligned} G(s) &= \frac{k(z_3 s + 1)}{(z_1 s + 1)(z_2 s + 1)} \cdot \frac{k(z_1 s + 1)(z_2 s + 1)}{k(z_3 s + 1)(z_c + \theta)s} \\ &= \frac{z_1 z_2 s^2 + (z_1 + z_2)s + 1}{k s (z_3 s + 1)(z_c + \theta)} \quad \text{--- (1)} \end{aligned}$$

z1

$$= \frac{1}{k(z_c + \theta)} \cdot (z_1 + z_2 - z_3) \left[ \frac{(z_1 s + 1)(z_2 s + 1)}{(z_3 s + 1)s(z_1 + z_2 - z_3)} \right]$$

For PID controller (parallel configuration)

$$G_c = k_c \left[ 1 + \frac{1}{z_I s} + \frac{z_D s}{\alpha z_I s + 1} \right] \quad \alpha z_I = z$$

$$= k_c \left[ \frac{z_I z s^2 + z_I z_D s^2 + (z_I + z)s + 1}{z_I s(\alpha z_I s + 1)} \right]$$

$$= k_c \left[ \frac{(z_D + z)s^2 + (1 + z/z_I)s + 1/z_I}{s(z_I s + 1)} \right] \quad \text{--- (11)}$$

Comparing (1) & (11) &  $z = z_3$

$$k_c = \frac{z_1 z_2}{k(z_c + \theta)} \quad k_c (z_D + z_3) = \frac{z_1 z_2}{k(z_c + \theta)}$$

$$\frac{z_1 + z_2}{k(z_c + \theta)} = k_c \left( 1 + z_3/z_I \right)$$

$$\frac{1}{k(z_c + \theta)} = k_c / z_I$$

Solving

$$z_I = z_1 + z_2 - z_3$$

$$k k_c = \frac{z_1 + z_2 - z_3}{z_c + \theta}$$

$$z_D = \frac{z_1 z_2 - z_3(z_1 + z_2 - z_3)}{z_1 + z_2 - z_3}$$

$$k k_c \left( 1 + \frac{z_3}{z_I} \right) = \frac{z_1 + z_2}{z_c + \theta}$$

$$\frac{k k_c}{z_I} = \frac{1}{z_c + \theta}$$

$$\frac{z_1 + z_2}{z_c + \theta} = k k_c \left[ 1 + \frac{z_3}{z_1 + z_2 - z_3} \right]$$

$$k k_c (z_D + z_3) = \frac{z_1 z_2}{z_c + \theta}$$

$$(z_1 + z_2 - z_3)(z_D + z_3) = z_1 z_2$$

$$b) \frac{k(z_3 s + 1)e^{-\theta s}}{z^2 s^2 + 2\zeta z s + 1}$$

$$\tilde{G}_1^+(s) = e^{-\theta s}$$

$$F = \frac{1}{z_c s + 1}$$

$$\tilde{G}_1^-(s) = \frac{k(z_3 s + 1)}{z^2 s^2 + 2\zeta z s + 1}$$

$$G_c = \frac{G_c^*}{1 - G_c^* \tilde{G}_1} \rightarrow \frac{\tilde{G}_1^{-1} F}{1 - \tilde{G}_1^{-1} F \tilde{G}_1^+} \tilde{G}_1^+$$

$$\frac{z^2 s^2 + 2\zeta z s + 1}{k(z_3 s + 1)} \left[ \cancel{1 - (1 - \theta s)} \right] \left[ 1 + z s - (1 - \theta s) \right]$$

$$\Rightarrow \frac{1}{k(z_c + \theta)} \frac{z^2 s^2 + 2\zeta z s + 1}{s(z_3 s + 1)} \Leftrightarrow \text{\$ PID controller (parallel config)} \quad G_c = K_c \left[ \frac{(z_0 + z_1) s^2 + \left(1 + \frac{z_1}{z_I}\right) s + 1/z_I}{s(z_3 + 1)} \right]$$

Comparing,

$$\frac{z^2}{k(z_c + \theta)} = K_c (z_0 + z_3)$$

$$\frac{2\zeta z}{k(z_c + \theta)} = K_c (1 + z_3/z_I)$$

$$\frac{1}{k(z_c + \theta)} = \frac{K_c}{z_I}$$

$$K_c = \frac{z_I}{z_c + \theta} = \frac{2\zeta z - z_3}{z_c + \theta}$$

$$\frac{2\zeta z}{z_c + \theta} = K_c (z_0 + z_I)$$

$$= \frac{(2\zeta z - z_3)(z_0 + z_I)}{z_c + \theta}$$

$$z_0 = \frac{z^2}{2\zeta z - z_3} - z_3$$

$$z_0 = \frac{z^2 - (2\zeta z - z_3)z_3}{2\zeta z - z_3}$$

$$z_I$$

gives

$$K_c = \frac{2\zeta z - z_3}{z_c + \theta} \quad \& \quad z_I = 2\zeta z - z_3$$

$$z_0 = \frac{z^2 - (2\zeta z - z_3)z_3}{2\zeta z - z_3}$$