Manan Mohnat
190110043

Q1 $q = C_v f(l) \sqrt{\dfrac{\Delta P_v}{g_s}}$, $g_s \to 1$, $f(l) = l$, for linear

$q = C_v \, l \sqrt{\Delta P_v}$

a) for configuration 1, $\Delta P = \Delta P_{v_1} + \Delta P_{v_2} = 20$ psi

$$= \left(\frac{q}{100\, l_1}\right)^2 + \left(\frac{q}{10\, l_2}\right)^2 = 20$$

q in terms of l_2 can be given as

$$q^2 \left(\frac{10^2 l_2^2 + 100^2 l_1^2}{100^2 l_1^2 \cdot 10^2 l_2^2} \right) = 20$$

$$q = \sqrt{\frac{20 \cdot 100^2 l_1^2 \cdot 10^2 l_2^2}{10^2 l_2^2 + 100^2 l_1^2}}$$

b) for config 2,

$\Delta P_{v_1} = \Delta P_{v_2} = 20$

$q = q_1 + q_2 \Rightarrow 100\, l_1 \sqrt{20} + 10\, l_2 \sqrt{20}$

For series, since the flow rate is dependent on the resistance offered by both the valves ~~now~~ the increase in l, which leads to decrease in R_2, leads to increase in overall flow rate.

for large l_1, l_2 almost acts as an independent linear valve while for lower l_1, influence can be seen on q v/s l_2.

In parallel configuration, ~~more flow rate will be through~~
The more ~~the~~ ~~l_2~~ ~~open~~ the valve is, the greater is the flow
$C_{v_1} > C_{v_2}$ ∴ ~~flow~~ Resistance of $V_1 < V_2$ and flow through $V_1 > V_2$, so the impact of l_2 on q is less, while change in l_1 is huge.

Also flow in parallel is more than in series.

mixture.

CL 302 HW-4

Q2 $Sp\,gr = 1.11$

$q = 200$ gal/min $= 45.42\ m^3/h$

$P_T = k$

$\Delta P_c = C_1 q^2 = 30$ psi $= 206843$ Pa

$C_1 = \frac{30}{(200)^2} \Rightarrow \boxed{7.5\times10^{-4}}$ gal psi $\frac{min^2}{gal^2}$

$\Delta P_v + \Delta P_c = \Delta P$

$C_1 q^2$

Take Pressure Linear Equal $f(x) = 0.5$ Flow thru the stem position

$q = C_v f(x) \sqrt{\frac{\Delta P_v}{g_s}}$

$P_c = C_1 q^2 \therefore C_1 = \frac{206843\ kg/m^3\ h^2}{45.42 \times 45.42\ m^3\ m^3} \Rightarrow 100.26\ \frac{kg\,h^2}{m^8}$

$Sp\,gr = 1.11$

$q = 200$ gal/min $= 0.012618\ m^3/s \Rightarrow 1.26\times10^{-2}\ m^3/s$

$\Delta P_c = 30$ psi $= 2.07\times10^{5}\ kg\ m^{-1}\ s^{-2}$

$$C_1 = \frac{P_c}{q^2} = \frac{2.07\times10^{5}}{1.26\times10^{-2}\times1.26\times10^{-2}} \cdot \frac{kg \cdot s^2}{m\,s^2\ m^3\,m^3} = \frac{kg}{m^7}\ 1.3\times10^{9}$$

(a) = 5 psi = 3.44×10^{4} Pa

(b) = 30 psi = 2.07×10^{5} kg Pa

(c) = 90 psi = 6.2×10^{5} Pa

Azeotropes or constant boiling mixture.

CL 302 HW-4

$\frac{R\ kg\ m^2}{s}$

Q2 $Sp\ gr = 1.11$

$q = 200$ gal/min $= 45.42\ m^3/h$

$P_T = k$

$\Delta P_c = C_1 q^2 = 30\ psi = 206843\ Pa$

$C_1 = \frac{30}{(200)^2} \Rightarrow \boxed{7.5 \times 10^{-4}}\ \frac{gal\ psi\ min^2}{gal^2}$

$\Delta P_v + \Delta P_c = \Delta P$; $C_1 q^2$

To Do

Plot Flow Rate v/s stem position

linear

Equal %

$f(l) = 0.5$

$q = C_v f(l) \sqrt{\frac{\Delta P_v}{g_s}}$

(a) $P_c = C_1 q^2 \therefore C_1 = \frac{206843}{45.42 \times 45.42} \frac{kg/m^3 \cdot h^2}{m^3 \cdot m^3} \Rightarrow 100.26\ \frac{kg\,h^2}{m^9}$

$Sp\ gr = 1.11$

$q = 200$ gal/min $= 0.012618\ m^3/s \Rightarrow 1.26 \times 10^{-2}\ m^3/s$

$\Delta P_c = 30\ psi = 2.07 \times 10^5\ kg\ m^{-1}\ s^{-2}$

$C_1 = \frac{P_c}{q^2} = \frac{2.07 \times 10^5}{1.26 \times 10^{-2} \times 1.26 \times 10^{-2}} \cdot \frac{kg \cdot s s}{m s^2 \cdot m^3 \cdot m^3} = \frac{kg}{m^7}\ 1.3 \times 10^9$

(a) $= 5\ psi = 3.44 \times 10^4\ Pa$

(b) $= 30\ psi = 2.07 \times 10^5$ kg Pa

(c) $= 90\ psi = 6.2 \times 10^5\ Pa$

taking $R = 10$

9.4

let the total pressure be k

$\Delta P_v + \Delta P_L = k$ $\quad \cancel{\Delta P_v = 1}$

$$q = C_v \, f(l) \sqrt{\frac{\Delta P_v}{g_s}} \quad \therefore \quad C_v = \frac{q}{f(l)} \sqrt{\frac{\Delta P_v \, g_s}{\Delta P_v}}$$

$$C_v = \frac{200}{0.5} \left(\frac{\Delta \bar{P}_v}{1.11} \right)^{-0.5} \Rightarrow$$

$$\Delta P_v = (\Delta \bar{P}_v + \Delta \bar{P}_c) - \Delta P_c = (30 + \bar{P}_v) - \frac{3}{4000} q^2$$

$$\cancel{q = 400 \left(\frac{\Delta \bar{P}_v}{1.11} \right)^{-0.5}}$$

$$q = \left[400 \sqrt{\left(\frac{1.11}{\Delta \bar{P}_v} \right)} \right] f(l) \sqrt{\frac{1.11}{30 + \Delta \bar{P}_v - 3/4000 \, q^2}}$$

$f(l) = l$ for linear

&

$f(l) = R^{l-1}$

R can be assumed as nothing

a) $\Delta \bar{P}_v = 5$ and ~~f6~~

$$q = 400 \sqrt{\frac{1.11}{5}} \; l \sqrt{\frac{1.11}{30 + 5 - \frac{3}{4000} q^2}}$$

$$l = \frac{q}{188.5} \left(\frac{35 - 0.00075 q^2}{1.11} \right)^{-0.5}$$

for equal %,

$$q = 400 \sqrt{\frac{1.11}{5}} \cdot 10^{l-1} \sqrt{\frac{1.11}{35 - 0.00075 q^2}}$$

9.4

$$l = 1 + \frac{\ln\left[\frac{q}{188.5}\left(\frac{35 - 0.00071\,q^2}{1.11}\right)^{-1/2}\right]}{\ln 10}$$

b) taking inspiration from a) $p = 30$

for linear,

$$l = \frac{q}{76.94}\left(\frac{60 - 0.00075\,q^2}{1.11}\right)^{-1/2}$$

for equal %

$$l = 1 + \frac{\ln\left|\frac{q}{76.94}\left(\frac{60 - 0.00075\,q^2}{1.11}\right)^{-1/2}\right.}{\ln 10}$$

c) $p = 90$

for linear

$$l = \frac{q}{44.22}\left(\frac{120 - 0.00075\,q^2}{1.11}\right)^{-0.5}$$

for equal %

$$l = 1 + \frac{\ln\left[\frac{q}{44.22}\left(\frac{120 - 0.00075\,q^2}{1.11}\right)^{-0.5}\right]}{\ln 10}$$

Conclusions :

for a)

linear value is not linear, Eq% is linear for small range

for b)

linear value is linear initially (for low l)

& eql %. ~~for~~ is ~~linear~~ more liner than a)

for c)

linear value is more linear for wider values and also equal % is linear for $l > 0.5$

As $\Delta \bar{P}_v \uparrow$, it can handle flowrates above $\bar{q}$ more easily. And is higher for equal % than linear.

$\Delta \bar{P}_v \uparrow$, costs $\uparrow$

range of linearity $\uparrow$ around nominal flow rate as $\Delta p \uparrow$

$\Delta p_v \uparrow$, cost $\uparrow$

Q3 # 9.9

$T\ ^{\circ}C$	Measurement, mA
0	4.0
100	8.1
200	11.9
300	16.1
400	20.0

$$\frac{T_m}{T} = \frac{K}{\tau s+1}$$

$$K = \frac{20-4}{400-0} = 0.04\,\frac{mA}{^{\circ}C}$$

from code, $\Delta\tau$ between thermometer & transmitter = $\boxed{1.262\ \text{min} \Rightarrow 75.72\,s}$

given τ_{thermo} = 20 s

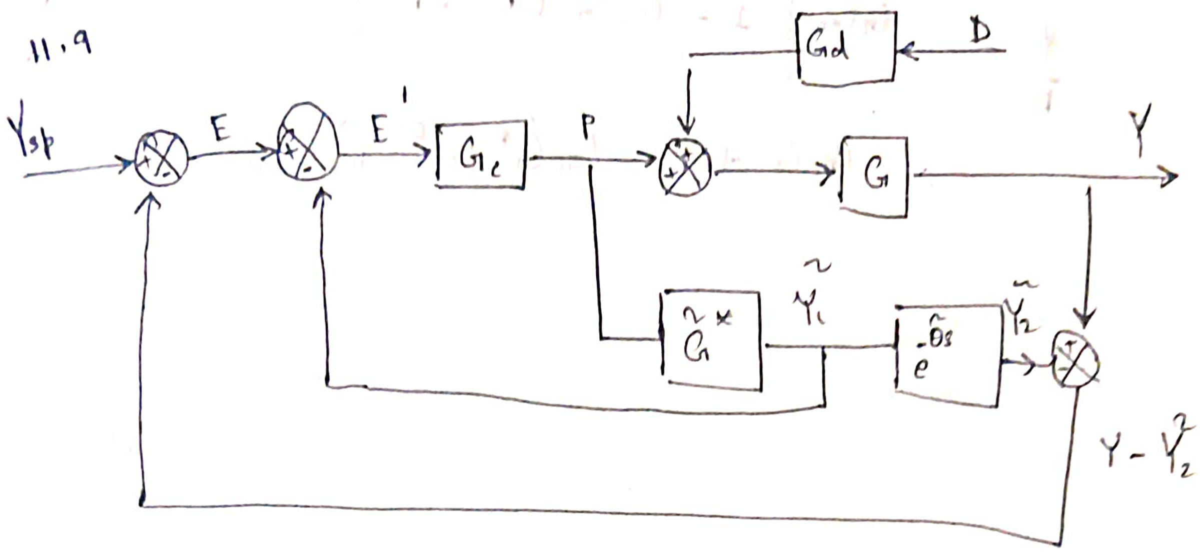
$\therefore\ \tau_{trans}$ = 75.72 + 20 = 95.72 s

range of linearity ↑ around nominal flow rate as Δp ↑

Δp_v ↑ , cost ↑

Q4)

11.9



$$\tilde{Y}_2 = \tilde{Y}_1 e^{-\tilde{\theta}s} \quad \text{—— (V)}$$

$$Y_{sp} - (Y - \tilde{Y}_2) = E \quad \text{—— (IV)}$$

$$E - \tilde{Y}_1 = E' \quad \text{—— (III)}$$

$$P = G_c E' \quad \text{—— (II)}$$

$$(P + G_d D)\, G = Y \quad \text{—— (I)}$$

$$P \tilde{G}^* = \tilde{Y}_1 \quad \text{—— (VI)}$$

(IV) in (III)

$$\longrightarrow Y_{sp} - Y + \tilde{Y}_2 - \tilde{Y}_1 = E'$$

(V) in

$$Y_{sp} - Y + \tilde{Y}_1 \left[e^{-\tilde{\theta}s} - 1 \right] = E'$$

(VI) in

$$Y_{sp} - Y + \underset{G_c E'}{P}\tilde{G}^* \left[e^{-\tilde{\theta}s} - 1 \right] = E'$$

$$Y_s - Y = E' \left[1 - G_c \tilde{G}^* \left[e^{-\tilde{\theta}s} - 1 \right] \right]$$

II in I

$$(G_c E' + G_d D)\, G = Y$$

Assuming $Y_s = 0$

$$\frac{G_c G \,(Y_s - Y)}{\left[1 - G_c \tilde{G}^* \left[e^{-\tilde{\theta}s} - 1 \right] \right]} - Y = -G_{\phi} G_d D$$

$$Y \left[\frac{G_c G + 1 - G_c G^* \left[e^{-\tilde{\theta}s} - 1 \right]}{1 - G_c G^* \left[e^{-\tilde{\theta}s} - 1 \right]} \right] = G_g G_d D$$

$$\frac{Y}{D} = \frac{G G_D \left[1 - G_c \hat{G}^* \left(e^{-\hat{\theta} s} - 1 \right) \right]}{G_c G + 1 - G_c \tilde{G}^* \left(e^{-\hat{\theta} s} - 1 \right)}$$

Qs)

11.10

$\tilde{Y}_{sp} = K_m Y_{sp}$ $\quad \rightarrow E K_c = P$

$E = K_m Y_{sp} - Y K_m$ $\quad Y_{sp} = 0$

Final Ans $\boxed{K_c > 1}$

$(D + G_1 P) G_3 = Y_3$ $\quad P = -Y K_m K_c$

$Y_3 + P G_2 = Y$

$\left(D + G_1 (-Y K_m K_c) \right) G_3 + (-Y K_m K_c) G_2 = Y$

$D G_3 - Y K_m K_c G_1 G_3 - Y K_m K_c G_2 = Y$

$$\frac{Y(s)}{D(s)} = \cancel{1 + K_m K} \frac{G_3}{1 + K_m K_c G_1 G_3 + K_m K_c G_2}$$

a)
$$= \frac{G_3}{1 + K_m K_c (G_1 G_3 + G_2)}$$

b) $G_1(s) = 5$ $\quad G_2(s) = \frac{4}{2s+1}$ $\quad K_m = 1$ $\quad G_3(s) = \frac{1}{s-1}$

$$\Rightarrow \frac{1/(s-1)}{1 + K_c\left[\frac{5}{s-1} + \frac{4}{2s+1}\right]} \Rightarrow \frac{1}{\cancel{(s-1)}\left[\frac{2s^2 - s - 1 + K_c(14s + 1)}{\cancel{(s-1)}(2s+1)}\right]}$$

1	2	$K_c - 1$
2	$14K_c - 1$	
3	$K_c - 1$	

$$\frac{2s+1}{2s^2 + (14K_c - 1)s + (K_c - 1)}$$

for stable, ~~14Kc-1~~ the product of roots > 0 and sum < 0

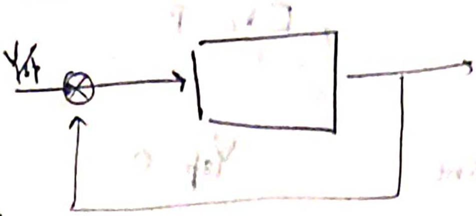
$\therefore -\frac{(14K_c - 1)}{2} < 0 \Rightarrow K_c > 1/14$ & $\frac{K_c - 1}{2} > 0 \Rightarrow K_c > 1$

$\therefore \boxed{K_c > 1}$

Q6

11.20

$$G_{OL}(s) = 0.5 K_c e^{-3s}/(10s+1)$$



$$Y = (\overset{U}{Y \mp Y_{sp}})\, G$$

$$\Rightarrow Y(1+G) = UG$$

$$G' \quad \frac{Y}{U} = \frac{G}{1+G} = \frac{0.5K_c e^{-3s}}{(10s+1) + 0.5K_c e^{-s}}$$

a) Pade approx $e^{-3s} \approx \dfrac{1-1.5s}{1+1.5s}$

$$\therefore G' = \frac{0.5\, K_c\, e\left(\frac{1-1.5s}{1+1.5s}\right)}{(10s+1) + 0.5K_c\left(\frac{1-1.5s}{1+1.5s}\right)}$$

for stability, real parts of roots of $(10s+1)(1+1.5s) + 0.5K_c(1-1.5s) = 0$

should be <0

$$\Rightarrow 15s^2 + (11.5 - 0.75K_c)s + (1 + 0.5K_c) = 0$$

for stability,

$$11.5 - 0.75K_c > 0$$

$$\Rightarrow K_c < 15.33$$

and $1 + 0.5K_c > 0$ or $K_c > -2$

therefore $-2 < K_c < 15.33$

b) $s = j\omega$, $e^{-3j\omega} = \cos(3\omega) - j\sin(3\omega)$,

focusing only on denominator part, as it only dictates stability,

$$(10j\omega + 1) + 0.5k_c\left[\cos(3\omega) - j\sin(3\omega)\right] = 0$$

for roots $1 + 0.5k_c\cos(3\omega) = 0$

and $10\omega - 0.5k_c\sin(3\omega) = 0$

$\Rightarrow \omega = 0$ ~~or~~ gives $k_c = -2$

$\therefore k_c > -2$

or $\tan(3\omega) = -10\omega$

$\omega =$ ~~-0.58~~ or 0.58 (through desmos)

$$1 + 0.5k_c\cos(3\times 0.58) = 0$$

$$k_c = -\frac{1}{0.5\cos(1.74)} = 11.87$$

Stable $k_c < 11.87$

~~-2~~ $\therefore 0 < k_c < 11.87$

$$\cancel{-2}\ 0 < k_c < 11.87$$

c) $e^{-3s} \approx 1 - 3s$

real parts roots < 0

$$(10s + 1) + 0.5k_c(1 - 3s) = 0$$

$$10s - 1.5sk_c + 0.5k_c = 0$$

$$s(10 - 1.5k_c) = -0.5k_c$$

$$s = \frac{-0.5k_c}{10 - 1.5k_c}$$

$$\frac{0.5k_c}{1.5k_c - 10} < 0$$

$$\boxed{10 < k_c < 6.67}$$

d) $e^{-8s} = \frac{1}{1+3s}$

$$(10s+1) + 0.5k_c\left(\frac{1}{1+3s}\right) = 0$$

$$(10s^2 + 13s + 1) + 0.5k_c = 0$$

$$10s^2 + 13s + 1 + 0.5k_c$$

for stability,

$$1 + 0.5k_c > 0$$

$$k_c > -2$$

e)

$$(10s+1)\left[1 + \frac{3}{2}s + \frac{3}{4}s^2\right] + 0.5k_c\left[1 - \frac{3}{2}s + \frac{3}{4}s^2\right] = 0$$

$$10s + 15s^2 + 7.5s^3 + 1 + 1.5s + 0.75s^2 + 0.5k_c - 0.75k_c s + 0.375k_c s^2 = 0$$

$$7.5s^3 + (15.75 + 0.375k_c)s^2 + (11.5 - 0.75k_c)s + 1 + 0.5k_c =$$

$$15.75 + 0.375k_c > 0$$

$$\underline{k_c > -42}$$

$$11.5 - 0.75k_c > 0$$

$$\underline{k_c < 15.33}$$

&

$$1 + 0.5k_c > 0$$

$$\underline{k_c > -2}$$

$$\therefore \quad -2 < k_c < 15.33$$

No, both gave the same bounds.

b and e gave the closest responses against b

Routh Array.

$7.5 \quad 11.5 - 0.75k_c$

$15.75 + 0.375k_c \quad 1 + 0.5k_c$

$\cancel{b_1 = 1+0.5} \quad b_2 = 0$

$$b_1 = \frac{(15.75 + 0.375k_c)(11.5 - 0.75k_c) - 7.5(1 + 0.5k_c)}{15.75 + 0.375k_c}$$

$$b_1 = \frac{181.125 - 0.28125k_c^2 - 7.5k_c - 7.5 - 3.75k_c}{15.75 + 0.375k_c}$$

$c_1 = 1 + 0.5k_c$

$$b_1 > 0 \Rightarrow \frac{181.125 - 0.28125k_c^2 - 11.25k_c - 7.5}{15.75 + 0.375k_c} > 0$$

for $15.75 + 0.375k_c > 0$ $\cancel{k_c > 0}$, $\rightarrow$ $181.125 - 0.28125k_c^2 - 11.25k_c - 7.5 > 0$

$\Rightarrow k_c > -42$

$\Rightarrow k_c < 11.896$ or $k_c > -51.896$

$\rightarrow$ $-42 < k_c < 11.896$

if k_c ~~<~~ -

if $k_c < -42$

then $k_c < -51.896$ or $k_c > 11.896$

$\rightarrow$ $k_c < -51.896$

intersection of the cases, gives solution

e) $-2 < k_c < 11.896$

→ 2/2 Pade gives a better approximation as the upper limit is closer to actual answer.

→ 2/2 Pade gives the best approximation (e) against b

a) $k_c \in (-2, 15.33)$

b) $k_c \in (0, 11.67)$

c) $k_c \in (-2, 6.67)$

d) $k_c \in (-2, \infty)$

e) $k_c \in (-2, 11.896)$

Among a) c) & d)

a) gives better

c) gives better amongst c & d (the two taylors)