Homework #3

Instructions:

- Total of 7 questions:
- HW2 will count for 5% of the overall marks.
- Grading here will be out of 35+30+5+5+5+5+5=90 marks
- The HW should be submitted on MS Team. The deadline is Sunday Midnight.
- TA help session can be arranged based on your requests.

Deriving empirical models: [35] 5 marks each for sub questions.

1) Take the following TF:

$$G(s) = \frac{(1-5s)(1-2s)\,e^{-0.5s}}{(12s+1)(6s+1)(0.3s+1)} \text{ Equation [1]}$$

All the step responses (from a to e) must be in single plot with proper color, line style (solid, dash, dash dot and dot) with legends. All pole zero plots (from a to e) must appear in one page so that the comparison can be made easily.

a. Plot the pole zero diagram of G(s). plot the step response in time domain, (use 60 seconds as final time for plotting)

Comparison of Eqn. 1 to the FOPTD

b. Use taylor series approximation to get FOPTD model. Compare the step response to the full model. Plot pole zero diagram.
c. Use Skogestad method to get FOPTD model. Compare the step response to the full model. Plot pole zero diagram.
d. Use the inflection point method to derive FOPTD model (use t=14 as inflection point). Compare the step response to the full model. Plot pole zero diagram.
e. Use the method described in the lecture (excel minimization) to derive the FOPTD model (obtain θ and τ) by minimizing the *sum of squared error* between full solution derived in part a and the FOPTD model (use t_{final} as 60 s). You can also use other softwares. Copy paste the code in case you use Matlab/python. In case you use excel, copy paste the screenshot of the excel sheet. Compare the time domain response. Plot pole zero diagram.

Comparison of Eqn. 1 to the second order plus time delay with numerator dynamics (eqn 2)

f. Can you propose a second order plus time delay with numerator dynamics model that gives you inverse response.

$$G(s) = \frac{(1-\tau_a s)\ e^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)} \text{Equation [2]}$$

   i. Hint: Get rid of one numerator term $(1-2s)$ and one denominator term $(0.3s+1)$ using either Taylor approximation or Skogestad method. See which way or doing the approximation leads to a better fit.

g. Use excel sheet NL regression to obtain the four parameters $\tau_a, \tau_1, \tau_2, \theta$ in Equation 2 by minimizing the '*sum of squared error*' between the true step response (part a) and the response predicted by Equation 2.
You can also use other software. Do copy paste the code in case you use Matlab/python. In case you use excel, copy paste the screenshot of the excel sheet

Compare the step response ***(compare a, f and g, with appropriate color, legend and line style)*** and pole zero plot to the original response ***(compare a, f and g) with appropriate color and legend***.

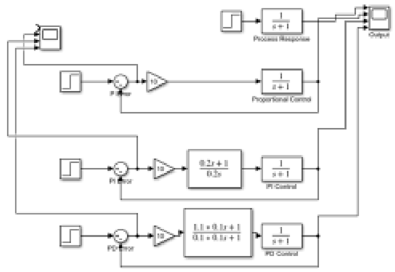
2: **[30] 5 marks each for sub questions.**

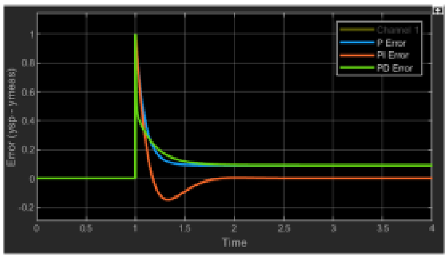
Take the following example shown in the class:

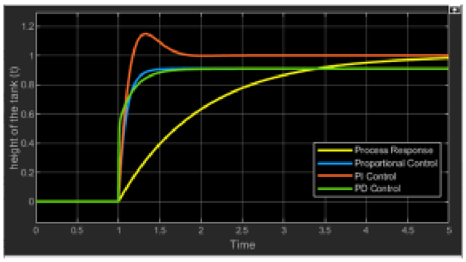
Simulink Exercise:

file uploaded

- Process TF
  - $G(s) = \frac{1}{s+1}$
- P Control
  - $\frac{P'}{E} = K_c$, where $K_c = 10$
- PI Control
  - $\frac{P'}{E} = K_c \frac{(\tau_I s+1)}{\tau_I s}$ where $\tau_I = 0.2$
- PD Control
  - $\frac{P'}{E} = K_c \left(1 + \frac{\tau_D s}{\alpha \tau_D s+1}\right)$ where $\tau_D = 0.1, \alpha = 0.1$







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a. The Simulink code is already uploaded on Team, take the code, and see if you can increase the value of K_c to arbitrary large without making the set point change response unstable. (i.e. can K_c be 50, 500 …). Plot the response of $K_c = 1000$. Is the response stable for all P, PD and PI controllers with such a high K_c?

b. Add a time delay term of 0.1 s with each TF. See what value of Kc you can use without making the response unstable? Mention the values of Kc for each P, PD and PI control where system becomes unstable. Also copy paste the screenshot of the Simulink code for each case.

Take the following three second order TFs with you own choice of poles in place of the 1st order TF.

c. Case 1: underdamped system: generate error as well as y_m plots as shown in the figure
   i. System response (only y_m plots)
   ii. P Control
   iii. PI Control
   iv. PD Control

d. Case 2: Critically damped system: generate error as well as y_m plots as shown in the figure
   i. System response (only y_m plots)
   ii. P Control
   iii. PI Control
   iv. PD Control

e. Case 3: Overdamped system: generate error as well as y_m plots as shown in the figure above.
   i. System response (only y_m plots)
   ii. P Control
   iii. PI Control
   iv. PD Control

Also copy paste the screenshot of the Simulink code for each case.

f. Can you make K_c arbitrary large without making the response unstable?

5 marks each for the remaining questions.

8.1 An electronic PI temperature controller has an output p of 12 mA when the set point equals the nominal process temperature. The controller response to step change in the temperature set point of 3 mA (equivalent to a change of 5°F) is shown below:

t, s	p, mA
0−	12
0+	10
20	9
60	7
80	6

Determine the controller gain K_c (mA/mA) and the integral time, τ_I. Is the controller reverse-acting or direct-acting?

8.2 The physically realizable form of the PD transfer function is given in the first equation of Exercise 8.1.

(a) Show how to obtain this transfer function with a parallel arrangement of two much simpler functions in Fig. E8.2:

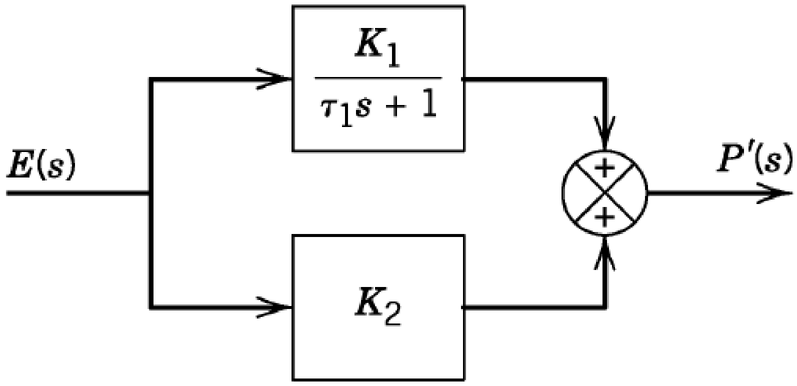

Figure E8.2

(b) Find expressions for K_1, K_2, and τ_1 that can be used to obtain desired values of K_c, τ_D, and α.

(c) Verify the relations for $K_c = 3$, $\tau_D = 2$, $\alpha = 0.1$.

8.3 The parallel form of the PID controller has the transfer function given by Eq. 8-14. Many commercial analog controllers can be described by the series form given by Eq. 8-15.

(a) For the simplest case, $\alpha \rightarrow 0$, find the relations between the settings for the parallel form ($K_c^\dagger$, $\tau_I^\dagger$, $\tau_D^\dagger$) and the settings for the series form (K_c, τ_I, τ_D).

(b) Does the series form make each controller setting (K_c, τ_I, or τ_D) larger or smaller than would be expected for the parallel form?

(c) What are the magnitudes of these interaction effects for $K_c = 4$, $\tau_I = 10$ min, $\tau_D = 2$ min?

8.7 An electronic PID temperature controller is at steady state with an output of 12 mA. The set point equals the nominal process temperature initially. At $t = 0$, the set point is increased at the rate of 0.5 mA/min (equivalent to a rate of 2°F/min). If the current settings are

$$\begin{aligned} K_c &= 2 \text{ (dimensionless)} \\ \tau_I &= 1.5 \text{ min} \\ \tau_D &= 0.5 \text{ min} \end{aligned}$$

(a) Derive an expression for the controller output $p(t)$.

(b) Repeat (a) for a PI controller.

(c) Plot the two controller outputs and qualitatively discuss their differences.

8.6 If the input Y_m to a PI controller changes stepwise ($Y_m(s) = 2/s$) and the controller output changes initially as in Fig. E8.6, what are the values of the controller gain and integral time?

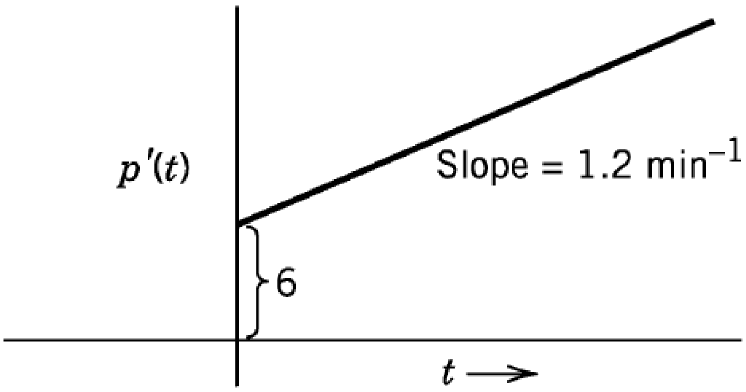

Figure E8.6