Homework #2

Instructions:

- Total of 9 questions:
- HW2 will count for 5% of the overall marks.
- Grading here will be out of 90 marks
- The HW should be submitted on MS Team. The deadline is Saturday 5^{th} Feb Midnight.
- TA help session can be arranged based on your requests.

Some Practice on Laplace Transform:

1. Prove Initial and final value theorem. [**2+2**]

2. Solving each of the following s-domain function into partial fraction, Assuming each represent a TF (not TF$\times U$), comment on the nature of response under step change (bounded vs unbounded, (2) overdamped vs underdamped, (2) integrating system vs having steady state), also calculate the poles and zeros, plot poles with symbol $\times$ and zero with **o** in the plot. Solve by hand. **[3+3+3+3]**

   a. Be careful with repeated roots.

(a) $Y(s) = \dfrac{6(s+1)}{s^2(s+1)}$

(b) $Y(s) = \dfrac{12(s+2)}{s(s^2+9)}$

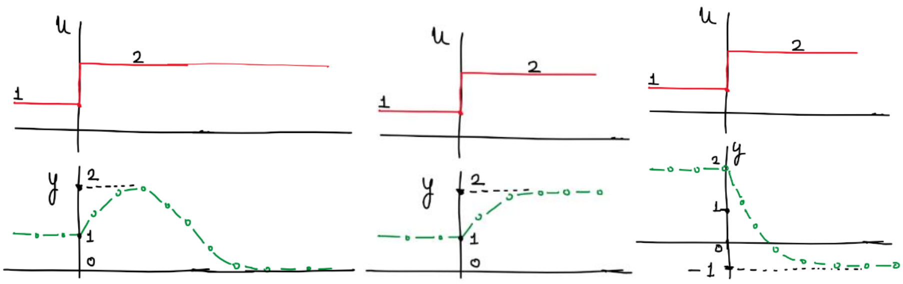
(c) $Y(s) = \dfrac{(s+2)(s+3)}{(s+4)(s+5)(s+6)}$

(d) $Y(s) = \dfrac{1}{[(s+1)^2+1]^2(s+2)}$

3. Does the function f(t)=1/(t-1) have laplace transform? **[4]**
   a. Hint: what are the fundamental conditions for the existence of LT?

Some Practice on Dynamics:

4. For the following output response based on input change, can you calculate steady state gain? **[3+2]**
   a.



   b. From you day-to-day life, give example of a system with negative steady state gain?

Sensor having second order dynamics as opposed to first order dynamics:

5. Take Problem 5.7 with a change mentioned below (tutorial problem): reproduced below for convenience. [3+4+4+4]

5.7 Appelpolscher has just left a meeting with Stella J. Smarly, IGC's vice-president for process operations and development. Smarly is concerned about an upcoming extended plant test of a method intended to improve the yields of a large packed-bed reactor. The basic idea, which came from IGC's university consultant and was recently tested for feasibility in a brief run, involves operating the reactor cyclically so that nonlinearities in the system cause the time-average yield at the exit to exceed the steady-state value. Smarly is worried about the possibility of sintering the catalyst during an extended run, particularly in the region of the "hotspot" (axially about one-third of the way down the bed and at the centerline) where temperatures invariably peak. Appelpolscher, who plans to leave the next day on a two-week big game photo safari, doesn't want to cancel his vacation. On the other hand, Smarly has told him he faces early, unexpected retirement in Botswana if the measurement device (located near the hot spot) fails to alert operating people and the reactor catalyst sinters. Appelpolscher likes Botswana but doesn't want to retire there. He manages to pull together the following data and assumptions before heading for the airport and leaves them with you for analysis with the offer of the use of his swimming pool while he is gone. What do you report to Smarly?

Data:

Frequency of cyclic operation = 0.1 cycles/min

Amplitude of thermal wave (temperature) at the measurement point obtained experimentally in the recent brief run = 15 °C

Average operating temperature at the measurement point, T_{mean} = 350 °C

~~Time constant of temperature sensor and thermowell = 1.5 min~~

Temperature at the reactor wall = 200 °C

Temperature at which the catalyst sinters if operated for several hours = 700 °C

Temperature at which the catalyst sinters instantaneously = 715 °C

Assumptions:

The reactor operational cycle is approximately sinusoidal at the measurement point.

The thermowell is located near the reactor wall so as to measure a "radial average" temperature rather than the centerline temperature.

The approximate relation is

$$T = \frac{T_{center} + 2T_{wall}}{3}$$

which also holds during transient operation.

Rather than the sensor and the thermowell having first order dynamics, consider the case when they are having second order critically damped dynamics with the following transfer function $G(s) = \frac{1}{(1.5s+1)^2}$. Keeping everything else same,

a. Calculate the steady stage temperature at the center, near sensor and wall. (already solved in tutorial)
b. What will be the maximum and minimum temperature where the sensor is located (note that sensor is located off center) when the cycles are in operation?
c. what will be the maximum and minimum temperature at the center of the reactor?
d. What should be the frequency of operation so that the temperature of the sensor will not go over 715 °C.

Second order system:

6. Can you give an example of underdamped second order system having physical meaning. (of course apart from what is covered during the course, e.g the manometer tube or mass m attached to spring). **[3]**
    a. Note: I would not like it if all of you give me same answer since that will suggest cheating.
7. A second order over damped system can be assumed to made of two first order system (i.e. two tanks in series, or 1st order tank dynamics with 1st order sensor dynamics), can a similar thinking be applied on a second order underdamped system. Can you think of an example where two physical systems having first order dynamics will produce a underdamped second order system? **[2]**

Learning Objective:

Step and frequency response of second order sytem, in the context of accuracy of sensing element:

8. Consider the following problem: partially covered in tutorial as well:

1. Manometer problem:

5.10 The dynamic behavior of the liquid level in each leg of a manometer tube, responding to a change in pressure, is given by

$$\frac{d^2h'}{dt^2} + \frac{6\mu}{R^2\rho}\frac{dh'}{dt} + \frac{3}{2}\frac{g}{L}h' = \frac{3}{4\rho L}p'(t)$$

where $h'(t)$ is the level of fluid measured with respect to the initial steady-state value, $p'(t)$ is the pressure change, and R, L, g, ρ, and μ are constants.

(a) Rearrange this equation into standard gain-time constant form and find expressions for K, τ, ζ in terms of the physical constants.

(b) For what values of the physical constants does the manometer response oscillate?

(c) Would changing the manometer fluid so that ρ (density) is larger make its response more oscillatory, or less? Repeat the analysis for an increase in μ (viscosity).

a. Draw the figure of a manometer tube and briefly describe the workings in one paragraph. **[4]**

b. Use the following values of parameters (each having different viscosity) to calculate the numerical values of $K, \tau,$ and ζ. **[3]**
    i. Case 1 [rho = 2000., **mu = 0.645e-1**, R = 0.5e-2, L = 1, g = 9.8]
    ii. Case 2 [rho = 2000., **mu = 2e-1**, R = 0.5e-2, L = 1, g = 9.8]
    iii. Case 3 [rho = 2000., **mu = 1e-2**, R = 0.5e-2, L = 1, g = 9.8]

c. For each case, comment wrt underdamped, over-damped or critically damped system.**[3]**

d. Step response: [10]

   i. Calculate the step response (solve by hand) of when p'(t)=1e4 Pa for each case. For the cases where you have underdamped system calculate the value of Decay ratio, overshoot, and period (using the formulaes given in the text).

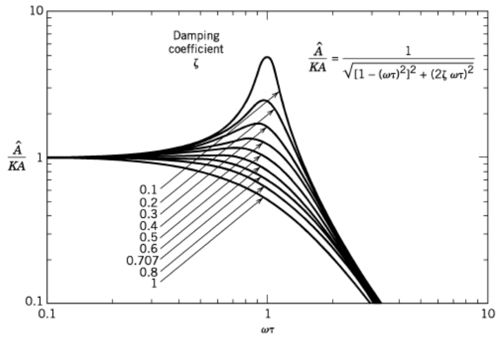
ii. Display the step change response of the three system in single plot, choose the final time appropriately when transients have died down). (zero marks if plots are not clearly visible with proper legends, axis title, and units, use different color and line style for each plot)

e. Frequency Response: [10]

i. For the underdamped case (case 3), plot the response of a sinusoidal input having amplitude of 1e4 Pa for three different frequencies obeying the following expressions.

1. $\omega\tau = 0.2$ (calculate ω since you have the value of τ from the underdamped model)
2. $\omega\tau = 1$
3. $\omega\tau = 2.5$

see if the ratio of amplitudes of output to input (scaled with steady state gain) after the initial transient has died down, follows the figure below.



ii. Plot the frequency response (input, and three output on the same plot), with proper legend and axis titles)

a. You can use Matlab/Maple/Python

iii. Comment on the phase difference

iv. Comment on the implication of the such sinusoidal. When is the reading from manometer be trusted?

v. Can you calculate the frequencies below which the manometer will make less than 5% error in its measurement of the pressure amplitude

Learning Objective:

Tank in series example and invoking the thinking about approximating the higher order TF into first or second order TF with time delays:

9. Plot the step response of the following 6 systems on single plot from t=0..40 (use Matlab/other softwares to plot): **[5+5+5]**

a. $\frac{1}{(3s+1)^n}$, for $n = 1, 2, 3, 4\ 5\ 6$..

b. Can the response of n=6 be approximated by first order system with time delay ($e^{-t_w s}/(\tau s + 1)$)? What features of the original step responses for n=6 cannot be captured by such approximations? Using hit and trial in matlab, can you approximately get the values of t_w, and τ that matches the response of n=6 reasonably. Plot the step response of the approximation and original step response (for n=6 case) on the same plot. (Use proper legend, color and linestyle for the plots)

c. Can the response of n=6 be approximated by second order system with time delay ($e^{-t_w s}/(\tau^2 s^2 + 2\tau\zeta s + 1)$)? Using hit and trial in matlab, can you approximately get the values of t_w, ζ and τ that matches the response of n=6. Plot the step response of the approximation and original step response (for n=6 case) on the same plot. (Use proper legend, color and linestyle for the plots)