Q1)

a) $$G(s) = \frac{(1-5s)(1-2s)e^{-0.5s}}{(12s+1)(6s+1)(0.3s+1)}$$

Taylor expansion $$e^{-0.5s} \approx (1-0.5s)$$

b) Taylor series approximation. FOPTD -

$$\frac{1}{12s+1} \cdot \left(e^{-0.5-2-5-6-0.3}\right)^{s} \Rightarrow \frac{1}{12s+1} \cdot e^{-13.8s}$$

$\tau = 12$

plot on matlab.

c) Skogestad FOPTD $\tau_2 = 6$

$$\frac{1}{12s+1} \; \frac{1}{\left[\left(12+\frac{6}{2}\right)s+1\right]} \cdot e^{(-0.5-2-5-3-0.3)s}$$

$$\Rightarrow \frac{1}{(15s+1)} e^{-10.8s}$$

d) $G(s) = \dfrac{Ke^{-\theta s}}{\tau s+1}$ ~~given~~ found $\theta = 10.3$

$m = 0.0453$ $m = \dfrac{KM}{\tau}$, from graph, $K = 1$,

$\therefore \tau = 1/m = 22.1$

f) $\dfrac{(1-\tau_a s)e^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)}$

$\tau_a = 5$
$\tau_1 = 12$ $\therefore$ by taylor approx,
$\tau_2 = 6$

$$\frac{(1-5s)\, e^{-0.5s-2s-0.3s}}{(12s+1)(6s+1)}$$

$$\Rightarrow \frac{1-5s}{(12s+1)(6s+1)} e^{-2.8s}$$

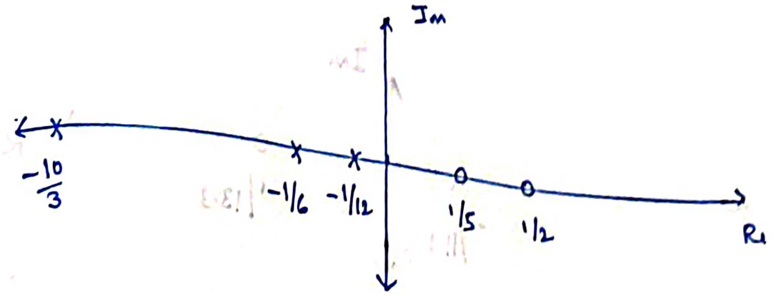
by skogestad,

$$\frac{(1-5s)e^{-0.5s-2s-0.15s}}{(12s+1)((6+0.15)s+1)}$$

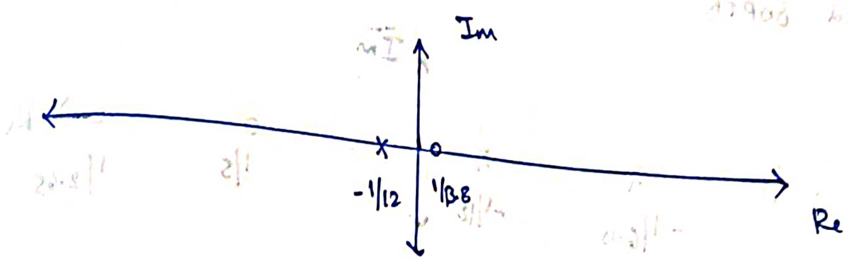
$$\Rightarrow \frac{(1-5s)e^{-2.65s}}{(12s+1)(6.15s+1)}$$

On zooming the graph,
Skogested gives better.

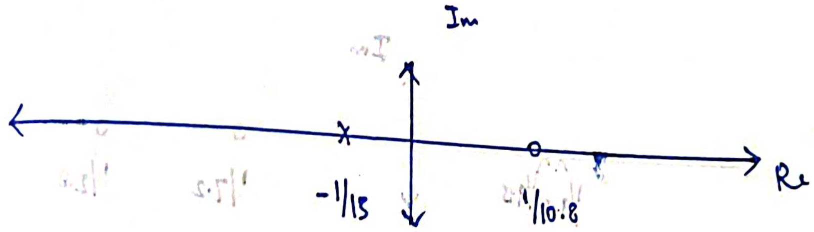
a)


Im
−10/3
−1/6
−1/12
1/5
1/2
Re

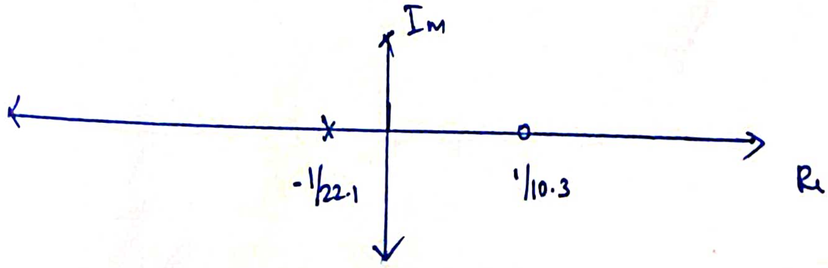
b)


Im
-1/12
1/13.8
Re

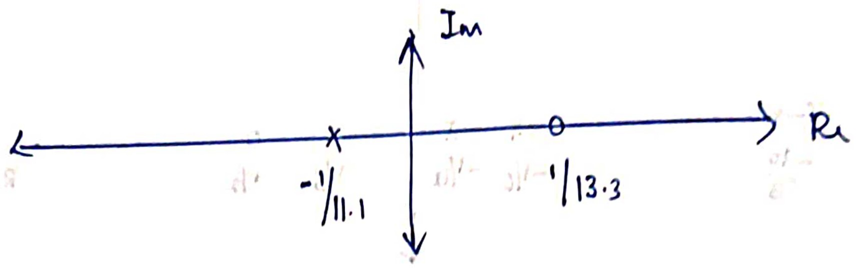
c)


Im
-1/15
1/10.8
Re

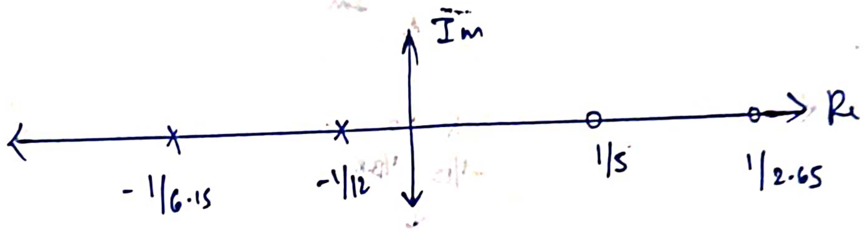
d)


Im
-1/22.1
1/10.3
Re

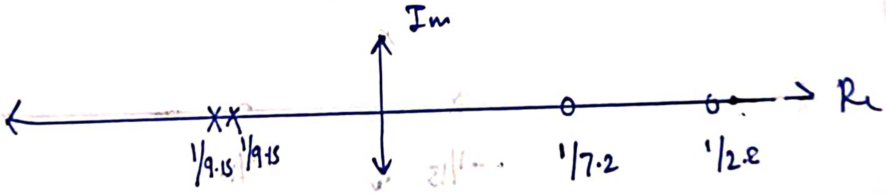
e)



f) Skogerted SOPTD



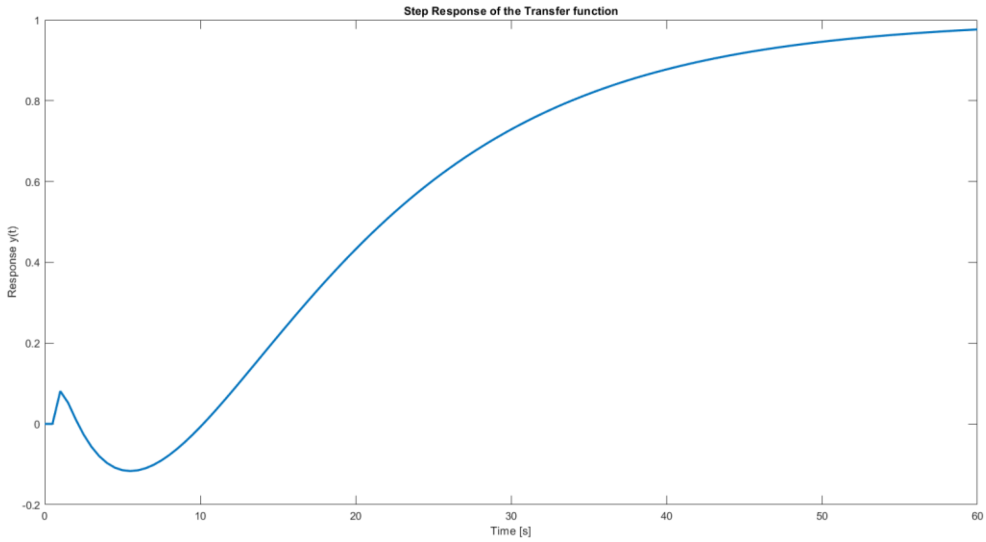
g) Non Linear

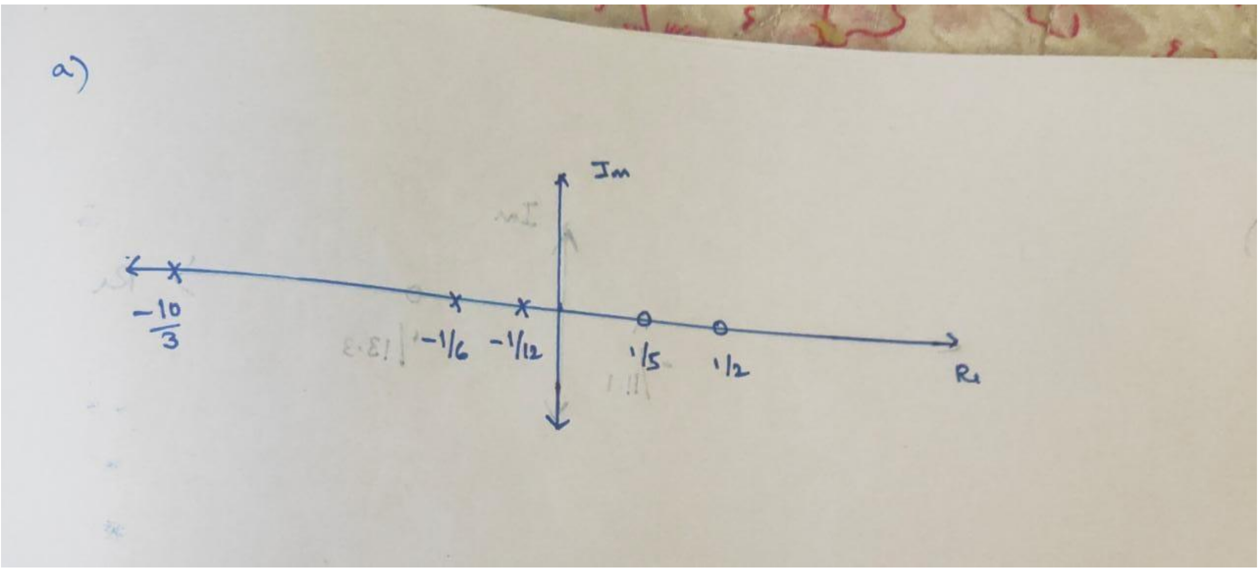


the two poles are almost equal

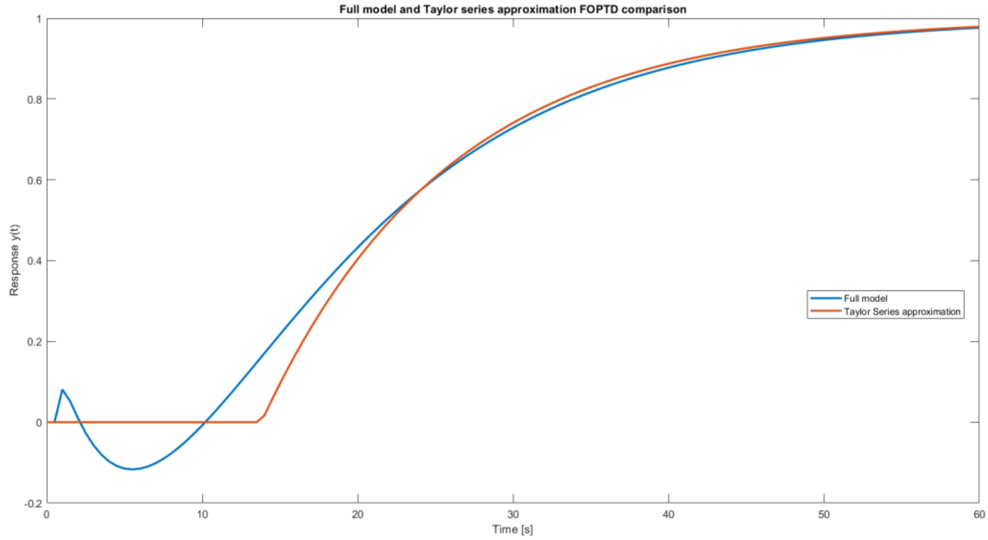
Q1) (Approximated time delay by taylor for the zeros)

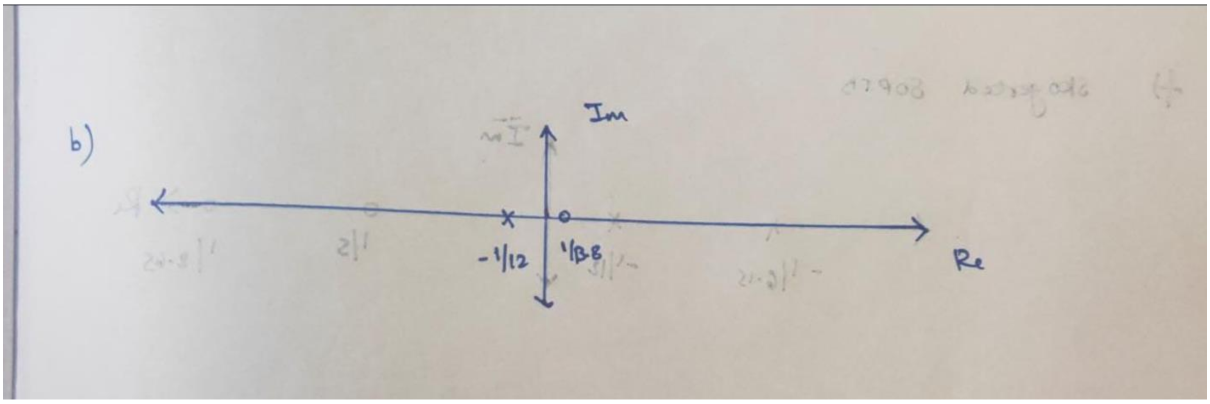
a)



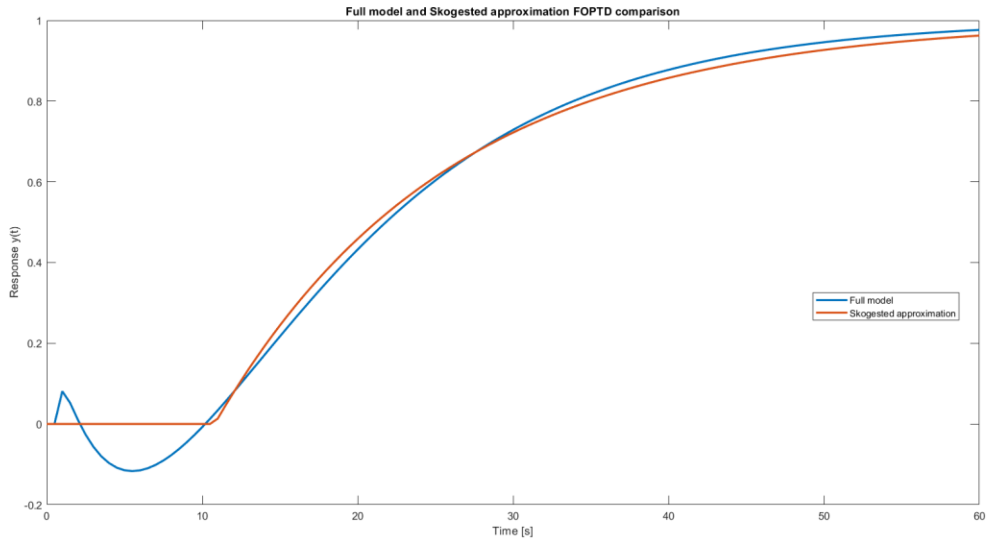


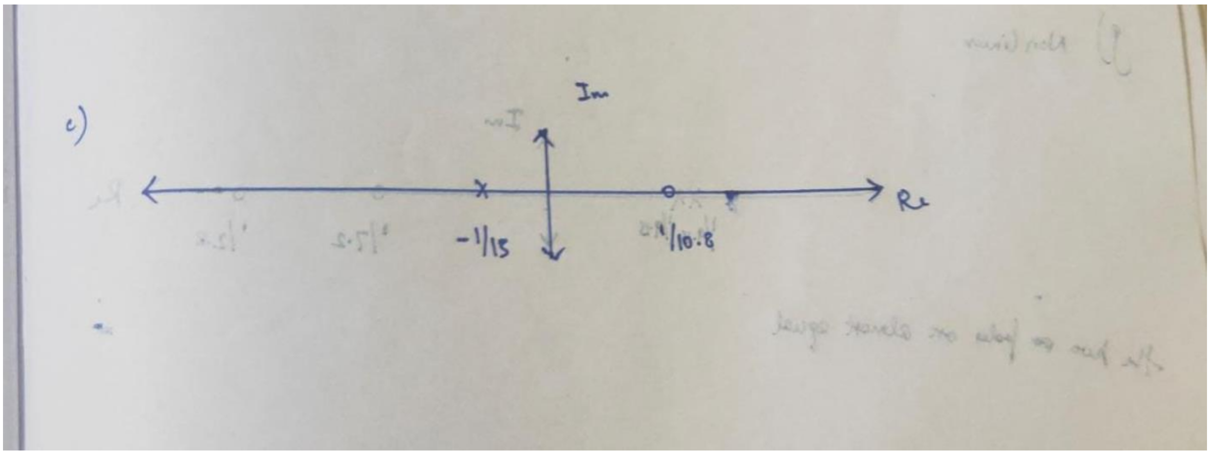
b)


Full model and Taylor series approximation FOPTD comparison
Response y(t)
Time [s]
Full model
Taylor Series approximation

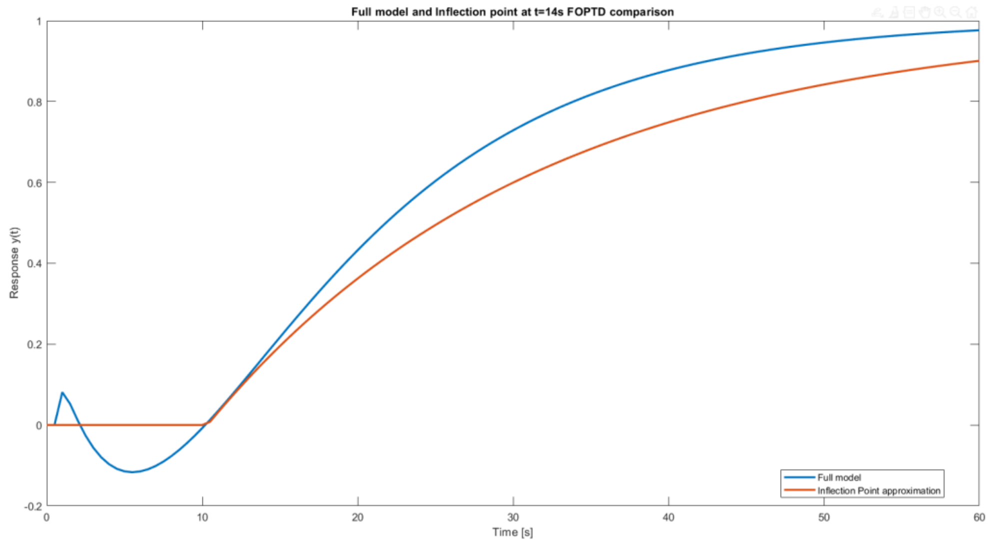

b)
Im
Re
-1/12
1/13.8

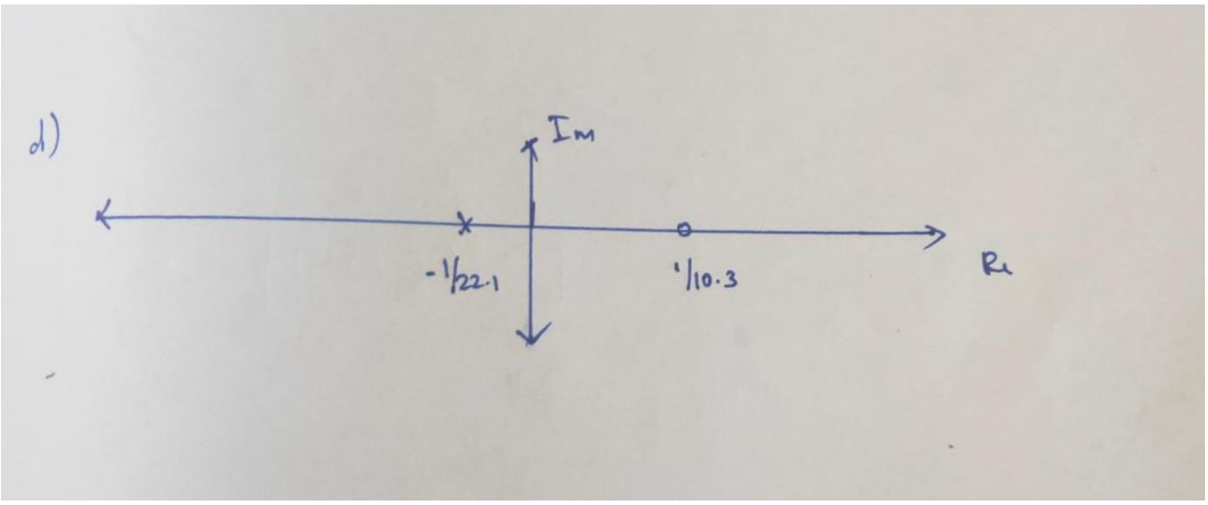
c)

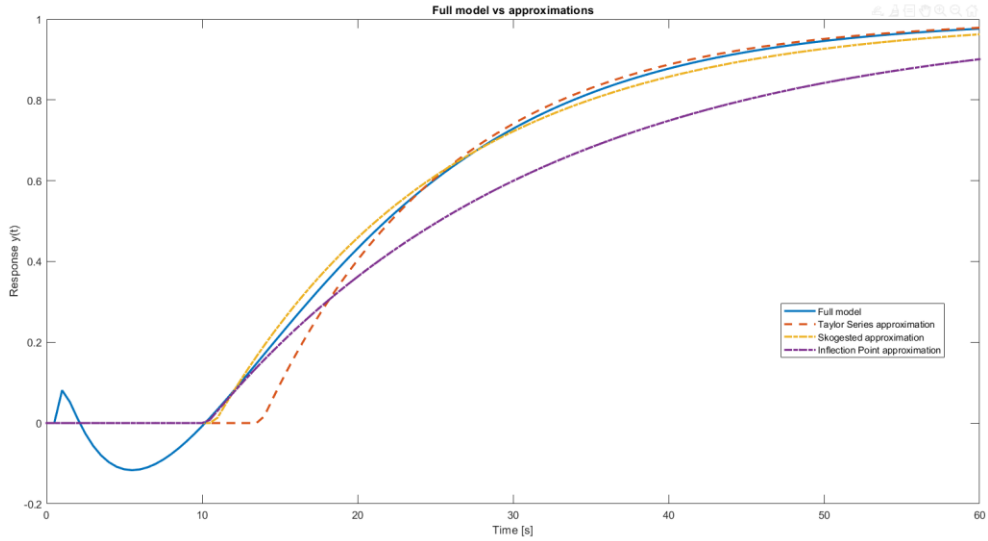

Full model and Skogested approximation FOPTD comparison
Response y(t)
Time [s]
Full model
Skogested approximation


c)
Im
Re
-1/15
1/10.8

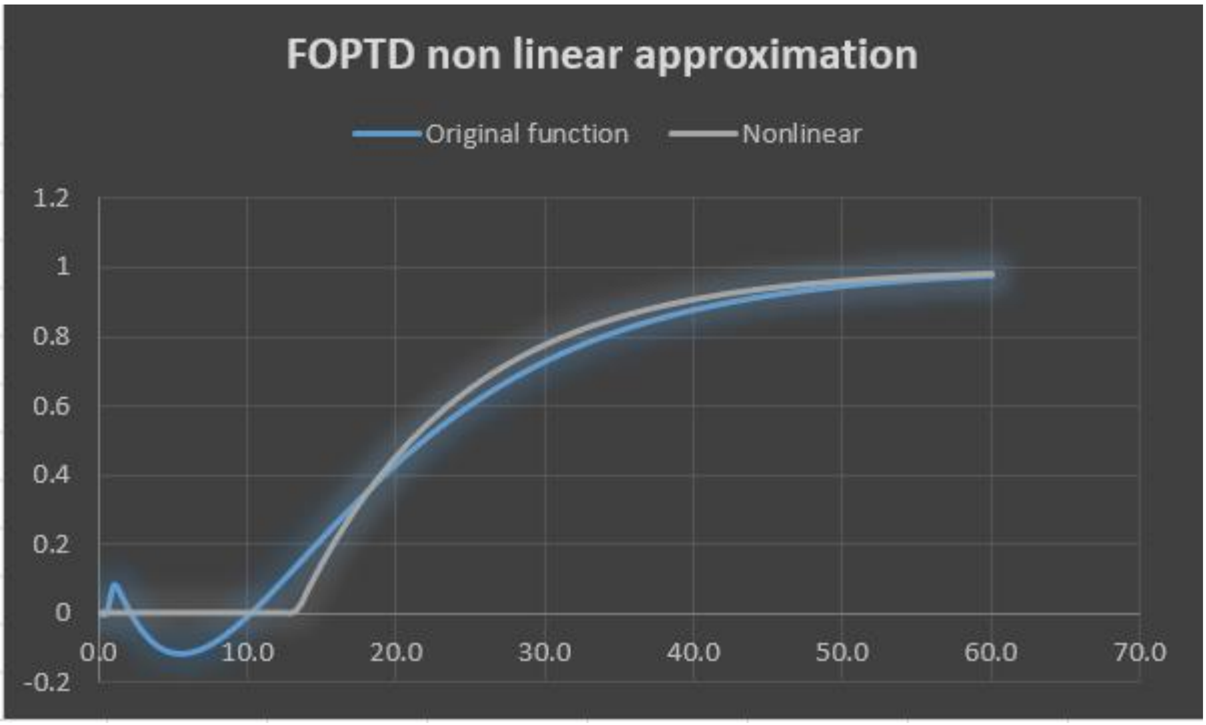
d)

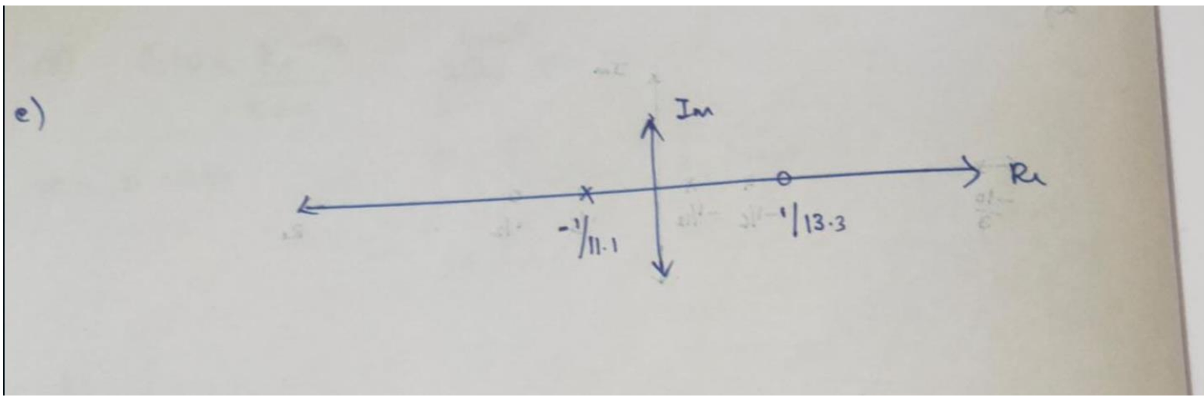

Full model and Inflection point at t=14s FOPTD comparison
Response y(t)
Time [s]
Full model
Inflection Point approximation
1
0.8
0.6
0.4
0.2
0
-0.2
0
10
20
30
40
50
60

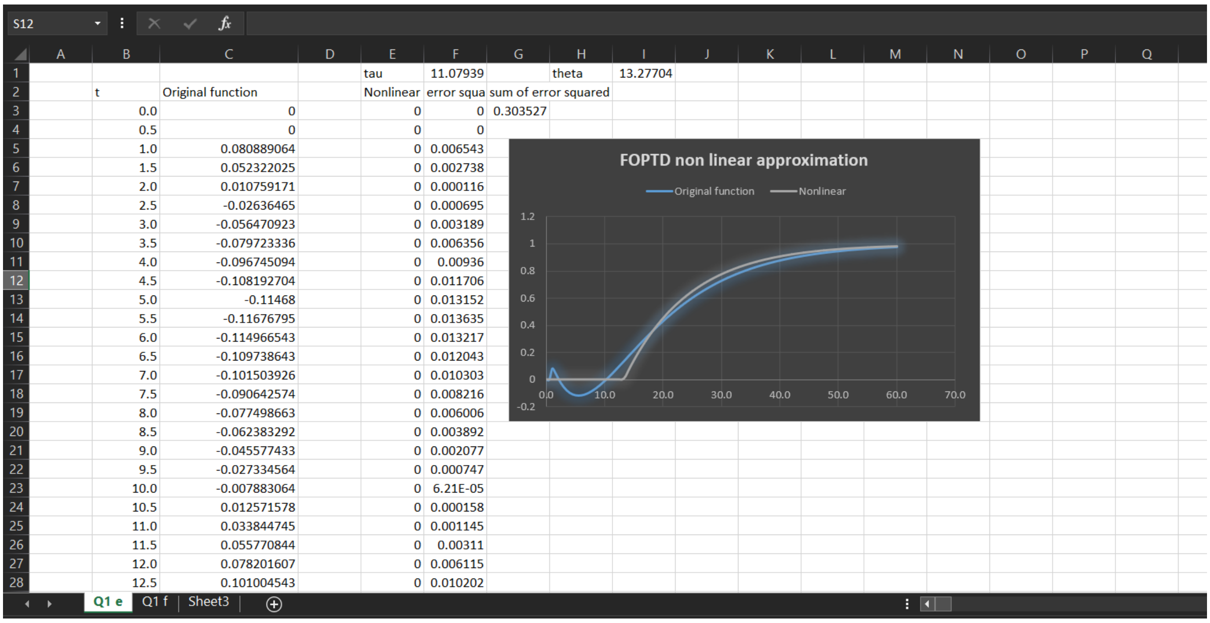

d)
Im
Re
-1/22.1
1/10.3


Full model vs approximations
Response y(t)
Time [s]
Full model
Taylor Series approximation
Skogested approximation
Inflection Point approximation

e)

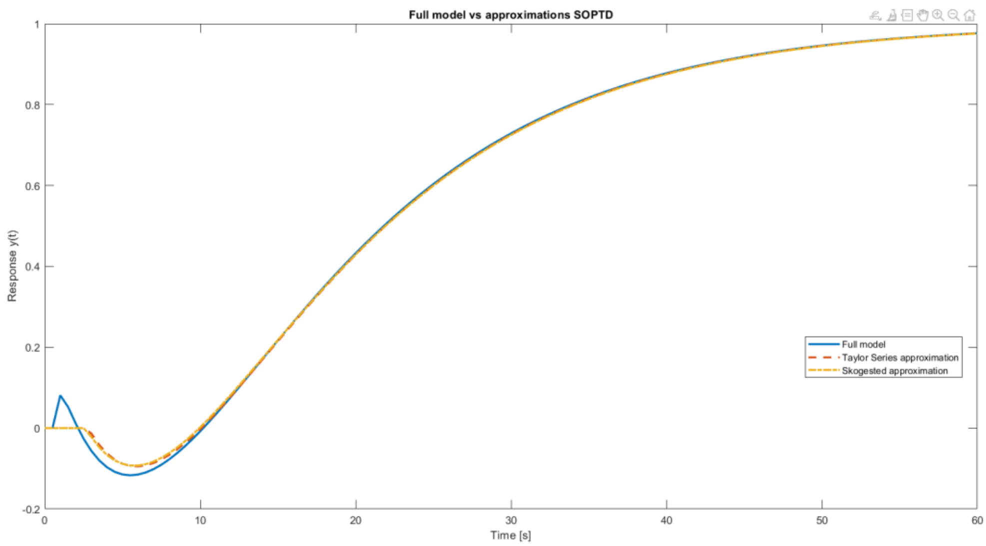

FOPTD non linear approximation
Original function
Nonlinear

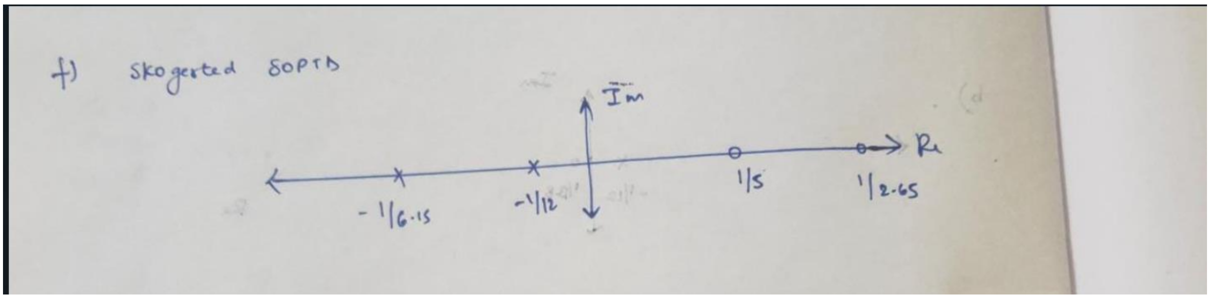

e)
Im
Re
-1/11.1
1/13.3



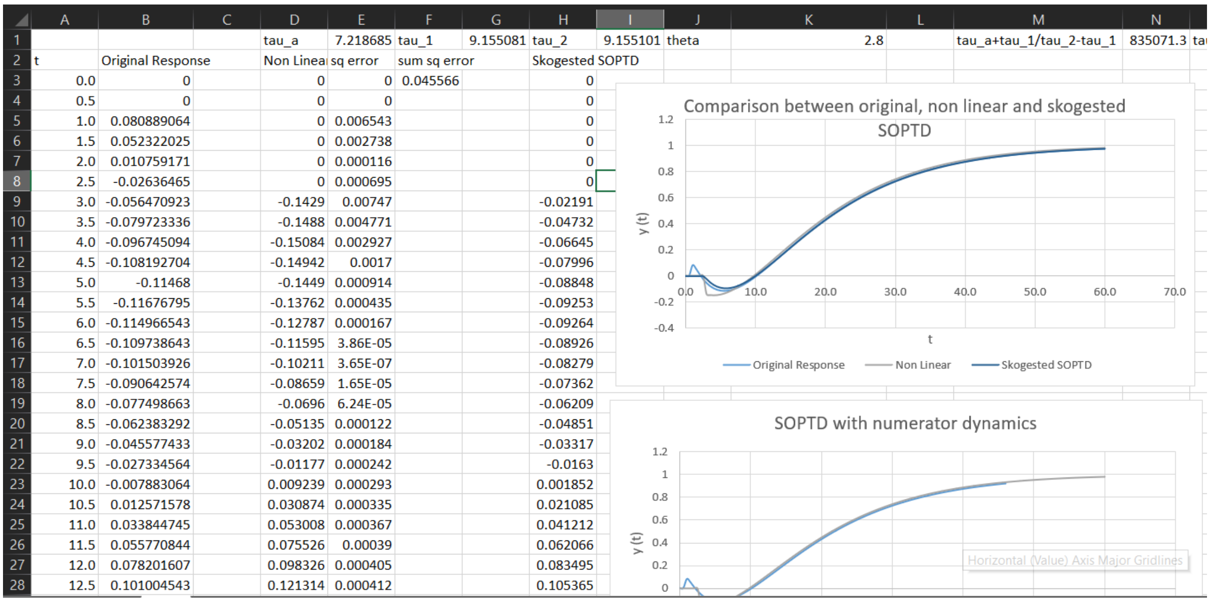
	A	B	C	D	E	F	G	H	I
1					tau	11.07939		theta	13.27704
2		t	Original function		Nonlinear	error squa	sum of error squared		
3		0.0	0		0	0	0.303527		
4		0.5	0		0	0			
5		1.0	0.080889064		0	0.006543			
6		1.5	0.052322025		0	0.002738			
7		2.0	0.010759171		0	0.000116			
8		2.5	-0.02636465		0	0.000695			
9		3.0	-0.056470923		0	0.003189			
10		3.5	-0.079723336		0	0.006356			
11		4.0	-0.096745094		0	0.00936			
12		4.5	-0.108192704		0	0.011706			
13		5.0	-0.11468		0	0.013152			
14		5.5	-0.11676795		0	0.013635			
15		6.0	-0.114966543		0	0.013217			
16		6.5	-0.109738643		0	0.012043			
17		7.0	-0.101503926		0	0.010303			
18		7.5	-0.090642574		0	0.008216			
19		8.0	-0.077498663		0	0.006006			
20		8.5	-0.062383292		0	0.003892			
21		9.0	-0.045577433		0	0.002077			
22		9.5	-0.027334564		0	0.000747			
23		10.0	-0.007883064		0	6.21E-05			
24		10.5	0.012571578		0	0.000158			
25		11.0	0.033844745		0	0.001145			
26		11.5	0.055770844		0	0.00311			
27		12.0	0.078201607		0	0.006115			
28		12.5	0.101004543		0	0.010202			

f)

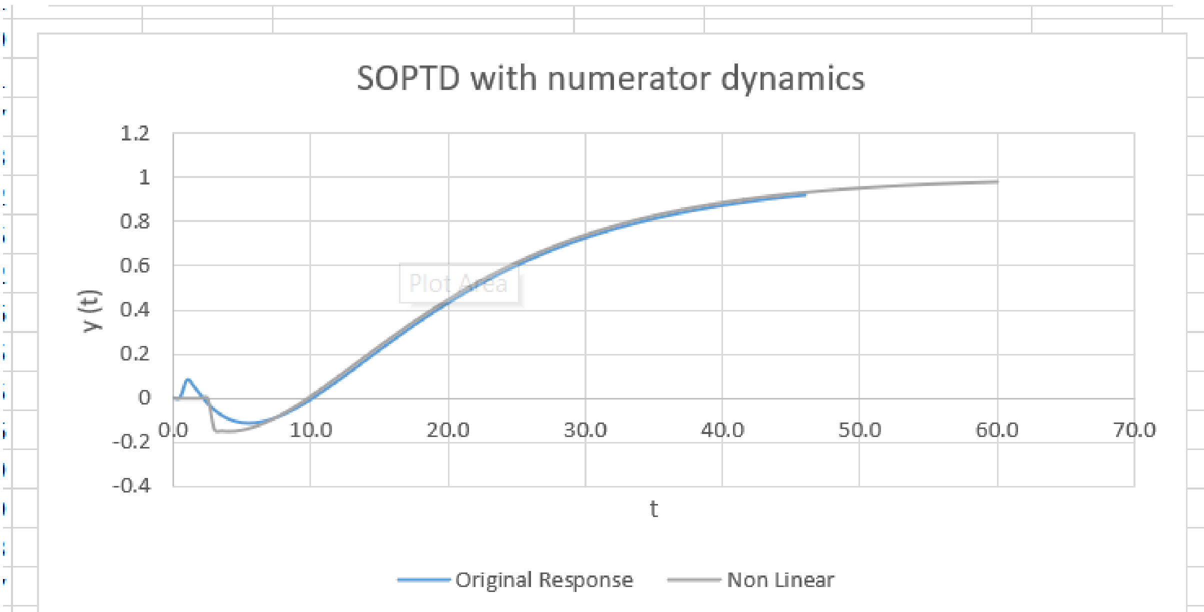




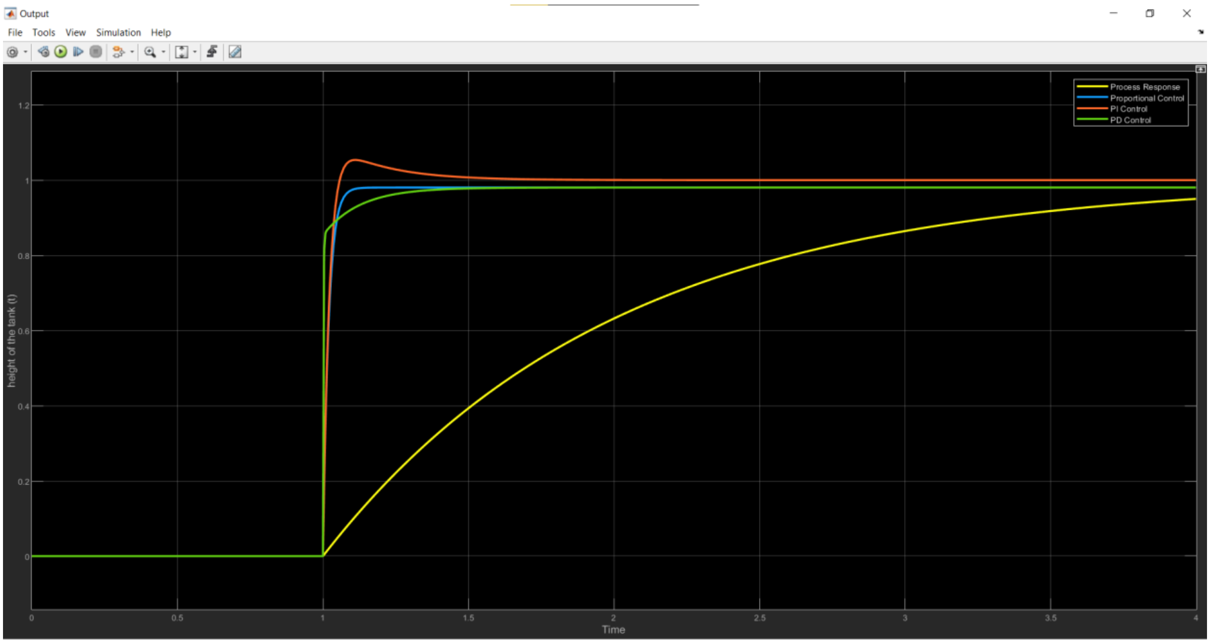
g)



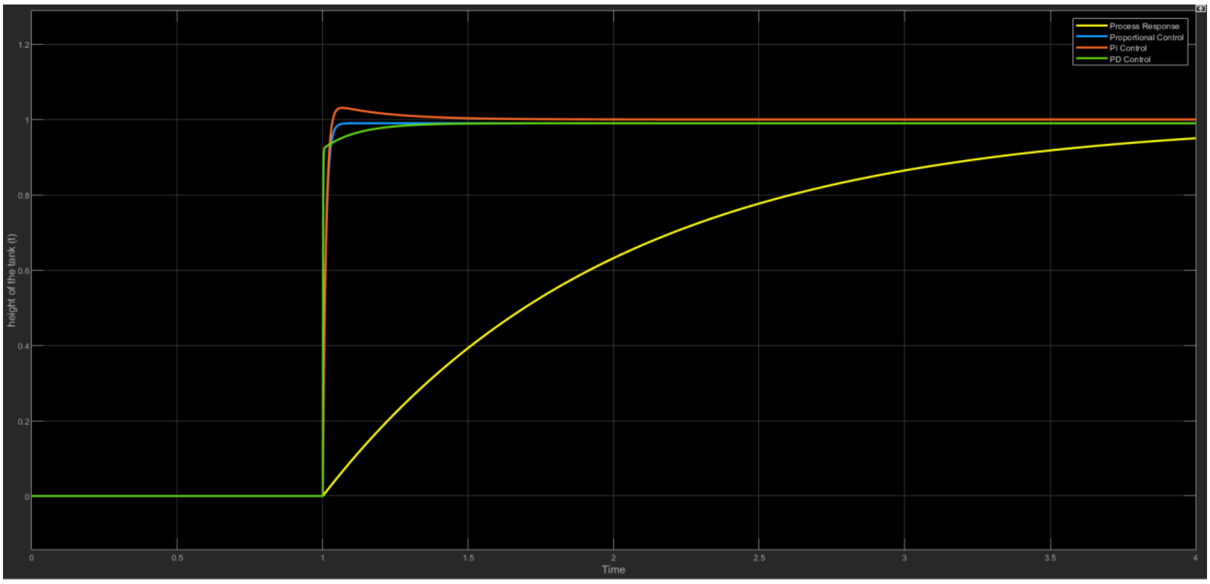
	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1				tau_a	7.218685	tau_1	9.155081	tau_2	9.155101	theta	2.8		tau_a+tau_1/tau_2-tau_1	835071.3
2	t	Original Response		Non Linea	sq error	sum sq error		Skogested SOPTD						
3	0.0	0		0	0	0.045566		0						
4	0.5	0		0	0			0						
5	1.0	0.080889064		0	0.006543			0						
6	1.5	0.052322025		0	0.002738			0						
7	2.0	0.010759171		0	0.000116			0						
8	2.5	-0.02636465		0	0.000695			0						
9	3.0	-0.056470923		-0.1429	0.00747			-0.02191						
10	3.5	-0.079723336		-0.1488	0.004771			-0.04732						
11	4.0	-0.096745094		-0.15084	0.002927			-0.06645						
12	4.5	-0.108192704		-0.14942	0.0017			-0.07996						
13	5.0	-0.11468		-0.1449	0.000914			-0.08848						
14	5.5	-0.11676795		-0.13762	0.000435			-0.09253						
15	6.0	-0.114966543		-0.12787	0.000167			-0.09264						
16	6.5	-0.109738643		-0.11595	3.86E-05			-0.08926						
17	7.0	-0.101503926		-0.10211	3.65E-07			-0.08279						
18	7.5	-0.090642574		-0.08659	1.65E-05			-0.07362						
19	8.0	-0.077498663		-0.0696	6.24E-05			-0.06209						
20	8.5	-0.062383292		-0.05135	0.000122			-0.04851						
21	9.0	-0.045577433		-0.03202	0.000184			-0.03317						
22	9.5	-0.027334564		-0.01177	0.000242			-0.0163						
23	10.0	-0.007883064		0.009239	0.000293			0.001852						
24	10.5	0.012571578		0.030874	0.000335			0.021085						
25	11.0	0.033844745		0.053008	0.000367			0.041212						
26	11.5	0.055770844		0.075526	0.00039			0.062066						
27	12.0	0.078201607		0.098326	0.000405			0.083495						
28	12.5	0.101004543		0.121314	0.000412			0.105365						



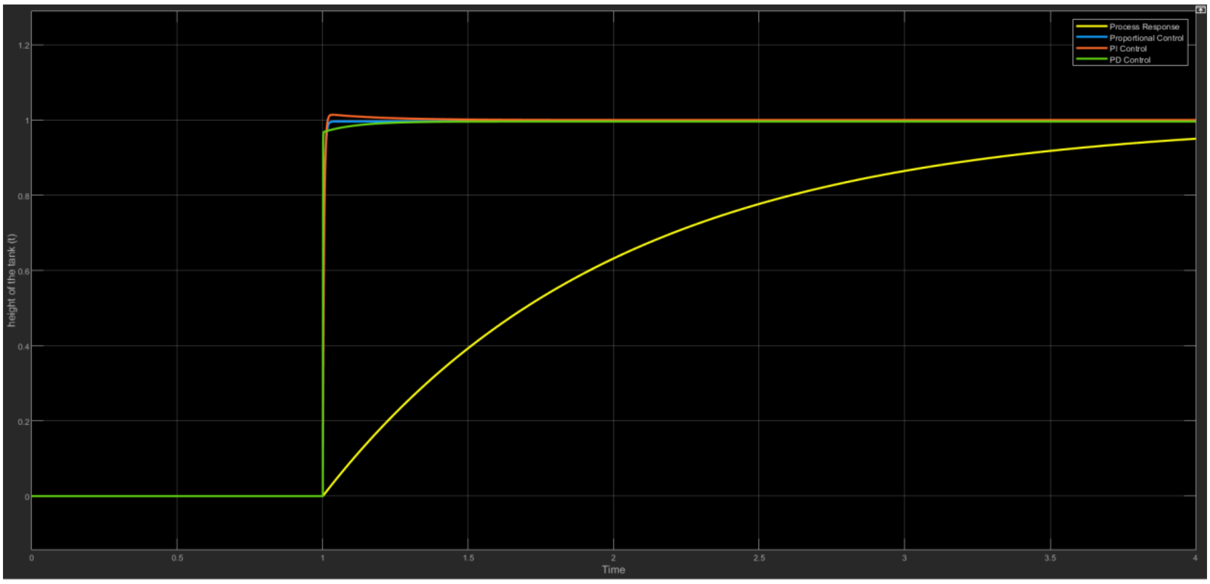
Q2)


Output
File Tools View Simulation Help
Process Response
Proportional Control
PI Control
PD Control
height of the tank (t)
Time

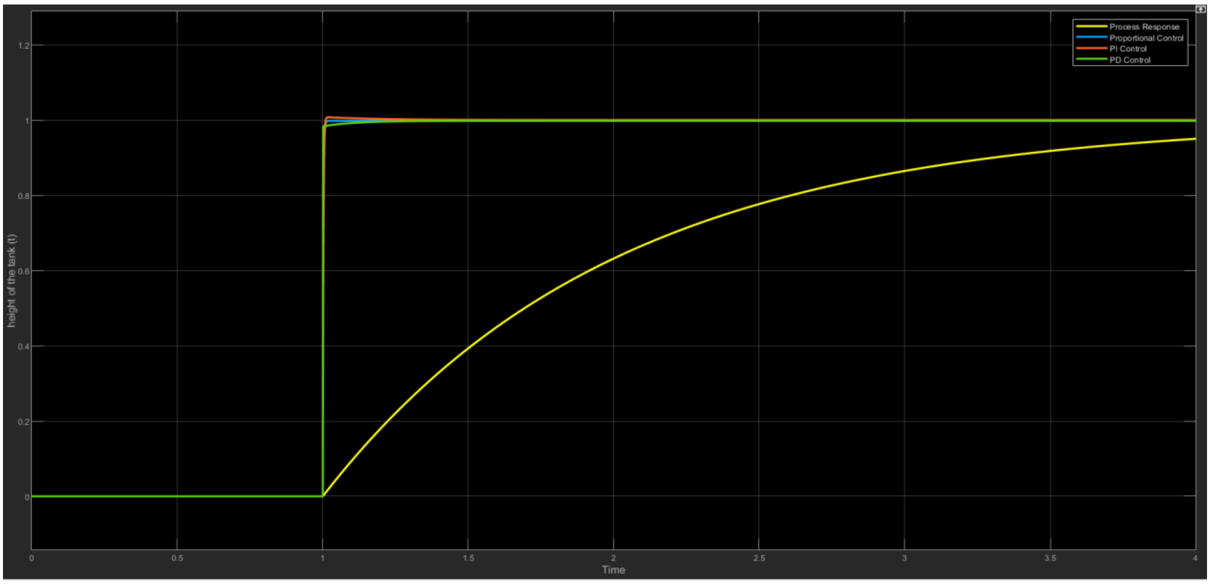
K=50


Process Response
Proportional Control
PI Control
PD Control
height of the tank (t)
Time

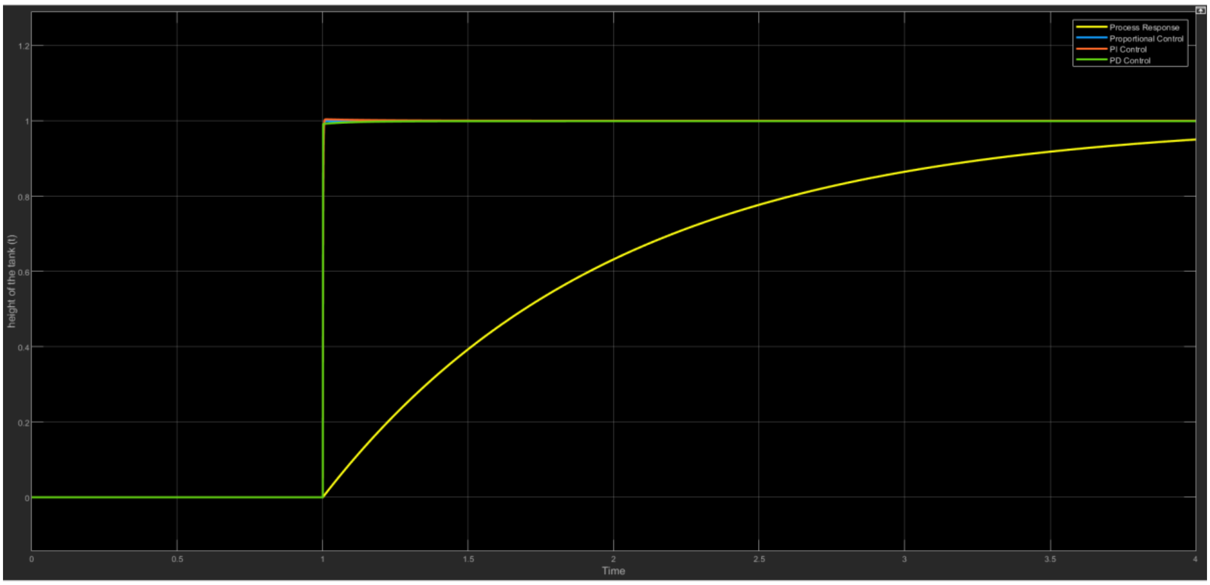
K=100


Process Response
Proportional Control
PI Control
PD Control
height of the tank (t)
Time

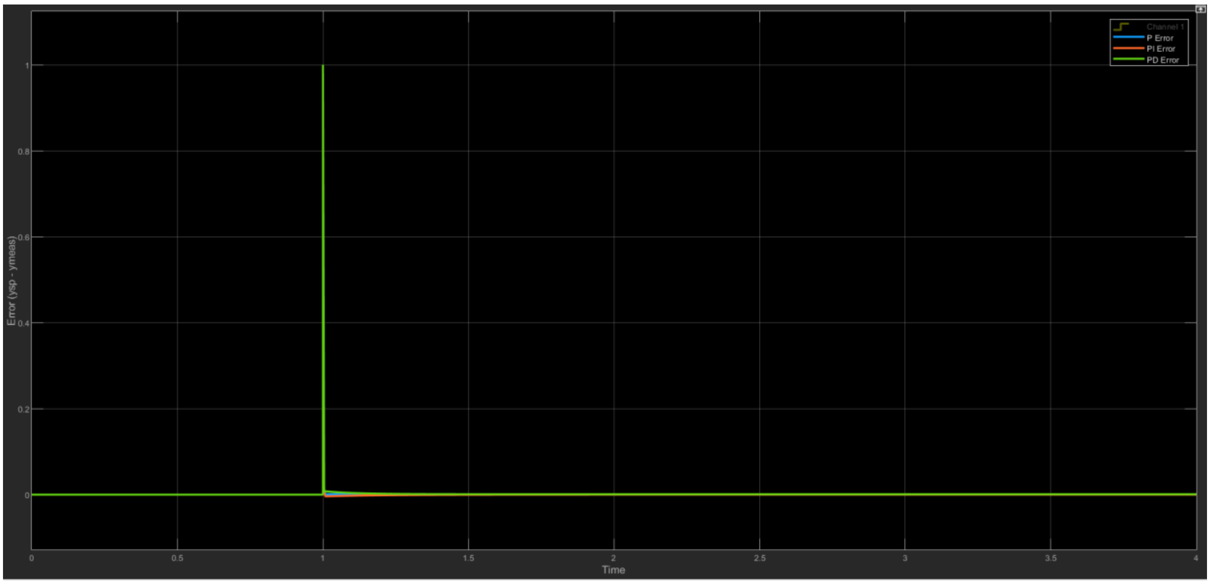
K=250


Process Response
Proportional Control
PI Control
PD Control
height of the tank (t)
Time

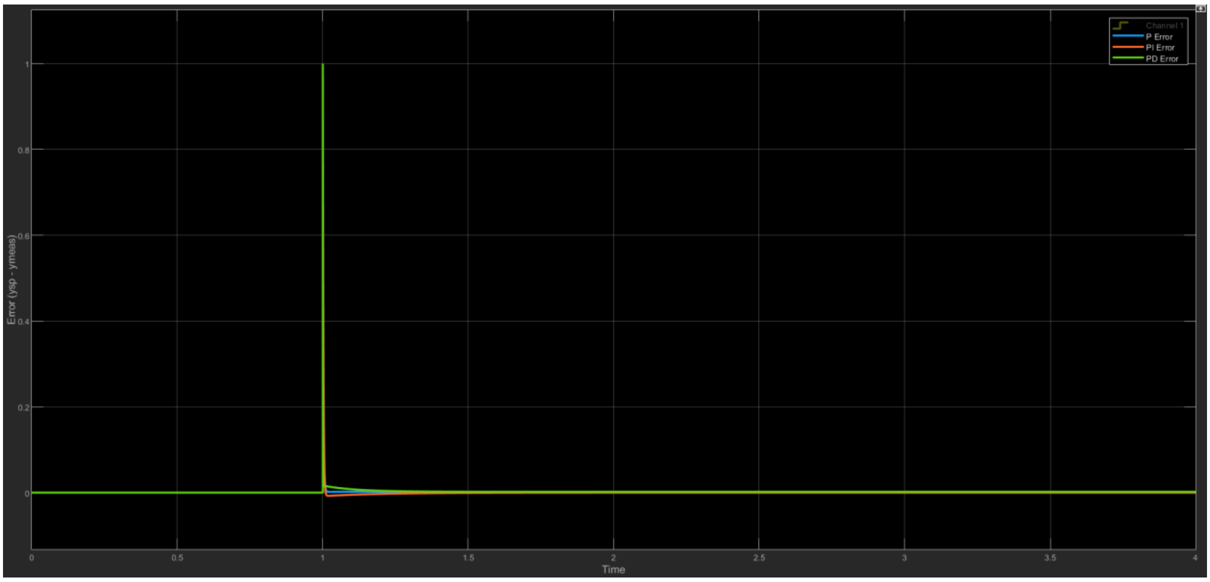
K=500


Process Response
Proportional Control
PI Control
PD Control
height of the tank (l)
Time
1.2
1
0.8
0.6
0.4
0.2
0
0
0.5
1
1.5
2
2.5
3
3.5
4

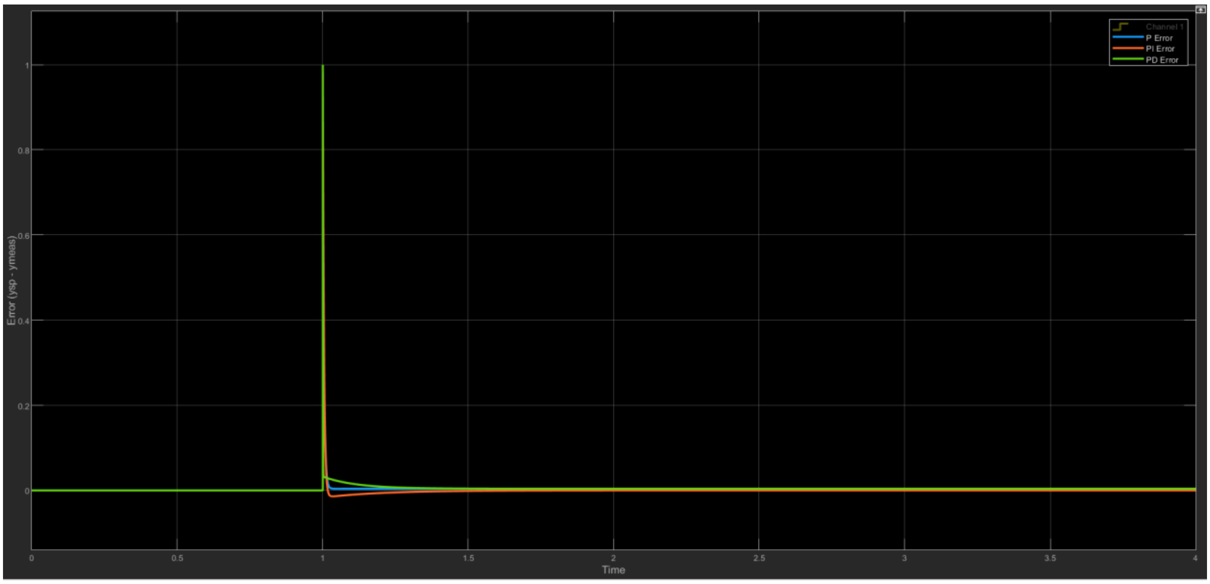
K=1000


Channel 1
P Error
PI Error
PD Error
Error (ysp - ymeas)
Time
1
0.8
0.6
0.4
0.2
0
0
0.5
1
1.5
2
2.5
3
3.5
4

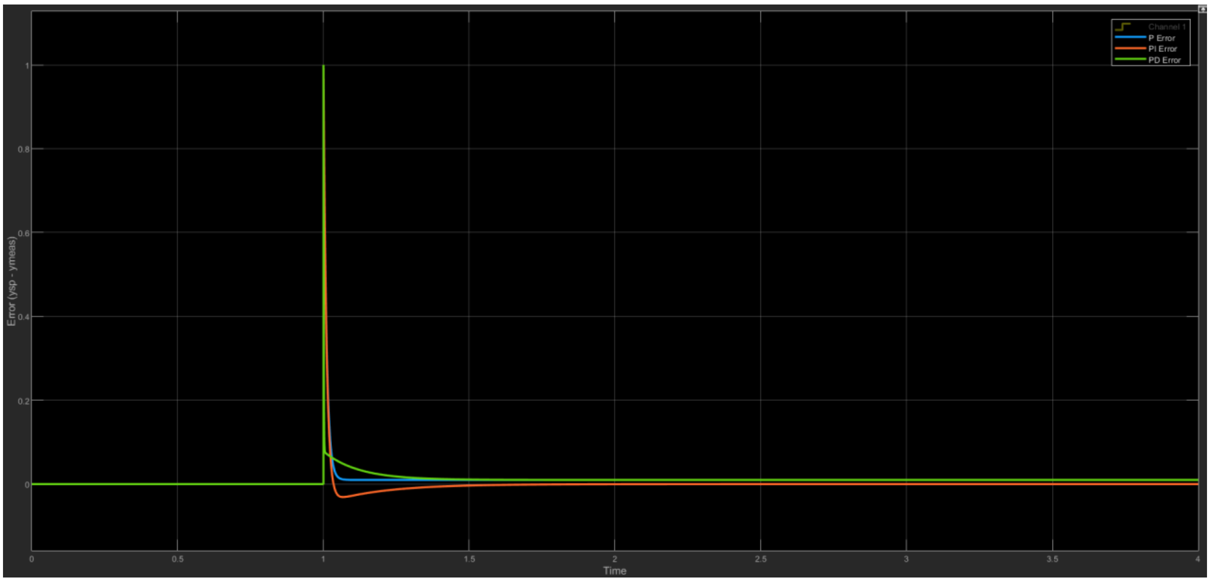
K=1000


Channel 1
P Error
PI Error
PD Error
Error (ysp - ymeas)
Time

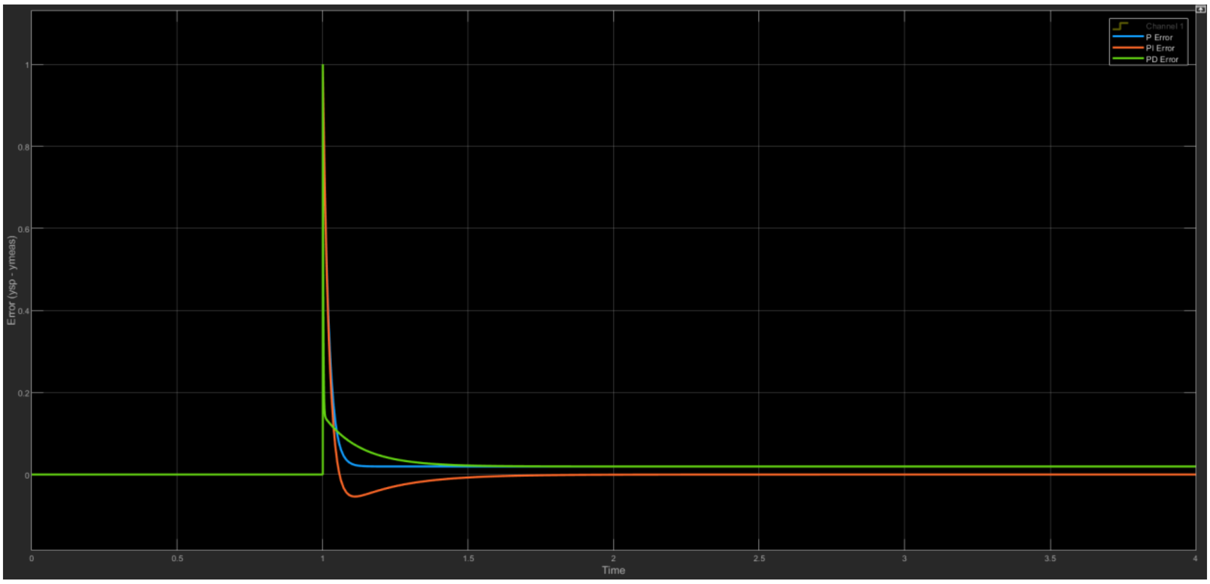
K=500


Channel 1
P Error
PI Error
PD Error
Error (ysp - ymeas)
Time

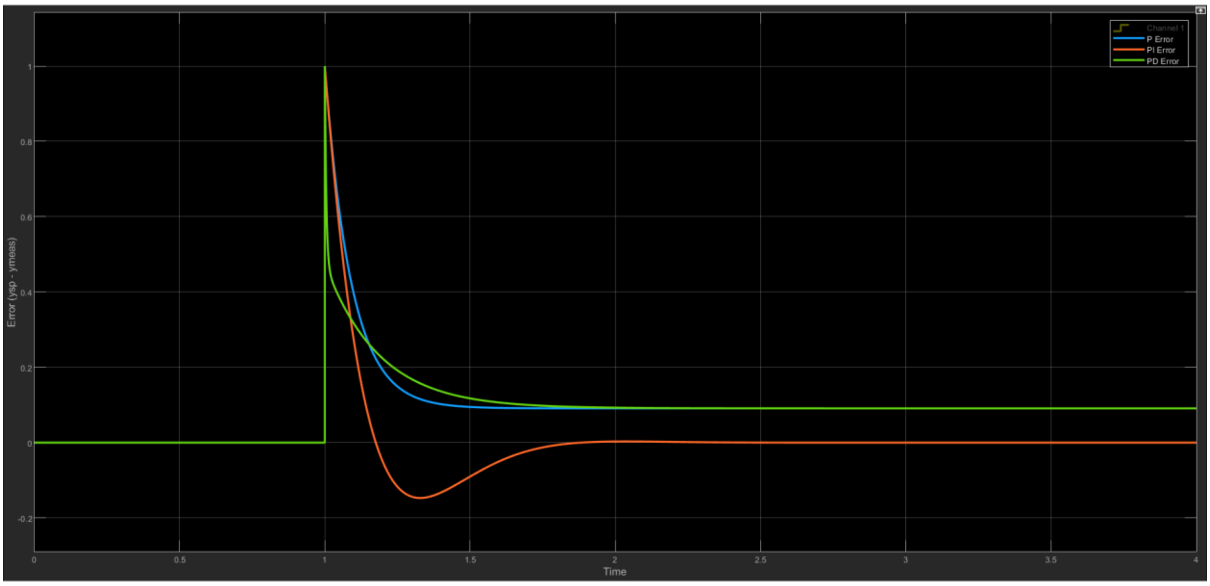
K=250


Channel 1
P Error
PI Error
PD Error
Error (ysp - ymeas)
Time

K=100

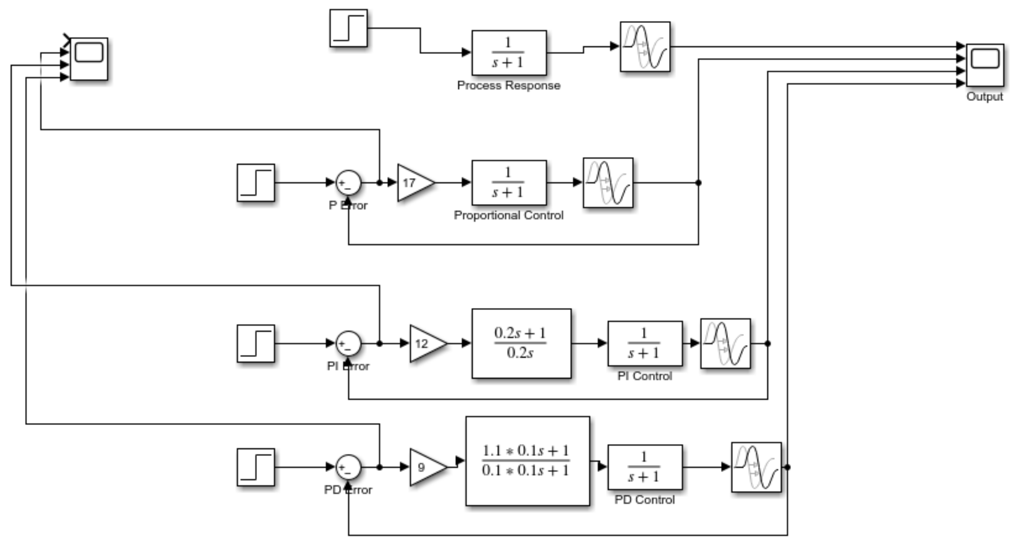

Channel 1
P Error
PI Error
PD Error
Error (ysp - ymeas)
Time

K=50



K=10

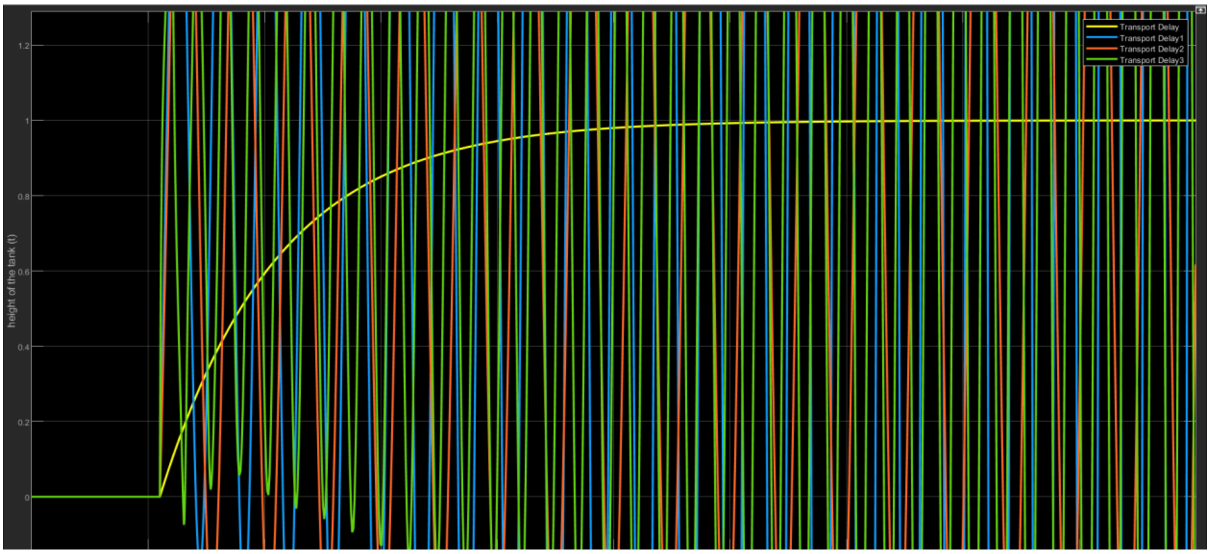
b) P, PI, PD controller together from top to bottom respectively



For P controller the value of Kc >= 16 gives unstable output

For PI controller, the value of Kc >= 12 gives unstable output

For PD controller, the value of Kc >=9 gives unstable output

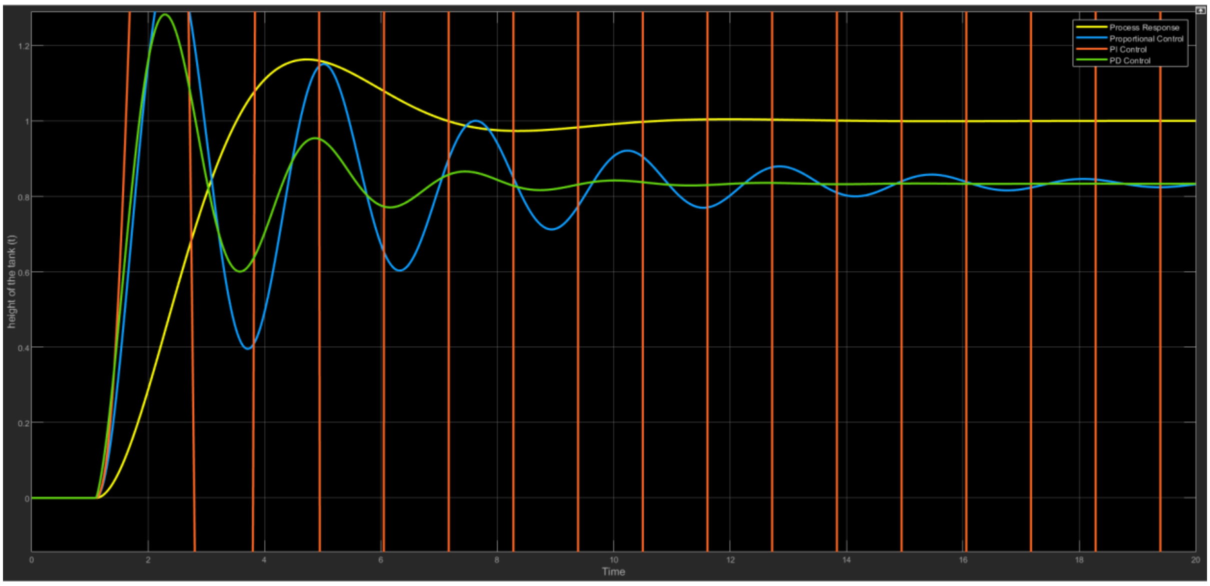


All are unstable outputs

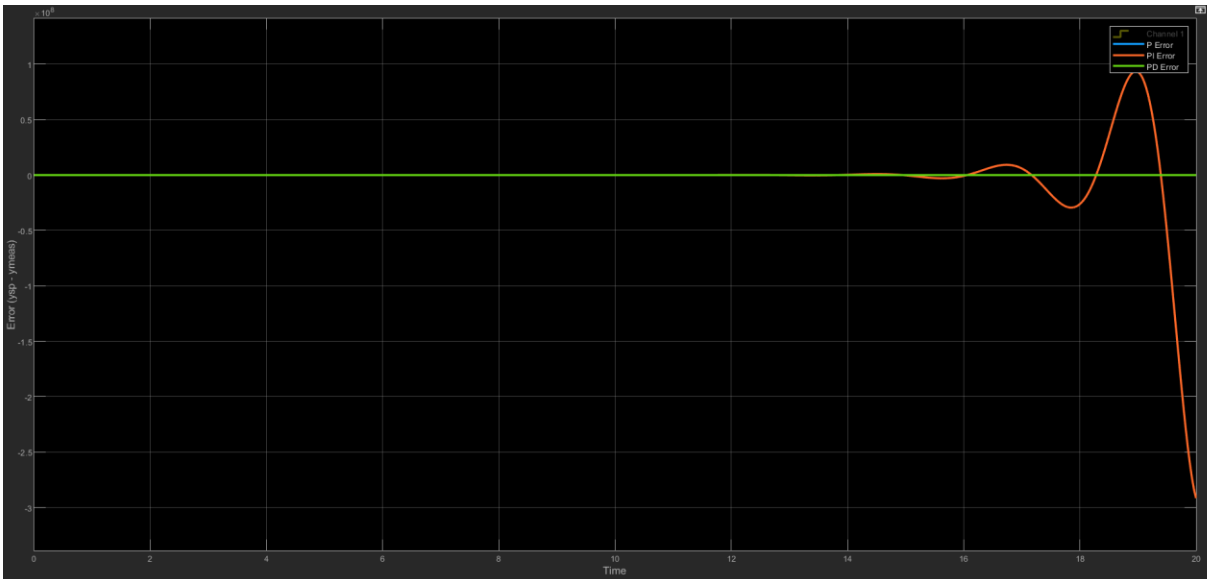
c)

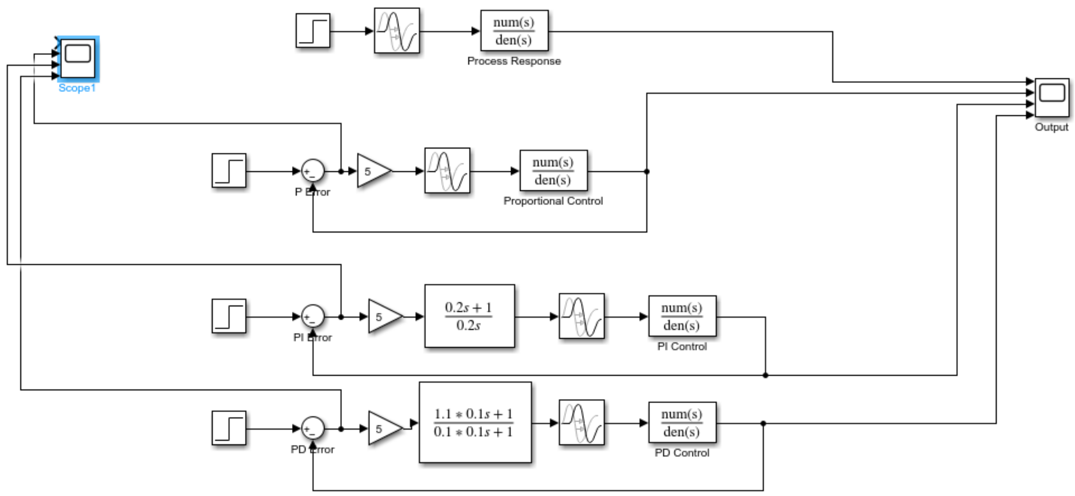
Case 1 – Underdamped (1/s^2 + s + 1)

Output

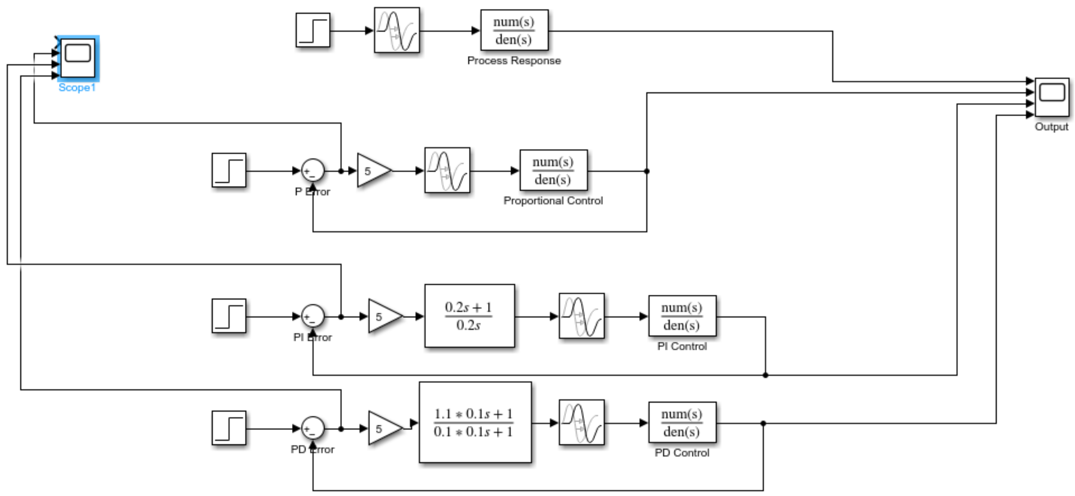


Error

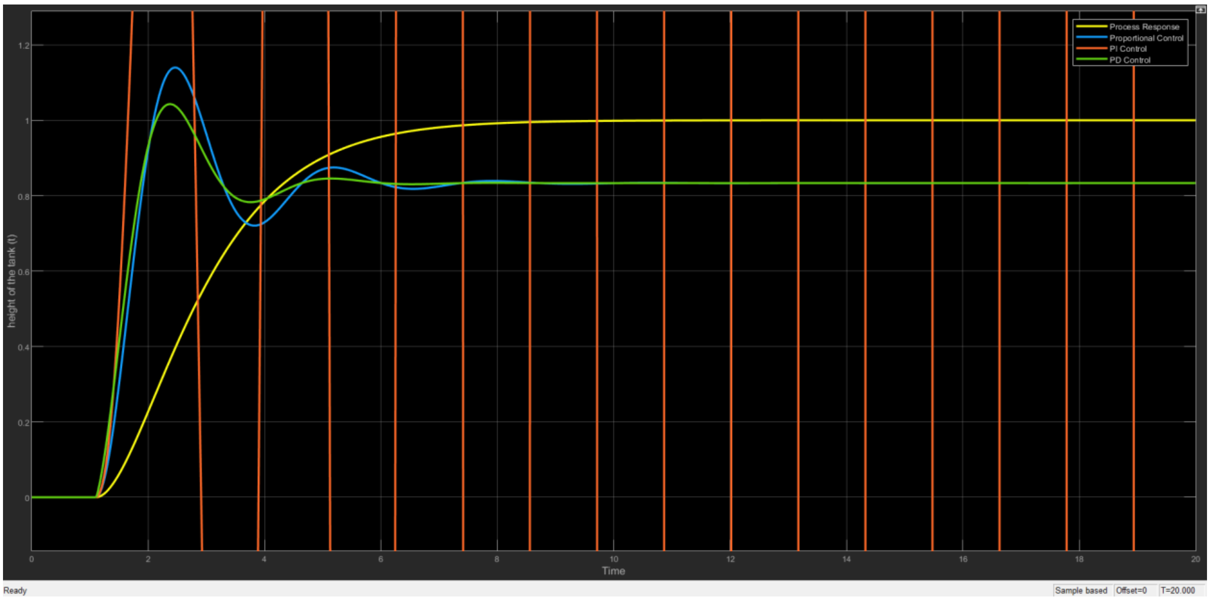




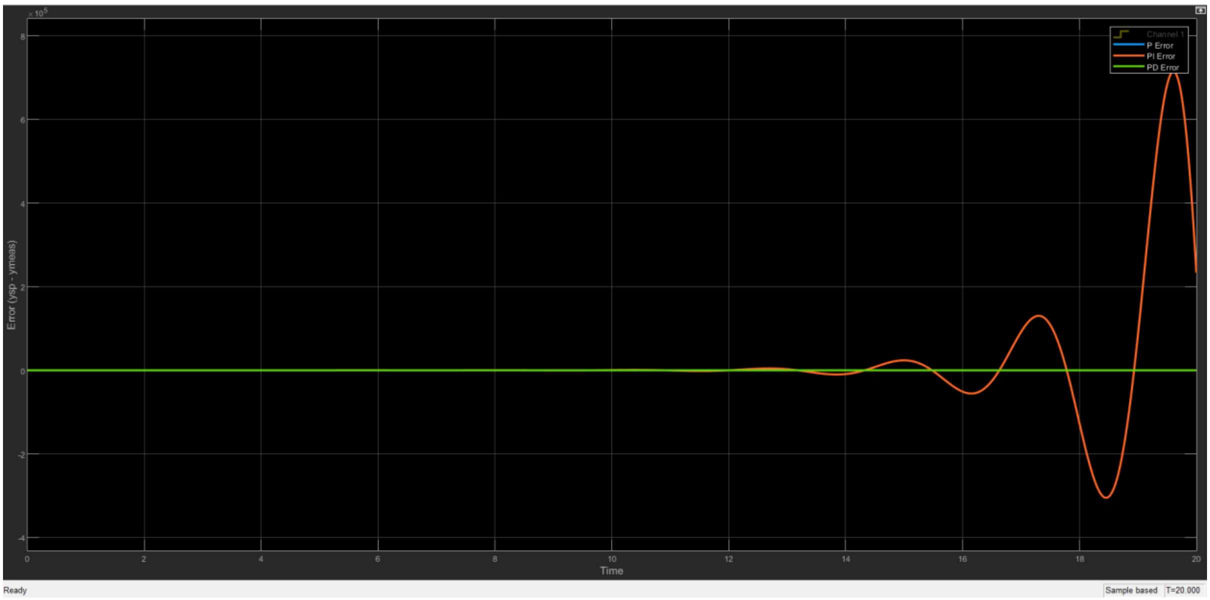
Case 2 : Critically damped (1/(s+1)^2)



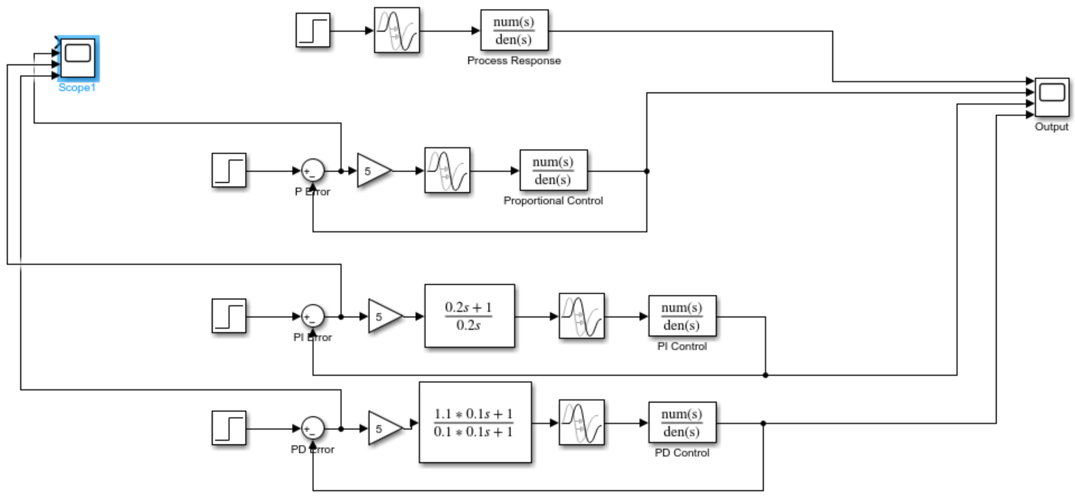
Output



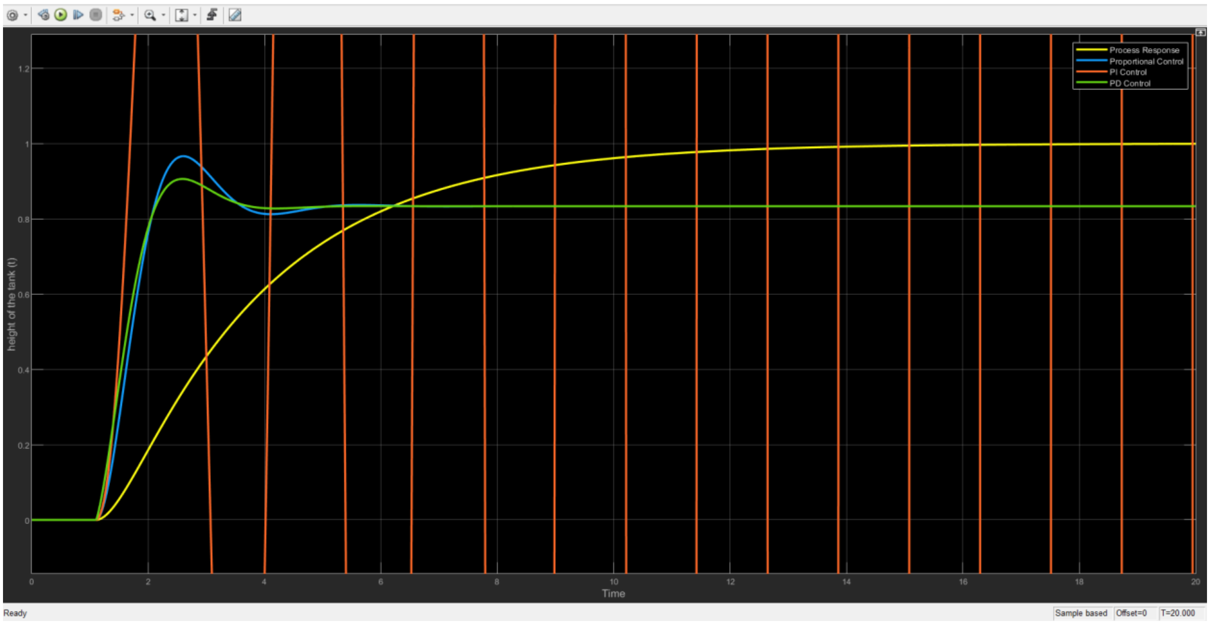
Error



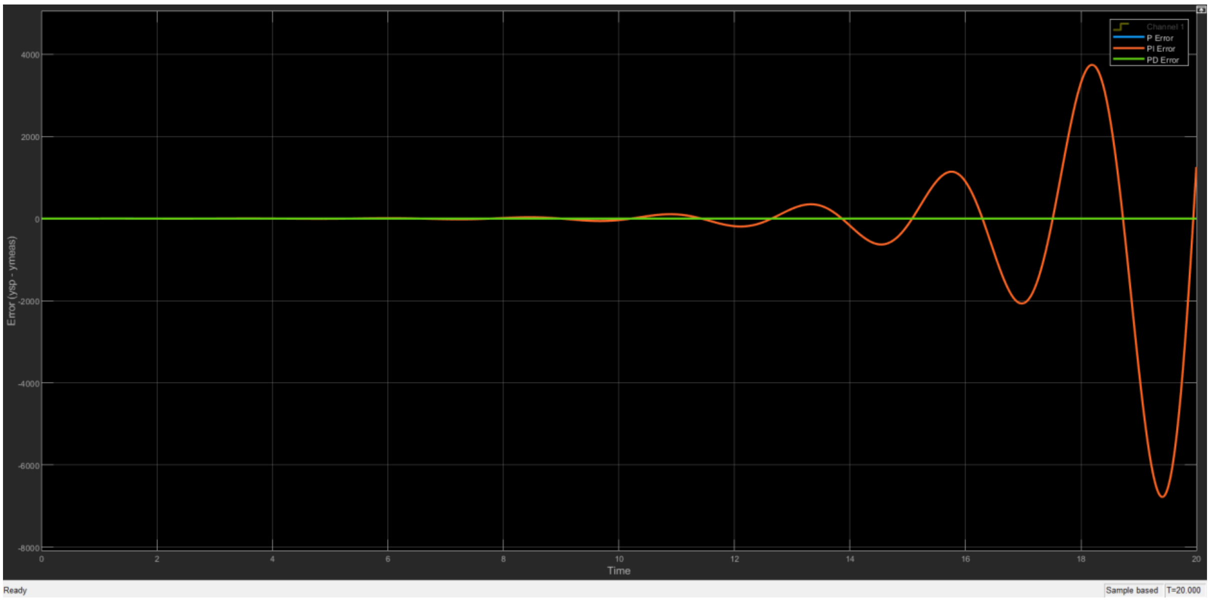
Case 3 : Over Damped (1/(s+1)(s+2))



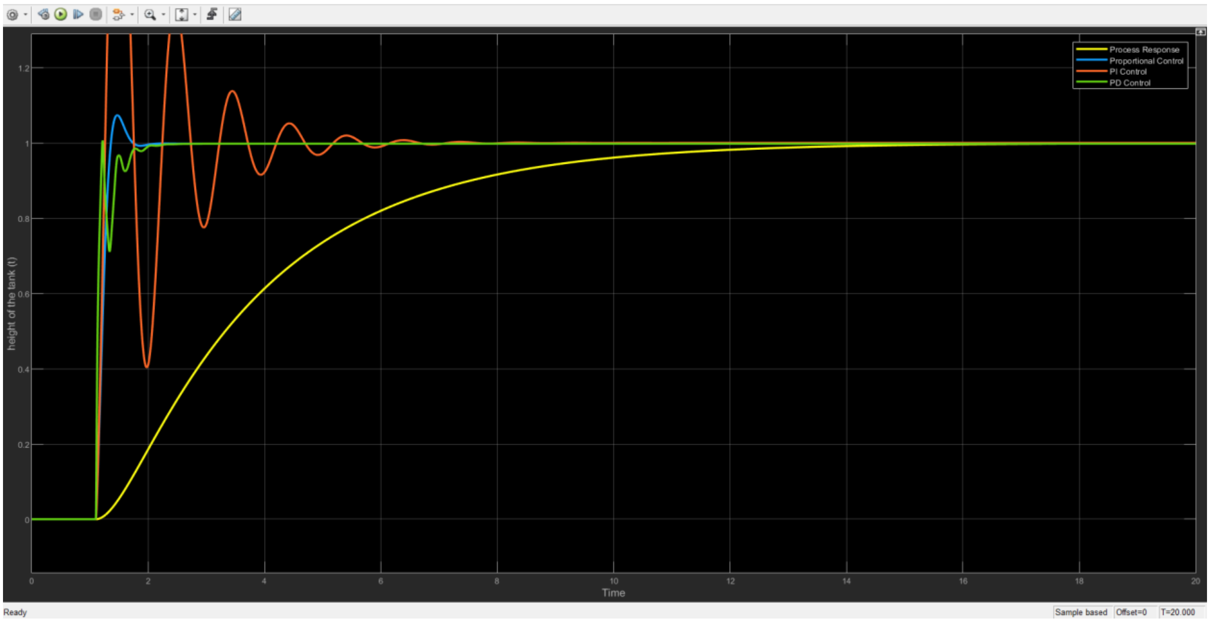
Output



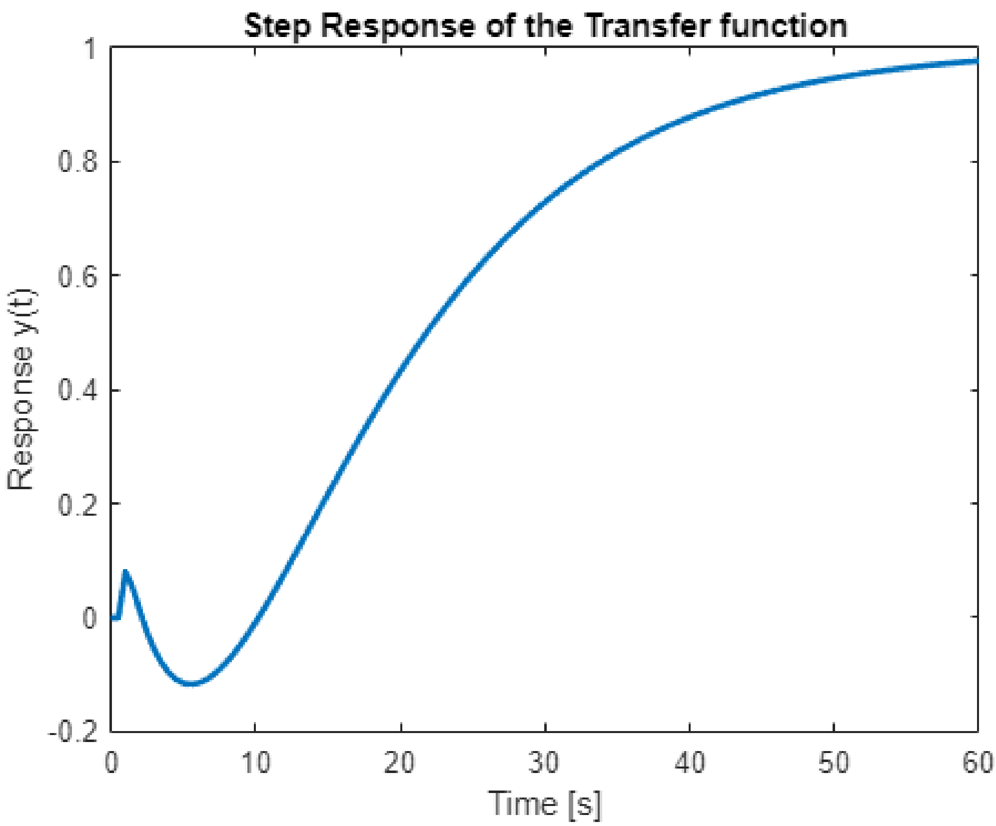
Error



f) Yes, For underdamped systems, the Kc = 500 is also stable for the transfer function of 1/(s^2+100s+1) but for 1/s^2+10s+1, the response became unstable

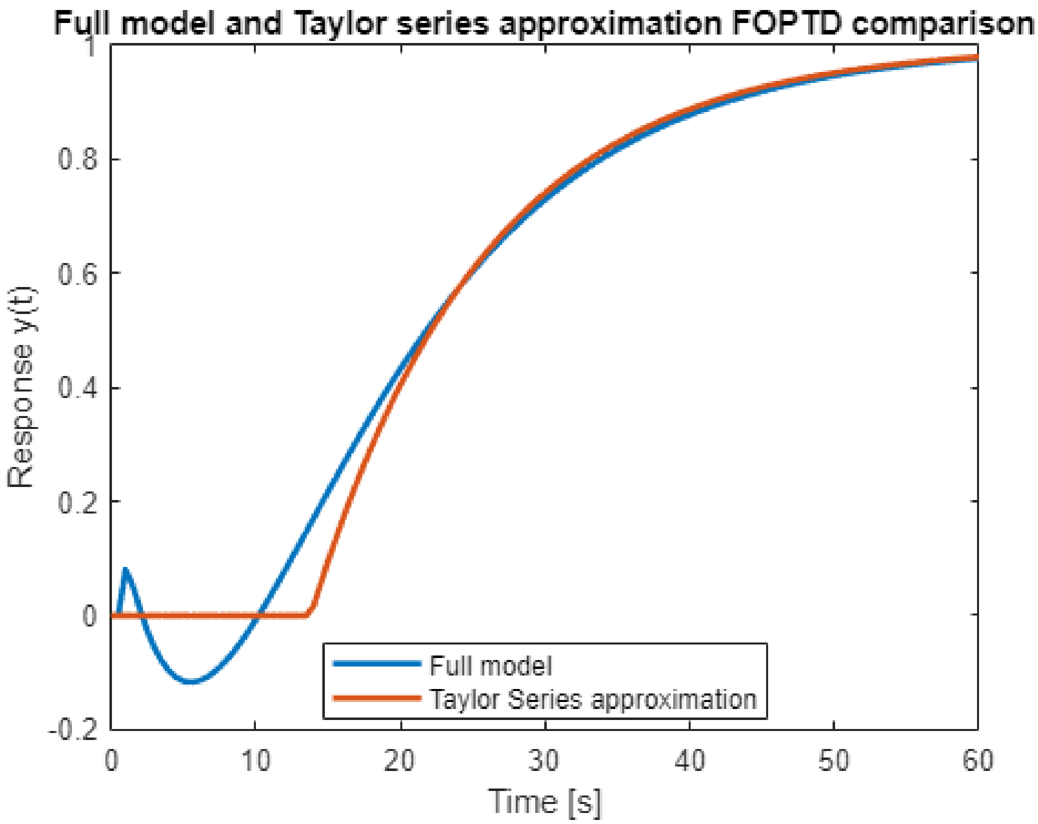


```
% defining variables
syms s
G = ((1-5*s)*(1-2*s)*exp(-0.5*s))/((12*s+1)*(6*s+1)*(0.3*s+1)); % the transfer function
U = 1/s; % step input
y=ilaplace(G*U); % respose of step input
dy=diff(y);
t=0:0.5:60;
t=t'; % for easier copying
y=subs(y);
doubley= double(y); % made for copying
    plot(t,y,'linewidth',2)
title("Step Response of the Transfer function")
ylabel("Response y(t)")
xlabel("Time [s]")
```

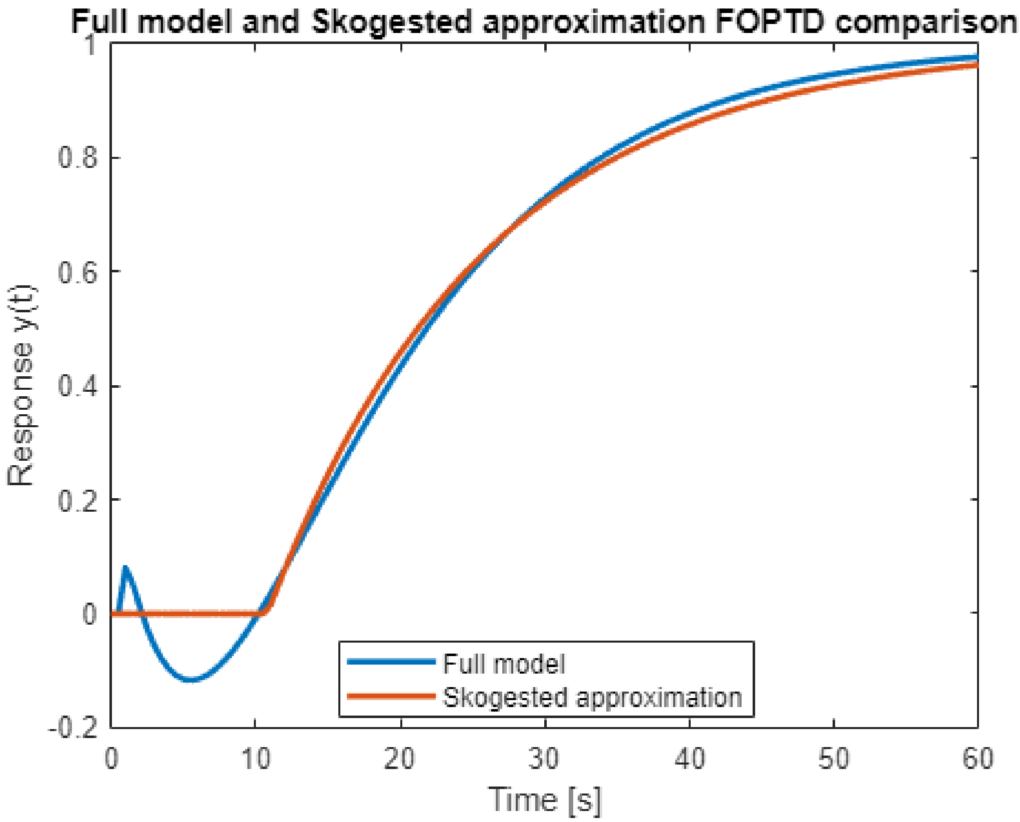



```
%b
G_b = exp(-13.8*s)/(12*s+1); % obtained by solving on paper
y_b=ilaplace(G_b*U);
y_b=subs(y_b);
plot(t,y,'linewidth',2)
hold on
plot(t,y_b,'linewidth',2)
hold off
ylabel("Response y(t)")
xlabel("Time [s]")
title("Full model and Taylor series approximation FOPTD comparison")
```

```
legend("Full model","Taylor Series approximation", Location="best")
```




```
%c
G_c = exp(-10.8*s)/(15*s+1); % obtained by solving on paper
y_c=ilaplace(G_c*U);
y_c=subs(y_c);
plot(t,y,'linewidth',2)
hold on
plot(t,y_c,'linewidth',2)
hold off
ylabel("Response y(t)")
xlabel("Time [s]")
title("Full model and Skogested approximation FOPTD comparison")
legend("Full model","Skogested approximation", Location="best")
```




```
%d
t_d=14;
m = subs(dy,t_d);
```

m =

$$\frac{595\,e^{-\frac{9}{8}}}{2106} - \frac{220\,e^{-\frac{9}{4}}}{513} + \frac{12190\,e^{-45}}{20007}$$

```
p=double(y(find(t==t_d)))
```

p = 0.1705

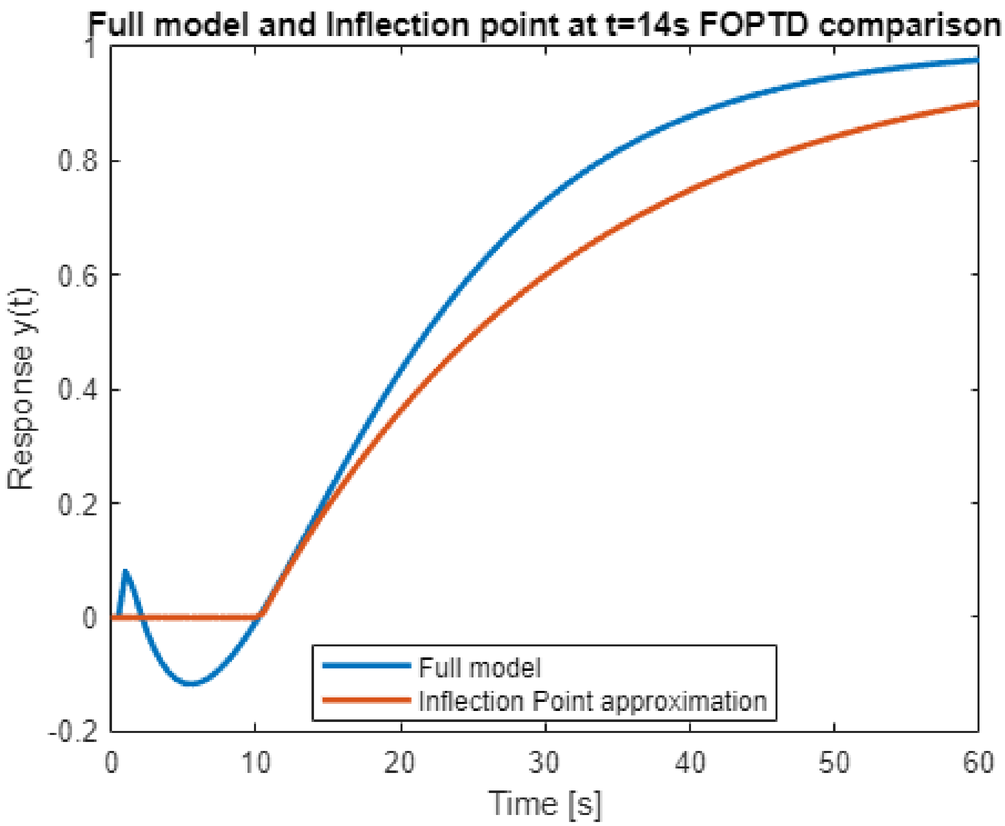
```
theta = (t_d*m-p)/m
```

theta =

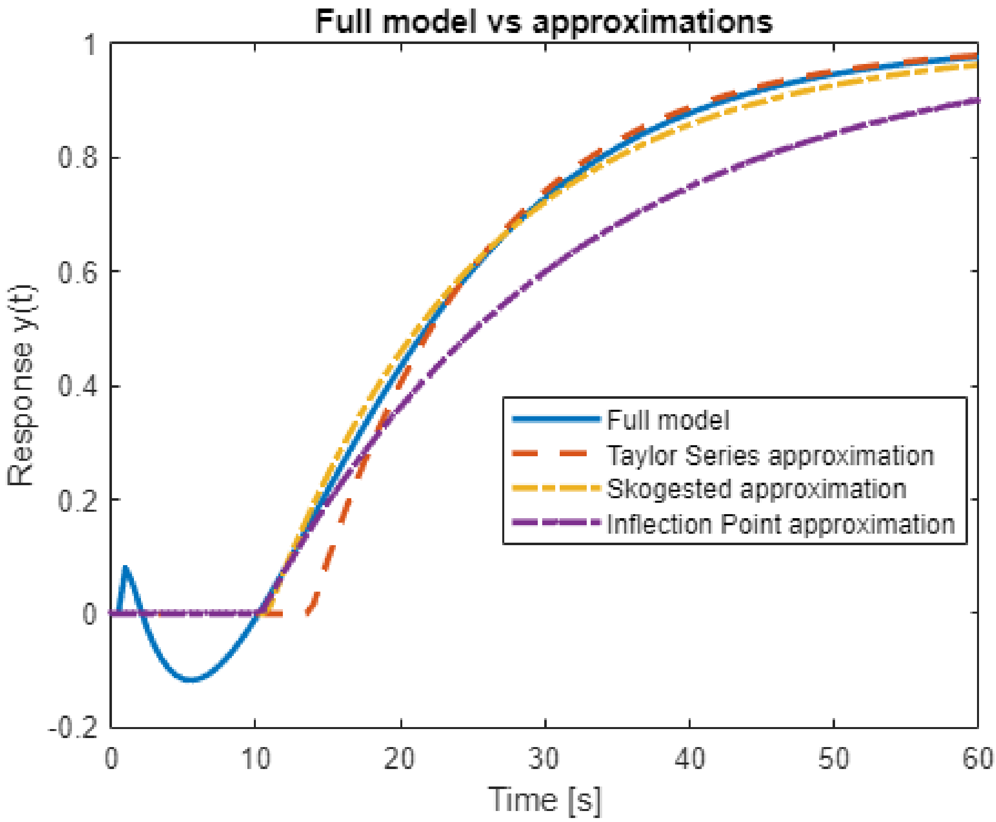
$$-\frac{\frac{3080\,e^{-\frac{9}{4}}}{513} - \frac{4165\,e^{-\frac{9}{8}}}{1053} - \frac{170660\,e^{-45}}{20007} + \frac{6143957457824883}{36028797018963968}}{\frac{595\,e^{-\frac{9}{8}}}{2106} - \frac{220\,e^{-\frac{9}{4}}}{513} + \frac{12190\,e^{-45}}{20007}}$$

```
tau_d=1/m;
G_d = exp(-theta*s)/(tau_d*s+1);
y_d=ilaplace(G_d*U);
y_d=subs(y_d);
plot(t,y,'linewidth',2)
```

```
hold on
plot(t,y_d,'linewidth',2)
hold off
ylabel("Response y(t)")
xlabel("Time [s]")
title("Full model and Inflection point at t=14s FOPTD comparison")
legend("Full model","Inflection Point approximation", Location="best")
```

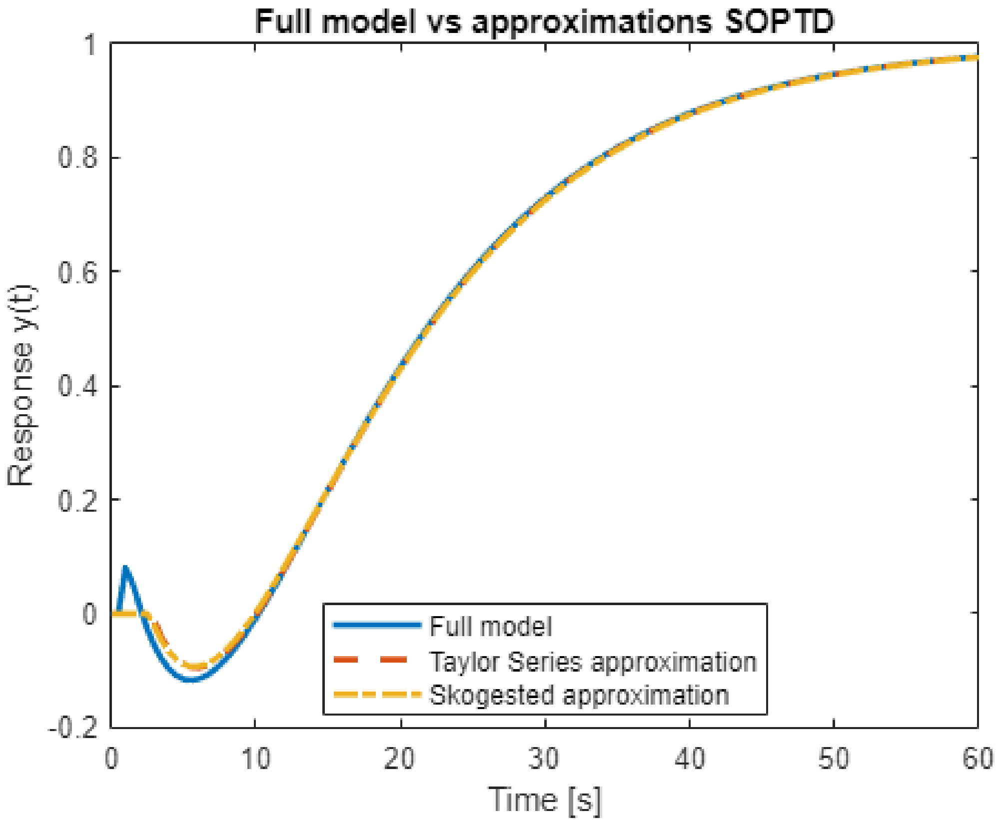



```
plot(t,y,'linewidth',2)
hold on
plot(t,y_b,'--','linewidth',2)
plot(t,y_c,'-.','linewidth',2)
plot(t,y_d,'-..','linewidth',2)
hold off
title("Full model vs approximations")
ylabel("Response y(t)")
xlabel("Time [s]")
legend("Full model","Taylor Series approximation","Skogested approximation","Inflection Point a
```

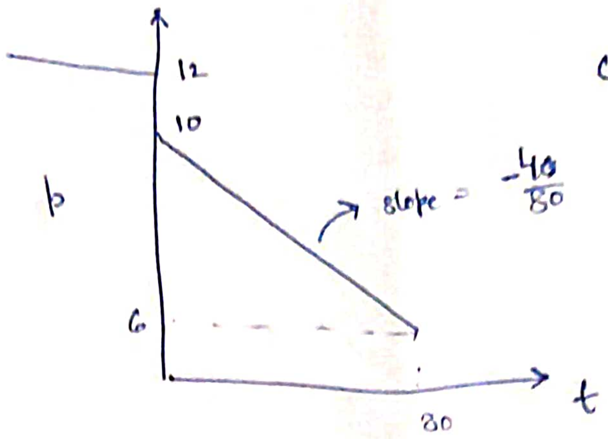



```
%e in excel

% f
G_2t = ((1-5*s)*exp(-2.8*s))/((12*s+1)*(6*s+1));
G_2s = ((1-5*s)*exp(-2.65*s))/((12*s+1)*(6.15*s+1));
y_2t=ilaplace(G_2t*U);
y_2t=subs(y_2t);
y_2s=ilaplace(G_2s*U);
y_2s=subs(y_2s);
y2s=double(y_2s);
plot(t,y,'linewidth',2)
hold on
plot(t,y_2t,'--','linewidth',2)
plot(t,y_2s,'-.','linewidth',2)
hold off
title("Full model vs approximations SOPTD")
ylabel("Response y(t)")
xlabel("Time [s]")
    legend("Full model","Taylor Series approximation","Skogested approximation", Location="best
```


Full model vs approximations SOPTD
Response y(t)
Time [s]
Full model
Taylor Series approximation
Skogested approximation
1
0.8
0.6
0.4
0.2
0
-0.2
0
10
20
30
40
50
60

801



Given step change = 3mA

$E(s) = 3/s$

for PI, $\frac{p'(s)}{E(s)} = K_c\left(1 + \frac{1}{\tau_I s}\right)$

$E(s) = M/s$

$p'(s) = \frac{K_c M}{s} + \frac{K_c M}{\tau_I s^2}$

$p'(t) = K_c M + \frac{K_c M}{\tau_I} t$

At $t=0$, $p'(t) = -2$

$M = 3$

$K_c = -2/3$

slope $= -1/20 = \frac{K_c M}{\tau_I}$

$\tau_I = 40$ min

Q # 8.2

(a)

$$\frac{P'(s)}{E(s)} = \frac{K_1}{\tau_I s + 1} + K_2$$

$$= \frac{(K_1 + K_2) + s K_2 \tau_I}{\tau_I s + 1}$$

$$(K_1 + K_2)\left[\frac{1 + \frac{K_2}{K_1 + K_2} \tau_I s}{\tau_I s + 1}\right]$$

b) for PD, $\frac{P'(s)}{E(s)} = K_c\left[\frac{1 + \tau_D s}{\alpha \tau_D s + 1}\right]$

by comparing

$K_c = K_1 + K_2$

$\cancel{\tau_D = \frac{K_2}{K_1 + K_2} \tau_1}$

$\cancel{\alpha = 1}$

$\alpha \tau_D = \tau_1$

$\tau_D = \frac{K_2 \tau_1}{K_1 + K_2}$

$\Rightarrow \alpha K_2 = K_1 + K_2$

$K_2 = \frac{K_c}{\alpha}$ & $K_1 = K_c\left[1 - \frac{1}{\alpha}\right]$

$\tau_D = \frac{\tau_1}{\alpha}$

Q 8.3

a) $\boxed{\text{Eq. 8.14}}$ $$\frac{P'(s)}{E(s)} = K_c^{+}\left[1 + \frac{1}{\tau_I^{+} s} + \tau_D^{+} s\right]$$ parallel form

$\boxed{\text{Eq. 8.15}}$

$$\frac{P'(s)}{E(s)} = K_c\left(\frac{\tau_I s + 1}{\tau_I s}\right)\left(\frac{\tau_D s + 1}{\alpha \tau_D s + 1}\right)$$

Expanding #8.15 $$K_c\left[1 + \frac{\tau_D}{\tau_I} + \frac{1}{\tau_I s} + \tau_D s\right]$$

Comparing s^0 coeff

$$K_c^{+} = K_c\left[1 + \frac{\tau_D}{\tau_I}\right]$$

Comparing s^1 coeff

$$\frac{K_c^{+}}{\tau_I^{+}} = \frac{K_c}{\tau_I}$$

$$\tau_I^{+} = \tau_I\left[1 + \frac{\tau_D}{\tau_I}\right]$$

$$K_c^{+}\tau_D^{+} = K_c\tau_D$$

$$\tau_D^{+} = \frac{\tau_D}{1 + \frac{\tau_D}{\tau_I}}$$

b) Since $\tau_D > 0$ & $\tau_I > 0$

$$\left(1 + \frac{\tau_D}{\tau_I}\right) \geq 1$$

$$K_c^{+} \geq K_c \quad , \quad \tau_I^{+} \geq \tau_I \quad , \quad \tau_D^{+} \leq \tau_D$$

c) $K_c = 4$ $\quad \tau_I = 10$ min $\quad \tau_D = 2$ min

$$K_c^{+}/K_c = 1 + \frac{2}{10} = 1.2$$

$$K_c^{+} = 4.8$$

$$\tau_I^{+} = 10 \times 1.2 = 12 \text{ min}$$

$$\tau_D^{+} = \frac{2}{1.2} = 1.67 \text{ min}$$

Q 8.7 ~~Outp~~ Output = 12 mA

$y_{sp} = T_0$

$\check{y}_m = T_0$

$K_c = 2$

$\tau_I = 1.5$ min

$\tau_D = 0.5$ min

$e(t) = 0.5t$

$E(t) = \frac{0.5}{s^2}$

PID controller, p(t) is given by

$$p(t) = \underbrace{\bar{p}(t)}_{12\,mA} + K_c\left[e(t) + \frac{1}{\tau_I}\int_0^t e(t^*)\,dt + \tau_D \frac{de(t)}{dt}\right]$$

$$p(t) = 12\,mA + K_c\left[0.5t + \frac{1}{1.5}\cdot\frac{0.5t^2}{2} + 0.5\times 0.5\right]$$

$$p(t) = 12\,mA + t + \frac{t^2}{2} + 0.5 \qquad \text{for } t > 0$$

$p(t) = 12$ mA for $t < 0$

(b) for PI, $\tau_D = 0$

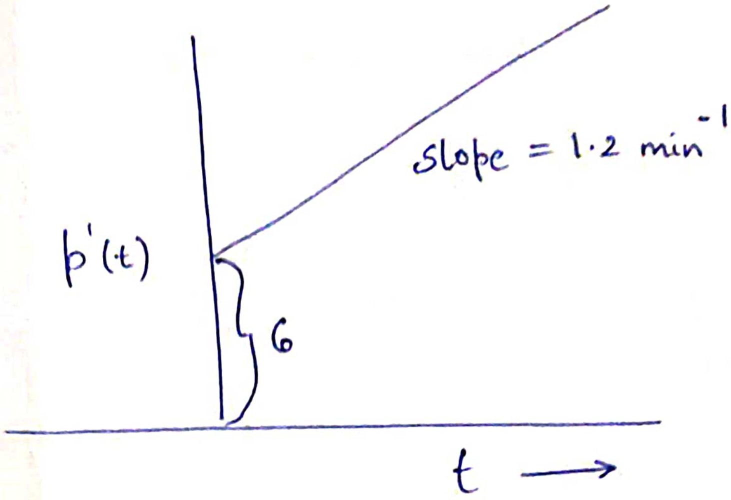
$\therefore\ p(t) = 12 + t + \frac{t^2}{3}$

(c) plot done separately.

Q1

Q2

Q 8.6 $Y_m(s) = \frac{2}{s}$



$y_{sp} = 0$

$y_m(t) = 2$

$e(t) = y_{sp} - y_m = -2$

$= \cancel{0}$

for PI controller,

$$p'(t) = K_c \left[e(t) + \frac{1}{\tau_I} \int_0^t e(t)\,dt \right]$$

$$= K_c \left[-2 + \frac{1}{\tau_I} \left[-2t \right]_0^t \right]$$

$$= -2K_c - \frac{2t\,K_c}{\tau_I}$$

at $t = 0$,

$p'(t) = 6 = -2K_c$

$K_c = -3$

$\text{slope} = 1.2 = -\frac{2K_c}{\tau_I}$

$\tau_I = \frac{-2 \times -3}{1.2} = 5 \text{ min}$