Manan Mohnot
19011 0043

Q1

Q a) $\tilde{G}(s) = \dfrac{k(\tau_3 s+1)e^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)}$

a) $\tilde{G}_c(s) = \tilde{G}^+(s) \cdot \tilde{G}^-(s)$

$G_c^*(s) = \underbrace{\dfrac{1}{\tau_c s+1}}_{F} \cdot \tilde{G}_c^-(s)$

$$G(s) = \frac{G_c^*(s)}{1 - G_c^*(s)\tilde{G}(s)} = \frac{(\tilde{G}_c^-(s))^{-1} F}{1-(\cancel{\tilde{G}_c^-(s)})^{-1} F \; \tilde{G}_c^+(s) \cdot \cancel{\tilde{G}_c^-(s)}}$$

$$= \frac{\tilde{G}_c \quad F}{(\tilde{G}_c^-)[1 - F\tilde{G}_c^+(s)]}$$

substituting.

$\cancel{G(s)} \neq \tilde{G}_c^+(s) - e^{-\theta s}$

$\tilde{G}_c^-(s) = \dfrac{k(\tau_3 s+1)}{(\tau_1 s+1)(\tau_2 s+1)}$

$$G(s) = \frac{1}{\left(\frac{(\tau_1 s+1)(\tau_2 s+1)}{k(\tau_3 s+1)}\right)^{-1}\left[1+\tau_c s - e^{-\theta s}\right]}$$

$e^{-\theta s} \simeq 1 - \theta s$

$$G(s) = \frac{\cancel{k(\tau_3 s+1)}}{\cancel{(\tau_1 s+1)(\tau_2 s+1)}(\tau_c + \theta) s} \quad \frac{k(\tau_1 s+1)(\tau_2 s+1)}{k(\tau_3 s+1)(\tau_c+\theta)s}$$

4

$$= \frac{1}{k(\tau_1+\theta)} \cdot (\tau_1+\tau_2-\tau_3)\left[\frac{(\tau_1 s+1)(\tau_2 s+1)}{(\tau_3 s+1)\, s \,(\tau_1+\tau_2-\tau_3)}\right]$$

Q1 b) $\dfrac{K(\tau_3 s+1)e^{-\theta s}}{\tau^2 s^2+2\xi\tau s+1}$

$$\tilde{G}^+(s) = e^{-\theta s}$$

$$\tilde{G}^-(s) = \frac{k(\tau_3 s+1)}{\tau^2 s^2+2\xi\tau s+1}$$

$$F = \frac{1}{\tau_c s+1}$$

$$G_c = \frac{G_c^*}{1-G_c^*\tilde{G}} \longrightarrow \frac{\tilde{G}_{c-}^{-1} F}{1-\tilde{G}_c^{-1} F \tilde{G}_+ \cancel{\tilde{G}_c}}$$

$$\frac{\tau^2 s^2+2\xi\tau s+1}{k(\tau_3 s+1)\left[\cancel{1+\tau-(1-\theta s)}\right]\left[1+\tau s-(1-\theta s)\right]}$$

$$\Rightarrow \frac{1}{k(\cancel{s}\tau+\theta)}\;\frac{\tau^2 s^2+2\xi\tau s+1}{s(\tau_3 s+1)}$$

Q2) $G(s) = \dfrac{1}{(s+1)(0.2s+1)(0.04s+1)(0.008s+1)}$

a) 1) SOPTD $\dfrac{ke^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)}$

largest neglected time constant in denominator $= 0.04$

$\therefore \theta = 1 + \frac{0.04}{2} + 0.008 = 0.028$

$\tau_1 = 1$ $\qquad K = 1$

$\tau_2 = 0.2 + \frac{0.04}{2} = 0.22$

$$\tilde{G}(s) = \frac{e^{-0.028s}}{(s+1)(0.22s+1)}$$

taking Condition I from table 12.1, with $\tau_3 = 0$

$$K_c = \frac{\tau_1 + \tau_2 - \tau_3}{k(\tau_c + \theta)} = \frac{1.22}{\tau_c + 0.028}$$

$$\tau_I = \tau_1 + \tau_2 - \tau_3 = 1.22$$

$$\tau_D = \frac{\tau_1\tau_2 - (\tau_1+\tau_2-\tau_3)\tau_3}{\tau_1+\tau_2-\tau_3} = 0.18$$

for $\tau_c = 0.148$ as $\theta/\tau_1 = 0.248 < 0.5$ hence $\tau_c =$ FOPTD θ

i.e default value, table 12.3 $\Rightarrow$ $\underline{\underline{K_c = 6.93}}$

Q2 a) ii) FOPDT $\dfrac{ke^{-\theta s}}{(\tau_1 s+1)}$

largest neglected time const - 0.2

$\theta = 1 + 0.008 + 0.04 + 0.2/2 = 0.148$

$\tau_1 = 1 + 0.2/2 = 1.1$

$k = 1$

$$\tilde{G}(s) = \frac{e^{-0.148 s}}{1.1 s + 1}$$

$$K_c = \frac{1}{k}\left(0.2 + 0.45\frac{\tau}{\theta}\right) = 3.54$$

$$\tau_I = \frac{0.4\theta + 0.8\tau}{\theta + 0.1\tau}\,\theta = 0.54$$

$$\tau_D = \frac{0.5\,\theta\tau}{0.3\theta + \tau} = 0.071$$

Q3

Q2) a) iii)

$\theta = 0.148$

$\tau = 1.1$

$k = 1$

$\theta/\tau = 0.1345$

$Y = A(\theta/\tau)^B$

for Disturbance.

for P, $Y = 9.0686$

$K_c = Y/k = 9.0686$

for I, $Y = 3.7$

~~$\tau/\tau_I = 3$~~ $\tau_I = \tau/Y = 0.2973$

for D, $Y = 0.0518$

$\tau_D = Y \cdot \tau = 0.0570$

for Set point,

for P, $Y = 5.3087$

$K_c = 5.31$

for I, $Y = 0.7763$

$\tau_I = 1.42$

for D, $Y = 0.0478$

$\tau_D = 0.053$

Q 2) a) iv)

from tweaking on Simulink,

$K_{cu} = 30.1$
$P_{cu} = 417$

$\therefore \; K_c = 0.6\,K_{cu} = 18.06$

$\tau_I = P_{cu}/2 = 0.28$

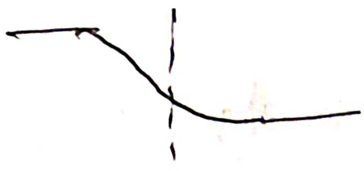
$\tau_D = P_{cu}/8 = 0.071$

b) Implemented on Simulink.

c) IMC settings provide the best result.
AMIGO most conservative
ITAE & ZN settings are the most aggressive
& produce damped oscillatory respons.
AMIGO also damped oscillatory response.

Q3

phase diagram indicates the presense of time delay.
(tail approaches infinity)



$AR_{OL}\big|_{w=0.01} = 3 \Rightarrow K = 3 \times 0.01 = 0.03$

2 → take this value

At $w = 0.1$, slope increases, ⇒ real zero $\left(\frac{s}{0.1}+1\right)$

A $w = 2$, slope decreases, ⇒ real pole ⇒ $\frac{s}{2}+1$

A $w = 20$, slope decreases ⇒ real pole ⇒ $\left(\frac{s}{20}+1\right)$

$$\frac{\cancel{\left(\frac{10s}{0.1}+1\right)}}{\cancel{(s+1)\left(\frac{s}{10}+1\right.}}$$

this implies, there are two real poles & 1 real zero
the phase starts from $-90°$ implies presence of s in denominator

$$\frac{K(\tau_a s+1)}{s(\tau_b s+1)(\tau_c s+1)}$$

$\therefore \tau_a = 1/0.1$

$\tau_b = 1/2$

$\tau_c = 1/20$

$$\therefore G(s) = \frac{0.03\,(10s+1)}{s(0.5s+1)(0.05s+1)}$$

$(\ldots s+1)\left(-\frac{s}{5}+1\right)$

b)

Q3

b) $\phi_g = PM - 180^\circ = 30 - 180^\circ = -150^\circ$

for $\phi_g = -150^\circ$, $\omega_g \approx 38.93$ rad/s

At $\omega = \omega_g$ $\frac{AR_{OL}}{K_c} = 0.007$

$AR_{OL}\big|_{\omega=\omega_g} = 1$

$\therefore K_c = 1/0.007 = 142.86$

ω_c is not defined for this

hence ~~Phase~~ Gain Margin is not applicable.

Q4)

a) $G_{OL} = \dfrac{1}{(4s+1)(2s+1)} \cdot k_c$

second order transfer function, the phase approaches $-\cancel{270}\ 180°$ but never attains it, hence the system is stable for all values of k_c i.e can't be unstable.

b) $G_{OL} = \dfrac{1}{(4s+1)(2s+1)} \cdot \dfrac{k_c(5s+1)}{5s}$

$+90-90-90-90$
$= -180°$

$\phi = -\tan^{-1}(4\omega) - \tan^{-1}(2\omega) + \tan^{-1}(5\omega) - \pi/2 \rightarrow$ min value $= -180°$

Never attains phase angle $-180°$, hence stable, can't be unstable

c) $G_{OL} = \dfrac{s+1}{(4s+1)(2s+1)} \dfrac{k_c(2s+1)}{s}$

$\phi = \tan^{-1}(\omega) + \tan^{-1}(2\omega) - \tan^{-1}(4\omega) - \tan^{-1}(2\omega) - \pi/2$

$\min(\phi) = +90+90-90-90-90 \quad = -90°$

Never attains $-180°$, hence system is stable, can't be unstable

Q4) d) $G_{OL} = \frac{1-s}{(4s+1)(2s+1)} \; k_c$

$\phi = \tan^{-1}(-\omega) \cancel{+} - \tan^{-1}(4\omega) - \tan^{-1}(2\omega)$

$\min(\phi) \simeq -270^\circ$

attains phase of -180°, hence the system can be unstable for large values of K_c

e) $G_{OL} = \frac{e^{-s}}{4s+1} \cdot k_c$

$\phi = -\tan^{-1}(4\omega) + \tan^{-1}\left(-\frac{\sin\omega}{\cos\omega}\right)$

$= -\tan^{-1}(\cancel{\infty}\, 4\omega) - \omega$

$\min(\phi) \rightarrow \infty$

$\therefore$ System can be unstable due to time delay at high frequencies. ~~(when it attains 180°~~

Q5 $G_c = k_c\left(\frac{2s+1}{0.1s+1}\right)$ $G_p = \frac{0.4}{s(5s+1)}$

$G_v = \frac{2}{0.5s+1}$ $G_d = \frac{3}{5s+1}$

$G_m = 1$

a) $G_{OL} = G_c G_v G_p G_m = k_c\left(\frac{2s+1}{0.1s+1}\right)\left(\frac{0.4}{s(5s+1)}\right)\left(\frac{2}{0.5s+1}\right)\cdot 1$

$AR_{OL}(\omega_c) = |G_{OL}(j\omega_c)|$

$\Rightarrow k_c \frac{\sqrt{4\omega^2+1}}{\sqrt{0.01\omega^2+1}} \quad \frac{0.4}{\omega\sqrt{25\omega^2+1}} \quad \frac{2}{\sqrt{0.25\omega^2+1}}$

$\phi_{OL}(\omega_c) = \angle G_{OL}(j\omega_c)$

$\phi = 0 + \tan^{-1}(2\omega) + 0 + 0 - \tan^{-1}(0.1\omega) - \pi/2 - \tan^{-1}(5\omega)$
$- \tan^{-1}(0.5\omega)$

$G_{OL} = \frac{k_c(1.6s + 0.8)}{\frac{s^4}{4} + \frac{61s^3}{20} + \frac{28s^2}{5} + s}$

b) $\phi_g = PM - 180 = 30^\circ - 180^\circ = -150^\circ$

for $\phi_g = -150^\circ$, $\omega_g = 1.72$ rad/min

1.71822 $\approx$ 1.72 rad/mins

$\therefore$ At $\omega = \omega_g$, $\frac{AR_{OL}}{k_c} = \cancel{\frac{-16.8645}{10}} = 0.144$ & $\because AR_{OL}|_{\omega=\omega_g} = 1$

$k_c = 1/0.144 = 6.94$

Q5 c) for $\phi = -180^\circ$, $w_c = 4.06$ rad/~~min~~s

$$\left.\frac{AR_{OL}}{k_c}\right|_{w=w_c} = 0.03245$$

$$A_c = AR_{OL}|_{w=w_c} = 0.325$$

$$GM = 1/A_c = 3.08$$

11) a) $$G_{OL} = \frac{k_c(1.6s+0.8)}{\frac{s^4}{4} + \frac{61s^3}{20} + \frac{28s^2}{5} + s} \cdot e^{-s}$$

$$e^{-s} \qquad e^{-jw} = \cos w\theta - j\sin w$$

AR_{OL} : same as before

$$\phi_{OL} = \tan^{-1}(2w) - \tan^{-1}(0.1w) - \pi/2 - \tan^{-1}(5w) - \tan^{-1}(0.5w) + \cancel{\tan^{-1}\left(-\frac{\sin w}{\cos w}\right)} \; -w$$

b) w_g for $\phi_g = -150^\circ = 0.38$ rad/s

At $w = w_g$, $\frac{AR_{OL}}{k_c} = 1.19$ $\quad \because AR_{OL}|_{w=w_g} = 1$

$k_c = 1/1.19 = 0.84$

c) for $\phi = -180^\circ$, $w_c = 0.8$ rad/s

$$\left.\frac{AR_{OL}}{k_c}\right|_{w=w_c} = 0.42$$

$$A_c = AR_{OL}|_{w=w_c} = 4.2 \qquad GM = 1/A_c = 0.24$$

Q6 $G_p(s) = \dfrac{e^{-s}}{s+1}$

$G_d(s) = \dfrac{2}{(s+1)(5s+1)}$

$G_v = G_m = G_t = 1$

a) $G_f = -\dfrac{G_d}{G_t G_v G_p}$

for steady state conditions,

$G_p = K_p = 1$

$G_d = K_L = 2$ $= -\dfrac{2}{1 \cdot 1 \cdot 1} = -2$

$G_v = G_m = G_t = 1$

$G_f = -2$

b) Dynamic Analysis

$G_f = -\dfrac{G_d}{G_v G_t G_p} \Rightarrow \dfrac{-\dfrac{2}{(s+1)(5s+1)}}{(1)(1)\dfrac{e^{-s}}{(s+1)}}$

$\rightarrow \dfrac{2}{(5s+1)(s+1)} e^{+s} \cdot (s+1)$ from Example 15.3

$\approx \dfrac{2((1+1)s+1)}{(5s+1)(s+1)} \rightarrow -\dfrac{2(2s+1)}{(5s+1)(s+1)}$

Q6

c) $\tilde{G} = G_v G_p G_m = \dfrac{e^{-s}}{s+1}$ $\theta = 1$ $\tau = 1$ $k = 1$ $\tau_c = 2$

from table 12.1, $\underline{G}$

taylor

$k_c = \dfrac{1}{\tau_c + \theta}$ $\tau_I = \tau$ $\tau_D = -$

$k_c = \frac{1}{3}$ $\tau_I = 1$

$$G_c = \frac{1}{3}\left[1 + \frac{1}{s}\right] \approx \frac{s+1}{3s}$$

d) $G_c = 0$, δ : $d(t) = u(t)$

$D(s) = 1/s$

Closed loop

$10s - 10s^2$
$+2 - 2s$
$+10s - 8s - 2$

$$Y = G_d D + [G_p G_v G_f G_t] D$$

$$= \frac{2}{(s+1)(5s+1)} \cdot \frac{1}{s} + \left[\frac{e^{-s}}{s+1} \cdot 1 \cdot (-2) \cdot (1)\right] \cdot \frac{1}{s}$$

$$=$$

$$Y(s) = \frac{2 + -2(5s+1)e^{-s}}{(s+1)(5s+1)e^{s}} = \frac{2 - (10s+2)e^{-s}}{(s+1)(5s+1)\cancel{e^{-s}}\,s}$$

$$e^{-s} \approx 1 - \theta\, 1s = \frac{2 - (10s+2)(1-s)}{(s+1)(5s+1)(s)} \Rightarrow \frac{10s^2 - 8s}{(s+1)(5s+1)(s)} \Rightarrow \frac{10s - 8}{(s+1)(5s+1)}$$

$$\approx$$

Q6

d) $Y(s) = \dfrac{10s-8}{(s+1)(5s+1)}$

$$y(t) = L^{-1}\left\{ \frac{9}{2(s+1)} - \frac{25}{2(5s+1)} \right\} = \frac{9}{2}e^{-t} - \frac{25}{2}e^{-\cancel{25t}\, t/5}$$

$$= 1/2\left[9e^{-t} - 25e^{-t/5}\right]$$

Now

for $G_f = \dfrac{-2(2s+1)}{(s+1)(5s+1)(s+1)}$

$$Y = \left(\frac{2}{(s+1)(5s+1)} + \frac{e^{-s}}{(s+1)} \cdot 1 \cdot \left(\frac{-2(2s+1)}{(5s+1)(s+1)} \right) \right) \frac{1}{s}$$

$$= \frac{2s+2 \mp 2e^{-s}(2s+1)}{(s+1)(5s+1)(s+1)\, s}$$

$2s+1-2s^2-s$

$-2+4s^2-2s$

taylor approximation

$e^{-s} \approx (1-s)$

$$Y(s) \quad \frac{2s+2 - 2(1-s)(2s+1)}{(s+1)^2(5s+1)\, s} = \frac{4s^2}{(s+1)^2(5s+1)\cancel{s}}$$

$$y(s) = L^{-1}\left\{ \frac{4s}{(s+1)^2(5s+1)} \right\} = L^{-1}\left\{ \frac{1}{4(s+1)} + \frac{1}{(s+1)^2} - \frac{5}{4(5s+1)} \right\}$$

$$= 1/4\, e^{-t} + e^{-t}t - 1/4\, e^{-t/5}$$

Q6) e) $\quad G_c \neq 0 \quad G_c = \frac{s+1}{3s}$

$$\frac{Y}{D} = \frac{G_d + G_t G_f G_v G_p}{1 + G_c G_p G_m G_v}$$

y for (a) (c) $\therefore G_f \, l = (-2)$

$$Y(s) = \frac{\frac{2}{(s+1)(5s+1)} + \frac{e^{-s}}{s+1}(-2)}{1 + \frac{s+1}{3s}\cdot 1 \cdot \frac{e^{-s}}{s+1}}$$

$5s+1-5s^2-s$

$5s^2-4s-1$

$10s^2-8s-\cancel{2}+\cancel{2}$

$e^{-s} \approx (1-s)$

$$= \frac{2 - 2(1-s)(5s+1)}{\frac{(s+1)(5s+1)}{3s}\left[3s\,\cancel{(s+1)} + \cancel{(s+1)}\,1-s\right]}$$

$$Y(s) \Rightarrow \frac{(10s^2-8s)3s}{(s+1)(5s+1)(2s+1)}\cdot\frac{1}{s}$$

$$\cancel{y(t) = L^{-1}(Y(s)) = L^{-1}\left\{3 - \frac{27}{2(s+1)} + \frac{13}{2s+1} - \frac{5}{2(5s+1)}\right\}}$$

$$\cancel{y(t) = 3\delta(t) - \frac{27}{2}e^{-t} + \frac{13}{2}e^{-t/2} - \frac{1}{2}e^{-t/5}}$$

$$y(t) = L^{-1}\left\{\frac{27}{2(s+1)} - \frac{26}{(2s+1)} + \frac{25}{2(5s+1)}\right\} = \frac{27}{2}e^{-t} - 13e^{-t/2} + 5/2\, e^{-t/5}$$

Q6) e) (b) & (c)

$$Y(s) = \frac{\frac{2}{(s+1)(5s+1)} + \frac{e^{-s}}{s+1} \cdot \frac{-2(2s+1)}{(5s+1)(s+1)}}{1 + \frac{s+1}{3s} \cdot 1 \cdot \frac{e^{-s}}{s+1}}$$

$$Y(s) = \frac{\frac{4s}{(s+1)^2(5s+1)} \cdot 3\cancel{s}}{3s + 1 - s} \cdot \frac{1}{\cancel{s}}$$

$$y(t) = L^{-1}\left\{ \frac{-27}{4} \cdot \frac{1}{s+1} - \frac{3}{(s+1)^2} - \frac{25}{4(5s+1)} + \frac{16}{2s+1} \right\}$$

$$= \frac{-27}{4} e^{-t} - 3e^{-t}t - \frac{25}{4} e^{-t/5} + 8e^{-t/2}$$