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HW5

CL 302

Q1

a) $\tilde{G}(s) = \dfrac{k(\tau_3 s+1)e^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)}$

$\tilde{G}(s) = \tilde{G}^+(s) \cdot \tilde{G}^-(s)$

$G_c^*(s) = \dfrac{1}{\tau_c s+1} \cdot \tilde{G}_c^-(s)$

$\dfrac{1}{\tau_c s+1} \to F$

$$G(s) = \frac{G_c^*(s)}{1 - G_c^*(s)\tilde{G}(s)} = \frac{(\tilde{G}_c^{-}(s))^{-1} F}{1-(\tilde{G}_c^-(s))^{-1} F \, \tilde{G}_c^+(s) \cdot \cancel{\tilde{G}_c^-(s)}}$$

$$= \frac{F}{(\tilde{G}_c^-)[1 - F\,\tilde{G}_c^+(s)]}$$

substituting.

$\cancel{G(s)}$ ≠ $\tilde{G}_c^+(s) = e^{-\theta s}$

$\tilde{G}_c^-(s) = \dfrac{k(\tau_3 s+1)}{(\tau_1 s+1)(\tau_2 s+1)}$

$$G(s) = \frac{1}{\left(\frac{(\tau_1 s+1)(\tau_2 s+1)}{k(\tau_3 s+1)}\right)^{-1}\left[1+\tau_c s - e^{-\theta s}\right]}$$

$e^{-\theta s} \simeq 1-\theta s$ taylor approximation

$$G(s) = \frac{k\cancel{(\tau_3 s+1)}}{\cancel{(\tau_1 s+1)(\tau_2 s+1)}\,\cancel{(\tau_c+\theta)}s} \qquad \frac{(\tau_1 s+1)(\tau_2 s+1)}{k(\tau_3 s+1)(\tau_c+\theta)s}$$

$$= \frac{\tau_1\tau_2 s^2 + (\tau_1+\tau_2)s + 1}{ks(\tau_3 s+1)(\tau_c+\theta)} \quad — (1)$$

τ_I

$$= \frac{1}{k(\tau_c+\theta)} \cdot (\tau_1+\tau_2-\tau_3)\left[\frac{(\tau_1 s+1)(\tau_2 s+1)}{(\tau_3 s+1)\, s \; (\tau_1+\tau_2-\tau_3)}\right]$$

For PID controller (parallel configuration)

$$G_c = k_c\left[1 + \frac{1}{\tau_I s} + \frac{\tau_D s}{\alpha\tau_D s+1}\right] \qquad \alpha\tau_D = \tau$$

$$= k_c\left[\frac{\tau_I \tau s^2 + \tau_I\tau_D s^2 + (\tau_I+\tau)s + 1}{\tau_I s(\alpha\tau s+1)}\right]$$

$$= k_c\left[\frac{(\tau_D+\tau)s^2 + (1+\tau/\tau_I)s + 1/\tau_I}{s(\tau s+1)}\right] \qquad \text{—— (ii)}$$

Comparing (i) & (ii) & $\tau = \tau_3$

$$k_c = \cancel{\tau_1\tau_2} \qquad k_c(\tau_D+\tau_3) = \frac{\tau_1\tau_2}{k(\tau_c+\theta)}$$

$$\frac{\tau_1+\tau_2}{k(\tau_c+\theta)} = k_c(1+\tau_3/\tau_I)$$

$$\frac{1}{k(\tau_c+\theta)} = k_c/\tau_I$$

Solving

$$\tau_I = \tau_1+\tau_2-\tau_3$$

$$kk_c = \frac{\tau_1+\tau_2-\tau_3}{\tau_c+\theta}$$

$$\tau_D = \frac{\tau_1\tau_2 - \tau_3(\tau_1+\tau_2-\tau_3)}{\tau_1+\tau_2-\tau_3}$$

$$kk_c\left(1+\frac{\tau_3}{\tau_I}\right) = \frac{\tau_1+\tau_2}{\tau_c+\theta}$$

$$\frac{kk_c}{\tau_I} = \frac{1}{\tau_c+\theta}$$

$$\frac{\tau_1+\tau_2}{\tau_c+\theta} = kk_c\left[1+\frac{\tau_3}{\tau_1+\tau_2-\tau_3}\right]$$

$$kk_c(\tau_D+\tau_3) = \frac{\tau_1\tau_2}{\tau_c+\theta}$$

$$(\tau_1+\tau_2-\tau_3)(\tau_D+\tau_3) = \tau_1\tau_2$$

b) $\dfrac{K(\tau_3 s+1)e^{-\theta s}}{\tau^2 s^2+2\xi\tau s+1}$

$\tilde{G}^+(s) = e^{-\theta s}$ $\quad F = \dfrac{1}{\tau_c s+1}$

$\tilde{G}^-(s) = \dfrac{k(\tau_3 s+1)}{\tau^2 s^2+2\xi\tau s+1}$

$G_c = \dfrac{G_c^*}{1-G_c^*\tilde{G}} \longrightarrow \dfrac{\tilde{G}_{c-}^{-1} F}{1-\cancel{\tilde{G}_c^{-1}} F \tilde{G}_+ \cancel{\tilde{G}_c}}$

$\dfrac{\tau^2 s^2+2\xi\tau s+1}{k(\tau_3 s+1)\left[\cancel{1+\tau - (1-\theta s)}\right]}\left[1+\tau_c s-(1-\theta s)\right]$

$\Rightarrow \dfrac{1}{k(\tau_c+\theta)} \dfrac{\tau^2 s^2+2\xi\tau s+1}{s(\tau_3 s+1)} \Longleftrightarrow$ & PID controller (parallel config) $G_c = K_c\left[\dfrac{(\tau_D+\tau')s^2+(1+\frac{\tau'}{\tau_I})s+1/\tau_I}{s(\tau_3 s+1)}\right]$

Comparing,

$\dfrac{\tau'^2}{k(\tau_c+\theta)} = K_c(\tau_D+\tau_3)$ $\quad \dfrac{1}{k(\tau_c+\theta)} = \dfrac{k_c}{\tau_I}$

$\dfrac{2\xi\tau}{k(\tau_c+\theta)} = k_c(1+\tau_3/\tau_I)$

gives

$Kk_c = \dfrac{2\xi\tau-\tau_3}{\tau_c+\theta}$ $\quad$ & $\tau_I = 2\xi\tau-\tau_3$

$\tau_D = \dfrac{\tau^2-(2\xi\tau-\tau_3)\tau_3}{2\xi\tau-\tau_3}$

$kk_c = \dfrac{\tau_I}{\tau_c+\theta} = \dfrac{2\xi\tau-\tau_I}{\tau_c+\theta}$

$\dfrac{\tau^2}{\tau_c+\theta} = kk_c(\tau_D+\tau_I)$

$= \dfrac{(2\xi\tau-\tau_3)(\tau_D+\tau_3)}{\tau_c+\theta}$

$\tau_D = \dfrac{\tau^2}{2\xi\tau-\tau_3} - \tau_3$

$\tau_D = \dfrac{\tau^2-(2\xi\tau-\tau_3)\tau_3}{2\xi\tau-\tau_3}$

τ_I