Homework #5

Instructions:

- Total of 6 questions:
- HW5 will count for 5% (5%) of the overall marks.
- Grading here will be out of 20+30+10+10+20+20 = 110 marks
- The HW should be submitted on Moodle.
- TA help session can be arranged based on your requests.

1. **IMC: (20=10+10)**

IMC-Based PID Controller Settings for Gc(s)

Take the two following second order plus numerator dynamics with time delay TF. Derive the PID controller tuning parameter as given below.

I	$\dfrac{K(\tau_3 s+1)e^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)}$	$\dfrac{\tau_1+\tau_2-\tau_3}{\tau_c+\theta}$	$\tau_1+\tau_2-\tau_3$	$\dfrac{\tau_1\tau_2-(\tau_1+\tau_2-\tau_3)\tau_3}{\tau_1+\tau_2-\tau_3}$
J	$\dfrac{K(\tau_3 s+1)e^{-\theta s}}{\tau^2 s^2+2\zeta\tau s+1}$	$\dfrac{2\zeta\tau-\tau_3}{\tau_c+\theta}$	$2\zeta\tau-\tau_3$	$\dfrac{\tau^2-(2\zeta\tau-\tau_3)\tau_3}{2\zeta\tau-\tau_3}$

Note that τ_3 is a positive numbers.

2. **IMC: (30=20 (5+5+5+5)+5+5)**

12.7 A process including sensor and control valve can be modeled by a fourth-order transfer function:

$$G(s)=\frac{1}{(s+1)(0.2s+1)(0.04s+1)(0.008s+1)}$$

(a) Design PID controllers using two design methods:

(i) A SOPTD model using the model reduction approach proposed by Skogestad (Section 6.3) and the IMC tuning relation in Table 12.1.

(ii) Repeat part (i) for the AMIGO method and an FOPTD model obtained by model reduction.

Use skogestad method to derive FOPTD model.

(iii) Derive the PID controller parameter using ITAE approach using both criteria (disturbance rejection and set point tracking) using FOPTD approximation derived in part ii.

(iv) Figure out the ultimate gain (K_{cu}) and period (P_u) by playing around with P-controller on the G(s) in simulink. (i.e. start with p controller and G(s), in a set point change mode, increase K till you hit sustained oscillations) Derive the PID controller tuning parameters using ZN settings. Submit the screenshot of the Simulink code to find ultimate gain and period.

(b) Evaluate the 4 controllers by simulating the Closed loop response of the G(s) to a unit change in set point (at t=1), and in disturbance (at t=10). Choose parameters needed for the controller setting appropriately with justification. Note that the process model remains the same (G(s)) for all four controller settings.

(c) comment on the response.

Hint: For SOPTD, use the results derived in problem 1 (IMC Table) by setting $\tau_3=0$. Use the following table for Amigo setting on a FOPTD model (it is not there in all the version of the seborg book). Use Skogestad for FOPTD approximatin.

Table 12.6 AMIGO Tuning Rules for PID Controllers (Åström and Hägglund, 2006)*

Model: $G(s) = \frac{Ke^{-\theta s}}{\tau s + 1}$	**Model:** $G(s) = \frac{Ke^{-\theta s}}{s}$
$K_c = \frac{1}{K}\left(0.2 + 0.45\frac{\tau}{\theta}\right)$	$K_c = \frac{0.45}{K}$
$\tau_I = \frac{0.4\theta + 0.8\tau}{\theta + 0.1\tau}\theta$	$\tau_I = 8\theta$
$\tau_D = \frac{0.5\theta\tau}{0.3\theta + \tau}$	$\tau_D = 0.5\theta$

*Also only valid for $\theta > 0$.

Q3: 10

14.21 A Bode diagram for a process, valve, and sensor is shown in Fig. E14.21.

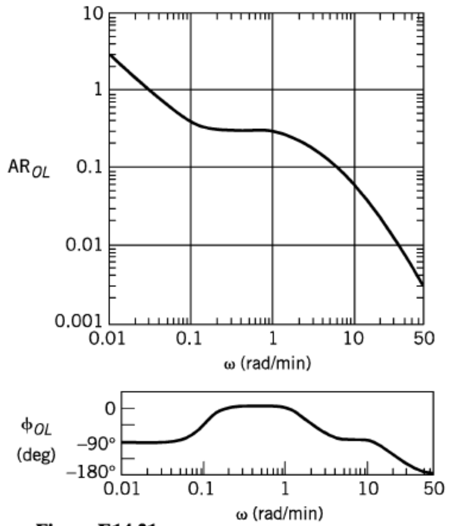

Figure E14.21

(a) Determine an approximate transfer function for this system.

(b) Suppose that a proportional controller is used and that a value of K_c is selected so as to provide a phase margin of 30°. What is the gain margin? What is the phase margin?

Note the transition frequencies (ω) which will be useful in getting the TF.

Q4: 10 (5×2)

14.15 Use arguments based on the phase angle in frequency response to determine if the following combinations of $G = G_v G_p G_m$ and G_c can become unstable for some value of K_c.

(a) $G = \dfrac{1}{(4s+1)(2s+1)}$ $\quad G_c = K_c$

(b) $G = \dfrac{1}{(4s+1)(2s+1)}$ $\quad G_c = K_c(1 + \dfrac{1}{5s})$

(c) $G = \dfrac{s+1}{(4s+1)(2s+1)}$ $\quad G_c = K_c \dfrac{(2s+1)}{s}$

(d) $G = \dfrac{1-s}{(4s+1)(2s+1)}$ $\quad G_c = K_c$

(e) $G = \dfrac{e^{-s}}{(4s+1)}$ $\quad G_c = K_c$

Q5: $20 = ((4 + 4 + 2) \times 2)$

14.24 Consider the feedback control system in Fig. 14.8, and the following transfer functions:

$$G_c = K_c\left(\frac{2s+1}{0.1s+1}\right) \qquad G_v = \frac{2}{0.5s+1}$$

$$G_p = \frac{0.4}{s(5s+1)} \qquad G_d = \frac{3}{5s+1}$$

$$G_m = 1$$

(a) Plot a Bode diagram for the open-loop transfer function.

(b) Calculate the value of K_c that provides a phase margin of 30°.

(c) What is the gain margin when $K_c = 10$?

Repeat part a,b and c for the case when $G_m = e^{-s}$

Q6: 20 (2+2+2+7+7)

Please note that the process TF has been changed compared to what appears in the book. Submit the Simulink screenshot and the comparison plot for par d and e. Discuss the responses.

15.5 The closed-loop system in Fig. 15.11 has the following transfer functions:

$$G_p(s) = \frac{e^{-s}}{s+1} \qquad G_d(s) = \frac{2}{(s+1)(5s+1)}$$

$$G_v = G_m = G_t = 1$$

(a) Design a feedforward controller based on a steady-state analysis.

(b) Design a feedforward controller based on a dynamic analysis.

(c) Design a feedback controller based on the IMC approach of Chapter 12 and $\tau_c = 2$.

(d) Simulate the closed-loop response to a unit step change in the disturbance variable using feedforward control only and the controllers of parts (a) and (b).

(e) Repeat part (d) for the feedforward-feedback control scheme of Fig. 15.11 and the controllers of parts (a) and (c) as well as (b) and (c).