Hindsight Expectation Maximization for Goal-conditioned Reinforcement Learning

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Abstract

We propose a graphical model framework for goal-conditioned reinforcement learning (RL), with an expectation maximization (EM) algorithm that operates on the lower bound of the RL objective. The E-step provides a natural interpretation of how 'learning in hindsight' techniques, such as hindsight experience replay (HER), to handle extremely sparse goal-conditioned rewards. The M-step reduces policy optimization to supervised learning updates, which greatly stabilizes end-to-end training on high-dimensional inputs such as images. We show that the combined algorithm, hindsight expectation maximization (hEM) significantly outperforms model-free baselines on a wide range of goal-conditioned benchmarks with sparse rewards.

1 Introduction

In goal-conditioned reinforcement learning (RL), an agent seeks to achieve a goal through interactions with the environment. At each step, the agent receives a reward, which ideally reflects how well it is achieving its goal. Traditional RL methods leverage these rewards to learn good policies. As such, the effectiveness of these methods rely on how informative the rewards are.

This sensitivity of traditional RL algorithms has led to a flurry of activity around reward shaping [1]. This limits the applicability of RL, as reward shaping is often specific to an environment and task — a practical obstacle to wider applicability. Binary rewards, however, are trivial to specify. The agent receives a strict indicator of success when it has achieved its goal; until then, it receives precisely zero reward. Such a sparsity of reward signals renders goal-conditioned RL extremely challenging for traditional methods [2].

How can we navigate such binary reward settings? Consider an agent that explores its environment but fails to achieve its goal. One idea is to treat, *in hindsight*, its exploration as having achieved some other goal. By relabeling a 'failure' relative to an original goal as a 'success' with respect to some other goal, we can imagine an agent succeeding frequently at many goals, in spite of failing at the original goals. This insight motivates hindsight experience replay (HER) [2], an intuitive strategy that enables off-policy RL algorithms, such as [3, 4], to function in sparse binary reward settings.

The statistical simulation of rare events occupies a similar setting. Consider estimating an expectation of low-probability events using Monte Carlo sampling. The variance of this estimator relative to its expectation is too high to be practical [5]. A powerful approach to reduce variance is importance sampling (IS) [6]. The idea is to adapt the sampling procedure such that these rare events occur frequently, and then to adjust the final computation. Could IS help in binary reward RL settings too?

Main idea. We propose a probabilistic framework for goal-conditioned RL that formalizes the intuition of hindsight using ideas from statistical simulation. We equate the traditional RL objective to maximizing the evidence of our probabilistic model. This leads to a new algorithm, hindsight

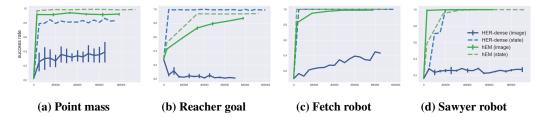


Figure 1: Training curves of hindsight expectation maximization (hEM) and HER on four goal-conditioned RL benchmark tasks. Inputs are either state-based (solid lines) or image-based (dashed lines). The y-axis shows the success rates and the x-axis shows the training time steps. All curves are calculated based on averages over 3-5 random seeds. hEM consistently outperforms HER across all baselines.

expectation maximization (hEM), which maximizes a tractable lower bound of the original objective [7]. A central insight is that the E-step naturally interprets hindsight replay as a special case of IS.

Figure 1 compares hem to Her [2] on four goal-conditioned RL tasks with low-dimensional state and high-dimensional images as inputs. While hem performs consistently well on both inputs, Her struggles with image-based inputs. This is due to how Her leverages hindsight replay within a temporal difference (TD)-learning procedure; performance degrades sharply with the dimensionality of the inputs (as observed previously in [8, 9]; also see Section 4). In contrast, hem leverages hindsight experiences through the lens of IS, thus enabling better performance in high dimensions.

The rest of this section presents a quick background on goal-conditioned RL and probabilistic inference. Expert readers may jump ahead to Section 2, which presents our graphical model.

Goal-conditioned RL background. A Markov decision process (MDP) can be simply extended to incorporate multiple goals. Consider an agent that interacts with an environment in episodes. At the beginning of each episode, a goal $g \in \mathcal{G}$ is fixed. At a discrete time $t \geq 0$, an agent in state $s_t \in \mathcal{S}$ takes action $a_t \in \mathcal{A}$ receives a reward $r(s_t, a_t, g) \in \mathbb{R}$ and transitions to its next state $s_{t+1} \sim p(\cdot \mid s_t, a_t) \in \mathcal{S}$. This process is independent of goals. A policy $\pi(a \mid s, g) : \mathcal{S} \times \mathcal{G} \mapsto \mathcal{P}(\mathcal{A})$ defines a map from state and goal to distributions over actions. Given a distribution over goals $g \sim p(\cdot)$, we consider the undiscounted episodic return $J(\pi) := \mathbb{E}_{g \sim p(\cdot)} \big[\mathbb{E}_{\pi} \big[\sum_{t=0}^{T-1} r(s_t, a_t, g) \big] \big]$. When rewards are independent from goals $r(s_t, a_t, g) \equiv r(s_t, a_t)$ and we recover classical RL [10].

Probabilistic inference background. Consider data as observed random variables $x \in \mathcal{X}$. Each measurement x is a discrete or continuous random variable. A likelihood $p_{\theta}(x \mid z)$ relates each measurement to latent variables $z \in \mathcal{Z}$ and unknown, but fixed, parameters θ . The full probabilistic generative model specifies a prior over the latent variable p(z). Bayesian inference requires computing the posterior $p(z \mid x)$ — an intractable task for all but a small class of simple models.

Variational inference approximates the posterior by matching a tractable density $q_{\phi}(z \mid x)$ to the posterior. The following calculation specifies this procedure:

$$\log p(x) = \log \mathbb{E}_{z \sim p(\cdot)} \left[p_{\theta}(x \mid z) \right]$$

$$= \log \mathbb{E}_{z \sim q_{\phi}(\cdot \mid x)} \left[p_{\theta}(x \mid z) \frac{p(z)}{q_{\phi}(z \mid x)} \right] \ge \mathbb{E}_{z \sim q_{\phi}(\cdot \mid x)} \left[\log p_{\theta}(x \mid z) \frac{p(z)}{q_{\phi}(z \mid x)} \right]$$

$$= \mathbb{E}_{z \sim q_{\phi}(\cdot \mid x)} \left[\log p_{\theta}(x \mid z) \right] - \mathbb{KL}[q_{\phi}(\cdot \mid z) \parallel p(z)] =: L(p_{\theta}, q).$$

$$(3)$$

(For a detailed derivation, please see [7].) Equation (3) defines the evidence lower bound (ELBO) $L(p_{\theta},q)$. Matching the tractable density $q_{\phi}(z\mid x)$ to the posterior thus turns into maximizing the ELBO via expectation maximization (EM) [11] or stochastic gradient ascent [12, 13]. Figure 2(a) presents a graphical model of the above. For a fixed set of θ parameters, the optimal variational distribution q is the true posterior $\arg\max_q L(p_{\theta},q) \equiv p(z\mid x) := p(z)p_{\theta}(x)/p(x)$. From an IS perspective, note how the variational distribution $q_{\phi}(z\mid x)$ serves as a proposal distribution in place of p(z).

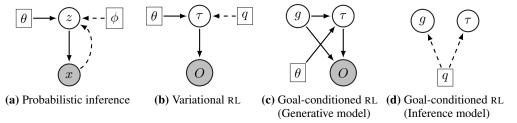


Figure 2: Graphical models. Solid lines represent generative models and dashed lines represent inference models. Circles represent random variables and squares represent parameters. Shading indicates that the random variable is observed.

2 A Probabilistic Model for Goal-conditioned Reinforcement Learning

Probabilistic modeling and control enjoy strong connections, especially in linear systems [14, 15]. Two recent frameworks connect probabilistic inference to general RL: Variational RL [16–19] and RL as inference [20–22]. We situate our probabilistic model by first presenting Variational RL below. Appendix A presents a detailed comparison to RL as inference.)

Variational RL. Begin by defining a trajectory random variable $\tau \equiv (s_t, a_t)_{t=1}^{T-1}$ to encapsulate a sequence of state and action pairs. The random variable is generated by a factorized distribution $a_t \sim \pi_\theta(\cdot \mid s_t), s_{t+1} \sim p(\cdot \mid s_t, a_t)$, which defines the joint distribution $p_\theta(\tau) \coloneqq \prod_{t=0}^{T-1} \pi_\theta(a_t \mid s_t) \ p(s_{t+1} \mid s_t, a_t)$. Conditional on τ , define the distribution of a binary optimality variable as $p(O=1 \mid \tau) \propto \exp(\sum_{t=0}^{T-1} r(s_t, a_t)/\alpha)$ for some $\alpha>0$, where we assume $r(s_t, a_t) \geq 0$ without loss of generality. Optimizing the standard RL objective corresponds to maximizing the evidence of $\log p(O=1)$, where all binary variables are treated as observed and equal to one. Positing a variational approximation to the posterior over trajectories gives the following lower bound,

$$\log p(O=1) \ge \mathbb{E}_{q(\tau)} \left[\log p(O=1 \mid \tau) \right] - \mathbb{KL} \left[q(\tau) \parallel p_{\theta}(\tau) \right] =: L(\pi_{\theta}, q). \tag{4}$$

Figure 2(b) shows a combined graphical model for both the generative and inference models of Variational RL. Equation (4) is typically maximized using EM (e.g., [23, 18, 19]), by alternating updates between θ and $q(\tau)$. Note that Variational RL does not model goals.

2.1 Probabilistic goal-conditioned reinforcement learning

To extend the Variational RL framework to incorporate goals, introduce a goal variable g and a prior distribution $g \sim p(\cdot)$. Conditional on a goal g, the trajectory variable $\tau \equiv (s_t, a_t)_{t=0}^{T-1} \sim p(\cdot \mid \theta, g)$ is sampled by executing the policy $\pi_{\theta}(a \mid s, g)$ in the MDP. Similar to Variational RL, the joint distribution factorizes as $p(\tau \mid \theta, g) \coloneqq \prod_{t=0}^{T-1} \pi_{\theta}(a_t \mid s_t, g) p(s_{t+1} \mid s_t, a_t)$. Now, define a goal-conditioned binary optimality variable G, such that f(G) = f(G) = f(G) = f(G). The following proposition shows the equivalence between inference in this model and traditional goal-conditioned RL.

Proposition 1. (Proof in Appendix B.) Maximizing the evidence of the probabilistic model is equivalent to maximizing returns in the goal-conditioned RL problem, i.e.,

$$\arg\max_{\theta} \log p(O=1) = \arg\max_{\theta} J(\pi_{\theta}). \tag{5}$$

2.2 Challenges with direct optimization

Equation (5) implies that algorithms to maximize the evidence of such probabilistic models could be readily applied to goal-conditioned RL. Unlike typical probabilistic inference settings, the evidence here could technically be directly optimized. Indeed, $p(O=1) \equiv J(\pi_{\theta})$ could be maximized via traditional RL approaches e.g., policy gradients [24]. In particular, the REINFORCE gradient estimator [25] of Equation (5) is given by as $\eta_{\theta} = \sum_{t \geq 0} \sum_{r' \geq t} r(s_{t'}, a_{t'}, g) \nabla_{\theta} \log \pi_{\theta}(a_t \mid s_t, g) \approx \nabla_{\theta} J(\pi_{\theta})$,

where $g \sim p(\cdot)$ and $(s_t, a_t)_{t=0}^{T-1}$ are sampled on-policy. The direct optimization of $\log p(O=1)$ consists in gradient ascents $\theta \leftarrow \theta + \eta_{\theta}$. This poses a practical challenge in goal-conditioned RL. To see why, consider the following example.

Illustrative example. Consider a one-step MDP with T=1 where $\mathcal{S}=\{s_0\}$, $\mathcal{A}=\mathcal{G}$, $r(s,a,g)=\mathbb{I}[a=g]$. Assume that there are a finite number of actions and goals $|\mathcal{A}|=|\mathcal{G}|=k$.

The following theorem shows the difficulty in building a practical estimator for $\nabla_{\theta} J(\pi_{\theta})$.

Theorem 1. (Proof in Appendix B.2.) Consider the example above. Let the policy $\pi_{\theta}(a \mid s, g) = softmax(L_{a,g})$ be parameterized by logits $L_{a,g}$ and let $\eta_{a,g}$ be the one-sample REINFORCE gradient estimator of $L_{a,g}$. Assume a uniform distribution over goals p(g) = 1/k for all $g \in \mathcal{G}$. Assume that the policy is randomly initialized (e.g. $L_{a,g} \equiv L, \forall a, g$ for some L). Let MSE[x] be the mean squared error $MSE[x] := \mathbb{E}[(x - \mathbb{E}[\eta_{a,g}])^2]$. It can be shown that the relative error of the estimate $\sqrt{MSE[\eta_{a,g}]}/\mathbb{E}[\eta_{a,g}] = k(1+o(1))$ grows approximately linearly with k for all $\forall a \in \mathcal{A}, g \in \mathcal{G}$.

The above theorem shows that in the simple setup above, the relative error of the REINFORCE gradient estimator grows linearly in k. This implies that to reduce the error with traditional Monte Carlo sampling would require $m \approx k^2$ samples, which quickly becomes intractable as k increases. Though variance reduction methods such as control variates [26] could be of help, it does not change the sup-linear growth rate of samples (see comments in Appendix B.2). The fundamental bottleneck is that dense gradients $r(s,a,g)\nabla_{\theta}\log\pi_{\theta}(a\mid s,g)\neq 0$ are rare event with a probability of 1/k, which makes it difficult to accurately estimate with on-policy measures [5]. This example hints at similar issues with more realistic cases and motivates an IS approach to address the problem.

2.3 Tractable lower bound

Consider a variational inequality similar to Equation (3) with a variational distribution $q(\tau, q)$

$$\log p(O=1) = \log \mathbb{E}_{q(\tau,g)} \left[p(O=1 \mid \tau, g) \frac{p(g)p(\tau \mid \theta, g)}{q(\tau, g)} \right]$$
(6)

$$\geq \mathbb{E}_{q(\tau,g)} \left[\log p(O=1 \mid \tau, g) \frac{p(g)p(\tau \mid \theta, g)}{q(\tau, g)} \right]$$
 (7)

$$= \mathbb{E}_{q(\tau,g)} \left[\log p(O=1 \mid \tau,g) \right] - \mathbb{KL} \left[q(\tau,g) \parallel p(g)p(\tau \mid \theta,g) \right] \eqqcolon L(\pi_{\theta},q). \tag{8}$$

This variational distribution corresponds to the inference model in Figure 2(d). As with typical graphical models, instead of maximizing $\log p(O=1)$, consider maximizing its ELBO $L(\pi_{\theta},q)$ with respect to both θ and variational distribution $q(\tau,g)$. Our key insight lies in the following observation: the bottleneck of the direct optimization of $\log p(O=1)$ lies in the sparsity of $p(O=1\mid \tau,g)=\sum_{t=0}^{T-1} r(s_t,a_t,g)$, where (τ,g) are sampled with the on-policy measure $g\sim p(\cdot), \tau\sim p(\cdot\mid \theta,g)$. The variational distribution $q(\tau,g)$ serves as a IS proposal in place of $p(\tau\mid \theta,g)p(g)$. If $q(\tau,g)$ puts more probability mass on (τ,g) pairs with high returns (high $p(O=1\mid \tau,g)$), the rewards become dense and learning becomes feasible. In the next section, we show how hindsight replay [2] provides an intuitive and effective way to select such a $q(\tau,g)$.

3 Hindsight Expectation Maximization

The EM-algorithm [11] for Equation (8) alternates between an E- and M-step: at iteration t, denote the policy parameter to be θ_t and the variational distribution to be q_t .

E-step:
$$q_{t+1} = \arg\max_{q} L(\pi_{\theta_t}, q)$$
, M-step: $\theta_{t+1} = \arg\max_{\theta} L(\pi_{\theta}, q_{t+1})$. (9)

This ensures a monotonic improvement in the ELBO $L(\pi_{\theta_{t+1}}, q_{t+1}) \ge L(\pi_{\theta_t}, q_t)$. We discuss these two alternating steps in details below, starting with the M-step.

M-step: Optimization for π_{θ} . Fixing the variational distribution $q(\tau, g)$, to optimize $L(\pi_{\theta}, q)$ with respect to θ is equivalent to

$$\max_{\theta} \mathbb{E}_{q(\tau,g)} \left[\log p(\tau \mid \theta, g) \right] \equiv \max_{\theta} \mathbb{E}_{q(\tau,g)} \left[\sum_{t=0}^{T-1} \log \pi_{\theta}(a_t \mid s_t, g) \right]. \tag{10}$$

The right hand side of Equation (10) corresponds to a supervised learning problem where learning samples come from $q(\tau, g)$. Prior studies have adopted this idea and developed policy optimization algorithms in this direction [18, 19, 27–29]. In practice, the M-step is carried out partially where θ is updated with gradient steps instead of optimizing Equation (10) fully.

E-step: Optimization for $q(\tau,g)$. The choice of $q(\tau,g)$ should satisfy two desirable properties: **(P.1)** it leads to monotonic improvements in $\log p(O=1) \equiv J(\pi_\theta)$ or a lower bound thereof; **(P.2)** it provide dense learning signals for the M-step. The posterior distribution $p(\tau,g\mid O=1)$ achieves **(P.1)** and **(P.2)** in a near-optimal way, in that it is the maximizer of the E-step in Equation (9), which monotonically improves the ELBO. The posterior also provides dense reward signals to the M-step because $p(\tau,g\mid O=1)\propto p(O=1\mid \tau,g)$. In practice, one chooses a variational distribution $q(\tau,g)$ as an alternative to the intractable posterior by maximizing Equation (8). Below, we show it is possible to achieve **(P.1)(P.2)** even though the E-step is not carried out fully. By plugging in $p(O=1\mid \tau,g)=\sum_{t=0}^{T-1}r(s_t,a_t,g)/\alpha$, we write the ELBO as

$$L(\pi_{\theta}, q) = \underbrace{\mathbb{E}_{q(\tau, g)} \left[\frac{\sum_{t=0}^{T-1} r(s_t, a_t, g)}{\alpha} \right]}_{\text{first term}} + \underbrace{-\mathbb{KL}[q(\tau, g) \parallel p(g)p(\tau \mid \theta, g)]}_{\text{second term}}.$$
 (11)

We now examine alternative ways to select the variational distribution $q(\tau, g)$.

Prior work. State-of-the-art model-free algorithms such as MPO [18, 19] applies a factorized variational distribution $q_{\rm ent}(\tau,g)=p(g)\Pi_{t=0}^{T-1}q_{\rm ent}(a_t\mid s_t,g)$. The variational distribution is defined by local distributions $q_{\rm ent}(a\mid s,g):=\pi_{\theta}(a\mid s,g)\exp(\hat{Q}^{\pi_{\theta}}(s,a,g)/\eta)$ for some temperature $\eta>0$ and estimates of Q-functions $\hat{Q}^{\pi_{\theta}}(s,a,g)$. The design of $q_{\rm ent}(\tau,g)$ could be interpreted as initializing $q_{\rm ent}(a\mid s,g)$ with $\pi_{\theta}(a\mid s,g)$ which effectively maximizes the *second term* in Equation (11), then taking one improvement step of the *first term* [18]. This distribution satisfies (**P.1**) because the combined EM-algorithm corresponds to entropy-regularized policy iteration, and retains monotonic improvements in $J(\pi_{\theta})$. However, it does not satisfy (**P.2**): when rewards are sparse $r(s,a,g)\approx 0$, estimates of Q-functions are sparse $Q^{\pi_{\theta}}(s,a,g)\approx 0$ and leads to uninformed variational distributions $q_{\rm ent}(a\mid s)\propto \pi_{\theta}(a\mid s)\exp(Q^{\pi_{\theta}}(s,a,g)/\eta)\approx \pi_{\theta}(a\mid s,g)$ for the M-step. In fact, when η is large and the update to $q(a\mid s)$ from $\pi_{\theta}(a\mid s,g)$ becomes infinitesimal, the E-step is equivalent to policy gradients [26, 24], which suffers from the sparsity of rewards as discussed in Section 2.

Hindsight variational distribution. Maximizing the *first term* of the ELBO is challenging when rewards are sparse. This motivates choosing a $q(\tau,g)$ which puts more weights on maximizing the *first term*. Now, we formally introduce hindsight variational distribution $q_h(\tau,g)$, the sampling distribution employed equivalently in HER [2]. Sampling from this distribution is implicitly defined by an algorithmic procedure:

- **Step 1.** Collect an on-policy trajectory or sample a trajectory from a replay buffer $\tau \sim \mathcal{D}$.
- **Step 2.** Find the g such that the trajectory is rewarding, in that $R(\tau,g)$ is high or the trial is successful. Return the pair (τ,g) .

Note that **Step 2** can be conveniently carried out with access to the reward function r(s,a,g) as in [2]. Contrary to $q_{\text{ent}}(\tau,g)$, this hindsight variational distribution maximizes the *first term* in Equation (11) by construction. This naturally satisfies (**P.2**) as $q_h(\tau,g)$ provides highly rewarding samples (τ,g) and hence dense signals to the M-step. The following theorem shows how $q_h(\tau,g)$ improves the sampling performance of our gradient estimates

Theorem 2. (Proof in Appendix B.3.) Consider the illustrative example in Theorem 1. Let $\eta^h(a,g) = r(s,b,g')\nabla_{L_{a,g}}\log\pi(b\mid s,g')/k$ be the normalized one-sample REINFORCE gradient estimator where (b,g') are sampled from the hindsight variational distribution with an on-policy buffer. Then the relative error $\sqrt{\text{MSE}[\eta^h_{a,g}]}/\mathbb{E}[\eta_{a,g}] = \sqrt{k}(1+o(1))$ grows sub-linearly for all $\forall a \in \mathcal{A}, g \in \mathcal{G}$.

Theorem 2 implies that to further reduce the relative error of the hindsight estimator $\eta^h(a,g)$ with traditional Monte Carlo sampling would require $m \approx (\sqrt{k})^2 = k$ samples, which scales linearly with the problem size k. This is a sharp contrast to $m \approx k^2$ from using the on-policy REINFORCE gradient

estimator. The above result shows the benefits of IS, where under $q_h(\tau, g)$ rewarding trajectory-goal pairs are given high probabilities and this naturally alleviates the issue with sparse rewards. The following result shows that $q_h(\tau, g)$ also satisfies (**P.1**) under mild conditions.

Theorem 3. (Proof in Appendix B.4.) Assume p(g) to be uniform without loss of generality and a tabular representation of policy π_{θ} . At iteration t, assume that the partial E-step returns $q_t(\tau,g)$ and the M-step objective in Equation (10) is optimized fully. Also assume the variational distribution to be the hindsight variational distribution $q_t(\tau,g) \coloneqq q_h(\tau,g)$. Let $\tilde{p}_t(g) \coloneqq \int_{\tau} q_t(\tau,g) d\tau$ be the marginal distribution of goals. The performance is lower bounded as $J(\pi_{\theta_{t+1}}) \ge |\sup(\tilde{p}_t(g))|/|\mathcal{G}| =: \tilde{L}_t$. When the replay buffer size \mathcal{D} increases over iterations, the lower bound improves $\tilde{L}_{t+1} \ge \tilde{L}_t$.

3.1 Algorithm

We now present hEM, a combination of the above E- and M-steps. The algorithm maintains a policy $\pi_{\theta}(a \mid s,g)$. At each iteration, hEM collects N trajectory-goal pairs by first sampling a goal $g \sim p(\cdot)$ and then rolling out a trajectory τ . All trajectories are stored into a replay buffer \mathcal{D} [3]. At training time, hEM carries out a partial E-step by sampling (τ,g) pairs from $q_h(\tau,g)$. For the partial M-step, the policy is updated through several gradient ascents on Equation (10) with the Adam optimizer [30]. Importantly, hEM is an off-policy RL algorithm *without* value functions, which also makes it agnostic to reward functions. The pseudocode is summarized in Algorithm 1. Please refer to Appendix C for full descriptions of the algorithm.

Algorithm 1 Hindsight Expectation Maximization (hEM)

- 1: **INPUT** policy $\pi_{\theta}(a \mid s, g)$.
- 2: **while** t = 0, 1, 2... **do**
- 3: Sample goal $g \sim p(\cdot)$ and trajectory $\tau \sim p(\cdot \mid \theta, g)$ by executing π_{θ} in the MDP. Save data (τ, g) to a replay buffer \mathcal{D} .
- 4: **E-step.** Sample from $q_h(\tau, g)$: sample $\tau \equiv (s_t, a_t)_{t=0}^{T-1} \sim \mathcal{D}$ and find rewarding goals g.
- 5: **M-step.** Update the policy by a few gradient ascents $\theta \leftarrow \theta + \nabla_{\theta} \log \sum_{t=0}^{T-1} \log \pi_{\theta}(a_t \mid s_t, g)$.
- 6: end while

3.2 Connections to prior work

Hindsight experience replay. The core of HER lies in the hindsight goal replay [2]. Similar to hEM, HER samples trajectory-goal pairs from the hindsight variational distribution $q_h(\tau,g)$ and minimize the Q-learning loss $\mathbb{E}_{(\tau,g)\sim q_h(\cdot)}[\sum_{t=0}^{T-1}(Q_{\theta}(s_t,a_t,g)-r(s_t,a_t,g)-\gamma\max_{a'}Q_{\theta}(s_i,a',g))^2]$. The development of hEM in Section 3 formalizes this choice of the sampling distribution $q(\tau,g)\coloneqq q_h(\tau,g)$ as partially maximizing the ELBO during an E-step. Compared to hEM, HER learns a critic $Q_{\theta}(s,a,g)$. We will see in the experiments that such critic learning tends to be much more unstable when rewards are sparse and inputs are high-dimensional, as was also observed in [8, 9].

Hindsight policy gradient. In its vanilla form, the hindsight policy gradient (HPG) considers on-policy stochastic gradient estimators of the RL objective [24] as $\mathbb{E}_{p(g)p(\tau|\theta,g)}[R(\tau,g)\nabla_{\theta}\log p(\tau\mid\theta,g)]$. Despite variance reduction methods such as control variates [26, 24], the unbiased estimators of HPG do not address the rare event issue central to sparse rewards MDP, where $R(\tau,g)\nabla_{\theta}\log p(\tau\mid\theta,g)$ taking non-zero values is a rare event under the on-policy measure $p(g)p(\tau\mid\theta,g)$. Contrast HPG to the unbiased IS objective in Equation (8): $\mathbb{E}_{q(\tau,g)}[R(\tau,g)\nabla_{\theta}\log p(\tau\mid\theta,g)\cdot\frac{p(g)p(\tau\mid\theta,g)}{q(\tau,g)}]$, where the proposal $q(\tau,g)$ ideally prioritizes the rare events [5] to generate rich learning signals. hEM further avoids the explicit IS ratios with the variational approach that leads to an ELBO [7].

4 Experiments

We evaluate the empirical performance of hEM on a wide range of goal-conditioned RL benchmark tasks. These tasks all have extremely sparse binary rewards which indicate success of the trial. The evaluation criterion is the success rate at test time. Since hEM builds on pure model-free concepts, we focus on the model-free state-of-the-art algorithm HER [2] as a comparison. In some cases we

also compare with closely related HPG [24]: however, we find that even as HPG adopts more dense rewards, its performance evaluated as the success rate is much more inferior than HER and hEM. For hyper-parameter details and additional results, please see Appendix C.

4.1 Simple examples

Flip bit. Taken from [2], the MDP is parameterized by the number of bits K. The state space and goal space $\mathcal{S} = \mathcal{G} = \{0,1\}^K$ and the action space $\mathcal{A} = \{1,2,\ldots,K\}$. Given s_t , the action flips the bit at location a_t . The reward function is $r(s_t,a_t)=\mathbb{I}[s_{t+1}=g]$, the state is flipped to match the target bit string. The environment is difficult for traditional RL methods as the search space is of exponential size $|\mathcal{S}| = 2^K$. In Figure 3(a), we present results for HER (taken from Figure 1 of [2]), hem and HPG. Observe that hem and HER consistently perform well even when K=50 while the performance of HPG drops drastically as the underlying spaces become enormous. See Figure 3(b) for the training curves of hem and HPG for K=50; note that HPG does not make any progress.

Continuous navigation. As a continuous analogue of the Flip bit MDP, consider a K-dimensional navigation task with a point mass. The state space and goal space coincide with $\mathcal{X} = \mathcal{G} = [-1,1]^K$ while the actions $\mathcal{A} = [-0.2, 0.2]^K$ specify changes in states. The reward function is $r(s_t, a_t) = \mathbb{I}[\parallel s_{t+1} - g \parallel < 0.1]$ which indicates success when reaching the goal location. Results are shown in Figure 3(c) where we see that as K increases, the search space quickly explodes and the performance of HER degrades drastically. The performance of hEM is not greatly influenced by increases in K. See Figure 3(d) for the comparison of training curves between hEM and HER for K = 40. HER already learns much more slowly compared to hEM and degrades further when K = 80.

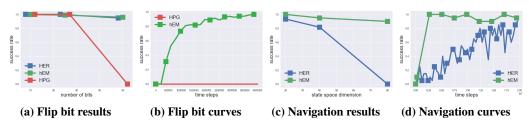


Figure 3: Summary of results for Flip bit and continuous navigation MDP. Plots (a) and (c) show the final performance after training is completed. Plots (b) and (d) show the training curves for Flip bit K=50 and navigation K=40 respectively. hEM consistently outperforms HER and HPG across these tasks.

4.2 Goal-contitioned reaching tasks

To assess the performance of hEM in contexts with richer system dynamics, we consider a wide range of goal-conditioned reaching tasks. We present details of their state space \mathcal{X} , goal space \mathcal{G} and action space \mathcal{A} in Appendix C. These include **Point mass**, **Reacher goal**, **Fetch robot** and **Sawyer robot**, as illustrated in Figure 7 in Appendix C.

Across all tasks, the reward takes the sparse form $r(s,a,g)=\mathbb{I}[\text{success}]$. As a comparison, we also include a HER baseline where the rewards take the form $\tilde{r}(s,a,g)=-\mathbb{I}[\text{failure}]$. Such reward shaping does not change the optimality of policies as $\tilde{r}=r-1$ but makes the reward more 'dense' and is more suitable for learning by neural network based Q-functions. We observe that this makes a significant difference in the performance of HER. We dnote this HER baseline as 'HER-dense'.

From the result in Figure 4, we see that hEM performs significantly better than HER with binary rewards (HER-sparse). The performance of hEM quickly converges to optimality while HER struggles at learning good Q-functions. However, when compared with HER-dense, hEM does not achieve noticeable gains. Such an observation confirms the importance of reward shaping to HER.

4.3 Image-based tasks

We further assess the performance of hEM when policy inputs are high-dimensional images (see Figure 8 for illustrations). Across all tasks, the state inputs are by default images $s \in \mathbb{R}^{w \times w \times 3}$ where $w \in \{48, 84\}$ while the goal is still low-dimensional. See Appendix C for the network architectures.

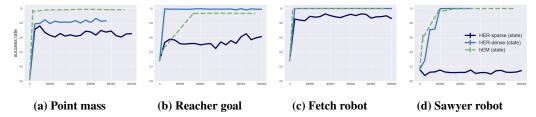


Figure 4: Training curves of hEM and HER on four goal-conditioned RL benchmark tasks with state-based inputs and sparse binary rewards. The y-axis shows the success rates and the x-axis shows the training time steps. All curves are calculated based on averages over 3-5 random seeds. Standard deviations are small across seeds.

We focus on the comparison between hEM and HER-dense in Figure 5, as the performance of HER with binary rewards is inferior as seen Section 4.2. We see that for image-based tasks, HER-dense significantly underperforms hEM. While HER-dense makes slow progress for most cases, hEM achieves stable learning across all tasks. We speculate this is partly due to the common observations [8, 9] that TD-learning directly from high-dimensional image inputs is challenging. For example, prior work [31] has applied a variational autoencoder approach [12] to reduce the dimension of the image inputs for downstream TD-learning. On the contrary, hEM only requires optimization in a supervised learning style, which is much more stable with end-to-end training on image inputs.

Further, image-based goals are much easier to specify in certain contexts [31]. We evaluate hEM on image-based goals for the Sawyer robot and achieve similar performance as the state-based goals. See results in Figure 9 in Appendix C.

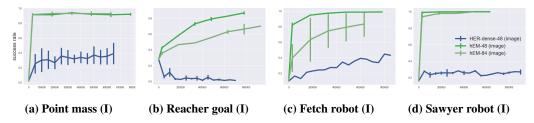


Figure 5: Training curves of hEM and HER on four goal-conditioned RL tasks with image-based inputs. Standard deviations are small across seeds. All curves are calculated based on averages over 3-5 random seeds. 'hEM-48' refers to image inputs with w=48. hEM achieves stable learning regardless of the input sizes, though larger sizes do slow the learning rate.

4.4 Ablation study

hem collects N trajectories at each iteration, which is set to N=20 in previous experiments (except for Fetch robot (I) and Reacher (I) where N=80). In certain cases, we find that the performance of hem critically depends on the size of N. In Figure 9 in Appendix C we provide ablation results on the Flip bit MDP with K=50 and image-based Fetch robot task. Both tasks are challenging, both due to an enormous state space or the high dimensionality of the images. In general, we find that larger N leads to better performance. Similar observations have been made for HER [2], where increasing the number of parallel workers generally improves training performance.

5 Conclusion

We present a probabilistic framework for goal-conditioned RL. This framework motivates the development of hEM, a simple and effective off-policy RL algorithm. Our formulation draws formal connections between hindsight goal replay [2] and IS for rare event simulation. hEM combines the stability of supervised learning updates via the M-step and the hindsight replay technique via the E-step. We show improved performance over a variety of benchmark RL tasks, especially in high-dimensional input settings with sparse binary rewards.

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A Details on Graphical Models for Reinforcement Learning

In this section, we review details of the RL as inference framework [32, 22] and highlight its critical differences from *Variational RL*.

The graphical model for RL as inference is shown in Figure 6(c). The framework also assumes a trajectory variable $\tau \equiv (s_t, a_t)_{t=0}^{T-1}$ which encompasses the state and action sequences. Conditional on the trajectory variable τ , the optimality variable is defined as $p(O=1\mid\tau)\propto \exp(\sum_{t=0}^{T-1}r(s_t,a_t)/\alpha)$ for $\alpha>0$. Under this framework, the trajectory variable has a prior $a_t\sim p(\cdot)$ where $p(\cdot)$ is usually set to be a uniform distribution over the action space \mathcal{A} .

The policy parameter θ comes into play with the inference model. The framework asks the question: what is the posterior distribution $p(\tau \mid O=1)$. To approximate this intractable posterior distribution, consider the variational distribution $q(\tau) \coloneqq \prod_{t=0}^{T-1} \pi_{\theta}(a_t \mid s_t) p(s_{t+1} \mid s_t, a_t)$. Searching for the best approximate posterior by minimizing the KL-divergence $\mathbb{KL}[q(\tau) \parallel p(\tau \mid O=1)]$, it can be shown that this is equivalent to maximum-entropy RL [33–35]. It is important to note that RL as inference does not contain trainable parameters for the generative model.

Contrasting this to *Variational* RL and the graphical model for goal-conditioned RL in Figure 2: the policy dependent parameter θ is part of a generative model. The variational distribution $q(\tau,g)$, defined separately from θ , is the inference model. In such cases, the variational distribution $q(\tau,g)$ is an auxiliary distribution which aids in the optimization of θ by performing partial E-steps.

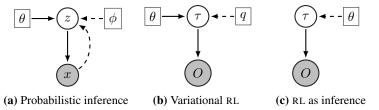


Figure 6: Plot (c) shows the graphical model for RL *as inference* [32, 22]. Solid lines represent generative models and dashed lines represent inference models. Circles represent random variables and squares represent parameters. Filled circles represent observed random variables. This graphical model does not have trainable parameters for the generative model. The policy dependent parameter θ is in the inference model.

B Details on proof

B.1 Proof of Proposition Proposition 1

The proof follows from the observation that $p(O=1) = \mathbb{E}_{g \sim p(\cdot), \pi}[p(O=1 \mid \tau, g)] = J(\pi_{\theta})$, and taking the log does not change the optimal solution.

B.2 Proof of Theorem 1

Recall that we have a one-step MDP setup where $\mathcal{A}=\mathcal{G}$ and $|\mathcal{A}|=k$. The policy $\pi(a\mid s,g)=\operatorname{softmax}(L_{a,g})$ is parameterized by logits $L_{a,g}$. When the policy is initialized randomly, we have $L_{a,g}\equiv L$ for some L and $\pi(a\mid s,g)=1/k$ for all a,g. Assume also $p(g)=1/k, \forall g$.

The one-sample REINFORCE gradient estimator for the component $L_{a,g}$ is $\eta_{a,g} = r(s,b,g')\log_{L_{a,g}}\pi(b\mid s,g')$ with $g'\sim p(\cdot)$ and $b\sim \pi(\cdot\mid s,g')$. Further, we can show

$$\mathbb{E}[\eta_{a,g}] = \frac{1}{k^2} \delta_{a,g} - \frac{1}{k^3}, \ \mathbb{V}[\eta_{a,g}] = (\frac{1}{k^2} + \frac{2}{k^5} - \frac{2}{k^3} - \frac{1}{k^4}) \delta_{a,g} + \frac{1}{k^4} - \frac{1}{k^6},$$

where $\delta_{a,b}$ are dirac-delta functions, which mean $\delta_{a,b} = 1$ if a = b and $\delta_{a,b} = 0$ otherwise. Taking the ratio, we have the squared relative error (note that the estimator is unbiased and MSE consists purely of the variance)

$$\frac{\text{MSE}[\eta_{a,g}]}{\mathbb{E}[\eta_{a,g}]^2} = \frac{(k^4 + o(k^4))\delta_{a,g} + (k^2 + o(k^2))}{(k^2 + o(k^2))\delta_{a,g} + 1)}$$

The expression takes different forms based on the delta-function $\delta_{a,g}$. However, in either case (either $\delta_{a,g}=1$ or $\delta_{a,g}=0$), it is clear that $\frac{\mathrm{MSE}[\eta_{a,g}]}{\mathbb{E}[\eta_{a,g}]^2}=k^2(1+o(1))$, which directly reduces to the result of the theorem.

Comment on the control variates. We also briefly study the effect of control variates. Let X,Y be two random variables and assume $\mathbb{E}[Y]=0$. Then compare the variance of $\mathbb{V}[X]$ and $\mathbb{V}[X+\alpha Y]$ where α is chosen optimally to minimize the variance of the second estimator. It can be shown that with the best α^* , the ratio of variance reduction is $(\mathbb{V}[X]-\mathbb{V}[X+\alpha^*Y])/\mathbb{V}[X]=\rho^2:=\operatorname{Cov}^2[X,Y]/\mathbb{V}[X]\mathbb{V}[Y]$. Consider the state-based control variate for the REINFORCE gradient estimator, in this case $-\alpha \cdot \nabla_{L_{a,g}} \pi(b \mid s,g')$ where α is chosen to minimize the variance of the following aggregate estimator

$$\eta_{a,g}(\alpha) = r(s,b,g') \log_{L_{a,g}} \pi(b \mid s,g') - \alpha \log_{L_{a,g}} \pi(b \mid s,g').$$

Note that in practice, α is chosen to be state-dependent for REINFORCE gradient estimator of general MDPs and is set to be the value function $\alpha \coloneqq V^\pi(s)$. Such a choice is not optimal [36] but is conveniently adopted in practice. Here, we consider an optimal α^* for the one-step MDP. The central quantity is the squared correlation ρ^2 between $r(s,b,g')\log_{L_{a,g}}\pi(b\mid s,g')$ and $\log_{L_{a,g}}\pi(b\mid s,g')$. With similar computations as above, it can be shown that $\rho^2\approx 1$ for $b\neq g'$ and $\rho^2\approx \frac{1}{k^2}$ otherwise. This implies that for k out of k^2 logits parameters, the variance reduction is significant; yet for the rest of the k^2-k parameters, the variance reduction is negligible. Overall, the analysis reflects that conventional control variantes do not address the issue of sup-linear growth of relative errors as a result of sparse gradients.

B.3 Proof of Theorem 2

Similar to the proof of Theorem 1, we can show that the normalized one-step REINFORCE gradient estimator $\eta_{a,g}^h = r(s,b,g')\nabla_{L_{a,g}}\log\pi(b\mid s,g')/k$ with $(b,g')\sim q_h(\tau,g)$ has the following property

$$\frac{\text{MSE}[\eta_{a,g}]}{\mathbb{E}[\eta_{a,g}]^2} = \frac{(k^3 + o(k^3))\delta_{a,g} + (k + o(k^2))}{(k^2 + o(k))\delta_{a,g} + 1)}.$$

This implies the result of the theorem.

B.4 Proof of Theorem 3

Without loss of generality we assume p(g) is a uniform measure, i.e. $p(g) = 1/|\mathcal{G}|$. If not, we could always find a transformation g = f(g') such that g' takes a uniform measure [6] and treat g' as the goal to condition on.

Let $|\mathcal{G}|<\infty$ and recall $\operatorname{supp}(\tilde{p}(g))$ to be the support of $\tilde{p}(g)$. The uniform distribution assumption deduces that $|\operatorname{supp}(\tilde{p}(g))|=\int_{g\in\operatorname{supp}(\tilde{p}(g))}dg$. At iteration t, under tabular representation, the M-step update implies that π_{θ} learns the optimal policy for all g that could be sampled from $g(\tau,g)$, which effectively corresponds to the support of $\tilde{p}(g)$. Formally, this implies $\mathbb{E}_{p(\tau|\theta_{t+1},g)}[R(\tau,g)]=1$ for $\forall g\in\operatorname{supp}(\tilde{p}_t(g))$. This further implies

$$J(\pi_{\theta_{t+1}}) \coloneqq \int \mathbb{E}_{p(\tau|\theta_{t+1},g)}[R(\tau,g)]p(g)dg \geq \int_{g \in \operatorname{supp}(\tilde{p}_t(g))} 1 \cdot p(g)dg = |\operatorname{supp}(\tilde{p}_t(g))|/|\mathcal{G}|.$$

C Additional Experiment Results

C.1 Details on Benchmark tasks

All reaching tasks are built with physics simulation engine MuJoCo [37]. We build customized point mass environment; the Reacher and Fetch environment is partly based on OpenAI gym environment [38]; the Sawyer environment is based on the multiworld open source code https://github.com/vitchyr/multiworld.

All simulation tasks below have a maximum episode length of T=50. The episode terminates early if the goal is achieved at a certain step. The sparse binary reward function is $r(s, a, g) = \mathbb{I}[\text{success}]$,

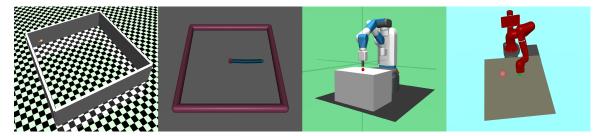


Figure 7: Illustration of tasks. From left to right: Point mass, Reacher, Fetch robot and Sawyer Robot. On the right is the image-based input for Fetch robot. For additional information on the tasks, see Appendix C.

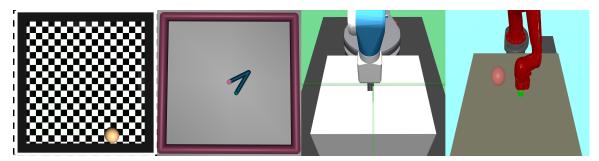


Figure 8: Illustration of image-based inputs for different reaching tasks in the main paper. Images are down-sampled to be of size $w \times w \times 3$ as inputs, where $w \in \{48, 84\}$.

which indicates the success of the transitioned state $s' = f(s, a)^1$. Below we describe in details the setup of each task, in particular the success criterion.

- Point mass [39]. The objective is to navigate a point mass through a 2-D room with obstacles to the target location. $|\mathcal{S}|=4, |\mathcal{G}|=2$ and $|\mathcal{A}|=2$. The goals $g\in\mathbb{R}^2$ are specified as a 2-D point on the plane. Included in the state s are the 2-D coordinates of the point mass, denoted as $s_{xy}\in\mathbb{R}^2$. The success is defined as $d(z(s_{xy}),z(g))\leq d_0$ where $d(\cdot,\cdot)$ is the Euclidean distance, z(g) is a element-wise normalization function $z(x):=(x-x_{\min})/(x_{\max}-x_{\min})$ where x_{\max},x_{\min} are the boundaries of the wall. The normalized threshold is $d_0=0.02\cdot\sqrt{2}$.
- Reacher [38]. The objective is to move via joint motors the finger tip of a 2-D Reacher robot to reach a target goal location. $|\mathcal{S}| = 11, |\mathcal{G}| = 2$ and $|\mathcal{A}| = 2$. As with the above point mass environment, the goals $g \in \mathbb{R}^2$ are locations of a point at the 2-D plane. Included in the state s are 2-D coordinates of the finger tip location of the Reacher robot s_{xy} . The success criterion is defined identically as the point mass environment.
- Fetch robot [38, 2]. The objective is to move via position controls the end effector of a fetch robot, to reach a target location in the 3-D space. |S| = 10, |G| = 3 and |A| = 3. This task belongs to the standard environment in OpenAI gym [38] and we leave the details to the code base and [2].
- Sawyer robot [31, 40]. The objective is to move via motor controls of the end effector of a sawyer robot, to reach a target location in the 3-D space. $|\mathcal{S}| = |\mathcal{G}| = |\mathcal{A}| = 3$. This task belongs to the multiworld code base.

Details on image inputs. For the customized point mass and Reacher environments, the image inputs are taken by cameras which look vertically down at the systems For the Fetch robot and Sawyer robot environment, the images are taken by cameras mounted to the robotic systems. See Figure 8 for an illustration of the image inputs.

¹For such simulation environments, the transition $s' \sim p(\cdot \mid s, a)$ is deterministic so we equivalently write s' = f(s, a) for some deterministic function f.

C.2 Details on Algorithms and Hyper-parameters

Hindsight Expectation Maximization. In our implementation, we take the policy network $\pi_{\theta}(a \mid s,g)$ to be a state-goal conditional Gaussian distribution $\pi_{\theta}(a \mid s,g) = \mathcal{N}(\mu_{\theta}(s,g),\sigma^2)$ with a parameterized mean $\mu_{\theta}(s,g)$ and a global standard deviation σ^2 . The mean is takes the concatenated vector [x,g] as inputs, has 3-5 hidden layers each with 3-5 hidden units interleaved with relu(x) non-linear activation functions, and outputs a vector $\mu_{\theta}(s,g) \in \mathbb{R}^{|\mathcal{A}|}$.

hEM alternates between data collection using the policy and policy optimization with the EM-algorithms. During data collection, the output action is perturbed by a Gaussian noise $a' = \mathbb{N}(0, \sigma_a^2) + a, a \sim \pi_\theta(\cdot \mid s, g)$ where the scale is $\sigma_a = 0.5$. Note that injecting noise to actions is a common practice in off-policy RL algorithms to ensure sufficient exploration [3, 4]. The baseline hEM collects data with N=30 parallel MPI actors, each with k=20 trajectories. When sampling the hindsight goal given trajectories, we adopt the *future* strategy specified in HER [2]: in particular, at state s, future achieved goals are uniformly sampled at trainig time as $q_h(\tau,g)$. All parameters are optimized with Adam optimizer [30] with learning rate $\alpha=10^{-3}$. By default, we run M=30 parallel MPI workers for data collection and training, at each iteration hEM collects N=20 trajectories from the environment. For image-based reacher and Fetch robot, hEM collects N=80 trajectories.

Hindsight Experience Replay. By design in [2], HER is combined with off-policy learning algorithms such as DQN or DDPG [3, 4]. We describe the details of DDPG. The algorithm maintains a Q-function $Q_{\theta}(s,a,g)$ parameterized similarly as a universal value function [41]: the network takes as inputs the concatenated vector [x,a,g], has 3-5 hidden layers with h=256 hidden units per layer interleaved with $\mathrm{relu}(x)$ non-linear activation functions, and outputs a single scalar. The policy network $\pi_{\theta}(s,g)$ takes the concatenated vector [x,g] as inputs, has the same intermediate architecture as the Q-function network and outputs the action vector $\pi_{\theta}(s,g) \in \mathbb{R}^{|\mathcal{A}|}$. We take the implementation from OpenAI baseline [42], all missing hyper-parameters are the default hyper-parameters in the code base. Across all tasks, HER is run with M=20 parallel MPI workers as specified in [42].

Image-based architecture. When state or goal are image-based, the Q-function network/policy network applies a convolutional network to extract features. For example, let $s,g \in \mathbb{R}^{w \times w \times 3}$ where $w \in \{48,84\}$ be raw images, and let $f_{\theta}(s), f_{\theta}(g)$ be the features output by the convolutional network. These features are concatenated before passing through the fully-connected networks described above. The convolutional network has the following architecture: $[32,8,4] \to \text{relu} \to [64,4,2] \to \text{relu} \to [64,3,2] \to \text{relu}$, where $[n_f,r_f,s_f]$ refers to: n_f number of feature maps, r_f feature patch dimension and s_f the stride.

C.3 Ablation study

Ablation study on the effect of N. hEM collects N trajectories at each training iteration. We vary $N \in \{5, 10, 20, 40, 80\}$ on two challenging domains: Flip bit K = 50 and Fetch robot (image-based) and evaluate the corresponding performance. See Figure 9. We see that in general, large N tends to lead to better performance. For example, when N = 5, hEM learns slowly on Flip bit; when N = 80, hEM generally achieves faster convergence and better asymptotic performance across both tasks. We speculate that this is partly because with large N the algorithm can have a larger coverage over goals (larger support over goals in the language of Theorem 3). With small N, the policy might converge prematurely and hence learn slowly. Similar observations have been made for HER, where they find that the algorithm performs better with a large number of MPI workers (effectively large N).

Ablation on image-based goals. To further assess the robustness of hEM against image-based inputs, we consider Sawyer robot where both states and goals are image-based. This differs from experiments shown in Figure 5 where only states are image-based. In Figure 9(c), we see that the performance of hEM does not degrade even when goals are image-based and is roughly agnostic to the size of the image. Contrast this with HER, which does not make significant progress even when only states are image-based.

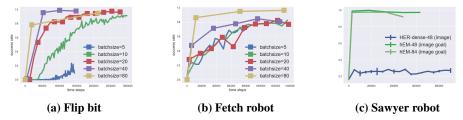


Figure 9: Ablation study. Plot (a) and (b): The effect of the data collection size N. Plot (c): The effect of image-based inputs for both states and goals. 'hEM-48' refers to image-based inputs with size $48 \times 48 \times 3$.

C.4 Comparison between hEM and HPG

We do not list HPG as a major baseline for comparison in the main paper, primarily due to a few reasons: by design, the HPG agent tackles discrete action space (see the author code base https://github.com/paulorauber/hpg), while many goal-conditioned baselines of interest [2, 31, 40] are continuous action space. Also, in [24] the author did not report comparison to traditional baselines such as HER and only report cumulative rewards instead of success rate as evaluation criterion. Here, we compare hEM with HPG on a few discrete benchmarks provided in [24] to assess their performance.

Details on HPG. The HPG is based on the author code base. [24] proposes several HPG variants with different policy gradient variance reduction techniques [26] and we take the HPG variant with the highest performance as reported in the paper. Throughout the experiment we set the learning rate to be 10^{-3} and other hyper-parameters take default values.

Benchmarks. We compare hEM and HPG on Flip bit K=25,50 and the four room environment. The details of the Flip bit environment could be found in the main paper. The four room environment is used as a benchmark task in [24], where the agent navigates a grid world with four rooms to reach a target location within episodic time limit. The agent has access to four actions, which moves the agent in four directions. The trial is successful only if the agent reaches the goal in time.

Results. We show results in Figure 10. For the Flip bit K=25, HPG and hEM behave similarly: both algorithms reach the near-optimal performance quickly and has similar convergence speed; when the state space increases to K=50, HPG does not make any progress while the performance of hEM steadily improves. Finally, for the four room environment, we see that though the performance of HPG initially increases quickly as hEM, its success rate quickly saturates to a level significantly below the asymtotpic performance of hEM. These observations show that hEM performs much more robustly and significantly better than HPG, especially in challenging environments.

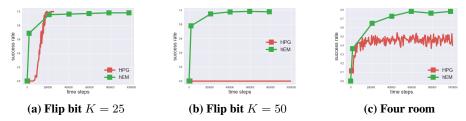


Figure 10: Comparison between hEM and HPG. HPG performs well on Flip bit MDP with K=25, but when K=50 its performance drops drastically. HPG also underperforms hEM on the four room environment where it makes fast progress initially but saturates to a low sub-optimal level.