Iteration-Promoting Variable Step Size Least Mean Square Algorithm for Accelerating Adaptive Channel Estimation

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Abstract: Invariable step size based least-mean-square error (ISS-LMS) was considered as a very simple adaptive filtering algorithm and hence it has been widely utilized in many applications, such as adaptive channel estimation. It is well known that the convergence speed of ISS-LMS is fixed by the initial step-size. In the channel estimation scenarios, it is very hard to make tradeoff between convergence speed and estimation performance. In this paper, we propose an iteration-promoting variable step size based least-mean-square error (IPVSS-LMS) algorithm to control the convergence speed as well as to improve the estimation performance. Simulation results show that the proposed algorithm can achieve better estimation performance (3dB) than previous ISS-LMS while without sacrificing convergence speed as well as computational complexity.

Keywords: Adaptive channel estimation, variable step-size (VSS), least mean square (LMS), adaptive filtering algorithm.

1. INTRODUCTION

Broadband signal transmission is becoming one of the mainstream techniques in the next generation wireless communication systems [1][2]. The channel becomes severely frequency-selective and accurate channel state information (CSI) of such a channel is required for coherent detection (or demodulation). One of the effective approaches is the adaptive channel estimation (ACE) using invariable step size least mean square error (ISS-LMS) algorithm [3], which has low complexity and can be easily implemented at the receiver. In the channel estimation scenarios, step-size of the ACE is the parameters to balance the estimation critical performance and convergence speed in different signal-noise-ratio (SNR) regimes. Hence, ISS-LMS using the only one step-size may be hard to adjust estimation performance and convergence speed under different SNR scenarios. In other words, ISS-LMS using bigger step-size can achieve faster convergence speed but obtain worse estimation performance, and vice versa.

Motivated by the research background, variable step size based least mean square error (VSS-LMS) algorithm is expected to effectively balance estimation performance and convergence speed. In last decade, instantaneous updating error based VSS-LMS algorithms have been proposed [4]–[11]. In contract to ISS-LMS, these already proposed VSS-LMS algorithms can get extra performance gain by adjusting their step-sizes while at the cost of slow convergence speed and/or high computational time. Because the step-size of these methods depend on updating channel estimation error, i.e., step-size is enlarged/reduced if the estimation error big/small. Indeed, the merit of these VSS-LMS algorithms is that the step-size can be adjusted easily in accordance with the updating error.

In this paper, hence, a simple iteration-promoting variable step-size based VSS-LMS algorithm (IPVSS-LMS) is proposed. On the one hand, the step-size is devised to reduce gradually as increasing number of adaptive iterations. On the other hand, fast convergence speed is also kept by adopting a hard threshold parameter that can terminate the proposed algorithm.

Unlike conventional VSS-LMS algorithms [4]-[11] as well as ISS-LMS [3], the proposed IPVSS-LMS algorithm can efficiently balance updating estimation performance and convergence speed. Hence, the proposed algorithm can achieve lower steady-state mean square error (MSE) estimation performance than previous algorithms [3]–[9] but without scarifying any convergence speed, which is almost the same as the ISS-LMS. The main work of this paper is summarized as follows. Firstly, the iteration-promoting variable step-size is devised for IPVSS-LMS algorithm. The equivalence of different proposed step-sizes is briefly discussed. Secondly, suitable threshold parameter is selected to control the termination of the algorithm's updating. In addition, steady-state MSE performance of the proposed algorithm is also derived. Finally, computer simulation results are given to confirm the effectiveness of the IPVSS-LMS algorithm.

The remainder of the rest paper is organized as follows. A system model is first described and then the

However, the tracking procedure to the updating error may generate additional computational burden. In large updating error scenario, the VSS approaches to initial step-size; while in small updating error scenario, the VSS reduces to a parameter which is decided by some threshold. The detailed theory analysis will be given in section 3. Aforementioned theoretical fact implies that step-size can set empirically to reduce itself in somewhat suitable stable range but without increasing needless computation complexity.

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drawback of ACE using ISS-LMS algorithm is pointed out in Section 2. In Section 3, iteration-promoting VSS-LMS algorithm is proposed to accelerate the convergence speed of ACE as well as to improve accuracy of the channel estimation. Computer simulation results are presented in Section 4 to show the performance of the proposed algorithm. Finally, we conclude the paper in Section 5.

Notation: Throughout the paper, matrices and vectors are represented by boldface upper case letters and boldface lower case letters, respectively; the superscripts $(\cdot)^T$, $(\cdot)^H$, $Tr(\cdot)$ and $(\cdot)^{-1}$ denote the transpose, the Hermitian transpose, the trace and the inverse operators, respectively; $E\{\cdot\}$ denotes the expectation operator.

2. PROBLME FORMULATION

Consider a baseband-equivalent frequency-selective fading wireless communication system where the channel $\boldsymbol{w} = [w_0, w_1, \cdots, w_{N-1}]^T$ is N dimensional signal vector and each channel tap satisfies random Gaussian distribution as $\mathcal{CN}(0,1)$. Assume that an input training signal $\boldsymbol{x}(n)$ is used to probe the unknown sparse channel. At the receiver side, the corresponding observed signal y(n) is given by

$$y(n) = \boldsymbol{w}^{T} \boldsymbol{x}(n) + z(n) \tag{1}$$

where $x(n) = [x(n), x(n-1), \dots, x(n-N+1)]^T$ denotes the length-N vector of input signal x(n); z(n) is an additive white Gaussian noise (AWGN), which is assumed to be independent with x(n); The objective of ACE is to adaptively estimate the unknown channel vector w using the training signal vector x(n) and the observed signal y(n).

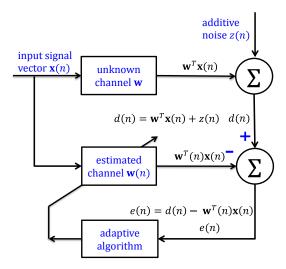


Fig.1 A framework of adaptive channel estimation.

According to (1), standard ISS-LMS based ACE method (see Fig. 1) is overviewed as follows. The cost

function of ISS-LMS [3] is constructed as

$$L(n) = (1/2)e^{2}(n)$$
 (2)

where e(n) denotes n-th update error as

$$e(n) = y(n) - \boldsymbol{w}^{T}(n)\boldsymbol{x}(n)$$

$$= z(n) + (\boldsymbol{w} - \boldsymbol{w}(n))^{T}\boldsymbol{x}(n)$$

$$= z(n) + \boldsymbol{v}^{T}(n)\boldsymbol{x}(n)$$
(3)

where v(n) = w - w(n) denotes channel estimation error and w(n) represents n-th channel estimator. According to (2), the update equation is derived as

$$w(n+1) = w(n) + \mu \frac{\partial L(n)}{\partial w(n)}$$

$$= w(n) + \mu e(n)x(n)$$
(4)

where $\mu \in (0, 1/\lambda_{\text{max}})$ is the ISS; λ_{max} is the maximum eigenvalue of covariance matrix $\mathbf{R}_{xx} = E[\mathbf{x}(n)\mathbf{x}^T(n)]$. Under the independence assumption, in [12], the steady-state MSE of ISS-LMS estimator $\mathbf{w}(n)$ is derived as

$$\kappa_{\mu}(\infty) = \lim_{n \to \infty} E\left\{ \left[(\boldsymbol{w} - \boldsymbol{w}(n))^{T} \boldsymbol{x}(n) \right]^{2} \right\}$$

$$= \frac{Tr \left[\boldsymbol{R}_{xx} \left(I - \mu \boldsymbol{R}_{xx} \right)^{-1} \right] \sigma_{n}^{2}}{2 - Tr \left[\boldsymbol{R}_{xx} \left(I - \mu \boldsymbol{R}_{xx} \right)^{-1} \right]}$$

$$\geq \frac{\lambda_{\max} \sigma_{n}^{2}}{2 - 3\mu \sigma_{n}^{2}}$$
(5)

where σ_n^2 denotes the variance of the noise variable of the z(t). One can easily find that the lower bound of steady-state MSE performance depends highly on the step-size μ . If μ approach to zero, then (5) can be further written as

$$\lim_{\mu \to 0} \kappa_{\mu}(\infty) \ge \lim_{\mu \to 0} \frac{\lambda_{\max} \sigma_n^2}{2 - 3\mu\sigma_n^2}$$

$$\ge \frac{\lambda_{\max} \sigma_n^2}{2}.$$
(6)

Hence, ISS-LMS using smaller step-size μ may achieve lower steady-state MSE performance. The idea case is that ISS-LMS adopts big step-size to achieve fast convergence speed while utilizes small step-size to get lower MSE performance. It is difficult to change the step-size adaptively during the gradient descend. This practical problem motivates us to develop iteration-promoting VSS-LMS algorithm in next section.

3. PROPOSED IPVSS-LMS ALGORITHM

According to (4), the update equation of IPVSS-LMS algorithm can be written as

$$w(n+1) = w(n) - \mu(n) \frac{\partial L(n)}{\partial w(n)}$$

$$= w(n) + \mu(n)e(n)x(n)$$
(7)

where IPVSS $\mu(n)$ is devised as

$$\mu(n) = \begin{cases} \phi, & \text{if } \mu(n) \le \phi \\ \mu/n, & \text{if } \mu(n) > \phi \end{cases}$$
 (8)

where ϕ denotes hard threshold parameter to ensure convergence if IPVSS $\mu(n)$ is enough small. Hence, step-size in (8) can realize two functions: 1) in large estimation error scenario, IPVSS can accelerate the convergence speed; and 2) in quasi-steady-state scenario, the estimation performance is accurate enough and then hard threshold ϕ can ensure fast convergence speed as well.

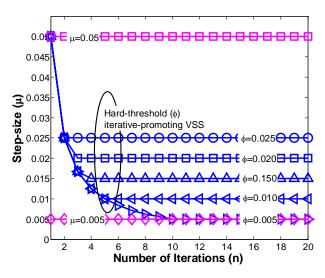


Fig. 2 IPVSS v.s. hard-threshold.

For a better understanding, we briefly discuss the differences of the proposed IPVSS-LMS with other VSS-based adaptive filtering algorithms, e.g., VSS-LMS [6][13] and normalized least mean fourth algorithm (NLMF) [10][11]. The step-size of VSS-LMS [6][13] is decided by

$$\mu_{vss}(n+1) = \mu_0 \cdot \frac{p^T(n+1)p(n+1)}{p^T(n+1)p(n+1) + C},$$
(9)

where C is a positive threshold parameter which is related to $\sigma_n^2 \operatorname{Tr}\{[\boldsymbol{x}(n)\boldsymbol{x}^T(n)]^{-1}\}$ and can be written as $C \sim \mathcal{O}(1/\operatorname{SNR})$, where SNR is the received signal noise ratio (SNR). According to Eq. (9), the range of VSS is

given by $\mu_{vss}(n+1) \in (0, \mu_0)$, where μ_0 is the maximal step-size. To the adaptive algorithm stability, the maximal step-size is less than 2 [3]. It is worth mentioning that p(n) in Eq. (9) is defined as

$$p(n+1) = \eta p(n) + (1-\eta) \frac{x(n)e(n)}{\|x(n)\|_{2}^{2}}, \quad (10)$$

where $\eta \in [0,1)$ is the smoothing factor for controlling the VSS and estimation error. If $\eta = 0$, then (9) can be rewritten as

$$\mu_{vss}(n+1) = \mu_0 \cdot \frac{p^T(n+1)p(n+1)}{p^T(n+1)p(n+1) + C}$$

$$= \mu_0 \cdot \frac{e^2(n)}{e^2(n) + C \|x(n)\|_2^2}.$$
(11)

In the initial updating, if $e^2(n) \gg ||x(n)||_2^2$, then (11) can approach to

$$\lim_{e^{2}(n)\gg\|\boldsymbol{x}(n)\|_{2}^{2}} v_{vss}(n+1) = \lim_{e^{2}(n)\gg\|\boldsymbol{x}(n)\|_{2}^{2}} \frac{\mu_{0}e^{2}(n)}{e^{2}(n) + C\|\boldsymbol{x}(n)\|_{2}^{2}}$$

$$= \mu_{0},$$
(12)

which is equivalent to IPVSS in the case of n=1. However, the update estimation error and additional computational complexity which is still required for updating VSS in (9). Unlike it, IPVSS in (7) is only calculated by the iteration numbers and a threshold. Hence, the proposed algorithm is very comparably simple as standard ISS-LMS algorithm [3]. Similarly, the proposed IPVSS-LMS is equivalent to step-size of NLMF [10][11] in the case of large estimation error (e.g., $e^2(n) \gg \|x(n)\|_2^2$). That is

$$\lim_{e^{2}(n)\gg\|\mathbf{x}(n)\|_{2}^{2}}\mu_{f} = \lim_{e^{2}(n)\gg\|\mathbf{x}(n)\|_{2}^{2}}\frac{\mu_{0}e^{2}(n)}{e^{2}(n)+\|\mathbf{x}(n)\|_{2}^{2}}$$

$$= \mu_{0}.$$
(13)

According to above discussions, both (12) and (13) insinuate that step-size could be set as big to speed up fast convergence speed in the initial updating stage where $e^2(n) \gg \|x(n)\|_2^2$. Hence, our proposed IPVSS is logical and then steady-state MSE is derived as

$$\kappa_{\mu}(\infty) = \lim_{n \to \infty} E\left\{ \left[\left(\boldsymbol{w} - \boldsymbol{w}(n) \right)^{T} \boldsymbol{x}(n) \right]^{2} \right\}$$

$$= \frac{Tr \left[\boldsymbol{R}_{xx} \left(I - \mu(n) \boldsymbol{R}_{xx} \right)^{-1} \right] \sigma_{n}^{2}}{2 - Tr \left[\boldsymbol{R}_{xx} \left(I - \mu(n) \boldsymbol{R}_{xx} \right)^{-1} \right]}$$

$$\geq \frac{\lambda_{\max} \sigma_{n}^{2}}{2 - 3\mu(n) \sigma_{n}^{2}} \geq \frac{\lambda_{\max} \sigma_{n}^{2}}{2 - 3\mu_{\min} \sigma_{n}^{2}}$$
(14)

In (14), the initial step-size $\mu(n)$ is the same as ISS-LMS while it decreases as the iteration time (n), as shown in Fig. 2. In the first stage, the main demand is fast convergence speed which is decide by the

iteration-promoting step-size (μ/n). In the second stage, the key performance indicator is steady-state MSE performance which is decided by the hard threshold or the minimum step-size (μ_{\min}). Hence, the proposed algorithm can be applied in ACE to obtain fast convergence speed as well as to achieve lower steady-state MSE performance.

4. COMPUTER SIMULATION

To validate the effectiveness of the proposed method, steady-state MSE standard is adopted to compare the results via M = 1000 independent Monte-Carlo runs. Here, the MSE metric is defined as

Average
$$MSE\{w(n)\} = \frac{1}{M} \sum_{i=1}^{M} ||w - w_i(n)||_2^2$$
 (15)

The received SNR is defined as P_0/σ_n^2 , where P_0 is the received power of the pseudo-random noise (PN)-sequence for training signal. Parameters for computer simulation are given in Tab. I.

Tab. I. Simulation parameters.

| Parameters | Values | |
|--|-------------------------------------|--|
| Training signal | Pseudorandom Binary sequence | |
| Channel length | N = 16 | |
| Distribution of each channel coefficient | Random Gaussian $\mathcal{CN}(0,1)$ | |
| Received SNR | 0dB∼20dB | |
| Step-size of standard ISS-LMS | 0.05 and 0.005 | |
| Step-size of VSS-LMS | 0.05 | |
| Hard threshold for VSS-LMS | 0.005, 0.01, 0.015, 0.02, 0.025 | |

4.1. Performance comparisons of IPVSS-LMS in different SNR regimes

Average MSE performance of the proposed method is evaluated in Figs. 3-7 under different SNR regimes, i.e. 0dB~20dB. To confirm the effectiveness of the proposed method, they are compared with standard ISS-LMS algorithm [3]. In the case of different SNR regimes, the proposed algorithm always achieves better performance with respect to average MSE while faster convergence speed with respect to iteration times than ISS-LMS. Since the proposed IPVSS-LMS algorithm adopts the iteration-promoting step-size (i.e., $\mu(n)$) to achieve better performance while utilizing a minimum step-size (threshold) to ensure convergence efficiently. Let us take the Fig. 6 for example to further illustrate the advantages of the proposed algorithm. Two performance curves of standard LMS are depicted by using two step-sizes (0.005 and 0.05) as for benchmarks.

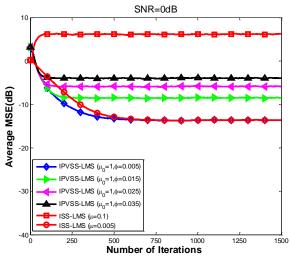


Fig. 3 MSE performance comparisons (SNR=0dB).

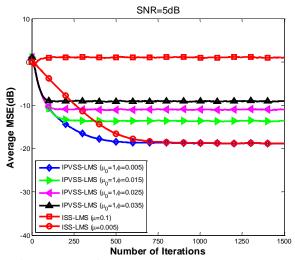


Fig. 4 MSE performance comparisons (SNR=5dB).

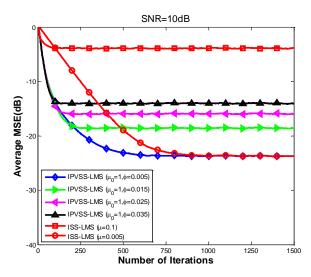


Fig. 5 MSE performance comparisons (SNR=10dB).

On the one hand, if the threshold is set as $\phi = 0.005$, the steady-state performance curve of VSS-LMS is the same as ISS-LMS using step-size $\mu = 0.005$. It is obviously observed that the convergence speed of IPVSS-LMS is faster than ISS-LMS. On the other hand, if the threshold is set as $\phi = 0.025$, the convergence speed of VSS-LMS is almost same as ISS-LMS using step-size $\mu = 0.05$. While the steady-state MSE performance curve is lower than ISS-LMS.

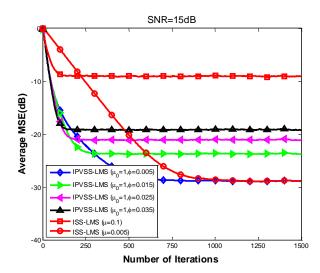


Fig. 6 MSE performance comparisons (SNR=15dB).

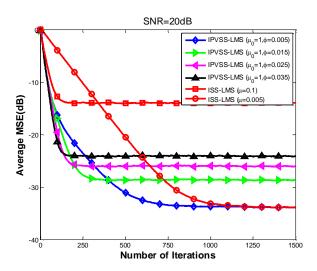


Fig. 7 MSE performance comparisons (SNR=20dB).

4.2. Comparing IPVSS-LMS with other methods in MSE and computation complexity

Without loss of generality, the average MSE performance curves of IPVSS-LMS and VSS-LMS are depicted as in Fig. 8.where the hard threshold is set as $\phi=0.035$. On the one hand, IPVSS-LMS can achieve faster convergence speed than conventional one. On the

second hand, IPVSS-LMS can keep almost the same MSE performance as conventional one. To further confirm the proposed method, its computational complexity is compared with the VSS-LMS [6]. It is worth noting that the computational complexity is the arithmetic complexity, which includes additions and multiplications. The complexities of the IPVSS-LMS algorithm and conventional one are shown in Table II. From Table II, we can see that the computational complexity of our IPVSS-LMS algorithm is lower than conventional VSS-LMS which is due to the calculation of updating step-size.

Tab. II. Computational complexity.

| Algorithm | Multiplications | Additions |
|-------------|-----------------|-----------|
| ISS-LMS [3] | 2 <i>N</i> | 2N + 1 |
| VSS-LMS [6] | 6N + 6 | 5N - 1 |
| IPVSS-LMS | 2N + 1 | 2N + 1 |

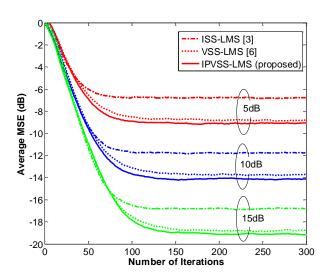


Fig. 8 MSE performance comparisons.

5. CONCLUSIONS

This paper proposed an IPVSS-LMS algorithm to balance between the convergence speed and steady-state MSE performance. We first derived the update equation of the proposed IPVSS-LMS algorithm and devised suitable threshold to control step-size efficiently. In addition, we confirmed that equivalence of the three type algorithms, i.e., IPVSS-LMS, VSS-LMS and NLMF algorithms, in big estimation error scenarios. Simulation results show that the proposed algorithm can achieve better estimation performance than previous ISS-LMS while without increasing convergence speed as computational complexity. In future work, steady-state performance analysis of the proposed IPVSS-LMS will be studied. In addition, this proposed

method will be applied in multiple-antenna wireless communications systems.

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