Exercise Sheet - Bayesian Learning

#1 $P(M=W) = 0.7$

$P(M=L) = 1 - 0.7 = 0.3$

$P(Fans = T \mid M = W) = 0.9$

$P(Fans = T \mid M = L) = 0.6$

Using Baye's Rule

$P(M=W \mid Fans=T)$

$$= \frac{P(Fans=T \mid M=W)\,P(M=W)}{P(Fans=T \mid M=W)\,P(M=W) + P(Fans=T \mid M=L)\,P(M=L)}$$

$$= \frac{0.9 \times 0.7}{0.9 \times 0.7 + 0.6 \times 0.3}$$

$$= \frac{0.63}{0.81} = 0.778$$

Thus, chances that Manchester has won is 77.8%.

#2 A → Nurse Forget to give medicine

¬A → Nurse gave the medicine

B → Mr Smith died

$P(A) = 0.3$

$P(\neg A) = 0.7$

$P(B|A) = 0.8$ $\quad P(B|\neg A) = 0.1$

Probability that Mr Smith died because medicine was not given,

$$P(A|B) = \frac{P(B|A) \times P(A)}{P(B|A) \times P(A) + P(B|\neg A) \times P(\neg A)}$$

$$= \frac{0.8 \times 0.3}{0.8 \times 0.3 + 0.1 \times 0.7}$$

$$= \frac{0.24}{0.31}$$

$$= 0.77429$$

Thus, there are 77.4% chances that Mr Smith died because Nurse forgets to give him medicine.

#3 $P(G=T) = 0.1$ $\qquad$ $P(T=P \mid G=T) = 0.8$

$P(C=T) = 0.3$ $\qquad$ $P(T=P \mid C=T) = 0.4$

$P(G=F, C=F) = 0.6$ $\qquad$ $P(T=P \mid G=F, C=F) = 0.2$

Probability that Gold is found in Campus if test came out Positive

$$P(G=T \mid T=P) = \frac{P(G=T)\,P(T=P \mid G=T)}{P(G=T)P(T=P \mid G=T) + P(C=T)P(T=P \mid C=T) + P(T=P \mid G=F, C=F)\,P(G=F, C=F)}$$

$$= \frac{0.8 \times 0.1}{0.8 \times 0.1 + 0.3 \times 0.4 + 0.2 \times 0.6}$$

$$= \frac{0.08}{0.32} = 0.25$$

Thus, there are 25% of chances to find gold in the Campus if test Comes out positive

#4 $P(M=T) = 0.05$ $P(M=F) = 0.95$

$P(T=P \mid M=T) = 0.95$ $P(T=N \mid M=T) = 0.05$

$P(T=N \mid M=F) = 0.7$ $P(T=P \mid M=F) = 0.3$

1. $P(M=T \mid T=P)$

$$= \frac{P(T=P \mid M=T)\, P(M=T)}{P(T=P \mid M=T)\, P(M=T) + P(T=P \mid M=F)\, P(M=F)}$$

$$= \frac{0.95 \times 0.05}{0.95 \times 0.05 + 0.3 \times 0.95}$$

$$= \frac{0.04785}{0.285}$$

$$= 0.14285$$

There are 14.28% chances of patient having Meningitis after getting positive Test Report.
As, chances are less it can be considered patient does not have disease

2. Now, if doctor conducts the 2nd test and found to be Negative, the probabilities associated are changed as

$P(M=T \mid T_1=P, T_2=N) =$

$$\frac{P(T_1=P, T_2=N \mid M=T)\, P(M=T)}{P(T_1=P, T_2=N \mid M=T)\, P(M=T) + P(T_1=P, T_2=N \mid M=F)\, P(M=F)}$$

As Test 2 (T_2) is independent of Test 1 (T_1) so we can Reduce the probability as

$P(M=T \mid T_1=P, T_2=N) =$

$$\frac{P(T_1=P \mid M=T)\,P(T_2=N \mid M=T)\,P(M=T)}{P(T_1=P \mid M=T)\,P(T_2=N \mid M=T)\,P(M=T)+P(T_1=P \mid M=F)\,P(T_2=N \mid M=F)\,P(M=F)}$$

$$= \frac{0.95 \times 0.05 \times 0.05}{0.95 \times 0.05 \times 0.05 + 0.3 \times 0.7 \times 0.95}$$

$$= \frac{0.002375}{0.116375} = 0.020$$

Thus, after Test 2 the Probability of having Meningitis reduces to 0.02 i.e. 2% from 14% and probability of not having meningitis become 98%.

#5 $P(R=T) = 0.8$ $\qquad$ $P(R=F) = 0.2$

$\cancel{P(R}\ P(T=P \mid R=T) = 0.75$ $\qquad$ $P(T=N \mid R=T) = 0.25$

$P(T=P \mid R=F) = 0.15$ $\qquad$ $P(T=N \mid R=F) = 0.85$

As Test came negative the probability that there will be no rain is given as $P(R=F \mid T=N)$

$P(R=F|T=N) =$

$$\frac{P(T=N|R=F)\,P(R=F)}{P(T=N|R=F)\,P(R=F) + P(T=N|R=T)\,P(R=T)}$$

$$= \frac{0.85 \times 0.2}{0.85 \times 0.2 + 0.25 \times 0.8}$$

$$= \frac{0.17}{0.37} = 0.459$$

Calcutions shows that their are about 46% chance that there will be No rain the following day and contrary to this there are 54% chance of Rain the following day. So Mr. Jones should not predict 'no Rain' the Next day.

#6 A → Someone have Chikungunya

B → Someone have Joint pain

T → Test

$P(A) = 0.0001$ $\qquad$ $P(\neg A) = 0.999$

$P(B|A) = 0.64$ $\qquad$ $P(\neg B|A) = 0.36$

$P(B|\neg A) = 0.6$ $\qquad$ $P(\neg B|\neg A) = 0.4$

$P(T=P|A) = 0.99$ $\qquad$ $P(T=N|A) = 0.01$

$P(T=P|\neg A) = 0.04$ $\qquad$ $P(T=N|\neg A) = 0.96$

1 Doctor can make its decision in the following manner

$P(A|B, T=P) =$

$$\frac{P(B, T=P|A)\,P(A)}{P(B, T=P|A)\,P(A) + P(B, T=P|\neg A)\,P(\neg A)}$$

Joint pain and Test is independent thus we have

$$\frac{P(B|A)\,P(T=P|A)\,P(A)}{P(B|A)\,P(T=P|A)\,P(A) + P(B|\neg A)\,P(T=P|\neg A)\,P(\neg A)}$$

$$= \frac{0.64 \times 0.99 \times 0.0001}{0.64 \times 0.99 \times 0.0001 + 0.6 \times 0.04 \times 0.999}$$

$= 0.0026$

The chances of fred Being infected is 0.26%, which is very low to decide it as infected.

2. If the 2nd test is also positive than we have

$$P(A|B, T_1 = P, T_2 = P)$$

$$= \frac{P(B|A)\,P(T_1 = P|A)\,P(T_2 = P|A)\,P(A)}{P(B|A)\,P(T_1 = P|A)\,P(T_2 = P|A)\,P(A) + P(B|\neg A)\,P(T_1 = P|\neg A)\,P(T_2 = P|\neg A)\,P(\neg A)}$$

Assumption is made that Joint pain, Test 1 and Test 2 are independent

$$= \frac{0.64 \times (0.99)^2 \times 0.0001}{0.64 \times (0.99)^2 \times 0.0001 + 0.6 \times (0.04)^2 \times 0.999}$$

$$= 0.0613$$

Thus after the 2nd test there are 6% chances now that fred might be infected. Now the Bilief about chickanguniya has changed to 6% from prior belief of 0.01%

3. Now if 3rd test is conducted, the chances of having chichunguniya can be seen as

$$P(A \mid B, T_1 = P, T_2 = P, T_3 = P)$$

$$= \frac{(0.99)^3 \times 0.64 \times 0.0001}{(0.99)^3 \times 0.64 \times 0.0001 + 0.6 \times (0.004)^3 \times 0.99}$$

$$= 0.6118$$

Thus it can be seen that after 3rd test comes positive the chances of having chinkungunya increases to 61.18% and that of not having it is 38.82%

Thus After (3 tests) the belief in having chickungunya is greater than not having ~~chick~~ chickungunya.

#7 $P(H=old \mid C=Y) = \frac{2}{5}$

$P(H=old \mid C=N) = \frac{0}{5} = 0$

$P(T > 2.5 \mid C=Y) = \frac{2}{5}$

$P(T > 2.5 \mid C=N) = \frac{3}{5}$

$P(Age \leq 55 \mid C=Y) = \frac{2}{5}$

$P(Age \leq 55 \mid C=N) = \frac{3}{5}$

We need

$P(C=Y \mid D)$ where D is Handset (H), Time Since Customer (T), Age

$$P(C=Y \mid D) = \frac{P(C=Y) * P(D \mid C=Y)}{P(C=Y) * P(D \mid C=Y) + P(C=N) * P(D \mid C=N)}$$

Seeing $P(H=old \mid C=N) = 0$ we have

$P(D \mid C=N) = 0$

$$\therefore P(C=Y \mid D) = \frac{\cancel{P(C=Y) * P(D \mid C=Y)}}{\cancel{P(C=Y) * P(D \mid C=Y)}}$$

$= 1$