CHAPTER 8

DERIVATIVES ANALYSIS AND VALUATION

LEARNING OUTCOMES

After going through the chapter student shall be able to understand

- Forward/ Future Contract
- Options
- Swaps
- Commodity Derivatives

1. INTRODUCTION

Derivative is a product whose value is to be derived from the value of one or more basic variables called bases (underlying assets, index or reference rate). The underlying assets can be Equity, Forex, and Commodity.

The underlying has a marketable value which is subject to market risks. The importance of underlying in derivative instruments is as follows:

- All derivative instruments are dependent on an underlying to have value.
- The change in value in a forward contract is broadly equal to the change in value in the underlying.
- In the absence of a valuable underlying asset the derivative instrument will have no value.
- On maturity, the position of profit/loss is determined by the price of underlying instruments. If the price of the underlying is higher than the contract price the buyer makes a profit. If the price is lower, the buyer suffers a loss.

Main users of Derivatives are as follows:

	Users	Purpose
(a)	Corporation	To hedge currency risk and inventory risk.
(b)	Individual Investors	For speculation, hedging and yield enhancement.
(c)	Institutional Investor	For hedging asset allocation, yield enhancement and to avail arbitrage opportunities.
(d)	Dealers	For hedging position taking, exploiting inefficiencies and earning dealer spreads.

The basic differences between Cash and the Derivative market are enumerated below:-

(a) In cash market tangible assets are traded whereas in derivative market contracts based on tangible or intangibles assets like index or rates are traded.

(b) In cash market, we can purchase even one share whereas in Futures and Options minimum lots are fixed.

(c) Cash market is more risky than Futures and Options segment because in "Futures and Options" risk is limited upto 20%.

(d) Cash assets may be meant for consumption or investment. Derivative contracts are for hedging, arbitrage or speculation.

(e) The value of derivative contract is always based on and linked to the underlying security. However, this linkage may not be on point-to-point basis.

(f) In the cash market, a customer must open securities trading account with a securities depository whereas to trade futures a customer must open a future trading account with a derivative broker.

(g) Buying securities in cash market involves putting up all the money upfront whereas buying futures simply involves putting up the margin money.

(h) With the purchase of shares of the company in cash market, the holder becomes part owner of the company. While in future it does not happen.

The most important derivatives are forward, futures and options.

2. FORWARD CONTRACT

Consider a Punjab farmer who grows wheat and has to sell it at a profit. The simplest and the traditional way for him is to harvest the crop in March or April and sell in the spot market then. However, in this way the farmer is exposing himself to risk of a downward movement in the price of wheat which may occur by the time the crop is ready for sale.

In order to avoid this risk, one way could be that the farmer may sell his crop at an agreed-upon rate now with a promise to deliver the asset, i.e., crop at a pre-determined date in future. This will at least ensure to the farmer the input cost and a reasonable profit.

Thus, the farmer would sell wheat forward to secure himself against a possible loss in future. It is true that by this way he is also foreclosing upon him the possibility of a bumper profit in the event of wheat prices going up steeply but then more important is that the farmer has played safe and insured himself against any eventuality of closing down his source of livelihood altogether. The transaction which the farmer has entered into is called a **forward transaction** and the contract which covers such a transaction is called a **forward contract.**

A forward contract is an agreement between a buyer and a seller obligating the seller to deliver a specified asset of specified quality and quantity to the buyer on a specified date at a specified place and the buyer, in turn, is obligated to pay to the seller a pre-negotiated price in exchange of the delivery.

This means that in a forward contract, the contracting parties negotiate on, not only the price at which the commodity is to be delivered on a future date but also on what quality and quantity to be delivered and at what place. No part of the contract is standardised and the two parties sit across and work out each and every detail of the contract before signing it.

For example, in case a gold bullion forward contract is being negotiated between two parties, they would negotiate each of the following features of the contract:

- the weight of the gold bullion to be delivered,
- the fineness of the metal to be delivered,
- the place at which the delivery is to be made,
- the period after which the delivery is to be made, and
- the price which the buyer would pay.

Suppose a buyer L and a seller S agrees to do a trade in 100 tolas of gold on 31 Dec 2013 at ₹ 30,000/tola. Here, ₹ 30,000/tola is the 'forward price of 31 Dec 2013 Gold'. The buyer L is said to be in long position and the seller S is said to be in short position. Once the contract has been entered into, L is obligated to pay S ₹ 30 lakhs on 31 Dec 2013, and take delivery of 100 tolas of gold. Similarly, S is obligated to be ready to accept ₹ 30 lakhs on 31 Dec 2013, and give 100 tolas of gold in exchange.

3. FUTURE CONTRACT

A Future Contract is an agreement between two parties that commits one party to buy an underlying financial instrument (bond, stock or currency) or commodity (gold, soyabean or natural gas) and one party to sell a financial instrument or commodity at a specific price at a future date. The agreement is completed at a specified expiration date by physical delivery or cash settlement or offset prior to the expiration date. In order to initiate a trade in futures contracts, the buyer and seller must put up "good faith money" in a margin account. Regulators, commodity exchanges and brokers doing business on commodity exchanges determine margin levels.

Suppose A buyer “B” and a Seller “S” enter into a 5,000 kgs Corn Futures contract at ₹ 5 per kg. Assuming that on the second day of trading the settlement price is ₹ 5.20 per kg. Settlement price is generally the representative price at which the contracts trade during the closing minutes of the trading period and this price is designated by a stock exchange as the settlement price. In case the price movement during the day is such that the price during the closing minutes is not the representative price, the stock exchange may select a price which it feels is close to being a representative price, e.g., average of the high and low prices which have occurred during a trading day. This price movement has led to a loss of ₹ 1,000 to S while B has gained the corresponding amount.

Thus, the initial margin account of S gets reduced by ₹ 1,000 and that of B is increased by the same amount. While the margin accounts, also called the equity of the buyer and the seller, get adjusted at the end of the day in keeping with the price movement, the futures contract gets replaced with a new one at a price which has been used to make adjustments to the buyer and seller’s equity accounts. In this case, the settle price is ₹ 5.20, which is the new price at which next day’s trading would start for this particular futures contract. Thus, each future contract is rolled over to the next day at a new price. This is called marking-to-market.

Difference between forward and future contract is as follows:

S. No.	Features	Forward	Futures
1.	Trading	Forward contracts are traded on personal basis or on telephone or otherwise.	Futures Contracts are traded in a competitive arena.
2.	Size of Contract	Forward contracts are individually tailored and have no standardized size	Futures contracts are standardized in terms of quantity or amount as the case may be
3.	Organized exchanges	Forward contracts are traded in an over the counter market.	Futures contracts are traded on organized exchanges with a designated physical location.
4.	Settlement	Forward contracts settlement takes place on the date agreed upon between the parties.	Futures contracts settlements are made daily via. Exchange’s clearing house.
5.	Delivery date	Forward contracts may be delivered on the dates agreed upon and in terms of actual delivery.	Futures contracts delivery dates are fixed on cyclical basis and hardly takes place. However, it does not mean that there is no actual delivery.

6.	Transaction costs	Cost of forward contracts is based on bid – ask spread.	Futures contracts entail brokerage fees for buy and sell order.
7.	Marking to market	Forward contracts are not subject to marking to market	Futures contracts are subject to marking to market in which the loss or profit is debited or credited in the margin account on daily basis due to change in price.
8.	Margins	Margins are not required in forward contract.	In futures contracts every participants is subject to maintain margin as decided by the exchange authorities
9.	Credit risk	In forward contract, credit risk is born by each party and, therefore, every party has to bother for the creditworthiness.	In futures contracts the transaction is a two way transaction, hence the parties need not to bother for the risk.

4. PRICING/ VALUATION OF FORWARD/ FUTURE CONTRACTS

The difference between the prevailing spot price of an asset and the futures price is known as the Basis, i.e.,

Basis = Spot price – Futures price

In a normal market, the spot price is less than the futures price (which includes the full cost-of-carry) and accordingly the basis would be negative. Such a market, in which the basis is decided solely by the cost-of-carry is known as a contango market.

Basis can become positive, i.e., the spot price can exceed the futures price only if there are factors other than the cost of carry to influence the futures price. In case this happens, then basis becomes positive and the market under such circumstances is termed as a backwardation market or inverted market.

Basis will approach zero towards the expiry of the contract, i.e., the spot and futures prices converge as the date of expiry of the contract approaches. The process of the basis approaching zero is called convergence.

The relationship between futures prices and cash prices is determined by the cost-of-carry. However, there might be factors other than cost-of-carry, especially in stock futures in which there may be various other returns like dividends, in addition to carrying costs, which may influence this relationship.

The cost-of-carry model in for futures/ forward, is as under:-

Future price = Spot price + Carrying cost – Returns (dividends, etc).

This is also called as Theoretical minimum price or arbitrage free price as calculated above.

Let us take an example to understand this relationship.

Example

The price of ACC stock on 31 December 2010 was ₹ 220 and the futures price on the same stock on the same date, i.e., 31 December 2010 for March 2011 was ₹ 230. Other features of the contract and related information are as follows:

Time to expiration	-	3 months (0.25 year)
Borrowing rate	-	15% p.a.
Annual Dividend on the stock	-	25% payable before 31.03. 2011
Face Value of the Stock	-	₹ 10

Based on the above information, the futures price for ACC stock on 31 December 2010 should be:

= 220 + (220 x 0.15 x 0.25) – (0.25 x 10) = 225.75

Thus, as per the 'cost of carry' criteria, the futures price is ₹ 225.75, which is less than the actual price of ₹ 230 on 31 March 2011. This would give rise to arbitrage opportunities and consequently the two prices will tend to converge.

How Will the Arbitrager Act?

He will buy the ACC stock at ₹ 220 by borrowing the amount @ 15 % for a period of 3 months and at the same time sell the March 2011 futures on ACC stock. By 31st March 2011, he will receive the dividend of ₹ 2.50 per share. On the expiry date of 31st March, he will deliver the ACC stock against the March futures contract sales.

The arbitrager's inflows/outflows are as follows:

Sale proceeds of March 2011 futures		₹ 230.00
Dividend		₹ 2.50
	Total (A)	₹ 232.50
Pays back the Bank		₹ 220.00
Cost of borrowing		₹ 8.25
Total (B)		₹ 228.25
Balance (A) – (B)		₹ 4.25

Thus, the arbitrager earns ₹ 4.25 per share without involving any risk.

In financial forward contracts, the cost of carry is primarily the interest cost.

Let us take a very simple example of a fixed deposit in the bank. ₹ 100 deposited in the bank at a rate of interest of 10% would be come ₹ 110 after one year. Based on annual compounding, the amount will become ₹ 121 after two years. Thus, we can say that the forward price of the fixed deposit of ₹ 100 is ₹ 110 after one year and ₹ 121 after two years.

As against the usual annual, semi-annual and quarterly compounding, which the reader is normally used to, continuous compounding are used in derivative securities. In terms of the annual compounding, the forward price can be computed through the following formula:

$A = P\ (1+r/100)^t$

Where, A is the terminal value of an amount P invested at a rate of interest of r % p.a. for t years.

However, in case there are multiple compounding in a year, say n times per annum, then the above formula will read as follows:

$A = P\ (1+r/n)^{nt}$

And in case the compounding becomes continuous, i.e., more than daily compounding, the above formula can be simplified mathematically and rewritten as follows:

$A = Pe^{rt}$

Where

e = Called epsilon, is a mathematical constant and has a value of - 2.718.

r = Risk-free Rate of Interest

t = Time Period

This function is available in all mathematical calculators and is easy to handle.

The above formula gives the future value of an amount invested in a particular security now. In this formula, we have assumed no interim income flow like dividends etc

Example

Consider a 3-month maturity forward contract on a non-dividend paying stock. The stock is available for ₹ 200. With compounded continuously risk-free rate of interest (CCRRI) of 10 % per annum, the price of the forward contract would be:

$A = 200 \times e^{(0.25)(0.10)}$ = ₹ 205.06

In case there is cash income accruing to the security like dividends, the above formula will read as follows:

$A = (P-I)e^{rt}$

Where I is the present value of the income flow during the tenure of the contract.

Example

Consider a 4-month forward contract on 500 shares with each share priced at ₹ 75. Dividend @ ₹ 2.50 per share is expected to accrue to the shares in a period of 3 months. The CCRRI is 10% p.a. The value of the forward contract is as follows:

Dividend proceeds	=	$500 \times 2.50 = 1250$
	=	$1250e^{-(3/12)(0.10)} = 1219.13$
Value of forward contract	=	$(500 \times 75 - 1219.13)\ e^{(4/12)(0.10)}$
	=	$36280.87 \times e^{0.033}$
	=	₹ 37498.11

However, in case the income accretion to the securities is in the form of percentage yield, y, as in the case of stock indices arising on account of dividend accruals to individual stocks constituting the index, the above formula will read as follows:

$A = Pe^{n(r-y)}$

Correlation between Forward and Futures Prices

For contracts of the same maturity, the forward and futures contracts tend to have the same value subject to the interest rates remaining fixed. In case the interest rates are fluid, the value of a futures contract would differ from that of a forward contract because the cash flows generated from marking to the market in the case of the former would be available for reinvestment at variable rates on a day-to-day basis. However, market imperfections like transaction costs, taxes and asset indivisibilities bring futures prices close enough to the forward prices to safely assume the two prices to be practically the same.

5. TYPES OF FUTURES CONTRACTS

5.1 Single Stock Futures

A single stock futures contract is an agreement to buy or sell shares or stock such as Microsoft, Intel, ITC, or Tata Steel at a point in the future. The buyer has an obligation to purchase shares or stock and the seller has an obligation to sell shares or stock at a specific price at a specific date in the future. Thus a stock futures contract is a standardized contract to buy or sell a specific stock at a future date at an agreed price. Single-stock futures contracts are completed via offset or the delivery of actual shares at expiration. Margin on a single-stock futures contract is expected normally to be 20% of notional value.

Each Stock Future contract is standardized and includes basic specifications.

The terms of the contract call for delivery of the stock by the seller at some time specified in the future. However, most contracts are not held to expiration. The contracts are standardized, making them highly liquid. To get out of an open long (buying) position, the investor simply takes an offsetting short position (sells). Conversely, if an investor has sold (short) a contract and wishes to close it out, he or she buys (goes long) the offsetting contract.

5.2 Index Futures

A contract for stock index futures is based on the level of a particular stock index such as the S&P 500 or the Dow Jones Industrial Average or NIFTY or BSE Sensex. The agreement calls for the contract to be bought or sold at a designated time in the future. Just as hedgers and speculators buy and sell futures contracts based on future prices of individual stocks they may—for mostly the same reasons—buy and sell such contracts based on the level of a number of stock indexes.

Stock index futures may be used to either speculate on the equity market's general performance or to hedge a stock portfolio against a decline in value. Unlike commodity futures or individual stocks, stock index futures are not based on tangible goods, thus all settlements are in cash. Because settlements are in cash, investors usually have to meet liquidity or income requirements to show that they have money to cover their potential losses.

Stock index futures are traded in terms of number of contracts. Each contract is to buy or sell a fixed value of the index. The value of the index is defined as the value of the index multiplied by the specified monetary amount. In Nifty 50 futures contract traded at the National Stock Exchange, the contract specification states:

1 Contract = 50 units of Nifty 50 * Value of Nifty 50

If we assume that Nifty 50 is quoting at 8000 , the value of one contract will be equal to ₹ 4,00,000 (50*8000). The contract size of 50 units of Nifty 50 in this case is fixed by National Stock Exchange where the contract is traded.

Example

Consider the following:

Current value of index	-	₹ 1400
Dividend yield	-	6%
CCRRI	-	10%

To find the value of a 3 month forward contract.

$$A = Pe^{t(r-y)}$$

$$= ₹\ 1400 \times e^{(3/12)(0.10 - .06)} = ₹\ 1400 \times 1.01005 = ₹\ 1,414.07$$

5.2.1 Trading Mechanism in Stock Futures

While trading in futures contracts (both stock as well as futures) both buyers and sellers of the contract have to deposit an initial margin with their brokers based on the value of contact entered. The rules for calculation of margins to be deposited with the brokers are framed by the stock exchanges.

Another major feature regarding the margin requirements for stock as well index futures is that the margin requirement is continuous. Every business day, the broker will calculate the margin requirement for each position. The investor will be required to post additional margin funds if the account does not meet the minimum margin requirement.

The investor can square off his position in the futures contract before expiry or wait till expiry date when the contracts will automatically stand as squared off at the closing price on the expiry date. In Indian stock market the expiry date is the last Thursday of the relevant month to which the future contract belongs.

Example–Margin Requirements

In a stock future contract on ITC stock at ₹ 120, both the buyer and seller have a margin requirement of 20% or ₹ 24. If ITC stock goes up to ₹ 122, the account of the long contract is credited with ₹ 200 (₹ 122-₹ 120 = ₹ 2 X 100 = ₹ 200) and the account of the seller (seller) is debited by the same ₹ 200. This indicates that investors in futures must be very vigilant - they must keep close track of market movements.

5.2.2 Purpose of Trading in Futures

Trading in futures is for two purposes namely:

(a) Speculation and

(b) Hedging

(a) Speculation – For simplicity we will assume that one contract= 100 units and the margin requirement is 20% of the value of contract entered. Brokerage and transaction costs are not taken into account.

Example- Going Long on a Single Stock Futures Contract

Suppose an investor is bullish on McDonald's (MCD) and goes long on one September stock future contract on MCD at ₹ 80. At some point in the near future, MCD is trading at ₹ 96. At that point, the investor sells the contract at ₹ 96 to offset the open long position and makes a ₹ 1600 gross profit on the position.

This example seems simple, but let's examine the trades closely. The investor's initial margin requirement was only ₹ 1600 (₹ 80 x 100 = ₹ 8,000 x 20% = ₹ 1600). This investor had a 100% return on the margin deposit. This dramatically illustrates the leverage power of trading futures. Of course, had the market moved in the opposite direction, the investor easily could have experienced losses in excess of the margin deposit.

The pay off table for the above transaction can be depicted as follows:-

Particulars	Details	Inflow/(outflow){In ₹}
Initial Payoff - Margin (Refundable at maturity)	₹ 8000 x 20% = ₹ 1600	(₹ 1600)

Pay off upon squaring off the contract	Profit (₹ 96 - ₹ 80) x 100 = ₹ 1600 Initial Margin = ₹ 1600	₹ 3200
Net Payoff		₹ 1600

Example- Going Short on a Single Stock Futures Contract

An investor is bearish in Kochi Refinery (KR) stock for the near future and goes short an August stock future contract on KR at ₹ 160. KR stock performs as the investor had guessed and drops to ₹ 140 in July. The investor offsets the short position by buying an August stock future at ₹ 140. This represents a gross profit of ₹ 20 per share, or a total of ₹ 2,000.

Again, let's examine the return the investor had on the initial deposit. The initial margin requirement was ₹ 3,200 (₹ 160 x 100 = ₹ 16,000 x 20% = ₹ 3,200) and the gross profit was ₹ 2,000. The return on the investor's deposit was more than 60% - a terrific return on a short-term investment.

Particulars	Details	Inflow/(outflow){In ₹}
Initial Payoff - Margin (Refundable at maturity)	₹ 160 x 100 x 20% = ₹ 3200	(₹ 3200)
Pay off upon squaring off the contract	Profit (₹ 160 - ₹ 140) x 100 = ₹ 2000 Initial Margin = ₹ 3200	₹ 5200
Net Payoff		₹ 2000

Example- Going Long on an Index Futures Contract

Suppose an investor has a bullish outlook for Indian market for the month of October 2014. He will go for a long position one October 2014 Nifty Index Future Contract. Assuming that he enters into long positions when Nifty is trading at 8000 and one month later he squares off his position when the value of Nifty rises to 8500 his payoff will be as under. (Assuming that one contract= 50 units of Nifty and margin requirement is 20% of the value of the contract)

Particulars	Details	Inflow/(outflow){In ₹}
Initial Payoff - Margin (Refundable at maturity)	(8000 x 50 x 20%) = ₹ 80,000	(₹ 80,000)
Pay off upon squaring off the contract	Profit (8500 - 8000) x 50 = ₹ 25,000 Initial Margin = ₹ 80,000	₹ 1,05,000
Net Payoff		₹ 25,000

Example- Going Short on an Index Futures Contract

Suppose an investor has a bearish outlook for Indian banking sector for the month of October 2014. He will go for a short position for one October 2014 Bank Nifty Future Contract. Assuming that he enters into short positions when Bank Nifty is trading at 25000 and one month later he squares off his position when the value of Bank Nifty declines to 24000 his payoff will be as under. (Assuming that one contract = 10 units of Bank Nifty and margin requirement is 20% of the value of the contract)

Particulars	Details	Inflow/(outflow){In ₹}
Initial Payoff – Margin (Refundable at maturity)	(25000 x 10 x 20%) = ₹ 50,000	(₹ 50,000)
Pay off upon squaring off the contract	Profit (25000-24000) x 10 = ₹ 10,000 Initial Margin = ₹ 50,000	₹ 60,000
Net Payoff		₹ 10,000

(b) Hedging – Hedging is the practice of taking a position in one market to offset and balance against the risk adopted by assuming a position in a contrary or opposing market or investment. In simple language, hedging is used to reduce any substantial losses/gains suffered by an individual or an organization. To hedge, the investor takes a stock future position exactly opposite to the stock position. That way, any losses on the stock position will be offset by gains on the future position.

Example- Using single stock future as a Hedge

Consider an investor who has bought 100 shares of Tata Steel (TS) at ₹ 300. In July, the stock is trading at ₹ 350. The investor is happy with the unrealized gain of ₹ 50 per share but is concerned that in a stock as volatile as TS, the gain could be wiped out in one bad day. The investor wishes to keep the stock at least until September, however, because of an upcoming dividend payment.

To hedge, the investor sells a ₹ 350 September stock future contract - whether the stock rises or declines, the investor has locked in the ₹ 50-per-share gain.In September on maturity date of the futures contract (last Thursday of September) , the investor sells the stock at the market price and buys back the future contract.

The pay-off at various price levels of Tata Steel is as under:-

Particulars	September Closing price of Tata Steel= ₹ 300	September Closing price of Tata Steel= ₹ 350	September Closing price of Tata Steel= ₹ 400
Initial Payoff Cost of scrip in cash market Margin Payment on futures contract Total Initial Payoff (outflow)	₹ 300 x 100 = ₹ 30000 ₹ 350 x 100 x 20% = ₹7000 ₹ 37000	₹ 300 x 100 = ₹ 30000 ₹ 350 x 100 x 20% = ₹7000 ₹ 37000	₹ 300 x 100 = ₹ 30000 ₹ 350 x 100X20% = ₹7000 ₹ 37000
Pay-off at maturity (September end)	Sale proceeds of TS in cash market= ₹300x 100 = ₹	Sale proceeds of TS in cash market= ₹350x 100 = ₹	Sale proceeds of TS in cash market= ₹400x 100 = ₹ 40000

Total Pay-off at maturity (Inflow)	30000 Margin refund on futures contract = ₹ 7000 Gain on futures contract(inflow) = (₹ 350 - ₹ 300) x 100 = ₹ 5000 ₹ 42000	35000 Margin refund on futures contract = ₹ 7000 No profit /loss on futures contract = (₹ 350 - ₹ 350) x100 = ₹ 0 ₹ 42000	Margin refund on futures contract = ₹ 7000 Loss on futures contract(outflow) = (₹ 350 -₹ 400) x 100 = - ₹ 5000 ₹ 42000
Net Payoff	₹ 5000	₹ 5000	₹ 5000

Hence it can be observed in the above table that in any case the investor has locked in a profit of ₹ 5000 via hedging.

In a similar manner as illustrated above index futures can also be used as a hedge. The difference would be that instead of single stock futures the investor would enter into a position into an Index Futures Contract according to the risk potential of the investor. Index Futures are also used to hedge a Portfolio of shares and number of contracts depends upon the β of the portfolio.

5.2.3 Marking to Market

It implies the process of recording the investments in traded securities (shares, debt-instruments, etc.) at a value, which reflects the market value of securities on the reporting date. In the context of derivatives trading, the futures contracts are marked to market on periodic (or daily) basis. Marking to market essentially means that at the end of a trading session, all outstanding contracts are repriced at the settlement price of that session. Unlike the forward contracts, the future contracts are repriced every day. Any loss or profit resulting from repricing would be debited or credited to the margin account of the broker. It, therefore, provides an opportunity to calculate the extent of liability on the basis of repricing. Thus, the futures contracts provide better risk management measure as compared to forward contracts.

Suppose on 1st day we take a long position, say at a price of ₹ 100 to be matured on 7th day. Now on 2nd day if the price goes up to ₹ 105, the contract will be repriced at ₹ 105 at the end of the trading session and profit of ₹ 5 will be credited to the account of the buyer. This profit of ₹ 5 may be drawn and thus cash flow also increases. This marking to market will result in three things – one, you will get a cash profit of ₹ 5; second, the existing contract at a price of ₹ 100 would stand cancelled; and third you will receive a new futures contract at ₹ 105. In essence, the marking to market feature implies that the value of the futures contract is set to zero at the end of each trading day.

5.2.4 Advantages of Futures Trading Vs. Stock Trading

Stock index futures is most popular financial derivatives over stock futures due to following reasons:

1. It adds flexibility to one's investment portfolio. Institutional investors and other large equity holders prefer the most this instrument in terms of portfolio hedging purpose. The stock systems do not provide this flexibility and hedging.
2. It creates the possibility of speculative gains using leverage. Because a relatively small amount of margin money controls a large amount of capital represented in a stock index contract, a small change in the index level might produce a profitable return on one's investment if one is right about the direction of the market. Speculative gains in stock futures are limited but liabilities are greater.
3. Stock index futures are the most cost-efficient hedging device whereas hedging through individual stock futures is costlier.
4. Stock index futures cannot be easily manipulated whereas individual stock price can be exploited more easily.
5. Since, stock index futures consists of many securities, so being an average stock, is much less volatile than individual stock price. Further, it implies much lower capital adequacy and margin requirements in comparison of individual stock futures. Risk diversification is possible under stock index future than in stock futures.
6. One can sell contracts as readily as one buys them and the amount of margin required is the same.
7. In case of individual stocks the outstanding positions are settled normally against physical delivery of shares. In case of stock index futures they are settled in cash all over the world on the premise that index value is safely accepted as the settlement price.
8. It is also seen that regulatory complexity is much less in the case of stock index futures in comparison to stock futures.
9. It provides hedging or insurance protection for a stock portfolio in a falling market.

5.2.5 Uses/Advantages of Stock Index Futures

Investors can use stock index futures to perform myriad tasks. Some common uses are:

(1) Investors commonly use stock index futures to change the weightings or risk exposures of their investment portfolios. A good example of this is investors who hold equities from two or more countries. Suppose these investors have portfolios invested in 60 percent U.S. equities and 40 percent Japanese equities and want to increase their systematic risk to the U.S. market and reduce these risks to the Japanese market. They can do this by buying U.S. stock index futures contracts in the indexes underlying their holdings and selling Japanese contracts (in the Nikkei Index).

(2) Stock index futures also allow investors to separate market timing from market selection decisions. For instance, investors may want to take advantage of perceived immediate increases in an equity market but are not certain which securities to buy; they can do this by purchasing stock index futures. If the futures contracts are bought and the present value of the money used to buy them is invested in risk-free securities, investors will have a risk exposure equal to that of the market. Similarly, investors can adjust their portfolio holdings at a more leisurely pace. For example, assume the investors see that they have several undesirable stocks but do not know what holdings to buy to replace them. They can sell the unwanted stocks and, at the same time, buy stock index futures to keep their exposure to the market. They can later sell the futures contracts when they have decided which specific stocks they want to purchase.

(3) Investors can also make money from stock index futures through index arbitrage, also referred to as program trading as it is carried out through use of computers. Basically, arbitrage is the purchase of a security or commodity in one market and the simultaneous sale of an equal product in another market to profit from pricing differences. Investors taking part in stock index arbitrage seek to gain profits whenever a futures contract is trading out of line with the fair price of the securities underlying it. Thus, if a stock index futures contract is trading above its fair value, investors could buy a basket of stocks composing the index in the correct proportion—such as a mutual fund comprised of stocks represented in the index—and then sell the expensively priced futures contract. Once the contract expires, the equities could then be sold and a net profit would result. While the investors can keep their arbitrage position until the futures contract expires, they are not required to. If the futures contract seems to be returning to fair market value before the expiration date, it may be prudent for the investors to sell early.

(4) Investors often use stock index futures to hedge the value of their portfolios. Provide hedging or insurance protection for a stock portfolio in a falling market. To implement a hedge, the instruments in the cash and futures markets should have similar price movements. Also, the amount of money invested in the cash and futures markets should be the same. To illustrate, while investors owning well-diversified investment portfolios are generally shielded from unsystematic risk (risk specific to particular firms), they are fully exposed to systematic risk (risk relating to overall market fluctuations). A cost-effective way for investors to reduce the exposure to systematic risk is to hedge with stock index futures, similar to the way that people hedge commodity holdings using commodity futures. Investors often use short hedges when they are in a long position in a stock portfolio and believe that there will be a temporary downturn in the overall stock market. Hedging transfers the price risk of owning the stock from a person unwilling to accept systematic risks to someone willing to take the risk.

To carry out a short hedge, the hedger sells a futures contract; thus, the short hedge is also called a "sell-hedge."

Example

Consider investors who own portfolios of securities valued at $.1.2 million with a dividend of 1 percent. The investors have been very successful with their stock picks. Therefore, while their portfolios' returns move up and down with the market, they consistently outperform the market by 6 percent. Thus, the portfolio would have a beta of 1.00 and an alpha of 6 percent. Say that the investors believe that the market is going to have a 15 percent decline, which would be offset by the 1 percent received from dividends. The net broad market return would be -14 percent but, since they consistently outperform the market by 6 percent, their estimated return would be -8 percent. In this instance, the investors would like to cut their beta in half without necessarily cutting their alpha in half. They can achieve this by selling stock index futures. In this scenario, the S&P 500 index is at 240. The contract multiplier is $500, and therefore each contract represents a value of $120,000. Since the investors want to simulate the sale of half of their $1.2 million portfolios, they must sell five contracts (5 × $120,000 = $600,000). Thus, their portfolios would be affected by only half of the market fluctuation. While the investors could protect their portfolios equally well by selling half of their shares of stock and buying them again at short time later, using a short hedge on stock index futures is much cheaper than paying the capital gains tax plus the broker commissions associated with buying and selling huge blocks of stock.

At the extreme, stock index futures can theoretically eliminate the effects of the broad market on a portfolio. Perfect hedges are very unusual because of the existence of basis risk. The basis is the difference between the existing price in the futures market and the cash price of the underlying securities. Basis risk occurs when changes in the economy and the financial situation have different impacts on the cash and futures markets.

(5) Stock index futures add flexibility to his or her portfolio as a hedging and trading instrument.

(6) Create the possibility of speculative gains using leverage. Because a relatively small amount of margin money controls a large amount of capital represented in a stock index contract, a small change in the index level might produce a profitable return on one’s investment if he or she is right about the market's direction.

(7) Maintain one’s stock portfolio during stock market corrections. One may not need "insurance" for all the time, but there are certain times when one would like less exposure to stocks. Yet, one doesn't want to sell off part of a stock portfolio that has taken him or her a long time to put together and looks like a sound, long-term investment program.

(8) One of the major advantages of futures markets, in general, is that one can sell contracts as readily as he or she can buy them and the amount of margin required is the same. Mutual funds do not specialize in bear market approaches by short selling stocks but, and also it is not possible for individuals to short sell stocks in a falling market to make money.

(9) Transfer risk quickly and efficiently. Whether one is speculating, looking for insurance protection (hedging), or temporarily substituting futures for a later cash transaction, most

stock index futures trades can be accomplished quickly and efficiently. Many mutual funds require investors to wait until the end of the day to see at what price they were able to purchase or sell shares. With today's volatility, once-a-day pricing may not give one the maneuverability to take positions at exactly the time he or she wants. Stock index futures give individual the opportunity to get into or out of a position whenever he or she wants.

6. OPTIONS

An Option may be understood as a privilege, sold by one party to another, that gives the buyer the right, but not the obligation, to buy (call) or sell (put) any underlying say stock, foreign exchange, commodity, index, interest rate etc. at an agreed-upon price within a certain period or on a specific date regardless of changes in underlying's market price during that period.

The various kinds of stock options include put and call options, which may be purchased in anticipation of changes in stock prices, as a means of speculation or hedging. A put gives its holder an option to sell, shares to another party at a fixed price even if the market price declines. A call gives the holder an option to buy, or call for, shares at a fixed price even if the market price rises.

6.1 Stock Options

Stock options involve no commitments on the part of the buyers of the option contracts individual to purchase or sell the stock and the option is usually exercised only if the price of the stock has risen (in case of call option) or fallen (in case of put option) above the price specified at the time the option was given. One important difference between stocks and options is that stocks give you a small piece of ownership in the company, while options are just contracts that give you the right to buy or sell the stock at a specific price by a specific date. Investing in options provide limited risk, high potential reward and smaller amount of capital required to control the same number of shares which can be done via investing through cash market.

6.2 Stock Index Option

It is a call or put option on a financial index. Investors trading index options are essentially betting on the overall movement of the stock market as represented by a basket of stocks.

Index options can be used by the portfolio managers to limit their downside risk. Suppose the value of the index is S. Consider a manager in charge of a well diversified portfolio which has a β of 1.0 so that its value mirrors the value of the index. If for each 100S rupees in the portfolio, the manager buys one put option contract with exercise price X, the value of the portfolio is protected against the possibility of the index falling below X. For instance, suppose that the manager's portfolio is worth ₹ 10,00,000 and the value of the index is 10000. The portfolio is worth 100 times the index. The manager can obtain insurance against the value of the portfolio dropping below ₹ 900,000 in the next two months by buying 1 put option contracts with a strike price of ₹ 9000. To illustrate how this would work, consider the situation where the index drops to 8500. The portfolio will be worth ₹ 850000 (100 x 8500). However, the payoff from the options will be 1 x (₹ 9000 – ₹ 8500) x 100 = ₹ 50000, bringing the total value of the portfolio up to the insured value of ₹ 9,00,000.

6.3 Parties to the Options

There are always two types of entities for an option transaction buyer and a seller (also known as writer of the option). So, for every call or put option purchased, there is always someone else selling/buying it. When individuals sell options, they effectively create a security that didn't exist before. This is known as writing an option and explains one of the main sources of options, since neither the associated company nor the options exchange issues options. When you write a call, you may be obligated to sell shares at the strike price any time before the expiration date. When you write a put, you may be obligated to buy shares at the strike price any time before expiration. The price of an option is called its premium. The buyer of an option cannot lose more than the initial premium paid for the contract, no matter what happens to the underlying security. So, the risk to the buyer is never more than the amount paid for the option. The profit potential, on the other hand, is theoretically unlimited

6.4 Premium for Options

In return for the premium received from the buyer, the seller of an option assumes the risk of having to deliver (if a call option) or taking delivery (if a put option) of the shares of the stock. Unless that option is covered by another option or a position in the underlying stock (opposite to the position taken via selling the option contracts), the seller's loss can be unlimited, meaning the seller can lose much more than the original premium received.

6.5 Types of Options

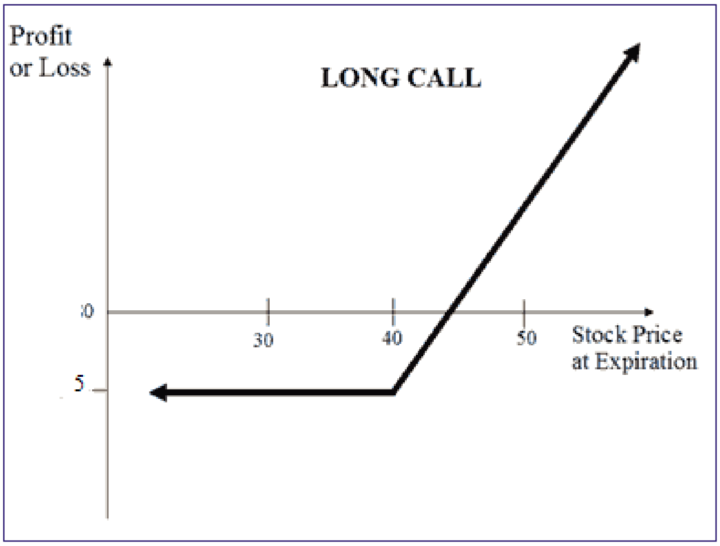
You should be aware that there are two basic styles of options: American and European. An American, or American-style, option can be exercised at any time between the date of purchase and the expiration date. Most exchange-traded options are American style and all stock options are American style. A European, or European-style, option can only be exercised on the expiration date. In Indian Market most of the options are European style options.

6.6 Pay-off scenarios

The possible pay-off under various scenarios are as follows:

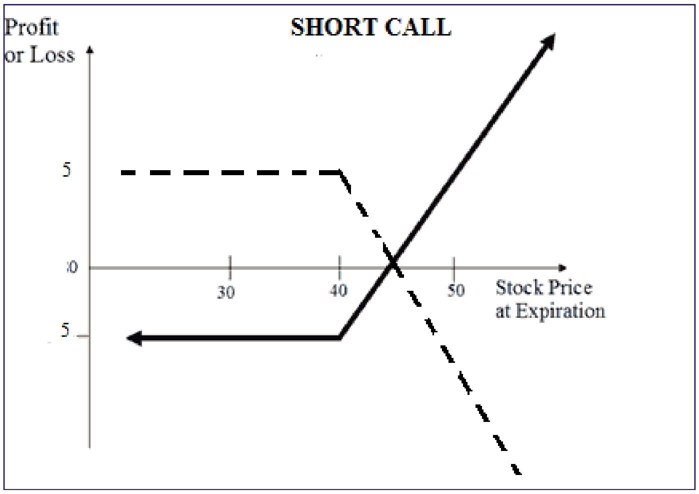
6.6.1 Pay-off for a Call Buyer

Also, called Long Call. For example, Mr. X buys a call option at strike price of ₹ 40 in exchange of a premium of ₹ 5. In case if actual price of the stock at the time of exercise is less than ₹ 40, Mr. X would not exercise his option his loss would be ₹ 5. Mr. X would exercise his option at any price above ₹ 40. In such situation his loss would start reducing and at the price of ₹ 45 there will be Break Even at the price of ₹ 45.



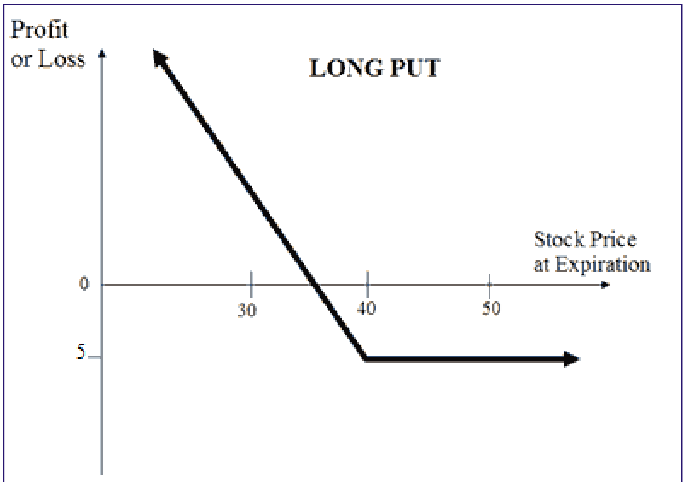
6.6.2 Pay-off for a Call Seller

Also, called Short Call. The pay-off profile of Call Seller shall be the mirror image of the Long Call as shown below in dotted line.



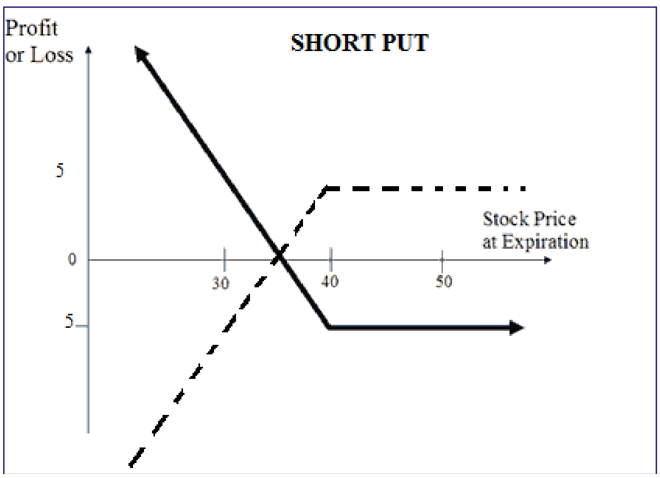
6.6.3 Pay-off for a Put Buyer

Also, called Long Put. For example, Mr. X buys a put option at strike price of ₹ 40 in exchange of a premium of ₹ 5. In case if actual price of the stock at the time of exercise is less than ₹ 40, Mr. X would exercise his option his gain would be (Spot Price – Exercise Price – Premium). Mr. X would exercise his option at any price below ₹ 40. The break- even Break Even price will be ₹ 35 and Mr. X would not exercise his option for any price above ₹ 40.



6.6.4 Pay-off for a Put Seller

Also, called Short Put. For example, The pay-off profile of Put Seller shall be the mirror image of the Long Put as shown below in dotted line.



6.7 Comparison with Single Stock Futures

Investing in stock futures differs from investing in equity options contracts in several ways:

- *Nature:* In options, the buyer of the options has the right but not the obligation to purchase or sell the stock. However while going in for a long futures position, the investor is obligated to square off his position at or before the expiry date of the futures contract.
- *Movement of the Market*: Options traders use a mathematical factor, the delta that measures the relationship between the options premium and the price of the underlying stock. At times, an options contract's value may fluctuate independently of the stock price. In contrast, the future contract will much more closely follow the movement of the underlying stock.

- *The Price of Investing*: When an options investor takes a long position, he or she pays a premium for the contract. The premium is often called a sunk cost. At expiration, unless the options contract is in the money, the contract is worthless and the investor has lost the entire premium. Stock future contracts require an initial margin deposit and a specific maintenance level of cash for mark to market margin

7. OPTION VALUATION TECHNIQUES

We have already been introduced to characteristics of both European and American Options. Assuming a European Call Option on a non dividend paying stock it is easy to see that its value at expiration date shall either be zero or the difference between the market price and the exercise price, whichever is higher. It may be noted that the value of an Option cannot be negative. An investor is required to pay a premium for acquiring such an Option. In case this premium is less than the value of the Option, the investor shall make profits, however, in case the premium paid is more than the value, the investor shall end up losing money. Note that, while measuring these gains or losses, Time Value of Money and Transaction Costs have been ignored. The opposite picture emerges for the Writer.

The Value of an Option with one period to expire: Simply speaking, the theoretical value of an Option should be the difference between the current stock price and the exercise price. In case the stock price is less than the exercise price the theoretical value shall be zero. However, as long as there is time to expiration it is possible for a zero theoretical value Option to have some actual positive Market value. This is because there may be a possibility of the stock price rising at which point of time the Option may be exercised advantageously.

7.1 Binomial Model

The binomial model breaks down the time to expiration into potentially a very large number of time intervals, or steps. This requires the use of probability and future discrete projections through which a tree of stock prices is initially produced working forward from the present to expiration.

To facilitate understanding we shall restrict ourselves to a European Option having a one year time branching process where at the end of the year there are only two possible values for the common stock. One is higher and the other lower than the current value. Assume that the probability of the two values to materialize is known. In such a situation, a hedged position can be established by buying the stock and by writing Options. This shall help offset price movements. At each step, it is assumed that the stock price will either move up or down. The pricing of the Options should be such that the return equals the risk-free rate.

The above mentioned is an example of Binomial Distribution. When the number of high and low value projections for the concerned stock are numerous, the tree shall represent all possible paths that the stock price could take during the life of the option.

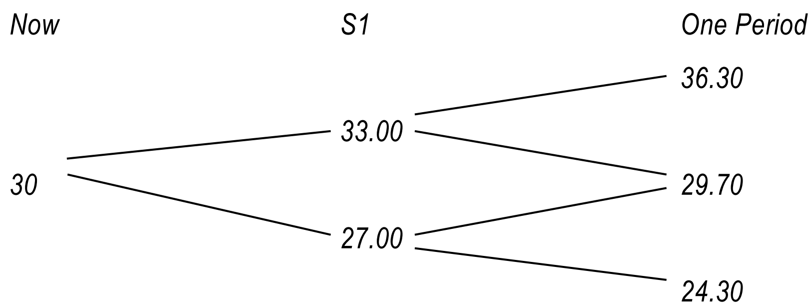
At the end of the tree - i.e. at expiration of the option - all the terminal option prices for each of the final possible stock prices are known as they simply equal their intrinsic values.

The big advantage the binomial model has over the Black-Scholes model is that it can be used to accurately price American options. This is because with the binomial model it's possible to check at every point in an option's life (i.e. at every step of the binomial tree) for the possibility of early exercise (e.g. where, due to e.g. a dividend, or a put being deeply in the money the option price at that point is less than its intrinsic value).

Where an early exercise point is found it is assumed that the option holder would elect to exercise, and the option price can be adjusted to equal the intrinsic value at that point. This then flows into the calculations higher up the tree and so on.

Illustration 1

Following is a two sub-periods tree of 6-months each for the share of CAB Ltd.:



Using the binomial model, calculate the current fair value of a regular call option on CAB Stock with the following characteristics: X = ₹ 28, Risk Free Rate = 5 percent p.a. You should also indicate the composition of the implied riskless hedge portfolio at the valuation date.

Solution

u = 33.00/30.00 = 36.30/33.00 = 1.10 d = 27.00/30.00 = 24.30/27.00 = 0.90

$r = (1 + .05)^{1/2} = 1.0247$

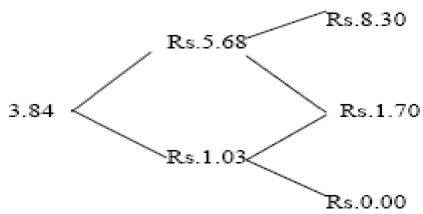
$p = \dfrac{r-d}{u-d} = \dfrac{1.0247-0.90}{1.10-0.90}$ = 0.1247/0.20 = 0.6235 (Prob. of increase in Price of Share)

Prob. of decrease in Price of Share = 1 – 0.6235 = 0.3765

C_{uu} = Max [0, 36.30 – 28] = 8.30

C_{ud} = Max [0, 29.70 – 28] = 1.70

C_{dd} = Max [0, 24.30 – 28] = 0



$$C_U = \frac{(0.6235)(8.30)+(0.3765)(1.70)}{1.025} = \frac{5.175+0.640}{1.025} = 5.815/1.025 = ₹\ 5.673$$

$$C_d = \frac{(0.6235)(1.70)+(0.3765)(0.00)}{1.025} = \frac{1.05995}{1.025} = ₹\ 1.0341$$

$$C_o = \frac{(0.6235)(5.673) + (0.3765)(1.0341)}{1.025} = \frac{3.537 + 0.3893}{1.025} = ₹\ 3.83$$

The composition of the implied risk-less hedge portfolio at valuation date is called Delta (Δ) and it is calculated as follows:

$$\frac{C_u - C_d}{S_U - S_d}$$

Where,

C_u = Pay-off from Call Option if price of Stock goes up

C_d = Pay-off from Call Option if price of Stock goes down

S_u = Upward price of the Stock

S_d = Downward price of the Stock

Accordingly, the Risk-less Portfolio shall require Δ Share shall be required for writing off one Call Option. The Δ shall be computed as follows:

$$\Delta = \frac{5 - 0}{33 - 27} = \frac{5}{6}$$

Thus, $\frac{5}{6}$ shares shall be held or purchased for writing one Call Option.

7.2 Risk Neutral Method

The "risk-neutral" technique can also be used to value derivative securities. It was developed by John Cox and Stephen Ross in 1976. The basic argument in the risk neutral approach is that since the valuation of options is based on arbitrage and is therefore independent of risk preferences; one should be able to value options assuming any set of risk preferences and get the same answer as by using Binomial Model. This model is a simple model.

Using this model, we can derive the risk neutral probabilities and apply the same probabilities in the binomial model.

Example

Suppose the price of the share of Company X is ₹ 50. In one year it is expected either to go up to ₹ 60 or go down to ₹ 40. The risk free rate of interest is 5%.

Let p be the probability that the price will increase then (1-p) will be probability of price decrease. The value of the stock today must be equal to the present value of the expected price after one year discounted at risk-free rate as follows:

$$50 = \frac{60p + 40(1-p)}{1.05}$$

On solving we shall get the value of p= 0.625. With this value we can find out the present value of the expected payout as follows:

$$\frac{10(0.625) + 0(1 - 0.625)}{1.05} = 5.95$$

It may however be noted that the discounting can also be made on daily basis as shown in following illustration.

Illustration 2

The current market price of an equity share of Penchant Ltd is ₹ 420. Within a period of 3 months, the maximum and minimum price of it is expected to be ₹ 500 and ₹ 400 respectively. If the risk free rate of interest be 8% p.a., what should be the value of a 3 months Call option under the "Risk Neutral" method at the strike rate of ₹ 450 ? Given $e^{0.02}$ = 1.0202

Solution

Let the probability of attaining the maximum price be p

$(500 - 420) \times p+(400 - 420) \times (1-p) = 420 \times (e^{0.02}-1)$

or, $80p - 20(1 - p) = 420 \times 0.0202$

or, $80p – 20 + 20p = 8.48$

or, $100p = 28.48$

$p= 0.2848$

The value of Call Option in ₹ = $\frac{0.2848x(500-450)}{1.0202} = \frac{0.2848x50}{1.0202}$ =13.96

7.3 Black-Scholes Model

The Black-Scholes model is used to calculate a theoretical price of an Option. The Black-Scholes price is nothing more than the amount an option writer would require as compensation for writing a call and completely hedging the risk of buying stock. The important point is that the hedger's view about future stock prices is irrelevant. Thus, while any two investors may strongly disagree on the rate of return they expect on a stock they will, given agreement to the assumptions of volatility and the risk-free rate, always agree on the fair value of the option on that underlying asset. This key concept underlying the valuation of all derivatives -- that fact that the price of an option is independent of the risk preferences of investors -- is called risk-neutral valuation. It means that all derivatives can be valued by assuming that the return from their underlying assets is the risk-free rate.

The model is based on a normal distribution of underlying asset returns.

The following assumptions accompany the model:

1. European Options are considered,
2. No transaction costs,
3. Short term interest rates are known and are constant,
4. Stocks do not pay dividend,
5. Stock price movement is similar to a random walk,
6. Stock returns are normally distributed over a period of time, and
7. The variance of the return is constant over the life of an Option.

The original formula for calculating the theoretical option price (OP) is as follows:

$$OP = SN(d_1) - Xe^{-rt}N(d_2)$$

Where:

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r + \frac{v^2}{2}\right)t}{v\sqrt{t}}$$

$$d_2 = d_1 - v\sqrt{t}$$

The variables are:

S = current stock price

X = strike price of the option

t = time remaining until expiration, expressed as a percent of a year

r = current continuously compounded risk-free interest rate

v = annual volatility of stock price (the standard deviation of the short-term returns over one year).

In = natural logarithm

$N(x)$ = standard normal cumulative distribution function (Area under Normal Curve)

e = the exponential function

Understanding the formula

$N(d_1)$ represents the hedge ratio of shares of stock to Options necessary to maintain a fully hedged position.

Consider the Option holder as an investor who has borrowed an equivalent amount of the exercise price at interest rate r. $Xe^{-rt}N(d_2)$ represents this borrowing which is equivalent to the present value of the exercise price times an adjustment factor of $N(d_2)$

N(d2) in the above formulae represents the probability that price in spot market on expiration would be higher than the exercise price of the call option.

The main advantage of the Black-Scholes model is speed -- it lets you calculate a very large number of option prices in a very short time.

The Black-Scholes model has one major limitation that it cannot be used to accurately price options with an American-style exercise as it only calculates the option price at one point of time -- at expiration. It does not consider the steps along the way where there could be the possibility of early exercise of an American option.

Illustration 3

(i) The shares of TIC Ltd. are currently priced at ₹ 415 and call option exercisable in three months' time has an exercise rate of ₹ 400. Risk free interest rate is 5% p.a. and standard deviation (volatility) of share price is 22%. Based on the assumption that TIC Ltd. is not going to declare any dividend over the next three months, is the option worth buying for ₹ 25?

(ii) Calculate value of aforesaid call option based on Block Scholes valuation model if the current price is considered as ₹ 380.

(iii) What would be the worth of put option if current price is considered ₹ 380.

(iv) If TIC Ltd. share price at present is taken as ₹ 408 and a dividend of ₹ 10 is expected to be paid in the two months time, then, calculate value of the call option.

Given

$\ln(1.0375) = 0.03681$

$\ln(0.95) = -0.05129$

$\ln(0.9952) = -0.00481$

$e^{0.0125} = 1.012578$

$e^{0.008333} = 1.0084$

Solution

(i) Given: TIC Ltd. Current Price = ₹ 415

Exercise rate = 400

Risk free interest rate is = 5% p.a.

SD (Volatility) = 22%

Based on the above bit is calculated value of an option based on Block Scholes Model:

$$d_1 = \frac{l_n\left(\frac{415}{400}\right) + \left[.05 + \frac{1}{2}(.22)^2\right].25}{.22\sqrt{.25}}$$

$$= \frac{.03681 + .01855}{.11} = .5032727$$

$$d_2 = \frac{l_n\left(\frac{415}{400}\right) + \left[.05 - \frac{1}{2}(.22)^2\right].25}{.22\sqrt{.25}}$$

$$= \frac{.03681 + .00645}{.11} = .3932727$$

$N(d_1) = N(.50327) = 1 - .3072 = .6928$

$N(d_2) = N(.39327) = 1 - .3471 = .6529$

$$\text{Value of Option} = 415(.6928) - \frac{400}{e^{(.05)(.25)}}(.6529)$$

$$= 287.512 - \frac{400}{1.012578}(.6529) = 287.512 - 257.916 = ₹\ 29.60$$

NB : N(0.39327) can also be find as under :

Step 1: From table of area under normal curve find the area of variable 0.39 i.e. 0.6517.

Step 2: From table of area under normal curve find the area of variable 0.40.

Step 3: Find out the difference between above two variables and areas under normal curve.

Step 4 : Using interpolation method find out the value of 0.00327. Which is as follows:

$$\frac{0.0037}{0.01} \times 0.00327 = 0.0012$$

Step 5: Add this value, computed above to the N(0.39). Thus N (0.39327) = 0.6517 + 0.0012 = 0.6529

Since market price of ₹ 25 is less than ₹ 27.60 (Block Scholes Valuation model) indicate that option is underpriced, hence worth buying.

(ii) If the current price is taken as ₹ 380 the computations are as follows:

$$d_1 = \frac{l_n\left(\frac{380}{400}\right) + \left[.05 + \frac{1}{2}(.22)^2\right].25}{.22\sqrt{.25}} = \frac{-0.05129 + .01855}{.11} = -0.297636$$

$$d_2 = \frac{l_n\left(\frac{380}{400}\right) + \left[.05 - \frac{1}{2}(.22)^2\right].25}{.22\sqrt{.25}} = \frac{-0.05129 + .00645}{.11} = -0.407636$$

$$V_o = V_s\, N(d_1) - \frac{E}{e^{rt}} N(d_2)$$

$N(d_1) = N(-0.297636) = .3830$

$N(d_2) = N(-0.407636) = .3418$

$$380\,(.3830) - \frac{400}{e^{(.05)\,(.25)}} \times (.3418)$$

$$145.54 - \frac{400}{1.012578}(.3418) = 145.54 – 135.02 = ₹\ 10.52$$

(iii) Value of call option = ₹ 10.52

Current Market Value = ₹ 415

Present Value of Exercise Price = $\frac{400}{1.0125} = 395.06$ or $\frac{400}{1.012578} = 395.03$

Value of Put Option can be find by using Put Call Parity relationship as follows:

$V_p = -V_s + V_c + PV(E)$

$V_p = -380 + 10.52 + 395.06 = 25.58$

= ₹ 25.58 Ans

or -380 + 10.52 + 395.03 = 25.55

= ₹ 25.55

(iv) Since dividend is expected to be paid in two months time we have to adjust the share price and then use Block Scholes model to value the option:

Present Value of Dividend (using continuous discounting) = Dividend $\times$ e^{-rt}

= ₹ 10 $\times$ $e^{-.05 \times .16666}$

= ₹ 10 $\times$ $e^{-.008333}$

= ₹ 9.917 (Please refer Exponential Table)

Adjusted price of shares is ₹ 408 – 9.917 = ₹ 398.083

This can be used in Block Scholes model

$$d_1 = \frac{l_n\left(\frac{398.083}{400}\right) + \left[.05 + \frac{1}{2}(.22)^2\right].25}{.22\sqrt{.25}} = \frac{-.00481 + .01855}{.11} = .125$$

$$d_2 = \frac{l_n\left(\frac{398.083}{400}\right) + \left[.05 - \frac{1}{2}(.22)^2\right].25}{.22\sqrt{.25}} = \frac{-.00481 + .00645}{.11} = .015$$

$N(d_1) = N(.125) = .5498$

$N(d_2) = N(.015) = .5060$

$$\text{Value of Option} = 398.083\ (.5498) - \frac{400}{e^{(.05)\ (.25)}}\ (.5060)$$

$$218.866 - \frac{400}{e^{.0125}}\ (.5060)$$

$$218.866 - \frac{400}{1.012578}\ (.5060) = 218.866 - 199.8858 = ₹\ 18.98$$

7.4 Greeks

The Greeks are a collection of statistical values (expressed as percentages) that give the investor a better overall view of how a stock has been performing. These statistical values can be helpful in deciding what options strategies are best to use. The investor should remember that statistics show trends based on past performance. It is not guaranteed that the future performance of the stock will behave according to the historical numbers. These trends can change drastically based on new stock performance.

Before we discuss these statistical measures let us discuss the factors that affects the value of option as these statistical measures are related to changes in the in these factors.

7.4.1 Factors Affecting Value of an Option

There are a number of different mathematical formulae, or models, that are designed to compute the fair value of an option. You simply input all the variables (stock price, time, interest rates, dividends and future volatility), and you get an answer that tells you what an option should be worth. Here are the general effects the variables have on an option's price:

(a) ***Price Movement of the Underlying:*** The value of calls and puts are affected by changes in the underlying stock price in a relatively straightforward manner. When the stock price goes up, calls should gain in value and puts should decrease. Put options should increase in value and calls should drop as the stock price falls.

(b) ***Time till expiry:*** The option's future expiry, at which time it may become worthless, is an important and key factor of every option strategy. Ultimately, time can determine whether your option trading decisions are profitable. To make money in options over the long term, you need to understand the impact of time on stock and option positions.

With stocks, time is a trader's ally as the stocks of quality companies tend to rise over long periods of time. But time is the enemy of the options buyer. If days pass without any significant change in the stock price, there is a decline in the value of the option. Also, the value of an option declines more rapidly as the option approaches the expiration day. That is good news for the option seller, who tries to benefit from time decay, especially during that final month when it occurs most rapidly.

(c) ***Volatility in Stock Prices:*** Volatility can be understood via a measure called Statistical (sometimes called historical) Volatility, or SV for short. SV is a statistical measure of the past price movements of the stock; it tells you how volatile the stock has actually been over a given period of time.

But to give you an accurate fair value for an option, option pricing models require you to put in what the future volatility of the stock will be during the life of the option. Naturally, option traders don't know what that will be, so they have to try to guess. To do this, they work the options pricing model "backwards" (to put it in simple terms). After all, you already know the price at which the option is trading; you can also find the other variables (stock price, interest rates, dividends, and the time left in the option) with just a bit of research. So, the only missing number is future volatility, which you can calculate from the equation.

(d) ***Interest Rate***- Another feature which affects the value of an Option is the time value of money. The greater the interest rates, the present value of the future exercise price are less.

Now let us discuss these measures.

7.4.2 Delta

A by-product of the Black-Scholes model is the calculation of the delta. It is the degree to which an option price will move given a small change in the underlying stock price. For example, option price (with a delta of 0.5) will move half a rupee for every full rupee movement in the underlying stock.

A deeply out-of-the-money call will have a delta very close to zero; a deeply in-the-money call will have a delta very close to 1.

The formula for a delta of a European call on a non-dividend paying stock is:

Delta = N (d_1) (see Black-Scholes formula above for d1)

Call Deltas are positive; Put Deltas are negative, reflecting the fact that the Put option price and the underlying stock price are inversely related. The Put Delta is equal to (Call Delta – 1).

As discussed earlier the delta is often called the Hedge Ratio. If you have a portfolio short 'n' options (e.g. you have written n calls) then n multiplied by the delta gives you the number of shares (i.e. units of the underlying) you would need to create a riskless position - i.e. a portfolio which would be worth the same whether the stock price rose by a very small amount or fell by a very small amount. In such a "delta neutral" portfolio any gain in the value of the shares held due to a rise in the share price would be exactly offset by a loss on the value of the calls written, and vice versa.

Note that as the Delta changes with the stock price and time to expiration the number of shares would need to be continually adjusted to maintain the hedge. How quickly the delta changes with the stock price are given by 'Gamma'.

In addition to Delta there are some other "Greeks" which some find useful when constructing option strategies.

7.4.3 Gamma

It measures how fast the Delta changes for small changes in the underlying stock price i.e. the Delta of the Delta. If you are hedging a portfolio using the Delta-hedge technique then you will want to keep gamma as small as possible, the smaller it is the less often you will have to adjust the hedge to maintain a delta neutral position. If gamma is too large, a small change in stock price could wreck your hedge. Adjusting gamma, however, can be tricky and is generally done using options i.e. it cannot be done by selling or buying underlying asset rather by selling or buying options.

7.4.4 Theta

The change in option price given a one day decrease in time to expiration. Basically, it is a measure of time decay. Unless you and your portfolio are travelling at close to the speed of light the passage of time is constant and inexorable. Thus, hedging a portfolio against time decay, the effects of which are completely predictable, would be pointless.

7.4.5 Rho

The change in option price given a one percentage point change in the risk-free interest rate. It is sensitivity of option value to change in interest rate. Rho indicates the absolute change in option value for a one percent change in the interest rate. For example, a Rho of .060 indicates the option's theoretical value will increase by .060 if the interest rate is decreased by 1.0.

7.4.6 Vega

Sensitivity of option value to change in volatility. Vega indicates an absolute change in option value for a one percent change in volatility. For example, a Vega of .090 indicates an absolute change in the option's theoretical value will increase by .090 if the volatility percentage is increased by 1.0 or decreased by .090 if the volatility percentage is decreased by 1.0. Results may not be exact due to rounding. It can also be stated as the change in option price given a one percentage point change in volatility. Like delta and gamma, Vega is also used for hedging.

8. COMMODITY DERIVATIVES

Trading in commodity derivatives first started to protect farmers from the risk of the value of their crop going below the cost price of their produce. Derivative contracts were offered on various agricultural products like cotton, rice, coffee, wheat, pepper etc.

The first organized exchange, the Chicago Board of Trade (CBOT) -- with standardized contracts on various commodities -- was established in 1848. In 1874, the Chicago Produce Exchange - which is now known as Chicago Mercantile Exchange (CME) was formed.

CBOT and CME are two of the largest commodity derivatives exchanges in the world.

8.1 Necessary Conditions to Introduce Commodity Derivatives

The commodity characteristic approach defines feasible commodities for derivatives trading based on an extensive list of required commodity attributes. It focuses on the technical aspects of the underlying commodity. The following attributes are considered crucial for qualifying for the derivatives trade:

1) a commodity should be durable and it should be possible to store it;
2) units must be homogeneous;
3) the commodity must be subject to frequent price fluctuations with wide amplitude; supply and demand must be large;
4) supply must flow naturally to market and there must be breakdowns in an existing pattern of forward contracting.

The first attribute, durability and storability, has received considerable attention in commodity finance, since one of the economic functions often attributed to commodity derivatives markets is the temporal allocation of stocks. The commodity derivatives market is an integral part of this storage scenario because it provides a hedge against price risk for the carrier of stocks.

Since commodity derivatives contracts are standardized contracts, this approach requires the underlying product to be homogeneous, the second attribute, so that the underlying commodity as defined in the commodity derivatives contract corresponds with the commodity traded in the cash market. This allows for actual delivery in the commodity derivatives market.

The third attribute, a fluctuating price, is of great importance, since firms will feel little incentive to insure themselves against price risk if price changes are small. A broad cash market is important because a large supply of the commodity will make it difficult to establish dominance in the market place and a broad cash market will tend to provide for a continuous and orderly meeting of supply and demand forces.

The last crucial attribute, breakdowns in an existing pattern of forward trading, indicates that cash market risk will have to be present for a commodity derivatives market to come into existence. Should all parties decide to eliminate each and every price fluctuation by using cash forward contracts for example, a commodity derivatives market would be of little interest.

A commodity derivative must reflect the commercial movement of a commodity both loosely and broadly enough, so that price distortions will not be a result of specifications in the contract. To warrant hedging, the contract must be as close a substitute for the cash commodity as possible. Hedging effectiveness is an important determinant in explaining the success of commodity derivatives and as a result considerable attention has been paid to the hedging effectiveness of commodity derivatives.

The total set of customer needs concerning commodity derivatives is differentiated into instrumental needs and convenience needs (see Figure 1). Customers will choose that "service-product" (futures, options, cash forwards, etc.) which best satisfy their needs, both instrumental and convenience, at an acceptable price.

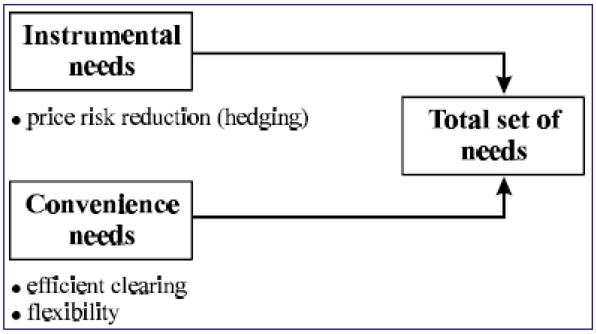


FIGURE 1

Instrumental needs are the hedgers' needs for price risk reduction. Hedgers wish to reduce, or, if possible, eliminate portfolio risks at low cost. The instrumental needs are related to the core service of the commodity derivatives market, which consists of reducing price variability to the customer. Not only do hedgers wish to reduce price risk, they also desire flexibility in doing business, easy access to the market, and an efficient clearing system. These needs are called convenience needs. They deal with the customer's need to be able to use the core service provided by the exchange with relative ease. The extent to which the commodity derivatives exchange is able to satisfy convenience needs determines the process quality. The service offering is not restricted to the core service, but has to be complemented by so-called peripheral services.

8.2 Investing in Commodity Derivatives

Commodity derivatives, which were traditionally developed for risk management purposes, are now growing in popularity as an investment tool. Most of the trading in the commodity derivatives market is being done by people who have no need for the commodity itself.

They just speculate on the direction of the price of these commodities, hoping to make money if the price moves in their favour.

The commodity derivatives market is a direct way to invest in commodities rather than investing in the companies that trade in those commodities.

For example, an investor can invest directly in a steel derivative rather than investing in the shares of Tata Steel. It is easier to forecast the price of commodities based on their demand and supply forecasts as compared to forecasting the price of the shares of a company which depend on many other factors than just the demand and supply of the products they manufacture and sell or trade in.

Also, derivatives are much cheaper to trade in as only a small sum of money is required to buy a derivative contract.

Let us assume that an investor buys a tonne of soybean for ₹ 8,700 in anticipation that the prices will rise to ₹ 9,000 by June 30, 2013. He will be able to make a profit of ₹ 300 on his investment, which is 3.4%. Compare this to the scenario if the investor had decided to buy soybean futures instead.

Before we look into how investment in a derivative contract works, we must familiarise ourselves with the buyer and the seller of a derivative contract. A buyer of a derivative contract is a person who pays an initial margin to buy the right to buy or sell a commodity at a certain price and a certain date in the future.

On the other hand, the seller accepts the margin and agrees to fulfill the agreed terms of the contract by buying or selling the commodity at the agreed price on the maturity date of the contract.

Now let us say the investor buys soybean futures contract to buy one tonne of soybean for ₹ 8,700 (exercise price) on November 30, 2013. The contract is available by paying an initial margin of 10%, i.e. ₹ 870. Note that the investor needs to invest only ₹ 870 here.

On November 30, 2013, the price of soybean in the market is, say, ₹ 9,000 (known as Spot Price -- Spot Price is the current market price of the commodity at any point in time).

The investor can take the delivery of one tonne of soybean at ₹ 8,700 and immediately sell it in the market for ₹ 9,000, making a profit of ₹ 300. So the return on the investment of ₹ 870 is 34.5%. On the contrary, if the price of soybean drops to ₹ 8,400 the investor will end up making a loss of 34.5%.

If the investor wants, instead of taking the delivery of the commodity upon maturity of the contract, an option to settle the contract in cash also exists. Cash settlement comprises exchange of the difference in the spot price of the commodity and the exercise price as per the futures contract.

At present, the option of cash settlement lies only with the seller of the contract. If the seller decides to make or take delivery upon maturity, the buyer of the contract has to fulfill his obligation by either taking or making delivery of the commodity, depending on the specifications of the contract.

In the above example, if the seller decides to go for cash settlement, the contract can be settled by the seller by paying ₹ 300 to the buyer, which is the difference in the spot price of the commodity and the exercise price. Once again, the return on the investment of ₹ 870 is 34.5%.

The above example shows that with very little investment, the commodity futures market offers scope to make big bucks. However, trading in derivatives is highly risky because just as there are high returns to be earned if prices move in favour of the investors, an unfavourable move results in huge losses.

The most critical function in a commodity derivatives exchange is the settlement and clearing of trades. Commodity derivatives can involve the exchange of funds and goods. The exchanges have a separate body to handle all the settlements, known as the clearing house.

For example, the holder of a futures contract to buy soybean might choose to take delivery of soya bean rather than closing his position before maturity. The function of the clearing house or clearing organisation, in such a case, is to take care of possible problems of default by the other party involved by standardising and simplifying transaction processing between participants and the organisation.

Certain special characteristics/benefits of Commodity derivatives trading are:

- To complement investment in companies that use commodities;
- To invest in a country's consumption and production;
- No dividends, only returns from price increases.

In spite of the surge in the turnover of the commodity exchanges in recent years, a lot of work in terms of policy liberalisation, setting up the right legal system, creating the necessary infrastructure, large-scale training programs, etc. still needs to be done in order to catch up with the developed commodity derivative markets.

8.3 Commodity Market

Commodity markets in a crude early form are believed to have originated in Sumer where small baked clay tokens in the shape of sheep or goats were used in trade. Sealed in clay vessels with a certain number of such tokens, with that number written on the outside, they represented a promise to deliver that number.

In modern times, commodity markets represent markets where raw or primary products are exchanged. These raw commodities are traded on regulated, commodity exchanges in which they are bought and sold in standardized contracts.

Some of the advantages of commodity markets are:

- Most money managers prefer derivatives to tangible commodities;
- Less hassle (delivery, etc);
- Allows indirect investment in real assets that could provide an additional hedge against inflation risk.

8.4 Commodity Futures

Almost all the commodities were allowed to be traded in the futures market from April 2003. To make trading in commodity futures more transparent and successful, multi-commodity exchanges at national level were also conceived and these next generation exchanges were allowed to start futures trading in commodities on-line.

The process of trading commodities is also known as Futures Trading. Unlike other kinds of investments, such as stocks and bonds, when you trade futures, you do not actually buy anything or own anything. You are speculating on the future direction of the price in the commodity you are trading. This is like a bet on future price direction. The terms "buy" and "sell" merely indicate the direction you expect future prices will take.

If, for instance, you were speculating in corn, you would buy a futures contract if you thought the price would be going up in the future. You would sell a futures contract if you thought the price would go down. For every trade, there is always a buyer and a seller. Neither person has to own any corn to participate. He must only deposit sufficient capital with a brokerage firm to insure that he will be able to pay the losses if his trades lose money.

On one side of a transaction may be a producer like a farmer. He has a field full of corn growing on his farm. It won't be ready for harvest for another three months. If he is worried about the price going down during that time, he can sell futures contracts equivalent to the size of his crop and deliver his corn to fulfill his obligation under the contract. Regardless of how the price of corn changes in the three months until his crop will be ready for delivery, he is guaranteed to be paid the current price.

On the other side of the transaction might be a producer such as a cereal manufacturer who needs to buy lots of corn. The manufacturer, such as Kellogg, may be concerned that in the next three months the price of corn will go up, and it will have to pay more than the current price. To protect against this, Kellogg can buy futures contracts at the current price. In three months Kellogg can fulfill its obligation under the contracts by taking delivery of the corn. This guarantees that regardless of how the price moves in the next three months, Kellogg will pay no more than the current price for its commodity.

In addition to agricultural commodities, there are futures for financial instruments and intangibles such as currencies, bonds and stock market indexes. Each futures market has producers and consumers who need to hedge their risk from future price changes. The speculators, who do not actually deal in the physical commodities, are there to provide liquidity. This maintains an orderly market where price changes from one trade to the next are small.

Rather than taking delivery or making delivery, the speculator merely offsets his position at some time before the date set for future delivery. If price has moved in the right direction, he will earn profit, if not, he will lose.

Advantages of Commodity Futures

Some of the advantages of commodity futures are:

- Easiest and cheapest way to invest in commodities
- 3 Major Categories like Agricultural products (soft commodities) –fibers, grains, food, livestock; Energy – crude oil, heating oil, natural gas; and Metals – copper, aluminum, gold, silver, platinum

8.5 Commodity Swaps

Producers need to manage their exposure to fluctuations in the prices for their commodities. They are primarily concerned with fixing prices on contracts to sell their produce. A gold producer wants to hedge his losses attributable to a fall in the price of gold for his current gold inventory. A cattle farmer wants to hedge his exposure to changes in the price of his livestock.

End-users need to hedge the prices at which they can purchase these commodities. A university might want to lock in the price at which it purchases electricity to supply its air conditioning units for the upcoming summer months. An airline wants to lock in the price of the jet fuel it needs to purchase in order to satisfy the peak in seasonal demand for travel.

Speculators are funds or individual investors who can either buy or sell commodities by participating in the global commodities market. While many may argue that their involvement is fundamentally destabilizing, it is the liquidity they provide in normal markets that facilitates the business of the producer and of the end-user.

Why would speculators look at the commodities markets? Traditionally, they may have wanted a hedge against inflation. If the general price level is going up, it is probably attributable to increases in input prices. Or, speculators may see tremendous opportunity in commodity markets. Some analysts argue that commodity markets are more technically-driven or more likely to show a persistent trend.

8.5.1 Types of Commodity Swaps

There are two types of commodity swaps: fixed-floating or commodity-for-interest.

(a) Fixed-Floating Swaps: They are just like the fixed-floating swaps in the interest rate swap market with the exception that both indices are commodity based indices.

General market indices in the international commodities market with which many people would be familiar include the S&P Goldman Sachs Commodities Index (S&PGSCI) and the Commodities Research Board Index (CRB). These two indices place different weights on the various commodities so they will be used according to the swap agent's requirements.

(b) Commodity-for-Interest Swaps: They are similar to the equity swap in which a total return on the commodity in question is exchanged for some money market rate (plus or minus a spread).

8.5.2 Valuing Commodity Swaps

In pricing commodity swaps, we can think of the swap as a strip of forwards, each priced at inception with zero market value (in a present value sense). Thinking of a swap as a strip of at-the-money forwards is also a useful intuitive way of interpreting interest rate swaps or equity swaps.

Commodity swaps are characterized by some peculiarities. These include the following factors for which we must account:

(i) The cost of hedging;

(ii) The institutional structure of the particular commodity market in question;

(iii) The liquidity of the underlying commodity market;

(iv) Seasonality and its effects on the underlying commodity market;

(v) The variability of the futures bid/offer spread;

(vi) Brokerage fees; and

(vii) Credit risk, capital costs and administrative costs.

Some of these factors must be extended to the pricing and hedging of interest rate swaps, currency swaps and equity swaps as well. The idiosyncratic nature of the commodity markets refers more to the often limited number of participants in these markets (naturally begging questions of liquidity and market information), the unique factors driving these markets, the inter-relations with cognate markets and the individual participants in these markets.

8.6 Hedging with Commodity Derivatives

Many times when using commodity derivatives to hedge an exposure to a financial price, there is not one exact contract that can be used to hedge the exposure. If you are trying to hedge the value of a particular type of a refined chemical derived from crude oil, you may not find a listed contract for that individual product. You will find an over-the-counter price if you are lucky.

They look at the correlation (or the degree to which prices in the individual chemical trade with respect to some other more liquid object, such as crude oil) for clues as to how to price the OTC product that they offer you. They make assumptions about the stability of the correlation and its volatility and they use that to "shade" the price that they show you.

Correlation is an un-hedgable risk for the OTC market maker, though. There is very little that he can do if the correlation breaks down.

For example, if all of a sudden the price for your individual chemical starts dropping faster than the correlation of the chemical's price with crude oil suggests it should, the OTC dealer has to start dumping more crude oil in order to compensate.

It is a very risky business. The OTC market maker's best hope is to see enough "two-way" business involving end-users and producers so that his exposure is "naturally" hedged by people seeking to benefit from price movement in either direction.

Commodity swaps and commodity derivatives are a useful and important tool employed by most leading energy, chemical and agricultural corporations in today's world.

Note: Please note other forms of Swaps such as Currency Swap and Interest Rate Swap have been discussed in the respective chapters.

TEST YOUR KNOWLEDGE

Theoretical Questions

1. What are the reasons for stock index futures becoming more popular financial derivatives over stock futures segment in India?
2. Write short note on Marking to market.
3. State any four assumptions of Black Scholes Model.
4. Define the term Greeks with respect to options.

Practical Questions

1. The 6-months forward price of a security is ₹ 208.18. The borrowing rate is 8% per annum payable with monthly rests. What should be the spot price?
2. The following data relate to Anand Ltd.'s share price:

Current price per share	₹ 1,800
6 months future's price/share	₹ 1,950

   Assuming it is possible to borrow money in the market for transactions in securities at 12% per annum, you are required:

   (i) to calculate the theoretical minimum price of a 6-months forward purchase; and

   (ii) to explain arbitrate opportunity.
3. On 31-8-2011, the value of stock index was ₹ 2,200. The risk free rate of return has been 8% per annum. The dividend yield on this Stock Index is as under:

Month	Dividend Paid p.a.
January	3%
February	4%
March	3%
April	3%
May	4%

June	3%
July	3%
August	4%
September	3%
October	3%
November	4%
December	3%

Assuming that interest is continuously compounded daily, find out the future price of contract deliverable on 31-12-2011. Given: $e^{0.01583} = 1.01593$

4. Calculate the price of 3 months PQR futures, if PQR (FV ₹10) quotes ₹220 on NSE and the three months future price quotes at ₹230 and the one month borrowing rate is given as 15 percent per annum and the expected annual dividend is 25 percent, payable before expiry. Also examine arbitrage opportunities.

5.

BSE	5000
Value of portfolio	₹ 10,10,000
Risk free interest rate	9% p.a.
Dividend yield on Index	6% p.a.
Beta of portfolio	1.5

We assume that a future contract on the BSE index with four months maturity is used to hedge the value of portfolio over next three months. One future contract is for delivery of 50 times the index.

Based on the above information calculate:

(i) Price of future contract.

(ii) The gain on short futures position if index turns out to be 4,500 in three months.

6. The share of X Ltd. is currently selling for ₹ 300. Risk free interest rate is 0.8% per month. A three month futures contract is selling for ₹ 312. Develop an arbitrage strategy and show what your riskless profit will be 3 months hence assuming that X Ltd. will not pay any dividend in the next three months.

7. A Mutual Fund is holding the following assets in ₹ Crores :

Investments in diversified equity shares	90.00
Cash and Bank Balances	10.00
	100.00

The Beta of the equity shares portfolio is 1.1. The index future is selling at 4300 level. The Fund Manager apprehends that the index will fall at the most by 10%. How many index futures he should short for perfect hedging? One index future consists of 50 units.

Substantiate your answer assuming the Fund Manager's apprehension will materialize.

8. A trader is having in its portfolio shares worth ₹ 85 lakhs at current price and cash ₹ 15 lakhs. The beta of share portfolio is 1.6. After 3 months the price of shares dropped by 3.2%.

   Determine:

   (i) Current portfolio beta

   (ii) Portfolio beta after 3 months if the trader on current date goes for long position on ₹ 100 lakhs Nifty futures.

9. Which position on the index future gives a speculator, a complete hedge against the following transactions:

   (i) The share of Right Limited is going to rise. He has a long position on the cash market of ₹ 50 lakhs on the Right Limited. The beta of the Right Limited is 1.25.

   (ii) The share of Wrong Limited is going to depreciate. He has a short position on the cash market of ₹ 25 lakhs on the Wrong Limited. The beta of the Wrong Limited is 0.90.

   (iii) The share of Fair Limited is going to stagnant. He has a short position on the cash market of ₹ 20 lakhs of the Fair Limited. The beta of the Fair Limited is 0.75.

10. Ram buys 10,000 shares of X Ltd. at a price of ₹ 22 per share whose beta value is 1.5 and sells 5,000 shares of A Ltd. at a price of ₹ 40 per share having a beta value of 2. He obtains a complete hedge by Nifty futures at ₹ 1,000 each. He closes out his position at the closing price of the next day when the share of X Ltd. dropped by 2%, share of A Ltd. appreciated by 3% and Nifty futures dropped by 1.5%.

    What is the overall profit/loss to Ram?

11. On January 1, 2013 an investor has a portfolio of 5 shares as given below:

Security	Price	No. of Shares	Beta
A	349.30	5,000	1.15
B	480.50	7,000	0.40
C	593.52	8,000	0.90
D	734.70	10,000	0.95
E	824.85	2,000	0.85

The cost of capital to the investor is 10.5% per annum.

You are required to calculate:

(i) The beta of his portfolio.

(ii) The theoretical value of the NIFTY futures for February 2013.

(iii) The number of contracts of NIFTY the investor needs to sell to get a full hedge until February for his portfolio if the current value of NIFTY is 5900 and NIFTY futures have a minimum trade lot requirement of 200 units. Assume that the futures are trading at their fair value.

(iv) The number of future contracts the investor should trade if he desires to reduce the beta of his portfolios to 0.6.

No. of days in a year be treated as 365.

Given: In (1.105) = 0.0998 and $e^{(0.015858)}$ = 1.01598

12. Details about portfolio of shares of an investor is as below:

Shares	No. of shares (lakh)	Price per share	Beta
A Ltd.	3.00	₹ 500	1.40
B Ltd.	4.00	₹ 750	1.20
C Ltd.	2.00	₹ 250	1.60

The investor thinks that the risk of portfolio is very high and wants to reduce the portfolio beta to 0.91. He is considering two below mentioned alternative strategies:

(i) Dispose off a part of his existing portfolio to acquire risk free securities, or

(ii) Take appropriate position on Nifty Futures which are currently traded at 8125 and each Nifty points is worth ₹ 200.

You are required to determine:

(1) portfolio beta,

(2) the value of risk free securities to be acquired,

(3) the number of shares of each company to be disposed off,

(4) the number of Nifty contracts to be bought/sold; and

(5) the value of portfolio beta for 2% rise in Nifty.

13. On April 1, 2015, an investor has a portfolio consisting of eight securities as shown below:

Security	Market Price	No. of Shares	Value
A	29.40	400	0.59
B	318.70	800	1.32

C	660.20	150	0.87
D	5.20	300	0.35
E	281.90	400	1.16
F	275.40	750	1.24
G	514.60	300	1.05
H	170.50	900	0.76

The cost of capital for the investor is 20% p.a. continuously compounded. The investor fears a fall in the prices of the shares in the near future. Accordingly, he approaches you for the advice to protect the interest of his portfolio.

You can make use of the following information:

(1) The current NIFTY value is 8500.

(2) NIFTY futures can be traded in units of 25 only.

(3) Futures for May are currently quoted at 8700 and Futures for June are being quoted at 8850.

You are required to calculate:

(i) the beta of his portfolio.

(ii) the theoretical value of the futures contract for contracts expiring in May and June.

Given ($e^{0.03}$ =1.03045, $e^{0.04}$ = 1.04081, $e^{0.05}$ =1.05127)

(iii) the number of NIFTY contracts that he would have to sell if he desires to hedge until June in each of the following cases:

(A) His total portfolio

(B) 50% of his portfolio

(C) 120% of his portfolio

14. Sensex futures are traded at a multiple of 50. Consider the following quotations of Sensex futures in the 10 trading days during February, 2009:

Day	High	Low	Closing
4-2-09	3306.4	3290.00	3296.50
5-2-09	3298.00	3262.50	3294.40
6-2-09	3256.20	3227.00	3230.40
7-2-09	3233.00	3201.50	3212.30
10-2-09	3281.50	3256.00	3267.50

11-2-09	3283.50	3260.00	3263.80
12-2-09	3315.00	3286.30	3292.00
14-2-09	3315.00	3257.10	3309.30
17-2-09	3278.00	3249.50	3257.80
18-2-09	3118.00	3091.40	3102.60

Abshishek bought one sensex futures contract on February, 04. The average daily absolute change in the value of contract is ₹ 10,000 and standard deviation of these changes is ₹ 2,000. The maintenance margin is 75% of initial margin.

You are required to determine the daily balances in the margin account and payment on margin calls, if any.

15. Mr. A purchased a 3 month call option for 100 shares in XYZ Ltd. at a premium of ₹ 30 per share, with an exercise price of ₹ 550. He also purchased a 3 month put option for 100 shares of the same company at a premium of ₹ 5 per share with an exercise price of ₹ 450. The market price of the share on the date of Mr. A's purchase of options, is ₹ 500. Calculate the profit or loss that Mr. A would make assuming that the market price falls to ₹ 350 at the end of 3 months.

16. The market received rumour about ABC corporation's tie-up with a multinational company. This has induced the market price to move up. If the rumour is false, the ABC corporation stock price will probably fall dramatically. To protect from this an investor has bought the call and put options.

    He purchased one 3 months call with a striking price of ₹ 42 for ₹ 2 premium, and paid Re.1 per share premium for a 3 months put with a striking price of ₹ 40.

    (i) Determine the Investor's position if the tie up offer bids the price of ABC Corporation's stock up to ₹ 43 in 3 months.

    (ii) Determine the Investor's ending position, if the tie up programme fails and the price of the stocks falls to ₹ 36 in 3 months.

17. Equity share of PQR Ltd. is presently quoted at ₹ 320. The Market Price of the share after 6 months has the following probability distribution:

Market Price	₹ 180	260	280	320	400
Probability	0.1	0.2	0.5	0.1	0.1

    A put option with a strike price of ₹ 300 can be written.

    You are required to find out expected value of option at maturity (i.e. 6 months)

18. You as an investor had purchased a 4 month call option on the equity shares of X Ltd. of ₹ 10, of which the current market price is ₹ 132 and the exercise price ₹ 150. You expect the price to range between ₹ 120 to ₹ 190. The expected share price of X Ltd. and related probability is given below:

Expected Price (₹)	120	140	160	180	190
Probability	.05	.20	.50	.10	.15

Compute the following:

(i) Expected Share price at the end of 4 months.

(ii) Value of Call Option at the end of 4 months, if the exercise price prevails.

(iii) In case the option is held to its maturity, what will be the expected value of the call option?

19. Mr. X established the following strategy on the Delta Corporation's stock :

(1) Purchased one 3-month call option with a premium of ₹ 30 and an exercise price of ₹ 550.

(2) Purchased one 3-month put option with a premium of ₹ 5 and an exercise price of ₹ 450.

Delta Corporation's stock is currently selling at ₹ 500. Determine profit or loss, if the price of Delta Corporation's :

(i) remains at ₹ 500 after 3 months.

(ii) falls at ₹ 350 after 3 months.

(iii) rises to ₹ 600.

Assume the option size is 100 shares of Delta Corporation.

20. The equity share of VCC Ltd. is quoted at ₹ 210. A 3-month call option is available at a premium of ₹ 6 per share and a 3-month put option is available at a premium of ₹ 5 per share. Ascertain the net payoffs to the option holder of a call option and a put option separately.

(i) the strike price in both cases in ₹ 220; and

(ii) the share price on the exercise day is ₹ 200,210,220,230,240.

Also indicate the price range at which the call and the put options may be gainfully exercised.

21. Sumana wanted to buy shares of EIL which has a range of ₹ 411 to ₹ 592 a month later. The present price per share is ₹ 421. Her broker informs her that the price of this share can sore up to ₹ 522 within a month or so, so that she should buy a one-month CALL of EIL. In order to be prudent in buying the call, the share price should be more than or at least ₹ 522 the assurance of which could not be given by her broker.

Though she understands the uncertainty of the market, she wants to know the probability of attaining the share price ₹ 592 so that buying of a one-month CALL of EIL at the execution price of ₹ 522 is justified. Advice her. Take the risk-free interest to be 3.60% and $e^{0.036} = 1.037$.

22. Mr. Dayal is interested in purchasing equity shares of ABC Ltd. which are currently selling at ₹ 600 each. He expects that price of share may go upto ₹ 780 or may go down to ₹ 480 in three months. The chances of occurring such variations are 60% and 40% respectively. A call option on the shares of ABC Ltd. can be exercised at the end of three months with a strike price of ₹ 630.

    (i) What combination of share and option should Mr. Dayal select if he wants a perfect hedge?

    (ii) What should be the value of option today (the risk free rate is 10% p.a.)?

    (iii) What is the expected rate of return on the option?

23. Consider a two-year call option with a strike price of ₹ 50 on a stock the current price of which is also ₹ 50. Assume that there are two-time periods of one year and in each year the stock price can move up or down by equal percentage of 20%. The risk-free interest rate is 6%. Using binominal option model, calculate the probability of price moving up and down. Also draw a two-step binomial tree showing prices and payoffs at each node.

24. The current market price of an equity share of Penchant Ltd is ₹ 420. Within a period of 3 months, the maximum and minimum price of it is expected to be ₹ 500 and ₹ 400 respectively. If the risk free rate of interest be 8% p.a., what should be the value of a 3 months Call option under the "Risk Neutral" method at the strike rate of ₹ 450?

    Given $e^{0.02} = 1.0202$

25. From the following data for certain stock, find the value of a call option:

Price of stock now	=	₹ 80
Exercise price	=	₹ 75
Standard deviation of continuously compounded annual return	=	0.40
Maturity period	=	6 months
Annual interest rate	=	12%

Given

Number of S.D. from Mean, (z)	Area of the left or right (one tail)
0.25	0.4013
0.30	0.3821
0.55	0.2912

0.60 0.2743

$e^{0.12x0.5}$ = 1.062

In 1.0667 = 0.0646

ANSWERS/ SOLUTIONS

Answers to Theoretical Questions

1. Please refer paragraph 5.2.4
2. Please refer paragraph 5.2.3
3. Please refer paragraph 7.3
4. Please refer paragraph 7.4

Answers to the Practical Questions

1. Calculation of spot price

The formula for calculating forward price is:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Where A = Forward price

P = Spot Price

r = rate of interest

n = no. of compounding

t = time

Using the above formula,

$208.18 = P (1 + 0.08/12)^6$

Or 208.18 = P x 1.0409

P = 208.18/1.0409 = 200

Hence, the spot price should be ₹ 200.

2. **Anand Ltd**

(i) Calculation of theoretical minimum price of a 6 months forward contract-

Theoretical minimum price = ₹ 1,800 + (₹ 1,800 x 12/100 x 6/12) = ₹ 1,908

(ii) Arbitrage Opportunity-

The arbitrageur can borrow money @ 12 % for 6 months and buy the shares at ₹ 1,800. At the same time he can sell the shares in the futures market at ₹ 1,950. On the expiry date 6 months later, he could deliver the share and collect ₹ 1,950 pay off ₹ 1,908 and record a profit of ₹ 42 (₹ 1,950 – ₹ 1,908)

3. The duration of future contract is 4 months. The average yield during this period will be:

$$\frac{3\%+3\%+4\%+3\%}{4} = 3.25\%$$

As per Cost to Carry model the future price will be

$F = Se^{(r_f - D)t}$

Where	S	= Spot Price
	r_f	= Risk Free interest
	D	= Dividend Yield
	t	= Time Period

Accordingly, future price will be

$= ₹\ 2,200\ e^{(0.08-0.0325)\times 4/12} = ₹\ 2,200\ e^{0.01583}$

$= ₹\ 2,200 \times 1.01593 = ₹\ 2235.05$

4. Future's Price = Spot + cost of carry – Dividend

F = 220 + 220 × 0.15 × 0.25 – 0.25** × 10 = 225.75

** Entire 25% dividend is payable before expiry, which is ₹2.50.

Thus, we see that futures price by calculation is ₹225.75 which is quoted at ₹230 in the exchange.

(i) Analysis:

Fair value of Futures less than Actual futures Price:

Futures Overvalued Hence it is advised to sell. Also do Arbitraging by buying stock in the cash market.

Step I

He will buy PQR Stock at ₹220 by borrowing at 15% for 3 months. Therefore, his outflows are:

Cost of Stock	220.00
Add: Interest @ 15 % for 3 months i.e. 0.25 years (220 × 0.15 × 0.25)	8.25
Total Outflows (A)	228.25

Step II

He will sell March 2000 futures at ₹230. Meanwhile he would receive dividend for his stock.

Hence his inflows are	230.00
Sale proceeds of March 2000 futures	2.50
Total inflows (B)	232.50

Inflow – Outflow = Profit earned by Arbitrageur

= 232.50 – 228.25 = 4.25

5. **(i)** Current future price of the index = 5000 + 5000 (0.09-0.06) $\frac{4}{12}$ = 5000+ 50= 5,050

∴ Price of the future contract = ₹ 50 x 5,050 = ₹ 2,52,500

(ii) Hedge ratio = $\frac{1010000}{252500} \times 1.5$ = 6 contracts

Index after there months turns out to be 4500

Future price will be = 4500 + 4500 (0.09-0.06) $\times \frac{1}{12}$ = 4,511.25

Therefore, Gain from the short futures position is = 6 x (5050 – 4511.25) x 50

= ₹1,61,625

Note: Alternatively we can also use daily compounding (exponential) formula.

6. The appropriate value of the 3 months futures contract is –

Fo = ₹ 300 $(1.008)^3$ = ₹ 307.26

Since the futures price exceeds its appropriate value it pays to do the following:-

Action	Initial Cash flow	Cash flow at time T (3 months)
Borrow ₹ 300 now and repay with interest after 3 months	+ ₹ 300	- ₹ 300 $(1.008)^3$ =- ₹ 307.26
Buy a share	- ₹ 300	ST
Sell a futures contract (Fo = 312/-)	0	₹ 312 – ST
Total	₹ 0	₹ 4.74

Such an action would produce a risk less profit of ₹ 4.74.

7. Number of index future to be sold by the Fund Manager is:

$$\frac{1.1 \times 90,00,00,000}{4,300 \times 50} = 4,605$$

Justification of the answer:

Loss in the value of the portfolio if the index falls by 10% is ₹ $\frac{11}{100}$x90 Crore = ₹ 9.90 Crore.

Gain by short covering of index future is: $\frac{0.1 \times 4{,}300 \times 50 \times 4{,}605}{1{,}00{,}00{,}000}$ = 9.90 Crore

This justifies the answer. Further, cash is not a part of the portfolio.

8. (i) Current Portfolio Beta

Current Beta for share portfolio = 1.6

Beta for cash = 0

Current portfolio beta = 0.85 x 1.6 + 0 x 0.15 = 1.36

(ii) Portfolio beta after 3 months:

Beta for portfolio of shares = $\frac{\text{Change in value of portfolio of share}}{\text{Change in value of market portfolio (Index)}}$

$$1.6 = \frac{0.032}{\text{Change in value of market portfolio (Index)}}$$

Change in value of market portfolio (Index) = (0.032 / 1.6) x 100 = 2%

Position taken on 100 lakh Nifty futures : Long

Value of index after 3 months = ₹ 100 lakh x (1.00 - 0.02)

= ₹ 98 lakh

Mark-to-market paid = ₹ 2 lakh

Cash balance after payment of mark-to-market = ₹ 13 lakh

Value of portfolio after 3 months = ₹85 lakh x (1 - 0.032) + ₹13 lakh

= ₹95.28 lakh

Change in value of portfolio = $\frac{₹100 \text{ lakh} - ₹95.28 \text{ lakh}}{₹100 \text{ lakh}}$ = 4.72%

Portfolio beta = 0.0472/0.02 = 2.36

9.

Sl. No. (1)	Company Name (2)	Trend (3)	Amount (₹) (4)	Beta (5)	(₹) (6) [(4) x (5)]	Position (7)
(i)	Right Ltd.	Rise	50 lakh	1.25	62,50,000	Short
(ii)	Wrong Ltd.	Depreciate	25 lakh	0.90	22,50,000	Long
(iii)	Fair Ltd.	Stagnant	20 lakh	0.75	15,00,000	Long
					25,00,000	Short

10. No. of the Future Contract to be obtained to get a complete hedge

$$= \frac{10000 \times ₹22 \times 1.5 - 5000 \times ₹40 \times 2}{₹1000}$$

$$= \frac{₹3,30,000 - ₹4,00,000}{₹1000} = 70 \text{ contracts}$$

Thus, by purchasing 70 Nifty future contracts to be long to obtain a complete hedge.

Cash Outlay

= 10000 x ₹ 22 – 5000 x ₹ 40 + 70 x ₹ 1,000

= ₹ 2,20,000 – ₹ 2,00,000 + ₹ 70,000 = ₹ 90,000

Cash Inflow at Close Out

= 10000 x ₹ 22 x 0.98 – 5000 x ₹ 40 x 1.03 + 70 x ₹ 1,000 x 0.985

= ₹ 2,15,600 – ₹ 2,06,000 + ₹ 68,950 = ₹ 78,550

Gain/ Loss

= ₹ 78,550 – ₹ 90,000 = - ₹ 11,450 (Loss)

11. (i) Calculation of Portfolio Beta

Security	Price of the Stock	No. of shares	Value	Weightage w_i	Beta B_i	Weighted Beta
A	349.30	5,000	17,46,500	0.093	1.15	0.107
B	480.50	7,000	33,63,500	0.178	0.40	0.071
C	593.52	8,000	47,48,160	0.252	0.90	0.227
D	734.70	10,000	73,47,000	0.390	0.95	0.370
E	824.85	2,000	16,49,700	0.087	0.85	0.074
			1,88,54,860			0.849

Portfolio Beta = 0.849

(ii) Calculation of Theoretical Value of Future Contract

Cost of Capital = 10.5% p.a. Accordingly, the Continuously Compounded Rate of Interest In (1.105) = 0.0998

For February 2013 contract, t= 58/365= 0.1589

Further F= Se^{rt}

F= ₹ 5,900$e^{(0.0998)(0.1589)}$

F= ₹ 5,900$e^{0.015858}$

F= ₹ 5,900X1.01598 = ₹ 5,994.28

Alternatively, it can also be taken as follows:

= ₹ 5900 $e^{0.105\times 58/365}$

= ₹ 5900 $e^{0.01668}$

= ₹ 5900 x 1.01682 = ₹ 5,999.24

(iii) When total portfolio is to be hedged:

$$= \frac{\text{Value of Spot Position requiring hedging}}{\text{Value of Future Contract}} \times \text{Portfolio Beta}$$

$$= \frac{1,88,54,860}{5994.28 \times 200} \times 0.849 \;\; = 13.35 \text{ contracts say 13 or 14 contracts}$$

(iv) When total portfolio beta is to be reduced to 0.6:

$$\text{Number of Contracts to be sold} = \frac{P(\beta_P - \beta_P^{'})}{F}$$

$$= \frac{1,88,54,860\,(0.849 - 0.600)}{5994.28 \times 200} \;\; = 3.92 \text{ contracts say 4 contracts}$$

12.

Shares	No. of shares (lakhs) (1)	Market Price of Per Share (2)	× (2) (₹ lakhs)	% to total (w)	ß (x)	wx
A Ltd.	3.00	500.00	1500.00	0.30	1.40	0.42
B Ltd.	4.00	750.00	3000.00	0.60	1.20	0.72
C Ltd.	2.00	250.00	500.00	0.10	1.60	0.16
			5000.00	1.00		1.30

(1) Portfolio beta 1.30

(2) Required Beta 0.91

Let the proportion of risk free securities for target beta 0.91 = p

$$0.91 = 0 \times p + 1.30\ (1 - p)$$

$$p = 0.30 \text{ i.e. } 30\%$$

Shares to be disposed off to reduce beta (5000 × 30%) ₹ 1,500 lakh and Risk Free securities to be acquired.

(3) Number of shares of each company to be disposed off

Shares	% to total (w)	Proportionate Amount (₹ lakhs)	Market Price Per Share	No. of Shares (Lakh)
A Ltd.	0.30	450.00	500.00	0.90
B Ltd.	0.60	900.00	750.00	1.20
C Ltd.	0.10	150.00	250.00	0.60

(4) Number of Nifty Contract to be sold

$$\frac{(1.30\text{-}0.91) \times 5000 \text{ lakh}}{8{,}125 \times 200} = 120 \text{ contracts}$$

(5) 2% rises in Nifty is accompanied by 2% x 1.30 i.e. 2.6% rise for portfolio of shares

	₹ Lakh
Current Value of Portfolio of Shares	5000
Value of Portfolio after rise	5130
Mark-to-Market Margin paid (8125 × 0.020 × ₹ 200 × 120)	39
Value of the portfolio after rise of Nifty	5091
% change in value of portfolio (5091 – 5000)/ 5000	1.82%
% rise in the value of Nifty	2%
Beta	0.91

13. (i) Beta of the Portfolio

Security	Market Price	No. of Shares	Value	β	Value x β
A	29.40	400	11760	0.59	6938.40
B	318.70	800	254960	1.32	336547.20
C	660.20	150	99030	0.87	86156.10
D	5.20	300	1560	0.35	546.00
E	281.90	400	112760	1.16	130801.60
F	275.40	750	206550	1.24	256122.00

G	514.60	300	154380	1.05	162099.00
H	170.50	900	153450	0.76	116622.00
			994450		1095832.30

Portfolio Beta = $\frac{10,95,832.30}{9,94,450} = 1.102$

(ii) Theoretical Value of Future Contract Expiring in May and June

$F = Se^{rt}$

$F_{May} = 8500 \text{ x } e^{0.20 \text{ x } (2/12)} = 8500 \text{ x } e^{0.0333}$

$e^{0.0333}$ shall be computed using Interpolation Formula as follows:

$e^{0.03}$	= 1.03045
$e^{0.04}$	= 1.04081
$e^{0.01}$	= 0.01036
$e^{0.0033}$	= 0.00342
$e^{0.0067}$	= 0.00694

$e^{0.0333}$ = 1.03045 + 0.00342 = 1.03387 or 1.04081 – 0.00694 = 1.03387

According the price of the May Contract

8500 X 1.03387 = ₹ 8788

Price of the June Contract

$F_{May} = 8500 \text{ x } e^{0.20 \text{ x } (3/12)} = 8500 \text{ x } e^{0.05} = 8500 \text{ x } 1.05127 = 8935.80$

(iii) No. of NIFTY Contracts required to sell to hedge until June

$= \frac{\text{Value of Position to be hedged}}{\text{Value of Future Contract}} \times \beta$

(A) Total portfolio

$\frac{994450}{8850 \times 25} \times 1.102 = 4.953$ say 5 contracts

(B) 50% of Portfolio

$\frac{994450 \times 0.50}{8850 \times 25} \times 1.102 = 2.47$ say 3 contracts

(C) 120% of Portfolio

$$\frac{994450 \times 1.20}{8850 \times 25} \times 1.102 = 5.94 \text{ say 6 contracts}$$

14. Initial Margin = μ + 3σ

Where μ = Daily Absolute Change

σ = Standard Deviation

Accordingly

Initial Margin = ₹ 10,000 + ₹ 6,000 = ₹ 16,000

Maintenance margin = ₹ 16,000 x 0.75 = ₹ 12,000

Day	Changes in future Values (₹)	Margin A/c (₹)	Call Money (₹)
4/2/09	-	16000	-
5/2/09	50 x (3294.40 - 3296.50) = -105	15895	-
6/2/09	50 x (3230.40 - 3294.40) = -3200	12695	-
7/2/09	50 x (3212.30 - 3230.40) = -905	16000	4210
10/2/09	50 x (3267.50 - 3212.30) = 2760	18760	-
11/2/09	50 x (3263.80 - 3267.50) = -185	18575	-
12/2/09	50 x (3292 - 3263.80) =1410	19985	-
14/2/09	50 x (3309.30 - 3292) = 865	20850	-
17/2/09	50 x (3257.80 - 3309.30) = -2575	18275	-
18/2/09	50 x (3102.60 - 3257.80) = -7760	16000	5485

15. Since the market price at the end of 3 months falls to ₹ 350 which is below the exercise price under the call option, the call option will not be exercised. Only put option becomes viable.

	₹
The gain will be:	
Gain per share (₹450 – ₹ 350)	100
Total gain per 100 shares	10,000
Cost or premium paid (₹ 30 x 100) + (₹ 5 x 100)	3,500
Net gain	6,500

16. Cost of Call and Put Options

= (₹ 2 per share) x (100 share call) + (₹ 1 per share) x (100 share put)

= ₹ 2 x 100 + 1 x 100

= ₹ 300

(i) Price increases to ₹43. Since the market price is higher than the strike price of the call, the investor will exercise it.

Ending position	= (- ₹ 300 cost of 2 option) + (₹ 1 per share gain on call) x 100
	= - ₹ 300 + 100
Net Loss	= - ₹ 200

(ii) The price of the stock falls to ₹36. Since the market price is lower than the strike price, the investor may not exercise the call option.

Ending Position	= (- ₹300 cost of 2 options) + (₹4 per stock gain on put) x 100
	= - ₹300 + 400
Gain	= ₹100

17. Expected Value of Option

(300 – 180) X 0.1	12
(300 – 260) X 0.2	8
(300 – 280) X 0.5	10
(300 – 320) X 0.1	Not Exercised*
(300 – 400) X 0.1	Not Exercised*
	30

* If the strike price goes beyond ₹ 300, option is not exercised at all.

In case of Put option, since Share price is greater than strike price Option Value would be zero.

18. (i) Expected Share Price

= ₹120X 0.05 + ₹140X 0.20 + ₹160X 0.50 + ₹180X 0.10 + ₹190X 0.15

= ₹6 + ₹28 + ₹80 + ₹18 + ₹28.50 = ₹160.50

(ii) Value of Call Option

= ₹150 - ₹150 = Nil

(iii) If the option is held till maturity the expected Value of Call Option

Expected price (X)	Value of call (C)	Probability (P)	CP
₹ 120	0	0.05	0
₹ 140	0	0.20	0
₹ 160	₹ 10	0.50	₹ 5

₹ 180	₹ 30	0.10	₹ 3
₹ 190	₹ 40	0.15	₹ 6
Total			₹ 14

Alternatively, it can also be calculated as follows:

Expected Value of Option

(120 – 150) X 0.1	Not Exercised*
(140 – 150) X 0.2	Not Exercised*
(160 – 150) X 0.5	5
(180 – 150) X 0.1	3
(190 – 150) X 0.15	6
	14

* If the strike price goes below ₹ 150, option is not exercised at all.

19. **(i)** Total premium paid on purchasing a call and put option

= (₹30 per share × 100) + (₹5 per share × 100).

= 3,000 + 500 = ₹3,500

In this case, X exercises neither the call option nor the put option as both will result in a loss for him.

Ending value = - ₹3,500 + zero gain = - ₹3,500

i.e Net loss = ₹3,500

(ii) Since the price of the stock is below the exercise price of the call, the call will not be exercised. Only put is valuable and is exercised.

Total premium paid = ₹3,500

Ending value = – ₹3,500 + ₹[(450 – 350) × 100] = – ₹3,500 + ₹10,000 = ₹6,500

∴ Net gain = ₹6,500

(iii) In this situation, the put is worthless, since the price of the stock exceeds the put's exercise price. Only call option is valuable and is exercised.

Total premium paid = ₹3,500

Ending value = -3,500 +[(600 – 550) × 100]

Net Gain = -3,500 + 5,000 = ₹1,500

20. Net payoff for the holder of the call option

					(₹)
Share price on exercise day	200	210	220	230	240
Option exercise	No	No	No	Yes	Yes
Outflow (Strike price)	Nil	Nil	Nil	220	220
Out flow (premium)	6	6	6	6	6
Total Outflow	6	6	6	226	226
Less inflow (Sales proceeds)	-	-	-	230	240
Net payoff	-6	-6	-6	4	14

Net payoff for the holder of the put option

					(₹)
Share price on exercise day	200	210	220	230	240
Option exercise	Yes	Yes	No	No	No
Inflow (strike price)	220	220	Nil	Nil	Nil
Less outflow (purchase price)	200	210	-	-	-
Less outflow (premium)	5	5	5	5	5
Net Payoff	15	5	-5	-5	-5

The call option can be exercised gainfully for any price above ₹226 (₹220 + ₹6) and put option for any price below ₹215 (₹220 - ₹5).

21. $p = \frac{e^{rt} - d}{u - d}$

$e^{rt} = e^{0.036}$

d = 411/421 = 0.976

u = 592/421 = 1.406

$$p = \frac{e^{0.036} - 0.976}{1.406 - 0.976} = \frac{1.037 - 0.976}{0.43} = \frac{0.061}{0.43} = 0.1418$$

Thus probability of rise in price 0.1418

22. **(i)** To compute perfect hedge we shall compute Hedge Ratio (Δ) as follows:

$$\Delta = \frac{C_1 - C_2}{S_1 - S_2} = \frac{150 - 0}{780 - 480} = \frac{150}{300} = 0.50$$

Mr. Dayal should purchase 0.50 share for every 1 call option.

(ii) Value of Option today

If price of share comes out to be ₹780 then value of purchased share will be:

Sale Proceeds of Investment (0.50 x ₹ 780)	₹ 390
Loss on account of Short Position (₹ 780 – ₹ 630)	₹ 150
	₹ 240

If price of share comes out to be ₹ 480 then value of purchased share will be:

Sale Proceeds of Investment (0.50 x ₹ 480)	₹ 240

Accordingly, Premium say P shall be computed as follows:

(₹ 300 – P) 1.025 = ₹ 240

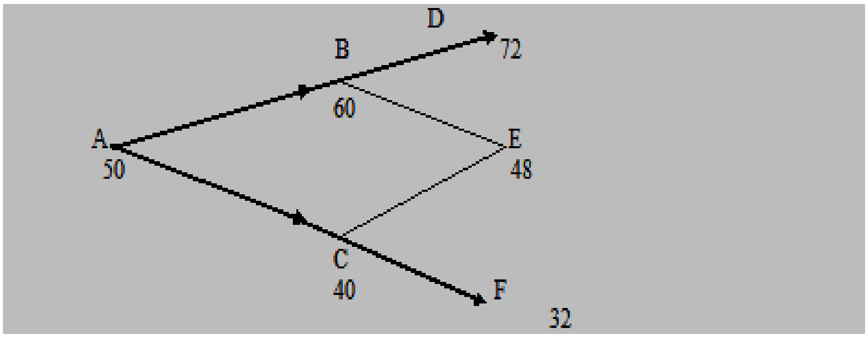
P = ₹65.85

(iii) Expected Return on the Option

Expected Option Value = (₹ 780 – ₹ 630) × 0.60 + ₹ 0 × 0.40 = ₹ 90

Expected Rate of Return = $\frac{90-65.85}{65.85}\times100$ = 36.67%

23. Stock prices in the two step Binominal tree

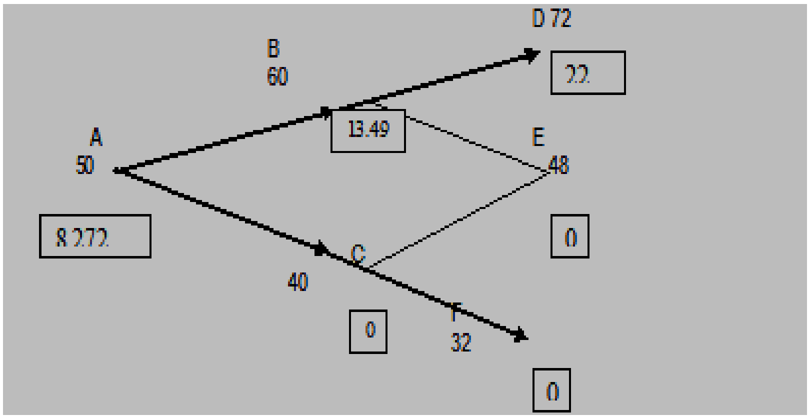


Using the single period model, the probability of price increase is

$$P = \frac{R-d}{u-d} = \frac{1.06-0.80}{1.20-0.80} = \frac{0.26}{0.40} = 0.65$$

therefore the p of price decrease = 1-0.65 = 0.35

The two step Binominal tree showing price and pay off



The value of an American call option at nodes D, E and F will be equal to the value of European option at these nodes and accordingly the call values at nodes D, E and F will be 22, 0 and 0 using the single period binomial model the value of call option at node B is

$$C = \frac{Cup + Cd(1-p)}{R} = \frac{22 \times 0.65 + 0 \times 0.35}{1.06} = 13.49$$

The value of option at node 'A' is

$$\frac{13.49 \times 0.65 + 0 \times 0.35}{1.06} = 8.272$$

24. Let the probability of attaining the maximum price be p

$(500 - 420) \text{ x } p+(400 - 420) \text{ x } (1-p) = 420 \text{ x } (e^{0.02}-1)$

or, $80p - 20(1 - p) = 420 \text{ x } 0.0202$

or, $80p – 20 + 20p = 8.48$

or, $100p = 28.48$

$p = 0.2848$

The value of Call Option in ₹ $= \frac{0.2848x(500-450)}{1.0202} = \frac{0.2848x50 + 0.7152 \times 0}{1.0202} = 13.96$

25. Applying the Black Scholes Formula,

Value of the Call option now:

The Formula $C = SN(d_1) - Ke^{(-rt)} N(d_2)$

$$d_1 = \frac{\ln(S/K) + (r + \sigma^2/2)t}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

Where,

C = Theoretical call premium

S = Current stock price

t = time until option expiration

K = option striking price

r = risk-free interest rate

N = Cumulative standard normal distribution

e = exponential term

σ = Standard deviation of continuously compounded annual return.

In = natural logarithim

$$d_1 = \frac{\ln(1.0667) + (12\% + 0.08)0.5}{0.40\sqrt{0.5}}$$

$$= \frac{0.0646 + (0.2)0.5}{0.40 \times 0.7071}$$

$$= \frac{0.1646}{0.2828}$$

= 0.5820

d_2 = 0.5820 – 0.2828 = 0.2992

$N(d_1)$ = N (0.5820)

$N(d_2)$ = N (0.2992)

Price = $SN(d_1) - Ke^{(-rt)} N(d_2)$

= 80 x $N(d_1)$ – (75/1.062) x $N(d_2)$

Value of option

$$= 80\ N(d_1) - \frac{75}{1.062} \times N(d_2)$$

$N(d_1)$ = N (0.5820) = 0.7197

$N(d_2)$ = N(0.2992) = 0.6176

$$\text{Price} = 80 \times 0.7197 - \frac{75}{1.062} \times 0.6176$$

= 57.57 – 70.62 x 0.6176

= 57.57 – 43.61

= ₹13.96

Teaching Notes:

Students may please note following important point:

Values of $N(d_1)$ and $N(d_2)$ have been computed by interpolating the values of areas under respective numbers of SD from Mean (Z) given in the question.

It may also be possible that in question paper areas under Z may be mentioned otherwise e.g. Cumulative Area or Area under Two tails. In such situation the areas of the respective Zs given in the question will be as follows:

Cumulative Area

Number of S.D. from Mean, (z)	*Cumulative Area*
0.25	0.5987
0.30	0.6179
0.55	0.7088
0.60	0.7257

Two tail area

Number of S.D. from Mean, (z)	*Area of the left and right (two tail)*
0.25	0.8026
0.30	0.7642
0.55	0.5823
0.60	0.5485