

Gate21.IN.45

EE23BTECH11062 - V MANAS

Question:

A sinusoid ($\sqrt{2}\sin(t)$) $\mu(t)$, where $\mu(t)$ is the step input, is applied to a system with transfer function $G(s) = \frac{1}{1+s}$. The amplitude of the steady state output is (GATE 2021 IN 45)

Solution:

Parameter	Description	Value
x(t)	input signal	$\sqrt{2}\sin(t) \cdot \mu(t)$
G(s)	Transfer function	$\frac{1}{1+s}$
ω_0	angular frequency of input signal	1
y(t)	output signal	

TABLE I
VARIABLES USED

$$y(t) = \sqrt{2}|G(j\omega_0)|_{\omega=\omega_0}\sin(t - \angle G(j\omega_0)_{\omega=\omega_0})u(t) \quad (5)$$

$$\Rightarrow y(t) = \sqrt{2}|G(j\omega)|_{\omega=1}\sin(t - \angle G(j\omega)_{\omega=1})u(t) \quad (6)$$

$$|G(j\omega)|_{\omega=1} = \frac{1}{\sqrt{2}}, \angle G(j\omega)_{\omega=1} = -45^\circ \quad (7)$$

$$y(t) = \sqrt{2} \times \frac{1}{\sqrt{2}} \sin(t - 45^\circ)u(t) \quad (8)$$

$$y(t) = \sin(t - 45^\circ)u(t) \quad (9)$$

So, the amplitude of steady state output is 1

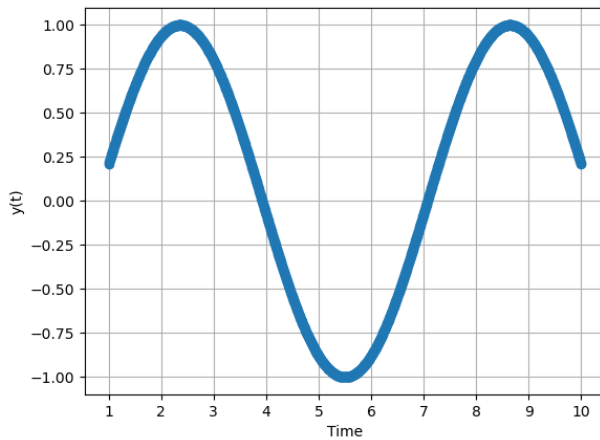


Fig. 1. plot of y(t)

$$G(s) = \frac{1}{s+1} \quad (1)$$

$$G(j\omega) = \frac{1}{j\omega + 1} \quad (2)$$

$$|G(j\omega)| = \frac{1}{\sqrt{\omega^2 + 1}} \quad (3)$$

$$\angle G(j\omega) = -\tan^{-1}(\omega) \quad (4)$$