

10.5.3.15

EE23BTECH11062 - V MANAS

Question:

A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: ₹200 for the first day, ₹250 for the second day, ₹300 for the third day, etc., the penalty for each succeeding day being ₹50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days?

Solution:

Variable	Description	Value
$x(n)$	General term	$(x(0) + n \times d)u(n)$
$x(0)$	First term of AP	200
d	common difference in the AP	50
$y(n)$	sum of $n+1$ terms of AP	

TABLE I
VARIABLES USED

$$x(n) = (200 + 50n)u(n) \quad (1)$$

$$X(z) = \frac{200}{1 - z^{-1}} + \frac{200z^{-1}}{(1 - z^{-1})^2} \quad |z| > |1| \quad (2)$$

$$y(n) = x(n) * u(n) \quad (3)$$

$$Y(z) = X(z) U(z) \quad (4)$$

$$\Rightarrow Y(z) = \left(\frac{200}{1 - z^{-1}} + \frac{50z^{-1}}{(1 - z^{-1})^2} \right) \left(\frac{1}{1 - z^{-1}} \right) \quad (5)$$

$$= \frac{200}{(1 - z^{-1})^2} + \frac{50z^{-1}}{(1 - z^{-1})^3} \quad (6)$$

Contour integration to find z transform

$$y(29) = \frac{1}{2\pi j} \oint_c Y(Z) z^{28} dz \quad (7)$$

$$= \frac{1}{2\pi j} \oint_c \frac{(200 - 150z^{-1})z^{28}}{(1 - z^{-1})^3} \quad (8)$$

$$(9)$$

pole at 1 repeated 3 times

$$m = 3 \quad (10)$$

$$R = \frac{1}{(m-1)!} \lim_{z \rightarrow a} \frac{d^{m-1}}{dz^{m-1}} ((za)^m f(z)) \quad (11)$$

$$= \frac{1}{(2!)} \lim_{z \rightarrow 1} \frac{d^2}{dz^2} \left((z-1)^3 \frac{(200 - 150z^{-1})z^{28}}{(1 - z^{-1})^3} \right) \quad (12)$$

$$= \lim_{z \rightarrow 1} \frac{d^2}{dz^2} (100 - 75z^{-1}) z^{31} \quad (13)$$

$$\Rightarrow y(n) = 27750 \quad (14)$$

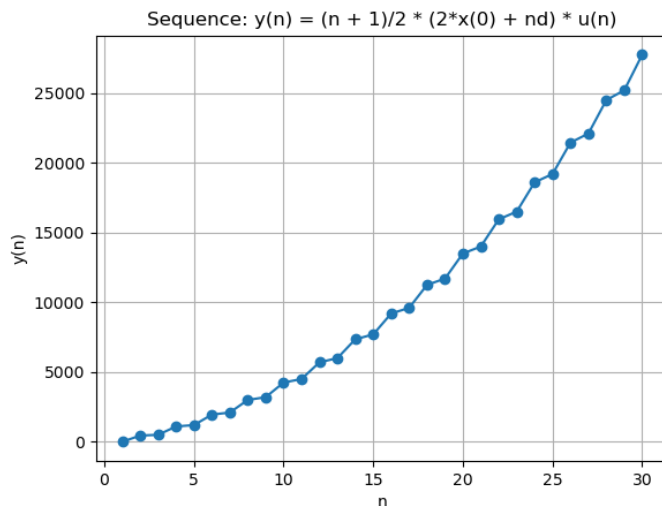


Fig. 1. Stem Plot of $x(n)$