10.5.3.15

EE23BTECH11062 - V MANAS

Question:

A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: ₹200 for the first day, ₹250 for the second day, ₹300 for the third day, etc., the penalty for each succeeding day being ₹50 more than for the preceding day.How much money the contractor has to pay as penalty, if he has delayed the work by 30 days?

Solution:

Variable	Description	Value
x(n)	General term	$(x(0) + n \times d)$u(n)
x(0)	First term of AP	200
d	common difference in the AP	50
y(n)	sum of n+1 terms of AP	

TABLE 1
VARIABLES USED
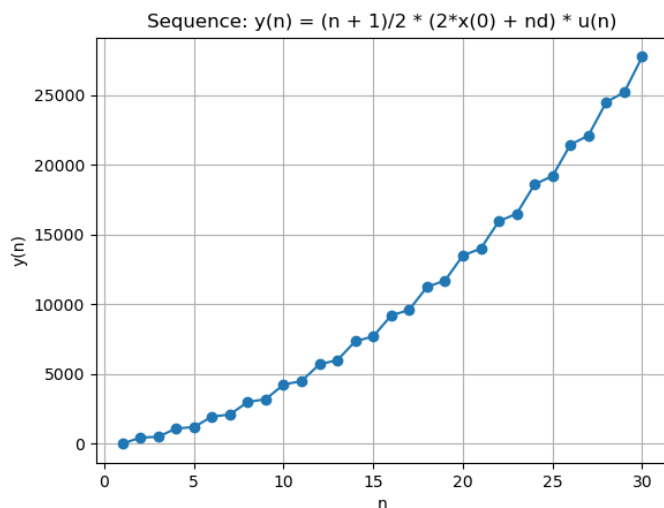

Fig. 1. Stem Plot of y(n)

$$x(n) = (200 + 50n)u(n) \tag{1}$$

$$X(z) = \frac{200}{1 - z^{-1}} + \frac{200z^{-1}}{(1 - z^{-1})^2} \qquad |z| > |1| \tag{2}$$

$$y(n) = x(n) * u(n) \tag{3}$$

$$Y(z) = X(z)\,U(z) \tag{4}$$

$$\implies Y(z) = \left(\frac{200}{1 - z^{-1}} + \frac{50z^{-1}}{(1 - z^{-1})^2} \right)\left(\frac{1}{1 - z^{-1}} \right) \tag{5}$$

$$= \frac{200}{(1 - z^{-1})^2} + \frac{50z^{-1}}{(1 - z^{-1})^3} \tag{6}$$

Contour integration to find z transform

$$y(29) = \frac{1}{2\pi j} \oint_c Y(Z)\, z^{28} dz \tag{7}$$

$$= \frac{1}{2\pi j} \oint_c \frac{(200 - 150z^{-1})z^{28}}{(1 - z^{-1})^3} \tag{8}$$

$$\tag{9}$$

pole at 1 repeated 3 times

$$m = 3 \tag{10}$$

$$R = \frac{1}{(m - 1)!} \lim_{z \to a} \frac{d^{m-1}}{dz^{m-1}} ((za)^m f(z)) \tag{11}$$

$$= \frac{1}{(2!)} \lim_{z \to 1} \frac{d^2}{dz^2} \left((z - 1)^3 \frac{(200 - 150z^{-1})z^{28}}{(1 - z^{-1})^3} \right) \tag{12}$$

$$= \lim_{z \to 1} \frac{d^2}{dz^2} \left(100 - 75z^{-1} \right) z^{31} \tag{13}$$

$$\implies y(n) = 27750 \tag{14}$$