11.9.5.3

EE23BTECH11062 - V MANAS

Question:
Let the sum of $n, 2n, 3n$ terms of an AP be S_1, S_2 and S_3, respectively, show that $S_3 = 3(S_2 - S_1)$
Solution:

Variable	Description
x(0)	First term of AP
d	common difference in the AP
n	number of terms in AP

TABLE I
Variables Used

By performing inverse Z transform on S_1(z) using contour integration

$$S_1 = \frac{1}{2\pi j}\oint_c S(Z)\, z^{n-1} dz \tag{1}$$

$$S_1 = \frac{1}{2\pi j}\oint_c \left(\frac{x(0)z^{n-1}}{(1-z^{-1})^2} + \frac{dz^{n-2}}{(1-z^{-1})^3}\right) dz \tag{2}$$

For R1 the pole has been repeated twice

$$R_1 = \frac{1}{1!}\lim_{z\to 1}\frac{d}{dz}\left((z-1)^2\frac{x(0)z^{n+1}}{(z-1)^2}\right) \tag{3}$$

$$= x(0)(n+1)\lim_{z\to 1} z^n \tag{4}$$

$$= x(0)(n+1) \tag{5}$$

For R2 the pole has been repeated thrice

$$R_2 = \frac{1}{2!}\lim_{z\to 1}\frac{d^2}{dz^2}\left((z-1)^3\frac{dz^{n+1}}{(z-1)^3}\right) \tag{6}$$

$$= \frac{d(n+1)}{2}\lim_{z\to 1}\frac{d}{dz}(z^n) \tag{7}$$

$$= \frac{d(n+1)(n)}{2}\lim_{z\to 1}(z^{n-1}) \tag{8}$$

$$= \frac{d(n+1)(n)}{2} \tag{9}$$

$$\implies R = R_1 + R_2$$

$$\implies S_1 = \frac{n+1}{2}(2x(0) + nd)u(n) \tag{10}$$

similarly,

$$\implies S_2 = \frac{2n+1}{2}(2x(0) + 2nd)u(n) \tag{11}$$

$$\implies S_3 = \frac{3n+1}{2}(2x(0) + 3nd)u(n) \tag{12}$$

$$\implies RHS = 3(S_2 - S_1)$$

$$RHS = 3\left(\frac{2n+1}{2}(2x(0) + 2nd)u(n) - \frac{n+1}{2}(2x(0) + nd)u(n)\right) \tag{13}$$

$$= \frac{3n+1}{2}(2x(0) + 3nd)u(n) \tag{14}$$

$\therefore$ LHS=RHS shows that $S_3 = 3(S_2 - S_1)$