**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans : B (0.2676)

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans : We have µ = 38 , σ =6

1. Probability of employees greater than age of 44= P(x > 44)

P(x > 44) = 1- P(x ≤ 44)

Z=(x – μ)/σ = (x - 38)/6

P(x ≤ 44) = P(Z ≤ (44 – 38)/6) = P(Z ≤ 1) = 84.1345%

Probability that the employee will be greater than age of 44 = 100 – 84.1345%

The probability of number of employees between 38 – 44 years of age = P(x < 44) – 0.5

= 84.1345 – 0.5 = 34.1345%

More employees at the processing center are older than 4 than between 38 and 44

This statement is TRUE.

1. Probability of employees less than age of 30 = P(x < 30).

Z = (x – μ)/ σ = (30 – 38)/6

P(x ≤ 30) = P(Z ≤ (30 -38)/6) P(Z ≤ -1.3333) = 9.12%

Under age 30 = 0.0912\*400 = 36.48 that is 36 employees.

The given statement is TRUE.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans : X1 ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are iid normal random variables

2X 2 = (2μ , 4σ2 )

X1 + X2 = (2μ , 2σ2 )

2X1 – (X1 + X2) = (0 , 6σ2)

2X1 – (X1 + X2) ~ N(0 , 6σ2)

The difference says that rwo variables are identically independently distributed.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans : Z = (x - 100)/20

      a = -(20\*2.576) + 100= 48.5

      b = (20\*2.576) + 100= 151.5

Option D.48.5 ,151.5 is correct.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Ans :

