CT5 – P MAS – 16

Mock Exam A Solutions

ActEd Study Materials: 2016 Examinations

Subject CT5

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Mock Exam A – Solutions

Markers: This document sets out one approach to solving each of the questions. Please give credit for other valid approaches. Some of the final answers are sensitive to rounding. Again, please give appropriate credit.

Solution 1

A decrement has a selective effect if individuals who do and do not experience it will experience different levels of the primary decrement of interest (usually mortality or morbidity) in the future. [1]

For example, ill-health retirement has a selective effect, as it results in lighter mortality among the remaining (*ie* active) members of a pension scheme. [1]
[Total 2]

Award 1 mark for any suitable example. Other possibilities include:

- *Married men usually experience lighter mortality and morbidity than those of the same age who do not get married, so the decrement of marriage has a selective effect.*
- *Withdrawal from a pension scheme is sometimes associated with voluntary resignation. This tends to select those with lighter mortality as people in poor health are less likely to move to a new job.*
- *Those migrating to a new country might experience different levels of mortality and morbidity to those who do not migrate.*

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Solution 2

For the non-unit reserve, we need to zeroise negative cashflows. The non-unit reserve per policy in force at time 3 is:

$${}_3V = \frac{161}{1.02} = 157.84 \qquad [½]$$

So, the new cashflow at time 2, would be:

$$-\left[70 + p_{54} \times {}_3V\right] = -\left[70 + (1 - 0.003976) \times 157.84\right] = -227.21 \qquad [½]$$

Similarly:

$${}_2V = \frac{227.21}{1.02} = 222.76 \qquad [½]$$

So, the new cashflow at time 1, would be:

$$-175 - p_{[52]+1} \times {}_2V = -\left[-175 + (1 - 0.003452) \times 222.76\right] = -46.99 \qquad [½]$$

Finally:

$${}_1V = \frac{46.99}{1.02} = 46.07 \qquad [½]$$

Along with the unit reserve at time 1, we get the total reserve at this time to be:

$$1,051 + 46 = 1,097 \qquad [½]$$

[Total 3]

Solution 3

(a) ***Definition***

This expression denotes the probability that a life aged exactly 50, who entered the population as a select life one year ago, at age 49 exact, dies between his 60th and 64th birthdays. [1½]

(b) ***Calculation***

Using standard life table functions, we have:

$$ {}_{10|4}q_{[49]+1} = \frac{l_{60} - l_{64}}{l_{[49]+1}} = \frac{9,287.2164 - 8,934.8771}{9,711.3524} = 0.036281 \qquad [1½] $$

[Total 3]

Alternative solution

Students could have approached the question by breaking it down into a survival part and a mortality part:

$$ {}_{10|4}q_{[49]+1} = {}_{10}p_{[49]+1} \times {}_4q_{60} = \frac{l_{60}}{l_{[49]+1}}\left[1 - \frac{l_{64}}{l_{60}}\right] = \frac{9,287.2164}{9,711.3524}\left[1 - \frac{8,934.8771}{9,287.2164}\right] $$
$$ = 0.956326 \times 0.037938 = 0.036281 $$

Solution 4

We will need to construct an equation of value:

$$ EPV(\text{premiums}) = EPV(\text{benefits}) $$

Assume P is the net annual premium required.

(1) ***EPV of temporary sickness benefit***

This is:

$$ 6,000 \int_0^{20} v^t \, {}_tp_{45}^{HT} \, dt \qquad [1] $$

(2) ***EPV of permanent sickness benefit***

While permanently sick, £12,000 *pa* is paid. So the expected present value is:

$$12,000\int_0^{20} v^t\,{}_tp_{45}^{HS}\,dt$$ [½]

(3) ***EPV of death benefit***

On death at time t, a benefit of $P\times t$ would be payable, regardless of which state is then currently occupied (this is because the waived premiums are included in the benefit amount). As death can occur while in any of the three other states, the expected present value is:

$$P\int_0^{20} t\,v^t\left[{}_tp_{45}^{HH}\mu_{45+t}+{}_tp_{45}^{HT}\nu_{45+t}+{}_tp_{45}^{HS}\phi_{45+t}\right]dt$$ [1½]

(4) ***EPV of premiums***

Premiums are only payable by healthy lives. So the expected present value is:

$$P\int_0^{20} v^t\,{}_tp_{45}^{HH}\,dt$$ [½]

So by setting $(1)+(2)+(3)=(4)$, we obtain:

$$P=\frac{6,000\int_0^{20} v^t\,{}_tp_{45}^{HT}\,dt+12,000\int_0^{20} v^t\,{}_tp_{45}^{HS}\,dt}{\int_0^{20} v^t\,{}_tp_{45}^{HH}\,dt-\int_0^{20} t\,v^t\left[{}_tp_{45}^{HH}\mu_{45+t}+{}_tp_{45}^{HT}\nu_{45+t}+{}_tp_{45}^{HS}\phi_{45+t}\right]dt}$$ [½]

[Total 4]

Alternatively, students can use $e^{-\delta t}$ in place of v^t.

Solution 5

(i) ***Present value random variable***

If the male dies first, the present value is $250,000v^{T_y}$ *if* $T_x < T_y < T_x + 5$, *and 0 otherwise. Similarly, if the female dies first, the present value is* $250,000v^{T_x}$ *if* $T_y < T_x < T_y + 5$, *and 0 otherwise.*

So we could write:

$$\begin{cases} 250,000v^{T_y} & \text{if } T_x < T_y < T_x + 5 \\ 250,000v^{T_x} & \text{if } T_y < T_x < T_y + 5 \\ 0 & \text{otherwise} \end{cases}$$

The first two expressions can be combined so as to produce the following present value:

$$\begin{cases} 250,000v^{\max(T_x,T_y)} & \text{if } \max\left(T_x,T_y\right) < \min\left(T_x,T_y\right) + 5 \\ 0 & \text{otherwise} \end{cases}$$

The present value can therefore be written as:

$$\begin{cases} 250,000v^{\max(T_x,T_y)} & \text{if } \left|T_x - T_y\right| < 5 \\ 0 & \text{otherwise} \end{cases}$$

[3]

Alternative solution

Alternatively, students could use the following approach:

The benefit will be discounted by T_y *years (ie from the moment of* (y) *'s death) if:*

- $T_y \le 5$ *and* (x) *dies before time* T_y
- $T_y > 5$ *and* (x) *dies before time* T_y *but after time* $T_y - 5$.

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Similarly, the benefit will be discounted by T_x years (ie from the moment of (x)'s death) if:

- $T_x \leq 5$ *and* (y) *dies before time* T_x
- $T_x > 5$ *and* (y) *dies before time* T_x *but after time* $T_x - 5$.

The present value can therefore be written as:

$$\begin{cases} 250{,}000v^{T_y} & \text{if } T_x < T_y < 5 \text{ or } 0 < T_y - 5 < T_x < T_y \\ 250{,}000v^{T_x} & \text{if } T_y < T_x < 5 \text{ or } 0 < T_x - 5 < T_y < T_x \\ 0 & \text{otherwise} \end{cases}$$ *[3]*

Markers: full marks should be awarded for any correct present value formula or equivalent expression (provided it includes the T_x and T_y terms appropriately). Deduct ½ mark for each error or omission.

(ii) ***Expected present value of the revised contract***

If (y) dies within the first 5 years, the benefit is payable on the death of (y) at time t provided (x) is dead by time t. The expected present value is then:

$$250{,}000\int_0^5 v^t \,{}_tp_y\, \mu_{y+t}\, {}_tq_x\, dt$$

If (y) dies after 5 years, then the benefit is payable on the death of (y) at time t provided (x) died between time $t-5$ and t. The expected present value of this is:

$$250{,}000\int_5^\infty v^t \,{}_tp_y\, \mu_{y+t}\left({}_tq_x - {}_{t-5}q_x\right) dt$$

The expected present value is:

$$250{,}000\left[\int_0^5 v^t \,{}_tp_y\, \mu_{y+t}\, {}_tq_x\, dt + \int_5^\infty v^t \,{}_tp_y\, \mu_{y+t}\left({}_tq_x - {}_{t-5}q_x\right) dt\right]$$ [2]

Alternative solutions

Alternatively, we could give this expression as:

$$250{,}000\left[\int_0^5 v^t\,{}_tp_y\,\mu_{y+t}\;{}_tq_x\,dt+\int_5^\infty v^t\,{}_tp_y\,\mu_{y+t}\;{}_{t-5}p_x\;{}_5q_{x+t-5}\,dt\right]$$

or $$250{,}000\left[\int_0^\infty v^t\,{}_tp_y\,\mu_{y+t}\;{}_tq_x\,dt-\int_0^\infty v^{t+5}\,{}_{t+5}p_y\,\mu_{y+t+5}\;{}_tq_x\,dt\right]$$ *[2]*

Another way to think about this is as follows. If (x) dies at time s and (y) dies at time t then we have:

$$250{,}000\int_0^\infty {}_sp_x\,\mu_{x+s}\int_s^{s+5} v^t\;{}_tp_y\,\mu_{y+t}\,dt\,ds$$ *[2]*

Markers: in each case, award one mark for each of the two integrals. Deduct ½ mark for each error or omission. Note that no marks are available for simplifying the expression further, as this was not asked for in the question.

Solution 6

Expected present value

The expected present value of this deferred whole life assurance is:

$$50{,}000\;{}_{15|}\bar{A}_{[45]}=50{,}000v^{15}\;{}_{15}p_{[45]}\,\bar{A}_{60}$$ [1]

Using AM92 Select mortality and an interest rate of 6% *pa*, this gives a value of:

$$\begin{aligned}50{,}000\;{}_{15|}\bar{A}_{[45]}&=50{,}000\times1.06^{-15}\times\frac{l_{60}}{l_{[45]}}\times1.06^{0.5}A_{60}\\&=50{,}000\times1.06^{-15}\times\frac{9{,}287.2164}{9{,}798.0837}\times1.06^{0.5}\times0.32692\\&=50{,}000\times0.133122\\&=£6{,}656.12\end{aligned}$$ [1]

Standard deviation

The variance of benefits under this policy is given by the expression:

$$50{,}000^2\left({}_{15|}{}^2\bar{A}_{[45]}-\left({}_{15|}\bar{A}_{[45]}\right)^2\right) \qquad [1]$$

where the 2 superscript on the first term in the brackets indicates that the function is evaluated at an interest rate of $1.06^2-1=12.36\%$.

From the expected present value calculation we know that ${}_{15|}\bar{A}_{[45]}=0.133122$. Also, using AM92 Select mortality and an interest rate of 6% *pa*, we have:

$$\begin{aligned}{}_{15|}{}^2\bar{A}_{[45]}&=1.1236^{-15}\times\frac{l_{60}}{l_{[45]}}\times1.1236^{0.5}\times{}^2A_{60}\\&=1.1236^{-15}\times\frac{9{,}287.2164}{9{,}798.0837}\times1.1236^{0.5}\times0.14098\\&=0.024662\end{aligned} \qquad [1]$$

So the variance is:

$$50{,}000^2\left(0.024662-(0.133122)^2\right)=17{,}351{,}625 \qquad [½]$$

and the standard deviation is:

$$\sqrt{17{,}351{,}625}=£4{,}166 \qquad [½]$$

[Total 5]

Solution 7

If we first ignore the condition that the first death has to occur within 10 years, then we have an annuity that commences on the first death and ceases on the second death. The annuity would then be payable for the duration of the last survivor status, less the duration of the joint life status. The EPV of this is:

$$20,000\left(\bar{a}^{m\ f}_{\overline{65:61}} - \bar{a}^{m\ f}_{65:61}\right)$$

We now need to adjust this, to allow for the fact that we only want to include payments where the first death occurs in the first 10 years. So we need to ***deduct*** *the EPV of payments that would occur if the first death happens* ***after*** *10 years, ie if both lives survive the first 10 years. This EPV is:*

$$20,000\ {}_{10}p^{m\ f}_{65:61}\, v^{10}\left(\bar{a}^{m\ f}_{\overline{75:71}} - \bar{a}^{m\ f}_{75:71}\right)$$

So the required EPV is:

$$EPV = 20,000\left[\bar{a}^{m\ f}_{\overline{65:61}} - \bar{a}^{m\ f}_{65:61} - {}_{10}p^{m\ f}_{65:61}\, v^{10}\left(\bar{a}^{m\ f}_{\overline{75:71}} - \bar{a}^{m\ f}_{75:71}\right)\right] \qquad [3]$$

Now:

$$\bar{a}^{m\ f}_{\overline{65:61}} - \bar{a}^{m\ f}_{65:61} \approx \ddot{a}^{m\ f}_{\overline{65:61}} - \tfrac{1}{2} - \ddot{a}^{m\ f}_{65:61} + \tfrac{1}{2} = \ddot{a}^{m\ f}_{\overline{65:61}} - \ddot{a}^{m\ f}_{65:61} \qquad [½]$$

$$= \ddot{a}^{m}_{65} + \ddot{a}^{f}_{61} - 2\ddot{a}^{m\ f}_{65:61} = 13.666 + 16.311 - 2\times 12.560 = 4.857 \qquad [1½]$$

Similarly:

$$\bar{a}^{m\ f}_{\overline{75:71}} - \bar{a}^{m\ f}_{75:71} \approx \ddot{a}^{m}_{75} + \ddot{a}^{f}_{71} - 2\ddot{a}^{m\ f}_{75:71} = 9.456 + 12.535 - 2\times 8.238 = 5.515 \qquad [½]$$

and:

$$v^{10}\ {}_{10}p^{m}_{65}\ {}_{10}p^{f}_{61} = 1.04^{-10} \times \frac{8,405.160}{9,647.797} \times \frac{9,299.747}{9,828.163} = 0.556908 \qquad [1]$$

Therefore:

$$EPV = 20,000\left[4.857 - 0.556908 \times 5.515\right] = £35,713 \qquad [½]$$

[Total 7]

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Possible alternative solution

We have a benefit payable to the female if the male dies first, or a benefit payable to the male if the female dies first. We start by considering the former. If you can't remember the formula for evaluating this type of benefit, you can work it out using integrals. If the annuity were payable continuously at the rate of 1 pa, then the expected present value of the benefit payable to the female following the death of the male is:

$$\int_0^{10} v^t \, {}_tp_{65}^m \, \mu_{65+t}^m \, {}_tp_{61}^f \, \bar{a}_{61+t}^f \, dt$$

We can think of this as follows:

- *the male dies at time t, with the female still alive*
- *the female receives an annuity for the rest of her life*
- *discount this benefit back to time 0*
- *integrate over all possible values of t.*

Then write:

$$\int_0^{10} = \int_0^{\infty} - \int_{10}^{\infty}$$

Integrating from 0 to ∞ *gives* $\bar{a}_{65|61}$. *In addition:*

$$\int_{10}^{\infty} v^t \, {}_tp_{65}^m \, \mu_{65+t}^m \, {}_tp_{61}^f \, \bar{a}_{61+t}^f \, dt = \int_0^{\infty} v^{s+10} \, {}_{s+10}p_{65}^m \, \mu_{75+s}^m \, {}_{s+10}p_{61}^f \, \bar{a}_{71+s}^f \, ds$$

$$= v^{10} \, {}_{10}p_{65}^m \, {}_{10}p_{61}^f \int_0^{\infty} v^s \, {}_sp_{75}^m \, \mu_{75+s}^m \, {}_sp_{71}^f \, \bar{a}_{71+s}^f \, ds$$

$$= v^{10} \, {}_{10}p_{65}^m \, {}_{10}p_{61}^f \, \bar{a}_{75|71}^{m\ f}$$

So we have the required formula:

$$\bar{a}_{65|61}^{m\ f} - v^{10} \, {}_{10}p_{65}^m \, {}_{10}p_{61}^f \, \bar{a}_{75|71}^{m\ f}$$

The expected present value of the benefit payable to the female following the death of the male is:

$$20{,}000\left[\bar{a}_{65|61}^{m\ f} - v^{10} \, {}_{10}p_{65}^m \, {}_{10}p_{61}^f \, \bar{a}_{75|71}^{m\ f}\right] \qquad [1½]$$

The values of the factors in this expression are:

$$\bar{a}^{m\ f}_{65|61} = \bar{a}^{f}_{61} - \bar{a}^{m\ f}_{65:61} \approx \ddot{a}^{f}_{61} - \ddot{a}^{m\ f}_{65:61} = 16.311 - 12.560 = 3.751$$

$$\bar{a}^{m\ f}_{75|71} = \bar{a}^{f}_{71} - \bar{a}^{m\ f}_{75:71} \approx \ddot{a}^{f}_{71} - \ddot{a}^{m\ f}_{75:71} = 12.535 - 8.238 = 4.297$$ *[1]*

and:

$$v^{10}\ {}_{10}p^{m}_{65}\ {}_{10}p^{f}_{61} = 1.04^{-10} \times \frac{8,405.160}{9,647.797} \times \frac{9,299.747}{9,828.163} = 0.556908$$ *[1]*

So the expected present value of this benefit is:

$$20,000\left[3.751 - 0.556908 \times 4.297\right] = £27,159$$ *[½]*

Similarly, the expected present value of the benefit payable to the male following the death of the female is:

$$20,000\left[\bar{a}^{f\ m}_{61|65} - v^{10}\ {}_{10}p^{m}_{65}\ {}_{10}p^{f}_{61}\ \bar{a}^{f\ m}_{71|75}\right]$$ *[1½]*

We have:

$$\bar{a}^{f\ m}_{61|65} = \bar{a}^{m}_{65} - \bar{a}^{m\ f}_{65:61} \approx \ddot{a}^{m}_{65} - \ddot{a}^{m\ f}_{65:61} = 13.666 - 12.560 = 1.106$$

$$\bar{a}^{f\ m}_{71|75} = \bar{a}^{m}_{75} - \bar{a}^{m\ f}_{75:71} \approx \ddot{a}^{m}_{75} - \ddot{a}^{m\ f}_{75:71} = 9.456 - 8.238 = 1.218$$ *[½]*

So the expected present value of this benefit is:

$$20,000\left[1.106 - 0.556908 \times 1.218\right] = £8,554$$ *[½]*

The expected present value of the combined benefit is therefore:

$$27,159 + 8,554 = £35,713$$ *[½]*

[Total 7]

Solution 8

(i)(a) ***Uniform distribution of deaths assumption***

The total number of deaths between the ages of 50 and 60 is:

$$9,941.923 - 9,826.131 = 115.792 \qquad [½]$$

If deaths occur uniformly between the ages of 50 and 60, we would expect to get $\frac{115.792}{2} = 57.896$ deaths between the ages of 50 and 55 (and the other 57.896 deaths between the ages of 55 and 60). [½]

So, under this assumption, the probability of dying between the ages of 50 and 55 is:

$${}_5q_{50} = \frac{57.896}{9,941.923} = 0.005823 \qquad [½]$$

[Total 1½]

Alternative solutions

Students could have used:

$${}_5q_{50} = \frac{1}{2} \times {}_{10}q_{50} = \frac{1}{2} \times \frac{115.792}{9,941.923} = 0.005823 \qquad [1½]$$

Or, one other possibility is:

$$\begin{aligned} {}_5q_{50} = 1 - {}_5p_{50} = 1 - \frac{l_{55}}{l_{50}} &= 1 - \frac{0.5l_{50} + 0.5l_{60}}{l_{50}} \\ &= 1 - \frac{0.5 \times 9,941.923 + 0.5 \times 9,826.131}{9,941.923} \\ &= 0.005823 \end{aligned} \qquad [1½]$$

(i)(b) ***Constant force of mortality assumption***

Assuming a constant force of mortality of μ between the ages of 50 and 60, we have:

$${}_{10}p_{50} = e^{-10\mu} \quad \text{and} \quad {}_5p_{50} = e^{-5\mu} = \left({}_{10}p_{50}\right)^{½}$$

Using the figures given in the question:

$$_{10}p_{50} = \frac{9,826.131}{9,941.923} = 0.988353$$ [½]

So $$_{5}p_{50} = \sqrt{\frac{9,826.131}{9,941.923}} = \sqrt{0.988353} = 0.994160$$ [½]

and $$_{5}q_{50} = 1 - 0.994160 = 0.005840$$ [½]

[Total 1½]

Alternative solution

Students could have used the equation:

$$_{10}p_{50} = 0.988353 = e^{-10\mu}$$ *[½]*

to calculate $\mu = 0.001172$ *and use this value in the formula:*

$$_{5}q_{50} = 1 - e^{-5\mu}$$ *[1]*

(ii) ***Number of survivors at exact age 55***

Under the UDD assumption, the number of survivors at exact age 55 is:

$$9,941.923 - 57.896 = 9,884.027$$ [1]

Alternative solution

Students could have used:

$$0.5l_{50} + 0.5l_{60} = 0.5 \times 9,941.923 + 0.5 \times 9,826.131 = 9,884.027$$ *[1]*

Under the constant force assumption, the number of survivors is:

$$9,941.923 \times {}_{5}p_{50} = 9,941.923\sqrt{\frac{9,826.131}{9,941.923}} = 9,883.857$$ [1]

[Total 2]

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(iii) ***Comment***

The two assumptions give very similar answers. [½]

The actual number of survivors in PMA92C20 is 9,904.805. [½]

This is higher than the figures given by both the UDD and the constant force assumptions. [½]

This means that in PMA92C20 there were more deaths between the ages of 55 and 60 than there were between the ages of 50 and 55. [½]

So, mortality is higher between 55 and 60 than it is between 50 and 55, which means that the force of mortality is increasing between 50 and 60. [½]

The UDD assumption implies that the force of mortality is increasing between the ages of 50 and 60 (whereas the constant force assumption obviously says that the force of mortality is the same at all ages between 50 and 60). [½]

However, it appears that the actual force of mortality is increasing faster than it would under the UDD assumption. [½]

So neither of the two assumptions is very appropriate. [½]

These assumptions should really only be applied over single years of age [½]
[Maximum 4]

Solution 9

(i) ***Part years count proportionately, no maximum***

The expected present value of the past service benefit is:

$$\frac{27}{60}\times 40{,}000\times\frac{{}^zM_{50}^{ra}}{{}^sD_{50}}=\frac{27}{60}\times 40{,}000\times\frac{128{,}026}{16{,}460}=140{,}004.1 \qquad [1]$$

The expected present value of the future service benefit is:

$$\frac{1}{60}\times 40{,}000\times\frac{{}^z\bar{R}_{50}^{ra}}{{}^sD_{50}}=\frac{1}{60}\times 40{,}000\times\frac{1{,}604{,}000}{16{,}460}=64{,}965.6 \qquad [1½]$$

So the expected present value of the pension is £204,970. [½]

[Total 3]

(ii) ***Complete years only count, no maximum***

The past service value is unchanged from before, while the future service value is now:

$$\frac{1}{60}\times 40{,}000\times\frac{\left({}^zM_{51}^{ra}+\ldots+{}^zM_{64}^{ra}+{}^zM_{65}^{ra}\right)}{{}^sD_{50}} \qquad [2]$$

Each ${}^zM_x^{ra}$ function here corresponds to the contribution made by each future year of service.

$$=\frac{1}{60}\times 40{,}000\times\frac{(10\times 128{,}026)+69{,}733+61{,}926+55{,}385+49{,}949+45{,}467}{{}^sD_{50}}$$

$$=\frac{1}{60}\times 40{,}000\times\frac{1{,}562{,}720}{16{,}460}=63{,}293.6 \qquad [1½]$$

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Alternatively, you could notice that ${}^{z}M_{x+1}^{ra} = {}^{z}\overline{M}_{x}^{ra} - \tfrac{1}{2}\,{}^{z}C_{x}^{ra}$ *for* $x \le 64$ *and* ${}^{z}C_{x}^{ra} = 0$ *for* $x < 60$. *So:*

$$\sum_{t=1}^{15} {}^{z}M_{50+t}^{ra} = {}^{z}\overline{R}_{50}^{ra} - \tfrac{1}{2}\left({}^{z}C_{50}^{ra} + \cdots + {}^{z}C_{64}^{ra}\right) = {}^{z}\overline{R}_{50}^{ra} - \tfrac{1}{2}\left({}^{z}M_{50}^{ra} - {}^{z}C_{65}^{ra}\right)$$

Calculating in this way gives $\sum_{t=1}^{15} {}^{z}M_{50+t}^{ra} = 1{,}562{,}720.5$.

So the expected present value of the whole pension is:

$$140{,}004.1 + 63{,}293.6 = £203{,}298$$ [½]

[Total 4]

(iii) ***Part years count proportionately, maximum of 40/60ths***

The expected present value of the future service benefit is:

$$\frac{1}{60} \times 40{,}000 \times \frac{{}^{z}\overline{R}_{50}^{ra} - {}^{z}\overline{R}_{63}^{ra}}{{}^{s}D_{50}} = \frac{1}{60} \times 40{,}000 \times \frac{1{,}604{,}000 - 100{,}374}{16{,}460}$$

$$= 60{,}900.2$$ [1½]

So the expected present value of the pension is:

$$140{,}004.1 + 60{,}900.2 = £200{,}904$$ [½]

[Total 2]

Solution 10

Calculating the reserves

The calculations in this question are quite sensitive to rounding, and valid alternative methods can give answers that are up to ± 0.25 *different from the answers given.*

We need to calculate the reserves at times 1 and 2, which means that first we need to calculate the net premium, using the equivalence principle.

The expected present value of the benefits is:

$$150{,}000 A^{1}_{62:\overline{3}|} = 150{,}000\left(A_{62} - \frac{D_{65}}{D_{62}} A_{65}\right)$$

$$= 150{,}000\left(0.48458 - \frac{689.23}{802.40} \times 0.52786\right)$$

$$= 4{,}675.36 \qquad [½]$$

Alternatively you could obtain this (except for rounding differences) by deducting the pure endowment value from the full endowment value in the Tables.

So, if NP is the annual net premium:

$$NP = \frac{150{,}000 A^{1}_{62:\overline{3}|}}{\ddot{a}_{62:\overline{3}|}} = \frac{4{,}675.36}{2.857} = £1{,}636.46 \qquad [½]$$

using the value for the annuity at 4% from the *Tables*.

The (prospective) reserve is found by calculating the EPV of future benefits less the EPV of future premiums:

$${}_1V = 150{,}000 A^{1}_{63:\overline{2}|} - 1{,}636.46\ddot{a}_{63:\overline{2}|} \qquad [½]$$

From the *Tables*, $\ddot{a}_{63:\overline{2}|} = 1.951$. Calculating the term assurance function:

$$A^{1}_{63:\overline{2}|} = A_{63} - \frac{D_{65}}{D_{63}} A_{65} = 0.4989 - \frac{689.23}{763.74} \times 0.52786 = 0.022538 \qquad [½]$$

Again, this could alternatively have been obtained by deducting the pure endowment value from the full endowment value.

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So we have:

$${}_1V = 150,000 \times 0.022538 - 1,636.46 \times 1.951 = £187.93 \qquad [½]$$

For the reserve at the end of year 2, we can work from first principles rather than try to use any assurance functions. The benefit will be 150,000 paid at the end of the year if death occurs, and the premium is payable immediately, so:

$${}_2V = 150,000vq_{64} - 1,636.46 = 150,000 \times \frac{0.012716}{1.04} - 1,636.46 = 197.58 \qquad [½]$$

Profit test

Let the premium be P. The figures for the profit test are:

Year	1	2	3
Premium	P	P	P
Expenses	–400	–50	–50
Interest (on $P-E$) (6%)	$0.06(P-400)$	$0.06(P-50)$	$0.06(P-50)$
Death benefit[1]	$-1,074.60$	$-1,622.25$	$-1,907.40$
Reserve brought forward	0	187.93	197.58
Interest on reserves (6%)	0	11.28	11.85
Reserve carried forward[2]	–186.58	–195.44	0

[3]

Mark scheme:
Award ½ mark for the premium, expenses and interest rows
Award ½ mark for each death benefit (total 1½ marks)
Award 1 mark for the reserve figures

Notes on calculations:

(1) Death benefit

Year 1: $150,000q_{[62]} = 150,000 \times 0.007164 = 1,074.60$

Year 2: $150,000q_{[62]+1} = 150,000 \times 0.010815 = 1,622.25$

Year 3: $150,000q_{64} = 150,000 \times 0.012716 = 1,907.40$

(2) Reserves

The reserve that needs to be set up at the end of year 1 is the reserve for the start of year 2 for those that survive the first year:

$$187.93\,p_{[62]} = 187.93 \times (1 - 0.007164) = 186.58$$

The reserve that needs to be set up at the end of year 2 is the reserve for the start of year 3 for those that survive the second year:

$$197.58\,p_{[62]+1} = 197.58 \times (1 - 0.010815) = 195.44$$

Summing the columns, this gives us a profit vector of:

Year 1 $1.06(P-400) - 1{,}261.18 = 1.06P - 1{,}685.18$

Year 2 $1.06(P-50) - 1{,}618.48 = 1.06P - 1{,}671.48$

Year 3 $1.06(P-50) - 1{,}697.97 = 1.06P - 1{,}750.97$ [½]

To get the net present value, we need the profit signature, which is the profit per policy in force at inception. This is obtained by multiplying the profit vector by the survival probability to the start of that year:

Year 1 $1.06P - 1{,}685.18$

Year 2 $\left[1.06P - 1{,}671.48\right] p_{[62]} = \left[1.06P - 1{,}671.48\right] \times (1 - 0.007164)$

$= 1.052406P - 1{,}659.51$

Year 3 $\left[1.06P - 1{,}750.97\right] {}_2p_{[62]} = \left[1.06P - 1{,}750.97\right] \dfrac{l_{64}}{l_{[62]}}$

$$= \left[1.06P - 1{,}750.97\right] \times \frac{8{,}934.8771}{9{,}097.7405} = 1.041024P - 1{,}719.62$$

[1]

The net present value is the discounted value of the profit signature using the risk discount rate:

$$\frac{1.06P - 1{,}685.18}{1.09} + \frac{1.052406P - 1{,}659.51}{1.09^2} + \frac{1.041024P - 1{,}719.62}{1.09^3}$$ [½]

Setting this equal to zero, we get:

$$2.662128P = 4,270.675$$

$$\Rightarrow \quad P = £1,604.23$$ [1]

[Total 9]

Solution 11

(i)(a) ***Child mortality rate***

South Africa:

$$\frac{41.920}{4,449.8} = 0.00942$$ [½]

England/Wales:

$$\frac{3.828}{3,086.2} = 0.00124$$ [½]

The figure for South Africa is almost 8 times higher than that in the UK. Even though both regions have developed economies, South Africa has a huge population living in poor, rural conditions, which is likely to affect child mortality. [1]

(i)(b) ***Factors affecting child mortality rate***

Markers, please award ½ mark for each sensible factor and ½ mark for each explanation of that factor, up to a maximum of 4 marks.

Nutrition: Poor nutrition in very early years can lead to children contracting disease and dying. Also poor nutrition in the mother could lead to poor milk quality. [1]

Housing: The physical quality of a house (state of repair, type of construction, heating, sanitation) and the way housing is used (overcrowding and shared cooking) can have an influence on infectious diseases. [1]

Climate and geography: Tropical diseases only occur in certain areas. Natural disasters affect mortality and may be correlated to particular climates and geographical locations. Differences can occur between urban and rural populations due to limited access to roads and hospitals. [1]

Education: If parents have limited education they may not understand how best to care for their children. [1]

Wealth or poverty: If parents are wealthy then they are going to be able to provide better surroundings and better care for their children than poor parents. [1]

Genetics: Some children may inherit a genetic condition that increases their likelihood of suffering an early death from a particular disease. [1]

[(Total 2) + (Maximum 4) = 6]

(ii) ***Crude rates (all ages)***

South Africa:

$$\frac{450.049}{44,820} = 0.01004$$ [½]

England/Wales:

$$\frac{530.373}{52,085} = 0.01018$$ [½]

The crude rate over all ages for England/Wales is marginally higher than that for South Africa. [1]

This conceals the fact that most of the England/Wales deaths occur at old ages, while most of the South Africa deaths occur at much younger ages. The age-specific mortality rates are therefore likely to be very different between the two countries (as illustrated already by the answer to part (i)(a)). [1]

[Total 3]

(iii)(a) ***Crude rates for limited age range***

South Africa (age 20-39):

$$\frac{131.093}{14,642.1} = 0.00895$$

England/Wales (age 20-39):

$$\frac{11.184}{14,623.6} = 0.000765$$ [1½]

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The figure for South Africa is over 10 times higher than the figure for England/Wales. This is contrary to the findings for the crude rates over all ages. [1]

(iii)(b) ***Directly standardised mortality rate***

The directly standardised mortality rate is given by:

$$\frac{\text{Expected deaths in standard population using regional mortality}}{\text{Total standard population}}$$

The expected number of deaths (in thousands) in the standard population using regional mortality is:

$$\frac{19.655}{4,294.5}\times 3,129.2+\frac{35.667}{3,934.9}\times 3,415.7$$

$$+\frac{39.203}{3,340.9}\times 3,978.1+\frac{36.568}{3,071.8}\times 4,100.6=140.778 \qquad [1]$$

So the directly standardised mortality rate is:

$$\frac{140.778}{14,623.6}=0.00963 \qquad [½]$$

This rate is even higher than the crude rate. This is because the population structure of England/Wales is more weighted towards the higher ages (in the range 20-39) than South Africa, and these are the ages at which the South African mortality rates are highest. [1]

[Total 5]

Solution 12

Level annual premium

Let the superscript *d* denote normal death, *z* denote zombie death and *m* denote mutation death. So:

$$\mu_{50}^{z} = \mu_{51}^{z} = \mu_{52}^{z} = 0.08 \quad \text{and} \quad \mu_{50}^{m} = \mu_{51}^{m} = \mu_{52}^{m} = 0.07$$

Using the relationship $\mu_x^d = -\log\left(1 - q_x^d\right)$, with AM92 mortality, we have:

$$\mu_{50}^{d} = -\log(1 - 0.002508) = 0.002511$$

$$\mu_{51}^{d} = -\log(1 - 0.002809) = 0.002813$$

$$\mu_{52}^{d} = -\log(1 - 0.003152) = 0.003157$$

[1 in total for these three values]

We can now use the relationship:

$$(aq)_x^k = \frac{\mu_x^k}{\mu_x^d + \mu_x^z + \mu_x^m}\left(1 - e^{-\left(\mu_x^d + \mu_x^z + \mu_x^m\right)}\right)$$

to find the dependent probabilities for each decrement $k = d, z, m$.

At age 50:

$$(aq)_{50}^{d} = \frac{0.002511}{0.002511 + 0.08 + 0.07}\left(1 - e^{-(0.002511+0.08+0.07)}\right) = 0.002329$$

$$(aq)_{50}^{z} = \frac{0.08}{0.002511 + 0.08 + 0.07}\left(1 - e^{-(0.002511+0.08+0.07)}\right) = 0.074198$$

and: $$(aq)_{50}^{m} = \frac{0.07}{0.002511 + 0.08 + 0.07}\left(1 - e^{-(0.002511+0.08+0.07)}\right) = 0.064923$$

[1 in total for these three values]

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At age 51:

$$(aq)_{51}^{d}=\frac{0.002813}{0.002813+0.08+0.07}\left(1-e^{-(0.002813+0.08+0.07)}\right)=0.002609$$

$$(aq)_{51}^{z}=\frac{0.08}{0.002813+0.08+0.07}\left(1-e^{-(0.002813+0.08+0.07)}\right)=0.074187$$

and: $$(aq)_{51}^{m}=\frac{0.07}{0.002813+0.08+0.07}\left(1-e^{-(0.002813+0.08+0.07)}\right)=0.064914$$

[1 in total for these three values]

At age 52:

$$(aq)_{52}^{d}=\frac{0.003157}{0.003157+0.08+0.07}\left(1-e^{-(0.003157+0.08+0.07)}\right)=0.002927$$

$$(aq)_{52}^{z}=\frac{0.08}{0.003157+0.08+0.07}\left(1-e^{-(0.003157+0.08+0.07)}\right)=0.074175$$

and: $$(aq)_{52}^{m}=\frac{0.07}{0.003157+0.08+0.07}\left(1-e^{-(0.003157+0.08+0.07)}\right)=0.064903$$

[1 in total for these three values]

We can summarise these values as follows:

x	$(aq)_x^d$	$(aq)_x^z$	$(aq)_x^m$
50	0.002329	0.074198	0.064923
51	0.002609	0.074187	0.064914
52	0.002927	0.074175	0.064903

We will also need the probabilities of remaining in the population during each year:

$$(ap)_{50}=1-(aq)_{50}^{d}-(aq)_{50}^{z}-(aq)_{50}^{m}$$

$$=1-0.002329-0.074198-0.064923=0.858550$$

$$(ap)_{51} = 1-(aq)_{51}^{d} - (aq)_{51}^{z} - (aq)_{51}^{m}$$
$$= 1 - 0.002609 - 0.074187 - 0.064914 = 0.858290$$

and: $$(ap)_{52} = 1-(aq)_{52}^{d} - (aq)_{52}^{z} - (aq)_{52}^{m}$$
$$= 1 - 0.002927 - 0.074175 - 0.064903 = 0.857995$$

[1 in total for these three values]

Letting the annual premium be P, the expected present value of the premiums is:

$$EPV(\text{prem}) = P\left[1 + (ap)_{50}\, v + {}_2(ap)_{50}\, v^2\right]$$
$$= P\left[1 + 0.858550v + 0.858550 \times 0.858290v^2\right]$$
$$= 2.528128P$$

[1]

Assuming that normal deaths occur on average mid-year, the expected present value of the benefit payable on normal death is:

$$EPV(\text{normal}) = 50{,}000\left[(aq)_{50}^{d}\, v^{0.5} + (ap)_{50}(aq)_{51}^{d}\, v^{1.5} + {}_2(ap)_{50}(aq)_{52}^{d}\, v^{2.5}\right]$$
$$= 50{,}000\left[0.002329v^{0.5} + 0.858550 \times 0.002609v^{1.5}\right.$$
$$\left. + \; 0.858550 \times 0.858290 \times 0.002927v^{2.5}\right]$$
$$= \$322.04$$

[1½]

The benefit payable on zombie death is a return of all premiums paid with no interest. Assuming that zombie deaths occur on average mid-year, the expected present value of the benefit payable on zombie death is:

$$EPV(\text{zombie}) = P\left[(aq)_{50}^{z}\, v^{0.5} + 2(ap)_{50}(aq)_{51}^{z}\, v^{1.5} + 3\,{}_2(ap)_{50}(aq)_{52}^{z}\, v^{2.5}\right]$$
$$= P\left[0.074198v^{0.5} + 2 \times 0.858550 \times 0.074187v^{1.5}\right.$$
$$\left. + \; 3 \times 0.858550 \times 0.858290 \times 0.074175v^{2.5}\right]$$
$$= 0.347266P$$

[1½]

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The benefit on mutation death is a return of all premiums paid, accumulated with interest at 3% *pa*. Assuming that mutation deaths occur on average mid-year, the expected present value of the benefit payable on mutation death is:

$$EPV(\text{mutation}) = 1.03^{0.5} P(aq)_{50}^{m} v^{0.5} + \left(1.03^{1.5} + 1.03^{0.5}\right) P(ap)_{50}(aq)_{51}^{m} v^{1.5}$$
$$+\left(1.03^{2.5} + 1.03^{1.5} + 1.03^{0.5}\right) P\,{}_2(ap)_{50}(aq)_{52}^{m} v^{2.5}$$

Since $v = 1.03^{-1}$, this simplifies to:

$$EPV(\text{mutation}) = P(aq)_{50}^{m} + (1+v) P(ap)_{50}(aq)_{51}^{m}$$
$$+\left(1+v+v^2\right) P\,{}_2(ap)_{50}(aq)_{52}^{m}$$
$$= P\begin{bmatrix} 0.064923 + (1+v)\times 0.858550 \times 0.064914 \\ \qquad + \left(1+v+v^2\right)\times 0.858550 \times 0.858290 \times 0.064903 \end{bmatrix}$$
$$= 0.314103P$$

[2]

The expected present value of the survival benefit is:

$$EPV(\text{survival}) = 30{,}000\,{}_3(ap)_{50} v^3$$
$$= 30{,}000 \times 0.858550 \times 0.858290 \times 0.857995 v^3$$
$$= \$17{,}357.77$$

[1]

The expected present value of the expenses is:

$$EPV(\exp) = 0.01 \times EPV(\text{prem})$$
$$= 0.01 \times 2.528128P$$
$$= 0.025281P$$

[½]

Using the principle of equivalence gives the equation:

$$EPV(\text{prem}) = EPV(\exp) + EPV(\text{normal}) + EPV(\text{zombie})$$
$$+ EPV(\text{mutation}) + EPV(\text{survival})$$

ie: $2.528128P = 0.025281P + 322.04 + 0.347266P + 0.314103P + 17{,}357.77$ [½]

Solving for P:

$$1.841478P = 17,679.81 \quad \Rightarrow \quad P = \$9,601$$ [1]

[Total 14]

Alternative solution

An alternative approach to this question is to use the dependent probabilities calculated to construct a multiple decrement table as follows.

Choosing a radix of $l_{50} = 100,000$*, and defining* d_x*,* z_x *and* m_x *to be the number of normal deaths, zombie deaths and mutation deaths between exact ages* x *and* $x+1$ *gives:*

Age, x	l_x	d_x	z_x	m_x
50	100,000	232.9	7,419.8	6,492.3
51	85,855.0	224.0	6,369.3	5,573.2
52	73,688.5	215.7	5,465.8	4,782.6
53	63,224.4			

Working with these gives, for instance:

$$EPV(\text{prem}) = P\left(1 + \frac{l_{51}}{l_{50}}v + \frac{l_{52}}{l_{50}}v^2\right)$$

$$= \frac{P}{100,000}\left(100,000 + 85,855.0v + 73,688.5v^2\right)$$

$$= 2.528128P$$

and:

$$EPV(\text{normal}) = 50,000\left(\frac{d_{50}}{l_{50}}v^{0.5} + \frac{d_{51}}{l_{50}}v^{1.5} + \frac{d_{52}}{l_{50}}v^{2.5}\right)$$

$$= \frac{50,000}{100,000}\left(232.9v^{0.5} + 224.0v^{1.5} + 215.7v^{2.5}\right)$$

$$= \$322.05$$

The values obtained using this approach are the same (up to rounding).

Solution 13

Note to markers: if students use slightly different methods throughout this question to the ones shown, then their answers will differ by a few pence. Please give full credit for all suitable methods even if the answers don't match exactly.

(i) ***Monthly premium***

Since the death benefit is payable at the end of the year of death, and bonuses are only added for those who survive until the end of the year, a life dying in any year will not benefit from that year's bonus. So the expected present value of the death benefit is:

$$EPVDB = 50{,}000\left[\frac{d_{[40]}}{l_{[40]}}v + 1.01923\frac{d_{[40]+1}}{l_{[40]}}v^2 + 1.01923^2\frac{d_{42}}{l_{[40]}}v^3 + \cdots + 1.01923^{19}\frac{d_{59}}{l_{[40]}}v^{20}\right] \qquad [1]$$

We can write this is:

$$EPVDB = \frac{50{,}000}{1.01923}\left[\frac{d_{[40]}}{l_{[40]}}(1.01923v) + \frac{d_{[40]+1}}{l_{[40]}}(1.01923v)^2 + \frac{d_{42}}{l_{[40]}}(1.01923v)^3 + \cdots + \frac{d_{59}}{l_{[40]}}(1.01923v)^{20}\right]$$

The expression in the bracket has the form of a term assurance benefit calculated at 4%, since $\frac{1.01923}{1.06} = \frac{1}{1.04}$. So we can write:

$$EPVDB = \frac{50{,}000}{1.01923}A^{\,1}_{[40]:\overline{20}|} @ 4\% \qquad [1]$$

Evaluating the term assurance function:

$$A^{\,1}_{[40]:\overline{20}|} = A_{[40]} - \frac{D_{60}}{D_{[40]}}A_{60} = 0.23041 - \frac{882.85}{2{,}052.54} \times 0.45640 = 0.03410 \qquad [1]$$

Alternatively you can look up the endowment assurance function in the Tables and then just subtract the D/D term. You get the same answer either way.

So the value of the death benefit is:

$$\frac{50,000}{1.01923} \times 0.03410 = 1,672.83 \qquad [½]$$

For the survival benefit, we want:

$$EPVSB = 50,000 \times (1.01923)^{20} \times v^{20} \times \frac{l_{60}}{l_{[40]}} = 50,000 \frac{D_{60}}{D_{[40]}} @ 4\% = 21,506.28$$

[1]

Note that you cannot just look up the value of an endowment assurance at 4%. Working out exactly how the bonuses accrue is an important part of these questions.

So using the equivalence principle, the equation of value is:

$$P\ddot{a}^{(12)}_{[40]:\overline{20}|} = 1,672.83 + 21,506.28 + 0.5P + 0.05P\ddot{a}_{[40]:\overline{20}|} \qquad [½]$$

where the annuities are calculated at 6%, and P is the annual amount of premium.

From the *Tables*:

$$\ddot{a}_{[40]:\overline{20}|} = 12.000$$

and $$\ddot{a}^{(12)}_{[40]:\overline{20}|} \approx 12.000 - \frac{11}{24}\left(1 - v^{20}\frac{l_{60}}{l_{[40]}}\right)$$

$$= 12.000 - \frac{11}{24}\left(1 - (1.06)^{-20}\frac{9,287.2164}{9,854.3036}\right) = 11.676 \qquad [1]$$

So we have:

$$11.676P = 1,672.83 + 21,506.28 + 0.5P + 0.05 \times 12.000P$$

This gives $P = 2,191.67$, and a monthly premium of £182.64. [1]

[Total 7]

This answer is quite sensitive to rounding.

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(ii) ***Gross premium prospective reserve***

We want the reserve at time 14, just after the addition of the fourteenth annual bonus, but before payment of the premium at the start of the fifteenth year. The EPV of the future death benefit at this point will have the form:

$$EPVFDB = 50,000\left[\frac{d_{54}}{l_{54}}1.01923^{14}v + \frac{d_{55}}{l_{54}}1.01923^{15}v^2 + \cdots + \frac{d_{59}}{l_{54}}1.01923^{19}v^6\right]$$

$$= 50,000 \times 1.01923^{13} \times \left[\frac{d_{54}}{l_{54}}(1.01923v)^1 + \frac{d_{55}}{l_{54}}(1.01923v)^2 + \cdots + \frac{d_{59}}{l_{54}}(1.01923v)^6\right]$$

$$= 50,000 \times 1.01923^{13} \times A^{1\,@4\%}_{54:\overline{6}|}$$

[1]

The term assurance is again calculated at 4%. Using the whole life functions in the tables, as before, the value of the death benefit is now:

$$EPVFDB = 50,000 \times 1.01923^{13} \times \left(A_{54} - \frac{D_{60}}{D_{54}}A_{60}\right)$$

$$= 50,000 \times 1.01923^{13} \times \left(0.37685 - \frac{882.85}{1,154.22} \times 0.45640\right)$$

$$= 1,777.65$$

[½]

The survival benefit is:

$$EPVFSB = 50,000 \times (1.01923)^{20} \times v^6 \times \frac{l_{60}}{l_{54}}$$

$$= 50,000 \times (1.01923)^{20} \times 1.06^{-6} \times \frac{9,287.2164}{9,595.9715}$$

$$= 49,931.59$$

[1]

So the prospective reserve is:

$$ {}_{14}V^{pro} = 1{,}777.65 + 49{,}931.59 + 0.05 \times 2{,}191.67\ddot{a}_{54:\overline{6}|} - 2{,}191.67\ddot{a}^{(12)}_{54:\overline{6}|} \qquad [½] $$

The annuities are:

$$ \ddot{a}_{54:\overline{6}|} = 5.156 \qquad [½] $$

and:

$$ \ddot{a}^{(12)}_{54:\overline{6}|} = 5.156 - \frac{11}{24}\left(1 - v^6 \frac{l_{60}}{l_{54}}\right) = 5.010 \qquad [½] $$

So the prospective reserve is:

$$ \begin{aligned} {}_{14}V^{pro} &= 1{,}777.65 + 49{,}931.59 + 0.05 \times 2{,}191.67 \times 5.156 - 2{,}191.67 \times 5.010 \\ &= 41{,}293.99 \end{aligned} \qquad [1] $$

[Total 5]

(iii) ***Mortality profit***

The reserve at the end of 2014 is 41,293.99. The death strain at risk will be:

$$ \begin{aligned} DSAR &= S - {}_{14}V \\ &= 50{,}000 \times 1.01923^{13} - 41{,}293.99 \\ &= 64{,}048.44 - 41{,}293.99 \\ &= 22{,}754.45 \end{aligned} \qquad [1] $$

Note that the sum assured payable if the policyholder dies in the 14th year of the policy is £50,000 plus 13 compound bonuses of 1.923%. This is payable at the end of the year of death.

The lives are aged 53 at the start of the 14th policy year (*ie* at time 13). The expected death strain is the death strain at risk, multiplied by the expected number of deaths, *ie*:

$$ \begin{aligned} EDS &= 488 \times q_{53} \times DSAR \\ &= 488 \times 0.003539 \times 22{,}754.45 \\ &= 39{,}297.66 \end{aligned} \qquad [1] $$

The actual death strain is the death strain at risk multiplied by the actual number of deaths:

$$ADS = 2 \times DSAR = 2 \times 22,754.45 = 45,508.90$$ [1]

So the mortality profit for this group of policyholders in calendar year 2014 is:

$$MP = EDS - ADS = 39,297.66 - 45,508.90 = -6,211.24$$

ie a loss of about £6,211. [1]

[Total 4]