CT4 – P MA – 16

Mock Exam A

ActEd Study Materials: 2016 Examinations

Subject CT4

Contents

Mock Exam A Questions

If you think that any pages are missing from this pack, please contact ActEd's admin team by email at **ActEd@bpp.com**.

How to use Mock Exam A

Guidance on how and when to use Mock Exam A is set out in the *Study Guide for the 2016 exams*. The recommended date and deadline date for submission of Mock Exam A are listed on a summary page at the back of this pack. **We strongly recommend that you work to the recommended date.**

Please note that we only accept the current version of Mock Exam A for marking, *ie* you can only submit this mock exam in the sessions leading to the 2016 exams. However, if you wish to submit your script for marking in 2016, but do not have the latest version of Mock Exam A, please let us know and we will send, free of charge, an up-to-date version for you to attempt and submit.

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Subject CT4: Mock Exam A

2016 Examinations

Time allowed: 3 hours

Instructions to the candidate

1. *Please:*

   – *attempt all of the questions, as far as possible under exam conditions*

   – ***begin your answer to each question on a new page***

   – ***leave at least 2cm margin on all borders***

   – *write in black ink using a medium-sized nib because we will be unable to mark illegible scripts*

   – *note that mock exam marking is not included in the price of the Course Materials. Please purchase Mock Exam Marking or Marking Vouchers before submitting your script.*

   – *note that we only accept the current version of mock exams for marking, ie you can only submit this mock exam in the sessions leading to the 2016 exams.*

2. *Please **do not**:*

   – *use headed paper*

   – *use highlighting in your script.*

At the end of the mock exam

If your script is being marked by ActEd, please follow the instructions on the reverse of this page.

In addition to this paper, you should have available actuarial tables and an electronic calculator.

Submission for marking

You should aim to submit this script for marking by the recommended submission date. The recommended and deadline dates for submission of this mock exam are listed on the summary page at the back of this pack and on our website at **www.ActEd.co.uk**.

Scripts received after the deadline date will not be marked, unless you are using a Marking Voucher. *It is your responsibility to ensure that scripts reach ActEd in good time*. ActEd will not be responsible for scripts lost or damaged in the post, or for scripts received after the deadline date. If you are using Marking Vouchers, then please make sure that your script reaches us by the Marking Voucher deadline date to give us enough time to mark and return the script before the exam.

When submitting your script, please:

- complete the cover sheet, including the checklist
- scan your script and cover sheet (and Marking Voucher if applicable) to a pdf document, then email it to: **ActEdMarking@bpp.com**
- **do not submit a photograph of your script**
- **do not include the question paper in the scan**.

In addition, please note the following:

- Please title the email to ensure that the subject and mock exam are clear *eg* "CT4 Mock Exam A No. 12345", inserting your ActEd Student Number for 12345.
- The mock exam should be scanned the **right way up** (so that it can be read normally without rotation) and as a single document. We cannot accept individual files for each page.
- Please set the resolution so that the script is legible and the resulting PDF is less than 3 MB in size. **The file size cannot exceed 4 MB**.
- Do not protect the PDF in any way (otherwise the marker cannot return the script to ActEd, which causes delays).
- Please include the "feedback from marker" sheet when scanning.
- Before emailing to ActEd, please check that your scanned mock exam includes all pages and conforms to the above.

Subject CT4: Mock Exam A

2016 Examinations

Please complete the following information:

Name:

Email address:

ActEd Student Number (see Note below):

Note: Your ActEd Student Number is printed on all personal correspondence from ActEd. Quoting it will help us to process your scripts quickly. If you do not know your ActEd Student Number, please email us at ActEd@bpp.com.

Your ActEd Student Number is not the same as your IFoA Actuarial Reference Number or ARN.

Number of following pages: _____

Please put a tick in this box if you have solutions and a cross if you do not: ☐

Please tick here if you are allowed extra time or other special conditions in the profession's exams: ☐

Time to do mock exam
(see Note below): _____ hrs _____ mins

Under exam conditions
(delete as applicable): yes / nearly / no

Note: If you take more than 3 hours, you should indicate how much you completed within this time so that the marker can provide useful feedback on your chances of success in the exam.

Score and grade for this mock exam (to be completed by marker):

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	**Total**	
$\frac{\_}{4}$	$\frac{\_}{4}$	$\frac{\_}{5}$	$\frac{\_}{6}$	$\frac{\_}{6}$	$\frac{\_}{7}$	$\frac{\_}{7}$	$\frac{\_\_}{12}$	$\frac{\_\_}{13}$	$\frac{\_\_}{16}$	$\frac{\_\_}{20}$	$\frac{\_\_\_}{100}$	=_____%

Grade: A B C D E **Marker's initials:** _____

Please tick the following checklist so that your script can be marked quickly. Have you:

[ ] Checked that you are using the latest version of the mock exam, *ie* 2016 for the sessions leading to the 2016 exams?
[ ] Written your full name and email address in the box above?
[ ] Completed your ActEd Student Number in the box above?
[ ] Recorded your attempt conditions?
[ ] Numbered all pages of your script (excluding this cover sheet)?
[ ] Written the total number of pages (excluding the cover sheet) in the space above?
[ ] Included your Marking Voucher or ordered Mock Exam Marking?

Please follow the instructions on the previous page when submitting your script for marking.

This page has been left blank in case you wish to submit your script by fax.

Feedback from marker

Notes on marker's section

The main objective of marking is to provide specific advice on how to improve your chances of success in the exam. The most useful aspect of the marking is the comments the marker makes throughout the script, however you will also be given a percentage score. Based on this score, the marker assigns a grade representing how this script would have fared in an exam. The grades are as follows:

A = Clear Pass B = Probable Pass C = Borderline D = Probable Fail E = Clear Fail

Please note that you can provide feedback on the marking of this mock exam at:

www.ActEd.co.uk/marking

This page has been left blank in case you wish to submit your script by fax.

1 The force of mortality at exact age x is defined as $\mu_x(ij)$, where:

$$i = \begin{cases} N & \text{if the person is a non-smoker} \\ S & \text{if the person is a smoker} \end{cases}$$

$$j = \begin{cases} M & \text{if the person is male} \\ F & \text{if the person is female} \end{cases}$$

You are given the following information, which applies to all ages x:

$$\frac{\mu_x(SF)}{\mu_x(NF)} = 2.32; \qquad \frac{\mu_x(NM)}{\mu_x(NF)} = 1.45; \qquad \frac{\mu_x(SM)}{\mu_x(NM)} = 1.86$$

Devise a Cox proportional hazards model for $\mu_x(ij)$, incorporating a baseline hazard function and three additional parameters.

Calculate the values of the three additional parameters and state the class of lives to which your baseline hazard rate applies. [4]

2 List the factors that should be considered when assessing the suitability of an actuarial model for its purpose. [4]

3 (i) Define the following types of stochastic process:

(a) a Poisson process with rate λ

(b) a white noise process. [3]

(ii) For both of the processes in (i), state whether:

(a) its time set is discrete, continuous or can be either

(b) its state space is discrete, continuous or can be either. [2]

[Total 5]

4 A time-homogeneous Markov jump process has 2 states, A and B. The force of transition per unit time from State A to State B is 0.4 and the force of transition per unit time from State B to State A is 0.1. Calculate the probability that the process is in State A at time 1 given that it is in State A at time 0. [6]

5 Under the Weibull model of mortality, the survival function is of the form:

$$S_x(t) = \exp\left(-\alpha t^{\beta}\right)$$

for $t \geq 0$.

(i) Derive an expression for μ_{x+t} under this model. [1]

In a mortality investigation involving n independent lives aged exactly x at the outset, we observe the following:

- Lives 1 to m are observed to die at durations $t_1, ..., t_m$ respectively
- Lives $m+1$ to n are censored at durations $t_{m+1}, ..., t_n$ respectively.

(ii) Show that, under the Weibull model, the log-likelihood function is:

$$\ln L = m \ln \alpha + m \ln \beta + (\beta - 1) \sum_{i=1}^{m} \ln t_i - \alpha \sum_{j=1}^{n} t_j^{\beta}$$ [3]

(iii) Assuming that $\beta = 1.5$, $m = 10$, $n = 20$ and $\sum_{j=1}^{20} t_j^{1.5} = 1,648.42$, estimate:

(a) the value of the parameter α

(b) the probability that a life survives for at least 10 years. [2]

[Total 6]

6 In a mortality investigation, 10,000 lives aged exactly x are observed for a period of one year or until their earlier death.

(i) List the data items that you would need in order to calculate estimates of the following mortality rates, and give formulae for those estimates.

(a) The probability of a life aged exactly x dying within one year.

(b) The force of mortality over the age range $(x, x+1)$. Assume the force of mortality is constant over that period. [3]

(ii) Suppose that the true force of mortality over the age range $(x, x+1)$ has a constant value μ.

(a) Give a formula for $\tilde{\mu}$, the estimator of μ, defining the symbols in your formula, and state the asymptotic distribution of $\tilde{\mu}$.

(b) Calculate $E[V]$, the total expected waiting time over the year of age x to $x+1$ for the 10,000 lives, given that $\mu = 0.05$. [4]

[Total 7]

7 A discrete-time stochastic process with two states, labelled 1 and 2, is being modelled as a time-homogeneous Markov chain with transition matrix:

$$\begin{pmatrix} p & 1-p \\ 1-q & q \end{pmatrix}$$

where $0 < p, q < 1$.

(i) Sketch the transition diagram for this process. [1]

(ii) Explain whether this Markov chain is irreducible and/or aperiodic. [2]

(iii) Derive an expression in terms of p and q for the stationary distribution of the Markov chain. [2]

The following data have been collected from the process:

Sequence of characters	11	12	21	22
Number of occurrences	62	59	59	69

(iv) Estimate the values of p and q using the data given above, and hence estimate the stationary distribution. [2]

[Total 7]

8 The director of the tourist board in the town where you live is analysing the sales in the gift shops at some of the historic monuments in the town centre. She has asked you to analyse the time it takes between an individual tourist or a party of tourists entering the shops and the first person in that party making a purchase.

You have designed a standard form to allow you to discreetly monitor the visitors whilst posing as a tourist. Here is your completed form for one of the monuments showing visitors entering the gift shops between 11am and noon on a particular day.

MONUMENT: The Awful Tower			
Time entered shop	*Time of purchase or leaving*	*Description of party*	*Outcome*
11:01	11:23	Romantic couple	Bought matching hats
11:06	11:25 / 11:20	Family (they split up, so treat as two groups)	Father bought joke book (11:25) Mother left carrying baby (11:20)
11:08	11:26	Rich ladies	Bought necklaces
11:12	11:27	Teenager (pickpocket?)	Bought baseball cap
11:25	11:28	Old ladies (wheelchair)	Taken to front of queue to buy souvenir book
11:25	11:30	Old man with stick	Left (looked unwell)
11:27	11:48	Father and son	Got bored and left
11:36	11:50	Group of pensioners	Bought postcards and stamps
11:38	11:55	School kids with teacher	Bought sweets
11:40	11:45	Couple eating chips	Left to go to the hotdog stand
11:50	12:05	Japanese tourists	Bought miniature tower models
11:55	12:10	Businessman	Left after receiving phone call
11:58	12:00	Lady with dog	Asked to leave (no pets allowed)
11:58	12:15	Girls with mobile phones	Bought T-shirts

(i) Identify three types of censoring that are present in your data and explain how they arise. [3]

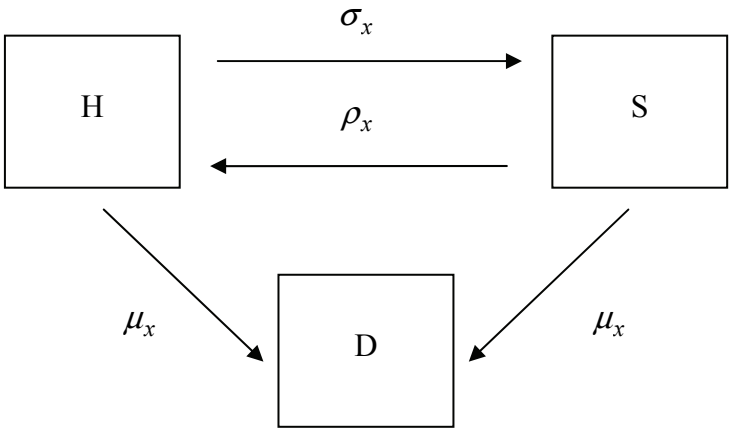
Let $S(t)$ denote the probability that the first person within a party of visitors to make a purchase will do so at least t minutes after entering the shop.

(ii) Prepare one or more tables or diagrams summarising the data in a form suitable for estimating values of $S(t)$. [4]

(iii) Use your table(s) or diagram(s) to estimate $S(20)$ using both the Kaplan-Meier and Nelson-Aalen models, stating any assumptions you make. [5]

[Total 12]

9 An insurance company uses the following Markov jump process model of transitions between the three states healthy, sick and dead, for a certain health insurance product, where x is the policyholder's age in years:



Under the insurance product, benefits are payable while the policyholder is sick.

(i) Discuss the possible limitations of the above model for its intended purpose. [3]

The insurance company has collected the following data.

The numbers of healthy (H) and sick (S) policyholders aged x last birthday

	Age last birthday					
	49		*50*		*51*	
Date	*H*	*S*	*H*	*S*	*H*	*S*
1.1.13	*3,475*	*54*	*4,418*	*72*	*5,421*	*111*
1.7.13	*3,642*	*65*	*4,227*	*67*	*6,052*	*98*
1.1.14	*3,955*	*39*	*4,331*	*95*	*5,108*	*97*
1.7.14	*3,720*	*42*	*5,011*	*70*	*5,876*	*102*
1.1.15	*3,784*	*60*	*4,977*	*81*	*5,715*	*143*

The numbers of transitions occurring between 1.1.2013 and 1.1.2015 inclusive, aged 50 nearest birthday at the time of transition:

- *420 transitions from healthy to sick*
- *12 transitions from healthy to dead*
- *390 transitions from sick to healthy*
- *31 transitions from sick to dead.*

(ii) State the rate interval implied by the age definition of the number of transitions in any particular category. [1]

(iii) Derive appropriate exposed to risk formulae that could be used to estimate the following transition rates from the data, defining any symbols that you use and stating all the assumptions that you make:

$$\mu_{50+f}, \quad \sigma_{50+f}, \text{ and } \rho_{50+f}$$

where f is a value that you should determine. Hence set out formulae for the estimates of the above three transition rates. [8]

(iv) Use the data to estimate the transition rate from sick to healthy at exact age 50. [1]

[Total 13]

10 The data below have been taken from a mortality investigation of a life insurer's male assured lives:

Age x last birthday	Observed deaths	Central exposed to risk, E_x^c	Graduated force of mortality, $\mathring{\mu}_x$
50	18	14,330	0.00108
51	16	14,908	0.00130
52	14	13,522	0.00183
53	30	12,098	0.00222
54	30	11,975	0.00278
55	50	12,058	0.00340
56	64	14,925	0.00396
57	67	13,002	0.00475
58	90	13,850	0.00558

where $\mathring{\mu}_x$ is assumed to be constant over the rate interval for age x last birthday.

(i) Explain why the insurer would not want to use crude mortality rates in its premium calculations. [2]

(ii) Explain why graduated rates have to pass certain tests before they can be used in financial calculations. [1]

(iii) An actuary has decided to carry out the cumulative deviations test on the above data. Explain the rationale behind this test. [4]

(iv) Carry out the cumulative deviations test on these data, stating your conclusion clearly. [3]

(v) Carry out the following additional tests on these data, and state your overall conclusion:

(a) chi-squared goodness-of-fit test

(b) grouping of signs test. [6]

[Total 16]

11 Boats of three sizes, big, medium-sized, and small, travel along a ship canal. Boats are assumed to arrive at a particular swing bridge over this canal according to a Poisson process, with average arrival rates (both directions combined) of:

- 20 big boats per year
- 600 medium-sized boats per year
- 46,000 small boats per year.

You can assume that there are 365¼ days in each year.

(i) Comment on the suitability of using a Poisson process to model the arrival times of the three different types of boat at the swing bridge. [3]

(ii) A man walks onto the bridge, at which moment there is no boat passing underneath. Calculate how long he expects to wait, in minutes, before the next boat arrives at the bridge. [1]

The swing bridge has to be opened for exactly 30 minutes to let a big boat through, and for exactly 15 minutes to let a medium-sized boat through. Small boats can pass underneath without the bridge being opened.

(iii) Use the assumed model to calculate the probability that the bridge has to be opened for more than 30 minutes on any particular day.

(You can assume for this purpose that no time efficiencies occur should a subsequent big or medium-sized boat arrive while the bridge is already open. For example, two medium sized boats arriving within the same 15 minute period will require the bridge to remain open for 30 minutes in total.) [5]

The canal takes boat traffic directly from the open sea to an inland port. The boats travel from the sea to the port and back again along the same canal, without changing direction except to turn around at the port itself. Boats can only enter and leave the canal at the one place where it joins the open sea.

At the port, there are three spaces for medium-sized boats to dock at the same time. The time a medium-sized boat stays in the dock follows an exponential distribution with a mean of 4 hours.

The number of spaces occupied by medium-sized boats, plus the number of medium-sized boats waiting to dock, is to be modelled as a Markov Jump process.

(iv) Draw the transition graph for this process, and write down the generator matrix of transition rates per hour. [3]

Hint: Note that the arrival rates given above are for arrivals in either direction.

(v) Write down the Kolmogorov backward differential equation for $p_{13}(t)$, where:

$p_{ij}(t) =$ probability that the process, in state i at a given point in time, is in state j t hours later

and, for example, state i means that the total number of boats in the dock plus those waiting to dock is equal to i. [2]

(vi) Express $p_{13}(2)$ as an integral:

(a) based on the relevant backward equation

(b) based on the relevant forward equation. [4]

(vii) At 2pm on a particular day all three docks are occupied by medium-sized boats, and no other boat is currently waiting to dock. Calculate the probability that all the docks will become empty before any new medium-sized boat arrives at the port. [2]

[Total 20]

END OF MOCK EXAM

For the session leading to the April 2016 exams – CT Subjects

Marking vouchers

Subjects	*Assignments*	*Mocks*
CT1, CT2, CT5, CT6	16 March 2016	22 March 2016
CT3, CT4, CT7, CT8	22 March 2016	30 March 2016

Series X Assignments

Subjects	*Assignment*	*Recommended submission date*	*Final deadline date*
CT1, CT2, CT5, CT6	**X1**	**18 November 2015**	13 January 2016
CT3, CT4, CT7, CT8		**25 November 2015**	20 January 2016
CT1, CT2, CT5, CT6	**X2**	**9 December 2015**	3 February 2016
CT3, CT4, CT7, CT8		**16 December 2015**	10 February 2016
CT1, CT2, CT5, CT6	**X3**	**20 January 2016**	24 February 2016
CT3, CT4, CT7, CT8		**27 January 2016**	2 March 2016
CT1, CT2, CT5, CT6	**X4**	**17 February 2016**	9 March 2016
CT3, CT4, CT7, CT8		**24 February 2016**	16 March 2016

Mock Exams

Subjects	*Recommended submission date*	*Final deadline date*
CT1, CT2, CT5, CT6	**9 March 2016**	22 March 2016
CT3, CT4, CT7, CT8	**16 March 2016**	30 March 2016

We encourage you to work to the recommended submission dates where possible.

We strongly recommend that you submit your mock exam electronically in order for us to return your marked script to you in plenty of time before your exam. If you submit your mock on the final deadline date you are likely to receive your script back less than a week before your exam.

For the session leading to the September/October 2016 exams – CT Subjects

Marking vouchers

Subjects	*Assignments*	*Mocks*
CT1, CT2, CT6, CT7	31 August 2016	7 September 2016
CT3, CT4, CT5, CT8	7 September 2016	14 September 2016

Series X Assignments

Subjects	*Assignment*	*Recommended submission date*	*Final deadline date*
CT1, CT2, CT6, CT7	**X1**	**8 June 2016**	6 July 2016
CT3, CT4, CT5, CT8		**15 June 2016**	13 July 2016
CT1, CT2, CT6, CT7	**X2**	**29 June 2016**	27 July 2016
CT3, CT4, CT5, CT8		**6 July 2016**	3 August 2016
CT1, CT2, CT6, CT7	**X3**	**20 July 2016**	10 August 2016
CT3, CT4, CT5, CT8		**27 July 2016**	17 August 2016
CT1, CT2, CT6, CT7	**X4**	**3 August 2016**	24 August 2016
CT3, CT4, CT5, CT8		**10 August 2016**	31 August 2016

Mock Exams

Subjects	*Recommended submission date*	*Final deadline date*
CT1, CT2, CT6, CT7	**24 August 2016**	7 September 2016
CT3, CT4, CT5, CT8	**31 August 2016**	14 September 2016

We encourage you to work to the recommended submission dates where possible.

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