CT3-01: Summarising data Page 25 **�**

***Chapter*** ***1*** ***Summary***

Numerical data can be discrete or continuous.

Categorical data can be dichotomous (attribute), nominal or ordinal.

Data can be presented either in tabular form (using a frequency table, a cumulative frequency table or a stem and leaf diagram) or in graphical form (using a lineplot, a dotplot, a boxplot, a bar chart or a histogram).

The location of a data set can be summarised using the mean, the median or the mode.

The spread of a data set can be summarised using the standard deviation, the range or the interquartile range.

The variance measures the spread squared.

Third moments can be used to summarise the skewness (ie the degree of asymmetry) of a data set.

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Page 26 CT3-01: Summarising data **�**

3

1

1

4 2

**�**

***Chapter*** ***1*** ***Formulae***

***Measures*** ***of*** ***Location***

n

x = x or

1

i

i=1

x = Âfixi i

M =(1 n+ 1)th value

2 2

***Measures*** ***of*** ***Spread***

R = max(xi)-min(xi)

i

i

IQR =Q3 -Q ,

1

alternatively

Q3 =(4 n+ 2)th value

Q3 =(3 n+ 3)th value

4 4

Q =(1 n+ 1)th value

Q =(1 n+ 1)th value

1

4 4

s2 = 1 Â(x - x)2 = 1 ÍÂx 2 -nx2˙ i=1 Îi=1 ˚

or Âfi -1Âfi(xi - x)2 = Âfi -1ÈÂfixi2 -nx2˘

Î ˚

***Measures*** ***of*** ***Skewness***

n

skewness = 1Â(xi - x)3 i=1

coeff of skew = skewness

n

where s2 = 1 n (xi - x)2 i=1

n

***Sample*** ***Moments***

n kth moment = x

1

i=1

kth moment about a = 1 n (xi -a)k i=1

kth central moment = 1 n (xi - x)k i=1

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CT3-02: Probability Page 25

***Chapter*** ***2*** ***Summary***

A set is a collection of objects, called elements. A is a subset of B, written AÃ B, if all the elements in A are contained in B. The complement of A, written A¢, is the set of all the elements not in A. The empty set is denoted ∆.

The union of A and B, written A»B, is the set of all elements in A or B or both. The intersection of A and B, written A«B, is the set of all elements in A and B.

Venn diagrams are used to represent sets and the relationships between them.

A sample space, S, is the set of all the possible outcomes from an experiment. An event is anything we might wish to observe from our experiment.

Probabilities are a numerical way of describing how likely an event is to happen. A formula for equally likely elements is given overleaf. Probabilities lie between 0 (impossible) and 1 (certain).

We can use the addition rule and the multiplication rule (see overleaf) to calculate probabilities. Tree diagrams are a helpful way of working out probabilities.

The conditional probabilities of A occurring given that B has already occurred is written P(A| B). The formula is given overleaf.

Events A and B are mutually exclusive if A«B =∆. Events A and B independent if P(A| B) = P(A).

E ,…,En is a partition of S if the Ei ’s are mutually exclusive and together make up the whole set S.

1

Bayes’ Theorem (see overleaf) allows us to ‘turnaround’ conditional probabilities, ie calculate P(Ei | A) given only information about P(A| Ei).

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Page 26 CT3-02: Probability

***Chapter*** ***2*** ***Formulae***

***Probabilities***

For equally likely elements:

number of elements in A number of elements in S

P(A) =

***Addition*** ***rule***

P(A» B) = P(A) + P(B) - P(A« B)

For mutually exclusive events A« B = ∆

P(A» B) = P(A) + P(B)

***Multiplication*** ***rule***

P(A« B) = P(A)P(B| A) or P(B)P(A| B)

For independent events P(A| B) = P(A) and P(B| A) = P(B), so:

P(A« B) = P(A)P(B)

***Conditional*** ***probabilities***

P(A| B) = P(A B B)

***Bayes’*** ***Theorem***

For a partition Ei, i =1,2,…,n

P(Ei | A) = P(Ei)P(A| Ei) , ÂP(Ej)P(A| Ej)

j=1

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i =1, 2,3,…,n

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CT3-03: Random Variables Page 27

***Chapter*** ***3*** ***Summary***

Random variables are used to model features of a population using probabilities.

A discrete random variable has a probability function (PF), P(X = x). This defines how the probability is split between the values the variable can take. The PF satisfies:

ÂP(X = x) =1 and P(X = x)≥0 x

A continuous random variable has a probability density function (PDF), fX (x). The PDF satisfies:

Ú fX (x) dx =1 and fX (x) ≥ 0 x

We can use the PDF to find probabilities as follows:

b

P(a < X <b) = Ú fX (x) dx. a

The cumulative distribution function (CDF), for both discrete and continuous random variables is given by:

FX (x) = P(X £ x)

For continuous random variables FX (x) = fX (x).

¢

Using formulae given overleaf, we can calculate the:

· mean m

· variance s2 · skewness m3

and other central and non-central moments of a random variable.

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Page 28 CT3-03: Random Variables

***Chapter*** ***3*** ***Formulae***

***Population*** ***Mean*** ***(Expectation)***

m = E(X) =ÂxP(X = x) or x

•

Ú xfX (x) dx -•

E[g(X)]=Âg(x)P(X = x) or x

•

Ú g(x)fX (x) dx -•

***Population*** ***Variance***

s2 = var(X) = E[(X - m)2]= E(X2)-E2(X)

***Population*** ***Skewness***

m3 = skew(X) = E[(X - m)3]= E[X3]-3mE[X2]+2m3 coefficient of skewness = m3

s

***Population*** ***Moments***

kth moment = E[Xk ]

kth moment about c = E[(X -c)k] kth central moment = E[(X - m)k ]

***Population*** ***median*** ***and*** ***IQR***

m such that P(X < m) = 0.5

IQR = q3 -q where P(X < q ) = 0.25 and P(X < q3) = 0.75

1 1

***Linear*** ***Functions*** ***of*** ***X***

E[aX +b]= aE[X]+b var[aX +b]= a2 var[X]

***Functions*** ***of*** ***a*** ***Random*** ***Variable***

Y =u(X) ﬁ fY (y) = fX Èu-1(y)˘

Î ˚

|  |
| --- |
| du-1(y) |
| dy |

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CT3-04: Probability Distributions Page 53 **�**

***Chapter*** ***4*** ***Summary***

Standard discrete distributions covered in this course are the discrete uniform, Bernoulli, binomial, geometric, negative binomial, hypergeometric and Poisson.

Waiting times between events in a Poi(l) distribution have a Exp(l) distribution.

Standard continuous distributions covered in this course are the continuous uniform, gamma, exponential, chi-square, normal, lognormal, beta, t and F.

The geometric and exponential distributions have the “memoryless” property:

P(X > x + n| X > n) = P(X > x)

The properties of the distributions are summarised overleaf.

The t-distribution with k degrees of freedom is defined as:

tk ∫ N(0,1) k

The F-distribution with m, n degrees of freedom is defined as:

Fm,n = cm m n

2

c

The Poisson process counts events occurring up to and including time t:

N(t) ~ Poi(lt)

It can be derived by considering events occurring in a small time interval h.

The waiting time between events in a Poisson process has an exponential distribution.

Random variables can be simulated by using the inverse transform method. First we take a random number, u, from U(0,1) then we set:

continuous

discrete

x = F-1(u)

x = xj where F(xj-1) <u £ F(xj)

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Page 54 **�**

**�**

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CT3-04: Probability Distributions

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CT3-05: Generating functions Page 29

***Chapter*** ***5*** ***Summary***

Generating functions are used to make finding moments of distributions easier.

The probability generating function (PGF) of a counting random variable is defined to be:

GX (t) = EÈtX ˘

Î ˚

The series expansion for PGFs is:

GX (t) = P(X = 0)+tP(X =1)+t2P(X = 2)+t3P(X = 3)+!

The moment generating function (MGF) of a random variable is defined to be:

MX (t) = EÈetX ˘

Î ˚

The series expansion for MGFs:

2 3

MX (t) =1+tE(X)+ 2!E(X )+ 3!E(X )+!

The cumulant generating function (CGF) of a random variable is defined to be:

CX (t) = lnMX (t)

Moments of a random variable can be found from its PGF, MGF or CGF using the formulae listed overleaf.

The uniqueness property means that if two variables have the same PGF, MGF or CGF then they have the same distribution.

If Y = a +bX , then:

GY (t) = taGX (tb), MY (t) = eatMX (bt) and CY (t) = at +CX (bt)

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Page 30 CT3-05: Generating functions

***Chapter*** ***5*** ***Formulae***

***Probability*** ***Generating*** ***Functions***

GX (t) = E(tX )=ÂtxP(X = x) x

E(X) = G¢ (1)

X

var(X) = G¢¢(1)+G¢ (1)-(G¢ (1))2

X X X

GX (t) = P(X = 0)+tP(X =1)+t2P(X = 2)+t3P(X = 3)+!

***Moment*** ***Generating*** ***Functions***

MX (t) = E(etX )= ÂetxP(X = x) or x

Úetx f (x) dx x

E(X) = M¢ (0)

X

var(X) = M¢¢(0)-(M¢ (0))2

X X

2 3

MX (t) =1+tE(X)+ 2!E(X )+ 3!E(X )+!

***Cumulant*** ***Generating*** ***Functions***

CX (t) = lnMX (t)

E(X) = CX (0)

¢

var(X) =C¢¢(0)

X

skew(X) =C¢¢¢(0)

X

***Linear*** ***Transformations***

Y = aX +b ﬁ MY (t) = ebtMX (at)

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CT3-06: Joint distributions Page 39

***Chapter*** ***6*** ***Summary***

Two discrete random variables X and Y have joint probability function (PF), P(X = x,Y = y). This defines how the probability is split between the different

combinations of the variables. The joint PF satisfies:

ÂÂP(X = x,Y = y) =1 and P(X = x,Y = y) ≥0 x y

Two continuous random variables X and Y have joint probability density function (PDF), fX,Y (x,y). The joint PDF satisfies:

ÚÚ fX,Y (x,y) dx dy =1 and fX,Y (x,y)≥ 0 x y

We can use the joint PDF to find probabilities as follows:

y2 x2

P(x < X < x2,y <Y < y2) = Ú Ú f (x,y) dx dy y x

1 1

1 1

The joint distribution function, for both discrete and continuous random variables is given by:

F(x,y) = P(X < x,Y < y)

2

For continuous random variables f (x,y) = ∂x∂ y F(x,y).

Using the formulae overleaf, we can calculate the:

· Marginal distributions, eg P(X = x) or fX (x)

· Conditional distributions, eg P(X = x|Y = y) or fX|Y=y(x| y) · Expectation of any function, E[g(X,Y)]

· Covariance, cov(X,Y)

· Correlation coefficient, r(X,Y) = corr(X,Y)

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Page 40 CT3-06: Joint distributions

The random variables X and Y are uncorrelated if and only if:

corr(X,Y) = 0 ¤ cov(X,Y) = 0 ¤ E(XY) = E(X)E(Y)

The random variables X and Y are independent if, and only if:

P(X = x,Y = y) = P(X = x)P(Y = y)

fX,Y (x,y) = fX (x)fY (y)

Independent random variables are always uncorrelated. Uncorrelated random variables are not necessarily independent.

There are rules connecting sums and products of expectations and sums of variances.

The convolution of the marginal probability (density) functions of X and Y is the probability (density) function of Z = X +Y . P(Z = z) or fZ (z) is given using the formulae on the formulae sheet. The convolution is written fZ = fX \* fY .

Sums of independent random variables make other random variables. The full list is given on the formulae sheet.

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CT3-06: Joint distributions Page 41

***Chapter*** ***6*** ***Formulae***

***Marginal*** ***probability*** ***(density)*** ***function***

P(X = x) = ∑P(X = x,Y = y) y

fX (x) = ∫ fX,Y (x,y) dy y

***Conditional*** ***probability*** ***(density)*** ***function***

P(X = x|Y = y) = P(X (Yx,Y ) y)

fX|Y=y(x,y) = fX,Y (x,y) Y

***Expectation***

E[g(X,Y)]= ∑∑g(x,y)P(X = x,Y = y) or x y

∫∫g(x,y)fX,Y (x,y) dxdy y x

***Covariance***

cov(X,Y) = E(X − E(X))(Y − E(Y)) = E(XY)− E(X)E(Y)

 

***Correlation***

corr(X,Y) = r(X,Y) =

cov(X,Y)

var(X)var(Y)

***Sums*** ***and*** ***products*** ***of*** ***moments***

E(X +Y) = E(X)+ E(Y)

E(XY) = E(X)E(Y)+Cov(X,Y)

= E(X)E(Y) if X,Y independent

The above are also true for functions g(X) and h(Y) of the random variables.

var(X +Y) = var(X)+ var(Y)+ 2cov(X,Y) = var(X)+ var(Y)

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if X,Y independent

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Page 42 CT3-06: Joint distributions

***Convolutions***

fZ = fX \* fY = ∑P(X = x)P(Y = z − x) or x

***Linear*** ***Combinations***

∫ fX (x)fY (z − x) dx x

For independent random variables X1,…,Xn :

E(c X1 +!+ cnXn)= c E(X1)+!+ cnE(Xn) var(c X1 +!+ cnXn)= c2 var(X1)+!+ c2 var(Xn)

1 1

2

1 1

***Linear*** ***Combinations*** ***of*** ***PGFs*** ***and*** ***MGFs***

For independent random variables X1,…,Xn :

Y = X1 +!+ Xn ⇒

⇒

GY (s) = GX1 (s)…GXn (s) =[GX (s)]n

MY (t) = MX1 (t)…MXn (t)

=[MX (t)]n

Xi's identical

Xi's identical

***Linear*** ***Combinations*** ***of*** ***random*** ***variables***

For independent distributions:

“Bernoulli(p)+!+ Bernoulli(p) ~ Bin(n, p)” “Bin(n,q)+ Bin(m,q) ~ Bin(n+ m,q)”

“Geo(p)+!+Geo(p) ~ NBin(k, p)” “NBin(k,q)+ NBin(m,q) ~ NBin(k + m,q)”

“Exp(l)+!+ Exp(l) ~ Ga(a,l)” “Ga(a,l)+Ga(d,l) ~ Ga( +d,l)”

a

“cm + cn ~ cm+n”

2 2 2

“Poi(l)+ Poi(µ) ~ Poi(l + µ)”

“N(µ1,s1 )± N(µ2,s2) ~ N(µ1 ± µ2,s1 +s2)”

2 2 2 2

Please note that some of the notation used for the linear combinations of random variables is non-standard and is used simply to convey the results in a concise format.

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CT3-07: Conditional expectation Page 15 **�**

***Chapter*** ***7*** ***Summary***

E(Y | X) is the mean of the conditional distribution of Y given X (which was defined in Chapter 6). The formulae for discrete and continuous distributions are given overleaf.

var(Y | X) is the variance of the conditional distribution of Y given X, it is given by:

var(Y | X) = E(Y2 | X)− E2(Y | X)

The unconditional mean and variance can be found from the conditional mean and variance using the formulae given overleaf and on page 16 of the Tables.

A quantity that is the sum of a random number of random quantities has a compound distribution:

S = X1 +!+ XN

We can find the mean, variance and MGF of a compound distribution using the formulae given overleaf.

We can find the skewness using the CGF.

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Page 16 CT3-07: Conditional expectation **�**

**�**

***Chapter*** ***7*** ***Formulae***

***Conditional*** ***Expectation***

E[Y | X = x]= i yiP[Y = yi | X = x]= i yi P[YP[ yi,X ] x]

∑ ∑

E[Y | X = x]= ∫y f (y| x) dy = ∫y f (x,y) dy y y

***Conditional*** ***Variance***

var[Y | X = x]= E[Y2 | X = x]− E2[Y | X = x]

***Relationships*** ***between*** ***unconditional*** ***and*** ***conditional*** ***moments***

E[Y]= E[E(Y | X)]

var[Y]= E[Var(Y | X)]+ var[E(Y | X)]

***Compound*** ***Distributions***

E(S) = E(N)E(X)

var(S) = E(N)Var(X)+ var(N)E2(X)

MS(t) = MN{logMX (t)}

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CT3-08: The Central Limit Theorem Page 13

***Chapter*** ***8*** ***Formulae***

***The*** ***Central*** ***Limit*** ***Theorem***

For X1,…,Xn iid RV’s with mean m and variance s2:

ÂXi ~ N(nm,ns2) ﬁ

X ~ NÊm,s2 ˆ ﬁ Ë ¯

ÂXi -nm ~ N(0,1) as nÆ • ns

X - m ~ N(0,1) as nÆ • s n

***Normal*** ***Approximations***

Bin(n, p) ~ N(np,npq)

Poi(l) ~ N(l,l)

Ga(a,l) ~ N(a , a ) c2 ~ N(k,2k)

l

k

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np >5, nq >5¸

with continuity correction l large ˛

a large

k large

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Page 22 CT3-09: Sampling and statistical inference **�**

**�**

***Chapter*** ***9*** ***Formulae***

***Moments*** ***of*** ***Statistics***

2 E(X) = µ Var(X) = n

E(S2) =s2 any distribution

var(S2) = ns 1 normal distribution only

4

***t-distribution***

tk º N(0,1) k

***F-distribution***

Fm,n º cm m n

2

***Sampling*** ***Distributions***

2

X ~ Nµ,  any distribution large n (or normal anyn)  

⇒

⇒

X − µ ~ N(0,1)

X − µ

~

t

S n n−1

s2 known

s2 unknown

(n−1)S2

s2 n−1

~

2 2

1 1 ~ n −1,n2 −1

F

1

2 2

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normal distribution only

normal distributions only

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CT3-10: Point estimation Page 35

***Chapter*** ***10*** ***Summary***

We have covered two methods here for estimating parameters.

The method of moments technique equates the population moments to the sample moments using the formulae detailed overleaf.

The method of maximum likelihood:

n

’

· find the likelihood L( ) = f (xi;q) i=1

q

· log L

· find q that solves ∂ lnL( ) = 0

q

2

· check for maximum ∂q2 lnL( ) < 0.

q

If the range of the distribution is a function of the parameter the maximum must be found from first principles.

Three properties of estimators are bias, mean square error (MSE) and consistency:

The bias of an estimator is given by E[g(X)]−q where g(X) is the estimator.

g(X) is an unbiased estimator of q if E[g(X)]= q .

The mean square error of an estimator is given by E[(g(X)−q)2] where g(X) is the estimator. An easier formula is var[g(X)]+bias2[g(X)]. An estimator is consistent if

MSE Æ 0 as nÆ •, where n is the size of the sample.

A good estimator has a small MSE, is unbiased and consistent.

The Cramér-Rao lower bound gives a lower bound for the variance of an unbiased estimator. It can be found using the formulae overleaf. It can be used to obtain confidence intervals.

The value of the CRLB depends on the parameter you are estimating. To use this formula, the likelihood must be expressed in terms of the correct parameter.

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Page 36 CT3-10: Point estimation

***Chapter*** ***10*** ***Formulae***

***Method*** ***of*** ***Moments***

1 parameter

2 parameters

n E(X) = Xi

i=1

E(X) = 1 n Xi i=1

E(X2) = 1 n X2

i

i=1

or var(X) = 1 n (Xi - X)2 i=1

alternatively E(X) = 1 n Xi i=1

var(X) = S2 = 1 n (Xi - X)2 i=1

***Method*** ***of*** ***Maximum*** ***Likelihood***

n

’

L( ) = f (xi;q) i=1

q

q that solves ∂ lnL( ) = 0

q

***Bias***

bias[g(X)]= E[g(X)]-q

***Mean*** ***Square*** ***Error***

MSE[g(X)]= E[(g(X)-q)2]= var[g(X)]+bias2[g(X)]

***Cramér-Rao*** ***Lower*** ***Bound***

CRLB( ) = - 2 1

q

EÍ lnL( ,X)˙ Î ˚

***Asymptotic*** ***Distribution*** ***of*** ***MLE***

ˆ ~ N( ,CRLB)

q

q

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Page 32 CT3-11: Confidence intervals

ˆ

***Chapter*** ***11*** ***Formulae***

***One*** ***sample*** ***normal*** ***distribution***

X − µ0 ~ N(0,1) s2 known

X − µ

s n n−1

~

t

s2 unknown

(n−1)S2

~

s2 n−1

0

***Two*** ***sample*** ***normal*** ***distribution***

(X1 − X2)−(µ1 − µ2) ~ N(0,1) s1 1 +s2 n2

s2 known

(X1 − X2)−(µ1 − µ2)

t

~

sp 1 n +1 n2 n +n2 −2

1

s2 unknown

2 2

1 1 ~ n −1,n2 −1

F

1

2 2

***One*** ***sample*** ***binomial***

ˆpq n ~ N(0,1) or X nˆnp ~ N(0,1)

***Two*** ***sample*** ***binomial***

(p − pˆ1 −(p − p2) ~ N(0,1) 1 n2

where p = x , p2 = x2 1 2

***One*** ***sample*** ***Poisson***

l −l ~ N(0,1) or l n

∑X − nl ~ N(0,1) nl

***Two*** ***sample*** ***Poisson***

( 1 − ˆ2)−( 1 −l2) ~ N(0,1) l l2

1 n2

where 1 = X1, ˆ2 = X2

l

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CT3-12: Hypothesis testing Page 39 **�**

***Chapter*** ***12*** ***Summary***

Statistical tests can be used to test assertions about populations.

The process of statistical testing involves setting up a null hypothesis and an alternative hypothesis, calculating a test statistic and using this to determine a p-value.

The probability of a Type I error is the probability of rejecting H0 when it is true. This is also called the size (or level) of the test. The probability of a Type II error is the probability of accepting H0 when it is false. The power of a test is the probability of rejecting H0 when it is false.

The “best” test can be found using the likelihood ratio criterion. This leads to the tests detailed on the formulae summary sheet.

The test for two normal means (unknown variances) requires that the variances are the same and uses the pooled sample variance:

sp = (n1 −1)s1 + (n2 −1)s2 1 2

2 2

2

c2 tests can be carried out to test for goodness of fit or to test whether two factors are independent (using contingency tables).

The statistic is ∑(O − Ei )2 . i

To find the number of degrees of freedom for the goodness of fit test, take the number of cells, subtract 1 if the total of the observed figures has been used in the calculation of the expected numbers (which is usually the case), and then subtract the number of parameters estimated.

To find the number of degrees of freedom for a contingency table calculate (r −1)(c−1). If the expected numbers in some cells are small, these should be

grouped. One degree of freedom is lost for each cell that is “lost”.

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Page 40 CT3-12: Hypothesis testing **�**

=

n

l

=

1

***Chapter*** ***12*** ***Formulae***

***One*** ***sample*** ***normal*** ***distribution***

X − µ0 ~ N(0,1) s2 known

X − µ

s n n−1

t

~

s2 unknown

(n−1)S2

~

s0 n−1

***Two*** ***sample*** ***normal*** ***distribution***

(X1 − X2)−(µ1 − µ2) ~ N(0,1) s1 1 +s2 n2

s2 known

(X1 − X2)−(µ1 − µ2)

t

~

sp 1 n +1 n2 n +n2 −2

1

s2 unknown

2 2

1 1 ~ n −1,n2 −1

F

1

2 2

***One*** ***sample*** ***binomial***

p0q00n ~ N(0,1) or Xnpnp0 ~ N(0,1) with continuity correction

***Two*** ***sample*** ***binomial***

( ˆ1 − pˆ ˆ−( ˆ ˆ− p2) ~ N(0,1)

1 n2

x + x2 n +n2

1

is the overall sample proportion

***One*** ***sample*** ***Poisson***

X −l0 ~ N(0,1) or

0

∑X − nl0 ~ N(0,1) with continuity correction 0

***Two*** ***sample*** ***Poisson***

( 1 − ˆ2)−( 1 −l2) ~ N(0,1) l l

1 n2

ˆ 1 1 +n2 ˆ2 n +n2

is the overall sample mean

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CT3-13: Correlation and regression Page 47

***Chapter*** ***13*** ***Summary***

A regression model, such as the simple linear regression model, can be used to model the response when an explanatory variable operates at a given level, or to model bivariate data points.

The sample correlation coefficient, r, measures the strength of the linear relationship between x and y. The formula is given overleaf and on page 25 of the Tables.

We can carry out hypothesis tests on the population correlation coefficient, r , using the t result or the Fisher-Z test. Both of these results are given overleaf and on page 25 of the Tables.

The linear regression model is given by:

i = a + b xi + ei where e ~ N(0,s2)

Y

i

The parameters a,b and s2 can be estimated using the formulae overleaf and on page 24 of the Tables.

Confidence intervals can be obtained for b and the predicted individual (or mean) response y using the formulae given overleaf and on pages 24 and 25 of the Tables.

The fit of the linear regression model can be analysed by:

Partitioning the total variance, SSTOT , into that which is explained by the model, SSREG, and that which is not, SSRES . The coefficient of determination, R2 , gives the percentage of this variance which is explained by the model. The formula is given

overleaf but not in the Tables.

Examining the residuals, e = yi - yi . We would expect them to be normally distributed about zero and to have no relationship with the x values. Both of these can be examined using diagrams.

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b

=

m

***Chapter*** ***13*** ***Formulae***

***Correlation***

r =

Sxy

SxxSyy

H0 :r = 0

otherwise

r n-2

1-r2 n-2

~ t

1ln1+r ~ N Ê1ln1+ r ,n-3ˆ

***Regression***

ˆ Sxy Sxx

and a = y -b x

ˆ

ˆ

s2 = 1 Â(yi - yi)2 = 1 ÊSyy - Sxy ˆ xx

2

ˆ

b -b

ˆ

~

t

2 n-2 xx

ˆ0 - m0 ~t Ê1 (x0 - x)2 ˆ 2

Ën Sxx ¯

and y0 - y0 ~t Ê1+ 1 + (x0 - x)2 ˆ ˆ2

ˆ

Ë xx ¯

***Fit*** ***of*** ***Model***

SSTOT = SSRES +SSREG

2 SSREG Sxy SSTOT SxxSyy

2

R

e = yi - yi

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CT3-14: Analysis of variance Page 33 **�**

***Chapter*** ***14*** ***Summary***

Analysis of variance (ANOVA) is testing for a difference between treatment means. The model is:

ij = m +ti + ij

Y

e

where ej ’s are independent and identically distributed N(0,s2).

i

The parameters m (total mean), ti (treatment effect) and s2 can be estimated using the formulae overleaf. The variance estimate is given on page 26 of the Tables.

The total variance (SST ) is split into variance within each treatment (SSR) and variance between treatment means (SSB):

SST = SSR + SSB

These can be calculated using the formulae given overleaf and on page 26 of the Tables.

The significance of the variance between treatment means (SSB) is established using the F test given overleaf and on page 26 of the Tables.

Residuals can be plotted to check the adequacy of the model – ie the normality and equality of variances assumptions.

Confidence intervals for means should use the results of Chapter 11, but with the variance estimate ˆ2 instead.

s

If it is found under ANOVA that the treatment means are not the same, we can analyse them further using the least significant difference approach. This is essentially testing each of the pairs of treatments to see if they have the same mean or not. The results of this can be shown on a diagram like the one below:

y2i < y3i < 1i < y4i

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**�**

***Chapter*** ***14*** ***Formulae***

***Parameter*** ***Estimates***

ˆ = yii = 1ÂÂyij i j

m

ti = yii - yii = 1 Âyij - 1ÂÂyij i j i j

ˆ

ˆ2 = n 1 k SSR

s

***Sum*** ***of*** ***Squares***

SST = ÂÂyij - yii

2

2

2 2 SSB = ii - ii

i

SSR = SST - SSB

***Statistical*** ***Test***

SSB k -1

SS

n-k k-1,n-k

~

F

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