AMRITA VISHWA VIDYAPEETHAM
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Galactic rotation curve

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24AIM111 Introductionto data structures &algorithms
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Introduction

The study of galactic rotation curves plays a crucial role in understanding the distribution of mass in galaxies and provides strong evidence for the existence of dark matter.

The project highlights the necessity of dark matter in explaining galaxy dynamics and provides an interactive framework to study how mass distribution affects rotational motion.

Objective

- To analytically model the rotational velocity of stars in a galaxy using Newtonian mechanics.
- To differentiate the contributions of visible matter and dark matter in the galactic rotation curve.
- To demonstrate the necessity of dark matter in explaining the observed nearly flat rotation curves.
- To visualize the impact of mass distribution on the orbital velocities of stars at different radii.

Title	Year/Author	Key Findings
ROTATION CURVES OF GALAXIES	Dr. L . Sriramkumar (2015)	The motion of stars in a spherically symmetric gravitational field is analyzed.
The Rotation Curve of the Milky Way Galaxy as Evidence for Dark Matter	Huma Jafree, Rebekah Polen et al. (2022)	observations confirm that the rotation curve remains flat ($v \approx$ constant), suggesting the presence of dark matter extending beyond the visible disk

Dark Matter and Spiral Galaxies	T. S. Van Albada and R. Sancisi (2020)	The distributions of bulge, disc, and halo ensure flat rotation curves, with the dark-to-luminous mass ratio remaining constant across galaxies.
Estimation of the local dark matter density using the rotation curve of the Milky Way	K. Malhana, K. Freese (2019)	The local dark matter density remains stable under reasonable variations of a spherical dark matter halo.
Rotation curves and the dark matter problem	Albert Bosma (2018)	Addresses the challenges in identifying dark matter and the ongoing efforts in astrophysics.

Problem Statement

- The observed galactic rotation curves of spiral galaxies do not follow the expected Keplerian decline at large radii.
- Instead of decreasing, the rotational velocity remains nearly constant, implying the presence of unseen mass, referred to as dark matter. Traditional Newtonian mechanics, when applied only to visible matter (stars, gas, and dust), fails to explain this behavior.
- This project aims to analyze the role of dark matter by developing an analytical model for galactic rotation curves using mass distribution functions.

Solution

This project analytically models galactic rotation curves by computing the contributions of visible matter and dark matter using Newtonian mechanics. While the disk and bulge follow an exponential profile, the dark matter halo (NFW model) explains the flat rotation curves observed at large radii, reinforcing the need for dark matter in galaxy dynamics.

Methodology

1.Define Mass Component

- Disk Mass (Mdisk): Stars and gas in the disk.
- Bulge Mass (Mbulge): Dense central region.
- Dark Matter Halo (ρhalo): Unseen, extended mass

2.Compute Velocity

- Use Newton's laws to determine rotational velocity v(r) from enclosed mass M(r).

3.Model Mass Distribution

- Visible mass: Exponential disk profile.
- Dark matter: Navarro-Frenk-White (NFW) profile.

4.Compute Rotation Curve

- Combine visible and dark matter contributions to get a nearly flat curve at large radii.

5.Visualization

- Plot rotation curves showing visible mass decline, dark matter dominance, and total observed curve.

Key Equations

Gravitational force and acceleration- based on Newton's laws

$$F = \frac{Gm_1m_2}{r^2}$$

$$a = \frac{F}{m} = \frac{GM}{r^2}$$

Equations of Circular Velocity for Stable Orbits

$$v(r) = \sqrt{\frac{GM(r)}{r}}$$

G= Gravitational constant

r= radius

M(r)=mass enclosed within radius

Navarro-Frenk-White (NFW) Profile for Dark Matter Halo Model

$$\rho(r) = \frac{\rho_0}{(r/r_s)(1 + r/r_s)^2}$$

$$V_{\mathrm{NFW}}(r) = V_0 \sqrt{\frac{\ln(1 + r/r_s)}{r/r_s}}$$

Where
ρ_0= density constant
V_0= scale velocity
r_s=scale radius

SNAPSHOT OF DATASET

Radius (kpc)	Observed Velocity (km/s)	Error in Velocity (km/s)	Gas Velocity (km/s)	Disk Velocity (km/s)
0.42	14.2	1.91	4.87	4.78
1.26	28.6	1.82	13.14	10.76
2.11	41	1.74	19.65	13.6
2.96	49	1.91	22.42	13.29
3.79	54.8	2.05	22.82	12.56
4.65	56.4	3.12	21.37	12.33
5.48	57.8	2.83	18.73	12.04
6.33	56.5	0.65	16.75	10.62

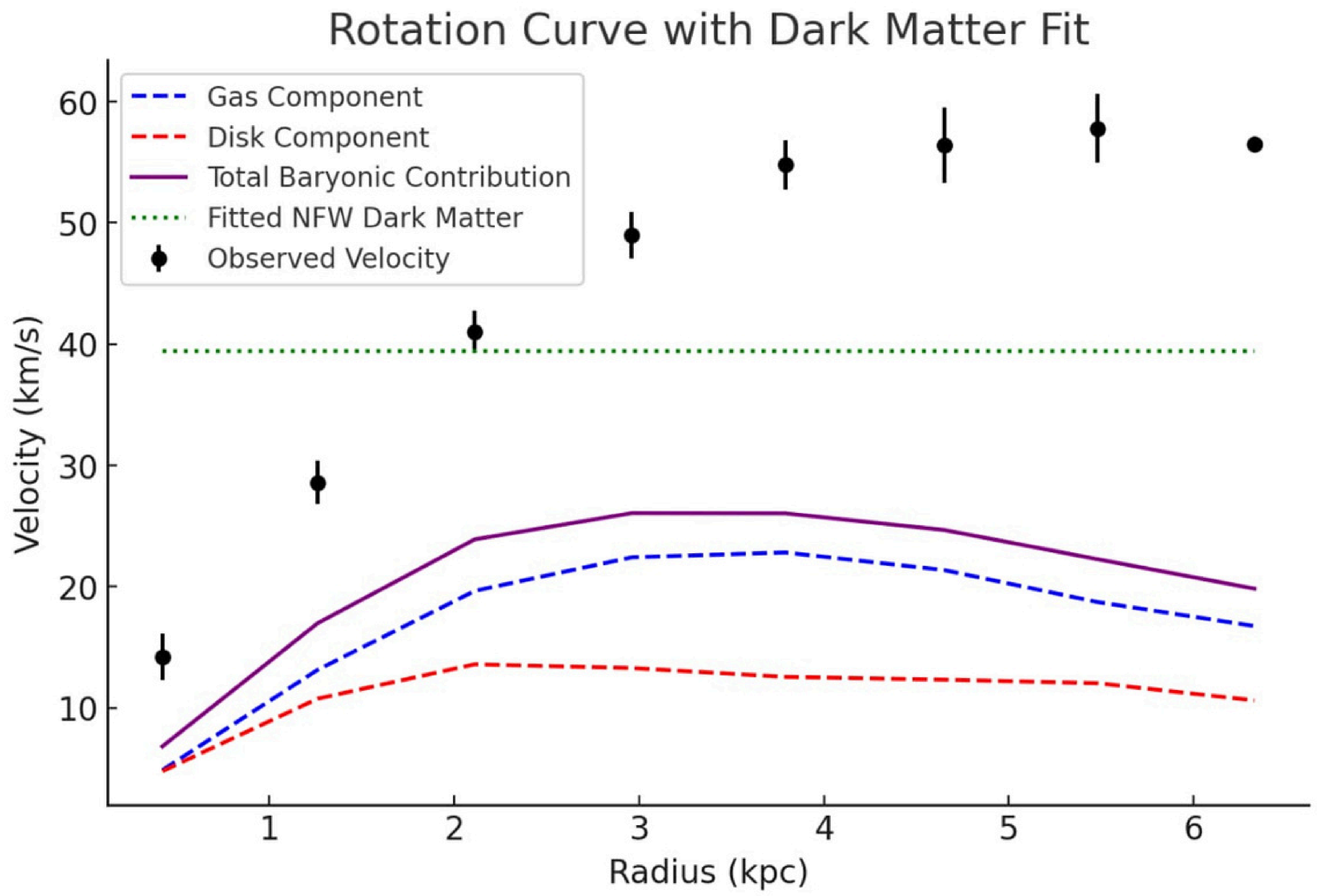
Dataset for UGCA 442

http://astroweb.cwru.edu/SPARC

Data Structures & Algorithm

- Brute force - for N-body simulation
- Euler's Method- updates positions and velocities at each time step
- Hernquist Profile- for dark matter acceleration
- Trust Region Reflective algorithm- for non-linear curve fitting

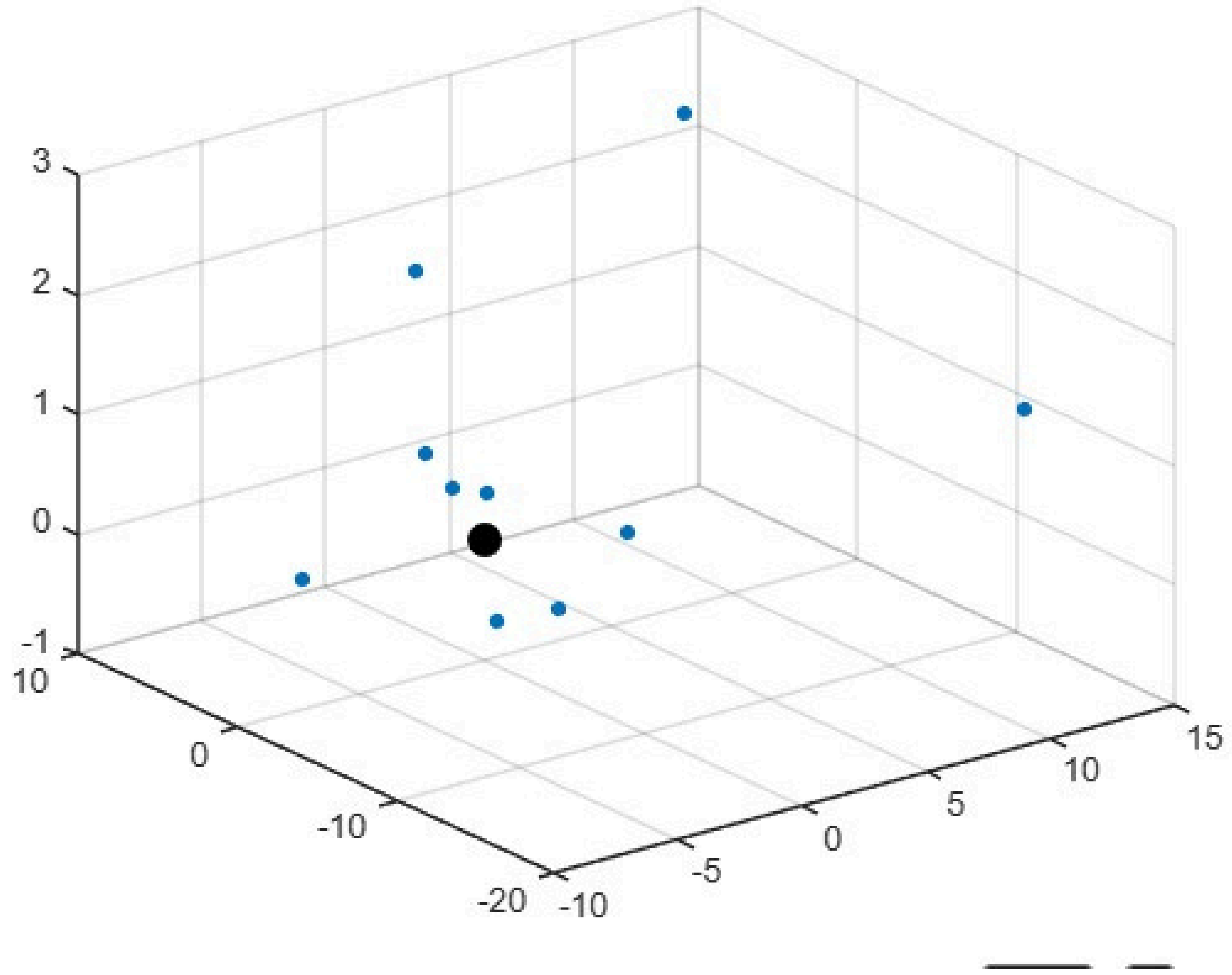
RESULTS



- If the galaxy followed Newtonian dynamics without dark matter, the velocities should decrease at larger radii.

- The observed velocity remains nearly constant or declines slowly at large radii, implying the presence of dark matter.

Simple N-body simulation



The simulation models the motion of stars in a simplified spiral galaxy, considering the effects of a supermassive dark matter halo.

Thank You