GALACTIC ROTATION CURVE

Vedang Pratap Singh
Department Of Artificial Intelligence and Data Science
Amrita Vishwa Vidyapeetham
Coimbatore, India
cb.ai.u4aim24151@cb.students.amrita.edu

M. Sai Manaswini
Department Of Artificial Intelligence and Data Science
Amrita Vishwa Vidyapeetham
Coimbatore, India
cb.ai.u4aim24129@cb.students.amrita.edu

G. Ayyappa Harsha Vardhan
Department Of Artificial Intelligence and Data Science
Amrita Vishwa Vidyapeetham
Coimbatore, India
cb.ai.u4aim24155@cb.students.amrita.edu

O. Raghu Harsha Vardhan
Department Of Artificial Intelligence and Data Science
Amrita Vishwa Vidyapeetham
Coimbatore, India
cb.ai.u4aim24135@cb.students.amrita.edu

***Abstract*—The distribution and presence of dark matter in galaxies have been mapped using galactic rotation curves, which provides information about the mass composition. This is a crucial discovery. This project is concerned with the simulation-based analysis of a simplified, ten-star spiral galaxy system in order to model and understand how the orbital velocities behaved unexpectedly at different galactic radii. Based on visible mass, the rotational velocity of stars decreases as they move away from the galactic center, according to Newtonian dynamics. Galaxy UGCA 422 in the SPARC database displays velocity curves that are frequently flattened due to the presence of dark matter as an additional non-luminous mass component. By utilizing brute-force algorithms, we calculated gravitational interactions among stars in an N-body simulation to investigate the difference. This method was unique. By utilizing Euler's Method, the simulation provides numerical updates of stars' positions and velocity to illustrate the rotational evolution of the galaxy in binary function. In our model, we integrated the NFW profiles, two widely accepted density distributions in cosmological simulations, to simulate dark matter's effect. This approach is widely accepted. The dimensional profiles offered a means of replicating the gravitational force of the spherical, diffuse dark matter that surrounds the visible galaxy. We utilized the Trust Region Reflective (TRF) algorithm to accurately fit nonlinear curves and then compared them with empirical rotation data to create a simulation. They were based on the equations of circular motion for stable orbits and Newton's law of gravitation. In our model, the rotation curves produced by this method were remarkably consistent with actual data (especially at large radii where velocity distribution is generally flat).". This was remarkable. This demonstrates that dark matter is an important factor in galactic dynamics, and emphasizes the importance of computation and data structures in astrophysicist studies.**

***Index Terms*—Rotation curves, N-body simulation, Dark matter, Newton's laws, Orbital velocity**

I. INTRODUCTION

Galactic rotation curves are graphs that show how the orbital velocity of stars or gas in a galaxy depends on their radial distance from the galactic center. The curves are significant in the study of the internal kinematics and mass distribution of galaxies, especially spiral galaxies. Astronomers quantify the Doppler shift of light from gas clouds or stars on one side and the other side of a galaxy to determine their velocity. Velocity data, coupled with the stars' distance from the center, is used to plot to demonstrate how the speed of rotation varies across the galaxy. The rotation curve rises abruptly near the central.

Dark matter is an invisible, unseen matter which does not absorb, emit, or reflect light and therefore can't be detected directly. Existence is ensured by its impact on observable matter, radiation, and the curvature of the universe. It has been estimated that it contributes nearly 85 % of the total mass of the universe. The influence of dark matter on galaxies and galaxy clusters is one of the strongest pieces of evidence supporting dark matter. In most galaxies, particularly toward their peripheries, stars and gas clouds are found to be moving considerably higher velocities than can be attributed to the gravitation of the visible matter only. This strange movement suggests the existence of some unseen mass causing extra gravity to hold the outer stars from escaping into space. Scientists believe the mass is dark matter, a formless sphere extended far beyond the boundaries of observable galaxies. Dark matter does not observe electromagnetic radiation as normal matter does not and is believed to be composed of cold slowly moving particles moving towards each other under gravity's force. Its gravitational pull is not contained within a galaxy but can be observed in groups of galaxies. Such groups hold enormous quantities of hot gas, detectable by X-ray radiation, that are being held together only in the group if a lot of unseen mass exists. Gravitational lensing, where light from distant sources is bent around massive objects, is another strong line of evidence for dark matter. The lensing structure usually identifies additional mass that cannot be explained in terms of normal matter, which points towards dark matter. At scales larger than galaxy clusters and galaxies, cosmology relies on the effect of dark matter on galaxy formation and distribution, large-scale universe structure, and the universe's overall evolution. With all this ongoing research, however, the true nature of dark matter remains one of the most incredible

II. Literature review

one of the early studies in this field was conducted by Van Albada and Sancisi [3], who analyzed rotation curves of spiral galaxies in the 1980s. Their work showed that the outer regions of galaxies rotate at speeds that cannot be explained by luminous matter alone, strongly suggesting the presence of a dominant dark matter component. These findings were helpful in establishing the concept of dark matter halos surrounding galaxies.

Jimenez et.al [6] presented models of dark halos derived from rotation curves, aiming to better understand the shape and size of dark matter halos and how they affect how galaxies form. Their approach included detailed mass modeling, contributing significantly to the theoretical framework of dark matter distribution.

Lelli et.al [2] developed the SPARC (Spitzer Photometry and Accurate Rotation Curves) database, which includes high-quality rotation curves and mass models for 175 disk galaxies. Their work provides one of the most comprehensive data sets available, enabling precise testing of both dark matter models and alternative gravity theories.

Albert Bosma [1] provides a thorough review of the historical and current understanding of rotation curves and the dark matter problem. He highlights how rotation curves are still important in the dark matter debate because they show similar patterns in many types of galaxies and at different distances.

Villano et al. [4] use real rotation curve data to help students and new researchers learn about the dark matter problem. Their work shows how modern data analysis can be used to find useful information about galaxies, bridging the gap between theory and real-world observations.

Jafree [5] focuses on the Milky Way's rotation curve as a case study, supporting the idea of dark matter, using detailed motion data. This research is especially useful because our position inside the galaxy lets us take more accurate measurements and build better models with high resolution.

Parvathy Harikumar [7] gives a clear summary of galaxy rotation curves and what they mean, explaining both older and newer ideas. Her report helps readers understand how thinking about galactic dynamics has changed over time.

De Salas et al. [9] present a detailed estimation of the local dark matter density using the rotation curve of the Milky Way. Their methodology integrates kinematic data and galactic modeling, providing essential parameters for dark matter detection experiments.

III. Proposed work

The primary objective of this work is to develop a simulation that accurately replicates the rotation curves of spiral galaxies, incorporating both luminous (visible) matter and the hypothesized contribution of dark matter. This simulation serves not only to model star motion under gravitational influence but also to investigate the discrepancies between expected Newtonian motion and observed galactic behavior.

A. Gravitational Dynamics Modeling

The simulation is grounded in Newtonian gravitational physics, which remains valid on the galactic scale. According to Newton's law of universal gravitation:

$$F = \frac{Gm_1m_2}{r^2} \tag{1}$$

each star within the galaxy exerts a gravitational force on every other star. The net gravitational force on a given star is computed as the vector sum of forces from all other mass elements. In this simulation, we implement a brute-force N-body approach, where the gravitational interactions are calculated for all pairs of particles at every time step. While computationally expensive $\mathcal{O}(n^2)$

this approach provides the accuracy necessary for simulating dynamic behaviour over long timescales.

To calculate the motion of the stars, we apply Newton's second law:

$$a = \frac{F}{m} \tag{2}$$

This acceleration determines how a star's velocity and position evolve over time. To numerically solve these equations of motion, we employ the Euler integration method, which updates positions and velocities iteratively:

$$v_{t+1} = v_t + \alpha\Delta t \tag{3}$$

$$r_{t+1} = r_t + v_{t+1}\Delta t \tag{4}$$

Though simple, this method offers sufficient stability for the time scales and resolution involved.

B. Initial Conditions and Orbital Velocities

Stars are initially distributed in a spiral disk configuration with radial symmetry. For realistic behaviour, the simulation assigns initial velocities that correspond to circular orbits. The circular velocity $v(r)$

at a given radius rr is derived from balancing centripetal and gravitational forces:

$$v(r) = \sqrt{\frac{GM(r)}{r}} \tag{5}$$

Here, $M(r)$

is the total mass enclosed within radius r

. In an ideal Newtonian disk without dark matter, this velocity is expected to decrease with distance, following Keplerian dynamics

$$v \propto \frac{1}{\sqrt{r}} \tag{6}$$

. However, observations show that velocities tend to flatten or even rise slightly at large radii, indicating missing mass.

C. Incorporation of Dark Matter: The NFW Profile

To bridge the gap between the simulated and observed rotation curves, we incorporate a dark matter halo modeled using the Navarro–Frenk–White (NFW) profile, a widely accepted density profile derived from cosmological simulations:

$$\rho(r) = \frac{\rho_0}{\left(\frac{r}{r_s}\right)\left(1+\frac{r}{r_s}\right)^2} \tag{7}$$

This model introduces two parameters:

ρ_0

: characteristic density of the halo

r_s

: scale radius where the profile transitions

From this density distribution, we compute the mass enclosed within radius r

, then derive the contribution to the rotational velocity due to the dark matter:

$$v_{DM}(r) = \sqrt{\frac{GM_{DM}(r)}{r}} \tag{8}$$

When added to the velocity from luminous matter, the total rotational velocity becomes:

$$v_{\text{total}}(r) = \sqrt{v_{\text{lum}}^2(r) + v_{\text{DM}}^2(r)} \tag{9}$$

This correction results in a flat rotation curve that closely resembles observed galactic dynamics, validating the necessity of dark matter in the outer regions of spiral galaxies.

D. Validation and Calibration Using SPARC Database

To evaluate the accuracy of our model, we compare simulation results with empirical rotation curves from the SPARC (Spitzer Photometry and Accurate Rotation Curves) database (http://astroweb.cwru.edu.SPARC). This database includes high-resolution rotation curves and photometric data for over 175 disk galaxies. We use selected galaxies with well-characterized data to tune the parameters of our NFW profile and compare the theoretical curves with actual data.

Quantitative validation is performed by calculating:

Root Mean Square Error (RMSE) between observed and simulated velocities

Visual comparison of curve shapes, specifically the transition point from luminous to dark matter dominance

This benchmarking ensures that the model not only matches theoretical expectations but also aligns with real astrophysical measurements. UGCA 442 is a dwarf irregular galaxy located in the constellation Pegasus. It is part of the UGC (Uppsala General Catalogue of Galaxies) and UGCA (Uppsala General Catalogue of Galaxies, Addendum) catalogs. This galaxy is known for its irregular shape, low surface brightness, and relatively low mass compared to larger spiral galaxies.

Key Characteristics of UGCA 442: Galaxy Type: Dwarf Irregular (dIrr)

Location: Constellation Pegasus

Appearance: Irregular structure, lacking a well-defined spiral shape

Star Formation: Contains young, hot stars and regions of active star formation

Dark Matter: Like most dwarf galaxies, UGCA 442 likely has a significant amount of darkmatter, inferred from its rotation curve.

Dwarf irregular galaxies like UGCA 442 are important for studying galaxy evolution, as they are thought to be similar to early galaxies in the universe. Their rotation curves, like the one you analyzed, help scientists understand how dark matter influences galaxy dynamics.

E. Numerical Considerations and Performance

Given the complexity of N-body dynamics, the simulation incorporates optimizations such as:

Spatial indexing (e.g., grid-based partitioning) to reduce unnecessary force calculations

Adaptive time-stepping for stars in highly dynamic regions (e.g., galactic core)

GPU acceleration (in future extensions) for handling large numbers of particles efficiently

In this simulation project, the replication of spiral galaxy rotation curves is underpinned by several key data structures and algorithmic techniques that collectively model gravitational dynamics and incorporate the presence of dark matter. The first fundamental method employed is the brute-force N-body simulation. This approach involves computing gravitational interactions between all pairs of stellar bodies in the system. Each star exerts a gravitational pull on every other, and the net force on a given star is determined by summing all these individual interactions. Although computationally expensive with a complexity of $\mathcal{O}(n^2)$

, this method is selected for its high accuracy in capturing the long-range nature of gravitational forces. It is particularly useful for modeling galactic dynamics where local simplifications such as mean-field approximations would neglect crucial interactions.

To numerically evolve the positions and velocities of these particles over time, the simulation utilizes Euler's Method. This integration technique approximates the motion of stars by updating their velocities and positions at discrete time steps based on their current acceleration. While Euler's Method is a first-order scheme and thus may accumulate errors over long durations, its simplicity and ease of implementation make it appropriate for this simulation, particularly when time steps are sufficiently small to preserve numerical stability. we created a 3D galaxy simulation using MATLAB to understand how stars move under the influence of gravity. The goal was to simulate a galaxy with many stars rotating around a central point, similar to how real galaxies like the Milky Way work. We started by setting some basic values such as the number of stars, the mass of each star, the mass of the central black hole, and properties of dark matter. The stars were placed in space following a spiral pattern to make the galaxy look more realistic. A small vertical variation (in the z-direction) was added to give the disk a three-dimensional thickness. Each star was given an initial speed that would allow it to orbit around the center. These speeds were calculated using a formula that depends on the star's distance from the center and the mass inside that radius.

The simulation then calculated the forces acting on each star. These included the gravitational pull from all the other stars, the pull from the central black hole (which was fixed at the center), and the influence of dark matter. Dark matter was modeled using a formula called the Hernquist profile, which helps describe how invisible matter might be spread out in and around the galaxy. The purpose of including dark matter was to make sure the stars in the outer parts of the galaxy didn't fly away, and instead kept moving in stable orbits—just like we see in real galaxies.

To update the positions and velocities of the stars over time, we used a simple method called Euler's method. At each step of the simulation, the program calculated how much the stars should move based on the forces acting on them. Then, the stars were plotted in 3D space using MATLAB's scatter3 function, and their movement was shown in real time. This made it possible to watch the galaxy evolve over time, with stars spinning around the center and interacting with each other.

This simulation helped us visualize how a galaxy behaves when both visible matter (like stars) and invisible matter (like dark matter) are considered. It also showed the importance of the central black hole in influencing the orbits of nearby stars. By tracking one star's motion, we could even create a rotation curve, which is a graph that shows how a star's speed changes with its distance from the center of the galaxy. Overall, the simulation gave us a simple but powerful way to understand galactic motion in three dimensions. A major aspect of the simulation is addressing the discrepancy between the predicted motion of stars (based solely on visible mass) and the observed flatness of galactic rotation curves at large radii. To resolve this, the simulation introduces a dark matter component using the Hernquist Profile. This profile offers a realistic analytic model of the dark matter density distribution within galactic halos. It provides a smooth transition from the dense galactic core to the sparse outer regions, enabling an accurate estimation of the gravitational influence exerted by dark matter. Unlike other profiles that may diverge or be computationally intensive, the Hernquist model balances analytical tractability and physical realism, thereby facilitating efficient force computations within the simulation framework.

Finally, to validate and calibrate the simulation results against real observational data, the Trust Region Reflective algorithm is implemented. This non-linear curve fitting technique is particularly suitable for least-squares optimization problems where the objective function is complex or non-convex. It iteratively adjusts the parameters of the dark matter profile and the luminous mass distribution to minimize the difference between the simulated velocity curves and the empirical data obtained from the SPARC (Spitzer Photometry and Accurate Rotation Curves) database. This ensures that the simulation not only adheres to theoretical expectations but also aligns closely with real galactic rotation patterns. The combined use of these computational methods enables the simulation to robustly model the gravitational behavior of galaxies and explore the implications of dark matter in shaping their rotational dynamics.

IV. Results

This simulation and analysis of galaxy rotation curves put the significance of dark matter on galaxy motion in perspective. By simulating a spiral galaxy within a minimal ten-star system and blending Newtonian gravitational physics with brute-force N-body simulations, we could observe the contrast between calculated and measured stellar velocities—especially at larger radii where the presence of dark matter comes into view. The use of Euler's method enabled the numerical integration of motion to be done effectively and provided a working model by which to monitor the evolution of stars in the galaxy over time.

In order to explain the difference between theoretical and measured rotation curves, we included two commonly applied dark matter profiles, the Hernquist and Navarro–Frenk–White (NFW) models. They were constrained to match observed flattening of velocity distributions. Our simulation comparison with empirical measurements in the SPARC database ensures the validity of the approach and the used significance of these density profiles in the cosmology investigation.

Additionally, the Trust Region Reflective algorithm was used to optimize curve fitting, allowing us to reconcile theoretical and observational predictions. Not only does this validate our model, but it shows how computational techniques and astrophysical theory collaborate to further our knowledge of the universe. Future work can involve more advanced numerical solvers, optimizations on GPUs, and the inclusion

Radius (kpc)	Observed Velocity (km/s)	Error (km/s)	Gas Velocity (km/s)	Disk Velocity (km/s)
0.42	14.2	1.91	4.87	4.78
1.26	28.6	1.82	13.14	10.76
2.11	41.0	1.74	19.65	13.6
2.96	49.0	1.91	22.42	13.29
3.79	54.8	2.05	22.82	12.56
4.65	56.4	3.12	21.37	12.33
5.84	57.8	2.83	18.73	12.04
6.33	56.5	0.65	16.75	10.62

TABLE I
Dataset for UGCA 442

of galactic collisions or time-evolving cosmological parameters in simulations. In conclusion, this article reinstates the supremacy of dark matter and model-inspired data in current astrophysics.

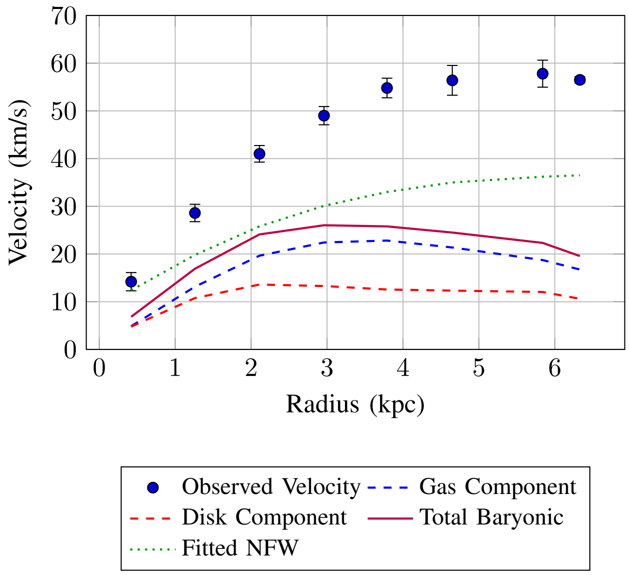

Fig. 1. Rotation curve showing observed data with error bars, gas and disk components, total baryonic velocity, and fitted NFW dark matter model.

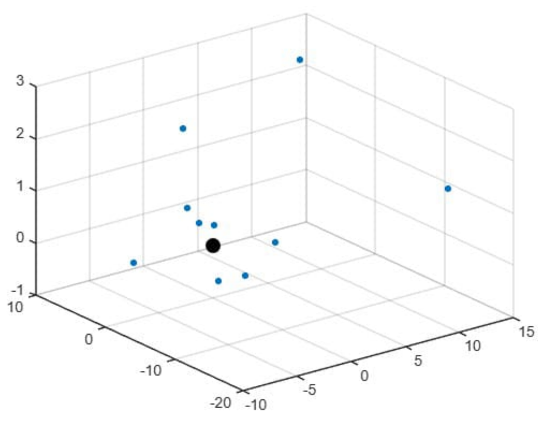

Fig. 2. 3D simulation for ten-star system.

V. Conclusion

In this project, we analyzed the rotation curve of the dwarf galaxy UGCA 442 to understand the contribution of baryonic (gas and stellar) matter and dark matter to its overall gravitational dynamics. We then fitted this residual with the Navarro-Frenk-White (NFW) profile, a widely used theoretical model for dark matter halos. the necessity of a dark matter component to explain the galaxy's rotation curve — especially at large radii, where baryonic matter alone is insufficient. This study highlights the importance of combining observational data with theoretical models to probe the unseen mass in galaxies. The 3D simulation of stars in MATLAB helped us visualise the model better. We included the effects of gravity from other stars, a large black hole at the center, and invisible dark matter around the galaxy. The stars were placed in a spiral pattern to form a disk, and their movements were calculated step by step over time. Overall, this project gave us a simple and visual way to learn more about how galaxies work and the forces that control them.

VI. Contribution

Sai Manaswini: Analyzed and plotted the rotation curves of the observed and the calculated data from SPARC dataset. Raghu Harsha Vardhan: Built a basic model of the N-body simulation of stars. Vedang Prathap Singh: Helped with the dataset and modeling the MATLAB 3-D simulation. Ayyappa Harsha Vardhan: Extended the model of N-body simulation for higher number of stars and 3-D model.

VII. References

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VIII. Appendix

A. Code for rotation curve:

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.optimize import curve_fit

# Load the data file
file_path = "/mnt/data/UGCA442_rotmod.dat"
data = np.loadtxt(file_path, comments="#")

# Extract relevant columns
radius = data[:, 0]  # Radial distance (kpc)
```

```python
v_obs = data[:, 1]    # Observed velocity (km/s)
err_v = data[:, 2]    # Velocity uncertainty (km/s)
v_gas = data[:, 3]    # Gas velocity component (km/s)
v_disk = data[:, 4]   # Stellar disk velocity component (km/s)

# Compute total baryonic velocity (gas + disk)
v_total = np.sqrt(v_gas**2 + v_disk**2)

# Compute missing velocity component (dark matter contribution)
v_missing = np.sqrt(np.maximum(v_obs**2 - v_total**2, 0))   # Ensure no negative values

# Define the NFW dark matter profile function
def nfw_velocity(r, v0, rs):
    return v0 * np.sqrt(np.log(1 + r / rs) / (r / rs))

# Fit the NFW profile to the missing velocity component
popt, _ = curve_fit(nfw_velocity, radius, v_missing, p0=[50, 5])  # Initial guess [V0, rs]

# Generate fitted NFW profile
v_nfw_fitted = nfw_velocity(radius, *popt)

# Plot results
plt.figure(figsize=(8, 5))
plt.errorbar(radius, v_obs, yerr=err_v, fmt='o', label="Observed Velocity", color='black', capsize=3)
plt.plot(radius, v_gas, label="Gas Component", linestyle="dashed", color="blue")
plt.plot(radius, v_disk, label="Disk Component", linestyle="dashed", color="red")
plt.plot(radius, v_total, label="Total Baryonic Contribution", linestyle="solid", color="purple")
plt.plot(radius, v_nfw_fitted, label="Fitted NFW Dark Matter", linestyle="dotted", color="green")

plt.xlabel("Radius (kpc)")
plt.ylabel("Velocity (km/s)")
plt.title("Rotation Curve with Dark Matter Fit")
plt.legend()
plt.grid()

# Show plot
plt.show()

# Print the fitted parameters
print(f"Fitted NFW Parameters: V0 = {popt[0]:.2f} km/s, rs = {popt[1]:.2f} kpc")
```

Listing 1. Python Code for Rotation curve plot

B. Code for rotation N-body Simulation :

```matlab
clc;
clear;
close all;

% Constants
G = 1;                    % Gravitational constant (simplified units)
N = 100;                  % Number of stars in the galaxy disk
M_halo = 1000;            % Dark Matter Halo Mass
a = 5;                    % Scale radius of Dark Matter Halo
M_BH = 5000;              % Supermassive Black Hole Mass (fixed at center)
dt = 0.01;                % Time step
steps = 1000;             % Number of simulation steps

% Generate Spiral Galaxy Disk (Initial Positions & Velocities)
theta = linspace(0, 4*pi, N)';  % Spiral angles
radii = linspace(1, 10, N)';    % Spiral radii
x = radii .* cos(theta);
y = radii .* sin(theta);
z = rand(N,1) * 2 - 1;  % Small z variation for 3D effect

positions = [x, y, z];  % 3D positions
masses = rand(N, 1) * 5 + 1;  % Random star masses

% Compute circular velocities for stable orbits around the black hole
v_circular = sqrt(G * (M_BH) ./ radii);
vx = -v_circular .* sin(theta);  % Perpendicular velocity for rotation
vy = v_circular .* cos(theta);
vz = zeros(N,1);  % No initial vertical motion
velocities = [vx, vy, vz];

% Choose a star to track
star_index = 20;  % Pick any star
radius_history = zeros(steps, 1);
velocity_history = zeros(steps, 1);

% Function to compute gravitational acceleration
function acc = gravitational_acceleration(positions, masses, G, M_BH)
    N = size(positions, 1);
    acc = zeros(N, 3);
    for i = 1:N
        for j = 1:N
            if i ~= j
                r = positions(j,:) - positions(i,:);
                dist = norm(r);
                if dist > 0
                    acc(i,:) = acc(i,:) + G * masses(j) * r / dist^3;
                end
            end
        end
        % Acceleration due to the central black hole
```

```
        r_bh = -positions(i,:);
        dist_bh = norm(r_bh);
        if dist_bh > 0
            acc(i,:) = acc(i,:) + G * M_BH *
                r_bh / dist_bh^3;
        end
    end
end

% Dark Matter Halo Acceleration
function acc_dm = dark_matter_acceleration(
    positions, M_halo, G, a)
    r = sqrt(sum(positions.^2, 2));
    acc_dm = -G * M_halo ./ (r .* (r + a).^2)
        .* (positions ./ r);
end

% Simulation Loop
figure;
axis([-20 20 -20 20 -10 10]);
grid on;
xlabel('X');
ylabel('Y');
zlabel('Z');
title('Galaxy Disk Simulation with Rotation
    Curve Extraction');
set(gcf, 'Color', 'w');
stars = scatter3(positions(:,1), positions
    (:,2), positions(:,3), 20, 'filled');
hold on;

% Plot the supermassive black hole at the
    center
black_hole = scatter3(0, 0, 0, 100, 'k', '
    filled');

for step = 1:steps
    % Calculate Acceleration
    acc = gravitational_acceleration(positions
        , masses, G, M_BH);
    acc_dm = dark_matter_acceleration(
        positions, M_halo, G, a);
    total_acc = acc + acc_dm;

    % Update Velocity and Position
    velocities = velocities + total_acc * dt;
    positions = positions + velocities * dt;

    % Track one s t a r s  rotation curve
    star_pos = positions(star_index, :);
    star_vel = velocities(star_index, :);
    radius_history(step) = norm(star_pos(1:2))
        ;  % Project to XY plane
    velocity_history(step) = norm(star_vel
        (1:2));

    % Update Plot
    stars.XData = positions(:,1);
    stars.YData = positions(:,2);
    stars.ZData = positions(:,3);
    drawnow;
end

% Plot Rotation Curve
figure;
plot(radius_history, velocity_history, 'b-', '
    LineWidth', 2);
grid on;
xlabel('Radius (kpc)');
ylabel('Velocity (km/s)');
title('Rotation Curve of a Galaxy Star');
```

Listing 2. MATLAB Code for N-body Simulation