=: Stochastic Geometry Based Performance Study on 5G NOMA Scheme :=

⇨ System Model :=

↪ Transmission distance btw. UE_i and its associated Base-Station is denoted by r_i.

↪ r_i is independent and identically distributed.

↪ PDF is defined as, $f_{r_i}(r_i) = e^{-\lambda \pi r_i^2} \cdot 2\pi\lambda r_i$

↪ $C_i = \dfrac{h_i r_i^{-\alpha}}{I_i}$ = channel Gain of UE_i normalised by interference.

↳ $h_i \sim \exp(1)$ = Rayleigh Fading Gain

↳ $r_i^{-\alpha}$ = Path Loss

↳ $I_i = \sum_{j \in \phi / b_0} g_j R_{j,i}^{-\alpha} P_{total}$ = Sum of inter-cell interference from all other BS's.

- g_j = Rayleigh Gain of Interfacing channel.

- $R_{j,i}$ = Transmission Distance from BS j to UE_i

★ Principle of NOMA :=

↪ Gain : $C_1 \leq C_2 \leq \dots\dots \leq C_M$

↪ Power : $P_1 \geq P_2 \geq \dots\dots \geq P_M$.

7. Coverage Probability And Average Achievable Rate :

- We start our analysis with a 2-UE NOMA case.
- The transmitted signal at BS is coded as the composite signal from UE_1 and UE_2.

$$\boxed{\therefore x = \sqrt{P_1}\, x_1 + \sqrt{P_2}\, x_2.}$$

- Also, Received Signal at UE_i,

$$\boxed{y_i = \sqrt{h_i r_i^{-\alpha}}\, x + I_i}$$

★ Coverage Probability :

- Coverage Probability is defined as $P[SIR > T]$, which means that the instantaneous SIR of any UE is greater than a certain threshod T.
- For 2-UE case,

$$SIR_1 = \frac{h_1 r_1^{-\alpha} P_1}{I_1 + h_1 r_1^{-\alpha} P_2} \qquad SIR_2 = \frac{h_2 r_2^{-\alpha} P_2}{I_2}$$

- UE_1 does not need to perform interference cancellation and directly treats x_2 as interference since it comes the first in the decoding order.

UE_2 first decodes x_1 and remove it from the received composite signal y_2, based on which UE_2 can further decode x_2.

★ **Channel Gain Distribution :-**

↳ To calculate coverage probability, we need to first derive the channel gain distribution.

$$\boxed{F_C(c) = P[C \leq c] = 1 - P[C > c]}$$

$$\therefore P[C > c] = E_r[P[C > c \mid r]]$$

$$= \int_{r>0} P\left[\frac{h r^{-\alpha}}{I} > c \mid r\right] f_r(r)\, dr.$$

$$\left[\because E[X] = \int_{r>0} r f_r(r)\, dr\right].$$

$$= \int_{r>0} E_I\left[P[h > c I r^{\alpha} \mid r, I]\right] f_r(r)\, dr$$

$$= \int_{r>0} E_I\left[\exp(-c r^{\alpha} I) \mid r, I\right] f_r(r)\, dr$$

$$[\because h \sim \exp(1)]$$

$$\therefore P[C > c] = \int_{r>0} \mathcal{L}_I(c r^{\alpha}) f_r(r)\, dr \qquad \text{...... (a).}$$

$[\because \mathcal{L}_I(s) = E_I[e^{-sI}]$ is the Laplace Transform of RV I evaluated on $s.]$

$$\rightarrow \mathcal{L}_I(s) = E_{\phi,g}\Big[\exp\Big(-s \underbrace{\sum_{j\in\phi/b_o} g_j R_j^{-\alpha} P_{Total}}_{I}\Big)\Big]$$

$$= E_\phi\Big[\prod_{j\in\phi/b_o} E_{g_j}\big[\exp(-s g_j R_j^{-\alpha} P_{Total})\big]\Big]$$

$$= \exp\Big(-2\pi\lambda \int_r^\infty \big\{1 - E_g\big[\exp(-s g v^{-\alpha} P_{Total})\big]\big\} v\,dv\Big)$$

$$\Big[\because \text{From PGFL, } E\Big[\prod_{x\in\phi} f(x)\Big] = \exp\Big(-\lambda \int_{R^2} (1-f(x))\,dx\Big)$$

$$= \exp\Big(-2\pi\lambda \int_r^\infty \Big\{1 - \frac{1}{1+s v^{-\alpha} P_{Total}}\Big\} v\,dv\Big)$$

$$[\because g \sim \exp(1)]$$

$$= \exp\Big(-2\pi\lambda \int_r^\infty \frac{1}{1+\frac{v^\alpha}{sP_{Total}}} v\,dv\Big)$$

$$= \exp\Big(-\pi\lambda (sP_{Total})^{2/\alpha} \int_{\frac{r^2}{(sP_{Total})^{2/\alpha}}}^\infty \frac{1}{1+u^{\alpha/2}}\,du\Big) \quad \text{......(b)}$$

$$\Big[\because \text{change of variable, } u = \frac{v^2}{(sP_{Total})^{2/\alpha}}\Big].$$

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substituting $s = Cr^{\alpha}$ back into Eq.^{n} (b),

$$\mathcal{L}_I(Cr^{\alpha}) = \exp\left(-\pi\lambda r^2 (CP_{Total})^{2/\alpha} \int_{(CP_{Total})^{-2/\alpha}}^{\infty} \frac{1}{1+u^{\alpha/2}}\, du\right)$$

..... (c)

→ Combining (a) and (c), we get,

$$P[c > C] = \int_{r>0} \exp\left(-\pi\lambda r^2 (CP_{Total})^{2/\alpha} \int_{(CP_{Total})^{-2/\alpha}}^{\infty} \frac{1}{1+u^{\alpha/2}}\, du\right) * e^{-\lambda\pi r^2}\, 2\pi\lambda r\, dr$$

$$= \int_{v>0} \exp\left[-v\lambda\pi\left(1 + (CP_{Total})^{2/\alpha} \int_{(CP_{Total})^{-2/\alpha}}^{\infty} \frac{1}{1+u^{\alpha/2}}\, du\right)\right] \pi\lambda\, dv.$$

$[\because v = r^2 \quad \therefore dv = 2r\,dr]$.

$$\boxed{\therefore P[c > C] = \frac{1}{1 + (CP_{Total})^{2/\alpha} \int_{(CP_{Total})^{-2/\alpha}}^{\infty} \frac{1}{1+u^{\alpha/2}}\, du}}$$

$\left[\because \int e^{-ax} = \frac{e^{-ax}}{-\cancel{a}}, \quad e^{-\infty} = 0, \quad e^{0} = 1\right]$.

Hence,
$$\boxed{F_C(C) = 1 - \frac{1}{1 + (CP_{Total})^{2/\alpha} \int_{(CP_{Total})^{-2/\alpha}}^{\infty} \frac{1}{1+u^{\alpha/2}}\, du}}$$

→ For $\alpha = 4$,

$$F_c(c) = 1 - \frac{1}{1 + \sqrt{cP_{Total}}\left[\frac{\pi}{2} - \tan^{-1}\left[\frac{1}{\sqrt{cP_{Total}}}\right]\right]}.$$

→ Also, $$f_c(c) = \frac{dF_c(c)}{dc} = \frac{\mu(c)'v(c) + \mu(c)v'(c)}{1 + 2\mu(c)v(c) + \mu(c)^2 v(c)^2}$$

↳ $\mu(c) = (c\,P_{Total})^{2/\alpha}$ $\therefore \mu'(c) = \frac{2}{\alpha} P_{Total}^{2/\alpha} c^{2/\alpha - 1}$

↳ $$v(c) = \int_{(cP_{Total})^{-2/\alpha}}^{\infty} \frac{1}{1+u^{\alpha/2}}\,du$$

$$\therefore v'(c) = \frac{2P_{Total}}{\alpha (cP_{Total})^{2/\alpha}\left[1 + cP_{Total}\right]}$$

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Coverage Probability of UE_1 & UE_2 :-

For a target SIR value of T, the coverage probability of UE_1 is given by,

$$P[SIR_1 > T] = 1 - P[SIR_1 \leq T]$$

$$= 1 - P\left[\frac{h_1 r_1^{-\alpha} P_1}{I_1 + h_1 r_1^{-\alpha} P_2} \leq T\right]$$

$$= 1 - P\left[\frac{1}{\frac{I_1}{h_1 r_1^{-\alpha} P_1} + \frac{P_2}{P_1}} \leq T\right]$$

($\because$ divided with '$h_1 r_1^{-\alpha} P_1$')

$$\boxed{= 1 - P\left[\frac{I_1}{h_1 r_1^{-\alpha} P_1} \geq \frac{1}{T} - \frac{P_2}{P_1}\right]}$$

→ Due to NOMA inter-user interference, SIR_1 has an upper bound as, $\lim_{I_1 \to 0} SIR_1 = \frac{P_1}{P_2}$.

$\therefore$ Hence, $P[SIR_1 > T] = 0$, when $T \geq P_1/P_2$.

. Also, when $T < P_1/P_2$ we have,

$$P[SIR_1 > T] = 1 - P\left[\frac{h_1 r_1^{-\alpha}}{I_1} \leq \frac{1}{\left(\frac{1}{T} - \frac{P_2}{P_1}\right)P_1}\right]$$

$$= 1 - P\left[C_1 \leq \frac{1}{P_1\left(\frac{1}{T} - \frac{P_2}{P_1}\right)}\right]$$

$$\boxed{\therefore P[SIR_1 > T] = 1 - F_{C_1}\left(\frac{1}{\frac{P_1}{T} - P_2}\right)}$$

↳ For UE_2, assume that NOMA inter-user interference from UE_1 is completely eliminated by SIC at UE_2.

↳ There is no such a limitation on SIR_2 as on SIR_1.

$$\boxed{\therefore P[SIR_2 > T] = 1 - F_{C_2}\left(\frac{T}{P_2}\right)}$$

★ **Average Achievable Rate for 2-UE case :-**

↳ We assume all UEs use an modulation and coding so that they can achieve 'Shanon Bound' for their instantaneous SIR, i.e. $\ln(1+SIR)$.

τ_i = Average Achievable Rate of UE_i.

→ For UE_1, $\tau_1 = E[\ln(1+SIR_1)]$.

$$= \int_{c_1>0} E\left[\ln\left(1+\frac{h_1 r_1^{-\alpha} P_1}{I_1 + h_2 r_1^{-\alpha} P_2}\right)\right] f_{c_1}(c_1)\, dc_1$$

$$= \int_{c_1>0}\int_{t>0} P\left[\ln\left(1+\frac{h_1 r_1^{-\alpha} P_1}{I_1 + h_1 r_1^{-\alpha} P_2}\right) > t\right] dt \cdot f_{c_1}(c_1)\, dc_1$$

$$\left[\because E[X] = \int_{t>0} P(X>t)\, dt, \text{ for } x>0\right]$$

$$= \int_{c_1>0}\int_{t=0}^{\ln\left(\frac{P_1}{P_2}+1\right)} P\left[\frac{h_1 r_1^{-\alpha} P_1}{I_1 + h_1 r_1^{-\alpha} P_2} > e^t - 1\right] dt \cdot f_{c_1}(c_1)\, dc_1$$

[∵ Taking logirithim]

$$\left[\because \text{As} \lim_{I_1 \to 0} \ln\left(1 + \frac{h_1 r_1^{-\alpha} P_1}{I_1 + h_1 r_1^{-\alpha} P_2}\right) = \ln\left(\frac{P_1}{P_2} + 1\right)\right].$$

$$\boxed{\tau_1 = \int_{t=0}^{\ln\left(\frac{P_1}{P_2}+1\right)} \left\{1 - F_{C_1}\left(\frac{1}{\frac{P_1}{e^t - 1} - P_2}\right)\right\} dt}$$

Similarly, $\boxed{\tau_2 = \int_{t>0}^{\infty} \left\{1 - F_{C_2}\left(\frac{e^t - 1}{P_2}\right)\right\} dt}$

⇒ **For M-UE :-**

$$SIR_i = \frac{h_i r_i^{-\alpha} P_i}{I_i + h_i r_i^{-\alpha} \sum_{j=i+1}^{M} P_j}$$

$$\boxed{\therefore SIR_i = \frac{1}{\frac{1}{C_i P_i} + \frac{\sum_{j=i+1}^{M} P_j}{P_i}}}$$

→ Points to Notice:

i) $\sum_{j=i+1}^{M} P_j = 0$ when $i = M$

ii) $T > \frac{P_i}{\sum_{j=i+1}^{M} P_j}$, coverage Probability of UE_i is a

$$\boxed{P[SIR_i > T] = \begin{cases} 0, & \text{if } T > \dfrac{P_i}{\sum_{j=i+1}^{M} P_j} \\ 1 - F_{c_i}\left(\dfrac{1}{\frac{P_i}{T} - \sum_{j=i+1}^{M} P_j}\right), & \text{otherwise.} \end{cases}}$$

$$\boxed{\tau_i = \int_{t=0}^{\ln\left(1+\frac{P_i}{\sum_{j=i+1}^{M} P_j}\right)} \left\{1 - F_{c_i}\left(\frac{1}{\frac{P_i}{e^t - 1} - \sum_{j=i+1}^{M} P_j}\right)\right\} dt}$$

Also, mean value $= \sum_{i=1}^{M} \tau_i / M$

★ Comparison with ~~N~~OMA :-

i) $SIR_i^{OMA} = c_i P_{total}$

ii) $P[SIR_i^{OMA} > T] = 1 - F_{c_i}\left(\frac{T}{P_{Total}}\right)$

iii) $\tau_i^{OMA} = \frac{1}{M} \int_{t>0} \left(1 - F_{c_i}\left(\frac{e^t - 1}{P_{Total}}\right)\right) dt.$