Manay Batavia

EDUCATION

Indian Institue of Technology Bombay

Mumbai, India

Integrated Master's of Science (B.S.+M.Sc.) in Mathematics

July 2017 - Present

- Cumulative Performance Index (CPI): 9.80/10, currently Department Rank 1.
- Minor in Computer Science and Engineering, Minor CPI: 9.60/10.

Publications, Preprints and Working Papers

- 3. A. Agarwal, M. Batavia, Z. Lihn, H. Lu, A. Maithani, A. Pradhan, S. Rath, A. Seceleanu, *Betti Numbers of Quadratic Ideals of Low Embedding Dimension*, in preparation.
- 2. M. Batavia, O. Javadekar, Sai Krishna P. M. S, A. Maithani, P. D. Poduri, S. Qadri, with an appendix by H. Ananthnarayan, *Some Results on Free Resolutions*, in preparation.
- S. Banerjee, M. Batavia, B. Kane, M. Kyranbay, D. Park, S. Saha, H. C. So, P. Varyani, Fermat's Polygonal Number Theorem for Repeated Generalised Polygonal Numbers, Journal of Number Theory, 2020, doi: 10.1016/j.jnt.2020.05.024.

RESEARCH EXPERIENCE

Master's Thesis: Homological Algebra

July 2021 – Present

Guide: Prof. Ananthnarayan Hariharan, Department of Mathematics

IIT Bombay

- Covered the preliminaries on graded modules, graded resolutions, Gröbner bases, initial submodules, etc.
- Explored Schreyer's algorithm for computing the projective resolution of a finitely generated module over a polynomial ring in finitely many variables.
- Studied La Scala and Stillman's modification of Schreyer's algorithm to ensure that the projective resolution computed is minimal.
- Learned about the relations between the Betti numbers of an ideal and those of its initial ideal, elegantly proved using a homogenization technique and Farkas's lemma.
- Learned about the lexsegment ideal and Macaulay's theorem on the complete characterization of the possible Hilbert functions of a standard graded algebra over a field.
- Explored the technique of polarisation, to reduce a monomial ideal to a squarefree monomial ideal.
- Plan to study Stanley-Reisner ideals and the Bigatti-Hulett theorem, making use of the above concepts of polarisation and the lexsegment ideal respectively.
- Plan to explore local cohomology and injective resolutions.

Betti Tables of Quadratic Ideals in Characteristic Two

June - August 2021

Guide: Prof. Alexandra Seceleanu, Department of Mathematics

University of Nebraska-Lincoln

This research was done as part of the Polymath Jr. 2021 program.

- Succeeded in explaining the resolutions, and hence, the Betti tables of all quadratic ideals with three generators in a polynomial ring in three variables over a field of characteristic two.
- The approach involved classification of all such ideals into certain categories and subcategories, followed by appropriate application of results on Koszul complexes and mapping cones, among others.
- Attained results which are easily applicable to larger ideals and larger polynomial rings.
- Familiarised myself with the computer algebra system Macaulay2, which was required for the significant amount of computation in the project.
- Presented our results at the Young Mathematicians Conference 2021, organised by the Ohio State University.
- Currently compiling the work done in this project for publication.

Drinfeld Modular Forms

May - July 2021

Guide: Prof. Giovanni Rosso, Department of Mathematics and Statistics

Concordia University, Montreal

• Versed myself with the theory of additive polynomials over a finite field from David Goss's text "Basic Structures of Function Field Arithmetic".

- Learned about Carlitz and Drinfeld modules, their morphisms and their adjoints.
- Proceeded to read Richard Pink's trifecta of papers on the Galois representations associated to a Drinfeld module in special characteristic.

Parameterized Verification under Sequential Consistency

March – June 2020

Guide: Prof. Roland Meyer, Institute of Theoretical Computer Science

Technische Universität Braunschweig

- Worked on proving that the paremeterized reachability problem under sequential consistency (SC) is EXPTIME-complete.
- The approach involved reduction of Boolean pushdown to parameterized program under SC and vice versa, drawing on techniques used by Farzan and Vandikas in their paper on automated hypersafety verification.

Generalization of Fermat's Polygonal Number Theorem

May – July 2019

Guide: Prof. Ben Kane, Department of Mathematics

The University of Hong Kong

- Succeeded in finding minimal number of terms for which various weighted sums of generalized polygonal numbers are universal.
- The approach involved using Siegel and Weil's work on local representations to conclude a special universality of the sum of five generalised polygonal numbers, and consequently treating the smaller cases using a variety of methods.
- The work done resulted in the above mentioned publication in the Journal of Number Theory.

Reading Projects

Lie Groups and Lie Algebras

January - May 2021

Guide: Prof. Dipendra Prasad, Department of Mathematics

 $IIT\ Bombay$

- $\bullet \ \ {\it Read and presented sections of the book "Symmetries, Representations and Invariants" (Goodman, Wallach).}$
- In addition to acclimatisation with Lie groups and Lie algebras, the primary objective was to explore the regular representations of classical connected groups and their Lie algebras.

Dedekind Domains December 2020

Guide: Prof. Ananthnarayan Hariharan, Department of Mathematics

IIT Bombay

Automata Theory

January – May 2020

Guide: Prof. Krishna S., Department of Computer Science and Engineering

IIT Bombay

- Explored a number of references and research papers related to automata theory, such as Dexter Kozen's "Automata and Computability" and the research papers Automated Hypersafety Verification (by Farzan and Vandikas) and Analysis of Boolean Programs (by Godefroid and Yannakakis).
- Studied the concepts of pushdown automata, context free grammar, Boolean programs, looping tree automata and reductions, among others.

Gröbner Bases, the Hilbert Basis Theorem and Elimination Theory

December 2019

Guide: Prof. Ananthnarayan Hariharan, Department of Mathematics

 $IIT\ Bombay$

Bilinear Forms and Linear Groups

December 2018

Guide: Prof. Ravi Raghunathan, Department of Mathematics

IIT Bombay

Course Projects

Decoding Cyclic Codes: The Cooper Philosophy | MA5105: Coding Theory

October 2021

- Learned about Cooper's ideas on decoding BCH cyclic codes using the theory of Gröbner bases.
- Read and presented the ideas outlined by Chen, Reed, Helleseth and Truong in their work on the use of Gröbner bases to decode binary cyclic codes up to the true minimum distance.

Combinatorial Algorithm for Computing the Determinant | CS711: Computational Ring Theory Nov 2020

• Read and presented the purely combinatorial algorithm proposed by Mahajan and Vinay to compute the determinant over arbitrary commutative rings. In the same paper, they also proved that the complexity class GapL characterizes the complexity of computing the determinant of matrices over the integers.

Speech Command Classification | CS419: Introduction to Machine Learning

April 2020

- Developed a Google Speech Command dataset classifier using deep learning.
- Programmed a convolution based neural network for time series data analysis and some preprocessing.

Texture Synthesis by Non-Parametric Sampling | CS663: Digital Image Processing

November 2019

• Implemented Efros and Leung's work on texture synthesis via MATLAB.

Digital Phase Meter | *EE112: Introduction to Electronics*

November 2019

• Designed and implemented a complex circuit using basic ICs which can measure phase difference between two input AC signals and display it with accuracy upto 1ms on a 8-segment display.

ACADEMIC ACHIEVEMENTS

- Department Rank 1 in mathematics at IIT Bombay.
- Awarded Advanced Performance (AP) grade in Group and Ring Theory (MA419), Complex Analysis (MA412), Introduction to Numerical Analysis (MA214) and Logic for CS and Automata Theory (CS228) at IIT Bombay (awarded to exceptional performers).
- All India rank of 174 in IIT JEE Advanced (out of 0.17 million candidates).
- All India rank of 8 in the B.Math entrance exam for Indian Statistical Institute (ISI).
- Acquired a score in the 99.89th percentile in JEE Mains (out of 1.19 million candidates).
- All India rank of 349 in Kishore Vaigyanik Protsahan Yojana (KVPY), organised by the Department of Science and Technology, Government of India (out of 0.1 million candidates).
- Awarded the Mitacs Globalink scholarship for a research internship in Canada.
- Awarded by Jamnabai Narsee School for highest in ICSE examination.

TEACHING EXPERIENCE

Teaching Assistant: Calculus I | MA109, IIT Bombay

Nov 2021 - Present

Teaching Assistant: Introduction to Calculus | MA105, IIT Bombay

July - Nov 2019

- Entrusted with conducting weekly tutorial sessions for a batch of 40-45 freshmen.
- Responsible for evaluating students' performance in weekly examinations.

Public Talks

M.Sc. Thesis Stage $1 \mid IIT Bombay$	Nov 2021
Decoding Cyclic Codes: The Cooper Philosophy $Coding\ Theory\ Course\ Seminar,\ IIT\ Bombay$	${\rm Oct}\ 2021$
Betti Tables of Quadratic Ideals in Characteristic Two Young Mathematicians Conference	Aug 2021
Explicit Factorisation over a Dedekind Domain Algebraic Number Theory, IIT Bombay	$\mathrm{Apr}\ 2021$
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Combinatorial Computation of the Determinant Computational Ring Theory Course Seminar, IIT Bomba	y Nov 2020
Discrete Valuation Rings Ring and Module Theory Course Seminar, IIT Bombay	Nov 2019

Relevant Coursework

Advanced Instructional Course in Commutative Algebra

Jan - May 2021

Instructors: Prof. Dilip Patil, Prof. Indranath Sengupta, Prof. Jugal Verma

National Centre for Mathematics

- Pitched at a graduate level.
- Flat and faithfully flat modules, faithfully flat descent, Tor and Ext functors, injective, projective and global dimension, regular local rings, flat extensions of regular local rings, Serre's normality criterion.
- Krull dimension, Noether's normalization lemma, Hilbert-Samuel functions and polynomials, normal rings, finiteness of integral closure, complete local rings, formal smoothness, Cohen's structure theorem for complete local rings.
- Regular sequences, quasi-regular sequences, depth and grade, Cohen-Macaulay rings and modules, homological characterization of injective dimension, Gorenstein rings, injective modules, Matlis duality.

Other Graduate Level Courses in Mathematics | IIT Bombay

- Algebra 1: Course in algebra intended for first year Ph.D. students at IIT Bombay.
- Combinatorics 1: Course in combinatorics intended for first year Ph.D. students at IIT Bombay.
- Advanced Commutative Algebra: Exploratory course on homological algebra.
- Topics in Algebra 2: Topics ranging from hyperplane arrangements to category theory to combinatorics.

Other Courses in Mathematics: Algebraic Number Theory, Analytic Number Theory, Coding Theory, Representation Theory of Finite Groups, Commutative Algebra, Ring ad Module Theory, Algebraic Topology, Functional Analysis, Partial Differential Equations, Measure Theory, Applied Stochastic Processes, Probability, Topology, Complex Analysis. Computer Science: Computational Ring Theory, Logic for CS and Automata Theory, Data Structures and Algorithms, Design and Analysis of Algorithms, Introduction to Machine Learning, Digital Image Processing.

TECHNICAL SKILLS

Programming Languages: C++, Python, MATLAB.

Software: Macaulay2, IATFX, SolidWorks, AutoCAD, XCircuit.

Positions of Responsibility

Department Academic Mentor | IIT Bombay

April 2019 – Present

- Delivered multiple seminars aimed at freshmen and sophomore students in the mathematics department, advising them on their academic choices and future avenues.
- Responsible for mentoring 12 sophomore students to ensure their academic and holistic development.
- Co-founded the Mathematics D-AMP blog, intended to be a one-stop destination for academic resources, course reviews, internship experiences and other resources useful for undergraduate math students at IIT Bombay.

National Service Scheme (NSS) Volunteer | IIT Bombay

July 2017 - May 2018

- Dedicated over 80 hours towards various activities directed toward sustaining the environment, including creating awareness and recycling plastic waste, among other ventures.
- Visited a Government school and imparted environmental education to underprivileged children.

Extra-Curricular Activities

- Mentored 12 students as part of the Summer of Science program at IIT Bombay, providing them with necessary guidance in group and ring theory, topology and real analysis.
- Finished 3rd in Math Bazinga 2018 an annual mathematics competition open to all students in IIT Bombay.
- Developed an autonomous Arduino robot which navigates through mazes, as part of a competition at IIT Bombay.
- Finished in the top 5 (out of 400+ participants) in a cricket trivia competition at EnB Buzz, IIT Bombay.