Real-Time Energy Storage Management for Renewable Integration in Microgrid: An Off-Line Optimization Approach

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***Abstract*—Microgrid is a key enabling solution to future smart grids by integrating distributed renewable generators and storage systems to efficiently serve the local demand. However, due to the random and intermittent characteristics of renewable energy, new challenges arise for the reliable operation of microgrids. To address this issue, we study in this paper the real-time energy management for a single microgrid system that constitutes a renewable generation system, an energy storage system, and an aggregated load. We model the renewable energy offset by the load over time, termed net energy profile, to be practically predictable, but with finite errors that can be arbitrarily distributed. We aim to minimize the total energy cost (modeled as sum of time-varying strictly convex functions) of the conventional energy drawn from the main grid over a finite horizon by jointly optimizing the energy charged/discharged to/from the storage system over time subject to practical load and storage constraints. To solve this problem in real time, we propose a new off-line optimization approach to devise the online algorithm. In this approach, we first assume that the net energy profile is perfectly predicted or known ahead of time, under which we derive the optimal off-line energy scheduling solution in closed-form. Next, inspired by the optimal off-line solution, we propose a sliding-window based online algorithm for real-time energy management under the practical setup of noisy predicted net energy profile with arbitrary errors. Finally, we conduct simulations based on the real wind generation data of the Ireland power system to evaluate the performance of our proposed algorithm, as compared with other heuristically designed algorithms, as well as the conventional dynamic programming based solution.**

***Index Terms*—Convex optimization, distributed storage, energy management, microgrid, online algorithm, renewable energy, smart grid.**

NOMENCLATURE

Indices and Numbers

i	Time slot index.
N	Total number of slots for energy scheduling.

Variables

C_i	Energy charged at time slot i.
D_i	Energy discharged at time slot i.
G_i	Energy drawn from the main grid at time slot i.
S_i	State of the energy storage system at the beginning of time slot i.
RE_i	Renewable energy at time slot i, which comprises of a predictable component $\overline{RE}_i$ and a prediction error component $\delta_{RE,i}$, i.e., $RE_i = \overline{RE}_i + \delta_{RE,i}$.
DE_i	Demand energy at time slot i, which comprises of a predictable component $\overline{DE}_i$ and a prediction error component $\delta_{DE,i}$, i.e., $DE_i = \overline{DE}_i + \delta_{DE,i}$.
Δ_i	Net energy profile at time slot i, which comprises of a predictable component $\overline{\Delta}_i$ and a prediction error component δ_i, i.e., $\Delta_i = \overline{\Delta}_i + \delta_i$.
$\underline{\nu}_i, \overline{\nu}_i, \omega$	Lagrange dual variables.

Constants

α_c	Charging efficiency of the storage system.
α_d	Discharging efficiency of the storage system.
$S_{\min}$	Minimum storage level of the storage system.
$S_{\max}$	Storage capacity.
$\overline{S}$	Minimum storage level at the end of time slot N.
a_i, b_i, c_i	Cost coefficients in the quadratic cost function.
M	Size of the sliding-window.
T	Given threshold in the threshold based online algorithm.
β	Number of quantized levels of the storage system.
K	Number of independent realizations.

Functions

$f_i(\cdot)$	Conventional generation cost function at time slot i.
$\mathcal{L}(\cdot)$	Lagrangian function.
$g(\cdot)$	Dual function.
$J(\cdot)$	Cost-to-go function in Bellman equations.

Manuscript received January 16, 2014; revised May 30, 2014 and July 21, 2014; accepted September 6, 2014. Date of publication September 30, 2014; date of current version December 17, 2014. Paper no. TSG-00034-2014.

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Digital Object Identifier 10.1109/TSG.2014.2359004

I. INTRODUCTION

DISTRIBUTED renewable energy generations (such as wind and solar) have been recognized as an environmentally and economically beneficial solution for future smart grids by greatly reducing both the carbon dioxide emissions of conventional fossil fuel based generation, and the energy transmission losses from generators to far apart loads. In order to efficiently integrate renewable energy to the gird, the concept of microgrids has drawn significant interests. By integrating and controlling a networked group of distributed renewable generators and storage systems, each microgrid supplies power to local users in a small geographical area more cost-effectively. In practice, microgrids can operate either with connection to the main grid or independently in an islanded mode [1], depending on their renewable generation capacity and load demand.

However, due to the random and intermittent characteristics of practical renewable energy coupled with the uncertainty of load demands, new challenges arise for the design of reliable and stable operation of microgrids. For example, the mismatch between renewable generation and load demand may lead to demand outage (in the case of insufficient renewable energy) or result in energy waste (in the opposite case of excessive renewable energy). To address this problem, various methods such as using supplement conventional generation [2] and enabling energy cooperation among neighboring microgrids [3]–[5] have been proposed. Moreover, energy storage is a practically appealing solution to smooth out the power fluctuations in the renewable energy generation, thus improving both the reliability and efficiency of microgrids.

In this paper, we investigate the real-time energy management problem for a single microgrid system consisting of a renewable generation system, an energy storage system, and an aggregated load. The main results of this paper are summarized as follows.

1) We model the renewable energy offset by the load over time, termed net energy profile, to be practically predictable but with finite errors that can be arbitrarily distributed. Under this setup, we aim to minimize the total energy cost (modeled as sum of time-varying strictly convex functions) of the conventional energy drawn from the main grid over a finite horizon by jointly optimizing the energy charged/discharged to/from the storage system over time subject to practical load and storage constraints.
2) To solve the formulated problem in real time, we propose a new off-line optimization approach to devise the online algorithm. Specifically, our proposed online algorithm is based on combining the optimal off-line solution by assuming perfect knowledge of the net energy profile with a "sliding-window" based sequential optimization. This is in contrast to the conventional sliding-window (or model predictive control as in [6]) based algorithm that uses dynamic programming to solve the optimization problem within each window, for which the prediction error of the net energy profile needs to follow a certain stochastic process with known distribution.
3) Finally, we conduct extensive simulations based on the real wind generation data of the Ireland power system [7] to evaluate the performance of our proposed algorithms. It is shown that our proposed sliding-window based online algorithm outperforms three heuristically designed online algorithms. Under the special case where the energy prediction errors are modeled as a stochastic process with known distribution, it is also shown that our proposed online algorithm achieves a performance very close to the performance upper bound by the optimal dynamic programming based solution for this case.

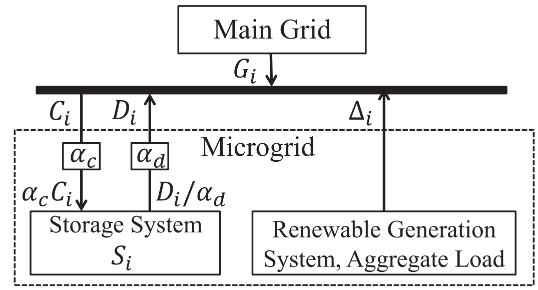

Fig. 1. System model.

There have been rich prior works [8]–[20] which studied the energy management problem in power systems with renewable energy integration and/or energy storage. The off-line energy management problem was studied in [8]–[13] under the ideal assumption that the generated renewable energy and the load demand are either deterministic or known *a priori* before scheduling. The prior works [14]–[18] investigated the real-time or online energy management problem under the stochastic demand and/or renewable energy generation by considering either a simplified energy storage model with infinite capacity [14] or assuming a stationary stochastic process with known distributions for the demand and/or renewable energy generation [15]–[18]. Furthermore, an optimal online energy management policy was proposed in [19] solely based on the current demand, renewable generation, and storage information under a simplified time-invariant linear cost model for conventional energy generation. Last but not least, Fathi and Bevrani [20] studied the online energy management problem for multiple microgrids without considering energy storage.

In contrast to the above prior works,[1] the main contribution of this paper is to devise a new online algorithm for the real-time energy management of microgrid systems by innovatively combining the off-line optimal solution with the sliding-window based sequential optimization, which practically works well under arbitrary error realizations in the net energy profile. It is worth pointing out that the conventional dynamic programming approach cannot be applied for practical scenarios where the distributions of future net energy prediction errors are not known to the microgrid, while our proposed online algorithm works with arbitrary error realizations.

II. System Model and Problem Formulation

We consider a power system consisting of one main grid and one single microgrid. The system model of our interest is thus depicted in Fig. 1, where a microgrid is shown to connect to the main grid and is composed of three major elements, i.e., a

[1]A preliminary conference version of this paper has also been presented in [21]. As compared to [21], this paper provides more detailed proofs and discussions for our proposed solutions, and includes more substantial simulation results to corroborate our analysis.

renewable generation system, an energy storage system, and an aggregate load. We first have the following assumptions for our system model.

1) We consider a time-slotted system with slot index $i, 1 \leq i \leq N$, where N denotes the total number of slots for energy scheduling.
2) For convenience of analysis, we consider a quasi-static time-varying energy model, in which the energy rate (including that of the generated renewable energy, the demand energy, the energy charged/discharged to/from the storage system, or the energy drawn from the main grid) is constant within each slot, but may change from one slot to another.
3) The duration of each slot is normalized to a unit time unless specified otherwise; thus, we can use power and energy interchangeably in this paper for a given slot.

Next, we define each element of the microgrid system in detail as follows.

A. Energy Storage Model

We denote the energy charged (discharged) to (from) the storage in slot i as $C_i \geq 0$ ($D_i \geq 0$). In practice, there are energy losses during both the charging and discharging processes, which can be specified by the charging and discharging efficiency parameters, denoted by $0 < \alpha_c < 1$ and $0 < \alpha_d < 1$, respectively. Then, by denoting the state (stored energy) of the storage system at the beginning of each time slot i as $S_i \geq 0$, we obtain the following equations for modeling the storage dynamics:

$$S_{i+1} = S_i + \alpha_c C_i - \frac{1}{\alpha_d} D_i, \ i = 1, \dots, N. \tag{1}$$

Note that S_1 is the initial energy storage at the beginning of slot 1, while S_{N+1} is the final energy stored at the end of the N-slot scheduling period. Furthermore, practical energy storage systems always have finite capacity and also cannot be discharged completely; as a result, we denote the storage capacity as $S_{\max} \geq 0$ and a minimum storage level as $S_{\min} \geq 0$, to avoid deep discharging. Then, we obtain the following constraints for the states of the storage system:

$$S_{\min} \leq S_i \leq S_{\max}, \quad i = 2, \dots, N+1 \tag{2}$$

where $S_{\min} \leq S_1 \leq S_{\max}$ is assumed by default. In addition, the final energy storage S_{N+1} needs to be kept above a given threshold $\overline{S}$ with $S_{\min} \leq \overline{S} \leq S_{\max}$, to achieve reliable and efficient energy scheduling for the next N-slot scheduling period. As a consequence, we have

$$S_{N+1} \geq \overline{S}. \tag{3}$$

Note that in practice there are other costs related to the energy storage system such as installment cost, operational cost, and aging cost, which should be taken into account for the long-term battery management. However, these factors are ignored in this paper for our investigation of real-time energy storage scheduling over a relatively short time horizon.

B. Load and Renewable Energy Model

In each slot i, the demand energy in a microgrid is denoted as $DE_i \geq 0$, while the generated renewable energy is given by $RE_i \geq 0$. For convenience, we define the net energy profile over time as $\Delta_i = RE_i - DE_i$, $i = 1, \dots, N$, which specifies the mismatch between the renewable energy supply and demand over time. Note that Δ_i can be zero, positive (representing a supply surplus), or negative (representing a supply deficit). We assume that both RE_i's and DE_i's are predictable in general but with finite prediction errors, due to their randomness in practice. Suppose that the predictable demand and renewable energy values are denoted as $\overline{DE}_i$ and $\overline{RE}_i$, respectively, in slot i. We then have $DE_i = \overline{DE}_i + \delta_{DE,i}$ and $RE_i = \overline{RE}_i + \delta_{RE,i}$, where $\delta_{DE,i}$ and $\delta_{RE,i}$ denote the prediction errors for the demand and renewable energy in slot i, respectively, which can be modeled by arbitrary (deterministic or stochastic) sequences over $i = 1, \dots, N$. Hence, we model the net energy profile for the microgrid as

$$\Delta_i = \overline{\Delta}_i + \delta_i, \ i = 1, \dots, N \tag{4}$$

where $\overline{\Delta}_i = \overline{RE}_i - \overline{DE}_i$ and $\delta_i = \delta_{RE,i} - \delta_{DE,i}$. Under this model, we further assume that at any slot $i \in \{1, \dots, N\}$, the exact net energy profile over time $k \leq i$, i.e., $\Delta_1, \dots, \Delta_i$, as well as the predictable net energy profile for time $k > i$, i.e., $\overline{\Delta}_{i+1}, \dots, \overline{\Delta}_N$, are perfectly known to the microgrid, whereas the prediction errors for time $k > i$, i.e., $\delta_{i+1}, \dots, \delta_N$, are unknown.

We assume that the microgrid should always meet the load demand by discharging from its storage and/or drawing energy from the main grid. Let the energy drawn from the main grid in slot i be denoted by $G_i \geq 0$. We then have the following energy neutralization constraints over time as:

$$G_i + \Delta_i + D_i \geq C_i, \quad i = 1, \dots, N. \tag{5}$$

Note that in case of energy surplus $\Delta_i > 0$, part of the energy may be curtailed due to the limited capacity of the energy storage system. In this case, (5) needs to hold with a strict inequality.

C. Conventional Generation Cost

In this paper, we focus on the cost of the conventional energy drawn from the main grid by ignoring other costs such as the operational cost of energy storage in microgrid, etc. We consider a general time-varying cost model for conventional generation and specifically model the costs over time by a sequence of functions of G_i, denoted by $f_i(G_i)$, $i = 1, \dots, N$, each of which is assumed to be known *a priori* to the microgrid[2] and have the following properties.

1) $f_i(G_i)$ is a strictly convex function [22] over $G_i \geq 0$.
2) $f_i(G_i)$ is a strictly positive and monotonically increasing function over $G_i \geq 0$.
3) $f_i(G_i)$ is continuous and differentiable over $G_i \geq 0$, where $F_i(G_i) \triangleq f_i'(G_i)$ is the differential of $f_i(G_i)$ of which the inverse function, $F_i^{-1}(\cdot)$, uniquely exists.

[2]We assume that the microgrid and the main grid belong to the same operator with the common objective to minimize the cost of energy delivered to microgrid users. In this case, the main grid informs the microgrid the cost function $f_i(G_i)$'s through an existing communication link connecting them.

One commonly adopted function of $f_i(G_i)$ satisfying all the above properties is the quadratic cost function for thermal generators [23] given as

$$f_i(G_i) = a_i G_i^2 + b_i G_i + c_i \tag{6}$$

where $a_i > 0$, $b_i \geq 0$, and $c_i \geq 0$ are given cost coefficients for slot i; in this case, we can further obtain $F_i(G_i) = 2a_i G_i + b_i$ and $F_i^{-1}(G_i) = (G_i - b_i)/2a_i$.

With the aforementioned models, we proceed to optimize the decision variables $\{C_i, D_i, G_i\}_{i=1}^N$ to minimize the cost of the total energy drawn from the main grid, i.e., $\sum_{i=1}^N f_i(G_i)$, while satisfying the given storage and load constraints. We thus formulate the optimization problem as follows:

$$\begin{aligned}
\text{(P1)}: \min_{\{C_i,D_i,G_i\}} \quad & \sum_{i=1}^{N} f_i(G_i) \\
\text{s.t.} \quad & (1)-(3), (5) \\
& C_i \geq 0,\ D_i \geq 0,\ G_i \geq 0,\ i = 1, \ldots, N.
\end{aligned}$$

Due to the unknown prediction error δ_k's in (4) at each slot i with $k > i$, (P1) is in general a challenging problem to solve. One commonly used method to solve problems of similar structure to (P1), is via the technique of dynamic programming, which provides the optimal online solution if δ_i's are modeled as a stochastic process with known distribution (e.g., a stationary or cyclostationary stochastic process). However, due to the notorious "curse of dimensionality" problem, the optimal solution by dynamic programming in general has an exponentially growing complexity with the number of decision variables as $N \to \infty$. Furthermore, in practical systems, the renewable energy generated and/or the load demand cannot be exactly modeled by stationary or cyclostationary processes; as a result, it may not be practically valid to model δ_i's as such a process with known distributions. Therefore, this motivates our work to propose an alternative optimization approach for solving (P1) online or in real-time. First, we derive the optimal off-line solution of (P1), by assuming that the net energy profile $\{\Delta_1, \ldots, \Delta_N\}$ is perfectly known ahead of time with no prediction errors, i.e., $\delta_i = 0$, $i = 1, \ldots, N$, in (4). Next, based on the developed off-line solution of (P1), we further propose an online algorithm for real-time energy management under the practical setup with noisy predicted net energy profile, uniquely exists subject to arbitrary error sequence of δ_i's. Note that as a by-product, the off-line optimization always provides a performance upper bound [or a lower bound on the objective value of (P1)] for any online algorithms under the same net energy profile realization Δ_i's.

III. Off-Line Optimization

In this section, we consider the off-line optimization of (P1) by assuming the net energy profile $\{\Delta_1, \ldots, \Delta_N\}$ are known at the beginning of slot $i = 1$. For convenience, we express (P1) more explicitly as follows:

$$\min_{\{C_i,D_i,G_i\}} \sum_{i=1}^{N} f_i(G_i)$$

$$\text{s.t.} \quad S_1 + \alpha_c \sum_{k=1}^{i} C_k - \frac{1}{\alpha_d} \sum_{k=1}^{i} D_k \geq S_{\min}, i = 1, \ldots, N \tag{7}$$

$$S_1 + \alpha_c \sum_{k=1}^{i} C_k - \frac{1}{\alpha_d} \sum_{k=1}^{i} D_k \leq S_{\max}, i = 1, \ldots, N \tag{8}$$

$$S_1 + \alpha_c \sum_{k=1}^{N} C_k - \frac{1}{\alpha_d} \sum_{k=1}^{N} D_k \geq \overline{S} \tag{9}$$

$$G_i + \Delta_i + D_i \geq C_i, i = 1, \ldots, N \tag{10}$$

$$C_i \geq 0,\ D_i \geq 0,\ G_i \geq 0, i = 1, \ldots, N. \tag{11}$$

It is easy to verify that (P1) is a convex optimization problem, since the objective function is convex and all its constraints are affine [22]. Thus, (P1) can be solved by standard convex optimization techniques such as the interior point method [22]. However, in order to draw more insights from the optimal solution, in this paper we apply the Lagrange duality method to obtain a closed-form optimal solution for (P1). First, we derive the dual function of (P1) by minimizing its Lagrangian. Next, we solve the dual problem to derive the optimal dual variables using the subgradient based method. Finally, with the optimal dual variables at hand, we obtain the optimal solution to (P1).

Let the Lagrange dual variables associated with the constraints in (7)–(9) be denoted by $\underline{\nu}_i$, $\overline{\nu}_i$, $i = 1, \ldots, N$, and ω, respectively, and define

$$\nu_i = \sum_{k=i}^{N} \left(\underline{\nu}_k - \overline{\nu}_k \right),\ i = 1, \ldots, N. \tag{12}$$

Then, the Lagrangian of (P1) is expressed as

$$\begin{aligned}
&\mathcal{L}(\omega, \{\underline{\nu}_i\}, \{\overline{\nu}_i\}, \{C_i\}, \{D_i\}, \{G_i\}) \\
&\quad = \sum_{i=1}^{N} f_i(G_i) + \sum_{i=1}^{N} (\nu_i + \omega) \left(\frac{D_i}{\alpha_d} - \alpha_c C_i \right) - \omega S_1 + \omega \overline{S} \\
&\qquad - \left(\sum_{i=1}^{N} \underline{\nu}_i \right) (S_1 - S_{\min}) - \left(\sum_{i=1}^{N} \overline{\nu}_i \right) (S_{\max} - S_1).
\end{aligned} \tag{13}$$

Accordingly, the dual function of $\mathcal{L}(\cdot)$ is given by

$$\begin{aligned}
&g\left(\omega, \{\underline{\nu}_i\}, \{\overline{\nu}_i\}\right) \\
&\quad = \min_{\{C_i,D_i,G_i\}} \mathcal{L}(\omega, \{\underline{\nu}_i\}, \{\overline{\nu}_i\}, \{C_i\}, \{D_i\}, \{G_i\}) \\
&\qquad\quad \text{s.t.} \quad (10), (11).
\end{aligned} \tag{14}$$

Thus, the dual problem of (P1) is given by

$$\text{(D1)}: \max_{\omega \geq 0, \{\underline{\nu}_i \geq 0\}, \{\overline{\nu}_i \geq 0\}} g\left(\omega, \{\underline{\nu}_i\}, \{\overline{\nu}_i\}\right). \tag{15}$$

Since (P1) is convex and satisfies the Slater's condition [22], strong duality holds between (P1) and (D1) [22]; as a result, we can solve (P1) optimally by solving (D1) equivalently. In the following, we first obtain $g(\omega, \{\underline{\nu}_i\}, \{\overline{\nu}_i\})$ with given $\omega \geq 0$, $\underline{\nu}_i \geq 0$, and $\overline{\nu}_i \geq 0$, $i = 1, \ldots, N$, by solving the minimization problems in (14), and then search over ω, $\{\underline{\nu}_i\}$, and $\{\overline{\nu}_i\}$ to maximize $g(\omega, \{\underline{\nu}_i\}, \{\overline{\nu}_i\})$ as shown in (15).

First, we have the following lemma.

Lemma 1: In order for $g(\omega, \{\underline{\nu}_i\}, \{\overline{\nu}_i\})$ to be bounded from below, it must hold that $\nu_i \geq -\omega$, $i = 1, \ldots, N$.

Proof: Please refer to Appendix A. ■

From Lemma 1, we need to solve the problem in (14) under given ω, $\{\underline{\nu}_i\}$ and $\{\overline{\nu}_i\}$ satisfying $\nu_i \geq -\omega$, $i = 1, \ldots, N$. In this case, by removing the irrelevant constant terms in (13), the minimization problem in (14) can be explicitly expressed as

$$\min_{\{C_i, D_i, G_i\}} \sum_{i=1}^{N} f_i(G_i) + \sum_{i=1}^{N} (\nu_i + \omega)\left(\frac{D_i}{\alpha_d} - \alpha_c C_i\right) \quad \text{s.t.} \quad (10), (11). \tag{16}$$

Note that the optimization problem in (16) can be decomposed over time into N independent optimization problems, each of which is expressed as follows for one $i \in \{1, \ldots, N\}$:

$$\begin{aligned} \min_{C_i, D_i, G_i} \ & f_i(G_i) + (\nu_i + \omega)\left(\frac{D_i}{\alpha_d} - \alpha_c C_i\right) \\ \text{s.t.} \ & G_i + \Delta_i + D_i \geq C_i \\ & C_i \geq 0, \ D_i \geq 0, \ G_i \geq 0. \end{aligned} \tag{17}$$

By denoting $\{C_i^*, D_i^*, G_i^*\}$ as the optimal solution for the minimization problem in (17), we then have the following lemma.

Lemma 2: There always exists an optimal solution for (17) satisfying that $C_i^* \cdot D_i^* = 0, i = 1, \ldots, N$.

Proof: Please refer to Appendix B. ■

Lemma 2 is intuitive since it cannot be optimal for the energy storage system to charge and discharge at the same time slot given $0 < \alpha_c < 1$ and $0 < \alpha_d < 1$.

With Lemmas 1 and 2, we are now ready to obtain the optimal solution to (17), shown as follows.

Proposition 1: The optimal solution to (17) is given by

$$C_i^* = \left[F_i^{-1}\big(\max(F_i(0), \alpha_c\omega + \alpha_c\nu_i)\big) + \Delta_i\right]^+ \tag{18}$$

$$D_i^* = \left[-F_i^{-1}\big(\max(F_i(0), \omega/\alpha_d + \nu_i/\alpha_d)\big) - \Delta_i\right]^+ \tag{19}$$

$$G_i^* = \left[C_i^* - D_i^* - \Delta_i\right]^+ \tag{20}$$

where $[x]^+ \triangleq \max(0, x)$.

Proof: Please refer to Appendix C. ■

From Proposition 1, we can obtain $g(\omega, \{\underline{\nu}_i\}, \{\overline{\nu}_i\})$ with any set of $\omega \geq 0$, $\underline{\nu}_i \geq 0$, $\overline{\nu}_i \geq 0$, and $\nu_i \geq -\omega$, $\forall i \in \{1, \ldots, N\}$. Next, we maximize $g(\omega, \{\underline{\nu}_i\}, \{\overline{\nu}_i\})$ over ω, $\{\underline{\nu}_i\}$, and $\{\overline{\nu}_i\}$ to solve the dual problem (D1) given in (15). Note that $g(\omega, \{\underline{\nu}_i\}, \{\overline{\nu}_i\})$ is concave but not necessarily differentiable [22]. Nevertheless, it can be verified that the subgradient of $g(\omega, \{\underline{\nu}_i\}, \{\overline{\nu}_i\})$ always exists [24], which can be expressed as $\overline{S} - (S_1 + \alpha_c \sum_{k=1}^{N} C_k^* - (1/\alpha_d)\sum_{k=1}^{N} D_k^*)$, $S_{\min} - (S_1 + \alpha_c \sum_{k=1}^{i} C_k^* - (1/\alpha_d)\sum_{k=1}^{i} D_k^*)$, and $(S_1 + \alpha_c \sum_{k=1}^{i} C_k^* - (1/\alpha_d)\sum_{k=1}^{i} D_k^*) - S_{\max}$ at ω, $\underline{\nu}_i$, and $\overline{\nu}_i$, respectively, $i = 1, \ldots, N$. Therefore, (D1) can be solved by subgradient based methods such as the ellipsoid method [24], for which the optimal (dual) solution can be obtained as $\omega^\star$, $\{\underline{\nu}_i^\star\}$, and $\{\overline{\nu}_i^\star\}$. With $\omega^\star$, $\{\underline{\nu}_i^\star\}$, and $\{\overline{\nu}_i^\star\}$, the optimal value of (D1) must be the same as that of (P1) due to strong duality. However, the corresponding solution obtained from Proposition 1, i.e., $\{C_i^*, D_i^*, G_i^*\}$, cannot be directly applied as the optimal solution of (P1), since it is in general nonunique and thus may not even be a feasible solution for (P1). Therefore, to obtain the optimal solution for (P1), we have the following proposition, which provides an optimal closed-form solution of (P1) in terms of the optimal dual variables $\omega^\star$, $\{\underline{\nu}_i^\star\}$, and $\{\overline{\nu}_i^\star\}$.

TABLE I

OPTIMAL OFF-LINE ALGORITHM FOR PROBLEM (P1)

Algorithm 1
a) Initialize ω, $\{\underline{\nu}_i\}$, and $\{\overline{\nu}_i\}$ with $\omega \geq 0$, $\underline{\nu}_i \geq 0$, $\overline{\nu}_i \geq 0$, and $\nu_i \geq -\omega$, $i = 1, \ldots, N$.
b) **Repeat:**
1) Compute $\{C_i^*\}$, $\{D_i^*\}$ and $\{G_i^*\}$ by using (18), (19) and (20), respectively, with given ω, $\{\underline{\nu}_i\}$, and $\{\overline{\nu}_i\}$;
2) Compute the subgradient of $g(\omega, \{\underline{\nu}_i\}, \{\overline{\nu}_i\})$ as $\overline{S} - (S_1 + \alpha_c \sum_{k=1}^{N} C_k^* - \frac{1}{\alpha_d}\sum_{k=1}^{N} D_k^*)$, $S_{\min} - (S_1 + \alpha_c \sum_{k=1}^{i} C_k^* - \frac{1}{\alpha_d}\sum_{k=1}^{i} D_k^*)$, and $(S_1 + \alpha_c \sum_{k=1}^{i} C_k^* - \frac{1}{\alpha_d}\sum_{k=1}^{i} D_k^*) - S_{\max}$, for ω, $\underline{\nu}_i$, and $\overline{\nu}_i$, respectively, $i = 1, \ldots, N$; update ω, $\{\underline{\nu}_i\}$, and $\{\overline{\nu}_i\}$ accordingly based on the ellipsoid method.
c) **Until** ω , $\{\underline{\nu}_i\}$, and $\{\overline{\nu}_i\}$ all converge within a prescribed accuracy.
d) Set $\omega^\star \leftarrow \omega$, $\underline{\nu}_i^\star \leftarrow \underline{\nu}_i, \overline{\nu}_i^\star \leftarrow \overline{\nu}_i$, $i = 1, \ldots, N$.
e) Initialize $i \leftarrow 1$.
f) **Repeat:**
1) Compute $C_i^\star$, $D_i^\star$ and $G_i^\star$ by using (21), (22) and (23), respectively;
2) Compute $S_i^\star = S_1 + \alpha_c \sum_{k=1}^{i-1} C_k^\star - \frac{1}{\alpha_d}\sum_{k=1}^{i-1} D_k^\star$;
3) Set $i \leftarrow i + 1$.
g) **Until** $i = N + 1$.

Proposition 2: The optimal solution to (P1) is given by

$$C_i^\star = \min\left(\left[F_i^{-1}\left(\max\left(F_i(0), \alpha_c\omega^\star + \alpha_c\nu_i^\star\right)\right) + \Delta_i\right]^+, \frac{S_{\max} - S_i^\star}{\alpha_c}\right) \tag{21}$$

$$D_i^\star = \min\left(\left[-F_i^{-1}\left(\max\left(F_i(0), \omega^\star/\alpha_d + \nu_i^\star/\alpha_d\right)\right) - \Delta_i\right]^+, \alpha_d\left(S_i^\star - S_{\min}\right)\right) \tag{22}$$

$$G_i^\star = \left[C_i^\star - D_i^\star - \Delta_i\right]^+ \tag{23}$$

for $i = 1, \ldots, N$, where $\nu_i^\star$ is defined in (12) with the given $\{\underline{\nu}_i^\star\}$ and $\{\overline{\nu}_i^\star\}$, and $S_i^\star = S_1 + \alpha_c \sum_{k=1}^{i-1} C_k^\star - (1/\alpha_d)\sum_{k=1}^{i-1} D_k^\star$.

Proof: Please refer to Appendix D. ■

Notice that in Proposition 2, (21)–(23) need to be computed iteratively from $i = 1$ to $i = N$. In summary, one algorithm for solving (P1) is given in Table I as Algorithm 1, in which steps a)–d) compute the optimal dual solution $\omega^\star$, $\{\underline{\nu}_i^\star\}$, and $\{\overline{\nu}_i^\star\}$ in (D1), while steps e)-g) obtain the optimal solution $\{C_i^\star\}$, $\{D_i^\star\}$ and $\{G_i^\star\}$ for (P1).

IV. SLIDING-WINDOW BASED ONLINE ALGORITHM

In the previous section, we have studied the off-line optimization under the ideal assumption that the net energy profile Δ_i's are perfectly known *a priori* by deriving the optimal off-line energy scheduling solution for (P1) which provides a performance upper bound for all online energy management algorithms in general. Inspired by the optimal off-line solution, in this section, we propose an online algorithm for (P1) under the practical setup of noisy net energy profile prediction, that is, at each slot i, only the past and current net energy profile, i.e., $\Delta_1, \ldots, \Delta_i$, and the predicable part in the future net energy profile, i.e., $\overline{\Delta}_{i+1}, \ldots, \overline{\Delta}_N$, are known to the microgrid, whereas the future prediction errors $\delta_{i+1}, \ldots, \delta_N$, are unknown. Our proposed online algorithm is based on combining the off-line solution for (P1) with a sliding-window based sequential optimization, which is applicable in practical scenarios where the prediction errors are modeled by arbitrary (unknown) sequences.

We define a parameter M with $1 \leq M \leq N$ as the size of the sliding-window. At each slot i, we regard the online optimization as a finite-horizon energy management problem over a window of M slots, with an initial energy state given by S_i, and an available net energy profile over this window as $\Delta_i, \overline{\Delta}_{i+1}, \ldots, \overline{\Delta}_{i+M-1}$. Note that except slot i, for the future $M-1$ slots in the online optimization at slot i,[3] we have used the predictable net energy profile $\overline{\Delta}_{i+1}, \ldots, \overline{\Delta}_{M+i-1}$ instead of the exact one $\Delta_{i+1}, \ldots, \Delta_{M+i-1}$ (since they are unknown yet) by ignoring their predictions errors. For the online optimization at slot i, we denote the decision variables over the window of size M as $\{C_j^{(i)}, D_j^{(i)}, G_j^{(i)}\}_{j=1}^M$. Then, we formulate the online optimization problem at slot i similarly to (P1), by replacing N and S_1 in (P1) by M and S_i, $f_1(\cdot), \ldots, f_N(\cdot)$ in (P1) by $f_i(\cdot), \ldots, f_{i+M-1}(\cdot)$, $\Delta_1, \Delta_2 \ldots, \Delta_N$ in (P1) by $\Delta_i, \overline{\Delta}_{i+1} \ldots, \overline{\Delta}_{i+M-1}$, $\overline{S}$ in (P1) by $\overline{S}_{i+M-1}$, and finally $\{C_i, D_i, G_i\}_{i=1}^N$ in (P1) by $\{C_j^{(i)}, D_j^{(i)}, G_j^{(i)}\}_{j=1}^M$. We also set $\overline{S}_{i+M-1} = \overline{S}$ if $i+M-1=N$ and $\overline{S}_{i+M-1} = 0$ otherwise. More explicitly, we formulate the online problem at slot i as

$$
\begin{aligned}
\min_{\left\{C_j^{(i)}, D_j^{(i)}, G_j^{(i)}\right\}_{j=1}^M} \quad & \sum_{j=1}^{M} f_{i+j-1}\left(G_j^{(i)}\right) \\
\text{s.t.} \quad & S_i + \alpha_c \sum_{k=1}^{j} C_k^{(i)} - \frac{1}{\alpha_d} \sum_{k=1}^{j} D_k^{(i)} \geq S_{\min},\ j=1,\ldots,M \\
& S_i + \alpha_c \sum_{k=1}^{j} C_k^{(i)} - \frac{1}{\alpha_d} \sum_{k=1}^{j} D_k^{(i)} \leq S_{\max},\ j=1,\ldots,M \\
& S_i + \alpha_c \sum_{k=1}^{M} C_k^{(i)} - \frac{1}{\alpha_d} \sum_{k=1}^{M} D_k^{(i)} \geq \overline{S}_{i+M-1} \\
& G_1^{(i)} + \Delta_i + D_1^{(i)} \geq C_1^{(i)} \\
& G_j^{(i)} + \overline{\Delta}_j + D_j^{(i)} \geq C_j^{(i)},\ j=2,\ldots,M \\
& C_j^{(i)} \geq 0,\ D_j^{(i)} \geq 0,\ G_j^{(i)} \geq 0,\ j=1,\ldots,M.
\end{aligned} \tag{24}
$$

Equation (24) can be solved by Algorithm 1 directly by a change of variables/parameters as specified above, with the optimal solution denoted by $\{C_j^{(i)\star}, D_j^{(i)\star}, G_j^{(i)\star}\}_{j=1}^M$. Then our proposed sliding-window based online algorithm sets the decision variables at time i as

$$
\begin{aligned}
& C_i^{\text{online}} = C_1^{(i)\star},\ D_i^{\text{online}} = D_1^{(i)\star} \\
& G_i^{\text{online}} = G_1^{(i)\star},\ i=1,\ldots,N.
\end{aligned} \tag{25}
$$

In summary, the above proposed online algorithm is given in Table II as Algorithm 2.

Remark 1: The sliding-window size M is a key design parameter for our proposed online algorithm. Specifically, larger M is desirable for the case with small prediction error δ_i's to fully exploit the benefit of long-term prediction, while smaller M is preferable for the case where the prediction errors are large so that the predicable net energy profile is rendered less useful as the window size is increased. On the other hand, when the storage capacity is large, larger M is preferable in order to fully utilize the storage capacity. As a result, in case of small prediction errors, larger M always performs better regardless of the storage capacity, while for the case of large prediction errors, the opposite is true unless the storage capacity is large enough so that the gain of using the larger window size to exploit the storage capacity compensates the loss due to more prediction errors.

TABLE II
SLIDING-WINDOW BASED ONLINE ALGORITHM FOR PROBLEM (P1)

Algorithm 2
a) Initialize $i \leftarrow 1$.
b) **Repeat:**
1) For slot i, solve problem (24) by Algorithm 1, and obtain its solution as $\{C_j^{(i)\star}, D_j^{(i)\star}, G_j^{(i)\star}\}_{j=1}^M$;
2) Set $C_i^{\text{online}} = C_1^{(i)\star}$, $D_i^{\text{online}} = D_1^{(i)\star}$, $G_i^{\text{online}} = G_1^{(i)\star}$, and update $S_{i+1} = S_i + \alpha_c C_i^{\text{online}} - \frac{1}{\alpha_d} D_i^{\text{online}}$;
3) Set $i \leftarrow i+1$.
c) **Until** $i = N+1$.

V. ALTERNATIVE ONLINE ALGORITHMS

In this section, we present four alternative online algorithms to provide performance benchmarks for our proposed sliding-window based online algorithm. First, we introduce three heuristically designed online algorithms, namely "threshold based," "myopic," and "energy halving," respectively. Next, under the special setup where the prediction errors follow a stochastic process with known distribution, we consider the dynamic programming based algorithm to solve (P1), which is optimal in this case and thus serves as the performance upper bound for our proposed online algorithm.

A. Threshold Based Online Algorithm

At each slot i, the decision variables C_i, D_i, and G_i are determined only based on the energy state S_i, the net energy profile element Δ_i at the current slot, and a given threshold T. Specifically, for the case of $\Delta_i > T$, the energy storage is charged by $\Delta_i - T$ until it reaches its capacity $S_{\max}$; whereas for the case of $\Delta_i \leq T$, the energy storage is first discharged by $T - \Delta_i$ to meet the load demand until it reaches its minimum level ($S_{\min}$ for $i < N$ and $\overline{S}$ for $i = N$), and then the energy from the main grid is drawn to meet the residual load

[3] The window of size M will exceed the N-slot horizon if $i+1-M > N$. In this case, we make use of the prediction values in the next N-slot period, i.e., $\overline{\Delta}_{N+1}, \overline{\Delta}_{N+2}, \ldots, \overline{\Delta}_{N+M-1}$ for energy management of slots $i = N - M + 2, \ldots, N$ in the current N-slot period.

(if any). Thus, at the first $N-1$ slots, the threshold based online algorithm sets

$$C_i^{\mathrm{thr}} = \min\left([\Delta_i - T]^+, \frac{S_{\max} - S_i}{\alpha_c}\right)$$
$$D_i^{\mathrm{thr}} = \min\left([T - \Delta_i]^+, \alpha_d(S_i - S_{\min})\right)$$
$$G_i^{\mathrm{thr}} = \left[C_i^{\mathrm{thr}} - D_i^{\mathrm{thr}} - \Delta_i\right]^+ \tag{26}$$

where $1 \le i < N$; while at the last time slot $i = N$, it sets

$$C_N^{\mathrm{thr}} = \max\left(\min\left([\Delta_N - T]^+, \frac{S_{\max} - S_N}{\alpha_c}\right), \frac{\overline{S} - S_N}{\alpha_c}\right)$$
$$D_N^{\mathrm{thr}} = \min\left([T - \Delta_N]^+, \alpha_d\left(S_N - \overline{S}\right)\right)$$
$$G_N^{\mathrm{thr}} = \left[C_N^{\mathrm{thr}} - D_N^{\mathrm{thr}} - \Delta_N\right]^+. \tag{27}$$

This algorithm was shown to be optimal when $\overline{S} = 0$, $T = \mathbb{E}[1/N \sum_{i=1}^{N} \Delta_i]$, the scheduling time horizon is infinite, i.e., $N \to \infty$, and $S_{\max} \to \infty$ [14].

B. Myopic Online Algorithm

This algorithm is equivalent to the threshold based online algorithm, by setting $T = 0$. Note that the myopic online algorithm was shown to be optimal in [19] when the cost function $f_i(\cdot)$'s are all linear and also time-invariant over i, and furthermore $\overline{S} = 0$.

C. Energy Halving Online Algorithm

This algorithm performs similarly as the myopic algorithm, except that only up to half of the stored energy can be used in the case of energy supply deficit at the first $N-1$ slots. For time slot N, the algorithm performs the same as that in (27), with $T = 0$. Please refer to [25] for the detail of this algorithm. In the energy halving algorithm, since at least half of the stored energy is available for future use in the first $N-1$ slots, any energy deficit in future slots is more likely to be compensated by storage as compared to the myopic online algorithm. By more conservatively balancing the current and future renewable energy availability, this algorithm can achieve lower energy cost than the myopic algorithm, as will be shown later by simulations.

D. Dynamic Programming Based Online Algorithm

At last, we consider a special case where the prediction errors, $\delta_1, \ldots, \delta_N$, follow a stochastic process with known distribution. Under this special case, we apply the celebrated dynamic programming method to solve (P1) optimally. Specifically, the dynamic programming based online algorithm aims to minimize the expected cost of the total energy drawn from the main grid, i.e., $\sum_{i=1}^{N} \mathbb{E}[f_i(G_i)]$, subject to (7)–(11). We thus have the following proposition.

Proposition 3: Given Δ_1 and S_1, the optimal value achieved by minimizing $\sum_{i=1}^{N} \mathbb{E}[f_i(G_i)]$ subject to (7)–(11), is given by $J_1(\Delta_1, S_1)$, which can be computed recursively based on the following Bellman equations, starting from $J_N(\Delta_N, S_N)$, $J_{N-1}(\Delta_{N-1}, S_{N-1})$, and so on until $J_1(\Delta_1, S_1)$

$$\begin{aligned} J_N(\Delta_N, S_N) = \min_{C_N, D_N, G_N} & f_N(G_N) \text{ s.t. } \overline{S} \\ & \le S_N + \alpha_c C_N - \frac{1}{\alpha_d} D_N \le S_{\max} G_N \\ & + \Delta_N + D_N \ge C_N \; C_N \\ & \ge 0,\; D_N \ge 0,\; G_N \ge 0 \end{aligned} \tag{28a}$$

$$\begin{aligned} J_i(\Delta_i, S_i) = \min_{C_i, D_i, G_i} & f_i(G_i) + \overline{J}_{i+1}\left(S_i + \alpha_c C_i - \frac{1}{\alpha_d} D_i\right) \\ & \text{s.t. } S_{\min} \le S_i + \alpha_c C_i - \frac{1}{\alpha_d} D_i \le S_{\max} G_i \\ & + \Delta_i + D_i \ge C_i \; C_i \\ & \ge 0,\; D_i \ge 0,\; G_i \ge 0 \end{aligned} \tag{28b}$$

for $i = 1, \ldots, N-1$, where

$$\overline{J}_{i+1}\left(S_i + \alpha_c C_i - \frac{1}{\alpha_d} D_i\right) = \mathbb{E}_{\Delta_{i+1}}\left[J_{i+1}\left(\Delta_{i+1}, S_i + \alpha_c C_i - \frac{1}{\alpha_d} D_i\right)\right] \tag{29}$$

and $\mathbb{E}_{\Delta_i}[\,\cdot\,]$ denotes the expectation over Δ_i. An optimal policy is accordingly given by $\pi^\star = \left\{C_i^{\mathrm{DP}}(\Delta_i, S_i), D_i^{\mathrm{DP}}(\Delta_i, S_i), G_i^{\mathrm{DP}}(\Delta_i, S_i)\right\}_{i=1}^{N}$, where $C_i^{\mathrm{DP}}(\Delta_i, S_i)$, $D_i^{\mathrm{DP}}(\Delta_i, S_i)$, and $G_i^{\mathrm{DP}}(\Delta_i, S_i)$ is the optimal solution to (28).

Proof: The proof follows directly by applying Bellman equations [26], and thus is omitted here for brevity. ■

In Proposition 3, we need to solve the problems given in (28) to obtain the optimal online policy. Since it is difficult to derive the closed-form expressions for $\overline{J}_{i+1}(S_{i+1})$, $i = 1, \ldots, N-1$, we take an alternative Monte Carlo approach to solve the problems in (28) by assuming that the storage state at each time slot i, $i = 1, \ldots, N$, can only be chosen from $\beta+1$ quantized levels with equal difference, denoted in the set $\mathcal{S} = \{0, S_{\max}/\beta, \ldots, S_{\max}\}$. First, consider time slot $i = N$. For any given $S_N \in \mathcal{S}$, we can derive the optimal solution to (28a) by comparing S_N with the threshold $\overline{S}$, which is expressed as

$$C_N^{\mathrm{DP}} = \left[\left(\overline{S} - S_N\right)/\alpha_c\right]^+, \; D_N^{\mathrm{DP}} = \left[\alpha_d\left(S_N - \overline{S}\right)\right]^+$$
$$G_N^{\mathrm{DP}} = \left[C_N^{\mathrm{DP}} - D_N^{\mathrm{DP}} - \Delta_N\right]^+. \tag{30}$$

Given the optimal solution in (30), we then obtain $\overline{J}_N(S_N)$ by averaging $J_N(\Delta_N, S_N)$ (using Monte Carlo simulations) over $K > 0$ independent realizations of Δ_N. By performing this procedure for all storage levels, we can obtain $\overline{J}_N(S_N)$, $\forall S_N \in \mathcal{S}$, which is stored for the next iteration. Next, consider time slot $i = N-1$. For any given $S_{N-1} \in \mathcal{S}$, we compute the optimal solution to (28b) numerically to obtain $J_{N-1}(\Delta_{N-1}, S_{N-1})$ given the stored values of $\overline{J}_N(S_N)$'s and accordingly, compute $\overline{J}_{N-1}(\Delta_{N-1})$ via Monte Carlo simulations. Similarly, for time slots $i = 1, \ldots, N-2$, we solve the corresponding problems in (28b) to recursively obtain $\overline{J}_{N-2}(\Delta_{N-2}, S_{N-2})$, $\overline{J}_{N-3}(\Delta_{N-3}, S_{N-3})$, and so on until $\overline{J}_1(\Delta_1, S_1)$. It is worth noting that the accuracy of the Monte Carlo method is determined by the parameters β and K. If β and K are chosen to be large enough, the above solution can approximate the optimal solution by dynamic programming closely.

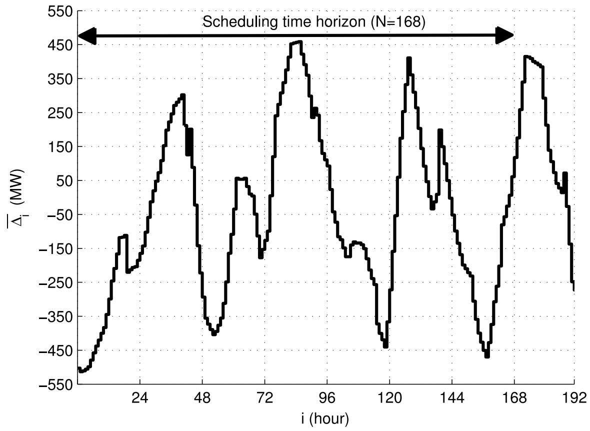

Fig. 2. Hourly predictable net energy profile over one week period.

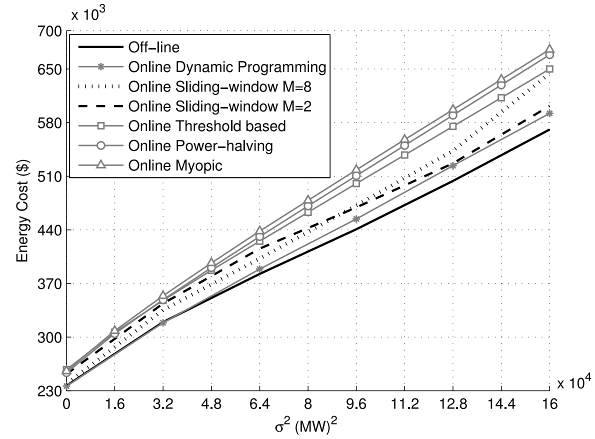

Fig. 3. Energy cost versus the variance of prediction error.

In the above algorithm, we solve the problems in (28) off-line to obtain $\overline{J}_i(S_i)$, $S_i \in \mathcal{S}$, $i = 1, \ldots, N$, which need to be stored in a lookup table for real-time energy management implementation. In each slot i, $1 \leq i \leq N$, given the energy state S_i and the future expected energy cost $\overline{J}_{i+1}(\Delta_{i+1}, S_{i+1})$, we can search from the lookup table to obtain the corresponding online policy for the current slot, which is C_i^{DP}, D_i^{DP} and G_i^{DP}. It is worth noting that in this special case of known prediction error distributions, the dynamic programming based algorithm obtains the performance upper bound for all online algorithms, including our proposed sliding-window based online algorithm as well as the three heuristic algorithms previously introduced.

VI. Simulation Results

In this section, we provide simulation results to evaluate the performance of our proposed algorithms by considering a scheduling period of one week, where we set $N = 168$ with each slot representing 1 h. We assume a quadratic time-invariant cost function given in (6), where $a_i = 0.03125$ \$/MW^2, $b_i = 1$ \$/MW, and $c_i = 0$, $\forall i \in \{1, \ldots, N\}$ [23]. We also set the parameters of the storage system in the microgrid as $\alpha_c = 0.7$, $\alpha_d = 0.8$, $S_1 = 0$, $S_{\min} = 0$, $\overline{S} = 0$, and $S_{\max} = 400$ MW. The predictable net energy profile $\{\overline{\Delta}_i\}$ is taken as the hourly predicted wind energy generation over one week period (from 27 June, 2013 to 3 July, 2013) in the Ireland power grid [7] offset by a time-invariant demand load of $\overline{DE}_i = 600$ MW, $\forall i = 1, \ldots, N$, as shown in Fig. 2. Furthermore, we assume that the prediction error δ_i's follow independent and identical Gaussian distributions with zero mean and variance σ^2.

First, we compare the performance of our proposed sliding-window based online algorithm with that of the three heuristically designed online algorithms. For the threshold based online algorithm we set $T = \sum_{i=1}^{168} \Delta_i / 168 = -64.03$ MW. In addition, since the sequence of prediction error δ_i's is assumed as a stationary stochastic process with known Gaussian distribution, we apply the dynamic programming based algorithm to obtain the performance upper bound (or lower bound on the total cost) for all considered online algorithms. For the proposed sliding-window based online algorithm, we consider two window sizes of $M = 2$ and $M = 8$. For the dynamic programming based algorithm, we set $\beta = 100$ and $K = 100$ to obtain accurate approximate solutions.

Fig. 3 shows the average energy cost versus the prediction error variance σ^2. First, it is observed that the energy cost of all considered algorithms increases with increasing σ^2, which is due to the fact that larger σ^2 corresponds to more substantial energy fluctuations, thus resulting in a higher average energy cost (since energy deficit may not be fully compensated by energy surplus due to limited storage capacity). It is also observed that for our proposed sliding-window based online algorithm, the case of $M = 8$ outperforms that of $M = 2$ when σ^2 is small, while the opposite is true when σ^2 becomes sufficiently large. This result is expected since the storage capacity is not large compared to the net energy profile in this example and thus in the case of large prediction errors, the algorithm with $M = 2$ performs better than that with the larger window size $M = 8$, as explained in Remark 1. Furthermore, it is observed that our proposed sliding-window based online algorithm achieves its cost very close to the minimum cost by the optimal dynamic programming based algorithm, and also outperforms notably over the other three heuristic online algorithms. Finally, the off-line optimization is observed to perform the best over all online algorithms since it is under the ideal assumption that the net energy profile is completely known (i.e., the prediction errors are known ahead of time).

Fig. 4 shows the performance of the dynamic programming based algorithm when prediction errors in the net energy profile deviate from the distribution presumed, as compared to the off-line optimization and our proposed online algorithm. In this example, we construct the lookup table for the dynamic programming algorithm by assuming that the prediction errors follow i.i.d. Gaussian distribution with zero mean and variance of 16×10^4 (MW)^2. However, the actual prediction errors for simulations are generated from another i.i.d. Gaussian distribution, with mean μ varying in the range of $[0,\ 75]$ MW and variance of 1.6×10^4 (MW)^2. It is observed that the performance of dynamic programming under imperfect knowledge of the error distribution is considerably worse than that of our proposed sliding-window based online algorithm with window size $M = 8$. Particularly, as μ increases, the performance gap of dynamic programming becomes more notable.

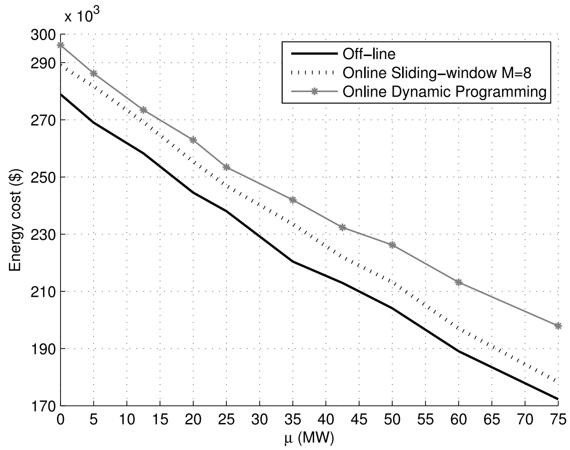

Fig. 4. Performance comparison with dynamic programming under imperfect knowledge of the distribution of prediction errors.

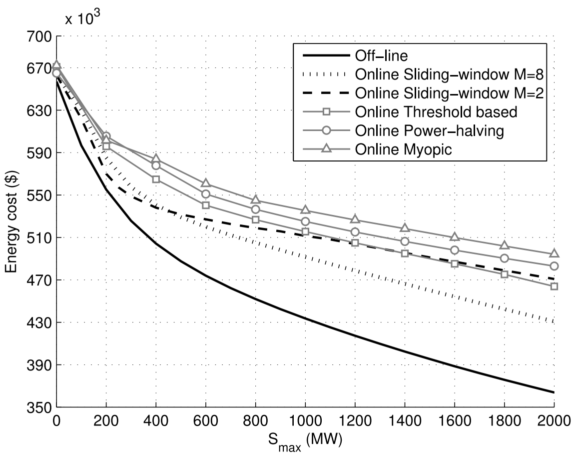

Fig. 5. Energy cost versus storage capacity.

Next, we investigate the impact of storage capacity $S_{\max}$ on the total energy cost. By fixing the variance of the prediction errors as $\sigma^2 = 12.8 \times 10^4$ (MW)^2, Fig. 5 shows the average energy cost versus the storage capacity for the sliding-window based online algorithm (with window sizes of $M = 2$ or $M = 8$), the three heuristic online algorithms, and the off-line optimization. It is observed that the sliding-window based online algorithm with window size of $M = 2$ or $M = 8$ as well as the three heuristic online algorithms perform the same as the off-line optimization when there is no storage, i.e., $S_{\max} = 0$. This is intuitive, since in this case, the prior knowledge of the net energy profile cannot be utilized for scheduling without storage. It is also observed that as $S_{\max}$ increases, the average energy cost for all considered algorithms decreases, while that of the off-line optimization decreases faster than the online algorithms due to the more complete net energy profile information available for scheduling optimization. Finally, it is observed that the sliding-window based online algorithm with $M = 2$ outperforms that of $M = 8$ when $0 < S_{\max} < 450$ MW, while the opposite is true when $S_{\max} > 450$ MW. This result is expected, as explained in Remark 1.

VII. Conclusion

This paper studies the finite-horizon real-time energy storage scheduling for a single microgrid system to minimize the energy cost of the conventional energy drawn from the main grid by jointly optimizing the energy charged/discharged to/from the storage system over time subject to practical load and storage constraints. Under a practical model in which the net energy profiles are predictable but with finite errors, we propose a new sliding-window based online algorithm for real-time energy management by innovatively combining with a well-structured off-line optimization solution, and demonstrate the significant benefits of our proposed online algorithm in practical power systems by simulations. It is hoped that our results will provide a new approach to optimally integrating renewable energy and managing energy storage in practical microgrid systems.

Appendix A
Proof of Lemma 1

Suppose that $\nu_j < -\omega$ holds for some $j \in \{1, \ldots, N\}$. In this case, by letting $G_j = 0$, $C_j \to \infty$, $D_j = C_j - \Delta_j$, and $G_i = C_i = D_i = 0, \forall i \neq j$, it can be shown from (17) that $\mathcal{L}(\omega, \{\underline{\nu}_i\}, \{\overline{\nu}_i\}, \{C_i\}, \{D_i\}, \{G_i\}) \to -\infty$ with (10) and (11) satisfied. Thus, $\nu_i < -\omega$ cannot be true for $g(\omega, \{\underline{\nu}_i\}, \{\overline{\nu}_i\})$ to be bounded from below. This lemma thus follows.

Appendix B
Proof of Lemma 2

Suppose that the optimal solution to (17) is unique and there exists a slot j with both $C_j^* > 0$ and $D_j^* > 0$, i.e., $C_j^* \cdot D_j^* \neq 0$, $j \in \{1, \ldots, N\}$. Then, we can construct another solution $\{\overline{C}_i, \overline{D}_i, G_i^*\} \neq \{C_i^*, D_i^*, G_i^*\}$ as $\overline{C}_i = C_i^* - \min(C_i^*, D_i^*)$ and $\overline{D}_i = D_i^* - \min(C_i^*, D_i^*), \forall i$, which satisfies $\overline{C}_i \cdot \overline{D}_i = 0$. Since $\nu_i \geq -\omega, \forall i$, from Lemma 1, it can be verified that $\mathcal{L}(\omega, \{\underline{\nu}_i\}, \{\overline{\nu}_i\}, \{\overline{C}_i\}, \{\overline{D}_i\}, \{G_i^*\}) \leq \mathcal{L}(\omega, \{\underline{\nu}_i\}, \{\overline{\nu}_i\}, \{C_i^*\}, \{D_i^*\}, \{G_i^*\})$, i.e., a lower objective value can be achieved by the newly constructed solution, which contradicts that $\{C_i^*, D_i^*, G_i^*\}$ is uniquely optimal for (17). This lemma is thus proved.

Appendix C
Proof of Proposition 1

Since the problem in (17) is convex and satisfies the Slater's condition, the Karash–Kuhn–Tucker (KKT) conditions are both necessary and sufficient for its optimality [22]. Let γ_i be the dual variable corresponding to the constraint $G_i + \Delta_i + D_i \geq C_i$, and λ_i^C, λ_i^D, and λ_i^G be the dual variables associated with the constraints of $C_i \geq 0$, $D_i \geq 0$, and $G_i \geq 0$, respectively. Suppose that the optimal solutions to (17) are given by C_i^*, D_i^*, and G_i^*, while the optimal (dual) solutions to (17) are denoted as $\gamma_i^*, \lambda_i^{C*}, \lambda_i^{D*}$, and λ_i^{G*}. Then, the KKT conditions for (17) can be expressed as follows:

$$G_i^* + \Delta_i + D_i^* - C_i^* \geq 0 \tag{31}$$

$$C_i^* \geq 0,\ D_i^* \geq 0,\ G_i^* \geq 0 \tag{32}$$

$$\gamma_i^* \geq 0,\ \lambda_i^{C*} \geq 0,\ \lambda_i^{D*} \geq 0,\ \lambda_i^{G*} \geq 0 \tag{33}$$

$$\gamma_i^* \left(C_i^* - \Delta_i - D_i^* - G_i^*\right) = 0 \tag{34}$$

$$\lambda_i^{C*}C_i^* = 0,\ \lambda_i^{D*}D_i^* = 0,\ \lambda_i^{G*}G_i^* = 0 \tag{35}$$

$$\gamma_i^* - \alpha_c\omega - \alpha_c\nu_i - \lambda_i^{C*} = 0 \tag{36}$$

$$\gamma_i^* - \frac{1}{\alpha_d}\omega - \frac{1}{\alpha_d}\nu_i + \lambda_i^{D*} = 0 \tag{37}$$

$$\gamma_i^* - F_i\left(G_i^*\right) + \lambda_i^{G*} = 0 \tag{38}$$

where (31)–(33) are for the primal and dual feasibility, (34) and (35) for the complimentary slackness, and (36)–(38) follows due to the fact that the gradient of the Lagrangian of (17) must vanish at optimal solution $\{C_i^*, D_i^*, G_i^*\}$.

With the KKT conditions in (31)–(38), we are now ready to prove the proposition by considering the nontrivial case of $\omega + \nu_i > 0$.[4] In this case, it follows from (36) that $\gamma_i^* = \alpha_c(\omega + \nu_i) + \lambda_i^{C*} > 0$. Together with (34), we have

$$C_i^* - \Delta_i - D_i^* - G_i^* = 0. \tag{39}$$

From (39) together with the fact that $C_i^* \cdot D_i^* = 0$ in Lemma 2, we then prove this proposition by studying the following three cases: 1) $C_i^* = 0$ and $D_i^* > 0$; 2) $C_i^* > 0$; and $D_i^* = 0$; and 3) $C_i^* = 0$ and $D_i^* = 0$, respectively.

First, consider the case of $C_i^* = 0$ and $D_i^* > 0$. In this case, it follows from (35) that $\lambda_i^{D*} = 0$, and thus (37) is simplified as $\gamma_i^* = 1/\alpha_d(\omega + \nu_i)$. Accordingly, from (38) we have

$$G_i^* = F_i^{-1}\left(1/\alpha_d(\omega + \nu_i) + \lambda_i^{G*}\right). \tag{40}$$

Using (40) together with (32) and (35), the optimal G_i^* is obtained as

$$\begin{aligned} G_i^* &= \left[F_i^{-1}(1/\alpha_d(\omega + \nu_i))\right]^+ \\ &= F_i^{-1}\left(\max\left(F_i(0),\ 1/\alpha_d(\omega + \nu_i)\right)\right). \end{aligned} \tag{41}$$

Accordingly, from (39) we have the optimal D_i^* as

$$D_i^* = -F_i^{-1}\left(\max\left(F_i(0),\ 1/\alpha_d(\omega + \nu_i)\right)\right) - \Delta_i. \tag{42}$$

Combining $C_i^* = 0$, D_i^* in (42) and G_i^* in (41), the optimal solution to (17) is obtained for the case of $C_i^* = 0$ and $D_i^* > 0$. It remains to show that this solution is indeed as given in (18)–(20). Note that $D_i^* > 0$ is given in this case, and thus it must hold from (42) that $-F_i^{-1}\left(\max\left(F_i(0),\ 1/\alpha_d(\omega + \nu_i)\right)\right) - \Delta_i > 0$. Therefore, D_i^* in (42) is the same as that in (19). For C_i^* in (18), since it can be verified that

$$\begin{aligned} &F_i^{-1}\left(\max(F_i(0), \alpha_c\omega + \alpha_c\nu_i)\right) + \Delta_i \\ &\quad \le F_i^{-1}\left(\max\left(F_i(0), 1/\alpha_d\left(\omega + \alpha_c\nu_i\right)\right)\right) + \Delta_i < 0. \end{aligned}$$

It follows that C_i^* in (18) corresponds to $C_i^* = 0$. By combing $C_i^* = 0$ and D_i^* in (42), it can be verified that G_i^* in (41) is consistent with that in (20). As a result, the solution in Proposition 1 is proved to be the optimal solution to (17) for the case of $C_i^* = 0$ and $D_i^* > 0$.

Second, consider the case of $C_i^* > 0$ and $D_i^* = 0$. In this case, it follows from (35) that $\lambda_i^{C*} = 0$, and thus (36) is simplified as $\gamma_i^* = \alpha_c(\omega + \nu_i)$. Accordingly, from (38) we have

$$G_i^* = F_i^{-1}\left(\alpha_c(\omega + \nu_i) + \lambda_i^{G*}\right). \tag{43}$$

[4]If $\omega + \nu_i = 0$, then it is easy to verify that the optimal solution to (17) is given by $C_i^* = [\Delta_i]^+$, $D_i^* = [-\Delta_i]^+$, and $G_i^* = 0$. This can be shown to coincide with the optimal solution given in (18)–(20).

Using (43) together with (32) and (35), the optimal G_i^* is obtained as

$$\begin{aligned} G_i^* &= \left[F_i^{-1}(\alpha_c(\omega + \nu_i))\right]^+ \\ &= F_i^{-1}\left(\max\left(F_i(0), \alpha_c(\omega + \nu_i)\right)\right). \end{aligned} \tag{44}$$

Accordingly, from (39) we have the optimal C_i^* as

$$C_i^* = F_i^{-1}\left(\max\left(F_i(0),\ \alpha_c(\omega + \nu_i)\right)\right) + \Delta_i. \tag{45}$$

Combining C_i^* in (45), $D_i^* = 0$ and G_i^* in (44), the optimal solution to (17) is obtained for the case of $C_i^* > 0$ and $D_i^* = 0$. Using the fact of $C_i^* > 0$ in this case and following the similar argument as in the previous case of $C_i^* = 0$ and $D_i^* > 0$, we can show that this solution is consistent with that given in (18)–(20). Therefore, the solution in Proposition 1 is proved to be optimal to (17) for the case of $C_i^* > 0$ and $D_i^* = 0$.

Finally, consider the case of $C_i^* = 0$ and $D_i^* = 0$. In this case, it follows from (39) that $G_i^* = -\Delta_i$. We show in the next that this solution is exactly the same as that given in (18)–(20). Note that this case only occurs when $\Delta_i \le 0$ (due to $G_i^* \ge 0$). From (38), we have $\gamma_i^* \le \gamma_i^* + \lambda_i^{G*} = F_i(-\Delta_i)$. Therefore, it must hold that

$$\begin{aligned} &F_i^{-1}\left(\max(F_i(0), \alpha_c\omega + \alpha_c\nu_i)\right) + \Delta_i \\ &\quad = F_i^{-1}\left(\max\left(F_i(0), \gamma_i^* - \lambda_i^{C*}\right)\right) + \Delta_i \\ &\quad \le F_i^{-1}\left(\max\left(F_i(0), \gamma_i^*\right)\right) + \Delta_i \\ &\quad \le F_i^{-1}\left(\max\left(F_i(0), F_i(-\Delta_i)\right)\right) + \Delta_i \le 0 \end{aligned} \tag{46}$$

where the first equality is true due to (36), the last two inequalities hold since $\gamma_i^* \le F_i(-\Delta_i)$ and $\Delta_i \le 0$, respectively. Therefore, C_i^* in (18) corresponds to $C_i^* = 0$. Similarly, we can also show $-F_i^{-1}\left(\max(F_i(0), \omega/\alpha_d + \nu_i/\alpha_d)\right) - \Delta_i \le 0$. As a result, D_i^* in (19) corresponds to $D_i^* = 0$. With the derived C_i^* and D_i^*, it follows that G_i^* in (20) is the same as $G_i^* = -\Delta_i$ derived above. Therefore, the solution in Proposition 1 is optimal to (17) for the case of $C_i^* = 0$ and $D_i^* = 0$.

By combining the above three cases, Proposition 1 is thus proved.

Appendix D
Proof of Proposition 2

Since (P1) is convex and satisfies the Slater's condition, KKT conditions are both necessary and sufficient conditions for its optimality. The KKT conditions for (P1) can be expressed as in (31)–(38) for all $i \in \{1, \ldots, N\}$ by replacing $\{C_i^*, D_i^*, G_i^*\}$ and $\{\lambda_i^{C*}, \lambda_i^{D*}, \lambda_i^{G*}, \gamma_i^*, \omega, \nu_i\}$ with $\{C_i^\star, D_i^\star, G_i^\star\}$ and $\{\lambda_i^{C\star}, \lambda_i^{D\star}, \lambda_i^{G\star}, \gamma_i^\star, \omega^\star, \nu_i^\star\}$, respectively, together with the following additional complementary slackness and feasibility conditions:

$$S_1 + \alpha_c\sum_{k=1}^{i} C_k^\star - \frac{1}{\alpha_d}\sum_{k=1}^{i} D_k^\star \ge S_{\min}$$

$$S_1 + \alpha_c\sum_{k=1}^{i} C_k^\star - \frac{1}{\alpha_d}\sum_{k=1}^{i} D_k^\star \le S_{\max}$$

$$S_1 + \alpha_c\sum_{k=1}^{N} C_k^\star - \frac{1}{\alpha_d}\sum_{k=1}^{N} D_k^\star \ge \overline{S}$$

$$\underline{\nu}_i^\star \left(S_{\min} - \left(S_1 + \alpha_c \sum_{k=1}^{i} C_k^\star - \frac{1}{\alpha_d} \sum_{k=1}^{i} D_k^\star\right)\right) = 0$$
$$\overline{\nu}_i^\star \left(-S_{\max} + \left(S_1 + \alpha_c \sum_{k=1}^{i} C_k^\star - \frac{1}{\alpha_d} \sum_{k=1}^{i} D_k^\star\right)\right) = 0$$
$$\omega^\star \left(\overline{S} - \left(S_1 + \alpha_c \sum_{k=1}^{N} C_k^\star - \frac{1}{\alpha_d} \sum_{k=1}^{N} D_k^\star\right)\right) = 0$$

for $i = 1, \ldots, N$. By applying the above KKT conditions and following the similar procedures for the proof of Proposition 1, this proposition is thus proved.

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