(20 points) Getting familiar with common distributions from the exponential family (you can submit this in a separate file and zip your submission)

Write out the mathematical formula for each of the distributions below, stating which variable(s) represents the parameter(s) of the distribution. Indicate whether it models discrete or continuous variables.

Also, in one or two sentences, describe when this distribution would come in handy during a modeling task. They are:

(a) Gaussian; (b) Bernoulli; (c) Binomial; (d) Multinomial; (e) Exponential; (f) Poisson.

(a) GAUSSIAN (Normal Distribution)

$f(x|\mu,\sigma^2) = (1/\sqrt{(2\pi\sigma^2)}) * \exp(-(x-\mu)^2/(2\sigma^2))$

Parameters: $\mu$ (mean), $\sigma^2$ (variance)

Variable type: Continuous

Models continuous data that clusters around a central value, commonly used for measurement errors, heights, weights, and many natural phenomena.

(b) BERNOULLI Distribution

$P(X=k|p) = p^k * (1-p)^(1-k)$, where $k \in \{0,1\}$

Parameters: p (probability of success)

Variable type: Discrete

Models binary outcomes (success/failure, yes/no), such as coin flips, binary classification problems, or any single trial with two possible outcomes.

(c) BINOMIAL Distribution

$P(X=k|n,p) = C(n,k) * p^k * (1-p)^(n-k)$, where $C(n,k) = n!/(k!(n-k)!)$

Parameters: n (number of trials), p (probability of success per trial)

Variable type: Discrete

This distribution models the number of successes in a fixed number of independent Bernoulli trials. It could maybe be used to analyze the results of surveys or quality control checks.

(d) MULTINOMIAL Distribution

$P(X_1=k_1,\ldots,X_m=k_m|n,p_1,\ldots,p_m) = (n!/(k_1!\ldots k_m!)) * \prod(p_i\text{^}k_i)$

Parameters: n (number of trials), $p_1,\ldots,p_m$ (probabilities for each category)

Variable type: Discrete

Models outcomes when there are more than 2 categories, such as rolling a die multiple times or classification problems with multiple classes.

(e) EXPONENTIAL Distribution

$f(x|\lambda) = \lambda * \exp(-\lambda x)$ for $x \geq 0$

Parameters: λ (rate parameter)

Variable type: Continuous

Models time between events in a Poisson process, such as time between customer arrivals, lifetime of electronic components, or waiting times.

(f) POISSON Distribution

$P(X=k|\lambda) = (\lambda\text{^}k * \exp(-\lambda))/k!$ for $k = 0,1,2,\ldots$

Parameters: λ (average rate of occurrence)

Variable type: Discrete

Models count data or rare events occurring in fixed intervals, such as

number of emails received per hour, traffic accidents per day, or defects per unit.