MEEN 673 **Spring Semester 2023**

Nonlinear Finite Element Analysis

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ASSIGNMENT No. 1

(Tests the understanding of material from Chapter 4 of the textbook)

Date: 19 Jan 2023 **Due: 5pm on 2nd Feb. 2023**

Problem 1 (10 points): Consider the nonlinear differential equation

$$-\frac{d}{dx}\left[a(x,u)\frac{du}{dx}\right]+b(x,u)\frac{du}{dx}+c(x,u)u=f(x),\quad 0<x<L, \tag{1}$$

subjected to the boundary conditions

$$\left[a\frac{du}{dx}\right]_{x=0}=Q_0,\quad u(L)=u_L, \tag{2}$$

where

$$a=a_0+a_1u+a_2\frac{du}{dx},\quad b=b_0+b_1u,\quad c=c_0+c_1u^p \tag{3}$$

and a_0, a_1, a_2, b_0, b_1, c_0, c_1, and f are only functions of x, and Q_0, u_L, and p are real numbers. **Develop the weak form and the associated finite element model of the equation over an element, $\Omega_e=(x_a^e,x_b^e)$. Compute the tangent coefficient matrix associated with the finite element model developed.**

PROGRAM 1: (100 points) Complete FEM1DUNSYMU* for finite element analysis of problems governed by single–variable, second–order, nonlinear equations. Your program should have the direct iteration as well as the Newton iteration options. You may verify your program using the problem described by the following equations:

$$-\frac{d}{dx}\left[a(x,u)\frac{du}{dx}\right]=f(x),\quad 0<x<1;\quad \frac{du}{dx}(0)=0\ ,\quad u(1)=\sqrt{2}. \tag{4}$$

For computational purpose, use the following data:

$$a(x,u)=u,\quad c(x,u)=0,\quad f(x)=-1. \tag{5}$$

Once the program is successfully verified, **solve the boundary-value problem described by Problem 4.5 of the textbook using the developed computer program.** Investigate convergence of the finite element solution with different number of linear and quadratic elements using 2, 4, and 8 linear elements and 1, 2, and 4 quadratic elements. Use a convergence tolerance of $\varepsilon=10^{-3}$ and $ITMAX=10$.

You must submit (on or before the due date): (1) an electronic copy of the part of the main computer program that pertains to nonlinear analysis and the subroutine for element calculations, (2) input for the problem(s) you are assigned to solve, and (3) plots/tables of the numerical results obtained with the two iterative methods for the problem(s).

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*The Fortran source code FEM1DUNSYMU is provided to you in Learning Management System (LMS). It has missing statements in the main program (need statements to calculate error and check for convergence of solution) and subroutine ELMATRCS1D (see Box 4.5.2 on page 196 of the book) that you must include to complete the program. The program already has subroutines for 1D interpolation functions, the imposition of boundary conditions, and solution of unsymmetric algebraic equations. The subroutine ECHO has the sole purpose of reproducing the input file. Those who wish to write their codes in other programming languages may convert the Fortran source codes to the programming language you choose to use.