MEEN 673 **Spring Semester 2023**

Nonlinear Finite Element Analysis

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ASSIGNMENT No. 3

(Tests the understanding of material from Chapter 6 on 2D Problems)

Date: 12 Feb. 2023 **Due: midnight on 25th (Saturday) Feb. 2023**

Note: Please submit a pseudo-code (this is how you begin to think of the implementation) with your assignment.

Consider a nonlinear partial differential equation, in a single unknown u, of the form

$$-\frac{\partial}{\partial x}\left(a_{11}\frac{\partial u}{\partial x}\right)-\frac{\partial}{\partial y}\left(a_{22}\frac{\partial u}{\partial y}\right)+a_{00}u=f(x,y)\ \text{ in }\ \Omega \tag{1}$$

where a_{11}, a_{22}, a_{00} are, in general, functions of x, y, u, and possibly its derivatives ($\partial u/\partial x, \partial u/\partial y$). Develop a computer program to study linear as well as nonlinear solutions of Eq. (1) and its special cases. Include both direct iteration and Newton's iteration methods in the program. Assume that a_{11}, a_{22}, and a_{00} are of the form

$$\begin{aligned}
a_{00} &= A00 + A0X*X + A0Y*Y\\
a_{11} &= A10 + A1X*X + A1Y*Y + A1U*U + A1UX*(DU/DX) + A1UY*(DU/DY)\\
a_{22} &= A20 + A2X*X + A2Y*Y + A2U*U + A2UX*(DU/DX) + A2UY*(DU/DY)
\end{aligned}\tag{2}$$

Your program should account for linear and quadratic rectangular Lagrange family of elements. The Fortran source codes of **FEM2DUNSI.FOR** (which includes the subroutine to impose boundary conditions on banded unsymmetric systems of equations, **BNDRYUNSYM**, and associated solver, **SOLVRUNS**) and typical input data file are placed on CANVAS.

In particular, revise **FEM2DUNSI.FOR** (an incomplete program) by completing subroutines **ELMATRCS2D** and **POSTPROC2D** for the linear and nonlinear partial differential equation in a single variable of Chapter 6, and solve the following two heat transfer problems correspond to heat conduction in a rectangular, isotropic medium (u denotes temperature, T, above a reference temperature). See Box 6.6.1 on page 277 of the text book.

Problem 1. The domain is of dimensions $a = 0.18$ m and $b = 0.1$ m along the x and y coordinates, respectively; internal heat generation is zero $f = 0$; the conductivity $a_{11} = a_{22} = k$ is of the form

$$k = k_0\left[1+\beta\left(T-T_0\right)\right] \tag{3}$$

where k_0 is the *constant thermal conductivity* and β is the *temperature coefficient of thermal conductivity*. Take $k_0 = 0.2$ W/(m. K), $T_0 = 0$, and $\beta = 2\times 10^{-3}$ (K^{-1}). The boundary conditions are

$$T(0,y) = 500\text{ K},\quad T(a,y) = 300\text{ K},\quad \frac{\partial T}{\partial y} = 0 \text{ at } y = 0, b \tag{4}$$

Problem 2. All data is the same as in **Problem 1**, except for the following changes

$$a = 0.2 \text{ m}, \quad b = 0.1 \text{ m}, \quad \beta = 0.2 \ (\text{K}^{-1}) \tag{5}$$

$$T(0, y) = 500 \text{ K}\ , \quad T(a, y) = 300 \text{ K}\ , \quad \frac{\partial T}{\partial y} = 0 \text{ at } y = 0, \quad T(x, b) = 500 - 1000x \ \text{ K} \tag{6}$$

Use 4×4 and 8×8 linear element meshes, and 2×2 and 4×4 nine-node quadratic element meshes to analyze the two problems. Use a convergence tolerance of $\varepsilon = 10^{-3}$ and a reasonable value of $ITMAX$. Submit: (1) any notes pertinent to the development of your code, (2) input for the problems solved, and (3) plots/tables of the numerical results obtained for the two problems.

REMINDER: Test No. 1 is scheduled for **Feb. 16, 2023**. Part 1 is to be done during the class time (75 minutes) and Part 2 solution must be returned by **6pm on Feb. 17, 2023**. Part 1 covers theoretical developments from Chapters 4, 5, and 6 while Part 2 covers problems that can be solved by Programs 1 and 2 (i.e., 1-D problems) and its modifications.
