Time Complexity Analysis of LUP Decomposition Solve Procedure

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Problem Statement

Solve the following recurrence relation arising from the LUP decomposition solve procedure:

$$T(n) = \sum_{i=1}^{n} \left[O(1) + \sum_{j=1}^{i-1} O(1) \right] + \sum_{i=1}^{n} \left[O(1) + \sum_{j=i+1}^{n} O(1) \right]$$

Step 1: Simplifying Inner Summations

First Summation

Consider:

$$\sum_{j=1}^{i-1} O(1)$$

This executes $i - 1$ times, hence:

$$\sum_{j=1}^{i-1} O(1) = O(i)$$

Therefore:

$$O(1) + \sum_{j=1}^{i-1} O(1) = O(i)$$

Second Summation

Consider:

$$\sum_{j=i+1}^{n} O(1)$$

This executes $n - i$ times, hence:

$$\sum_{j=i+1}^{n} O(1) = O(n - i)$$

Therefore:

$$O(1) + \sum_{j=i+1}^{n} O(1) = O(n-i)$$

Step 2: Substituting Back into the Recurrence

Substitute the simplified terms:

$$T(n) = \sum_{i=1}^{n} O(i) + \sum_{i=1}^{n} O(n-i)$$

Step 3: Evaluating the Summations

First Term

$$\sum_{i=1}^{n} O(i) = O\left(\sum_{i=1}^{n} i\right) = O(n^2)$$

Second Term

$$\sum_{i=1}^{n} O(n-i) = O\left(\sum_{k=0}^{n-1} k\right) = O(n^2)$$

Step 4: Final Result

Adding both terms:

$$T(n) = O(n^2) + O(n^2) = O(n^2)$$

Conclusion

$$\boxed{T(n) = O(n^2)}$$

Thus, the time complexity of the solve procedure in LUP decomposition is quadratic in the size of the matrix.