

The Complexity Class Co-NP

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Definition of Co-NP

The complexity class **Co-NP** consists of all decision problems whose **complements** belong to the class **NP**.

Formally,

$$\text{Co-NP} = \{L \mid \bar{L} \in \text{NP}\}$$

where \bar{L} denotes the complement of language L .

Equivalently, a decision problem is in Co-NP if for every **NO-instance**, there exists a certificate that can be verified in polynomial time.

Intuitive Explanation

- In NP, a “YES” answer can be verified efficiently.
- In Co-NP, a “NO” answer can be verified efficiently.

Thus, Co-NP problems are those for which proving that a solution does *not* exist is easy to verify.

Type of Problems in Co-NP

Problems in Co-NP typically involve:

- Verifying that **no solution exists**
- Confirming that a structure **does not satisfy** a given property
- Certifying the **invalidity** or **unsatisfiability** of a condition

Examples of Co-NP Problems

1. UNSAT

The complement of the Boolean satisfiability problem (SAT).

- Question: Is a given Boolean formula **unsatisfiable**?
- A certificate is an argument showing that no assignment satisfies the formula.

2. TAUTOLOGY

- Question: Is a Boolean formula true for **all** possible truth assignments?
- This is the complement of the SAT problem.

3. Non-Hamiltonian Graph

- Question: Does a given graph **not** contain a Hamiltonian cycle?
- A certificate can demonstrate the impossibility of such a cycle.

Relationship Between NP and Co-NP

It is known that:

$$P \subseteq NP \cap Co-NP$$

However, it is still an open problem in computer science whether:

$$NP = Co-NP$$

Most researchers believe that:

$$NP \neq Co-NP$$

Conclusion

Co-NP is the class of decision problems whose NO-instances can be verified in polynomial time.

These problems focus on efficiently certifying the *absence* of solutions rather than their existence.