Phys 740 Assignment 2: Celestial Mechanics

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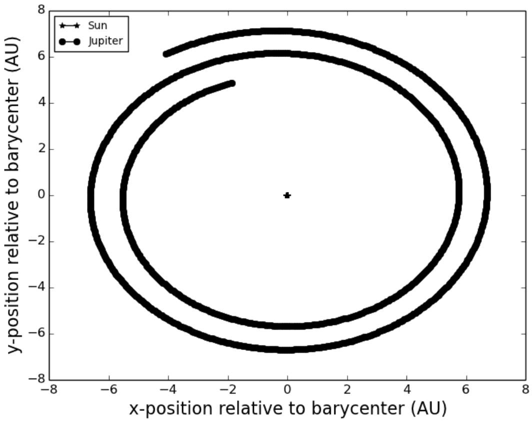
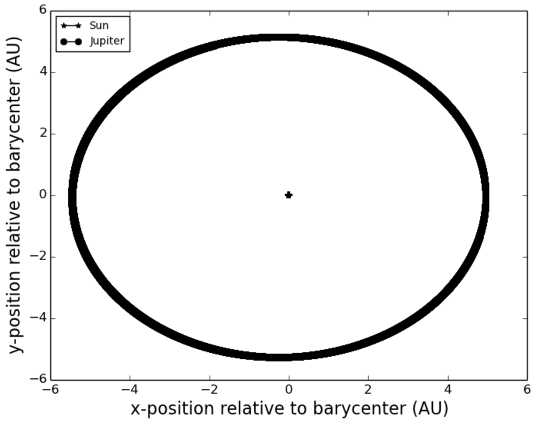
Abstract

In this assignment I wrote an N-Body code that calculates the motions of bodies influenced by the gravitational force. The code is written in python and uses a second order leapfrog method to solve for the equations of motion along with an $O(\frac{n^2}{2})$ algorithm to calculate the gravitational acceleration between the bodies. Using this N-Body code I first plotted the trajectory, energy, and angular momentum for the Sun-Jupiter system and that of the full solar system to ensure that the code was functioning correctly. I then analyzed some more interesting systems including a binary star system, a ternary star system, and our standard solar system with an additional planet between Mars and Jupiter. I discovered that each of these systems can be stable given an appropriate set of initial conditions.

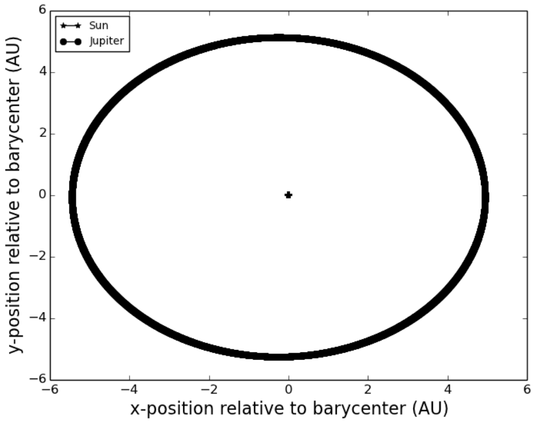
Sun-Jupiter System



(a) Timestep of 5.8 days

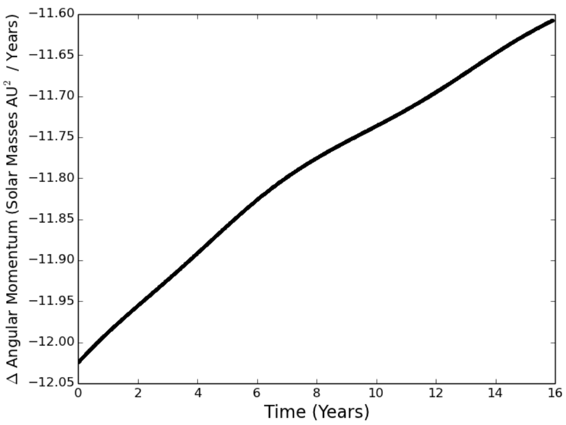


(b) Timestep of 1.39 hours

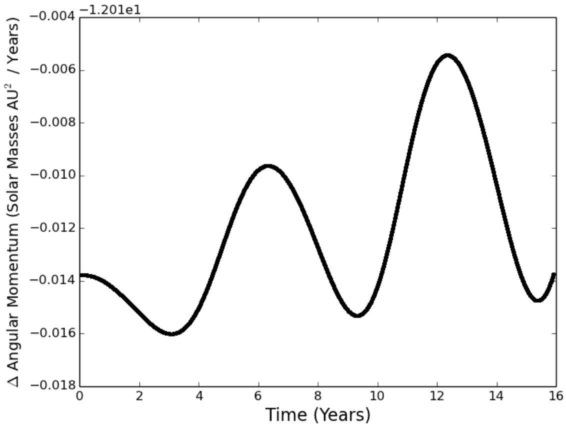


(c) Timestep of 8.37 minutes

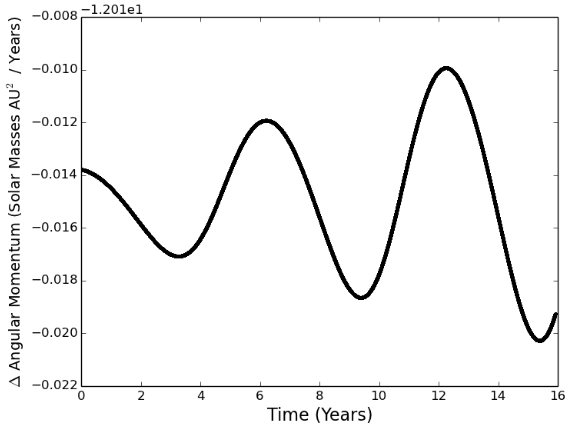
Figure 1: **Trajectories of Sun-Jupiter System with Varying Time Steps** - In these plots we see the motion of Jupiter around the sun. Even though the true motion of the bodies in this system is three dimensional, the plane in which Jupiter orbits the sun does not change over time and we are therefore able to view this motion in a two dimensional plot. All three plots were generated using data from a run of 16 Earth-years of time. In this time Jupiter orbits the Sun about three times. The data for each plot was generated using successively lower time step in the leap-frog integration. Notice that with a time step of 5.8 days (plot (a)) Jupiter's orbit is not stable, however at 1.39 hours it becomes stable. Plot (c) illustrates the fact that a significantly lower time step does not cause any instability.



(a) Timestep of 5.8 days

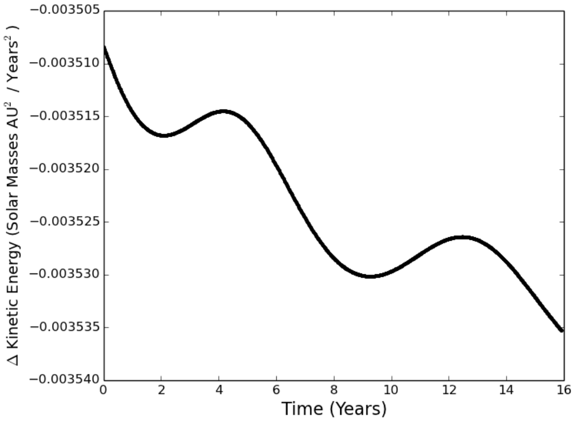


(b) Timestep of 1.39 hours

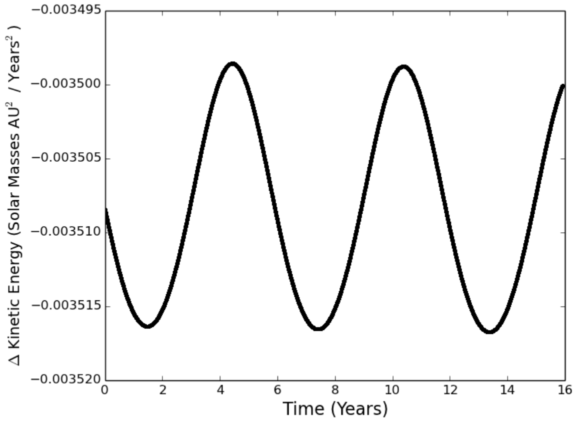


(c) Timestep of 8.37 minutes

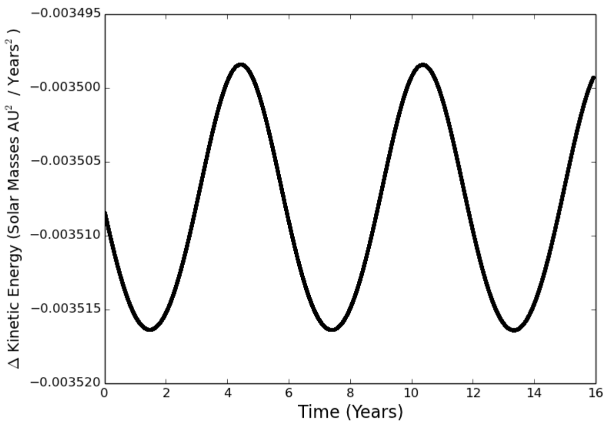
Figure 2: **Change in Angular Momentum of Sun-Jupiter System with Varying Time Steps** - Data about the change in relative angular momentum of the runs presented in Figure 1 is presented here. The relative change in angular momentum was calculated as a change from the initial angular momentum. In other words, $\Delta L = L(t) - L(0)$. Note that L here implies the total angular momentum of the system. Notice that the y-axes of plots (b) and (c) have a multiplicative factor of 12.01. As expected the change in angular momentum is sinusoidal. The slow increase in amplitude of the sinusoid is likely due to the fact that the integration method is only second order. Plot (a) shows that a time-step of 5.8 days is far too large to properly allow the integrator to conserve angular momentum. Recall from figure 1 that a time step of 8.37 minutes did not show much change in the trajectory when compared to a time step of 1.39. However, notice that there is a clear difference in plots (b) and (c) here. Plot (c) shows much better conservation of angular momentum, implying that there is indeed value in using a smaller time step.



(a) Timestep of 5.8 days



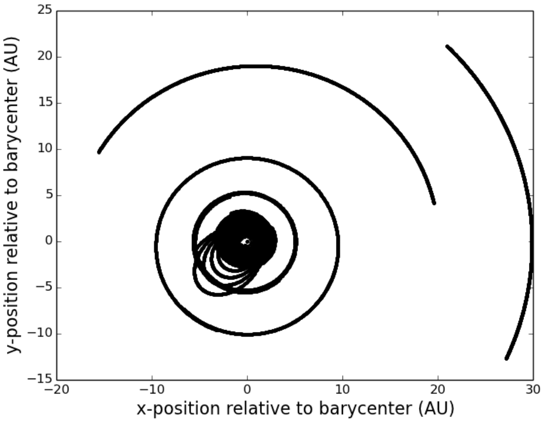
(b) Timestep of 1.39 hours



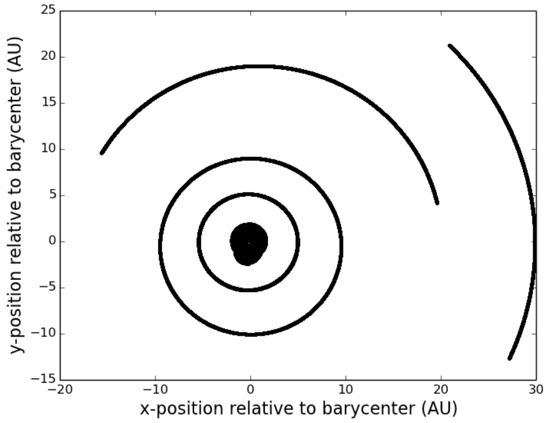
(c) Timestep of 8.37 minutes

Figure 3: **Change in Kinetic Energy of Sun-Jupiter System with Varying Time Steps** - Data about the relative change in kinetic energy of the runs presented in Figure 1 is presented here. The relative change in energy was calculated as follows, $\Delta T = T(t) - T(0)$. Note that T here refers to the total kinetic energy of the system, not just that of a single body. As expected the kinetic energy of the system fluctuates sinusoidally. As with previous figure of the Sun-Jupiter system, plot (a) shows that a time step of 5.8 days is too large to allow the integration method to properly conserve kinetic energy. Also notice that unlike figure 2, going from a time step of 1.39 hours to a time step of 8.37 minutes does not allow the integration method to capture the conservation of energy any better.

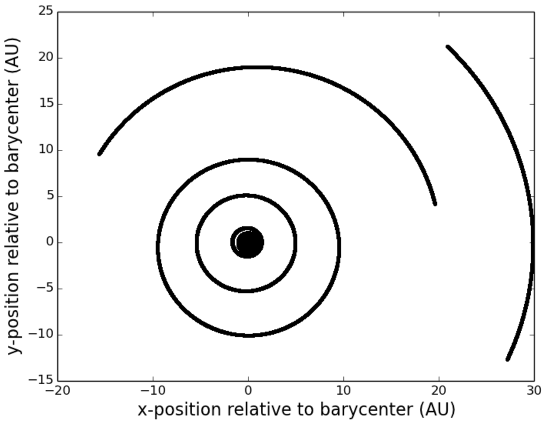
Solar System



(a) Timestep of 13.9 hours

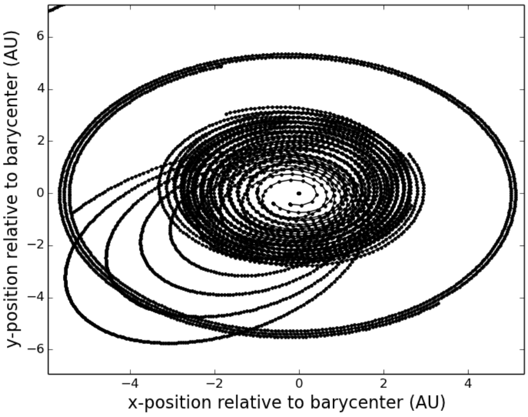


(b) Timestep of 1.39 hours

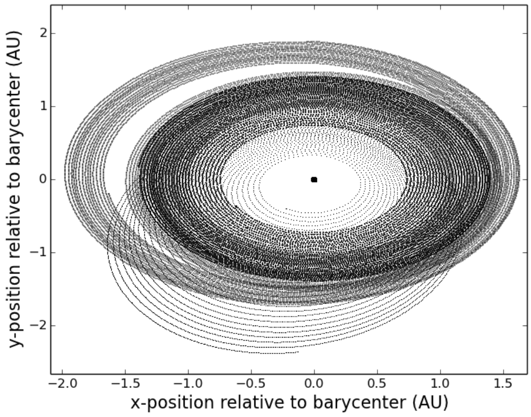


(c) Timestep of 8.37 minutes

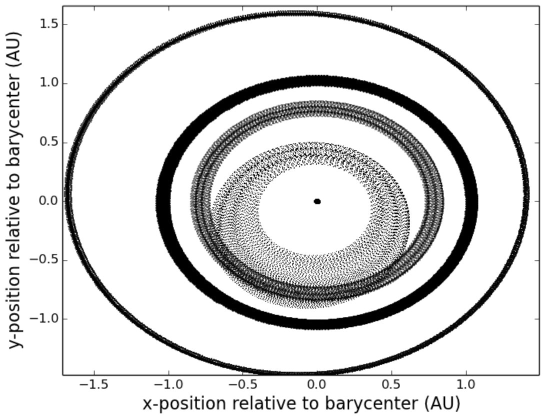
Figure 4: **Trajectories of Solar System with Varying Time Steps** - The order of the depicted bodies from inner most to outer most here is Sun, Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune. This calculation was run for 16 earth years simulation time. Notice how the inner planets have very unstable orbits at larger time steps, this is worsened by the fact that the leap frog solver is only a second order numerical method. A higher order method would take longer to run, but would perform more accurately for a given time step. See figure 5 for a closer look at the inner planets in this run.



(a) Timestep of 13.9 hours

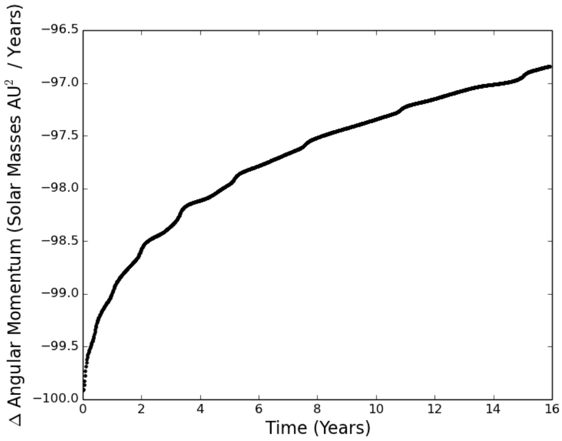


(b) Timestep of 1.39 hours

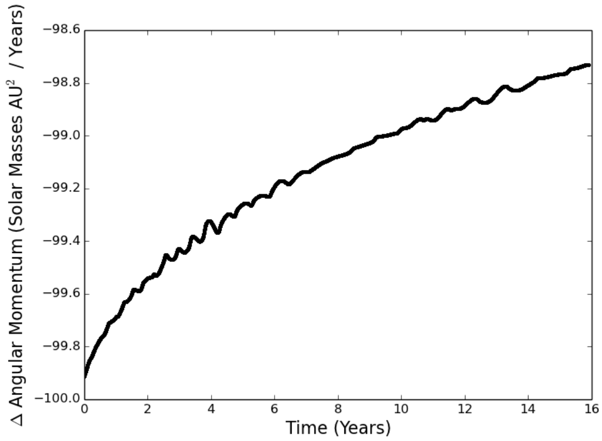


(c) Timestep of 8.37 minutes

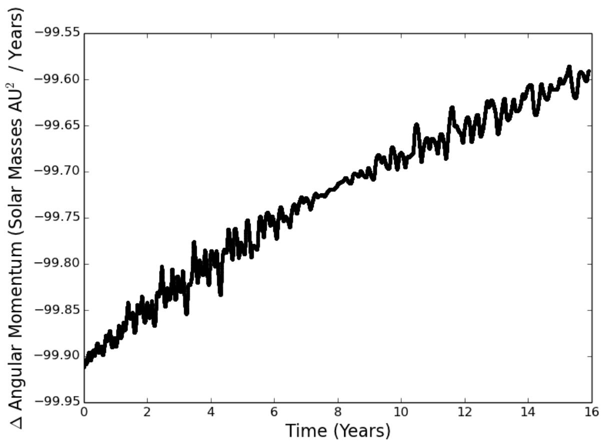
Figure 5: **Trajectories of Inner Planets of Solar System with Varying Time Steps** - The order of the planets is of course the same as that in figure 4, however the outermost body in plot (a) is Jupiter and in plots (b) and (c) is Mars. From this zoomed in view it is very clear that a small time step is very important for proper motion of the inner planets. In plot (a) it isn't even possible to discern which planet is which, also a problem in plot (b). In plot (c) we can discern the planets much better, but notice Mercury is still not orbiting correctly. Unfortunately, running the code a time step small enough to properly evaluate Mercury's motion would require several more hours (perhaps days) of computer run time. This leads us to generally exclude mercury's motion for longer calculations. Its likely that using a higher order method would fix this problem.



(a) Timestep of 13.9 hours

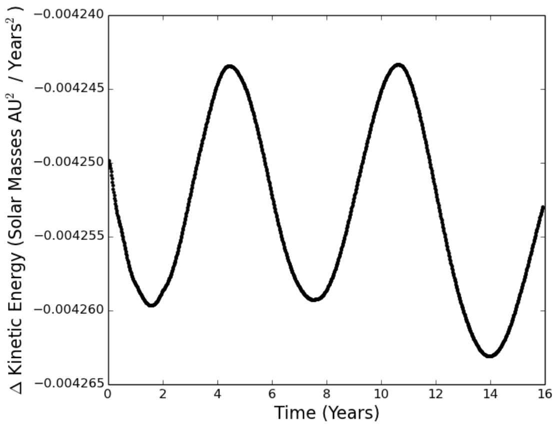


(b) Timestep of 1.39 hours

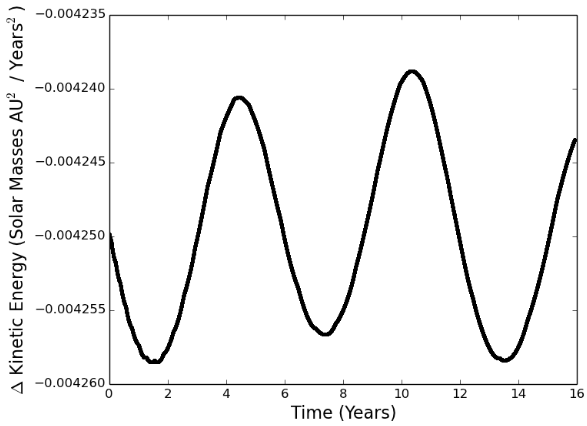


(c) Timestep of 8.37 minutes

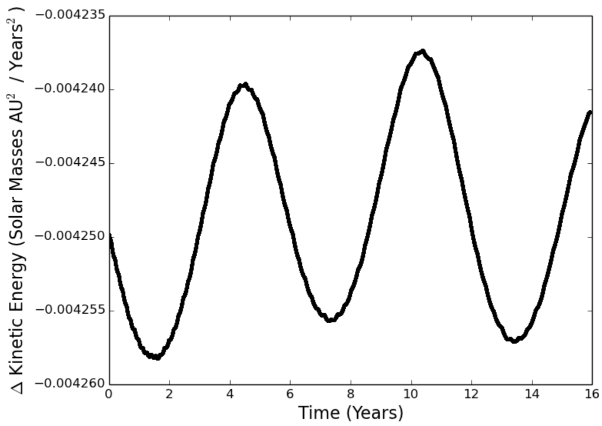
Figure 6: **Change in Angular Momentum of Solar System with Varying Time Steps** - Angular momentum data of the Solar System calculation is presented here. Angular momentum here is calculated in the same way that it was for figure 2, $\Delta L = L(t) - L(0)$. Where L is the total angular momentum of the system. As expected, as smaller time step lets the code come closer to properly conserving angular momentum. However, it is very clear from these plots that the inclusion of Mercury in the calculation makes it extremely difficult for angular momentum to be conserved as it should. As mentioned before, a very small time step would need to be used, one that would take far too long to run the simulation.



(a) Timestep of 13.9 hours



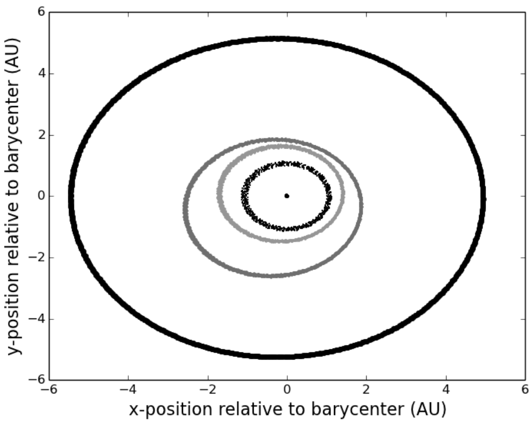
(b) Timestep of 1.39 hours



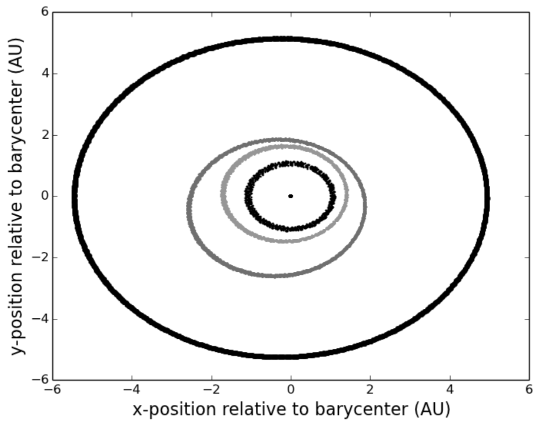
(c) Timestep of 8.37 minutes

Figure 7: **Change in Kinetic Energy of Solar System with Varying Time Steps** - Kinetic energy here is defined and calculated in the same way as in Figure 3, $\Delta T = T(t) - T(0)$ with T the total kinetic energy of the system, not just a single body. One would expect these plots, especially plot (a), to be erratic similar to figure 6a. However, it is important to keep in mind that even with a large time step, the motion of the outer planets is mostly correct. Since kinetic energy is $\frac{1}{2}mv^2$, most of the mass comes from the large bodies such as the Sun, Jupiter, and Saturn all of which have stable orbits even with a large time step. This is why even plot (a) looks nearly the same as plot (c).

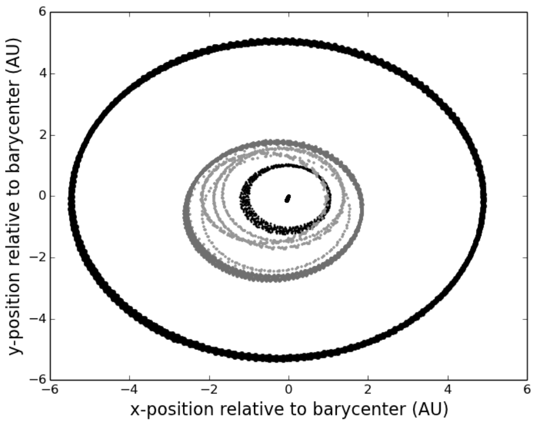
Modified Solar System



(a) Planet M of Mars mass

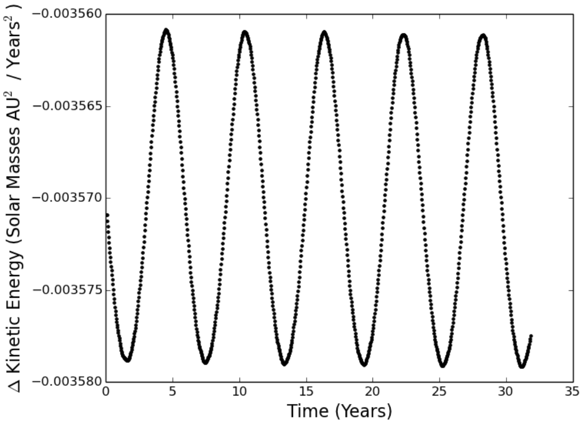


(b) Planet M of Uranus mass

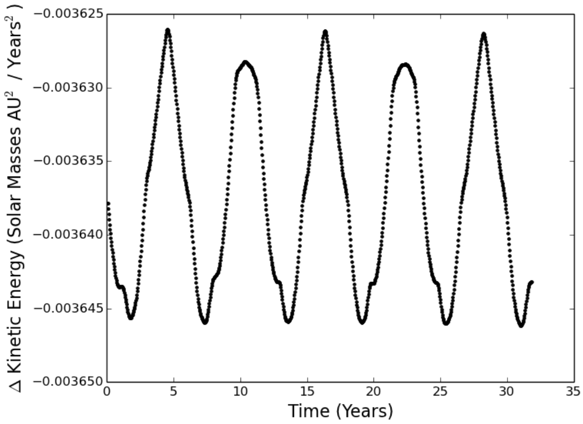


(c) Planet M of Jupiter Mass

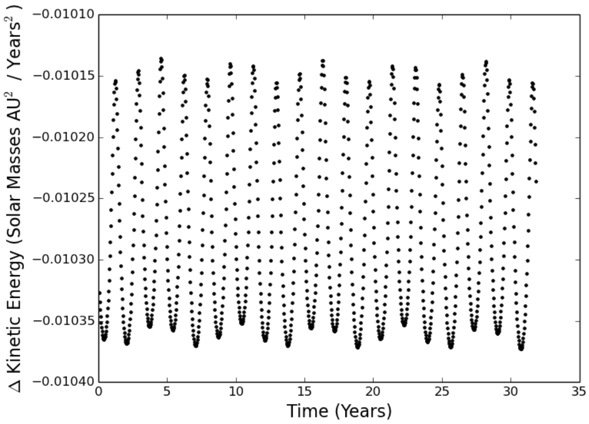
Figure 8: **Trajectories of Modified Solar System with varying Planet M masses -** The order of the depicted bodies from inner most to outer most here is Sun, Earth, Mars, Planet M, Jupiter. This calculation was run for 32 earth years simulation time. To speed up simulation run time Mercury, Venus, Saturn, Uranus, and Neptune were excluded as they would not have had much of an influence on what is being studied here. A body labeled Planet M was added in between Jupiter and Mars. Notice that it is primarily Mars' orbit that is perturbed by the presence of Planet M. As Planet M's mass is increased, the perturbation on Mars' orbit increases greatly.



(a) Planet M of Mars mass



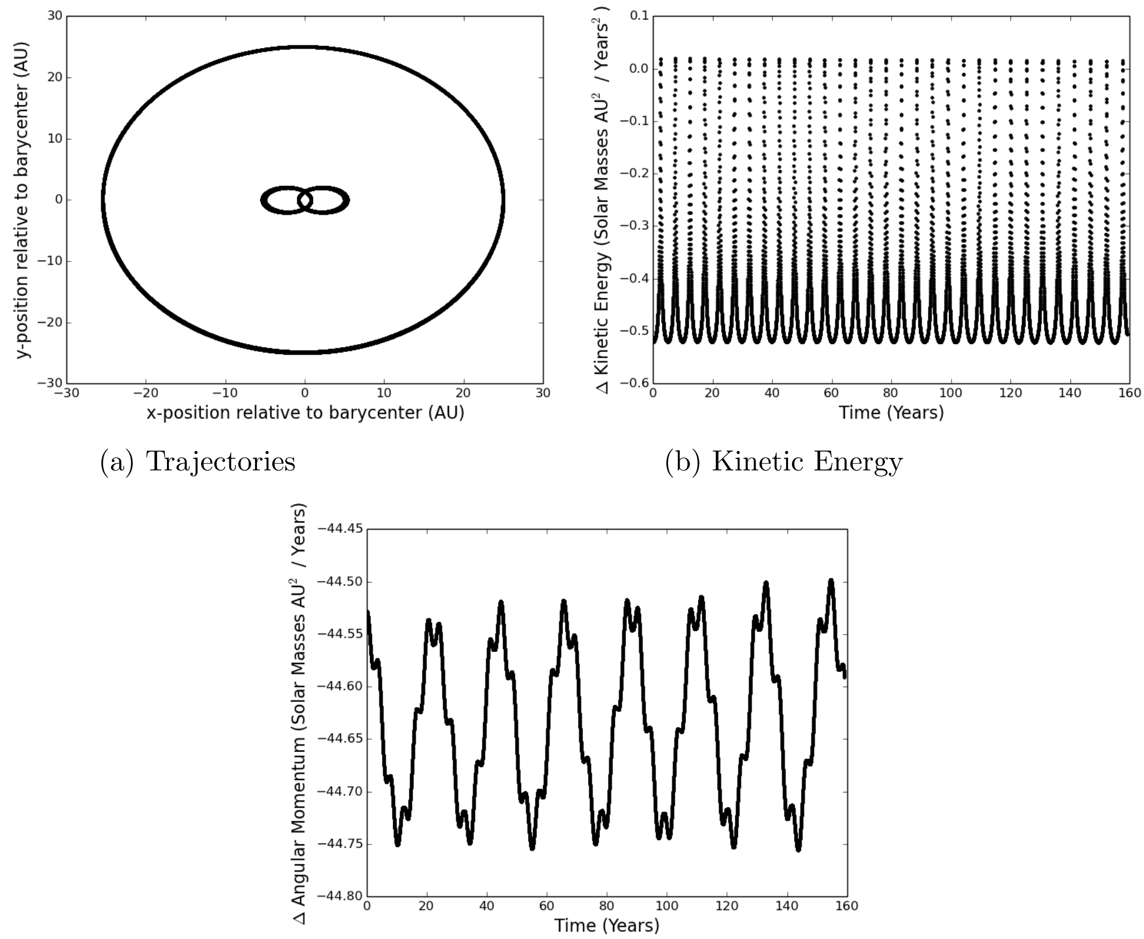
(b) Planet M of Uranus mass



(c) Planet M of Jupiter Mass

Figure 9: **Kinetic Energy of Modified Solar System with varying Planet M masses** - These plots present kinetic energy data of the simulations shown in figure 9. These kinetic energy plots were calculated in the same as the plots in figure 7. Notice that even though figure 8b does not sure much perturbation in the trajectory of Mercury's orbit, figure 9b shows that there is indeed a large variation in the kinetic energy. It is likely that after a longer period of time Mercury would like escape even for a Planet M of Uranus mass.

Binary Star System



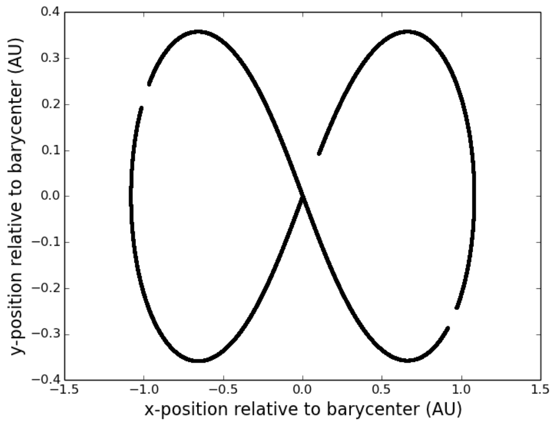
(a) Trajectories

(b) Kinetic Energy

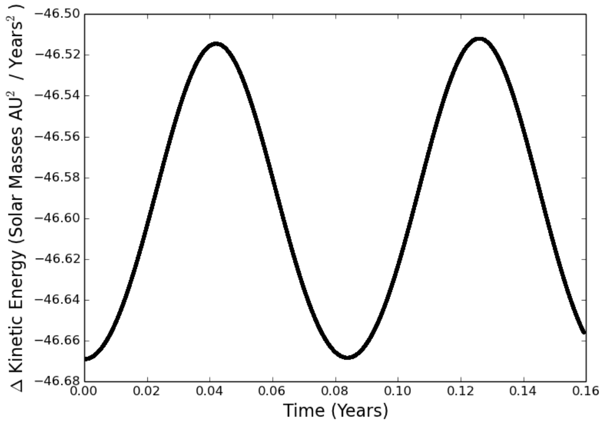
(c) Angular Momentum

Figure 10: **Analysis of a Binary Star System with 1 Planet** - In this simulation a binary star system with 1 planet in orbit around it was simulated for 160 earth years. The binary star system is of 2 solar mass stars orbiting about 10 AU apart their greatest separation. For stability, the orbiting body had to be placed about 25 AU away from the reduced mass of the binary star system. Attempts were made to place a second body in orbit, but a stable orbit could not be achieved for an appreciable mass.

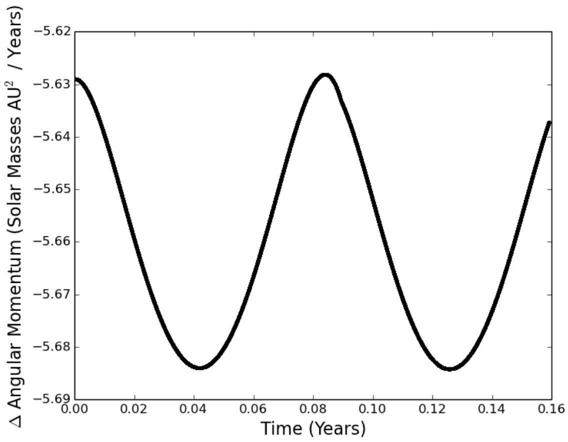
Ternary Star System



(a) Trajectories

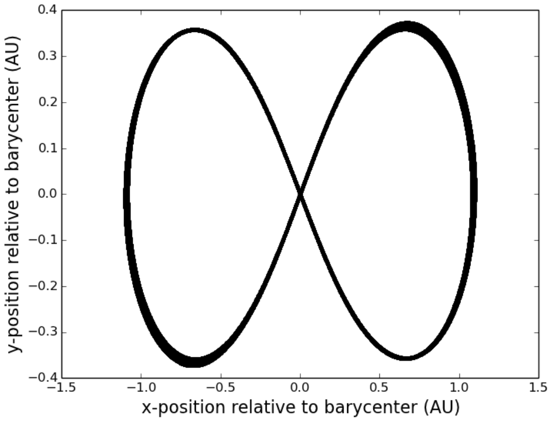


(b) Kinetic Energy

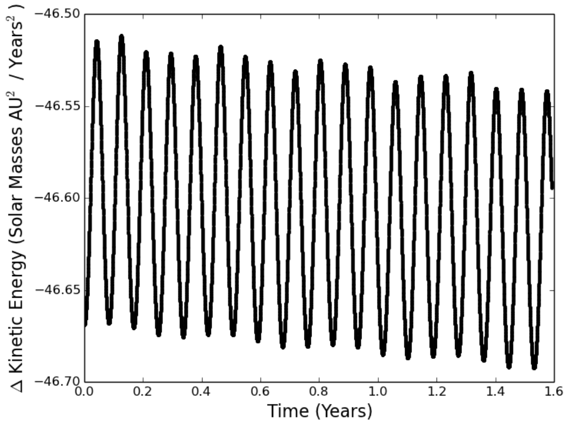


(c) Angular Momentum

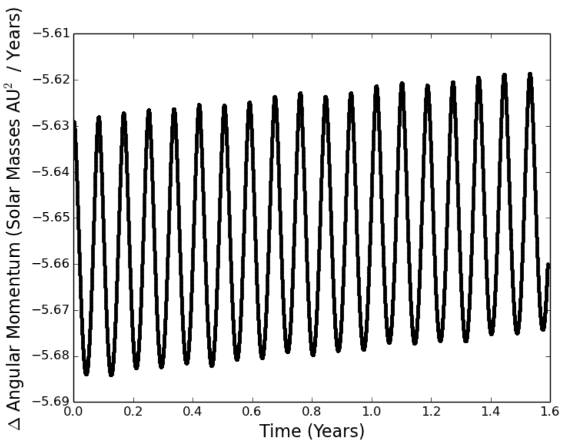
Figure 11: **Analysis of a Ternary Star System** - Simulation data of a stable ternary star system is presented here. Each star is 1 solar mass. The run shown here is only about 2 earth years simulation time to properly depict the 3 individual stars. See Figure 12 for a longer run. The stars orbit each other in a clear figure-8 pattern and do so in a stable manner.



(a) Trajectories



(b) Kinetic Energy



(c) Angular Momentum

Figure 12: **Analysis of a Ternary Star System** - An analysis of the same system as in figure 10, but run for 16 earth years simulation time. This longer run shows that over time the net kinetic energy is decreasing while the net angular momentum is increasing. This is most likely due to a slow increase in the separation between the stars. This system will likely not be stable for very long periods of time.