Assignment 04 Manav_r

Q.2. Find the z-transform of $3^{|k+1|}$ and find its region of convergence on z-plane

→ $f(k) = 3^{|k+1|}$

$$|k+1| = \begin{cases} -(k+1) & ; k < -1 \\ (k+1) & ; k \geq -1 \end{cases}$$

$$\therefore f(k) = \begin{cases} 3^{-(k+1)} & ; k < -1 \\ 3^{(k+1)} & ; k \geq -1 \end{cases}$$

$$Z[f(k)] = \sum_{k=-\infty}^{\infty} f(k) z^{-k}$$

$$= \sum_{-\infty}^{-2} 3^{-(k+1)} z^{-k} + \sum_{-1}^{\infty} 3^{(k+1)} z^{-k}$$

$$= \sum_{2}^{\infty} 3^{-(-k+1)} z^{k} + \sum_{0}^{\infty} 3^{k-1+1} z^{-(k-1)}$$

$$\therefore Z[f(k)] = 3z^2 \cdot \frac{1}{1-3z} + z \cdot \frac{1}{1-3/z} \quad \because \left[\sum_{0}^{\infty} z^n = \frac{1}{1-z}\right]$$

$$\therefore Z[f(k)] = \frac{3z^2}{1-3z} + \frac{3z}{z-3} \quad \text{for } |3| < |z| < \frac{1}{|3|}$$

Q.7. Find z-transform of $\sin(3k+5)$, $k \geq 0$ and find its region of convergence on z-plane

→ $f(k) = \sin(3k+5)$, $k \geq 0$

$$= \frac{1}{2i}\left[e^{i(3k+5)} - e^{-i(3k+5)}\right]; k \geq 0$$

$$= \frac{e^{5i}}{2i}\left[e^{i3k} - e^{-i3k}\right]$$

$$\therefore Z[e^{i3k}] = \sum_{k=0}^{\infty} e^{i3k} z^{-k} = \sum_{k=0}^{\infty} \left(\frac{e^{i3}}{z}\right)^k = \frac{z}{z-e^{i3}}$$

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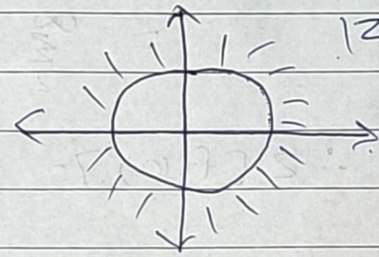
Q.7. for $\left|\frac{e^{i3}}{z}\right|<1 \Rightarrow |e^{i3}|<|z| \Rightarrow |z|>1.$

Similarly.

$Z[e^{-i3k}] = \frac{z}{z-e^{-i3}}$ for $|e^{-i3}|<|z| \Rightarrow |z|>1.$

$\therefore Z[f(k)] = \frac{e^{5i}}{2i}\left[\frac{z}{z-e^{i3}} - \frac{z}{z-e^{-i3}}\right]$

$\therefore Z[f(k)] = \frac{ze^{5i}\sin 3}{z^2-2z\cos 3+1}$ for $|z|>1$



Q.9. Find Z-transform of $\frac{k}{k-1}$, $k\geq 1$ and find its ROC

$\rightarrow$ $f(k) = \frac{k}{k-1}$, $k\geq 1$ $\therefore Z[f(k)] = \sum_{k=1}^{\infty} \frac{k}{k-1} z^{-k}$

$= \frac{1}{z}\sum_{k=0}^{\infty} \frac{k+1}{k} z^{-k}$

$\therefore Z[f(k)] = \frac{z}{z(z-1)} + \frac{1}{z}\sum_{k=0}^{\infty} \frac{z^{-k}}{k}$ for $|z|>1.$

Let $f_1(k) = z^{-k}$

$\therefore Z[f_1(k)] = \sum_{k=0}^{\infty} z^{-k}\cdot z^{-k} = \sum_{k=0}^{\infty} z^{-2k} = \frac{z^2}{z^2-1}$

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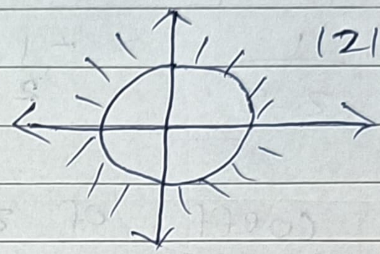
Q.9. $\therefore Z[f(k)] = \dfrac{1}{z-1} + Z\left\{\dfrac{f_1(k)}{k}\right\}$

$= \dfrac{1}{z-1} + (-1)\displaystyle\int \dfrac{1}{z}\, Z[f_1(k)]\,dz$

$= \dfrac{1}{z-1} - \dfrac{1}{2}\log(z^2-1)$ for $|z^2|>1$

$\therefore Z\left[\dfrac{k}{k-1}\right] = \dfrac{1}{z-1} - \dfrac{1}{2}\log(z^2-1)$ for $|z|>1$.



Q.11. Find inverse z-transform of

$\dfrac{z+1}{(z-2)^2}$ i) $|z|>|2|$ ii) $|z|<|2|$

$\rightarrow$ $f(z) = \dfrac{z+1}{(z-2)^2} = \dfrac{1}{z-2} + \dfrac{3}{(z-2)^2}$

1. for $|z|>|2|$

$\dfrac{1}{z-2} = \dfrac{1}{z}\cdot\dfrac{1}{(1-2/z)} = \dfrac{1}{z}\displaystyle\sum_{k=0}^{\infty}\left(\dfrac{2}{z}\right)^k$

$= \displaystyle\sum_{k=0}^{\infty} 2^k z^{-k-1}$ for $|z|>|2|$.

$\dfrac{3}{(z-2)^2} = \dfrac{3}{z^2(1-2/z)^2} = \dfrac{3}{z^2}\displaystyle\sum_{k=0}^{\infty}(k+1)\left(\dfrac{2}{z}\right)^k$

$= \displaystyle\sum_{k=0} 3(k+1)2^k \cdot z^{-k-2} \Rightarrow |z|>|2|$

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Q11. coeff. of z^{-k-1} for $1/(z-2) = 2^{k}$, $k \geq 0$

$\therefore$ coeff. of z^{-k} for $1/(z-2) = 2^{k-1}$; $k \geq 1$.

coeff. of z^{-k-2} for $3/(z-2)^2 = 3(k+1)2^{k}$; $k \geq 0$

$\therefore$ coeff. z^{-k} for $3/(z-2)^2 = 3(k-1)\,2^{k-2}$; $k \geq 2$

$$\therefore z^{-1}[f(z)] = \begin{cases} 2^{k-1} & ; k \geq 1. \\ 3(k-1)2^{k-2} & ; k \geq 2 \end{cases}$$

ii

for $|z| < |2|$

$$\frac{1}{z-2} = \frac{-1}{2} \frac{1}{(1-z/2)} = \frac{-1}{2} \sum_{k=0}^{\infty} \left(\frac{z}{2}\right)^{k}$$

for $|z| < |2|$.

coeff. of $z^{k} = -2^{-k-1}$; $k \geq 0$

$\therefore$ coeff of $z^{-k} = -2^{k-1}$; $k \leq 0$

$$\frac{3}{(z-2)^2} = \frac{3}{z^2} \frac{1}{(1-z/2)^2} = \frac{3}{z^2} \sum_{k=0}^{\infty} (k+1)\left(\frac{z}{2}\right)^{k}$$

for $|z| < |2|$

coeff. of $z^{k} = 3(k+1)\,2^{-k-2}$; $k \geq 0$

$\therefore$ coeff. of $z^{-k} = 3(1-k)\,2^{k-2}$; $k \leq 0$

$\therefore z^{-1}[f(z)] = 3(1-k)\,2^{k-2} - 2^{k-1}$; $k \leq 0$

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Q.16. Find inverse z-transform of

$\frac{2z-3}{z^2-3z-4}$ for 1.) $|z|<1$ 2.) $1<|z|<4$ 3.) $|z|>4$

→ Let $f(z) = \frac{2z-3}{z^2-3z-4} = \frac{A}{z-4} + \frac{B}{z+1}$

$\therefore 2z-3 = A(z+1) + B(z-4)$

$\therefore A=1$ and $B=1$.

$\therefore f(z) = \frac{1}{z-4} + \frac{1}{z+1}$

1. for $|z|<1$

$\frac{1}{z-4} = \frac{-1}{4}\frac{1}{(1-z/4)} = \cancel{-4}\ \frac{-1}{4}\sum_{k=0}^{\infty}\left(\frac{z}{4}\right)^k$

$= \sum_{k=0}^{\infty} \frac{-1}{4^{k+1}} z^k \Rightarrow |z|<4.$

coeff. of $z^k = \frac{-1}{4^{k+1}}$, $k \geq 0$.

$\therefore$ coeff of $z^{-k} = \frac{-1}{4^{-k+1}} = -4^{k-1}$, $k \leq 0$

$\frac{1}{z+1} = \sum_{k=0}^{\infty} (-1)^k z^k$ for $|z|<1$

coeff of $z^k = (-1)^k$, $k \geq 0$

$\therefore$ coeff of $z^{-k} = (-1)^{-k}$; $k \leq 0$

$\therefore z^{-1}[f(z)] = -4^{k-1} + (-1)^{-k}$; $k \leq 0$

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2 For $1<|z|<4$.

$$\frac{1}{z+1} = \frac{1}{z[1+1/z]} = \frac{1}{z}\sum_{k=0}^{\infty} (-1)^k (1/z)^k$$

$$= \sum_{k=0}^{\infty} (-1)^k z^{-k-1} \Rightarrow 1<|z|$$

Coeff of $z^{-k-1} = (-1)^k$; $k \geq 0$.

$\therefore$ coeff of $z^{-k} = (-1)^{k-1}$; $k-1 \geq 0 \Rightarrow k \geq 1$

$$\therefore z^{-1}[f(z)] = \begin{cases} (-1)^{k-1} & ; k \geq 1. \\ -4^{k-1} & ; k \leq 0 \end{cases}$$

3. For $|z|>4$.

$$\frac{1}{z-4} = \frac{1}{z} \cdot \frac{1}{[1-4/z)} = \frac{1}{z}\sum_{k=0}^{\infty} \left(\frac{4}{z}\right)^k$$

$$= \sum_{k=0}^{\infty} 4^k z^{-k-1} \quad \text{for } \left|\frac{4}{z}\right| <1 \Rightarrow 4<|z|.$$

coeff of $z^{-k-1} = 4^k$; $k \geq 0$

$\therefore$ coeff of $z^{-k} = 4^{k-1}$; $k-1 \geq 0 \Rightarrow k \geq 1$.

$\therefore z^{-1}[f(z)] = (-1)^{k-1} + 4^{k-1}$; $k \geq 1$.

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