{Learn, Create, Innovate};

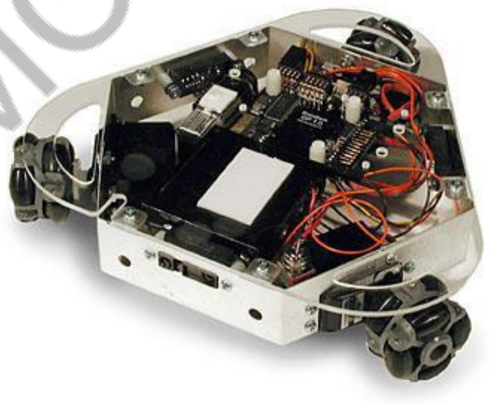
Mobile Robots

Kinematic Models

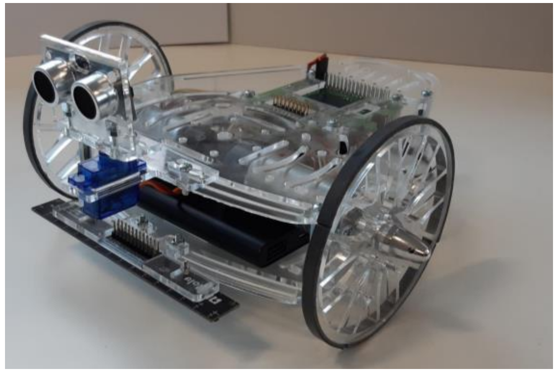
Manchester Robotics

Mobile Robots

- There exists many types of wheeled robotic platforms
  - Differential-Drive robots
  - Omnidirectional robots
  - Ackermann-steering robots
  - and many others…
- In this course we will focus on differential drive robots, also known as “differential wheeled robots”.



Holonomic Robot
Acroname ©.

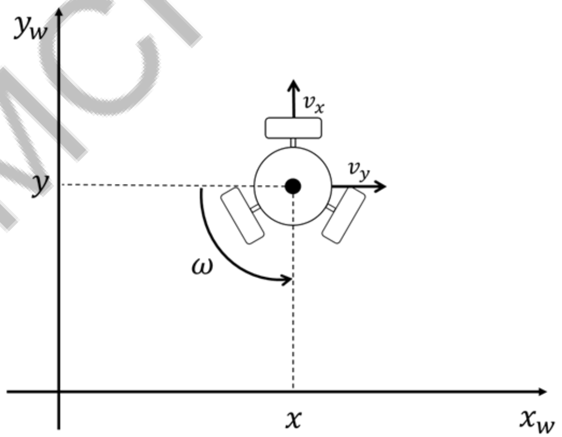


Differential-drive
Puzzlebot ©.

Mobile Robots

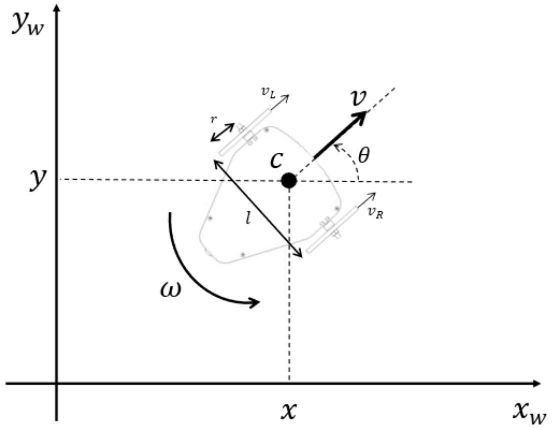
Mobile robots can be classified as "Holonomic" or "Nonholonomic".

- This classification depends on the relationship between controllable and total degrees of freedom of a robot.



Holonomic Robot

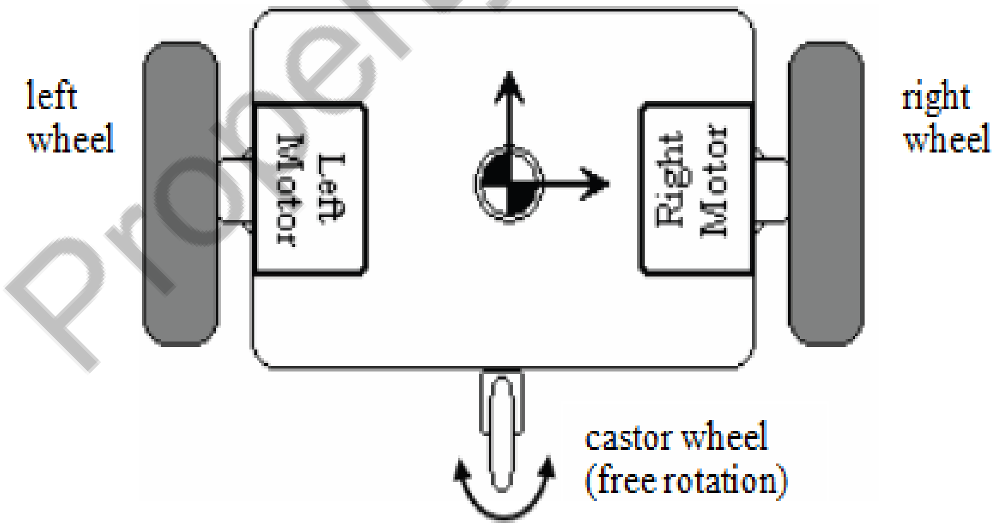
- Holonomic Robots: If the controllable degree of freedom is equal to total degrees of freedom, then the robot
- Nonholonomic Robots: If the controllable degree of freedom is less than the total degrees of freedom. Such systems are therefore called underactuated. Differential Drive Systems fall into this category.



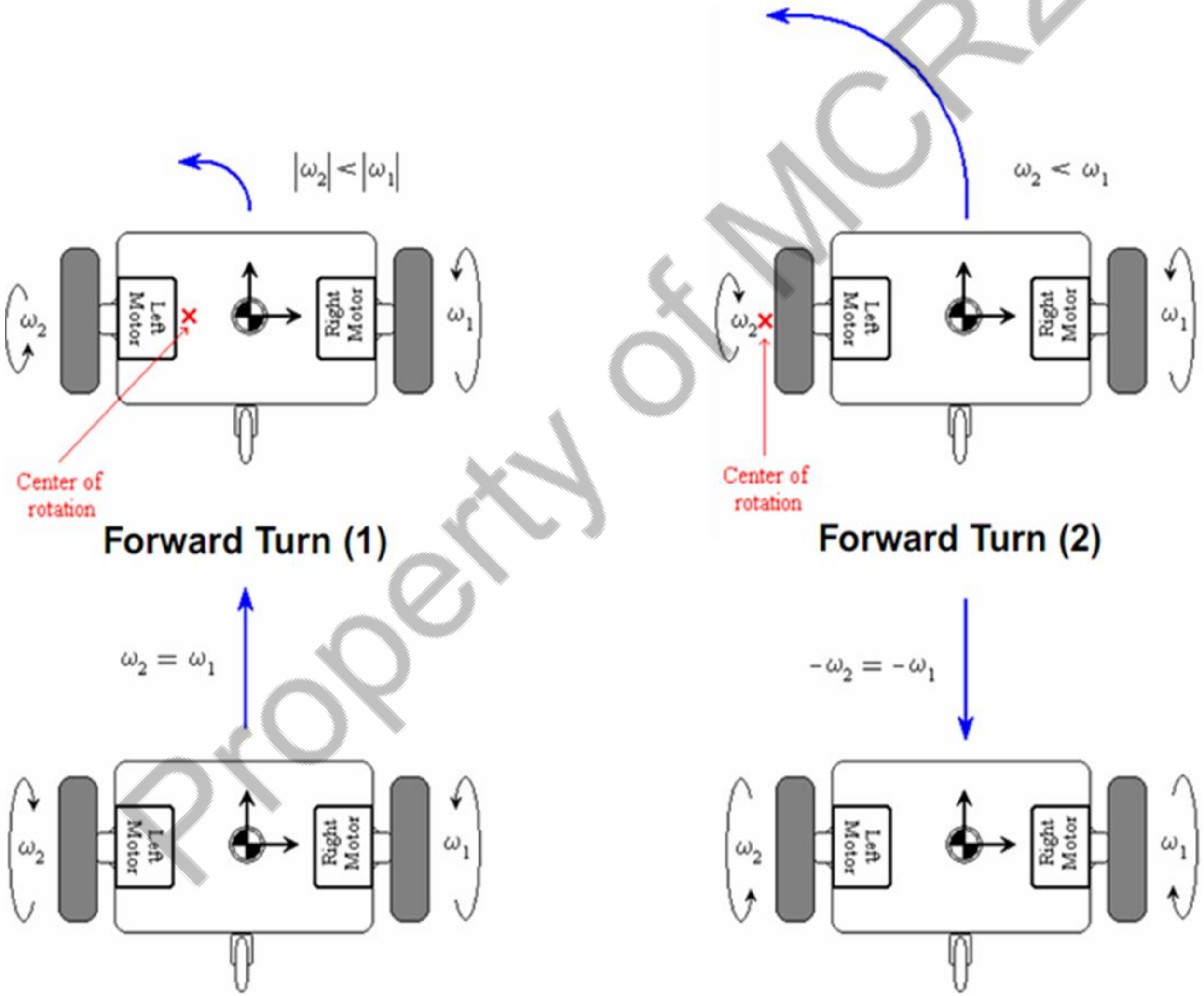
Nonholonomic Robot

Differential drive robots

- Also known as differential wheeled robots, these are mobile robots whose movement is based on two separately driven wheels placed on either side of the robot body. It can thus change its direction by varying the relative rate of rotation of its wheels, thereby requiring no additional steering motion.



Differential drive kinematics



Robot Modelling (Kinematics)

- In engineering, a System is an entity that consists of interconnected components, built with a desired purpose. For this case, the system is said to be the differential drive robot.
- Systems can be modelled depending on their dynamical behavior. This models describe the behavior of a system using mathematical concepts and language.
    - Outputs depend on the present and past values of the inputs.
    - Changes over time.
    - Sometimes called dynamic systems or sequential systems.
    - Mathematically described with differential or difference equations.
- Dynamic Modelling: Considers different time varying phenomena and the interaction between motions, forces and material properties.
- Kinematic modelling: Studies the motion of a robot or mechanism under a set of constraints, regardless of the forces that cause it. Represent the relationship between the robot motor speeds (inputs) and the robot state.

DC Brushed Motor with encoder

$$In \rightarrow \boxed{\frac{K}{s^2 + 2\zeta\omega_n s + \omega_n^2}} \rightarrow Out$$

2nd Order System

The Unicycle

- In order to derive the kinematics of a differential drive robot, it is instructive to first consider a simpler system; the unicycle.

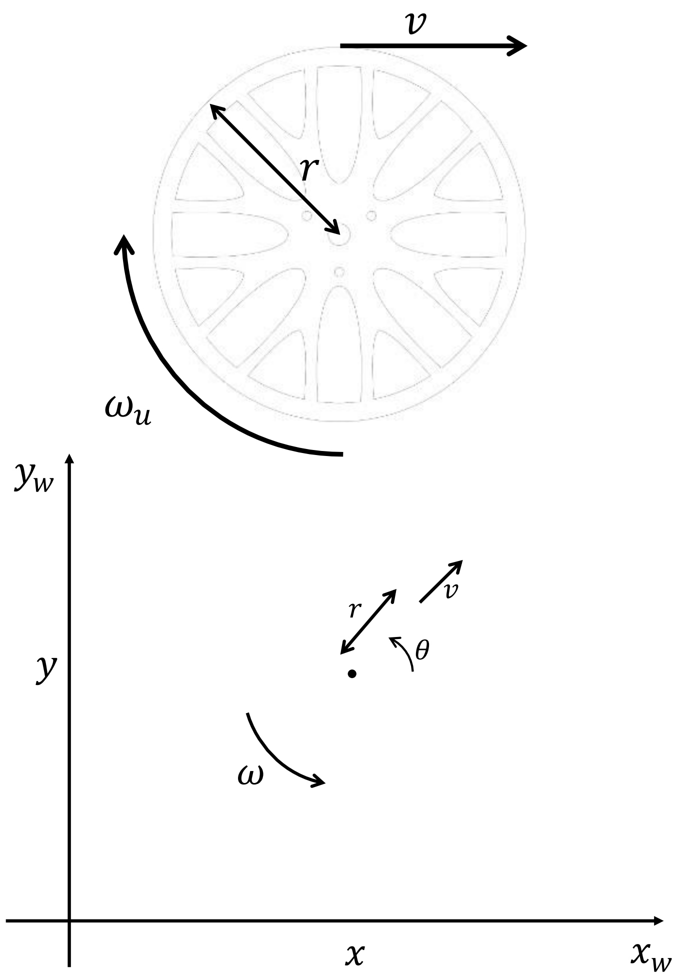
- Ignoring balancing concerns, there are two action variables, i.e., direct inputs to the system in the x_w, y_w plane.

$$\begin{cases} \dot{x} = v \cdot \cos\theta \\ \dot{y} = v \cdot \sin\theta \\ \dot{\theta} = \omega \end{cases}$$

  where $v = \omega_u\, r$ is the linear velocity of the unicycle, ω_u is the wheel angular velocity, r is wheel radius whereas the second one is the steering velocity denoted by ω.

- It can be seen, that velocity component in the direction perpendicular to the direction of movement is always zero this is known as the non-slip condition

$$\dot{x} \cdot sin\theta - \dot{y} \cdot cos\theta = 0$$



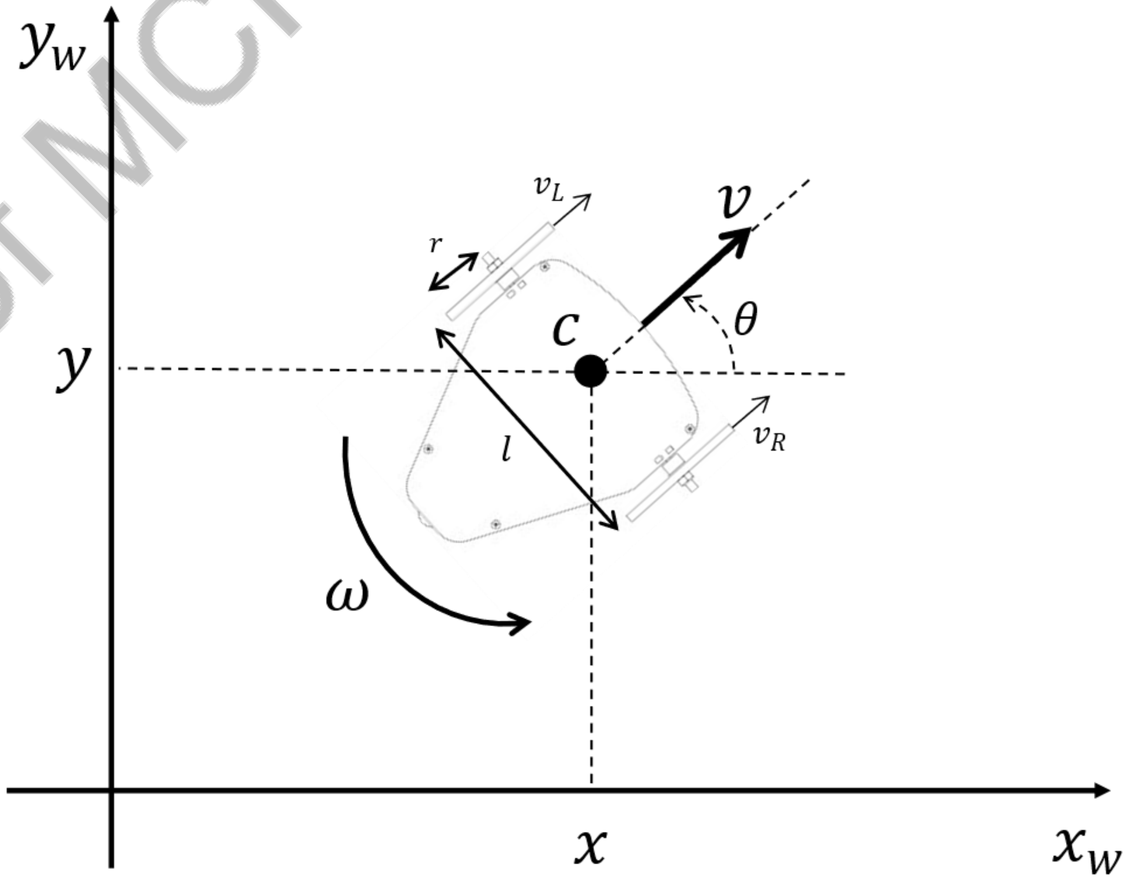
Unicycle side and top views

Differential Drive Kinematics

- For the case of a differential drive system, we can extend the idea of the Unicycle.
- To do this it is necessary to estimate the forward velocity v and the angular velocity ω.
- For this case, the resultant forward velocity v through C (centre of mass) may be reasoned as an average of the two forward wheel velocities given by

$$v = \left(\frac{v_R + v_L}{2}\right) = r\left(\frac{\omega_R + \omega_L}{2}\right)$$

where r is the radius of the wheel and ω_R, ω_L are the angular velocities of the left and right wheels, respectively.



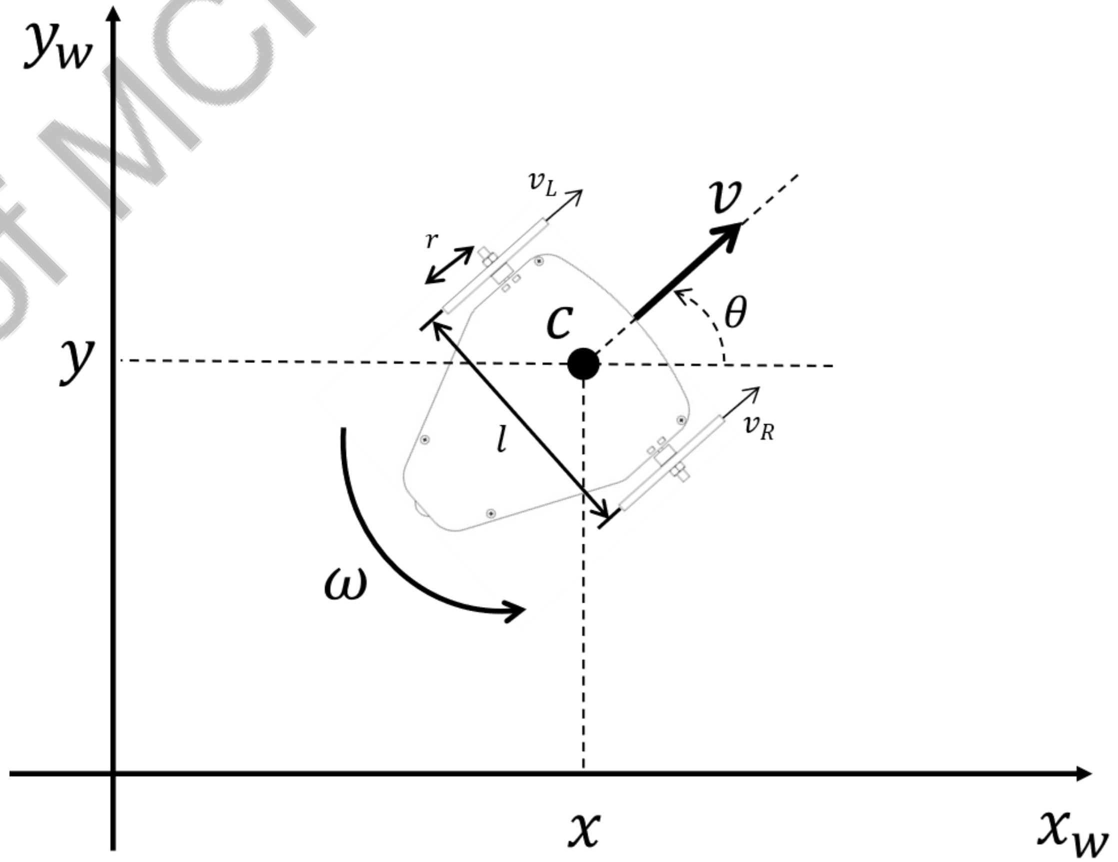
Differential Drive Robot
Puzzlebot ©.

Differential Drive Kinematics

- The resultant angular velocity ω (steering velocity), may also be reasoned as proportional to the difference between wheel velocities but inversely proportional to distance between the wheels, i.e.,

$$\omega = \left(\frac{v_R - v_L}{l}\right) = r \cdot \left(\frac{\omega_R - \omega_L}{l}\right)$$

where r is the radius of the wheel, l is the distance between wheels (wheelbase) and ω_R, ω_L are the angular velocities of the left and right wheels, respectively.



Differential Drive Robot
Puzzlebot ©.

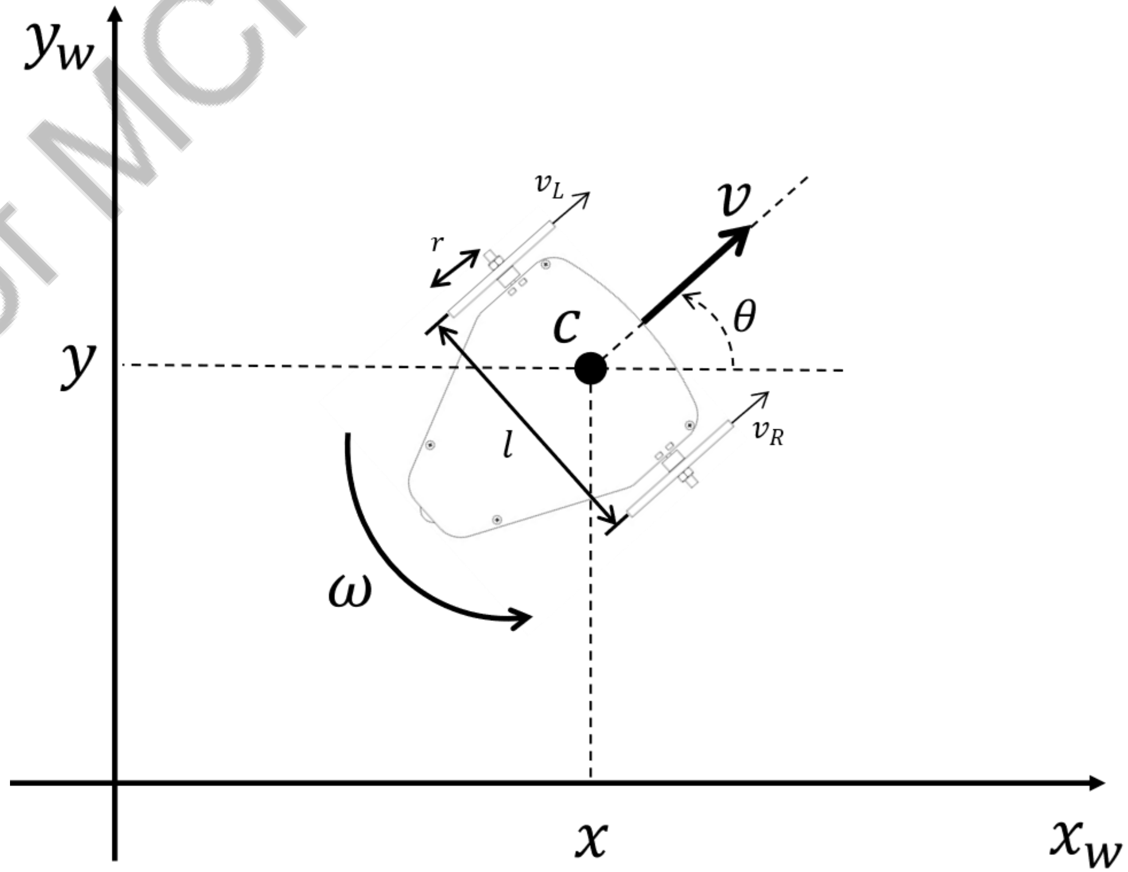
Differential Drive Kinematics

- Thus, just like the unicycle, the configuration transition equations may be given as

$$\begin{cases} \dot{x} = v \cdot cos\theta \\ \dot{y} = v \cdot sin\theta \\ \dot{\theta} = \omega \end{cases}$$

- Replacing v and ω we obtain

$$\begin{cases} \dot{x} = r\left(\frac{\omega_R + \omega_L}{2}\right) \cdot \cos\theta \\ \dot{y} = r\left(\frac{\omega_R + \omega_L}{2}\right) \cdot \sin\theta \\ \dot{\theta} = r\left(\frac{\omega_R - \omega_L}{l}\right) \end{cases}$$



Differential Drive Robot
Puzzlebot ©.

Differential Drive Kinematics

- In general, in the mobile robotic community, the state of a robot is denoted by $\mathbf{s}$, namely the *pose* or *posture*, for this case it consists of the robot position and orientation with respect to a frame of reference (world frame).

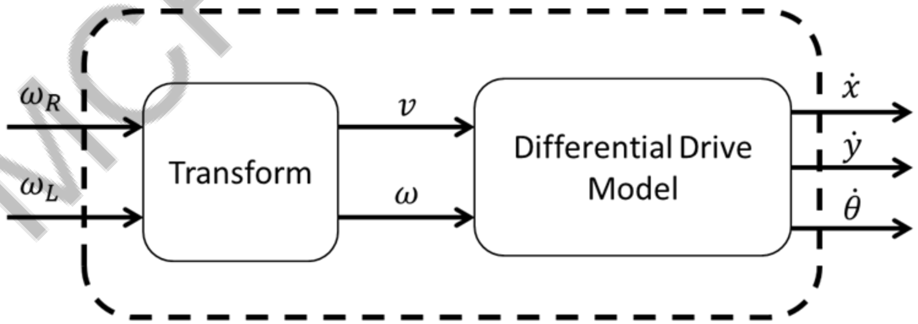
$$\mathbf{s} = \begin{bmatrix} s_x & s_y & s_\theta \end{bmatrix}^T$$

- The kinematic model of a differential drive robot can then be stated in terms of the robot *pose* $(\mathbf{s})$, as follows

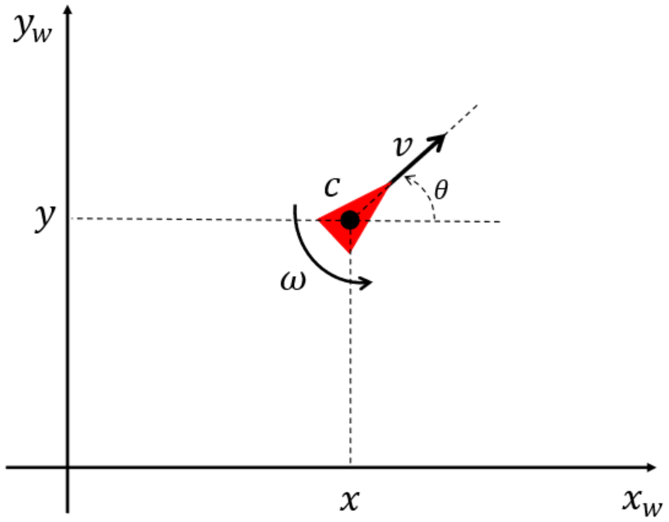
$$\frac{d}{dt}\begin{bmatrix} s_x \\ s_y \\ s_\theta \end{bmatrix} = \begin{bmatrix} \cos(s_\theta) & 0 \\ \sin(s_\theta) & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} v \\ \omega \end{bmatrix}$$

- where, the inputs to the system $\mathbf{u} = \begin{bmatrix} v & \omega \end{bmatrix}^T$, are given by

$$\begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} \frac{r}{2} & \frac{r}{2} \\ \frac{r}{l} & -\frac{r}{l} \end{bmatrix}\begin{bmatrix} \omega_R \\ \omega_L \end{bmatrix}$$



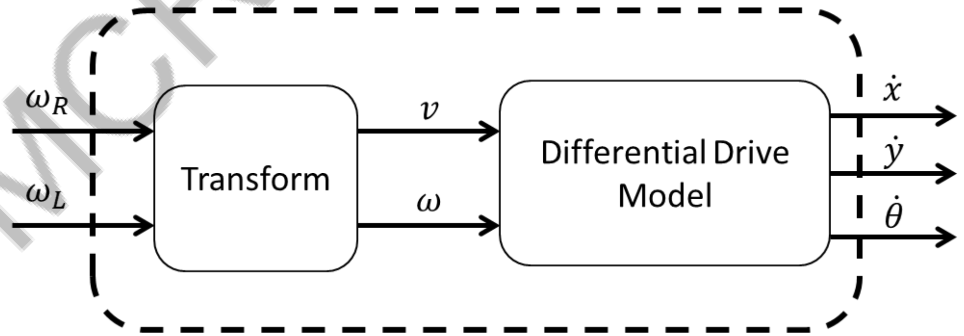
Kinematic Model



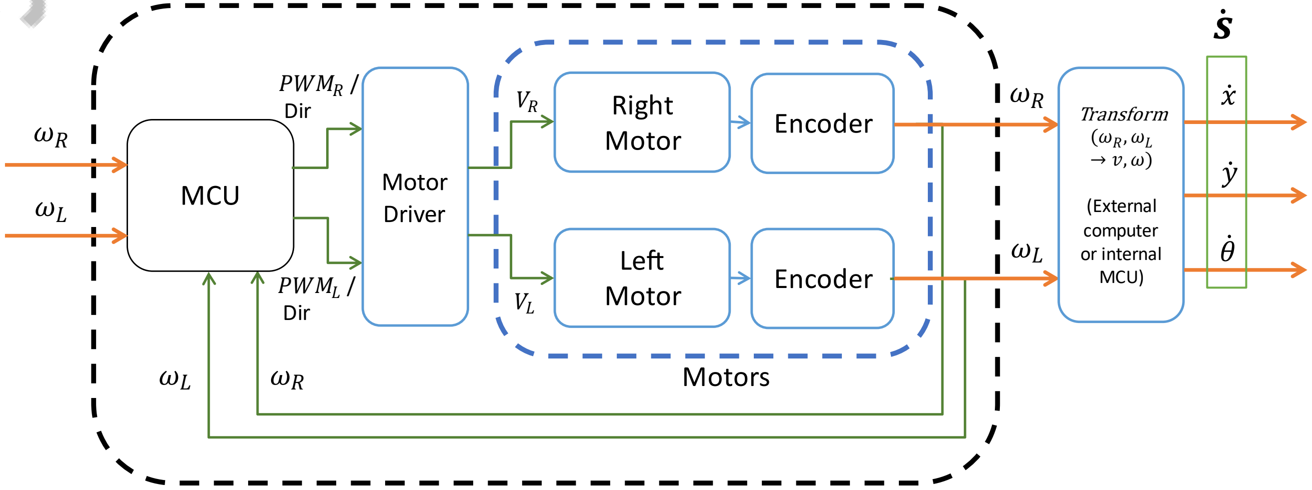
Differential Drive Robot Representation.

The Mobile Robot

- The real robot, in comparison with the kinematic model, requires the usage of sensors and actuators to work.
- For the real robot, a closed control loop for each of the motors is required, in order to reach the required velocities, set by the user.
- In robotics this is called low level control. For the case of a wheeled mobile robot is a common practice to implement a PID control.
- It can be observed that the inputs and outputs are the same, but the inner loop controller and actuators will determine how close the Real Robot will resemble the Kinematic Model.



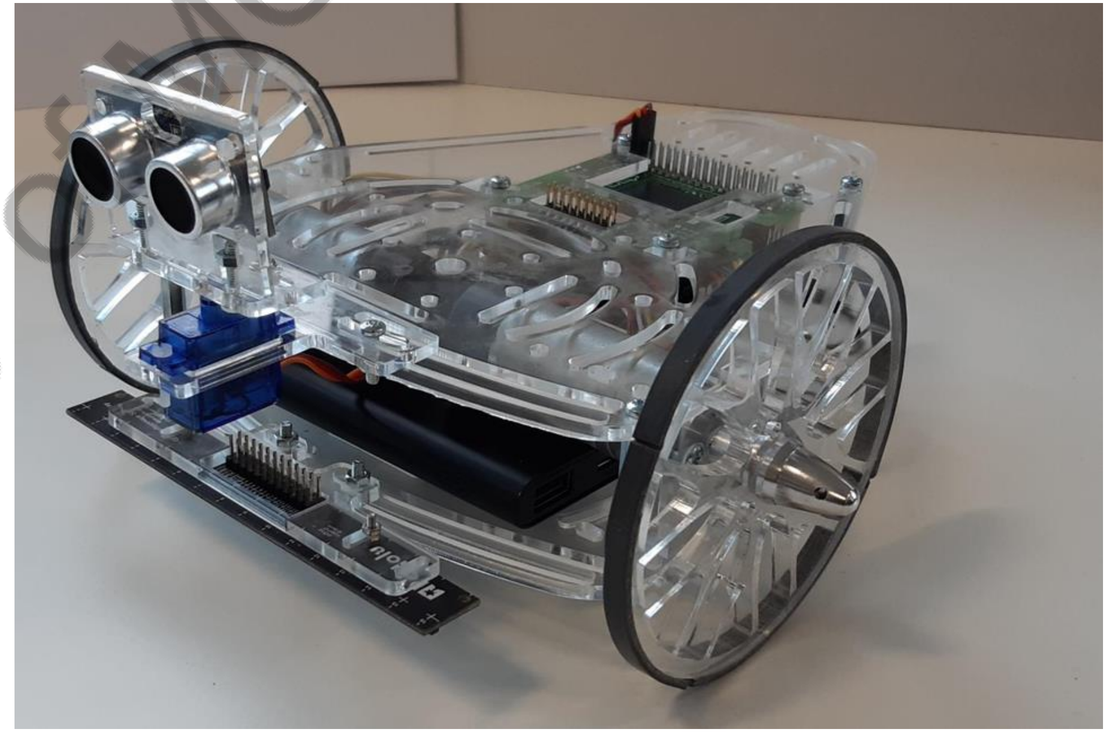
Kinematic Model



Real Robot Diagram*

Sensors and Actuators for Differential Drive Robots

- As stated before, the kinematic model of a differential drive does not consider the physical characteristics and forces of the robot.
- This type of model is used when testing high level robotic algorithms (control) such as path planning, dead reckoning localization, trajectory tracking, AI, etc.
- Differential drive robots in reality, have sensors and actuators (motors, encoders, etc.) that allow us to control them to reach the correct speed required to correctly perform its functions.



Differential Drive Robot Sensors and Actuators.

Thank you

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