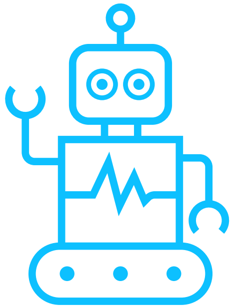
Dynamical Systems

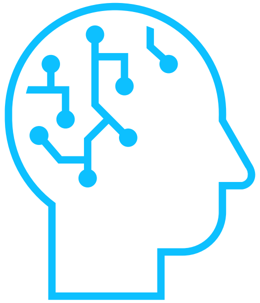
A Review…

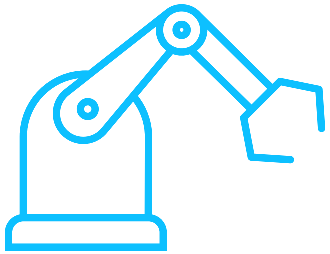
{Learn, Create, Innovate};

Systems: general aspects

- In general, the notion of a **system** is used in many fields of activity. With this notion, we want to delimitate a form of existence in a well-defined space.
- Some examples of dynamic systems: the democratic system, the education system of a country, the nervous system, the automatic temperature regulation system, a mobile robot, a robotic manipulator, etc.

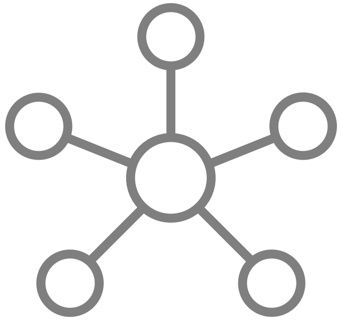


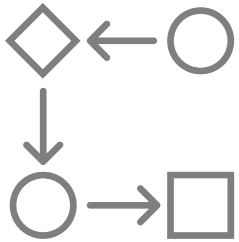




Systems: general aspects

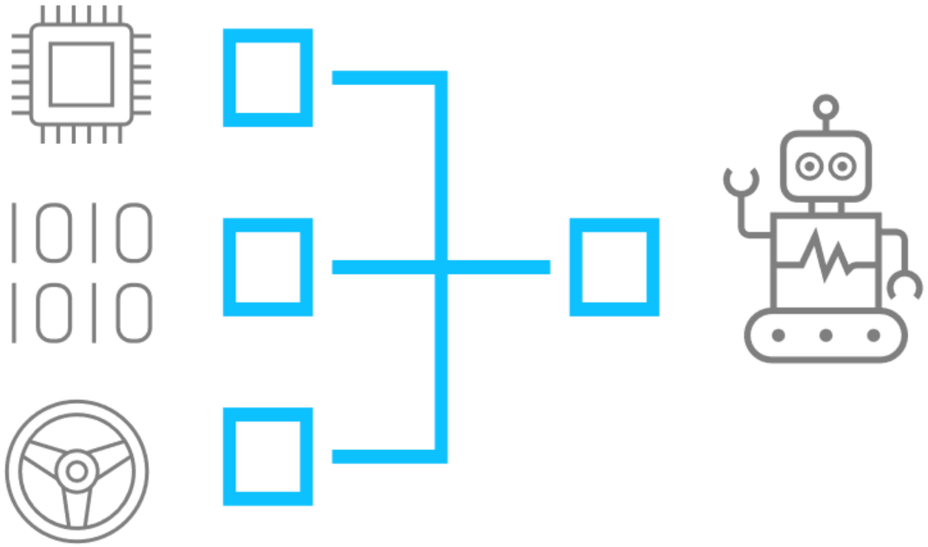
- The notion of system helps us, in a first instance, to delimitate, for example:
  - A state management mechanism
  - A way of education at the national level
  - Part of the components that contribute to the integration of the human body into the environment
  - The elements necessary to obtain a constant temperature in an enclosure
  - The elements of a mobile robot, the elements of a robotic manipulator, etc





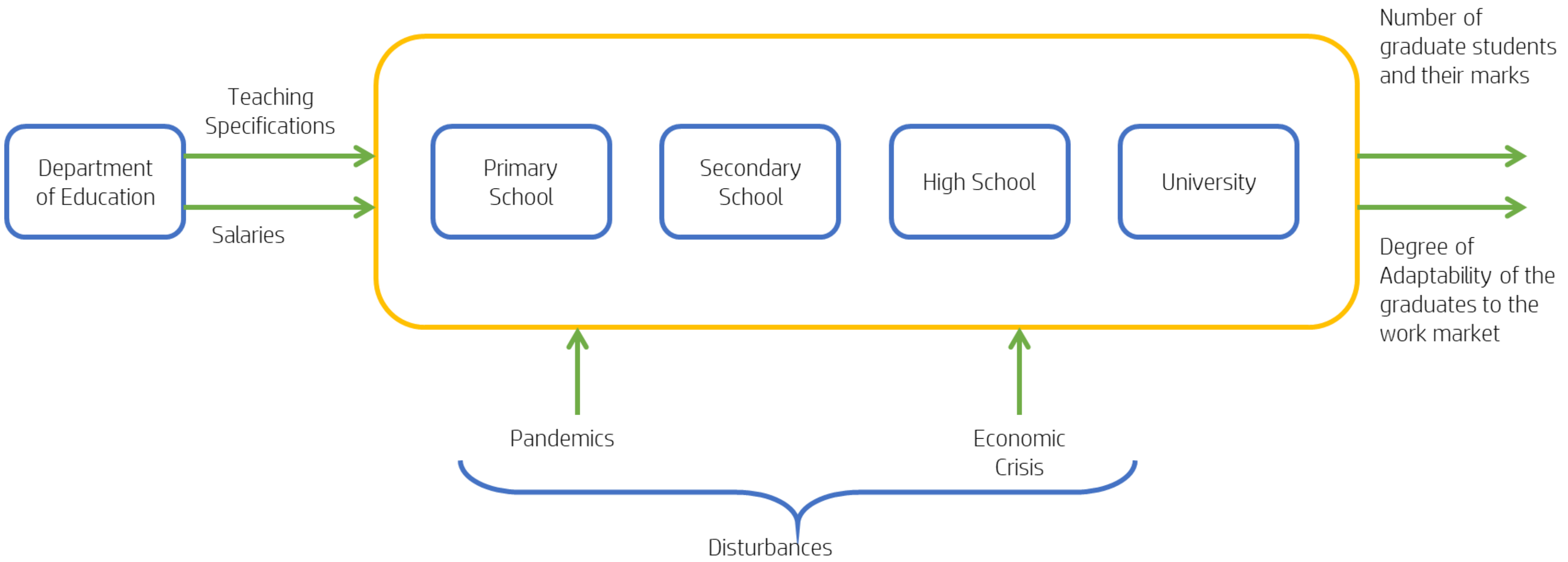
Systems: general aspects

- Generally speaking, a dynamic system is a structure that has multiple connections between its component parts.
- Another characteristic is that its elements are structured according to the same criteria or to achieve the same goal.
- In many situations a system can contain subsystems that can be regarded as independent systems.



Systems general aspects

For a better understanding, consider one of the examples stated above, namely the education system. The block diagram of the mentioned system is represented in the following figure:

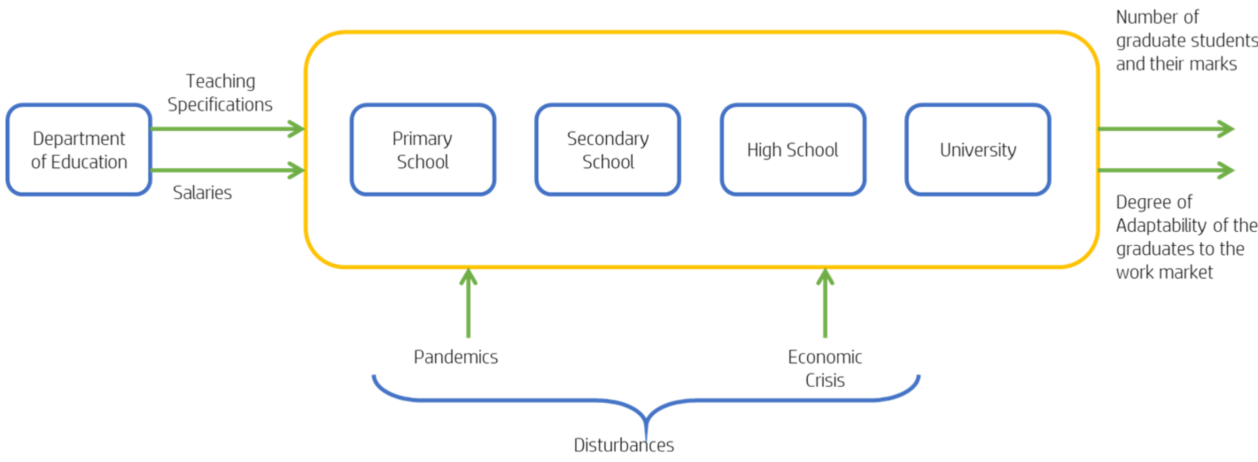


This is just a representation of a fictitious education system any similarity with reality is purely coincidental. In reality, the education system is more complex and presents other inputs, outputs and disturbances.

Systems general aspects

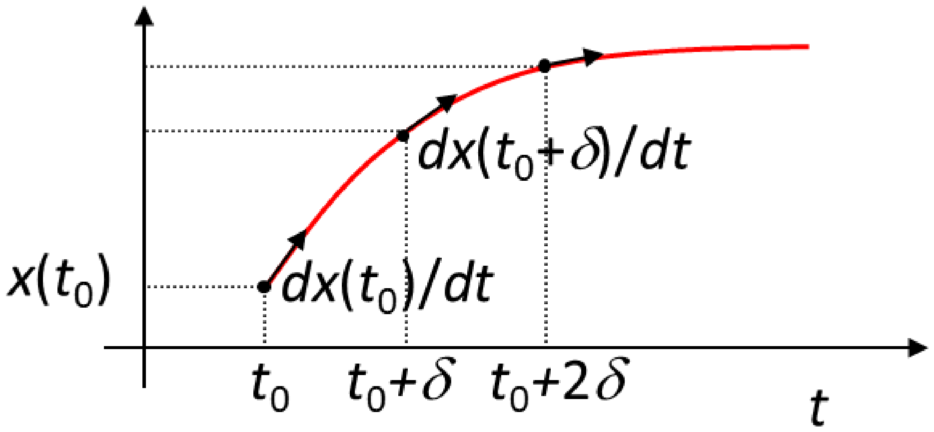
Aspects regarding the dynamic nature of the education system:

- The output performance depends on the structure of the teaching specifications over a period of time that includes the current year and also a number of previous years
- The salary has an important contribution to the quality of the teaching. Good salary attracts good teachers
- The relations between schools and universities have a decisive role in terms of the quality and continuity of the educational process
- The professionalism of the academics
- The facilities and the labs of each education institution contribute to student formation
- Another aspect regarding the output performance is related to the presence of disturbances, which can have negative consequences if a rejection mechanism is not applied.



Dynamical Systems

- **Definition**: A **dynamic system** consists of an abstract **state space**, whose coordinates describe the state at any instant; and a **dynamical rule** that specifies the **immediate future** of all state variables, given only the **present values** of those same state variables.
- Dynamical systems are usually represented by ODE's State space, difference equation, etc.



$$\{f(\boldsymbol{x}), \boldsymbol{x}(t_0), \boldsymbol{u}(t)\} \rightarrow \boldsymbol{x}(t)$$

$f(\boldsymbol{x}(t))$: Dynamical Rule

$\boldsymbol{x}(t)$: States

$\boldsymbol{x}(0)$: Initial values

$\boldsymbol{u}(t)$: Inputs

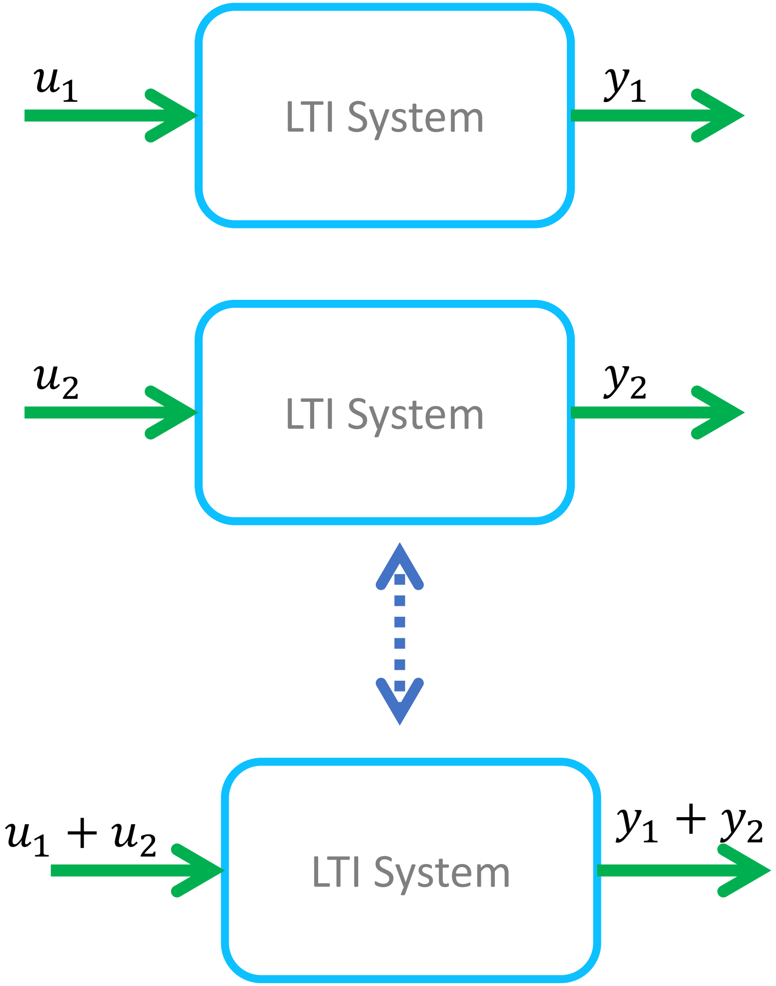
Dynamical Systems

$$\{f(\boldsymbol{x}), \boldsymbol{x}(t_0), \boldsymbol{u}(t)\} \rightarrow \boldsymbol{x}(t)$$

- In a nutshell it tells us what happens at a particular time t, given some rules and initial values.
- There are many ways to solve differential equations, such as complementary functions (CF) and Particular Integrals (PI) and Laplace.
  - Complementary function:
    - Assume the solution is exponential.
    - Assumes the system is homogeneous $(\boldsymbol{u}(t) = 0)$
    - Tries to find an exponential to satisfy the transient part.
  - Particular Integral:
    - Tries to find the steady state behaviour
    - Analysis of the output when $\boldsymbol{t} \rightarrow \infty$
  - Laplace Transform: Simplifies this process by analysing the system in the frequency domain, where some operation become simpler such as convolution that becomes a multiplication.
- The key is not to know how to solve it, is more important to know how to analyse it!

Linear Time Invariant Systems

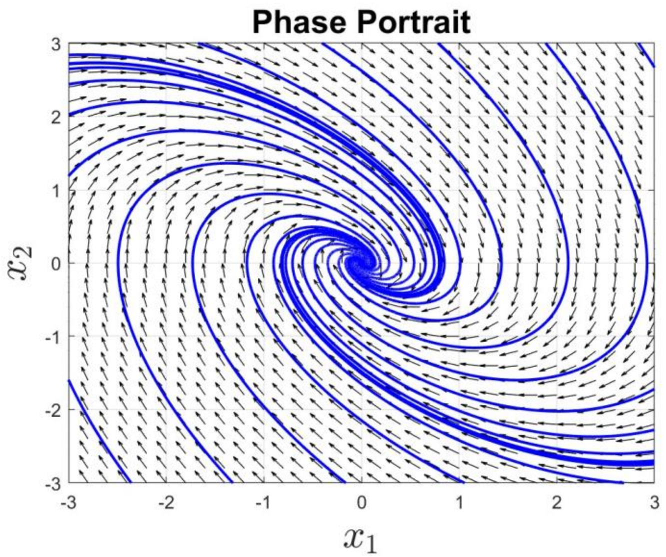
- Linear Time-Invariant Systems (LTI) are a subset of systems that comply with two main characteristics:
- Time-invariance: The system does not change with time. In other words, the output does not depend on when an input is applied.
- Linearity: A System whose output for a combination of inputs is the same as a linear combination of each individual input (superposition principle).

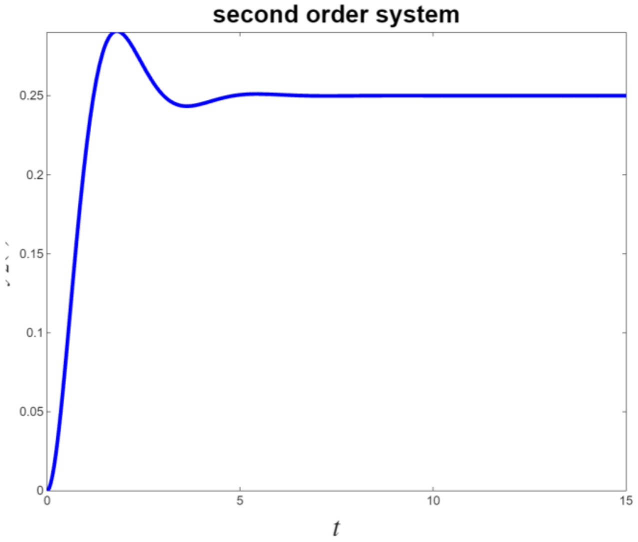


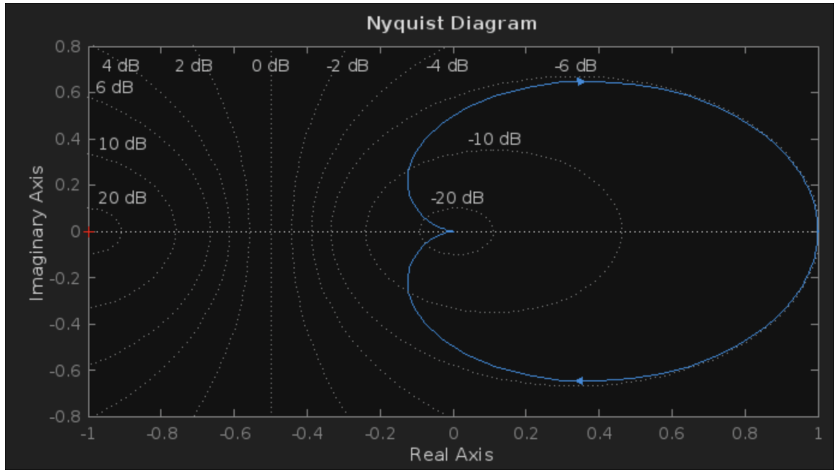
Representation of LTI Dynamical Systems

Representations

- Different representations exist for LTI systems.
- Each representation conveys different information
- Depending on the usage, some provide more important information than others.
- All of them are used when designing advanced controllers.
- In this section, we will introduce four different representations (keep in mind that there exists many more).







Representation of LTI Dynamical Systems

ODE Representation:

$$\ddot{y} + 2\,\dot{y} + 4\,y = u$$

- Useful when analysing the system in the time domain.
- Useful to solve the differential equations i.e., obtain $y(t)$.
- One solution method is the use of *complementary functions* and *particular integrals* to obtain $y(t)$.
- Some ODE representation can be arbitrarily complex (non-linear) and may not be possible to solve.

File: ex_ode

```
%% Init simulation
clc
clear
close all

%% Use symbolic toolbox to define a 1st order ODE

% dy/dt + Ay(t) = u(t)

% Define variables
syms y(t);

%Input Definition
u=dirac(t); %Impulse

%First order system
Dy=diff(y,t); %Define Derivatives
f1 = Dy(t) + (1)*y(t) == u; %Define Functions

%% Solve ODE
y = simplify(dsolve(f1, [y(0)==0.0]))

%% Plot
figure(1)
fplot(y,[0,15])
```

Representation of LTI Dynamical Systems

Impulse Response Representation

$$\dot{y}(t) + ky(t) = \delta(t), y(0) = 0$$

$$y(t) = \mathrm{e}^{-\mathrm{kt}}$$

- A LTI dynamic system is characterized its impulse response signal

$$y(t) = \int_{-\infty}^{\infty} g(t-\tau)u(\tau)\,d\tau$$

- Use convolution to calculate $y(t)$ given $\{u(t), g(t), y(0)\}$
- Impulse response of a first order system is equivalent to set initial conditions $y(0) = y_0$
- Impulse response for second order system is equivalent to set initial conditions for $\dot{y}\,(0) = y_0$

File: ex_impulse

```
%% Init simulation
clc
clear
close all
%% Use symbolic toolbox to define a 1st order ODE
% dy/dt + Ay(t) = u(t)
% Define variables
syms y1(t) y2(t);
%Input Definition
u=0.0;
u2 = dirac(t-0.000001);
%First order system
Dy1=diff(y1,t);
Dy2=diff(y2,t);
f1 = Dy1(t) + (1)*y1(t) == u;
f2 = Dy2(t) + (1)*y2(t) == u2;
% Solve ODE's
y1 = simplify(dsolve(f1, [y1(0)==1]))
y2 = simplify(dsolve(f2,[y2(0)==0]))
% Plot
figure(1)
fplot(y1,[0,15],'LineWidth',3,'color','b')
hold
fplot(y2,[0,15],"--",'LineWidth',3,'color','r')
xlabel ('$t$','interpreter','latex','FontSize',22)
ylabel ('$y_1(t), y_2(t)$','interpreter','latex','FontSize',22)
title ('First order systems','FontSize',20)
```

Representation of LTI Dynamical Systems

Transfer Function Representation

$$G(s) = \frac{1}{s+k}$$

- Depicts the Laplace transform of the impulse response of the system.
- **Laplace transform** of the LTI system $g(t)$

$$G(s) = \int_{-\infty}^{\infty} g(t)e^{-st}\,dt$$

- Convolution becomes a **multiplication** in the s-domain

$$Y(s) \;=\; G(s)U(s).$$

- Gives valuable information of the system's behaviour using the poles and zeroes' analysis.
- Provides Frequency response analysis.

```
File: ex_laplace
%% Init simulation
clc
clear
close all
%% Use symbolic toolbox to define a 1st order ODE in Laplace

% dy/dt + Ay(t) = u(t), u(t)=dirac(t)
% G(s)=1/s+A

% Define variables
syms s t;
%Define TF and invert laplace
Y = ((1)/(s+1));
y = simplify(ilaplace(Y))

% Plot
figure(1)
fplot(y,[0,15],'LineWidth',3,'color','b')
hold
xlabel ('$t$','interpreter','latex','FontSize',22)
ylabel ('$y(t)$','interpreter','latex','FontSize',22)
title ('First order systems','FontSize',20)
```

Representation of LTI Dynamical Systems

State-Space Representation

$$\dot{y}(t) = y(t) + u(t)$$

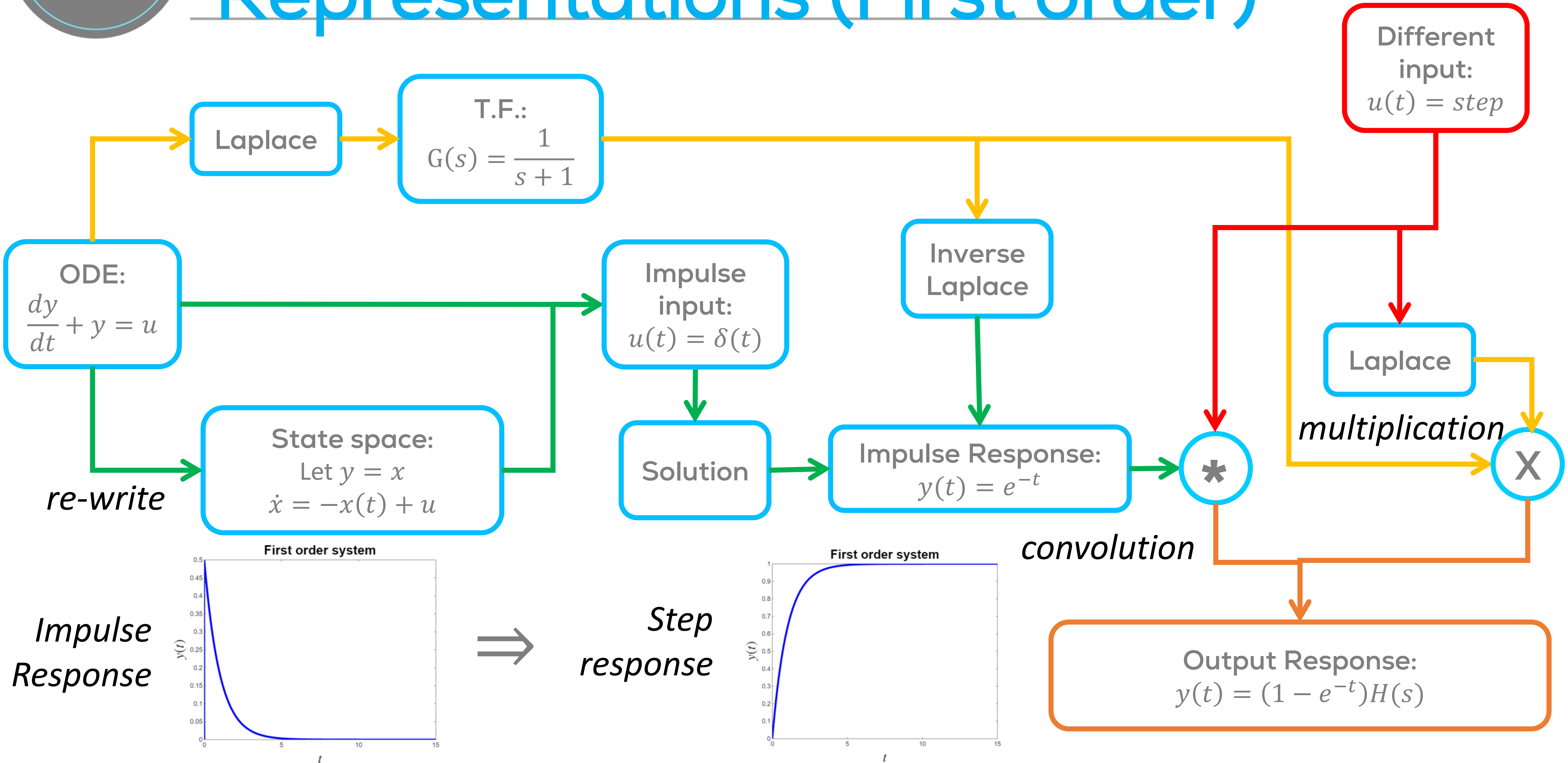
- State space representation allows the user to observe the relationship between the states.
- Like ODEs, state space representation complex (non-linear) but it may not be easy or even possible to calculate $y(t)$
- For LTI systems, the solution is given by

$$\mathbf{x}(t) = e^{\mathbf{A}t}\mathbf{x}(0) + \int_0^t e^{\mathbf{A}(t-\tau)}\mathbf{B}\mathbf{u}(\tau)\,d\tau$$

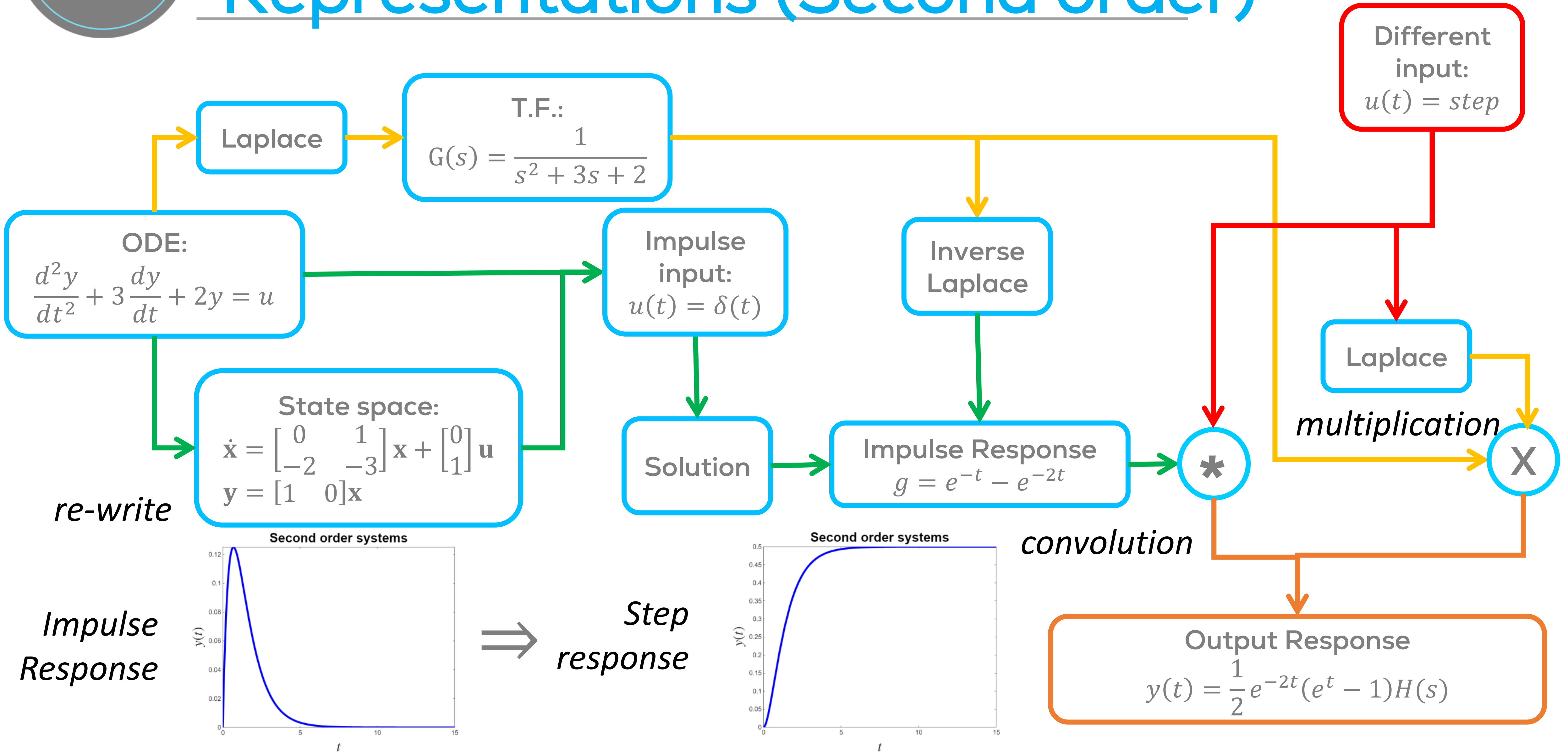
File: ex_ss

```
%% Dynamical System
% dy/dt + k*y(t) = u(t)
%% Initialise simulation
clc
clear
close all
%% Simulation Parameters
y_0=1; % Initial Conditions
dt=0.001; % Sampling time
tf=15; % Final time
%% Euler Approximation configuration
% Vector initialisation
t=0:dt:tf;
y= zeros(length(t),1);
y(1)=y_0;
%% Euler Approximation of the Solution
for k=1:length(t)-1
y(k+1)=y(k)+dt*(-y(k));
t(k+1)=t(k)+dt;
end
%% Plotting
figure(1);
grid on
hold on
axis([0 15 0 2])
% Plot Solution
plot(t,y,'LineWidth',3,'color','r')
%labels
xlabel ('$t$','interpreter','latex','FontSize',22)
ylabel ('$y$','interpreter','latex','FontSize',22)
title ('First Order System','FontSize',20)
```

Examples of LTI Representations (First order)



Examples of LTI Representations (Second order)



Reflections on LTI Representations

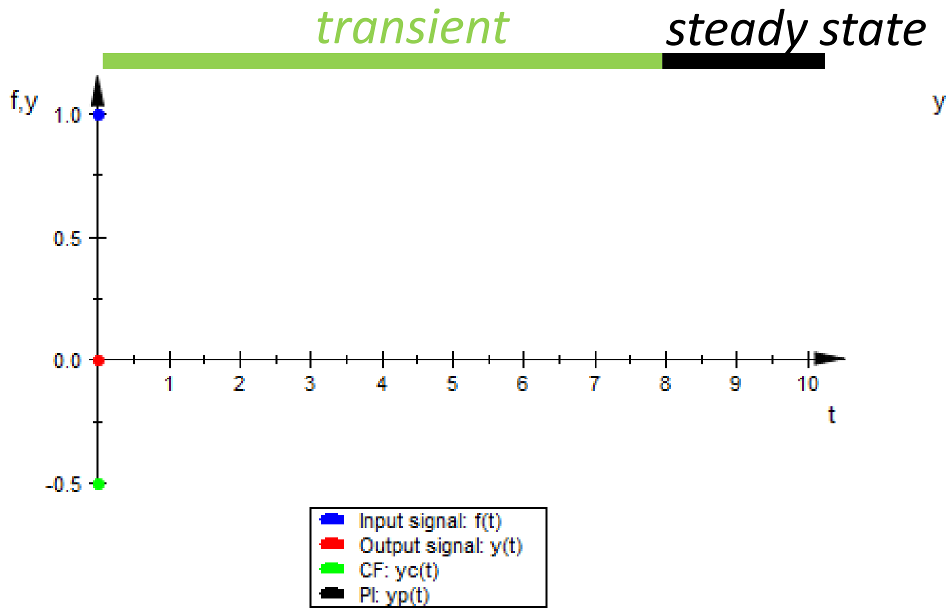
All the LTI ODE dynamic system representations are **equivalent**. However, some representations are "more natural"

- **ODEs** Are useful because they depict the problem in terms of "forces".
- **Impulse response signal** is equivalent to the complementary function or transient response component. Similar to the **transient response**. Solved using convolution.
- **Transfer functions** are used to simplify the block (control) analysis as **convolution** is simply **multiplication**. Solved using Laplace.
- **State space** can be easily generalized to non-linear & multi-input, multi-output systems. Concepts like controllability, observability and recursive filters can be represented. Solved using (matrix-based) convolution.

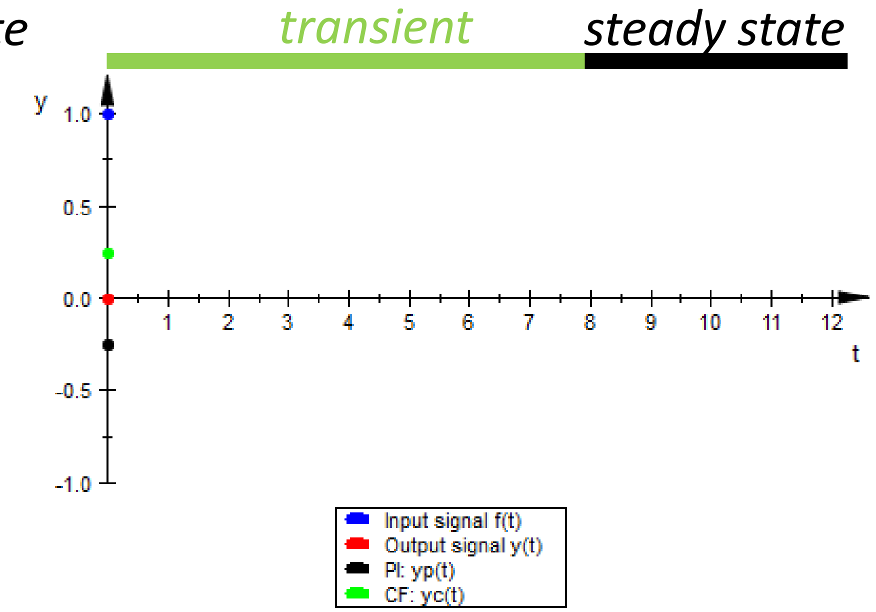
Transients and steady state

- For simple inputs like steps or impulses ($u(t)$), the user can focus in the **transient** and **steady state** **phase**.
- For pulsed or periodic input signals, the output of a stable, LTI system will converge to a regular, **steady state** pattern after the initial **transient** phase has finished.

$$4\frac{\partial}{\partial t}y(t)+2\,y(t)=1,\qquad y(0)=0$$



$$\frac{\partial^2}{\partial t^2}y(t)+\frac{\partial}{\partial t}y(t)+2\,y(t)=\cos(2\,t),$$



Poles

- The **transient** phase of **LTI ODEs** and **exponential** signals are **synonymous**:

$$Y(s) = G(s)U(s)$$

- Roughly, if G has poles $\{s1, \dots, sn\}$ and U has poles $\{p1, \dots, pm\}$, then Y's poles are the union of the two sets, i.e.

$$denom\{Y(s)\} = (s - s_1)\cdots(s - s_n)(s - p_1)\cdots(s - p_m)$$

$$Y(s) = \frac{A_1}{s - s_1} + \cdots + \frac{A_n}{s - s_n} + \frac{B_1}{s - p_1} + \cdots + \frac{B_m}{s - p_m}$$

- Taking inverse Laplace transforms

$$y(t) = A_1 e^{s_1 t} + \cdots + A_n e^{s_n t} + B_1 e^{p_1 t} + \cdots + B_m e^{p_m t}$$

Transient response, a "scaled" version of $g(t)$

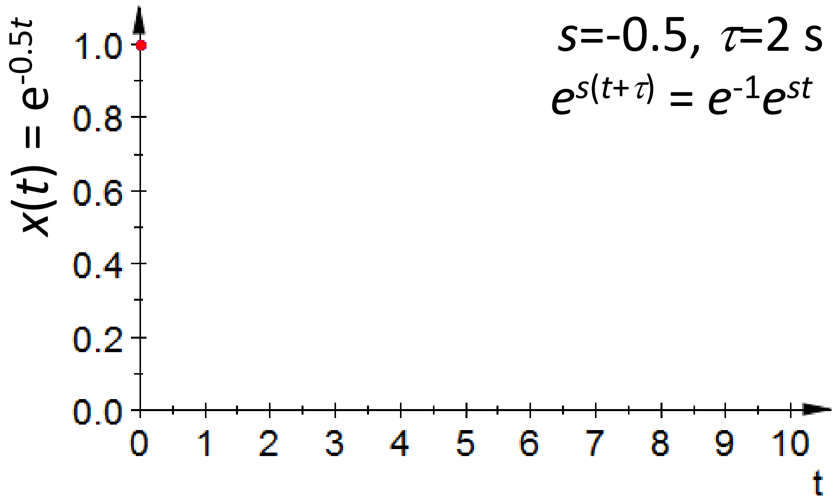
Steady state response, a "scaled" version of $u(t)$

Real, Imaginary & Complex Exponentials

A. Real exponent

s=-0.5, τ=2 s

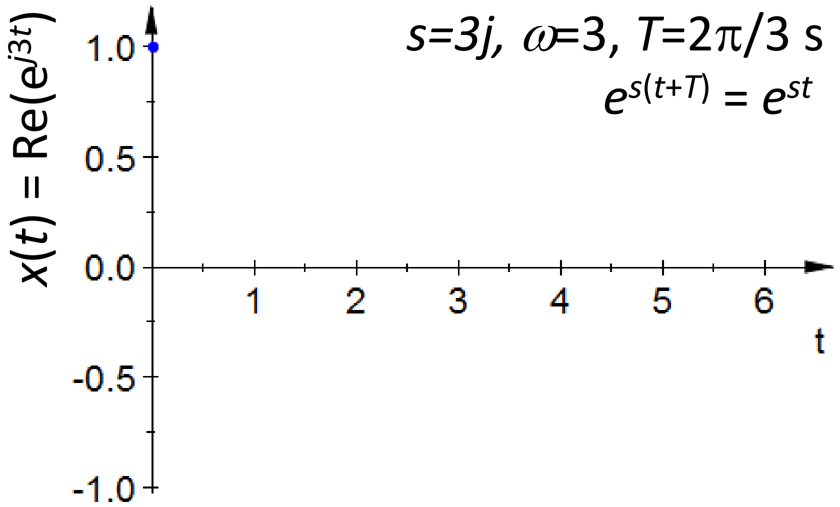
$e^{s(t+\tau)} = e^{-1}e^{st}$



B. Imaginary exponent

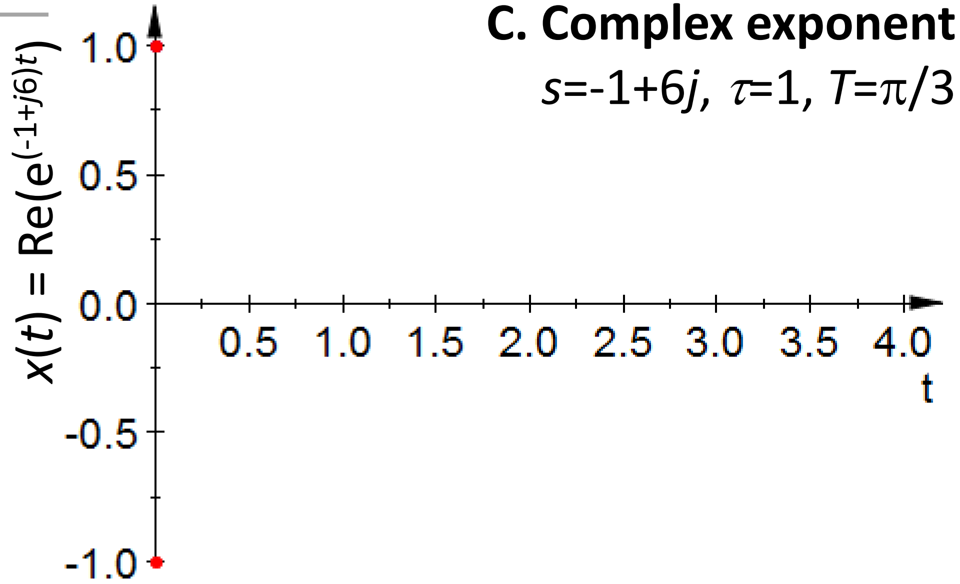
s=3j, ω=3, T=2π/3 s

$e^{s(t+T)} = e^{st}$



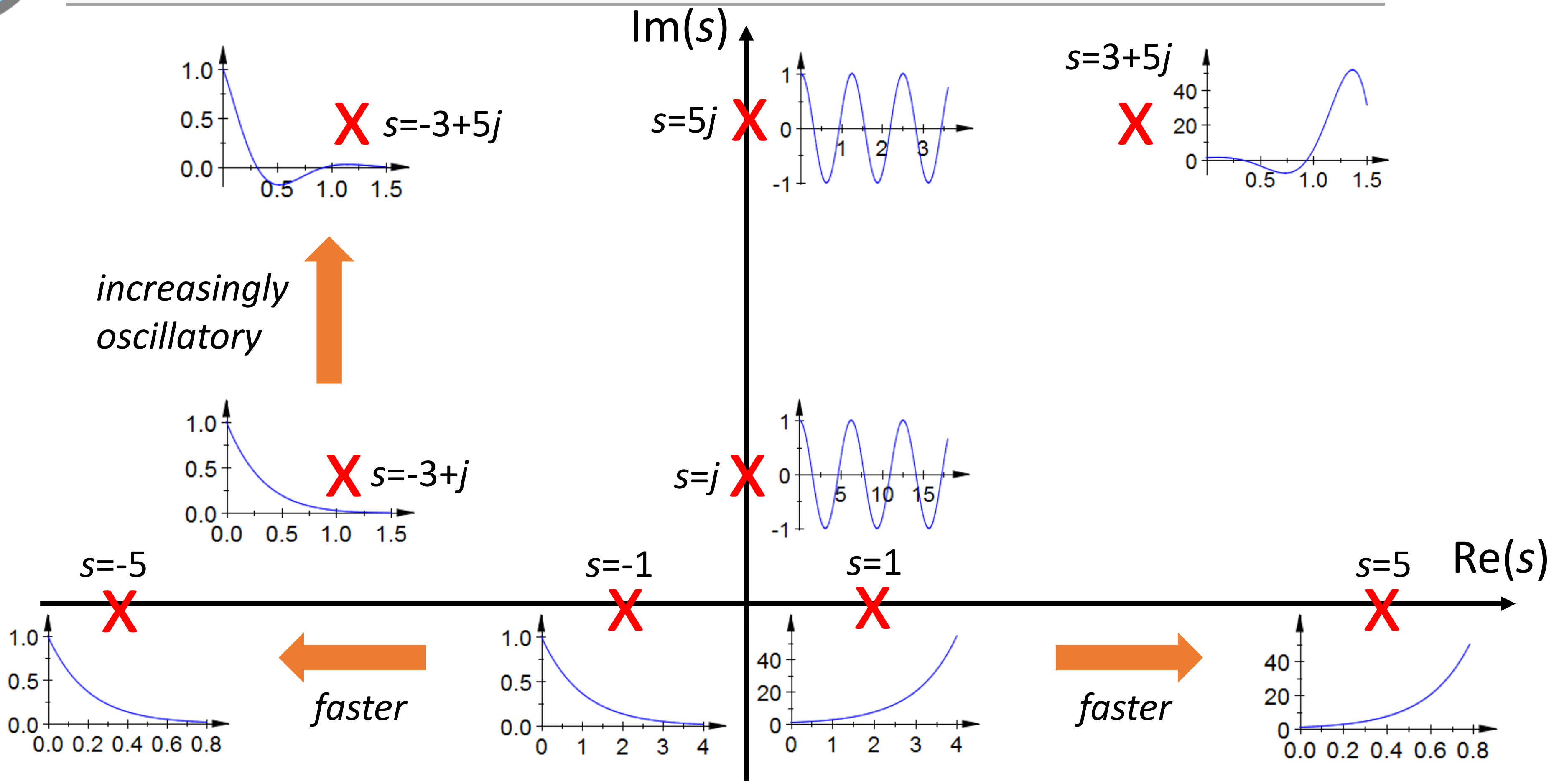
C. Complex exponent

s=-1+6j, τ=1, T=π/3



- In exponential signals, e^{st}, the **exponent s** simply **scales time**.
- For **real** valued exponents, scaling time corresponds to a **time constant**
- For **imaginary** valued exponents, scaling time corresponds to a **time period**.
- Complex valued exponents combine both elements

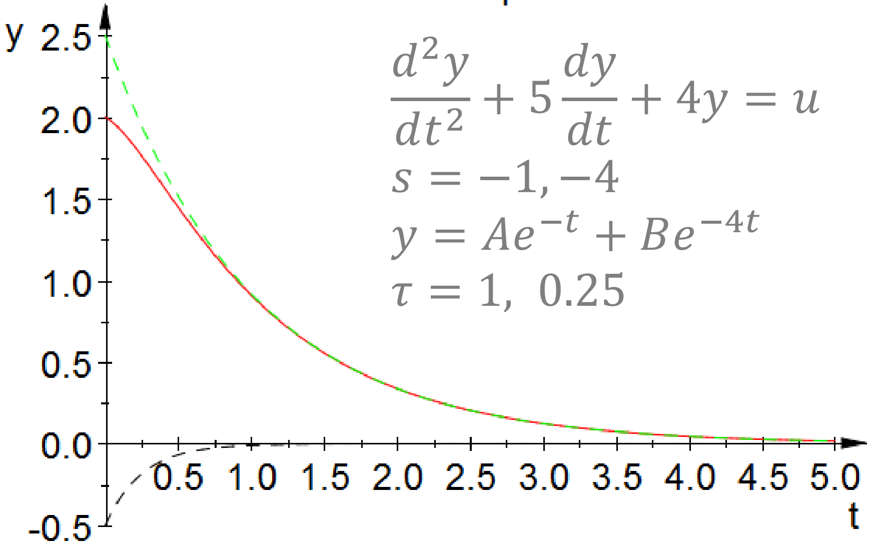
s-plane - e^{st}



Stability means having poles in the left half plane means that the transient response has exponentials with negative real parts, i.e. they decay.

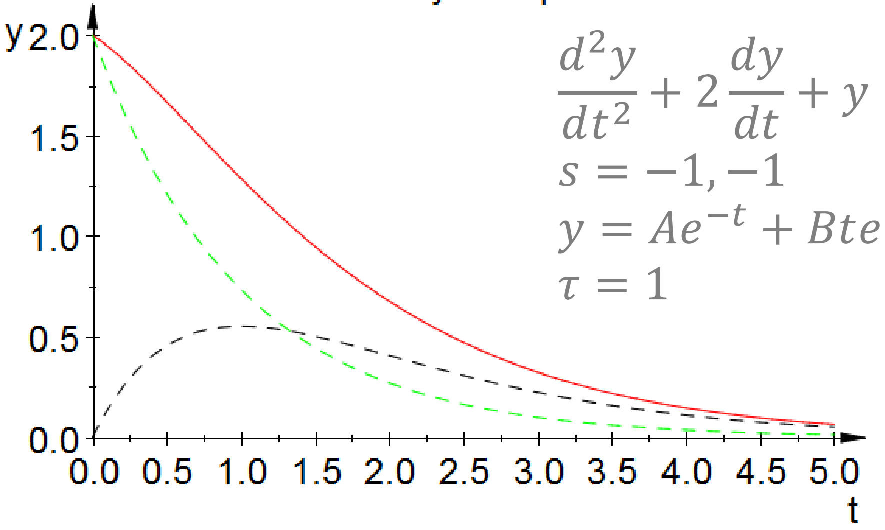
Transient – Pole, 2nd Order, Unforced (CF) Examples

1. Overdamped



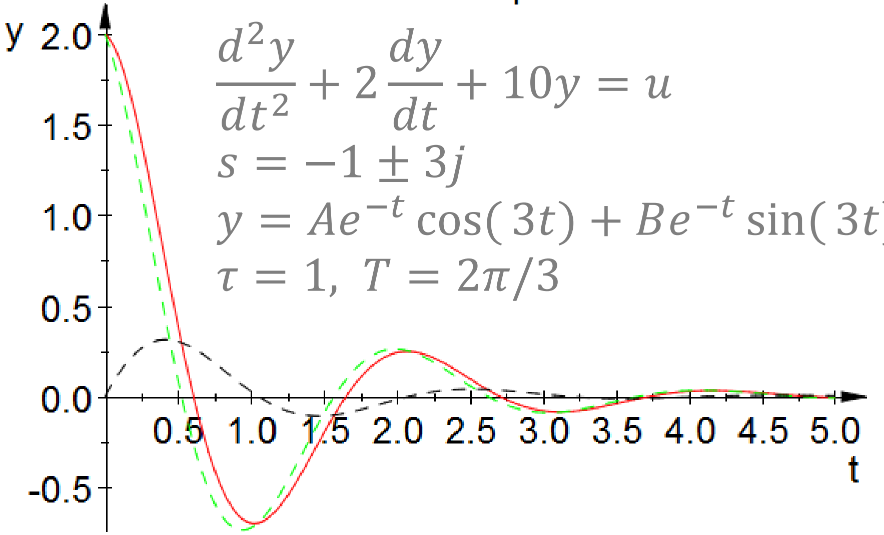
$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 4y = u$$
$$s = -1, -4$$
$$y = Ae^{-t} + Be^{-4t}$$
$$\tau = 1,\ 0.25$$

2. Critically Damped



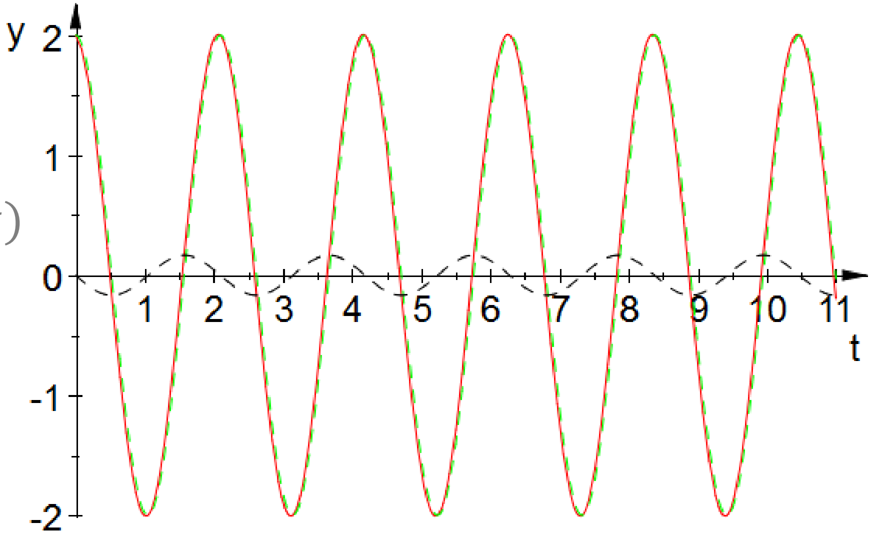
$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = u$$
$$s = -1, -1$$
$$y = Ae^{-t} + Bte^{-t}$$
$$\tau = 1$$

3. Underdamped



$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 10y = u$$
$$s = -1 \pm 3j$$
$$y = Ae^{-t}\cos(3t) + Be^{-t}\sin(3t)$$
$$\tau = 1,\ T = 2\pi/3$$

4. Resonant



$$\frac{d^2y}{dt^2} + 9y = u,$$
$$y = A\cos(3t) + B\sin(3t),$$
$$s = \pm 3j,$$
$$T = 2\pi/3$$

Zeros

A **zero** represents is a value of s for which **numerator** of the **transfer function** equals **zero**. It operates on / directly affects the input

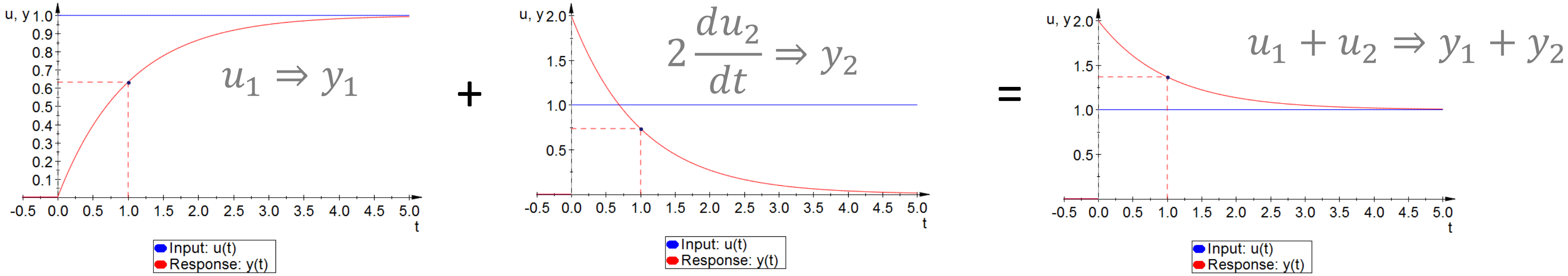
Three interpretations can be given:

1. Pre-filter on the input given by the system actuators.
2. The corresponding input signal $u = e^{st}$ will produce an identically zero output (filter design?)
3. The zeros determine the **constant multipliers** associated with the **exponentials** in the **transient response** $y = Ae^{-t} + Be^{-4t}$
   - A related interpretation is that of **pole-zero cancellation**. You can view this as cancelling a common factor in the transfer function or as the corresponding exponential having a **zero constant multiplier** and hence the effective dynamical order is reduced by 1
4. Analysis for these systems can be done using the superposition principle. In other words how will the system react to the sum of the inputs. Zeros can cause unexpected (non-oscillatory) overshoot or non-minimum phase behaviour.

Zeros: 1st & 2nd Order Examples

$$\frac{dy}{dt} + y = 2\frac{du}{dt} + u$$

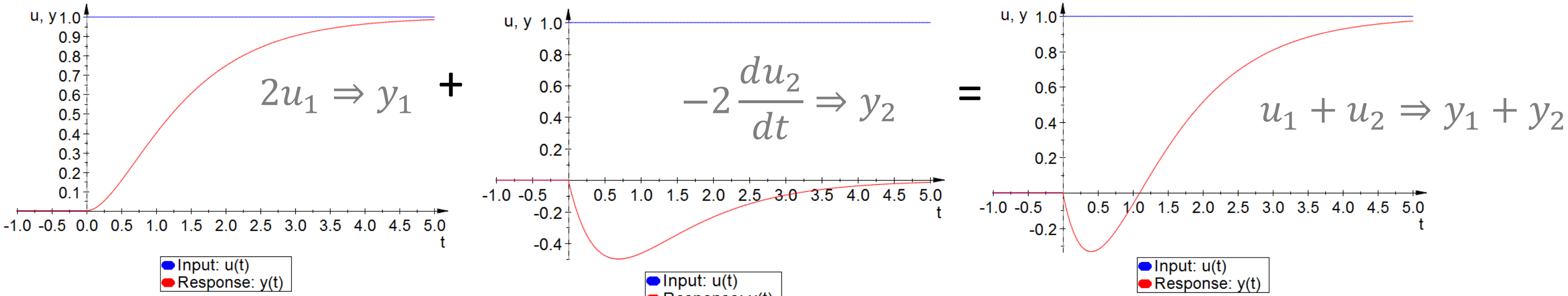
1st order, step response (biproper)



- Transfer function is biproper which causes an initial output jump

2nd order, step response (non minimum phase)

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = -2\frac{du}{dt} + 2u$$



- The non minimum phase causes a negative initial response

Example

Linear system

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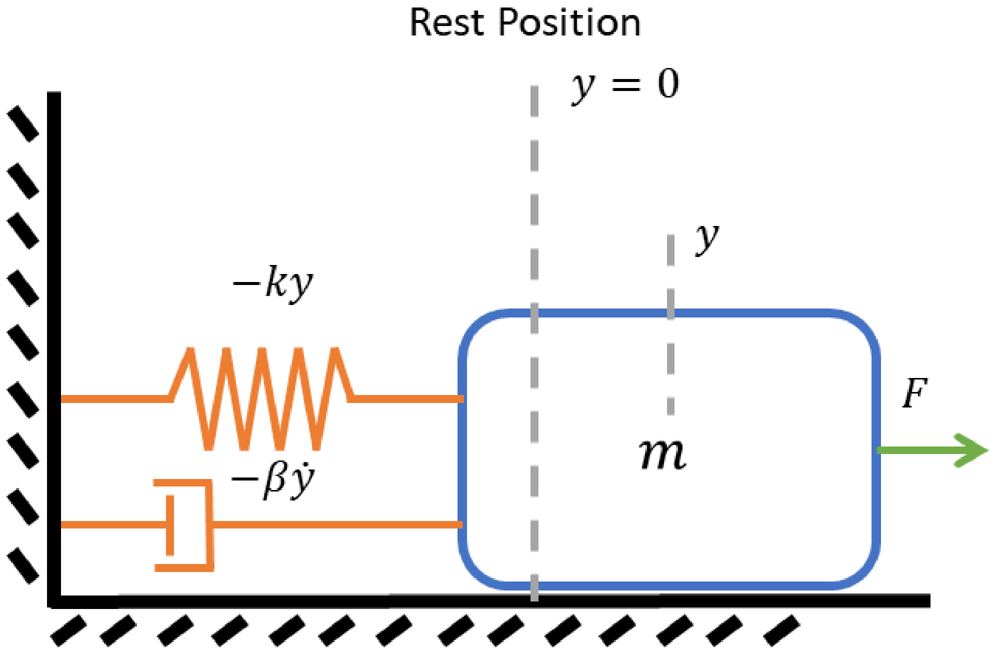
Representation of a linear system

Worked example

- Let us consider an Ideal Mass-Spring-Damper system where an external force F is applied on the mass. The output of the system is the position of the mass y.

Q: Which are the states (the set of coordinates) that describe the dynamics of this mechanical system?

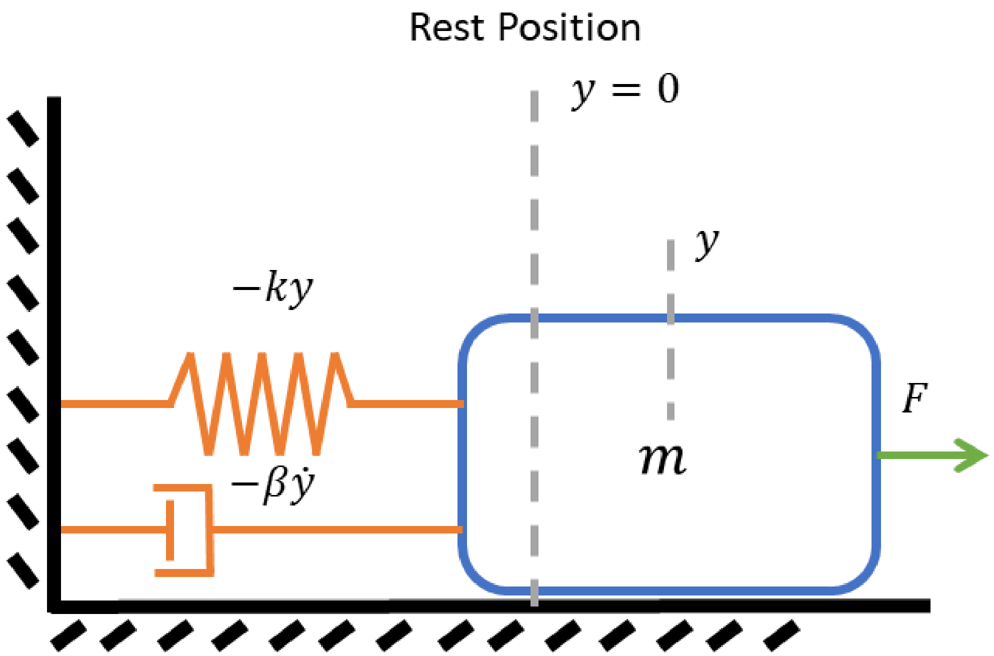


State-space representation of a linear system

Worked example

- Applying Newton’s second law, the dynamics of the system are given by:

$$\sum_i F_i = ma = m\,\ddot{y} \tag{18}$$



State-space representation of a linear system

Worked example

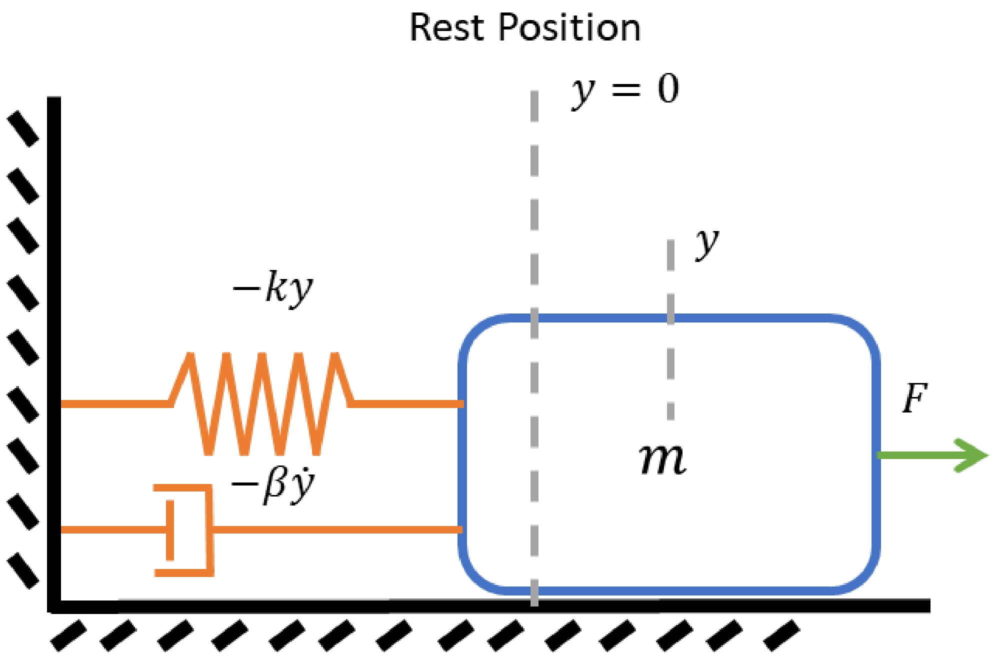
- There are three forces in the direction of y: the spring force ($-ky$), the damper force ($-\beta\,\dot{y}$), and the external force (F).

$$F + (-ky) + (-\beta\,\dot{y}) = m\,\ddot{y} \tag{19}$$

$$\ddot{y} + \frac{\beta}{m}\,\dot{y} + \frac{k}{m}y = \frac{F}{m} \tag{20}$$

- Overall dynamic equation is given by

$$\underbrace{\boldsymbol{M}(\boldsymbol{x})\,\ddot{\boldsymbol{x}}}_{\text{"Acceleration" Force}} + \underbrace{\boldsymbol{c}(\boldsymbol{x},\dot{\boldsymbol{x}})\,\dot{\boldsymbol{x}}}_{\text{"Velocity" Force}} + \underbrace{\boldsymbol{g}(\boldsymbol{x})}_{\text{"Position/ gravity" Force}} = \underbrace{\boldsymbol{u}}_{\text{"Input" Force}}$$



Solving ODEs in MATLAB

- Calculate the step response of the LTI ODE.

$$\ddot{y} + \frac{\beta}{m}\dot{y} + \frac{k}{m}y = \frac{1}{m}F$$

Where $\beta = 2, m = 1,\ k = 1,\ F = H(t)$, with zero initial conditions

$$\ddot{y} + 2\,\dot{y} + 1\,y = H(t)$$

a) Using MATLAB symbolic toolbox

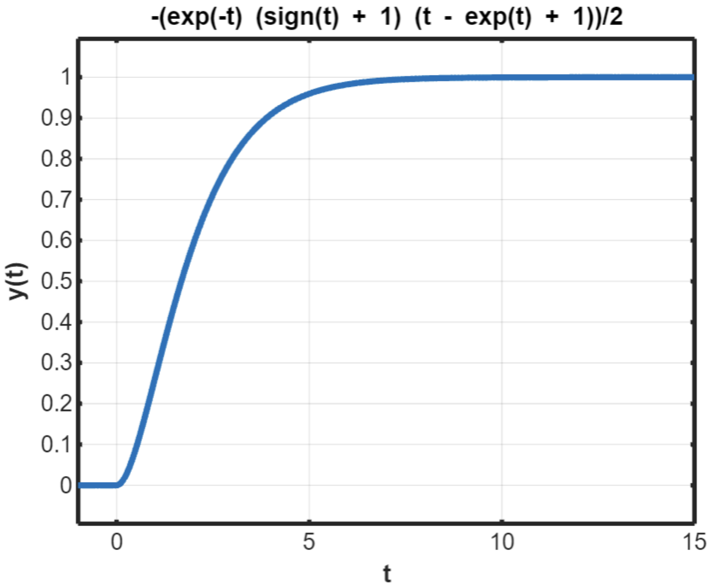
```
m = 1.0; k = 1.0; b = 2.0; f = 1;
syms y(t);
Dy = diff(y,t);
D2y = diff(y,t,2);

y = simplify(dsolve(D2y(t) + (b/m)*Dy(t) +
(k/m)*y(t) == (f/m)*heaviside(t), [y(0)==0,
Dy(0)==0]));

ezplot(y, [-1 15]);
```

b) Using MATLAB symbolic Toolbox and Lapace

```
m = 1.0; k = 1.0; b = 2.0; f = 1;
syms U Y y s t;
U = laplace(heaviside(t));
Y = U*((1/m)/(s^2+(b/m)*s+(k/m)));
y = simplify(ilaplace(Y));
figure(2)
ezplot(y, [0 15]);
```

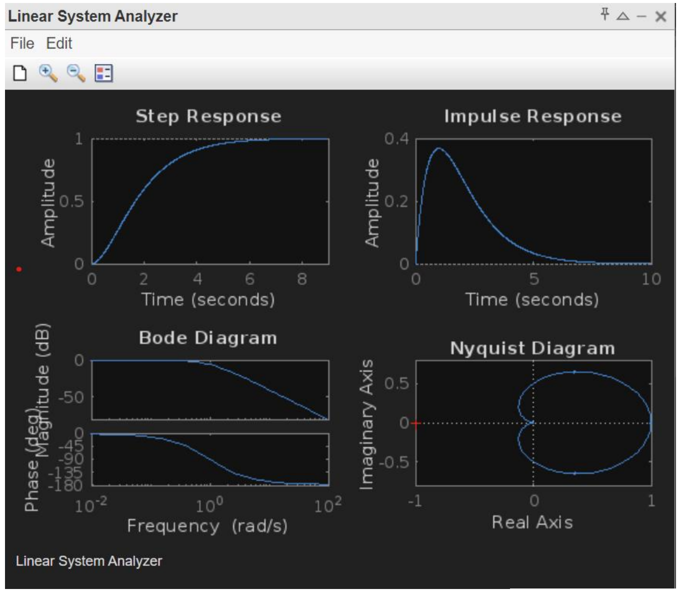


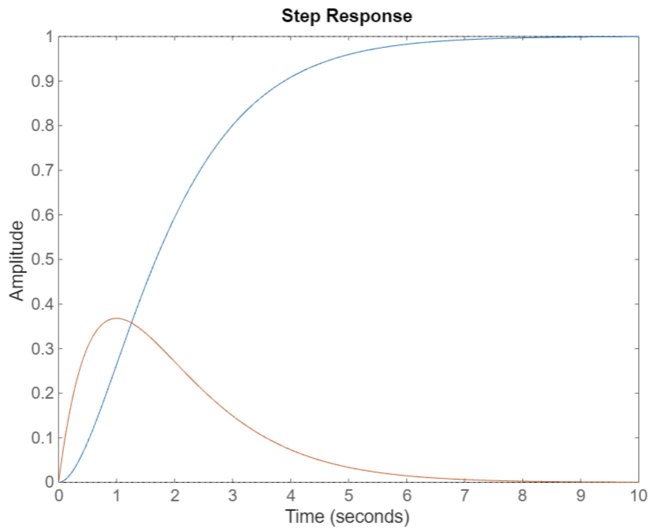
Dynamic Simulation using MATLAB

- Simulate (not solve) the response of the ODE in MATLAB.
- The solution is a vector of output values $y(i)$ at discrete time intervals $t(i)$

a) Control Systems Toolbox

```
%% System Parameters
m = 1.0; k = 1.0; b = 2.0; f = 1;
%% Solution using Control System Toolbox
g = tf([(1/m)],[1 (b/m) (k/m)]);
figure(3)
step(g);
hold
impulse(g);
% Step and impulse response
ltiview(g);  % Handy GUI for analyzing systems
```

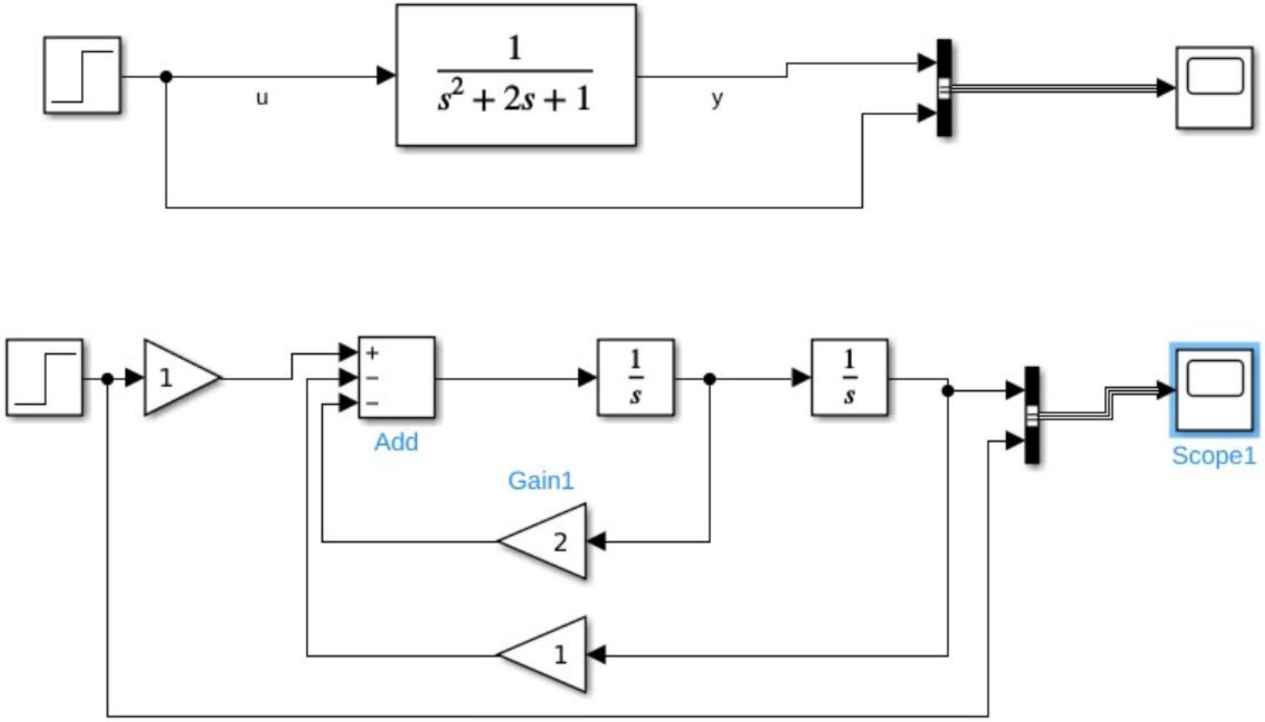


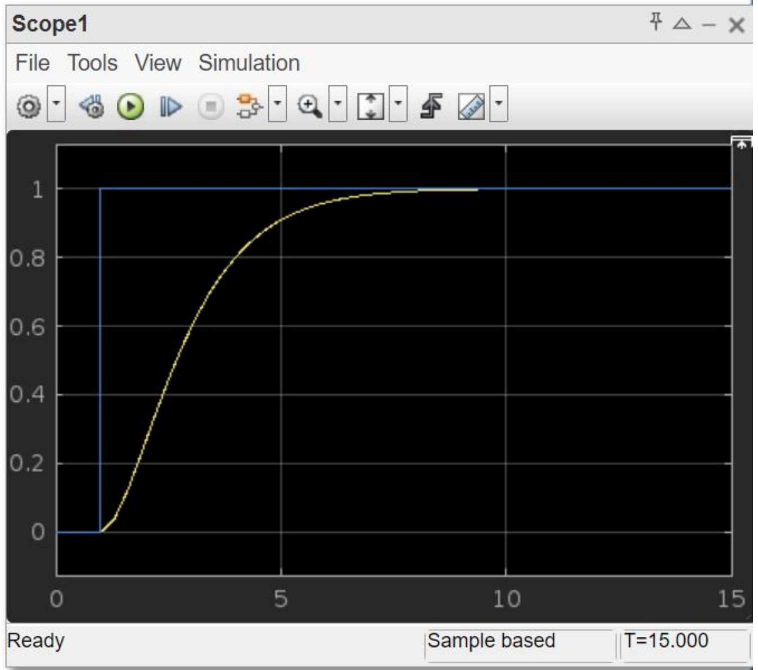


Dynamic Simulation using MATLAB

b) Using **Simulink**

- b) Using TF
- c) Using Math Operations blocks





State-space representation of a linear system

Worked example

- Let us define the set of states as:

$$\begin{aligned} x_1 &= y \\ x_2 &= \dot{y} \end{aligned} \tag{21}$$

- Then we can find a state-space representation of this system. From the definition of both coordinates, it is trivial that $\dot{x}_1 = x_2$, then (20) can be rewritten in term of x_1, x_2, and $\dot{x}_2$.

$$\dot{x}_2 + \frac{\beta}{m} x_2 + \frac{k}{m} x_1 = \frac{F}{m} \tag{22}$$

$$\ddot{y} + \frac{\beta}{m} \dot{y} + \frac{k}{m} y = \frac{F}{m} \tag{23}$$

- As a result, the system is described by two first order differential equations:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{\beta}{m} x_2 - \frac{k}{m} x_1 + \frac{1}{m} F \end{cases} \tag{24}$$

State-space representation of a linear system

Worked example

- We rewrite these two equations using matrices and the state $\boldsymbol{x} = (x_1, x_2)$.

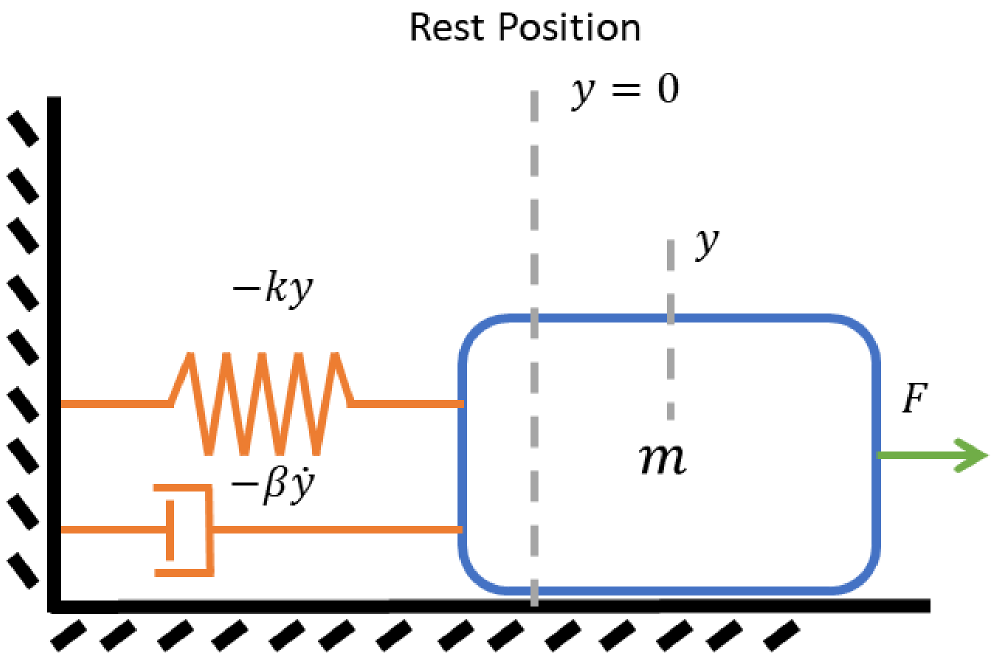
$$\dot{x}_1 = 0x_1 + x_2 + 0F \qquad (25)$$

$$\dot{x}_2 = -\frac{\beta}{m}x_2 - \frac{k}{m}x_1 + \frac{1}{m}F \qquad (26)$$

$$\dot{\boldsymbol{x}} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{\beta}{m} \end{bmatrix}\boldsymbol{x} + \begin{bmatrix} 0 \\ 1/m \end{bmatrix}F \qquad (27)$$

- Using (21), the output equation is given by:

$$y = x_1 + 0x_2 + 0F = \begin{bmatrix} 1 & 0 \end{bmatrix}\boldsymbol{x} + 0F \qquad (28)$$



State-space representation of a linear system

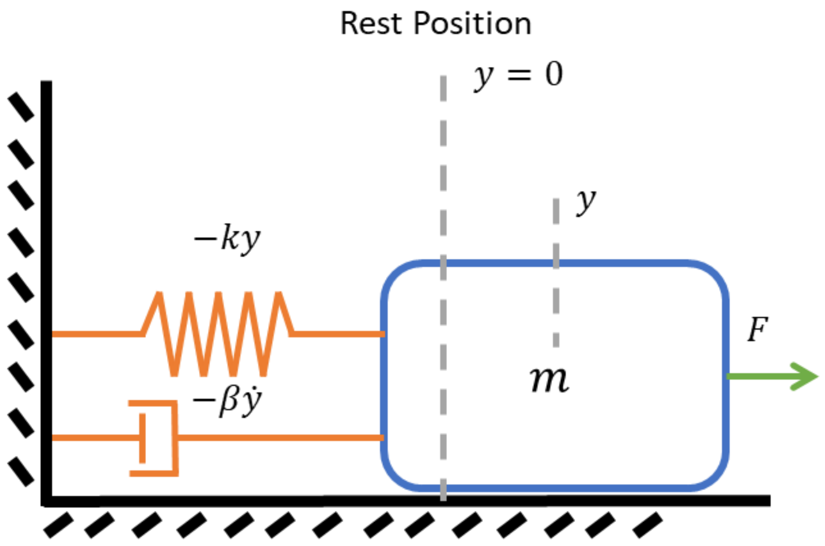
Worked example

In summary, the state-space representation of an ideal mass-spring-damper is given by:

$$A = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{\beta}{m} \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 1/m \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix} \qquad D = 0$$

$$\boxed{\begin{aligned} \dot{\boldsymbol{x}} &= \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{\beta}{m} \end{bmatrix}\boldsymbol{x} + \begin{bmatrix} 0 \\ 1/m \end{bmatrix}F \\ y &= \begin{bmatrix} 1 & 0 \end{bmatrix}\boldsymbol{x} \end{aligned}} \qquad (29)$$



Solving ODEs in MATLAB

a) Making a simple ODE solver based on a numerical method such as Euler's method

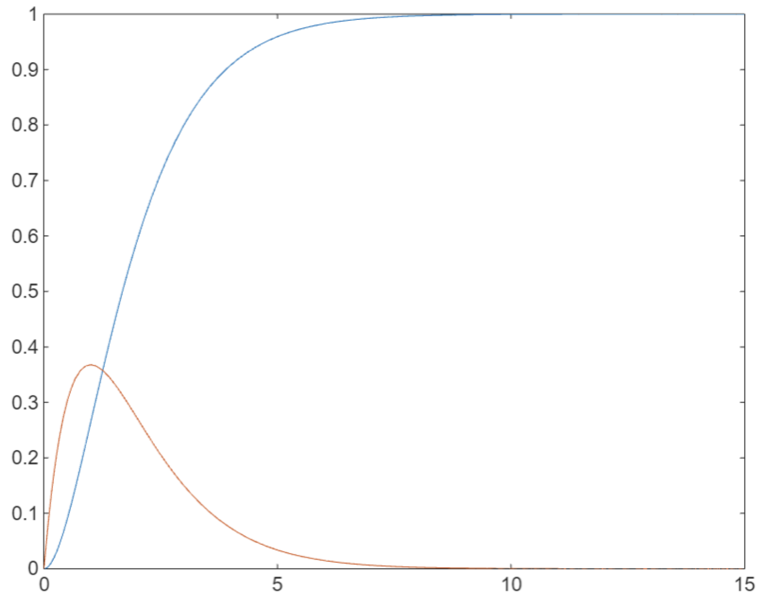
```
%% Parameters
b_damper=2.0;k_spring=1.0;mass=1.0;force=1.0;

%% Simulation Parameters
x1_0=0;x2_0=0;
dt=0.001;
tf=15

%% Euler Approximation configuration
t=0:dt:tf;
x1= zeros(length(t),1);
x2= zeros(length(t),1);
x1(1)=x1_0;x2(1)=x2_0;

%% Euler Approximation of the Solution
for k=1:length(t)-1
x1(k+1)=x1(k)+dt*(x2(k));
x2(k+1)=x2(k)+dt*(-(b_damper/mass)*x2(k)-(k_spring/mass)*x1(k)+(1/mass)*force);
end

%% Plotting
plot(t,x1,'LineWidth',3,'color','b')
hold
plot(t,x2,'LineWidth',3,'color','r')
```

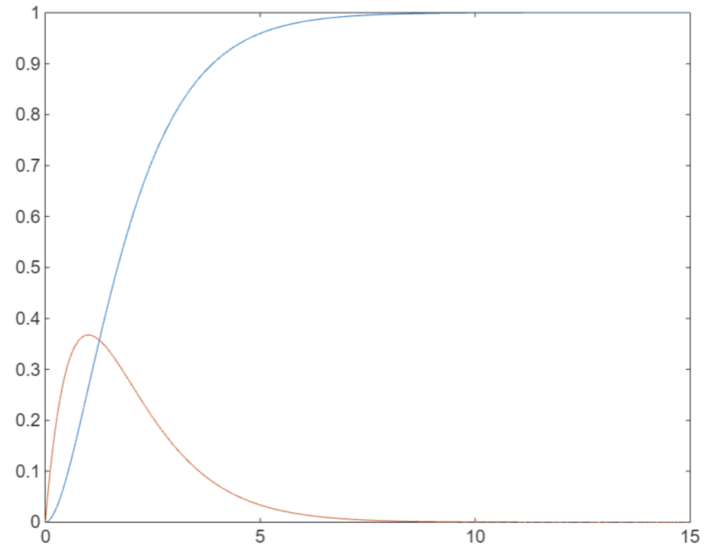


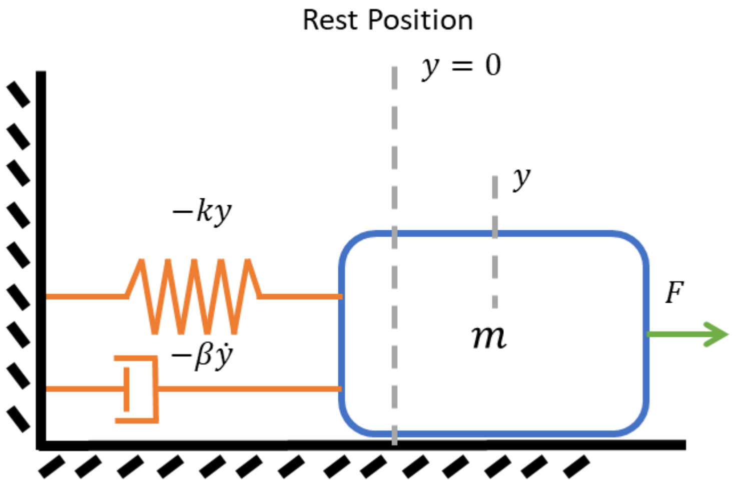
Solving ODEs in MATLAB

b) Using MATLAB ODE integrator

```
%% Initialise Sim
clc
clear all
close all
%% System Parameters
m = 1.0; k = 1.0; b = 2.0; f = 1;
gt = linspace(0,15,1000);
g = heaviside(gt);
tspan = [0 15];
[t,y] = ode45(@(t,y) mass_spring_system(t,y,m,k,b,f,g,gt),
tspan,[0 0]);
figure(6)
plot(t,y)
```

```
function dydt = mass_spring_system(t,y,m,k,b,f,g,gt)
g = interp1(gt,g,t); % Interpolate the data set (gt,g) at time t
dydt = [y(2); -(b/m)*y(2)-(k/m)*y(1)+(1/m)*g];
end
```





Manipulator: A dynamic control problem

- Each joint or link has two states $\{q, \dot{q}\}$: angular position and velocity
- Each joint uses a single motor that produces a torque (input)
- The torques produce joint accelerations
- A 6 DOF manipulator will have 12 states and 6 inputs.
- The non-linear dynamic model is of the form

$$M(q)\,\ddot{q} + c(q,\dot{q})\,\dot{q} + g(q) = \tau$$

- Understanding the system's dynamic model is crucial for designing joint controllers that calculate the joint torques/voltages and make the manipulator follow a reference trajectory.

ABB Fanta can challenge

https://www.youtube.com/watch?v=SOESSCXGhFo

Thank you

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