Dual Link Manipulator

Linearisation

{Learn, Create, Innovate};

Linearisation of a State Space System

- For a dynamical system of the form

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u})$$

$$\mathbf{y} = \mathbf{h}(\mathbf{x}, \mathbf{u})$$

- We can linearise it (local model to be analysed) about $\{\mathbf{x}^*, \mathbf{u}^*\}$ using a 1^{st} order **multivariate Taylor series****.

$$\Delta\dot{\mathbf{x}} = \frac{\partial \mathbf{f}(\mathbf{x}^*, \mathbf{u}^*)}{\partial \mathbf{x}} \Delta\mathbf{x} + \frac{\partial \mathbf{f}(\mathbf{x}^*, \mathbf{u}^*)}{\partial \mathbf{u}} \Delta\mathbf{u}$$

$$\Delta\mathbf{y} = \frac{\partial \mathbf{h}(\mathbf{x}^*, \mathbf{u}^*)}{\partial \mathbf{x}} \Delta\mathbf{x} + \frac{\partial \mathbf{h}(\mathbf{x}^*, \mathbf{u}^*)}{\partial \mathbf{u}} \Delta\mathbf{u}$$

- The linear state space model can then be described as:

$$\Delta\dot{\mathbf{x}} = \mathbf{A}\,\Delta\mathbf{x} + \boldsymbol{B}\,\Delta\mathbf{u}$$

$$\Delta\mathbf{y} = \mathbf{C}\,\Delta\mathbf{x} + \mathbf{D}\,\Delta\mathbf{u}$$

*** Named after the mathematician Brook Taylor (1685-1731). This is not related to Taylor Swift nor a TV series about Taylor Swift.*

- Where the matrices are defined as follows

$$\boldsymbol{A} = \frac{\partial \mathbf{f}(\mathbf{x}^*, \mathbf{u}^*)}{\partial \mathbf{x}}, \qquad \boldsymbol{B} = \frac{\partial \mathbf{f}(\mathbf{x}^*, \mathbf{u}^*)}{\partial \mathbf{u}}$$

$$\boldsymbol{C} = \frac{\partial \mathbf{h}(\mathbf{x}^*, \mathbf{u}^*)}{\partial \mathbf{x}}, \qquad \boldsymbol{D} = \frac{\partial \mathbf{f}(\mathbf{x}^*, \mathbf{u}^*)}{\partial \mathbf{u}}$$

- The states and control are now incremental

$$\Delta\mathbf{x} = \mathbf{x} - \mathbf{x}^*, \quad \Delta\mathbf{u} = \mathbf{u} - \mathbf{u}^*$$

**Usually the incremental variables are implicit (remove Δ)

SLM Linearisation Example

- The SLM dynamical model is given by

$$\ddot{q} + \frac{mga\,cos(q)}{J + ma^2} = \frac{1}{J + ma^2}\,\tau$$

- Linearising the model using using a 1^{st} order **multivariate Taylor series** around the operating point $\{q^*, \tau^*\}$ gives the same linearized system as previously obtained:

$$\Delta\ddot{q} - \frac{mga\sin(q^*)}{(J + ma^2)}\Delta q = \frac{1}{(J + ma^2)}\Delta\tau$$

- This can be expressed in state space form as

$$\begin{bmatrix}\Delta\dot{q}\\ \Delta\ddot{q}\end{bmatrix} = \begin{bmatrix}0 & 1\\ \frac{mga\sin(q*)}{J+ma^2} & 0\end{bmatrix}\begin{bmatrix}\Delta q\\ \Delta\dot{q}\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{J+ma^2}\end{bmatrix}\Delta\tau$$

SLM Linearisation Example

- The SLM state space dynamical model is given by

$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u})$

$$\begin{bmatrix} \dot{q} \\ \ddot{q} \end{bmatrix} = \begin{bmatrix} \dot{q} \\ (\tau - mga\cos(q))/(J + ma^2) \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}, \quad u = \tau$$

- The linearised model is then given by

$$\frac{\mathrm{d}}{\mathrm{dt}}(\Delta \boldsymbol{q}) = \mathbf{A}\,\Delta \boldsymbol{q} + \boldsymbol{B}\,\Delta\tau$$

$$\mathbf{A} = \begin{bmatrix} \partial f_1/\partial x_1 & \partial f_1/\partial x_2 \\ \partial f_2/\partial x_1 & \partial f_2/\partial x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ mga\sin(q^*)/(J + ma^2) & 0 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} \partial f_1/\partial u_1 \\ \partial f_2/\partial u_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1/(J + ma^2) \end{bmatrix}$$

- The same equation is obtained using this method.
- This is called Jacobian.

Definition (Jacobian): Given a vectorial function $f: \mathbb{R}^n \to \mathbb{R}^n$, then the Jacobian matrix $\mathcal{J}_f \in \mathbb{R}^{n\times n}$ is defined by:

$$\mathcal{J}_f = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix}$$

Linearisation around an operating point (generalisation)

Let us consider the nonlinear system given by:

$$\dot{x} = f(x, u)$$

$$y = h(x, u)$$

Then if $f(x_0, u_0) = 0$, the point (x_0, u_0) is referred to as an operating point. Under this condition, we can perform a linearisation of the system. Let us define the new input, state, and output as the variation around x_0, u_0, and y_0.

$$\Delta u = u - u_0$$

$$\Delta x = x - x_0$$

$$\Delta y = y - y_0$$

Linearisation around an operating point (generalisation)

Using the Jacobian, it is possible to linearise $f(x,u)$ and $h(x,u)$ around the point (x_0,u_0) and we get:

$$f(x,u) \simeq f(x_0,u_0) + \mathcal{J}_f^x(x_0,u_0)(x-x_0) + \mathcal{J}_f^u(x_0,u_0)(u-u_0)$$

$$h(x,u) \simeq h(x_0,u_0) + \mathcal{J}_h^x(x_0,u_0)(x-x_0) + \mathcal{J}_h^u(x_0,u_0)(u-u_0)$$

where $f(x_0,u_0) = 0$ and $h(x_0,u_0) = y_0$.

The linearised system is given by:

$$\frac{d}{dt}(\Delta x) \simeq \mathcal{J}_f^x(x_0,u_0)\Delta x + \mathcal{J}_f^u(x_0,u_0)\Delta u$$

$$\Delta y \simeq \mathcal{J}_h^x(x_0,u_0)\Delta x + \mathcal{J}_h^u(x_0,u_0)\Delta u$$

where the superscripts x and u in the Jacobian matrices indicate the parameter that is considered as a variable.

- In other words

$$\frac{\mathrm{d}}{\mathrm{dt}}(\Delta\mathbf{x}) = \mathbf{A}\,\Delta\mathbf{x} + \boldsymbol{B}\,\Delta\mathbf{u}$$

$$\boldsymbol{y} = \mathbf{C}\,\Delta\mathbf{x} + \mathbf{D}\,\Delta\mathbf{u}$$

- where

$$\mathbf{A} = \mathcal{J}_f^x(x_0,u_0), \qquad \mathbf{B} = \mathcal{J}_f^u(x_0,u_0)$$

$$\mathbf{C} = \mathcal{J}_h^x(x_0,u_0), \qquad \mathbf{D} = \mathcal{J}_h^u(x_0,u_0)$$

Dual Link Manipulator Model

DLM Manipulator (Model)

- The dual-link manipulator model is given by:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u})$$

$$\frac{d}{dt}\begin{bmatrix}\mathbf{q} \\ \dot{\mathbf{q}}\end{bmatrix} = \begin{bmatrix}\dot{\mathbf{q}} \\ \mathbf{M}(\mathbf{q})^{-1}(\boldsymbol{\tau} - \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} - \mathbf{g}(\mathbf{q}))\end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix}\mathbf{q} \\ \dot{\mathbf{q}}\end{bmatrix}, \quad \mathbf{u} = \boldsymbol{\tau}$$

where

$$M(\boldsymbol{q}) = \begin{bmatrix} m_1 a^2 + M_1 l_1^2 + m_2(l_1^2 + d^2 + 2 l_1 d C_2) + M_2(l_1^2 + l_2^2 + 2 l_1 l_2 C_2) & m_2 d(l_1 C_2 + d) + M_2 l_2(l_1 C_2 + l_2) \\ m_2 d(l_1 C_2 + d) + M_2 l_2(l_1 C_2 + l_2) & m_2 d^2 + M_2 l_2^2 \end{bmatrix}$$

$$\boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \begin{bmatrix} -2 l_1 S_2(m_2 d + M_2 l_2)\dot{q}_2 & -l_1 S_2(m_2 d + M_2 l_2)\dot{q}_2 \\ l_1 S_2(m_2 d + M_2 l_2)\dot{q}_1 & 0 \end{bmatrix}$$

$$\mathbf{g}(\mathbf{q}) = g\begin{bmatrix} m_1 a C_1 + M_1 l_1 C_1 + m_2(l_1 C_1 + d C_{12}) + M_2(l_1 C_1 + l_2 C_{12}) \\ C_{12}(m_2 d + M_2 l_2) \end{bmatrix}$$

$$\boldsymbol{\tau} = \begin{bmatrix}\tau_1 \\ \tau_2\end{bmatrix}$$

Dual Link Manipulator Model

DLM Manipulator (Model)

- Expanding the model in MATLAB:

$$\frac{d}{dt}\begin{bmatrix}\mathbf{q}\\ \dot{\mathbf{q}}\end{bmatrix} = \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dfrac{\left(m_2\, d^2 + M_2\, l_2^2\right)\sigma_3}{\sigma_1} + \dfrac{\sigma_4\,\sigma_2}{\sigma_1} \\ -\dfrac{\sigma_2\,\sigma_3}{\sigma_1} - \dfrac{\sigma_4\,\left(M_1\, l_1^2 + M_2\, l_1^2 + M_2\, l_2^2 + a^2\, m_1 + d^2\, m_2 + l_1^2\, m_2 + 2\, M_2\, l_1\, l_2 \cos(q_2) + 2\, d\, l_1\, m_2 \cos(q_2)\right)}{\sigma_1} \end{pmatrix}$$

where

$$\sigma_1 = -M_2^2\, l_1^2\, l_2^2 \cos(q_2)^2 + M_2^2\, l_1^2\, l_2^2 + m_1\, M_2\, a^2\, l_2^2 + M_2\, d^2\, l_1^2\, m_2 - 2\, M_2\, d\, l_1^2\, l_2\, m_2 \cos(q_2)^2 + M_2\, l_1^2\, l_2^2\, m_2 + M_1\, M_2\, l_1^2\, l_2^2 + m_1\, a^2\, d^2\, m_2 - d^2\, l_1^2\, m_2^2 \cos(q_2)^2 + d^2\, l_1^2\, m_2^2 + M_1\, d^2\, l_1^2\, m_2$$

$$\sigma_2 = m_2\, d^2 + l_1\, m_2 \cos(q_2)\, d + M_2\, l_2^2 + M_2\, l_1 \cos(q_2)\, l_2$$

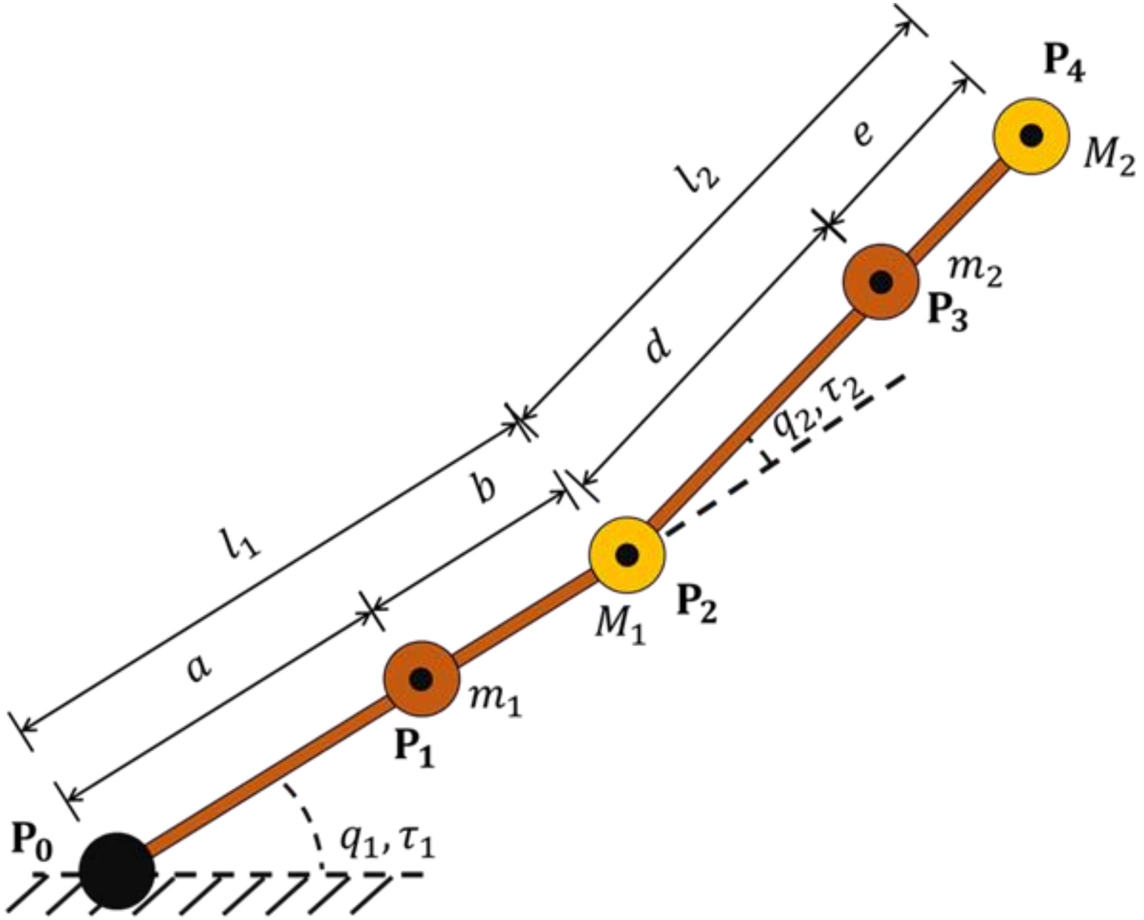
$$\sigma_3 = l_1 \sin(q_2)\, \sigma_5\, \dot{q}_2^{\,2} + 2\, l_1\, \dot{q}_1 \sin(q_2)\, \sigma_5\, \dot{q}_2 + \tau_1 - g\, \left(M_2\, \left(l_2 \cos(q_1 + q_2) + l_1 \cos(q_1)\right) + m_2\, \left(d \cos(q_1 + q_2) + l_1 \cos(q_1)\right) + a\, m_1 \cos(q_1) + M_1\, l_1 \cos(q_1)\right)$$

$$\sigma_4 = l_1 \sin(q_2)\, \sigma_5\, \dot{q}_1^{\,2} - \tau_2 + g \cos(q_1 + q_2)\, \sigma_5$$

$$\sigma_5 = M_2\, l_2 + d\, m_2$$

DLM Linearisation Example

- Let the parameters of the DLM to be:
  - $m1 = 3\ kg$
  - $m2 = 3\ kg$
  - $M1 = 1.5\ kg$
  - $M2 = 1\ kg$
  - $a = 0.2\ m$
  - $b = 0.2\ m$
  - $d = 0.2\ m$
  - $e = 0.2\ m$
  - $g = 9.8\ ms^{-2}$



DLM Nonlinear Model

- Substituting the values in the model using MATLAB:

$$\frac{d}{dt}\begin{bmatrix} q_1 \\ q_2 \\ \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \\ -\frac{7\,\sigma_1}{25\,\sigma_3} - \frac{\sigma_4\,\sigma_2}{\sigma_3} \\ \frac{25\,\sigma_4\,\sigma_1}{4\cos(q_2)^2 - 7} + \frac{25\left(\frac{4\cos(q_2)}{5} + \frac{32}{25}\right)\sigma_2}{4\cos(q_2)^2 - 7} \end{pmatrix}$$

where

$$\sigma_1 = \frac{2\sin(q_2)\,\dot{q}_2^{\,2}}{5} + \frac{4\,\dot{q}_1\sin(q_2)\,\dot{q}_2}{5} + \tau_1 - \frac{49\cos(q_1+q_2)}{5} - \frac{686\cos(q_1)}{25}$$

$$\sigma_2 = \frac{2\sin(q_2)\,\dot{q}_1^{\,2}}{5} - \tau_2 + \frac{49\cos(q_1+q_2)}{5}$$

$$\sigma_3 = \frac{4\cos(q_2)^2}{25} - \frac{7}{25}$$

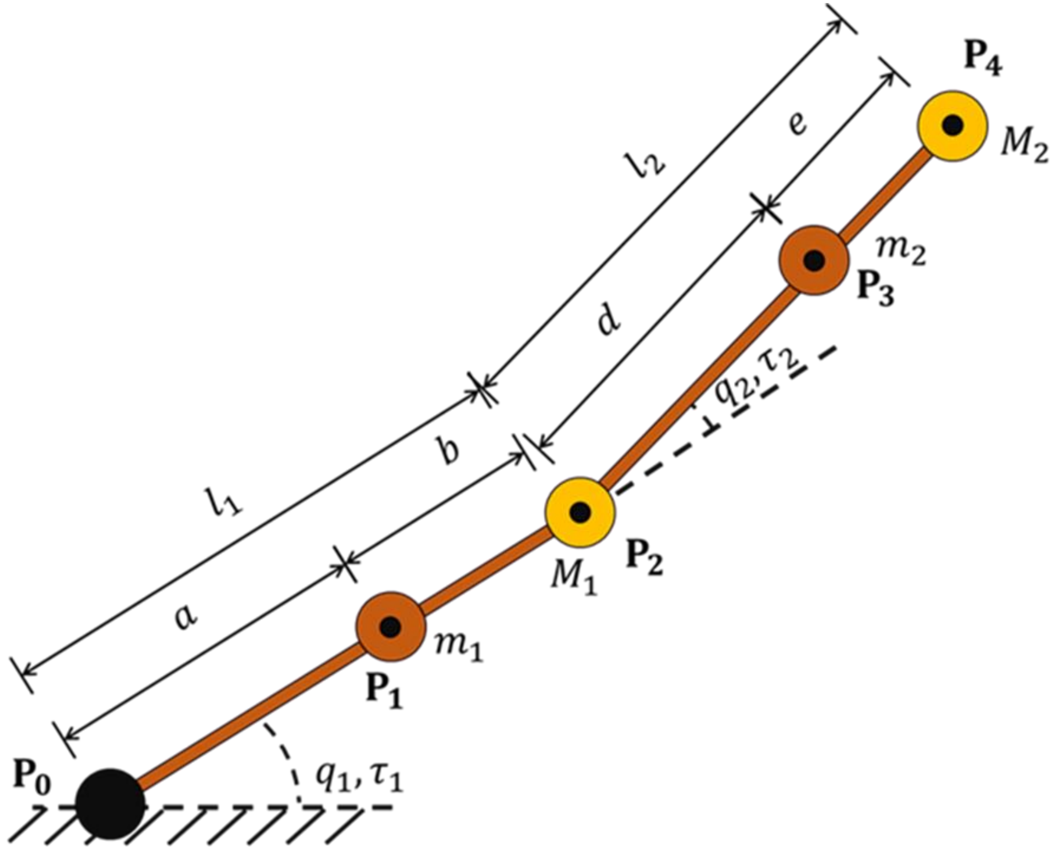
$$\sigma_4 = \frac{2\cos(q_2)}{5} + \frac{7}{25}$$

While the non-linear, state space equations appear complex there are a few observations to make:

- The denominator is the determinant of the mass-inertia matrix, always positive and always invertible

DLM Linearisation Example

- Having obtained the DLM Nonlinear model, it is time now to linearise it around an operating point.
- This can be done using the concept of the Jacobian.
- In this case, we will use MATLAB symbolic toolbox to make the linearisation easier.



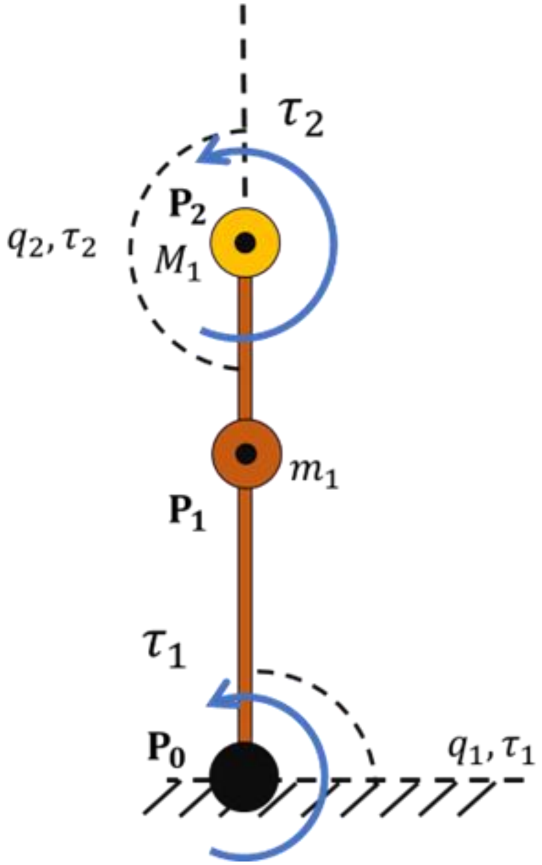
DLM Linearisation Example

- For this example, the linearisation of the Dual Link Manipulator will be about the static upright position:

$$\mathbf{q}^* = \left[\frac{\pi}{2}, \pi, 0, 0\right]^T$$

$$\boldsymbol{\tau}^* = [0,0]^T$$

- This is in order to analyse the dynamics and design linear feedback controllers.

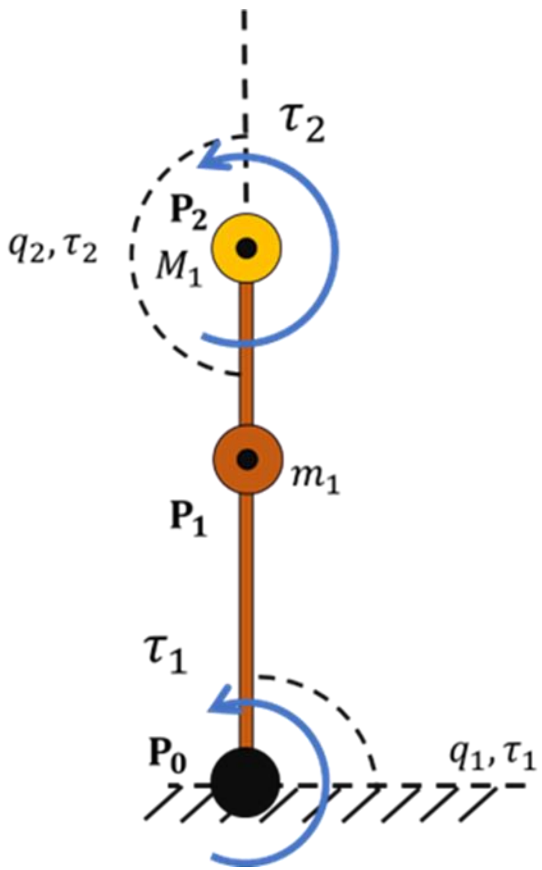


DLM Linearisation Example

- In general, using linear feedback methods (PID) to perform joint control i.e., $\mathbf{r} \rightarrow \mathbf{q}$ for a manipulator or a humanoid robot, isn't optimal, as the dynamics are inherently non-linear

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \boldsymbol{\tau}$$

- However, we observed that for a single link manipulator, a PID controller could give acceptable performance for all orientations.



DLM Linearisation Example

- The linear system of a DLM will have the following form

$$\Delta\dot{\boldsymbol{q}} = \mathbf{A}\,\Delta\mathbf{q} + \boldsymbol{B}\,\Delta\boldsymbol{\tau}$$

$$\Delta\mathbf{y} = \mathbf{C}\,\Delta\mathbf{q} + \mathbf{D}\,\Delta\boldsymbol{\tau}$$

- Using the Jacobian, to linearise the system and substituting the values we obtain the $\mathbf{A}$ Matrix:

$$\mathbf{A}(\mathbf{q}^*, \dot{\mathbf{q}}^*, \boldsymbol{\tau}^*) =$$

$$\begin{pmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\frac{49\,(25\sin(q_1+2\,q_2) - 73\sin(q_1))}{5\,(10\cos(2\,q_2) - 25)} & \frac{280\,\dot{q}_1^{\,2}\cos(q_2)^3 + 1000\,\dot{q}_1^{\,2}\cos(q_2)^2 - 70\,\dot{q}_1^{\,2}\cos(q_2) - 700\,\dot{q}_1^{\,2} + 560\,\dot{q}_1\,\dot{q}_2\cos(q_2)^3 - 140\,\dot{q}_1\,\dot{q}_2\cos(q_2) + 280\,\dot{q}_2^{\,2}\cos(q_2)^3 - 70\,\dot{q}_2^{\,2}\cos(q_2) - 24500\sin(q_1)\cos(q_2)^2 + 4116\cos(q_1)\cos(q_2)\sin(q_2) - 1000\,\tau_2\sin(q_2)^3 + 2750\,\tau_2\sin(q_2) + 17150\sin(q_1) - 700\,\tau_1\sin(2\,q_2) + 700\,\tau_2\sin(2\,q_2)}{25\,(\sigma_7 - 7)^2} & -\frac{4\sin(q_2)\left(\frac{7\,\dot{q}_1}{25} + \frac{7\,\dot{q}_2}{25} + \frac{2\,\dot{q}_1\cos(q_2)}{5}\right)}{\sigma_6} & \frac{\sin(q_2)\,(28\,\dot{q}_1 + 28\,\dot{q}_2)}{20\sin(q_2)^2 + 15} \\
\frac{49\left(\sin(q_1+q_2) + \frac{14\sin(q_1)}{5}\right)\sigma_1}{\sigma_6} - \frac{49\sin(q_1+q_2)\,\sigma_2}{\sigma_6} & \frac{25\,\sigma_1\left(\frac{2\cos(q_2)\,\dot{q}_2^{\,2}}{5} + \frac{4\,\dot{q}_1\cos(q_2)\,\dot{q}_2}{5} + \sigma_4\right)}{\sigma_7 - 7} - \frac{25\left(\sigma_4 - \frac{2\,\dot{q}_1^{\,2}\cos(q_2)}{5}\right)\sigma_2}{\sigma_7 - 7} - \frac{2\sin(q_2)\,\left(10\sin(q_2)\,\dot{q}_2^{\,2} + 20\,\dot{q}_1\sin(q_2)\,\dot{q}_2 + 25\,\tau_1 - 245\cos(q_1+q_2) - 686\cos(q_1)\right)}{20\cos(q_2)^2 - 35} - \frac{4\sin(q_2)\,(\sigma_3 - 5\,\tau_2 + \sigma_5)}{\sigma_7 - 7} + \frac{100\sin(2\,q_2)\,\sigma_1\left(\frac{2\sin(q_2)\,\dot{q}_2^{\,2}}{5} + \frac{4\,\dot{q}_1\sin(q_2)\,\dot{q}_2}{5} + \tau_1 - \frac{\sigma_5}{5} - \frac{686\cos(q_1)}{25}\right)}{(\sigma_7 - 7)^2} + \frac{100\sin(2\,q_2)\,\sigma_2\left(\frac{\sigma_3}{5} - \tau_2 + \frac{\sigma_5}{5}\right)}{(\sigma_7 - 7)^2} & \frac{4\sin(q_2)\left(\frac{32\,\dot{q}_1}{25} + \frac{7\,\dot{q}_2}{25} + \frac{4\,\dot{q}_1\cos(q_2)}{5} + \frac{2\,\dot{q}_2\cos(q_2)}{5}\right)}{\sigma_6} & \frac{\sin(q_2)\,(20\,\dot{q}_1 + 20\,\dot{q}_2)\,\sigma_1}{\sigma_7 - 7}
\end{pmatrix}$$

where

$$\sigma_1 = \frac{2\cos(q_2)}{5} + \frac{7}{25}$$

$$\sigma_2 = \frac{4\cos(q_2)}{5} + \frac{32}{25}$$

$$\sigma_3 = 2\sin(q_2)\,\dot{q}_1^{\,2}$$

$$\sigma_4 = \frac{49\sin(q_1+q_2)}{5}$$

$$\sigma_5 = 49\cos(q_1+q_2)$$

$$\sigma_6 = 5\left(\frac{\sigma_7}{25} - \frac{7}{25}\right)$$

$$\sigma_7 = 4\cos(q_2)^2$$

- Verifying the odd entry is possible
- Computer algebra was the right choice …
- This is just for two links …

DLM Linearisation Example

- The $\mathbf{B}$ Matrix is obtained using MATLAB as follows

$$\mathbf{B}(\mathbf{q}^*) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ -\dfrac{7}{4\cos(q_2)^2 - 7} & \dfrac{10\cos(q_2) + 7}{4\cos(q_2)^2 - 7} \\ \dfrac{10\cos(q_2) + 7}{4\cos(q_2)^2 - 7} & -\dfrac{20\cos(q_2) + 32}{4\cos(q_2)^2 - 7} \end{pmatrix}$$

- Only depends on the relative joint angle q_2, as the lower submatrix is simply the inverse of the mass–inertia matrix
- The torque signals enter the state space equations multiplied by the inverse mass/inertia matrix.
- The original non-linear dynamics are affine: $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u}$
- The output of the system is q_1 and q_2, therefore the matrix $\mathbf{C}$ can be defined as follows

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

DLM Linearisation Example

- Linearising around the previously defined operating point:

$$\mathbf{q}^* = \left[\frac{\pi}{2}, \pi, 0, 0\right]^T$$

$$\boldsymbol{\tau}^* = [0,0]^T$$

- The model then becomes (MIMO):

$$\Delta\dot{\boldsymbol{q}} = \mathbf{A}\,\Delta\mathbf{q} + \boldsymbol{B}\,\Delta\boldsymbol{\tau}$$

$$\Delta\mathbf{y} = \mathbf{C}\,\Delta\mathbf{q} + \mathbf{D}\,\Delta\boldsymbol{\tau}$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{784}{25} & -\frac{98}{3} & 0 & 0 \\ -\frac{539}{25} & -49 & 0 & 0 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{7}{3} & 1 \\ 1 & 4 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Linearised MIMO system

- Converting from SS to TF using MATLAB it is possible to obtain the MIMO model of the system.

$$G_1 = \frac{2.33s^2 + 81.67}{s^4 + 17.64s^2 - 2241}$$

$$G_2 = \frac{s^2 + 81.67}{s^4 + 17.64s^2 - 2241}$$

$$G_3 = \frac{s^2 - 81.67}{s^4 + 17.64s^2 - 2241}$$

$$G_4 = \frac{4s^2 - 147}{s^4 + 17.64s^2 - 2241}$$

- The matrix described the interactions of the inputs and outputs of the system.

$$\begin{bmatrix} q_1(s) \\ q_2(s) \end{bmatrix} = \begin{bmatrix} \frac{2.33s^2 + 81.67}{s^4 + 17.64s^2 - 2241} & \frac{s^2 - 81.67}{s^4 + 17.64s^2 - 2241} \\ \frac{s^2 + 81.67}{s^4 + 17.64s^2 - 2241} & \frac{4s^2 - 147}{s^4 + 17.64s^2 - 2241} \end{bmatrix} \begin{bmatrix} \tau_1(s) \\ \tau_2(s) \end{bmatrix}$$

DLM Linearisation Example

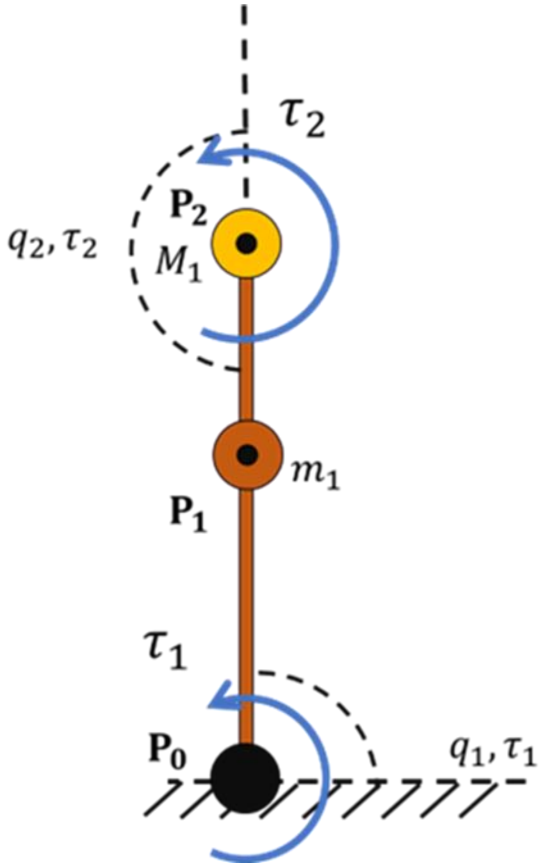
- Analysing the poles of the linearised system
- This corresponds to analysing the eigenvalues of the matrix $\mathbf{A}$.

Link 1   Link 2

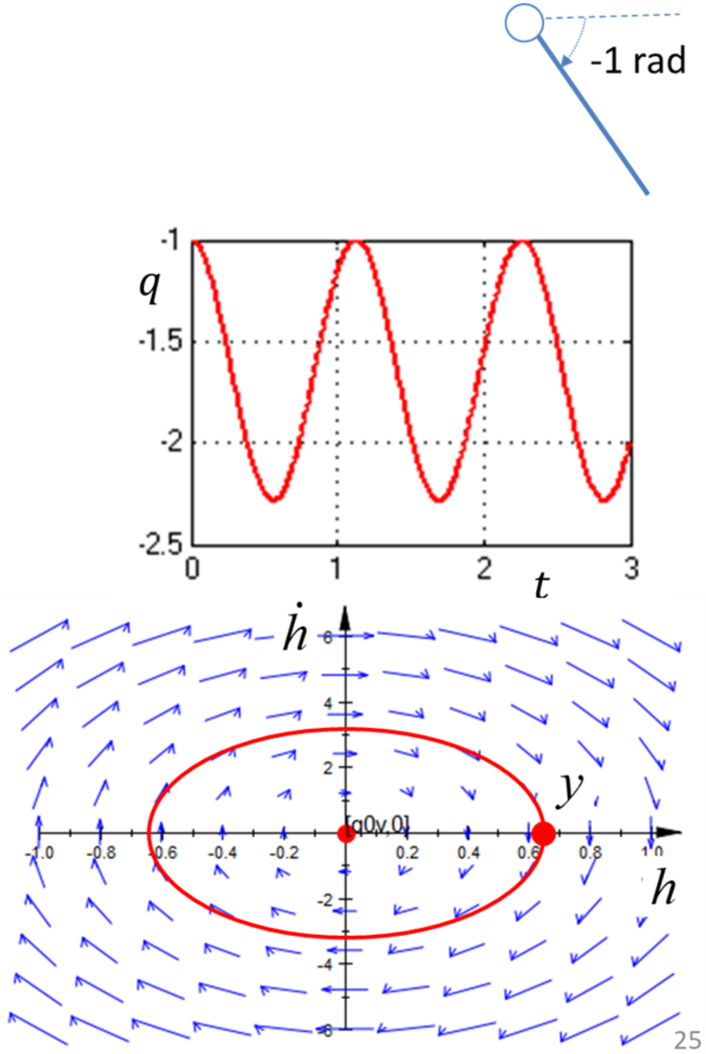
$$\lambda(\mathbf{A}) = \{-6.2716, \quad 6.2716, \quad 7.54i, \quad -7.54i\}$$

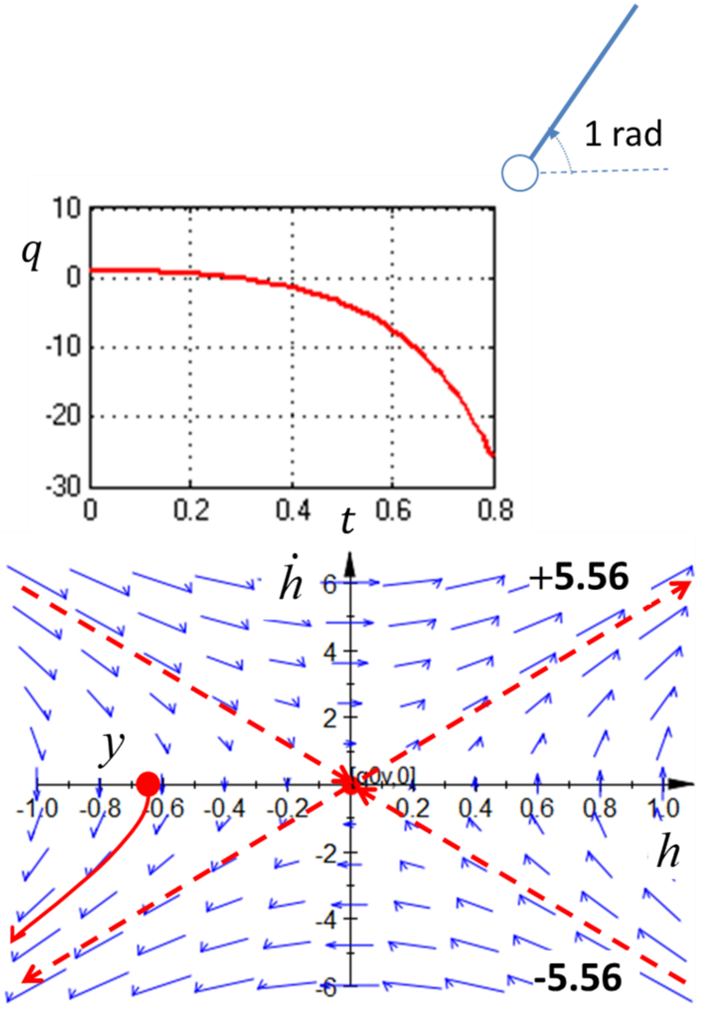
$$\tau = 0.15 \;, \omega = 7.55$$

- Analysing the poles it can be observed:
- Oscillatory behaviour for the Link 2 (SLM in the downright position)
- Unstable for the link 1 (SLM in the upright position)



DLM Linearisation Example



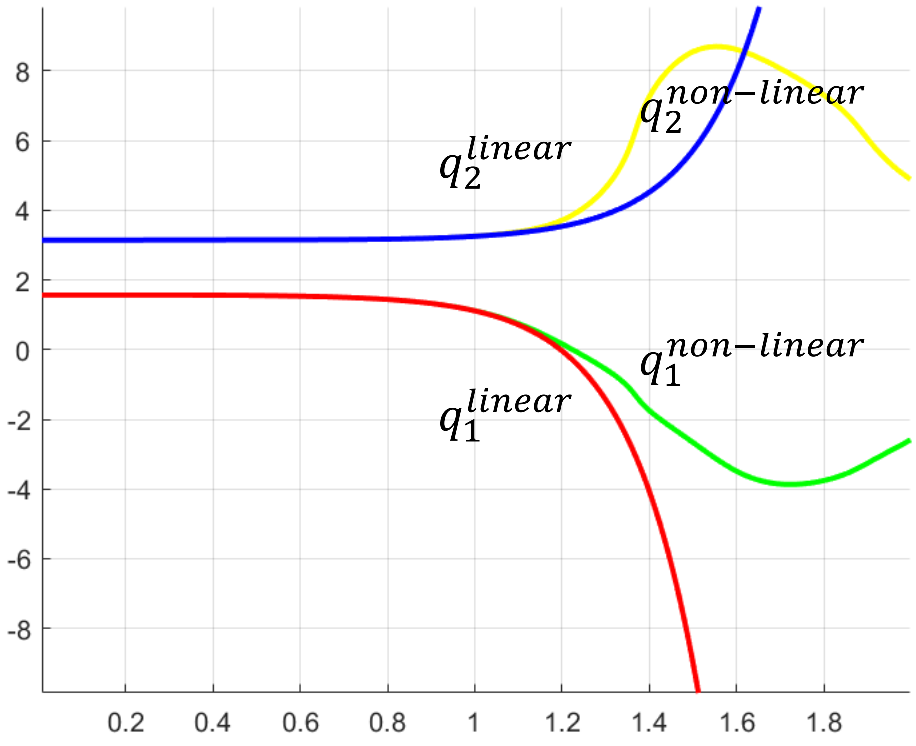


Linearized v Non-linear Comparison Simulation

Compare a linearized and a non-linear (unforced) simulation in MATLAB.

- Linearization point $\{\boldsymbol{q}^*, \boldsymbol{\tau}^*\}=\{[\pi/2,\pi,0,0], [0,0]\}$
- Initial conditions
  - Non-linear: $\mathbf{q}(0)=[\pi/2-0.003,\pi+0.003,0,0]$
  - Linear: $\Delta\mathbf{q}(0)=[-0.003,0.003,0,0]$
- As can be seen the unforced, non-linear plant behaves like before, the links begin to **circle** around in an **unpredictable fashion**
- The linearized model initially agrees quite well (for between ½ or 1 s) and then **exponentially diverges**, both the **states and errors**.
- This should be expected for an **unstable**, linear model

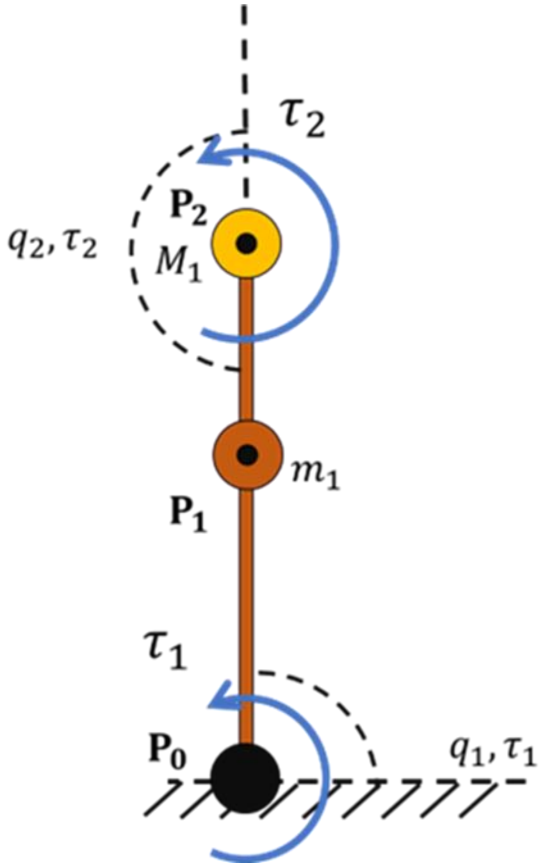
The evolution of the linear and nonlinear joint angles and the corresponding errors are shown below



Joint Angle Evolution

Conclusions

- The linearized model is a good approximation to the non-linear model for the DLM and can be used for linear, feedback control (PID) design
- One key insight is that the eigenvalues consist of an stable/unstable pair (Link 1) and an oscillatory pair (Link 2).
- In subsequent section, design a linear (PID) joint controller for each joint.



Thank you

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