Dual Link Manipulator

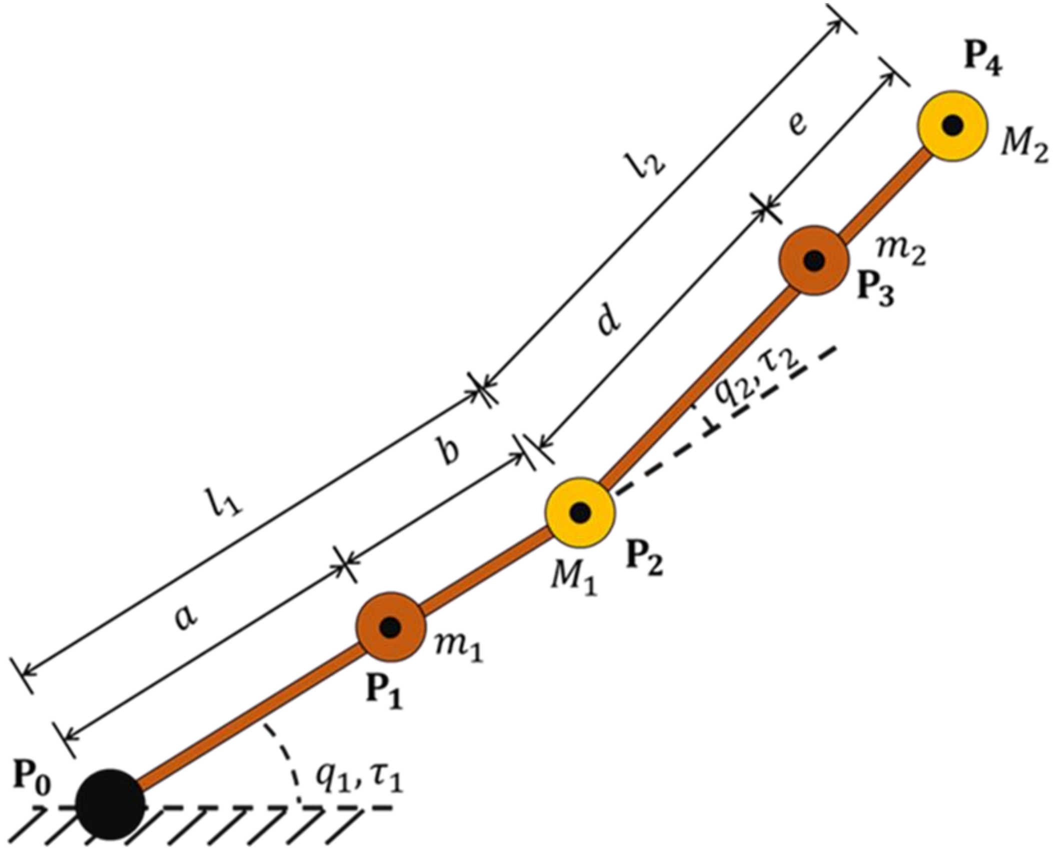
Joint Control

{Learn, Create, Innovate};

PID Joint Control

In this section we'll see how to design (PID) joint controllers for the link 1 and link 2 of the Dual Link Manipulator.

- Centralized & independent joint control.
- Link PID joint control for a torque, τ_2, located at $\mathbf{P}_2$.
- PID joint control for a torque, τ_1, located at $\mathbf{P}_0$.
- For each PID controller design:
  - Independent joint linearization
  - Unforced dynamics analysis
  - Design parameters using pole placement



Linear Centralized & Decentralized Control

Centralized (full state) control $\{\mathbf{q}, \dot{\mathbf{q}}\} \to \boldsymbol{\tau}$

- Full state / sensor information is available calculate the control signal (torque) applied to each joint
- Can be linear or non-linear.
- Potential to achieve precise joint control for high performance manoeuvres, although this requires accurate models

Decentralized (PID design for each joint) $\{q_i, \dot{q}_i\} \to \tau_i$

- Simpler to implement as it assumes each joint only has access to its own sensor readings
- Controller performance necessarily limited
- Performance / controller parameters depends on the position of the other (unknown) joints
- Discussed in this section

TIPAS MACHINE

Dual Link Manipulator

PID Joint Control

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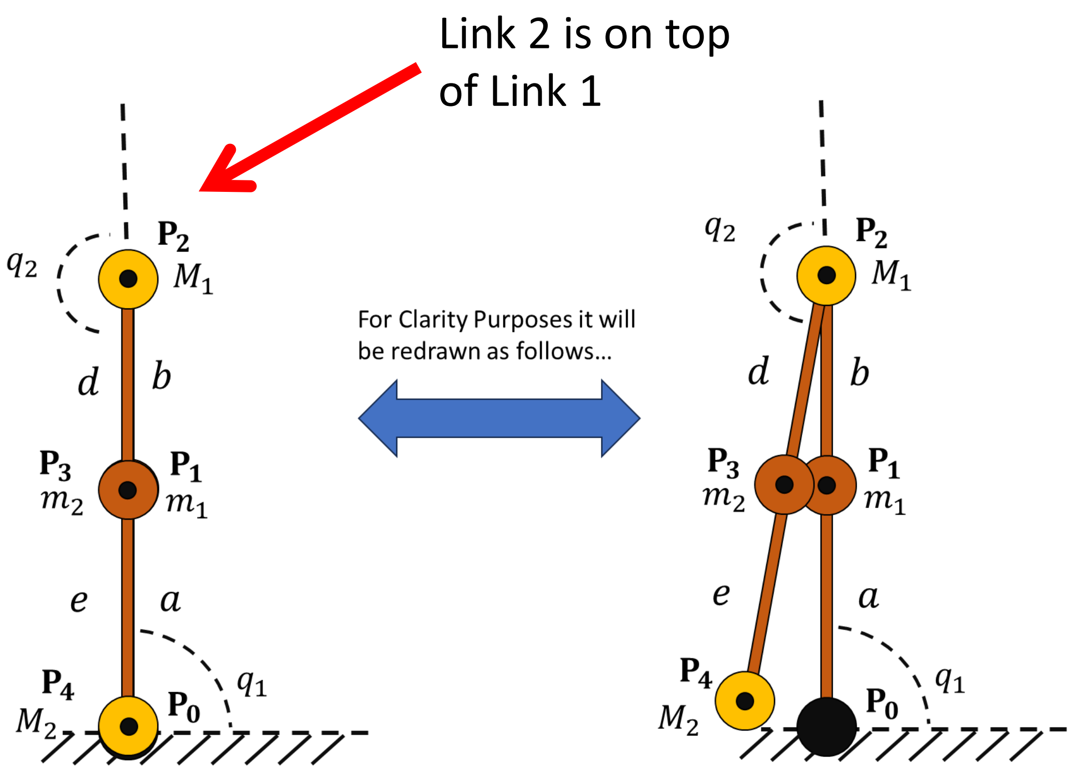
PID Joint Control of the Link 2

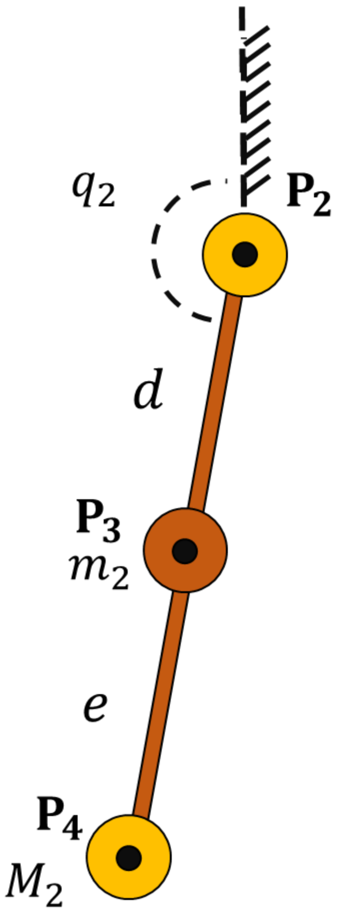
Introduction

- For this exercise a position (set-point or tracking) controller for the link 2 about $\mathbf{P}_2$ will be developed (without motor dynamics).
- Consider Link 1 Fixed, vertical and stationary
- Link 2 contains two masses $\mathbf{m}_2 \rightarrow \mathbf{P}_3$ and $\mathbf{M}_2 \rightarrow \mathbf{P}_4$
- Angle of link 2 is measured from the vertical q_2



PID Joint Control of the Link 2

- The system can be redrawn as a Single Link Manipulator (SLM), since the link 1 is fixed.
- Now the analysis becomes the same as with the simple pendulum system.



PID Joint Control of the Link 2

- Using Euler-Lagrange

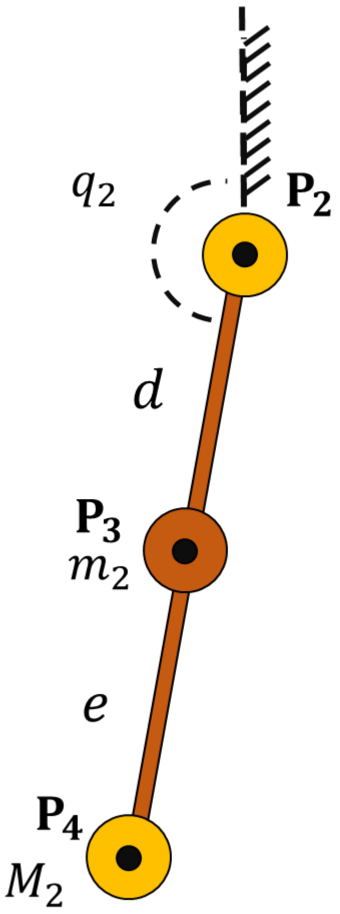
$$l_2 = d + e$$

$$\mathbf{P}_3 = d\begin{bmatrix}\sin q_2\\ \cos q_2\end{bmatrix} = d\begin{bmatrix}S_1\\ C_2\end{bmatrix}$$

$$\mathbf{P}_4 = l_2\begin{bmatrix}\sin q_2\\ \cos q_2\end{bmatrix} = l_2\begin{bmatrix}S_1\\ C_2\end{bmatrix}$$

Potential Energy Can then be defined:

$$PE = g\sum m_i h_i = g(m_2 d C_2 + M_2\, l_2 C_2)$$



PID Joint Control of the Link 2

- The kinetic Energy can be defined

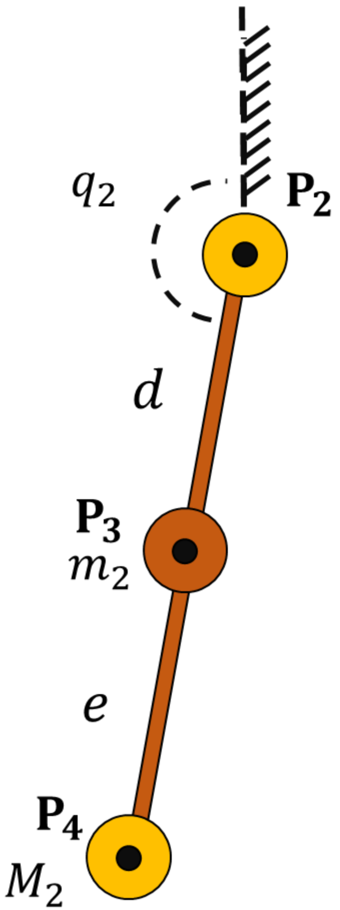
$$\dot{\mathbf{P}}_3 = d\begin{bmatrix} C_1 \\ -S_2 \end{bmatrix}\dot{q}_2 \qquad \dot{\mathbf{P}}_4 = l_2\begin{bmatrix} C_1 \\ -S_2 \end{bmatrix}\dot{q}_2$$

$$\left\|\dot{\mathbf{P}}_3\right\|_2^2 = d^2\dot{q}_2^2 \qquad \left\|\dot{\mathbf{P}}_4\right\|_2^2 = l_2^2\dot{q}_2^2$$

$$KE = \sum \frac{1}{2}m_i v_i^2 + \frac{1}{2}J_i\,\dot{q}_i^2$$

$$KE = \frac{1}{2}(m_2 d^2\dot{q}_2^2 + J_1\,\dot{q}_2^2 + M_2 l_2^2\dot{q}_2^2 + J_2\dot{q}_2^2)$$

$$KE = \frac{1}{2}(m_2 d^2 + J_1 + M_2 l_2^2 + J_2)\dot{q}_2^2$$



PID Joint Control of the Link 2

- The Lagrangian is obtained as:

$$L = KE - PE$$

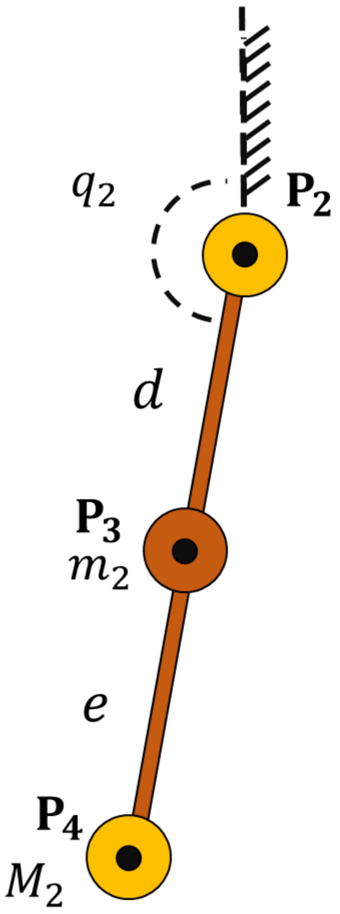
$$L = \frac{1}{2}(m_2 d^2 + J_1 + M_2 l_2 + J_2)\,\dot{q}_2^2 - g(m_2 d C_2 + M_2\, l_2 C_2)$$

$$\frac{\partial L}{\partial q} = g(m_2 d + M l_2) S_2$$

$$\frac{\partial L}{\partial \dot{q}} = -(m_2 d^2 + J_1 + M_2 l_2^2 + J_2)\,\dot{q}_2$$

$$Let\ J_1 = J_2 = 0$$

$$\frac{\mathrm{d}}{\mathrm{dt}}\left[\frac{\partial L}{\partial \dot{q}}\right] = (m_2 d^2 + M_2 l_2^2)\,\ddot{q}_2$$



PID Joint Control of the Link 2

- The Link 2 Model can then be defined as:

$$(m_2 d^2 + M_2 l_2^2)\,\ddot{q}_2 - g S_2 (m_2 d + M l_2) = \tau$$

- Linearising around $\mathrm{x}^* = [\pi, 0],\ \Rightarrow\ \ \tau^* = 0$

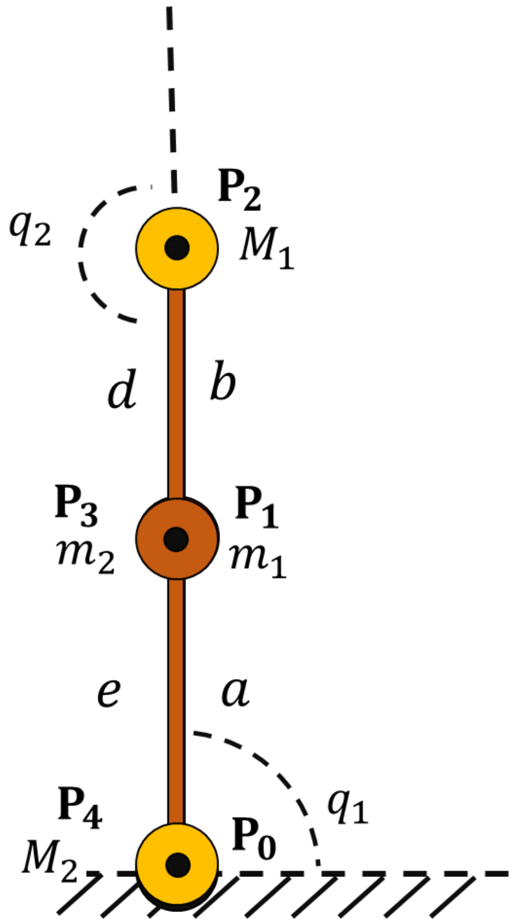
$$\Delta\ddot{q}_2 + \frac{g(m_2 d + M_2 l_2)}{m_2 d^2 + M_2 l_2^2}\Delta q_2 = \frac{1}{m_2 d^2 + M_2 l_2^2}\Delta\tau$$

$$\Delta\ddot{q}_2 = \frac{1}{m_2 d^2 + M_2 l_2^2}\left(-g(m_2 d + M_2 l_2)\,\Delta q_2 + \Delta\tau\right)$$

- Redefine $\Delta q = \mathrm{q}$ and $\Delta\tau = \tau$
- Using the laplace transform on the linearised system

$$\frac{q(s)}{\tau(s)} = \frac{1}{(m_2 d^2 + M_2 l_2^2)s^2 + g(m_2 d + M_2 l_2)}$$

$$s = \pm j\sqrt{g\frac{m_2 d + M_2 l_2}{m_2 d^2 + M_2 l_2^2}}$$



PID Joint Control of the Link 2

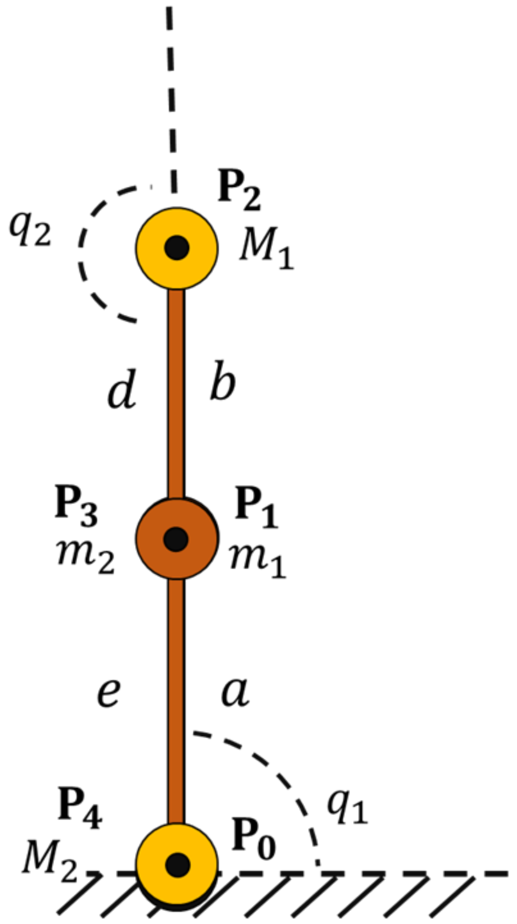
- Poles on the imaginary axis
- Sinusoidal Dynamics
- Angular frequency

$$\omega = \sqrt{g\frac{m_2 d + M_2 l_2}{m_2 d^2 + M_2 l_2^2}} \quad \frac{rad}{s}$$

- Time Period

$$T = \frac{2\pi}{\omega}$$

- Same behaviour of a Pendulum



PID Joint Control of the Link 2

PID Design

- The Linear system is given by

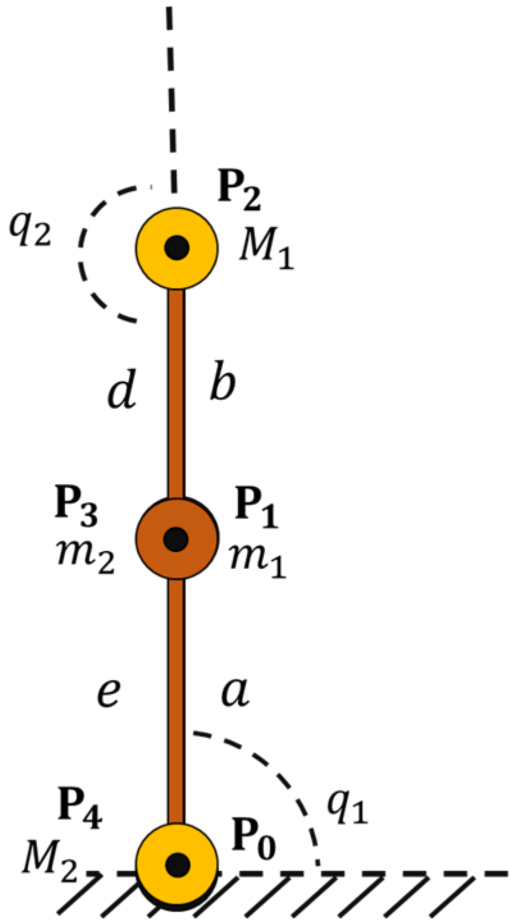
$$\ddot{q}_2 + \frac{g(m_2 d + M_2 l_2)}{m_2 d^2 + M_2 l_2^2} q_2 = \frac{1}{m_2 d^2 + M_2 l_2^2} \tau$$

- Let $\sigma_1 = m_2 d + M_2 l_2$ and $\sigma_2 = m_2 d^2 + M_2 l_2^2$
- Let the PID Controller to be described by

$$\tau = K_p\, e + K_i \int e\, dt + K_d\, \dot{e}$$

- Substituting

$$\ddot{q}_2 + g \frac{\sigma_1}{\sigma_2} q_2 = \frac{1}{\sigma_2} \left(K_p\, e + K_i \int e\, dt + K_d\, \dot{e}\right)$$



PID Joint Control of the Link 2

PID Design

- Derivating and using Laplace,

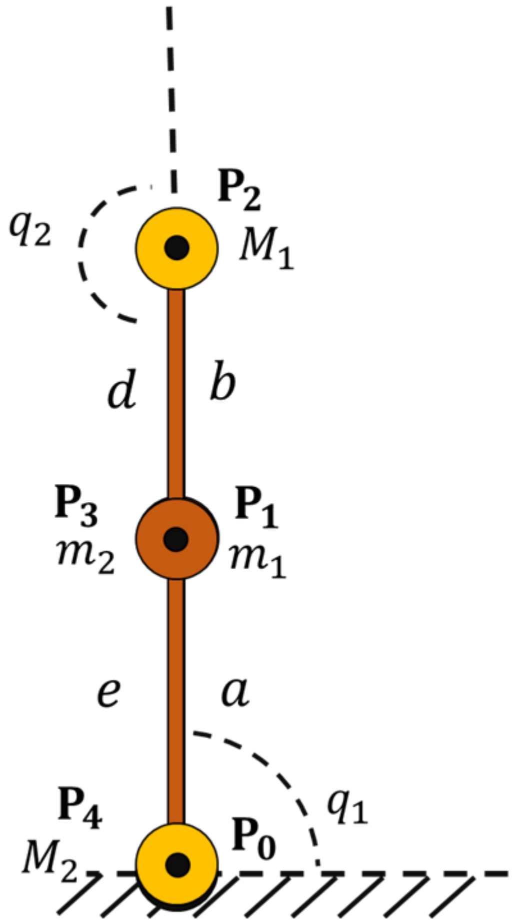
$$\ddot{q}_2 + g\frac{\sigma_1}{\sigma_2}q_2 = \frac{1}{\sigma_2}\left(K_p\, e + K_i\int e\, dt + K_d\, \dot{e}\right)$$

- Let the error be defined as $e = \mathrm{r} - \mathrm{q}_2$
- Deriving and Transform using Laplace

$$q_2(s)\left(s^3 + \frac{K_d}{\sigma_2}s^2 + \left(\frac{\sigma_1}{\sigma_2}g + \frac{K_p}{\sigma_2}\right)s + \frac{K_i}{\sigma_2}\right) = \frac{1}{\sigma_2}(K_d s^2 + K_p s + K_i)r(s)$$

- The denominator is then given by

$$s^3 + \frac{K_d}{\sigma_2}s^2 + \left(\frac{\sigma_1}{\sigma_2}g + \frac{K_p}{\sigma_2}\right)s + \frac{K_i}{\sigma_2}$$



PID Joint Control of the Link 2

PID Design

- The system must be operating on the downright region
- We can neglect $\dot{r}$ in $\dot{e}$ to eliminate the zero
- Choosing K_p, K_i, K_d changes the location of the poles
- The chosen poles are

$$s = -10, -15, -20$$

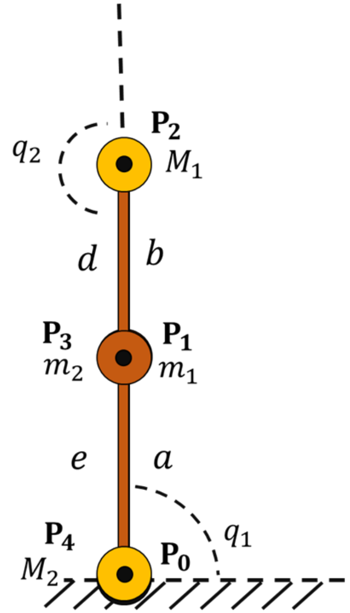
- Time constant $\tau = 0.1, 0.067, 0.05\ s$ respectively

$$(s+10)(s+15)(s+20) = s^3 + 45s^2 + 650\,s + 3000$$

$$\frac{K_d}{\sigma_2} = 45, \qquad \frac{\sigma_1}{\sigma_2}g + \frac{K_p}{\sigma_2} = 650, \qquad \frac{Ki}{\sigma_2} = 3000$$

$$K_d = 45\sigma_2, \qquad K_P = \sigma_2\left(650 - \frac{\sigma_1}{\sigma_2}g\right), \qquad Ki = 3000\sigma_2$$

- Low Time constant for manipulators
- Different poles to make the system "robust"
- Relatively large values of PID gains due to the "fast" response needed.



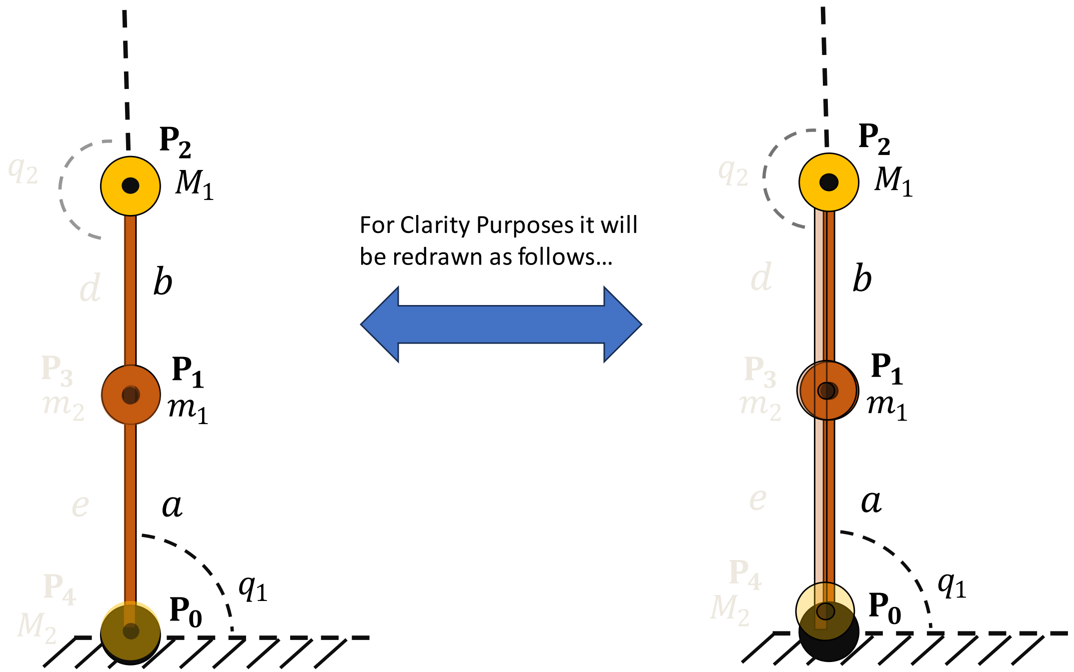
PID Joint Control of the Link 1

PID Design for Link 1

- Similar Procedure as for Link 2
- Assume Link 2 is rigidly coupled to Link 1 (one on top of the other Link2 does not move like a pendulum with extra weight) $q_2 = \pi$ and $\mathbf{P}_1 \equiv \mathbf{P}_3$
- Assume $m_1 = m_2$ (link masses) are equal
- The model using Euler-Lagrange is

$$(2m_1 a^2 + M_1 l_1^2)\,\ddot{q}_1 + g(2m_1 a + M_1 l_1)\cos(q_1) = \tau$$

- Linearising around $\mathrm{x}^* = \left[\frac{\pi}{2}, 0\right] \Rightarrow \tau^* = 0$



PID Joint Control of the Link 1

PID Design for Link 1

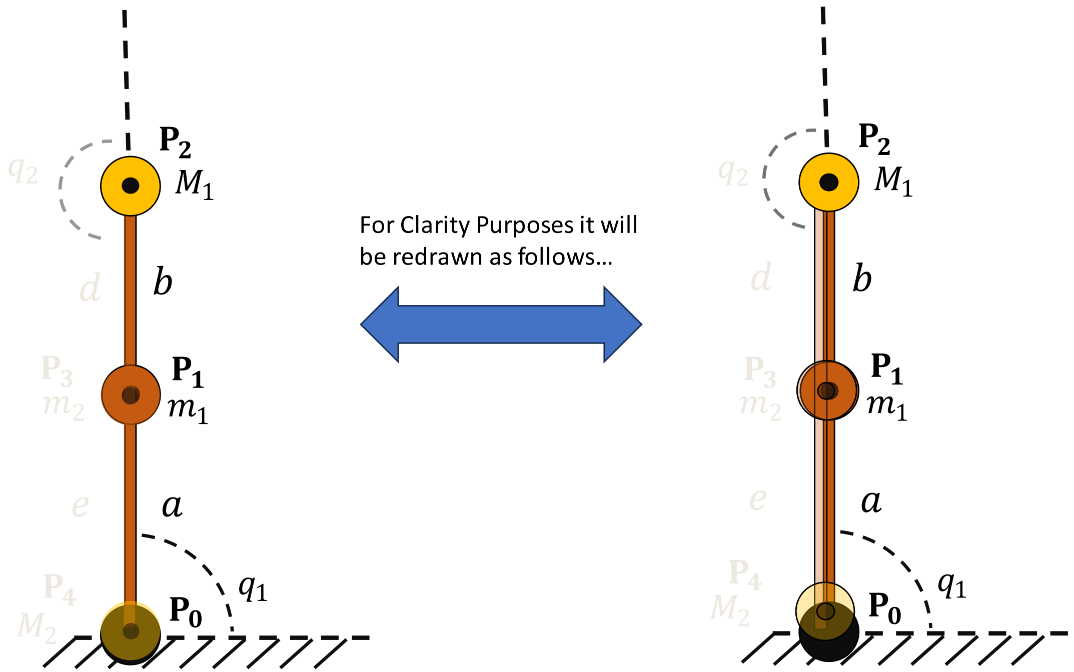
- Linearising around $\mathrm{x}^* = \left[\frac{\pi}{2}, 0\right] \Rightarrow \tau^* = 0$
- Let $\Delta q_1 = q_1$ and $\Delta\tau = \tau$

$$(2m_1 a^2 + M_1 l_1^2)\,\ddot{q}_1 + g(2m_1 a + M_1 l_1) q_1 = \tau$$

- The natural (autonomous, unforced) response has poles at

$$s \pm \sqrt{\frac{g(2m_1 a + M_1 l_1)}{(2m_1 a^2 + M_1 l_1^2)}}$$

- Unstable dynamics
- Time constant $\tau = \frac{1}{s} sec$ (not the torque)



PID Joint Control of the Link 1

- The chosen poles are

$$s = -10, -15, -20$$

- Time constant $\tau = 0.1, 0.067, 0.05\ s$ respectively

$$(s+10)(s+15)(s+20) = s^3 + 45s^2 + 650\ s + 3000$$

- Let the PID Controller to be described by

$$\tau = K_p\ e + K_i \int e\ dt + K_d\ \dot{e}$$

- After Laplace, the denominator of the TF for the linearised system

$$s^3 + \frac{K_d}{\sigma_2} s^2 + \left(\frac{K_P}{\sigma_2} - \frac{\sigma_1}{\sigma_2} g\right) s + \frac{K_i}{\sigma_2}$$

$$\sigma_1 = g(2m_1 a + M_1 l_1)$$

$$\sigma_2 = (2m_1 a^2 + M_1 l_1^2)$$

- The gains are

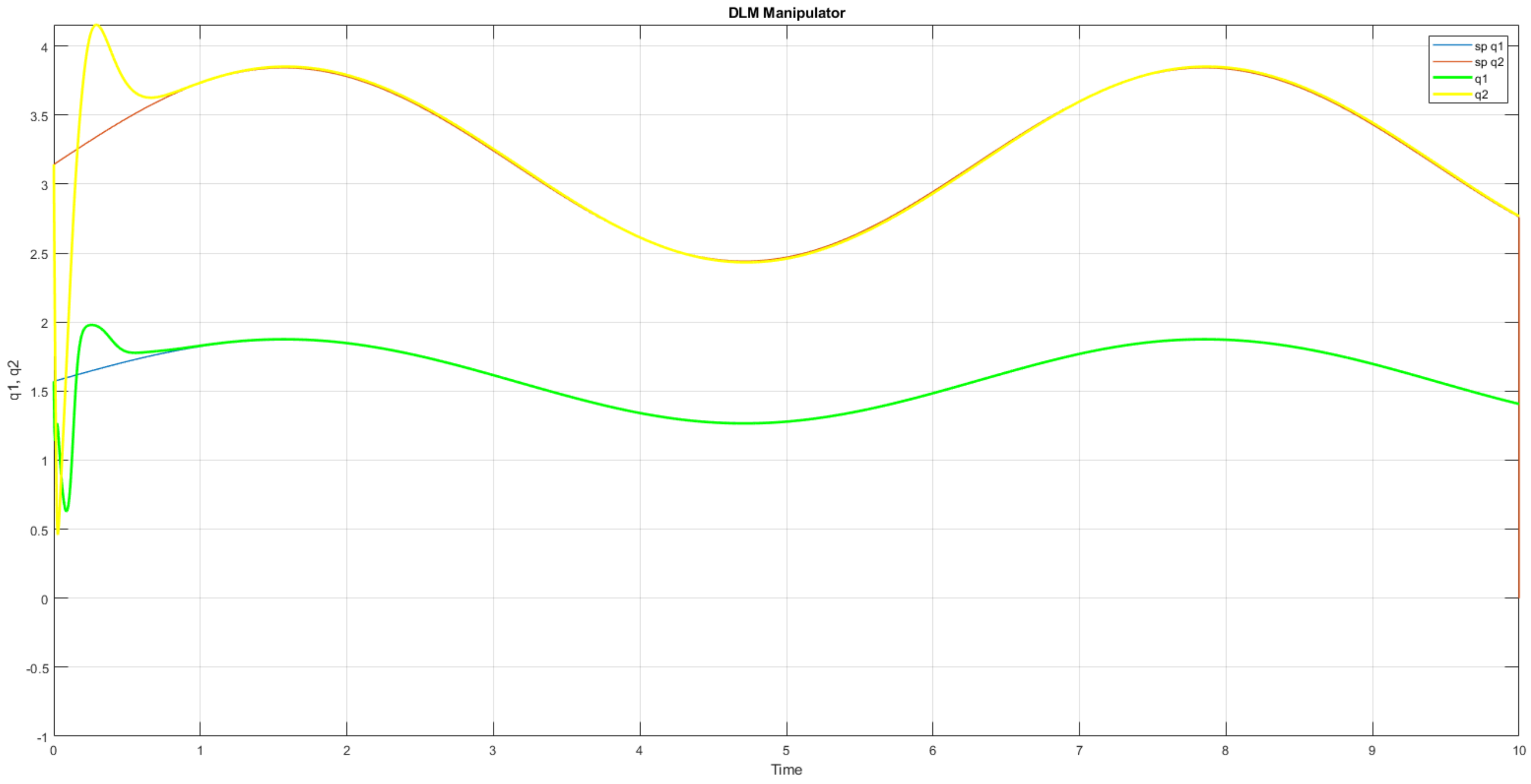
$$K_d = 45\sigma_2, \qquad K_P = \sigma_2\left(650 + \frac{\sigma_1}{\sigma_2} g\right), \qquad Ki = 3000\sigma_2$$

- Larger gains due to the larger inertia

- Design for the worst-case scenario (one link on op of the other)

Results

- Model parameters
- % Model Parameters
- m_1 = 3;
- M_1 = 1.5;
- m_2 = 3.0;
- M_2 = 1.0;
- a = 0.2;
- b = 0.2;
- d = 0.2;
- e = 0.2;
- l_1 = a+b;
- l_2 = d+e;
- g = 9.8;



Observations

- The controllers have assumptions
  - The other link is “fixed” w.r.t the link being controller (no interaction)
- The simulation and parameter selection was done with linearised model plants
- The design assumptions are not untypical for manipulators
- Joint gains are set heuristically (most of the time)
- Performance is validated, by performing tests on the real system, and investigating regions of poor performance
- Set points to be tested will lie at the limits of the linearised region.

Conclusions

- Independent PID was designed for a DLM.
- The joint angle values are not too large, so a linear analysis is valid.
- The interactions between joints is not too large, therefore and independent PID can be used.
- When using pole placement, we must be aware of the actuators (they must be able to withstand the input requirements by the controller)
- Torques for link 1 are larger than for ink 2 (as expected)

Dual Link Manipulator

Computer Torque Control (CTC)

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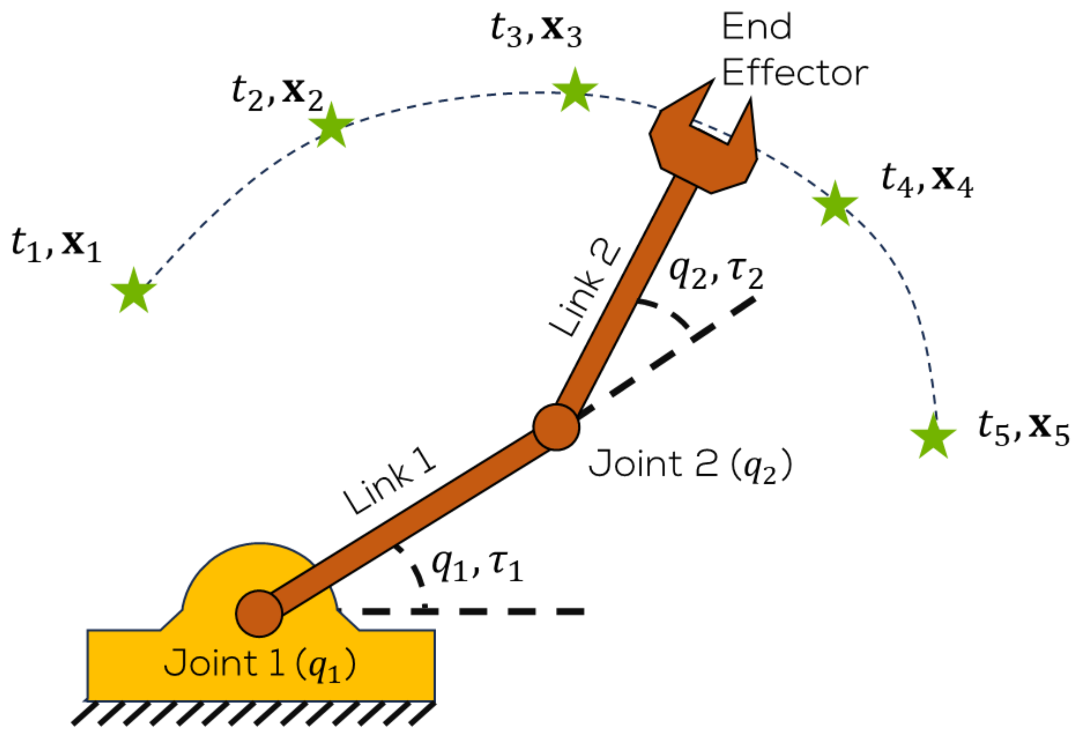
Computer Toque Control

- Manipulators follow trajectories from point A to point B
- Contrary to other types of setpoints, trajectories are
  - Smooth functions (at least twice derivable)
  - Describe position, velocities accelerations and time.
  - No discontinuities



Computer Toque Control

- PID control is reactive
- Integral requires time to build up before steady state is achieved
- Small errors cause the proportional part to react slower
- Mainly used for linear systems
- Much of the torque is used to compensate for gravity effects and to accelerate the joints.

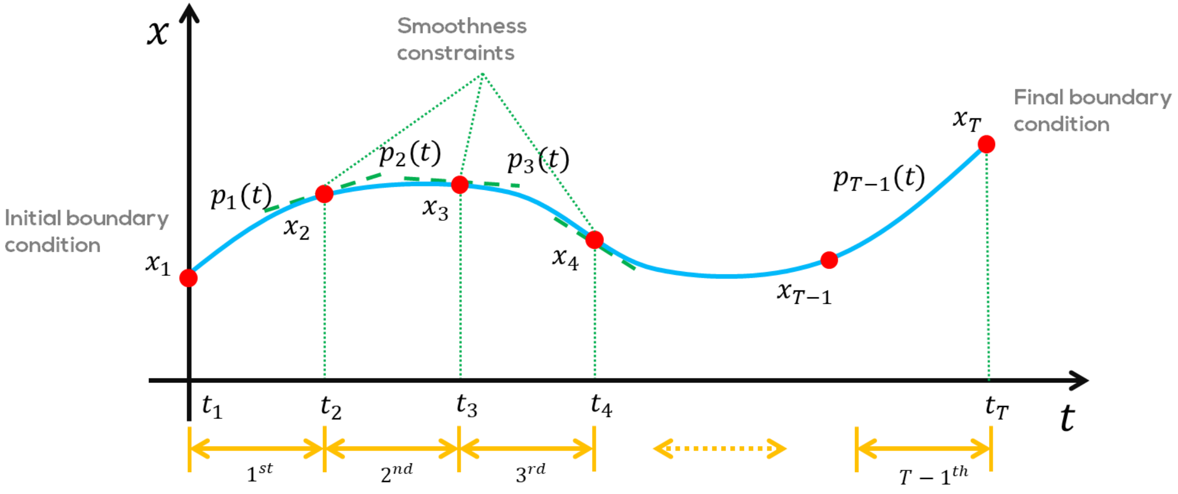
- We can use the model of the robot since we usually have a "decent" one

…. Maybe …

- We can use the model to estimate the best control signal fed into the system…

Computer Toque Control

- It's widely used in robotics
- Uses a trajectory as setpoint
- Since the trajectory is a smoot function and can be derived, the acceleration can be used as a setpoint.
- CTC is an example of a type of controllers used to perform feedback linearisation.
- These types of controllers compensates the nonlinear dynamics of the system by using the model information to try "linearise" the system.
- The user requires the full knowledge of the system dynamics (or at least have a good model)
- This structure allows the design of more robust controllers.



CTC Design

- Let the system be described by:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \boldsymbol{\tau}$$

- Let the tracking error be defined as:

$$\mathbf{e}(\mathrm{t}) = \mathbf{q}^*(\mathrm{t}) - \mathbf{q}(\mathrm{t})$$

Where $\mathbf{q}^*(\mathrm{t})$ are the desired angles

Therefore

$$\dot{\mathbf{e}}(\mathrm{t}) = \dot{\mathbf{q}}^*(\mathrm{t}) - \dot{\mathbf{q}}(\mathrm{t})$$
$$\ddot{\mathbf{e}}(\mathrm{t}) = \ddot{\mathbf{q}}^*(\mathrm{t}) - \ddot{\mathbf{q}}(\mathrm{t})$$

- Substitutng

$$\ddot{\mathbf{e}}(\mathbf{t}) = \ddot{\mathbf{q}}^*(\mathrm{t}) - \left(\mathbf{M}^{-1}\left(\boldsymbol{\tau} - \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} - \mathbf{g}(\mathbf{q})\right)\right)$$

$$\ddot{\mathbf{e}}(\mathbf{t}) = \ddot{\mathbf{q}}^*(\mathrm{t}) + \left(\mathbf{M}^{-1}\left(-\boldsymbol{\tau} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q})\right)\right)$$

- Define

$$\boldsymbol{\tau} = \mathbf{M}(\ddot{\mathbf{q}}^* - \mathbf{u}) + \mathbf{C}\dot{\mathbf{q}} + \mathbf{g}$$

- Then substituting on the error function

$$\ddot{\mathbf{e}}(\mathbf{t}) = \ddot{\mathbf{q}}^*(\mathrm{t}) + \left(\mathbf{M}^{-1}\left(-\mathbf{M}(\ddot{\mathbf{q}}^* - \mathbf{u}) - \mathbf{C}\dot{\mathbf{q}} - \mathbf{g} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{g}\right)\right)$$

$$\therefore \ddot{\mathbf{e}}(\mathbf{t}) = \mathbf{u}$$

- Selection of $\mathbf{u}$ controls the dynamics of the error

CTC Design

In short:

- CTC Inverts system's dynamics
  - (careful with nonminimum phase systems)
- The selection of $\mathbf{u}$ canbe done in many ways (Typically "PD" to make the error 2nd order):

$$\ddot{\mathbf{e}}(\mathbf{t}) = \mathbf{u}$$

- Let $\mathbf{u} = -\mathbf{K_d}\,\dot{\boldsymbol{e}} - \boldsymbol{K_p}\mathbf{e}$
- Then:

$$\ddot{\mathbf{e}}(\mathbf{t}) + \mathbf{K_d}\,\dot{\boldsymbol{e}} + \boldsymbol{K_p}\mathbf{e} = \mathbf{0}$$

- Poles of the error are determined by $\mathbf{K_d}$ and $\mathbf{K}_p$
- Error will converge to zero
- $\mathbf{K_d}$ and $\mathbf{K}_p$ are diagonal (decoupled error)
- Poles can be chosen

$$(\mathrm{s} - \mathrm{p}_1)(\mathrm{s} - \mathrm{p}_2) = 0$$

- Typical requirements: Fast non-oscillatory response (critically damped)

$$\mathrm{p}_1 = \mathrm{p}_2 = -1/\tau^*$$

  - τ^* is the desired time constant

CTC Design

Example

- Let $\tau^* = 0.1s$ (Desired time constant)

- Therefore, dynamics are given by:

$$(s + 10)(s + 10) = 0$$

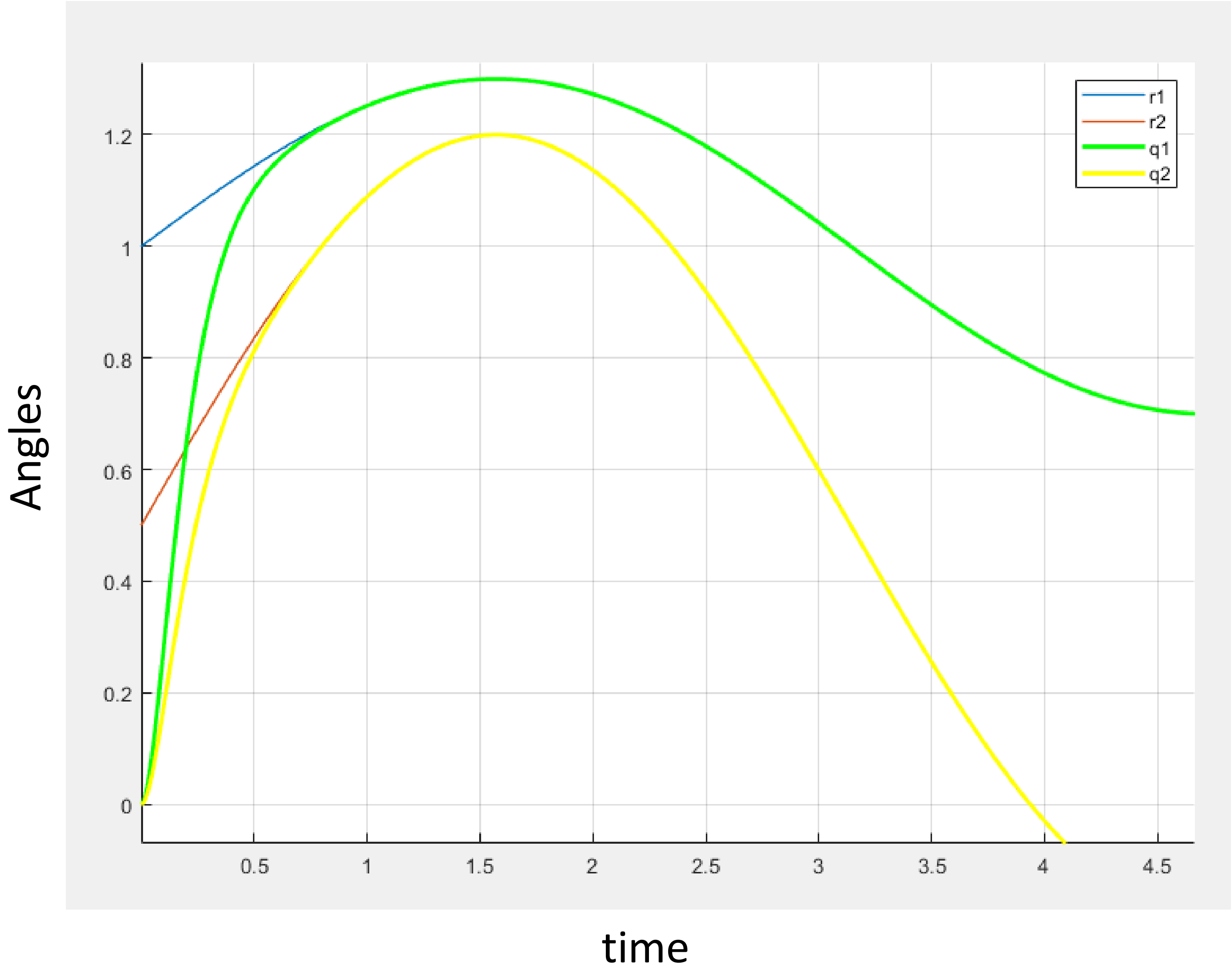
$$s^2 + 20s + 100$$

- Since:

$$\ddot{\mathbf{e}}(\mathbf{t}) + \mathbf{K_d}\,\dot{e} + \boldsymbol{K_p}\mathbf{e} = 0$$

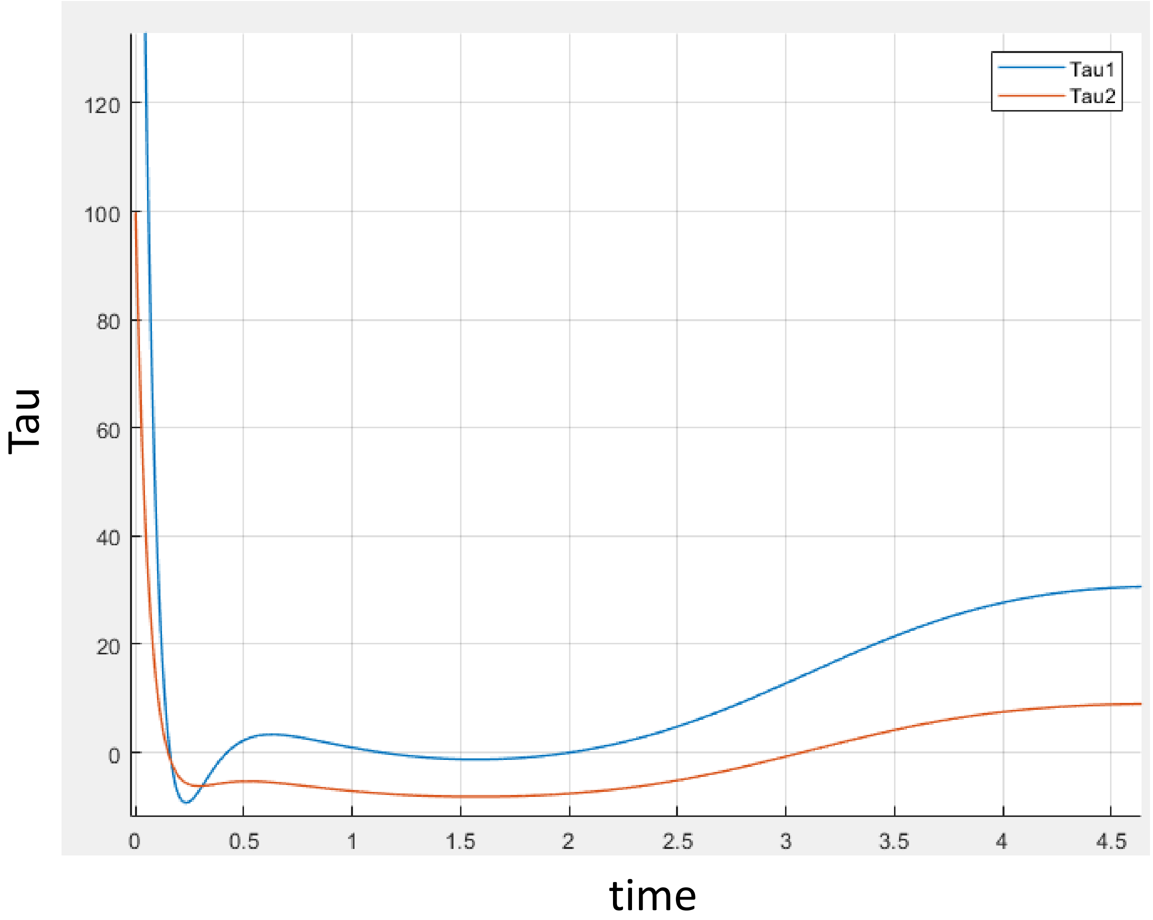
$$\mathbf{K_d} = 20$$

$$\mathbf{K}_{\boldsymbol{p}} = 100$$

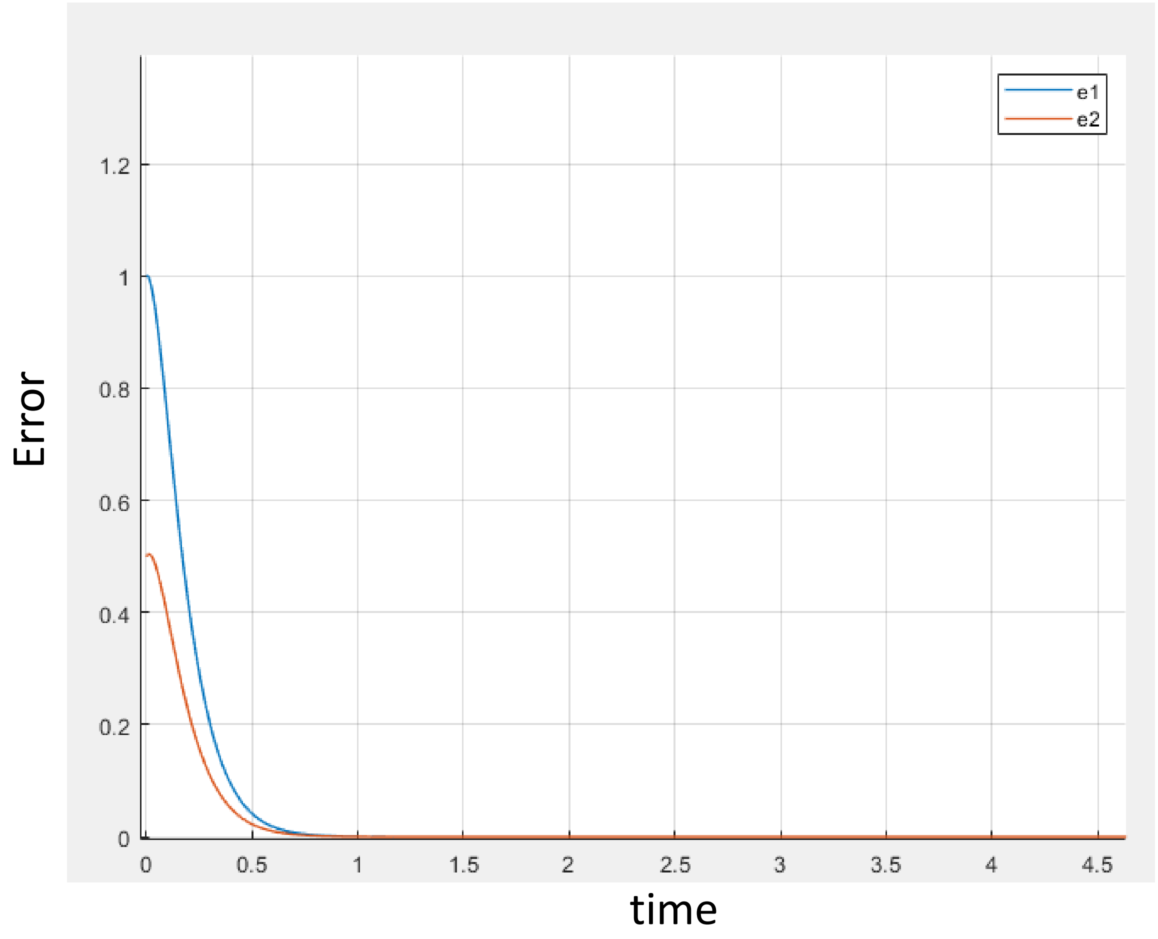


CTC Design

- Joint is tracking the set point
- Control torques look smaller
- Less propagation of interaction effects compared to joint PID
- Easier to design
- No linearizing assumptions
- Work at any operating point



CTC Design



- No interaction effects
- Errors go exponentially to zero after ~0.5 s (desired time constant is 0.1 s).
- Note that the initial errors (transient) are larger.
- The corresponding torques are smaller.

Conclusions

- Torque consists of desired reference accelerations, decoupled using the mass inertia matrix. This enables the control signal to respond instantly, rather than having to integrate error
- Easy to specify response in terms of desired error dynamics

What is the problem?

- We've assumed perfect model knowledge in the torque calculation. Feedback control takes the opposing view.
- Model dynamics error analysis would show that there exists an non-zero error with this type of control and extra feedback-type schemes must be used (outside the scope)
- Compared to joint PID control, the control scheme is more complex to implement (not so important nowadays), and a centralized control architecture (sensors, network, computing) must be used.

Thank you

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