Model Based Localisation

Worked Example

Problem Statement

Estimate the position of the robot for two-time steps, i.e., μ_1, μ_2 and Σ_1, Σ_2.

$$\boldsymbol{\mu}_0 = \begin{bmatrix} s_{x,0} \\ s_{y,0} \\ s_{\theta,0} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$ robot initial position

$$\boldsymbol{\Sigma}_0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$ initial covariance matrix

Assume the following conditions remain constant $\forall\, k$

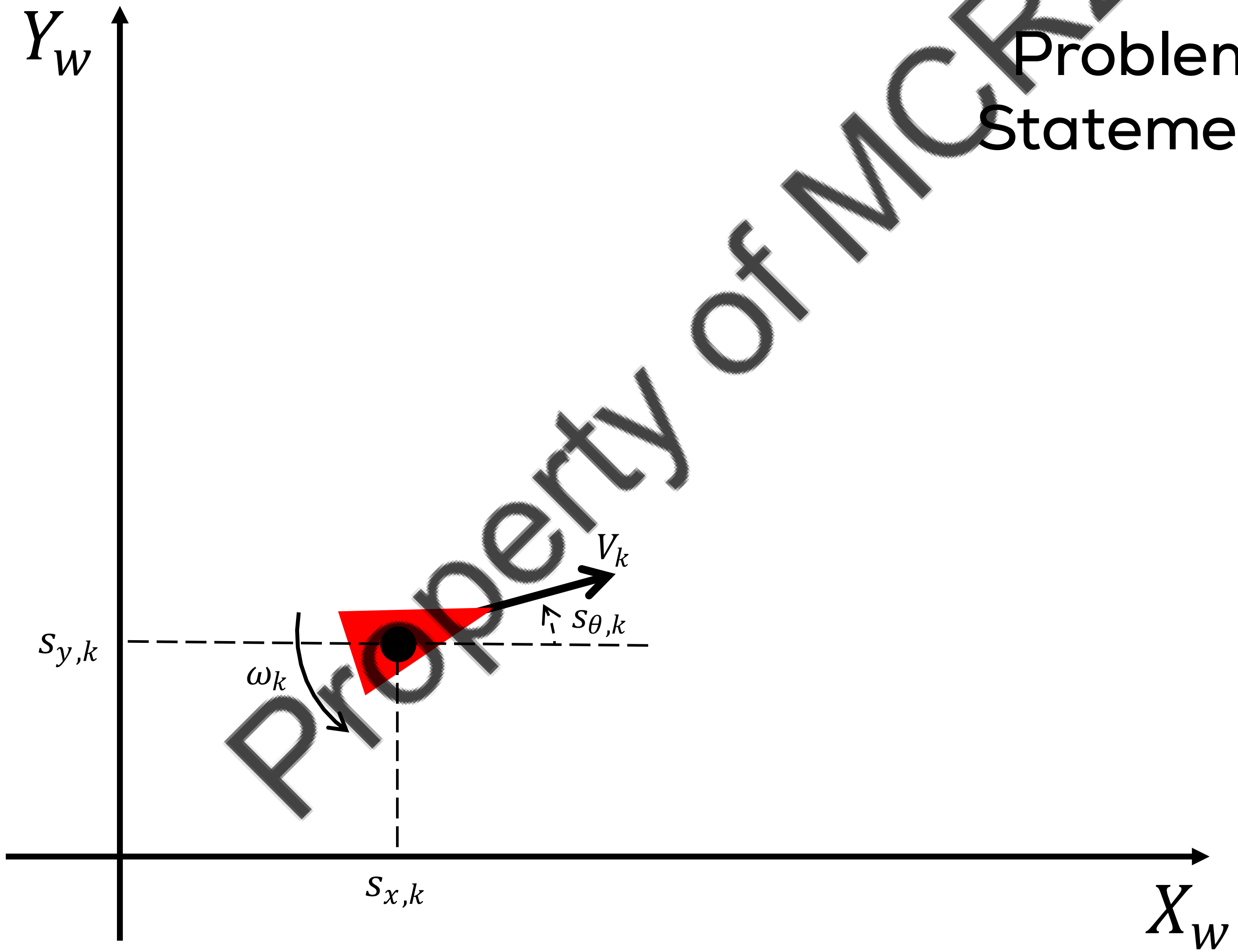
$$\boldsymbol{Q}_k = \begin{bmatrix} 0.5 & 0.01 & 0.01 \\ 0.01 & 0.5 & 0.01 \\ 0.01 & 0.01 & 0.2 \end{bmatrix}$$ motion model covariance matrix

$V_k = 1m/s$ mobile robot linear velocity

$\omega_k = 1rad/s$ mobile robot angular velocity

$\Delta t = 0.1s$ sampling time

Problem Statement



Problem Statement

Solution: Initial Conditions

Estimate the position of the robot for two timesteps, i.e., μ_1, μ_2 and Σ_1, Σ_2.

$$\boldsymbol{\mu}_0 = \begin{bmatrix} s_{x,0} \\ s_{y,0} \\ s_{\theta,0} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$ robot initial position

$$\boldsymbol{\Sigma}_0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$ initial covariance matrix

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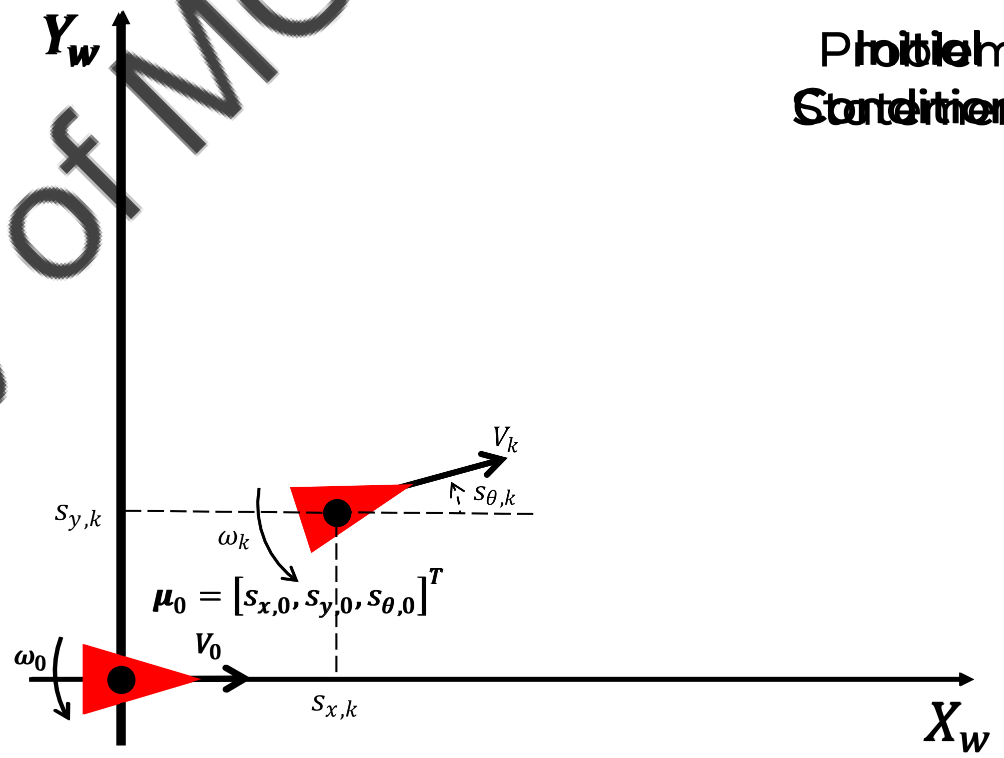
$\Delta t = 0.1s$ sampling time



Problem Statement

Initial Conditions

Solution: Iteration 1

Calculate the estimated position of the robot

$$\hat{\mu}_1 = \mathbf{h}(\boldsymbol{\mu}_0, \mathbf{u}_1)$$

$$\hat{\mu}_1 = \begin{bmatrix} \hat{s}_{x,1} \\ \hat{s}_{y,1} \\ \hat{s}_{\theta,1} \end{bmatrix} = \begin{bmatrix} s_{x,0} + \Delta t \cdot V_1 \cdot \cos(s_{\theta,0}) \\ s_{y,0} + \Delta t \cdot V_1 \cdot \sin(s_{\theta,0}) \\ s_{\theta,0} + \Delta t \cdot \omega_1 \end{bmatrix} = \begin{bmatrix} 0 + 0.1 \cdot 1 \cdot \cos(0) \\ 0 + 0.1 \cdot 1 \cdot \sin(0) \\ 0 + 0.1 \cdot 1 \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0 \\ 0.1 \end{bmatrix}$$

Calculate the linearized model to be used in the uncertainty propagation.

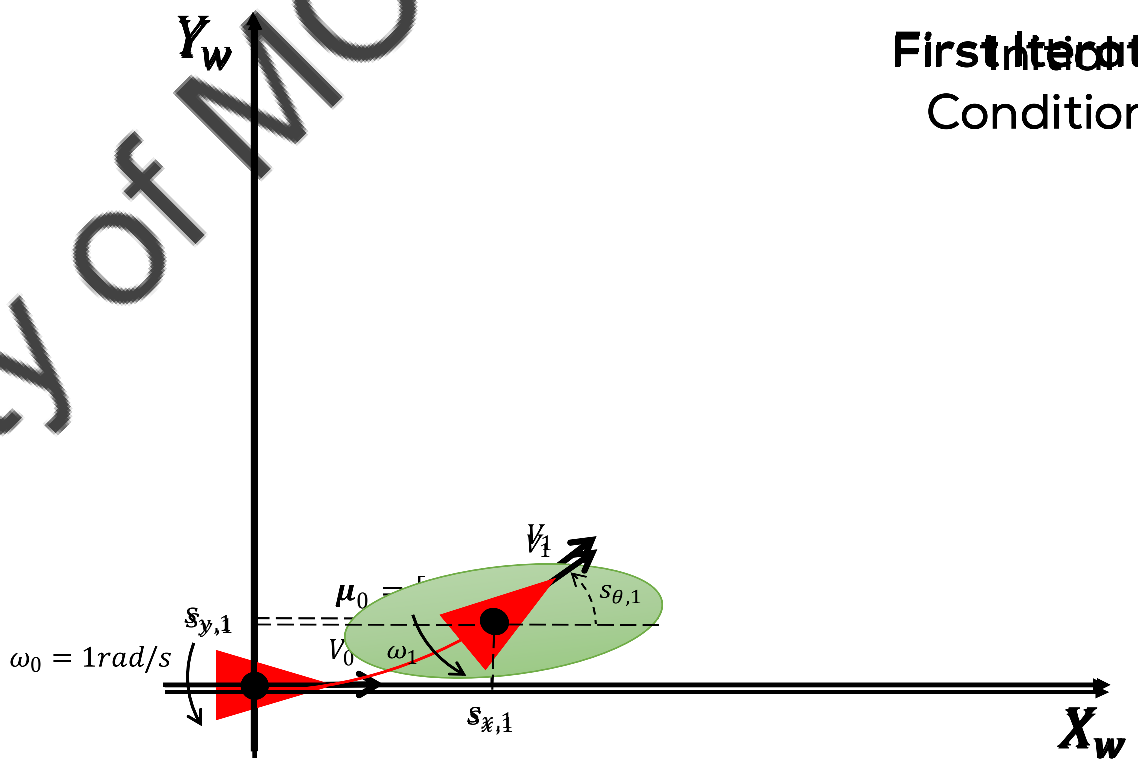
$$\boldsymbol{H}_1 = \begin{bmatrix} 1 & 0 & -\Delta t \cdot V_1 \cdot \sin(s_{\theta,0}) \\ 0 & 1 & \Delta t \cdot V_1 \cdot \cos(s_{\theta,0}) \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0.1 \\ 0 & 0 & 1 \end{bmatrix}$$

Calculate the propagation of the uncertainty

$$\hat{\boldsymbol{\Sigma}}_1 = \mathbf{H}_1 \cdot \boldsymbol{\Sigma}_0 \cdot \mathbf{H}_1^T + \mathbf{Q}_1$$

$$\hat{\boldsymbol{\Sigma}}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0.1 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0.1 & 1 \end{bmatrix} + \begin{bmatrix} 0.5 & 0.01 & 0.01 \\ 0.01 & 0.5 & 0.01 \\ 0.01 & 0.01 & 0.2 \end{bmatrix}$$

$$\hat{\boldsymbol{\Sigma}}_1 = \begin{bmatrix} 0.5 & 0.01 & 0.01 \\ 0.01 & 0.5 & 0.01 \\ 0.01 & 0.01 & 0.2 \end{bmatrix}$$

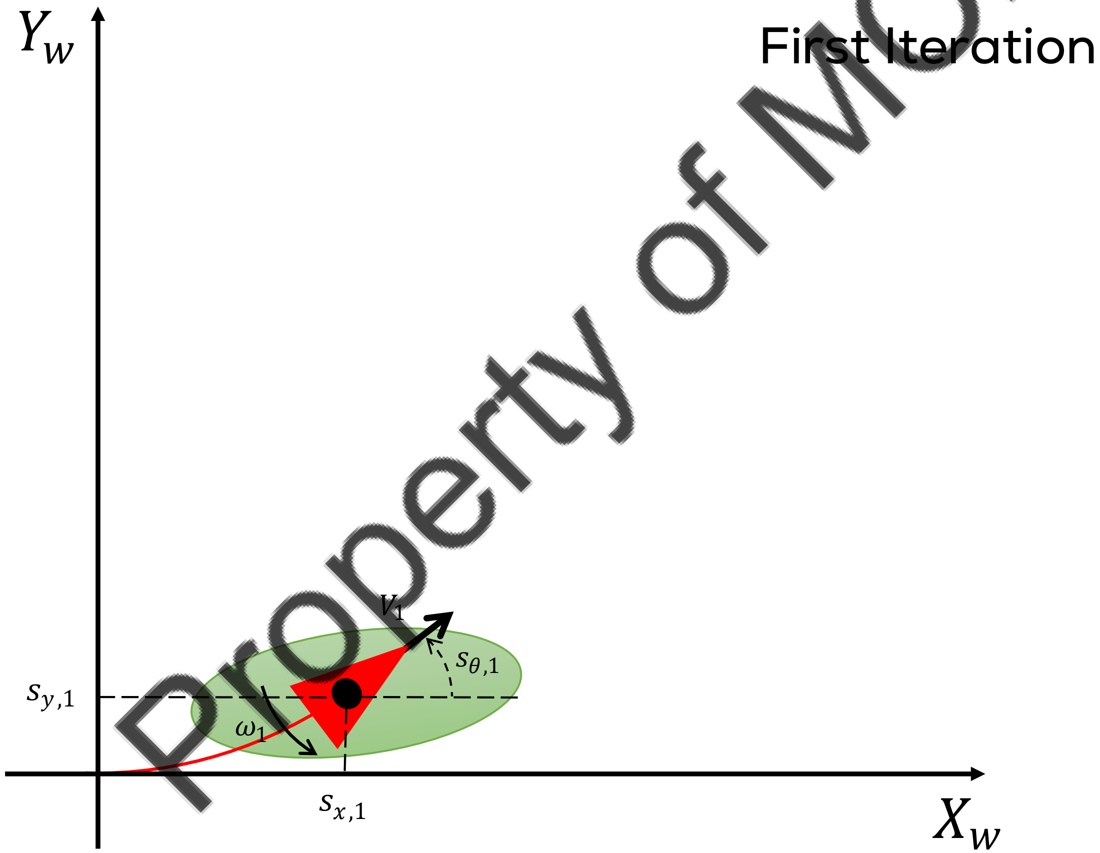


First Iteration

Initial Conditions

Figure Not To Scale

Solution: Iteration 1



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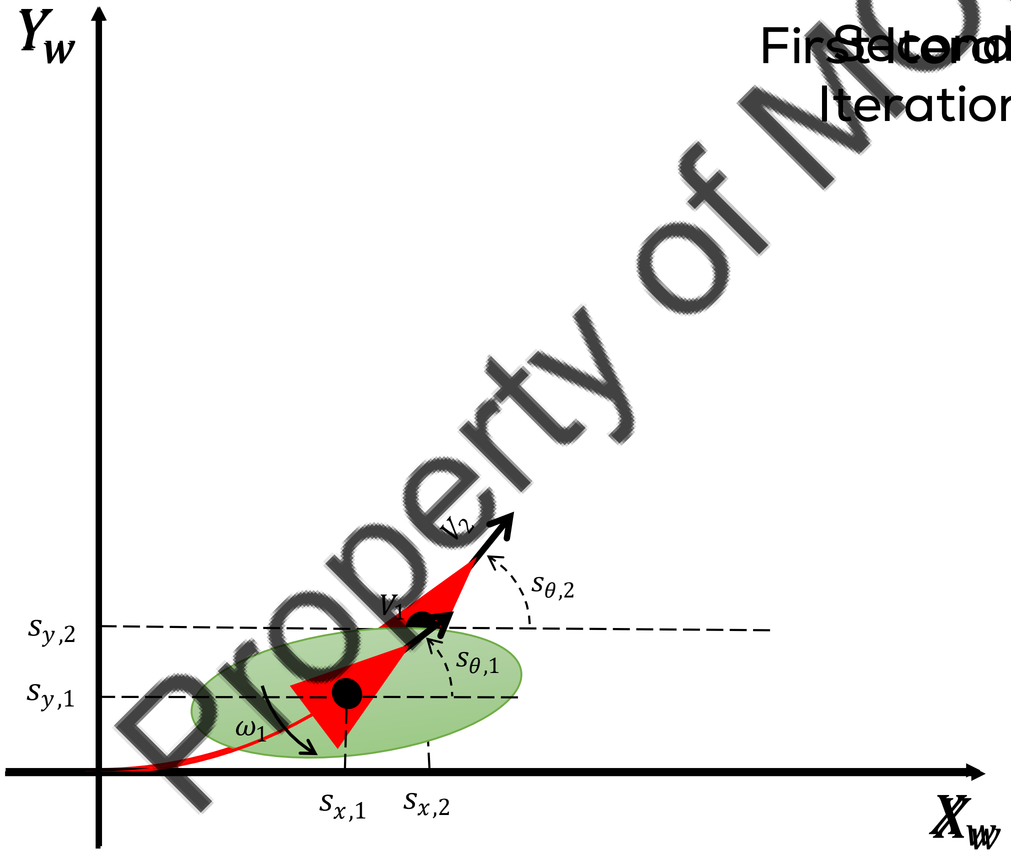
Solution: Iteration 2



First Iteration

Second Iteration

Figure Not To Scale

Figure Not To Scale

Solution: Iteration 2

$$\boldsymbol{\mu}_1 = \begin{bmatrix} s_{x,1} \\ s_{y,1} \\ s_{\theta,1} \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0 \\ 0.1 \end{bmatrix}$$

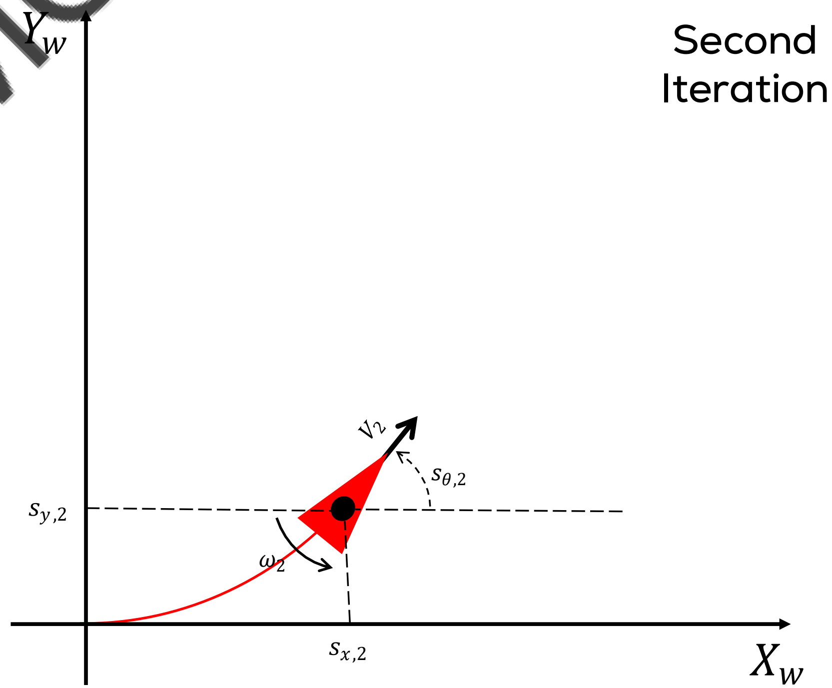
$$\boldsymbol{\Sigma}_1 = \begin{bmatrix} 0.5 & 0.01 & 0.01 \\ 0.01 & 0.5 & 0.01 \\ 0.01 & 0.01 & 0.2 \end{bmatrix}$$ covariance matrix (previous step)

$V_2 = 1m/s$ mobile robot linear velocity

$\omega_2 = 1rad/s$ mobile robot angular velocity

$\Delta t = 0.1s$ sampling time

Second Iteration



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Solution: Iteration 2

Calculate the estimated position of the robot

$$\hat{\boldsymbol{\mu}}_2 = \mathbf{h}(\boldsymbol{\mu}_1, \mathbf{u}_2)$$

$$\hat{\boldsymbol{\mu}}_2 = \begin{bmatrix} \hat{s}_{x,2} \\ \hat{s}_{y,2} \\ \hat{s}_{\theta,2} \end{bmatrix} = \begin{bmatrix} s_{x,1} + \Delta t \cdot V_2 \cdot \cos(s_{\theta,1}) \\ s_{y,1} + \Delta t \cdot V_2 \cdot \sin(s_{\theta,1}) \\ s_{\theta,1} + \Delta t \cdot \omega_2 \end{bmatrix} = \begin{bmatrix} 0.1 + 0.1 \cdot 1 \cdot \cos(0.1) \\ 0 + 0.1 \cdot 1 \cdot \sin(0.1) \\ 0.1 + 0.1 \cdot 1 \end{bmatrix} = \begin{bmatrix} 0.1995 \\ 0.01 \\ 0.2 \end{bmatrix}$$

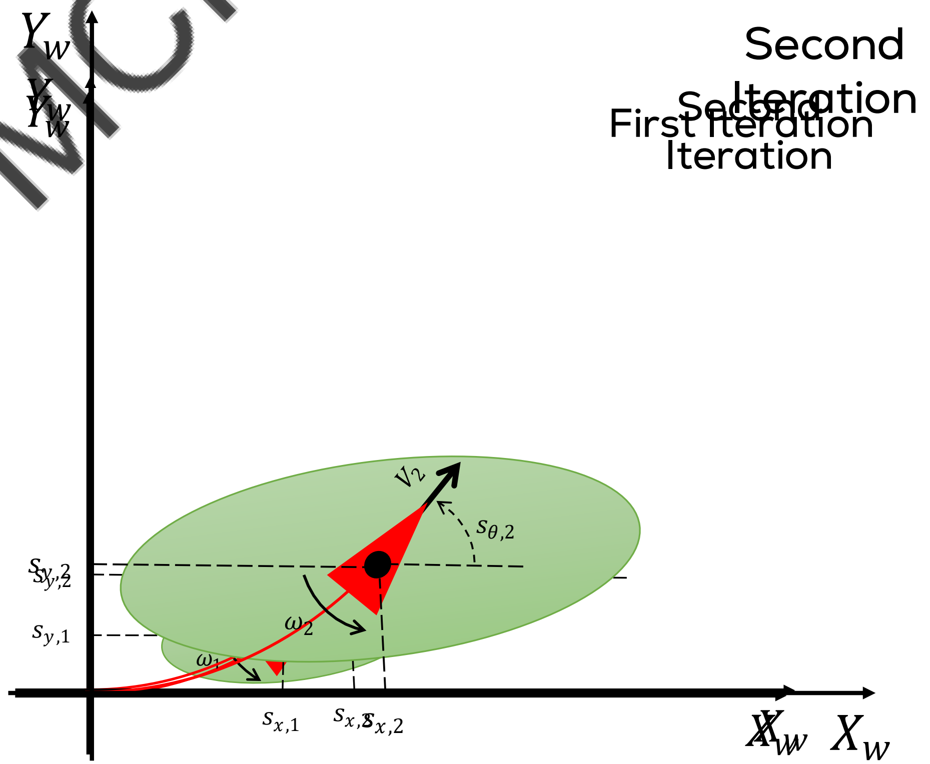
Calculate the linearized model to be used in the uncertainty propagation

$$\boldsymbol{H}_2 = \begin{bmatrix} 1 & 0 & -\Delta t \cdot V_2 \cdot \sin(s_{\theta,1}) \cdot \\ 0 & 1 & \Delta t \cdot V_2 \cdot \cos(s_{\theta,1}) \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -0.01 \\ 0 & 1 & 0.0995 \\ 0 & 0 & 1 \end{bmatrix}$$

Calculate the propagation of the uncertainty

$$\hat{\boldsymbol{\Sigma}}_2 = \mathbf{H}_2 \cdot \boldsymbol{\Sigma}_1 \cdot \mathbf{H}_2^T + \mathbf{Q}_2$$

$$\hat{\boldsymbol{\Sigma}}_2 = \begin{bmatrix} 0.998 & 0.0207 & 0.0180 \\ 0.0207 & 1.0040 & 0.0399 \\ 0.0180 & 0.0399 & 0.4000 \end{bmatrix}$$



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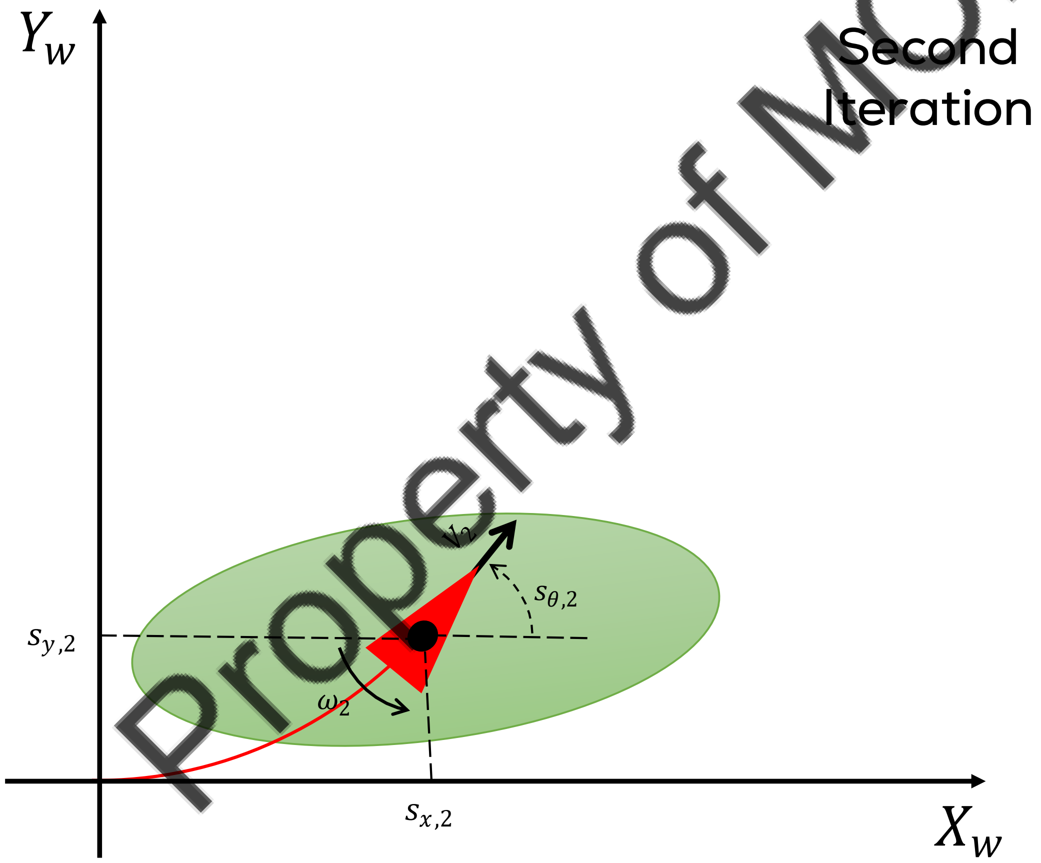
Solution: Iteration 2



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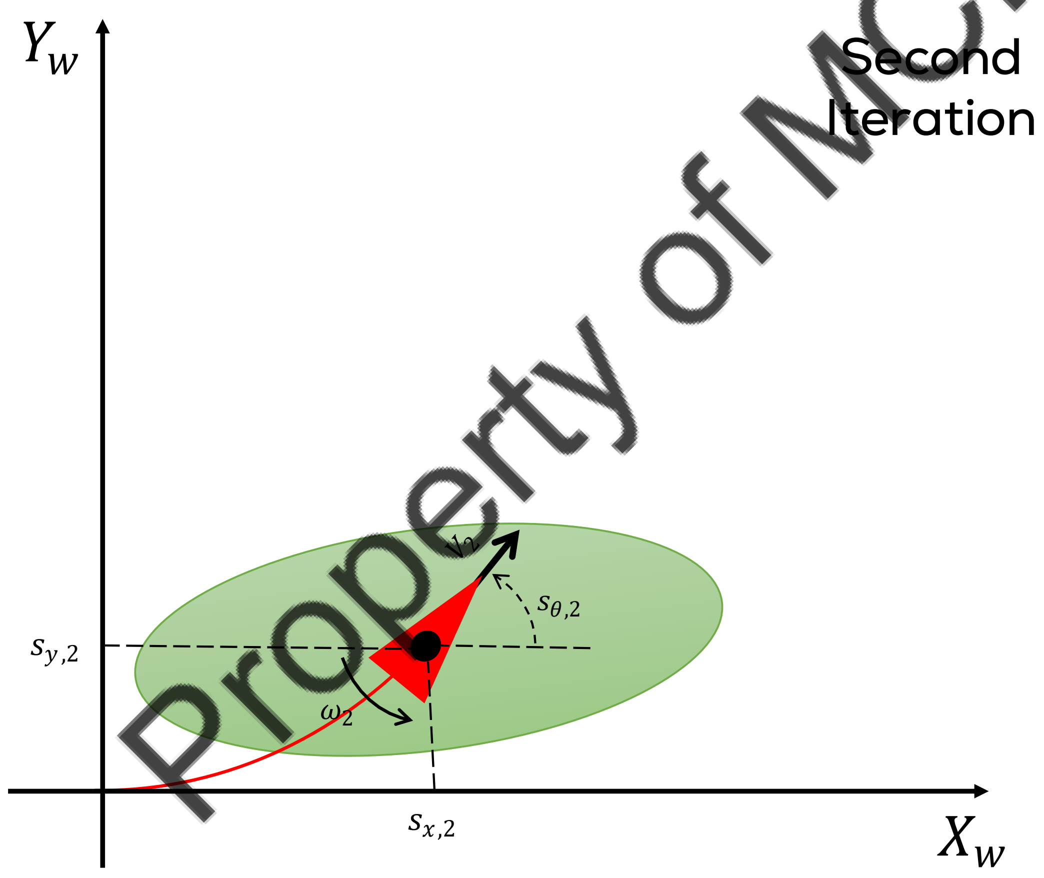
Solution: Iteration 2



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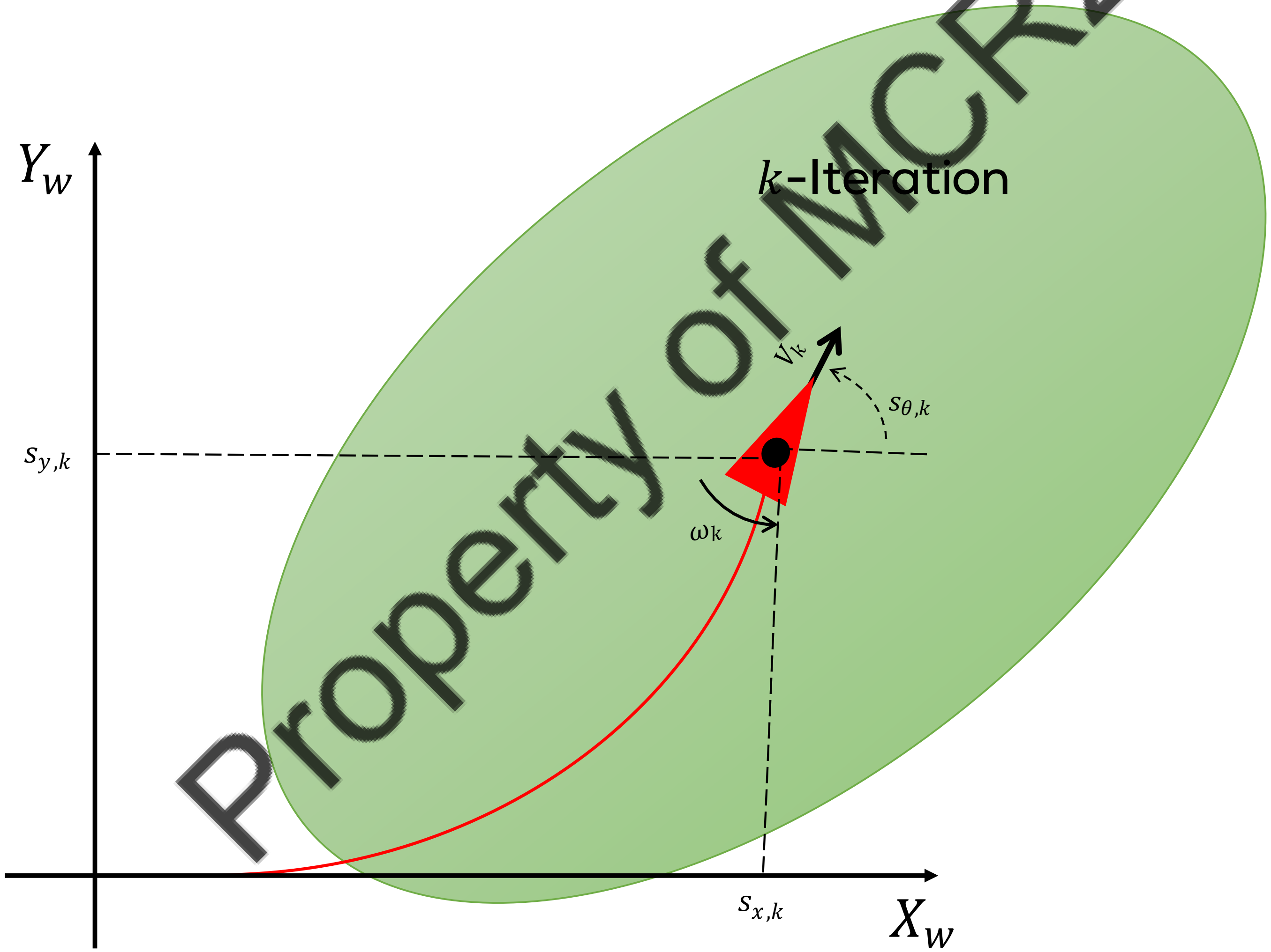
Solution: Iteration k



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Solution: Iteration k