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ROS2

Solving ODE's in ROS 2

Manchester Robotics

Introduction

Introduction

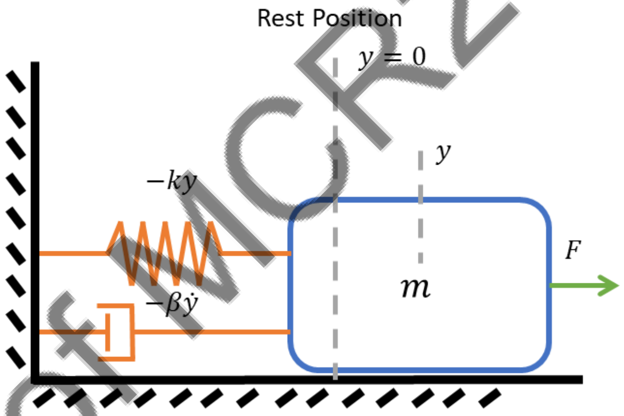
- Ordinary Differential Equations (ODEs) play a crucial role in robotics for modelling dynamic systems such as robot motion, control, and sensor behaviour.
- In this class, we will focus on solving second-order ODEs using **Euler's Method** and implement it in **ROS2**.

Dynamic Simulation

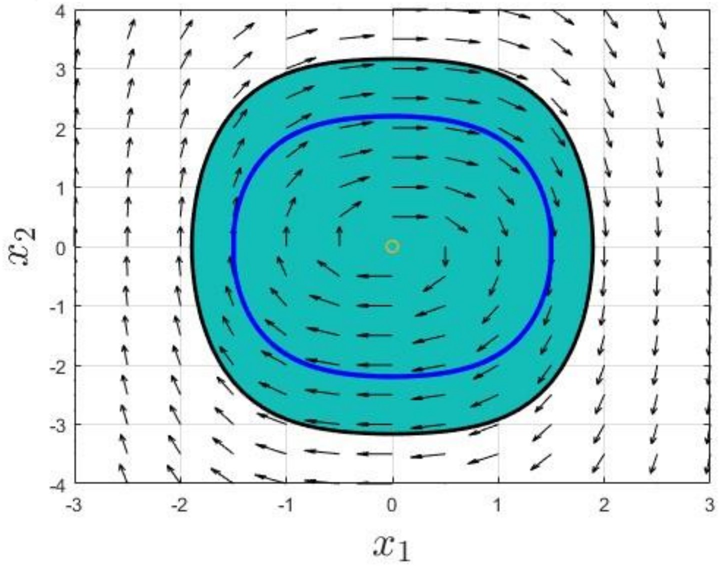
- **Dynamic simulation** (or dynamic system simulation) is using a computer program to model the time-varying behaviour of a dynamical system.
- Ordinary differential equations or partial differential equations typically describe the systems.
- The simulator solves these equations to determine the behaviour of state variables over a specified time.
- Creating a model of a dynamic system allows for predicting the values of the model-system state variables based on past state values.

Dynamic Simulation

- Simulation models are commonly obtained from discrete-time approximations of continuous-time mathematical models.
- Models can incorporate real-world constraints, like gear backlash, collisions, and rebound from a hard stop.
- As models are more complex, equations can become nonlinear, chaotic, with added disturbances and noise.



$$\dot{\boldsymbol{x}} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{\beta}{m} \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} 0 \\ 1/m \end{bmatrix} F$$



Dynamic Simulation

- Solving some nonlinear equations “by hand” can become difficult or almost impossible.
- Thanks to the advancement of computers, it is now possible to solve them using different computational algorithms.
- Dynamical models, in general, are solved through numerical integration methods to produce the transient behaviour of the state variables.
- Therefore, it can be said that a numerical simulation is done by stepping through a time interval and calculating the solution of the mathematical model solution through numerical integration.

Lorenz Attractor

Discrete-time dynamic models

Introduction

Discrete-time dynamic models

- A digital computer by its very nature, deals internally with discrete-time data or numerical values of functions at equally spaced intervals determined by the sampling period.
- Thus, discrete-time models such as difference equations are widely used in computer control applications.
- One way a continuous-time dynamic model can be converted to discrete-time form is by employing a finite difference approximation.

- Consider a nonlinear differential equation

$$\frac{dy(t)}{dt} = f(y,u) \qquad (30)$$

where y is the output variable and u is the input variable.

Discrete-time dynamic models

- This equation can be numerically integrated (for instance using Euler method) by introducing a finite difference approximation for the derivative.
- For example, the first-order, backward difference approximation to the derivative at $t = k\Delta t$ is:

$$\frac{dy(t)}{dt} \cong \frac{y(k) - y(k-1)}{\Delta t} \quad (31)$$

where Δt is the integration interval (the control engineers name it sampling time) specified by the user and $y(k)$ denotes the values of $y(k)$ at $t = k\Delta t$.

- So,

$$\frac{y(k) - y(k-1)}{\Delta t} \cong f(y(k-1), u(k-1)) \quad (32)$$

- or:

$$y(k) = y(k-1) + \Delta t \cdot f(y(k-1), u(k-1)) \quad (33)$$

- This is a first-order difference equation that can be used to predict $y(k)$ based on information at the previous time step $(k-1)$. This type of expression is called a recurrence relation.

Euler’s Method

- Euler's method is a numerical approach for solving differential equations by approximating solutions iteratively.
- For a first-order ODE:

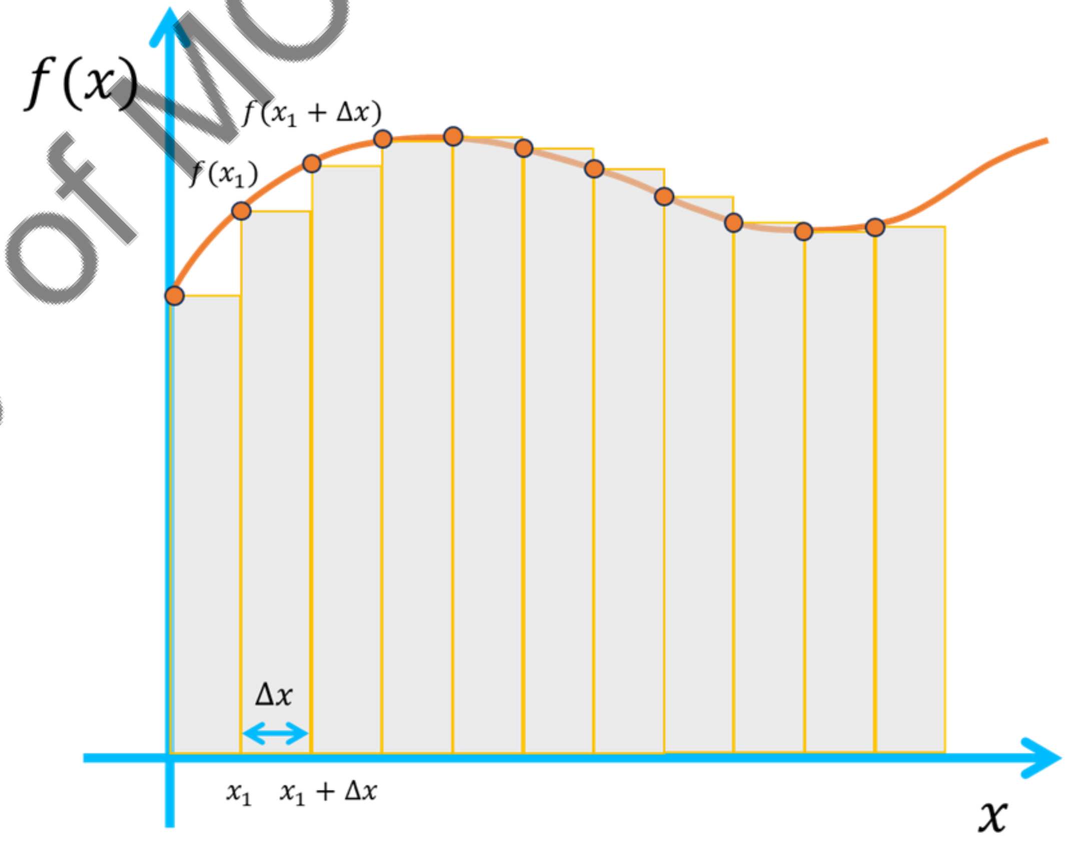
$$\dot{x} = f(x)$$

- Euler's approximation is given by:

$$x_{n+1} = x_n + \Delta x \cdot f(x_n)$$

- where: Δx is the step size, x_n is the current value and x_{n+1} is the next estimated value.

Euler’s Method



Discrete-time dynamic models

- For higher-order ODEs, we can use a generalisation of the Euler method that we used for solving first-order ODEs. To illustrate the method, let us consider a 2nd order ODE:

$$\frac{d^2\,y(t)}{dt^2} = f(t, y, \frac{dy(t)}{dt}) \qquad (34)$$

- Or:

$$\ddot{y} = f(t, y, \dot{y}) \qquad (35)$$

- For discretization, the idea is to write the second order system (ODE) as a system of two first order systems (ODEs) and then apply Euler's method to the first order equations.
- So, defining a new variable:

$$\begin{cases} y = x_1 \\ \dot{y} = x_2 = \dot{x}_1 \end{cases} \qquad (36)$$

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f(t, x_1, x_2) \end{cases} \qquad (37)$$

- We need initial conditions:

For Newton the initial conditions are the initial position and initial velocity.

$$\begin{cases} x_1(t_0) = 0 \\ x_2(t_0) = 0 \end{cases} \qquad (38)$$

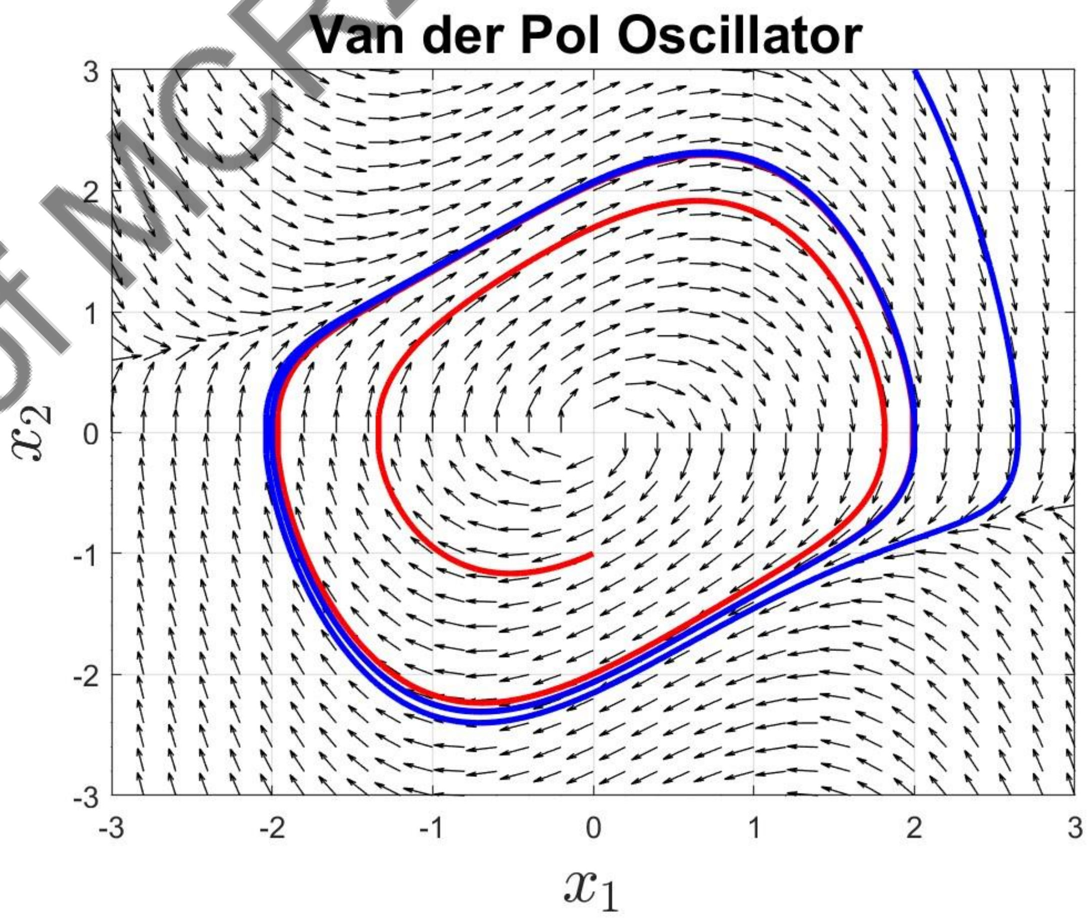
Discrete-time dynamic models

- Now, the idea is to solve both x_1 and x_2 simultaneously using Euler's method for both first order ODEs:

$$\begin{cases} \dfrac{x_1(k) - x_1(k-1)}{\Delta t} = x_2(k-1) \\ \dfrac{x_2(k) - x_2(k-1)}{\Delta t} = f((k-1), x_1, x_2) \end{cases} \quad (39)$$

$$\begin{cases} x_1(k) = x_1(k-1) + \Delta t \cdot x_2(k-1) \\ x_2(k) = x_2(k-1) + \Delta t \cdot f((k-1), x_1, x_2) \end{cases} \quad (40)$$

- This can be generalized to third order ODEs, or fourth order ODEs, as well as n order ODEs.



Activity

DC Motor Simulation

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motor_control package

Requirements

- Download the motor_control template package from Github.

Instructions

- Download the motor_control package from GitHub (inside Templates).
- Add it to your source directory inside your workspace

```
motor_control/
├── launch
│   └── motor_launch.py
├── LICENSE
├── motor_control
│   ├── dc_motor.py
│   ├── __init__.py
│   └── set_point.py
├── package.xml
├── resource
│   └── motor_control
├── setup.cfg
├── setup.py
└── test
    ├── test_copyright.py
    ├── test_flake8.py
    └── test_pep257.py
```

DC Motor Simulation

Motor Control package

- The package is composed of two nodes:
  - dc_motor node: Simulate a First Order System, representing a DC Motor.
  - set_point node: Providing an input for the system

```
motor_control/motor_control/dc_motor.py
motor_control/motor_control/set_point.py
```

- You can see the contents of each node by opening the file on any text editor (gedit, vscode, nano, vim, etc.)

DC Motor Node

- The DC Motor will be simulated using a First Oder system shown in here.

$$\tau \frac{dy(t)}{dt} + y(t) = Ku(t).$$

where, τ is the time constant, K is the system gain, $y(t)$ is the system output (speed rad/s) and $u(t)$ the input signal (volts).

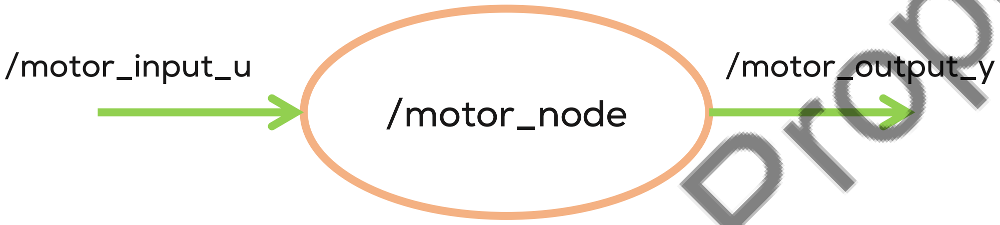
$$y[k+1] = y[k] + \left(-\frac{1}{\tau} \cdot y[k] + \frac{K}{\tau} \ u[k] \right) T_s$$

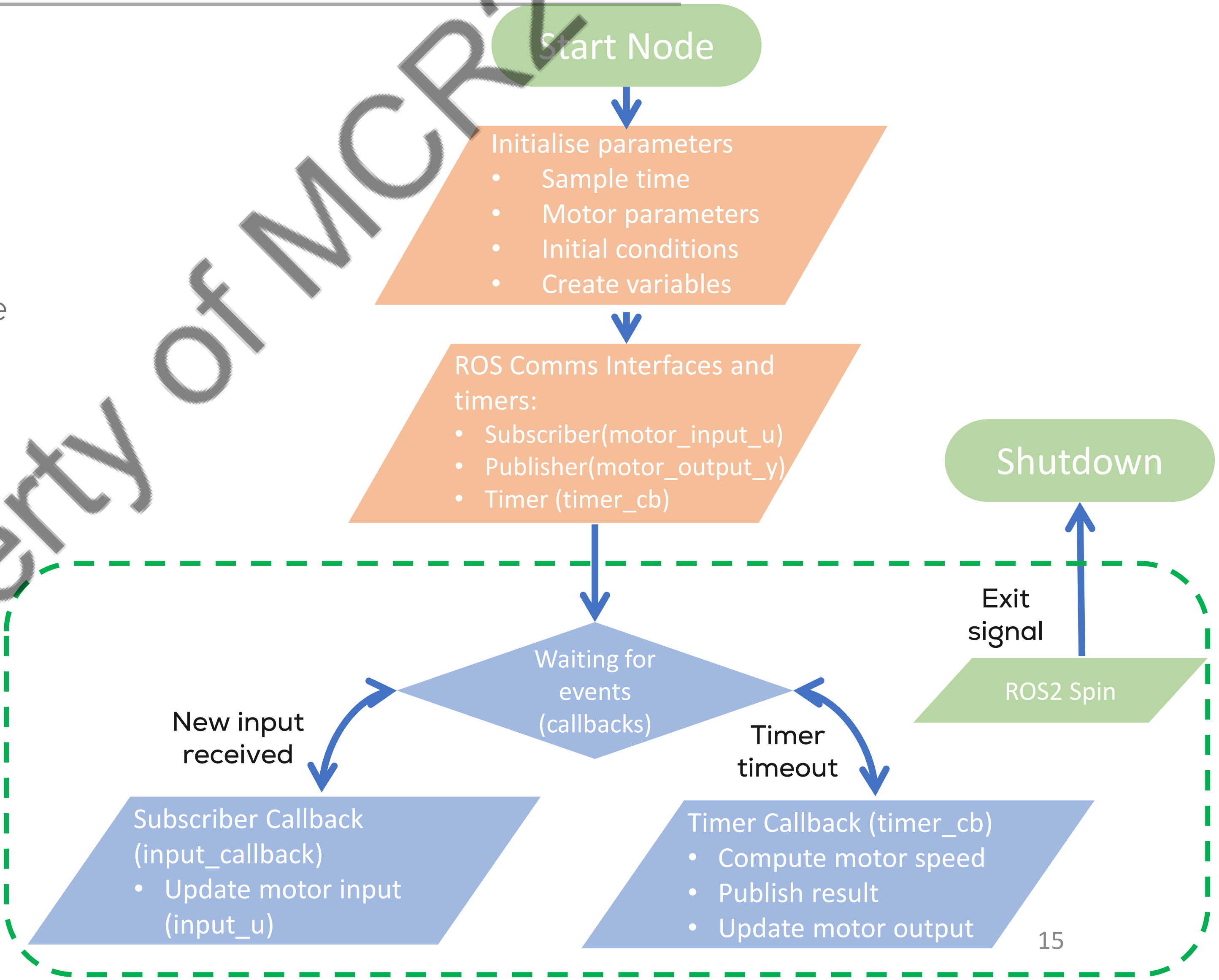
Where T_s is the sampling time.

DC Motor Simulation

DC Motor Node Structure

- The node subscribes to the topic "/motor_input_u" and publishes the vales of the motor speed on the topic "/motor_output_y".
- Both topics contain an interface (message) Float32





dc_motor.py

```python
# Imports
import rclpy
from rclpy.node import Node
from std_msgs.msg import Float32

#Class Definition
class DCMotor(Node):
    def __init__(self):
        super().__init__('dc_motor')

        # DC Motor Parameters
        self.sample_time = 0.02
        self.param_K = 1.75
        self.param_T = 0.5
        self.initial_conditions = 0.0

        #Set the messages
        self.motor_output_msg = Float32()

        #Set variables to be used
        self.input_u = 0.0
        self.output_y = self.initial_conditions

        #Declare publishers, subscribers and timers
        self.motor_input_sub = self.create_subscription(Float32, 'motor_input_u', 
self.input_callback,10)
        self.motor_speed_pub = self.create_publisher(Float32, 'motor_speed_y', 10)
        self.timer = self.create_timer(self.sample_time, self.timer_cb)

        #Node Started
        self.get_logger().info('Dynamical System Node Started \U0001F680')
```

Libraries

Initialise parameters

ROS publishers, subscribers Timers

```python
    #Timer Callback
    def timer_cb(self):
        #DC Motor Simulation
        #DC Motor Equation y[k+1] = y[k] + ((-1/τ) y[k] + (K/τ) 
u[k]) T_s
        self.output_y += (-1.0/self.param_T * self.output_y + 
self.param_K/self.param_T * self.input_u) * self.sample_time
        #Publish the result
        self.motor_output_msg.data = self.output_y
        self.motor_speed_pub.publish(self.motor_output_msg)

    #Subscriber Callback
    def input_callback(self, input_sgn):
        self.input_u = input_sgn.data

#Main
def main(args=None):
    rclpy.init(args=args)

    node = DCMotor()

    try:
        rclpy.spin(node)
    except KeyboardInterrupt:
        pass
    finally:
        node.destroy_node()
        rclpy.try_shutdown()

#Execute Node
if __name__ == '__main__':
    main()
```

Timer callback: Motor simulation

Subscriber callback

Main

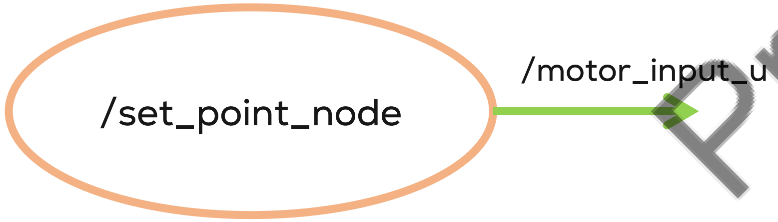
DC Motor Simulation

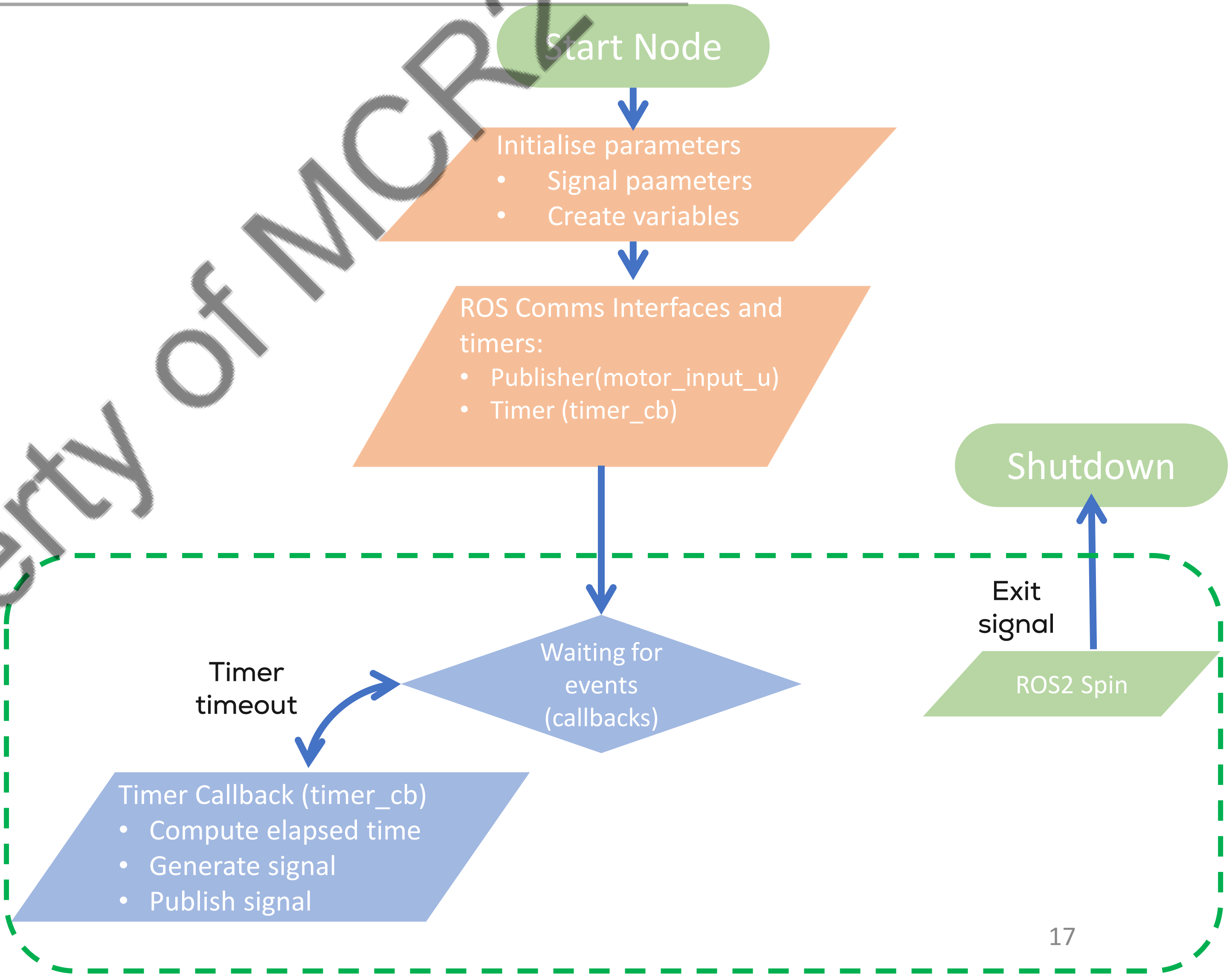
Set Point node structure

- The node publishes the vales of input signal on the topic "/motor_input_u".

$$u(t) = A\ sin(\omega t)$$

- The topic contain an interface (message) Float32





set_point.py

```python
# Imports
import rclpy
from rclpy.node import Node
import numpy as np
from std_msgs.msg import Float32

#Class Definition
class SetPointPublisher(Node):
    def __init__(self):
        super().__init__('set_point_node')

        # Retrieve sine wave parameters
        self.amplitude = 2.0
        self.omega  = 1.0

        #Create a publisher and timer for the signal
        self.signal_publisher = self.create_publisher(Float32,
'motor_input_u', 10)
        timer_period = 0.1 #seconds
        self.timer = self.create_timer(timer_period, self.timer_cb)

        #Create a messages and variables to be used
        self.signal_msg = Float32()
        self.start_time = self.get_clock().now()

        self.get_logger().info("SetPoint Node Started \U0001F680")
```

Libraries

Initialise parameters

ROS publishers, subscribers Timers

```python
    # Timer Callback: Generate and Publish Sine Wave Signal
    def timer_cb(self):
        #Calculate elapsed time
        elapsed_time = (self.get_clock().now() -
self.start_time).nanoseconds/1e9
        # Generate sine wave signal
        self.signal_msg.data = self.amplitude *
np.sin(self.omega * elapsed_time)
        # Publish the signal
        self.signal_publisher.publish(self.signal_msg)

#Main
def main(args=None):
    rclpy.init(args=args)

    set_point = SetPointPublisher()

    try:
        rclpy.spin(set_point)
    except KeyboardInterrupt:
        pass
    finally:
        set_point.destroy_node()
        rclpy.try_shutdown()

#Execute Node
if __name__ == '__main__':
    main()
```

Timer callback:

Signal Generator

Main

DC Motor Simulation

Instructions

- Compile the package using colcon

```
$ cd ~/ros2_ws
$ colcon build
$ source install/setup.bash
```

- Launch the package

```
$ ros2 launch motor_control motor_launch.py
```

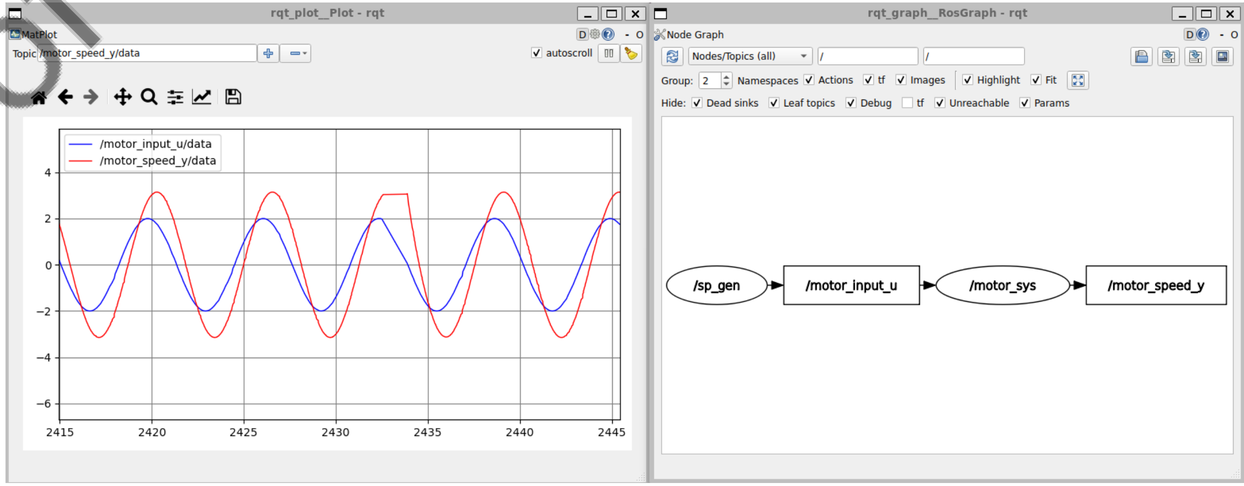
- Open two terminals run the rqt_graph and the rqt_plot

```
$ ros2 run rqt_plot rqt_plot
```

```
$ ros2 run rqt_graph rqt_graph
```

Results

- If everything goes well, you should see the following



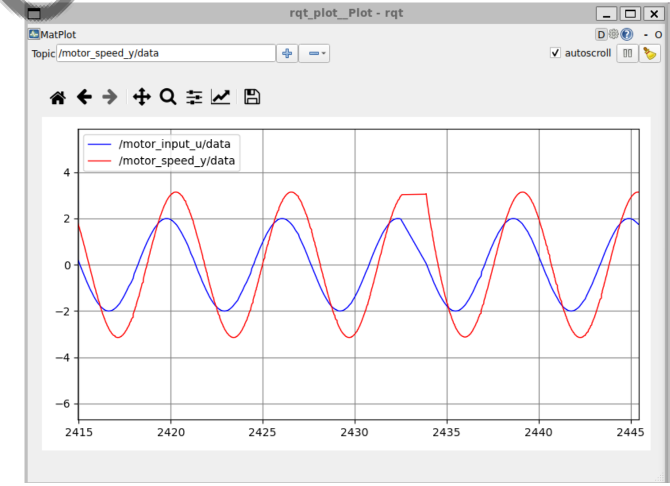
- Check the published topics

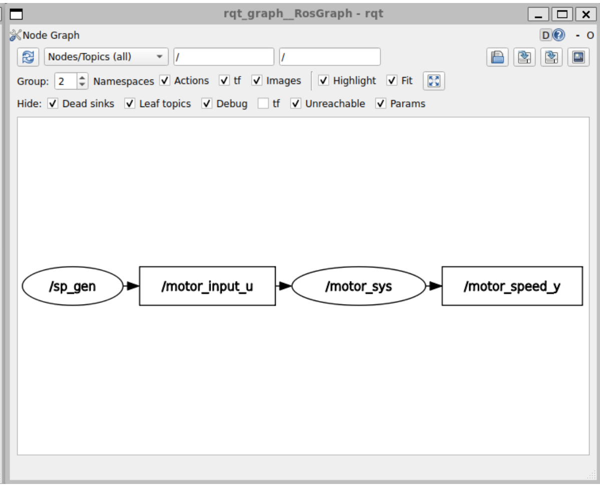
```
mario@MarioPC:~$ ros2 topic list
/motor_input_u
/motor_speed_y
/parameter_events
/rosout
```

motor_launch.py

```python
from launch import LaunchDescription
from launch_ros.actions import Node

def generate_launch_description():
    motor_node = Node(name="motor_sys",
                       package='motor_control',
                       executable='dc_motor',
                       emulate_tty=True,
                       output='screen',
                       )

    sp_node = Node(name="sp_gen",
                       package='motor_control',
                       executable='set_point',
                       emulate_tty=True,
                       output='screen',
                       )

    l_d = LaunchDescription([motor_node, sp_node])

    return l_d
```

- The launch file starts a motor_node and a set_point node.





Q&A

Questions?

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Thank You

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