{Learn, Create, Innovate};

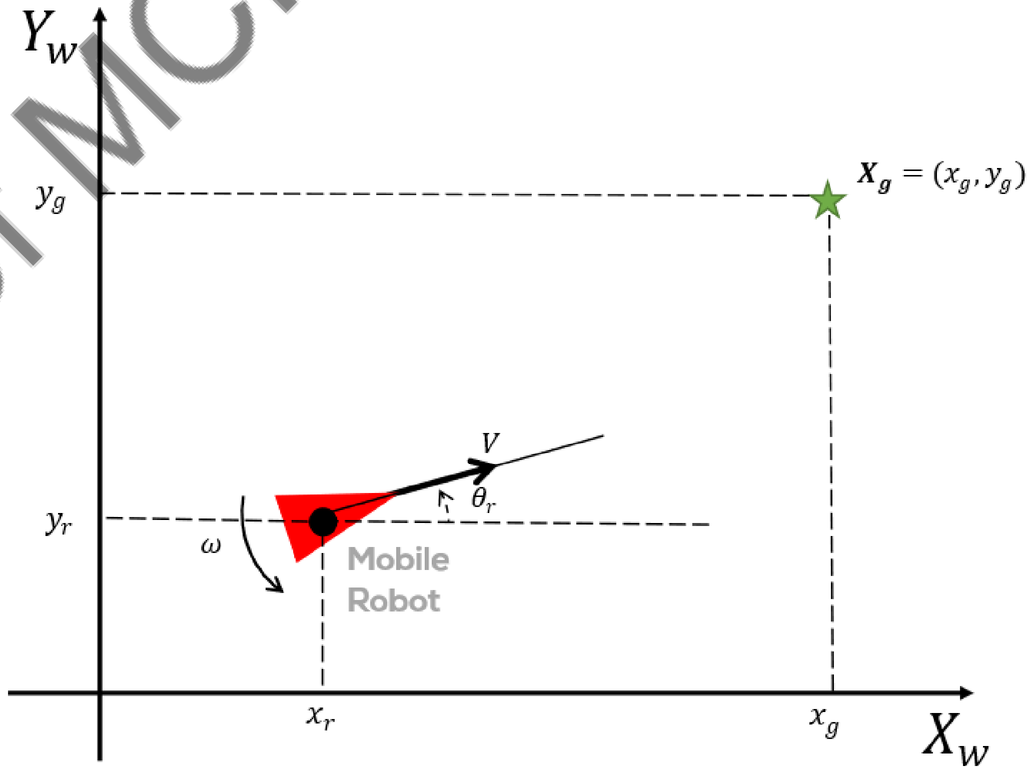
Autonomous Systems

Mobile Robot: Motion Control

MCR2 Manchester Robotics

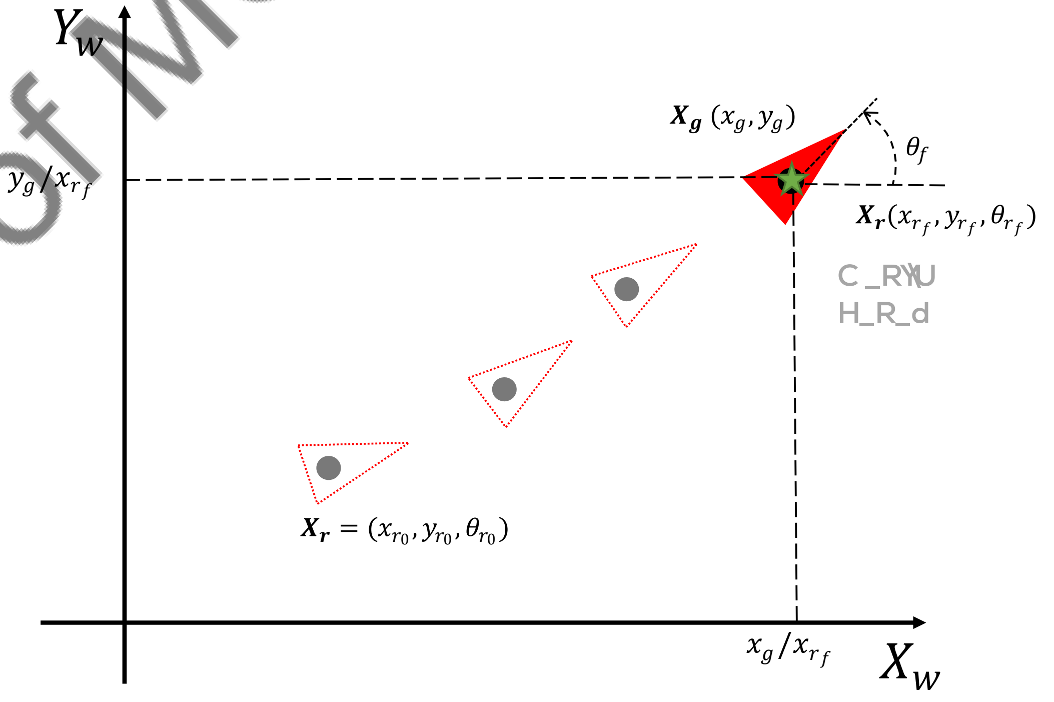
Motion Control

- The **motion control** for a mobile robot deals with the task of finding the control inputs that need to be applied to the robot such that a predefined goal can be reached in a finite amount of time.
- Control of differential drive robots has been studied from several points of view but essentially falls into one of the following three categories: **point-to-point navigation (or point stabilisation), trajectory tracking, and path following**.



Motion Control: Point Stabilisation

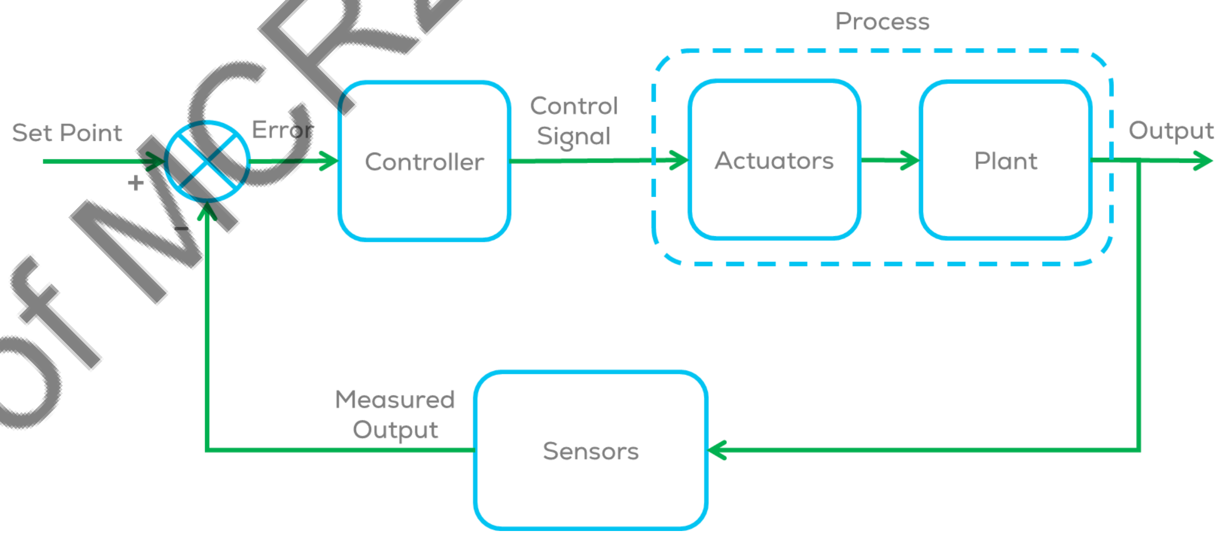
- The objective here is to drive the robot to a desired fixed state, say a fixed position and orientation.
- When the vehicle has nonholonomic constraints, point stabilisation presents a true challenge to control systems, specially when the vehicle has nonholonomic constraints.
    - Since that goal cannot be achieved with smooth time-invariant state-feedback control laws.
- This control technique will be used in this course.

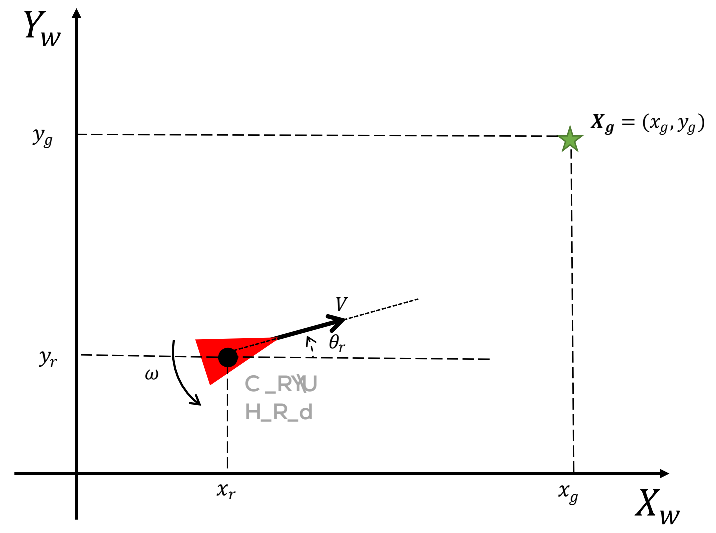


Y_w
y_g/x_{r_f}
$\boldsymbol{X_g}\ (x_g, y_g)$
θ_f
$\boldsymbol{X_r}(x_{r_f}, y_{r_f}, \theta_{r_f})$
C _RYU
H_R_d
$\boldsymbol{X_r} = (x_{r_0}, y_{r_0}, \theta_{r_0})$
x_g/x_{r_f}
X_w

The Control System

- When Designing a Controller a question arises…
- How can we transform the classical feedback control diagram, to fit the purpose of controlling the position of a mobile robot using Point Stabilisation?
- … We start by defining the main things of the control diagram: Set Point, Measured Output, Control Signal, etc…

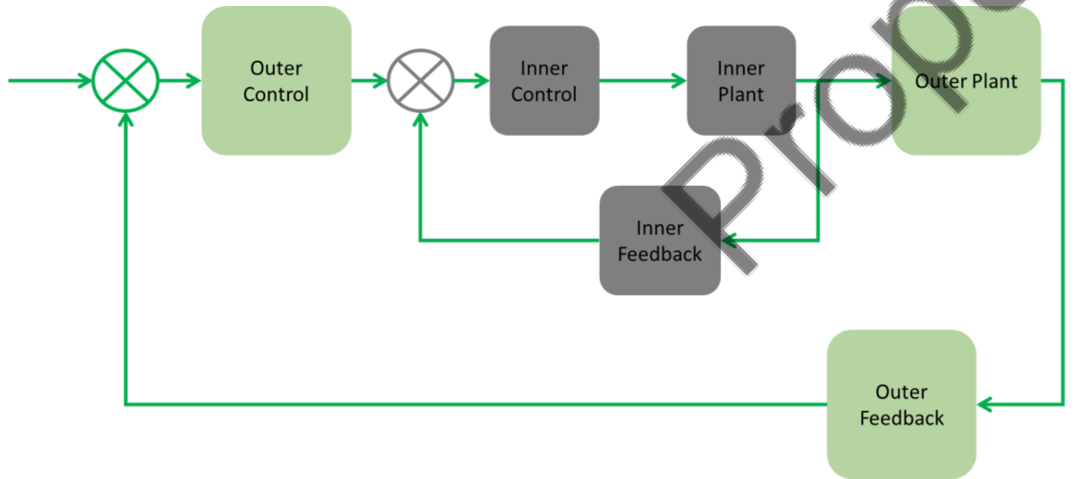


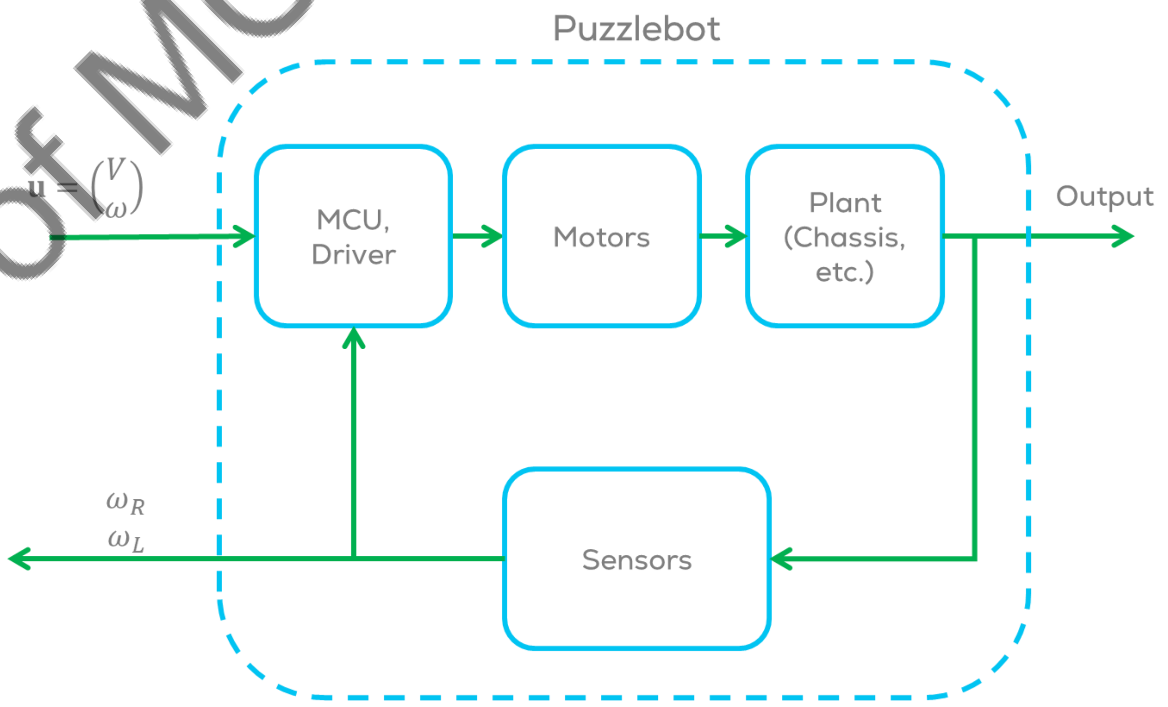


The Mobile Robot

Internal Controllers

- Mobile Robots usually contain internally some controllers to regulate the speed of the motors.
- These controllers must be taken into consideration when trying to control the robot using an external controller (cascade control)





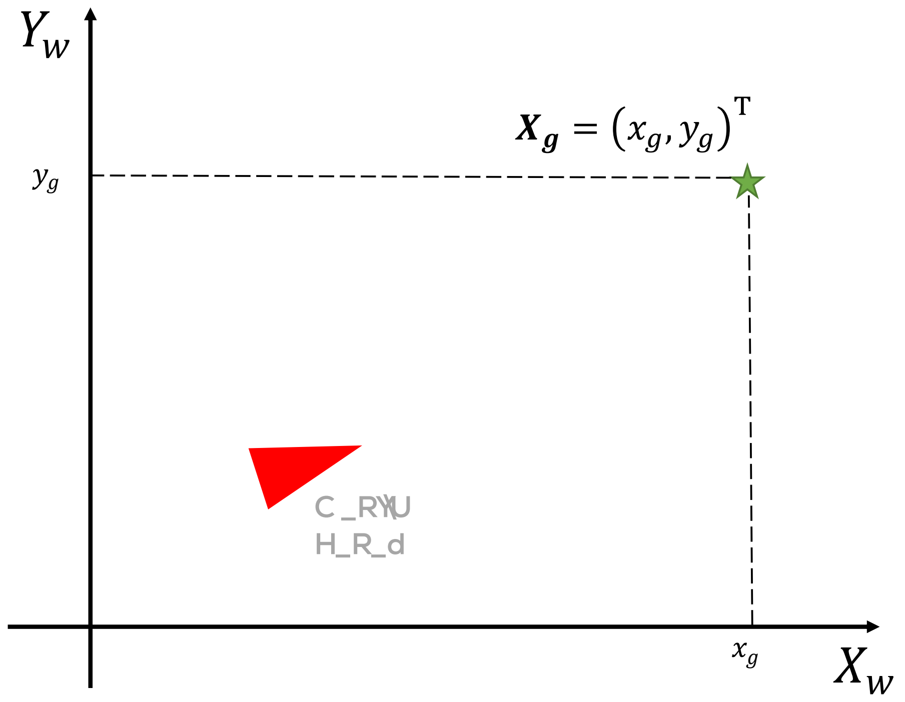
Point Stabilisation

Set Point

- The set point/goal can be defined in the simplest way as a set of coordinates in a multidimensional space. For instance, if the robot is moving in a two-dimensional space the goal is:

- $\boldsymbol{X}_g = \begin{pmatrix} x_g \\ y_g \end{pmatrix}$ if the position of the robot needs to be controlled, or

- $\boldsymbol{X}_g = \begin{pmatrix} x_g \\ y_g \\ \theta_g \end{pmatrix}$ if both position and heading need to be controlled.



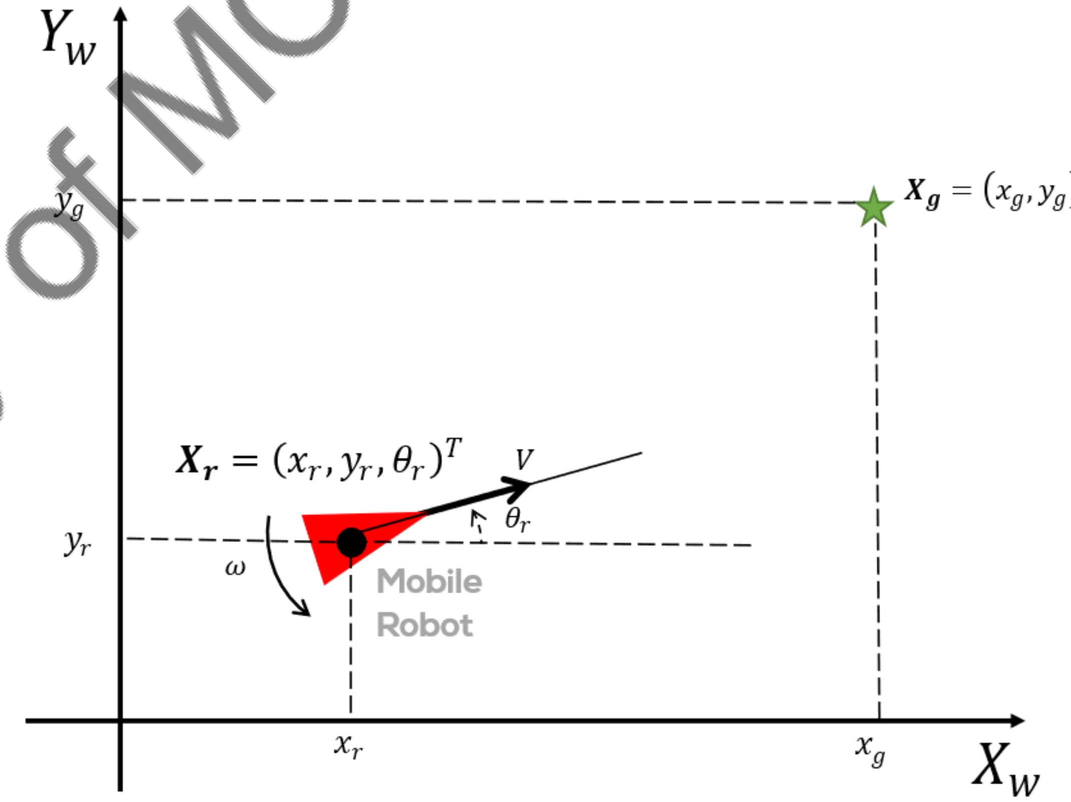
Point Stabilisation

Control Inputs

- The inputs to the robot are the linear and angular velocity of the robot

$$\mathbf{u} = \begin{pmatrix} V \\ \omega \end{pmatrix}.$$

- The linear velocity of the robot, V, is always oriented in the direction of x axis of the robot reference frame because of the non-holonomic constraint.
- The inputs are transformed into wheel velocities by the motors.



Y_W
y_g
$\boldsymbol{X_g} = (x_g, y_g)^T$
$\boldsymbol{X_r} = (x_r, y_r, \theta_r)^T$
V
θ_r
y_r
ω
Mobile Robot
x_r
x_g
X_W

Point Stabilisation

Error

- To Error is the difference between the set point and the measurement.

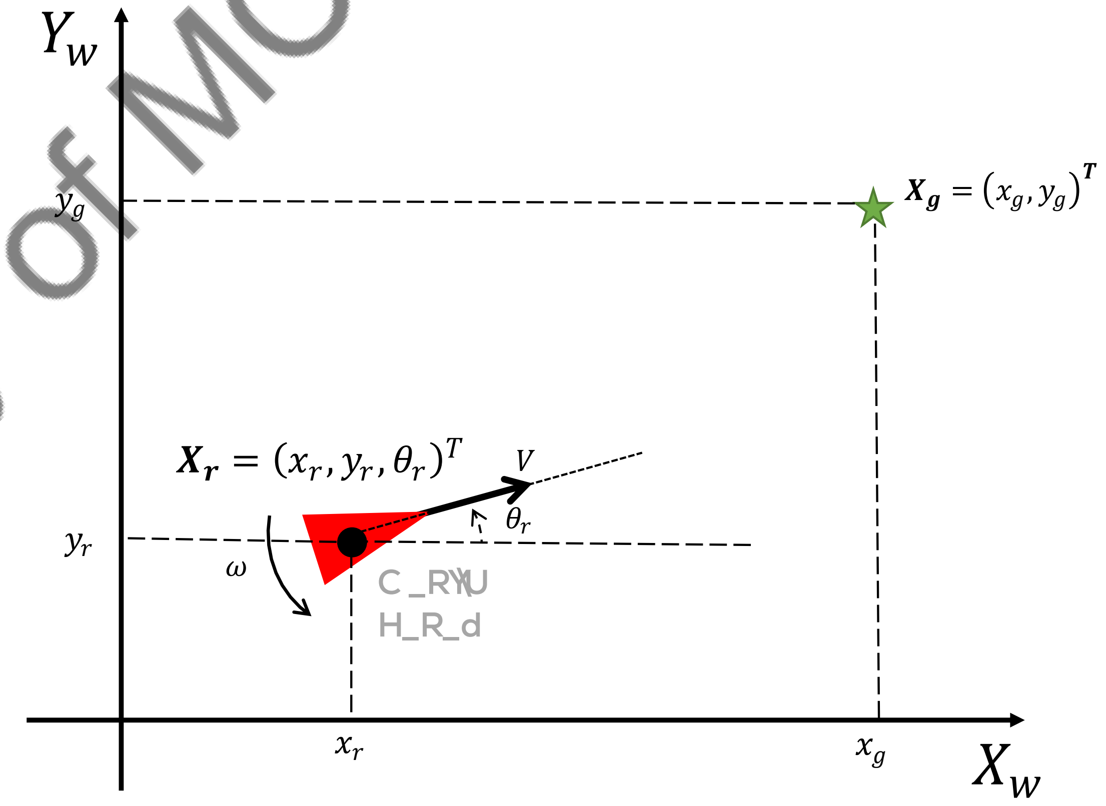
- In this case the set point was stablished to be $\boldsymbol{X}_g = \begin{pmatrix} x_g \\ y_g \\ \theta_g \end{pmatrix}$

- Therefore, for a non-holonomic robot moving in a 2D environment the error must be defined as:

$$\boldsymbol{e} = \boldsymbol{X}_g - \boldsymbol{X}_r$$

- Where, $\boldsymbol{X}_r$ represents the robot's pose represented by the vector

$$\boldsymbol{X}_r = \begin{pmatrix} x_r \\ y_r \\ \theta_r \end{pmatrix}.$$

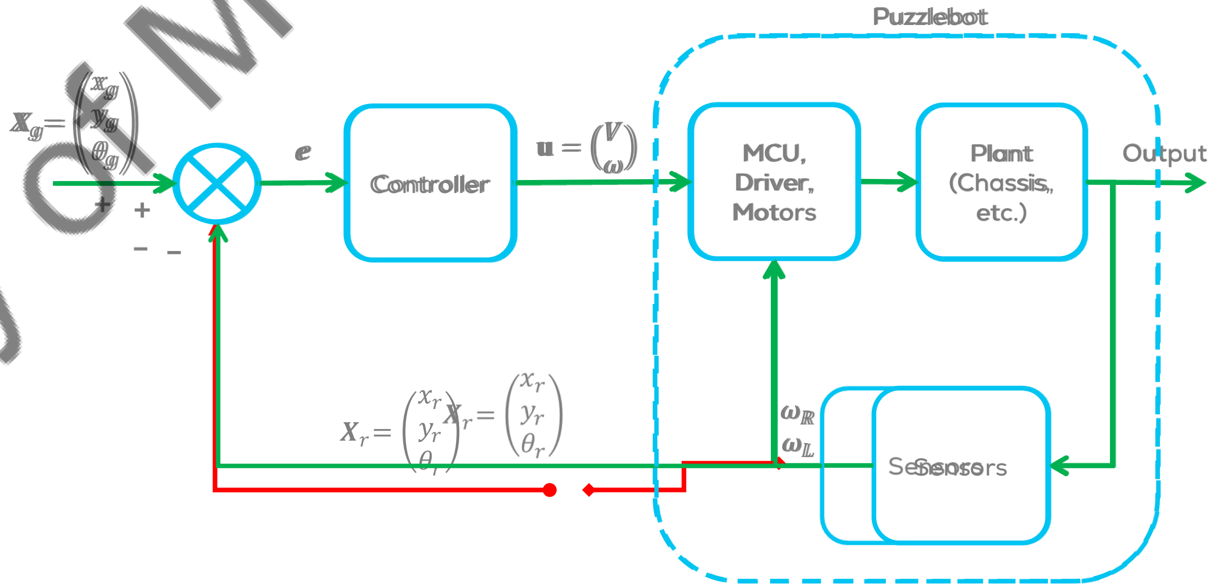


Y_w
y_g
$\boldsymbol{X_g} = (x_g, y_g)^T$
$\boldsymbol{X_r} = (x_r, y_r, \theta_r)^T$
V
θ_r
y_r
ω
C _RYU
H_R_d
x_r
x_g
X_w

The Control System

Control

- The control diagram can then be changed as follows…
- Just one problem…
- In reality, the Sensors of a Robot do not provide the position of the robot…
- Usually, the sensors of the mobile robot provide the information about the velocity of each wheel i.e., ω_R and ω_L.
- Another question arises… Can I use the information of the wheel's speed to get the position of my robot?
- YES… it's called Localisation!



Puzzlebot
$X_g = \begin{pmatrix} x_g \\ y_g \\ \theta_g \end{pmatrix}$
+
−
e
Controller
$u = \begin{pmatrix} V \\ \omega \end{pmatrix}$
MCU, Driver, Motors
Plant (Chassis, etc.)
Output
ω_R
ω_L
Sensors
$X_r = \begin{pmatrix} x_r \\ y_r \\ \theta_r \end{pmatrix}$

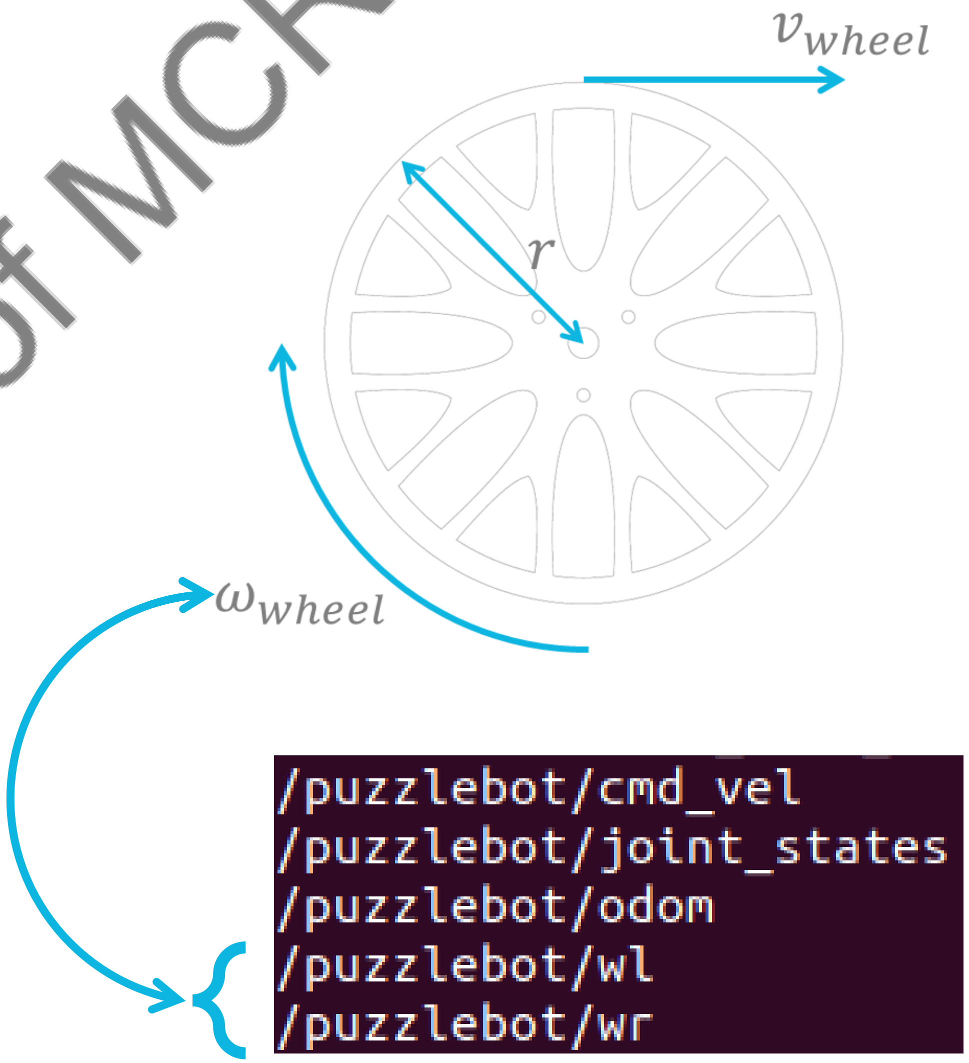
Localisation

Tangential Velocity

- The first step to get the position of a robot, is to get the tangential velocity of the wheel.
- The tangential speed of a wheel is given by

$$v_{wheel} = r \cdot \omega_{wheel}$$

- Where r is the radius and ω_{wheel} is the angular speed of the wheel.
- For this case, the robot has two wheels, therefore two tangential velocities must be calculated.
- On the Puzzlebot (and most robots) this is given by the topics `/wr` and `/wl`



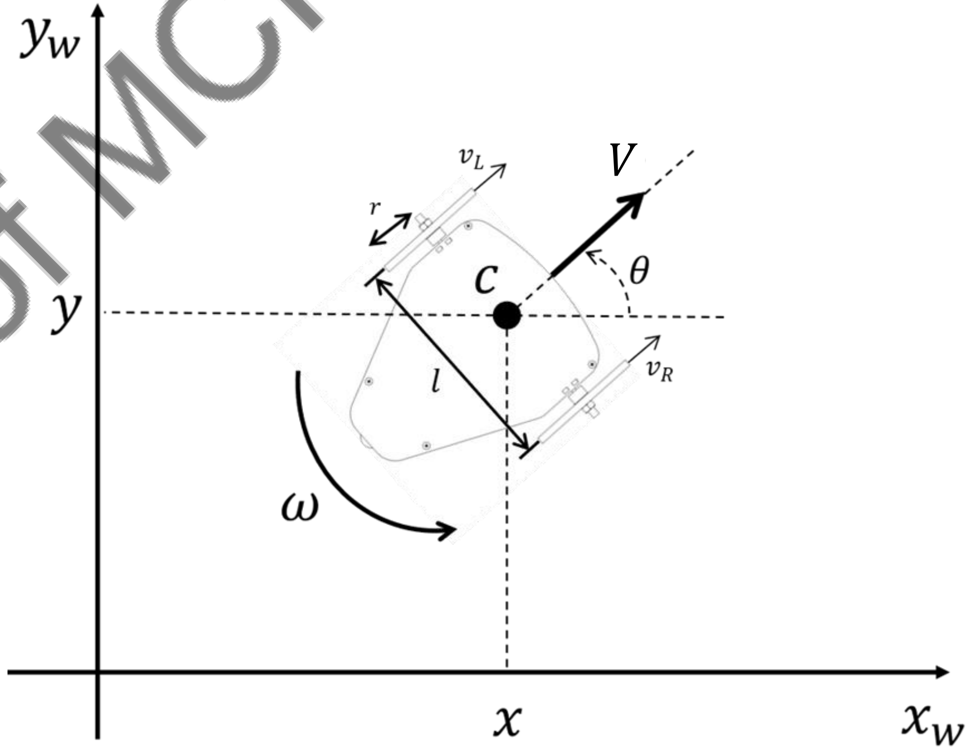
```
/puzzlebot/cmd_vel
/puzzlebot/joint_states
/puzzlebot/odom
/puzzlebot/wl
/puzzlebot/wr
```

Localisation

Robot Velocity

- Using the wheel velocities, it is possible to estimate the forward velocity V and the angular velocity ω.
- For this case, the resultant forward velocity V through C (centre of mass) may be reasoned as an average of the two forward wheel velocities given by

$$V = \frac{v_R + v_L}{2} = r\frac{\omega_R + \omega_L}{2}$$



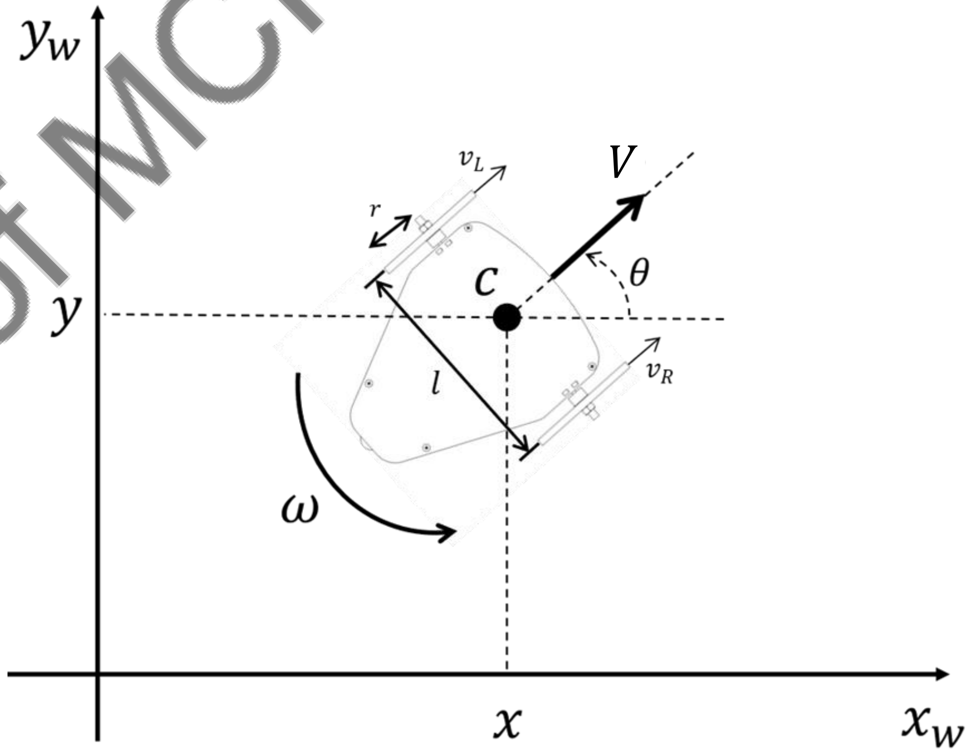
Differential Drive Robot
Puzzlebot ©.

Localisation

Robot Velocity

- The resultant angular velocity ω (steering velocity), may also be reasoned as proportional to the difference between wheel velocities but inversely proportional to distance between the wheels, i.e.,

$$\omega = \frac{v_R - v_L}{l} = r\frac{\omega_R - \omega_L}{l}$$



Differential Drive Robot
Puzzlebot ©.

Localisation

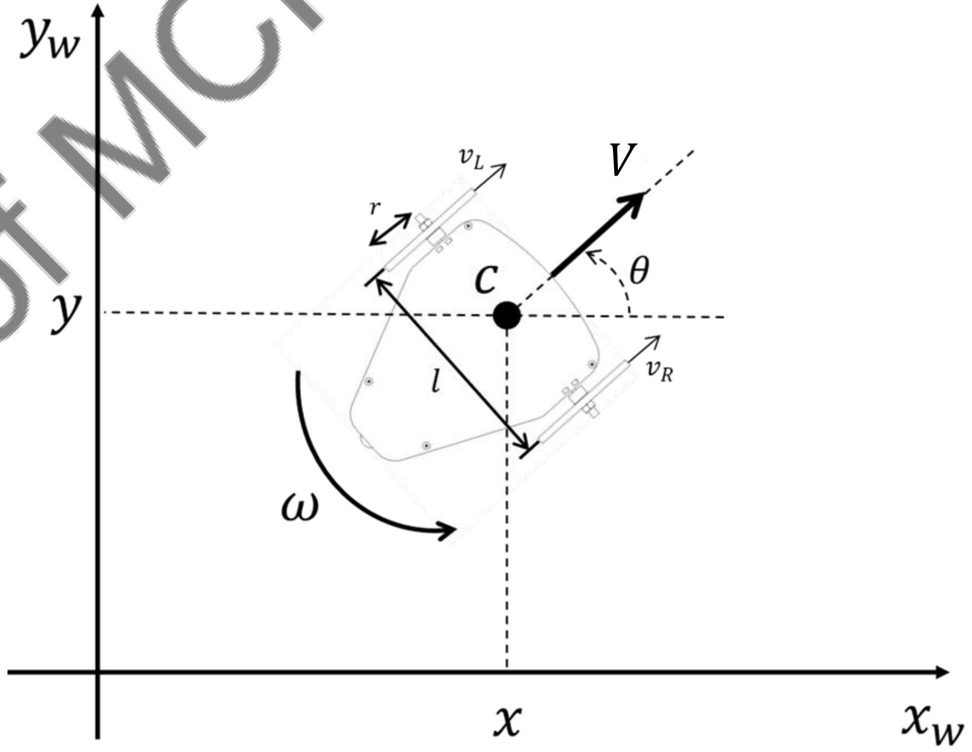
Robot Velocity

- Using the Differential Drive Kinematic Model, given by:

$$\begin{cases} \dot{x} = \mathrm{V} \cdot cos\theta \\ \dot{y} = \mathrm{V} \cdot sin\theta \\ \dot{\theta} = \omega \end{cases}$$

- It is possible to decompose the speed of the robot into its components

$$\begin{cases} \dot{x} = r\dfrac{\omega_R + \omega_L}{2} \cdot cos\theta \\ \dot{y} = r\dfrac{\omega_R + \omega_L}{2} \cdot sin\theta \\ \dot{\theta} = r\left(\dfrac{\omega_R - \omega_L}{l}\right) \end{cases}$$



Differential Drive Robot
Puzzlebot ©.

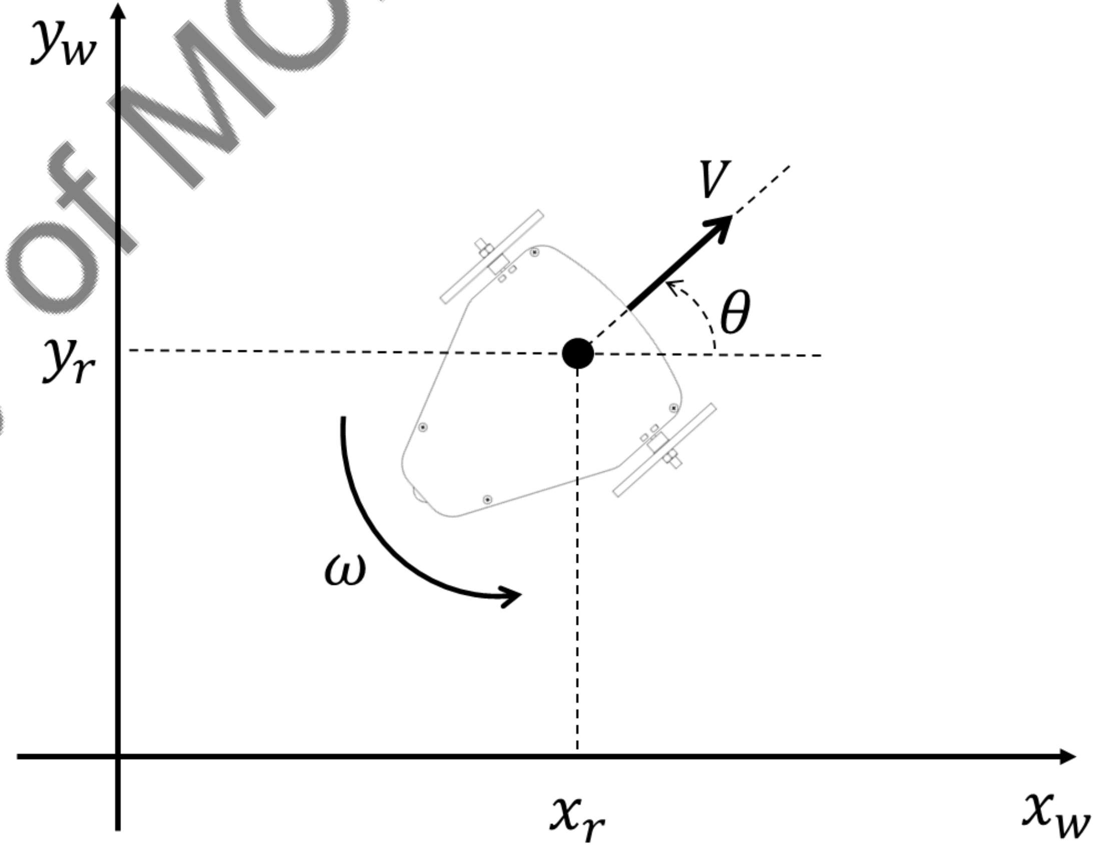
Localisation

Robot Position

- Discretising the model and solving it using Euler's Method:

$$\begin{cases} x_{r\,k+1} = x_{r\,k} + r\dfrac{\omega_R + \omega_L}{2}\cos\theta\ dt \\ y_{r\,k+1} = y_{r\,k} + r\dfrac{\omega_R + \omega_L}{2}\sin\theta\ dt \\ \theta_{r\,k+1} = \theta_{r\,k} + r\dfrac{\omega_R - \omega_L}{l}dt \end{cases}$$

- Where ω_R and ω_L are the speed given by the encoders
- Estimating the change in position a robot from its sensors is called "odometry".



Determining the Robot Position

$$\boxed{\theta_{r\,k+1}} = \boxed{\theta_{r\,k}} \boxed{+} r\frac{\boxed{\omega_R - \omega_L}}{\boxed{l}}\boxed{dt}$$

$$\boxed{x_{r\,k+1}} = \boxed{x_{r\,k}} \boxed{+} r\frac{\boxed{\omega_R + \omega_L}}{2}\boxed{dt}\cos\boxed{\theta_k}$$

$$\boxed{y_{r\,k+1}} = \boxed{y_{r\,k}} \boxed{+} r\frac{\boxed{\omega_R + \omega_L}}{2}\boxed{dt}\sin\boxed{\theta_k}$$

Robot Location:
(x_k, y_k, θ_k): Pose of the robot at timestep k (m, m, rad). Stored in memory, initial value 0

Robot Constants:
r: Wheel radius = 0.05 m
l: Distance between robot wheels = 0.19 m

Measured variables
(ω_R, ω_L): Wheel velocity (rad/s)
dt: Time between samples (s)

Values of θ can grow unbounded so they must be contained within a single circle (wrap2pi):
Either:

$$-\pi \leq \theta < \pi$$

Or:

$$0 \leq \theta < 2\pi$$

Point Stabilisation Control

- For the sake of simplicity, in this course, the goal is defined only by a 2D set of coordinates

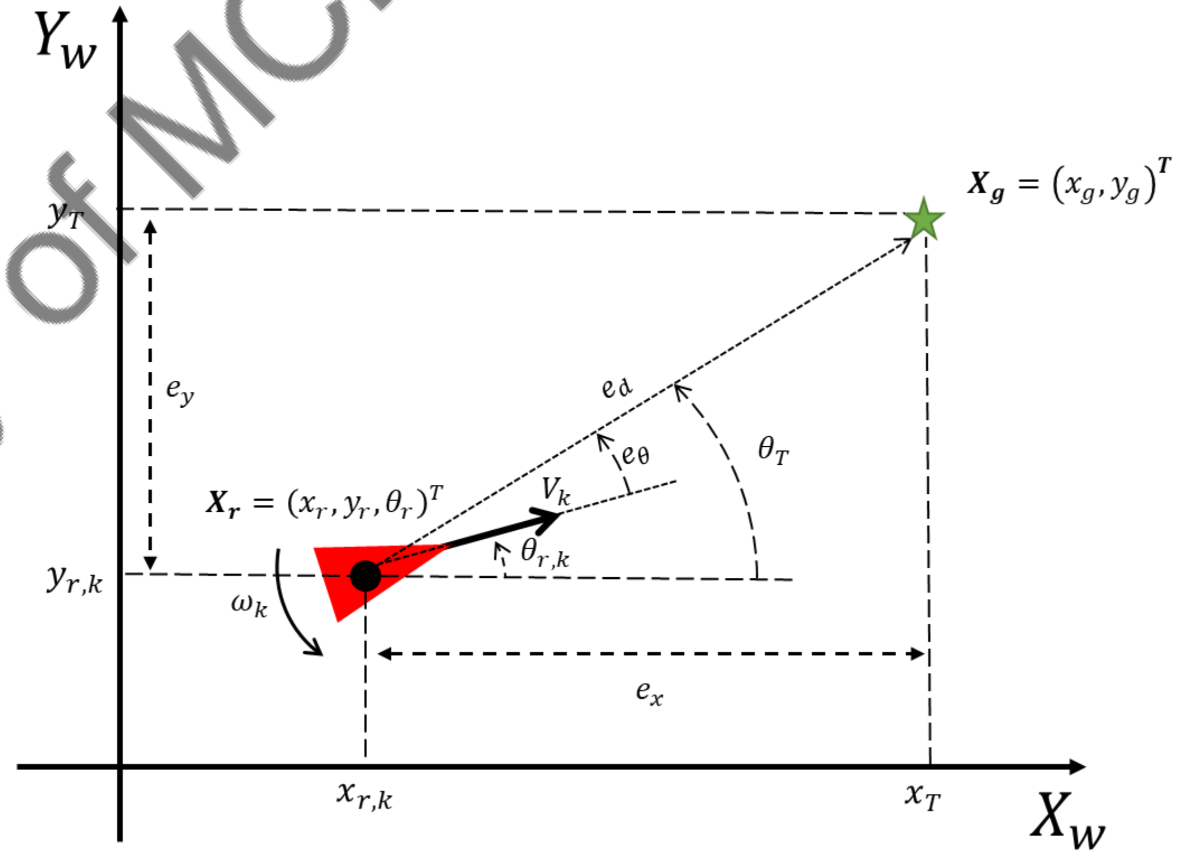
$$\boldsymbol{X}_g = \begin{pmatrix} x_g \\ y_g \end{pmatrix}.$$

- Then compute the errors by using the goal coordinates and robot position as in the following:

$$e_x = x_g - x_r$$

$$e_y = y_g - y_r$$

$$e_\theta = atan2(e_y, e_x) - \theta_r$$



Point Stabilisation

The robot position is assumed to be known and can be computed using various localisation techniques as Dead Reckoning. The equations for the error can be represented in vector format as follows:

$$\boldsymbol{e} = \begin{pmatrix} e_x \\ e_y \\ e_\theta \end{pmatrix}$$

The general form of the control law can be written as:

$$\begin{pmatrix} V \\ \omega \end{pmatrix} = \boldsymbol{K} \begin{pmatrix} e_x \\ e_y \\ e_\theta \end{pmatrix},$$

where

$$\boldsymbol{K} = \begin{pmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \end{pmatrix},$$ is the control matrix.

Point Stabilisation

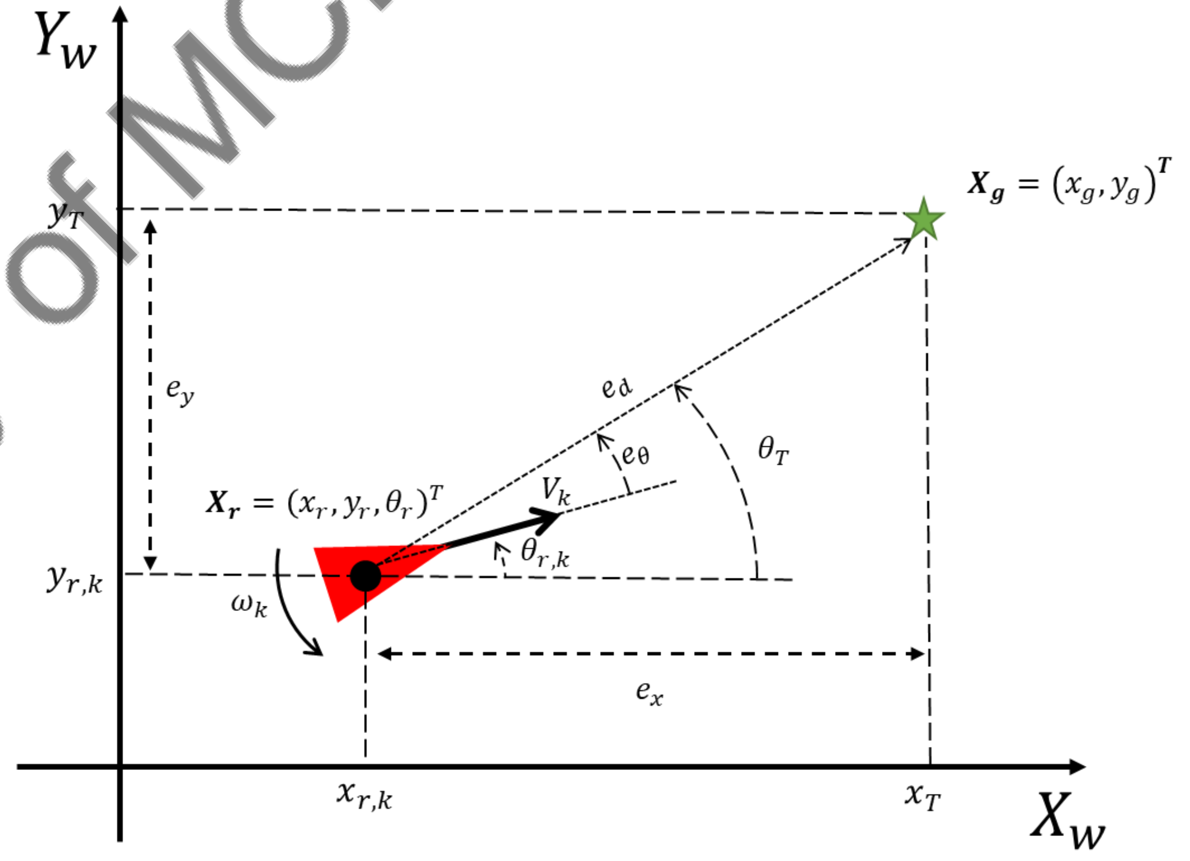
- For simplicity, the six controller gain parameters can be reduced to only two by defining the distance error:

$$e_d = \sqrt{e_x^2 + e_y^2}$$

- The control law can now be written as:

$$V = K_d e_d$$

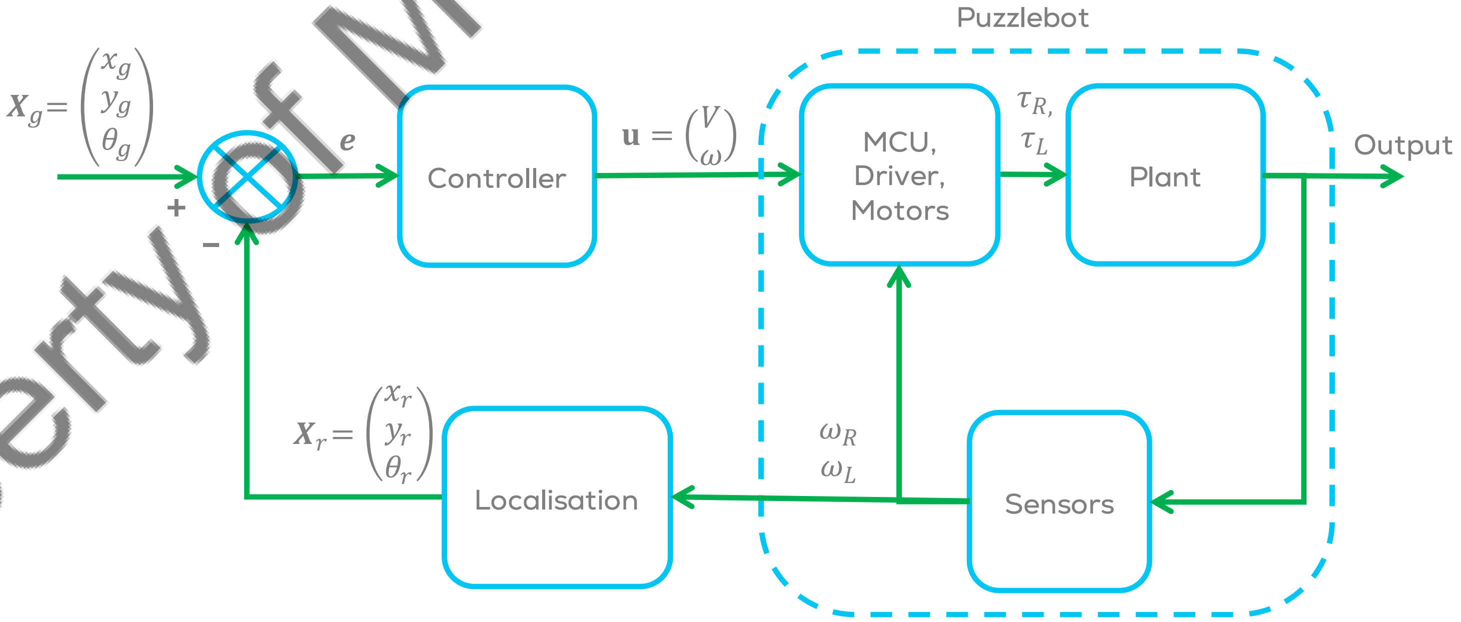
$$\omega = K_\theta e_\theta$$



The Control System

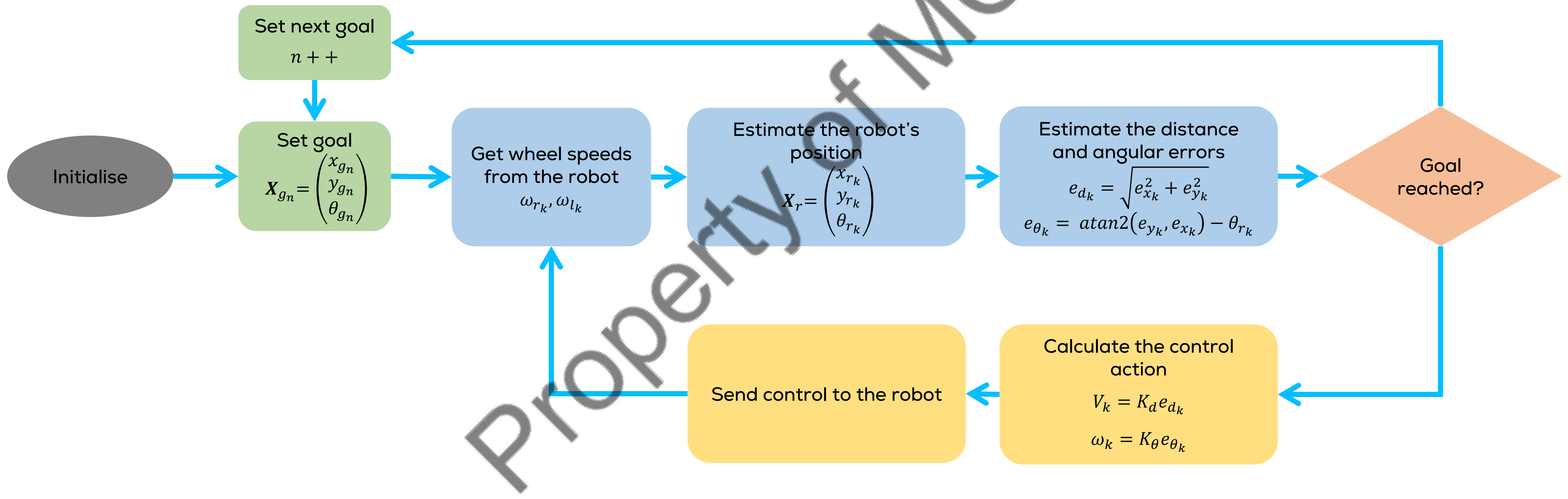
Control

- The control diagram can now be redefined as follows.
- It can be observed that the position can now be estimated using the encoder information.



$X_g = \begin{pmatrix} x_g \\ y_g \\ \theta_g \end{pmatrix}$
+
−
e
Controller
$\mathbf{u} = \begin{pmatrix} V \\ \omega \end{pmatrix}$
Puzzlebot
MCU, Driver, Motors
τ_R, τ_L
Plant
Output
Sensors
ω_R
ω_L
Localisation
$X_r = \begin{pmatrix} x_r \\ y_r \\ \theta_r \end{pmatrix}$

Robot Control Diagram



Set next goal
$n++$
Initialise
Set goal
$\boldsymbol{X}_{g_n} = \begin{pmatrix} x_{g_n} \\ y_{g_n} \\ \theta_{g_n} \end{pmatrix}$
Get wheel speeds from the robot
$\omega_{r_k}, \omega_{l_k}$
Estimate the robot's position
$\boldsymbol{X}_r = \begin{pmatrix} x_{r_k} \\ y_{r_k} \\ \theta_{r_k} \end{pmatrix}$
Estimate the distance and angular errors
$e_{d_k} = \sqrt{e_{x_k}^2 + e_{y_k}^2}$
$e_{\theta_k} = atan2(e_{y_k}, e_{x_k}) - \theta_{r_k}$
Goal reached?
Send control to the robot
Calculate the control action
$V_k = K_d e_{d_k}$
$\omega_k = K_\theta e_{\theta_k}$

Considerations: Accuracy

- It will not be possible to tune the controllers such that the robot moves perfectly into position (Proportional control Steady State Error).
  - You will need a threshold after which your algorithm decides it has successfully arrived.
  - Suggested initial threshold: 10 cm ($e_d < 0.1\ m$)
- For real robots, if you measure the position of the robot, it will likely not match up with the measurement computed from the encoders.
  - This is inevitable due to additive noise in the encoder readings.
  - The solution to this is to use sensors that can measure the position of the robot relative to its environment (another class).

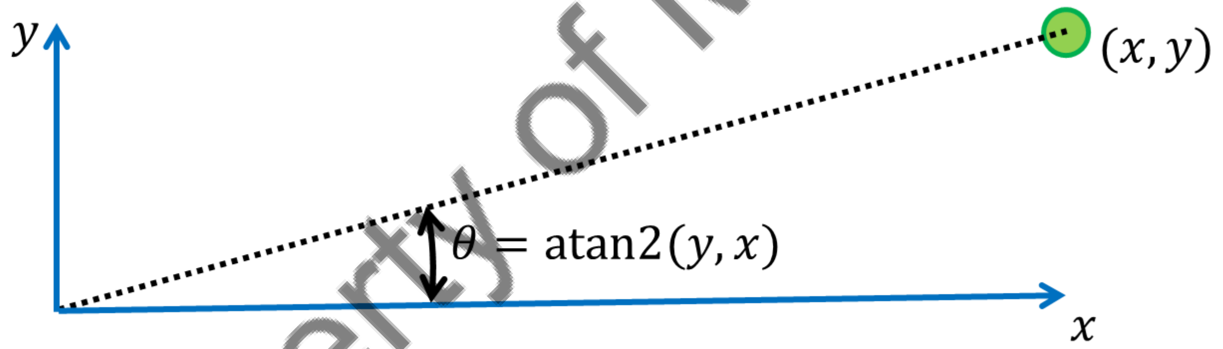
Considerations: Angle

- When estimating the angle of the robot i.e., θ_{r_k} the angle can grow unbounded,this will affect the error estimation and consequently de controller.
- To avoid this the estimated angle must be "wrapped to pi" or "wrappped to 2pi" to keep the angle bounded between $[-\pi,\pi]$ or $[0,2\pi]$.
- There are many ways to wrap and angle as an example the following function will wrap an angle between $[-\pi,\pi]$:

```
#wrap to pi function
def wrap_to_Pi(theta):
    result = np.fmod((theta + np.pi),(2 * np.pi))
    if(result < 0):
        result += 2 * np.pi
    return result - np.pi
```

Considerations: atan2

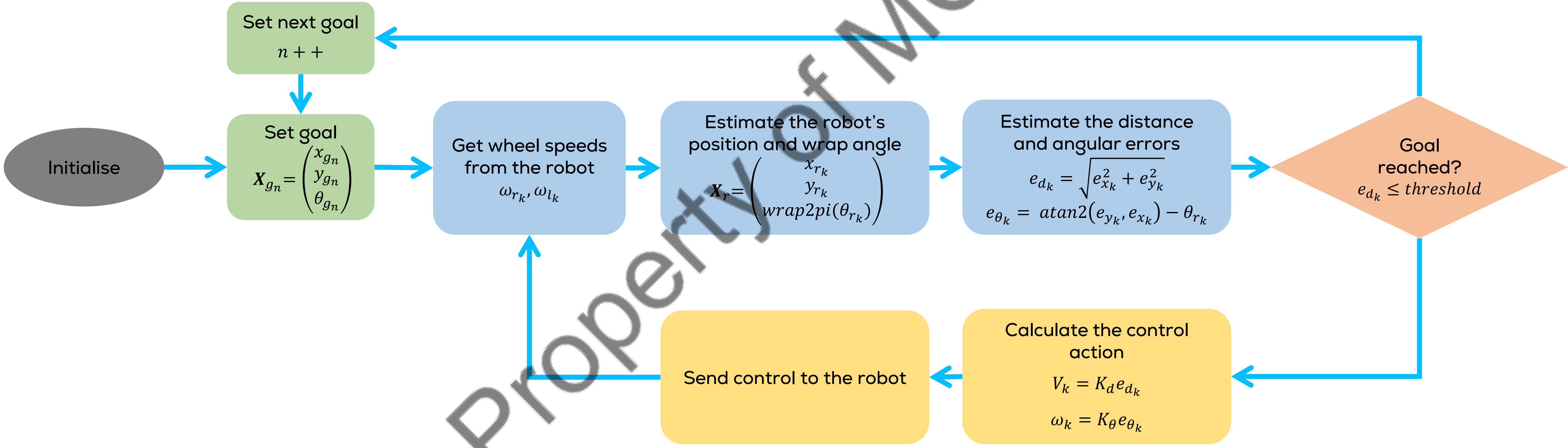
- The atan2 function is a special form of arctan or $\tan^{-1}$.
- It takes two arguments, y and x, and returns the angle to the x axis:



- It is included in most maths libraries, but it is recommended to use numpy, as numpy will be necessary later in the course

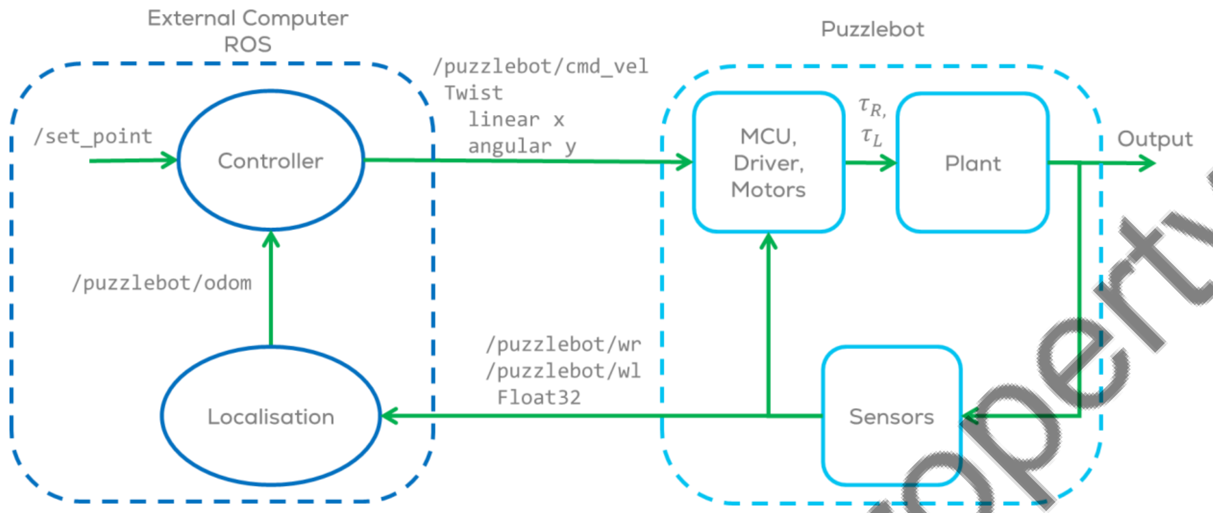
```
import numpy
theta = numpy.arctan2(y,x)
```

Robot Control Diagram

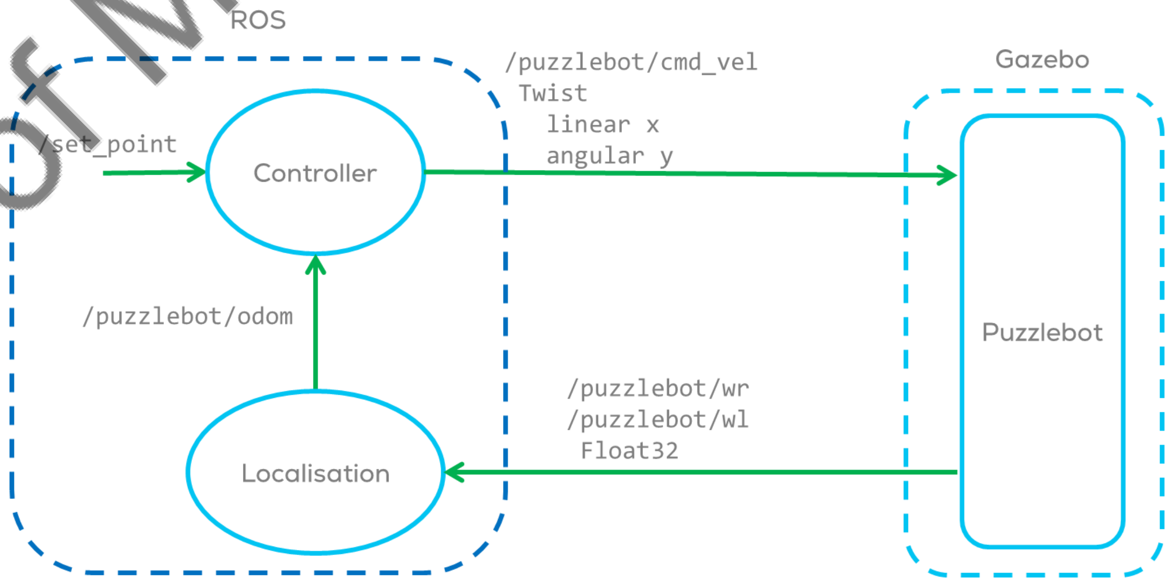

Initialise
Set next goal
$n++$
Set goal
$\boldsymbol{X}_{g_n} = \begin{pmatrix} x_{g_n} \\ y_{g_n} \\ \theta_{g_n} \end{pmatrix}$
Get wheel speeds from the robot
$\omega_{r_k}, \omega_{l_k}$
Estimate the robot's position and wrap angle
$\boldsymbol{X}_r = \begin{pmatrix} x_{r_k} \\ y_{r_k} \\ wrap2pi(\theta_{r_k}) \end{pmatrix}$
Estimate the distance and angular errors
$e_{d_k} = \sqrt{e_{x_k}^2 + e_{y_k}^2}$
$e_{\theta_k} = atan2(e_{y_k}, e_{x_k}) - \theta_{r_k}$
Goal reached?
$e_{d_k} \leq threshold$
Calculate the control action
$V_k = K_d e_{d_k}$
$\omega_k = K_\theta e_{\theta_k}$
Send control to the robot

The Control System in ROS

Real Robot

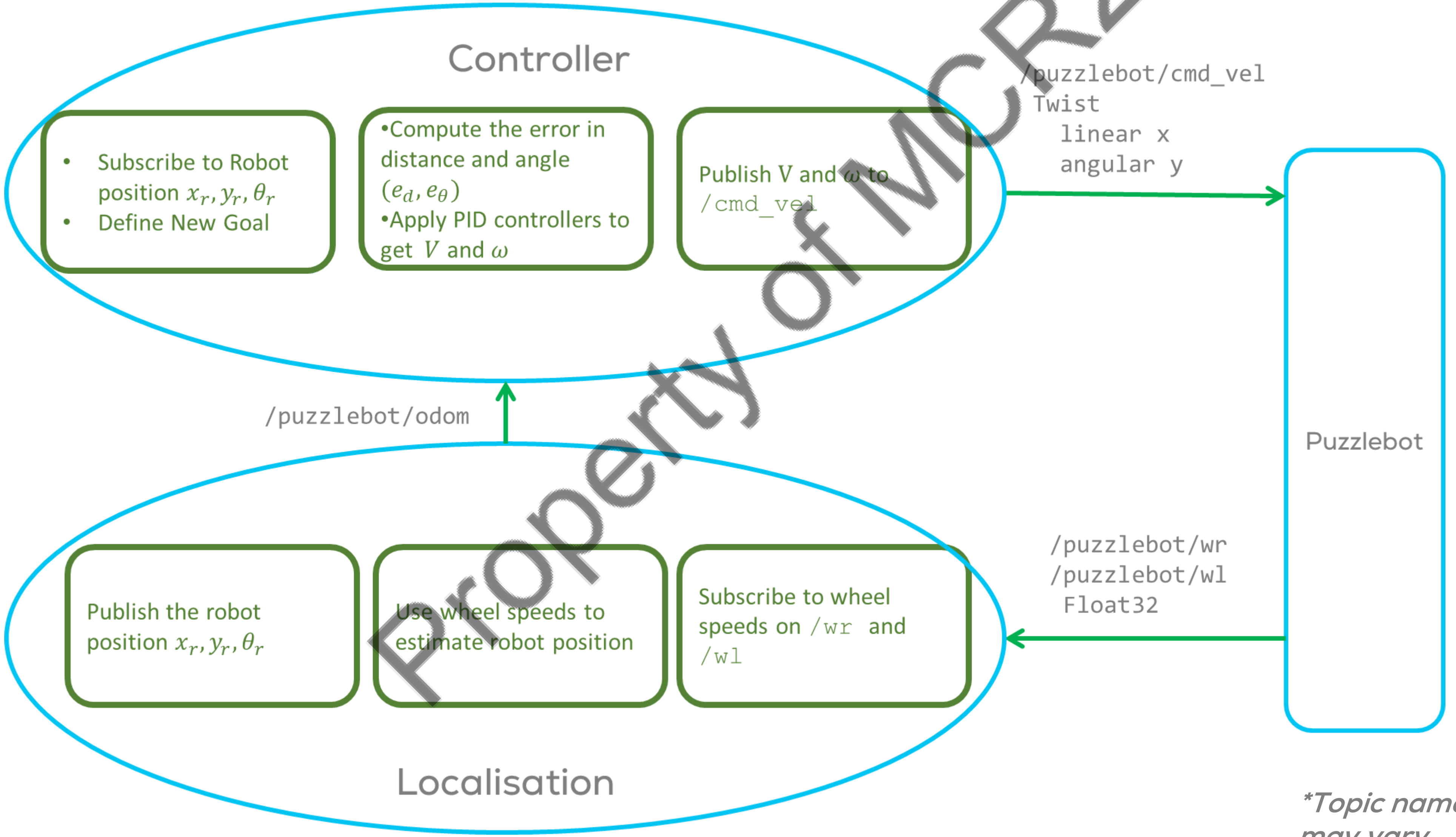


Gazebo sim



**Topic names and interfaces may vary*

ROS setup


Controller
Subscribe to Robot position x_r, y_r, θ_r
Define New Goal
•Compute the error in distance and angle (e_d, e_θ)
•Apply PID controllers to get V and ω
Publish V and ω to /cmd_vel
/puzzlebot/cmd_vel
Twist
linear x
angular y
Puzzlebot
/puzzlebot/odom
Publish the robot position x_r, y_r, θ_r
Use wheel speeds to estimate robot position
Subscribe to wheel speeds on /wr and /wl
/puzzlebot/wr
/puzzlebot/wl
Float32
Localisation

*Topic names and interfaces may vary

Thank you

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