Project Report – Faculty bar

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1 Introduction

1.1 Problem description

Consider a system that manages the orders of customers in a bar. Orders are issued to a cashier, which has two queues: one for normal users and one for VIP users. VIP users have non-preemptive priority over normal users. Users belonging to the same queue are served according to a FIFO policy. Orders can be of simple or compound type. Simple orders are completed directly by the cashier, with a service rate $r_{cashier}$. Composite orders are first served by the cashier, with a service rate $r_{cashier}$, and then are passed to the kitchen, which queues them and serves them in FIFO order, with a service rate $r_{kitchen}$: Both normal and VIP users can issue both types of orders.

1.2 Objectives

In this analysis, the following objectives will be investigated:

- relationship between the cashier service time and the overall experienced response time. In particular, the minimum cashier rate that guarantees a given customer QoS, in terms of maximum response time.
- advantages, in terms of response time, of being a VIP customer.
- adantages and disadvantages of introducing priority head-of-line queueing also in the kitchen.
- optimal value for the ratio between VIP and normal customers in order to achieve the best overall quality of service.
- demonstrate that queue lengths don't depend on the ratio between the service rates.

1.3 Performance indexes

In order to fulfill the above-stated objectives, the following performance indexes will be taken into consideration:

- response times for all four categories of users (normal/VIP, simple/compound order), in particular their mean and 90^{th} percentile.
- average queueing time for all queues.
- average queue length.

1.4 Scenarios

The following scenarios will be taken into consideration:

- constant interarrival times, constant service times;
- exponential distribution of interarrival and service times;

- “realistic” distribution of interarrival and service times. TODO

2 Model

2.1 Description

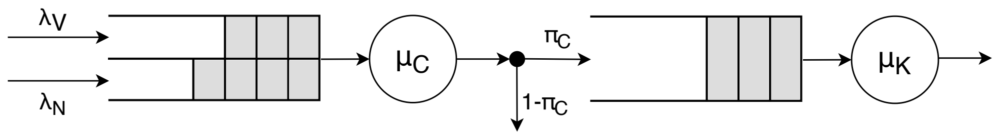

Figure 1: Schematic representation of the model of the bar.

In fig. 1 you can see the model of the bar, where:

- λ_V is the average arrival rate for VIP customers;
- λ_N is the average arrival rate for "normal" (i.e. non-VIP) customers;
- μ_C is the average service rate of the cashier ($r_{cashier}$);
- μ_K is the average service rate of the kitchen ($r_{kitchen}$);
- π_C is the ratio of compound orders over the total, i.e. the probability that an order is compound.

The cashier is modeled as a service center with two queues that are managed in an head-of-line-priority fashion.

The kitchen is modeled as a simple M/M/1 service center. Note that if VIP priority is introduced also in the kitchen, it will become identical to the cashier, mutatis mutandis.

2.2 Assumptions

The following assumptions were made when modeling the system:

- No renegation: customers cannot leave the queue.
- No jockeying: VIP customers cannot move to the normal customers' queue.
- Infinite queueing space: there is no upper bound in the number of customers in a queue.
- the interarrival times of VIP and normal customers are independent RVs.
- the type of an order (simple or compound) is independent of the other orders.
- the service rate is the same for normal and VIP customers.

2.3 Validation

The model exactly reflects the one dictated in the assignment.

2.4 Stochastic model for the exponential scenario

In the exponential scenario, we can formulate a stochastic model of the system using queueing theory.

The cashier priority queueing is, in fact, a well known queueing strategy (L. Kleinrock, 1976)[1]. In the case of two static priorities, the following equations for the average waiting time hold:

$$E[W_V^C] = \frac{\lambda_V + \lambda_N}{(\mu_C - \lambda_V)\mu_C} \tag{1}$$

$$E[W_N^C] = \frac{\lambda_V + \lambda_N}{(\mu_C - \lambda_V - \lambda_N)(\mu_C - \lambda_V)} \tag{2}$$

Note that, for $\lambda_V = 0$ (or $\lambda_N = 0$) the above equations are the same as for an M/M/1 system.

In order to obtain the average response times, we just need to add $\frac{1}{\mu_C}$ due to the linearity of the mean operator.

Furthermore, note that if we didn't make any distinction between orders and just saw them entering the cashier SC and exiting it, we would not be able to distinguish it from an M/M/1 SC with arrival rate $\lambda_V + \lambda_N$ and service rate μ_C just by looking at the distribution of the inter-departure times (we don't care about the order).

This result has been confirmed by the validation runs on the simulator. Please note that this would not have hold if service rate were different between normal and VIP customers.

Therefore, we can apply queueing network theory which tells us that the average arrival rate at the kitchen is the same as an M/M/1 SC with arrival rate equal to $\pi_C(\lambda_V + \lambda_N)$. Thus, the response time of the kitchen (independently of the customer type) is:

$$E[R^K] = \frac{1}{\mu_K - \pi_C(\lambda_V + \lambda_N)} \tag{3}$$

Please note that, if the kitchen had a priority queueing, we would not be able to follow the same reasoning above. However, simulation results suggest that the mean waiting time could be computed as in eqs. (1) and (2) considering $\pi_C\lambda_N$ and $\pi_C\lambda_V$ as the arrival rates.

[1] Leonard Kleinrock (1976). Head-of-the-Line Priorities. *Queueing systems, volume 2: Computer applications* (pp. 119-126). New York, Wiley.

Finally, we can write the average response times for the four classes of orders:

$$E[R_{N,S}] = E[W_N^C] + \frac{1}{\mu_C} \tag{4}$$

$$E[R_{N,C}] = E[W_N^C] + \frac{1}{\mu_C} + E[R^K] \tag{5}$$

$$E[R_{V,S}] = E[W_V^C] + \frac{1}{\mu_C} \tag{6}$$

$$E[R_{V,C}] = E[W_V^C] + \frac{1}{\mu_C} + E[R^K] \tag{7}$$

3 Implementation

3.1 Code overview

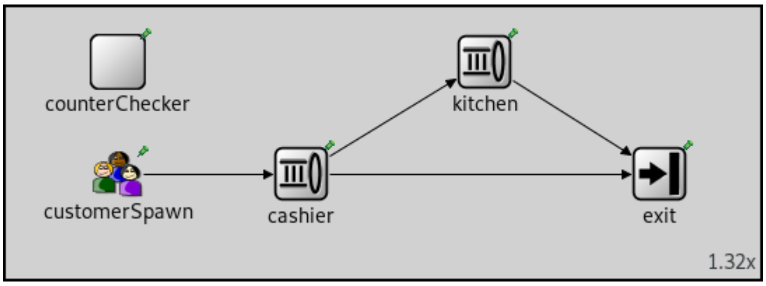

Figure 2: Diagram of the Omnet++ network.

The Omnet++ network is composed of 5 nodes:

- **customerSpawn**: generates the orders using a configurable distribution.
- **cashier**: serves orders and, upon completion, sends them to the *kitchen* if they are compound or to the *exit* otherwise. The type of order is determined in the *customerSpawn* node.
- **kitchen**: serves orders and sends them to the *kitchen* upon completion.
- **exit**: collects statistics about order response times and disposes of them.
- **counterChecker**: collects number of generated, exited and in-queue orders in order to check that no order is lost.

3.1.1 Validation

In order to verify that the implementation exaclty reflects our model, we carried out the following tests.

Test 1: Single-type orders with constant inter-arrival times Only orders of a single type are generated with constant inter-arrival times in order to check that the correct path is followed by the orders and that statistics collection is working correctly. Note that in this case there is no queueing.

Test 2: Simple and compound normal orders with constant inter-arrival times Only normal orders are generated, both simple and compound, with constant inter-arrival times in order to check that the orders are correctly routed between kitchen and exit with the set probability.

Test 3: Normal and VIP simple orders with constant inter-arrival times Only simple orders are generated, both normal and VIP, with constant inter-arrival times in order to check that the priority queueing works correctly. In order to do that, two normal orders are sent at time t=1 and t=2 respectively, a VIP order is sent at time t=2.5 and the service time is 2. It is expected the last VIP order to be serviced second.

Test 3b: Normal and VIP compound orders with constant inter-arrival times Only compound orders are generated, both normal and VIP, with constant inter-arrival times in order to check that the priority queueing works correctly. In order to do that, two normal orders are sent at time t=1 and t=2 respectively, a VIP order is sent at time t=2.5 and the service time is 2. It is expected the last VIP order to be serviced second. The test is repeated for both FIFO and priority queue in the kitchen.

Test 4: Simple orders with exponential inter-arrival times Simple orders are generated with exponential inter-arrival times and the mean response time for each kind is compared with the mathematical model. Several scenarios are considered namely:

- total arrival rate is kept constant to 1 while the proportion of normal and VIP orders varies from 0% to 100%, with a step of 25%.
- cashier service rate is varied from 1.1 to 2 with a step of 0.1.

Test 4b: Compound orders with exponential inter-arrival times Compound and simple orders are generated with exponential inter-arrival times. Compound orders are 10% of the total, while simple orders are ignored (they are only generated in order to create a realistic scenario). The mean response time for each kind of compound orders is compared with the mathematical model. Several scenarios are considered namely:

- total arrival rate is kept constant to 1 while the proportion of normal and VIP orders varies from 0% to 100%, with a step of 25%.
- kitchen service rate is varied from 0.15 to 0.5 with a step of 0.05.
- cashier service rate is kept constant to 1.5.

The test is repeated for both FIFO and priority queue in the kitchen. In the case of the priority queue, the mathematical model could not be used so it was checked that:

- mean response time for normal orders is higher than VIP orders.
- mean waiting time for normal orders is higher than VIP orders.
- performance indexes are in a continuous relationship with the simulation factors.

Validation results All the validation tests listed above have been passed by the implementation. Test 4/4b have been checked by automatically plotting, through a script, the performance metrics. fig. 3 shows an example of the plots. You can check all the plots in `simulation/jupyter/Test.ipynb`.

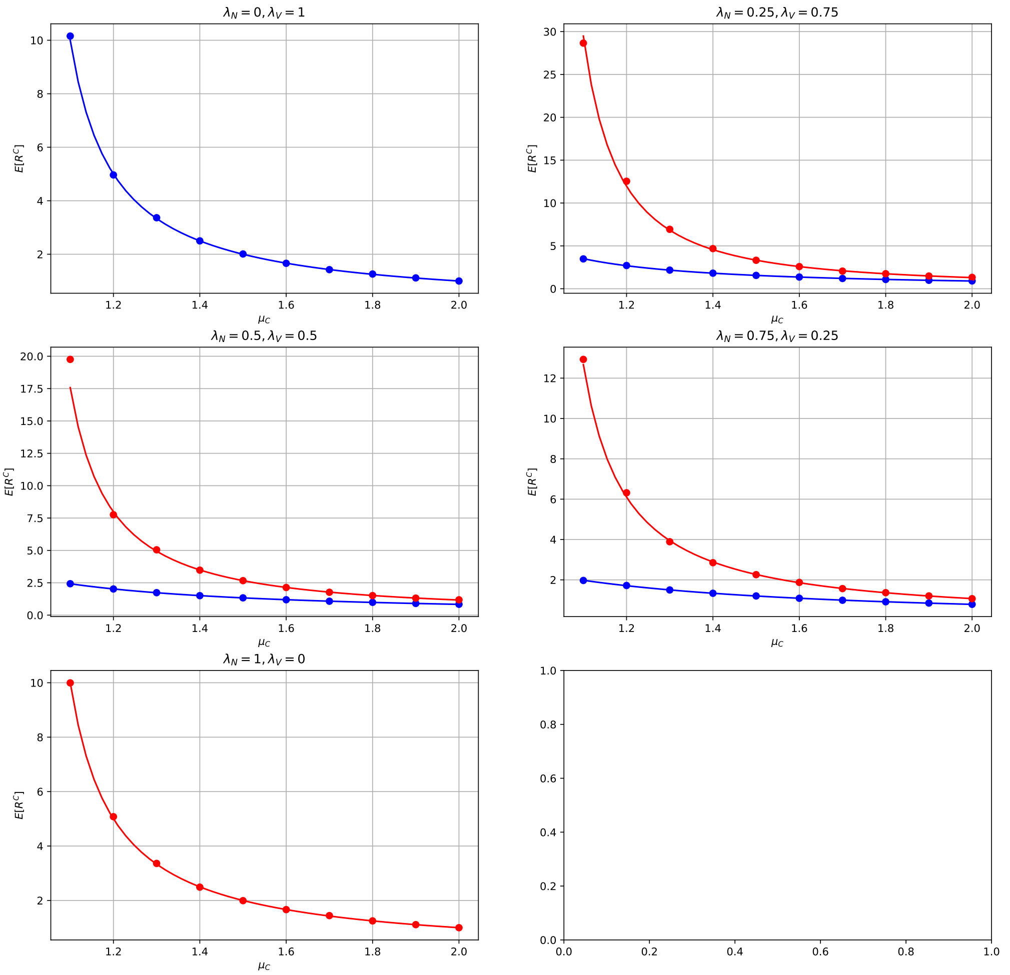

Figure 3: Validation of Test 4 showing the mean cashier response time for both VIP (in blue) and normal (in red) customers.

3.2 Calibration

4 Experiments

4.1 Design

In order to offer the best possible service, we need to properly tune every factor on which the system depends on:

- cashier service rate μ_C
- kitchen service rate μ_K
- odds of an order being a Simple one π_S= 1-π_C
- odds of an order being a Compound one π_C
- inter-arrival rate of Vip orders λ_V
- inter-arrival rate of Normal orders λ_N
- Kitchen queue type, "fifo" or "priority"

We'll try to summarize all the information that could be useful to a Bar's Administrator, mainly focusing on granting privileged services to Vip users. To analyze how much a VIP user gains, in terms of Response Time, compared to a Normal User, we added two new statistics called "simpleResponseTimeRatio" and "compoundResponseTimeRatio", respectively for simple orders and compound orders. They are computed as follows:

$$simpleResponseTimeRatio = 1 - \frac{E[R_{V,S}]}{E[R_{N,S}]} \tag{8}$$

$$compoundResponseTimeRatio = 1 - \frac{E[R_{V,C}]}{E[R_{N,C}]} \tag{9}$$

First of all we restricted the allowed values of the factors in order to have experiments that properly resembles a Bar. The chosen values are:

- μ_C = [1.5;2.0]
- μ_K = [0.45;0.6]
- π_S = 1-π_C
- π_C = [0.1; 0.3]
- π_V = [1.3-π_N]
- π_N = [0.1;1.2]
- KitchenQueueType = ["fifo""priority"]

In order to grasp which of the factors contributes to a certain phenomenon and what its contribution is we computed a number of 2^kr (r = 5) factorial analysis. We will show them one by one and then discuss the more peculiar aspects. These will be the factors taken into consideration:

- A = ratio between the amount of VIP and Normal users, by increasing it, the Normal arrival rate increases whereas Vip arrival rate decreases.
- B = odds of an order being a Compound one
- C = kitchen Rate
- D = cashier Rate

In the Omnetpp configuration that we utilized we considered the exponential scenario with 5 repetitions.

4.2 Fifo Scenario

4.2.1 simpleResponseTimeRatio

	q	SSx	Variation	Confidence Interval
A	-0.0463	0.172	7.82%	(-0.0471, -0.0455)
B	-0.000963	7.42e-05	0.00%	(-0.00178, -0.000143)
AC	-0.00123	0.000121	0.01%	(-0.00205, -0.000411)
D	-0.159	2.02	92.03%	(-0.16, -0.158)
AD	-0.00441	0.00155	0.07%	(-0.00523, -0.00359)
ABCD	-0.00109	9.44e-05	0.00%	(-0.00191, -0.000267)
Errors		0.00123	0.06%	

There's no particular interplay occurring between factors, we can clearly see that the simpleResponseTimeRatio depends on:

- The main reason why VIP users are privileged is due to the queueing of orders, if there's no queue the difference of Normal and Vip response times are negligible. This is exactly why the cashier rate has a negative contribution: with the increase of this factor there's less queueing and thus when a new normal order comes chances are that there will be no Vip users in queue so it can be served immediately.
- The ratio between the amount of VIP and Normal users (A) has a negative contribution, this is a little bit unexpected. The reason has a connection with the starvation of Normal users. Let's try to break it up in two cases:
  - Many VIP users, few Normal users: for obvious reasons VIP users are subdued to long queue so their response time will be lower. With the same reasoning the few Normal users will have to wait a long time before being served since, while being in queue, other VIP users will arrive, eventually inducing the normal order to starvation (with starvation we intend that there are very long response time but the orders will eventually be served).

- Few VIP users, many Normal users: now VIP users are very fast, but normal user are no more in starvation(long response time but much more reasonable). The Normal response time has lowered much more from the previous case, compared to the Vip response time, lowering the simpleResponseTimeRatio too.

4.2.2 cashierVipQueueLength

	q	SSx	Variation	Confidence Interval
A	-1.09	94.8	60.19%	(-1.09, -1.08)
D	-0.633	32.0	20.33%	(-0.637, -0.629)
AD	0.619	30.6	19.45%	(0.615, 0.623)
Errors		0.0307	0.02%	

We can observe that:

- if the Vip arrival rate decreases (factor A increases) the VIP queue length will also decrease accordingly.
- by increasing the cashier rate Vip queue length will also decrease.
- Here we can also see an interesting interplay by factor A and D, meaning that a good portion of the variation is explained by the interaction of those factors.

4.2.3 cashierNormalQueueLength

	q	SSx	Variation	Confidence Interval
A	1.08	93.7	28.86%	(1.07, 1.1)
D	-1.59	2.01e+02	61.88%	(-1.6, -1.57)
AD	-0.608	29.6	9.11%	(-0.624, -0.593)
Errors		0.44	0.14%	

Basically all the observation done for the cashierVipQueueLength remains with minor changes. We can observe that now factor A has a positive contribution, this was to be expected because by increasing factor A we increase the Normal arrival rate meaning that new Normal orders are subject to longer queue.

4.2.4 kitchenQueueLength

	q	SSx	Variation	Confidence Interval
A	0.0315	0.0793	0.02%	(0.00582, 0.0571)
B	1.67	2.23e+02	52.90%	(1.65, 1.7)
AB	0.0319	0.0814	0.02%	(0.00624, 0.0576)
C	-1.12	1.01e+02	23.97%	(-1.15, -1.1)
AC	-0.0336	0.0905	0.02%	(-0.0593, -0.00798)
BC	-1.1	96.0	22.74%	(-1.12, -1.07)
ABC	-0.0337	0.091	0.02%	(-0.0594, -0.00806)
Errors		1.21	0.29%	

In this particular scenario the kitchen has no priority queue. This means that all the orders coming from the cashier are treated in the same way, even if they are VIP. Of course, by that, we can expect that factor A would not bring any contribution to the variation and it's exactly what we can see from the results. Instead:

- factor B has high positive contribution. By increasing it (probability of compound orders increase) we can expect longer queue.
- of course by increasing factor C (kitchen rate) we can expect shorter queue in the kitchen.
- Here we can also see an interesting interplay by factor A and D, meaning that a good portion of the variation is explained by the interaction of those factors.

4.3 Priority Scenario

For what concern the cashier part of the system, all the observations done in the Fifo scenario still holds. In fact we can see that all the results are exactly the same, so we will not discuss this case any further.

4.3.1 kitchenVipQueueLength

	q	SSx	Variation	Confidence Interval
A	-0.553	24.4	24.86%	(-0.56, -0.546)
B	0.537	23.0	23.43%	(0.53, 0.544)
AB	-0.516	21.3	21.63%	(-0.523, -0.508)
C	-0.314	7.87	8.00%	(-0.321, -0.307)
AC	0.306	7.48	7.61%	(0.299, 0.313)
BC	-0.3	7.21	7.33%	(-0.307, -0.293)
ABC	0.294	6.9	7.02%	(0.287, 0.301)
Errors		0.091	0.09%	

- by increasing factor A (Normal Arrival Rate increases, Vip arrival Rate decreases) the VIP queue length tends to decrease.
- by increasing factor B (probability of compound orders), the total inter-arrival rate of the kitchen increases too so new orders are more subject to queueing.
- of course, by increasing the Kitchen Rate (factor C) the length of VIP Queue length decreases.
- of course by being a Pasta system (no orders are created or destroyed inside the system), the inter-arrival rate of the cashier is also the inter-departure rate and, by that, we can be sure that the cashier rate (factors D) brings no contribution to the Normal Queue Length in the kitchen.

4.3.2 kitchenNormalQueueLength

	q	SSx	Variation	Confidence Interval
A	0.584	27.3	9.88%	(0.562, 0.607)
B	1.13	1.03e+02	37.26%	(1.11, 1.16)
AB	0.547	24.0	8.68%	(0.525, 0.57)
C	-0.811	52.6	19.05%	(-0.834, -0.788)
AC	-0.339	9.22	3.34%	(-0.362, -0.317)
BC	-0.795	50.6	18.32%	(-0.818, -0.773)
ABC	-0.327	8.58	3.11%	(-0.35, -0.305)
Errors		0.943	0.34%	

We can observe that:

- by increasing factor A (Normal Arrival Rate increases) the Normal queue length tends to increase too.
- by increasing factor B (probability of compound orders), the total inter-arrival rate of the kitchen increases too so new orders are more subject to queueing.
- of course, by increasing the Kitchen Rate (factor C) the length of Normal Queue length decreases.
- of course by being a Pasta system (no orders are created or destroyed inside the system), the inter-arrival rate of the cashier is also the inter-departure rate and, by that, we can be sure that the cashier rate brings no contribution to the Normal Queue Length in the kitchen.
- all the interplays bring negligible contributions apart from the BC one which should be taken into consideration when tuning the system.

4.4 Vip Rate study

One of the things that should be taken into consideration when fine-tuning the system is the amount of allowed VIP users. Of course if all users are VIP there would be no benefit, all the observation discussed till now would makes no sense and chance are that the few Normal user that comes will never be served. So, in order to decide the correct percentage of allowed VIP users, that permits a reasonable response time for Normal User along with some benefits to the Vip ones, we executed a full factorial analysis with the following configuration:

By changing only the percentage of VIP users we can set a reasonable response time for Normal user (we opted for 15m) and establish the optimal percentage of Vip users that is between 10% and 30%.

We can also plot the simpleResponseTimeRatio with the same configuration and ensure that, in fact, Vip users are at least 80% faster than Normal users in that range.

Note that we have taken into consideration only the cashier Response Time because there's nothing special going on the kitchen, all users are treated equally. Basically the total Response Time can be seen as the cashier Response Time plus a constant.

All the above can be repeated for the Priority case to get similar results.

4.5 Data collection

5 Analysis