① $T(n) = 3T(n-1)+12n$ $\quad T(0)=5$ $(n=0, \Rightarrow 5)$

$\Rightarrow T(n) = 3T(n-1)+12n \rightarrow ①$

Put $n = n-1$

$T(n-1) = 3T(n-2)+12n-12 \rightarrow ②$

Put ② in ①

$T(n) = 3^2 T(n-2)+24n-12 \rightarrow ③$

Put $n = n-2$ in ①

$T(n-2) = 3T(n-3)+12n-36 \rightarrow ④$

Put ④ in ③

~~$T(n) = 3^3 T(n-3) + 12(3)n + 12(3)$~~

$\Downarrow$ K times

$$\boxed{T(n) = 3^k T(n-k) + 12(k)n + 12k}$$

$\cancel{T(0)} = \quad n-k=0 \Rightarrow \boxed{n=k}$

$T(n) = 3^n T(n-n) + 12(n^2) + 12n$

$T(n) = 3^n T(0) + 12(n^2) + 12n$

$T(2) = 3^2(5) + 12(4) + 12(2)$

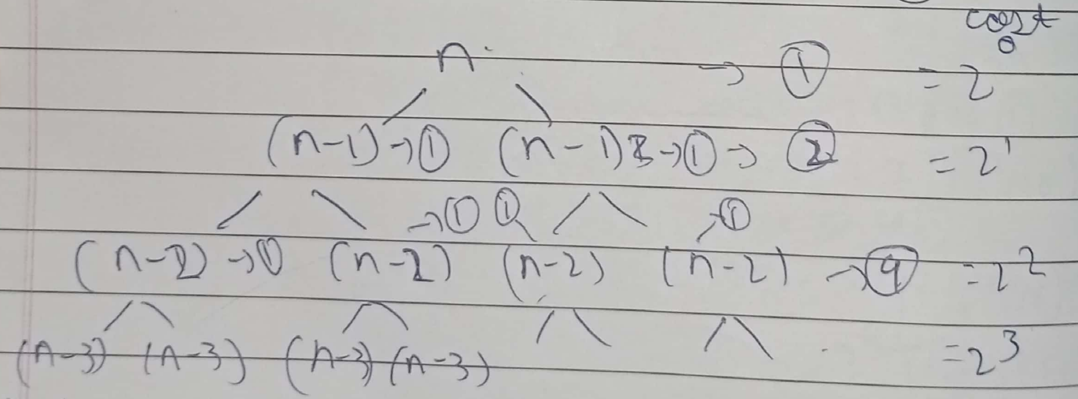
$= 9(5) + 48 + 24$

$$\boxed{T(2) = 117}$$

Recursive Tree

Q3) a) $T(n) = 2T(n-1)+1$

$= T(n-1) + T(n-1) + 1$ cost



cost LHS = RHS $(2^0 + 2^1 + 2^2 + \dots 2^k)$

L.HS	RHS
$n-k=1$	$n-k=1$
$\boxed{n-1=k}$	$\boxed{k=n-1}$

eqn $\Rightarrow 2^0 + 2^1 + 2^2 + \dots 2^{n-1}$

~~$= 1+4+9+16+\dots 2^{n-1}$~~

$= 1+2+4+8+16+\dots+2^{n-1}$

$= 2\left(\frac{1}{2}+1+2+4+8+\dots+2^n\right)$

$$eq^n = 2\left(\frac{1}{2} + a\frac{r^n - 1}{r-1}\right) \quad , a = 1, \; r = 2$$

$$= 2\left(\frac{1}{2} + \frac{2^n - 1}{1}\right)$$

$$= 2\left(\frac{1}{2} + 2^n - 1\right)$$

$$= 1 + 2^{n+1} - 2$$

$$\boxed{eq^n = 2^{n+1} - 1} \Rightarrow \boxed{O(2^n)}$$

b) $T(n) = 2T(n/2) + n = T(n/2) + T(n/2) + n$

cost

$$n \rightarrow n$$

$$n/2 \Leftarrow n/2 \quad n/2 \quad n/2 \rightarrow n$$

$$n/4 \Leftarrow n/4 \quad n/4 \quad n/4 \quad n/4 \Rightarrow n$$

k time

kn cost

LHS = RHS

$$= n/2^k = n$$

$$= \frac{n}{2^k} = n$$

$$= n = \log 2^k n$$

$$1 = k \log_2 2$$

$$\boxed{k = 1}$$

$$eq^n = kn$$

$$= n$$

$$= \boxed{O(n)}$$

II) Substitution

(a) $T(n) = T(n-1) + C$

$\Rightarrow T(n) = T(n-1) + C \rightarrow (1)$

Put $n = n-1$ in (1)

$T(n-1) = T(n-2) + 2C \rightarrow (2)$

Put $n = n-2$ in (1)

$T(n-2) = T(n-3) + 3C \rightarrow (3)$

Put (3) in (2)

$T(n) = T(n-3) + 3C$

↓ K time

$T(n) = T(n-k) + kC \rightarrow (4)$

$n - k = C$ $\boxed{k = n - C}$

$\boxed{\cancel{n = k + C}}$

Put in (4)

$T(n) = T(n - n + C) + (n - C)C$

$= T(C) + Cn - C^2$

$= Cn$ (Biggest)

$\boxed{T(n) = O(n)}$

(b) $T(n) = 2T(n/2) + n$

$\Rightarrow T(n) = 2T(n/2) + n \rightarrow (1)$

Put $n = n/2$ in (1)

$T(n/2) = 2T(n/2^2) + n/2 \rightarrow (2)$

Put (2) in (1)

$T(n) = 2\left[2T\left(\frac{n}{2^2}\right) + \frac{n}{2}\right] + n$

$= 4T\left(\frac{n}{4}\right) + \frac{2n}{2} + n$

$T(n) = 4T\left(\frac{n}{4}\right) + 2n \rightarrow (3)$

Put $n = n/4$ in (1)

$T(n/4) = 2T\left(\frac{n}{8}\right) + \frac{n}{4} \rightarrow (4)$

Put (4) in (3)

$T(n) = 4\left[2T\left(\frac{n}{8}\right) + \frac{n}{4}\right] + 2n$

$= 8T\left(\frac{n}{8}\right) + \frac{8n}{4} + 2n$

$= 2^3 T\left(\frac{n}{2^3}\right) + 2n + 2n$

$T(n) = 2^3 T\left(\frac{n}{2^3}\right) + 4n \rightarrow (5)$

Put $n = n/8$ in (1)

$T(n/8) = 2T(n/16) + \frac{n}{8} \rightarrow (6)$

Put (6) in (5)

$T(n) = 2^3\left[2T(n/16) + \frac{n}{8}\right] + 4n$

$= 2^4 T\left(\frac{n}{16}\right) + n + 4n$

$T(n) = 2^4 T\left(\frac{n}{2^4}\right) + 5n$

↓ K times

$T(n) = 2^k T\left(\frac{n}{2^k}\right) + (k+1)n$

$\boxed{\text{cost} = n} \rightarrow \boxed{k = n}$ $= 2^n \Rightarrow \boxed{O(2^n)}$

c) $T(n) = 2T(n/2) + c$

From Previous eq^n

$GR \Rightarrow T(n) = 2^k T\left(\frac{n}{2^k}\right) + (k+1)n$

$k = c \Rightarrow$ cost $= c$

$T(n) = 2^c T\left(\frac{n}{2^c}\right) + cn$

$\Downarrow$

biggest

$$\boxed{T(n) = O(n)}$$

d) $T(n) = T(n/2) + c$

$GR \Rightarrow T(n) = T\left(\frac{n}{2^k}\right) + kc$

$\Uparrow$ From Previous

$k = c \Rightarrow$

$$\boxed{T(n) = O(n)}$$