Indian Institute of Technology, Kanpur
Department of Earth Sciences

ESO213A: Fundamentals of Earth Sciences

Lecture 19. Concept of Stress

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Experimental Rock Deformation Laboratory

Aims of this lecture

- Concept of Mechanics, Force and Stress, dimensions and units
- Stress on a Surface and at a Point; Stress Tensor
- Stress Ellipse and Ellipsoid
- Compressive, Tensile and Shear stresses

Structural Geology and Mechanics

- We have learnt now that the rock-masses get strained (deformed) and to achieve the strain the rock-mass must have experienced some natural forces/pressures.

- If we consider the rocks are "*materials*", the study of the deformation of rocks under forces falls under the subject "***Mechanics***", which deals the science related to the behaviour of physical materials subjected to force and displacements. *Think of the term we use "Rock Mechanics".*

- Therefore, the study of the deformation of rocks can be included and explained under the broader subject: ***MECHANICS.***

Concept of Continuum

- A body is and remains CONTINUOUS under the action of external forces
  - Consisting of continuous material points
  - Neighboring points remain neighbors
  - Neglecting its atomistic structure

CONTINUUM MECHANICS

- A continuum, or continuous medium, is represented as a continuous aggregates of idealized material particles (elemental volumes). They are small enough that their position can be given in terms of points in some co-ordinate systems, yet large enough that local value of any variable does not depend on fluctuations at the atomic scale in the immediate neighbourhood of the point.

- In order to deal with the properties and mechanics of continuum, it is therefore necessary to refer the body to *a system of co-ordinates*.

Force

- An object, in motion or in equilibrium is a function of the object's mechanical interaction with the other objects. FORCE is the quantitative measure and description of the mechanical interaction.

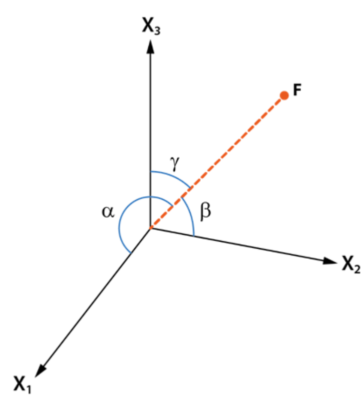
- A **force** is a *vector quantity* [*first-order tensor*] and has magnitude, direction and point of application

- SI unit of Force is **Newton** [1Newton is required to accelerate a 1kg mass at 1 meter/second^2] ……[$\boldsymbol{mlt^{-2}}$]

  Dyne [1Dyne is required to accelerate a 1gm mass at 1 cm/second^2]

Force

Being a vector, the components of the Force (***F:*** magnitude *F, and* α*,* β*,* γ *are the angles it makes with the axes of a Cartesian coordinate system*) in 3D:



$$F_{X_1} = F\cos\alpha \qquad F_{X_2} = F\cos\beta \qquad F_{X_3} = F\cos\gamma$$

and

$$F^2 = F_{X_1}^2 + F_{X_2}^2 + F_{X_3}^2$$

Force

- The forces acting on a body can be divided into two groups:
  - ***Internal forces****:* Internal forces represent the interaction between the particles in the body.
  - ***External forces****:* refer to the action of other bodies on the particles of a given body.

Body forces act on unit mass or unit volume of the body (e.g., gravity and magnetic forces).

Surface forces act on the surface of a body when it comes in contact with another body. The surface forces are mostly responsible for the deformation of rocks at various scales. *The surface forces acting on an area are often referred as traction*.

Stress on a surface - TRACTION

- The stress on a surface (traction) can be idealised in geological context in many different ways: on a fault plane, on the contact areas between adjacent grains, meteoritic impacts etc.
- In mechanics, the stress on a surface (traction, *T*) is defined as the ratio between the Reactive Force (*F*) and the Surface area (*S*), on which the force is acting.

$$\vec{T} = \frac{\vec{F}}{S}$$

- As, Force is a vector, the Stress on a surface (traction) is also a vector.
- SI unit of Stress is **Pascal (Pa)** = Newton/(Meter)^2 = 1 kg/m.s^2 ……[$\boldsymbol{ml^{-1}t^{-2}}$]

1 Pa = 10^{-5} bar = 0.000145 psi   1 MPa = 10 bar = 145 psi

Pressures in normal bicycle and car tyres are 0.6 and 0.24 MPa, respectively. Lithostatic Pressures at the lower-upper mantle boundary (670 km) ~28 GPa; at core-mantle boundary 330 GPa and at the center of the earth ~400 GPa.

Stress on a surface element

- ***Stress acting on a SURFACE:*** VECTOR (Traction)
- ***Stress acting at a POINT:*** TENSOR

To know more about this and the derivations, see here:

https://youtu.be/rKafI2wUgJ8?list=PLHyuArGIlyyR_2mObwQ3yng18LDnDqidp

Stress at a point – STRESS TENSOR

- For easier mathematical operations, imagine a very small parallelepiped around the ***P*** with dimensions ΔX_1, ΔX_2 and ΔX_3.
- The Tractions of the three positive faces of the parallelepiped can be resolved into their Cartesian components: one normal, and two tangential to the face on which the tractions acts.

$$T_{(1)i} = (\sigma_{11}, \tau_{12}, \tau_{13}) \Rightarrow \text{Face normal to } X_1$$

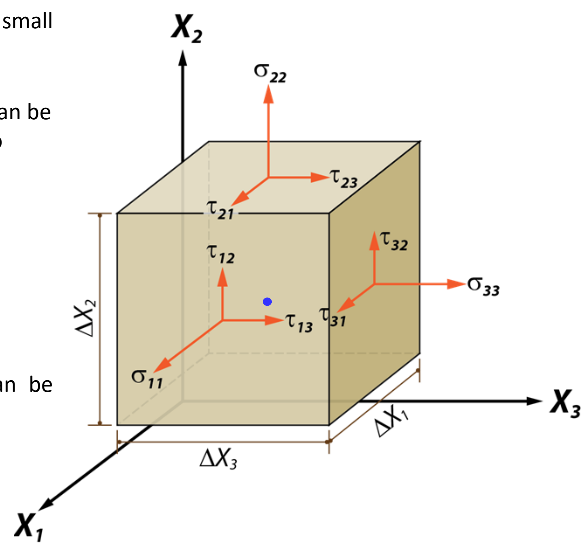
$$T_{(2)i} = (\tau_{21}, \sigma_{22}, \tau_{23}) \Rightarrow \text{Face normal to } X_2$$

$$T_{(3)i} = (\tau_{31}, \tau_{32}, \sigma_{33}) \Rightarrow \text{Face normal to } X_3$$

- The nine components of the tractions acting at a point can be expressed in index notation:

$$\sigma_{ij} = \begin{bmatrix} \sigma_{11} & \tau_{12} & \tau_{13} \\ \tau_{21} & \sigma_{22} & \tau_{23} \\ \tau_{31} & \tau_{32} & \sigma_{33} \end{bmatrix}$$

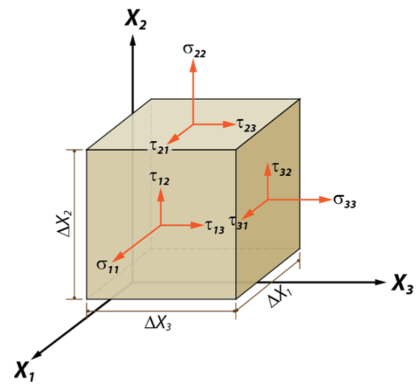
STRESS TENSOR



Stress at a point - Equilibrium

- The Conditions of equilibrium for *body and surface forces*

$$\left.\begin{aligned}\frac{\partial\sigma_{11}}{\partial x_1}+\frac{\partial\sigma_{21}}{\partial x_2}+\frac{\partial\sigma_{31}}{\partial x_3}+\rho\boldsymbol{X}_1=0\\ \frac{\partial\sigma_{12}}{\partial x_1}+\frac{\partial\sigma_{22}}{\partial x_2}+\frac{\partial\sigma_{32}}{\partial x_3}+\rho\boldsymbol{X}_2=0\\ \frac{\partial\sigma_{13}}{\partial x_1}+\frac{\partial\sigma_{23}}{\partial x_2}+\frac{\partial\sigma_{33}}{\partial x_3}+\rho\boldsymbol{X}_3=0\end{aligned}\right\}\quad \frac{\partial\sigma_{ji}}{\partial x_j}+\rho\boldsymbol{X}_i=0$$

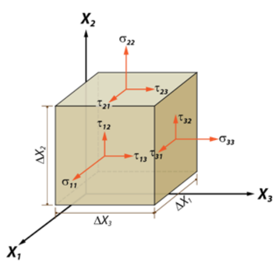


- The Conditions of equilibrium for *moments*

$$\sigma_{ij}=\sigma_{ji} \implies \frac{\partial\sigma_{ji}}{\partial x_j}+\rho\boldsymbol{X}_i=0;\frac{\partial\sigma_{ij}}{\partial x_j}+\rho\boldsymbol{X}_i=0$$

Check the derivations in Ghosh's book (Chapter 5)

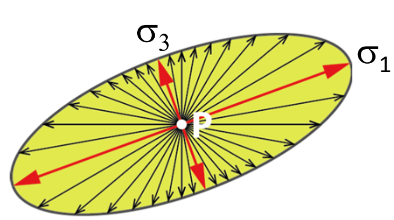
Stress Ellipse and Stress Ellipsoid



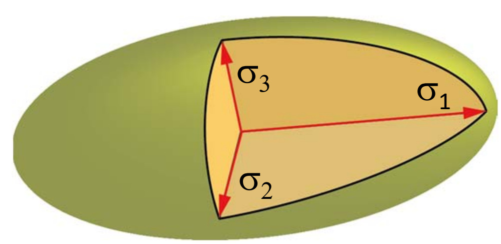
$$\sigma_{ij}=\begin{bmatrix}\sigma_{11} & \tau_{13}\\ \tau_{31} & \sigma_{33}\end{bmatrix}$$

$$\sigma_{ij}=\begin{bmatrix}\sigma_{11} & \tau_{12} & \tau_{13}\\ \tau_{21} & \sigma_{22} & \tau_{23}\\ \tau_{31} & \tau_{32} & \sigma_{33}\end{bmatrix}$$

Different pairs of planes would have different magnitude of stress vectors – when resolved around the point, it would produce an ellipse (2D) or ellipsoid (3D)



2D

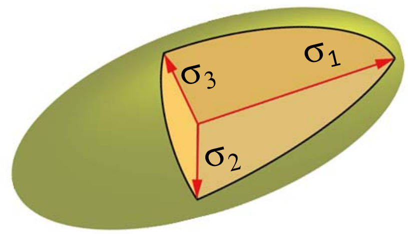


3D

Stress Ellipse and Stress Ellipsoid

- The geometric disposition of the stress ellipsoid (shape and orientation) *reveals the state of stress at a given point* in a rock-mass deforming or even in static-state.

- The largest, smallest and intermediate magnitudes (σ_1, σ_3, and σ_2 respectively) of the stress ellipsoid are known as **Principal Stress** *(eigenvalues)* of Stress of the stress ellipsoid, and the directions as **Principal Axes** *(eigenvectors)* of the stress ellipsoid.



Please remember, the stress and strain ellipsoids (and ellipses in 2D) are very similar physically and mathematically. However, they are different. **(A)** A stress ellipsoid may not lead to a strain ellipsoid (i.e., rocks are nor deforming); **(B)** The shape and orientation of the strain ellipsoid may be very different to those of a stress ellipsoid responsible for the strain.

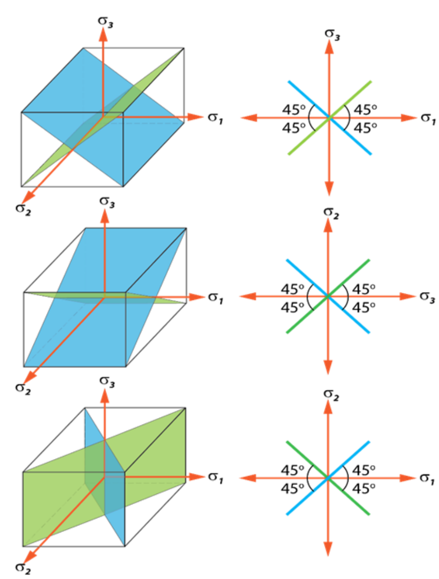
Maximum shear stresses and their orientations

The principal shearing stresses act on the following planes

1. A pair of planes intersecting along the σ_2 axis and inclined at ±45° with the σ_1 or the σ_3 axis; the absolute value of the shear stress on these planes is the greatest and has the value $\tau_2 = \pm\frac{1}{2}(\sigma_3 - \sigma_1)$.

2. A pair of planes intersecting along the σ_1 axis and inclined at + 45° with the σ_2 or σ_3 axis; the shear stress on these planes is $\tau_1 = \pm\frac{1}{2}(\sigma_2 - \sigma_3)$.

3. A pair of planes intersecting along the σ_3 axis and inclined at ±45° with the σ_1 or σ_2-axis; the shear stress on these planes is $\tau_3 = \pm\frac{1}{2}(\sigma_1 - \sigma_2)$.



Maximum shear stress and orientation

Photo: Romain Plateaux

Maximum shear stress and orientation

Photo: AGU Blog

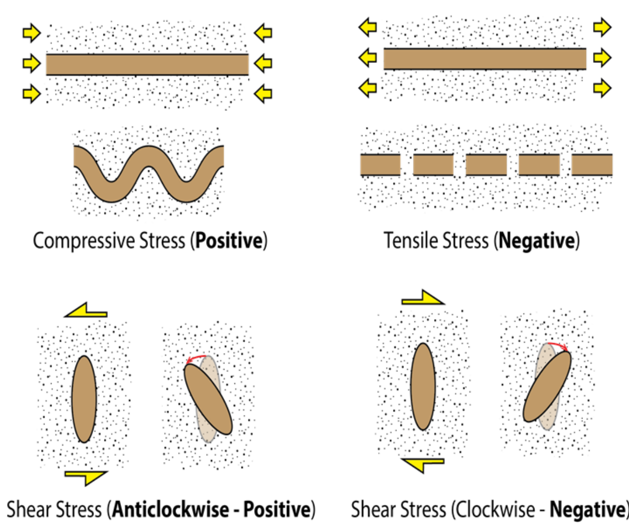
Photo: Santanu Misra

Maximum shear stress and orientation

Photo: GNS Science, NZ

Sign convention of Stress Axes

- **Compressive Stress**: The stress on rock-mass which tends to shrink/shorten the material along the direction of stress (e.g., folding, thrust-faults).
- **Tensile Stress**: The stress on rock-mass which tends to extend the material along the direction of stress (e.g., boudinage, normal-faults).
- **Shear Stress**: The stress on rock-mass which acts along / parallel to the surface (e.g., stress along fault-planes).



Additional Lectures

https://youtu.be/mP0ZT2fmzEM?list=PLHyuArGIIyyR_2mObwQ3yng18LDnDqidp

Next Lecture

Geological Structures