EE200A – PS8

Q.1)

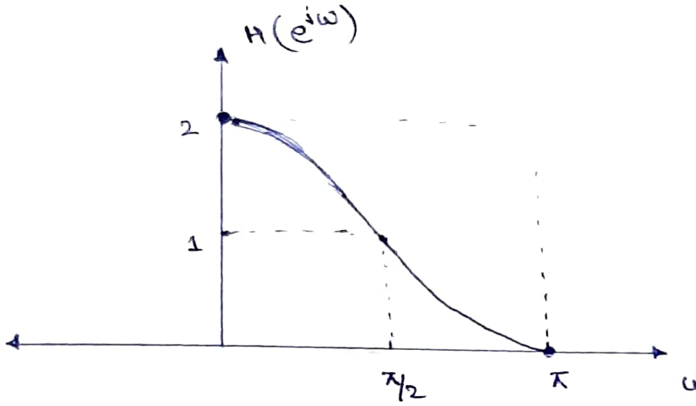
$$y[n] = x[n] + \frac{1}{2}\left(x[n+1] + x[n-1]\right)$$

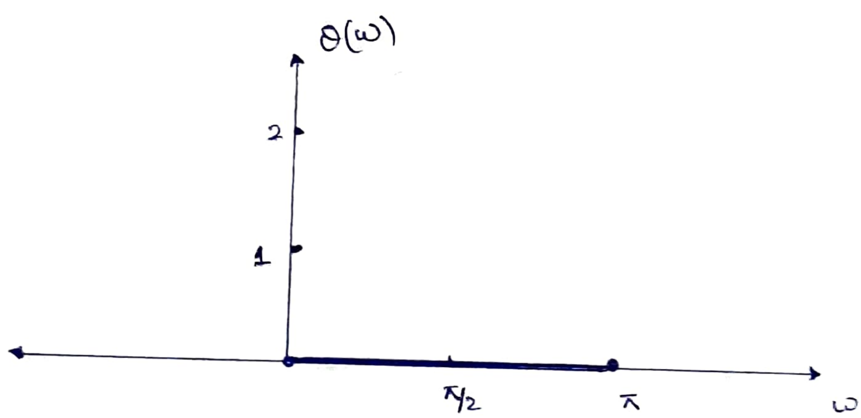
$$h[n] = \delta[n] + \frac{1}{2}\left(\delta[n+1] + \delta[n-1]\right)$$

$$H(e^{j\omega}) = 1 + \cos(\omega)$$

$$|H(e^{j\omega})| = |1 + \cos(\omega)|$$

$$\theta(\omega) = 0 \quad \text{as} \quad 1+\cos(\omega) \geq 0 \quad \forall\ \omega \in \mathbb{R}$$





Yes, it has a frequency selective characteristic as it allows frequencies near 0 to pass while stops frequencies near π.

Hence, a factor of 2 - interpolator acts as a low-pass filter.

Q.2) Note that the give equation is of the form:

$$y[n] + \sum_{l=1}^{N} q_l\, y[n-l] = \sum_{l=0}^{M} p_l\, x[n-l]$$

Hence, $p_0 = 0.1367$, $q_1 = -1.0148$

$p_1 = 0$, $q_2 = 0.7265$

$p_2 = -0.1367$

So, $$H(z) = \frac{0.1367 - 0.1367 z^{-2}}{1 - 1.0148 z^{-1} + 0.7265 z^{-2}}$$

$\because$ ROC of $H(z)$ contains unit circle,

$$H(e^{j\omega}) = \frac{0.1367 - 0.1367 e^{-2j\omega}}{1 - 1.0148 e^{-j\omega} + 0.7265 e^{-2j\omega}}$$

Yes, it does. It has a frequency selective characteristic. It is a band-pass filter because it allows frequencies around 1 to pass and blocks others.

Q.3] Similar to Q2 :

$p_0 = 0.8633$ | $q_1 = -1.0148$

$p_1 = -1.0148$ | $q_2 = 0.7265$

$p_2 = 0.8633$

So, $$H(z) = \frac{0.8633 - 1.0148z^{-1} + 0.8633z^{-2}}{1 - 1.0148z^{-1} + 0.7265z^{-2}}$$

$\because$ ROC of $H(z)$ includes unit circle:

So, $$H(e^{j\omega}) = \frac{0.8633 - 1.0148e^{-j\omega} + 0.8633e^{-2j\omega}}{1 - 1.0148e^{-j\omega} + 0.7265e^{-2j\omega}}$$

Yes, it has frequency selective characteristics. It acts like a band stop filter.

Q.4] Similar to Q2:

$p_0 = b$ | $q_1 = a$

$p_1 = c$

So, $H(z) = \dfrac{b + cz^{-1}}{1 + az^{-1}}$

$\because$ ROC of $H(z)$ includes unit circle, $H(e^{j\omega}) = \dfrac{b + ce^{-j\omega}}{1 + ae^{-j\omega}}$

Now, $|H(e^{j\omega})| = \left| \dfrac{b + ce^{-j\omega}}{1 + ae^{-j\omega}} \right|$

$= \left| \dfrac{be^{j\omega} + c}{e^{j\omega} + a} \right|$

For this to be a constant:

$|be^{j\omega} + c| = \lambda |e^{j\omega} + a|$ where $\lambda \in \mathbb{R}$ is a constant

$(be^{j\omega} + c)^2 = \lambda^2 (e^{j\omega} + a)^2$

$b^2 e^{2j\omega} + c^2 + 2bce^{j\omega} = \lambda^2 e^{2j\omega} + \lambda^2 a^2 + 2\lambda^2 a e^{j\omega}$

$(b^2 - \lambda^2) e^{2j\omega} + 2e^{j\omega}(bc - \lambda^2 a) + (c^2 - \lambda^2 a^2) = 0$

$\because$ the eqn is valid for all $\omega \in \mathbb{R}$, it must be that

$b^2 = \lambda^2 \rightarrow (1)$

$bc = \lambda^2 a \rightarrow (2)$ and

$c^2 = \lambda^2 a^2 \rightarrow (3)$

Substituting (1) in (2):

$bc = b^2 a$

$\Rightarrow b(ba - c) = 0$

$\Rightarrow b = 0$ OR $ba = c$

So, either $b=0$ and $c=0$ and $a \in \mathbb{R}$

OR, $ba=c$ with $b \neq 0$

Q.5) $h[n] = 3\delta[n] - 5\delta[n-1] + a\delta[n-2] + b\delta[n-3]$

$H(e^{j\omega}) = 3 - 5e^{-j\omega} + ae^{-2j\omega} + be^{-3j\omega}$

$h[n]$ is of length 4.

$M+1=4$

So, $M=3$ (odd)

Now, $h[n] = \pm h[3-n]$ should be satisfied

for $0 \le n \le 3$.

For $n=0$: $h[0] = 3$

$h[3] = b$

So, $b = \pm 3$

For $n=1$: $h[1] = -5$

$h[2] = a$

So, $a = \mp 5$

For $n=2$: $h[2] = a$

$h[1] = -5$

So. $b = \pm 3$ and $a = \pm 5$

If we take $h[n] = h[M-n]$, then

$b = 3$ and $a = -5$

If we take $h[n] = -h[M-n]$, then

$b = -3$ and $a = 5$

Q.6] $\tilde{x}[n] = 5\cos(\omega_1 n + 0.4) + 3\cos(\omega_2 n + 0.8)$

Now, output response for $5\cos(\omega_1 n + 0.4)$

is $= 5 \left| \dfrac{2 - 0.8e^{-j\omega_1}}{1 + 0.9e^{-j\omega_1}} \right| \cos(\omega_1 n + \theta(\omega_1) + 0.4)$

where $\theta(\omega_1)$ is the phase response for $H(e^{j\omega_1})$.

Now, $|H(e^{j\omega_1})| = 0.7925$

$\theta(\omega_1) = 0.6316$

Now, output response for $3\cos(\omega_2 n + 0.8)$

is $= 3 \left| \dfrac{2 - 0.8e^{-j\omega_2}}{1 + 0.9e^{-j\omega_2}} \right| \cos(\omega_2 n + \theta(\omega_2) + 0.8)$

$|H(e^{j\omega_2})| = 4.5204$

$\theta(\omega_2) = 1.2718$

So, $\tilde{y}[n] = 3.9625\cos(0.2\pi n + 1.0316)$

$+ 13.5612\cos(0.8\pi n + 2.0718)$

Q.4)

$$h_{HP}[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_{HP}(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{-\omega_c} e^{j\omega n}\, d\omega + \int_{\omega_c}^{\pi} e^{j\omega n}\, d\omega\right]$$

$$= \frac{1}{2\pi}\left[\frac{1}{jn}\left(e^{-j\omega_c n} - e^{-j\pi n}\right) + \frac{1}{jn}\left(e^{j\pi n} - e^{j\omega_c n}\right)\right]$$

$$= \frac{1}{2\pi j n}\left[\left(e^{j\pi n} - e^{-j\pi n}\right) - \left(e^{j\omega_c n} - e^{-j\omega_c n}\right)\right]$$

$$= \frac{1}{2\pi j n}\left[2j\sin(\pi n) - 2j\sin(\omega_c n)\right]$$

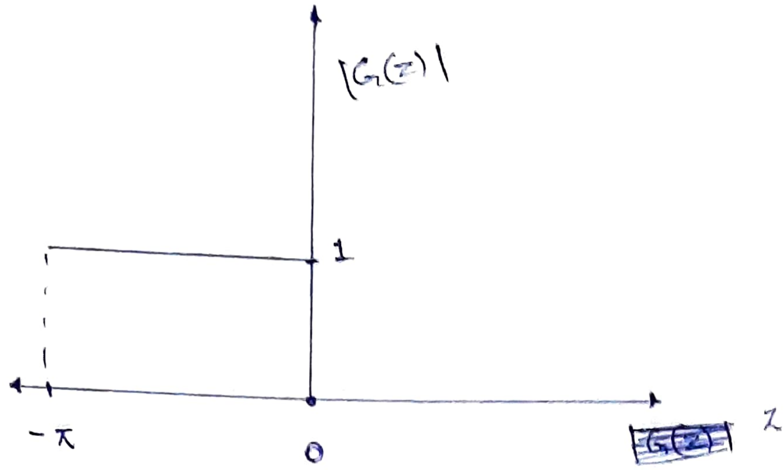
$$h_{HP}[n] = \frac{-1}{\pi n}\sin(\omega_c n), \quad -\infty < n < \infty, \; n \neq 0$$

$$h_{HP}[0] = \frac{1}{2\pi}\left[\int_{-\pi}^{\pi} H_{HP}(e^{j\omega})\, d\omega\right]$$

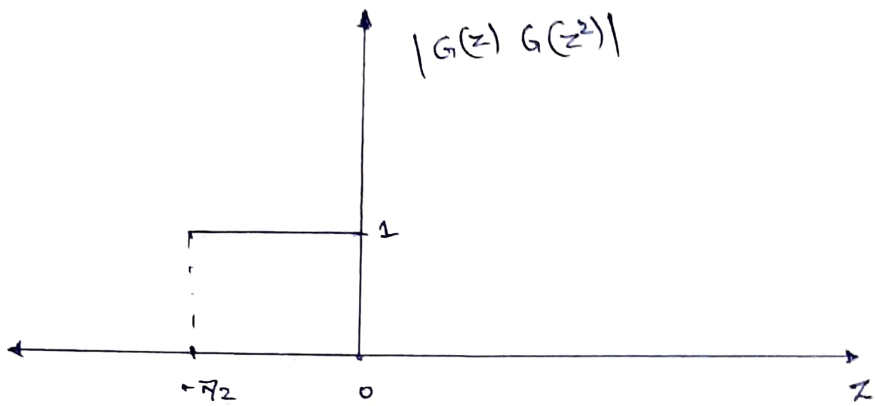
$$= \frac{1}{2\pi}\left[(\pi - \omega_c) + (\pi - \omega_c)\right]$$

$$h_{HP}[0] = \frac{\pi - \omega_c}{\pi}$$

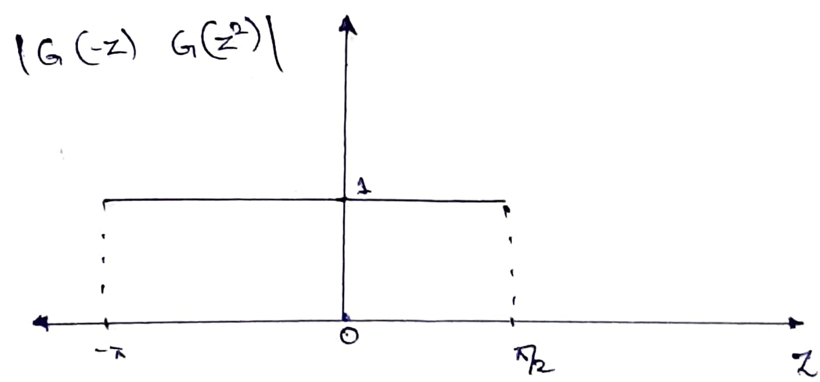
Q.8] (a.)



(b.)



(c.)



(d.)