Dartmouth College, Department of Economics: Economics 21, Summer'02

Topic 1a: Intertemporal Choice

Economics 21, Summer 2002
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Based Primarily on Varian, Ch. 10

Background

Discounting

Introduction: Discounting

- Puzzle: How much would you be willing to pay (today) for the right to be paid $165 in 1 year's time?
  - Suppose the interest rate is 10%, there is no uncertainty, and you are not wealth constrained.
- Thought experiment: How much would you have to put into a 10% interest paying bank account (today) in order to get $165 out in 1 year's time?

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Discounting, cont'd

- More generally: How much is it worth to you today to get an amount m in 1 year's time when the interest rate is r?
  - How is r usually measured? Examples:
    - a 10% interest rate means r = 0.1
    - a 3.655% interest rate means r = 0.03655
- Rephrase: How much do you have to put into an r interest paying account (today) in order to get m out in 1 year's time?
- This is called the *present value* of m.

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Discounting, cont'd

- How much do you have to put into an r interest paying account (today) in order to get m out in 1 year's time?
  - Suppose you put v into the bank today.
  - In one year, you will get $v + rv$, or $(1 + r)v$.
  - You want this to equal m, so $(1 + r)v = m$.
  - $v = m / (1 + r)$
- Therefore, you need to put $m / (1 + r)$ into the bank today (at interest rate r) in order to get m out in 1 year's time.
- The *present value* of m (next year) is $m / (1 + r)$.

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Discounting, cont'd

- How much do you have to put into an r interest paying account (today) in order to get m out in 2 year's time?
  - Suppose you put v into the bank today.
  - In one year, you will get $v + rv$, or $(1 + r)v$.
  - You put that into the bank again and in one further year you get $(1 + r)(1 + r)v$, or $(1 + r)^2 v$.
  - You want this to equal m, so $(1 + r)^2 v = m$.
  - $v = m / (1 + r)^2$
- Therefore, you need to put $m / (1 + r)^2$ into the bank today (at interest rate r) in order to get m out in 1 year's time.
- The *present value* of m (in two years) is $m / (1 + r)^2$.

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Discounting Income Streams

- Suppose you get m_1 this year, m_2 next year, m_3 in two years, etc.. What is the present value of this income "stream"?
  - ◆ You get m_1 now, so we need not discount it. (It is now worth to you m_1.)
  - ◆ You get m_2 in one year's time, so the present value of that portion of the income stream is $m_2 / (1 + r)$.
  - ◆ You get m_3 in two year's time, so the present value of that portion of the income stream is $m_3 / (1 + r)^2$.
  - ◆ ...
- The present value of the whole income stream is therefore: $m_1 + m_2 / (1 + r) + m_3 / (1 + r)^2 + \ldots$

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Buzz Group: Investment

- Firms base investment decision on the present value of the income stream that investment generates.
- The Fed raises interest rates: what will happen to firm's investment?
  - ◆ That is, will firms invest more or less?

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Bonds

- *Bonds* are financial instruments that promise a fixed payment each year ("coupon"), and after a certain number of years (on "maturity"), a fixed amount ("face value").
- <u>Example</u>: How much is a bond with maturity 05/15/05, face value of $100 and coupon of $6.75 worth to you now when the interest rate is 3.655%?
  - $v = m/(1 + r)$　　　$+ m/(1 + r)^2$　　　$+ (m+f)/(1 + r)^3,$
  - $v = 6.75/(1.03655)$　$+ 6.75/(1.03655)^2 + 106.75/(1.03655)^3,$
  - $v = 6.51$　　　　　$+ 6.28$　　　　　$+ 95.85,$
  - $v = 108.64.$

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Bonds, cont'd

- <u>Example, cont'd</u>: This bond has a present value of $108.64. How much should you therefore be just willing to pay for it?
  - That is, at what price should you expect this bond to trade?
  - Wall Street Journal (06/11/02):

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Perpetuities

- A *Perpetuity ("consol")* is a special bond: It pays a coupon every year *forever*.
- Its present value is:
$$v = \frac{m}{1+r} + \frac{m}{(1+r)^2} + \frac{m}{(1+r)^3} + ...$$
- How do we calculate this?
$$v = \frac{1}{1+r}\left[m + \frac{m}{1+r} + \frac{m}{(1+r)^2} + ...\right]$$
$$v = \frac{1}{1+r}[m + v]$$
$$v = m/r$$

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Intertemporal Choice

"Timed" Consumption Goods
Borrowing and Saving (Lending)
Comparative Statics

Intertemporal Choices

- We want to explain how consumers allocate their consumption over time.
  - This will explain why consumers:
    - » borrow (consume more today than their endowment today)
    - » save/lend (consume less today than their endowment today)

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Intertemporal Choices, cont'd

- Simplest setting: two time periods 1, 2.
  - Consumption in period 1: c_1 (in period 2: c_2)
    - » this is a "composite good" - sometimes we will just call it "money"
    - » and: remember Topic 1: What is a good? (here: location in time)
    - » and: normally two goods are enough (e.g. consumption when young/consumption when old, etc.)
  - Endowment (income) in period 1 (2): m_1 (m_2)
  - Interest rate r
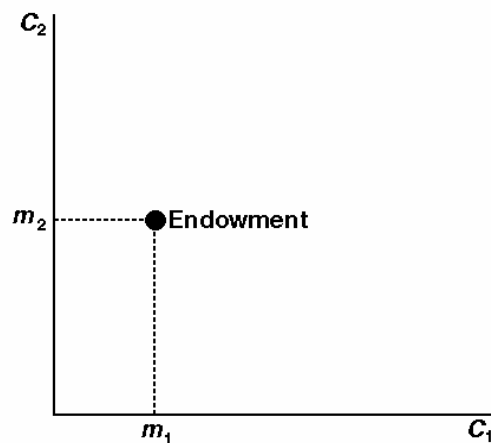- We have preferences over "timed" consumption.

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"Timed" Consumption

- You can actually buy "orange juice to be delivered in January 2002." And such goods are traded (*"futures"*).
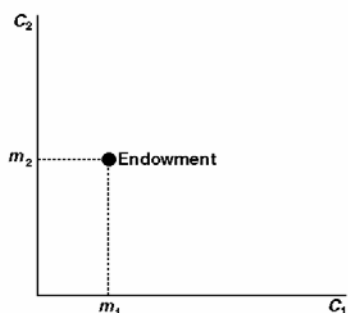 - ◆ <u>Example</u>: (Wall Street Journal 06/11/02)

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The Consumption "Space"

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The Budget Constraint

c_2

m_2 -------- ●Endowment

m_1 c_1

- ■ Consumption possibilities:
  - ◆ the endowment point
  - ◆ <u>saving</u>: consume 1 unit less than your endowment in period 1, so you can consume $1+r$ units more than your endowment in period 2
  - ◆ <u>borrowing</u>: consume 1 unit more than your endowment in period 1, but you have to pay back $1+r$ units in period 2 (i.e. consume $1+r$ units less than your endowment in period 2)
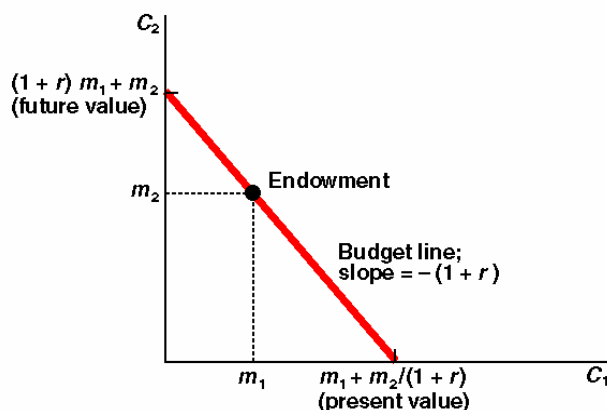
- ◆ You can think of $(1+r)$ as the price of present consumption in terms of future consumption.

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The Budget Constraint, cont'd

- ■ Another way to think about the budget constraint:
  - ◆ Over the two periods, whether you save or borrow in period 1, you cannot consume more than the total value of your endowment.
  - ◆ Or: you can consume in period 2 whatever your income in period 2, plus (minus) whatever you have saved (borrowed) in period 1, and the interest payment on this.
  - ◆ In notation: $c_2 = m_2 + (1 + r)(m_1 - c_1)$
  - ◆ Rewrite this: $c_2 = (1 + r) m_1 + m_2 - (1 + r) c_1$

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The Budget Constraint, cont'd

$(1 + r)\, m_1 + m_2$
(future value)

m_2 ...... Endowment

Budget line;
slope $= -(1 + r)$

m_1 $m_1 + m_2/(1 + r)$
(present value) C_1

- Budget line: $c_2 = (1 + r)\, m_1 + m_2 - (1 + r)\, c_1$

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The Budget Constraint, cont'd

- Another way of rewriting the budget constraint:
  - $c_2 = (1 + r)\, m_1 + m_2 - (1 + r)\, c_1$
  - $c_2 + (1 + r)\, c_1 = (1 + r)\, m_1 + m_2$
  - $c_1 + c_2 / (1+r) = m_1 + m_2 / (1 + r)$
  - The present value of consumption has to be equal to the present value of the endowment.
- Or:
  - $(1 + r)c_1 + c_2 = (1 + r)\, m_1 + m_2$
  - The future value of consumption has to be equal to the future value of the endowment.

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Intertemporal Choice

- Intertemporal choice: borrowing or saving (lending).

A Borrower　　　　　　B Lender
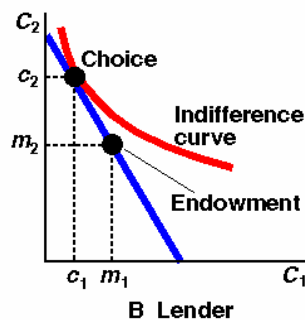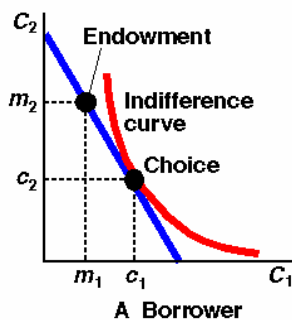
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Comparative Statics: Increasing r

- What happens to the budget line when the interest rate r increases?

- Intuitively: When you borrow 1 unit in period 1, you have to pay back 1 + r units in period 2. When r increases, your repayment increases (so your consumption in period 2 is lower).

- Or: 1 + r is the price of present (period 1) consumption in terms of future (period 2) consumption. If r increases, this price increases.
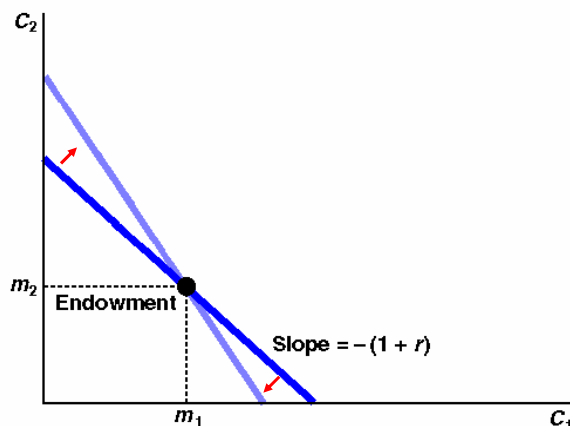
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Comparative Statics, cont'd

- Or: Recall the budget line equation:
  - ◆ $c_2 = (1 + r) m_1 + m_2 - (1 + r) c_1$
  - ◆ the slope of the budget line is $- (1 + r)$
  - ◆ when r increases, the slope becomes steeper
  - ◆ and: the vertical intercept becomes larger
- But remember: you will always be able to consume your endowment.

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Increasing r: the Budget Constraint

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Comparative Statics, cont'd

- When the interest rate rises, there are two effects:
  - ◆ Substitution Effect
    - » $1 + r$ is the price of present (period 1) consumption in terms of future (period 2) consumption; this price increases, so the consumer will want to consume less in period 1 (substitute away from c_1).
  - ◆ Wealth (previously: "Income") Effect
    - » A lender's wealth increases: she will want to consume more.
    - » A borrower's wealth decreases: she will want to consume less.
- An interest rate increase will lead a borrower to consume unambiguously less in period 1. For a lender, the effect on period 1 consumption is ambiguous.
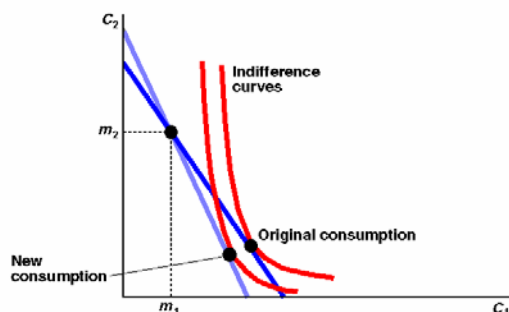
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Comparative Statics: A Lender
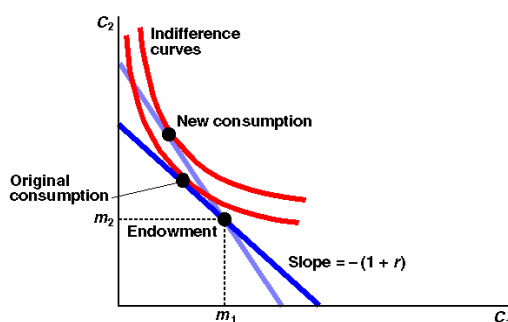
- The effect on a lender's period 1 consumption of an interest rate increase is ambiguous.
- But: a lender remains a lender after the interest rate increase (proof: by revealed preference).

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Comparative Statics: A Borrower

- A borrower consumes unambiguously less in period 1 when r increases.
- And: if a borrower remains a borrower, she is made unambiguously worse off by an increase in r.

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Inflation

- Now incorporate the possibility of inflation (changing price of consumption) into the budget constraint:
  - ◆ Suppose the price of consumption in the first period is p_1, and the price of consumption in the second period is p_2.

$$p_2 c_2 = p_2 m_2 + (1+r)(p_1 m_1 - p_1 c_1)$$

$$c_2 = m_2 + \frac{p_1}{p_2}(1+r)(m_1 - c_1)$$

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Inflation, cont'd

- So the budget constraint is:
$$c_2 = m_2 + \frac{p_1}{p_2}(1+r)(m_1 - c_1)$$
- The definition of inflation is:
$$\pi = \frac{p_2 - p_1}{p_1} = \frac{p_2}{p_1} - 1 \qquad \text{so } \frac{p_1}{p_2} = \frac{1}{1+\pi}$$
- And the budget constraint rewrites as:
$$c_2 = m_2 + \frac{1+r}{1+\pi}(m_1 - c_1)$$

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Inflation, cont'd

- So the budget constraint is:
$$c_2 = m_2 + \frac{1+r}{1+\pi}(m_1 - c_1)$$
  - ◆ The slope of the budget constraint is: $-\frac{1+r}{1+\pi} = -(1+\rho)$
- So if you give up one unit of consumption in period 1, you can get $(1+r)/(1+\pi)$ more consumption in period 2.
  - ◆ This is why ρ is called the *real* interest rate (it is in terms of goods, not money).
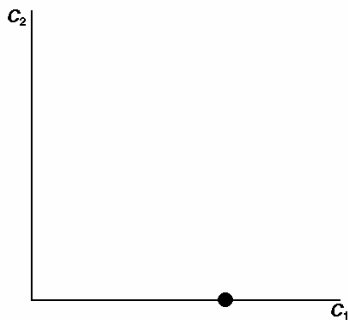
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Investment Decisions: An Application

Perfect Capital Markets and the
Fisher Separation Theorem:
Are Shareholders Myopic?

Making Investments

c_2

c_1

- Firms can make payouts to shareholders (dividends):
  - either: everything in period 1 and nothing in period 2:
    - » Produce nothing, pay out all capital in period 1.
  - or: invest:
    - » Invest (pay out less) in period 1, then produce and pay out in period 2:
    - » What are the possible investments?
    - » Production opportunity frontier

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Making Investments, cont'd
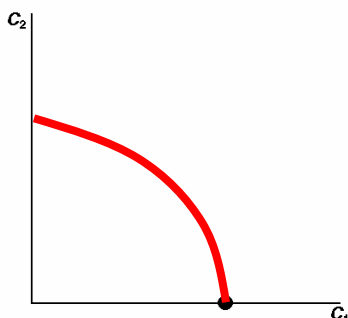
- How much should firms invest?
  - ◆ Shareholders may have different preferences over present and future consumption.
- With a perfect capital market (borrowing and lending rates equal), the investment decision is separate from consumption decisions (i.e. individual preferences).

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The Fisher Separation Theorem

- Given perfect and complete capital markets, the investment (or, production) decision is governed purely by the maximization of the present value of the firm (an objective criterion). This is separate from individuals' preferences over consumption (a subjective criterion).
- (One theme of economics: when markets operate smoothly, they equilibrate opposing preferences.)

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"Short-Termism"?

- Question for students:
- Shareholders are often accused of being "myopic," in the sense that they push companies to undertake investment strategies that pay off in the short term, without regard to the long-term performance of the company.
- From what you have learnt, do you think this view is correct?

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