TA201 Assignment 3

Q.1] During the combining process, the initial and the final mass must remain same.

Shape is given to be spherical.

Assume density of the spheres remains constant (We can assume this as the spheres are made of the same material).

Assume density $= \rho$ and

diameter of final sphere $= d$

Now, Initial mass = Final mass

$$\rho \times (2) \times \frac{4}{3}\pi (5\mu m)^3 = \rho \times \frac{4}{3}\pi \left(\frac{d}{2}\right)^3$$

$$250\,\mu m^3 = \left(\frac{d}{2}\right)^3$$

$$\Rightarrow \left(\frac{d}{2}\right)^3 = \left(5 \times (2)^{1/3}\,\mu m\right)^3$$

$$\Rightarrow \frac{d}{2} = 5 \times (2)^{1/3}\,\mu m$$

$$\Rightarrow \boxed{d = 10 \times (2)^{1/3}\,\mu m}$$

Interfacial energy is related to areas.

Hence, change in total energy w.r.t initial energy $= \frac{|\text{Final energy} - \text{Initial energy}|}{\text{Initial energy}}$

where $|x|$ represents absolute value of x.

$$\text{Initial energy} = (1.5\ J/m^2) \times 2 \times \left(4\pi(5\times10^{-6}\,m)^2\right)$$

$$\text{Final energy} = (1.5\ J/m^2) \times \left(4\pi\left(\tfrac{d}{2}\times10^{-6}\,m\right)^2\right)$$

So, $\text{change} = \dfrac{|\text{Final} - \text{Initial}|}{\text{Initial}}$

$$= \frac{|(5\times 2^{1/3})^2 - (5)^2\times 2|}{2\times(5)^2}$$

$$= \frac{|2^{2/3} - 2|}{2}$$

$$= 0.2063$$

In % $= 20.63\%$

$\approx 21\%$

Q.2] The engineering stress and strain in a tensile test are defined relative to the orignal area and length of the test specimen.

Assume a cylindrical shape of the material being test with initial area = A_0 and initial length = L_0.

So, $s = \frac{F}{A_0}$ where F = force applied in the test

and

$e = \frac{L - L_0}{L_0}$ where L = length of the material at any point.

True stress and strain are based on the instantaneous values of the variables involved.

So, $\sigma = \frac{F}{A}$, where F = force applied
A = actual (instantaneous area resisting the load)

Value of true strain in a tensile test can be estimated by dividing the total elongation into small increments, calculating the engineering strain for each increment on the basis of its starting length, and then adding up the strain values. In the limit, true strain is defined

$$\varepsilon = \int_{L_0}^{L} \frac{dL}{L} = \ln\left(\frac{L}{L_0}\right)$$

Now, $$e = \frac{L - L_0}{L_0}$$

$$\Rightarrow \quad e + 1 = \frac{L}{L_0}$$

So, $$\boxed{\varepsilon = \ln(1+e)}$$

Note that, we assume volume remains constant. Hence

$$A_0 L_0 = A L$$

$$A_0 L_0 = A L_0 (1+e)$$

So, $$\boxed{A = \frac{A_0}{(1+e)}}$$

Hence, $$\sigma = \frac{F}{A}$$

$$= \frac{F(1+e)}{A_0}$$

$$\boxed{\sigma = s(1+e)}$$

Q.3) Note that if the portion of the true stress-strain curve representing the plastic region were plotted on a log-log scale, we would get a linear relationship. Hence, the relationship between true stress and true strain in the plastic region can be expressed as:

$$\sigma = K \varepsilon^n$$ where

σ = true stress

K = strength coefficient

ε = true strain

n = strain hardening exponent

Necking or localized deformation begins at maximum load, where the increase in stress due to decrease in the cross-sectional area of the specimen becomes greater than the increase in the load-carrying ability of the metal due to strain hardening.

So, the condition for UTS is:

$$dF = 0$$

$$\Rightarrow d(\sigma A) = 0 \quad \text{as } F = \sigma A$$

$$\Rightarrow \sigma\, dA + A\, d\sigma = 0$$

$$\Rightarrow \frac{dA}{A} = \frac{-d\sigma}{\sigma} \longrightarrow (1)$$

But we also assume volume to be constant.

So, $A_o L_o = AL$

$$0 = L\,dA + A\,dL$$

$$\Rightarrow \frac{dA}{A} = -\frac{dL}{L}$$

$$\Rightarrow \frac{dA}{A} = -d\varepsilon \quad \rightarrow (2.)$$

So, from (1.) and (2.) :

$$\frac{d\sigma}{\sigma} = +d\varepsilon$$

$$\Rightarrow \frac{d\sigma}{d\varepsilon} = +\sigma \quad \rightarrow (3.)$$

But since $\sigma = K\varepsilon^n$

$$\Rightarrow \frac{d\sigma}{d\varepsilon} = Kn\varepsilon^{n-1} \quad \rightarrow (4.)$$

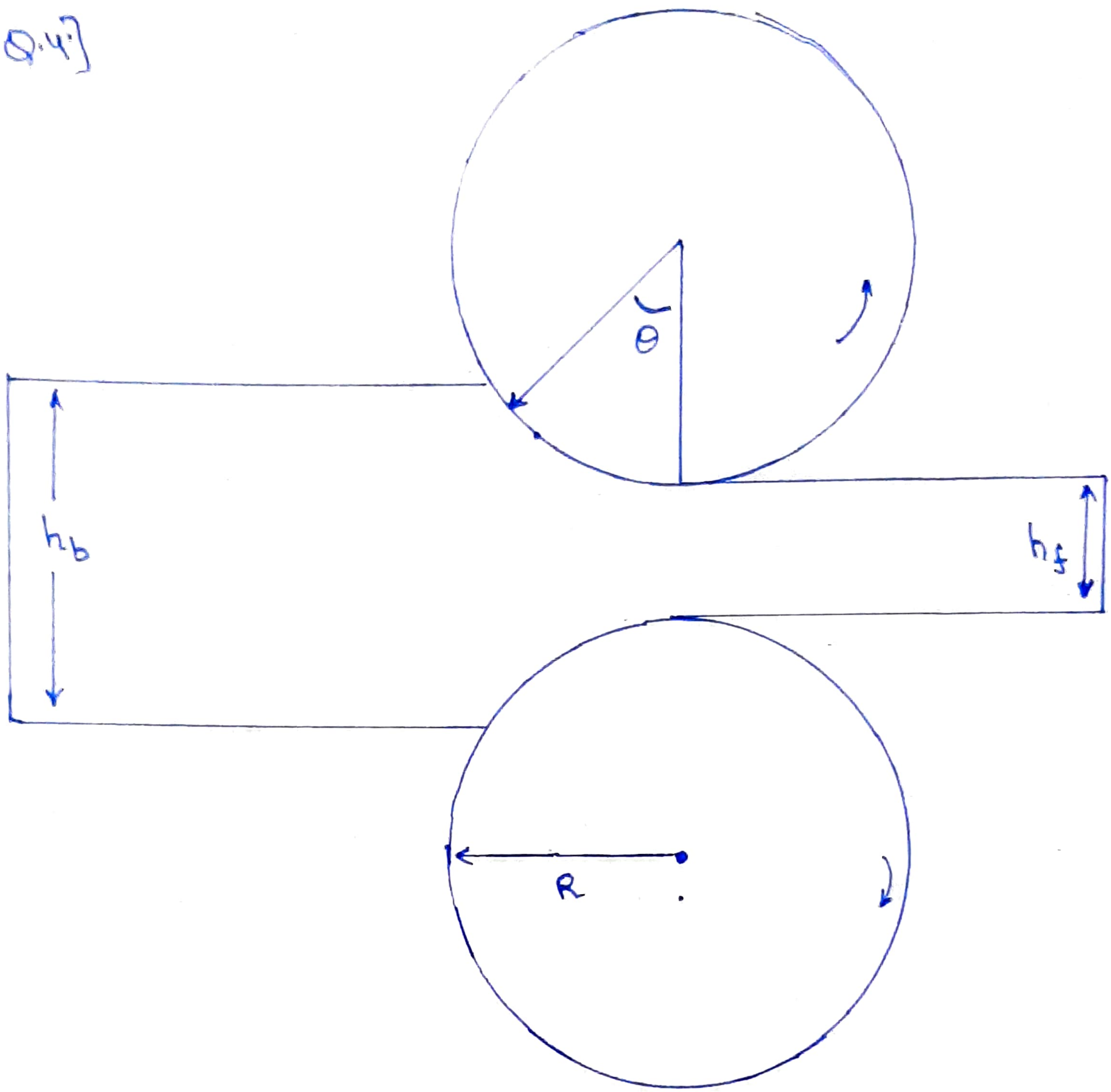
Hence, $Kn\,\varepsilon^{n-1} = \sigma$ from (3.) and (4.)

$$\Rightarrow Kn\,\varepsilon^{n-1} = K\varepsilon^n$$

$$\Rightarrow \boxed{n = \varepsilon}$$

So, the condition for UTS occurs when $\boxed{\varepsilon = n}$.

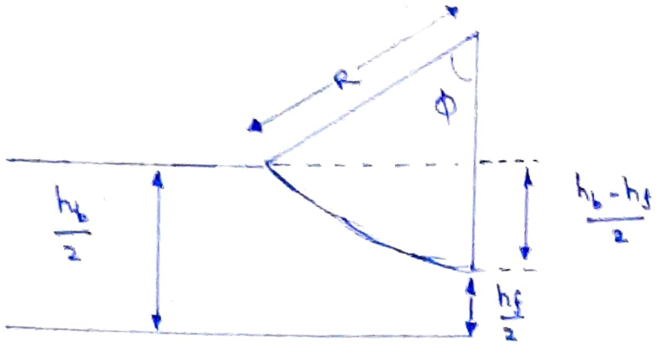
Q.4]



R = roll radius

h_b = initial thickness

h_f = final thickness.



$$h_b = h_f + 2R(1 - \cos\phi)$$

By Taylor's expansion :

$$\cos(\phi) = 1 - \frac{\phi^2}{2!} + \frac{\phi^4}{4!} \dots$$

$$h_b = h_f + R\phi^2$$

$$h_b - h_f = R\phi^2$$

Let $\quad$ draft $= d$
$\qquad\qquad = h_b - h_f$

So, $\quad d = R\phi^2$

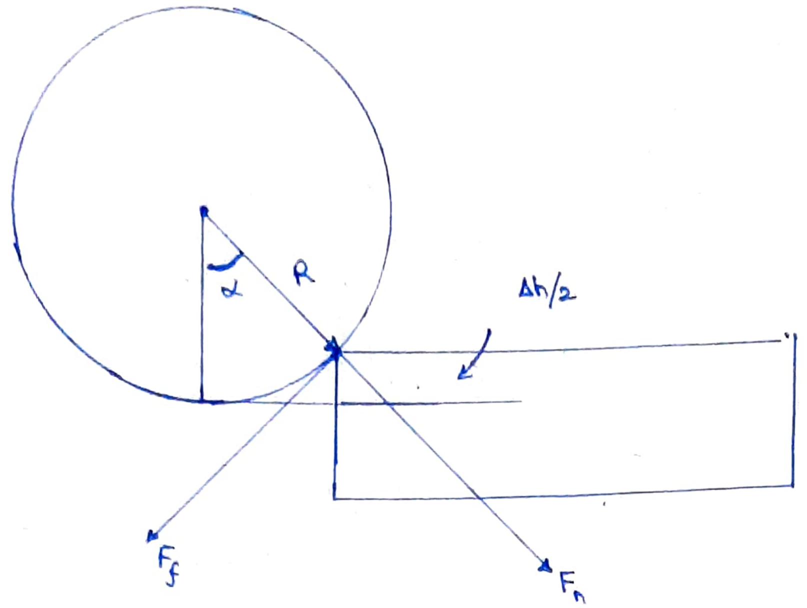
→ The material will be drawn into the nip if the horizontal component of the friction force F_f is larger, or atleast equal to the opposing horizontal component of the normal force (F_n).

$$F_f \cos(\alpha) \geq F_n \sin(\alpha)$$

$$F_f = \mu . F_n$$

$\tan\alpha = \mu$

where μ = friction coefficient.



Also, $\cos\alpha = \dfrac{R - \frac{\Delta h}{2}}{R} = 1 - \dfrac{\Delta h}{2R}$

and $\Delta h << R$, $\sin\alpha = \sqrt{1-\cos^2\alpha}$

$\sin\alpha = \sqrt{1 - 1 + \dfrac{\Delta h}{2R} - \left(\dfrac{\Delta h}{2R}\right)^2}$, $\sin\alpha \approx \sqrt{\dfrac{\Delta h}{R}}$

$\tan\alpha \approx \sqrt{\dfrac{\Delta h}{R - \Delta h}} \approx \sqrt{\dfrac{\Delta h}{R}}$

So, approximately, $(\tan\alpha)^2 = \dfrac{\Delta h}{R}$

Hence, maximum draft :

$(\Delta h)_{max} = \mu^2 R$

By above formula:

$$(\Delta h)_{max} = (0.1)^2 \times 500 \text{ mm}$$

$$= 5 \text{ mm}$$

Hence, Maximum reduction $= \dfrac{5}{200}$

$$= 2.5 \times 10^{-2}$$