MTH101 - Quiz 2A, 31.10.2013(17:20-17:45 hrs), Maximum Marks: 15

NAME: —————————— **Roll No.**—————— **Section:** ————

1. Let C denote the circular disc of radius 2 centered at $(4,0)$. Find the volume of the torus that is generated by revolving C about the y-axis using the Washer Method. [5]

2. Discuss the convergence/divergence of the integral $\int_1^\infty \frac{t^3(\sin t)}{e^t(\ln t)}dt$. [5]

3. Let $f : [1,10] \to \mathbb{R}$ be continuous. Show that there exists $c \in [1,e]$ such that $\int_1^e f(t)dt = cf(c)$. [5]

<u>Tentative Marking Scheme</u>

1. Note that the disc is bounded by the curves $x = 4 + \sqrt{4-y^2}$ and $x = 4 - \sqrt{4-y^2}$, $-2 \le y \le 2$.

   The required volume is $\pi \int_{-2}^{2} \left[(4+\sqrt{4-y^2})^2 - (4-\sqrt{4-y^2})^2\right] dy$. [3]

   $= 16\pi \int_{-2}^{2} \sqrt{4-y^2}dy$ [1]

   $= 16\pi \times$area of a semi circular region

   $= 32\pi^2$. [1]

2. Write $\int_1^\infty \frac{t^3(\sin t)}{e^t(\ln t)}dt = \int_1^c \frac{t^3(\sin t)}{e^t(\ln t)}dt + \int_c^\infty \frac{t^3(\sin t)}{e^t(\ln t)}dt$ for some $1 < c < \frac{\pi}{2}$.

   Note that $\left(\frac{t^3(\sin t)}{e^t}\right)\frac{1}{(\ln t)} \ge M\frac{1}{(\ln t)}$ on $[1,c]$ for some $M > 0$.

   Since $\int_1^c \frac{1}{(\ln t)}dt$ diverges by LCT with $\frac{1}{t-1}$, $\int_1^c \frac{t^3(\sin t)}{e^t(\ln t)}dt$ diverges.

   Therefore the given integral diverges.

3. Define $F(x) = \int_1^x f(t)dt$ and $G(x) = \ln x$ on $[1,e]$. [3]

   By CMVT, $\exists\ c \in [1,e]$ s.t. $\frac{F(e)-F(1)}{G(e)-G(1)} = \frac{F'(c)}{G'(c)}$. [2]

MTH101 - Quiz 2B, 31.10.2013(17:20-17:45 hrs), Maximum Marks: 15

NAME: —————————— **Roll No.**—————— **Section:** ———

1. Let $g : [1, 6] \to \mathbb{R}$ be continuous. Show that there exists $x_0 \in [1, e]$ such that $g(x_0) = \frac{1}{x_0} \int_1^e g(x)dx$. [5]

2. Let K denote the circular disc of radius 2 centered at $(0, 0)$. Using the Shell Method, find the volume of the torus that is generated by revolving K about the line $x = 4$. [5]

3. Discuss the convergence/divergence of the integral $\int_1^\infty \frac{(\cos x)x^4}{e^{2x}(\ln x)}dx$. [5]

<u>Tentative Marking Scheme</u>

1. Define $F(x) = \int_1^x g(t)dt$ and $G(x) = \ln x$ on $[1, e]$. [3]

   By CMVT, $\exists\ x_0 \in [1, e]$ s.t. $\frac{F(e)-F(1)}{G(e)-G(1)} = \frac{F'(x_0)}{G'(x_0)}$. [2]

2. The required volume is $\int_{-2}^2 2\pi(4 - x)(2\sqrt{4 - x^2})dx$. [3]

   $= 16\pi \int_{-2}^2 \sqrt{4 - x^2}dx$ [1]

   $= 16\pi \times$area of a semi circular region

   $= 32\pi^2$. [1]

3. See QUIZ 2A.