

Rules of disjunction.

$$\frac{\phi}{\phi \vee \psi} \quad \vee i_1$$

$$\frac{\psi}{\phi \vee \psi} \quad \vee i_2$$

$\phi \vee \psi$  - How can it be used in a proof.

$$\frac{\phi \vee \psi \quad \begin{array}{|c|} \hline \phi \\ \vdots \\ \lambda \end{array} \quad \begin{array}{|c|} \hline \psi \\ \vdots \\ \lambda \end{array}}{\lambda} \quad \vee e$$

Example  $p \vee q \vdash q \vee p$

1  $p \vee q$  Premise.

2	$p$	assumption
3	$q \vee p$	$\vee i_2$ 2

4	$q$	assumption
5	$q \vee p$	$\vee i_1$ 4

6  $q \vee p$   $\vee e$  1, 2-3, 4-5

Example

$$q \rightarrow r \vdash p \vee q \rightarrow p \vee r$$

1.	$q \rightarrow r$	premise
2	$p \vee q$	assumption
3	$p$	assumption
4	$p \vee r$	$\vee i_1$ 3
5	$q$	assumption
6	$r$	$\rightarrow_e$ 1, 5
7	$p \vee r$	$\vee i_2$ 6
8	$p \vee r$	$\vee e$ 2, 3-4, 5-7
9	$p \vee q \rightarrow p \vee r$	$\rightarrow_i$ 2-8

## Example

$$P \wedge (q \vee r) \vdash (P \wedge q) \vee (P \wedge r)$$

1.  $P \wedge (q \vee r)$  premise

2.  $P$   $\wedge E, 1$

3.  $q \vee r$   $\wedge E_2, 1$

4.  $q$  assumption

5.  $P \wedge q$   $\wedge I, 2, 4$

6.  $(P \wedge q) \vee (P \wedge r)$   $\vee I_1, 5$

7.  $r$  assumption

8.  $P \wedge r$   $\wedge I, 2, 7$

9.  $(P \wedge q) \vee (P \wedge r)$   $\vee I_2, 8$

10.  $(P \wedge q) \vee (P \wedge r)$   $\vee E, 3, 4-6, 7-9$

Rules for negation.

$$\frac{\perp}{\phi} \quad \perp e$$

$$\frac{\phi \quad \neg \phi}{\perp} \quad \neg e$$

$$\frac{\boxed{\begin{array}{c} \phi \\ \vdots \\ \perp \end{array}}}{\neg \phi}$$

Example .

$$P \rightarrow q, P \rightarrow \neg q \vdash \neg P$$

1.	$P \rightarrow q$	premise
2	$P \rightarrow \neg q$	premise
3	$P$	assumption.
4	$q$	$\rightarrow_e 1, 3$
5	$\neg q$	$\rightarrow_e 2, 3$
6	$\perp$	$\neg_e 4, 5$
7	$\neg P$	$\neg i 3-6$

## Example (John)

$$p \wedge \neg q \rightarrow r, \neg r, p \vdash q$$

1.  $p \wedge \neg q \rightarrow r$  premise

2.  $\neg r$  premise

3.  $p$  premise

4.  $\neg q$  assumption

5.  $p \wedge \neg q$   $\wedge i$  3,4

6.  $r$   $\rightarrow e$  1,5

7.  $\perp$   $\neg e$  6,2

8.  $\neg \neg q$   $\neg i$  4-7

9.  $q$   $\neg \neg e$  8

## Modus Tollens (MT)

$$\frac{\phi \rightarrow \psi \quad \neg \psi}{\neg \phi} \text{ MT}$$

1.  $\phi \rightarrow \psi$  premise

2.  $\neg \psi$  premise

3.  $\phi$  assumption

4.  $\psi$   $\rightarrow_e 1, 3$

5.  $\perp$   $\neg_e 4, 2$

6.  $\neg \phi$   $\neg_i 3-5$

Law of excluded middle

$$\vdash \phi \vee \neg \phi$$

1.	$\neg(\phi \vee \neg \phi)$	Assumption
2	$\phi$	Assumption
3	$\phi \vee \neg \phi$	$\vee i_1 2$
4	$\perp$	$\neg e 3, 1$
5	$\neg \phi$	$\neg i 2-4$
6	$\phi \vee \neg \phi$	$\vee i_2 5$
7	$\perp$	$\neg e 6, 1$
8	$\neg \neg(\phi \vee \neg \phi)$	$\neg i 1-7$
9	$\phi \vee \neg \phi$	$\neg \neg e 8$