Assignment - 5

Q[1] (a.) $$E_1 = \sum_{n=-\infty}^{\infty} |\mu[n]|^2$$

$$= \sum_{n=0}^{\infty} 1$$

$$= \infty$$

$$(\text{Avg. Power})_1 = \lim_{k \to \infty} \frac{1}{(2k+1)} \sum_{n=-k}^{k} |\mu[n]|^2$$

$$= \lim_{k \to \infty} \frac{1}{(2k+1)} \sum_{n=0}^{k} 1$$

$$= \lim_{k \to \infty} \frac{1}{(2k+1)} (k+1)$$

$$= \frac{1}{2}$$

August 2010

S	M	T	W	T	F	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

22 Wednesday

(b.)

$$E_2 = \sum_{n=-\infty}^{\infty} |n u[n]|^2$$

$$= \sum_{n=0}^{\infty} n^2$$

$$= \infty$$

$$(\text{Avg. Power})_2 = \lim_{k\to\infty} \left(\frac{1}{2k+1}\right) \sum_{n=-k}^{k} |n u[n]|^2$$

$$= \lim_{k\to\infty} \frac{1}{(2k+1)} \sum_{n=0}^{k} n^2$$

23 Thursday

$$= \lim_{k\to\infty} \frac{1}{(2k+1)} \frac{(k)(k+1)(2k+1)}{6}$$

$$= \lim_{k\to\infty} \frac{k^2+k}{6}$$

$$= \infty$$

(c.) $$E_3 = \sum_{n=-\infty}^{\infty} |4\sin(0.5\pi n)|^2$$

$$= \sum_{n=-\infty}^{\infty} 16\,|\sin(0.5\pi n)|^2$$

$$= 2\sum_{n=1}^{\infty} 16\left|\sin\left(\frac{\pi(2n-1)}{2}\right)\right|^2$$

$$= 32\sum_{n=1}^{\infty} 1$$

$$= \infty$$

$$(\text{Avg. Power})_3 = \lim_{k\to\infty} \frac{1}{(2k+1)} \sum_{n=-k}^{k} |4\sin(0.5\pi n)|^2$$

$$= 16 \lim_{k\to\infty} \frac{1}{(2k+1)}\, 2\sum_{n=1}^{k} \left|\sin\left(\frac{\pi(2n-1)}{2}\right)\right|^2$$

If k = even, then:

$$(\text{Avg. Power})_3 = 32 \lim_{k\to\infty} \left(\frac{1}{2k+1}\right)\left(\frac{k}{2}\right)$$

$$= \lim_{k\to\infty} \frac{16k}{2k+1}$$

$$= 8$$

27 Monday

If $k =$ odd, then:

$$(\text{Avg. Power})_3 = 32 \lim_{k\to\infty} \frac{1}{(2k+1)} \left(\frac{k+1}{2}\right)$$

$$= \lim_{k\to\infty} \frac{16(k+1)}{(2k+1)}$$

$$= 8$$

28 Tuesday

Q.2)
$$\tilde{s}[n] = \cos\left(\frac{7\pi n}{20}\right)$$

$$\omega_0 = \frac{7\pi}{20}$$

$$\left(\frac{7\pi}{20}\right) N_0 = 2\pi m$$

$$N = \frac{40}{7} m$$

For $m = 7$, $N_0 = 40$

So, Fundamental period = 40

Consider $\tilde{\omega}[n] = \cos\left(\left(\frac{7\pi}{20} + 2\pi\right) n\right)$

$$= \cos\left(\frac{7\pi n}{20} + 2\pi n\right)$$

$$= \cos\left(\frac{7\pi n}{20}\right) = \tilde{s}[n]$$

So, $\tilde{\omega}[n] = \cos\left(\frac{47\pi n}{20}\right)$ and

$\tilde{x}[n] = \cos\left(\frac{87\pi n}{20}\right)$ are

2 other sinusoidal sequences with the same fundamental period as $\tilde{s}[n]$.

1 Friday

Q.3] (a.)

$$x[n] = x[-1]\,\delta[n+1] + x[0]\,\delta[n] + x[1]\,\delta[n-1] + x[2]\,\delta[n-2] + x[3]\,\delta[n-3] + x[4]\,\delta[n-4]$$

$$x[n] = \sum_{k=-1}^{4} x[k]\,\delta[n-k]$$

(b.)

$$x[n] = x[-1](u[n+1] - u[n]) + x[0](u[n] - u[n-1]) + x[1](u[n-1] - u[n-2]) + x[2](u[n-2] - u[n-3]) + x[3](u[n-3] - u[n-4]) + x[4](u[n-4] - u[n-5])$$

2 Saturday

$$x[n] = \sum_{k=-1}^{4} x[k]\,(u[n-k] - u[n-k-1])$$

(c.) For unit sample representation:

3 Sunday

$$\text{Even}\{x[n]\} = \frac{x[n] + x[-n]}{2}$$

$$= \frac{\sum_{k=-1}^{4} x[k]\,\delta[n-k] + \sum_{k=-1}^{4} x[k]\,\delta[-n-k]}{2}$$

Monday 4

$$= \frac{\sum_{k=-4}^{4} x[k]\,\delta[n-k] + \sum_{k=-4}^{4} x[k]\,\delta[n+k]}{2}$$

$$= \frac{\sum_{k=-4}^{4} x[k]\left(\delta[n-k] + \delta[n+k]\right)}{2}$$

$$\text{Odd}\{x[n]\} = \frac{\sum_{k=-4}^{4} x[k]\,\delta[n-k] - \sum_{k=-4}^{4} x[k]\,\delta[-n-k]}{2}$$

Tuesday 5

$$= \frac{\sum_{k=-4}^{4} x[k]\left(\delta[n-k] - \delta[n+k]\right)}{2}$$

For unit step representation:

$$\text{Even}\{x[n]\} = \frac{x[n] + x[-n]}{2}$$

$$= \frac{\sum_{k=-4}^{4} x[k]\left(\mu[n-k] - \mu[n-k-1]\right) + \sum_{k=-4}^{4} x[k]\left(\mu[-n-k] - \mu[-n-k-1]\right)}{2}$$

6 Wednesday

$$\text{Odd}\{x[n]\} = \frac{x[n] - x[-n]}{2}$$

$$= \frac{\sum_{k=-4}^{4} x[k]\left(u[n-k] - u[n-k-1]\right) - \sum_{k=-4}^{4} x[k]\left(u[-n-k] - u[-n-k-1]\right)}{2}$$

Even sequence $= \{2, 5/2, -1, -3/2, 4, -3/2, -1, 5/2, 2\}$

Odd sequence $= \{-2, -5/2, 1, 3/2, 0, -3/2, -1, 5/2, 2\}$

7 Thursday

8·4·]

$$x[n] \longrightarrow \boxed{\uparrow 2} \longrightarrow x[n/2]$$

$$x[n/2] = n\,\mu[n/2]$$

$$x[n/2] \longrightarrow \boxed{z^{-1}} \longrightarrow x[n/2 - 1]$$

$$x[n/2] \longrightarrow \boxed{z} \longrightarrow x[n/2 + 1]$$

$$x[n/2 - 1] \xrightarrow{0.5} \frac{1}{2}\, x\left[\frac{n}{2} - 1\right]$$

$$x\left[\frac{n}{2} + 1\right] \xrightarrow{0.5} \frac{1}{2}\, x\left[\frac{n}{2} + 1\right]$$

$$y[n] = \frac{1}{2}\left\{ x\left[\frac{n}{2} - 1\right] + x\left[\frac{n}{2} + 1\right] \right\}$$

$$= \frac{1}{2}\left\{ (n-2)\,\mu\left[\frac{n}{2} - 1\right] + (n+2)\,\mu\left[\frac{n}{2} + 1\right] \right\}$$

$$\mu\left[\frac{n}{2} - 1\right] = \begin{cases} 1, & n \geq 2 \\ 0, & n < 2 \end{cases}$$

September 2010

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			1	2	3	4
5	6	7	8	9	10	11
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11 Monday

$$u\left[\frac{n}{2}+1\right] = \begin{cases} 1, & n \geq -2 \\ 0, & n < -2 \end{cases}$$

So, $y[n] = \frac{1}{2}\left\{(n-2) + (n+2)\right\} = n$

for $n \geq 2$

$y[n] = \frac{1}{2}\left\{(n+2)\right\} = \frac{n+2}{2}$

for $-2 \leq n < 2$

12 Tuesday

$y[n] = 0$ for $n < -2$

September 2010						
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15 Friday

Q5) $p_a[n] = p_a(t)\Big|_{t=nT} = p_a(nT), \; -\infty < n < \infty$

$T = \frac{1}{F_T} = \frac{1}{50} = 0.2$

So,

$p[n] = 3\cos\left(\frac{2\pi n}{5}\right) - 7\sin\left(\frac{\pi n}{2}\right) + 5\cos\left(\frac{8\pi n}{5}\right)$
$+ 4\cos\left(\frac{5\pi n}{2}\right)$

16 Saturday

$- 6\cos\left(\frac{22\pi n}{5}\right)$

Now, $\cos\left(\frac{22\pi n}{5}\right) = \cos\left(\frac{2\pi n + 20\pi n}{5}\right)$

$= \cos\left(\frac{2\pi n}{5} + 4\pi n\right)$

$= \cos\left(\frac{2\pi n}{5}\right)$

17 Sunday

$\cos\left(\frac{8\pi n}{5}\right) = \cos\left(\frac{-2\pi n}{5} + 2\pi n\right)$

$= \cos\left(\frac{-2\pi n}{5}\right) = \cos\left(\frac{2\pi n}{5}\right)$

So, $p[n] = 2\cos\left(\frac{2\pi n}{5}\right) + 4\cos\left(\frac{5\pi n}{2}\right) - 7\sin\left(\frac{\pi n}{2}\right)$

September 2010

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20 Wednesday

Q.6] (a.) $\tilde{x}[n] = \sin\left(\frac{\pi n}{10}\right)$

$$\left(\frac{\pi}{10}\right) N_o = 2\pi$$

$$N_o = 20$$

(b.) $\tilde{x}[n] = \sin\left(\frac{\pi n}{5}\right)$

$$\frac{\pi}{5} N_o = 2\pi$$

$$N_o = 10$$

21 Thursday

(c.) $\tilde{x}[n] = \sin\left(\frac{4\pi n}{5}\right)$

$$\left(\frac{4\pi}{5}\right) N_o = 4\pi$$

$$N_o = 5$$

(d.) $\tilde{x}[n] = \sin\left(\frac{18\pi n}{20}\right) = \sin\left(\frac{9\pi n}{10}\right)$

$$N \frac{9\pi}{10} = 2\pi k \Rightarrow N_o = 9$$