NNF, CNF.

Disjunctive normal form. (DNF)

literal - $p / \neg p$

α - DNF if it is a disjunction of conjunctions of literals

Ex - $(P_1 \wedge \neg P_2 \wedge P_3) \vee (P_4 \wedge P_5 \wedge \neg P_6)$

Theorem. For any $\alpha \in \Phi$, $\exists\, \alpha_1$ in CNF s.t $\models \alpha \equiv \alpha_1$.

Theorem. For any $\alpha \in \Phi$, $\exists\, \alpha_2$ in DNF s.t $\models \alpha \equiv \alpha_2$

Q1. Given $\alpha \in \Phi$, is α satisfiable? $\exists v$ s.t $v \models \alpha$

Q2. Given $\alpha \in \Phi$, is α valid? $\forall v$ does $v \models \alpha$

Theorem. Given α in CNF, we can efficiently check if α is valid.

Theorem. Given α is DNF, we can efficiently check if α is satisfiable.

Can Q1 be solved efficiently.

$$P \stackrel{?}{=} NP \quad [CS340].$$

CNF. $\alpha = (P_1 \wedge q_1) \vee (P_2 \wedge q_2) \vee \cdots \vee (P_n \wedge q_n)$ - n clauses.
What is the equivalent α_1 in CNF?

DNF. $\alpha = (P_1 \vee q_1) \wedge (P_2 \vee q_2) \wedge \cdots \wedge (P_n \vee q_n)$
What is the equivalent α_2 in DNF.

	SAT(Q_1)	VALIDITY(Q_2)
propositional formulas.	Hard	Hard
CNF	Hard	Easy
DNF	Easy	Hard

SAT solvers.

Graph Colouring Problem - Example.

Given $G = (V, E)$ $E \subseteq V \times V$

G is 3-colourable if $\exists f : V \to \{R, G, B\}$ s.t

- $\forall e = (u, v) \in E$ we have $f(u) \neq f(v)$.

[adjascent nodes get different colour].

Model this problem using a set of clauses $C(G)$.

For each $v \in V$, associate propositions R_v, B_v, G_v

For each $v \in V$,

- $R_v \vee B_v \vee G_v$.
- $R_v \to \neg G_v$, $R_v \to \neg B_v$, - -

$\neg R_v \vee \neg G_v$, $\neg R_v \vee \neg B_v$

} Every vertex gets exactly one colour.

For each edge $e = (u, v) \in E$

- $\neg R_u \vee \neg R_v$, $\neg G_u \vee \neg G_v$, $\neg B_u \vee \neg B_v$.

$C(G)$: α - Conjunction of all clauses in $C(G)$.

Theorem. G is three colourable iff α is satisfiable.