Correct options are (a), (b), (d)

3.3]	(a·)	It is a linear PDE.
		Order = 2
		Vederate.
	(p.)	It is a linear PDE
		Order = 2
	(c)	st is a linear PDE
		Order = 2
-	(d·)	It is a quasi-linear PDE.
		Order = 2
	(e·)	It is a quasi-linear PDE.
		Order = 2
	(4.)	It is a quasi-linear PDE.
		Order = 1
	(8.)	It is a linear PDE
-		Order = 3

Q.3 Problem 1: ux - suy = u in R2  $u(x_10) = 1$ From the usual notations: a (2,8,4) = 1 b(x1414) = -2 c(x, y, u) = u (urve T = x-axis General point is: (8,0,1). Now, by method of characteristics: 文(t)=1 => マ(t)= t+h, 7(F) = -5 = 7(F) = -5F + 45  $u'(e) = u(e) \Rightarrow u(e) = h_3 e^{t}$ Now, at t=0, the characteristic curve passes through (S.O.1). So, XI = S 45 = 0  $\lambda_3 = 1$ so, z(t) = t+s A(A) = -2F  $u(t) = e^{t}$ SO. U(218) =

Date Prage

 $\begin{cases} u_{x} + u_{y} = u'^{2} \\ u(x,0) = 0 \end{cases}$ 

From the usual notations: a(x,y,u) = 1

c(x, y, u) = 1

Now, by observation, we can see that u(x,y) = 0 is a trivial solution to the above problem.

By method of characteristics: x'(t) = 1 y'(t) = 1  $y'(t) = (u(t))^{1/2}$   $y'(t) = (u(t))^{1/2}$ 

Consider the 3rd ODE:  $\frac{d(u(t))}{dt} = (u(t))^{1/2}, \quad u(0) = 0.$ 

By observation,  $u(t) \equiv 0$  is a trivial solution. Also for  $t \geqslant 0$   $2\sqrt{u(t)} = t + \lambda_8 \quad , \quad u(0) = 0$   $\Rightarrow 2\sqrt{u(t)} = t$ 

is also a valid solution.

 $=) \quad u(t) = t^2$ 

Also, :: x(0) = S, y(0) = 0: x(t) = t + S and

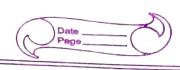
y(+)= +

so, by eliminating t:

u(x,y) = y² is also a

solution.

No the existence theorem is not violated. The theorem can be applied only when c(x18,4) is continuously differentiable in the required domain. But the derivative of Ju is not continuous at points where u = 0. Hence u(x,y) = y2 is valid only for yro. Offennice exxy)= 0 is a valid solution.



Q.5]

From the usual notations:  $a(x_iy_iu) = u$   $b(x_iy_iu) = i$   $c(x_iy_iu) = u^2$ 

General point on the initial curve is

By, method of characteristics: u'(t) = u(t)  $u'(t) = \underbrace{t}$   $u'(t) = (u(t))^{2}$ 

Mow, y'(t)=1, y(0)=0

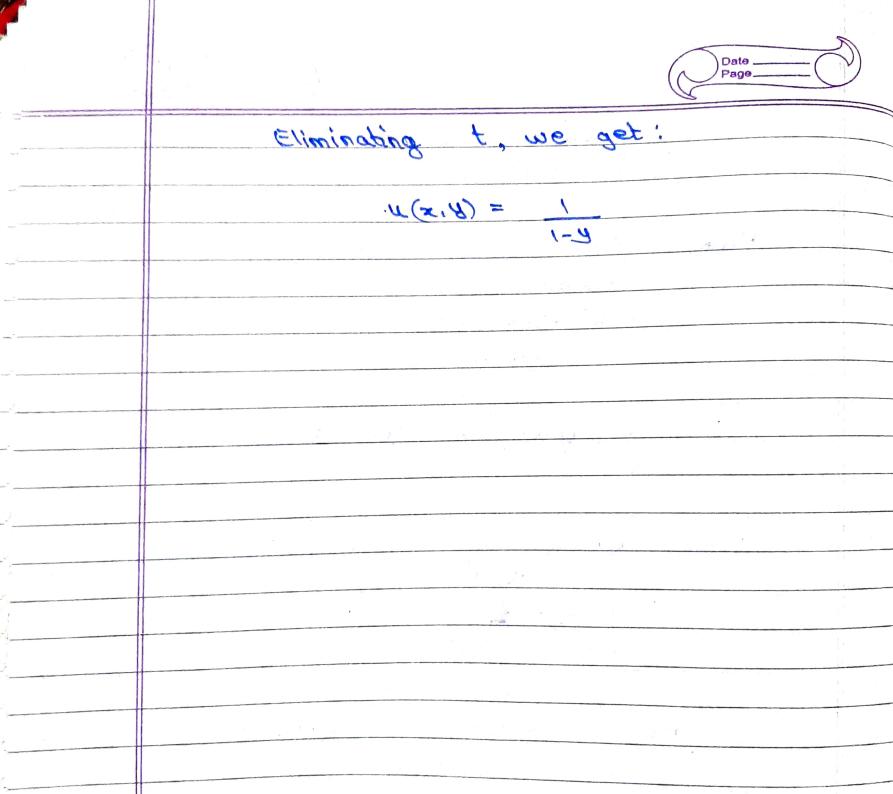
Now,  $u'(t) = (u(t))^2$ , u(0) = 1 $= \frac{-1}{u(t)} = .t + h_3$ , u(0) = 1

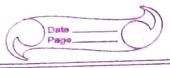
m(F)

=) 11(f) = 1

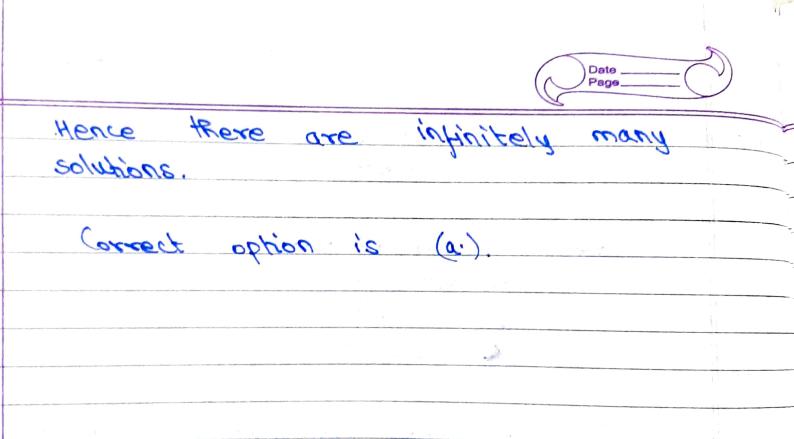
So, x'(t) = 1, x(0) = S

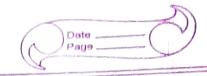
 $\Rightarrow x(t) = -\ln(11-t1) + \lambda_1 + x(0) = 8$   $\Rightarrow x(t) = -\ln(1t-11) + 8$ 





Q.6 From usual notabious: a(x18,14) = 1 b(x, 8, 4) = 1 € (x18,14) =0 General point on initial curve is (E.E.I). By method of characteristics: x(t)=1 = x(t)= t+h1 y'(t) = 1 =)  $y(t) = t + \lambda_2$  y'(t) = 0 =)  $y(t) = \lambda_3$ At t=0; characteristic curve passes through (sisil) x(t)= 645 So. A(F)= F+2 u(t)=1 so, u(x,y)=1 and x=yHence, we can in general write a general function u(x,y) = 1 + k(x-y)where RER, which sobieties the obove problem:  $\frac{3u}{2x} = k \qquad \frac{3u}{3u} = -k \Rightarrow u_x + u_y = 0$ and u(x1x) = 1





For problem 4: Q.7. Consider x = 4+s,

x= 4+52

are 2 different PCs.

parallel, the PCs don't intersect.

For problem 5: Consider  $x = -\ln|y-1| + S_1$ 

= - 10/8-1/ + 82

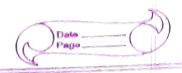
are 2 different PCs.

Now, - (x-s1) = In(y-11)

- (x-s2) = In(14-11)

50, |4-1| = e-(x-s2)

These are just shifted copies of each other. They will only intersect when si=sz. Hence, since si+sz, the curves do not intersect.



For the problem 4: [8.8]

let I be parametrized by

(5,0).

80, f(s) = s, h(s) = 080, f'(s) = 1, h'(s) = 0

Now, a (9(8), h(s), g(8)) = 1 and

b (50), h(s), q(e))= 1

Non - characteristics condition:

3'(s) a (f(s), h(s), 8(s)) + h'(s) b (f(s), h(s), 8(s))

Now,  $f'(s) \circ (f(s), h(s), g(s)) = 1$ 

h'(s) b(f(s), k(s), q(s)) = 0

Hence, the Non-characteristics condition

solistied.

For the toy problem 2:

let I be parametrized by

(012).

 $s_{0}, \quad f(s) = s \quad , \quad g'(s) = 1$ 

h(s)=0, h'(s)=0

Also, given: a(t(s), h(s), g(s)) = 1

b(g(s),h(s),g(s))=0

so, f'(s) a (f(s), h(s), g(s)) = 1

1, (B) P(26), 1(B), 8(B) = 0

Hence, the Non-characteristic condition

is satisfied.