$$-f'dt = dm(v+dv-v')$$

Bigger mass:

$$\vec{F}_{ext}\,dt + \vec{f}\,dt = m(\vec{v}+d\vec{v}) - m\vec{v}$$

add (1) and (2)

$$\vec{F}_{ext}\,dt = m\,d\vec{v} + dm(\vec{v}-\vec{v}')$$

$$m\frac{d\vec{v}}{dt} = \vec{F}_{ext} + (\vec{v}'-\vec{v})\frac{dm}{dt}$$

$m(t)$

Variable mass equation

If : $v'=0$, $m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt} = \vec{F}_{ext}$ $\longrightarrow$ Special case

Variable mass equation:

$$m\frac{d\vec{v}}{dt} = \vec{F}_{ext} + \vec{u}_{rel}\frac{dm}{dt}$$

$\vec{u}_{rel}$ is the rel. of dm w.r.t m

check the Galilean invariance

$\vec{u}_{rel}$ is the same in all inertial frames

29/01

Falling rain drop:

starts from $x=0$, v_0 | Rain drop gather moisture

$x=0$

$\downarrow x$

$+ve\ x$

$$\frac{dm}{dx} = am$$

water particles are at rest w.r.t ground

$$m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt} = \vec{F}_{ext}$$

$\vec{V}=0$

$$\longrightarrow \frac{dm}{dt} = \frac{dm}{dx}\frac{dx}{dt} = amv$$

$$m(t)\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt} = mg$$

$$m(t)\frac{d\vec{v}}{dt} + \vec{v}\,am\vec{v} = mg$$

$$m\frac{dv}{dt} + amv^2 = mg$$

$$\cancel{m}\frac{dv}{dt} + a\cancel{m}v^2 = \cancel{m}g$$

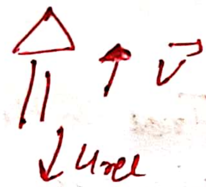
$$\frac{dv}{dt} + av^2 = g$$

$$v\frac{dv}{dx} = g - av^2$$ solve it to find the terminal velo

Rocket motion

free space

Drop $-mg$



$$m(t)\frac{d\vec{v}}{dt} - \vec{u}_{rel}\frac{dm}{dt} = -mg$$

$$m(t)\frac{dv}{dt} = \vec{u}_{rel}\frac{dm}{dt}$$

$$m(t)\frac{dv}{dt} = -u_{rel}\frac{dm}{dt} \quad \Big| \quad dv = -u_{rel}\frac{dm}{m}$$

$$v_f - v_0 = \Delta v = -u_{rel}\int_{m_0}^{m_1}\frac{dm}{m}$$

$$\Rightarrow M_1 = m(t) = m_0 e^{-\frac{\Delta v}{u_{rel}}}$$

Payload → higher, keeping Δv and u_0

Calculate Δv
Kleppner

(2) Include g and solve the equⁿ}

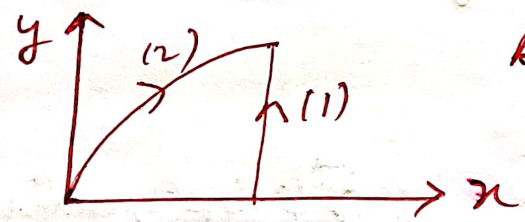
* Conservative Force Field

Total Mechanical energy is conserved

Work energy theorem

$$W = \int_1^2 \vec{F}\cdot d\vec{r} = \int m\frac{d\vec{v}}{dt}\cdot d\vec{r} = \int_1 m\vec{v}\cdot d\vec{v} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

Workdone is the change in Kinetic energy
Workdone depends on path



Air drag is there — Work is not state function
∴ path dependent

Conservative : Work done does not depend on path ;
Only on initial and final co-ordinates
potential

$\psi(x,y,z)$

Workdone in a closed loop is zero
↳ Conservative force field

$\vec{\nabla}\phi \rightarrow$ Mapping from a scalar field ψ to vector field $\vec{\nabla}\psi$

$$\left\{\begin{array}{l}\vec{\nabla}\times\vec{F}=0\\ \vec{F}=-\vec{\nabla}\psi\end{array}\right.$$

H.W
Show $\vec{\nabla}$ is a vector operator
S.D. Joglekar

$$\vec{\nabla}\psi = \hat{i}\frac{\partial\psi}{\partial x} + \hat{j}\frac{\partial\psi}{\partial y} + \hat{k}\frac{\partial\psi}{\partial z}$$

$$\vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

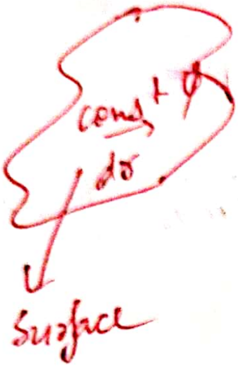
Magnitude : $|\vec{\nabla}\phi|$ =

Direction =

For. displacement

$d\vec{r} : d\varphi = 0$

~~[illegible]~~



$\nabla\psi \cdot d\vec{r}$

$$= \left(\hat{i}\frac{\partial\psi}{\partial x} + \hat{j}\frac{\partial\psi}{\partial y} + \hat{k}\frac{\partial\psi}{\partial z}\right)(\hat{i}dx + \hat{j}dy + \hat{k}dz)$$

$$= \frac{\partial\psi}{\partial x}dx + \frac{\partial\psi}{\partial y}dy + \frac{\partial\psi}{\partial z}dz$$

$$= d\Psi$$

$d\Psi = 0$, for $d\vec{r}$

$\therefore\ \vec{\nabla}\psi \cdot d\vec{r} = 0$ | $\vec{\nabla}\varphi$ is perpendicular to constant φ surface

$\Psi(r) = -\frac{GM}{r}$ $\quad -\vec{\nabla}\Psi = -\frac{GM}{r^2}\hat{r}$

31/01

$\rightarrow m \quad \rightarrow v$

$\longrightarrow V_a$ flat car on a track

Jumping out of the flat which is initially at rest;
with a speed u with respect to the flat car }

(i) all of them jumps out simultaneously

(N) : momentum conservation

$$P_{initial} = 0;$$

$$P_{final} = M V_a + Nm(V_a - u)$$

$$\Rightarrow \boxed{V_a = \frac{Nm}{Nm+M} u}.$$

(ii) They jump one by one

(ii) j^{th} fellow has jumped out $(j+1)^{th}$ jumping out.

$$P_{ini} = [\underbrace{(N-j)}_{\text{no.}}m + M]V_j \longrightarrow j^{th} \text{ person jumped out}$$

~~still on the flat~~

$$P_{final} = [(N-j-1)m + M]V_{j+1} + m(V_{j+1} - u) \rightarrow (2)$$

No external force ; $\Delta P = 0$

$$V_{j+1} = \left(\frac{m}{(N-j)m + M}\right) u + V_j$$

$$V_{\text{one by one}} = \left[\frac{m}{Nm+M} + \frac{m}{(N-1)m+M} + \frac{m}{m+M}\right] u$$

↳ check

and $V_a < V_{\text{one-by-one}}$ } Yes or no

Conservative force field:

1. Work done path indep. depends on initial and final coordinates → Work done in a closed loop is zero

2. Stokes Theorem:

$$\oint \vec{F}\cdot d\vec{l} = \int_\sigma (\vec{\nabla}\times\vec{F})\cdot d\vec{\sigma}$$

↳ always valid if loop doesnot include any singularity of $\vec{F}$

3. Conservative force:

$$\oint \vec{F}\cdot d\vec{l}$$

↳ for any arbitrary closed loop

$\vec{\nabla}\times\vec{F} = 0$

4. Curl (grad ψ) $= 0$

$$\vec{F} = -\text{grad}\,\Psi = -\vec{\nabla}\psi$$

↳ Scalar potential → $\nabla\psi$ is ⊥ to constant ψ-surface

5. $\vec{F}(r) = -\dfrac{GMm}{r^2}\hat{r}$, $\Psi(r) = -\dfrac{GM}{r}$

$\vec{F} = -mg\hat{k}$: $\Psi = mgz$

Prove: $\vec{\nabla}\times(\nabla\psi) = 0$, ⇒

$$\vec{\nabla}\times\vec{A} = \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x_1} & \frac{\partial}{\partial x_2} & \frac{\partial}{\partial x_3} \\ A_1 & A_2 & A_3 \end{pmatrix}$$

$\{1, 2, 3 \to x, y, z$

$$[\vec{\nabla}\times(\vec{\nabla}\Psi)]_i = \sum_{jk} \epsilon_{ijk}\,\partial_j(\nabla\Psi)_k$$

$\partial_j = \frac{\partial}{\partial x_j} \qquad j = 1, 2, 3$

$$= \sum_{j,k} \epsilon_{ijk} \underbrace{\partial_j \partial_k}_{\text{symmetric in } j,k} \psi$$

ϵ_{ijk}: anti-symmetric in j and k; j, k Summed over

$= 0$

Potential : Unit mass: $\psi = \int_{\infty}^{r} -\frac{GM}{r^2}\, dr = -\frac{GM}{r}$

attractive potential is negative.

Workdone : $\vec{F} = -\nabla\psi$ $\qquad \vec{F}$ can not have an explicit time dependence

$$\therefore \int_1^2 \vec{F}\cdot d\vec{r} = -\int \vec{\nabla}\psi \cdot d\vec{r} = -\int_1^2 d\psi = \psi_1 - \psi_2 \quad \text{—(1)}$$

$T_2 - T_1 = \psi_1 - \psi_2$

work-energy theorem: $W = T_2 - T_1$ —(2)

$\therefore \boxed{T_1 + \psi_1 = T_2 + \psi_2}$

Total mechanical energy is conserved

HW: (i) In rigid body

$\vec{v} = r\omega\,\hat{\phi} \qquad \vec{\nabla}\times\vec{v} = ?$

(ii) $F = 3x^2y\,\hat{i} + x^3\hat{j}$ $\rightarrow$ is it conservative

check curl $\vec{\nabla}\times\vec{F} = 0$

what is $\psi(x, y)$

(iii) Central field: $\vec{F} = f(r)\,\vec{r}$ } Energy and angular momentum conserved

$\vec{\nabla}\times f(r)\,\vec{r} = 0$

(iii) S.H.M $F = -kx \rightarrow$ Is ~~it~~ conservative or not.

curl is not defined in one dim.

I can define: $\psi(x) = \frac{1}{2}kx^2$ so that
$$\frac{-\partial\psi}{\partial x} = -kx$$

3/02

$\vec{V} = r\omega\hat{\phi}$; Calculate $\vec{\nabla}\times\vec{V}$

Conservative force field $\longrightarrow$ Stoke's Theorem

$\vec{F} = -\vec{\nabla}\psi \longrightarrow$ scalar potential

so that $\vec{\nabla}\times\vec{F} = 0$ $\quad$ $\vec{\nabla} =$ vector operator

irrotational vectors

One dimensional problem:

$\vec{F} = -kx\,\hat{i}$; $\quad \psi = \frac{1}{2}kx^2$

$\Rightarrow$ curl is not defined

Total energy (mechanical) conserved

$$T_1 + \psi_1 = T_2 + \psi_2$$

Potential ψ is a state function: depends on coordinates

$$W = \psi_1 - \psi_2 \Rightarrow \text{Path independent}$$

Explicit time dependence: Not there

$\vec{F} = 3x^2y\,\hat{i} + x^3\hat{j}$; what is $\psi = ?$

Central field: Energy and ang. momentum both conserved

$$\vec{F}(r) = f(r)\vec{r}\,; \quad \vec{\nabla}\times\vec{F}(r) = 0$$

Prove: $[\vec{\nabla}\times\vec{F}(r)]_i = [\vec{\nabla}\times f(r)\vec{r}]_i$

$$= \sum_{jk} \epsilon_{ijk}\,\partial_j\,(f(r)\vec{r})_k$$

$$\partial_j = \frac{\partial}{\partial x_j}$$

$r = \sqrt{x_1^2 + x_2^2 + x_3^2}$ $\qquad \frac{\partial}{\partial x_1}$ or $\frac{\partial}{\partial x_2}$ or $\frac{\partial}{\partial x_3}$

$= \sqrt{\sum x_i x_i}$

$$= \sum_{\substack{jk \\ j\neq k}} \varepsilon_{ijk}\, \partial_j x_k + \sum_{jk} \varepsilon_{ijk}\, x_k\, \partial_j [f(r)]$$

$\vec{r} = \hat{i}x_1 + \hat{j}x_2 + \hat{k}x_3$

$$= 0 + \sum \varepsilon_{ijk}\, x_k\, \partial_r f(r)\, \partial_j r$$

$$\frac{\partial}{\partial x_j} f(r) = \frac{\partial f(r)}{\partial r}\frac{\partial r}{\partial x_j}$$

$$= \sum \varepsilon_{ijk}\, x_k \{\partial_r f(r)\} \frac{x_j}{r}$$

$$= \sum_{jk} \varepsilon_{ijk}\, x_j x_k \left[\frac{1}{r}\{\partial_r f(r)\}\right]$$

Examples: 5.9 Kleppner ch-4 and 5 Kleppner

$\vec{F}(r) = \frac{A}{r}\hat{\phi}$; conservative?

Workdone:

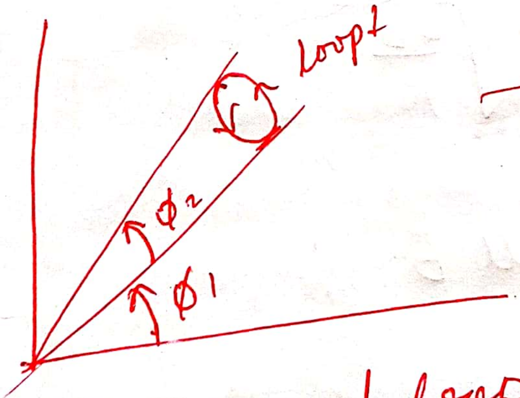
$$W = \int \vec{F}\cdot d\vec{r} = \int \vec{F}\cdot(dr\,\hat{r} + r\,d\phi\,\hat{\phi})$$

$$= A\int d\phi \longrightarrow \text{for loop 1}$$

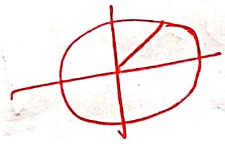
$$W = A(\phi_2-\phi_1) + A(\phi_1-\phi_2) = 0$$

↳ Is it conservative?

Closed loop:



Loop 2: workdone is $2\pi A$



W = for any closed loop enclosing the origin

$$W = 2\pi A$$

Origin: $\vec{\nabla}\times\left(\frac{A}{r}\hat{\phi}\right) = \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{rA}{r}\right) = 0$

↳ everywhere except for $r=0$

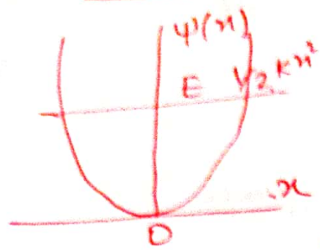
Hence, not a conservative force

One dimensional problems:

Harmonic oscillator:

$$E = \frac{1}{2} m\dot{x}^2 + \frac{1}{2} k x^2$$

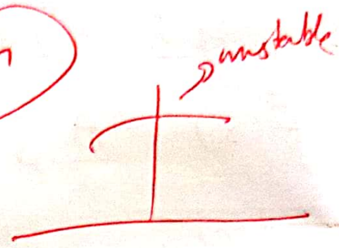
Bound State



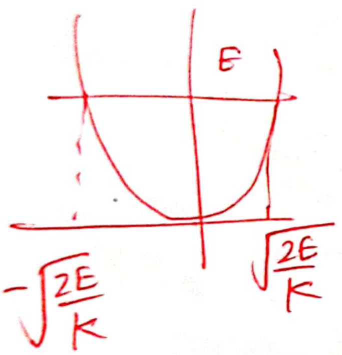
$$\left.\begin{array}{l} \dfrac{\partial \psi}{\partial x} > 0 \text{ for } x>0 \\ \dfrac{\partial \psi}{\partial x} < 0 \text{ for } x<0 \end{array}\right\}$$ Force is always restoring towards $x=0$

$x=0$ stable Equilibrium

$$\boxed{\frac{\partial^2 \psi}{\partial x^2} > 0}$$



For all E motion is bounded between classical turning pts.



$$\frac{1}{2} m\dot{x}^2 + \frac{1}{2} kx^2 = E$$

Classical turning pts (x_0) : K.E Vanishes

$$\Rightarrow \frac{1}{2} K x_0^2 = E$$

$$\Rightarrow x_0 = \pm \sqrt{\frac{2E}{K}}$$

Motion is bounded between CTPs for all E:

Time period:

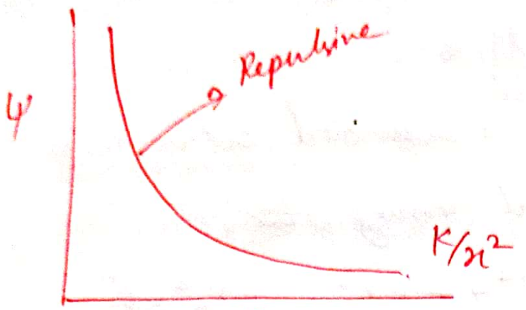
$$\frac{1}{2}\left(\frac{dx}{dt}\right)^2 + \frac{1}{2} k x^2 = E = \frac{1}{2} k x_0^2$$

$$\Rightarrow \left(\frac{dx}{dt}\right)^2 = k(x_0^2 - x^2)$$

$$t = \int_{x}^{x} \frac{dx}{\sqrt{K}\,(x_0^2 - x^2)^{1/2}}$$

$$= 2\times 2 \int_0^{x_0} \frac{dx}{\sqrt{k/m}\,(x_0^2 - x^2)^{1/2}} = \frac{2\pi}{\omega_0}$$

$$\omega_0 = \sqrt{\frac{k}{m}}$$



Scattering State

$V_0 \quad +\infty$

$\downarrow$

mass m projected with $\frac{1}{2} m V_0^2$ from $x = +\infty$

$$\psi(x) = \frac{k}{x^2}$$

E is conserved

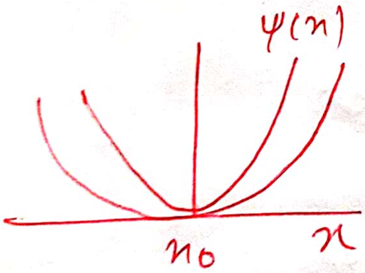
$\psi(x)$ diverges as $x \to 0$

what is the closest distance?

Ans $\frac{1}{2} m V_0^2 = \frac{k}{x_0^2}$

No bound state :

Harmonic Approximation : (stable equilibrium, minimum ψ at $x = x_0$)



"Small" motion close to x_0 so that $(x - x_0)$ is small

$$\psi(x) = \psi(x_0) + \left.\frac{\partial \psi}{\partial x}\right|_{x_0} (x - x_0) + \frac{1}{2} \left.\frac{\partial^2 \psi}{\partial x^2}\right|_{x_0} (x - x_0)^2 + \cdots$$

Expand

$x = x_0$

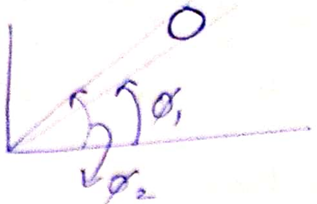
ψ_{bound}

$$\left.\frac{\partial \psi}{\partial x}\right|_{x_0} = 0 : \text{ No force acts}$$

$$\psi(x) = \frac{1}{2} \left.\frac{\partial^2 \psi}{\partial x^2}\right|_{x_0} (x - x_0)^2 > 0$$

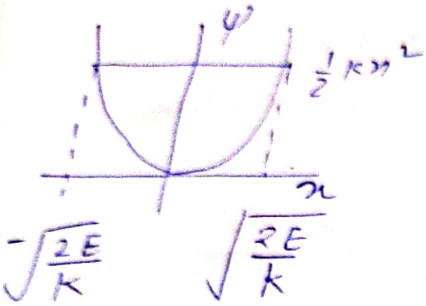
5/02/20

Recap :- 1. $\vec{F} = \frac{A}{r}\hat{\phi}$



Not conservative : $\vec{\nabla} \times \vec{F}$ is singular at the origin

2.12 Simple Harmonic Oscillator:



• Motion is bounded betw. two classical turning pts

$$\frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2 = \frac{1}{2}kx_0^2 = E$$

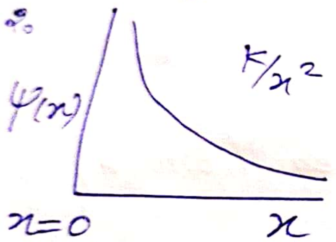
↳ CTP: K.E vanishes

bound state

Force is always directed towards $x = 0$

Restoring force

~~SA + 90~~



Closest distance : x_0

$$\boxed{\frac{k}{x_0^2} = \frac{1}{2}mv_0^2}$$

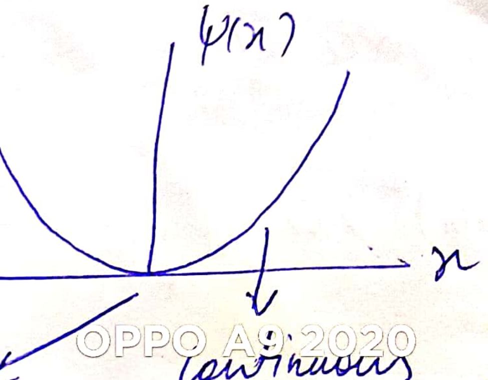
Repulsive force towards $x \to \infty$ } Particle goes back

One turning pt :

Scattering state

<u>Harmonic Approximation</u>: Focus on motion close to the stable equilibrium.

Pt : x_0



Expand $\psi(x)$ around $x = x_0$;

$$\psi(x) = \psi(x_0) + \left.\frac{\partial \psi}{\partial x}\right|_{x_0}(x - x_0) + \frac{1}{2}\left.\frac{\partial^2 \psi}{\partial x^2}\right|_{x_0}(x - x_0)^2 + \cdots$$

Extremum condition $= 0$

+ve because ψ is minimum

↳ Small amplitude motion

Set $\psi(x_0$ ...

$\psi(x) = \frac{1}{2}$ ...

Compare with ...

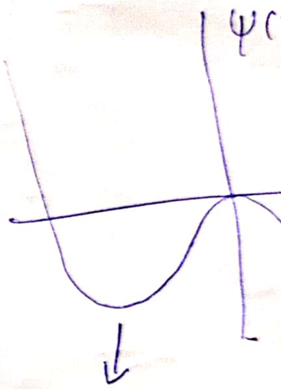
Motion close to a... equilibrium...

Oscillator with ...

Take a Potent...

$\psi($ ...

$x = 0$



$x = 0$, w... equ...

$W =$ ...

Lena...

Co...

Set $\psi(x_0) = 0$,

$$\psi(x) = \underbrace{\frac{1}{2}\left.\frac{\partial^2\psi(x)}{\partial x^2}\right|_{x_0}(x-x_0)^2} \rightarrow \text{Harmonic Oscillator:}$$

Compare with $\psi(x) = \frac{1}{2}kx^2$

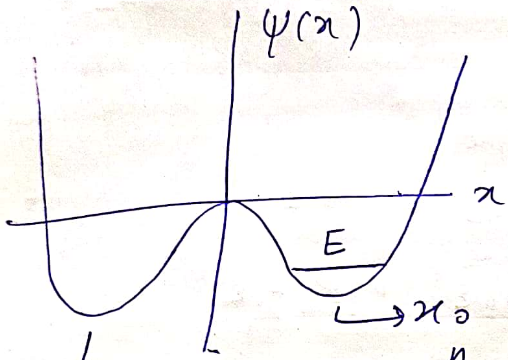
$k_{eff} = \left.\frac{\partial^2\psi(x)}{\partial x^2}\right|_{x_0}$ → +ive

Motion close to a stable equilibrium is like Harmonic Oscillator with $k_{eff} = \left.\frac{\partial^2\psi}{\partial x^2}\right|_{x_0}$

Take a Potential:

$$\psi(x) = -\frac{\alpha}{2}x^2 + \frac{\beta}{2}\underbrace{x^4}_{\text{Quartic term}}$$

$x = 0$, $\psi(x) = 0$, $x \rightarrow -x$, Symmetric



Continuous

{ $x=0$, unstable equilibrium }

$\frac{\partial\psi}{\partial x} = 0$, $-\alpha x + 2\beta x^3 = 0$

∴ Three extrema:

$x = 0, \sqrt{\frac{\alpha}{2\beta}}, -\sqrt{\frac{\alpha}{2\beta}}$

Amplitude of small oscillation: Expand $\psi(x)$ around $x = x_0 = \sqrt{\frac{\alpha}{2\beta}}$

$$k_{eff} = \left.\frac{\partial^2\psi}{\partial x^2}\right|_{x_0} = 2\alpha \quad \text{↳ check ?}$$

$$W = \sqrt{\frac{k_{eff}}{m}} = \sqrt{\frac{2k}{m}}$$

$$\psi(Z) = \frac{k_{eff}}{2}(x-x_0)^2$$

Lenard-Jones potential

Isolated 2 body problem: Can be reduced to 1-body problem in the CM frame in terms of relative

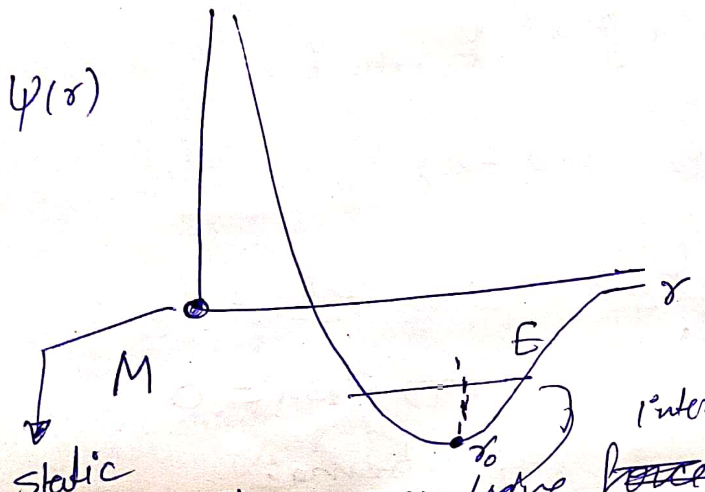
Co-ordinates $\vec{r} = \vec{r_1} - \vec{r_2}$ and $\mu = \frac{m_1 m_2}{m_1 + m_2}$

Simplification: Assume $u >> m$

- $u = M$
- CM falls on the mass M which is static m is moving in field of M:

$$\psi(r) = \left[\left(\frac{r_0}{r}\right)^{12} - 2\left(\frac{r_0}{r}\right)^{6} \right]$$

↳ repulsive ↳ attractive



interactive

- Strong repulsive ~~force~~ as $r \to 0$
- Weakly attractive ~~force~~ interactive for r large

{two classical turning points}

Central field: Energy and ang. mom. are conserved

Assume: $M >> m$ Energy of mass: (m)

{$u \simeq M$ static}

$$= \frac{1}{2} m \dot{r}^2 + \frac{l^2}{2mr^2} + \psi(r)$$

Equivalent one dim problem

Recall: Kepler's problem: $\psi(r) = -K/r$

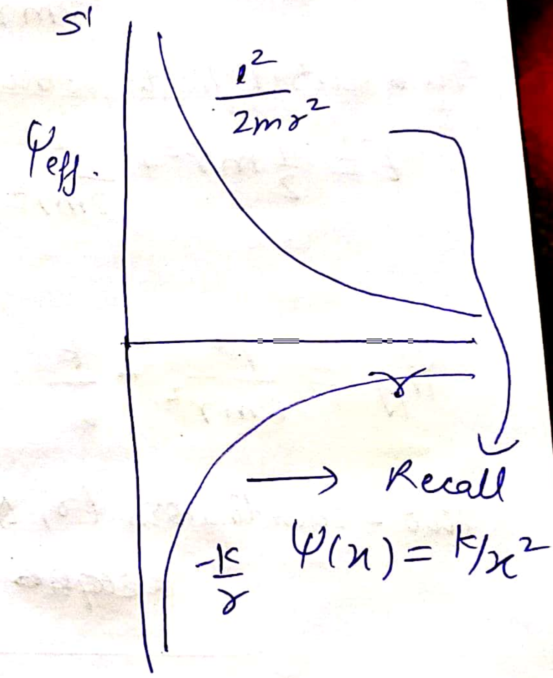
Area covered in time dt: $\frac{1}{2} r^2 d\phi$

$\therefore$ Areal velocity $= \boxed{\frac{1}{2} r^2 \dot{\phi}}$ → constant

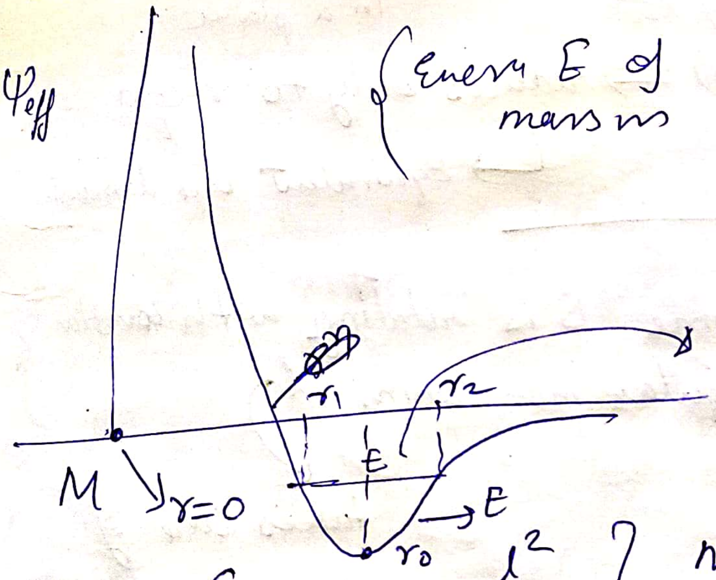
• Equivalent one dim problem: l is constant

$$\boxed{\psi_{eff} = \frac{l^2}{2mr^2} - \frac{k}{r}}$$

$$\psi_{eff} = \frac{l^2}{2mr^2} - \frac{k}{r^2}$$



S' : rotating with constant angular momentum l



$$\left\{ \begin{array}{ll} r \to 0, & \frac{l^2}{2mr^2} \\ r \to \infty & -k/r \end{array} \right\}$$

mass m is always equidistant from M

$$\boxed{r_0 = \frac{l^2}{mk}} \rightarrow \text{Circular orbit}$$

Minimise ψ_{eff}

$$\Rightarrow \left.\frac{\partial \psi_{eff}}{\partial r}\right|_{r_0} = 0$$

⇓ Check

$$\boxed{r_0 = \frac{l^2}{mk}}$$

$$E = \left[\frac{1}{2} m \dot{r}^2 + \frac{l^2}{2mr^2} - \frac{k}{r}\right] = \frac{l^2}{2mr_0^2} - \frac{k}{r_0} = -\frac{k}{2r_0}$$

In this case: CTPs as $\frac{1}{2}m\dot{r}^2 = 0$

So, $\boxed{\frac{l^2}{2mr_0^2} - \frac{k}{r_0} = E}$ → Two real solns' for r_0? an elliptic orbit

07/02

In polar co-ordinates:

$\boxed{\text{Chapter-9 Kleppner}}$

$$T = \frac{1}{2}m\dot{r}^2 + \frac{l^2}{2mr^2}$$

In a central field problem: → l is conserved

$$E = \frac{1}{2}m\dot{r}^2 + \frac{l^2}{2mr^2} - \psi(r)$$

- Areal velocity is constant, Keppler's law
- Motion is confined in a plane

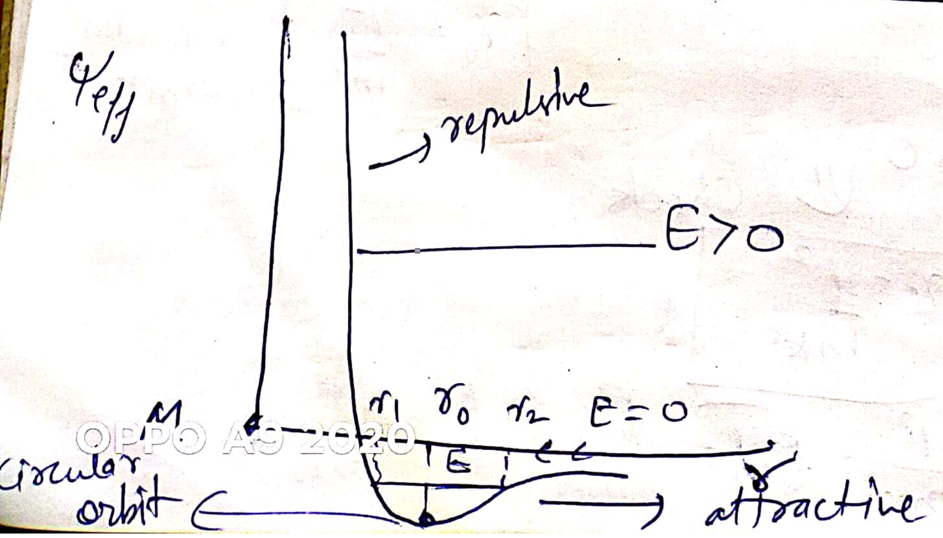
$$\psi_{eff} = \frac{l^2}{2mr^2} - \frac{k}{r}$$

Two-coordinators (r, θ) ⇒ reduces only to r-eqn
~~reequa~~ ↳ Equivalent one dimension

l is there: $\left.\begin{array}{l}\\ l \text{ is conserved}\end{array}\right\}$ $\left\{\text{frame S is rotating with constant angular momentum } l\right\}$ takes care of variation θ

$\boxed{\dot{\phi} \neq 0}$

We need orbit eqn: $r = r(\phi)$



$\psi_{eff} =$

$\frac{\partial \psi_{eff}}{\partial r}$

• m

but mo

$l \Rightarrow$

$r_0 =$

r_0

at

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r,

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E

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m is moving the field of a

$m << M$ (Simplifying assumption)

$\psi_{eff} = \frac{l^2}{2mr^2} - \frac{k}{r}$: Keppler problem

$\left.\frac{\partial \psi_{eff}}{\partial r}\right|_{r_0} = 0$; $r_0 = \frac{l^2}{mk}$

- m is always equidistant from M but moving with a constant angular momentum l

$l \Rightarrow$ Not static

$r_0 = \frac{l^2}{ml}$: Circular motion

$r_0 = \frac{(mr_0^2 w_0)^2}{mk} \Rightarrow \boxed{T^2 \propto r_0^3}$

at ~~r=r0~~ $r = r_0$; $E = \psi_{eff})_{r_0} = \frac{-k}{2r_0}$

Circular orbit: $\frac{1}{2}m\dot{r}^2 = 0$

at a particular energy E we get a min distance r_1 and a min distance r_2 therefore 2 eTPS therefore motion is an ellipse.

$E = \frac{1}{2}m\dot{r}^2 + \frac{l^2}{2mr^2} - \frac{k}{r}$ Turning pts:

S.H.O $\frac{1}{2}m\dot{x}^2 = 0$

$\boxed{\text{Turning pts : } \frac{1}{2}m\dot{r}^2 = 0}$

denoted by "r_0" $E = \frac{l^2}{2mr_0^2} - \frac{k}{r_0}$ —— (1)

$r_0 = \frac{-2mk \pm \sqrt{4m^2k^2 + 8mEl^2}}{4mE}$

If $\sqrt{4m^2k^2 + 8mEl^2} = 0$

$r_0 = \frac{-k}{2E}$: $E = -k/_{2r_0} \longrightarrow$ circular orbit

for elliptic orbit r_0 must have two real values

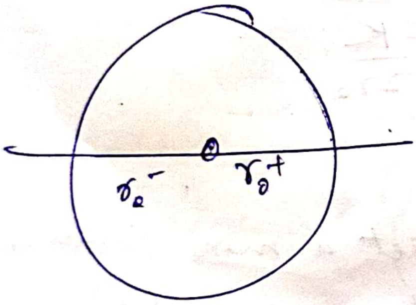
1. $E = 0$, $r_0 = \frac{l^2}{2mk}$ ——— from (1) → parabola
2. $E > 0$: No bound state, hyperbola
3. from (2), when do you have a closed orbit

$\cdot E$ is negative $\quad E = -|E|$

$$r_0 = \frac{2mk \mp \sqrt{4m^2k^2 - 8m|E|l^2}}{4m|E|} \quad \text{—(3)}$$

from (2) with $E = -|E|$

Two solns: r_0^+ and r_0^-



$$r_0^+ + r_0^- = 2a$$

$$r_0^+ + r_0^- = \boxed{\frac{4mk}{4m|E|} = 2a}$$

$$\Downarrow$$

$$E = -|E|$$
$$= -k/_{2a}$$

To calculate:

$$\vec{\nabla}\cdot\left(\frac{\vec{r}}{r^3}\right) = ? = 4\pi\,\delta^3(\vec{r})$$

↳ Revise

$$\vec{\nabla}\cdot\underbrace{\left(-\frac{GM\vec{r}}{r^3}\right)}_{\vec{F}} = -4\pi G\,\delta^3(\vec{r})$$

Gauss's theorem

$$\oint \vec{F}\cdot d\vec{\sigma} = -4\pi G \underbrace{M}_{\text{Mass}}$$

Divergence theorem

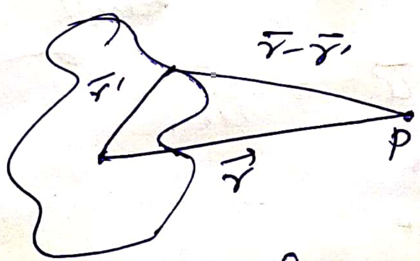
$$\int \vec{\nabla}\cdot\vec{F}\, d\tau = -4\pi G \int \rho\, d\tau \rightarrow \text{elementary volume}$$

$$\therefore \boxed{\vec{\nabla}\cdot\vec{F} = -4\pi G\rho}$$

$$\boxed{\nabla^2\psi = 4\pi G\rho} \qquad \vec{F} = -\vec{\nabla}\psi$$

An arbitrary mass distribution:

$$d\psi_r = -\frac{G\rho\, d\tau}{|\vec{r}-\vec{r}'|}$$



$$\Psi_p = -G\int \frac{\rho(\vec{r}')\,d\tau'}{|\vec{r}-\vec{r}'|} = -G\int \frac{\rho(\vec{r}')\,d\tau'}{\sqrt{r^2+r'^2-2rr'\cos\theta}}$$

(Principle of superposition)

$$\Psi = -G\int \frac{\rho(\vec{r}')\,d\tau'}{r\sqrt{1+\left(\frac{r'}{r}\right)^2-2\left(\frac{r'}{r}\right)\cos\theta'}}$$

$$\frac{r'}{r} \ll 1: \quad \psi_0 = -\frac{G}{r}\int \rho(r')\,d\tau' = -\frac{GM}{r} : \text{Monopole term}$$

(at large distance, can be considered as mass centred at a sphere : monopol)

Show that:

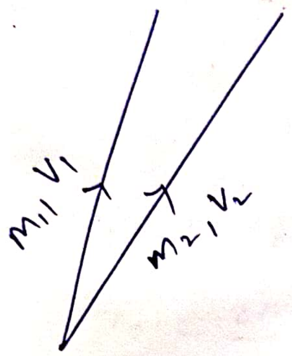
$$= -\frac{GM}{r} - \frac{G}{r^2}\int \rho(\vec{r}')\,d\tau'\,(r'\cos\theta)$$

↳ prove that for a spherically symmetric $\rho(\vec{r}')$: this term vanishes

Chapter-4 Kleppner

Scattering: Elastic Collision: Energy and momentum both conserved.

Lab frame and CM frame



$$\vec{V}_{cm} = \vec{V} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1+m_2}$$

$$\vec{v}_{1c} = \vec{v}_1 - \vec{V} = \frac{m_2(\vec{v}_1 - \vec{v}_2)}{m_1+m_2}$$

$$= \frac{m_2\vec{u}}{m_1+m_2}$$

$$m_1\vec{v}_{1c} = \frac{m_1 m_2}{m_1+m_2}\vec{u}$$ rel. velocity

$$= \mu\vec{u} = \vec{P}_{1c}$$

Ily. $m_2\vec{v}_{2c} = -\mu\vec{u} = \vec{P}_{2c}$

$$\boxed{\vec{P}_{1c} + \vec{P}_{2c} = 0}$$

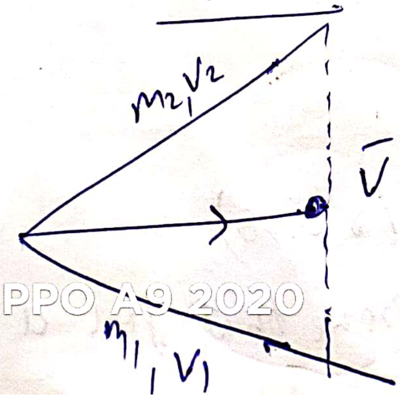
Lab frame:

$$m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1+m_2)\vec{V}$$

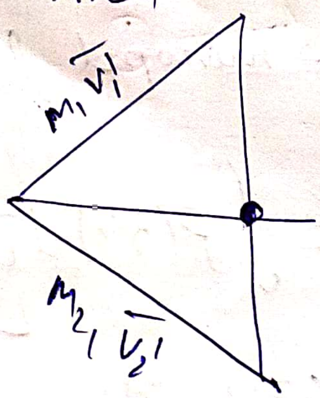
10/02

from the Lab frame:

Initial:



final

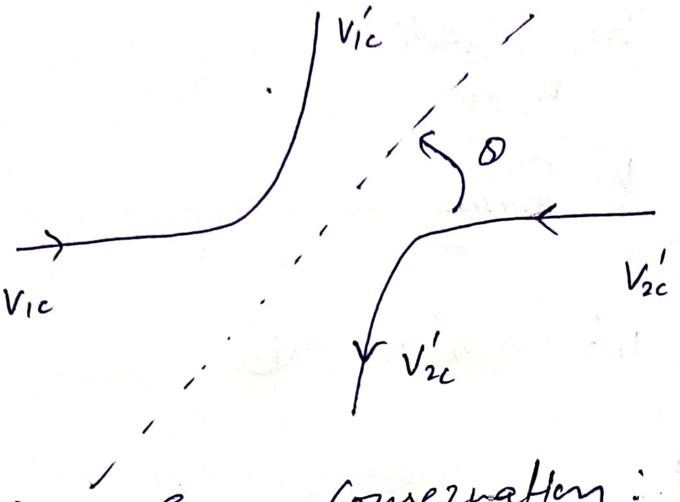


- v_1, v_2, v_1', v_2' ↳ complicated relationship
- Not in the same plane

Azimuthal symmetry. ϕ does not appear

In the CM frame:

$\vec{P}_{1c} + \vec{P}_{2c} = \vec{P}'_{1c} + \vec{P}'_{2c} = 0$ plane of scattering

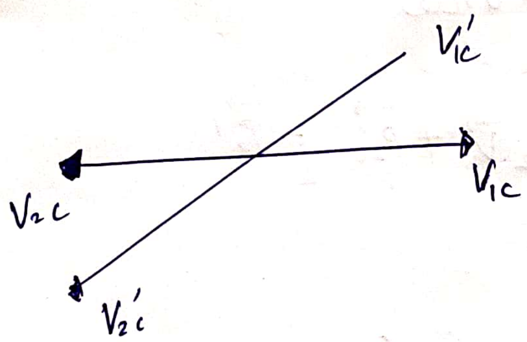


• Energy Conservation: KK (Chapter-4)

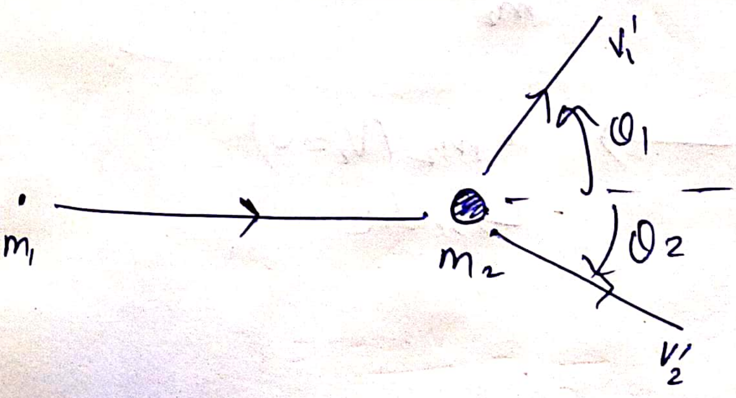
$v_{1c} = v'_{1c}$ and $v_{2c} = v'_{2c}$

In the CM frame:

• Velocity vectors simply rotate



Assume: M_2 is at rest: $V = \frac{m_1 \vec{v}_1}{m_1 + m_2}$



Lab frame:

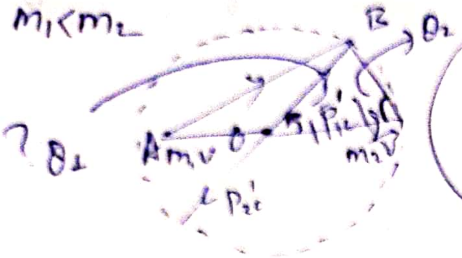
Experimental Situation:

Momentum Diagram: Draw a circle of radius;

$|P_{ic}'|$

$$m_1 \vec{V} = \frac{m_1^2 V_1}{m_1 + m_2}$$

$m_1 < m_2$



m_2 is static

$V_2 = 0$

$$\vec{V} = \frac{m_1 \vec{V_1}}{m_1 + m_2}$$

$$m_2 \vec{V} = \frac{m_1 m_2}{m_1 + m_2} \vec{V_1} = \mu \vec{V_1} \quad \text{--- (1)}$$

$$m_1 \vec{v_{1c}} = \mu \vec{u}_{rel} = \mu \vec{v},$$

$$\boxed{|\vec{v}_{1c}| = |V_{1c}'|}$$ → Energy conservation

$$\vec{P_{1c}'} = \mu \vec{u} = \mu \vec{V_1} \quad \text{--- (2)}$$

$$P_{1c} + P_{2c} = P_{1c}' + P_{2c}' = 0$$

Compare (1) and (2): C lies on the circle

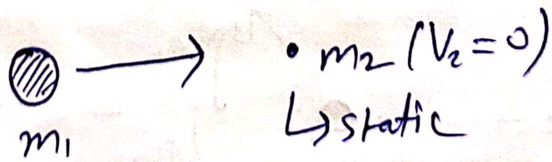
$$\vec{AB} = m_1 \vec{V} + \vec{P_{1c}'} = \vec{P_1}'$$

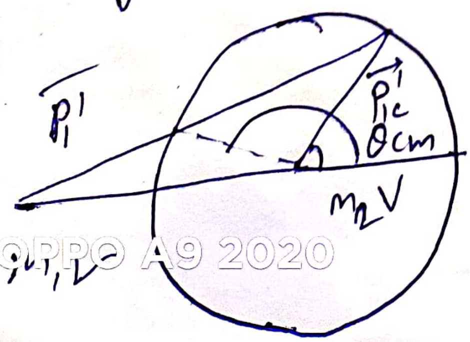
what is $\vec{BC} = m_2 \vec{V} - \vec{P_{1c}'} = m_2 \vec{V} + \vec{P_{2c}'} = \vec{P_2}'$

From this diagram: show:

$$\tan \theta_1 = \frac{P_{1c}' \sin \theta_{cm}}{m_1 V + P_{1c}' \cos \theta_{cm}} = \frac{\sin \theta_{cm}}{\frac{m_1}{m_2} + \cos \theta_{cm}}$$

If $m_1 > m_2$:





I have $\theta_{cm}^{(1)} < \pi/2$: forward scattering

$\theta_{cm}^{(2)} > \pi/2$: backward scattering