Phase II of Deletion in R-B Trees

Dfixup(T, x) // x is one more unit of black over and above its own normal color.

(RB, BB)

Only other possible violations of R-B properties in are the following

(i) x maybe root node

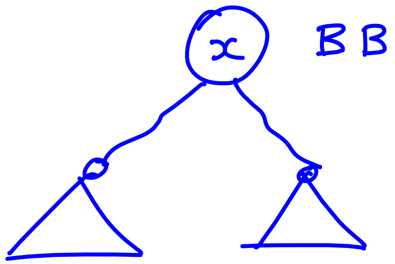
(ii) x may have red parent

<u>Case I:</u>

If x.col == red then x = black

If x == T. root then x = black

Restores RB properties



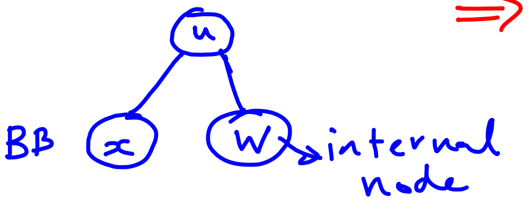
Drop an extra black from the root .

In the following cases x is BB (doubly black)

x is not the root node

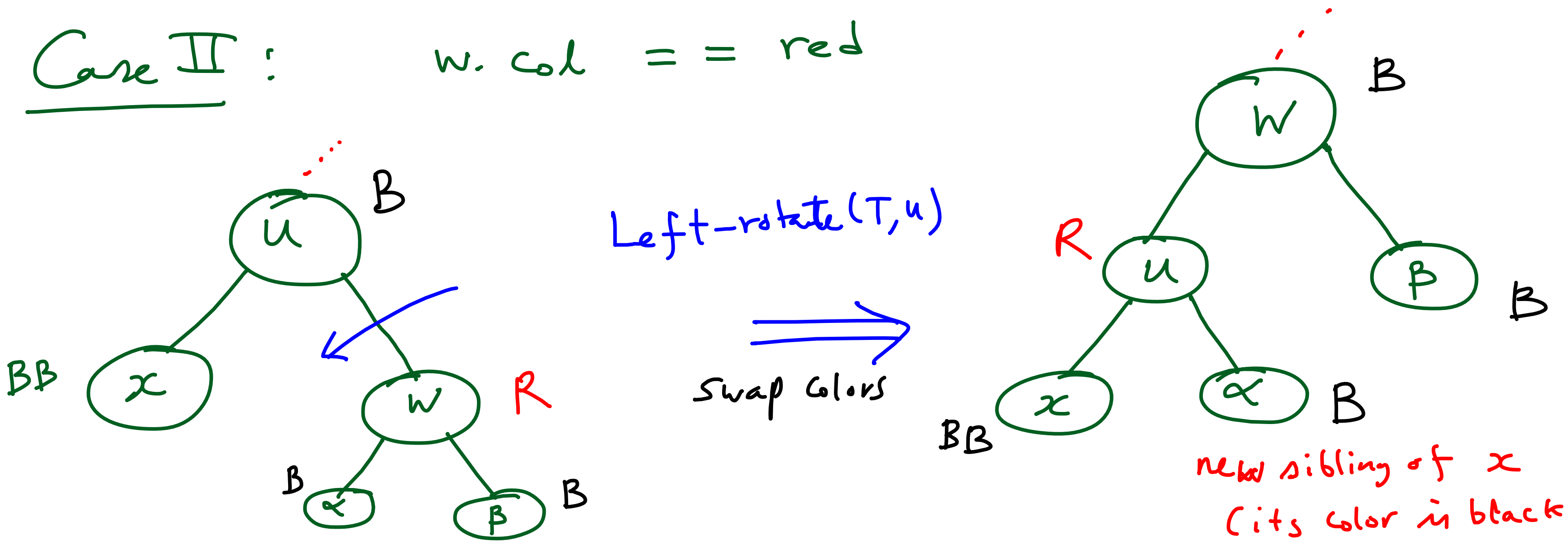
$\Rightarrow$ x has a parent say, u

$\Rightarrow$ x has a sibling w



BB count of x $\Rightarrow$ w is internal node

Case II : $w.col == red$



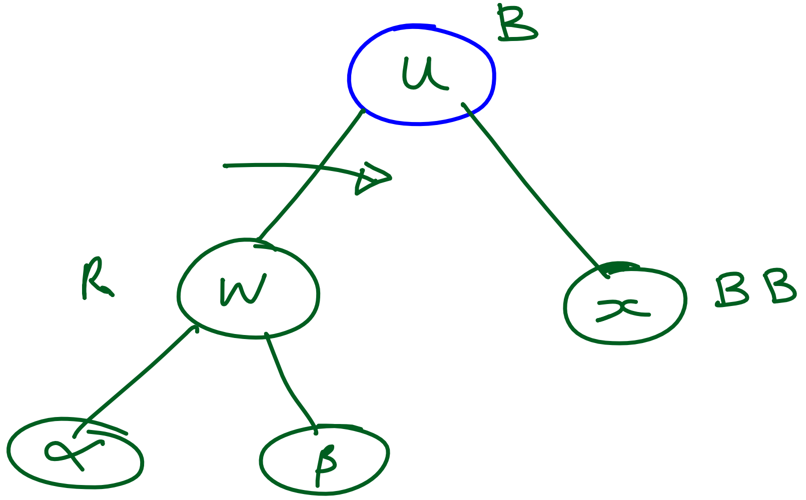
new sibling of x (its color is black)
new w

Exercise all path properties are maintained

(for ex:
$u - x \twoheadrightarrow \ell$
$u - \alpha \twoheadrightarrow \ell_1$
will have same no. of black nodes)

Symmetrically

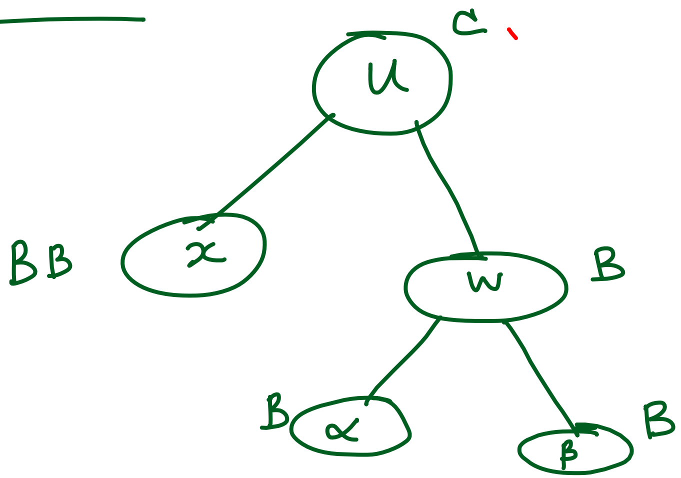
[symmetric version of case 2]

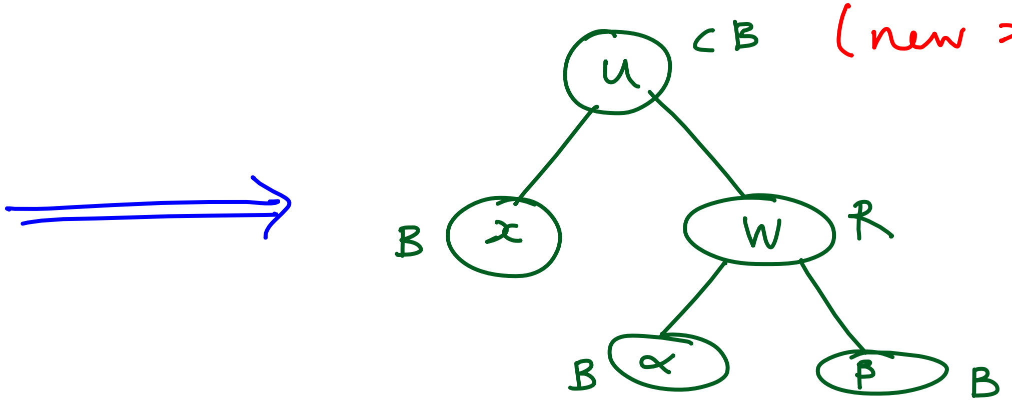


$W.col == black$

$(c+1 \equiv cB)$

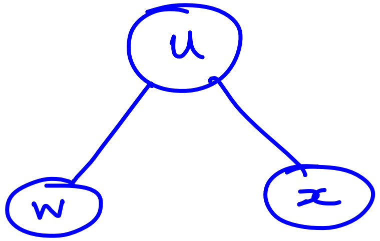
Case III





(new x)

Symmetrically

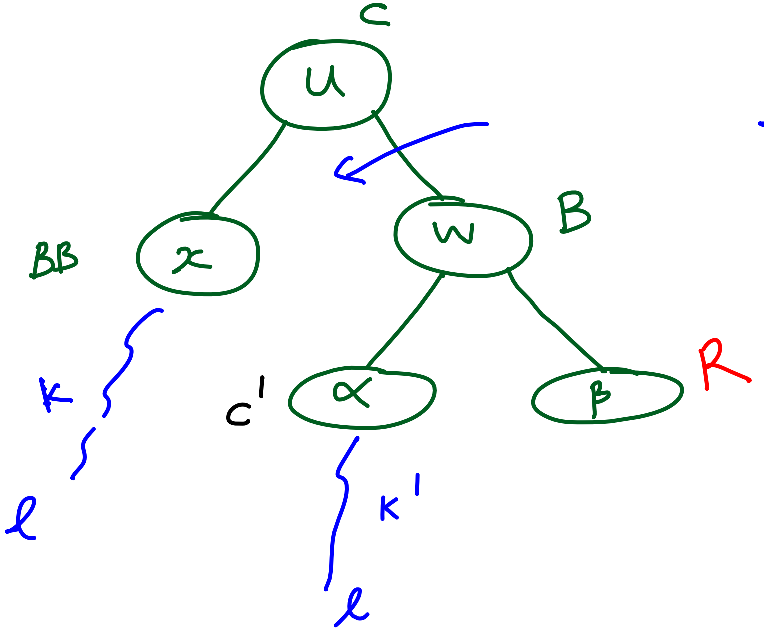


x is right child of its parent

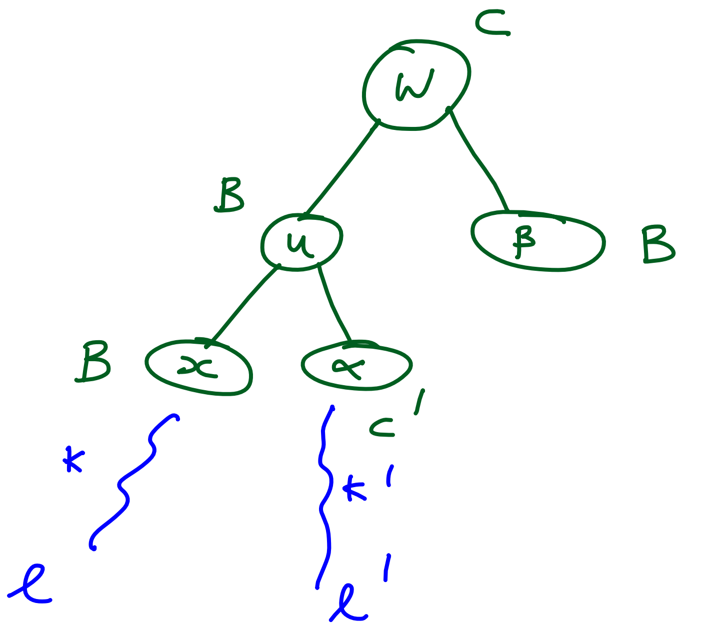
W.col == black, at least one of W's children is Red

Case IV

(c is considered as 0 if it is red and 1 if it is black)



$\Longrightarrow$

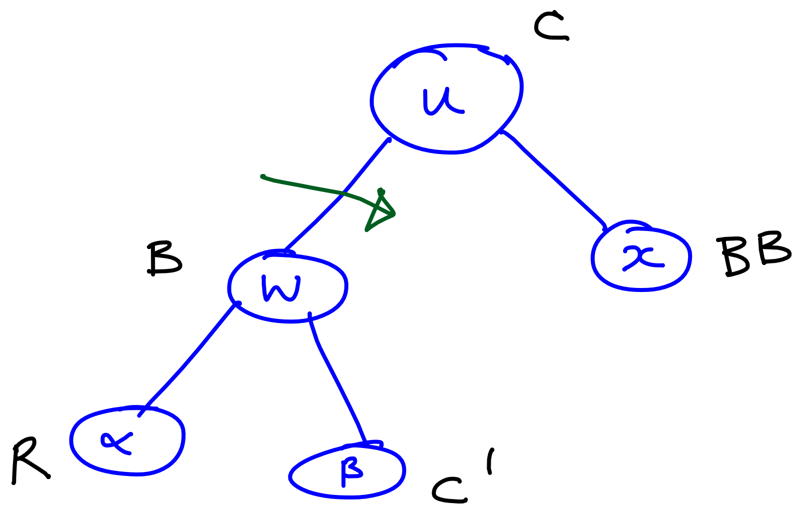


$$2 + k = 1 + c' + k'$$
$$1 + c' + k'$$

$$c + 2 + k$$
$$= c + 1 + k' + c'$$
$$\Rightarrow 1 + k = k' + c'$$

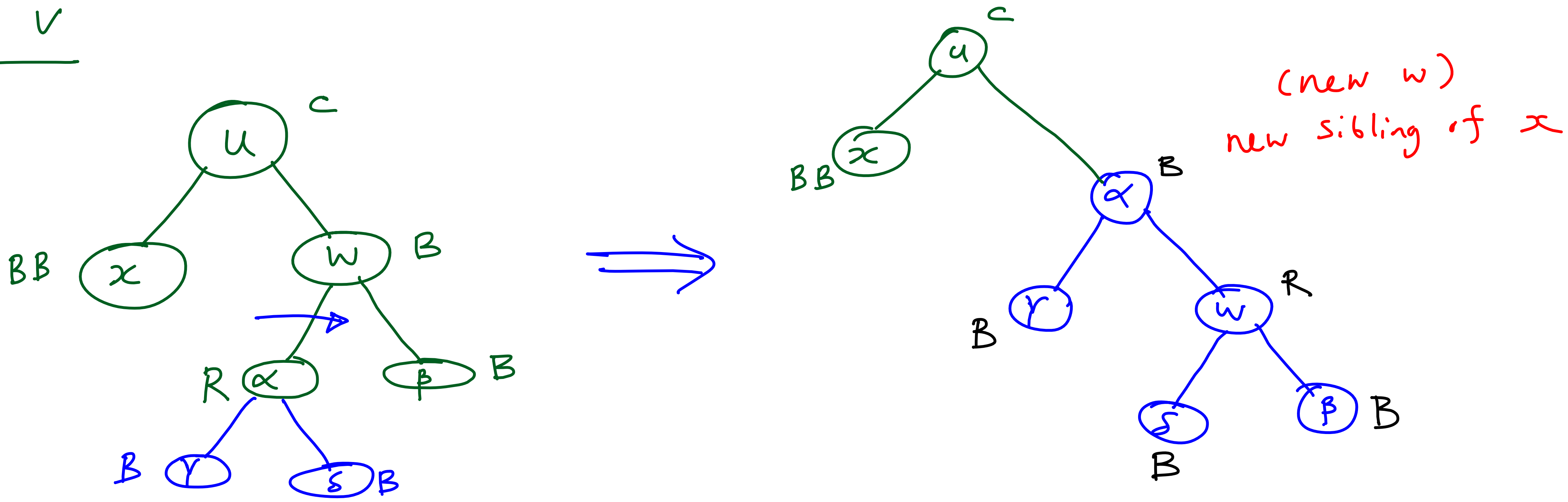
Finishes the fixing up (for Case IV)

Symmetric to Case IV

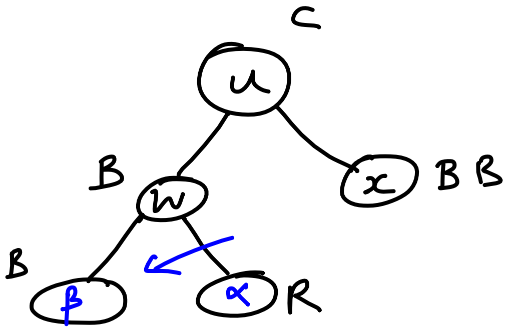


Exercise Show that right rotation (T, u) finishes this case. (Restores RB properties of T)

Case V



Symmetrically



is handled

(Ex: fill in the details)

Termination

Case I : Terminates immediately

Case IV : Terminates in the same call

Case V : Next call to Dfixup ends up in case-IV.

Case III : Next call to Dfixup is from a node with height more than height of x.

Case II : If next call is case III $\longrightarrow$ call following this is for red x (case I)

If next call is case IV $\longrightarrow$ terminates

If next call is case V $\longrightarrow$ call following that is in case IV $\longrightarrow$ terminates.

Time Complexity

$O(h)$ $h \equiv$ height of T.

Correctness

Invariant shown on page 1 is maintained to all calls of Dfixup(T, x)

Procedure terminates in case I or Case IV

T has all R-B properties

[Exercise: verify these]

(we have verified them at least informally while justifying various cases)

```
Dfixup(T, x)
  If (x.col == red) or (x == T.root)
        x.col = black
        return

  w = sibling(x)
  u = x.p
  If w.col == red
        if w == u.left
              Right-rotate(T, u)
        else Left-rotate(T, u)
        swapcol(w, u)
  elif w.left.col == black and w.right.col == black
        w.col = red
        x = u
  elif w == u.left and w.left.col == red
        Right-rotation(T, u)
        swapcol(u, w)
        w.left.col = black
        return
```

```
elif   w == u.right  and  (w.right).col == red
           Left-rotate(T, u)
           swapcol(w, u)
           (w.right).col == black
           return

elif   w == u.right        //(w.left).col = red

           swapcol(w, w.left)
           Right-rotate(T, w)
                                //(w.right).col = red
elif   w == u.left
           swapcol(w, w.right)
           Left-rotate(T, w)


Dfixup(T, x)
```