Department of Mathematics and Statistics, I.I.T. Kanpur

MTH101A - Quiz 2B Examination - 20.10.2011

Maximum Marks: 20 **Time: 17:30-18:00 hrs**

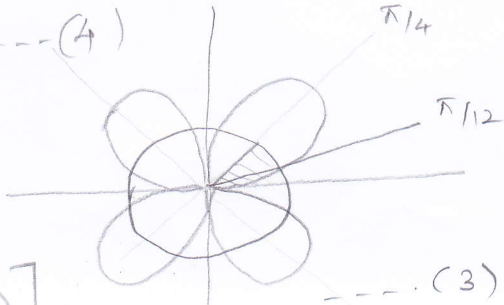
NAME : ——————— Roll No. : ———— Section : ———

1 Sketch the curves $r = -\sin(2\theta)$ and $r = 1/2$. Further, find the area of the region that is inside the curve $r = -\sin(2\theta)$ and also inside the circle $r = 1/2$. [10]

2 Let C be the (infinite) cylinder generated by revolving the line $y = x + \sqrt{6}$ about the line $y = x$. Let S be the solid sphere $x^2 + y^2 + z^2 \leq 4$. Find the volume of the portion of the sphere which lies inside the cylinder C. [10]

1. pt of intersection: $\sin 2\theta = 1/2 \Rightarrow 2\theta = \pi/6 \Rightarrow \theta = \pi/12$ --- (1)

$$\text{Area} = 8\left[\frac{1}{2}\int_0^{\pi/12}(\sin 2\theta)^2 + \frac{1}{2}\int_{\pi/12}^{\pi/4}\frac{1}{4}\,d\theta\right] \text{ --- (4)}$$



--- (3)

$$= 8\left[\frac{1}{2}\int_0^{\pi/12}\frac{1-\cos 4\theta}{2} + \frac{1}{2}\cdot\frac{1}{4}\left(\frac{\pi}{4}-\frac{\pi}{12}\right)\right]$$

$$= 8\left[\frac{1}{2}\int_0^{\pi/12}\frac{1}{2} - \frac{1}{2\cdot 2}\int_0^{\pi/12}\cos 4\theta + \frac{1}{8}\cdot\frac{\pi}{6}\right]$$

$$= \frac{\pi}{6} + \frac{\pi}{6} - \frac{8}{2\cdot 2\cdot 4}\left[\sin 4\cdot\pi/12 - \sin 0\right]$$

$$= \frac{\pi}{3} - \frac{1}{2}\sin\frac{\pi}{3}$$

$$= \pi/3 - 1/2\cdot\sqrt{3}/2 \text{ --- (2)}$$

Department of Mathematics and Statistics, I.I.T. Kanpur

MTH101A - Quiz 2A Examination - 20.10.2011

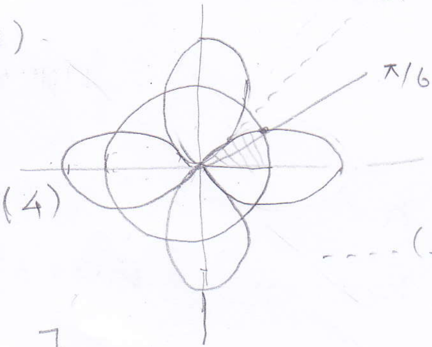
Maximum Marks: 20 **Time: 17:30-18:00 hrs**

NAME : ____________ Roll No. : ________ Section: ______

1 Sketch the curves $r = -\cos(2\theta)$ and $r = 1/2$. Further, find the area of the region that is inside the curve $r = -\cos(2\theta)$ and also inside the circle $r = 1/2$. [10]

2 Let C be the (infinite) cylinder generated by revolving the line $y = -x + \sqrt{6}$ about the line $y = -x$. Let S be the solid sphere $x^2 + y^2 + z^2 \leq 4$. Find the volume of the portion of the sphere which lies inside the cylinder C. [10]

1. $\cos 2\theta = 1/2 \Rightarrow 2\theta = \pi/3 \Rightarrow \theta = \pi/6$ ---- (1)



---- (3)

$$\text{Area} = 8\left[\frac{1}{2}\int_0^{\pi/6}\left(\frac{1}{2}\right)^2 d\theta + \frac{1}{2}\int_{\pi/6}^{\pi/4}(\cos 2\theta)^2 d\theta\right] \quad ....(4)$$

$$= 8\left[\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{\pi}{6} + \frac{1}{2}\int_{\pi/6}^{\pi/4}\frac{1+\cos 4\theta}{2}\,d\theta\right]$$

$$= 8\left[\frac{1}{8}\cdot\frac{\pi}{6} + \frac{1}{4}\left(\pi/4 - \pi/6\right) + \frac{1}{4}\int_{\pi/6}^{\pi/4}\cos 4\theta\, d\theta\right]$$

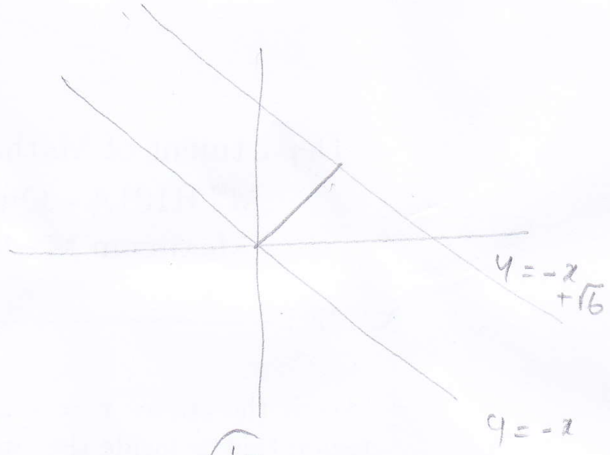
$$= \frac{\pi}{6} + \frac{\pi}{6} + \frac{8}{4\cdot 4}\left[\sin 4\cdot\frac{\pi}{4} - \sin 4\cdot\pi/6\right]$$

$$= \frac{\pi}{6} + \pi/6 - \frac{1}{2}\cos 30$$

$$= \frac{\pi}{6} + \pi/6 - \frac{1}{2}\cdot\frac{\sqrt{3}}{2}$$

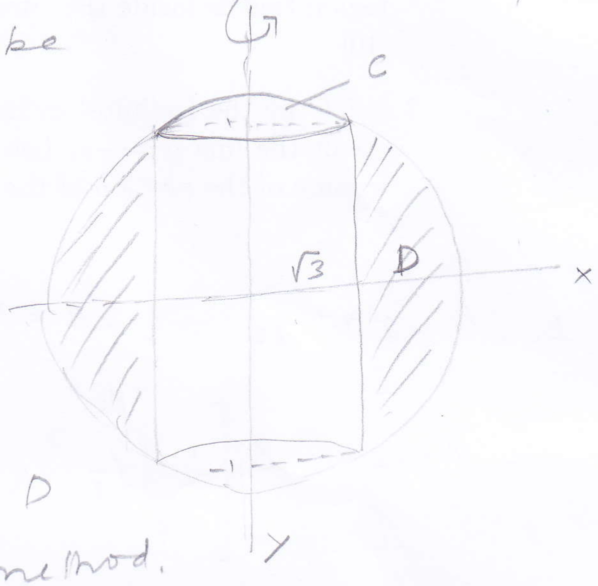
$$= \frac{\pi}{3} - \frac{\sqrt{3}}{4}. \quad ---- (2)$$

2. The distance between $(0,0)$ and the line $y = -x + \sqrt{6}$



$= \dfrac{\sqrt{6}}{\sqrt{2}} = \sqrt{3}$. ---- (2)

The axis of the cylinder can be considered as the y-axis.



Volume of the whole sphere is $\dfrac{4}{3}\pi r^3 = \dfrac{32}{3}\pi$. --(2)

we find the volume of the region generated by revolving D around y-axis by washer method.

The volume $= \displaystyle\int_{-1}^{1} \pi \left(f(y)^2 - 3 \right) dy$ ------(4)

$= \displaystyle\int_{-1}^{1} \pi \left(4 - y^2 - 3 \right) dy = \dfrac{4\pi}{3}$.

The required volume $= \dfrac{32}{3}\pi - \dfrac{4\pi}{3} = \dfrac{28\pi}{3}$ ----(2)

Alternate sol:

The required volume $=$ vol(cylinder) $+ 2$ volume (C).

$= 3\pi \cdot 2 + 2$ volume (C) ----(2)

Volume (C) $= \displaystyle\int_{1}^{2} \pi f(y)^2 = \int_{1}^{2} \pi \left(4 - y^2 \right) dy$. ---(4)

$= \displaystyle\int_{1}^{2} 4\pi - \int_{1}^{2} \pi y^2 dy = 4\pi - \left[\pi \frac{y^3}{3} \Big|_{1}^{2} \right]$

$= 4\pi - \pi \dfrac{8}{3} + \dfrac{\pi}{3} = \dfrac{12\pi - 8\pi + \pi}{3} = \dfrac{5}{3}\pi$

~~At~~ The required volume $= 6\pi + \dfrac{10}{3}\pi = 28\pi/3$ ---(2)