Indian Institute of Technology, Kanpur
Department of Earth Sciences

ESO213A: Fundamentals of Earth Sciences

Lecture 18. Concept of Strain

Santanu Misra
Department of Earth Sciences
Indian Institute of Technology, Kanpur
smisra@iitk.ac.in • *http://home.iitk.ac.in/~smisra/*

Experimental Rock Deformation Laboratory

Aims of this lecture

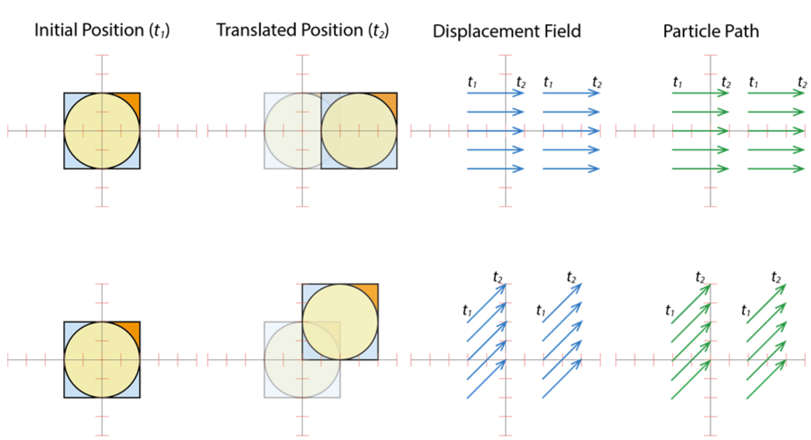
- Overview of Deformation (strain) – displacement, velocity vectors, particle paths
- Homogeneous and Heterogeneous Deformation
- Strain in 1D, 2D and 3D
- Strain Ellipse, Ellipsoid and description
- Coaxial (Pure Shear) and Non-coaxial (Simple Shear) Deformation

What is deformation?

- A change in form and/or shape.
- Deformation is the transformation from an initial to a final geometry by means of rigid body translation, rigid body rotation, distortion and/or volume change (dilation).
- Deformation relates the positions of particles before and after the deformation history, and the positions of points before and after deformation can be connected with vectors – Displacement Vectors.
- The actual path that each particle follows during the deformation history is referred to as Particle Paths.

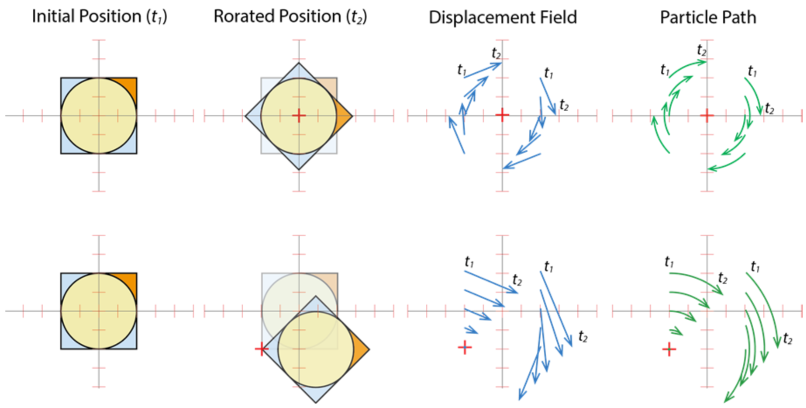
Rigid Body Translation

Rigid Body Translation moves every particle in the rock-mass in the same direction and the same distance. The displacement field consists of parallel vectors of equal length.



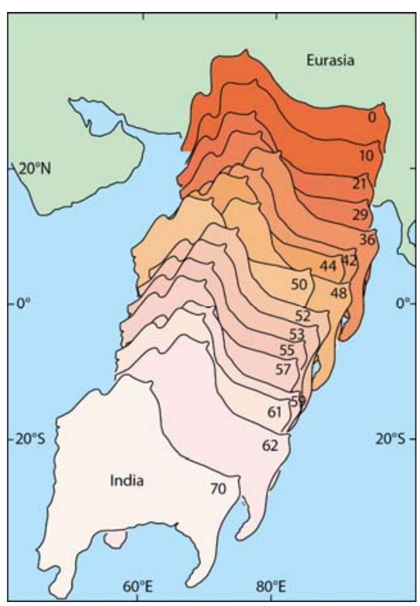
Rigid Body Rotation

Rigid Body Rotation involves a uniform physical rotation of a rock volume relative to an external coordinate system and along a rotation axis (+).



Rigid Body Motion of India

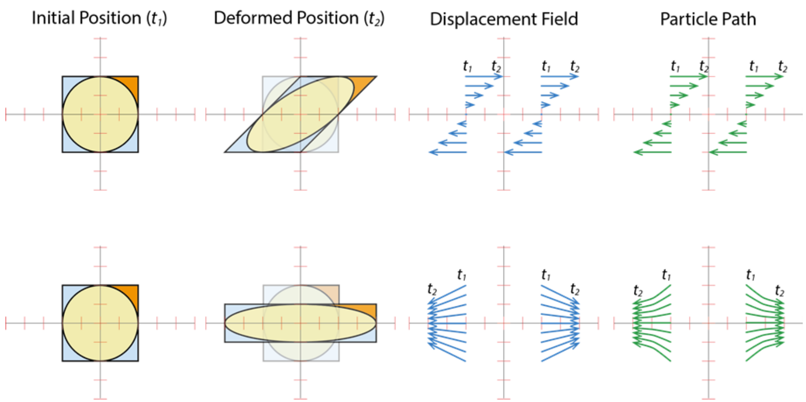
Think of the travel of India from the Southern Hemisphere to its present position.



Patrait & Achache, Nature (1984)

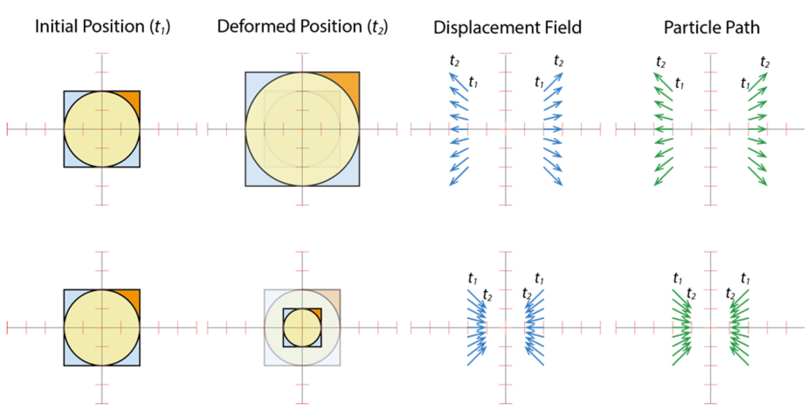
Distortion (Strain) – Constant Volume

Any non-rigid change in shape, without change in volume, is referred to as **Constant Volume Strain**. The particles in a rock have changed positions relative to each other involving either or both Translation and Rotation.



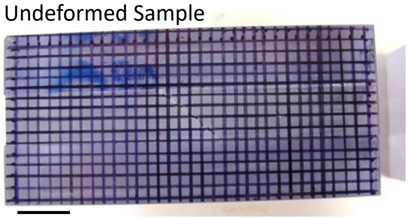
Distortion (Strain) – Volume Deformation

Even if the shape of a rock-mass is unchanged, it may shrink or expand during deformation. We therefore have to add volume change (area change in two dimensions) for a complete description of deformation. Volume change, also referred to as *dilation (positive or negative)*, is commonly considered to be a special type of strain, called **volumetric strain**.



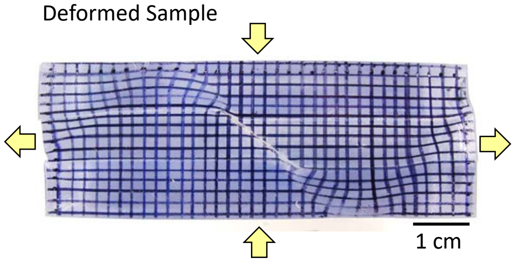
A few examples

Undeformed Sample

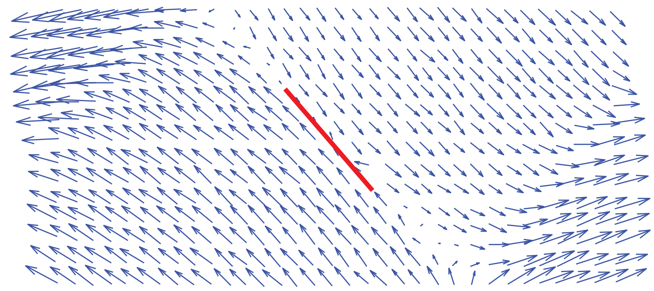


1 cm

Deformed Sample

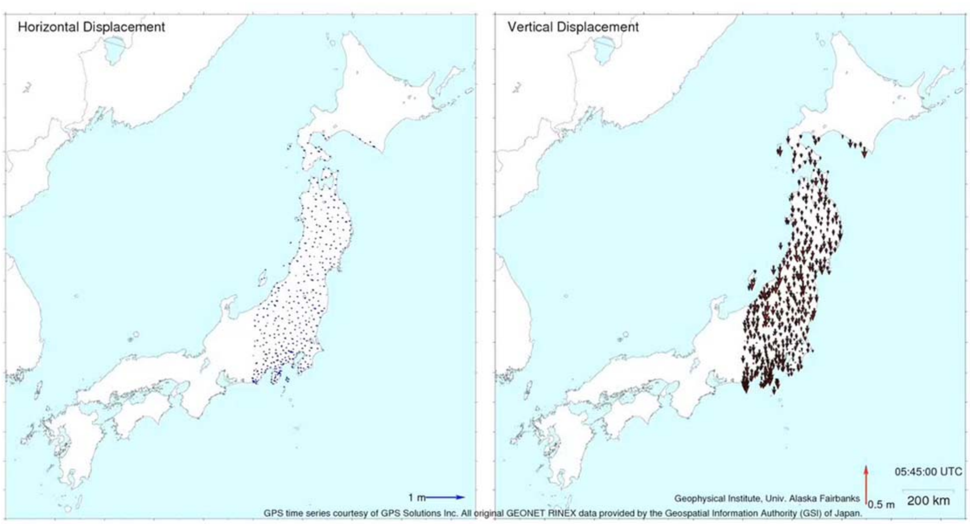


Displacement Field



Unpublished experiments: S. Misra

A few examples



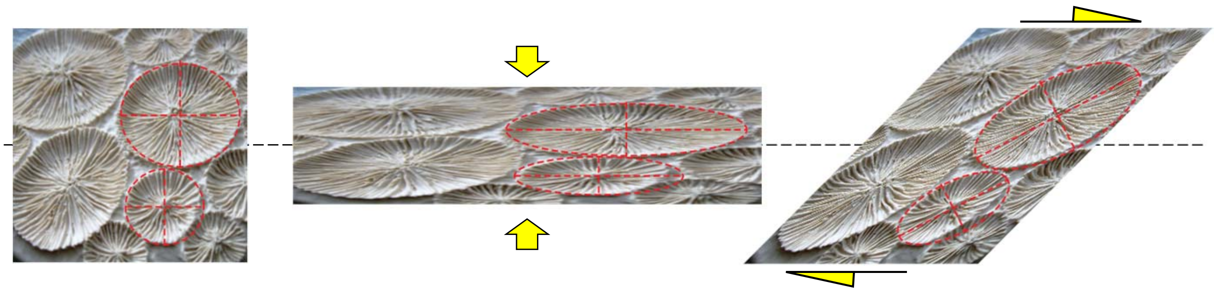
Horizontal and Vertical ground motion of 2011 Tōhoku Earthquake, Japan

Homogeneous Deformation

A given strain may have accumulated in an infinite number of ways.

Where the deformation applied to a rock volume is identical throughout that volume, the deformation is **Homogeneous**. [A circle will change to an ellipse]

For homogeneous deformation, *originally straight and parallel lines will be straight and parallel also after the deformation*. Also, *identically shaped and oriented objects will also be identically shaped and oriented after the deformation.*



Undeformed — Deformed (Pure Shear) — Deformed (Simple Shear)

Heterogeneous (inhomogeneous) Deformation

If the deformation is not homogeneous, it is then Heterogeneous.

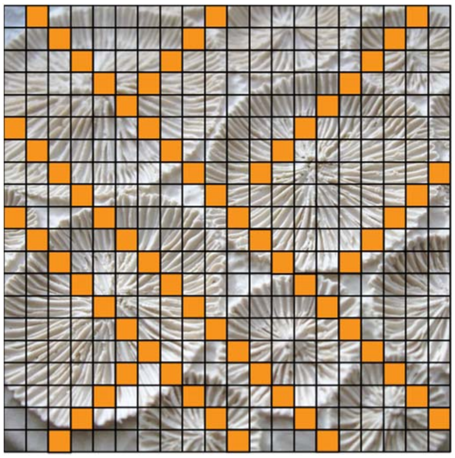
Where the deformation applied to a rock volume is **NOT** identical throughout that volume, the deformation is **Heterogeneous**.

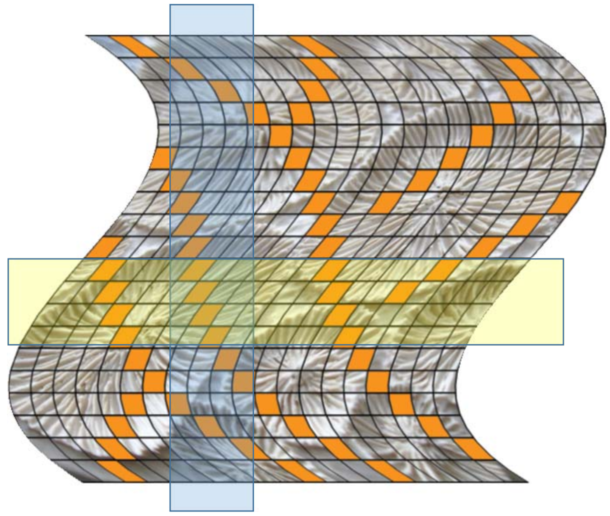
For Heterogeneous deformation, *originally straight and parallel lines will **NOT** be straight and parallel after the deformation*. Also, *identically shaped and oriented objects will also **NOT** be identically shaped and oriented after the deformation.*

Undeformed — Deformed — Deformed

Homogeneous and Heterogeneous Deformation

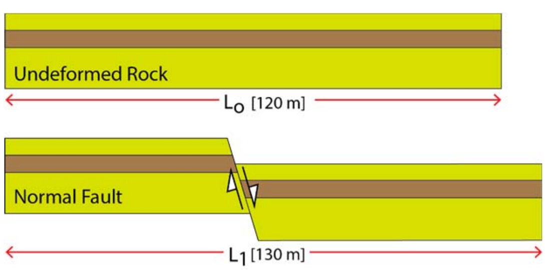
A deformation that is homogeneous on one scale may be considered heterogeneous on a different scale and direction.





Strain in One Dimension

LINEAR STRAIN



- **Elongation**: the ratio of the change in length to the initial length

$$\varepsilon = (L_1 - L_0)/L_0 = 0.083$$

- **Stretch**: the ratio of deformed and undeformed length

$$s = L_1/L_0 = (1+\varepsilon) = 1.083$$

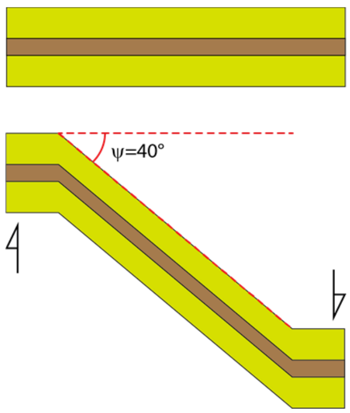
- **Quadratic Elongation**: the square of stretch

$$\lambda = s^2 = (1+\varepsilon)^2 = 1.174$$

These are used for different purposes in strain analysis; however, they are not independent

Strain in One Dimension

ANGULAR STRAIN



Angular shear, ψ, which describes the change in angle between two originally perpendicular lines in a deformed medium.

The shear strain γ is the tangent to the angular shear :

$\gamma = \tan \psi = 0.84$

Strain in Two Dimension

In two dimensions, during homogeneous deformation an imaginary circle on the surface of the deforming rock takes a shape of an ellipse.

The ellipse is known as **Strain Ellipse** and describes the amount of elongation/shortening in any direction in a plane of homogeneous deformation

A circle of unit radius (but it can be of any size) and flattened vertically parallel to the coordinate axis is homogeneously deformed into an ellipse with two major axes which, initially, were diameters of the circle. This **strain ellipse** is a two-dimensional, graphical concept to visualize the amount of ***linear*** and ***angular*** strain involved in the deformation of a rock.

The longest and shortest radii, known as the **Principal Strain Axes,** define the strain ellipse. In the considered coordinate-parallel flattening, these axes are vertical (short) and horizontal (long)

Strain in Two Dimension

Description of Strain Ellipse – Dimension and orientation

One needs only three parameters to describe the strain ellipse – the dimension (short and long) and the orientation of the principal strain axes.

The strain ellipse is conveniently described by a long (*X*) and a short (*Z*) axis (the principal strain axes).

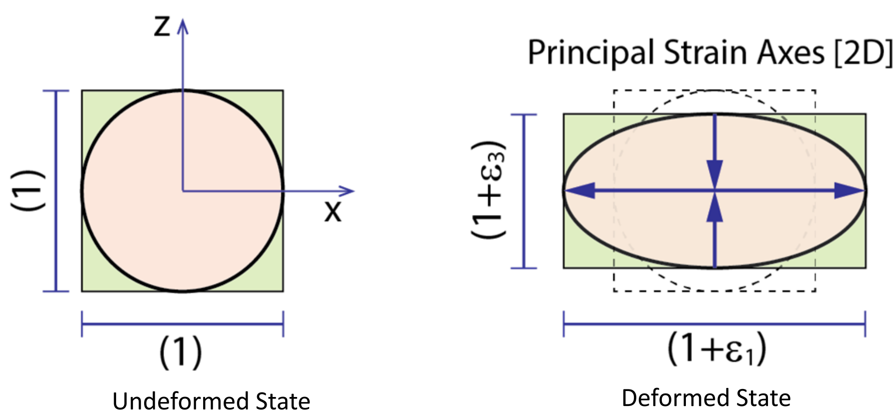
The length of the LONG principal strain axis (stretch axis) : $(1+\varepsilon_1) = \sqrt{\lambda_1}$

The length of the SHORT principal strain axis (shortening axis) : $(1+\varepsilon_3) = \sqrt{\lambda_3}$

The *orientation* is usually the anticlockwise angle between the abscissa and the longest principal strain axis

Strain in Two Dimension

Homogeneous Deformation of a circle into an ellipse with i) no area dilation and ii) no rotation



Strain in three-dimensions

- Strain in three-dimensions is very much analogous to strain in two-dimensions [we need to add Y].
- Homogeneous deformation without volume change in three-dimensions can be described as the change in shape of an imaginary or a material sphere.
- The sphere becomes an ellipsoid whose shape and orientation describe the strain. The equation describing this ellipsoid is:

$$\frac{x^2}{(1+\varepsilon_1)^2} + \frac{y^2}{(1+\varepsilon_2)^2} + \frac{z^2}{(1+\varepsilon_3)^2} = 1$$

- The three axes of the strain ellipsoid are the maximum (X), intermediate (Y) and minimum (Z) **principal strain axes.** They are also mutually perpendicular to each other.

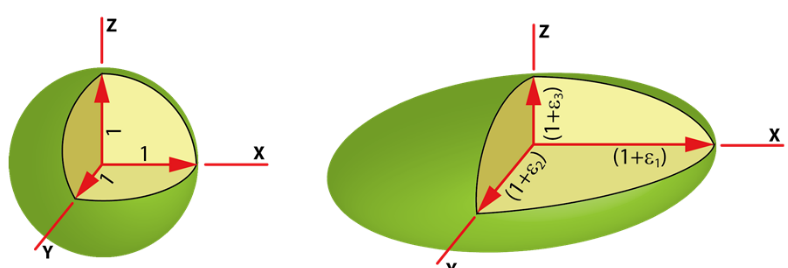
$X = 1 + \varepsilon_1 = \sqrt{\lambda_1}$ *[Stretching axis]*

$Y = 1 + \varepsilon_2 = \sqrt{\lambda_2}$ *[Intermediate axis]*

$Z = 1 + \varepsilon_3 = \sqrt{\lambda_3}$ *[Shortening axis]*

Strain in three-dimensions

Homogeneous deformation of a sphere



$X = Y = Z = 1$

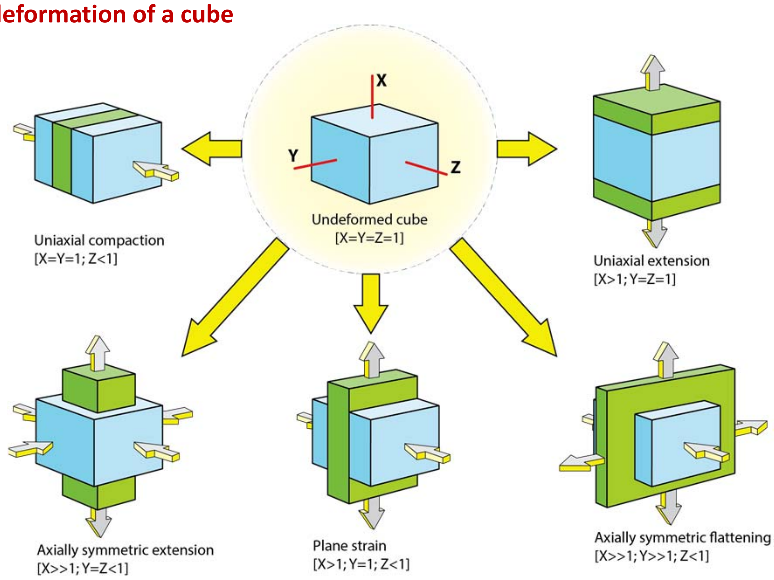
$X = 1 + \varepsilon_1 = \sqrt{\lambda_1}$ *[Stretching axis]*

$Y = 1 + \varepsilon_2 = \sqrt{\lambda_2}$ *[Intermediate axis]*

$Z = 1 + \varepsilon_3 = \sqrt{\lambda_3}$ *[Shortening axis]*

Strain in three-dimensions

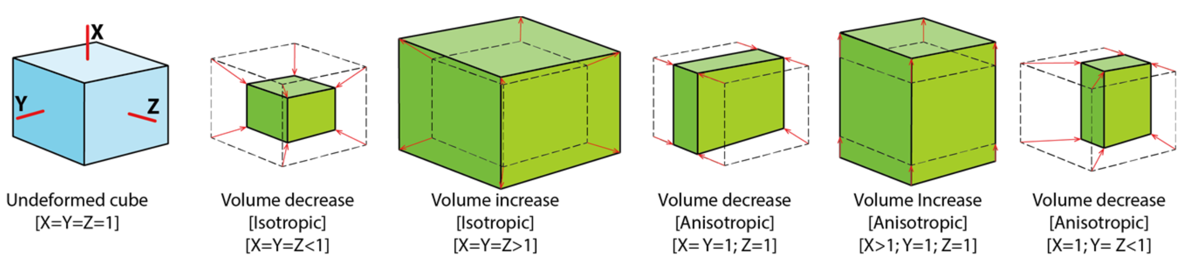
Homogeneous deformation of a cube



Strain in three-dimensions

The Volume Change

- A pure volume change or volumetric strain of an object is given by ***$\Delta=(V - V_0)/V_0$***, where ***V_0*** and ***V*** are volumes of the object before and after the deformation, respectively.
- The volume factor ***Δ*** is thus negative for volume decrease and positive for volume increase.
- ***Isotropic volume change*** happens when ***$X=Y=Z>1$*** (volume increase) or ***$X=Y=Z<1$*** (volume decrease)
- ***Anisotropic volume change*** happens when ***$XYZ\neq1$*** (also ***$X=Y=Z \neq 1$***)and two or all of ***X***, ***Y*** and ***Z*** are different.



Pure and Simple Shear

If we approximate, the overall strain is homogeneous and can be discussed in two dimensions (*plane strain*), where no area dilatation has taken place, then, there are two end members -

PURE SHEAR (coaxial deformation)

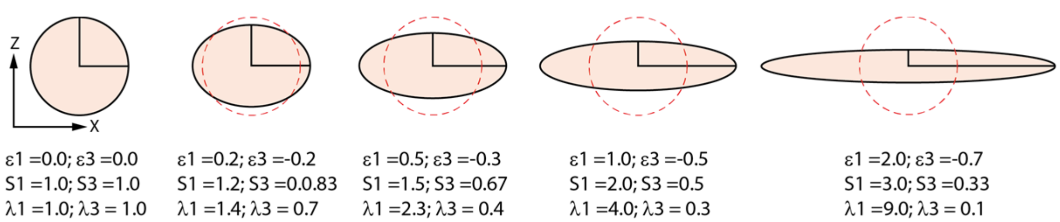
SIMPLE SHEAR (non-coaxial deformation)

The pure and Simple shear are strain regimes and they are the end members to better understand and approximate natural deformation. In nature, ***there is nothing Pure and nothing Simple***.

Strain in Two Dimension

Coaxial Deformation: The principal strain axes remain parallel to the same material lines throughout straining (i.e. the axes of the finite and infinitesimal strain ellipses remain parallel throughout the deformation). The **coaxial** deformation is **irrotational**.

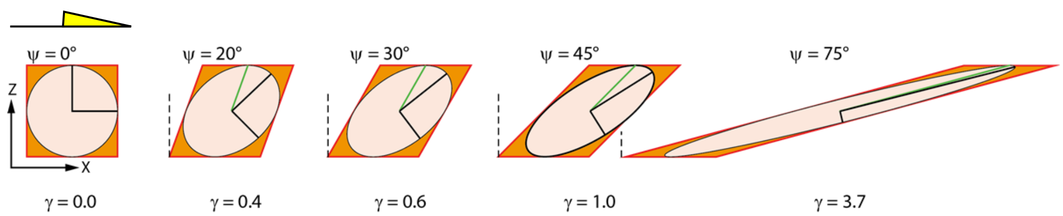
Pure Shear: A constant volume, coaxial and plane strain deformation. *All lines (except the principal strain axes) deflects towards the line of maximum extension*. In that case $(1+\varepsilon_1) = 1 / (1+\varepsilon_3)$.



Strain in Two Dimension

Non-coaxial Deformation: The axes of the finite and infinitesimal strain ellipses are not parallel. Detailed observation reveals that the principal axes of the strain ellipse rotate through different material lines at each infinitesimal strain increment: non-coaxial deformation is **rotational**.

Simple Shear: A constant volume, non-coaxial and plane strain deformation.
A square or rectangle subjected to simple shear changes to a parallelogram. The vertical sides of the square rotate but remain parallel to each other during deformation.



Next Lecture

Basics of Force and Stress