Indian Institute of Technology, Kanpur
Department of Earth Sciences

ESO213A: Fundamentals of Earth Sciences

Lecture 30. Mantle Dynamics

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Experimental Rock Deformation Laboratory

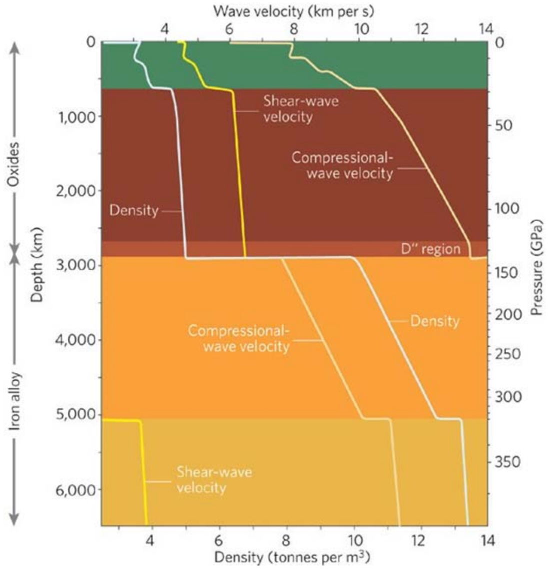
Aims of this lecture

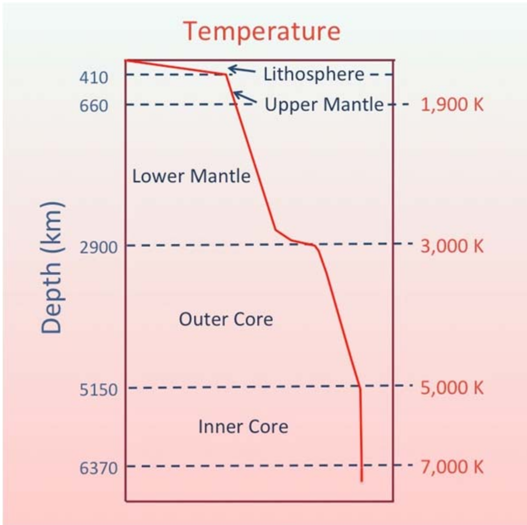
- Earth's Temperature profile
- Heat transfer in Earth's Mantle
- Mantle Convection

Reading:
- Marshak's Book (Chapter 2)
- Grotinger & Jordan's book (Chapter 14)

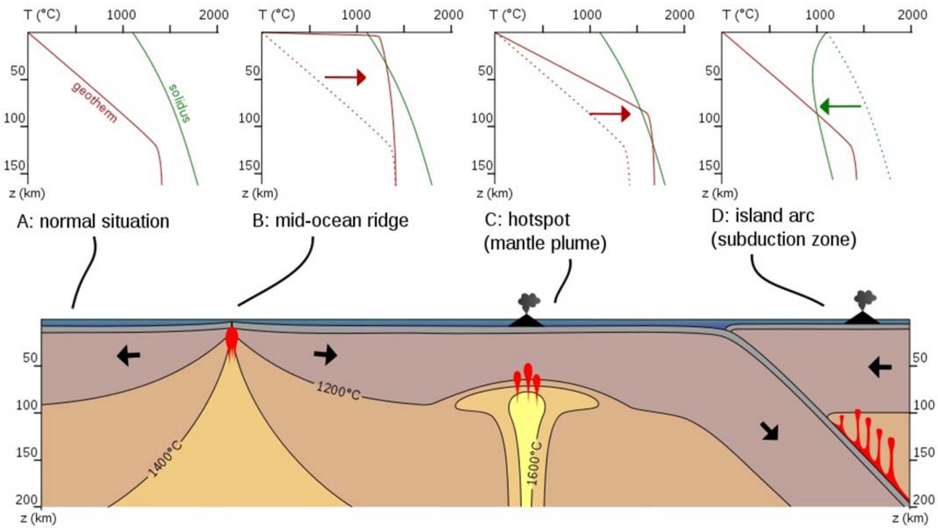
Geodynamics: *Turcotte and Schubert, Chambridge University Press (3rd Ed)*

As a whole





Crust and upper Mantle



Heat conduction through the lithosphere

- The Earth is cooling... losing internal energy.
- Heat is being released from the Earth's interior at a rate of about *44TW/year*. Averaged over the surface of the Earth, this amounts to a heat flow of about *70W/m²* through the crust .
- Heat energy diffuses through the crust and lithosphere by conduction according to Fourier's Law of Thermal Conduction.
- With magmas at volcanoes and spreading ridges, heat is being advected to the surface. Actually, this accounts for only a fraction of the heat that is brought to the surface and radiated through the atmosphere into space.

Fourier's Law of heat conduction

- A rate equation that allows determination of the conduction heat flux from knowledge of the temperature distribution in a medium

$$\vec{q}(\vec{r}) = -k\nabla T(\vec{r})$$

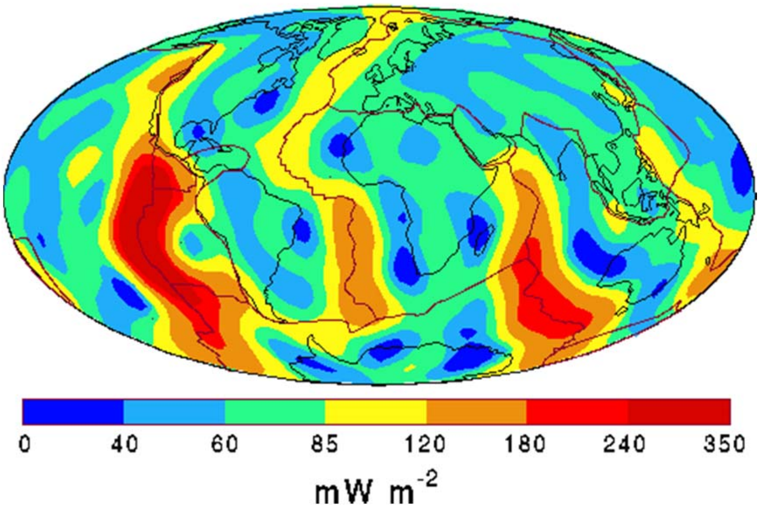
$\vec{q}$: *heat flux* $[W \cdot m^{-2}]$

k: *conductivity* $[W \cdot m^{-2} \cdot K^{-1}]$

T: *temperature* $[K]$

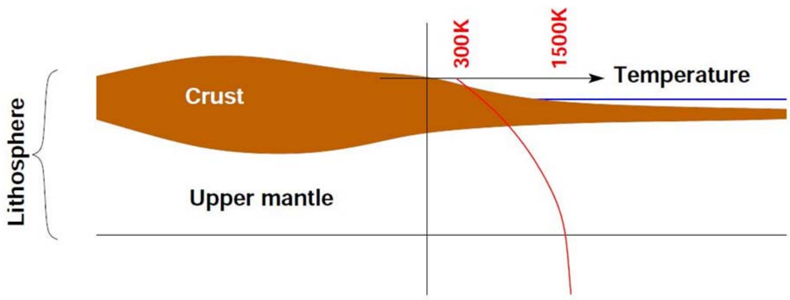
- Heat transfer is in the direction of decreasing temperature (basis for minus sign).
- Direction of heat transfer is perpendicular to lines of constant temperature (isotherms).
- Heat flux vector may be resolved into orthogonal components.
- Fourier's Law serves to define the thermal conductivity of the medium

Global Distribution of Heat Flow



http://geophysics.ou.edu

Temperature through the lithosphere and crust



- For a lithosphere with *100km* depth (i.e., an average gradient of *12K/km*), the average lithospheric thermal conductivity: $k \sim 4\ W\ m^{-2}\ K^{-1}$.
- Measured conductivities of surface rocks: $k \sim 2\text{-}3\ W\ m^{-2}\ K^{-1}$.

Thermal diffusion (conductive)

$$\nabla^2 T(\vec{r},t) + \frac{1}{k} h_i(\vec{r},t) = \frac{1}{D}\partial_t T(\vec{r},t)$$

$$\nabla^2 T(\vec{r},t) = \frac{1}{D}\partial_t T(\vec{r},t)$$

T: *temperature* $[K]$

$D = \frac{k}{\rho C_P}$: *thermal diffusivity* $[m^2 \cdot s^{-1}]$

ρ: *density* $[kg \cdot m^{-3}]$

C_p: *heat capacity at constant pressure* $[J \cdot kg^{-1} \cdot K^{-1}]$

$h_i = \partial_t Q_i$: *rate of heat input/volume* $[W \cdot m^{-3}]$

Q_i: *quantity of heat energy* $[J]$

Thermally driven mantle convection

- The contribution of diffusive cooling of the mantle is insignificant in comparison to convective heat transport through the mantle.
- The mantle behaves like a *viscous fluid* on long timescales; being a fluid, it can flow and can be driven into convection by a temperature gradient.
- Heat flows out of the depths of the cooling Earth transported through the mantle between the D'' layer and the base of the lithosphere by convective fluid motions rather than conduction. This is the more effective means of moving heat through a fluid.

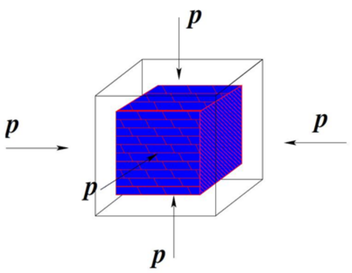
How does convection work?

- It's not conduction! More rapid heat transfer.
- Raise a parcel of hot rock.
- If constant entropy: Lower P => expands => larger volume => decreasing T
- This is known as adiabatic process.

Adiabatic compression

Consider a cube of mantle material under pressure

Energy is received due to work…



$$\Delta T \propto \Delta P$$

$$\propto T$$

$$\propto \frac{1}{\rho}$$

$$\propto \frac{\alpha P}{C_p}$$

T: Temperature
P: Pressure
ρ: Density
α: Thermal expansion
C_p: Heat capacity

$$\Delta T = \frac{\alpha P}{C_p} \frac{T \vec{g} \rho \Delta r}{\rho}$$

Adiabatic temperature gradient

- Need the change of temperature with pressure at constant entropy, ***S***

$$\left(\frac{\partial T}{\partial P}\right)_S = -\left(\frac{\partial T}{\partial S}\right)_P \left(\frac{\partial S}{\partial P}\right)_T$$

Maxwell's thermodynamic relation $\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P$

Coefficient of thermal expansion $\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P$

Specific heat $mC_p = T\left(\frac{\partial S}{\partial T}\right)_P$

Re-arranging $\left(\frac{\partial T}{\partial P}\right)_S = \frac{\alpha T}{\rho\, C_p}$

Adiabatic *T* gradient as a function of *P*

…. is this sufficient?

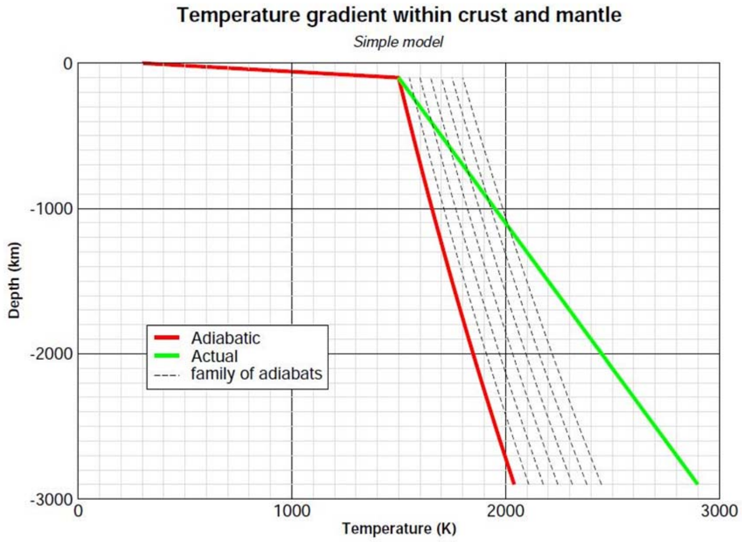
Adiabatic temperature gradient

- $$\left(\frac{\partial T}{\partial P}\right)_S = \frac{\alpha T}{\rho\, C_p}$$ best to have it as function of depth…

For the Earth $$\frac{dP}{dr} = -g\rho$$

Re-arranging $$\left(\frac{\partial T}{\partial r}\right)_S = \frac{\alpha\, T\, g}{C_p}$$

Mantle temperature?



If the actual temperature gradient exceeds the adiabatic gradient?

- If the actual temperature gradient (i.e., the increase of temperature with depth) exceeds the local adiabatic temperature gradient, then any infinitesimal displacement of a volume of mantle material will be enhanced through buoyancy, if displaced upwards or negative bouyancy if displaced downwards.
- **We have "*convection*"!**
- The process of convection removes heat from depth in the mantle to the base of the lithosphere where it is conducted out to the surface. The interior cools; the actual temperature gradient reduces.
- The process of convection pulls the entire mantle temperature toward the adiabatic gradient. If the temperature gradient falls to the adiabatic or below, convection ceases!
- If the temperature at the base of the lithosphere is *1500 K* as corresponds to Hawaiian lava eruptions, then the adiabatic gradient to top of the D'' layer would account for a base temperature in excess of about *2100 K* depending on the distributed thermal expansivity, α_p, and heat capacity, C_p, throughout the mantle.
- Heat "conducts" into the fluid mantle through the D'' boundary layer.

Mantle convection

- **In a fluid:**

  Occurs when density distribution deviates from equilibrium

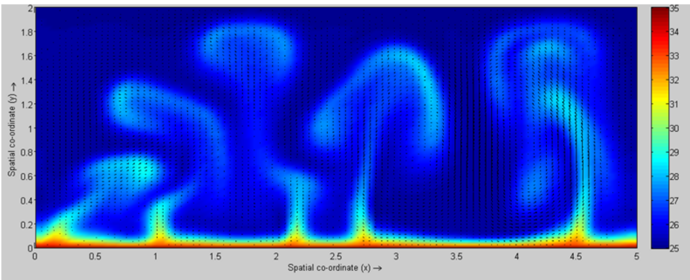
  Fluid may then flow to achieve equilibrium again

- **In a viscous solid heated from below:**

  Initially heat is transported by conduction into the fluid at the base

  Increased temperature reduces the density making the material at the base less dense than fluid above

  Once the buoyancy force due to the density contrast overcomes the inertia of the fluid convection begins

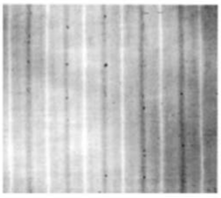


Rayleigh-Bernard convection

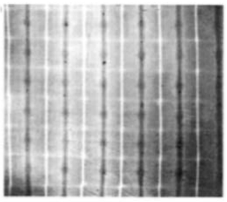
- **Newtonian viscous fluid:**

Stress = dynamic viscosity x strain rate

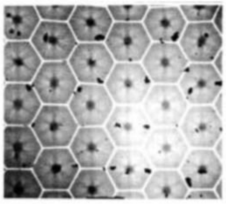
As fluid at the base heats up, initial convection is in 2D rolls



As heating proceeds, a second set of rolls forms perpendicular to the first – *bimodal*



Continued heating – *hexagonal* pattern

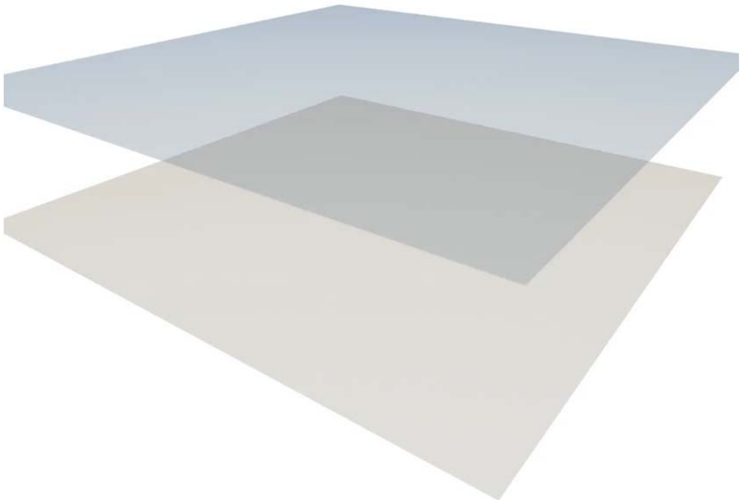


Spoke to irregular pattern

Read:
David White (1988); *The planforms and onset of convection with a temperature-dependent viscosity*; Journal of Fluid Mechanics, 191, 247-286.

Rayleigh-Bernard convection - examples

Rayleigh-Bernard convection - examples



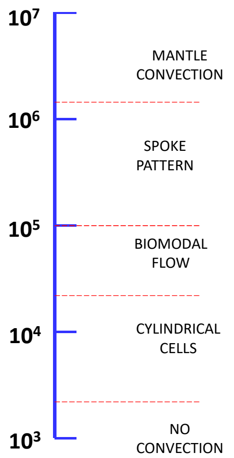
Rayleigh number

- Convection can be *driven* by internal or bottom heating. Surely, both contribute.
- The Rayleigh number measures the ratio of the forcing-to-retardation of the convection. A nondimensional number which describes the nature of heat transfer by suggesting the relationships between buoyancy and viscosity with a fluid
- For internal heating: $$Ra = \frac{g\rho^3 \alpha h_i d^5}{\eta k D}$$
- For bottom heating: $$Ra = \frac{g\rho^2 \alpha C_p T_{sx} d^3}{\eta k}$$

η is the local viscosity,
T_{sx}, the local adiabatic excess.
D is mantle thickness, base of lithosphere to D''.

- **The critical Rayleigh number:**
  - The point at which convection initiates
  - Approximately > 10^3 (dependent on geometry)
  - By knowing the material properties and physical geometry we can determine if there will be convection and the nature of that convection

Rayleigh number and convective mode



Rayleigh number of the mantle:

Upper mantle (thickness 700 km): 10^6

Lower mantle (thickness 2000 km): 3×10^7

Whole mantle (thickness 2700 km): 10^8

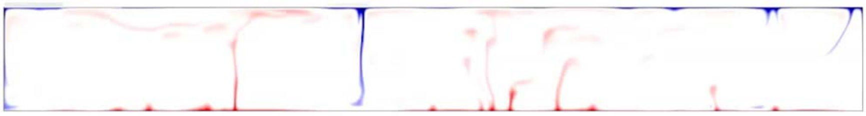
Mantle convection – temperature dependence of Viscosity

Ra	10^7
T of viscosity*	1000 x
η increase @ 660	0
Internal heating	0

Ra	10^7
T of viscosity*	100 x
η increase @ 660	0
Internal heating	0

Ra	10^7
T of viscosity*	10 x
η increase @ 660	0
Internal heating	0

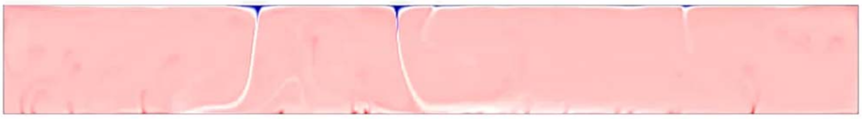
Ra	10^7
T of viscosity*	0 x
η increase @ 660	0
Internal heating	0



* Temperature dependence of Viscosity: --- x contrast between hottest and coldest

Mantle convection – Rayleigh number variation

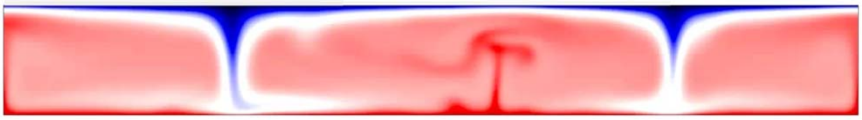
Ra 10^8
T of viscosity 1000 x
η increase @ 660 0
Internal heating 0



Ra 10^7
T of viscosity 1000 x
η increase @ 660 0
Internal heating 0

Ra 10^6
T of viscosity 1000 x
η increase @ 660 0
Internal heating 0

Ra 10^5
T of viscosity 1000 x
η increase @ 660 0
Internal heating 0



* Temperature dependence of Viscosity: --- x contrast between hottest and coldest

Mantle convection – variation due to internal heating

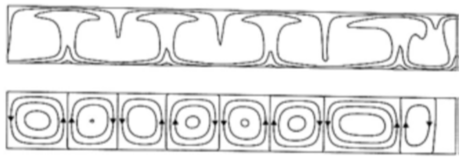
Ra 10^7
T of viscosity 1000 x
η increase @ 660 0
Internal heating 0

Ra 10^7
T of viscosity 1000 x
η increase @ 660 0
Internal heating 20

Ra 10^7
T of viscosity 1000 x
η increase @ 660 0
Internal heating 40

* Temperature dependence of Viscosity: --- x contrast between hottest and coldest

Mantle convection – the effect of heating - summary



Heat from below
T is fixed on upper boundary

Aspect ratio of 1
… not what we see on Earth

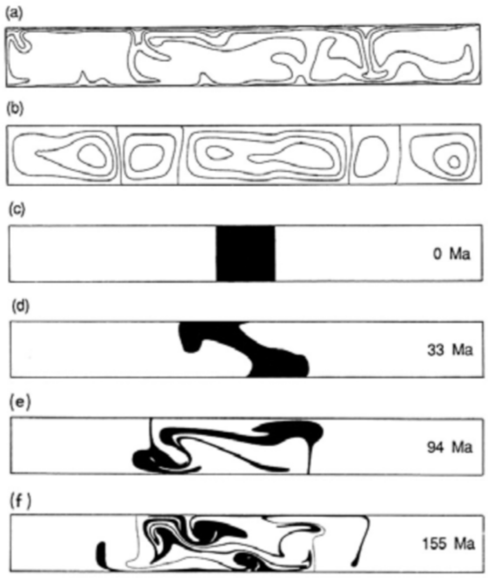
Heat from below
Constant heat flow across upper boundary

Large aspect ratio
… realistic?

Internal heating only

No upwelling sheets

Mantle convection – the effect of heating - summary

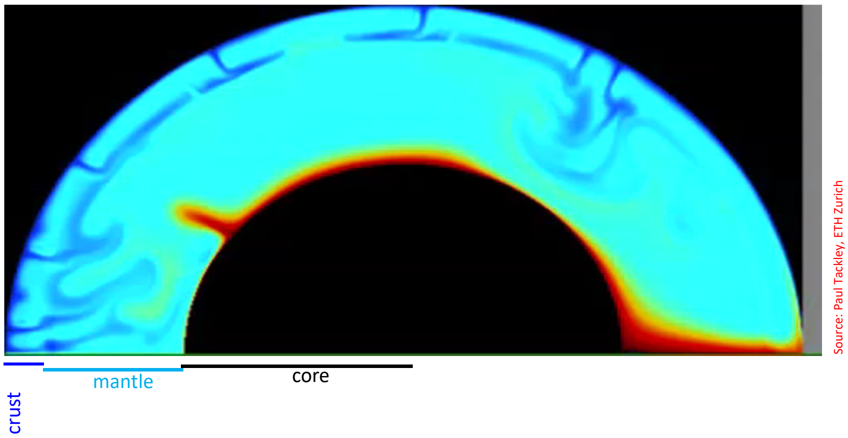


Isotopic ratios of oceanic basalts are very uniform but different from bulk earth values

The mantle is well mixed

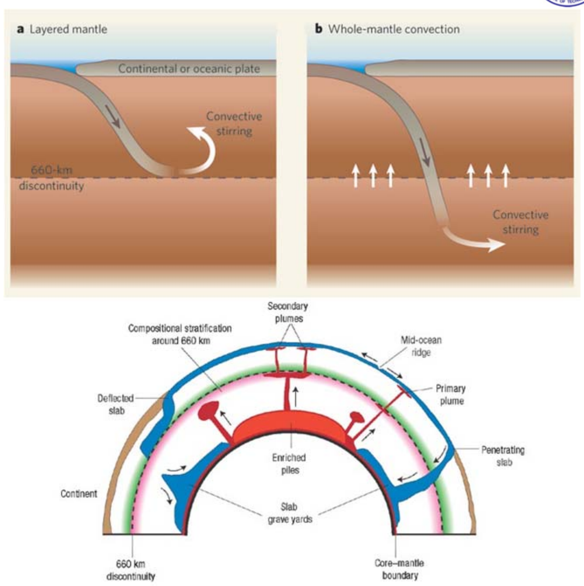
Any body smaller than 1000 km is reduced to less than 1 cm thick in 825 Ma !

Mantle convection – A near perfect combination



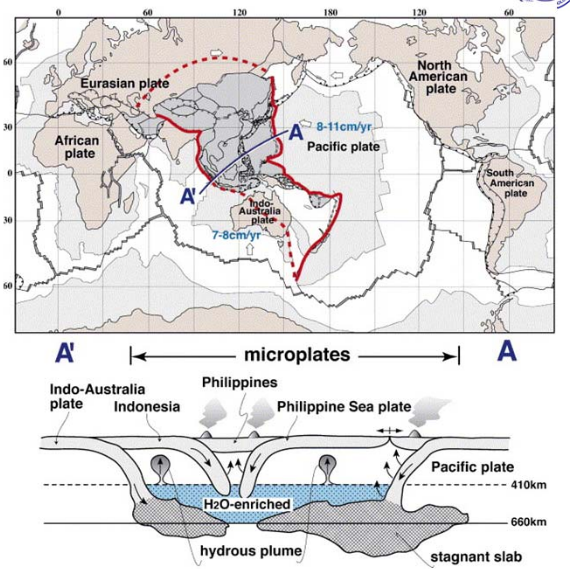
Layered mantle convection?

- There is/was a long-standing debate concerning the possibly layered convection in the mantle.
- There is general agreement that the 660 km phase change does retard sinking subducting plates and bouyant rising melts during convection
- There is also general agreement that plates can and do penetrate through 660 and that rising plumes and convective sheets rise through 660.
- Seismic tomography shows that we have a pooling of material around 660 as would be expected of layered convection while there remains sufficient penetration to involve the whole mantle in the convective process.



Stratified mantle

- In East Asia, the old Pacific plate subducts from the east, and the Indo-Australia plate from the south (double-sided subduction zone).
- The upper and lower mantle here are the coldest mantle regions in the world.
- This is the most active region on the Earth, indicating that the role of water is several orders of magnitude higher than that of the temperature in terms of lowering viscosity and drop of solidus. Note also the predominant occurrence of microplate in this region. Not only the fragmentation of continents but also the formation of small oceans constitutes the major reason for the dominant occurrence of microplates.
- The schematic cross-section of WPTZ is shown below to illustrate the stagnant slabs, hydrous MBL, and formation of hydrous plumes at 410 km depth by the breakdown reaction of hydrous wadsleyite enriched in incompatible elements



Some questions…

- Earth has mantle convection and plate tectonics – Why?
- When did it start? How long it will continue?
- What about other planets?