$$p \qquad\qquad\qquad\qquad \neg q$$

1. if the train arrives late and there is no taxi then John is late for his meeting.

$$r$$

1. $p$ and $\neg q$ implies $r$.

2. John is not late and train arrived late

$$\neg r \qquad\qquad\qquad\qquad p$$

$p \wedge \neg q \to r$, $\neg r$, $p$ Deduce $q$.

Natural Deduction.

Premises, Set of rules that allow us to draw a conclusion.

Proof Rules.

Premises $\phi_1, \ldots \phi_n$ Conclusion $\psi$.

$$\phi_1, \phi_2, \ldots \phi_n \vdash \psi$$

Expression is called a sequent.
It is valid if a proof for it can be found.

Rules of natural deduction.

Conjunction.

$$\frac{\phi \quad \psi}{\phi \wedge \psi} \wedge i \qquad \frac{\phi \wedge \psi}{\phi} \wedge e_1 \qquad \frac{\phi \wedge \psi}{\psi} \wedge e_2$$

Example. $p \wedge q,\ r \vdash q \wedge r$

$$\dfrac{\dfrac{p \wedge q}{q} \wedge e_2 \qquad r}{q \wedge r} \wedge i$$

Conjunction.

$$\frac{\phi \quad \psi}{\phi \wedge \psi}\ \wedge i \qquad \frac{\phi \wedge \psi}{\phi}\ \wedge e_1 \qquad \frac{\phi \wedge \psi}{\psi}\ \wedge e_2$$

Double Negation.

$$\frac{\neg\neg\phi}{\phi}\ \neg\neg e \qquad \frac{\phi}{\neg\neg\phi}\ \neg\neg i$$

Example. 1

$p, \neg\neg(q \wedge r) \vdash \neg\neg p \wedge r$

| | | |
|---|---|---|
| 1. | $p$ | premise. |
| 2 | $\neg\neg(q \wedge r)$ | premise |
| 3. | $\neg\neg p$ | $\neg\neg i$ 1 |
| 4. | $q \wedge r$ | $\neg\neg e$ 2 |
| 5 | $r$ | $\wedge e_2$ 4 |
| 6 | $\neg\neg p \wedge r$ | $\wedge i$ 3,5 |

Example 2.

$(p \wedge q) \wedge r, s \wedge t \vdash q \wedge s$

| | | |
|---|---|---|
| 1. | $(p \wedge q) \wedge r$ | premise |
| 2. | $s \wedge t$ | premise. |
| 3 | $p \wedge q$ | $\wedge e_1$ 1 |
| 4. | $q$ | $\wedge e_2$ 3 |
| 5. | $s$ | $\wedge e_1$ 2 |
| 6. | $q \wedge s$. | $\wedge i$ 4,5 |