

| S  | M  | T  | W  | T  | F  | S  |
|----|----|----|----|----|----|----|
|    | 1  | 2  | 3  |    |    |    |
| 4  | 5  | 6  | 7  | 8  | 9  | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| 18 | 19 | 20 | 21 | 22 | 23 | 24 |
| 25 | 26 | 27 | 28 | 29 | 30 |    |



# 2010

## MARCH

Wednesday **17**

### Assignment - 2

$$Q.1] \quad P(t) y''(t) + Q(t) y'(t) + R(t) y(t) + \lambda y(t) = 0$$

Multiplying above equation by  $\mu(t) > 0$  function:

$$\begin{aligned} P(t) \mu(t) y''(t) + Q(t) \mu(t) y'(t) + R(t) \mu(t) y(t) \\ + \lambda \mu(t) y(t) = 0 \end{aligned}$$

↪ ①

Thursday **18**

Consider

$$(P(t) y'(t))' + S(t) y(t) + \lambda R(t) y(t) = 0$$

$$\Rightarrow P'(t) y'(t) + P(t) y''(t) + S(t) y(t) + \lambda R(t) y(t) = 0$$

$$\Rightarrow P(t) y''(t) + P'(t) y'(t) + S(t) y(t) + \lambda R(t) y(t) = 0$$

↪ ②

Comparing ① and ② :

# MARCH



| S  | M  | T  | W  | T  | F  | S  |
|----|----|----|----|----|----|----|
| 1  | 2  | 3  | 4  | 5  | 6  |    |
| 7  | 8  | 9  | 10 | 11 | 12 | 13 |
| 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 |
|    |    |    |    |    |    | 28 |

February 2010

## 19 Friday

$$P(t) = R(t) \mu(t) \rightarrow ③$$

$$P'(t) = P'(t) \mu(t) + R(t) \mu'(t) \rightarrow ④$$

$$P'(t) = Q(t) \mu(t) \rightarrow ⑤$$

$$S(t) = R(t) \mu(t) \rightarrow ⑥$$

$$\lambda \gamma(t) = \lambda \mu(t) \rightarrow ⑦$$

From ④ and ⑤:

$$\mu'(t) P(t) = (Q(t) - P'(t)) \mu(t)$$

## 20 Saturday

$$\int \frac{\mu'(t)}{\mu(t)} dt = \int \frac{Q(t) - P'(t)}{P(t)} dt$$

$$\int \frac{1}{\mu(t)} \frac{d(\mu(t))}{dt} dt = \int \frac{(Q(t) - P'(t))}{P(t)} dt$$

$$\ln |\mu(t)| = \int \frac{(Q(t) - P'(t))}{P(t)} dt + C$$

## 21 Sunday

$$\mu(t) = e^{\int \frac{(Q(t) - P'(t))}{P(t)} dt} e^C$$

where  $\mu(t) > 0$  and

$C = \text{const. of integration}$

| S  | M  | T  | W  | F  | S     |
|----|----|----|----|----|-------|
|    | 1  | 2  | 3  |    |       |
| 4  | 5  | 6  | 7  | 8  | 9 10  |
| 11 | 12 | 13 | 14 | 15 | 16 17 |
| 18 | 19 | 20 | 21 | 22 | 23 24 |
| 25 | 26 | 27 | 28 | 29 | 30    |

April 2010



# 2010

## MARCH

$c = 0$  can be chosen  
w.l.o.g.

Monday 22

$$\text{So, } \mu(t) = e^{\int \frac{(Q(t) - P'(t))}{P(t)} dt}$$

From eqn ③, ⑥ and ⑦:

$$P(t) = P(t) e^{\int \frac{(Q(t) - P'(t))}{P(t)} dt}$$

$$S(t) = R(t) e^{\int \frac{(Q(t) - P'(t))}{P(t)} dt}$$

$$r(t) = e^{\int \frac{(Q(t) - P'(t))}{P(t)} dt} \quad (\lambda \neq 0)$$

Tuesday 23

w.l.o.g = without loss of generality

# 2010

## MARCH



| S | M  | T  | W  | T  | F  | S  |
|---|----|----|----|----|----|----|
|   | 1  | 2  | 3  | 4  | 5  | 6  |
|   | 7  | 8  | 9  | 10 | 11 | 12 |
|   | 13 | 14 | 15 | 16 | 17 | 18 |
|   | 19 | 20 | 21 | 22 | 23 | 24 |
|   | 25 | 26 | 27 | 28 |    |    |

February 2010

## 24 Wednesday

Q.2] (c.) , (d.) are the correct options.

## 25 Thursday

| S  | M  | T  | W  | T  | F  | S  |
|----|----|----|----|----|----|----|
|    | 1  | 2  | 3  |    |    |    |
| 4  | 5  | 6  | 7  | 8  | 9  | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| 18 | 19 | 20 | 21 | 22 | 23 | 24 |
| 25 | 26 | 27 | 28 | 29 | 30 |    |



2010  
MARCH

Friday 26

Q.3] (a) From Q1, we have

$$P(t) = t, \quad Q(t) = 2, \quad R(t) = 0$$

$$\text{So, } p(t) = P(t) e^{\int \frac{Q(t) - P'(t)}{P(t)} dt}$$

$$\begin{aligned} \text{Now, } \int \left( \frac{Q(t) - P'(t)}{P(t)} \right) dt &= \int \frac{2-1}{t} dt \\ &= \ln(|t|) \bullet \end{aligned}$$

$$\text{So, } e^{\ln(|t|)} = |t|$$

$$\text{For } t \geq 0, |t| = t$$

$$\begin{aligned} \text{So, } p(t) &= t \cdot t \\ &= t^2 \end{aligned}$$

Saturday 27

$$\begin{aligned} s(t) &= 0 \\ r(t) &= t \end{aligned}$$

So, the SLEVP is :

28 Sunday

$$(t^2 y'(t))' + \lambda t y(t) = 0$$

# 2010

## MARCH



February 2010

| S  | M  | T  | W  | T  | F  | S  |
|----|----|----|----|----|----|----|
|    | 1  | 2  | 3  | 4  | 5  | 6  |
| 7  | 8  | 9  | 10 | 11 | 12 | 13 |
| 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 |
|    | 28 |    |    |    |    |    |

## 29 Monday

For  $t < 0$ ,  $|t| = -t$

$$p(t) = -t^2$$

$$s(t) = 0$$

$$\tau(t) = -t$$

So, the SLEVP is:

$$(-t^2 y'(t))' - \lambda t y(t) = 0$$

$$(t^2 y'(t))' + \lambda t y(t) = 0$$

## 30 Tuesday

| S  | M  | T  | W  | T  | F  | S  |
|----|----|----|----|----|----|----|
| 1  | 2  | 3  | 4  | 5  | 6  | 7  |
| 8  | 9  | 10 | 11 | 12 | 13 | 14 |
| 15 | 16 | 17 | 18 | 19 | 20 | 21 |
| 22 | 23 | 24 | 25 | 26 | 27 | 28 |
| 29 | 30 | 31 |    |    |    |    |

August 2010



# 2010

Monday **12**

Q.4] (a)

Only possible eigenvalues are  
when  $\lambda > 0$

General solution is :

$$y(t) = A \cos(\sqrt{\lambda} t) + B \sin(\sqrt{\lambda} t)$$

For  $y(0) = 0 \Rightarrow A = 0$

$$y(L) = 0 \Rightarrow B \sin(\sqrt{\lambda} L) = 0$$

$$\Rightarrow \sqrt{\lambda} L = n\pi \quad \text{where } n \in \mathbb{N}$$

Tuesday **13**

$$\Rightarrow \sqrt{\lambda} = \frac{n\pi}{L} \quad \text{where } n \in \mathbb{N}$$

So,  $y(t) = B \sin\left(\frac{n\pi}{L} t\right), n \in \mathbb{N}$

**Friday**

**16**

$$8.4] (c) \quad \frac{y''(t)}{t} - \frac{y'(t)}{t^2} + (\lambda+1) \frac{y(t)}{t^3} = 0, \quad t \in (1, e^\lambda)$$

$$t^2 y''(t) - t y'(t) + (\lambda+1) y(t) = 0, \quad t \in (1, e^\lambda)$$

This is a Homogeneous Euler-Cauchy D.E. Hence, characteristic equation:

$$m^2 - 2m + (\lambda+1) = 0$$

$$\begin{aligned} m &= \frac{2 \pm \sqrt{4 - 4(\lambda+1)}}{2} \\ &= \frac{2 \pm \sqrt{-4\lambda}}{2} \end{aligned}$$

**17 Saturday**

[Variable  $x$  is used just for calculations]

Since, this is a RSLEVP,  $\lambda \in \mathbb{R}$ .

Case I:  $\lambda < 0$ .

$$\begin{aligned} \text{So, } m &= \frac{2 \pm 2\sqrt{-\lambda}}{2} \\ &= 1 \pm \sqrt{-\lambda} \end{aligned}$$

**18 Sunday** So,  $y(x) = A x^{1+\sqrt{-\lambda}} + B x^{1-\sqrt{-\lambda}}$

$$y(1) = 0 \Rightarrow 0 = A + B$$

$$y(e^\lambda) = 0 \Rightarrow 0 = A e^{\pi\sqrt{-\lambda}} + B e^{-\pi\sqrt{-\lambda}}$$

$$\begin{aligned} \Rightarrow A (e^{2\pi\sqrt{-\lambda}} - 1) &= 0 \\ \Rightarrow A &= 0 \end{aligned}$$

|          | S  | M  | T  | W  | T  | F  | S  |
|----------|----|----|----|----|----|----|----|
| May 2010 | 30 | 31 |    |    | 1  |    |    |
|          | 2  | 3  | 4  | 5  | 6  | 7  | 8  |
|          | 9  | 10 | 11 | 12 | 13 | 14 | 15 |
|          | 16 | 17 | 18 | 19 | 20 | 21 | 22 |
|          | 23 | 24 | 25 | 26 | 27 | 28 | 29 |



2010  
APRIL

$$\Rightarrow B = 0$$

Monday 19

Hence,  $y(x) = 0$

Hence,  $\lambda < 0$  is not possible.

Case II:  $\lambda = 0$

So,  $m = 1$  (Repeated)

So,  $y(x) = Ax + Bx\ln(x)$

$$y(1) = 0 \Rightarrow A = 0$$

$$y(e^x) = 0 \Rightarrow B = 0$$

$$\Rightarrow y(x) = 0$$

Hence,  $\lambda = 0$  is not possible.

| S  | M  | T  | W  | T  | F  | S  |
|----|----|----|----|----|----|----|
| 1  | 2  | 3  | 4  | 5  | 6  | 7  |
| 8  | 9  | 10 | 11 | 12 | 13 | 14 |
| 15 | 16 | 17 | 18 | 19 | 20 | 21 |
| 22 | 23 | 24 | 25 | 26 | 27 | 28 |
| 29 | 30 | 31 |    |    |    |    |



# 2010

JULY

Friday 16

Case III :

$$\lambda > 0$$

$$\text{So, } m = 1 \pm i\sqrt{\lambda}$$

$$\text{So, } y(x) = A x \cos(\sqrt{\lambda} \ln(x)) + B x \sin(\sqrt{\lambda} \ln(x))$$

$$y(1) = 0 \Rightarrow A = 0$$

$$y(e^\pi) = 0 \Rightarrow 0 = B \sin(\sqrt{\lambda} \pi)$$

$$\Rightarrow \sqrt{\lambda} \pi = n\pi, \quad n \in \mathbb{N}$$

$$\text{So, } \sqrt{\lambda} = n \rightarrow n \in \mathbb{N}$$

So,  $y_n(t) = Bt \sin(n \ln(t))$ ,  $n \in \mathbb{N}$   
is an eigenfunction for  
eigenvalue  $\lambda$

So,  $y_n(t) = Bt \sin(n \ln(t))$  is a  
solution for  $\lambda > 0$   
eigenvalues.

18 Sunday

| S  | M  | T  | W  | F  | S  |
|----|----|----|----|----|----|
| 1  | 2  | 3  | 4  |    |    |
| 5  | 6  | 7  | 8  | 9  | 10 |
| 12 | 13 | 14 | 15 | 16 | 17 |
| 19 | 20 | 21 | 22 | 23 | 24 |
| 26 | 27 | 28 | 29 | 30 |    |



# 2010

## AUGUST

Wednesday

# 4

Q.5]

Given:  $\{\sin(nx), \cos(mx)\}_{n,m \in \mathbb{N}}$  is  
a orthogonal family on  
 $(-\pi, \pi)$

Consider  $\sin(nx), \cos(mx)$  for any  
 $n, m \in \mathbb{N}$

So,  $\int_{-\pi}^{\pi} \sin(nx) \cos(mx) dx = 0$  is given.

Thursday

# 5

$$\text{Let } \pi t = x$$

$$\pi dt = dx$$

$$\int_{-1}^1 \sin(nt\pi) \cos(mt\pi) dt = 0$$

$$\Rightarrow \int_{-1}^1 \sin(nt\pi) \cos(mt\pi) dt = 0 \quad \text{for any } n, m \in \mathbb{N}$$

↪ (1)

# 6 Friday

Now, consider  $\sin(nx), \sin(mx)$  for any  $n, m \in \mathbb{N}$   
s.t  $n \neq m$

So,  $\int_{-\pi}^{\pi} \sin(nx) \sin(mx) dx = 0$  is given.

$$\text{Let } \pi t = x$$

$$\pi dt = dx$$

$$\int_{-\pi}^{\pi} \sin(nt\pi) \sin(mt\pi) dt = 0$$

# 7 Saturday

$\Rightarrow \int_{-\pi}^{\pi} \sin(nt\pi) \sin(mt\pi) dt = 0$  for any  $n, m \in \mathbb{N}$

s.t  $n \neq m$

②

# 8 Sunday

| S  | M  | T  | W  | T  | F  | S  |
|----|----|----|----|----|----|----|
|    | 1  | 2  | 3  | 4  |    |    |
| 5  | 6  | 7  | 8  | 9  | 10 | 11 |
| 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| 19 | 20 | 21 | 22 | 23 | 24 | 25 |
| 26 | 27 | 28 | 29 | 30 |    |    |

September 2010



2010  
AUGUST

Monday 9

Now, consider  $\int_{-\pi}^{\pi} \cos(nx) dx$

Now, consider  $\cos(nx)$  and  $\cos(mx)$  for any  $n, m \in \mathbb{N}$   
s.t.  $n \neq m$

$$\text{So, } \int_{-\pi}^{\pi} \cos(nx) \cos(mx) dx = 0 \text{ is given.}$$

$$\text{Let } t\pi = x$$

$$\pi dt = dx$$

$$\int_{-1}^1 \pi \cos(nt\pi) \cos(mt\pi) dt = 0$$

$$\Rightarrow \int_{-1}^1 \cos(nt\pi) \cos(mt\pi) dt = 0 \quad \text{for any } n, m \in \mathbb{N} \\ \text{s.t. } n \neq m$$

Tuesday 10

Combining ①, ② and ③, we get  
that if  $\{\sin(nx), \cos(nx)\}_{n \in \mathbb{N}}$  is orthogonal

on  $(-\pi, \pi)$  then  $\{\sin(n\pi x), \cos(n\pi x)\}_{n \in \mathbb{N}}$   
is orthogonal family on  $(-1, 1)$

| S  | M  | T  | W  | T  | F  | S  |
|----|----|----|----|----|----|----|
| 1  | 2  | 3  | 4  | 5  |    |    |
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| 13 | 14 | 15 | 16 | 17 | 18 | 19 |
| 20 | 21 | 22 | 23 | 24 | 25 | 26 |
| 27 | 28 | 29 | 30 |    |    |    |

June 2010



# 2010

**Friday** 7

Q6] Consider 2 DEs :

$$y''(t) + y(t) = 0 \rightarrow \textcircled{1} \text{ and}$$

$$y''(t) + (e^{t^2} + 1)y(t) = 0 \rightarrow \textcircled{2}$$

Now,  $e^{t^2} + 1 > 1 \quad \forall t \in \mathbb{R}$

Solution of DE  $\textcircled{1}$  is:

$$m^2 + 1 = 0$$

$$\Rightarrow m = \pm i$$

$$\text{So, } y(t) = A \sin(t) + \phi$$

w.l.o.g  $\phi = 0$  and  $A = 1$   
can be assumed.

$$y(t) = \sin(t)$$

$\sin(t)$  has  $\infty$  many zeros in  $\mathbb{R}$ .

Now, consider  $y_1(t)$  as a solution of  
 $\textcircled{2}$ . By Sturm Comparison Theorem:

Between any 2 zeros of  
 $\sin(t)$  there is atleast 1 zero  
of  $y_1(t)$ .

**9 Sunday**

$\therefore \sin(t)$  has  $\infty$  zeros, it follows  
that  $y_1(t)$  has  $\infty$  zeros in  $\mathbb{R}$ .

# 2010

**MAY**



| S  | M  | T  | W  | T  | F  | S  |
|----|----|----|----|----|----|----|
|    |    |    |    | 1  | 2  | 3  |
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| 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| 18 | 19 | 20 | 21 | 22 | 23 | 24 |
| 25 | 26 | 27 | 28 | 29 | 30 |    |

April 2010

## 10 Monday

[Variable  $x$  is used for intermediate calculations]

Q.7.] Consider the DE:

$$t^2 y''(t) + t y'(t) + \lambda y(t) = 0 \text{ on } (1, e)$$

$$\text{with } y(1) = y(e) = 0$$

This is a Homogeneous Euler Cauchy

D. Equation. Hence, characteristic eqn:

$$m^2 + \lambda = 0$$



Case 1:  $\lambda < 0$

$$\text{Then, } m = \pm \sqrt{-\lambda}$$

## 11 Tuesday

$$\text{So, } y(x) = A x^{-\sqrt{-\lambda}} + B x^{\sqrt{-\lambda}}$$

$$y(1) = 0 \Rightarrow 0 = A + B$$

$$y(e) = 0 \Rightarrow 0 = A e^{-\sqrt{-\lambda}} + B e^{\sqrt{-\lambda}}$$

$$\Rightarrow 0 = B e^{2\sqrt{-\lambda}} + A$$

$$\Rightarrow 0 = B (e^{2\sqrt{-\lambda}} - 1)$$

$$\Rightarrow B = 0$$

$$\Rightarrow A = 0 \Rightarrow y(x) = 0 \text{ on } (1, e)$$

Hence,  $\lambda < 0$  is not possible for eigenvalue.

| S  | M  | T  | W  | T  | F  | S  |
|----|----|----|----|----|----|----|
| 1  | 2  | 3  | 4  | 5  |    |    |
| 6  | 7  | 8  | 9  | 10 | 11 | 12 |
| 13 | 14 | 15 | 16 | 17 | 18 | 19 |
| 20 | 21 | 22 | 23 | 24 | 25 | 26 |
| 27 | 28 | 29 | 30 |    |    |    |

June 2019



# 2010

**MAY**

Wednesday **12**

Case 2:  $\lambda = 0$ 

$$\Rightarrow m = 0$$

Hence,  $y(x) = A + B \ln(x)$

$y(1) = 0 \Rightarrow A = 0$

$y(e) = 0 \Rightarrow B = 0$

Hence,  $y(x) = 0$  - on  $(1, e)$ 

Hence,  $\lambda = 0$  eigenvalue is  
not possible.

Case 3:  $\lambda > 0$ 

$$\Rightarrow m = \pm i\sqrt{\lambda}$$

Thursday **13**

So,  $y(x) = A \cos(\sqrt{\lambda} \ln(x)) + B \sin(\sqrt{\lambda} \ln(x))$

$y(1) = 0 \Rightarrow A = 0$

$y(e) = 0 \Rightarrow 0 = B \sin(\sqrt{\lambda})$

So,  $\sqrt{\lambda} = n\pi$ ,  $n \in \mathbb{N}$

So,  $y_n(t) = B \sin(n\pi \ln(\frac{t}{1}))$  are  
solutions, for eigenvalues  $\lambda = (n\pi)^2$ 
 $\therefore$  distinct eigenvalues's eigenfunctions  
are orthogonal to each other.

Hence, the family  $\{ \sin(n\pi \log(t)) \}_{n \in \mathbb{N}}$  is

an orthogonal family.

2010  
MAY



| S  | M  | T  | W  | T  | F  | S  |
|----|----|----|----|----|----|----|
|    |    |    | 1  | 2  | 3  |    |
| 4  | 5  | 6  | 7  | 8  | 9  | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| 18 | 19 | 20 | 21 | 22 | 23 | 24 |
| 25 | 26 | 27 | 28 | 29 | 30 |    |

April 2010

14 Friday



Consider  $e^{\int \sin^2(n\pi \ln(t)) dt}$

$$\text{let } \ln(t) = x$$

$$dt = t dx \\ = e^x dx$$

$$\text{so, } \int e^{\int \sin^2(n\pi \ln(t)) dt} dt = \int_0^1 \sin^2(n\pi x) e^x dx \\ = \frac{1}{2} \int_0^1 e^x (1 - \cos(2n\pi x)) dx$$

15 Saturday

$$= \frac{1}{2} \left[ \int_0^1 e^x dx - \int_0^1 e^x \cos(2n\pi x) dx \right]$$

$$= \frac{1}{2} \left[ (e-1) - \frac{e^x}{1 + (2n\pi)^2} (\cos(2n\pi x) + 2n\pi \sin(2n\pi x)) \Big|_0^1 \right]$$

16 Sunday

$$= \frac{1}{2} \left[ (e-1) - \left( \frac{1}{1 + 4n^2\pi^2} \right) [e - 1] \right]$$

$$= \frac{1}{2} \left[ \frac{(e-1) + 4n^2\pi^2(e-1) - (e-1)}{1 + 4n^2\pi^2} \right]$$

June 2010

| S  | M  | T  | W  | T  | F  | S  |
|----|----|----|----|----|----|----|
|    | 1  | 2  | 3  | 4  | 5  |    |
| 6  | 7  | 8  | 9  | 10 | 11 | 12 |
| 13 | 14 | 15 | 16 | 17 | 18 | 19 |
| 20 | 21 | 22 | 23 | 24 | 25 | 26 |
| 27 | 28 | 29 | 30 |    |    |    |



2010  
**MAY**

Monday **17**

$$= \frac{2n^2(\epsilon - 1)}{4n^2 + 1}$$

No, it is not orthonormal. It can be made orthonormal by

dividing  $\sin(n\pi \ln(t))$  by  $\sqrt{\frac{2n^2(\epsilon - 1)}{4n^2 + 1}}$

Tuesday **18**

| S  | M  | T  | W  | T  | F  | S  |
|----|----|----|----|----|----|----|
|    | 1  | 2  | 3  | 4  | 5  |    |
| 6  | 7  | 8  | 9  | 10 | 11 | 12 |
| 13 | 14 | 15 | 16 | 17 | 18 | 19 |
| 20 | 21 | 22 | 23 | 24 | 25 | 26 |
| 27 | 28 | 29 | 30 |    |    |    |



2010  
**MAY**

Friday **21**

Q.8.] Consider :

$$(P(t) \phi'(t))' + q(t) \phi(t) + \mu \sigma(t) \phi(t) = 0$$

Multiply by  $\phi(t)$  on both sides:

$$\phi(t) (P(t) \phi'(t))' + q(t) (\phi(t))^2 + \mu \sigma(t) (\phi(t))^2 = 0$$

Integrate from  $a$  to  $b$  on both sides:

$$\int_a^b \phi(t) (P(t) \phi'(t))' dt + \int_a^b q(t) (\phi(t))^2 dt + \int_a^b \mu \sigma(t) (\phi(t))^2 dt = 0$$

**Saturday 22**

$$\left. \phi(t) P(t) \phi'(t) \right|_a^b - \int_a^b P(t) (\phi'(t))^2 dt + \int_a^b q(t) (\phi(t))^2 dt + \int_a^b \mu \sigma(t) (\phi(t))^2 dt = 0$$

**23 Sunday**

$$\mu \int_a^b \sigma(t) (\phi(t))^2 dt = -P(t) \phi(t) \phi'(t) \Big|_a^b + \int_a^b (P(t) (\phi'(t))^2 - Q(t) (\phi(t))^2) dt$$

$$\mu = -P(t) \phi(t) \phi'(t) \Big|_a^b + \int_a^b (P(t) (\phi'(t))^2 - Q(t) (\phi(t))^2) dt$$

$$\int_a^b \sigma(t) (\phi(t))^2 dt$$

# 2010

## MAY



| S  | M  | T  | W  | T  | F  | S  |
|----|----|----|----|----|----|----|
|    |    |    |    |    | 1  | 2  |
|    |    |    |    |    | 3  |    |
| 4  | 5  | 6  | 7  | 8  | 9  | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| 18 | 19 | 20 | 21 | 22 | 23 | 24 |
| 25 | 26 | 27 | 28 | 29 | 30 |    |

April 2010

## 28 Friday

With  $q(t) = 0$  on  $(a, b)$  and  
 $y(a) = 0, y'(b) = 0$ :

$$\mu = \frac{-p(t)\phi(t)\phi'(t)|_a^b + \int_a^b p(t)(\phi'(t))^2 dt}{\int_a^b \sigma(t)(\phi(t))^2 dt}$$

$\therefore \phi(a) = 0$  and  $\phi'(b) = 0$ :

## 29 Saturday

$$\mu =$$

$$\frac{\int_a^b p(t)(\phi'(t))^2 dt}{\int_a^b \sigma(t)(\phi(t))^2 dt}$$

$\therefore$  it is a RSLEVP:

$p(t) > 0$  and  $\sigma(t) > 0$  on  $[a, b]$

## 30 Sunday

Hence,  $\mu \geq 0$

$$\text{For } \mu = 0 \Rightarrow \int_a^b p(t)(\phi'(t))^2 dt = 0$$

| S  | M  | T  | W  | T  | F  | S  |
|----|----|----|----|----|----|----|
| 1  | 2  | 3  | 4  | 5  |    |    |
| 6  | 7  | 8  | 9  | 10 | 11 | 12 |
| 13 | 14 | 15 | 16 | 17 | 18 | 19 |
| 20 | 21 | 22 | 23 | 24 | 25 | 26 |
| 27 | 28 | 29 | 30 |    |    |    |

June 2010



# 2010

**MAY**

**Monday 31**

$$\Rightarrow p(t)(\phi'(t))^2 = 0 \quad \text{on } (a, b)$$

$$\Rightarrow (\phi'(t))^2 = 0 \quad \text{on } (a, b)$$

$$\Rightarrow \phi'(t) = 0 \quad \text{on } (a, b)$$

$$\Rightarrow \phi(t) = \text{constant on } (a, b)$$

$$\therefore \phi(a) = 0$$

$$\Rightarrow \phi(t) = 0 \quad \text{on } (a, b)$$

But  $\phi(t) = 0$  on  $(a, b)$  is  
a trivial solution.

Hence,  $\mu = 0$  is not possible  
eigenvalue.

**June**

**Tuesday 1**

Hence,  $\mu > 0$

$\Rightarrow \mu$  is strictly positive