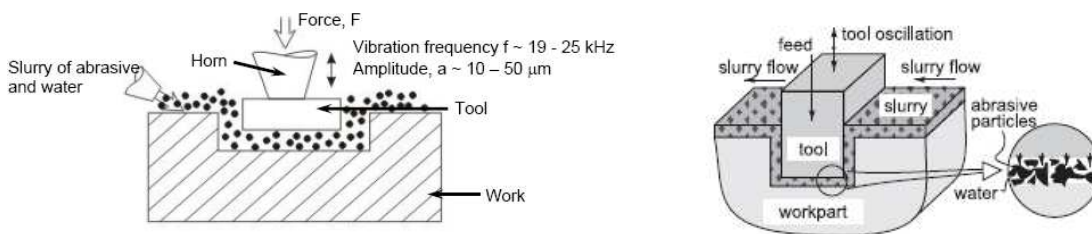# ULTRASONIC MACHINING

Definition:

Ultrasonic Machining is a non-traditional process, in which abrasives contained in a slurry are driven against the work by a tool oscillating at low amplitude (25-100 microns) and high frequency (15-30 kHz).

Process:

Ultrasonic machining is a mechanical type non-traditional machining process. It is employed to machine hard and brittle materials (both electrically conductive and non conductive material) having hardness usually greater than 40 HRC. The process was first developed in 1950s and was originally used for finishing EDM surfaces.


In ultrasonic machining, tool of desired shape vibrates at ultrasonic frequency (19 to 25 kHz.) with an amplitude of 15-50 Microns over work piece. Generally tool is pressed down with a feed force F. Between the tool and work, machining zone is flooded with hard abrasive particles generally in the form of water based slurry. As the tool vibrates over the work piece, abrasive particles acts as indenter and indent both work and tool material . Abrasive particles , as they indent , the work material would remove the material from both tool and work piece. In Ultrasonic machining material removal is due to crack initiation, propagation and brittle fracture of material. USM is used for machining hard and brittle materials, which are poor conductors of electricity and thus cannot be processed by Electrochemical machining (ECM) or Electro discharge machining (EDM).

The tool in USM is made to vibrate with high frequency on to the work surface in the midst of the flowing slurry. The main reason for using ultrasonic frequency is to provide better performance. Audible frequencies of required intensities would be heard as extremely loud sound and would cause fatigue and even permanent damage to the auditory apparatus.

Equipment:

Ultrasonic Machining consists of :
1. High Power sine wave generator
2. Magneto-strictve Transducer
3. Tool Holder
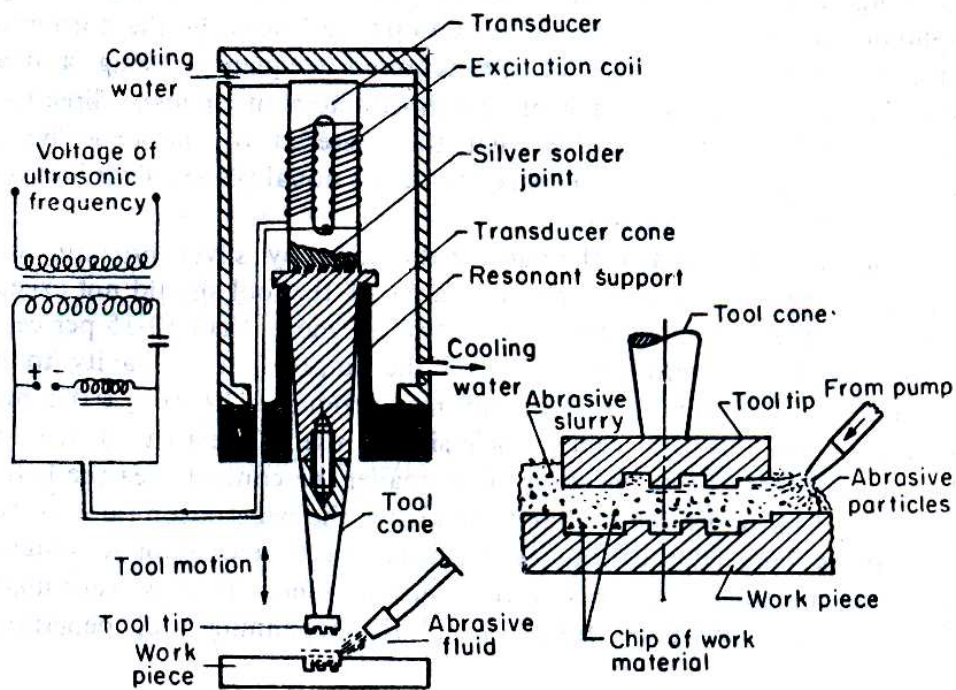4. Tool
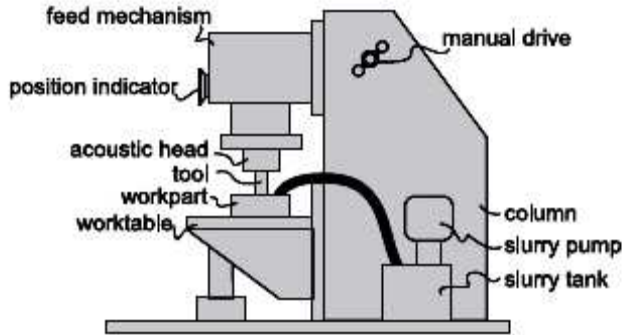
Principal components of an ultrasonic machine.

Fig. 2.1 Schematic representation of ultrasonic machining process

High power sine wave generator
This unit converts low frequency (60 Hz) electrical power to high frequency (20kHz) electrical power.

Transducer
The high frequency electrical signal is transmitted to traducer which converts it into high frequency low amplitude vibration. Essentially transducer converts electrical energy to mechanical vibration. There are two types of transducer used
1. Piezo electric transducer
2. Magneto-stricitve transducer.

Piezo electric transducer: These transducer generate a small electric current when they are compressed. Also when the electric current is passed though crystal it expands. When the current is removed , crystal attains its original size and shape. Such transducers are available up to 900 Watts. Piezo electric crystals have high conversion efficiency of 95%.

Magneto-strictive transducer: These also changes its length when subjected to strong magnetic field. These transducer are made of nickel , nickel alloy sheets. Their conversion efficiency is about 20-30%. Such transducers are available up to 2000 Watts. The maximum change in length can be achieved is about 25 microns.

Tool holder. OR Horn.
The tool holder holds and connects the tool to the transducer. It virtually transmits the energy and in some cases, amplifies the amplitude of vibration. Material of tool should have good acoustic properties, high resistance to fatigue cracking. Due measures should be taken to avoid ultrasonic welding between transducer and tool holder. Commonly used tool holders are Monel, titanium, stainless steel. Tool holders are more expensive, demand higher operating cost.
Tool holder can be classified as :

Amplifying Tool Holder	Non-Amplifying Tool Holder
They give as much as 6 times increased tool motion. It is achieved by stretching and relaxing the tool holder material. MRR = 10 times the non amplifying tool.	Non amplifying tool holders have circular cross section and give same amplitude at both ends.

Tool
Tools are made of relatively ductile materials like Brass, Stainless steel or Mild steel so that Tool wear rate (TWR) can be minimized. The value of ratio of TWR and MRR depends on kind of abrasive, work material and tool materials.

OPERATIONS OF
ULTRASONIC CUTTING.

As the tool vibrates with a specific frequency, an abrasive slurry (usually a mixture of abrasive grains and water of definite proportion) is made to flow through the tool work interface. The impact force arising out of vibration of the tool end and the flow of slurry through the work tool interface actually causes thousands of microscopic abrasive grains to remove the work material by abrasion. Material removal from the hard and brittle materials will be the form of sinking, engraving or any other precision shape.

(a) SCHEME   (b) STROKE MAGNIFICATION
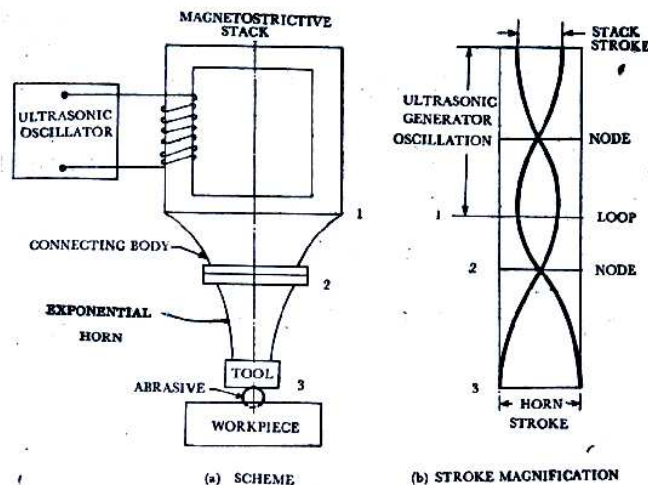
Process parameters

1. Amplitude of vibration (15 to 50 microns)
2. Frequency of vibration (19 to 25 kHz).
3. Feed force (F) related to tool dimensions
4. Feed pressure
5. Abrasive size
6. Abrasive material
    ** Al_2O_3, SiC, B_4C, Boron silicarbide, Diamond.
7. Flow strength of the work material
8. Flow strength of the tool material
9. Contact area of the tool
10. Volume concentration of abrasive in water slurry
11. Tool
    a. Material of tool
    b. Shape
    c. Amplitude of vibration
    d. Frequency of vibration
    e. Strength developed in tool
12. Work material
    a. Material
    b. Impact strength
    c. Surface fatigue strength

13. Slurry
    a. Abrasive – hardness, size, shape and quantity of abrasive flow
    b. Liquid – Chemical property, viscosity, flow rate
    c. Pressure
    d. Density

Process capability

1. Can Machine work piece harder than 40 HRC to 60 HRC like carbides, ceramics, tungsten glass that cannot be machined by conventional methods

2. Tolerance range 7 micron to 25 microns

3. Holes up to 76 micron have been drilled hole depth upto 51mm have been achieved easily. Hole depth of 152mm deep is achieved by special flushing techniques.

4. Aspect ratio 40:1 has been achieved

5. Linear material removal rate -0.025 to 25mm/min

6. Surface finish -0.25 micron to 0.75 micron

7. Non directional surface texture is possible compared to conventional grinding

8. Radial over cut may be as low as 1.5 to 4 times the mean abrasive grain size.

Applications

1. Machining of cavities in electrically non-conductive ceramics
2. Used to machine fragile components in which otherwise the scrap rate is high
3. Used for multistep processing for fabricating silicon nitride (Si_3N_4) turbine blades
4. Large number of holes of small diameter. 930 holes with 0.32mm has been reported (Benedict, 1973) using hypodermic needles
5. Used for machining hard, brittle metallic alloys, semiconductors, glass, ceramics, carbides etc.
6. Used for machining round, square, irregular shaped holes and surface impressions.
7. Used in machining of dies for wire drawing, punching and blanking operations
8. USM can perform machining operations like drilling, grinding and milling operations on all materials which can be treated suitably with abrasives.
9. USM has been used for piercing of dies and for parting off and blanking operations.
10. USM enables a dentist to drill a hole of any shape on teeth without any pain
11. Ferrites and steel parts , precision mineral stones can be machined using USM
12. USM can be used to cut industrial diamonds
13. USM is used for grinding Quartz, Glass, ceramics
14. Cutting holes with curved or spiral centre lines and cutting threads in glass and mineral or metallo-ceramics

Advantages

1. It can be used machine hard, brittle, fragile and non conductive material
2. No heat is generated in work, therefore no significant changes in physical structure of work material
3. Non-metal (because of the poor electrical conductivity) that cannot be machined by EDM and ECM can very well be machined by USM.
4. It is burr less and distortion less processes.
5. It can be adopted in conjunction with other new technologies like EDM,ECG,ECM.

Disadvantages

1. Low Metal removal rate
2. It is difficult to drill deep holes, as slurry movement is restricted.
3. Tool wear rate is high due to abrasive particles. Tools made from brass, tungsten carbide, MS or tool steel will wear from the action of abrasive grit with a ratio that ranges from 1:1 to 200:1
4. USM can be used only when the hardness of work is more than 45 HRC.

Material removal models in USM

Theoretical analysis and experimental results have revealed that USM is a form of abrasion and material removal in the form of small grains by four mechanisms
1. Throwing of abrasive grains
2. Hammering of abrasive grains
3. Cavitations in the fluid medium arising out of ultrasonic vibration of tool.
4. Chemical erosion due to micro –agitation

Material removal due to throwing and hammering is significant and MR due to cavitations and chemical erosion can be ignored.

Abrasive particles are assumed to be spherical in shape having diameter d_g. Abrasive particles move under high frequency vibrating tool. There are two possibilities when the tool hit the particle.

➢ If the size of the particle is small and gap between the tool and work is large, then particle will be thrown by tool to hit the work piece.

➢ If the size of the particle is large and gap between tool and work is small, then particle is hammered over the work surface.
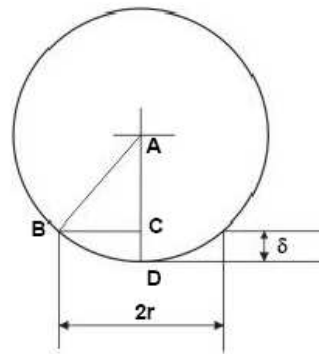
From the geometry

$AB^2 = AC^2 + BC^2$

$$\left(\frac{d_g}{2}\right) = \left(\frac{d_g}{2} - \delta\right)^2 + r^2$$

$$\left(\frac{d_g}{2}\right) - \left(\frac{d_g}{2} - \delta\right)^2 = r^2$$

$r^2 = -\delta^2 + d_g\delta$ Neglecting δ^2 term we can write

$r = \sqrt{d_g\delta}$

Volume of the material removed is the volume of the hemispherical crater due the fracture per grit per cycle.

Volume of the material removed

$$\Gamma = \frac{1}{2}\left[\frac{4}{3}\pi\, r^3\right] = \frac{1}{2}\left[\frac{4}{3}\pi\,(r)^{\frac{3}{2}}\right]$$

=K1 $\left(d_g\delta\right)^{\frac{3}{2}}$ where K1 is constant --(1)

Number of impacts (N) on the work piece by the grits in each cycle depends on number of grits beneath tool at any time. This is inversely proportional to diameter of grit.

N = $\left[\dfrac{K2}{d_g^{\ 2}} \right]$, where K2 is constant --(2)

All the abrasive particle under the tool need not be necessarily effective

MRR = $\begin{Bmatrix} \text{Volume of the material} \\ \text{removed} \\ \text{per grit per cycle.} \end{Bmatrix} X \begin{Bmatrix} \text{Number of} \\ \text{cycles per} \\ \text{second} \end{Bmatrix} X \begin{Bmatrix} \text{Number} \\ \text{of impact} \\ \text{per cycle} \end{Bmatrix} X \begin{Bmatrix} \text{Pr \textit{obability} of abrasive particle} \\ \text{under tool being effective} \end{Bmatrix}$

MRR = Γ f $\dfrac{K_2}{d_g^{\ 2}} \ K_3$ using $\Gamma = K1 \left(d_g \delta \right)^{\frac{3}{2}}$, we have

MRR = K1 K2 K3 f $\sqrt{\dfrac{\delta^3}{d_g}}$ This is the general Material removal equation -------(3)

Model 1 : Grain Throwing model

It is assumed that a particle is hit and thrown by the tool on to work surface. Assuming sinusoidal vibration. Displacement of the tool (y) is given in time period (t) and amplitude (a/2) of oscillation.

X = $\dfrac{a}{2} Sin \left(2 \pi f t \right)$

Velocity = $\dot{V}$ = $\dfrac{a}{2}$ x 2π f $Cos \left(2 \pi f t \right)$

Velocity = $\dot{V}_{Max}$ = π a f

Let us assume that grits also move with same velocity $\dot{V}_{Max}$, then we can write

Kinetic Energy = KE = $\left[\dfrac{1}{2} M \left(\dot{V}_{max} \right)^2 \right]$ = $\left[\dfrac{1}{2} \left(\dfrac{\pi}{6} d_g^{\ 3} \rho_g \right) \left(\pi a f \right)^2 \right]$ ------------------(4)

$\left[\begin{array}{l} NOTE : Here \text{ the diameter of the grit has to be taken to calculate the mass because we are} \\ \text{here calculating the kinetic energy of the grit} \end{array} \right]$

A grit penetrates to the depth equal to δ into the work piece. The work done by the grit is given by

W D by the grit = $\dfrac{1}{2} F \delta$ --(5)

Also we know the Flow strength of material = $\sigma_w = \dfrac{F}{\pi r^2}$ or F = $\sigma_w \pi r^2$ --------(6)

Using (6) in (5) we have,

W D by the grit = $\dfrac{1}{2} \sigma_w \pi \, d_g \delta \, \delta$ ---(7)

W D by the grit should be equal to the Kinetic energy of the particle

$$\frac{1}{2}\sigma_w \pi \, d_g \delta \, \delta \;=\; \left[\frac{1}{2}\left(\frac{\pi}{6} d_g^{\,3} \rho_g\right)(\pi \, a \, f)^2\right]$$

Simplifying we have

$$\delta = \pi \, a \, f \, d_g \sqrt{\frac{\rho_g}{6\sigma_w}}$$ --(8)

Using this (8) in the general equation (3), we have

Volumetric material removal rate due to throwing mechanism

$$\boxed{\mathbf{V_{throwing} \;=\; K1 \; K2 \; K3 \; d_g \; f^{\,5/2}} \left[\frac{\pi^2 \, a^2 \, \rho_g}{6 \, \sigma_w}\right]^{\frac{3}{4}}}$$ ----(9)

Model 2: Grain Hammering Model

When the gap between the tool and the work piece is smaller than the diameter of the grit it will result into partial penetration in the tool (δ_t) as well as in the work piece (δ_w). The values of (δ_t) and (δ_w) depends on the hardness of the tool and work piece material, respectively. Force F acts on abrasive particle only for a short time (ΔT) during the cycle time "T". During this time period, the abrasive particle is in contact with the tool and work piece both. The mean force (F_{avg}) on the grit can be expressed by
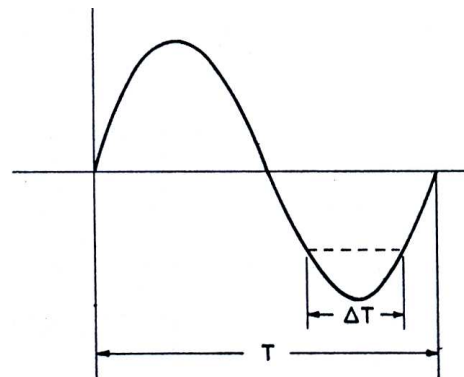
$$F_{avg} = \frac{1}{T}\int_{0}^{T} F(t)\, dt \text{ ---(10)}$$

Here F(t) is the force at any instant of time "t". Force on the grit by the tool starts increasing as soon as grit gets in contact with both tool and the work piece at the same time. It attains maximum value and then starts decreasing until attains the zero value. Hence the momentum equation can be written as

$$\frac{1}{T}\int_{0}^{T} F(t)\, dt = \left[\frac{F}{2}\right]\Delta T \text{ ---(11)}$$
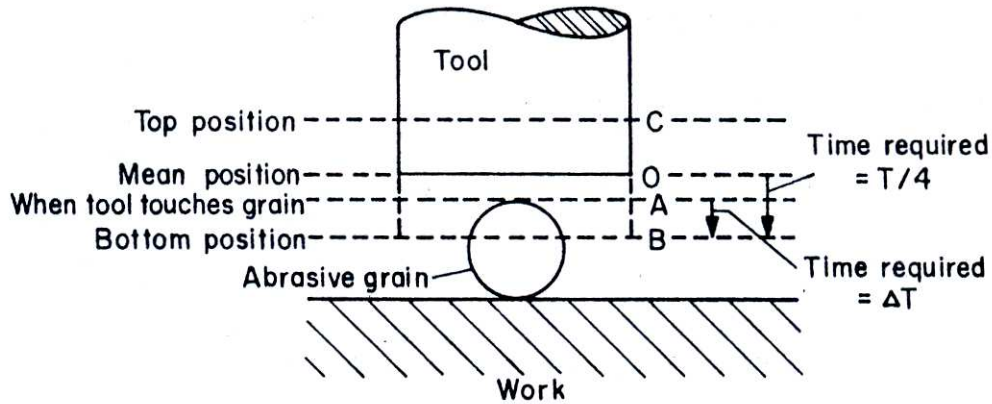
Total penetration due the hammering is given by

$$\delta = (\delta_w) + (\delta_t)$$

a/2 is amplitude of oscillation of the tool. The mean velocity of the tool during the quarter cycle is given by $\dfrac{(a/2)}{(T/4)}$ Therefore , time (ΔT) required to travel from A to B is given by the following equation

$$\Delta T = \frac{\delta T}{2a} \text{ ---(12)}$$

Using (12) in equation (11)

$$F_{avg} = \frac{1}{T}F\frac{\Delta T}{2} = \frac{1}{T}\frac{F}{2}\frac{\delta T}{2a} = \frac{F\delta}{4a}$$

$$F = F_{avg}\left(\frac{4a}{\delta}\right) \text{ ---(13)}$$

Let N be the number of grains under the tool , Stress acting on the tool (σ_t) and work piece (σ_w) can be found as follows.

$$\sigma_w = \frac{F}{N\left(\pi\, d_g \delta_w\right)} \quad \text{--(14)}$$

$$\sigma_t = \frac{F}{N\left(\pi\, d_g \delta_t\right)} \quad \text{--(15)}$$

$$\frac{\sigma_w}{\sigma_t} = \frac{\delta_t}{\delta_w}$$

From the equation (2), (13) and (14) we have

$$\sigma_w = \frac{F}{N\left(\pi\, d_g \delta_w\right)} = \frac{F_{avg}}{\delta}\, \frac{4\, a\, d_g^{\,2}}{K2(\pi\, d_g\, \delta_w)} = \frac{F_{avg}\, 4\, a\, d_g}{K2\, \pi\, d_g\, (\delta_w)^2\left(\dfrac{\delta_t}{\delta_w}+1\right)}$$

Writing $\dfrac{\delta_t}{\delta_w} = \lambda$, rearranging we have

$$\delta_w = \sqrt{\frac{4\, F_{avg}\, a\, d}{\sigma_w\, \pi\, K2\left(\lambda+1\right)}}$$

Volumetric material removal rate from the work piece due to hammering mechanism can be evaluated using the equation (3) as follows:

$$\textbf{MRR = K1\ K2\ K3\ f}\ \sqrt{\frac{\delta_w^{\,3}}{d_g}}$$

$$\boxed{\textbf{V}_{\textbf{Hammering}}\ \ \textbf{= K1\ K2\ K3}\ \left(d_g\right)^{\frac{1}{4}}\ \textbf{f}\ \left[\frac{4\, F_{avg}\, a\, d}{\sigma_w\, \pi\, K2\left(\lambda+1\right)}\right]^{\frac{3}{4}}}\quad \text{----------(16)}$$

PROBLEM 1 : Find out the approximate time required to machine a hole of diameter equal to 6.0 mm in a tungsten carbide plate (Flow strength of work material = 6.9 x 10 ^{9} N/m^2) of thickness equal to one and half times of hole diameter. The mean abrasive grain size is 0.015mm diameter. The feed force is equal to 3.5 N. The amplitude of tool oscillations is 25 microns and the frequency is equal to 25 kHz. The tool material is copper having flow strength= 1.5 x 10 ^{9} N/m^2. The slurry contains one part of abrasives to one part of water. Take the values of different constant as

K1 = 0.3 , K2 = 1.8x 10^{-6} (In SI units) and K3= 0.6 and abrasive slurry density = 3.8 g/cm^3. Also calculate the ratio of the volume removed by throwing mechanism to the volume removed by hammering mechanism.

Data Given:

Diameter of the hole = 6mm = 6 x10^{-3} m
Depth of hole = 1.5d = 9 x 10^{-3} m
Mean abrasive size (d_g) = 1.5 x 10^{-5} m
Feed force (F) = 3.5 N
Amplitude of oscillation = a/2 = 25x10^{-6} m
Frequency of oscillation = f = 25000 CPS
Flow strength of work material = $\sigma_w = 6.9x10^9 \text{ N/m}^2$

Flow strength of tool material = $\sigma_t = 1.5x10^9 \text{ N/m}^2$

Abrasive grain density ρ_g = 3.8x 10^3 kg/m^3

$$\lambda = \frac{\text{Flow Strength of work material}}{\text{Flow Strength of tool material}} = \frac{\sigma_w}{\sigma_t} = 4.6$$

K1 = 0.3 , K2 = 1.8x 10^{-6} (In SI units) and K3= 0.6

Solution

Grain Throwing Model:

Let us use equations we have developed for the Grain Throwing model

Penetration in work piece due to throwing is given by

$$\delta_{throwing} = \pi \text{ a f d}_g \sqrt{\frac{\rho_g}{6\sigma_w}} = \pi(50x10^{-6})x(2.5x10^4)x(1.5x10^{-5})\sqrt{\frac{3.8x10^3}{6x(6.9x10^9)}} = 1.78x10^{-5} \text{ mm}$$

Volume removed by throwing is given by

$$\textbf{MRR}_{\textbf{Throwing}} \textbf{ = K1 K2 K3 f} \sqrt{\frac{\delta_{throwing}^3}{d_g}}$$

Substituting all the values we have

$$\textbf{MRR}_{\textbf{Throwing}} = 0.3x1.8x10^{-6}x0.6x2.5x10^4 \sqrt{\frac{(1.78x10^{-5})^3}{1.5x10^{-2}}} = 4.97 \text{ x}10^{-3} \text{ mm}^3\text{/s}$$

Grain Hammering Model

Penetration in work piece due to hammering is given by

$$\delta_{Hammering} = \sqrt{\frac{4\,F_{avg}\,a\,d}{\sigma_w\,\pi\,K2\,(\lambda+1)}} = \sqrt{\frac{4\,X3.5x\,(2\,X215X10^{-6})x(1.5x10^{-5})}{(6.9x10^9)\pi\,(1.8x10^{-6})(4.6+1)}} = 2.192x10^{-4}mm$$

Volume removed by throwing is given by

$$\textbf{MRR}_{\textbf{Hammering}}\textbf{=K1 K2 K3 f}\sqrt{\frac{\delta_{Hammering}^{\;3}}{d_g}} = 0.3x1.8x10^{-6}x0.6x\sqrt{\frac{(2.192x10^{-4})^3}{1.5x10^{-2}}}x2.5x10^4$$

$$= 0.2146 \text{ mm}^3/s$$

Total MRR = MRR due to throwing action + MRR due to hammering action
$$= 4.97\ x10^{-3}\ \text{mm}^3/s + 0.2146\ \text{mm}^3/s$$
$$= 0.21957\ \text{mm}^3/s$$

Volume of the hole to be drilled $= \left(\frac{\pi}{4}x6^2\,x9\right) = 254.416\ \text{mm}^3$

Time required to drill a hole $= \dfrac{\text{Volume of the hole to be drilled}}{\text{Volumetric MRR due to both throwing and hammering action}}$

$$= \frac{245.416}{0.21957} = 1158.70 \text{ seconds} = 19.31 \text{ Minutes}$$

Ration of the volumetric MRR due to throwing and hammering is given by

$$\frac{MRR_{Throwing}}{MRR_{Hammering}} = \frac{0.00497}{0.2146} = 0.023$$

Thus, it is evident that the material removed by hammering is much more than by throwing (approximately 45 times) hence , for approximate calculations, MRR by throwing can be ignored.

Practice Problem
1. Cylindrical impression of 10mm diameter and 1mm deep is to be made on WC specimen. Feed force is constant and is equal to 5N. Average diameter of grains in slurry is 10 microns. Tool oscillates with the amplitude of 30 microns at 20 kHz. Abrasive and water ratio in the slurry is 1. Flow strength of WC work piece is 7000 N/mm^2 and that of the copper is 1500 N/mm^2. Calculate the time required to complete the job. Assume K1=0.3 K2 = 1.8mm^2 and K3=0.6. Make the assumption if necessary.
2. Derive an equation suggested by Shaw to obtain volumetric material removal rate considering both throwing and hammering mechanisms.

Machining characteristics

Following are the USM process criteria
1. Material removal rate
2. Geometrical accuracy
3. surface finish
4. Out of roundness

Process criteria are generally influenced by the process parameters
The characteristics of above process parameters on process criteria are as follows

1. Effect of amplitude on MRR

Increase in amplitude of vibration increases MRR. To maximize the amplitude of vibration concentrator should operate at resonance frequency. Under certain circumstances this limits also the maximum size of abrasive to be used.
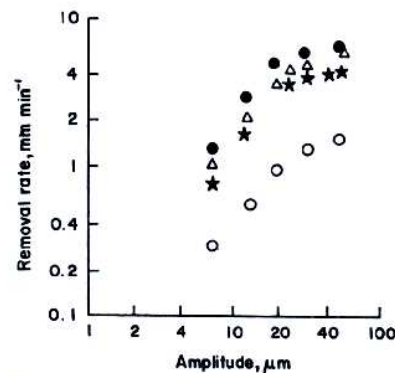
Fig. 3.6   Effects of amplitude of vibration on material removal rate during USM. Workpiece: glass; tool: steel; abrasive: B$_4$C (120 mesh size); pressure: • 0.20 MPa; Δ 0.16 MPa; * 0.10 MPa; ○ 0.04 MPa. [*Kremer et al.,* 1981].

2. Effect of Frequency on MRR

Frequency has significant effect on MRR. Frequency used for machining process must be resonant frequency to obtain the greatest amplitude at the tool tip and thus achieve the maximum utilization of the acoustic system.
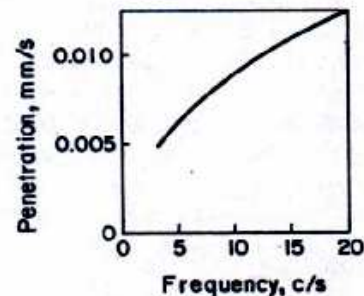

3. Effect of abrasive grain size

An increase in abrasive grain size results in higher MRR but poorer surface finish.
Maximum MRR is achieved when abrasive grain size is comparable with amplitude of vibration of the tool. Hardness of the abrasives and method of introducing the slurry has also effect on MRR.
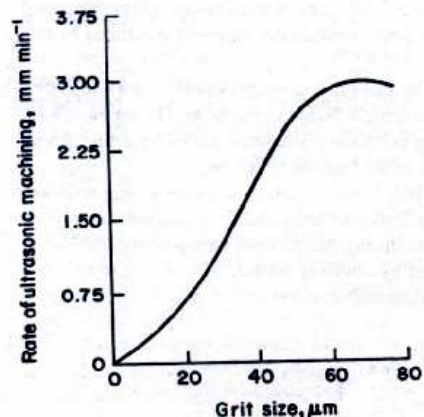
3. Effect of Applied static load (Feed force).

MRR increase with the feed force. Maximum MRR depends on the amplitude of vibrations. Surface finish is found to be little affected by the applied static load. Higher loads, contrary to expections, do not give give a rougher finish. Surface finish, In fact,improves because the grains are crushed to small size with higher loads.
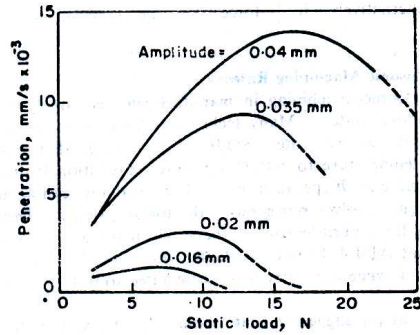

4. Effect of Slurry, Tool and Work Material.

MRR increases with slurry concentration. Slurry saturation occurs at 30 to 40% abrasive/water mixture.

Material Removal rate drops with increasing viscosity.

The pressure with which the slurry is fed into the cutting zone affects MRR . In some cases MRR can be increased even ten times by supplying the slurry at increased pressure.

The shape of the tool affects the MRR. Narrower rectangular tool gives more MRR compared to square cross section. Conical tool gives twice MRR compared to cylindrical tool.

The brittle behavior of material is important in determining the MRR. Brittle material can be cut at higher rates than ductile materials.
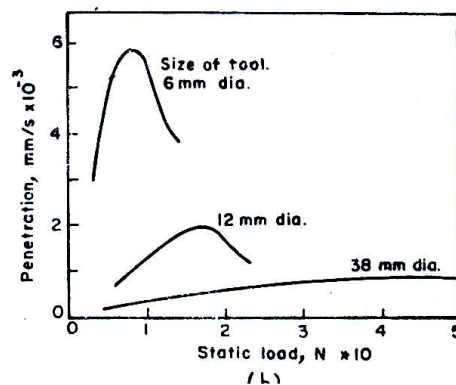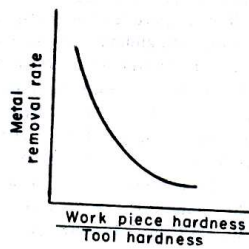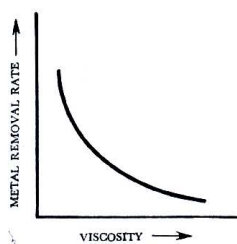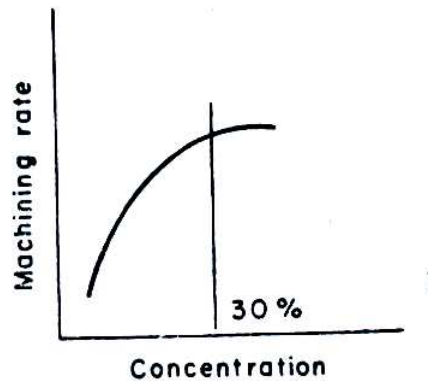
Design of Horn (Velocity Transformer)

The function of horn (also called concentrator) is to amplify and focus vibration of the transducer to an adequate intensity for driving the tool to fulfill the cutting operation. They are made of a hard, non magnetic and easily machinable steel having good fatigue strength like K-Monel,Metal bronze and Mild steel. Linearly tapered and exponentially taped horns have lengths equal to one-half of the wave length of sound in the metal of which they are made.

1 Design of Exponential Concentrator of Circular cross section.

Let us consider case of exponential horn with circular cross section. This type of the horn is used for cutting large diameter through holes in work piece.

K= transformation ratio – amplification ratio

$$K = \frac{D_o}{d} = \sqrt{\frac{S_o}{S_1}}$$

Where D_o is the larger diameter
d is the smaller diameter
S_o is the area of the larger section
S_1 is the area of the smaller section

Taper index for an exponential horn

$$\beta == \frac{\ln(K)}{L}$$

Fig. 2.4 Exponential types of tool cone used in ultrasonic machining

The value of K should be chosen taking into consideration machining condition and the properties of magnestrictive materials K = 3 to 4 for fine machining and 4 to 5 for rough machining.

The length of the horn or concentrator **L** is usually chosen as half wave (n=1) or full wave (n=2).

Length of the exponential Horn is given by

$$L = \frac{nC}{f} \sqrt{1 + \left\{ \frac{\ln(K)}{2\pi n} \right\}^2}$$

And the variation of D is given by

$$D = D_o \exp^{-\beta X}$$ - this is also called Law of change of shape.

A check must be made to ensure that operating frequency exceeds critical frequency f_c given by $f_C = \dfrac{\beta C}{2\pi}$

Problem 1
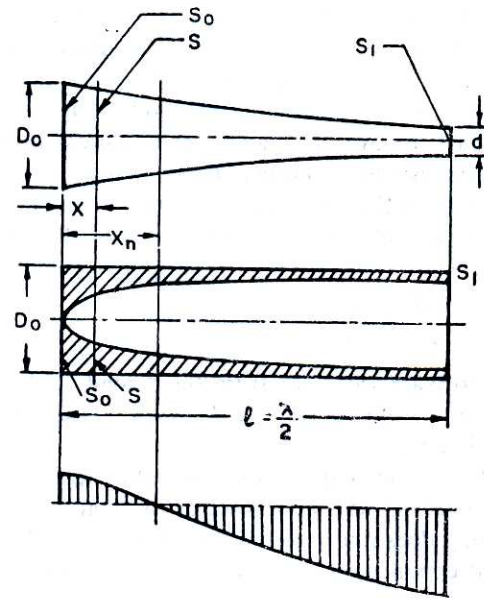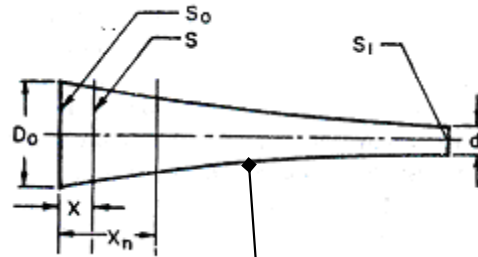
Design a HALF WAVE steel exponentially tapered concentrator with circular cross section to work at a frequency of 20 kHz. The transducer has larger diameter of 100mm and smaller diameter of 4mm. Take the velocity of sound in steel C= 5×10^6 mm/sec.

Data given

D_O = 100mm

d = 4mm

f = 20kHz = 20000 CPS.

C= 5×10^6 mm/sec.

n = ½ (because it is half wave horn)

1. Concentrator ratio or Amplification ratio or Transformation ratio.

$$K = \frac{D_o}{d} = \frac{100}{4} = 20$$

2. Length of exponential horn

$$L = \frac{n\,C}{f}\sqrt{1+\left\{\frac{\ln(K)}{2\pi\,n}\right\}^2} \;=\; \frac{1}{2}\,x\,\frac{5 \times 10^6}{20000}\sqrt{1+\left\{\frac{\ln(20)}{2\pi\,\tfrac{1}{2}}\right\}^2} \;=\; 172.7 \text{ mm}$$

Profile vary like this

3. Taper index for an exponential horn

$$\beta == \frac{\ln(K)}{L} = \frac{\ln(20)}{172.7} = 0.017344$$

4.Law of change of shape is given by

$$D = D_o\,\exp^{-\beta X} = 100\exp^{-0.017344}$$

5.Check the critical frequency

$$f_C = \frac{\beta\,C}{2\pi} = \frac{0.017344}{2}\,x\,\frac{5 \times 10^6}{\pi} = 13803 Hz > \text{ operating frequency of 20000 Hz}$$

Design is safe, therefore Concentrator will work satisfactorily.

Law of change of shape is given in following table.

X (mm)	0.017344X	$\exp^{(-0.017344X)}$	$100\exp^{(-0.017344X)}$
0	0	1	100.00
10	-0.173	0.8408	84.08
30	-0.520	0.5943	59.43
50	-0.867	0.4201	42.01
70	-1.214	0.2970	29.70
90	-1.561	0.2099	20.99
110	-1.908	0.1484	14.84
130	-2.255	0.1049	10.49
150	-2.602	0.0742	7.42
170	-2.948	0.0524	5.24
173.7	-3.013	0.0492	4.92 (4mm)

Law of change of shape

Diameter of horn / Lenght of horn
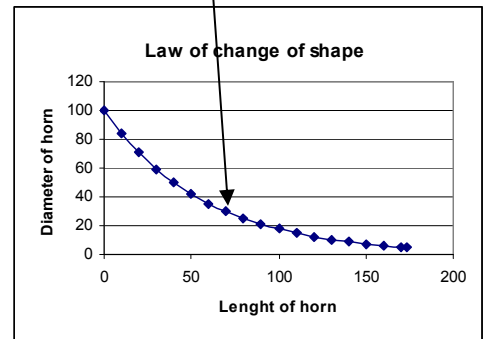
## 2 Design of Exponential Concentrator of Rectangular cross section.

Let us consider case of exponential horn with Rectangular cross section. This type of the horn is widely used in ultrasonic cutting.

K= transformation ratio – amplification ratio

$$K = = \sqrt{\frac{S_o}{S_1}} = \sqrt{\frac{a_o}{a_1}}$$

Where,
S_o is the area of the larger section
S_1 is the area of the smaller section
a_o and a_1 are widths at larger and smaller end.
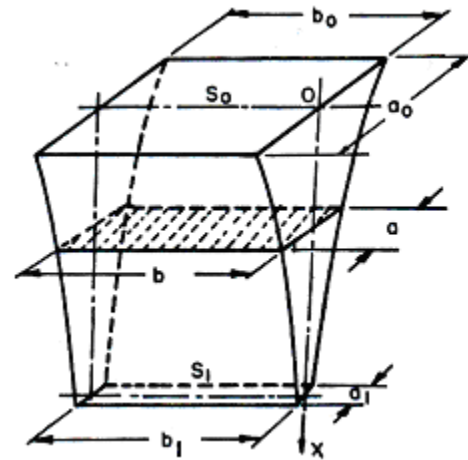
Taper index for an exponential horn

$$\beta == \frac{\ln(K)}{L}$$

The length of the horn or concentrator **L** is usually chosen as half wave (n=1) or full wave (n=2).

Length of the exponential Horn is given by

$$L = \frac{nC}{f}\sqrt{1+\left\{\frac{\ln(K)}{2\pi n}\right\}^2}$$

And the variation of D is given by

$$a = a_o \exp^{-2\beta X}$$ - this is also called Law of change of shape.

A check must be made to ensure that operating frequency exceeds critical frequency f_c given by $f_C = \frac{\beta C}{2\pi}$

Problem 2

Design a HALF WAVE steel exponentially tapered concentrator with rectangular cross section to work at a frequency of 20 kHz. The transducer has larger rectangular cross section of 100x100mm and smaller rectangular section of 4x4mm. Take the velocity of sound in steel $C= 5x10^6$ mm/sec.

Data given

$a_O = 100$mm
$a_1 = 4$mm
$f = 20$kHz $= 20000$ CPS.
$C= 5x10^6$ mm/sec.
$n = ½$ (because it is half wave horn)
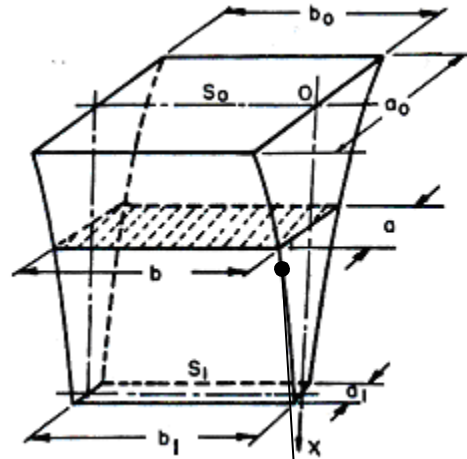
1. **Concentrator ratio or Amplification ratio or Transformation ratio.**

$$K = = \sqrt{\frac{S_o}{S_1}} = \sqrt{\frac{a_o}{a_1}} = \sqrt{\frac{100}{4}} = 4.47$$

2. **Length of exponential horn**

$$L = \frac{n\,C}{f}\sqrt{1+\left\{\frac{\ln(K)}{2\pi n}\right\}^2} = \frac{1}{2}x\frac{5x10^6}{20000}\sqrt{1+\left\{\frac{\ln(4.47)}{2\pi\,½}\right\}^2} = 138.47 \text{ mm``}$$

3. **Taper index for an exponential horn**

$$\beta == \frac{\ln(K)}{L} = \frac{\ln(4.47)}{138.47} = 0.0108138$$

4.Law of change of shape is given by

$$a = a_o\,\exp^{-2\beta X} = 100\exp^{-0.021627X}$$

Profile vary like this

5.Check the critical frequency

$$f_C = \frac{\beta\,C}{2\pi} = 1.5X\frac{0.0108138}{2}x\frac{5x10^6}{\pi} = 1.5X8606 > \text{operating frequency of 20000 Hz}$$

Design is safe, therefore Concentrator will work satisfactorily.
Law of change of shape is given in following table.

X (mm)	0.021627X	$\exp^{(-0.021627X)}$	$100\exp^{(-0.021627X)}$
0	0	1	100.00
10	-0.21627	0.8055	80.55
20	-0.43254	0.6489	64.89
40	-0.86508	0.4210	42.10
60	-1.29762	0.2732	27.32
80	-1.73016	0.1773	17.73
100	-2.1627	0.1150	11.50
120	-2.59524	0.0746	7.46
138.7	-2.9996649	0.0498	4.98 (4mm)

Law of change of shape

Diameter of horn

Lenght of horn
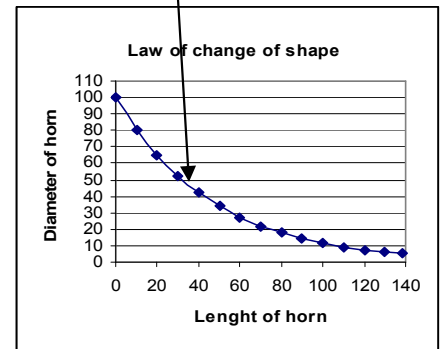
3 Design of Exponential Concentrator of hollow cylindrical cross section.

Let us consider case of exponential horn with hollow cylindrical cross section. This type of the horn is widely used trepanning that is cutting along contours of the desired opening.

K= transformation ratio – amplification ratio

$$K = \sqrt{\frac{(D_o)^2}{(D_a)^2 - (D_1)^2}}$$

Where,

D_o = diameter of the larger section(outside)
D_1 = diameter of the smaller section (inside)

Taper index for an exponential horn

$$\beta = \frac{\ln(K)}{L}$$

The length of the horn or concentrator **L** is usually chosen as half wave (n=1) or full wave (n=2).
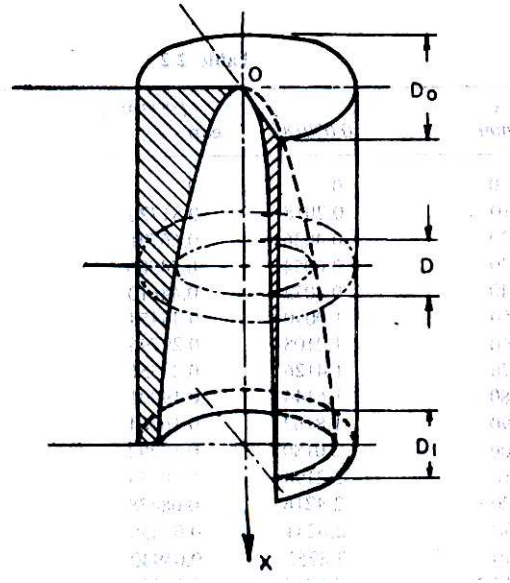
Length of the exponential Horn is given by

$$L = \frac{nC}{f} \sqrt{1 + \left\{ \frac{\ln(K)}{2\pi n} \right\}^2}$$

And the variation of D is given by

$$D = D_o \sqrt{1 - \left(\exp^{-2\beta X} \right)}$$ - this is also called Law of change of shape.

A check must be made to ensure that operating frequency exceeds critical frequency f_c given by $f_C = 1.5 \frac{\beta C}{2\pi}$

___

Jagadeesha T, Assistant Professor, MED, National Institute of Technology, Calicut

Problem 3

Design a steel exponentially tapered concentrator to work at a frequency of 20 kHz. The transducer has hollow cylindrical cross section of larger diameter =9mm and smaller diameter of 7mm.(thickness of tool =1mm) Take the velocity of sound in steel C= 5×10^6 mm/sec. take wave number =0.425.

Data given

D_O = 9mm ;D_1 = 7mm

f = 20kHz = 20000 CPS.;C= 5×10^6 mm/sec.

n = 0.425

1.K= transformation ratio – amplification ratio

$$K = \sqrt{\frac{(D_o)^2}{(D_a)^2 - (D_1)^2}} = \sqrt{\frac{(9)^2}{(9)^2 - (7)^2}} = 1.59099$$

Where,

D_o = diameter of the larger section(outside)

D_1 = diameter of the smaller section (inside)

2. Length of exponential horn

$$L = \frac{n\,C}{f}\sqrt{1 + \left\{\frac{\ln(K)}{2\pi n}\right\}^2} = 0.425 x \frac{5 \times 10^6}{20000}\sqrt{1 + \left\{\frac{\ln(1.59099)}{2\pi 0.425}\right\}^2} = 107.84 \text{ mm}$$

3. Taper index for an exponential horn

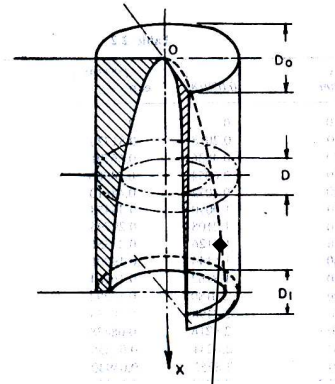$$\beta = \frac{\ln(K)}{L} = \frac{\ln(1.59099)}{107.84} = 0.0043058$$

4.Law of change of shape is given by

$$D = D_o\sqrt{1 - \left(\exp^{-2\beta X}\right)} = 9\sqrt{1 - \left(\exp^{-2x0.043058\,X}\right)}$$

5.Check the critical frequency

$$f_C = 1.5\frac{\beta\,C}{2\pi} = \frac{0.0043058}{2} x \frac{5x10^6}{\pi} = 1.5 X 3430 = 5150 > \text{ operating frequency of 20kHz}$$

Design is safe, therefore Concentrator will work satisfactorily.

Profile vary like this

X (mm)	0.008611x	1- $\exp^{(-0.008611X)}$	d (mm)
0	0	0	0
10	0.08611	0.2872	2.585
20	0.17222	0.39775	3.58
30	0.25833	0.47714	4.29
40	0.3444	0.5397	4.858
50	0.4305	0.5915	5.323
60	0.5166	0.6352	5.72
100	0.8611	0.7598	6.83
107.8	0.9282	0.7776	7

Law of change of shape

Diameter of horn

Length of horn