Q.1] Assume 2 inputs $x_1[n]$ and $x_2[n]$, and $y_1[n]$ and $y_2[n]$

as corresponding outputs.

So, $y_1[n] = x_1[n] - y_1[n-1]\left(y_1[n-1]-1\right)$

$y_2[n] = x_2[n] - y_2[n-1]\left(y_2[n-1]-1\right)$

Now, take $x_3[n] = \alpha x_1[n] + \beta x_2[n]$

Then, $y_3[n] = x_3[n] - y_3[n-1]\left(y_3[n-1]-1\right)$

$\Rightarrow y_3[n] = \alpha x_1[n] + \beta x_2[n] - y_3[n-1]\left(y_3[n-1]-1\right)$

Now, $\alpha y_1[n] + \beta y_2[n]$

$= \alpha x_1[n] + \beta x_2[n]$

$- \alpha y_1[n-1]\left(y_1[n-1]-1\right)$

$- \beta y_2[n-1]\left(y_2[n-1]-1\right)$

Clearly $y_3[n] \neq \alpha y_1[n] + \beta y_2[n]$.

Hence system is non-linear.

Notes

Now, consider input $x[n]$ and output $y[n]$.

Then $y[n] = x[n] - y[n-1](y[n-1]-1)$

Now, consider input $x_4[n] = x[n-n_0]$ and output $y_4[n]$.

$$y_4[n] = x_4[n] - y_4[n-1](y_4[n-1]-1)$$

$$\Rightarrow y[n-n_0] = x[n-n_0] - y[n-n_0-1](y[n-n_0-1]-1)$$

Hence, the system is time invariant

Now, $y[n] = x[n] - y[n-1](y[n-1]-1)$

Let $x[n] = \alpha \mu[n]$ and $y[-1] = 1$

So, $y[n] = \alpha\mu[n] - y[n-1](y[n-1]-1)$

As $n \rightarrow \infty$ $y[n] = y[n-1]$

as $\infty - 1 = \infty$

9.00

So, let $\lim_{n\to\infty} y[n] = \lambda$

10.00

Then $\lim_{n\to\infty} y[n-1] = \lambda$

11.00

So, $\lambda = \alpha - \lambda(\lambda - 1)$ $[\mu[+ve]=1]$

12.00

$\lambda = \alpha - \lambda^2 + \lambda$

$\lambda^2 = \alpha$

1.00

$\lambda = \sqrt{\alpha}$

2.00

So, as $n \to \infty$ $y[n] \to \sqrt{\alpha}$

3.00

4.00

5.00

6.00

7.00

Notes

Birthday / Anniversary

Q.2.] Suppose $y[n]$ is a convolution sum of finite length signals $x[n]$ and $h[n]$ of lengths L and M respectively. Then finite length sequence of $y[n]$ is $L+M-1$.

Length of $r[n]$ = $L+K+1$

Length of $v[n]$ = $N-M+1$

$$\text{length of } x[n] = (L+K+1) + (L+K+1) - 1$$
$$= 2(L+K)+1$$

First sample of $x[n]$ will be at $n = -2K$ and last at $n = 2L$

Hence for $x[n]$, $-2K \le n \le 2L$.

$$\text{Length of } y[n] = 2(N-M+1) - 1$$
$$= 2(N-M)+1$$

Similar to above, for $y[n]$

$2M \le n \le 2N$

Length of $w(n) = (L+K+1) + (N-M+1) - 1$

$= (L+K+N-M)+1$

First sample of $w[n]$ occurs at $n = M-K$ and last at $n = L+N$,

Hence for $w[n]$, $M-K \leq n \leq L+N$

Q.3]

$$\sum x[n] = 2-4+7+3$$
$$= 8$$

$$\sum v[n] = -5+11$$
$$= 6$$

$$\sum y[n] = 18-52+37+45$$
$$= 48$$

Hence $\left(\sum x[n]\right)\left(\sum v[n]\right) = \left(\sum y[n]\right)$

Now, we observe the sequences $\sum(-1)^n x[n]$, $\sum(-1)^n v[n]$, $\sum(-1)^n y[n]$.

Now, $\sum(-1)^n x[n] = 2+4-7+3 = 2$

$$\sum(-1)^n v[n] = 3-2-4+1-6$$
$$= -8$$

$$\sum(-1)^n y[n] = -16$$

S	M	T	W	T	F	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28					

FEB '11

January 2011

Week 02

Day 012 • 353

Date 12 • 01 • 2011

12

Wednesday

9.00 Hence $\left(\sum (-1)^n x[n]\right)\left(\sum (-1)^n v[n]\right) = \left(\sum (-1)^n y[n]\right)$

10.00

11.00

12.00

1.00

2.00

3.00

4.00

5.00

6.00

7.00

Notes

Birthday / Anniversary

FEB MAR APR MAY JUN

Q.4.] Note that :

$$r_{xy}[n] = \sum_{l=-\infty}^{\infty} x[l]\, y[l-n]$$

$$= \sum_{l=-\infty}^{\infty} x[l]\, y[l-n]$$

$$= \sum_{l=-\infty}^{\infty} x[l]\, y[-(n-l)]$$

$$r_{xy}[n] = x[n] \circledast y[-n] \rightarrow (1)$$

Now, $r_{yx}[n] = \sum_{l=-\infty}^{\infty} y[l]\, x[l-n]$

$$= \sum_{l=-\infty}^{\infty} x[l-n]\, y[l-n+n]$$

$$= \sum_{u=-\infty}^{\infty} x[u]\, y[u+n]$$

$$= r_{xy}[-n]$$

9.00

Q.5] Note that, in general:

$r_{xy}[-n] = r_{yx}[n]$ by the result in Q 4

10.00

11.00

By replacing y with x:

12.00

$r_{xx}[-n] = r_{xx}[n]$.

1.00

Hence, $r_{xx}[n]$ is an even sequence.

2.00

3.00

Now, $r_{xx}[n] = \sum_{l=-1}^{4} x[l]\,x[l-n]$

4.00

$r_{xx}[0] = \sum_{l=-1}^{4} x[l]\,x[l]$

5.00

$= 3^2 + 4^2 + 2^2 + 5^2 + 4^2$

6.00

$= 32 + 4 + 25 + 9$

$= 45 + 25$

7.00

$= 70$

Notes

Birthday / Anniversary

15

Saturday

January 2011

Week 02

Day 015 • 350

Date 15 • 01 • 2011

	S	M	T	W	T	F
JAN '11	30	31				
	2	3	4	5	6	7
	9	10	11	12	13	14
	16	17	18	19	20	21
	23	24	25	26	27	28

9.00

$$r_{xx}[1] = \sum_{l=-1}^{4} x[l]\,x[l-1]$$

10.00

$$= (4)(-3) + 5(-2) + 4(5)$$

$$= -12 - 10 + 20$$

11.00

$$= -2$$

12.00

$$r_{xx}[2] = \sum_{l=-1}^{4} x[l]\,x[l-2]$$

1.00

$$= 4(-2) + 4(-2)$$

$$= -16$$

2.00

$$r_{xx}[3] = \sum_{l=-1}^{4} x[l]\,x[l-3]$$

3.00

$$= (-3)(-2) + 4(5)$$

4.00

$$= 6 + 20$$

$$= 26$$

5.00

$$r_{xx}[4] = \sum_{l=-1}^{4} x[l]\,x[l-4]$$

6.00

$$= 5(-3) + 4(4)$$

7.00

$$= 1$$

Notes

Birthday / Anniversary

9.00

$$r_{xx}[5] = \sum_{l=-1}^{4} x[l]\,x[l-5]$$

10.00

$$= 4(-3)$$

$$= -12$$

11.00

Since $r_{xx}[n] = r_{xx}[-n]$

12.00

$r_{xx}[n] = \{-12, 1, 26, -16, -2, 70, -2, -16, 26, 1, -12\}$

1.00

Now, $r_{yy}[0] = \sum_{l=-3}^{2} y[l]\,y[l]$

2.00

$$= 1 + 9 + 4 + 36 + 49$$

3.00

$$= 99$$

4.00

$$r_{yy}[1] = \sum_{l=-3}^{2} y[l]\,y[l-1]$$

5.00

$$= 3 - 6 - 42$$

$$= -45$$

6.00

$$r_{yy}[2] = \sum_{l=-3}^{2} y[l]\,y[l-2]$$

7.00

$$= -2 - 12$$

$$= -14$$

Notes

Birthday / Anniversary

	S	M	T	W	T	F	S
JAN '11	30	31					1
	2	3	4	5	6	7	8
	9	10	11	12	13	14	15
	16	17	18	19	20	21	22
	23	24	25	26	27	28	29

9.00

$$r_{yy}[+3] = \sum_{l=-3}^{2} y[l]\, y[l-3]$$

10.00

$$= 6(3) + 14$$

$$= 32$$

11.00

$$r_{yy}[4] = \sum_{l=-3}^{2} y[l]\, y[l-4]$$

12.00

$$= 6 - 21$$

1.00

$$= -15$$

2.00

$$r_{yy}[5] = \sum_{l=-3}^{2} y[l]\, y[l-5]$$

3.00

$$= -7$$

4.00

Since $y[n] = y[-n]$

5.00

$$r_{yy}[n] = \{-7, -15, 32, -14, -45, 99, -45, -14, 32, -15, -7\}$$

6.00

7.00