Probability for CS

- This is an 8 weeks (modular) course, compulsory for CS students.
- Students already know some of the concepts. The course will consolidate those concepts & give many relevant examples.
  → Which is required in future courses like Machine learning, Randomized algorithms, Randomized methods in complexity, etc.

- I thank the previous instructors: Surender Baswana & Rajat Mittal.

- Probability, or chance, of an event E is $P(E) = \frac{\text{favorable possibilities}}{\text{all "}}$.

- Seeing this as a diagram:

$\Rightarrow P(E) = \frac{\text{event}}{\text{sample space}} = \frac{|E|}{|\Omega|}$.

Eg. 1: A fair coin when tossed gives Heads or Tails with equal probability.

$\Rightarrow P(\text{Head}) = |E|/|\Omega| = 1/2 \ (=50\%)$

▷ $0 \le P(E) \le 1$

▷ $P(E) + P(\overline{E}) = 1$

exactly one of the two happen!

Eg. 2: A dice has 6 sides. Roll two dices to get numbers X & Y. What's the probability that they are coprime?

- $P(X, Y \text{ coprime}) = \dfrac{① + ② + ③ + ④ + ⑤ + ⑥}{6 \times 6}$

$= \dfrac{6 + 3 + 4 + 3 + 6 + 2}{6 \times 6} = \dfrac{24}{6 \times 6} = \dfrac{2}{3} \cdot (\approx 67\%)$

Qn: Did we assume the two dices to be distinguishable or indistinguishable?

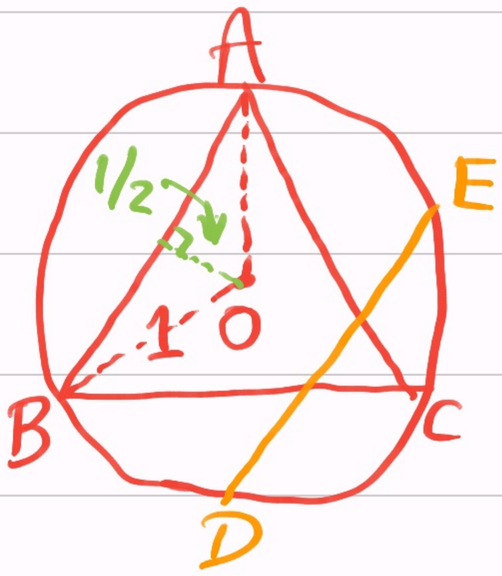
Eg.3: We pick random numbers $a, b \in \mathbb{N}$.
What's $P(\gcd(a,b) = 1) = ?$

- This is tricky because of the infinite sample space.

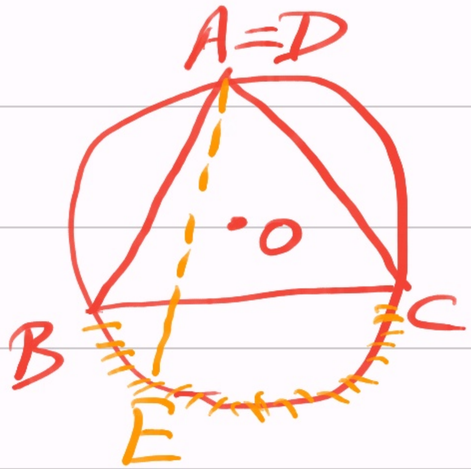
⇒ We have to be careful in defining $P(\cdot)$ value for an element or <u>subset</u>.

– So, here is a <u>paradox</u>:

<u>Ex.4</u>: An equilateral-triangle ABC in a unit circle centered at O.
Draw a <u>random</u> chord <u>DE</u>.
What is $P(|DE| \geq |AB|) = ?$



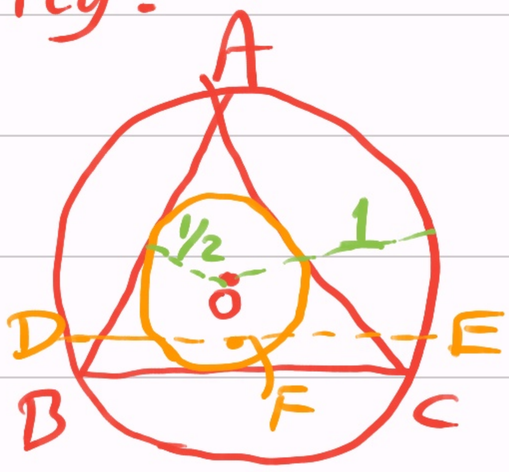
Process-1: Pick D, E randomly.



Wlog $D = A$. Then favorable E fall on the BC-arc.

$$\Rightarrow P(|DE| \geq |AB|) = \frac{|BC\text{-arc}|}{|\text{circle}|} = \frac{1}{3}.$$

Process-2: Pick the mid-point of DE randomly.



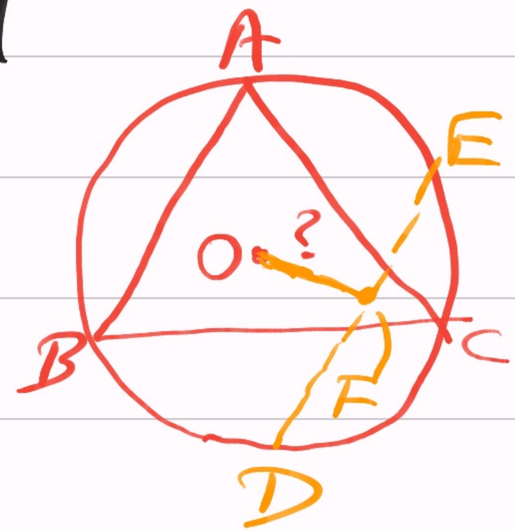
Favorable F fall in the inscribed circle. (of radius $= \sin 30° = 1/2$)

$$\Rightarrow P(|DE| \geq |AB|) = \frac{\pi (1/2)^2}{\pi\, 1^2} = 1/4.$$

Process-3: Randomly pick the distance $|OF|$.

$$\Rightarrow P(|DE| \geq |AB|) = \frac{\text{inscribed-circle-rad}}{1}$$

$$= 1/2 .$$



▷ There might be many ways to define $P(\cdot)$ on subsets E of an infinite sample space Ω.

↳ Probability is not merely counting!

— Now an eg. that is practical & often confused!

Eg.5: Say, a covid19 RT-PCR test is 80% accurate. I went for the test & "unfortunately" it came out positive. What's P(I have covid)=?

→ Very tempting to say 0.8 ; but very wrong!

- Exact answer requires the knowledge of:

1) How many people <u>have</u> covid19?

2) How many times is the test +ve, in general?

→(1) = True positives + False negatives.

→(2) = True positives + False positives.

▷ If (1) $\ll$ (2), then it's a "bad" test $\Rightarrow$
$P(\text{I have covid}) \ll 0.8$.

[Same as saying: False positives $\gg$ False negatives.]

- This makes testing, survey & prediction a complicated affair!
  → Prone to misinterpretation in the media.

- Lastly, probability is sometimes used to prove the existence of an object. (non-constructive proof?)
  → probabilistic method

Eg.6: Let $B \subseteq \mathbb{Z}$ with $|B| = n$. There exists $A \subseteq B$, with $|A| \geq n/3$, that is sum-free: i.e. no two elements of A sum up to an element in A.

Outline of the course

- Basic concepts from high-school, JEE....
- Random variables, Expectation, Moments, Distributions.
- Conditional probability, Concentration inequalities, Page rank algorithm, Probabilistic method.

Grading: Quiz x 3 – 45%
Assignment x 3 – 30%
Exam x 1 – 25%

(Plagiarism etc. will be severely punished!)