EE 200: Problem Set 3

1. Show that the analog system with an input/output relation given by

$$y(t) = \int_{t-t_0}^{t+t_0} x(\tau)d(\tau)$$

where $y(t)$ and $x(t)$ are, respectively, the output and input signals, is a linear, non-causal, and time-invariant system.

2. Evaluate the following convolution integrals:

   (a) $y_1(t) = [\mu(t) - \mu(t-1)] \circledast [\mu(t) - \mu(t-1)]$

   (b) $y_2(t) = \mu(t) \circledast e^{-\alpha t}\mu(t), \quad \alpha > 0$

3. The periodic convolution integral of two periodic signals $\tilde{g}(t)$ and $\tilde{h}(t)$ with fundamental period T_0 is given by

$$y(t) = \tilde{g}(t) \circledast \tilde{h}(t) = \int_0^{T_0} \tilde{g}(\tau)\tilde{h}(t-\tau)d\tau$$

Show that $y(t)$ is also a periodic signal with a fundamental period T_0.

4. The cross-correlation function $r_{xy}(\tau)$ of two real analog signals $x(t)$ and $y(t)$ is defined by

$$r_{xy}(\tau) = \int_{-\infty}^{\infty} x(\xi)y(\xi - \tau)d\xi$$

and is a measure of the similarity between two analog signals as function of time lag τ.

Evaluate the cross-correlation function for $x(t) = e^{-\alpha t}\mu(t)$, $y(t) = e^{-\beta t}\mu(t)$ $\alpha > 0,\ \beta > 0$.

5. The auto-correlation function $r_{xx}(\tau)$ of a real analog signal $x(t)$ is defined by

$$r_{xy}(\tau) = \int_{-\infty}^{\infty} x(\xi)x(\xi - \tau)d\xi$$

which is a cross-correlation of $x(t)$ with itself.

Evaluate the auto correlation function for $x(t) = \mu(t - \alpha) - \mu(t),\ \alpha > 0$.

6. Show that the inverse of a causal LTI analog system with an impulse response $g(t) = A\delta(t) + Be^{-\alpha t}\mu(t)$ is a causal LTI analog system with an impulse response given by

$$h(t) = \frac{1}{A}\delta(t) - \frac{B}{A^2}e^{-\left(\alpha + \frac{B}{A}\right)t}\mu(t)$$