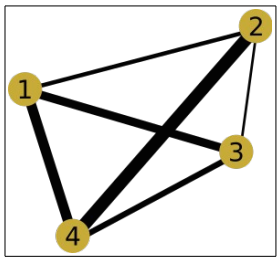
The Quadratic Assignment Problem

Martino Ferrari

1. Quadratic Assignment Problem

The quadratic assignment problem (QAP) is one of the fundamental combinatorial optimization problems.
It models one common logistic problem, and it is part of the *facility location* category of problems.



The QAP models the following situation:
There are a set of n facilities and a set of n locations. For each pair of locations, a distance is specified and for each pair of facilities a weight is specified (e.g., the amount of supplies transported between the two facilities). The problem is to assign all facilities to different locations with the goal of minimizing the sum of the distances multiplied by the corresponding flows.[1]

We can formalize the problem in this way:

$n \in \mathbb{N}$ the number of facilities

$D \in \mathbb{N}^n$ the distance matrix

$W \in \mathbb{N}^n$ the weight matrix

The space of solution S is composed of all the possible permutation of the n facilities.
And the fitness is defined as $f(\psi) = \sum_{i,\, j=1..n} W_{[i][j]} \cdot D_{[\psi_i][\psi_j]}$ where ψ_i is the position of the facility i in the solution ψ .

2. Introduction to the Taboo Search

The Taboo Search (TS) iterate in the neighborhood $N(\psi)$ of a solution ψ to find the next solution ψ' to explore, till some condition (e.g. the number of iterations) has been satisfied. The way it will chose the next solution can vary depending from the problem to solve.
The peculiarity of the method is that while is moving from a solution to next, it updates a taboo list (tl) of length l , in where are stored the solution visited, or more often the movements done, in the last l iterations.
The solutions or the movements in the taboo list are forbidden for the next l iterations.
So we can define $N^1(\psi)$ as $\nexists \psi' \in N^1(\psi) \rightarrow \psi' \in tl$, this is the list of the neighbor not forbidden of a solution ψ .
The taboo list is implemented to augment the diversification and explore more areas of the solutions space.
The size l of the taboo list (that is not necessary implemented like a list) is very important and it changes the behavior of the research. If l is small the algorithm will privilege the *intensification* (selecting neighbors with better fitness) over the *diversification* (exploring new solutions area, hoping to find better fitness). Instead if l is big the algorithm will privilege the *diversification* over the *intensification*.
If l is to small the research can fall in some local maximum/minimum, but if it's to big it could happen that all the neighbors are forbidden.

3. Taboo Search and the QAP

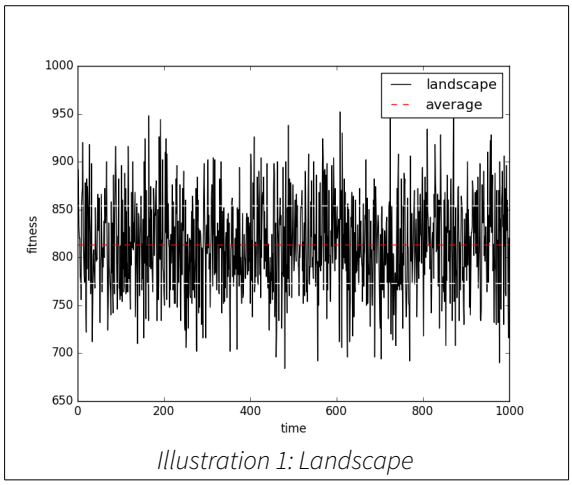


Illustration 1: Landscape

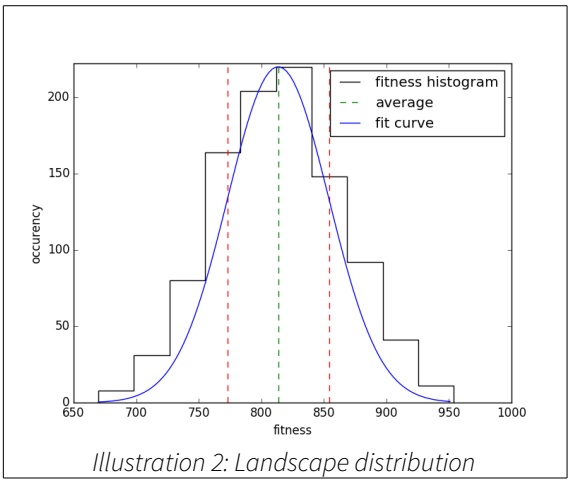


Illustration 2: Landscape distribution