TP4: Ant-System

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1 The Travelling Salesman Problem

Also this time we will study the Travelling Salesman Problem, for this reason I will use the same introduction did for the last TP.

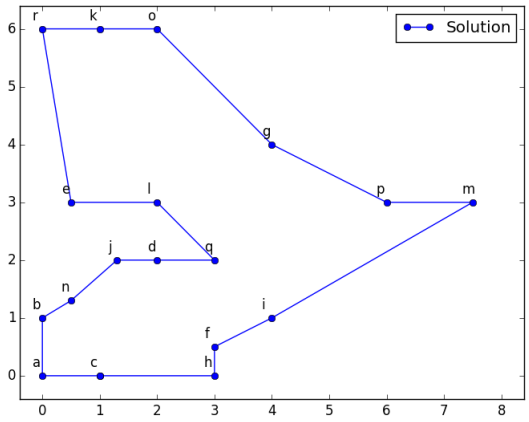


Figure 1. example of a TSP problem

The travelling salesman problem (**TSP**) is a classic and well-known problem, that consists of finding the shortest path connecting a set of cities all interconnected with direct roads (as in figure 1).

Given n cities a solution is $x_i = \{c_1, c_2, ..., c_n\}$ and the solutions space S has size $n!$.

For this reason it's often not feasible to explore S completely and it's only possible to find an acceptable solution trough an heuristic algorithm.

The neighbourhood $N(x_i)$ of a solution x_i is the set of all possible unique permutations of two cities of the itinerary of x_i, so the size of $N(x_i)$ is $\frac{n \cdot (n-1)}{2}$.

2 Ant System

The Ant System (**AS**) is a metaheurstic introduced in the 1992 by Dorigo inspired by Goss and Dereborg experience over the ants behaviour to optimize combinatorial problems.

The **AS** is a swarm intelligence (or collective/group intelligence) kind of algorithm, where many (very) simple individual are used to explore the research space S using the knowledge of all the group. This means that the collective intelligence and its perception are far better than the ones of the singular idividual.

Gross and later Deneubourg studied how ants where able to find the shortest path to the nurtriment source in a controlled enviroment, as showen in figure2.

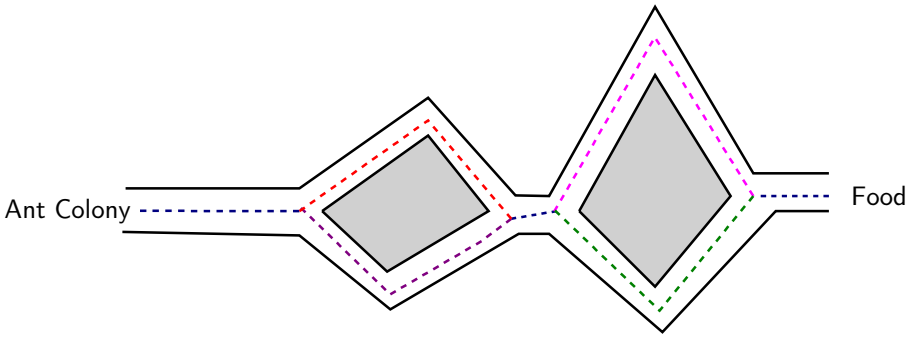


Figure 2. Example of enviroment used in the Gross Experience

This enviroment was build using pipelines with different lenght connecting the ant colony to the food source. The experience shows that a great majority (90% of the ants) ants after a short period of time was choosing the shortest path.

This happens because every time an ant is moving it releases a chemical called *pherormone*, and naturally the ants will follow the strongest path of pherormones. So as the frequency of movement on the shortest path is higher (as is shorter) also the the path of pherormones will be stronger.

Moreover the pherormone naturally evaporates, this implies that the pherormone traces left in the longer and less used path will disappear.

Gross and Deneubourg proposed the following mathematical modelling of the ant behaviour at a crossing:

Given

- L_m: the number of ants that already passed on the L (lower) path
- U_m: the number of ants that already passed on the U (upper) path

The probability P_U that the $(m+1)$th ant will choose the U path will be

$$P_U(m+1) = \frac{(U_m + k)^h}{(U_m + k)^h + (L_m + k)^h}$$

And the probability P_L that the $(m+1)$th ant will choose the L path will be

$$P_L(m+1) = 1 - P_U(m+1) = \frac{(L_m + k)^h}{(U_m + k)^h + (L_m + k)^h}$$

Where k and h are parameter representing the minimum quantity of pherormones need to influence the choise and the ant perception of the pherormones.

The results of the interactions between the ants are structures auto-organized in the time and in the space, where there are not difference of knwoledge or importance between the individuals and any of them as a global knowledge.

This kind of auto-organized system are very interesting because the individuals are very simple and they are in general very robust to the error and the disfuction, some times the performance can benifct from eventual errors, exploring new part of the solution space. More over this kind of system are easely parallesible.

3 The Ant System and the TSP

In our case we will study the adaptation of the **AS** for the **TSP** problem.

As for any other metaheuristics we have to define some elements:

- The research space S
- A start solution x_0
- The neighbourhood of a solution $N(x_i)$
- The exploration operator U
- The stop condition

The research space S will be the one already defined in the section 1 for the **TSP**.

The start solution x_0 is simply a random solution extracted from S.

The neighbourhood of a solution $N(x_i)$ is the entire solution space S, but it will be not explored, instead the successor x_{i+1} will be created by the exploration operator U.

As the **AS** is based on the collective knowledge of m ants at every iteration the algorithm will explore m solutions.

The exploration operator $U(t)$ of an ant will create the successor solution from the path of the ant itself.

Given $\eta_{(i,j)}$ the visiblity of the city j from the city i (the inverse of the distance) and $\tau_{(i,j)}(t)$ the pherormones accumulated between the city j and i at the time t, the ant will chose the next city with probability

$$P_{(i,j)}(t) = \begin{cases} 0 & \text{if } j \notin J \\ \dfrac{[\tau_{(i,j)}(t-1)]^{\alpha}[\eta_{(i,j)}]^{\beta}}{\sum_{l \in J} \{[\tau_{(i,l)}(t-1)]^{\alpha}[\eta_{(i,l)}]^{\beta}\}} & \text{if } j \in J \end{cases}$$

Where J is the set of all the city not yet visited from the ant.

α and β are parameters, bigger α will favorize the pherormone trails and so the intensification while bigger β will favorize the visibility and so the diversification.

It's very important to note that at a time t the m ants will move indipendently from the others, and they are influenced only from the pherormone trails leaved at the time $t-1$.

For this reason the pherormone trails are updated at the end of the iteration t with the following formula:

$$\forall (i,j) \in S \quad \tau_{(i,j)}(t) = (1-\rho)\tau_{(i,j)}(t-1) + \Delta\tau_{(i,j)}(t)$$

Whit $\Delta\tau_{(i,j)}(t)$ defined as:

$$\Delta\tau_{(i,j)}(t) = \sum_{k=0}^{m} \Delta\tau^k_{(i,j)}(t)$$

And $\Delta\tau^k_{(i,j)}(t)$ of the kth ant defined as:

$$\Delta\tau^k_{(i,j)}(t) = \begin{cases} 0 & \text{se } (i,j) \notin T^k(t) \\ \dfrac{Q}{L^k(t)} & \text{se } (i,j) \in T^k(t) \end{cases}$$

Where Q is a constant, $L^k(t)$ the total length of the itinerary found at the time t and $T^k(t)$ the set of the itinerary.

The end condition is simply a maximum number of iteration $t_{\max}$.

4 Implementation

Also this time I implemented the algorithm in C++ using extensively the object-oriented features of the language, reusing all the code I could from the previous TP.

I choose to code the problem using 3 different classes.

The first class represent the collective knowledge, that manage the τ matrix, $\Delta\tau$ matrix, the η matrix and the probability matrix, all $n \times n$ simmetric matrices.

The visibility η matrix is computed once for all, as the distance between the cities never change.

The τ and the probability matrices are updated at the end of every iteration while the $\Delta\tau$ is updated every time a ant ends its walk.

The probability matrix contains the value of $[\tau_{(i,j)}(t-1)]^\alpha[\eta_{(i,j)}]^\beta$ for every pairs of city, this permits to avoid to compute it for every ant and every city combination, but unluckly there is still the needs of normilize this number.

The second calss represent a single ant, and implement most of the logic presented before.

Finally the ant system class that contains the m ants and executes the sequence of operations.

For the α, β and ρ parameters I used the given values, respectively 1, 5 and 0.1.

For Q and τ_0, the intial pherormone value, I used L_{nn} and $\frac{1}{L_{nn}}$, where L_{nn} is the lenght of the solution of a greedy euristic. I presented the greedy euristic in the previous report.

Finally I will discuss in the next section of the value m and $t_{\max}$.

5 Results

This time I will skip the analysis of the landscape as I analyzed the **TSP** landscape in the previous TP.

However before comparing and analyze the performance of my **AS** implementation I made some experiment on how m and $t_{\max}$ impact the quality of the solution and the computational performance of the algorithm.

To do so I used the file *cities.dat*, of whom I discovered the global solution in the last TP, that has lenght of 27.5154.

Using a simple bash script I runned the code with different configuration of m and $t_{\max}$ the results are summarized in the plot of figure 3.

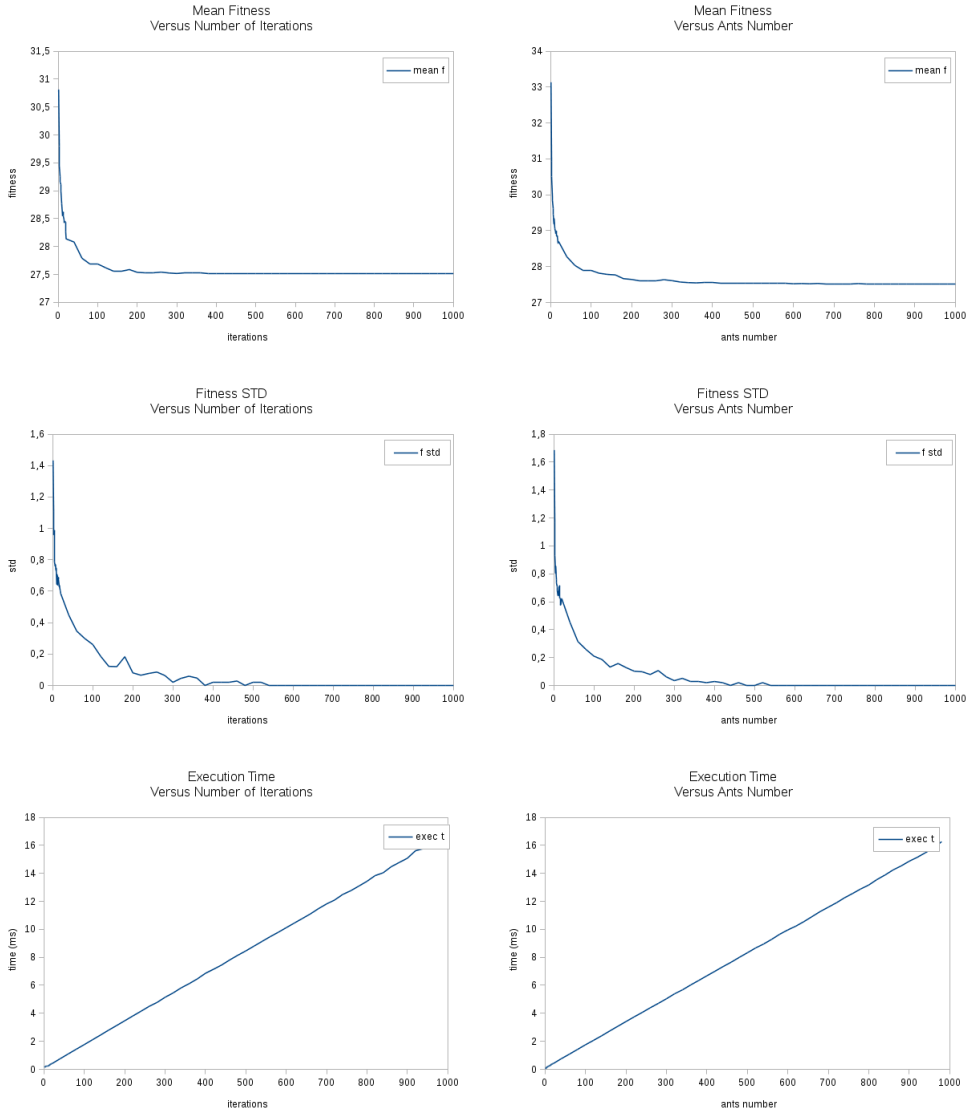


Figure 3. Mean fitness, std and execution times varing $t_{\max}$ and m

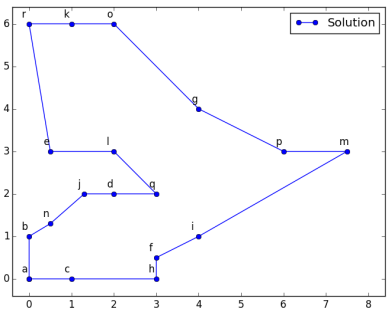
The left figures represent the variations of the mean fitness the standard distribution and the execution times of 100 execution of the algorithm with $m = 10$ ants (equal to the number of city) dependingi on the value of $t_{\max}$ (varied from 1 to 1000).

The right figure represent the variations of the same parameters with $t_{\max} = 10$ depending on the value of m (varied from 1 to 1000).

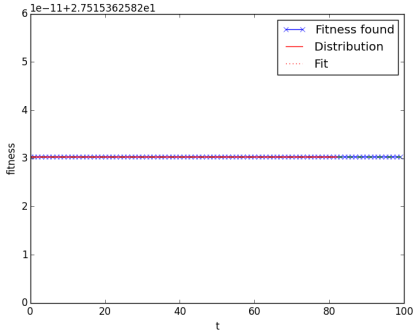
The impact of the parameter m and $t_{\max}$ is equal, and the evolution for both the execution time is linear while the solution quality improve much faster and converge around 540 iterations and 10 ants or 540 ants and 10 iterations.

For the next experiment I choose to use a combination of $m = 160$ ants and a $t_{\max} = 30$, the code executed 100 times over the file *cities.dat* with this values will result in a distribution with standard distribution 0 and execution time of around 9 ms.

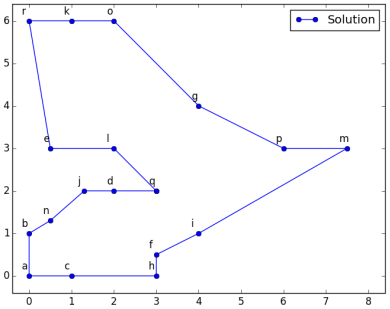
I was asked to compare the results of my **AS** algorithm with the **Greedy** and the simulated annealing (**SA**) metaheuristics, both implemented for the previous TP, over 10 runs. However I chose to run it 100 times to have more relevant data. The results for *cities.dat* were the following:



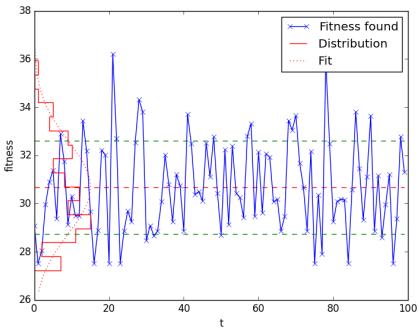
AS best solution



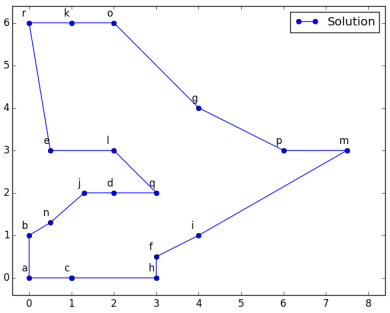
AS distribution



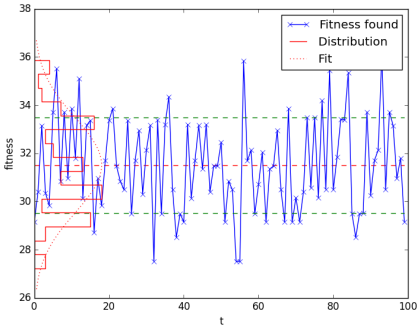
SA best soltution



SA distribution



Greedy best solution



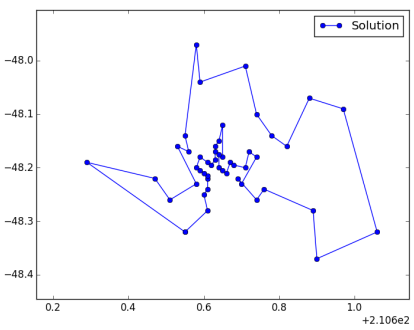
Greedy distribution

Figure 4. Results of 100 runs over *cities.dat*

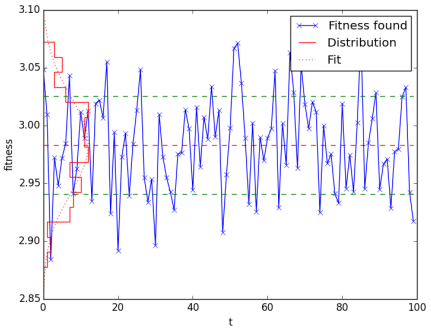
As previsible (knowing the results of the previous TP) the three algorithm are capable to found the optimum solution, however the **AS** outperform the other two, founding the optimum in the 100% of the cases, while the **SA** in the 7% and the **Greedy** only in the 3% of the cases.

The **AS** is performing very well, with an execution time of 8.0 ms for run, while the **SA** needs 15.6 ms and the **Greedy** needs only 0.1 ms.

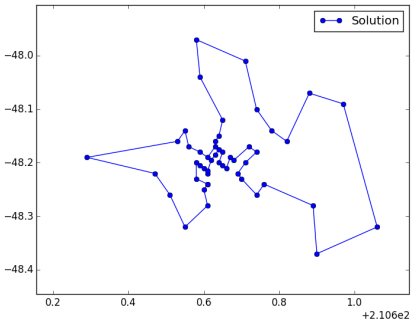
I made similar comparison for the file *cities2.dat*, the results were the following:



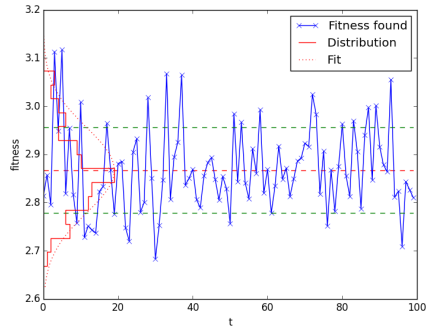
AS best solution



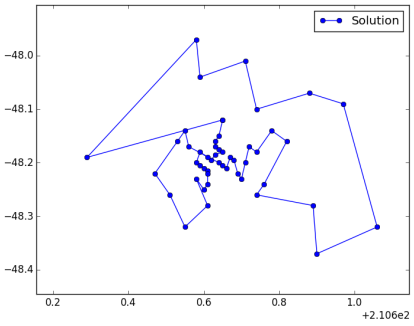
AS distribution



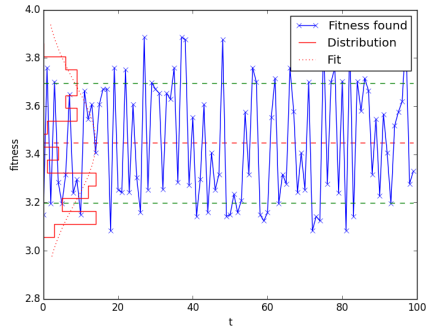
SA best soltution



SA distribution



Greedy best solution



Greedy distribution

Figure 5. Results of 100 runs over *cities2.dat*

In this case the **SA** is performing better then the **AS** and the **Greedy**. The best solutions of the algorithms, as shown in figure 5, are different. For the **SA** the best solution has a length of 2.68 while the one found by the **AS** algorithm has a length of 2.88 and finally the one found by the **Greedy** has lenght of 3.08.

However the distribution of the **SA** and of the **AS** are similar the first with average of 2.86 and standard deviation of 0.09 and the second with average of 2.98 and standard deviation of 0.04, while the **Greedy** has an average of 3.44 and standard deviation of 0.25.

Even the difference of execution time between the **AS** and the **SA** is smaller, with the first that needs only 42.3 ms and the second that needs 75.8 ms, with a ratio of 1.8 while with the file *cities.dat* the ratio was close to 2.0. Again the **Greedy** algorithm has an execution time far better of only 0.8 ms.

However I belive that the quality performance could be improved tuning the parameters of the **AS** to increase the diversification to explore new zones of the solution space S.

The following table will summarise the results of the previous experiments, plus the ones made over random problems of size 50, 60, 80 and 100 (always over 100 runs):

File	n	Algorithm	Best length	Average length	STD	Average test time
cities.dat	18	AS	27.52	27.52	0.00	8.0 ms
		SA	27.52	28.35	0.67	15.6 ms
		Greedy	27.52	31.79	2.17	0.1 ms
cities2.dat	49	AS	2.88	2.98	0.04	45.2 ms
		SA	2.68	2.86	0.09	75.8 ms
		Greedy	3.08	3.44	0.25	0.8 ms
cities50.dat	50	AS	74.82	77.54	1.23	49.0 ms
		SA	72.58	82.51	4.01	118.3 ms
		Greedy	78.09	87.03	5.17	0.7 ms
cities60.dat	60	AS	108.01	115.68	2.13	68.0 ms
		SA	109.15	119.55	4.71	135.0 ms
		Greedy	120.68	127.01	4.46	0.8 ms
cities80.dat	80	AS	147.05	152.48	2.02	109.1 ms
		SA	147.86	162.09	6.48	161.7 ms
		Greedy	144.74	157.56	7.97	1.1 ms
cities100.dat	100	AS	72.59	76.25	1.31	161.9 ms
		SA	77.46	90.79	5.89	159.5 ms
		Greedy	72.60	83.00	5.12	2.0 ms

Globally the **AS** has good quality performance, with comparable (or even better) result than the **SA** metaheuristic and it is in avareage faster than the **SA**. However as the **AS** stop condition is a maximum number of iteration the execution time grow linearly (or quadratic) with the complexity of the problem while the **SA** has a more complex stop condition that make the execution time much more variable and dependent from the landscape of S.

NB: I executed the codes over a different (and more powerfull) machine, and the time expressed in the table is the average test execution time and not the total execution time like in the last TP.

6 Project structure

compile.sh	is the compilation script
compile_basic.sh	is the compilation script without plot dependencies
src/	contains all the source codes (above the core codes)
→/tsp.*	is the code that defines the TSP solution, neighbours, etc
→/antsystem.*	is the code that defines the AS metaheursitic
→/annealing.*	is the code that defines the SA metaheuristic
→/greedy.*	is the code that defines the Greedy heuristic
bin/	contains all the binary, dat files and the compilation script
→/antsystem	runs the AS over a specified file
→/annealing	runs the SA over a specified file
→/greedy	runs the Greedy over a specified file
→/antsystem_ants.sh	script used to analyse the influence of m on the AS performance
→/antsystem_iters.sh	script used to analyse the influence of $t_{\max}$ on the AS performance
bin/csv	contains the csv generated by my scripts

All the executables if executed without arguments will show the basic usage.

The codes and report can also be found on my github repository: Metaheuristic.