Multimedia Security and Privacy

TP1: Basic Image Processing

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Submission

Please archive your report and codes in "Name_Surname.zip" (replace "Name" and "Surname" with your real name), and upload to "`Assignments/TP1: Basic Image Processing`" on `https://chamilo.unige.ch` before Thursday, March 8 2017, 23:59 PM. Note, **the assessment is mainly based on your report, which should include your answers to all questions and the experimental results**.

1 Introduction

This exercise will run through some of the basic image commands from the Matlab imaging tool box.

Exercise

If haven't done so, you should read the Matlab introduction on working with Matrices and Programming.

1. Read in the image `'peppers.png'`. The image is present in Matlab.
2. Display the histogram of the image.
3. Convert the image to grayscale.
4. Determine the global mean and the global variance of the image.
5. Determine the local mean and variance of an image and display them. See Matlab `blockproc` and `colfilt`. Use distinct non-overlapping blocks.

2 Noise

This exercise will showcase two noise types, Additive White Gaussian Noise (AWGN) and *salt & pepper* noise.

Additive White Gaussian Noise

AWGN is a type of *channel* in which the discrete channel output Y_i at some time event with index i is the sum on the input X_i and noise Z_i, where Z_i is independently and identically-distributed (i.i.d.) from a zero-mean Gaussian distribution with standard deviation σ (or variance σ^2). Formally:

$$\begin{aligned} Z_i &\sim \mathcal{N}(0,\sigma) \\ Y_i &= X_i + Z_i \sim \mathcal{N}(X_i,\sigma) \end{aligned}$$

Exercise

1. Write a function that generates an array of size $N \times M$ with Gaussian Noise, i.e. samples drawn from the distribution $\mathcal{N}(\mu,\sigma)$. See the Matlab function `randn`.

Salt & pepper Noise

Salt & pepper noise can be defined as follows. Let there be a $N \times M$ gray scale image whose datatype supports a value range of $\{s_{min}...s_{max}\}$. Let $\mathbf{y}$ denote the noisy image and $\mathbf{x}$ the original image. Then the observed gray level in image $\mathbf{y}$ at pixel location (i,j) is given by:

$$y_{i,j} = \begin{cases} s_{min} & \text{with probability } p \\ s_{max} & \text{with probability } q \\ x_{i,j} & \text{with probability } 1-p-q \end{cases} \tag{1}$$

Exercise

1. Implement a function that generates *salt & pepper* noise with parameters p and q.

Metrics

We will now define and implement two important metrics used in signal and image processing. The Mean Squared Error (MSE) and the Peak Signal to Noise Ratio (PSNR).

Mean Squared Error

The Mean Squared Error (MSE) between two images $\mathbf{x}$ and $\mathbf{y}$ is defined as:

$$\text{MSE} = \frac{1}{N \cdot M} \sum_{i=1}^{N} \sum_{j=1}^{M} (y[i,j] - x[i,j])^2,$$

where N and M are the width and the height of image $\mathbf{x}$ and $\mathbf{y}$.

Exercise

1. Write a function that determines the Mean Squared Error (MSE) between two images $\mathbf{x}$ and $\mathbf{y}$.
2. Read in a new copy of the image `cameraman.tif`, keep it in its original datatype and range, i.e. `uint8` and $\{0..255\}$.
3. Now read in a second copy of the image `cameraman.tif` but map it to `double` and $\{0..1\}$. See Matlab `im2double`. Compare the two images using the MSE. Can you explain the result?

Peak Signal to Noise Ratio

The Peak Signal to Noise Ratio (PSNR) is defined as:

$$\text{PSNR} = 10 \log_{10} \left(\frac{\alpha^2}{\text{MSE}(\mathbf{x}, \mathbf{y})} \right),$$

where α is the maximum value possible with the `type` of $\mathbf{x}$ and $\mathbf{y}$. The unit of the PSNR is dB.

Exercise

1. Write a function that implements the PSNR function.

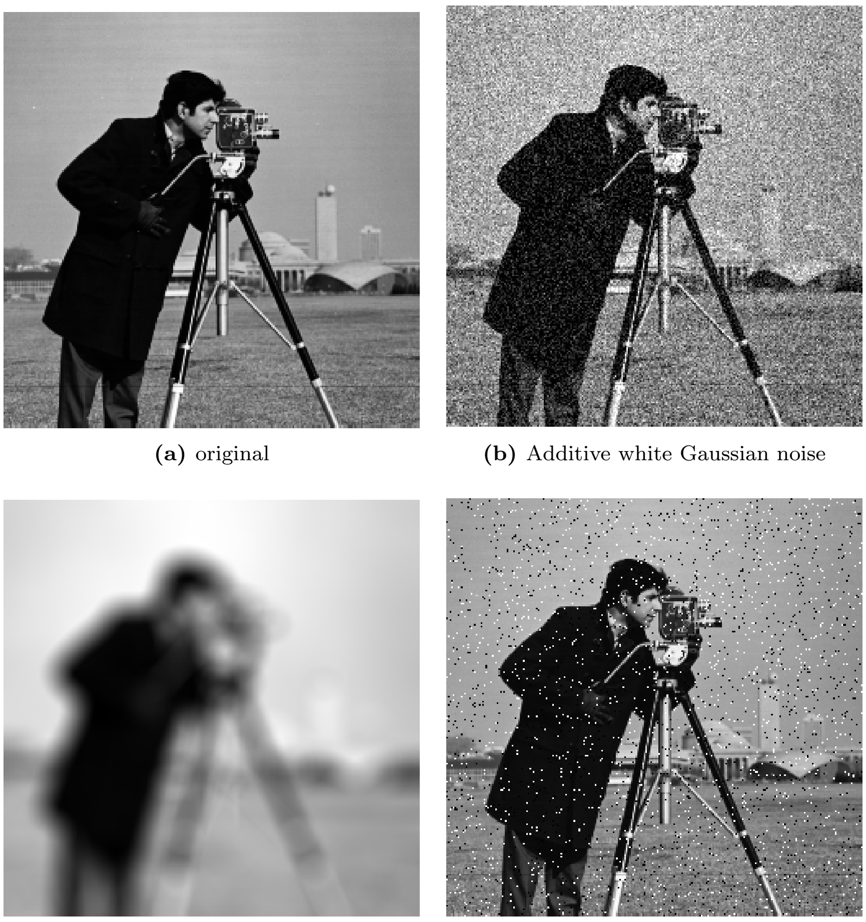
Testing

We will test how various noise strengths influence the PSNR value between the original image and the noisy copy.

Exercise

1. Refractor the PNSR definition such that the PSNR is expressed as a function of the noise variance σ_z^2. You may assume that $\sigma_z^2 = MSE(\mathbf{x}, \mathbf{y})$.
2. Add Gaussian noise to an image such that the PSNR ratio with the original image is 10dB, 20dB, 30dB and 40dB. Use `randn`, **not** `imnoise`.
3. Show the noisy images on the screen. How do they look?
4. Show the histograms for these noisy images, can you explain what you see?

5. Add salt & pepper Noise to an image until the PSNR ratio between the original and the noisy image is 40 dB. Visually compare it to the 40dB noisy image to which Gaussian noise was added. What can you conclude?



(a) original

(b) Additive white Gaussian noise

(c) Gaussian blur

(d) Salt and Pepper noise

Figure 1 – All these noisy images have approximately the same `MSE` value between them and the original, about 0.0135

3 Identification

In this mini project we will design and test a perceptual image hash function. Perceptual hash functions generate a short binary descriptor vector from an image. They are designed so that minor changes in the image itself, do not lead to a different descriptor value. They thus differ completely from their cryptographic counterparts who are designed to flip half of the output bits if a single one input bit changes.

Perceptual hashes are used, for example, to retrieve identical images, such as matching an image thumbnail against it original. See the reverse-image search engine `http://www.tineye.com/`.

Three principle things will be done

- Design and implementation of the perceptual hash function.
- Testing the uniqueness of the generated descriptor vectors for a small collection of images.

- Testing the influence of noise on the image on its resulting descriptor vector.

You will find 200 tiff images, each 256×256 on `Chamilo`.

Perceptual Hash

The basic algorithm can be implemented as follows:

Exercise

1. Read in an image and convert it to grayscale.
2. Determine the global mean of the image.
3. For each 32×32 subimage block, determine the local mean.
4. For each subimage block determine if the local mean is larger than the global mean. If so, the hash value for this particular block is 1 and 0 otherwise. The resulting descriptor is thus a 64 bit binary vector.

Hamming distance

Our algorithm outputs 64 bit binary descriptor vectors. They can thus be compared with the Hamming distance. In information theory, the Hamming distance h between two strings of equal length N is the number of positions at which the corresponding symbols are different. It can be implemented as:

$$h = \sum_{i=1}^{N} \oplus(a[i], b[i])$$

The probability of error P_b between between two strings of equal length N is:

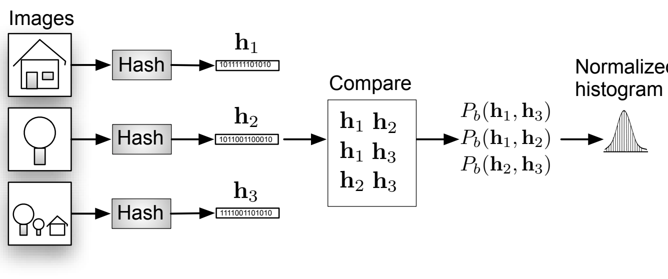
$$P_b = \frac{h}{N};$$

Exercise

1. Implement a function that determines the Hamming distance h and probability of error P_b between two binary vectors.
2. Implement a Matlab function that can read in 200 images sequentially and stores the 200 resulting binary descriptor vectors. See the Matlab `dir` function.

Inter and intra class distance

The inter-class distance is the distance, following some metric, between members of a non-identical class. In this case, all 200 images form their own class. To ascertain if our perceptual hash can distinguish all 200 images we will test all distances between them. If different images give the same descriptor vector, a so called *collision*, their difference will be zero. Consequently, these images can not be distinguished based on our hash function. Secondly, if the difference in hash value between two different images is very small, it might not be possible to distinguish an original image from a completely different image and an identical but slightly distorted copy. See Figure 2.
The intra-class distance is the distance between members of the same class. In our case, this is the distance between the descriptor value from an image and a noise distorted copy. We will thus measure how much the descriptor of an image changes if that image is corrupted by noise. See Figure 3.


Images
Hash
Hash
Hash
$\mathbf{h}_1$
1011111101010
$\mathbf{h}_2$
1011001100010
$\mathbf{h}_3$
1111001101010
Compare
$\mathbf{h}_1\ \mathbf{h}_2$
$\mathbf{h}_1\ \mathbf{h}_3$
$\mathbf{h}_2\ \mathbf{h}_3$
$P_b(\mathbf{h}_1, \mathbf{h}_3)$
$P_b(\mathbf{h}_1, \mathbf{h}_2)$
$P_b(\mathbf{h}_2, \mathbf{h}_3)$
Normalized histogram

(a)

Figure 2 – Inter-class test schematic

Implementation

All distances can be determined computational efficiently using matrix multiplication. Let matrix $\mathbf{X} \in \mathbb{R}^{L \times M}$ be the matrix with column vectors $\{\mathbf{x}^1, \ldots, \mathbf{x}^M\}$ where each vector $\mathbf{x}^m \in \mathbb{R}^L$ is a descriptor vector of length L from image m, $1 \leq m \leq M$. Similar, let matrix $\mathbf{Y}$ be the matrix formed from descriptor vectors, but originating from distorted images. In this particular TP column vector $\mathbf{x}^1$ is the 64 bit hash value from image 1, and $\mathbf{y}^1$ is the 64 bit hash value from the distorted image 1.
Intra and inter class distances can be ascertained as follows:

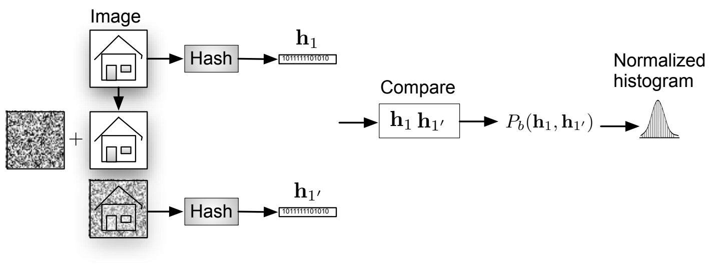
- Convert $\mathbf{X}$ and $\mathbf{Y}$ values from $\{0, 1\}$ to $\{-1, 1\}$
- Determine the *confusion* matrix: $\mathbf{Z} = \mathbf{X}^T\mathbf{Y}$.
- Convert to traditional hamming distances: $\forall z \in \mathbf{Z} \mid z := (z - L)./-2$
- The intra class distances can now be read of the diagonal of matrix $\mathbf{Z}$, the off diagonal elements form the inter class distances.

Exercise

1. Implement a function that determines the probability of error P_b between all pairs of two from the 200 binary descriptor values.
2. Build and show a *normalized* histogram of all found P_b values. Normalized means that all values from all histogram buckets should sum up to 1. You are thus estimating a probability mass function (PMF) of the inter-class distances.
3. What can you conclude from the histogram.

Exercise

1. Implement a function that takes all images one by one, makes a copy and distorts this copy with AWGN resulting in a PSNR of 35dB.
2. Determine the descriptor of both the original image and the distorted copy and determine the P_b between them. Do this for all 200 images.
3. Build and show a *normalized* histogram of all found intra P_b values.
4. What can you conclude from the histogram.



(a)

Figure 3 – Intra-class test schematic

5. Compare the estimated probability density graphs from the intra-class and inter-class distances. Do they overlap? What can you conclude from this?

Hints

- Start with testing with 20 images while debugging.
- Write out intermediate results to disk.
- Test your normalized histogram on samples from a known distribution first.