Watermark Performance Evaluation

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1 Non-blind watermark detection

Exercise 1.1

For the given hypothesis:

$$\begin{cases} \mathcal{H}_0: & v = x + z \\ \mathcal{H}_1: & v = x + w + z \end{cases}$$

where x is the host image, w is the watermark and z is a noise with distribution: $\mathcal{N}(0, \sigma_{noise}^2)$. Compute numerically the following property: P_f, P_m, P_d

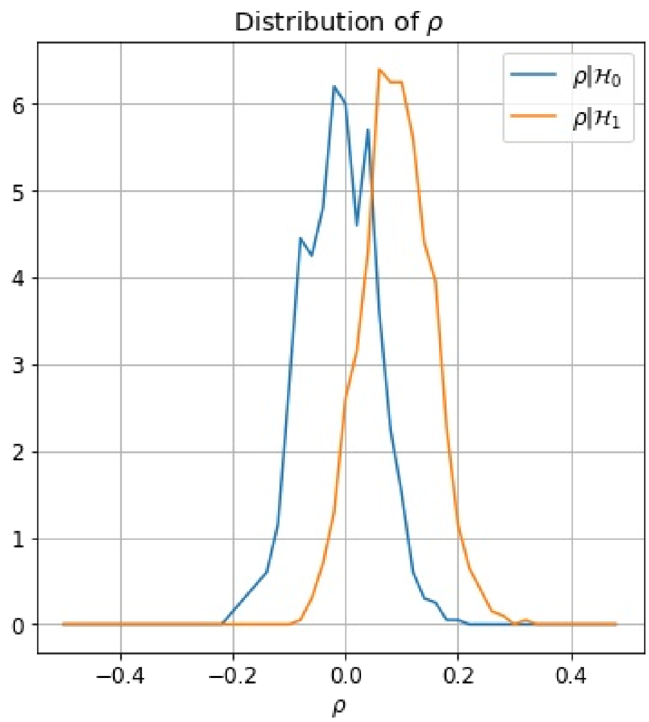

Figure 1. Distribution of ρ depending from H_0 and H_1

The numerical results were the following:

- $\mu_{\rho|H_0} = 0.002, \quad \sigma_{\rho|H_0} = 0.004$
- $\mu_{\rho|H_1} = 0.098, \quad \sigma_{\rho|H_1} = 0.004$

As expected the variance of the two distribution is very similar (asymptotically is the same) as in our case:

$$\bar{w}^2 = E[w\,w^T] = \frac{1}{N}\sum_{i=0}^{N} w[i]w[i] = \frac{N \cdot \theta_N}{N} = \theta_N$$

and so:

$$\sigma_{\rho|H_0}^2 = \sigma_{\rho|H_1}^2 = \sigma_\rho^2 = \frac{\theta_N \cdot \sigma_{x'}^2}{N}$$

Instead the mean of $\rho|\mathcal{H}_1$ depends from the intensity γ of the watermark:

$$\mu_{\rho|H_1} = \gamma \cdot \bar{w}^2 = \gamma \cdot \theta_N$$

While the $\mu_{\rho|H_0}$ depends only from the $E[Z]$.

Using this information is now possible to compute the threshold τ as well as displaying the different probabilities depending on the threshold.

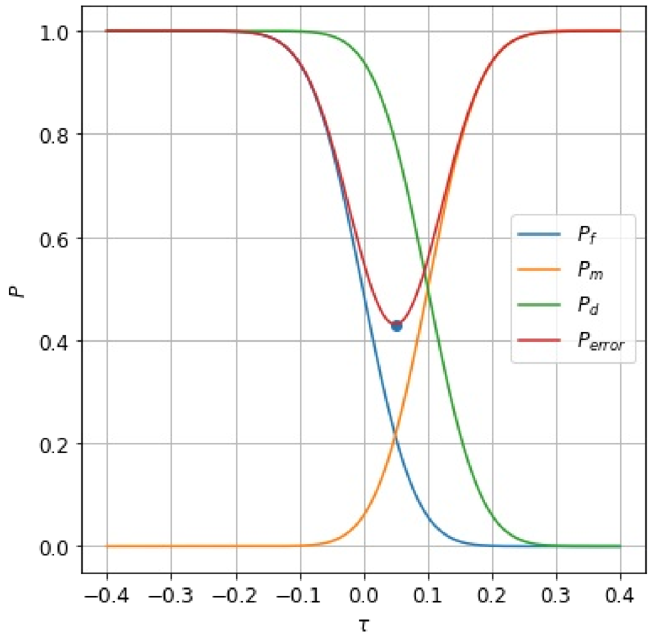

Figure 2. plot of P_f, P_m, P_d and P_{error} depending on τ and the chosen threshold

Using the Bayes hypothesis, the threshold τ can be found in the middle point (0.05) of the two distribution (as the two hypothesis are equiprobable and have same variance). This is also shown in the figure above, as is in the middle point where the P_{error} is minimal.

The resulting probabilities are:

- $\tau = 0.05$
- $P_f = 0.210$
- $P_m = 0.209$
- $P_{\text{error}} = 0.420$
- $P_d = 0.791$

Exercise 1.2

In this exercise we were asked to evaluate the performance of the *Non-Blind* detector at different noise and watermark configurations.

The next table will summarize the results:

	$\sigma^2_{\text{noise}} = 50$				$\sigma^2_{\text{noise}} = 100$			
	$\theta_N = 0.1$		$\theta_N = 0.3$		$\theta_N = 0.1$		$\theta_N = 0.3$	
	$\gamma = \pm 1$	$\gamma = \pm 5$	$\gamma = \pm 1$	$\gamma = \pm 5$	$\gamma = \pm 1$	$\gamma = \pm 5$	$\gamma = \pm 1$	$\gamma = \pm 5$
$\mu_{\rho\|H_0}$	0.003	-0.005	0.000	0.005	-0.001	-0.010	-0.006	-0.013
$\mu_{\rho\|H_1}$	0.097	0.500	0.303	1.499	0.091	0.505	0.310	1.497
σ^2_ρ	0.004	0.004	0.012	0.012	0.016	0.015	0.047	0.045

Table 1. $\mu_{\rho|H_0}$, $\mu_{\rho|H_1}$ and σ^2_ρ varying parameters of *non-blind* detector

Using this values is possible to display the ROC curve and the P_{error} plot of the different configurations:

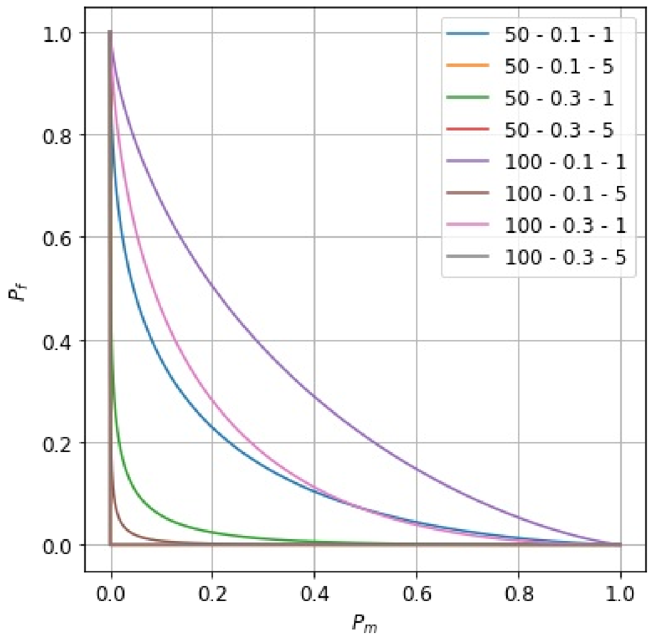

Figure 3. ROC for *non-blind* detector

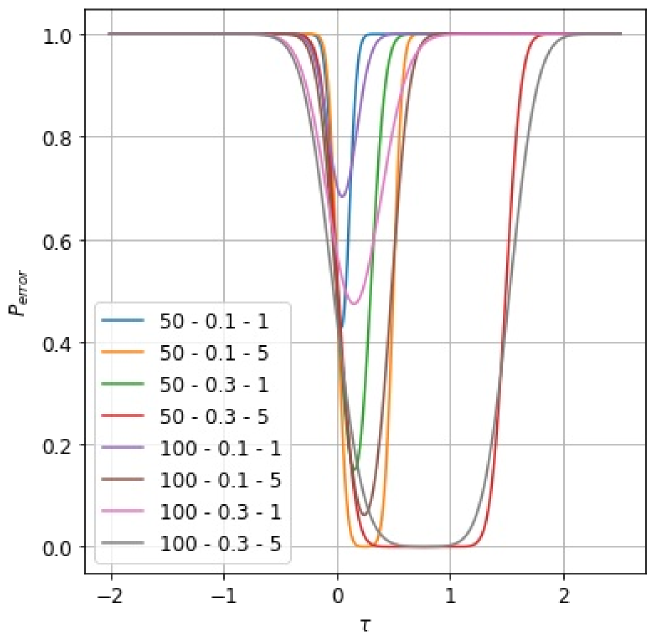

Figure 4. P_{error} for different configuration of the *non-blind* detector

In conclusion the parameters control respectively:

- θ_N: variate proportionally $\mu_{\rho|H_1}$ and σ_ρ^2

- γ: variate quadratic-ally $\mu_{\rho|H_1}$
- σ^2_{noise}: variate quadratic-ally σ^2_ρ

This is visualized in the following plots:

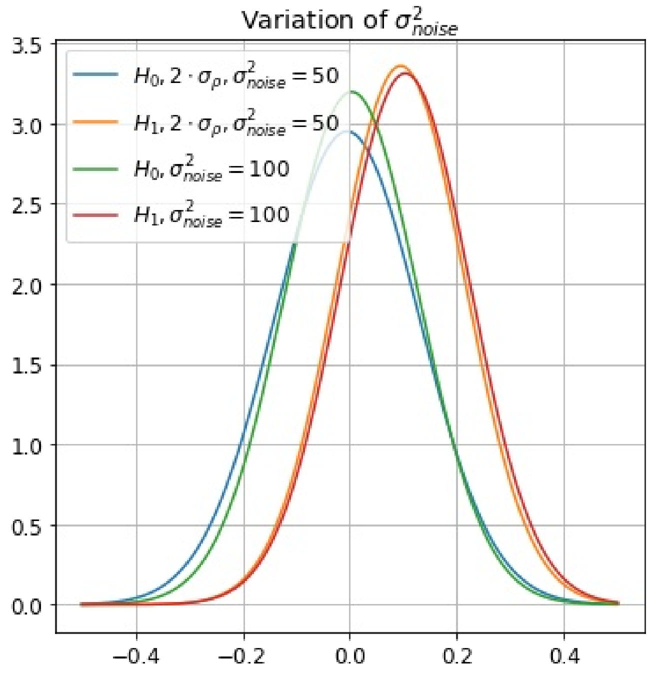

Figure 5. effect of the variation of σ^2_{noise} over σ_ρ and $\mu_{\rho|H_0}$, $\mu_{\rho|H_1}$

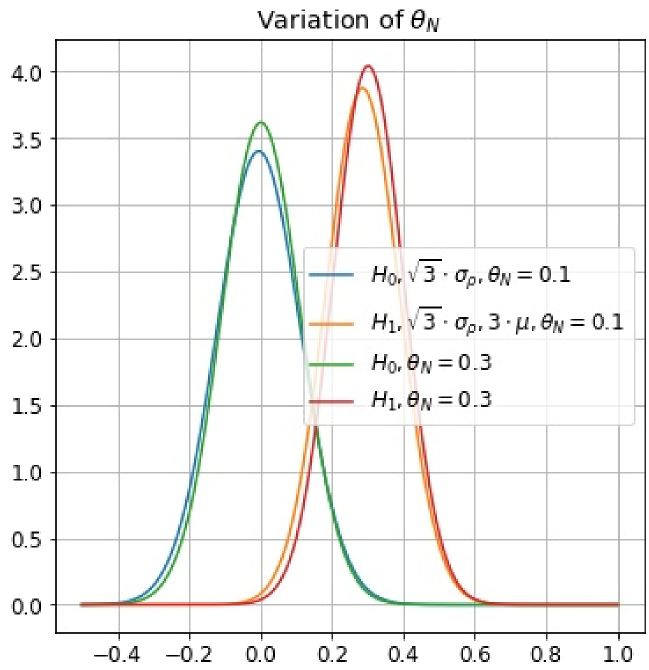

Figure 6. effect of the variation of θ_N over σ_ρ and $\mu_{\rho|H_0}$, $\mu_{\rho|H_1}$

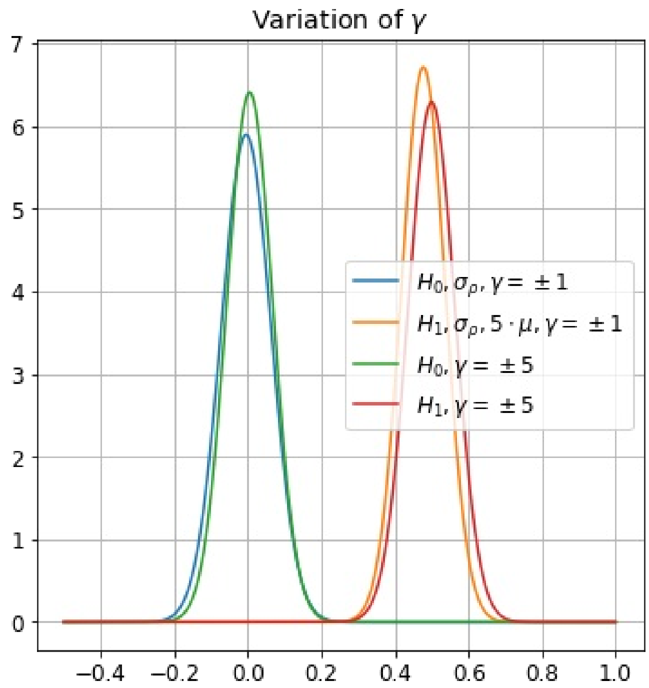

Figure 7. effect of the variation of γ over σ_ρ and $\mu_{\rho|H_0}$, $\mu_{\rho|H_1}$

2 Blind watermark detection

Exercise 2.1

In this exercise we will evaluate the performances of a blind watermark detector using the same configuration than in the previous exercise.

The main difference (a very important one) is the fact that the detector doesn't have access to the original image but instead have only the watermarked noised one. To be able to extract the watermark, the host image is estimated using a low pass filter (a median filter in my case).

The next table will summarize the results:

	$\sigma^2_{\text{noise}} = 50$				$\sigma^2_{\text{noise}} = 100$			
	$\theta_N = 0.1$		$\theta_N = 0.3$		$\theta_N = 0.1$		$\theta_N = 0.3$	
	$\gamma = \pm 1$	$\gamma = \pm 5$	$\gamma = \pm 1$	$\gamma = \pm 5$	$\gamma = \pm 1$	$\gamma = \pm 5$	$\gamma = \pm 1$	$\gamma = \pm 5$
$\mu_{\rho\vert H_0}$	0.000	-0.004	0.002	0.003	0.003	0.010	0.018	-0.013
$\mu_{\rho\vert H_1}$	0.092	0.442	0.260	1.319	0.098	0.444	0.269	1.320
σ^2_ρ	0.004	0.004	0.012	0.011	0.014	0.016	0.038	0.046

Table 2. $\mu_{\rho|H_0}$, $\mu_{\rho|H_1}$ and σ^2_ρ varying parameters of *blind* detector

Exercise 2.2

In this last exercise we will plot first the *ROC* of the *Blind* detector and evaluate the performance differences with the *Non-Blind* one.

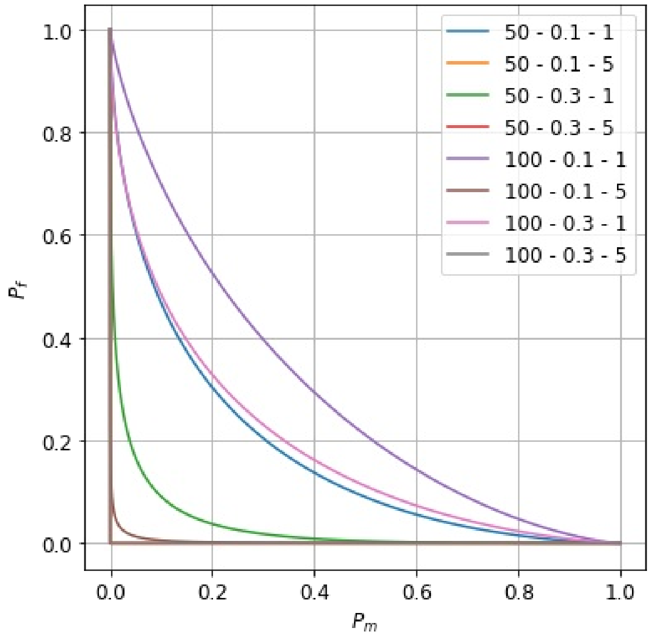

Figure 8. ROC of *blind* detector

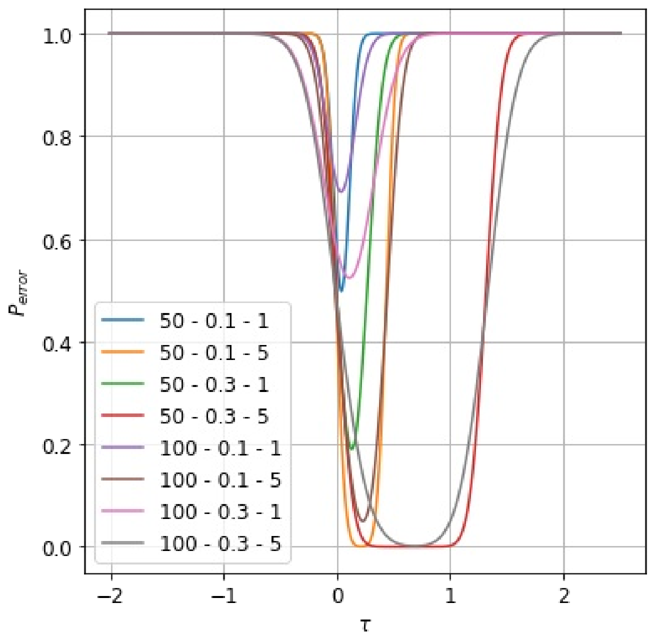

Figure 9. P_{error} for different configuration of the *blind* detector

As it's possible to see the *ROC* and the P_{error} plots are both very similar to the *Non-Blind* ones. The γ, θ_N and σ^2_{noise} has the same impact of before, however the fact that the original image is unknown has a small impact on the $\mu_{\rho|H_1}$, in average it is 10.1% smaller, and so close to $\mu_{\rho|H_0}$, increasing the probability of error.

Overall this small loss in performance doesn't impact to much in the detection of the watermark and with smarter filtering it could be possible to decrease the difference in performance between the two systems.