

Cry-Wolf Syndrome in Recommendation

(Authors' names blinded for peer review)

In this paper, we examine the effects of the cry-wolf syndrome in a setting where a manufacturer hires a forecaster to make recommendation on the capacity decision. In this context, we define cry-wolf as a behavioral syndrome in which the manufacturer becomes less compliant with a forecaster's valuable recommendation after the forecaster's prior recommendation is known to be *ex post* non-optimal. We find that, although the cry-wolf syndrome unequivocally leads to lower performance for the manufacturer when the forecaster's forecast ability is exogenous, cry-wolf can help boost the forecaster's investment in improving forecast ability when the ability can be endogenously determined by the forecaster and, thus, *benefit* the manufacturer. Moreover, cry-wolf can *improve* the performance of the system—the manufacturer and forecaster together—when the commission that the manufacturer pays to the forecaster is not low. Our findings are helpful for understanding the conditions under which cry-wolf can be detrimental or beneficial, and for indicating the need for caution with regard to actions curtailing the cry-wolf syndrome among managers.

Key words: behavioral operations management, cry-wolf, forecasting, managerial bias

1. Introduction

The cry-wolf syndrome refers to the phenomenon of decision-makers ignoring future recommendations after receiving a *false alarm*—that is, a recommendation that turns out to be *ex post* non-optimal (Green and Kolesar 2004, LeClerc and Joslyn 2015). This syndrome has been empirically observed in various contexts. For example, using data from the aircraft track and alert system, Wickens et al. (2009) find that air traffic controllers often fail to respond to true alerts from the alarm system, after receiving alarms that turn out to be *ex post* unnecessary. Using field data, LeClerc and Joslyn (2015) show that an individual's past experience with false weather warnings often leads to noncompliance with future weather warnings.

Interestingly, this cry-wolf syndrome has also been observed in controlled laboratory experiments. For example, Bolton and Katok (2017) conducted laboratory experiments in a classical cost-loss setting in which a decision-maker faces a loss with a likelihood of occurrence—referred as the loss-probability—in multiple rounds. In each round, the decision-maker must choose between a status quo option (*take-risk*) and the option to incur a cost (*take-cost*) to avoid a possible subsequent loss. In each round, the decision-maker also receives an *ex ante* optimal recommendation—whether to

take-cost or take-risk. Interestingly, Bolton and Katok (2017) find that decision-makers are susceptible to the cry-wolf syndrome in the sense that they do not necessarily follow the recommendations given to them. In particular, if a recommendation in an earlier round is a false alarm—that is, a loss occurs with the take-risk advice, or a loss does not occur, despite take-cost advice—then, decision makers will make a choice as if no recommendation were available.

Against this backdrop, we examine how the cry-wolf syndrome impacts the interaction between the decision-maker and the forecaster as well as the performance of the entire system. To this end, we deploy a cost-loss framework, with a recommendation receiver—manufacturer (*he*)—and a recommendation provider—forecaster (*she*). The manufacturer sells his product in a market where the demand can be either low or high, but the manufacturer has the capacity to meet only a low demand. In this context, the manufacturer risks leaving some demand unfulfilled, that is, a take-risk option. In order to avoid this possible loss from unfulfilled demand, the manufacturer can increase his capacity—by, for example, renting space or machines for production, training existing workers, or hiring new workers (Van Mieghem 2003, Qi et al. 2017)—thereby resulting in significant costs, that is, a take-cost decision.

A manufacturer can choose to commission a forecaster to provide recommendations regarding the capacity decision. The quality of these recommendations depends on the forecaster's ability to predict market trend—namely, her ability to utilize her private information to construct future market demand. The higher her ability, the better the recommendation. If the forecaster's *ex ante* recommendation is *ex post* non-optimal, the manufacturer will likely suffer from the cry-wolf syndrome, and may discontinue his relationship with the forecaster with some probability. The higher the probability, the greater the effect of the cry-wolf syndrome. We call this probability the *cry-wolf syndrome level* of the manufacturer. We find that in the case of an exogenous forecaster's ability, both the manufacturer and forecaster suffer from the cry-wolf syndrome—that is, the more pronounced the cry-wolf syndrome, the lower the performance of both the manufacturer and the forecaster.

When a forecaster strategically assesses her own forecast ability, she balances the benefits and costs associated with an investment in forecasting such as the purchase of forecasting software, to

decide the optimal forecast ability level. The motivation behind making a forecast investment is the compensation from the manufacturer: the higher her forecast investment, the better her forecast ability, and the higher the compensation if the manufacturer adopts the recommendation. The cost can be, for example, the cost of purchasing forecasting software.

We find that the forecaster's investment in her forecast ability depends on the manufacturer's susceptibility to the cry-wolf syndrome. In particular, as the manufacturer becomes more vulnerable to the cry-wolf syndrome, the forecaster may invest less in forecasting. This is because the manufacturer will terminate his business with the forecaster if her recommendation turns out to be *ex post* non-optimal. Nonetheless, if the commission that the forecaster charges the manufacturer is not low and the manufacturer's capacity cost is moderate, then the forecaster's ability can be higher as the manufacturer's cry-wolf syndrome becomes more severe. If this is the case, the cry-wolf syndrome can lead to *improved* performance for the manufacturer, unlike the setting of exogenous forecast ability in which the cry-wolf syndrome always lowers the manufacturer's performance.

More surprisingly, we find that cry-wolf can also improve the entire system's performance when the forecaster endogenously decides her forecast ability. To help put this into perspective, we consider a hypothetical centralized system comprising a forecaster and a manufacturer coordinated by an unbiased central planner. We show that such a system would have a lower performance if the system were managed by a biased central planner; similarly, the performance of such a system would decrease if one gets rid of the central planner, leaving the forecaster and manufacturer to make decisions independently. However, if both events occur simultaneously—the central planner were eliminated and the unbiased manufacturer were replaced with a biased manufacturer—then the system may be *better* off when the manufacturer is biased versus unbiased, depending on the commission that the manufacturer pays to the forecaster and the forecaster's operational efficiency of forecast investment (i.e., how efficiently the forecaster can improve her forecast ability). In the extreme scenario, cry-wolf could even serve as a force that drives the forecaster's ability to the level of its unbiased centralized benchmark or even higher.

We also verify the robustness of our key insights in two additional scenarios. First, we study a scenario where the manufacturer can endogenously decide the commission paid to the forecaster. For this case, we find that not only the manufacturer and the system, but also the forecaster can benefit from the cry-wolf syndrome. This is because this syndrome can help the forecaster earn a more generous commission from the manufacturer. Second, we study a scenario where the forecaster can improve her understanding of the market over time. Again, for this scenario, we find that both the manufacturer and system can benefit from the manufacturer's cry-wolf.

2. Literature Review

The literature has studied forecast and its related behavioral biases in different contexts (see, for example, Grushka-Cockayne et al. 2016, Regnier 2017 and the references therein). This literature often shows that the behavioral biases of the forecasters lead to lower performances for the decision-makers; thus, measurements must be developed for the decision-makers to overcome these biases. We add to this literature by studying a prevalent yet understudied cognitive bias of the decision-maker—namely, cry-wolf—and find that this syndrome can potentially benefit the decision-maker and even the system as a whole.

Our paper is also related to the extensive literature on information sharing (e.g., Lee et al. 2000, Cachon and Lariviere 2001). For example, Gümüs (2014) studies the information sharing between competitive suppliers and a monopoly buyer, and characterizes the condition where a credible information sharing is sustainable. Marschak et al. (2015) study the information sharing between a forecaster and a newsvendor, and the impact of the forecaster's information gathering on the newsvendor's order quantity and sales quantity. Spiliotopoulou et al. (2016) examine the coordination issue in a context where a central planner makes inventory decision while demand forecast information is from multiple regional managers. Pekgun et al. (2019) find that when a supplier serves two buyers and the supply is scarce, allocating more inventory to the buyer with better order forecast accuracy can significant improve buyers' order forecast accuracy. With Bayesian persuasion framework, Küçükgül et al. (2020) study a social information sharing between a platform and consumers, and Drakopoulos et al. (2021) study an information-provisioning and

pricing game between a retailer and consumers. A closely related paper by Alizamir et al. (2020) studies warning information sharing between a public agency and a stakeholder.

Note that although the majority of information sharing literature focuses on rational decision-makers as listed above, behavioral researchers have shown that real firms (and people) have limited information-processing power in information sharing. For example, Bolton and Katok (2017) recently conducted laboratory experiments in a classical cost-loss setting, and they find that subjects are vulnerable to the cry-wolf syndrome so that they do not necessarily follow the optimal recommendations that they receive. Thus, we study the information sharing between a forecaster and a manufacturer with the cry-wolf syndrome. By doing so, we make three key contributions.

First, and counter to our intuition, we find that the cry-wolf syndrome can benefit, rather than hurt, the manufacturer if the forecaster endogenously decides her forecast ability, even though the cry-wolf syndrome never benefits the manufacturer with an exogenous forecast ability from the forecaster. Second, and perhaps more interestingly, we demonstrate that cry-wolf is a bias that can even improve system-wide performance. Third, we find that, when the cry-wolf syndrome is present, forecast ability can be higher than it otherwise would be. Indeed, cry-wolf bias can help the manufacturer acquire a forecast ability that is equal to or even exceeds the *first-best* one—that is, the forecast ability of a centralized benchmark managed by an unbiased central planner. Consequently, we also enrich the body of literature that aims to improve forecast accuracy through various mechanisms (Cachon and Lariviere 2001, Chen 2003) by showing that cry-wolf—a behavioral syndrome of the manufacturer—can mitigate the need for designing such improvement mechanisms.

Overall, this paper contributes to the general realm of behavioral models in managerial decision-making. This emerging literature identifies and clarifies that individuals and firms tend to be biased in various ways (e.g., Zhao and Steckle 2010, Katok 2011, Li et al. 2016), although standard models assume that decision-makers are, on average, correct regarding their decisions. We follow suit by adopting an enduring theory of cognitive bias—the cry-wolf syndrome—as one plausible explanation to reconcile the adoption of prescribed recommendation with practical applications.

3. The Arrangement Between the Manufacturer and Forecaster

Consider a manufacturer who produces a product and then sells the product to customers. Customer demand is uncertain: it can be either low or high with equal probabilities. Here, we assume that the demand is binary to align with the experiment designed in Bolton and Katok (2017).¹ The customer demand distribution is known to both the manufacturer and the forecaster. The manufacturer's existing production capacity is assumed to be low—that is, the manufacturer is only capable of satisfying the low but not the high demand. Before demand realization, the manufacturer has an opportunity to alter his capacity. Specifically, the manufacturer has two options: *take-risk* and *take-cost*. Take-risk is a status-quo option in which the manufacturer maintains his existing capacity. By choosing this option, he risks leaving some demand unfulfilled if the realized demand is high. Given a high demand, we assume that the shortage cost is one, without loss of generality. When choosing the take-cost option, the manufacturer increases his production capacity to satisfy the high demand through means such as renting additional production equipment or hiring temporary workers, thereby enabling him to satisfy the high demand and avoid the risk of shortage. However, by doing so, the manufacturer incurs an extra cost $c(\geq 0)$ for increasing capacity. Figure 1 illustrates the manufacturer's decision. In this paper, we assume that $c \leq 1$ so as to eliminate the trivial case in which the manufacturer always chooses the take-risk option.

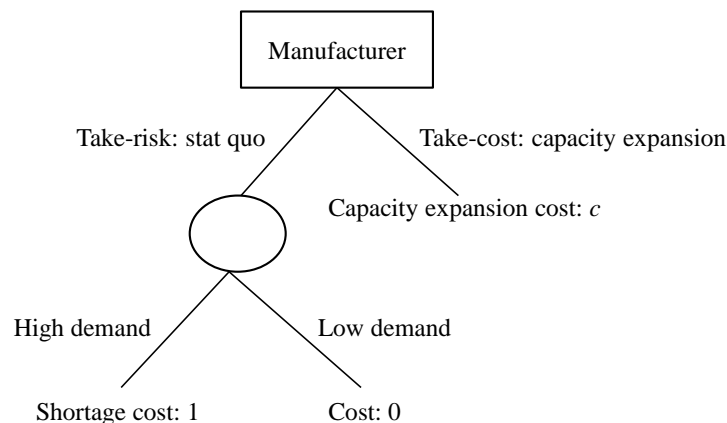


Figure 1 The manufacturer's take-risk and take-cost decisions.

¹ This treatment also facilitates the tractability of our analysis. Alternatively, one can assume that the demand is continuous. Then, the analysis is much more involved but without much additional managerial insights.

The manufacturer weighs the expected loss from the take-risk option against the cost c from the take-cost option, with the objective of minimizing his expected cost. Consequently, the manufacturer's optimal decision is take-risk if and only if $c > 1/2$. Otherwise, the manufacturer's optimal decision is take-cost. Overall, the expected cost for the manufacturer is $c \wedge (1/2)$, where $x \wedge y = \min\{x, y\}$. When $c = 1/2$, the manufacturer is indifferent between take-cost and take-risk. Note that, in this paper, the manufacturer aims to minimize his expected cost, and we abstract away from other possible costs (such as opportunity cost for idle capacities, which occurs when the manufacturer expands capacity but the realized demand is low). Alternatively, we can model the manufacturer as a profit maximizer or factor in other costs. For these cases, the key results remain robust.

3.1. Recommendation from a Forecaster

For better performance, the manufacturer can commission a forecaster. Based on her private signal of the market demand and the manufacturer's cost information, the forecaster provides an explicit recommendation—take-cost versus take-risk—to the manufacturer. The quality of recommendation depends on the forecaster's ability to process the private signal, which is described in the following manner. Let l and h refer to the true demand state being low or high, respectively, and let \tilde{l} and \tilde{h} denote the perceived signal indicating that the demand will be l or h , respectively. Let $P(k|j)$ denote the probability of observing signal $k \in \{\tilde{h}, \tilde{l}\}$ given that the underlying state is $j \in \{h, l\}$. We particularly specify $P(\tilde{h}|h) = \rho + \bar{\rho}/2$, $P(\tilde{l}|h) = \bar{\rho}/2$, $P(\tilde{l}|l) = \rho + \bar{\rho}/2$, and $P(\tilde{h}|l) = \bar{\rho}/2$, where $\rho \in [0, 1]$. Throughout the paper, we define $\bar{x} = 1 - x$ for any x . We call ρ the *forecast accuracy parameter*. It characterizes the forecaster's forecast ability to correctly classify true demand: as ρ increases, so does the forecast ability.

Our specification of the above probabilities satisfies the required properties: (i) At one extreme when $\rho = 1$, the forecaster's ability is perfect in the sense that $P(\tilde{h}|h) = P(\tilde{l}|l) = 1$ with certainty. At the other extreme when $\rho = 0$, the signal is no improvement over the initial demand information. (ii) The forecaster is unbiased in that $P(h) = P(\tilde{h}) = 1/2$ and $P(l) = P(\tilde{l}) = 1/2$. (iii) The consistency property $P(\tilde{h}) = P(h)P(\tilde{h}|h) + P(l)P(\tilde{h}|l)$ and analogously for $P(\tilde{l})$. From (i)-(iii), we have $P(h|\tilde{h}) = P(\tilde{h}|h)$ and $P(l|\tilde{l}) = P(\tilde{l}|l)$.

We assume that the forecaster knows the manufacturer's capacity cost c . With her private signal and forecast ability ρ , the forecaster foresees the manufacturer's expected cost:

$$\kappa(\rho) := \underbrace{P(\tilde{h})[P(h|\tilde{h}) \wedge c]}_{\text{high signal}} + \underbrace{P(\tilde{l})[P(h|\tilde{l}) \wedge c]}_{\text{low signal}}. \quad (1)$$

In other words, when the observed signal is high, occurring with a probability $P(\tilde{h})$, the manufacturer's expected cost is $P(h|\tilde{h}) \wedge c$, where $P(h|\tilde{h}) = 1 \times P(h|\tilde{h}) + 0 \times P(l|\tilde{h})$ is the expected cost if the manufacturer goes with the take-risk option while c is the cost if the manufacturer takes the other option. Similarly, when the observed signal is low, the manufacturer's expected cost is $P(h|\tilde{l}) \wedge c$.

Next, we show the impact of forecast accuracy on the manufacturer's expected cost $\kappa(\rho)$ in Lemma 1.

LEMMA 1. Define $\omega := |1/2 - c|$. Then,

a) The manufacturer's expected cost defined in (1) can be expressed as

$$\kappa(\rho) = \begin{cases} c & \text{if } c \leq 1/2 \text{ and } \rho \leq 2\omega, \\ (2c + \bar{\rho})/4 & \text{if } c \leq 1/2 \text{ and } \rho > 2\omega, \\ (2c + \bar{\rho})/4 & \text{if } c > 1/2 \text{ and } \rho > 2\omega, \\ 1/2 & \text{if } c > 1/2 \text{ and } \rho \leq 2\omega. \end{cases} \quad (2)$$

b) Moreover, $\kappa(\rho)$ always decreases in forecast accuracy ρ , that is, $\kappa'(\rho) \leq 0$.

Lemma 1a indicates that the manufacturer's expected cost depends on the capacity cost c and the forecast accuracy ρ . (i) If both the capacity cost and the forecast accuracy are low ($c \leq 1/2$ and $\rho \leq 2\omega$), then the manufacturer should increase the capacity regardless of the demand signal. (ii) If the capacity cost is low ($c \leq 1/2$) and the forecast accuracy is high ($\rho > 2\omega$), the manufacturer's best option depends on the observed signal: the manufacturer should choose take-cost for a high signal (or take-risk for a low signal). (iii) When both the capacity cost and the forecast accuracy are high ($c > 1/2$ and $\rho > 2\omega$), the manufacturer should choose take-cost (resp. take-risk) for a high (resp. low) signal. (iv) When the capacity cost is high ($c > 1/2$) and the forecast accuracy is low ($\rho \leq 2\omega$), the manufacturer should choose take-risk regardless of the demand signal. Intuitively, the manufacturer should take different options according to different received signals only when the forecast accuracy is sufficiently high, as in (ii) and (iii).

Lemma 1b indicates that a more accurate forecast never hurts the manufacturer. Take $\omega = 0$ (i.e., $c = 1/2$) as an example. Without the forecaster ($\rho = 0$), the manufacturer is indifferent between take-risk and take-cost. However, with the forecaster ($\rho > 0$), the manufacturer can choose an option (take-risk or take-cost) according to the private signal, which lowers the manufacturer's cost. So, it is beneficial for the manufacturer to adopt the forecaster's recommendation, and the manufacturer's expected cost $\kappa(\rho)$ becomes even lower as ρ increases.

In what follows, we first introduce the compensation contract between the manufacturer and the forecaster (Section 3.2) and then the cry-wolf syndrome (Section 4). We assume that the forecast accuracy ρ is exogenous and common knowledge. We later extend our analysis to the case with endogenous forecast accuracy (Section 5).

3.2. Compensation

Information is not free, and the manufacturer must compensate the information provider: the forecaster. We consider an outcome-based contract between the manufacturer and the forecaster, in which the manufacturer shares with the forecaster a portion of the benefits reaped from having improved information. Note that given the forecast accuracy ρ , the manufacturer's expected cost reduction due to the improved information from the forecaster is $\kappa(0) - \kappa(\rho)$. Thus, the forecaster's expected compensation paid by the manufacturer can be defined as

$$T(\rho) = t[\kappa(0) - \kappa(\rho)], \quad (3)$$

where $t \in [0, 1]$ is the *commission rate* for the forecaster. The commission rate can measure the relative bargaining power between the manufacturer and the forecaster. When the forecaster serves to a powerful manufacturer, the commission rate is likely to be low; when the power structure between the forecaster and the manufacturer is relatively balanced, the commission rate is moderate. We assume that the commission rate is exogenous in the main model, to focus on the impact of the cry-wolf syndrome. Later, we extend our analysis to the case of an endogenous commission rate (Section 6.1).

The linear contract defined in (3) has two advantages. First, it is parsimonious and has been used to model the compensation plan in similar contexts (e.g., Lal and Srinivasan 1993, Chen et al. 2016). In our context, this contract form allows us to obtain closed-form solutions and insights pertaining

to the combination of cry-wolf bias and forecast investment decision. Second, it is practical and outcome-based in that $T(\rho)$ measures the extent to which the forecaster helps the manufacturer save cost. In particular, $T(\rho)$ increases in ρ —that is, as the forecaster predicts demand more precisely, she receives more compensation as the manufacturer's expected cost becomes lower.

4. The Cry-Wolf Syndrome

We now introduce the notion of the cry-wolf syndrome into this setting. The cry-wolf syndrome arises because the forecaster's *ex ante* recommendation can be *ex post* non-optimal due to the uncertainty of demand, even though the recommendation is *ex ante* optimal to the manufacturer. In particular, when the forecaster recommends take-cost (take-risk) but the realized demand is low (high), such a recommendation is treated as a false alarm by the manufacturer. Consequently, the false alarm occurs with the following probability:

$$\delta(\rho) := \text{P}[\text{false alarm}] = \begin{cases} 1/2 & \text{if } c \leq 1/2 \text{ and } \rho \leq 2\omega, \\ \bar{\rho}/2 & \text{if } c \leq 1/2 \text{ and } \rho > 2\omega, \\ \bar{\rho}/2 & \text{if } c > 1/2 \text{ and } \rho > 2\omega, \\ 1/2 & \text{if } c > 1/2 \text{ and } \rho \leq 2\omega. \end{cases} \quad (4)$$

(i) When $c \leq 1/2$ and $\rho \leq 2\omega$, the forecaster always recommends take-cost to the manufacturer. Thus, the false alarm occurs if the realized demand is low, which occurs with probability $1/2$. (ii) When $c \leq 1/2$ and $\rho > 2\omega$, the forecaster's recommendation depends on the observed signal. If the forecaster observes the high (or low) signal, she recommends take-cost (or take-risk) to the manufacturer. Thus, the likelihood of a false alarm is $\text{P}(\tilde{h})\text{P}(l|\tilde{h}) + \text{P}(\tilde{l})\text{P}(h|\tilde{l}) = \bar{\rho}/2$, where the false alarm occurs with the probability $\text{P}(\tilde{h})\text{P}(l|\tilde{h})$ when the received signal is high (or $\text{P}(\tilde{l})\text{P}(h|\tilde{l})$ when the received signal is low) but the realized demand turns out to be low (or high). (iii) Similarly, when $c > 1/2$ and $\rho > 2\omega$, the likelihood of a false alarm is $\text{P}(\tilde{l})\text{P}(h|\tilde{l}) + \text{P}(\tilde{h})\text{P}(l|\tilde{h}) = \bar{\rho}/2$. (iv) When $c > 1/2$ and $\rho \leq 2\omega$, the forecaster always recommends take-risk to the manufacturer; thus, the likelihood of a false alarm is $1/2$. Note that the probability of the false alarm is decreasing in ρ . In other words, the higher the forecast accuracy, the lower the likelihood of a false alarm. This observation subsequently helps explain Propositions 2–5.

A false alarm can give rise to the cry-wolf syndrome. Bolton and Katok (2017) conducted laboratory experiments, in which a decision-maker receives a creditable recommendation based on

more precise information. However, they find that, even if the decision-maker knows that following the recommendation leads to a better performance, the decision-maker still exhibits cry-wolf bias. In order to incorporate this, we study a setting in which firms operate in two periods. In each period, the forecaster first observes the private market signal and updates her market belief in order to make the recommendation—either take-cost or take-risk—to the manufacturer. We assume that the forecast accuracy is fixed for both periods in the main model, and we further extend our analysis to the case in which the forecaster can improve the forecast accuracy across periods by learning-while-doing in one of the extensions (Section 6.2). The manufacturer makes his decision accordingly after receiving the recommendation from the forecaster. In the first period, knowing that the forecaster provides the recommendation that truthfully conveys the information inferred from her signal, the manufacturer simply follows the forecaster’s recommendation, which is optimal. However, if the recommendation in the first period is proven to be a false alarm, then the biased manufacturer dismisses the forecaster (and thus there is no recommendation from the forecaster in the second period) with probability α , where α is exogenous and common knowledge, and $\alpha \in [0, 1]$. The higher the α , the more severe the cry-wolf syndrome is. See Figure 2 for an illustration of the sequence of events.

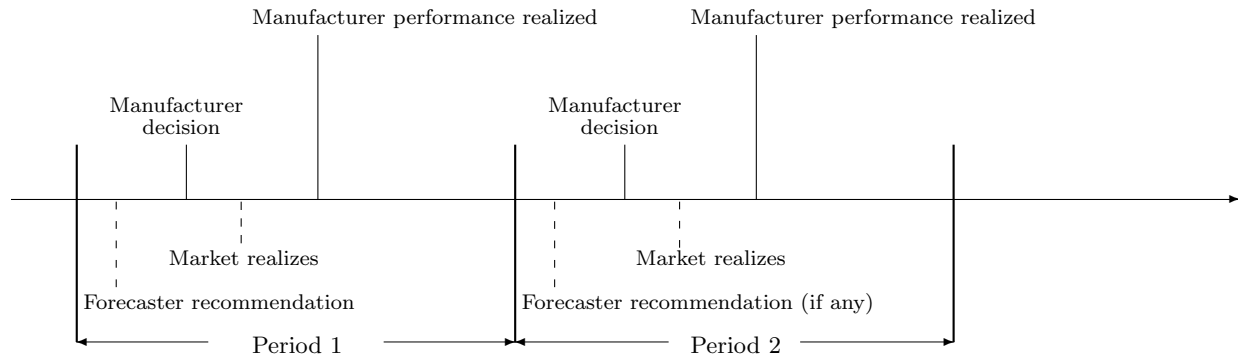


Figure 2 The sequence of events.

Taken together, the manufacturer’s expected cost is

$$\mathcal{C}(\alpha, \rho) = \underbrace{\kappa(\rho) + T(\rho)}_{\text{first period}} + \underbrace{(1 - \alpha)[\kappa(\rho) + T(\rho)] + \alpha \left[\bar{\delta}(\rho)[\kappa(\rho) + T(\rho)] + \delta(\rho)\kappa(0) \right]}_{\text{second period}}$$

$$\begin{aligned}
&= [2 - \alpha\delta(\rho)][\kappa(\rho) + T(\rho)] + \alpha\delta(\rho)\kappa(0) \\
&= [2 - \alpha\delta(\rho)][\bar{t}\kappa(\rho) + t\kappa(0)] + \alpha\delta(\rho)\kappa(0).
\end{aligned} \tag{5}$$

In the first period, the manufacturer's cost includes both $\kappa(\rho)$, as defined in (1), and the compensation $T(\rho)$ paid to the forecaster, as defined in (3). However, in the second period, the manufacturer may discontinue doing business with the forecaster if the prior forecast is perceived as a false alarm, which happens with the probability $\delta(\rho)$, as defined in (4).

LEMMA 2. a) *For a given α , the manufacturer's cost $\mathcal{C}(\alpha, \rho)$ decreases in forecast accuracy ρ .*
b) *Given an exogenous forecast accuracy ρ , $\mathcal{C}(\alpha, \rho)$ increases in α .*

Lemma 2a indicates that the manufacturer always benefits if the forecaster has a higher forecast ability. Furthermore, for a fixed ρ , Lemma 2b indicates that cry-wolf syndrome always drags down the manufacturer's performance. This is because regardless of whether or not there is a false alarm in the first period, it is always optimal for the manufacturer in the second period to commission and then follow the recommendation of the forecaster who has more precise information. Overall, Lemma 2 serves as a benchmark for our results presented in Section 5 below.

5. Strategic Forecaster

In this section, we study the case in which the forecaster can strategically adjust the forecast accuracy ρ before the first period to maximize her expected profit over both periods:

$$\begin{aligned}
\Pi(\alpha) &= \max_{\rho \in [0,1]} [T(\rho) + (1 - \alpha)T(\rho) + \alpha\bar{\delta}(\rho)T(\rho)] - \theta\rho^2 \\
&= \max_{\rho \in [0,1]} [2 - \alpha\delta(\rho)]T(\rho) - \theta\rho^2,
\end{aligned} \tag{6}$$

where $\theta(\geq 0)$ is the *efficiency parameter* that represents the operational efficiency of forecast investment. As θ is lower, the forecaster can improve her forecast accuracy more efficiently. Consistent with the literature (Heese and Swaminathan 2006, Chayet et al. 2011), the investment cost of offering a forecast is convex in quality so that it is increasingly expensive to offer a better forecast. Following this literature, we assume a quadratic function for the investment cost for mathematical tractability. We also assume $\theta \geq \theta_{min}$, where $\theta_{min} = \frac{t(4+\alpha)}{16}$ in order to ensure the regularity. The investment cost can be, for example, the expense of purchasing software, which will carry over for both periods. We assume that the forecast accuracy can be observed by the manufacturer.

PROPOSITION 1. Define $c_1 := \frac{2t\alpha - 2(4-\alpha)[4\theta - \sqrt{2\theta(8\theta - \alpha t)}]}{\alpha^2 t}$ if $\alpha > 0$ and $c_1 := \frac{1}{2} - \frac{t}{16\theta}$ if $\alpha = 0$. Then, the solution of (6) is

$$\hat{\rho} = \begin{cases} \frac{t[4-\alpha(2\theta+1)]}{16\theta-2t\alpha} & \text{if } c_1 \leq c \leq \bar{c}_1 \\ 0 & \text{otherwise.} \end{cases} \quad (7)$$

Proposition 1 indicates that when the capacity cost is either low ($c < c_1$) or high ($c > \bar{c}_1$), the forecaster should not invest in forecasting. For such cases, the manufacturer always prefers take-cost or always prefers take-risk. As a result, it is useless for the forecaster to invest in forecasting. Only if the capacity cost is moderate ($c_1 \leq c \leq \bar{c}_1$), does the forecast add value to the manufacturer, and consequently, the forecaster invests.

Proposition 1 implies that with moderate capacity cost ($c_1 \leq c \leq \bar{c}_1$), the forecast accuracy $\hat{\rho}$ increases in the commission rate t and decreases in the operational efficiency of forecast investment θ . Intuitively, the forecaster has a higher incentive to improve her forecast accuracy when she can do so efficiently or when she can receive a higher commission from the manufacturer. Next, we study the impact of cry-wolf syndrome on the forecast accuracy $\hat{\rho}$, which is more involved.

PROPOSITION 2. Define $t_1 := \min(2\theta, 1)$ and $c_{\rho 1} = 1 - \frac{t}{4\theta}$.

- a) When the commission rate t is low ($t \leq t_1$), the cry-wolf syndrome always leads to a lower forecast accuracy—that is, $\hat{\rho}$ decreases in α .
- b) However, when $t > t_1$, the cry-wolf syndrome leads to a higher forecast accuracy—that is, $\hat{\rho}$ increases in α , when the capacity cost is moderate ($\max(c_{\rho 1}, c_1) < c < \min(\bar{c}_{\rho 1}, \bar{c}_1)$).

Proposition 2a indicates that, when the commission rate is low, a more severe cry-wolf syndrome results in a lower level of forecast investment. Intuitively speaking, the forecaster foresees that the manufacturer may end the business with her in the second period, thereby making her forecast investment insane. In other words, the manufacturer's cry-wolf syndrome stifles the forecaster's investment in forecasting.

Interestingly, Proposition 2b indicates that the forecast accuracy level can also be higher as the manufacturer's syndrome becomes more severe. In other words, the cry-wolf syndrome *increases* the forecaster's incentive to invest in forecasting when the commission rate is not low ($t > t_1$) and the capacity cost is fairly moderate ($\max(c_{\rho 1}, c_1) < c < \min(\bar{c}_{\rho 1}, \bar{c}_1)$). The rationale underlying this

is explained in the following manner. When the commission rate is not low, and when the manufacturer benefits greatly from forecasting (i.e., when the manufacturer's capacity cost is moderate), the forecaster's revenue from the manufacturer is high, which provides the forecaster a high incentive to increase her forecast ability in order to reduce the probability of losing the revenue in the second period due to a false alarm. This cry-wolf syndrome's positive impact on the forecaster's investment incentive can dominate its negative impact due to the reduced investment return from the potential loss of the second-period revenue (the driver for Proposition 2a as discussed above) under the conditions specified in Proposition 2b, thereby leading to a higher forecast accuracy level.

A natural question after Proposition 2b is the extent to which the cry-wolf bias can increase the forecast accuracy level. As a benchmark, we study a centralized system where an unbiased central planner minimizes the total costs of the manufacturer and forecaster by choosing an appropriate forecast accuracy (i.e., *first-best* forecast accuracy):

$$\rho_c = \arg \min_{\rho \in [0,1]} \theta \rho^2 + 2\kappa(\rho). \quad (8)$$

LEMMA 3. $\rho_c \geq \hat{\rho}|_{\alpha=0}$.

Lemma 3 indicates that the centralized system invests more in forecasting than the decentralized system, when the manufacturer is unbiased ($\alpha = 0$). In other words, compared to the centralized system, the decentralized system underinvests in forecasting. This is because, in the decentralized system, the forecaster receives only a portion, instead of all, of the benefit of forecast improvement with the manufacturer, which decreases the forecaster's incentive for investment. This result reveals that the decentralized system is inefficient in stimulating forecast accuracy improvement. A natural question arises: Can the cry-wolf syndrome counterbalance the decentralized system inefficiency?

With the centralized benchmark, Proposition 3 below demonstrates that the cry-wolf syndrome can enhance the forecast accuracy to the level that is equal to or even higher than the first-best level ρ_c characterized in (8). See Figure 3 below for an illustration.

PROPOSITION 3. Define $t_2 := \min(\frac{8\theta}{\alpha(1-2\theta)+8\theta}, 1)(\geq t_1)$ and $c_{\rho 2} := 1 - \frac{1}{4\theta} + \frac{2\bar{t}}{\alpha t}(\geq c_{\rho 1})$. When the commission rate t is high ($t > t_2$), we obtain the following aspects:

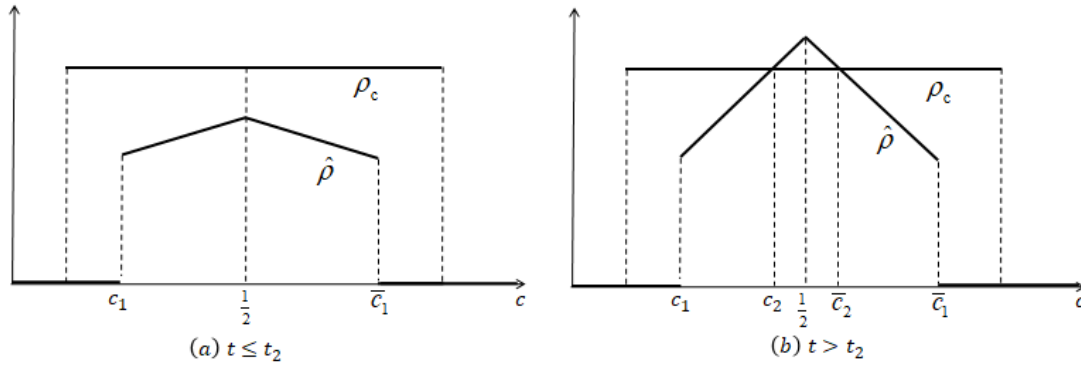


Figure 3 An illustration of Proposition 3.

- a) The cry-wolf syndrome leads to a forecast accuracy that is equal to the first-best forecast accuracy—that is, $\hat{\rho} = \rho_c$, when $\alpha = \frac{8\bar{t}\theta}{t[1-2(1+2\omega)\theta]}$.
- b) The cry-wolf syndrome can even lead to a forecast accuracy that is strictly higher than the first-best forecast accuracy—that is, $\hat{\rho} > \rho_c$, when the capacity cost is moderate such that $c_2 < c < \bar{c}_2$, where $c_2 = \max(c_{\rho_2}, c_1)$.

Consequently, the forecast accuracy of the biased decentralized system can reach (Proposition 3a) or even exceed (Proposition 3b) that of the unbiased centralized system under conditions with a higher commission rate ($t > t_2$, where $t_2 \geq t_1$) and a more moderate capacity cost ($c_2 < c < \bar{c}_2$, where $c_2 \geq \max(c_{\rho_1}, c_1)$) as compared to Proposition 2. In other words, cry-wolf is a bias that can offset the effect of decentralization on under-investment in forecasting; thus, the resulting investment of the biased decentralized system can be equivalent to or higher than its unbiased centralized benchmark. This result complements the information sharing literature by showing that cry-wolf syndrome can substitute contracts aimed at increasing the information provider's effort of improving forecast accuracy.

Given the equilibrium forecast accuracy $\hat{\rho}$ in (7), we evaluate the manufacturer's equilibrium cost:

$$\mathcal{C}(\alpha) := \mathcal{C}(\alpha, \hat{\rho}) \quad (9)$$

PROPOSITION 4. a) The cry-wolf syndrome always decreases the forecaster's equilibrium profit—that is, $\Pi'(\alpha) \leq 0$.

b) When the commission rate t is low ($t \leq t_1$), the manufacturer's equilibrium cost increases in

his level of cry-wolf syndrome—that is, $C'(\alpha) \geq 0$. However, when $t > t_1$, there exists a forecast efficiency threshold $\theta_1(> 0)$, such that the manufacturer's equilibrium cost can decrease as the level of cry-wolf syndrome increases—that is, $C'(\alpha) < 0$, if $\theta < \theta_1$.

Proposition 4a indicates that a more severe manufacturer's cry-wolf syndrome always hurts the forecaster. This is because a manufacturer with cry-wolf syndrome may discontinue the business with the forecaster in the second period, which reduces the forecaster's revenue. Proposition 4b demonstrates that, interestingly, the manufacturer can *benefit* from his cry-wolf syndrome. The cry-wolf syndrome impacts the manufacturer's performance in two opposite ways. The first (direct) impact is that the manufacturer's equilibrium expected cost, for a given forecast accuracy level, is higher when he is more biased (Lemma 2b). This is because the cry-wolf syndrome leads the manufacturer to irrationally discontinue doing business with the forecaster. Thus, the direct impact of a more severe cry-wolf syndrome on the manufacturer's performance is negative. The second (indirect) impact is that a more severe cry-wolf syndrome can lead to a higher forecast accuracy (Proposition 2), which in turn results in a lower expected cost for the manufacturer (Lemma 2a). Thus, the cry-wolf syndrome exerts an indirect, positive force on the manufacturer's expected performance. The overall impact of a more severe cry-wolf bias on the manufacturer's expected performance depends on the relative strength of these two forces; the positive impact—an increased forecast accuracy—is sufficiently significant to dominate the negative impact when the commission rate t is not low, such that $t > t_1$, and the forecaster can efficiently improve her forecast accuracy, such that $\theta < \theta_1$.

Proposition 5 questions whether the cry-wolf syndrome can benefit the manufacturer to the extent of benefiting the system—composed of the manufacturer and forecaster—as a whole. Next, we evaluate the system cost in the following manner:

$$\mathcal{S}(\alpha) := \mathcal{C}(\alpha) - \Pi(\alpha). \quad (10)$$

PROPOSITION 5. *When the commission rate t is low ($t \leq t_1$), the system cost increases in the level of cry-wolf syndrome—that is, $\mathcal{S}'(\alpha) \geq 0$. However, when $t > t_1$, there exists a forecast efficiency threshold $\theta_2(\leq \theta_1)$ such that the system cost can decrease as the level of cry-wolf syndrome increases—that is, $\mathcal{S}'(\alpha) < 0$, if $\theta < \theta_2$.*

As per Proposition 5, the cry-wolf syndrome can improve the system-wide performance, when the commission rate is not low ($t > t_1$) and the forecaster can more efficiently improve her forecast accuracy ($\theta < \theta_2$, where $\theta_2 \leq \theta_1$) as compared to Proposition 4b. First, the same as Proposition 4, the necessary condition for cry-wolf helps regardless of whether the manufacturer or the system is that the commission rate t is not low, which gives the forecaster a high incentive to improve her forecast ability. Second, cry-wolf can help the system when θ is low. For insight, the syndrome can enhance the system's—particularly, the forecaster's—forecast investment; the more efficiently the forecaster can improve the forecast accuracy, the more the system can benefit from a higher forecast accuracy due to the syndrome. Moreover, $\theta_2 \leq \theta_1$ implies that the cry-wolf syndrome improves system-wide performance under conditions similar to but tighter than those specified in Proposition 4b.

Recall that the cry-wolf syndrome always hurts the forecaster (Proposition 4a) while benefits the manufacturer, depending on the commission rate and the forecast efficiency (Proposition 4b). An improved system-wide performance identified in Proposition 5 shows that the cry-wolf syndrome can bring more benefit to the manufacturer as compared to its harm to the forecaster. This implies a potential win-win situation for the manufacturer and the forecaster if the manufacturer can compensate the forecaster for her loss, as shown in Section 6.1.

6. Model Extensions

In our analysis, we have assumed that (i) the commission rate between the manufacturer and the forecaster is exogenous, and (ii) the forecaster's capability of processing information solely depends on her forecast investment and remains the same over the two periods. In this section, we first extend our discussion to the case in which the commission rate is determined endogenously by the manufacturer before the forecaster chooses her forecast accuracy (Section 6.1). Then, we analyze the case in which the forecaster can learn over a period, so that her capability of processing information improves in the second period as compared to that in the first period (Section 6.2).

6.1. Endogenous Commission Rate

The manufacturer decides the commission rate t to minimize his expected cost $\mathcal{C}(\alpha)$ in (9), thereby expecting the forecaster's response as characterized in Proposition 1. Here we assume that the

operational efficiency of forecast investment θ is common knowledge. After obtaining the optimal commission rate, we can then evaluate the costs of the manufacturer and the system as a whole, and the profit of the forecaster. Because it is difficult to obtain the closed-form solution for the optimal commission rate (see Proposition 6 in Appendix), we resort to numerical experiments to investigate the insights. We plot the firms' performance and the system's performance in Figure 4 for two cases: a more costly forecasting investment ($\theta = 0.33$) and a less costly forecasting investment ($\theta = 0.22$).

As per Figure 4a, both the manufacturer and the forecaster suffer from the manufacturer's cry-wolf syndrome. Consequently, the cost of the system as a whole increases as α increases. This is consistent with our intuition that cry-wolf syndrome is harmful for firms. However, as per Figure 4b, both the manufacturer and the forecaster can benefit from the cry-wolf syndrome when the forecaster can more efficiently improve her forecast accuracy (θ is smaller). In other words, when the commission rate is endogenously determined by the manufacturer, cry-wolf syndrome can improve not only the manufacturer's and the system's performance, but also the forecaster's performance. This is different from the case with the exogenous commission rate, where the forecaster never benefits from the manufacturer's cry-wolf syndrome.

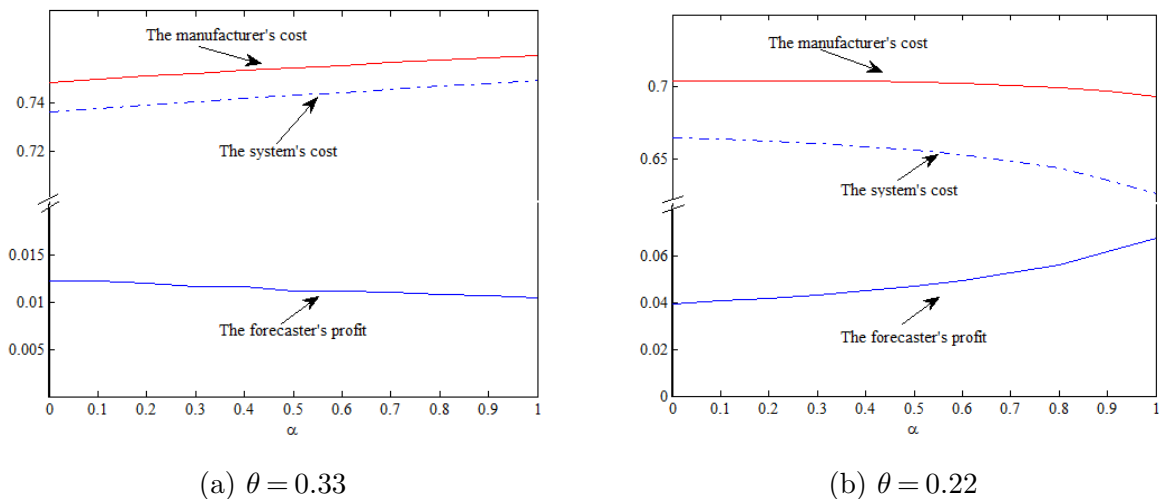


Figure 4 The performances of the manufacturer, the forecaster, and the system with $c = 0.4$.

For insight, recall that the manufacturer's cry-wolf syndrome may stifle the forecaster's investment in forecasting. The manufacturer would raise the commission rate to compensate the fore-

caster for her loss due to the cry-wolf syndrome, thereby benefiting the forecaster. Intuitively, the forecaster invests more to improve her forecast accuracy after the manufacturer raises the commission rate. However, with a relatively high θ , the forecaster's equilibrium profit decreases as the manufacturer becomes more biased even if the manufacturer increases the commission rate. It is only when θ is relatively low, the increasing compensation can eventually outweigh the forecaster's loss from the manufacturer's cry-wolf syndrome.

6.2. Learning

In practice, the forecaster may be able to improve the forecast accuracy across periods by learning-while-doing. In other words, given the forecaster's forecast accuracy ρ in the first period, she can improve the accuracy to $\rho + \varepsilon$ in the second period, where $\varepsilon(> 0)$ is exogenous and common knowledge. Consequently, the forecaster's profit is

$$\Pi_\varepsilon(\alpha) = \max_{\rho \in [0,1]} [T(\rho) + (1 - \alpha)T(\rho + \varepsilon) + \alpha\bar{\delta}(\rho)T(\rho + \varepsilon)] - \theta\rho^2.$$

With the forecaster's optimal decision on ρ , we can assess the manufacturer's cost

$$\begin{aligned} \mathcal{C}_\varepsilon(\alpha) = & \underbrace{\kappa(\rho) + T(\rho)}_{\text{first period}} \\ & + \underbrace{(1 - \alpha)[\kappa(\rho + \varepsilon) + T(\rho + \varepsilon)] + \alpha[\bar{\delta}(\rho)[\kappa(\rho + \varepsilon) + T(\rho + \varepsilon)] + \delta(\rho)\kappa(0)]}_{\text{second period}}. \end{aligned}$$

It is challenging to analytically examine how the manufacturer's and system's costs change as the level of cry-wolf syndrome increases, but our numerical experiments show that our main findings—namely, the cry-wolf can benefit the manufacturer and the entire system as a whole—remain. Moreover, as per Figure 5, the cost of the manufacturer and system decrease, and the forecaster's profit increases with the degree of learning-while-doing, ε . In other words, both the manufacturer and the forecaster benefit from learning-while-doing of the forecaster.

7. Conclusion

This paper examines the effects and implications of the cry-wolf syndrome in a recommendation context. In this context, if a manufacturer encounters a false alarm, then he may terminate his involvement with the forecaster, even if the recommendation from the forecaster is valuable to him. We find that if the manufacturer pays a high commission to the forecaster, the forecaster then has

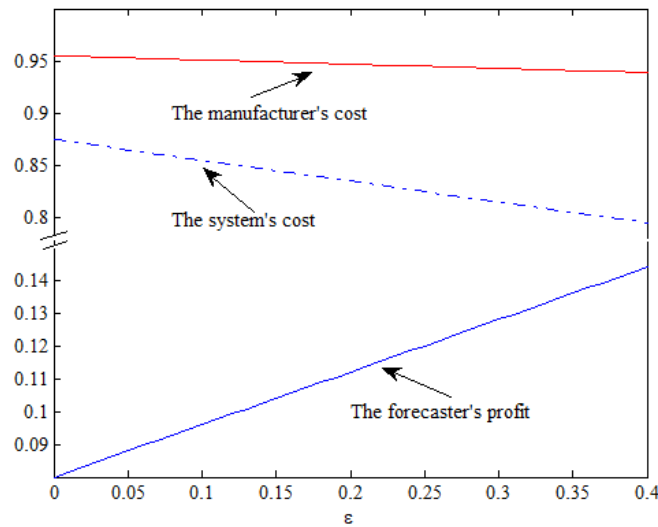


Figure 5 The performances of the manufacturer, the forecaster and the system with learning-while-doing.

Parameters: $c = 0.5, \alpha = 0.8, \theta = 0.4, t = 0.8$.

a greater incentive to invest in her forecasting ability when the manufacturer is more susceptible to the cry-wolf syndrome. Consequently, although the cry-wolf syndrome always leads to lower performance for the manufacturer in the case of an exogenous forecast accuracy, this may not be true when the forecast accuracy is endogenously determined by the forecaster. Indeed, depending on the commission, the cry-wolf syndrome can lead to a first-best forecast accuracy level for the system. Consequently, cry-wolf as a syndrome of the manufacturer can be a counterbalance for system inefficiency between the manufacturer and the forecaster, which will eventually benefit the overall system.

7.1. Managerial Insights

We expect that our analyses are of value to firms that adopt strategies in response to the cry-wolf syndrome. Prior behavioral research has suggested that the cry-wolf syndrome is detrimental for firms, and a variety of programs have consequently been designed to mitigate the cry-wolf syndrome (Bolton and Katok 2017). Despite studies that indicate a possible negative impact of the cry-wolf syndrome, our study offers a different perspective, complementing the literature by indicating that it can be desirable for a firm and the entire system when the information provider strategically

decides her forecast ability. This finding implies that caution should be exercised when deciding whether and when cry-wolf mitigation is beneficial.

Specifically, since the mitigation of cry-wolf syndromes can presumably be started by forecasters, manufacturers, or even by exogenous forces, our results suggest the following strategic guidance regarding such an intervention. (i) If exogenous forces, for example—a government agency—aim to enhance system performance by initiating cry-wolf reduction, then the mitigation program should ideally be implemented if the system is centralized. If the system is decentralized, the value of such a program depends on the commission agreement between the manufacturer and the forecaster (Proposition 5). (ii) If a forecaster can initiate such a program to reduce the cry-wolf syndrome, then the forecaster probably should implement it, especially when the forecaster cannot improve her forecast ability efficiently. When the commission between firms is low, the manufacturer should facilitate forecaster-led initiatives. However, when the commission is high, the manufacturer should consider resisting relinquishing control of these efforts to the forecaster (Proposition 4). (iii) If the manufacturer is the one who initiates these programs, the manufacturer should be aware that the programs may not be helpful when the commission is relatively high.

More generally, our results indicate that the mitigation of the cry-wolf syndrome in a decentralized system might benefit one firm at the expense of the other. This suggests that prior to implementing any cry-wolf mitigation initiative, the participant who might lose from cry-wolf mitigation might initiate efforts aimed at aligning firm incentives through coordinating contracts (e.g., revenue sharing contracts). Under contracts like these, the performance of the decentralized system in equilibrium would equal that of the centralized system, and each firm receive a percentage (Chen 2003). Then, we know not only that the performance of the decentralized system as a whole improves with a reduction in the cry-wolf syndrome, but also that the efforts to reduce cry-wolf will benefit both the forecaster and manufacturer.

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Appendix: Proofs

Proof of Lemma 1 a) From equation (1),

$$\kappa(\rho) = \frac{(\rho + \bar{\rho}/2) \wedge c}{2} + \frac{(\bar{\rho}/2) \wedge c}{2}.$$

Thus, we obtain equation (2).

b) Differentiating $\kappa(\rho)$ in (2) with respect to ρ , we have $\kappa'(\rho) \leq 0$ for all the four cases. Q.E.D.

Proof of Lemma 2 a) From (5),

$$\mathcal{C}(\alpha, \rho) = 2[t\kappa(0) + \bar{t}\kappa(\rho)] + \alpha\delta(\rho)\bar{t}[\kappa(0) - \kappa(\rho)].$$

From Lemma 1b and the fact that $\delta(\rho)$ decreases in ρ , we can conclude that $\mathcal{C}(\alpha, \rho)$ decreases in ρ .

b) From (5),

$$\begin{aligned} \frac{\partial \mathcal{C}(\alpha, \rho)}{\partial \alpha} &= \delta(\rho)\kappa(0) - \delta(\rho)[\kappa(\rho) + T(\rho)] \\ &= \delta(\rho)[\kappa(0) - \kappa(\rho) - t(\kappa(0) - \kappa(\rho))] \\ &\geq 0, \end{aligned}$$

where the second equality is from the definition of $T(\rho)$ and the inequality is from Lemma 1b. Q.E.D.

Proof of Proposition 1. The forecaster’s objective function is

$$\Pi(\alpha, \rho) := \begin{cases} -\theta\rho^2 & \text{if } c \leq 1/2 \text{ and } \rho \leq 2\omega \\ \frac{t(4-\alpha\bar{\rho})(\rho-2\omega)}{8} - \theta\rho^2 & \text{if } c \leq 1/2 \text{ and } \rho > 2\omega \\ \frac{t(4-\alpha\bar{\rho})(\rho-2\omega)}{8} - \theta\rho^2 & \text{if } c > 1/2 \text{ and } \rho > 2\omega \\ -\theta\rho^2 & \text{if } c > 1/2 \text{ and } \rho \leq 2\omega. \end{cases}$$

We first study the case where $c \leq 1/2$. (i) When $\rho \leq 2\omega$, $\Pi(\alpha, \rho) = -\theta\rho^2$ decreases in ρ and $\Pi(\alpha, \rho = 0) = 0$. (ii) When $\rho > 2\omega$, $\Pi(\alpha, \rho)$ is concave in ρ with the first-order solution $\rho = \frac{t[2-\alpha(\omega+0.5)]}{8\theta-t\alpha}$.

This solution is in the region $[0, 1]$ as our assumption $\theta \geq \frac{t(4+\alpha)}{16}$ implies that $8\theta - t\alpha \geq 0$ and $t[2 - \alpha(\omega + 0.5)] \leq 8\theta - t\alpha$. Consequently, the first-order solution is optimal when

$$\frac{t[2 - \alpha(\omega + 0.5)]}{8\theta - t\alpha} > 2\omega \text{ and } \Pi\left(\alpha, \rho = \frac{t[2 - \alpha(\omega + 0.5)]}{8\theta - t\alpha}\right) \geq 0,$$

which is equivalent to $c \geq c_1$, where $c_1 := \frac{2t\alpha - 2(4-\alpha)[4\theta - \sqrt{2\theta(8\theta - \alpha t)}]}{\alpha^2 t}$ if $\alpha > 0$ and $c_1 := \frac{1}{2} - \frac{t}{16\theta}$ if $\alpha = 0$, and we can verify that $c_1 < 1/2$. When $c < c_1$, $\rho = 0$ is optimal.

Similarly, when $c > 1/2$, we can show that $\rho = \frac{t[2 - \alpha(\omega + 0.5)]}{8\theta - t\alpha}$ is optimal when $c \leq \bar{c}_1$; otherwise, $\rho = 0$ is optimal. Then, we have (7). Q.E.D.

Proof of Proposition 2. Suppose $c_1 \leq c \leq 1/2$. From (7) and the proof of Proposition 1, we have $\frac{d\hat{\rho}}{d\alpha} = \frac{2t(t-4\bar{c}\theta)}{(8\theta - \alpha t)^2} > 0 \iff c > c_{\rho 1}$, where $c_{\rho 1} = 1 - \frac{t}{4\theta}$. Because $c_{\rho 1} < 1/2 \iff t > 2\theta$, we know that $\frac{d\hat{\rho}}{d\alpha} > 0$ if $t > \min(2\theta, 1)$ and $\max(c_{\rho 1}, c_1) < c \leq 1/2$.

Suppose $1/2 < c \leq \bar{c}_1$. Similarly, we can show that $\frac{d\hat{\rho}}{d\alpha} > 0$ if $t > \min(2\theta, 1)$ and $1/2 < c < \min(\bar{c}_{\rho 1}, \bar{c}_1)$.

Suppose $c < c_1$ or $c > \bar{c}_1$. We have $\frac{d\hat{\rho}}{d\alpha} = 0$. Let $t_1 := \min(2\theta, 1)$, $\underline{c} := \max(c_{\rho 1}, c_1)$, and $\bar{c} := \min(\bar{c}_{\rho 1}, \bar{c}_1)$. The desired result follows. Q.E.D.

Proof of Lemma 3. The solution of (8) is the same as $\arg \max_{\rho \in [0, 1]} -[\theta \rho^2 + 2\kappa(\rho)]$. In a way similar to the proof of Proposition 1, we can show that optimal forecast accuracy of the centralized system is $\rho_c = \frac{1}{4\theta}$ if $\frac{1}{2} - \frac{1}{16\theta} \leq c \leq \frac{1}{2} + \frac{1}{16\theta}$, and $\rho_c = 0$ otherwise. With $\alpha = 0$, from Proposition 1, the optimal forecast accuracy of the decentralized system is $\frac{t}{4\theta}$ if $\frac{1}{2} - \frac{t}{16\theta} \leq c \leq \frac{1}{2} + \frac{t}{16\theta}$, and 0 otherwise. The desired result follows after comparing the solutions of centralized and decentralized systems. Q.E.D.

Proof of Proposition 3. a) From Proposition 1, $\hat{\rho}$ achieves the maximal value when $\omega = 0$. For $\omega = 0$,

$$\hat{\rho} - \rho_c = \frac{t(4 - \alpha)}{2(8\theta - \alpha t)} - \frac{1}{4\theta} = \frac{(8\theta + \alpha - 2\alpha\theta)t - 8\theta}{4(8\theta - t\alpha)\theta},$$

which is non-negative if and only if $t \geq \frac{8\theta}{\alpha(1-2\theta)+8\theta}$. Define $t_2 := \min(\frac{8\theta}{\alpha(1-2\theta)+8\theta}, 1)$. It is easy to verify that $t_2 \geq t_1$ from their definitions.

Lastly, let $\hat{\rho} - \rho_c = 0$, we obtain $\alpha = \frac{8\bar{t}\theta}{t[1-2(1+2\omega)\theta]}$.

b) Note that when $c_1 \leq c \leq \bar{c}_1$, we have $\hat{\rho} - \rho_c = \frac{t[2 - \alpha(\omega + 1/2)]}{8\theta - t\alpha} - \frac{1}{4\theta} > 0 \iff \omega < \frac{1}{4\theta} - \frac{2\bar{t}}{\alpha t} - \frac{1}{2} \iff c_{\rho 2} < c < \bar{c}_{\rho 2}$, where $c_{\rho 2} := 1 - \frac{1}{4\theta} + \frac{2\bar{t}}{\alpha t}$. It is easy to show that $c_{\rho 2} \geq c_{\rho 1}$. Thus, we can conclude. Q.E.D.

Proof of Proposition 4. a) For the case $c_1 \leq c \leq \frac{1}{2}$, $\Pi'(\alpha) = \frac{\frac{t}{4}[\theta - \frac{t}{4}(1 + \frac{c\alpha}{2})][\frac{1}{2}(\theta - \frac{t}{4}) - c(\theta - \frac{t\alpha}{16})]}{(\theta - \frac{t\alpha}{8})^2}$. By solving $\Pi'(\alpha) < 0$ with respect to c , we have $\Pi'(\alpha) < 0$ when $c > \frac{2(4\theta - t)}{16\theta - t\alpha}$. We can show that $\frac{2(4\theta - t)}{16\theta - t\alpha} \leq c_1$ after

some algebra. Therefore, $\Pi'(\alpha) < 0$ when $c_1 \leq c \leq \frac{1}{2}$. Similarly, we have $\Pi'(\alpha) < 0$ when $\frac{1}{2} \leq c \leq \bar{c}_1$. When $c < c_1$ or $c > \bar{c}_1$, we have $\hat{\rho} = 0$ and thus $\Pi'(\alpha) = 0$.

b) From (9),

$$\mathcal{C}'(\alpha) = \frac{\partial \mathcal{C}(\alpha, \hat{\rho})}{\partial \alpha} + \frac{\partial \mathcal{C}(\alpha, \rho)}{\partial \rho} \Big|_{\rho=\hat{\rho}} \frac{d\hat{\rho}}{d\alpha}$$

Recall that Lemma 2 implies that $\frac{\partial \mathcal{C}(\alpha, \hat{\rho})}{\partial \alpha} \geq 0$, and $\frac{\partial \mathcal{C}(\alpha, \rho)}{\partial \rho} \leq 0$, and Proposition 2 implies that $\frac{d\hat{\rho}}{d\alpha} < 0$ when $t \leq t_1$. Thus, $\mathcal{C}'(\alpha) \geq 0$ when $t \leq t_1$.

Suppose $t > t_1$. In order to show that the manufacturer's equilibrium cost can decrease in α when θ is small, it is enough to show that this result holds in some case (e.g., $c = \frac{1}{2}$). Notice that $\mathcal{C}'(\alpha)|_{c=\frac{1}{2}} = \frac{t(1-t)(1-\frac{\alpha}{4})\Gamma_1}{32(\theta-\frac{t\alpha}{8})^3}$, where $\Gamma_1 = 2\theta^2 - \frac{3}{4}t\theta(1-\frac{\alpha}{4}) + \frac{1}{32}\alpha t^2(1+\frac{\alpha}{4})$. To find the condition for $\mathcal{C}'(\alpha)|_{c=\frac{1}{2}} < 0$, we need to find the condition for $\Gamma_1 < 0$. Note that $\frac{d^2\Gamma_1}{d\theta^2} > 0$ and $\Gamma_1|_{\theta=\theta_{min}} < 0$. We know that Γ_1 is convex in θ and has two solutions. Let θ_1 and θ'_1 denote the large and small root, respectively. We have $\theta'_1 < \theta_{min} < \theta_1$, and $\theta_1 = \frac{t[3(1+\frac{\alpha}{4})+\sqrt{(1+\frac{\alpha}{4})(9-\frac{7}{4}\alpha)}]}{16}$. Therefore, $\mathcal{C}'(\alpha)|_{c=\frac{1}{2}} < 0$ when $\theta < \theta_1$. Q.E.D.

Proof of Proposition 5. The result for $t \leq t_1$ is directly from Proposition 4, because a necessary condition for $\mathcal{S}'(\alpha) < 0$ is $\mathcal{C}'(\alpha) < 0$, i.e., $t > t_1$.

Suppose $t > t_1$. In order to show that the system cost can decrease in α when θ is small, it is enough to show that this result holds in some case (e.g., $c = \frac{1}{2}$). Notice that $\mathcal{S}'(\alpha)|_{c=\frac{1}{2}} = \frac{t(1-\frac{\alpha}{4})\Gamma_2}{32(\theta-\frac{t\alpha}{8})^3}$, where $\Gamma_2 = (2-t)\theta^2 + \frac{1}{32}\alpha t^2(1+\frac{\alpha}{4}) - \frac{1}{4}t\theta(3-2t+\frac{3\alpha}{4})$. To find the condition for $\mathcal{S}'(\alpha)|_{c=\frac{1}{2}} < 0$, we need to find the condition for $\Gamma_2 < 0$. Note that $\frac{d^2\Gamma_2}{d\theta^2} > 0$ and $\Gamma_2|_{\theta=\theta_{min}} < 0$. We know that Γ_2 is convex in θ and has two solutions. Let θ_2 and θ'_2 denote the large and small root, respectively. We have $\theta'_2 < \theta_{min} < \theta_2$, and $\theta_2 = \frac{t[3-2t+\frac{3\alpha}{4}+\sqrt{4t^2-4t(1+\frac{\alpha}{4})(3-\frac{\alpha}{2})+(1+\frac{\alpha}{4})(9-\frac{7}{4}\alpha)}]}{8(2-t)}$. Therefore, $\mathcal{S}'(\alpha)|_{c=\frac{1}{2}} < 0$ when $\theta < \theta_2$. Because the necessary condition for $\mathcal{S}'(\alpha)|_{c=\frac{1}{2}} < 0$ is $\mathcal{C}'(\alpha)|_{c=\frac{1}{2}} < 0$, we must have $\theta_2 \leq \theta_1$. Q.E.D.

PROPOSITION 6. Suppose $\alpha > 0$, $\theta \geq \frac{\alpha}{4(4-\alpha)}$ and $\max(\frac{2(1-4\theta)}{\alpha-16\theta}, 1) < c < \frac{1}{2}$. The optimal commission rate $\hat{t} = \hat{t}_1$ if $\mathcal{C}(\hat{t}_1) > 2c$, and the optimal commission rate $\hat{t} = 0$ otherwise, where $\mathcal{C}(\cdot)$ and \hat{t}_1 are defined in the proof.

Proof of Proposition 6. According to Proposition 1, the optimal forecast accuracy $\hat{\rho}(t) = \frac{t[2-\alpha(1-c)]}{8\theta-t\alpha}$ when $c_1(t) \leq c < \frac{1}{2}$, and $\hat{\rho}(t) = 0$ when $0 < c < c_1(t)$. It is easy to show that $c_1(t) \leq c$ is equivalent to $t \geq \frac{8\theta(1-2c)(4-\alpha)}{(2-c\alpha)^2} (> 0)$. Let $t_{min} := \min(\frac{8\theta(1-2c)(4-\alpha)}{(2-c\alpha)^2}, 1)$. If $t < t_{min}$, then $\hat{\rho}(t) = 0$ and the corresponding manufacturer's expected cost is $2c$. For $t \geq t_{min}$, substituting $\hat{\rho}(t) = \frac{t[2-\alpha(1-c)]}{8\theta-t\alpha}$

into the manufacturer's expected cost (5) and considering the manufacturer's expected cost as a function of t , we have

$$\mathcal{C}(t) = [2 - \alpha\delta(\hat{\rho}(t))][\kappa(\hat{\rho}(t)) + T(\hat{\rho}(t), t)] + \alpha\delta(\hat{\rho}(t))\kappa(0). \quad (11)$$

Note that $\frac{d^2\mathcal{C}(t)}{dt^2} = \frac{16[2-(1-c)\alpha]^2\theta^2[(t-3)\alpha+16\theta]}{(t\alpha-8\theta)^4}$. Because the sign of $\frac{d^2\mathcal{C}(t)}{dt^2}$ is the same as that of $(t-3)\alpha+16\theta$, and $(t-3)\alpha+16\theta$ increases in t , we know that the sign of $\frac{d^2\mathcal{C}(t)}{dt^2}$ changes at most once (from negative to positive). We further notice that $\frac{d\mathcal{C}(t)}{dt} = \frac{-t^3\alpha^2(c\alpha-2)^2+24\alpha\theta t^2(c\alpha-2)^2-64t\theta^2[16+\alpha(\alpha-4+4\alpha c^2-8c-2c\alpha)]+128\theta^2[(2+c\alpha-\alpha)^2+4\theta(1-2c)(4-\alpha)]}{8(t\alpha-8\theta)^3}$, $\frac{d\mathcal{C}(t)}{dt}|_{t=0} = -\frac{(2-\alpha+c\alpha)^2+4\theta(1-2c)(4-\alpha)}{32\theta} < 0$, and $\frac{d\mathcal{C}(t)}{dt}|_{t=1} = -\frac{(8\theta-2-16c\theta+c\alpha)(c\alpha^2-2\alpha+32\theta-8\theta\alpha)}{8(8\theta-\alpha)^2}$. Let $\Gamma_c = -(8\theta-2-16c\theta+c\alpha)(c\alpha^2-2\alpha+32\theta-8\theta\alpha)$. Solving the function $\Gamma_c = 0$ with respect to c , we have two solutions: $c_{11} = \frac{2(1-4\theta)}{\alpha-16\theta} < \frac{1}{2}$ and $c_{12} = \frac{2\alpha+8\alpha\theta-32\theta}{\alpha^2} < 0$ when $\theta \geq \frac{\alpha}{4(4-\alpha)}$. Moreover, Γ_c is convex in c for $\frac{d^2\Gamma_c}{dc^2} = 2\alpha^2(16\theta-\alpha) > 0$. Therefore, when $\max(\frac{2(1-4\theta)}{\alpha-16\theta}, 0) < c < \frac{1}{2}$, we have $\Gamma_c > 0$. That is, $\frac{d\mathcal{C}(t)}{dt}|_{t=1} > 0$. Thus, there exists a unique real solution t^* for $\frac{d\mathcal{C}(t)}{dt} = 0$, and $\max(t_{min}, t^*)$ minimizes (11). Let $\hat{t}_1 := \max(t_{min}, t^*)$. Without loss of generality, we assume that the manufacturer sets $t = 0$ if the manufacturer chooses not to incentivize the forecaster to improve forecast ability (recall that if $t < t_{min}$, then $\hat{\rho}(t) = 0$ and the resulting $\mathcal{C}(t) = 2c$). The desired result follows. Q.E.D.