

XV Taller de la División de Gravitación y Física Matemática

de la SMF

¿Puede la auto-interacción estabilizar los estados excitados de una estrella de bosones?

Armando A. Roque Estrada



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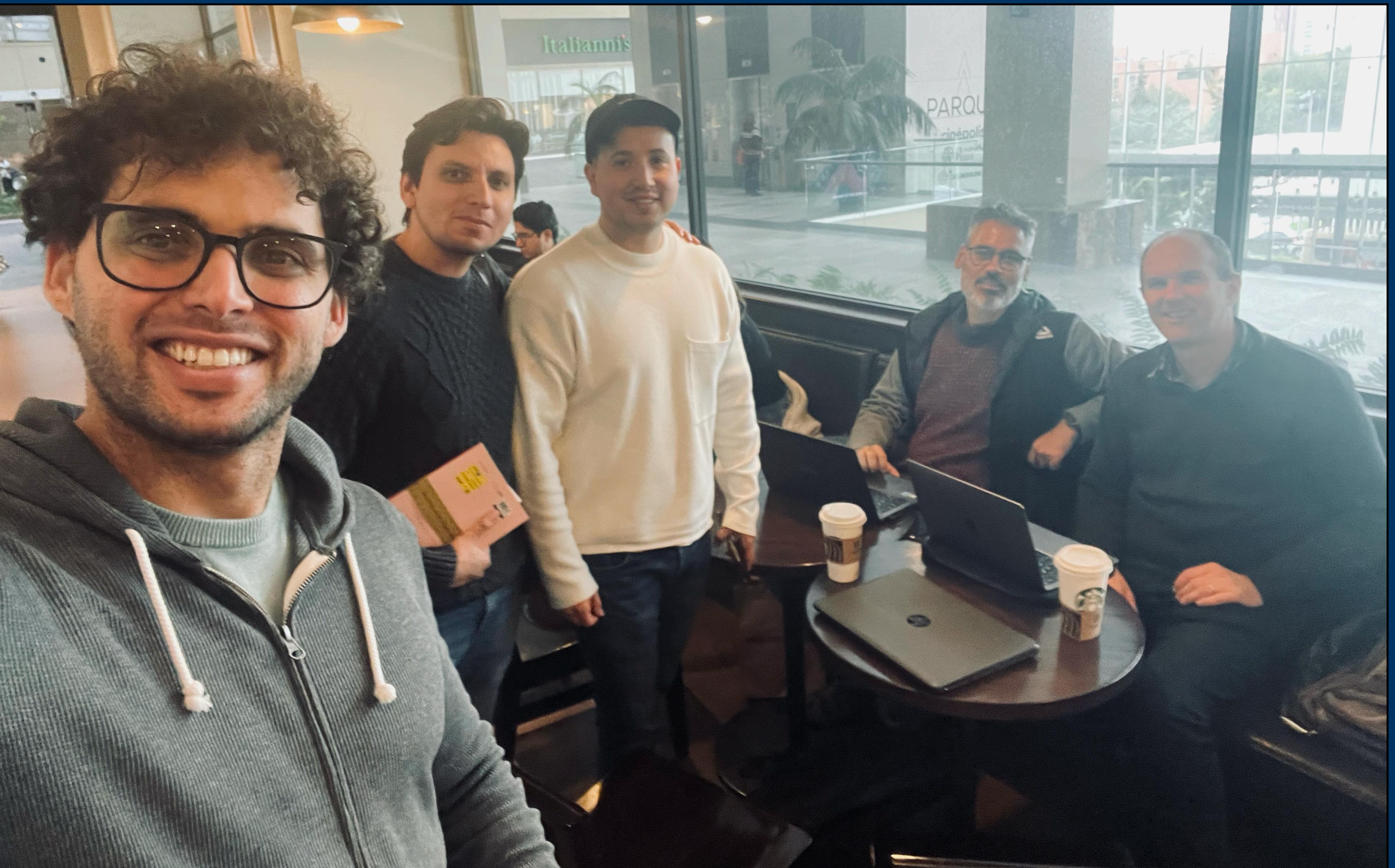
Colaboradores del proyecto:

MSc. Emmanuel Chávez Nambo

Dr. Alberto Diez-Tejedor

Dr. Olivier Sarbach





27 de septiembre

Estructura

- I. ¿Qué entendemos como una estrella solitónica?
- II. Algunos comentarios sobre su fenomenología y estabilidad
- III. Estrellas de bosones no-relativistas
- IV. Estabilidad
- IV. Comentarios finales

¿Qué es una estrella solitónica?

estrella

Gravedad + “Materia”

Primer intento:

John A. Wheeler

PHYSICAL REVIEW

VOLUME 97, NUMBER 2

JANUARY 15, 1955

Geons*

JOHN ARCHIBALD WHEELER

Palmer Physical Laboratory, Princeton University, Princeton, New Jersey

(Received September 8, 1954)

$$G_{ik} = (8\pi G/c^4)T_{ik},$$

$$T_i{}^k = (1/4\pi)(F_{i\alpha}F^{k\alpha} - \frac{1}{4}F_{\alpha\beta}F^{\alpha\beta}\delta_i{}^k),$$

Inestables

Segundo intento:



Gravedad

RG +

Campo Escalar
Complejo ψ

Sector de materia

Sistema
Einstein-Klein-
GordonEstrellas de
Bosones

La acción

$$S[g_{\mu\nu}, \psi] = \int d^4x \sqrt{-g} \left[\frac{1}{16\pi G} R - \nabla_\mu \psi \nabla^\mu \psi^* - V(\psi\psi^*) \right]$$

La física

“Presión”
repulsiva

Gravedad
atractiva

Equilibrio
hidrostático

Objetos
compactos
localizados



PHYSICAL REVIEW

VOLUME 172, NUMBER 5

25 AUGUST 1968

Klein-Gordon Geon*

DAVID J. KAUP†

University of Maryland, College Park, Maryland

(Received 4 March 1968)

A study of the spherically symmetric eigenstates of the Klein-Gordon Einstein equations (Klein-Gordon geons) reveals that these geons have properties that are uniquely different from other gravitating systems that have been studied. The equilibrium states of these geons seem analogous to other gravitating systems; but when the question of stability is considered from a thermodynamical viewpoint, it is shown that, in contrast with other systems, adiabatic perturbations are forbidden. The reason is that the equations of state for the thermodynamical variables are not algebraic equations, but instead are differential equations. Consequently, the usual concept of an equation of state breaks down when Klein-Gordon geons are considered. When the question of stability is reconsidered in terms of infinitesimal perturbations of the basic

PHYSICAL REVIEW

VOLUME 187, NUMBER 5

25 NOVEMBER 1969

Systems of Self-Gravitating Particles in General Relativity and the Concept of an Equation of State*

REMO RUFFINI†

Palmer Physical Laboratory, Princeton University, Princeton, New Jersey 08540
and

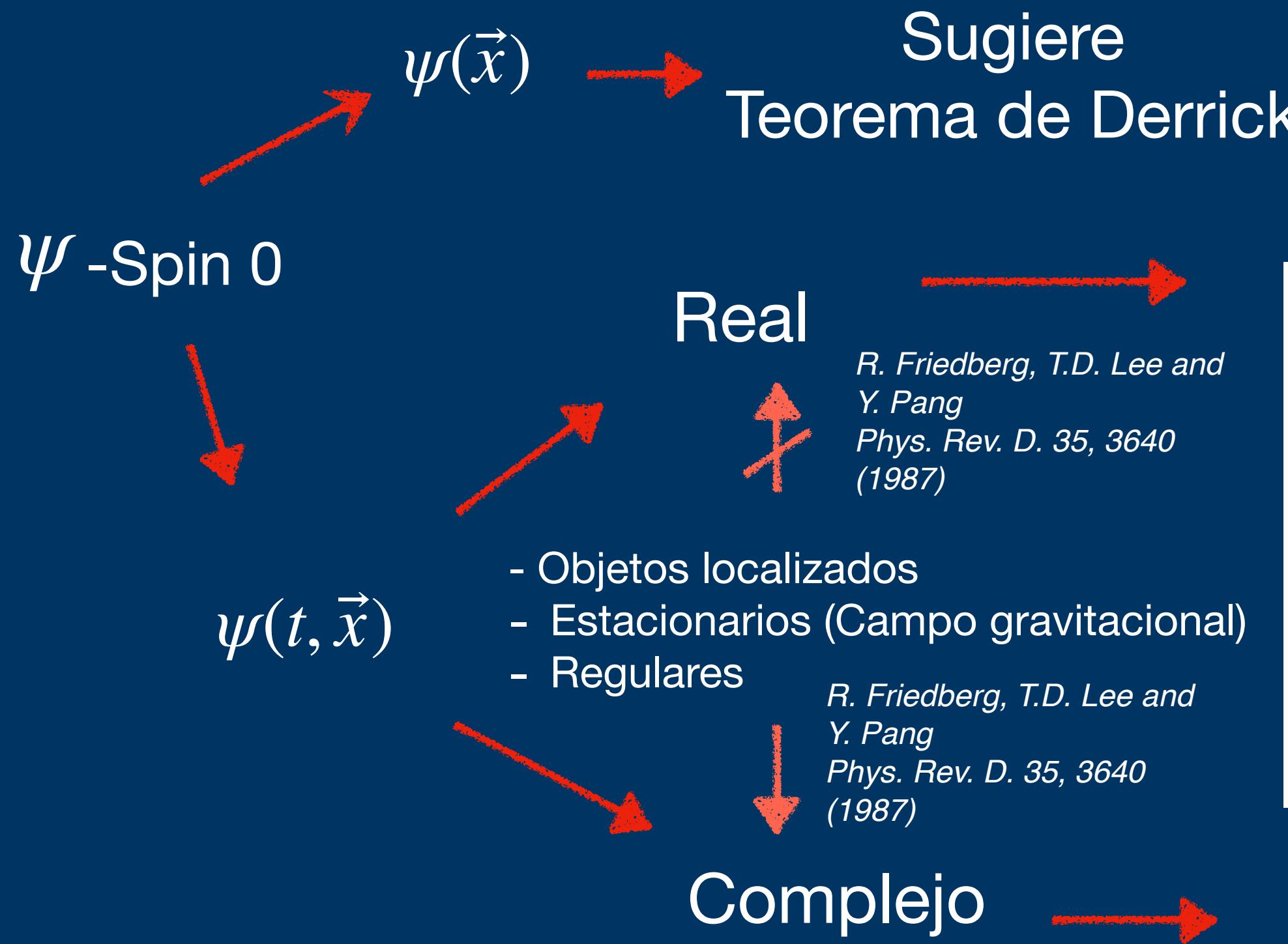
Institute for Advanced Study, Princeton, New Jersey 08540

AND

SILVANO BONAZZOLA‡

Facoltà di Matematica, Università di Roma, Roma, Italy
(Received 4 February 1969)

El campo escalar:



El potencial:

$$V[|\psi|^2] = m^2 |\psi|^2 + M^4 \sum_{n=2}^{\infty} \frac{v_{2n}}{(2n)} \left| \frac{\psi}{M} \right|^{2n}$$

termino de masa

términos de auto-interacción

$\sim |\psi|^4$

$\sim c_1 |\psi|^4 + c_2 |\psi|^6$

J. Math. Phys. 5, 1252–1254 (1964)

Soluciones localizadas y estacionarias de Eq. KG en 3D o dimensiones más altas son inestables (si $V[|\psi|^2] > 0$ no existen)

Dependiente del tiempo

$$N^2(t, r) = 1 + \sum_{j=0}^{\infty} N_{2j}(r) \cos(2j\omega_0 t),$$

$$g^2(t, r) = 1 + \sum_{j=0}^{\infty} g_{2j}(r) \cos(2j\omega_0 t),$$

$$\phi(t, r) = \sum_{j=1}^{\infty} \phi_{2j-1}(r) \cos[(2j-1)\omega_0 t].$$

VOLUME 66, NUMBER 13

PHYSICAL REVIEW LETTERS

1 APRIL 1991

Oscillating Soliton Stars

Edward Seidel

National Center for Supercomputing Applications, Beckman Institute, Urbana, Illinois 61801

Wai-Mo Suen

Department of Physics, Washington University, St. Louis, Missouri 63130

(Received 26 December 1990)

osciliones

Simetría $U(1)$

$$\psi(t, \vec{x}) = \sigma(r) e^{-i\omega t}$$

- Q-balls (no relativista)
- Mini-solion (boson) stars

- Campos con Spin 1: Estrellas de Proca

$$\mathcal{L}_M = -\frac{1}{2} F_{\mu\nu}^* F^{\mu\nu} - \boxed{m_0^2 A_\mu^* A^\mu} - \lambda (A_\mu^* A^\mu)^2,$$

$$A_\mu(t, \vec{x}) = e^{-i\omega t} (f(r), ig(r), 0, 0).$$

- Campos con Spin 2: teorías Bimetric

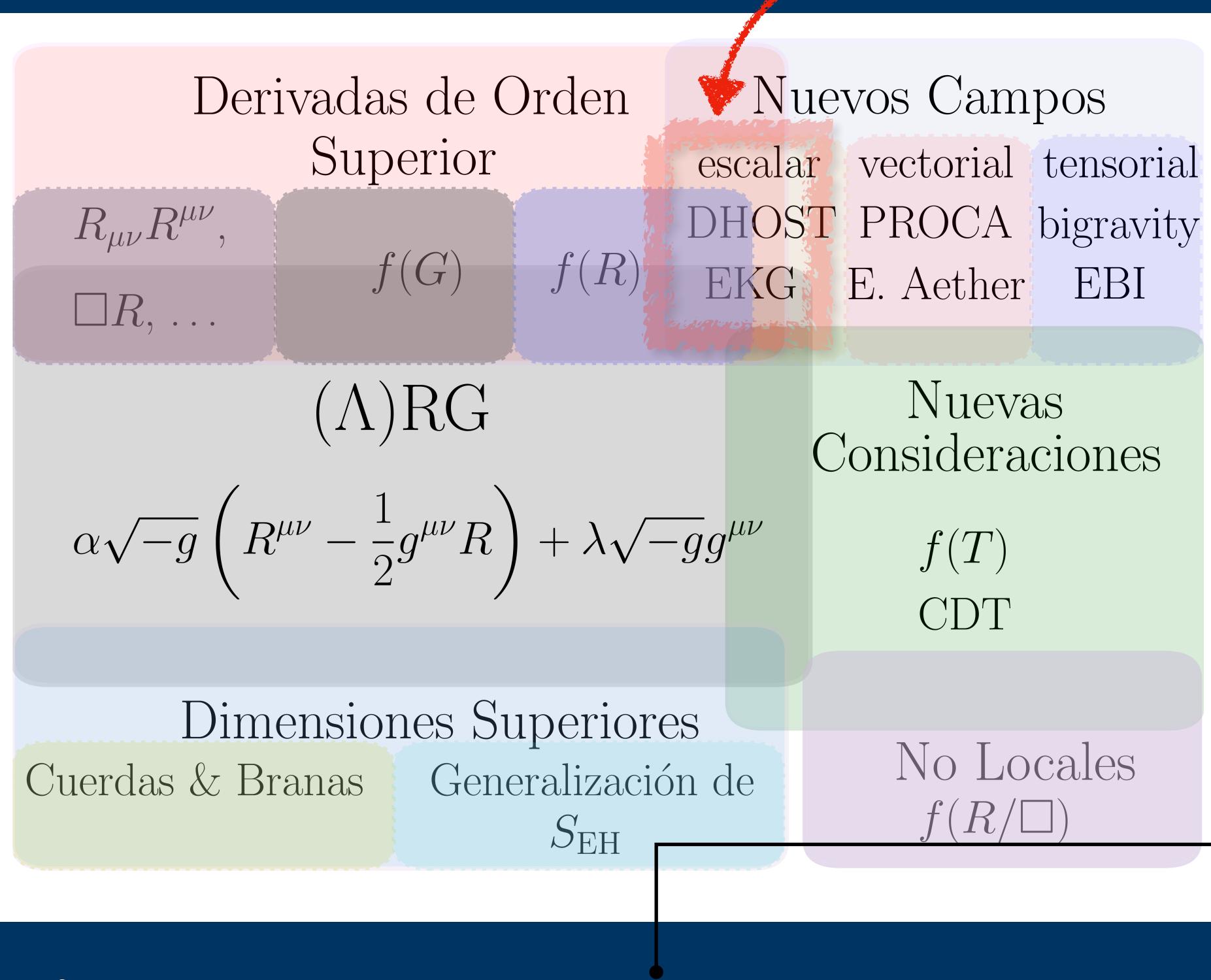
Gravedad modificada (escalar)

Grado de libertad escalar extra proveniente del sector gravitatorio

Ejemplo

El modelo gravedad (beyond) Horndeski

$$M_{Pl} = 1/\sqrt{8\pi G}$$



detección GW170817, y su contraparte electromagnética GRB170817A

$$c_{GW}/c - 1 \leq 7 \times 10^{-16}$$

modelos *viables* beyond Horndeski.

$$G_{5,X} = 0, \quad F_5 = 0, \quad 2G_{4,X} - XF_4 + G_{5,\phi} = 0$$

P. Creminelli, et. al., J.M. Ezquiaga, et. al.

$$S = S_{grav} + S_m = \int d^4x \sqrt{-g} \left(\sum_{i=2}^5 \mathcal{L}_i[g_{\mu\nu}, \phi] + \mathcal{L}_m[g_{\mu\nu}, \Psi] \right)$$

$$\mathcal{L}_2 := G_2[\phi, X]$$

$$\mathcal{L}_3 := G_3[\phi, X] \Box \phi$$

acoplamiento entre $\phi, g_{\mu\nu} \rightarrow c_{GW} \neq c$, (de manera general).

$$\mathcal{L}_4 := G_4[\phi, X]R - 2G_{4X}(\phi, X)(\Box\phi^2 - \phi^{\mu\nu}\phi_{\mu\nu})$$

$$\mathcal{L}_5 := G_5[\phi, X]G_{\mu\nu}\phi^{\mu\nu} + \frac{1}{3}G_{5X}(\phi, X)(\Box\phi^3 - 3\Box\phi\phi_{\mu\nu}\phi^{\mu\nu} + 2\phi_{\mu\nu}\phi^{\mu\sigma}\phi_{\sigma}^{\nu})$$

$$\mathcal{L}_4^{ext} := F_4[\phi, X]\epsilon^{\mu\nu\rho}_{\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\phi_{\mu}\phi_{\mu'}\phi_{\nu\nu'}\phi_{\rho\rho'}$$

$$\mathcal{L}_5^{ext} := F_5[\phi, X]\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\phi_{\mu}\phi_{\mu'}\phi_{\nu\nu'}\phi_{\rho\rho'}\phi_{\sigma\sigma'}$$

donde $\phi_{\mu} \equiv \nabla_{\mu}\phi$, $\phi_{\mu\nu} \equiv \nabla_{\nu}\nabla_{\mu}\phi$, $\Box\phi \equiv \phi_{\mu}^{\mu}$, $X \equiv g^{\mu\nu}\phi_{\mu}\phi_{\nu}$

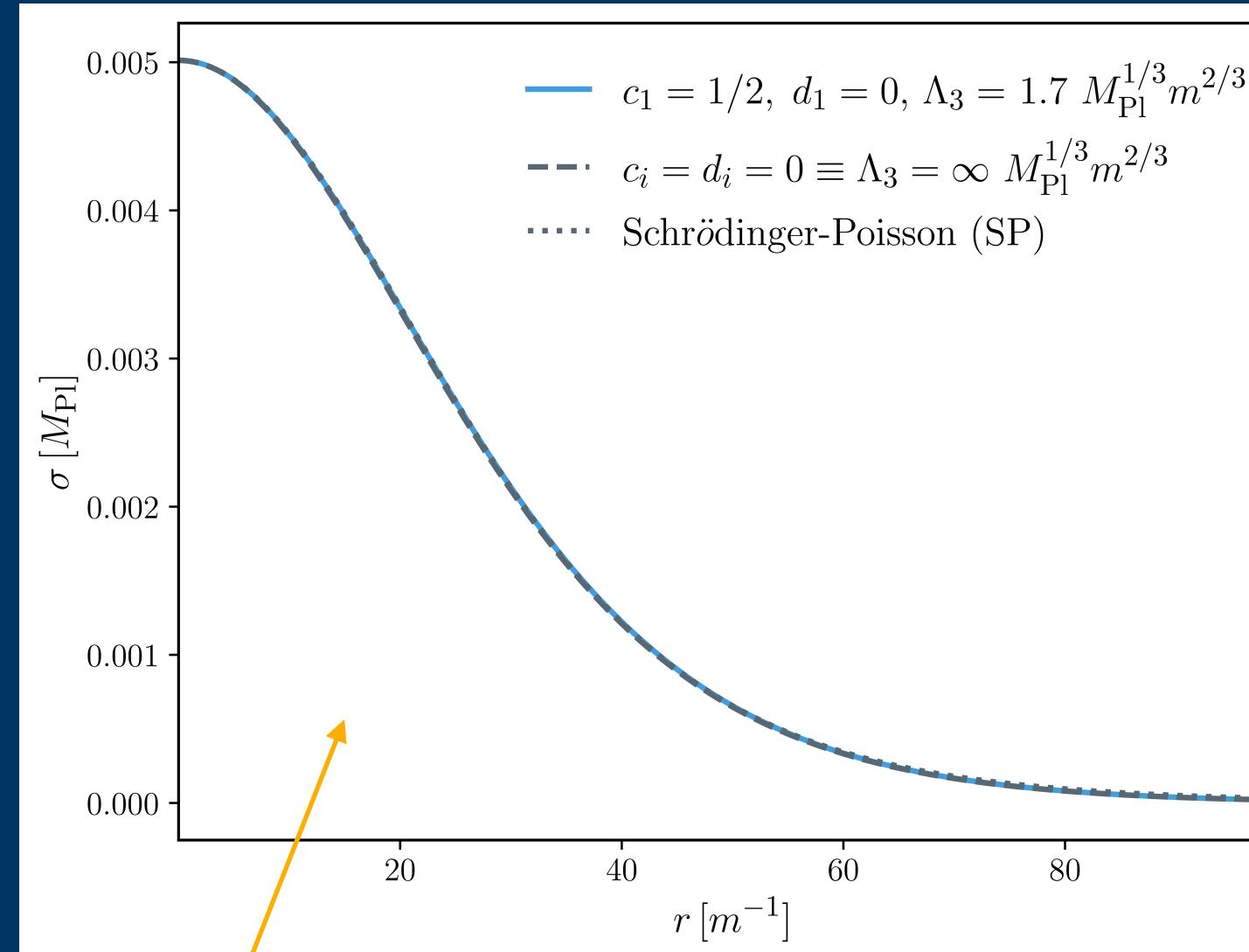
- ¿Son estrellas escalares?
- ¿Son estrellas de bosones?

Gravedad y solitones en (beyond) Horndeski

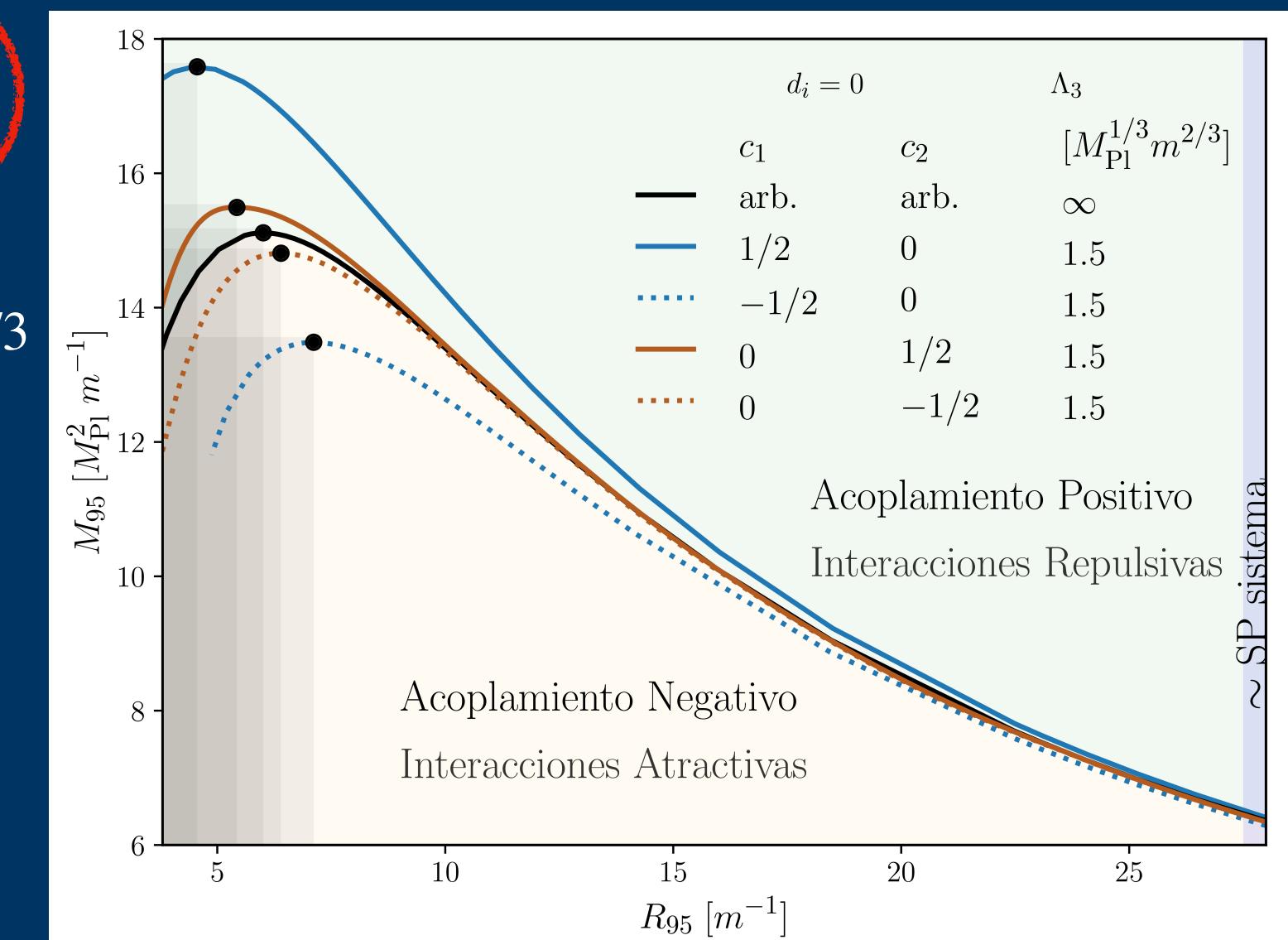
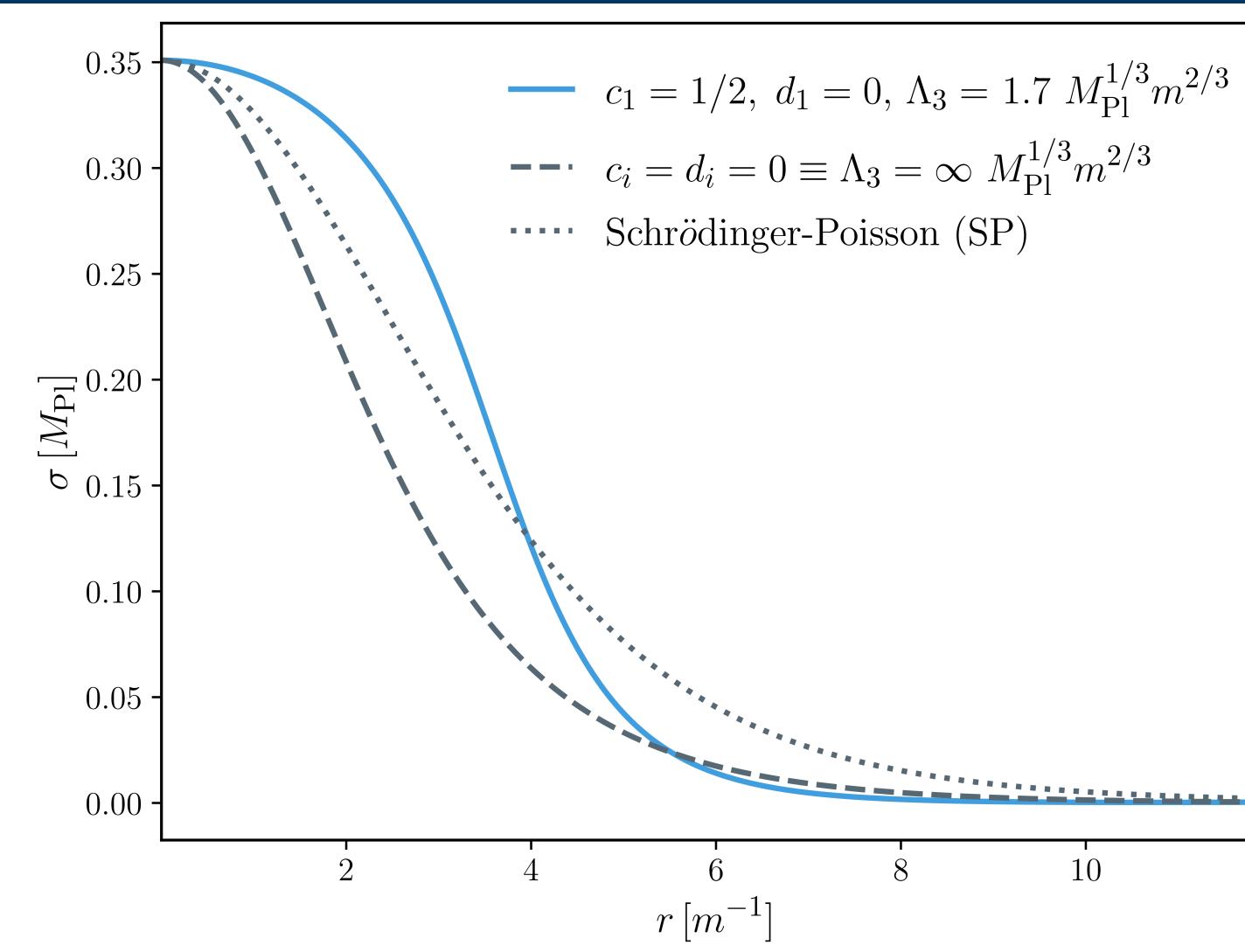
El modelo efectivo de bajas energías

$$\mathcal{L} = \frac{1}{2} M_{Pl}^2 R - X - m^2 \phi \bar{\phi} + \frac{M_{Pl}}{\Lambda_3^3} \left[c_1 X R - 2c_1 [\square \phi \square \bar{\phi} - \phi^{\mu\nu} \bar{\phi}_{\mu\nu}] + \frac{d_1}{X} \epsilon^{\mu\nu\rho}{}_\sigma \epsilon^{\mu'\nu'\rho'\sigma} \phi_\mu \bar{\phi}_{\mu'} \phi_{\nu\nu'} \bar{\phi}_{\rho\rho'} \right] + \dots$$

$\sigma_0 \ll \frac{M_{Pl}^{1/2} \Lambda_3^{3/2}}{m}$



$$\sigma_0 \ll M_{Pl} \equiv 1/\sqrt{8\pi G} = 2.431 \times 10^{18} GeV$$

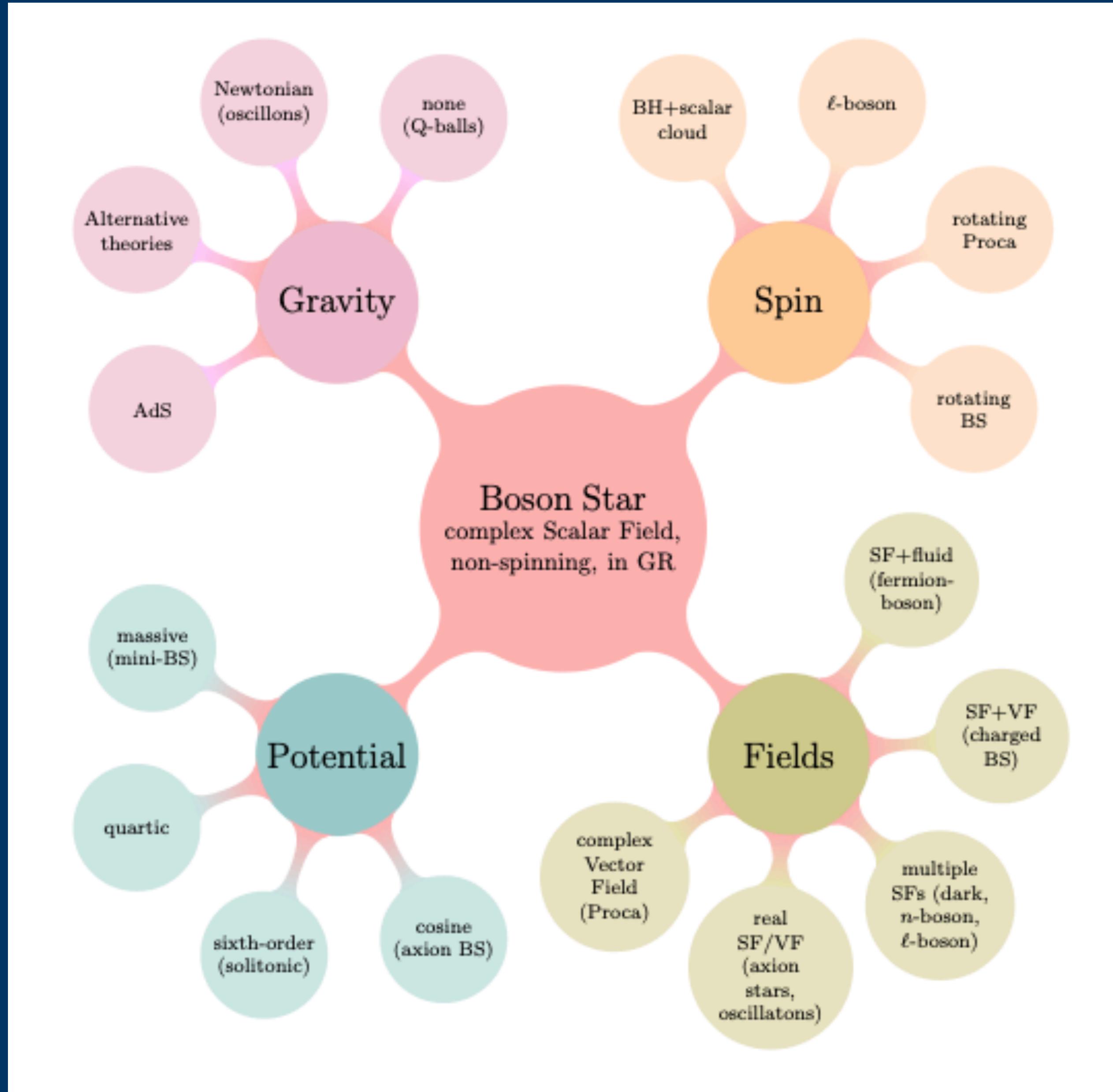


$$G_4[X] = \frac{c_0 \Lambda_2^8}{\Lambda_3^6} + \frac{c_1 X \Lambda_2^4}{\Lambda_3^6} + \frac{c_2 X^2}{\Lambda_3^6} + \frac{c_3 X^3}{\Lambda_3^6 \Lambda_2^4} + \dots$$

$$\Rightarrow F_4[X] = \frac{d_1 \Lambda_2^4}{\Lambda_3^6 X} + \frac{2d_2}{\Lambda_3^6} + \frac{3d_3 X}{\Lambda_3^6 \Lambda_2^4} + \dots$$

Landscape (“paisaje”):

arXiv:1202.5809v5



Parte de la Tabla II: arXiv:0801.0307v1

Compact Object	Self-Interaction $U(\Phi ^2)$ [$\alpha, \beta, \lambda, \lambda_{(2n+2)}, \Phi_0$ are constants]
Mini-BS:	$U_K = m^2 \Phi ^2$
Newtonian BS:	$U_N = m^2 \Phi ^2$
Self-interacting BS:	$U_{HKG} = m^2 \Phi ^2 - \alpha \Phi ^4 + \beta \Phi ^6$
BS:	$U_{CSW} = m^2 \Phi ^2 + \lambda \Phi ^4/2$
Non-topol. Soliton Star:	$U_{NTS} = m^2 \Phi ^2(1 - \Phi ^2/\Phi_0^2)^2$
General BS:	$U_{HKL} = U_{CSW} + \dots + \lambda_{(2n+2)} \Phi ^{2n+2}$
Sine-Gordon BS:	$U_{SG} = \alpha m^2 [\sin(\pi/2[\beta\sqrt{ \Phi ^2} - 1]) + 1]$
Cosh-Gordon BS:	$U_{CG} = \alpha m^2 [\cosh(\beta\sqrt{ \Phi ^2}) - 1]$
Liouville BS:	$U_L = \alpha m^2 [\exp(\beta^2 \Phi ^2) - 1]$

Fenomenología

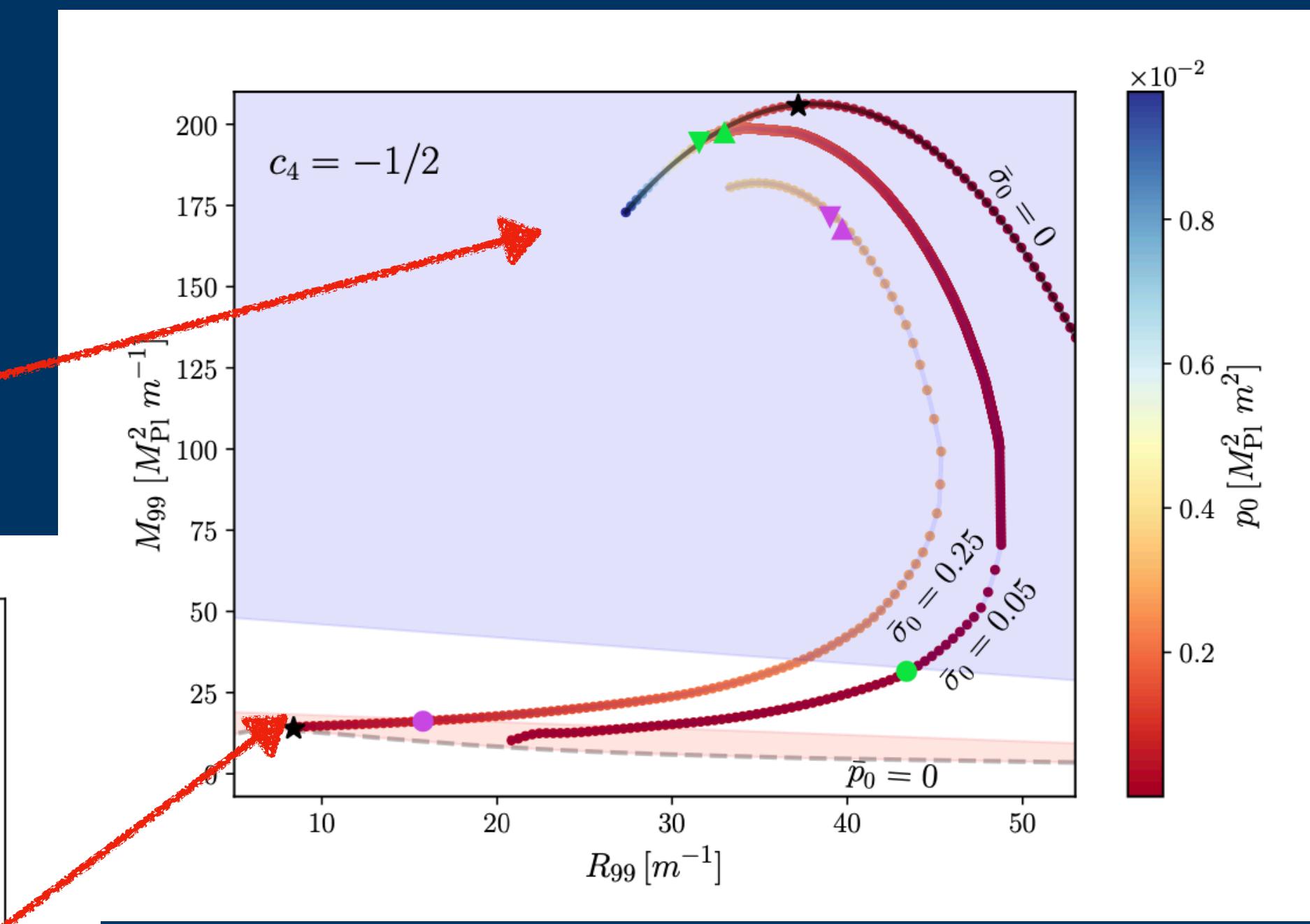
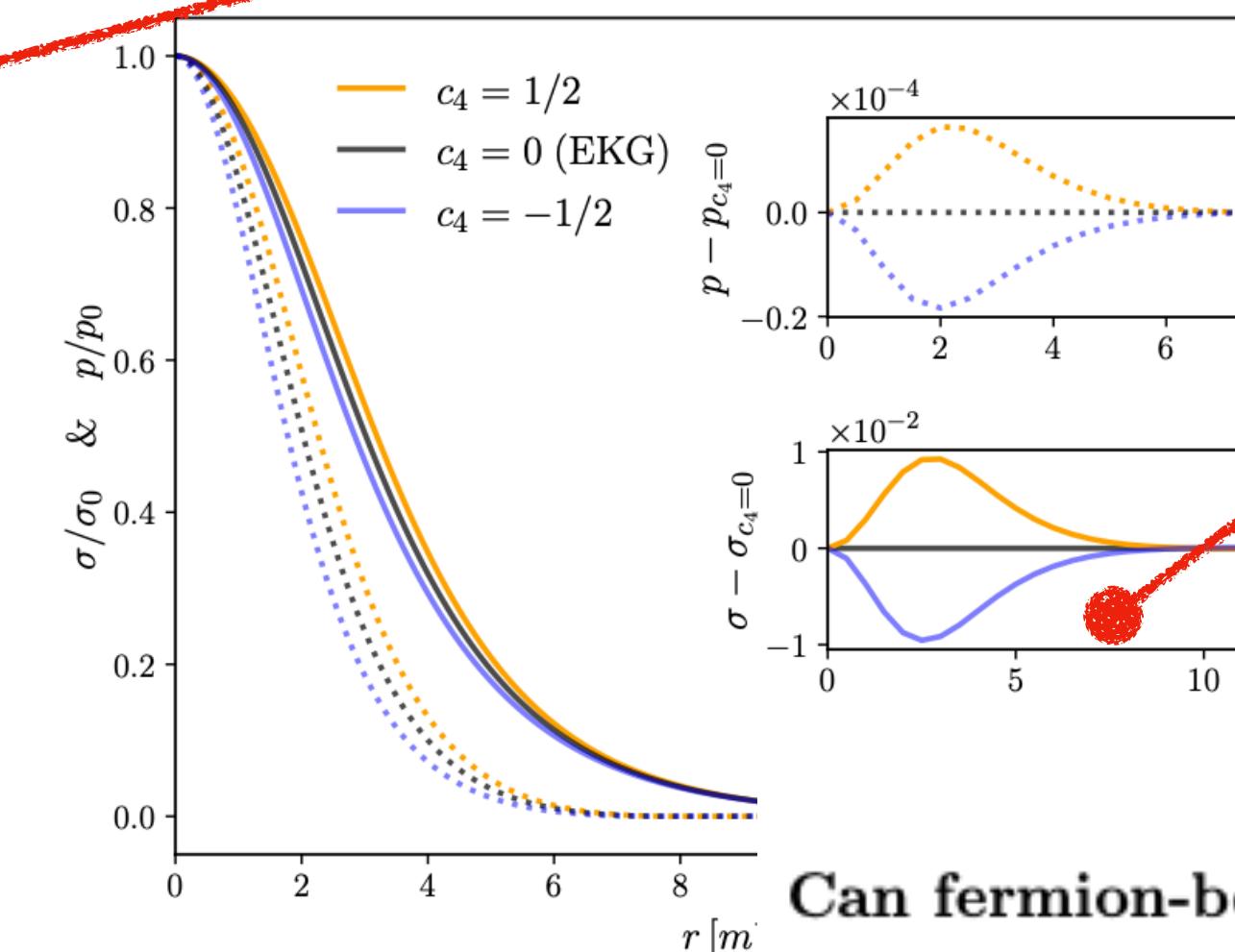
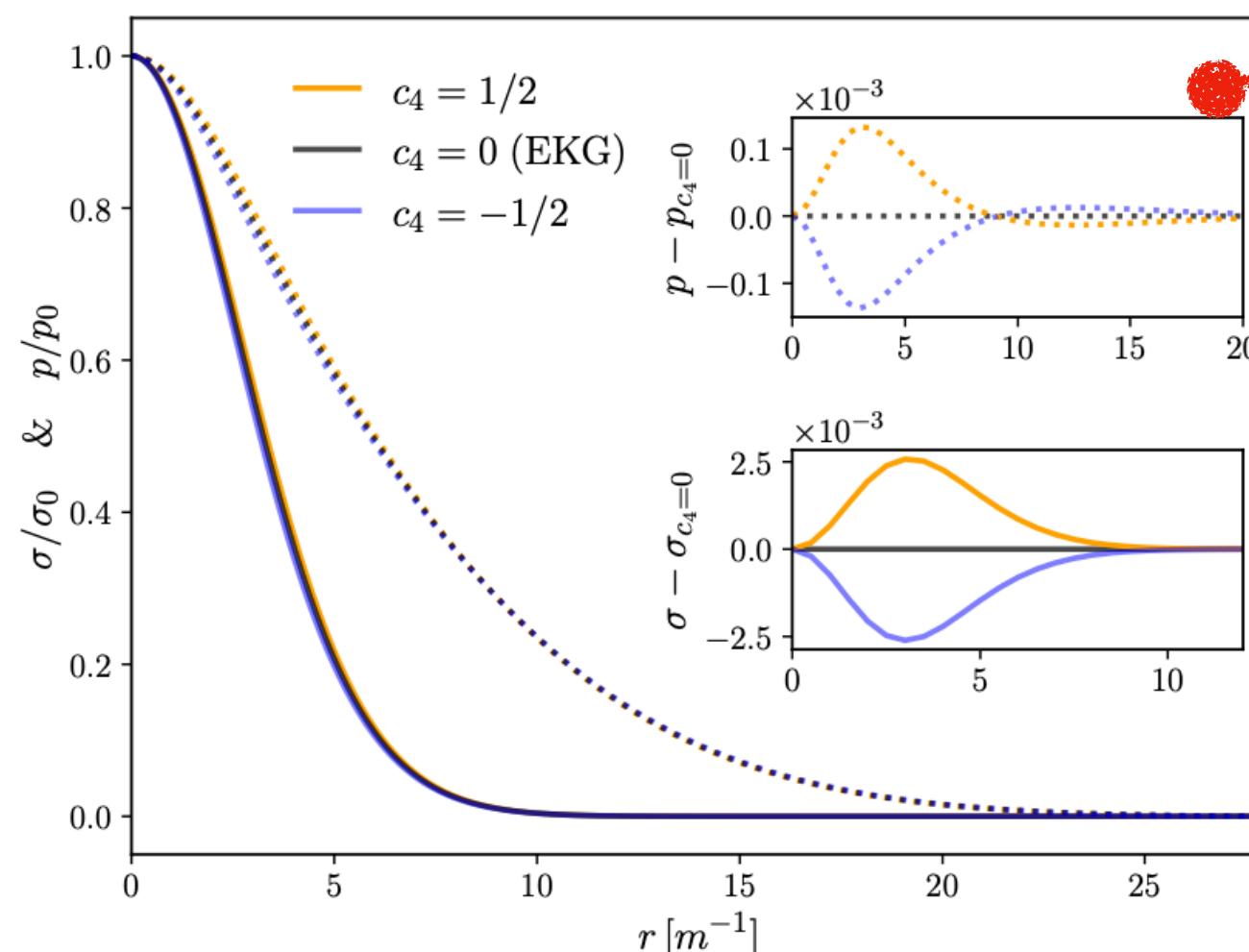
Objetos astrofísicos

Horndeski fermion-boson stars

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Universidad de Guanajuato, C.P. 37150, León, México

December 2021



Can fermion-boson stars reconcile multi-messenger observations of compact stars?

Fabrizio Di Giovanni,¹ Nicolas Sanchis-Gual,² Pablo Cerdá-Durán,¹ and José A. Font^{1,3}

¹Departamento de Astronomía y Astrofísica, Universitat de València,
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²Departamento de Matemática da Universidade de Aveiro and Centre for Research and Development
in Mathematics and Applications (CIDMA), Campus de Santiago, 3810-183 Aveiro, Portugal

³Observatori Astronòmic, Universitat de València,
Catedrático José Beltrán 2, 46980, Paterna (València), Spain
(Dated: October 28, 2021)

Halos de materia oscura

boson stars

$$\rho(r) = \begin{cases} \rho_{\text{core}}(r) & \text{for } r < r_\epsilon, \\ \rho_{\text{outer}}(r) & \text{for } r \geq r_\epsilon, \end{cases}$$

MNRAS 000, 000–000 (0000)

Preprint 22 August 2017

Compiled using MNRAS L^AT_EX style file v3.0

Unbiased constraints on ultralight axion mass from dwarf spheroidal galaxies

Alma X. González-Morales^{1,2*}, David J. E. Marsh³, Jorge Peñarrubia⁴,
L. Arturo Ureña-López²

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³Department of Physics, King's College London, Strand, London, WC2R 2LS, UK

⁴Institute for Astronomy, University of Edinburgh, Royal Observatory, Blackford Hill, Edinburgh EH9 3HJ, UK

ug 2017

22 August 2017

ℓ -boson stars

Journal of Cosmology and Astroparticle Physics
An IOP and SISSA journal

Bayesian analysis for rotational curves with ℓ -boson stars as a dark matter component

Atalia Navarro-Boullosa,^{a,b} Argelia Bernal^a and J. Alberto Vazquez^b

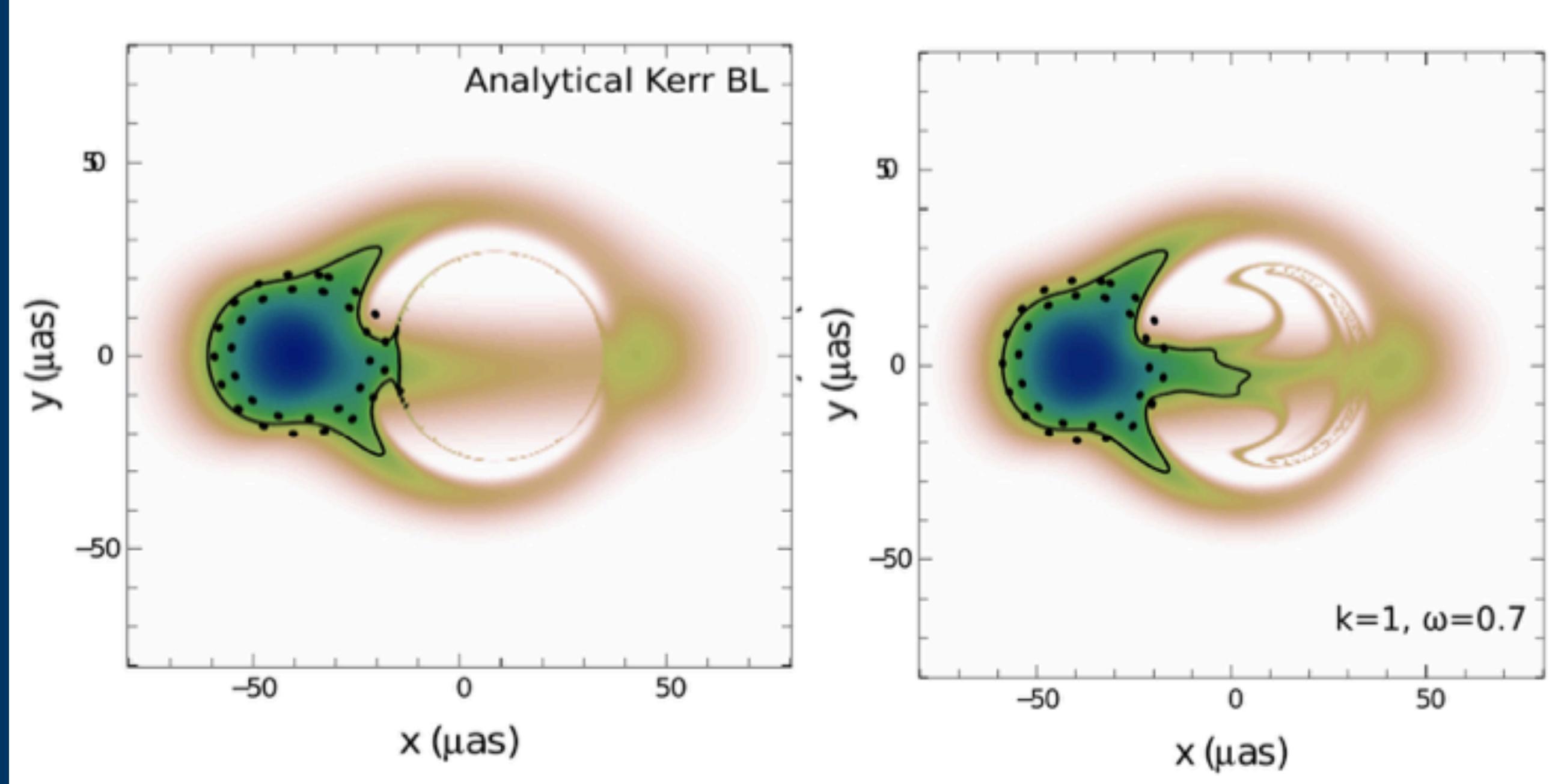
^aDepartamento de Física, DCI, Campus León, Universidad de Guanajuato, C.P. 37150, León, Guanajuato, Mexico

^bInstituto de Ciencias Físicas, Universidad Nacional Autónoma de México, Apdo. Postal 48-3, 62251 Cuernavaca, Morelos, Mexico

E-mail: atalia.navarro.boullosa@gmail.com, bernal.a@ugto.mx, javazquez@icf.unam.mx

Objetos ultracompactos

Disco de acreción Sgr A*: parecen muy similares



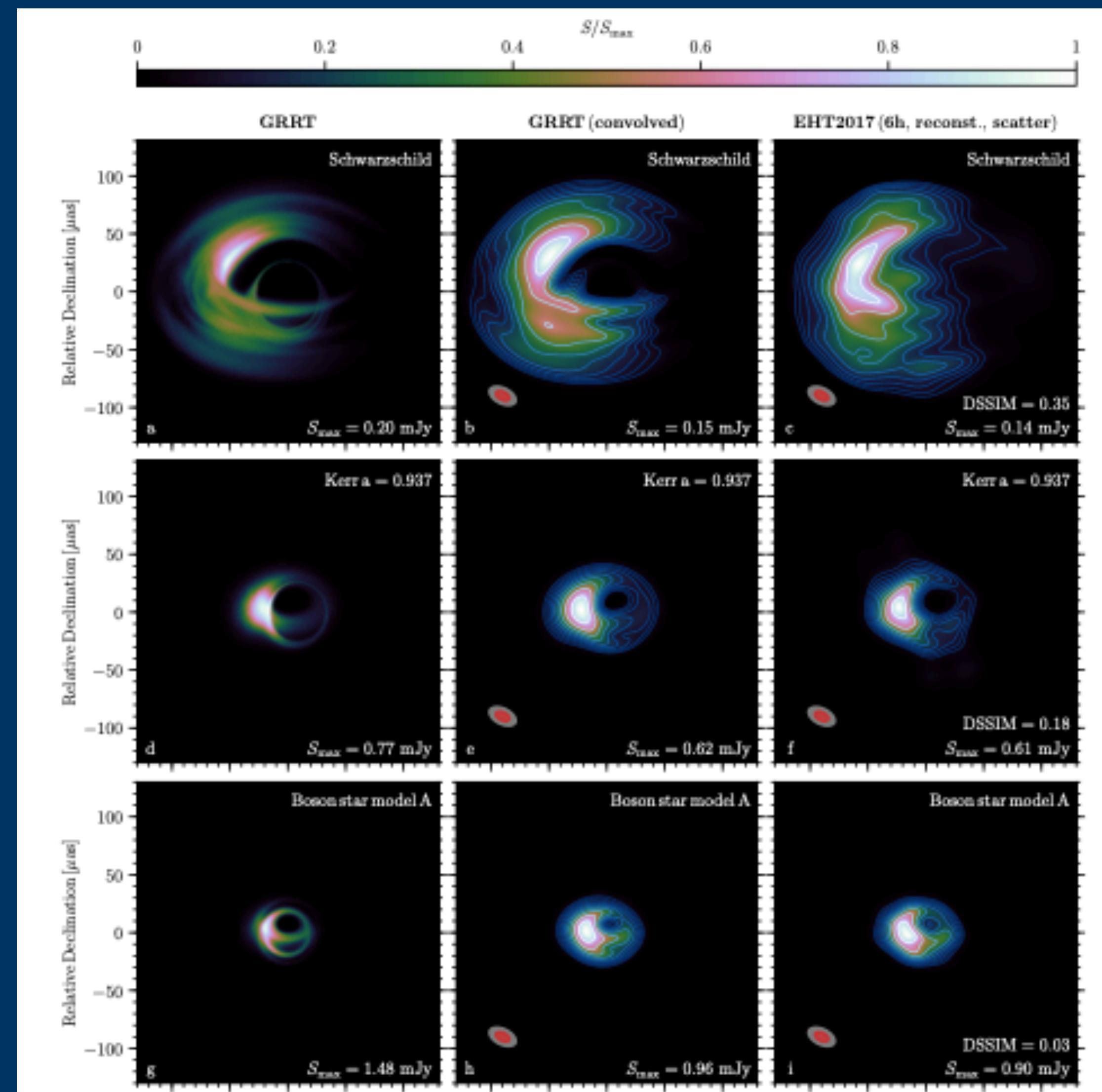
arXiv:1510.04170

Estas diferencias podrían desaparecer, o al menos disminuir,
al considerar una estrella Proca en lugar de una estrella de bosones

arXiv:2204.12949.

Disco de acreción Sgr A* (más actual)

arXiv:1809.08682

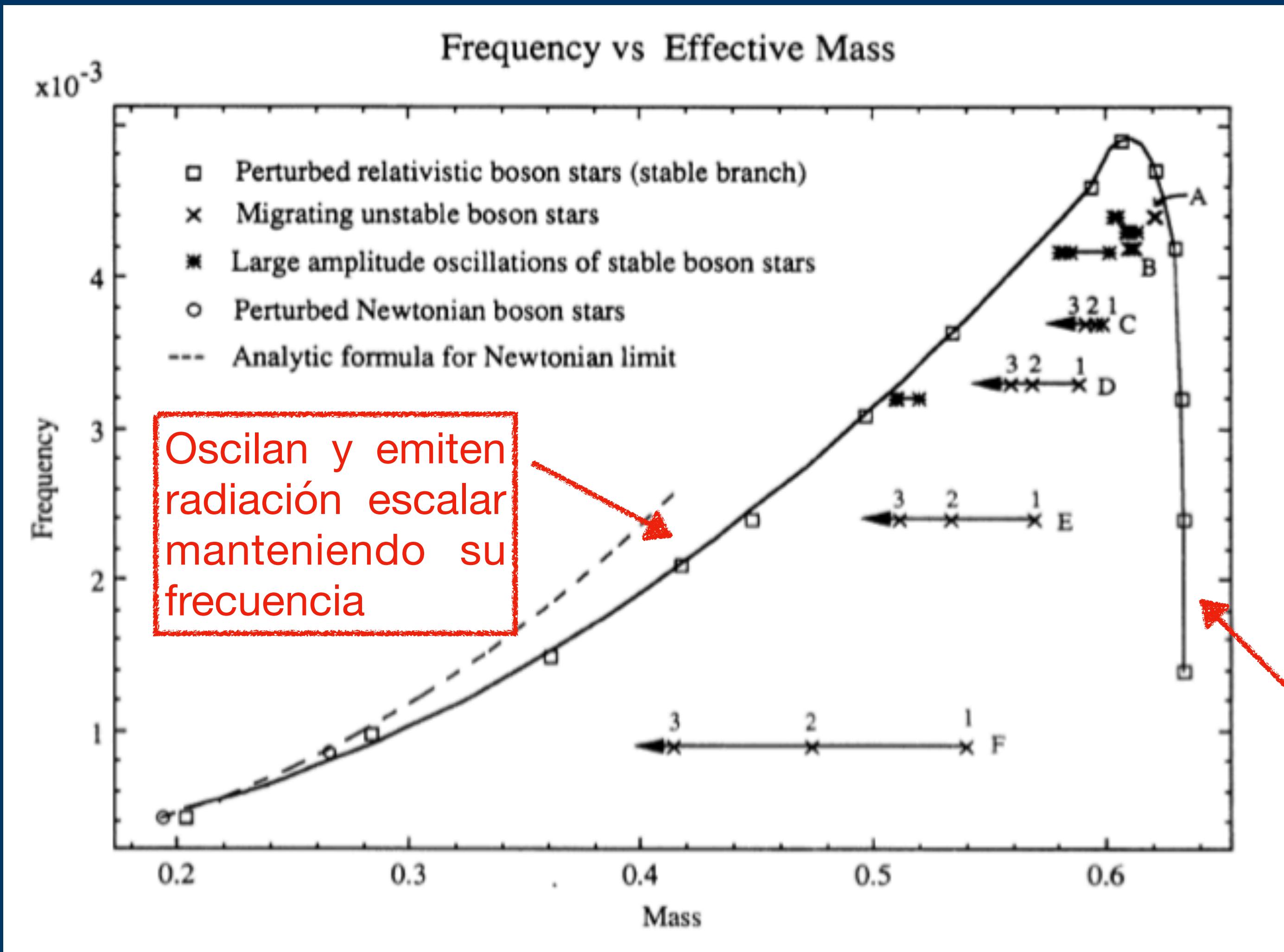


BS- más simétricos

Región oscura más pequeña (ausencia de Horiz. Event.)

Estabilidad

Término de masa: $L_m = -\nabla_\mu \psi^* \nabla^\mu \psi - m |\psi|^2$



Bajo perturbaciones lineales:

- Configuraciones en el estado base ($n = 0$) son estables si se encuentran a la izquierda de la configuración de máxima masa.
- Configuraciones en estados excitados ($n = 1, 2, \dots$) son inestables.

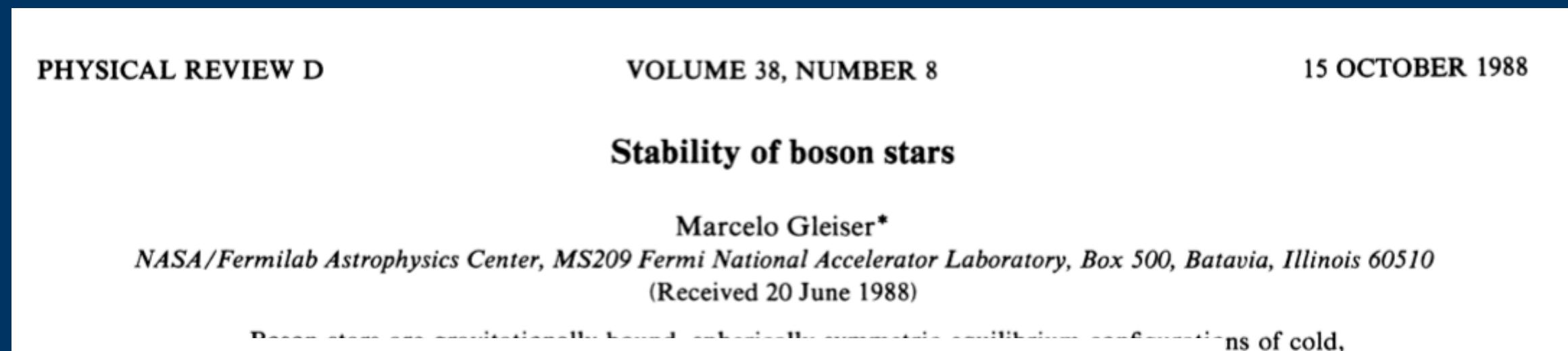
¿Qué ocurre si se incluye la autointeracción en el potencial?

Estabilidad cuando hay autointeracción

Autointeracción cuártica: $L_m = -\nabla_\mu \psi^* \nabla^\mu \psi - m |\psi|^2 - \lambda |\psi|^4$

Estado base ($n = 0$):

Sin importar la signatura de λ existe una rama estable para configuraciones en el estado base determinado por la configuración de máxima masa.



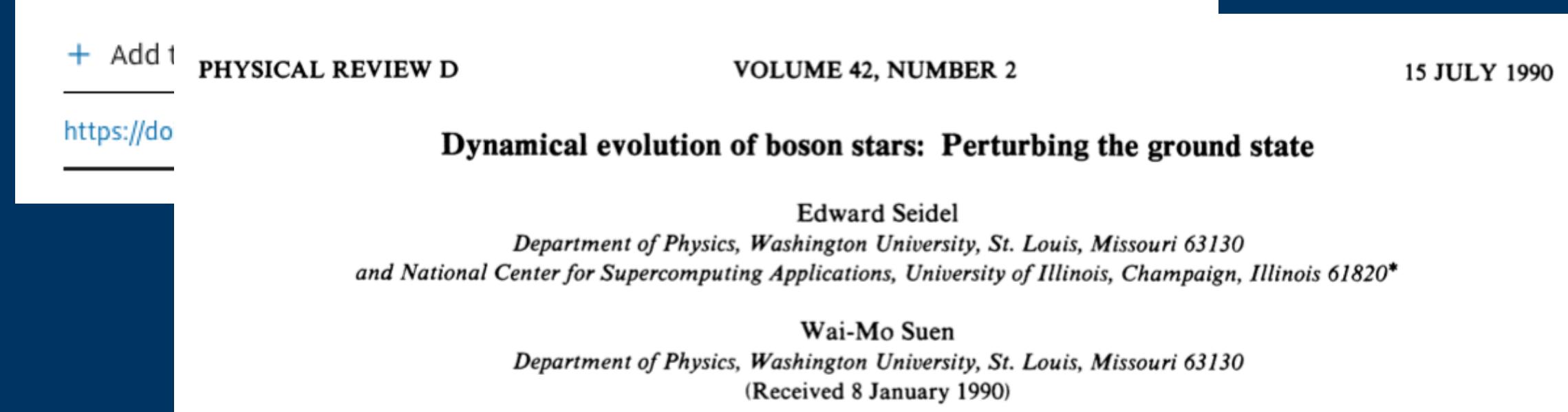
Gravitational stability of scalar matter

Marcelo Gleiser^a, Richard Watkins^b

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<https://doi.org/10.4236/ojs.201909091>



La signatura de λ indica si es repulsiva ($\lambda > 0$) o atractiva ($\lambda < 0$) la autointeracción

Estados excitados ($n \neq 0$):

Configuraciones en estados exitados ¿son inestables?

Para el caso atractivo, Sí, sin embargo para el caso repulsivo no es claro

Self-interactions can stabilize excited boson stars

Nicolas Sanchis-Gual¹, Carlos Herdeiro¹, and Eugen Radu¹

Dynamical Evolution of Boson Stars II: Excited States and Self-Interacting Fields

Jayashree Balakrishna¹, Edward Seidel^{2,3,4}, and Wai-Mo Suen^{1,5}

Stability of excited Bose stars

Ph. Jetzer

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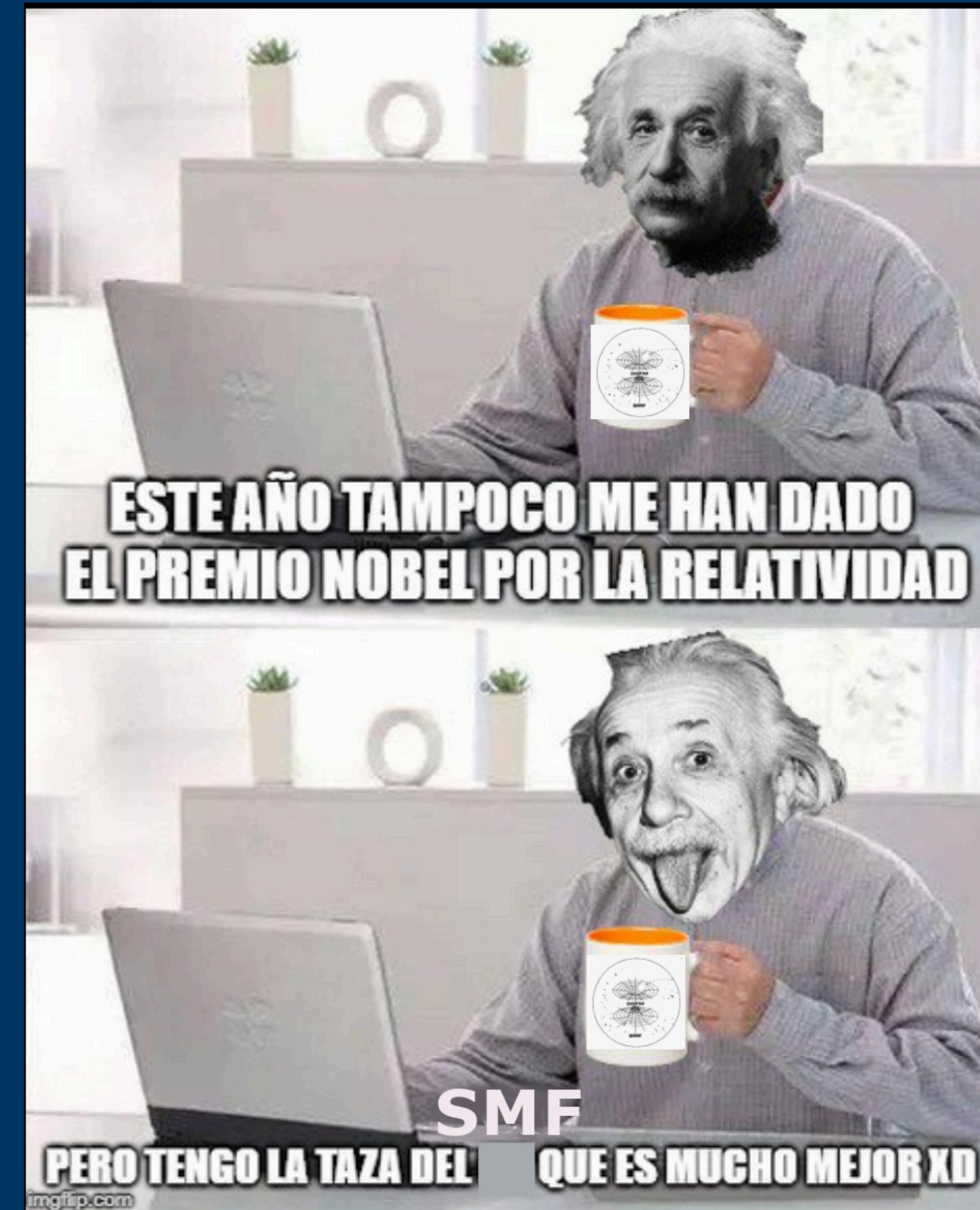
[https://doi.org/10.1016/0920-5632\(90\)90388-B](https://doi.org/10.1016/0920-5632(90)90388-B)

ose stars

**iiiDiferentes
conclusiones!!!**

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¿Tiempo de evolución? Aproximaciones



Adaptado de @ift_uam_csic

¿Tiempo de evolución?
Aproximaciones
...

Busquemos la respuesta en el:

Límite no relativista

Acción:

$$S[g_{\mu\nu}, \phi] = \int d^4x \sqrt{-g} \left(\frac{1}{16\pi G} R + \mathcal{L}_M \right),$$

$$\mathcal{L}_M = -\nabla_\mu \phi^* \nabla^\mu \phi - m_0^2 |\phi|^2 - \lambda |\phi|^4.$$

Solo nos interesa los grados de libertad escalar y consideramos el Newton gauge.

$$ds^2 = -[1 + 2\Phi(t, \vec{x})] dt^2 + [1 - 2\Psi(t, \vec{x})] \delta_{ij} dx^i dx^j,$$

Descomponemos el campo escalar (Ansatz)

$$\phi(t, \vec{x}) = \frac{1}{\sqrt{2m_0}} e^{-im_0 t} \psi(t, \vec{x}),$$

“Pesos” de las aproximaciones

$$\Phi \sim \Psi \sim \epsilon$$

$$\psi \sim \sqrt{M_{\text{Pl}}^2 m_0} \epsilon,$$

$$\partial_t \sim \epsilon^{1/2} \partial_i \sim \epsilon m_0,$$

$$S[\Phi, \Psi, \psi] = \int dt \int d^3x \left[\frac{1}{8\pi G} \Psi \Delta (2\Phi - \Psi) + \psi^* \left(i \frac{\partial}{\partial t} + \frac{1}{2m_0} \Delta - \frac{\lambda}{4m_0^2} |\psi|^2 \right) \psi - m_0 \Phi |\psi|^2 \right]$$

Sistema: Gross-Pitaevskii-Poisson

$$i \frac{\partial \psi}{\partial t} = -\frac{1}{2m_0} \Delta \psi \pm \frac{\pi \Lambda}{M_{\text{Pl}}^2} |\psi|^2 \psi + m_0 \mathcal{U} \psi,$$

$$\Delta \mathcal{U} = 4\pi G m_0 |\psi|^2$$

$$\Lambda := \frac{|\lambda| M_{\text{Pl}}^2}{2\pi m_0^2}$$

Comentarios heurísticos del potencial de autointeracción

Considerando que el potencial debe cumplir que:

- Preserva la simetría $U(1)$.
- Es analítico en $|\phi|^2$ en la vecindad $\phi = 0$.

$$-\frac{\lambda}{4m_0^2} |\psi|^4 \left[1 + \frac{v_6}{240v_4} \frac{|\psi|^2}{m_0 M^2} + \dots \right]$$

$\lambda = v_4/4!$

Tendremos que la forma más general tomaría una estructura como:

$$V(\phi) = M^4 \sum_{n=2}^{\infty} \frac{v_{2n}}{(2n)!} \left| \frac{\phi}{M} \right|^{2n},$$

cte. de acoplamiento adimensional

escala característica de energía

En el límite no relativista tendremos

Recordando: $\psi \sim \sqrt{M_{\text{Pl}}^2 m_0} \epsilon$,

Implica contribución del 2do término importante cuando

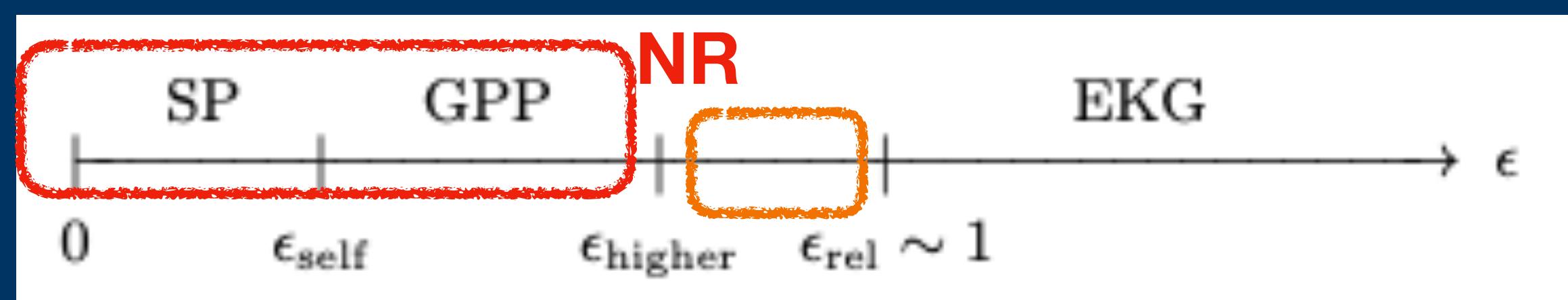
$$\epsilon \sim \epsilon_{\text{higher}} = \sqrt{240v_4/v_6} (M/M_{\text{Pl}}),$$

$$\epsilon_{\text{self}} = m_0^2 / (\lambda M_{\text{Pl}}^2),$$

Notar que:

- $n = 0$ es una constante.
- $n = 1$ sería equivalente al término de masa.
- $n = 2$ sería equivalente al término de autointeracción cuártico.

Resumen: en el límite NR los resultados de considerar el término cuártico son generales.



Estrellas de bosones no-relativistas:

Sistema: Gross-Pitaevskii-Poisson

Gross-Pitaevskii:

$$i\frac{\partial\psi}{\partial t} = -\frac{1}{2m_0}\Delta\psi \pm \frac{\pi\Lambda}{M_{\text{Pl}}^2}|\psi|^2\psi + m_0\mathcal{U}\psi,$$

Poisson:

$$\Delta\mathcal{U} = 4\pi Gm_0|\psi|^2$$

$$\Lambda := \frac{|\lambda|M_{\text{Pl}}^2}{2\pi m_0^2},$$

Funcional de Energía

$$\mathcal{E} = \int \left(\frac{1}{2m_0}|\nabla\psi|^2 \pm \frac{\pi\Lambda}{2M_{\text{Pl}}^2}|\psi|^4 + \frac{1}{2}m_0\mathcal{U}|\psi|^2 \right) d^3x,$$

Sistema adimensional:

$$i\frac{\partial\psi}{\partial t} = (-\Delta \pm |\psi|^2 + \mathcal{U})\psi,$$
$$\Delta\mathcal{U} = |\psi|^2,$$

$$\bar{t} := \frac{2m_0}{\Lambda}t, \quad \bar{x} := \frac{2m_0}{\Lambda^{1/2}}x,$$
$$\bar{\mathcal{U}} := \frac{\Lambda}{2}\mathcal{U}, \quad \bar{\psi} := \left(\frac{\pi\Lambda^2}{2M_{\text{Pl}}^2m_0}\right)^{1/2}\psi.$$

$$i\frac{\partial\psi}{\partial t} = \hat{\mathcal{H}}(\psi)\psi,$$

$$\hat{\mathcal{H}}(\psi) := -\Delta \pm |\psi|^2 + \Delta^{-1}(|\psi|^2),$$

$$M^{phys} = [M_{\text{Pl}}^2/(4\pi m_0 \Lambda^{1/2})]M \quad \xrightarrow{\hspace{1cm}} \quad M := 4\pi \int_0^\infty [\sigma^{(0)}(r)]^2 r^2 dr$$

Soluciones estacionarias

$$\psi(t, \vec{x}) = e^{-iEt} \sigma^{(0)}(\vec{x}),$$

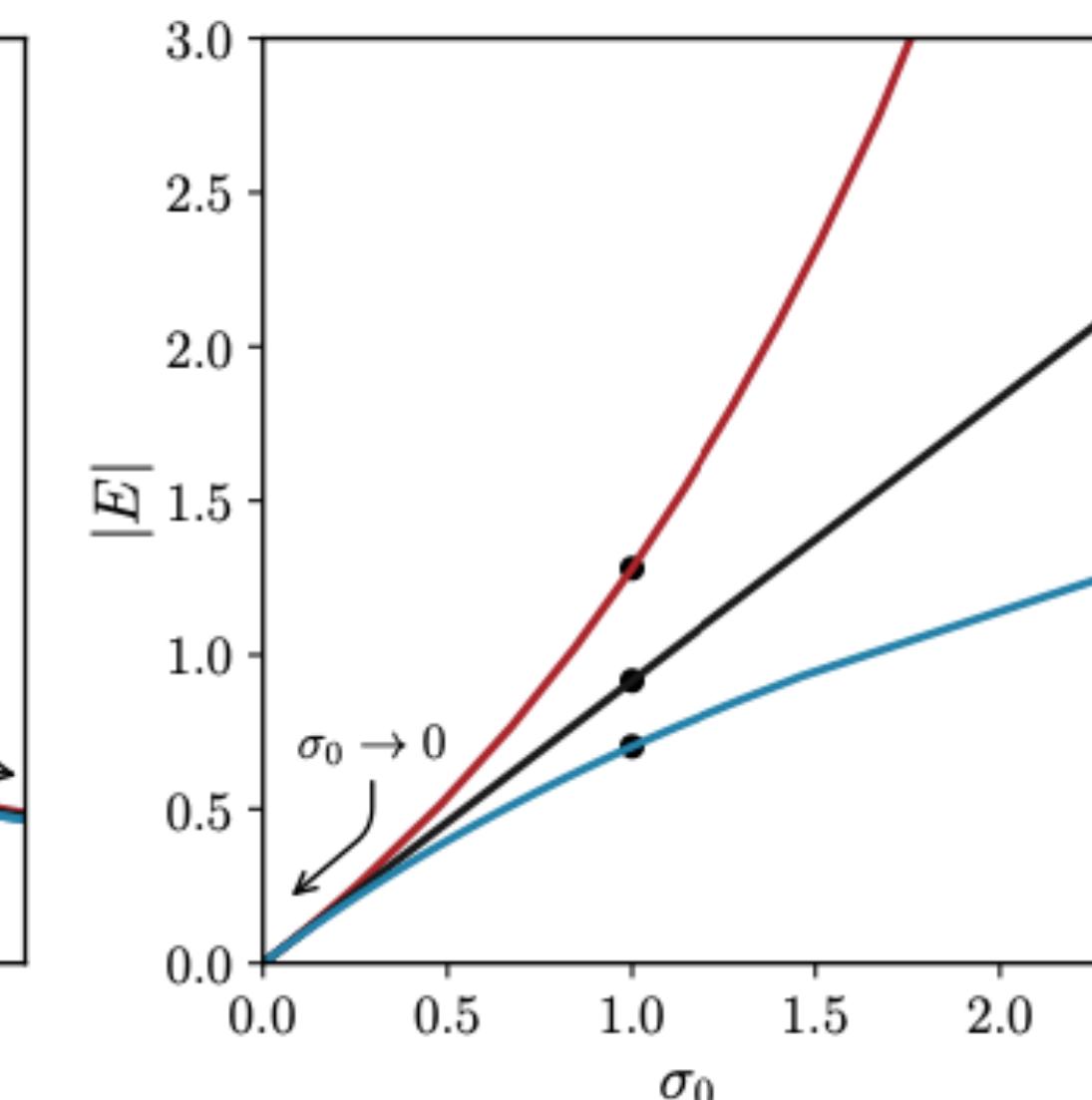
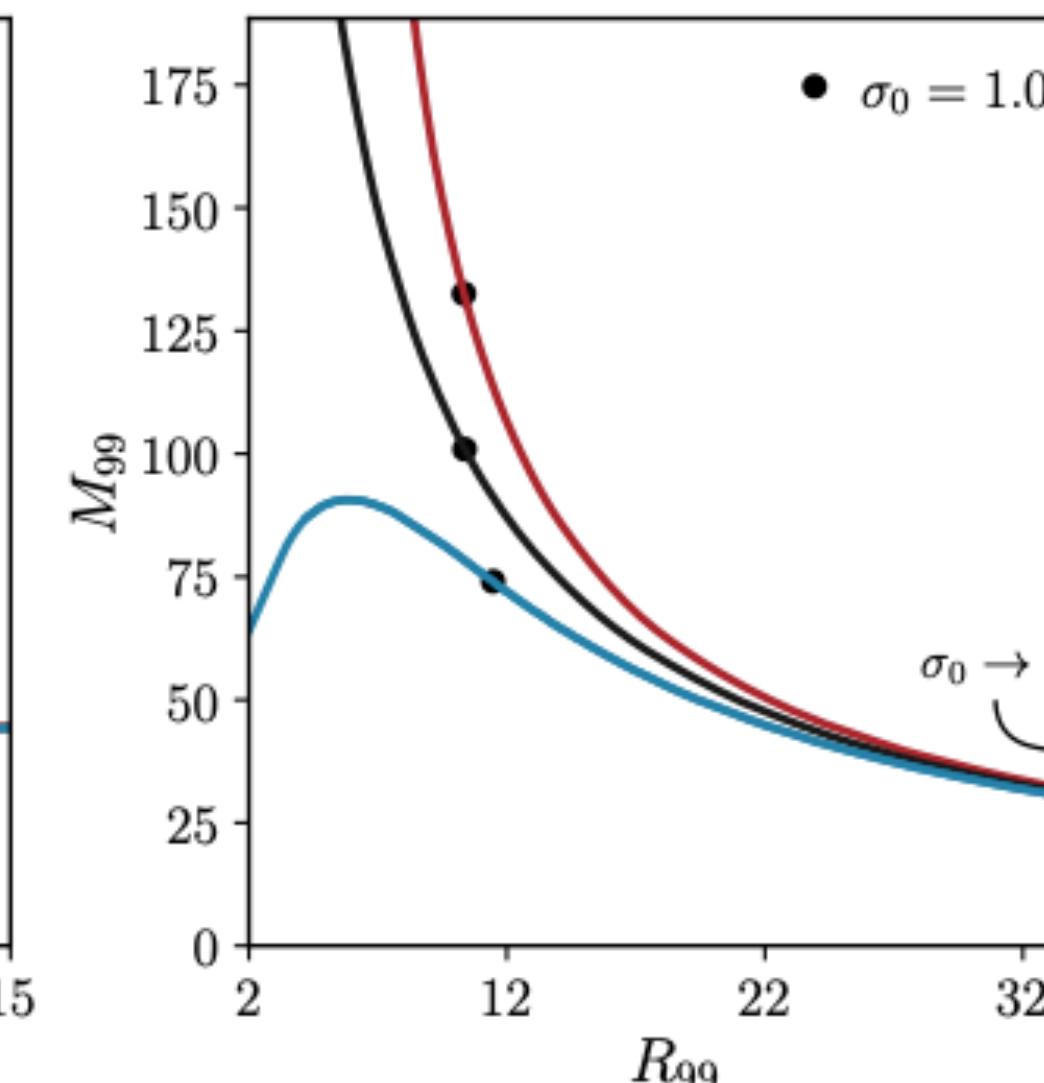
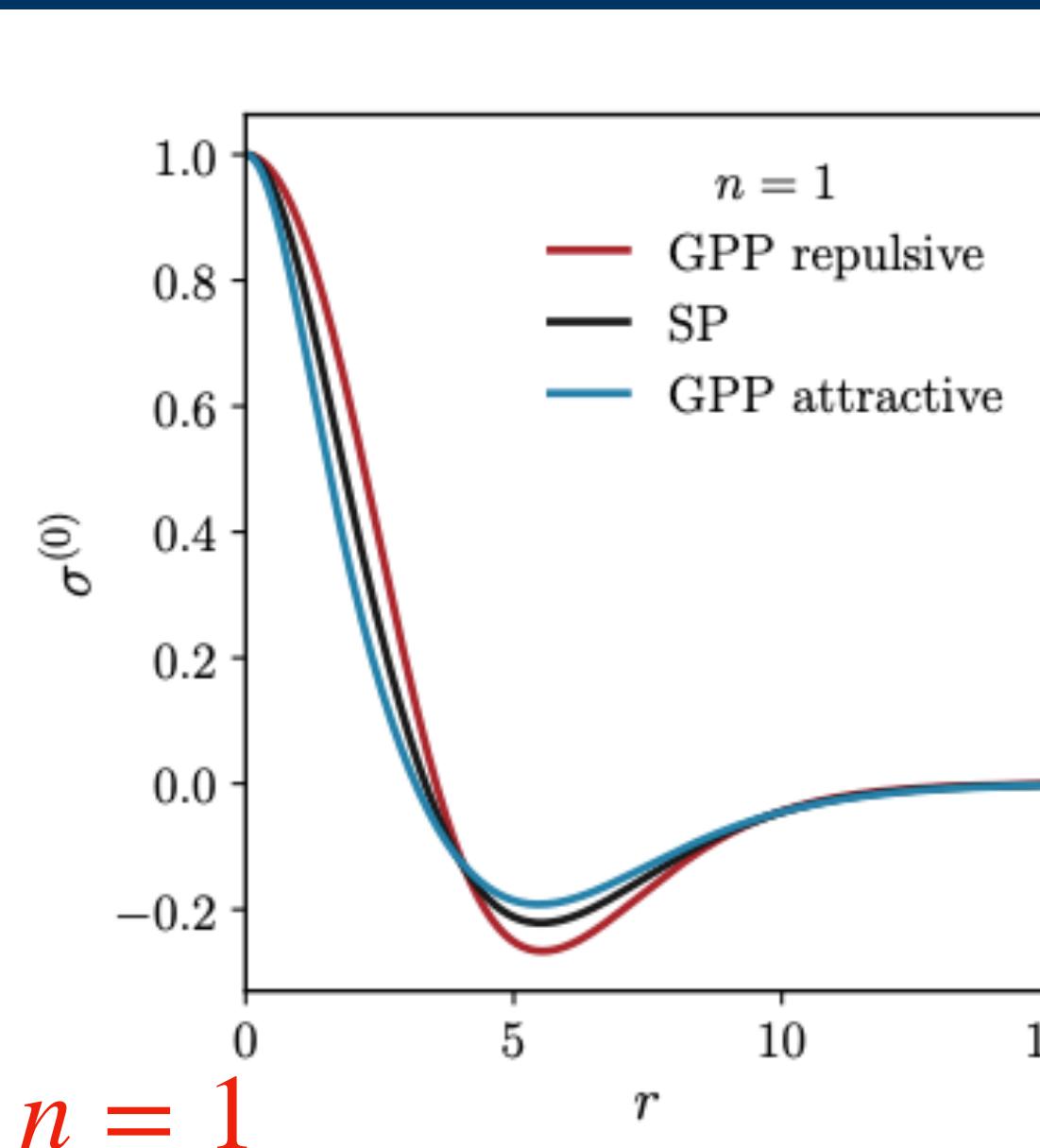
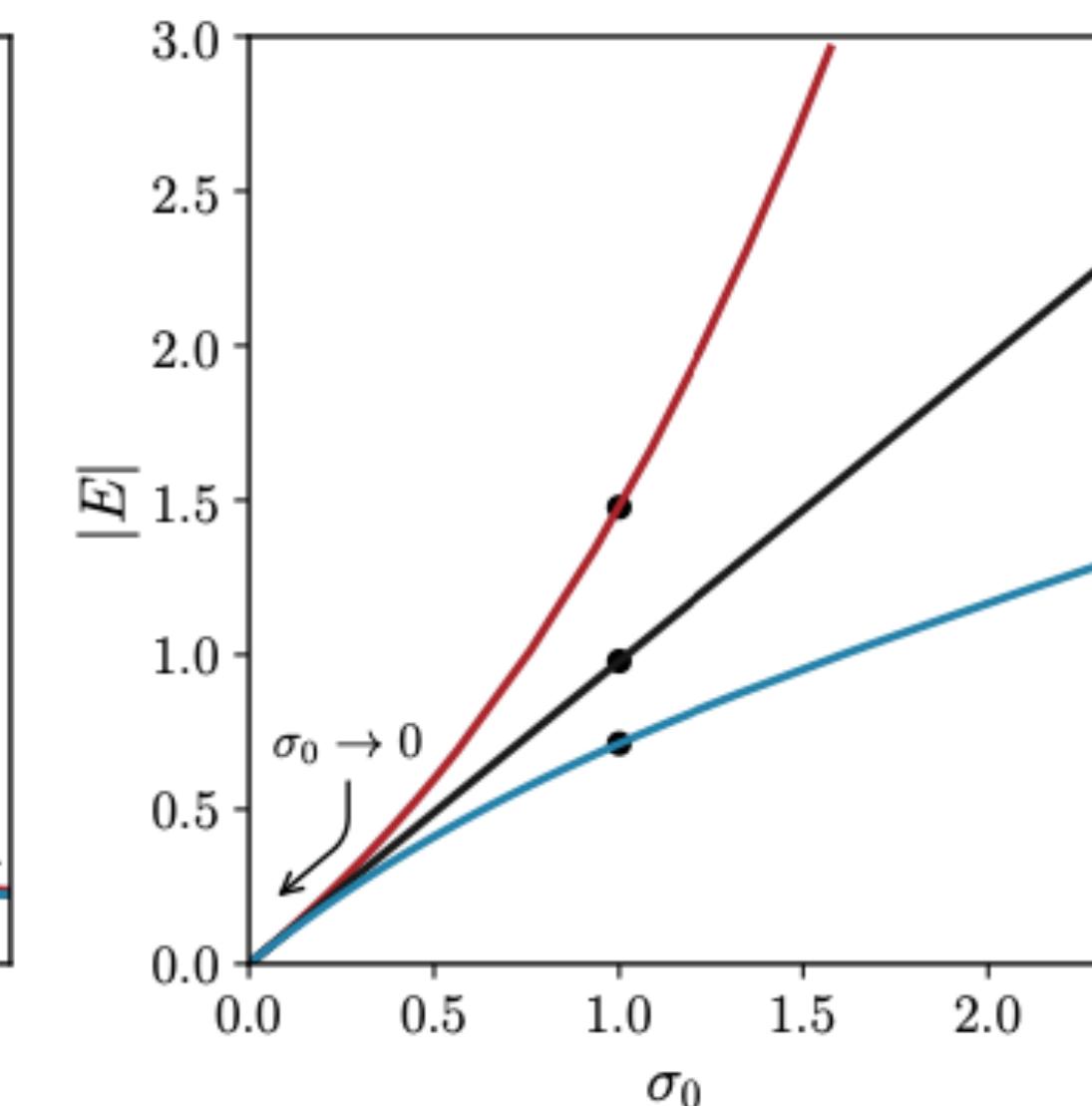
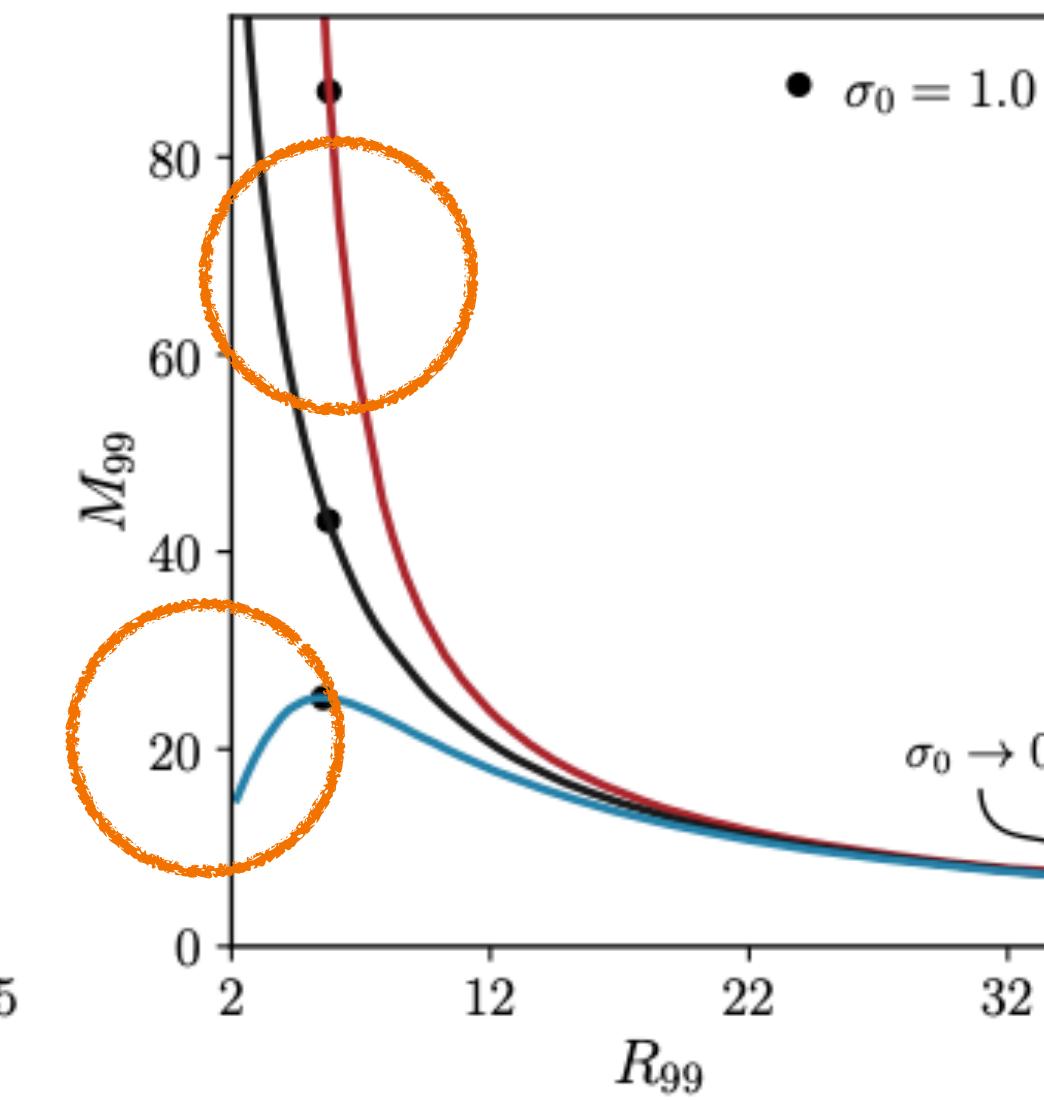
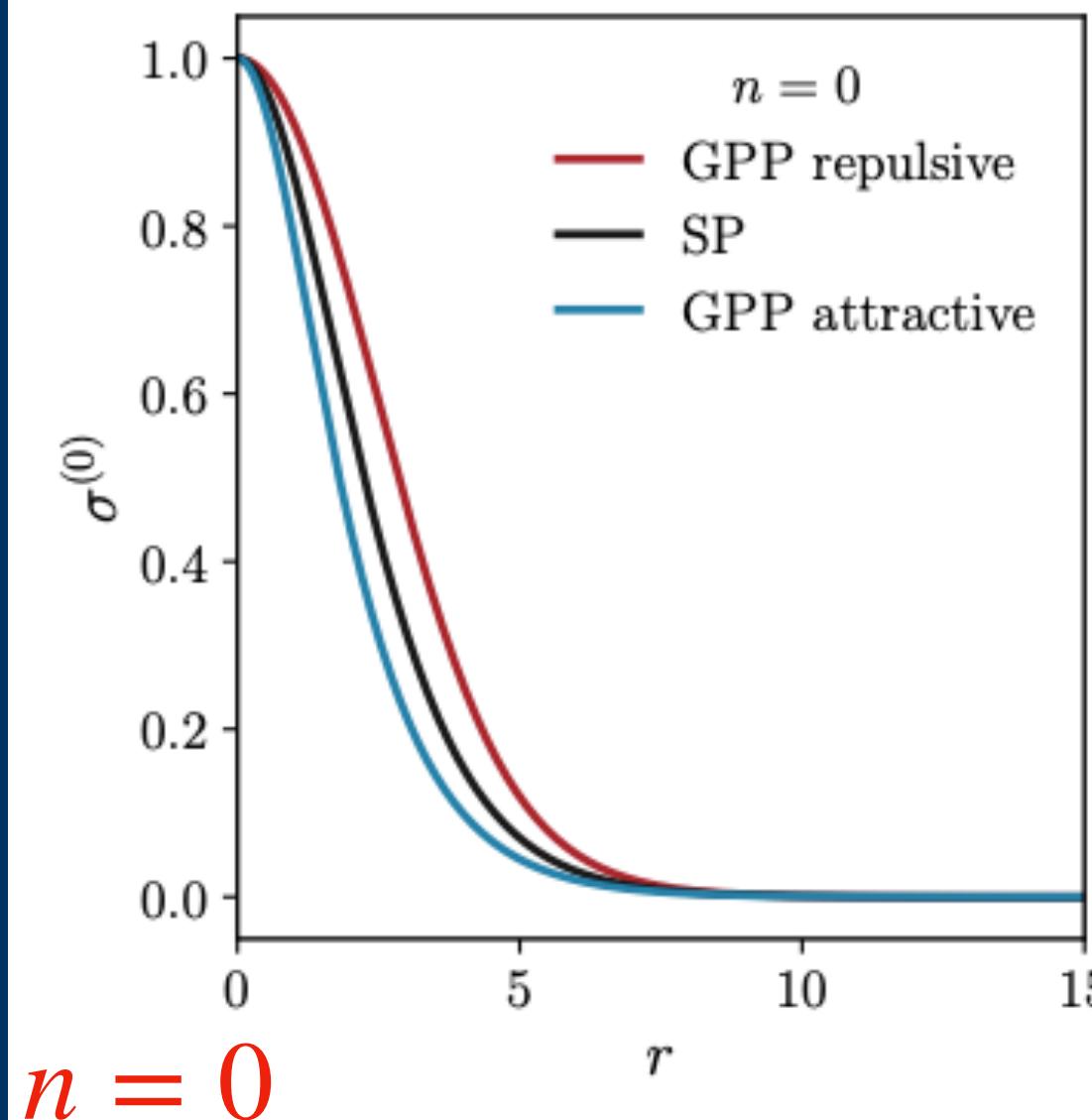
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Soluciones esféricamente simétricas

$$\sigma^{(0)}(\vec{x}) = \sigma^{(0)}(r)$$

=

$$E\sigma^{(0)} = \hat{\mathcal{H}}(\sigma^{(0)})\sigma^{(0)}.$$



Variación del funcional de Energía

En términos de las variables adimensionales

$$\mathcal{E}[\psi] = T[\psi] \pm F[n] - D[n, n], \quad n := |\psi|^2, \quad \longrightarrow$$

$$T[\psi] := \frac{1}{2} \int |\nabla \psi(\vec{x})|^2 d^3x,$$

$$F[n] := \frac{1}{4} \int n(\vec{x})^2 d^3x,$$

$$D[n, n] := \frac{1}{16\pi} \int \int \frac{n(\vec{x})n(\vec{y})}{|\vec{x} - \vec{y}|} d^3y d^3x.$$

Asumiendo un parámetro real y positivo $\nu > 0$

Escalamos

$$\psi_\nu(t, \vec{x}) := \nu^{3/2} \psi(t, \nu \vec{x}),$$

Número de partículas
invariante

$$\mathcal{E}[\psi_\nu] = \nu^2 T[\psi] \pm \nu^3 F[n] - \nu D[n, n],$$

Variaciones
en $\psi_{\nu=1}$

$$\frac{d}{d\nu} \mathcal{E}[\psi_\nu] \Big|_{\nu=1} = 2T[\psi] \pm 3F[n] - D[n, n],$$

$$\frac{d^2}{d\nu^2} \mathcal{E}[\psi_\nu] \Big|_{\nu=1} = 2T[\psi] \pm 6F[n].$$

Para solución estacionarias

$$D[n, n] = 2T[\psi] \pm 3F[n]. \quad \longrightarrow$$

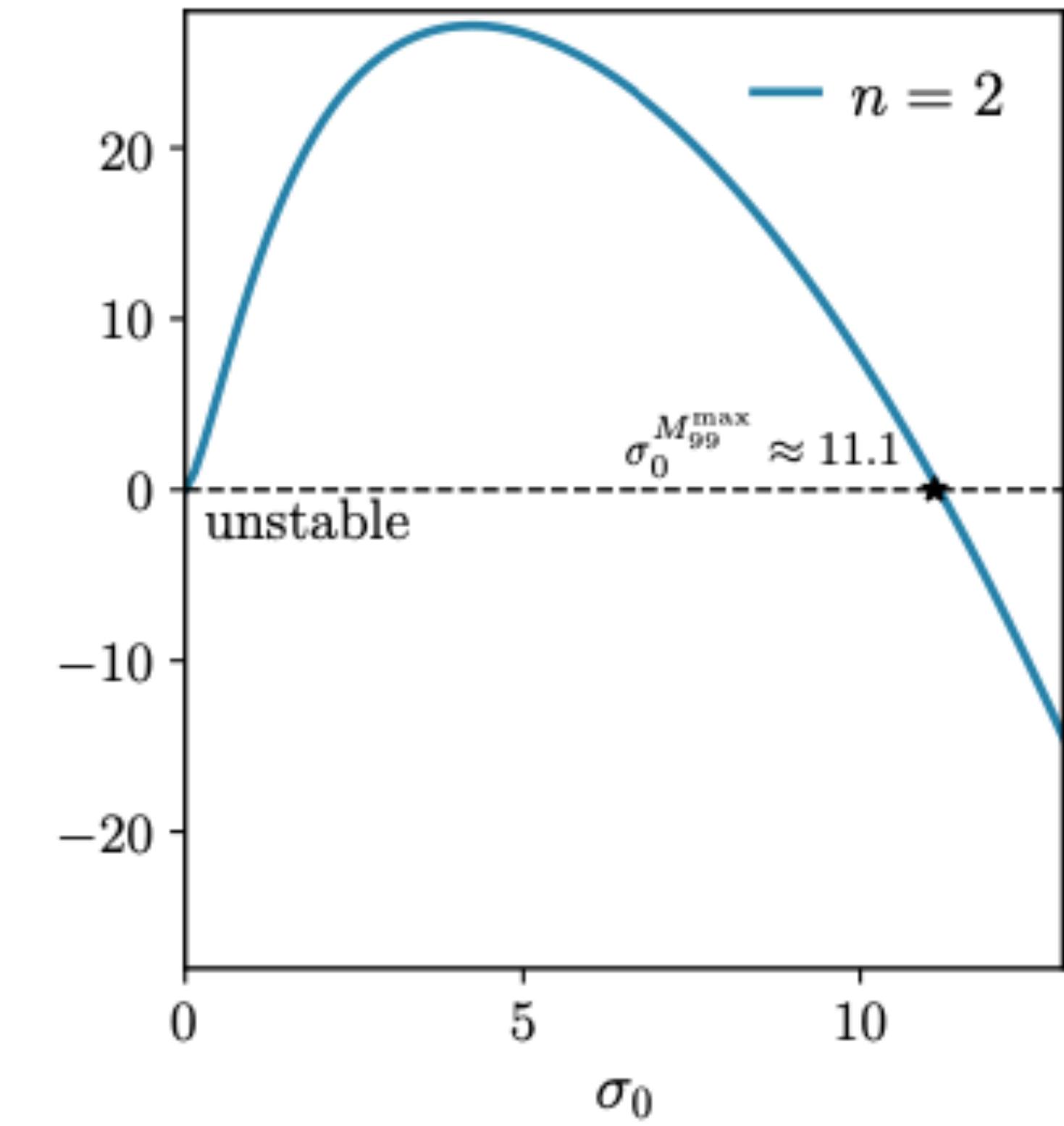
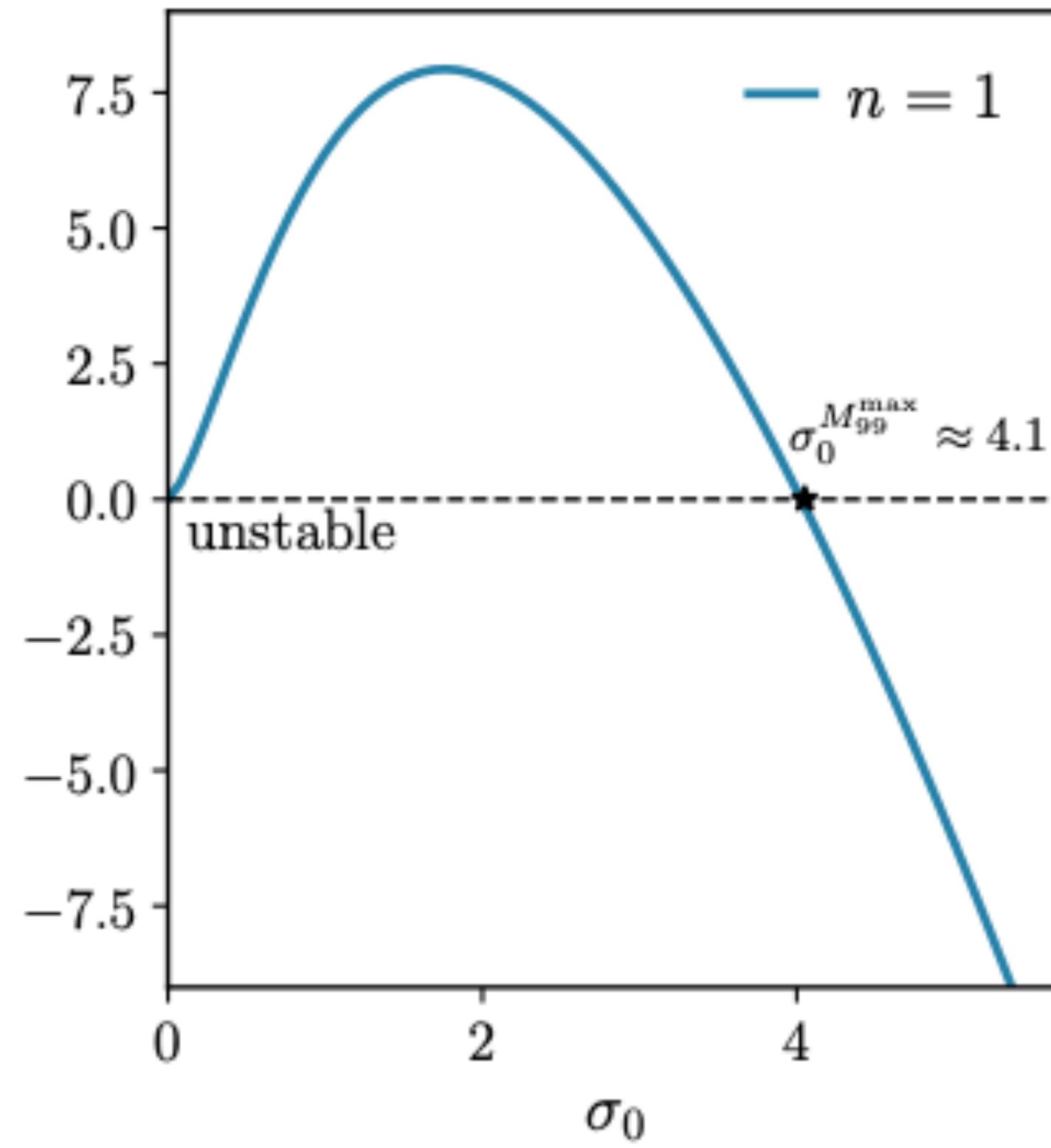
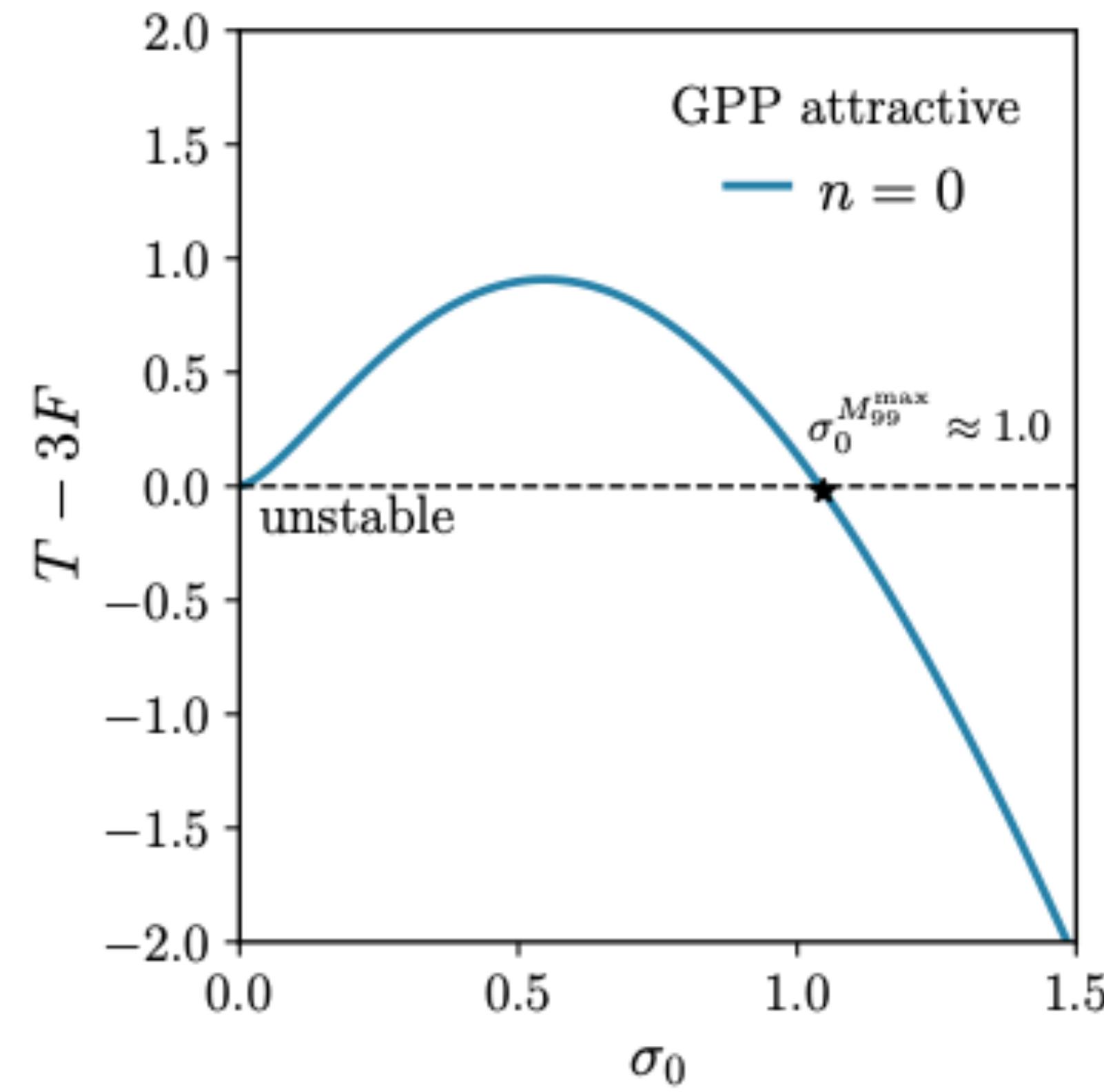
$$\mathcal{E}[\psi] = -T[\psi] \mp 2F[n].$$

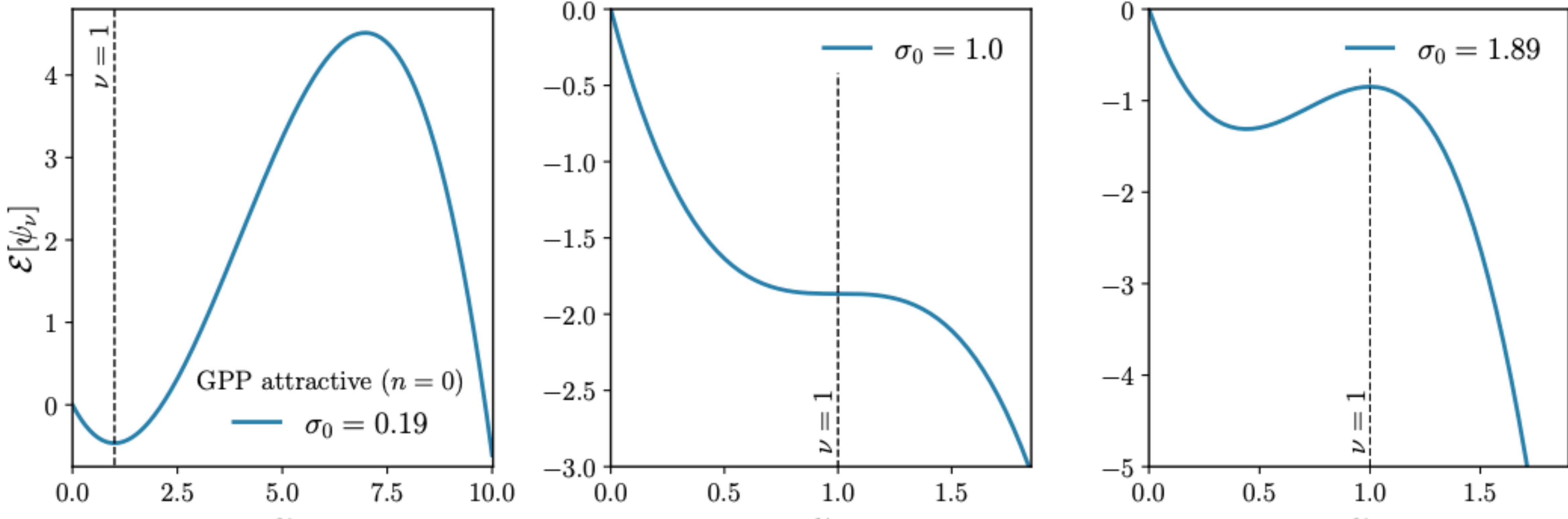
El punto crítico en $\psi_{\nu=1}$

$$\mathcal{E}[\psi_\nu] \text{ if } T - 3F > 0$$

Mínimo local

Caso Atractivo $\lambda = -1$





Notar que para el caso Repulsivo $\lambda = +1$

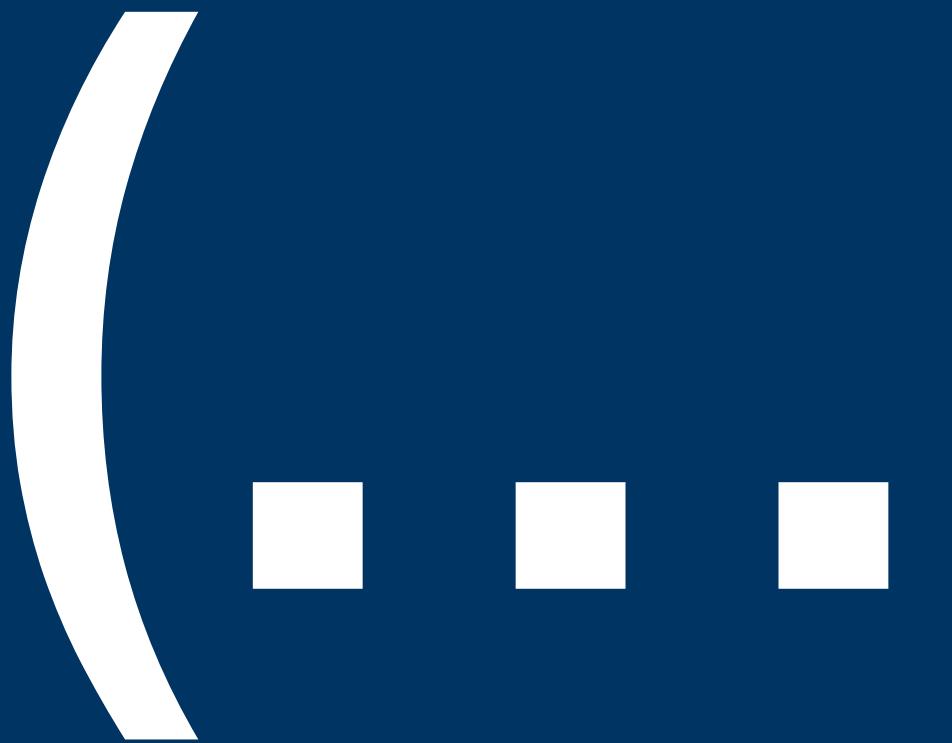
$$\begin{aligned} T[\psi] &:= \frac{1}{2} \int |\nabla \psi(\vec{x})|^2 d^3x, \\ F[n] &:= \frac{1}{4} \int n(\vec{x})^2 d^3x, \end{aligned} \quad > 0 \quad \xrightarrow{\hspace{1cm}} \quad$$

$$\left. \frac{d^2}{d\nu^2} \mathcal{E}[\psi_\nu] \right|_{\nu=1} = 2T[\psi] \pm 6F[n]. \quad > 0$$

Mínimo local

Perturbaciones lineales (Metodología)

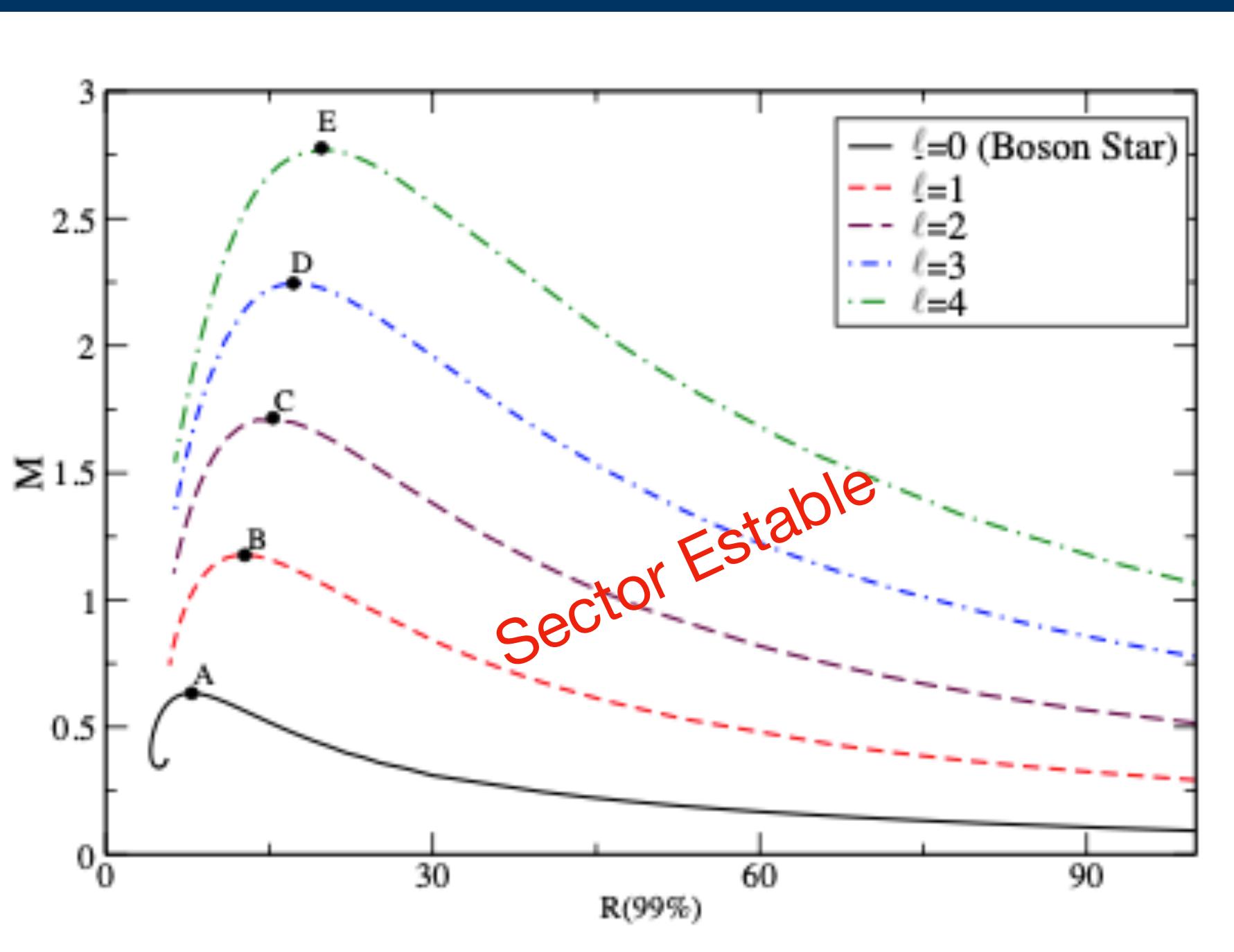
Abrimos Paréntesis



Antecedentes

Estabilidad de las ℓ -boson star

arXiv:1805.11488v2



¿Y las demás?

¿Es general?

On the linear stability of ℓ -boson stars with respect to radial perturbations

Miguel Alcubierre,¹ Juan Barranco,² Argelia Bernal,² Juan Carlos Degollado,³ Alberto Diez-Tejedor,² Miguel Megevand,⁴ Darío Núñez,¹ and Olivier Sarbach⁵

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tina

Dynamical evolutions of ℓ -boson stars in spherical symmetry

Miguel Alcubierre,¹ Juan Barranco,² Argelia Bernal,² Juan Carlos Degollado,³ Alberto Diez-Tejedor,² Miguel Megevand,⁴ Darío Núñez,¹ and Olivier Sarbach⁵

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Dynamical ℓ -boson stars: generic stability and evidence for non-spherical solutions

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(Dated: April, 2020)

Radial linear stability of nonrelativistic ℓ -boson stars

Armando A. Roque, Emmanuel Chávez Nambo, and Olivier Sarbach
Phys. Rev. D **107**, 084001 – Published 3 April 2023

Article

References

Citing Articles (3)

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ABSTRACT

We study the linear stability of nonrelativistic ℓ -boson stars, describing static, spherical configurations of the Schrödinger-Poisson system with multiple wave functions h of the angular momentum ℓ . In this work we restrict our analysis to time-dependent radial profiles of the $2\ell + 1$ wave functions, keeping their angular dependence. By combining analytic and numerical methods, we find that for each ℓ , the ground state is stable, whereas the n th excited states possess $2n$ unstable (exponentially in time) modes. Our results also indicate that all excited states correspond to saddle points of the corresponding functional of the theory.

Are nonrelativistic ground state ℓ -boson stars only stable for $\ell = 0$ and $\ell = 1$?

Emmanuel Chávez Nambo, Armando A. Roque, and Olivier Sarbach
Phys. Rev. D **108**, 124065 – Published 26 December 2023

Article

References

Citing Articles (1)

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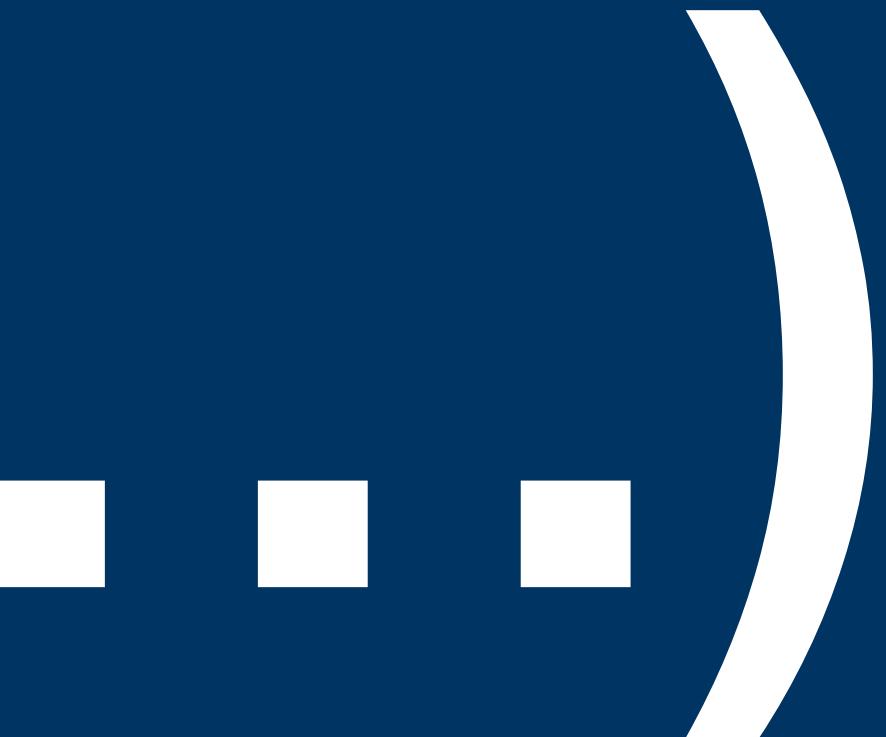
Export Citation

ABSTRACT

In previous work we analyzed the linear stability of nonrelativistic ℓ -boson stars with respect to radial modes and showed that ground state configurations are stable with respect to these modes, whereas excited states are unstable. In this work we extend the analysis to nonspherical linear mode perturbations. To this purpose, we expand the wave function in terms of tensor spherical harmonics which allows us to decouple the perturbation equations into a family of radial problems. By using a combination of analytic and numerical methods, we show that ground state configurations with $\ell > 1$ possess exponentially in time growing nonradial modes, whereas only oscillating modes are found for $\ell = 0$ and $\ell = 1$. This leads us to conjecture that nonrelativistic ℓ -boson stars in their ground state are stable for $\ell = 1$ as well as $\ell = 0$, while ground state and excited configurations with $\ell > 1$ are unstable.

Los resultados indican que las únicas configuraciones estables son $n = 0, \ell = 0, 1$.

Cerramos Paréntesis



Perturbaciones lineales

Consideremos una perturbación de la forma

$$\psi(t, \vec{x}) = e^{-iEt} \left[\sigma^{(0)}(\vec{x}) + \epsilon \sigma(t, \vec{x}) + \mathcal{O}(\epsilon^2) \right],$$

$$\begin{aligned} \sigma(t, \vec{x}) &= [A(\vec{x}) + B(\vec{x})] e^{\lambda t} + [A(\vec{x}) - B(\vec{x})]^* e^{\lambda^* t}, \\ A(\vec{x}) &= \sum_{LM} A_{LM}(r) Y^{LM}(\vartheta, \varphi), \end{aligned}$$

Sistema

$$i\lambda A_{LM} = (\hat{\mathcal{H}}_L^{(0)} - E) B_{LM},$$

$$i\lambda B_{LM} = (\hat{\mathcal{H}}_L^{(0)} - E) A_{LM} + 2\sigma^{(0)} \hat{K}_L [\sigma^{(0)} A_{LM}]$$

$$\hat{\mathcal{H}}_L^{(0)} := -\Delta_L \pm \sigma^{(0)2} + \Delta_s^{-1}(\sigma^{(0)2}),$$

$$\hat{K}_L := \pm 1 + \Delta_L^{-1},$$

$$\Delta_L := \Delta_s - L(L+1)/r^2$$

Sistema discretizado

$$r(A_{LM}, B_{LM})^T.$$

$$\begin{pmatrix} 0 & \tilde{\mathbb{D}}_N^2 \mp \Sigma_0^2 + U_L^{\text{eff}} \\ \tilde{\mathbb{D}}_N^2 \mp 3\Sigma_0^2 + U_L^{\text{eff}} - 2\alpha\Sigma_0(\tilde{\mathbb{D}}_N^2 - \mathbb{L})^{-1}\Sigma_0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{a}_L \\ \mathbf{b}_L \end{pmatrix} = -i\lambda \begin{pmatrix} \mathbf{a}_L \\ \mathbf{b}_L \end{pmatrix},$$

$$\Sigma_0 := \mathbf{diag} \left(\sigma^{(0)}(x_1), \sigma^{(0)}(x_2), \dots, \sigma^{(0)}(x_{N-1}) \right),$$

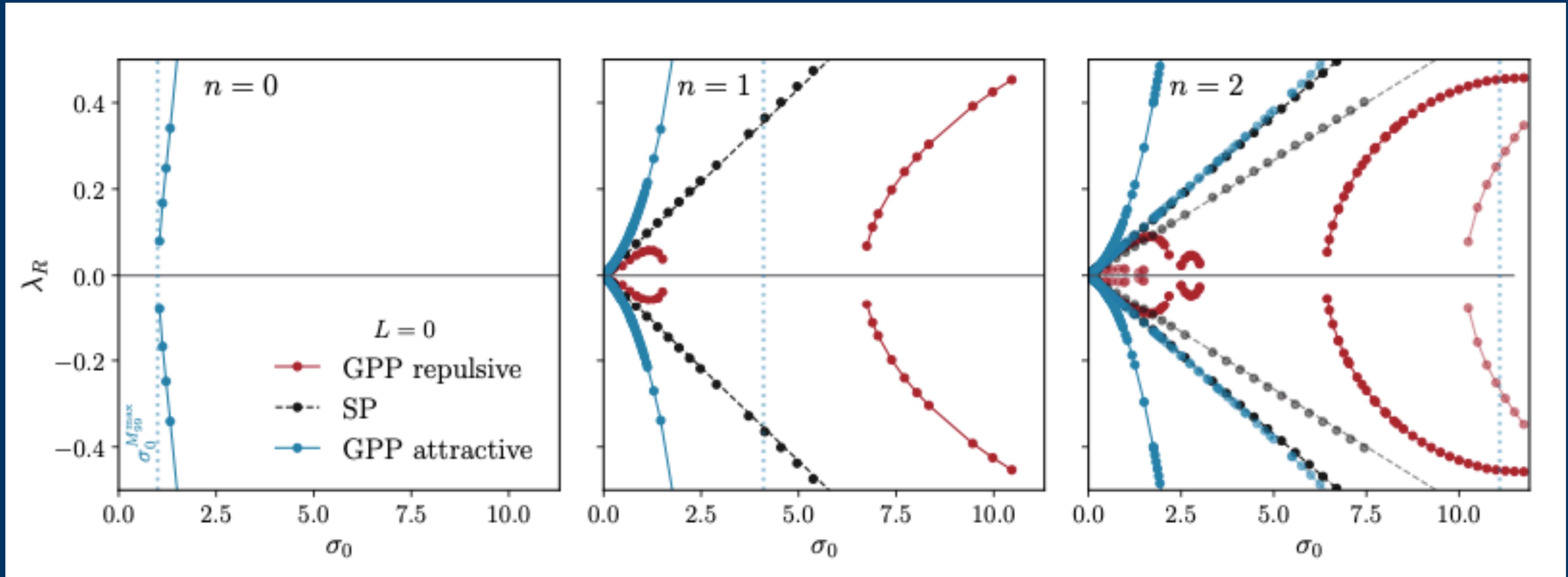
$$U_L^{\text{eff}} := \mathbf{diag} \left(U_L^{\text{eff}}(x_1), U_L^{\text{eff}}(x_2), \dots, U_L^{\text{eff}}(x_{N-1}) \right),$$

$$\mathbb{L} := \mathbf{diag} \left(\frac{L(L+1)}{x_1^2}, \frac{L(L+1)}{x_2^2}, \dots, \frac{L(L+1)}{x_{N-1}^2} \right),$$

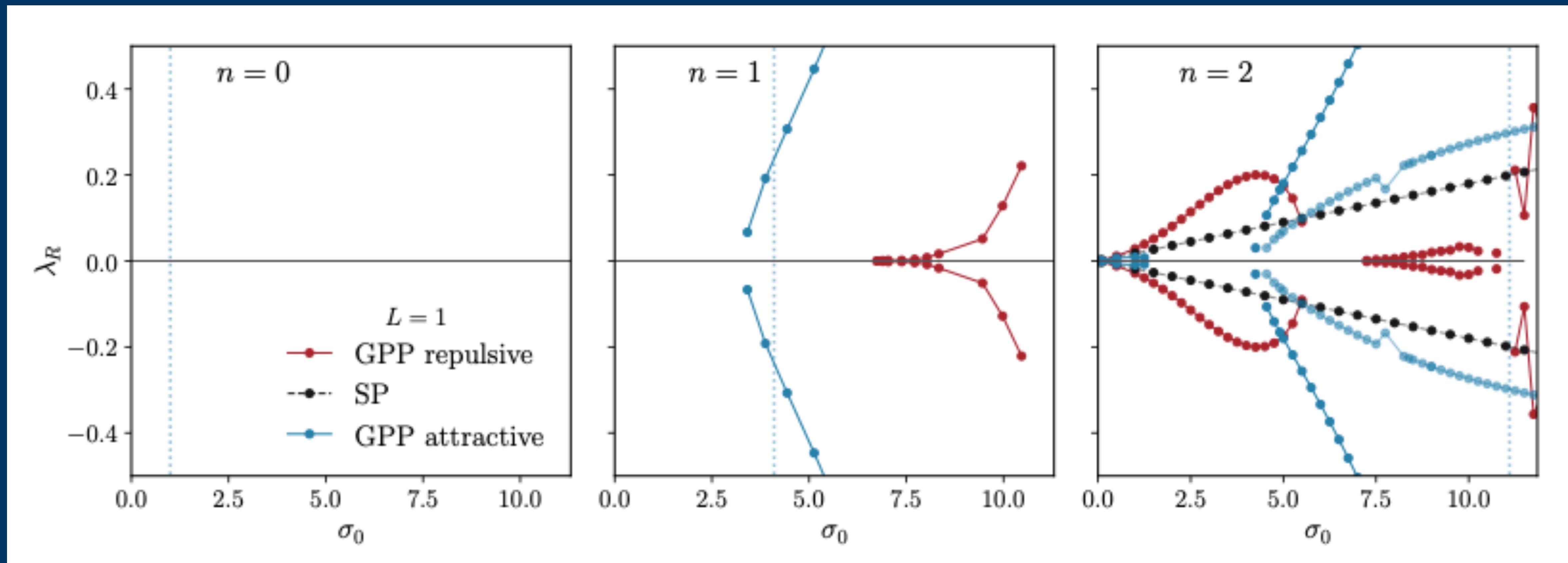
¡¡Un problema de Autovalores!!

$$\begin{pmatrix} \mathbf{a}_L \\ \mathbf{b}_L \end{pmatrix} := \begin{pmatrix} a_L(x_1), \dots, a_L(x_{N-1}), b_L(x_1), \dots, b_L(x_{N-1}) \end{pmatrix}^T$$

$L = 0$ (perturbación radial)

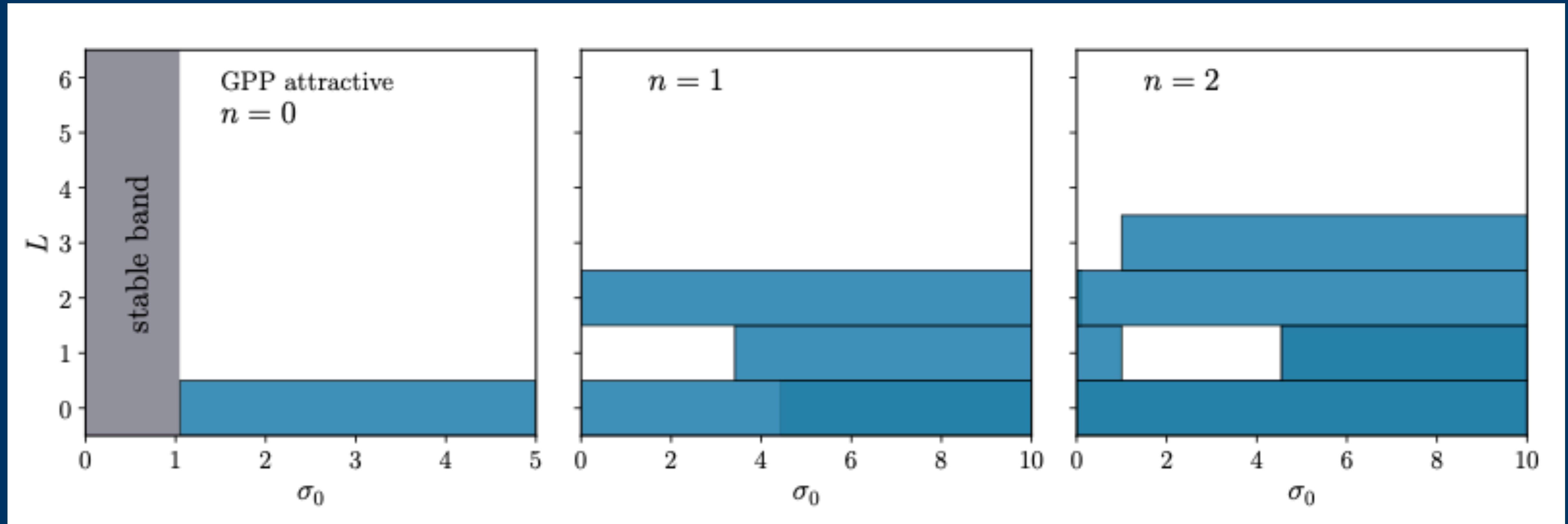


$L = 1$

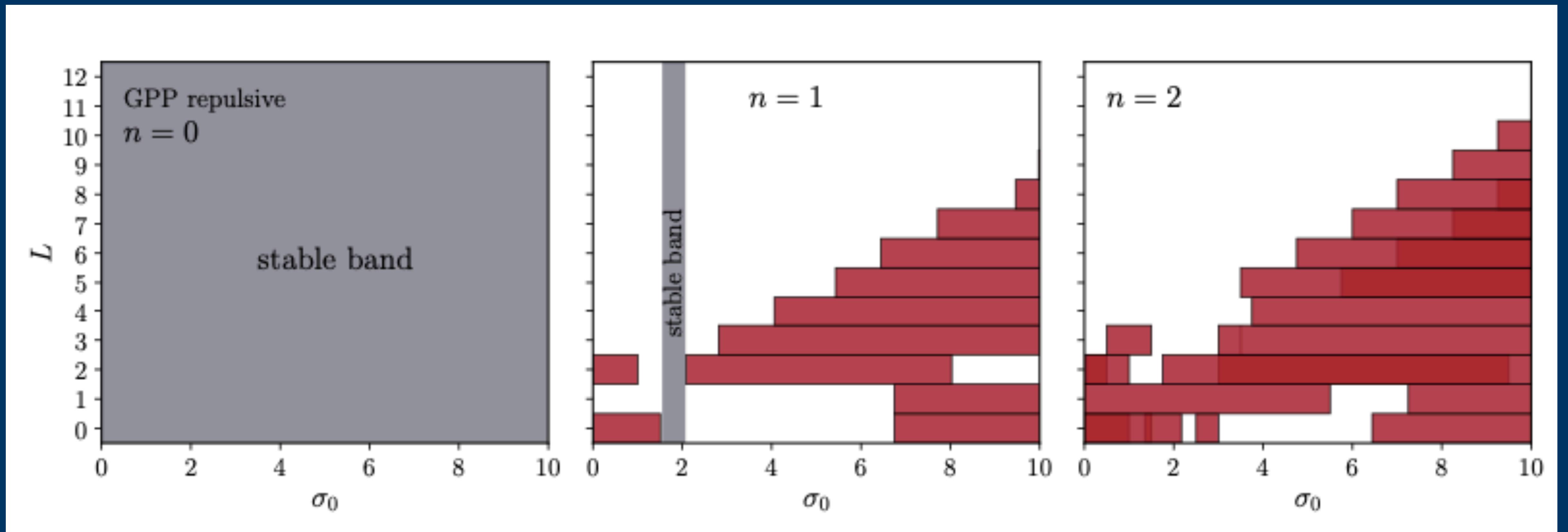


Regiones de estabilidad

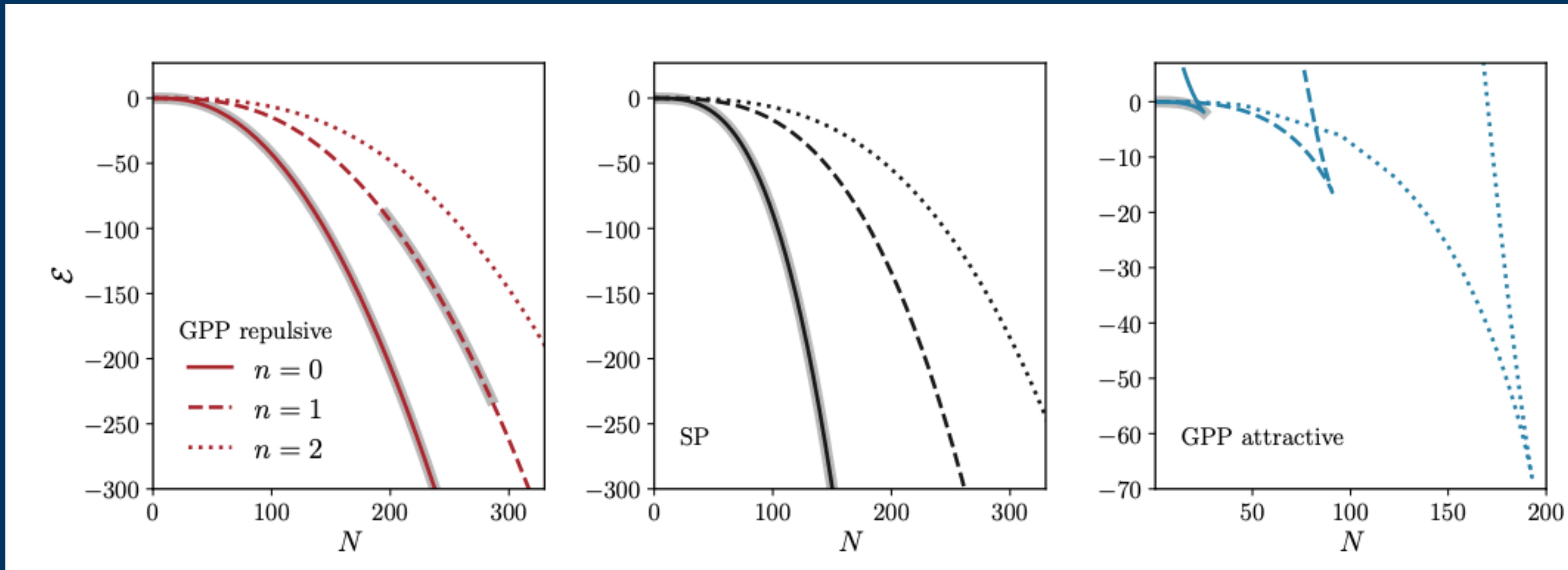
Caso Repulsivo



Caso Atractivo

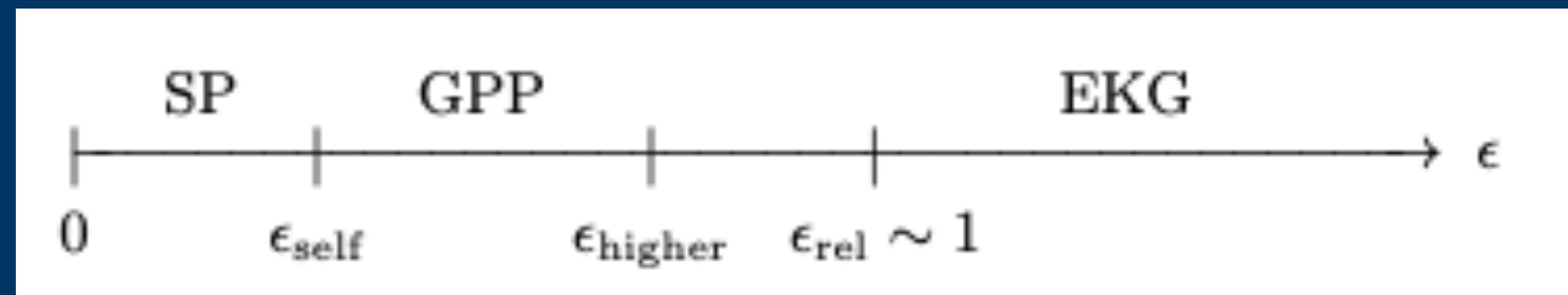


Funcional de energía



Resumen

- **Estados bases:** son estables para el caso repulsivo $\lambda = 1$ (similar al caso sin auto-interacción) existiendo para el caso atractivo $\lambda = -1$ una región estable en el límite $\psi \rightarrow 0$ donde la autointeracción es despreciable y el sistema GPP se reduce al sistema SP.



- **Estados exitados:**

Caso atractivo $\lambda = -1$ siempre es Inestables.

Caso repulsivo $\lambda = +1$ para el primer estado exitado existe una región finita de estabilidad ante perturbaciones lineales. Para el segundo estado no se encontró una región de estabilidad común.

Comentarios Finales

- Aunque el estudio se realizó para los primeros dos estados excitados, se espera un comportamiento **similar** para el resto de estados.
- Dada una configuración siempre ocurrirá que a partir de una cierto valor L , **solo** existen modos estables producto de una perturbación lineal.
- Es lógico pensar que la región estable **permanezca** una vez que este límite no sea válido.
- Como consecuencia de las variables usadas los resultados son válidos para cualquier valor de la autointeracción.

¡Gracias!

Artículo principal



Artículos discutidos



Repositorio numérico



Trabajos/Charlas/etc.
Grup. Cosmología y Gravitación UAZ

