Seminario Teórico

División de Ciencias e Ingenierías. UG

¿Puede la auto-interacción estabilizar los estados excitados de una estrella de bosones?

División de Ciencias e Ingenierías, UGTO, 2021

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Estructura

I. ¿Qué es una estrella de bosones?

II. Estrellas de bosones no-relativistas:

III. Estabilidad

IV. Comentarios finales

¿Qué es una estrella de bosones?

Ingredientes

Gravedad RG + Campo Escalar ψ → Sistema de Klein-Gordon

Sector de materia

La acción

$$S[g_{\mu\nu},\psi] = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}R - \boxed{\nabla_\mu\psi\nabla^\mu\psi - V(\psi)}\right]$$

La física

"Presión" repulsiva → Equilibrio hidrostático ← Gravedad

Equilibrio hidrostático → Objetos compactos localizados

PHYSICAL REVIEW VOLUME 172, NUMBER 5 25 AUGUST 1968

Klein-Gordon Geon*

DAVID J. KAUP†
University of Maryland, College Park, Maryland
(Received 4 March 1968)

A study of the spherically symmetric eigenstates of the Klein-Gordon Einstein equations (Klein-Gordon geons) reveals that these geons have properties that are uniquely different from other gravitating systems that have been studied. The equilibrium states of these geons seem analogous to other gravitating systems; but when the question of stability is considered from a thermodynamical viewpoint, it is shown that, in contrast with other systems, adiabatic perturbations are forbidden. The reason is that the equations of state for the thermodynamical variables are not algebraic equations, but instead are differential equations. Consequently, the usual concept of an equation of state breaks down when Klein-Gordon geons are considered. When the question of stability is reconsidered in terms of infinitesimal perturbations of the basic

PHYSICAL REVIEW VOLUME 187, NUMBER 5 25 NOVEMBER 1969

Systems of Self-Gravitating Particles in General Relativity and the Concept of an Equation of State*

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and
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(Received 4 February 1969)

A method of self-consistent fields is used to study the equilibrium configurations of a system of self-gravitating scalar bosons or spin-$\frac{1}{2}$ fermions in the ground state without using the traditional perfect-fluid approximation or equation of state. The many-particle system is described by a second-quantized free field, which in the boson case satisfies the Klein-Gordon equation in general relativity, $\nabla_\alpha\nabla^\alpha\phi=\mu^2\phi$, and in the fermion case the Dirac equation in general relativity $\gamma^\alpha\nabla_\alpha\psi=\mu\psi$ (where $\mu=mc/\hbar$). The coefficients of the metric $g_{\alpha\beta}$ are determined by the Einstein equations with a source term given by the mean value $\langle\phi|T_{\mu\nu}|\phi\rangle$ of the energy-momentum tensor operator constructed from the scalar or the spinor field. The state vector $\langle\phi|$ corresponds to the ground state of the system of many particles. In both cases, for completeness, a nonrelativistic Newtonian approximation is developed, and the corrections due to special and general relativity explicitly are pointed out. For N bosons, both in the region of validity of the Newtonian treatment (density from 10^{-80} to 10^{54} g cm^{-3}, and number of particles from 10 to 10^{40}) as well as in the relativistic region (density $\sim10^{54}$ g cm^{-3}, number of particles $\sim10^{40}$), we obtain results completely different from those of a traditional fluid analysis. The energy-momentum tensor is anisotropic. A critical mass is found for a system of $N\sim[(\text{Planck mass})/m]^2\sim10^{40}$ (for $m\sim10^{-25}$ g) self-gravitating bosons in the ground state, above which mass gravitational collapse occurs. For N fermions, the binding energy of typical particles is $G^2m^5N^{4/3}\hbar^{-2}$ and reaches a value $\sim mc^2$ for $N\sim N_{\mathrm{crit}}\sim[(\text{Planck mass})/m]^3\sim10^{57}$ (for $m\sim10^{-24}$ g, implying mass $\sim10^{33}$ g, radius $\sim10^6$ cm, density $\sim10^{15}$ g/cm^3). For densities of this order of

Nota: $c=\hbar=1$

El campo escalar:

ψ -Spin 0

$\psi(\vec{x})$ → Teorema de Derrick

Soluciones localizadas y estacionarias de Eq. KG en 3D o dimensiones más altas son inestables

$\psi(t, \vec{x})$

Real →

VOLUME 66, NUMBER 13 PHYSICAL REVIEW LETTERS 1 APRIL 1991

Oscillating Soliton Stars

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(Received 26 December 1990)

We study a new type of self-gravitating object, described by a soliton solution to the coupled system of the Einstein equation and a matter field equation. The solution describing the self-gravitating object is not static but, instead, is periodic, with both the spacetime geometry and the matter field oscillating in time. In particular, we show that a system described by a real scalar field $g^{\mu\nu}\phi_{;\mu\nu} - m^2\phi = 0$ admits such a solution, which is stable under perturbations. The existence of such objects could have important implications for astrophysics and cosmology.

PACS numbers: 95.30.Sf, 04.20.Jb

$$N^2(t,r) = 1 + \sum_{j=0}^{\infty} N_{2j}(r)\cos(2j\omega_0 t),$$

$$g^2(t,r) = 1 + \sum_{j=0}^{\infty} g_{2j}(r)\cos(2j\omega_0 t),$$

$$\phi(t,r) = \sum_{j=1}^{\infty} \phi_{2j-1}(r)\cos[(2j-1)\omega_0 t].$$

oscilones

Complejo →

Simetría $U(1)$

$$\psi(t, \vec{x}) = \sigma(r) e^{-i\omega t}$$

→

- Q-balls
- Boson stars

El potencial

$$V[|\psi|] = \boxed{m^2|\psi|^2} + M^4 \sum_{n=2}^{\infty} \frac{v_{2n}}{(2n)!} \boxed{\left|\frac{\psi}{M}\right|^{2n}}$$

término de masa

términos de auto-interacción

$\sim |\psi|^4$

$\sim c_1|\psi|^4 + c_2|\psi|^6$

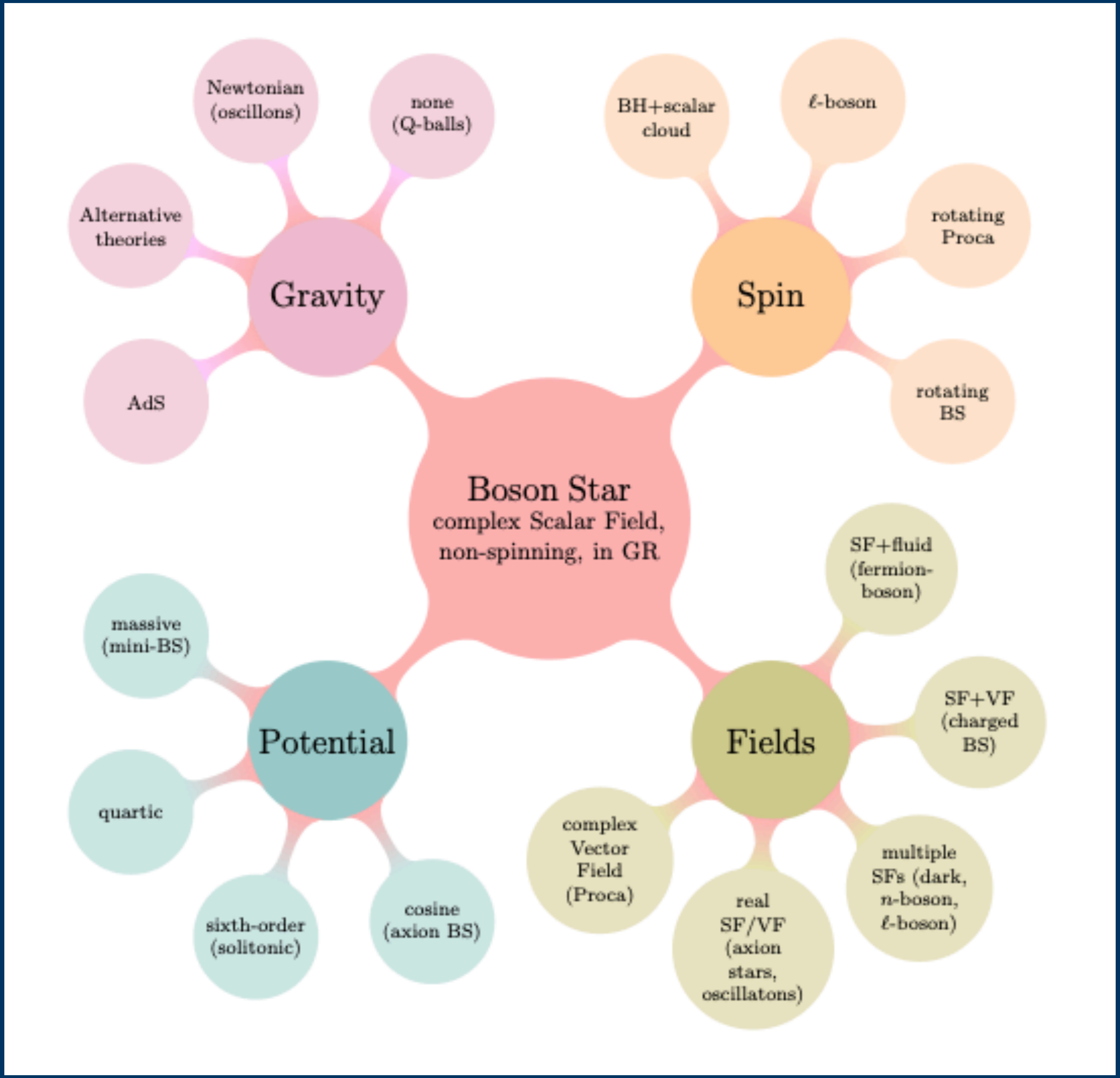
- Campos con Spin 1: Estrellas de Proca

$$\mathcal{L}_M = -\frac{1}{2}F^*_{\mu\nu}F^{\mu\nu} - m_0^2 A^*_\mu A^\mu - \lambda(A^*_\mu A^\mu)^2,$$

$$A_\mu(t, \vec{x}) = e^{-i\omega t}(f(r), ig(r), 0, 0).$$

- Campos con Spin 2: teorías Bimetric

Resumen:



arXiv:1202.5809v5

Fenomenología

Objetos astrofísicos

Horndeski fermion-boson stars

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December 2021

Abstract. We establish the existence of static and spherically sy
boson stars, in a low energy effective model of (beyond) Horndeski the
are in equilibrium, and are composed by a mixing of scalar and fermionic matters that only interact gravitationally one with each other. Properties such as mass, radius, and compactness are studied, highlighting the existence of two families of configurations

Can fermion-boson stars reconcile multi-messenger observations of compact stars?

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[2]*Departamento de Matemática da Universidade de Aveiro and Centre for Research and Development in Mathematics and Applications (CIDMA), Campus de Santiago, 3810-183 Aveiro, Portugal*
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(Dated: October 28, 2021)

Mixed fermion-boson stars are stable, horizonless, everywhere regular solutions of the coupled Einstein-(complex, massive) Klein-Gordon-Euler system. While isolated neutron stars and boson

Halos de materia oscura

MNRAS **000**, 000–000 (0000) Preprint 22 August 2017 Compiled using MNRAS LaTeX style file v3.0

Unbiased constraints on ultralight axion mass from dwarf spheroidal galaxies

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22 August 2017

ABSTRACT
It has been suggested that the internal dynamics of dwarf spheroidal galaxies (dSphs) can be used to test whether or not ultralight axions with $m_a \sim 10^{-22}$ eV are a preferred dark matter candidate. However, comparisons to theoretical predictions tend to be inconclusive for the simple reason that while most cosmological models consider

Axion dark matter, solitons, and the cusp-core problem

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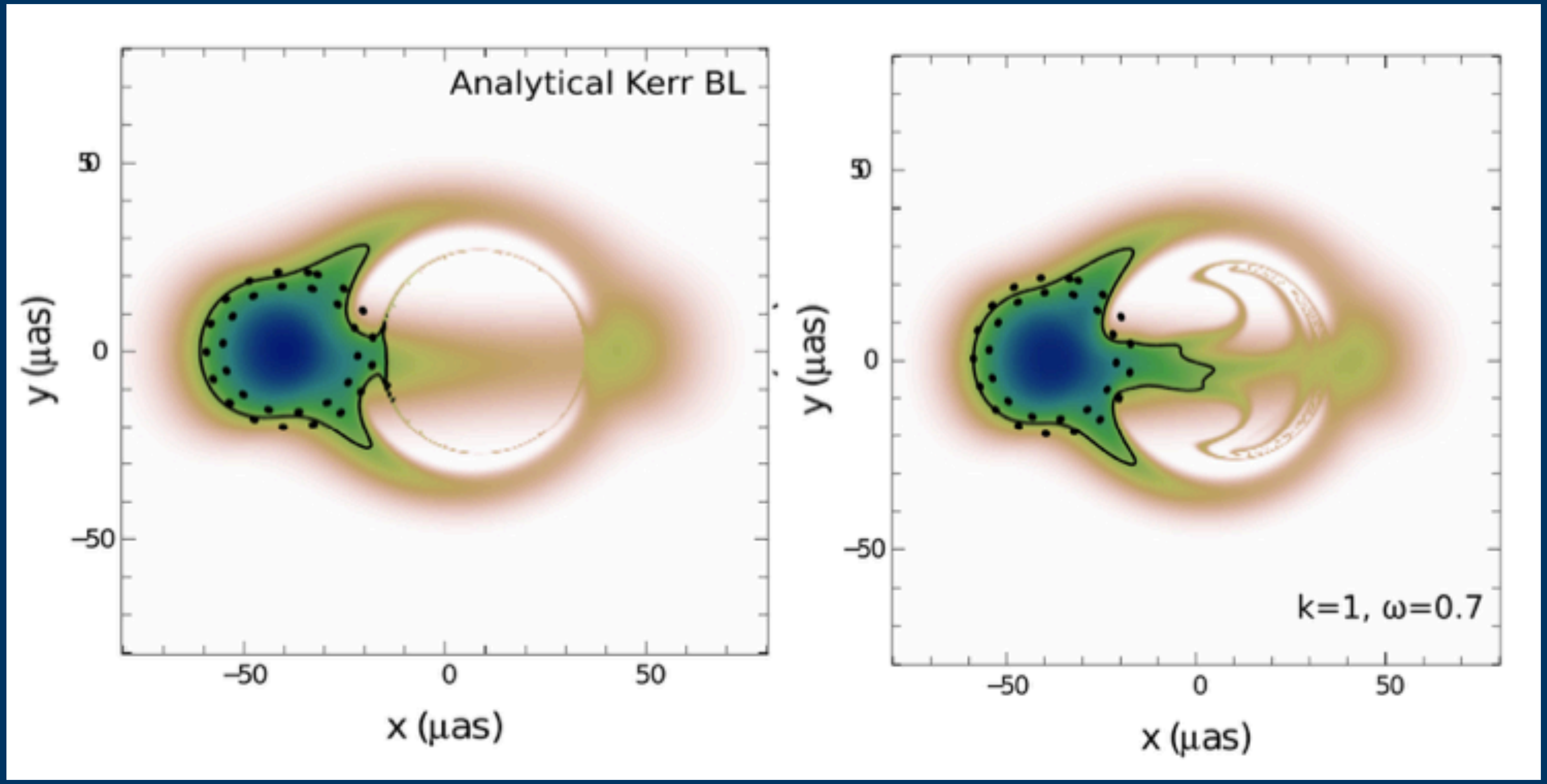
Draft version: 18 June 2015

ABSTRACT
Self-gravitating bosonic fields can support stable and localised (solitonic) field configurations. Such solitons should be ubiquitous in models of axion dark matter, with their characteristic mass and size depending on some inverse power of the axion mass, m_a. Using a scaling symmetry and the uncertainty principle, the soliton core size can be related to the central density and axion mass in a universal way. Solitons have a constant central density due to pressure-support, unlike the cuspy profile of cold dark matter (CDM). Consequently, solitons composed of ultra-light axions (ULAs) may resolve the 'cusp-core' problem of CDM. In DM halos, thermodynamics will lead to a CDM-like

Objetos ultracompactos

Disco de acreción Sgr A*: parecen muy similares

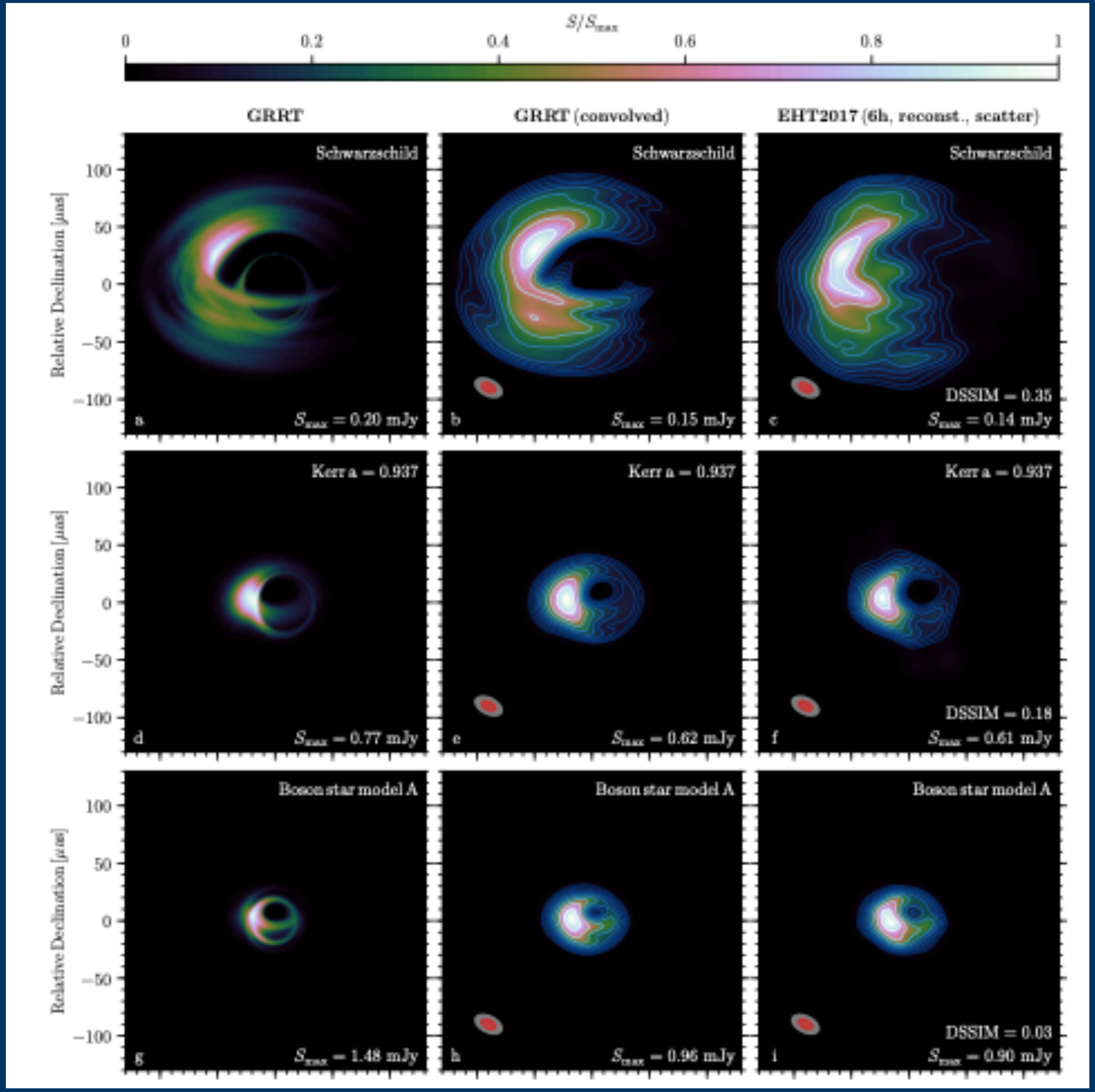


arXiv:1510.04170

Estas diferencias podrían desaparecer, o al menos disminuir,
al considerar una estrella Proca en lugar de una estrella de bosones
arXiv:2204.12949.

Disco de acreción Sgr A* (más actual)

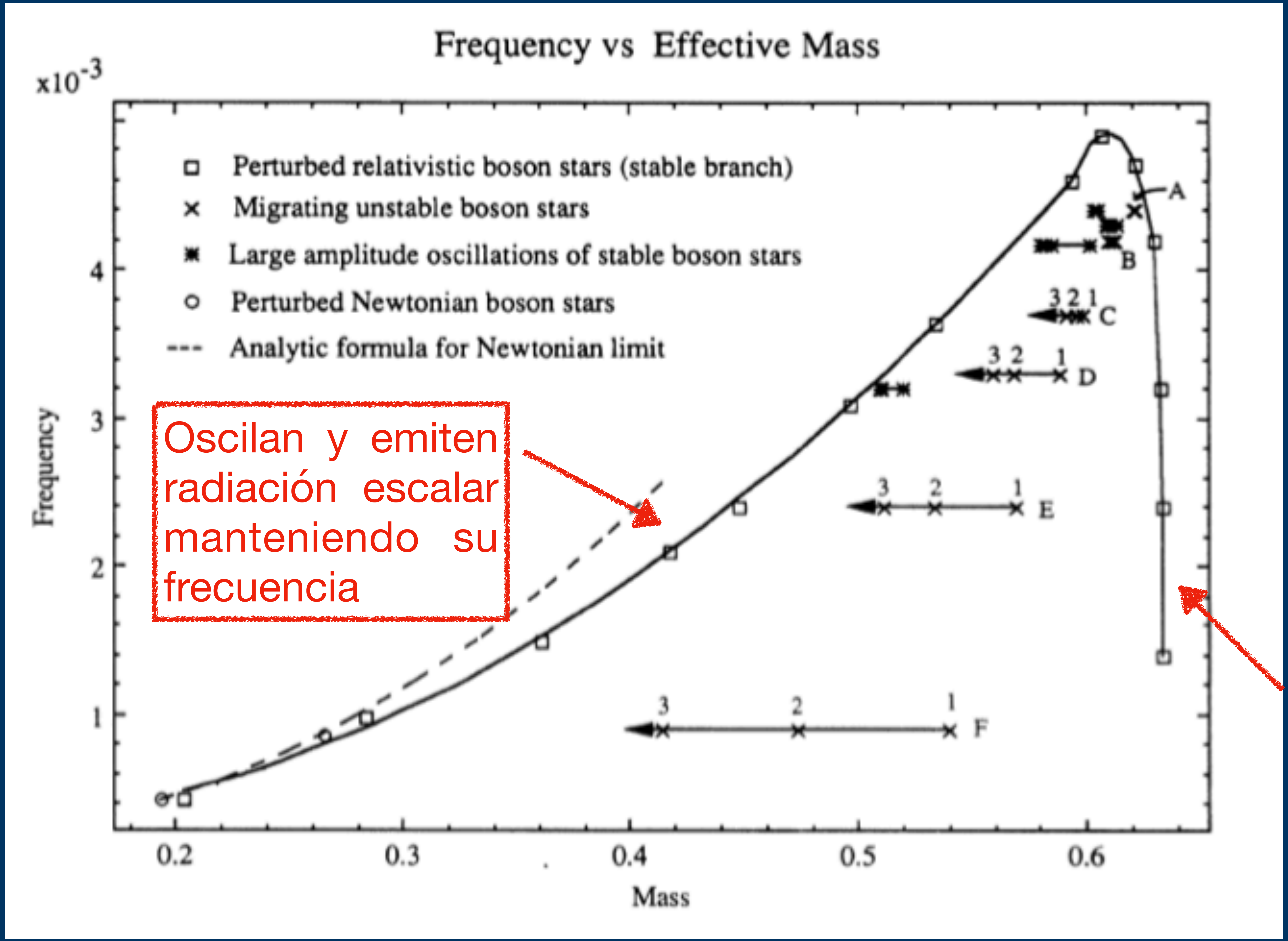
arXiv:1809.08682



BS- más simétricos
Región oscura más pequeña (ausencia de Horiz. Event.)

Estabilidad

Término de masa: $L_m = -\nabla_\mu \psi^* \nabla^\mu \psi - m|\psi|^2$



Edward Seidel and Wai-Mo Suen
Phys. Rev. D **42**, 384 – Published 15 July 1990

Bajo perturbaciones lineales:

- Configuraciones en el estado base (n=0) son estables si se encuentran a la izquierda de la configuración de máxima masa.

- Configuraciones en estados exitados (n=1, 2, …) son inestables.

Colapsan hasta convertirse en un BH o emigrar a una configuración estable (naturaleza de la perturbación)

¿Qué ocurre si se incluye la autointeracción en el potencial?

Estabilidad

Autointeracción cuartica: $L_m = -\nabla_\mu \psi^* \nabla^\mu \psi - m|\psi|^2 - \lambda |\psi|^4$

La signatura de λ indica si es repulsiva o atractiva la autointeracción

- Sin importar la signatura de λ existe una rama estable para configuraciones en el estado base (n=0) determinado por la configuración de máxima masa.

PHYSICAL REVIEW D VOLUME 38, NUMBER 8 15 OCTOBER 1988

Stability of boson stars

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(Received 20 June 1988)

configurations of cold,
ıs naturally present lo-
bility criteria for fluid

Gravitational stability of scalar matter

Marcelo Gleiser [a], Richard Watkins [b]

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https://doi.org/10.1016/0550-3213(89)90627-5

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PHYSICAL REVIEW D VOLUME 42, NUMBER 2 15 JULY 1990

Dynamical evolution of boson stars: Perturbing the ground state

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(Received 8 January 1990)

- Configuraciones en estados exitados ¿son inestables?

Para el caso atractivo, SÍ, sin embargo para el caso repulsivo no es claro

¿Tiempo de evolución?
Aproximaciones
...

Dynamical Evolution of Boson Stars
II: Excited States and Self-Interacting Fields

Jayashree Balakrishna[1], Edward Seidel[2,3,4], and Wai-Mo Suen[1,5]

Self-interactions can stabilize excited boson stars

Nicolas Sanchis-Gual[1], Carlos Herdeiro[1], and Eugen Radu[1]

Stability of excited Bose stars

Ph. Jetzer

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¡¡¡Diferentes conclusiones!!!

¿Tiempo de evolución?
Aproximaciones
...

Busquemos la respuesta en el:

Límite no relativista

Acción:

$$S[g_{\mu\nu}, \phi] = \int d^4x\sqrt{-g}\left(\frac{1}{16\pi G}R + \mathcal{L}_M\right),$$

$$\mathcal{L}_M = -\nabla_\mu\phi^*\nabla^\mu\phi - m_0^2|\phi|^2 - \lambda|\phi|^4.$$

Newton gauge (solo nos interesa los grados de libertad escalar)

$$ds^2 = -[1 + 2\Phi(t,\vec{x})]\,dt^2 + [1 - 2\Psi(t,\vec{x})]\,\delta_{ij}dx^i dx^j,$$

Descomponemos el campo escalar

$$\phi(t,\vec{x}) = \frac{1}{\sqrt{2m_0}}e^{-im_0 t}\psi(t,\vec{x}),$$

"Pesos"

$$\Phi \sim \Psi \sim \epsilon$$

$$\psi \sim \sqrt{M_{\rm Pl}^2 m_0 \epsilon},$$

$$\partial_t \sim \epsilon^{1/2}\partial_i \sim \epsilon m_0,$$

$$S[\Phi,\Psi,\psi] = \int dt \int d^3x \left[\frac{1}{8\pi G}\Psi\Delta\left(2\Phi - \Psi\right) + \psi^*\left(i\frac{\partial}{\partial t} + \frac{1}{2m_0}\Delta - \frac{\lambda}{4m_0^2}|\psi|^2\right)\psi - m_0\Phi|\psi|^2\right]$$

$$i\frac{\partial\psi}{\partial t} = -\frac{1}{2m_0}\Delta\psi \pm \frac{\pi\Lambda}{M_{\rm Pl}^2}|\psi|^2\psi + m_0\mathcal{U}\psi,$$

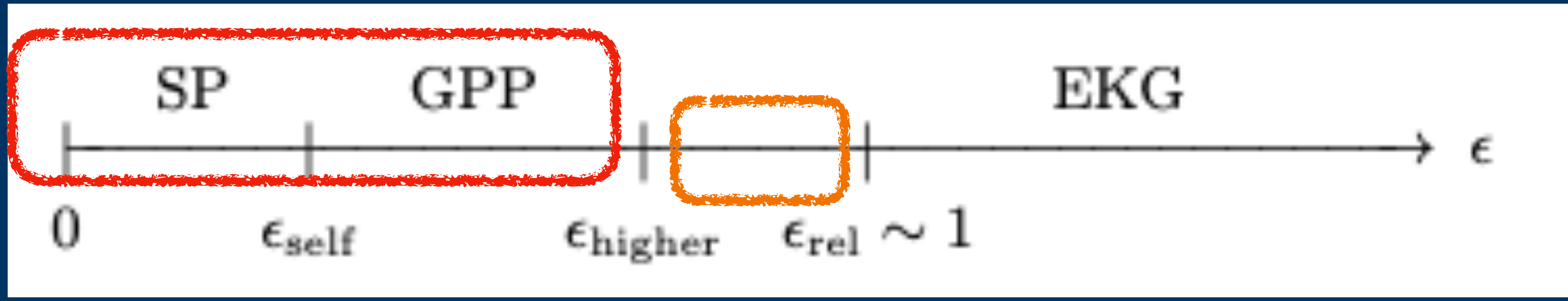
$$\Delta\mathcal{U} = 4\pi G m_0|\psi|^2$$

Comentarios heurísticos del potencial

Caso general

$$V(\phi) = M^4 \sum_{n=2}^{\infty} \frac{v_{2n}}{(2n)!} \left| \frac{\phi}{M} \right|^{2n},$$

Resumen: es general el resultado



En el límite no relativista
tendremos que:

$$-\frac{\lambda}{4m_0^2}|\psi|^4 \left[\boxed{1} + \boxed{\frac{v_6}{240 v_4} \frac{|\psi|^2}{m_0 M^2}} + \dots \right]$$

$$\boxed{\epsilon \sim \epsilon_{\rm higher} = \sqrt{240 v_4/v_6}(M/M_{\rm Pl}),} \ll \boxed{\epsilon_{\rm self} = m_0^2/(\lambda M_{\rm Pl}^2),}$$

Estrellas de bosones no-relativistas:

Sistema: Gross-Pitaevskii-Poisson

Gross-Pitaevskii:

$$i\frac{\partial\psi}{\partial t} = -\frac{1}{2m_0}\Delta\psi \pm \frac{\pi\Lambda}{M_{\mathrm{Pl}}^2}|\psi|^2\psi + m_0\mathcal{U}\psi,$$

Poisson:

$$\Delta\mathcal{U} = 4\pi G m_0|\psi|^2$$

Funcional de Energía

$$\mathcal{E} = \int\left(\frac{1}{2m_0}|\nabla\psi|^2 \pm \frac{\pi\Lambda}{2M_{\mathrm{Pl}}^2}|\psi|^4 + \frac{1}{2}m_0\mathcal{U}|\psi|^2\right)d^3x,$$

Sistema adimensional:

$$\bar{t} := \frac{2m_0}{\Lambda}t, \quad \bar{x} := \frac{2m_0}{\Lambda^{1/2}}x,$$
$$\bar{\mathcal{U}} := \frac{\Lambda}{2}\mathcal{U}, \quad \bar{\psi} := \left(\frac{\pi\Lambda^2}{2M_{\mathrm{Pl}}^2 m_0}\right)^{1/2}\psi.$$

$$i\frac{\partial\psi}{\partial t} = \left(-\Delta \pm |\psi|^2 + \mathcal{U}\right)\psi,$$
$$\Delta\mathcal{U} = |\psi|^2,$$

$$i\frac{\partial\psi}{\partial t} = \hat{\mathcal{H}}(\psi)\psi,$$
$$\hat{\mathcal{H}}(\psi) := -\Delta \pm |\psi|^2 + \Delta^{-1}(|\psi|^2),$$

Soluciones estacionarias

$$\psi(t,\vec{x}) = e^{-iEt}\sigma^{(0)}(\vec{x}),$$

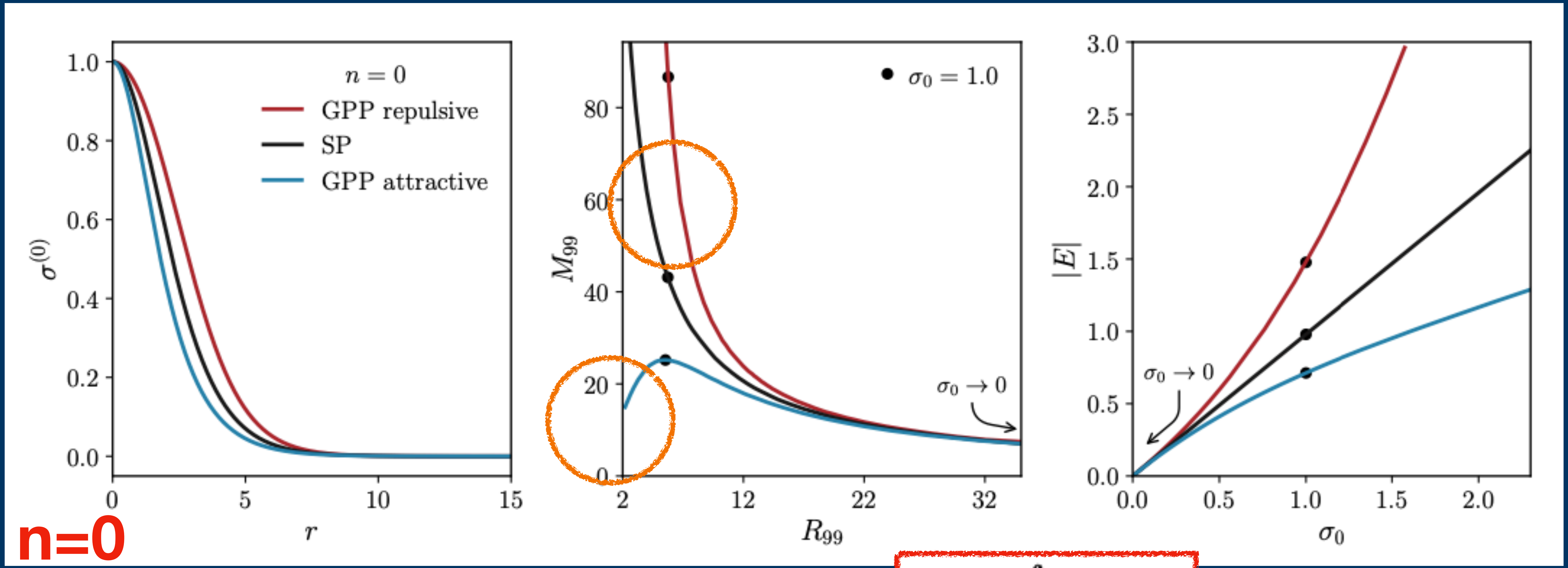
$$+$$

Soluciones esféricamente simétricas

$$\sigma^{(0)}(\vec{x}) = \sigma^{(0)}(r)$$

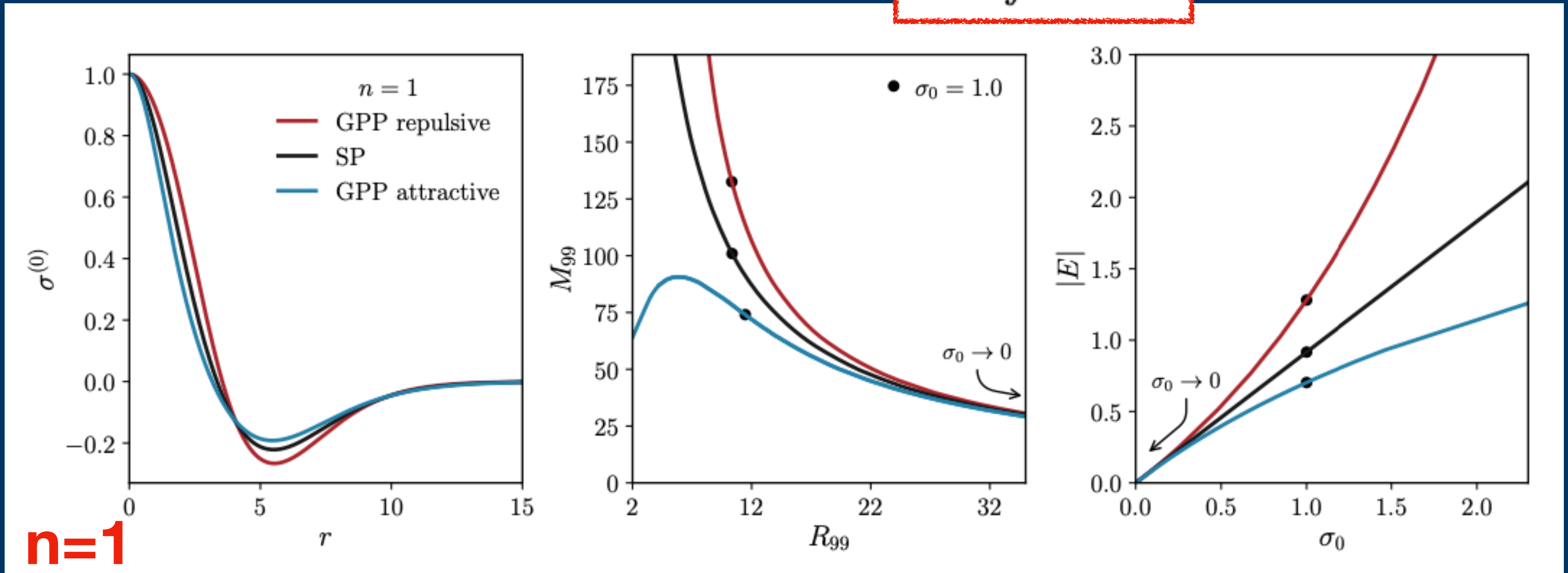
$$=$$

$$E\sigma^{(0)} = \hat{\mathcal{H}}(\sigma^{(0)})\sigma^{(0)}.$$



n=0

$$N = \int |\psi|^2 d^3x.$$



n=1

Variación del funcional de Energía

En terminos de las variables adimensionales

$$\mathcal{E}[\psi] = T[\psi] \pm F[n] - D[n,n], \quad n := |\psi|^2,$$

$$T[\psi] := \frac{1}{2}\int |\nabla\psi(\vec{x})|^2 d^3x,$$
$$F[n] := \frac{1}{4}\int n(\vec{x})^2 d^3x,$$
$$D[n,n] := \frac{1}{16\pi}\int\int \frac{n(\vec{x})n(\vec{y})}{|\vec{x}-\vec{y}|} d^3y d^3x.$$

Asumiendo un parámetro real y positivo $\nu > 0$

Escalamos

$$\psi_\nu(t,\vec{x}) := \nu^{3/2}\psi(t,\nu\vec{x}),$$

Número de partículas invariante

$$\mathcal{E}[\psi_\nu] = \nu^2 T[\psi] \pm \nu^3 F[n] - \nu D[n,n],$$

Variaciones en $\psi_{\nu=1}$

$$\frac{d}{d\nu}\mathcal{E}[\psi_\nu]\bigg|_{\nu=1} = 2T[\psi] \pm 3F[n] - D[n,n],$$
$$\frac{d^2}{d\nu^2}\mathcal{E}[\psi_\nu]\bigg|_{\nu=1} = 2T[\psi] \pm 6F[n].$$

Para solución estacionarias

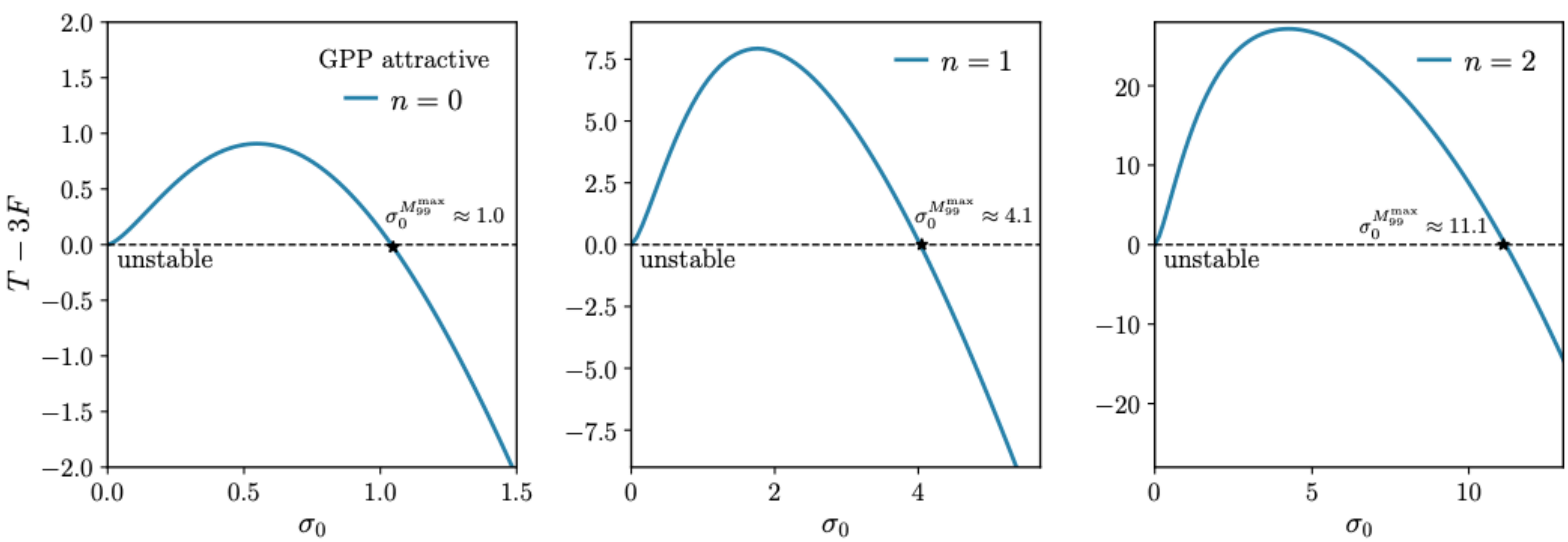
$$D[n,n] = 2T[\psi] \pm 3F[n].$$

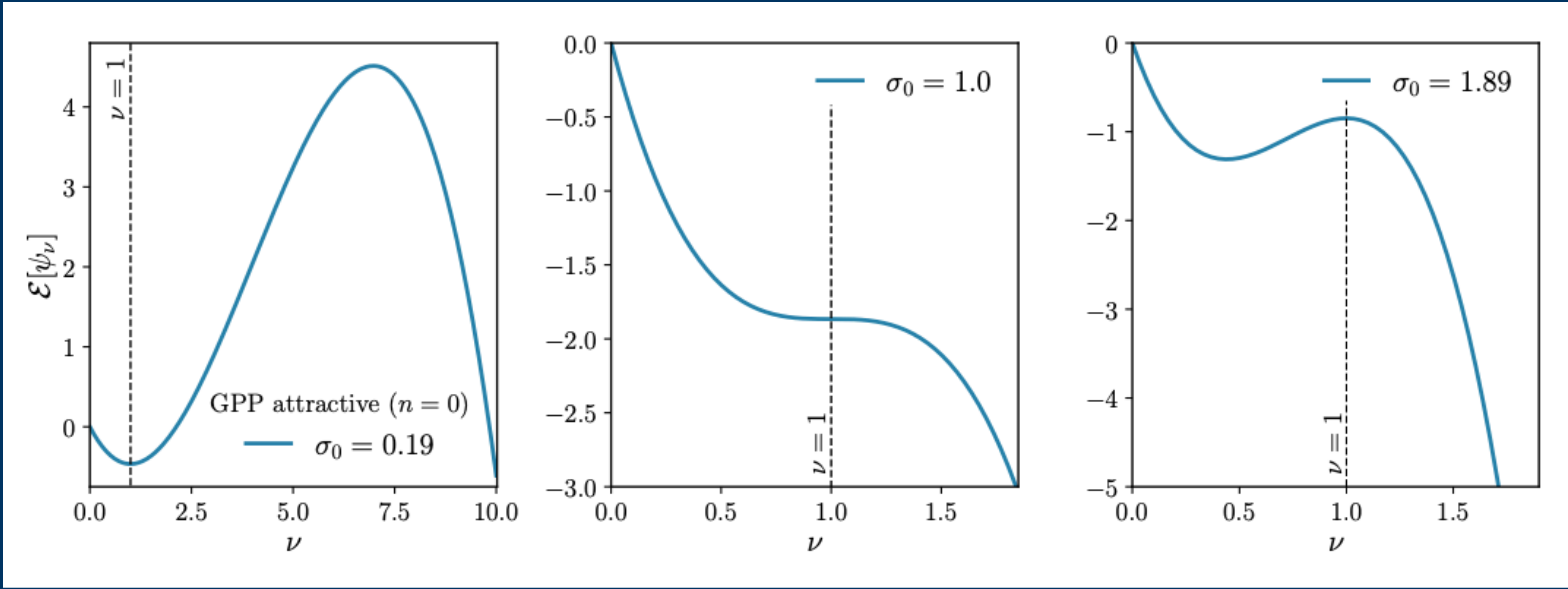
$$\boxed{\mathcal{E}[\psi] = -T[\psi] \mp 2F[n].}$$

El punto crítico en $\psi_{\nu=1}$

$\mathcal{E}[\psi_\nu]$ if $T - 3F > 0$ Mínimo local

Caso Atractivo $\lambda = -1$





Notar que para el caso Repulsivo $\lambda = +1$

$$T[\psi] := \frac{1}{2}\int |\nabla\psi(\vec{x})|^2 d^3x,$$
$$F[n] := \frac{1}{4}\int n(\vec{x})^2 d^3x,$$

> 0 $\longrightarrow$

$$\left.\frac{d^2}{d\nu^2}\mathcal{E}[\psi_\nu]\right|_{\nu=1} = 2T[\psi] \pm 6F[n].$$

> 0

Mínimo local

Perturbaciones lineales

Consideremos una perturbación de la forma

$$\psi(t,\vec{x}) = e^{-iEt}\left[\sigma^{(0)}(\vec{x}) + \epsilon\sigma(t,\vec{x}) + \mathcal{O}(\epsilon^2)\right],$$

$$\sigma(t,\vec{x}) = [A(\vec{x}) + B(\vec{x})]\, e^{\lambda t} + [A(\vec{x}) - B(\vec{x})]^*\, e^{\lambda^* t},$$

$$A(\vec{x}) = \sum_{LM} A_{LM}(r) Y^{LM}(\vartheta,\varphi),$$

Sistema

$$\begin{aligned} i\lambda A_{LM} &= \left(\hat{\mathcal{H}}_L^{(0)} - E\right) B_{LM}, \\ i\lambda B_{LM} &= \left(\hat{\mathcal{H}}_L^{(0)} - E\right) A_{LM} + 2\sigma^{(0)}\hat{K}_L\left[\sigma^{(0)} A_{LM}\right] \end{aligned}$$

$$\begin{aligned} \hat{\mathcal{H}}_L^{(0)} &:= -\triangle_L \pm \sigma^{(0)2} + \triangle_s^{-1}(\sigma^{(0)2}), \\ \hat{K}_L &:= \pm 1 + \triangle_L^{-1}, \end{aligned}$$

$$\triangle_L := \triangle_s - L(L+1)/r^2$$

Sistema discretizado

$r(A_{LM}, B_{LM})^T$

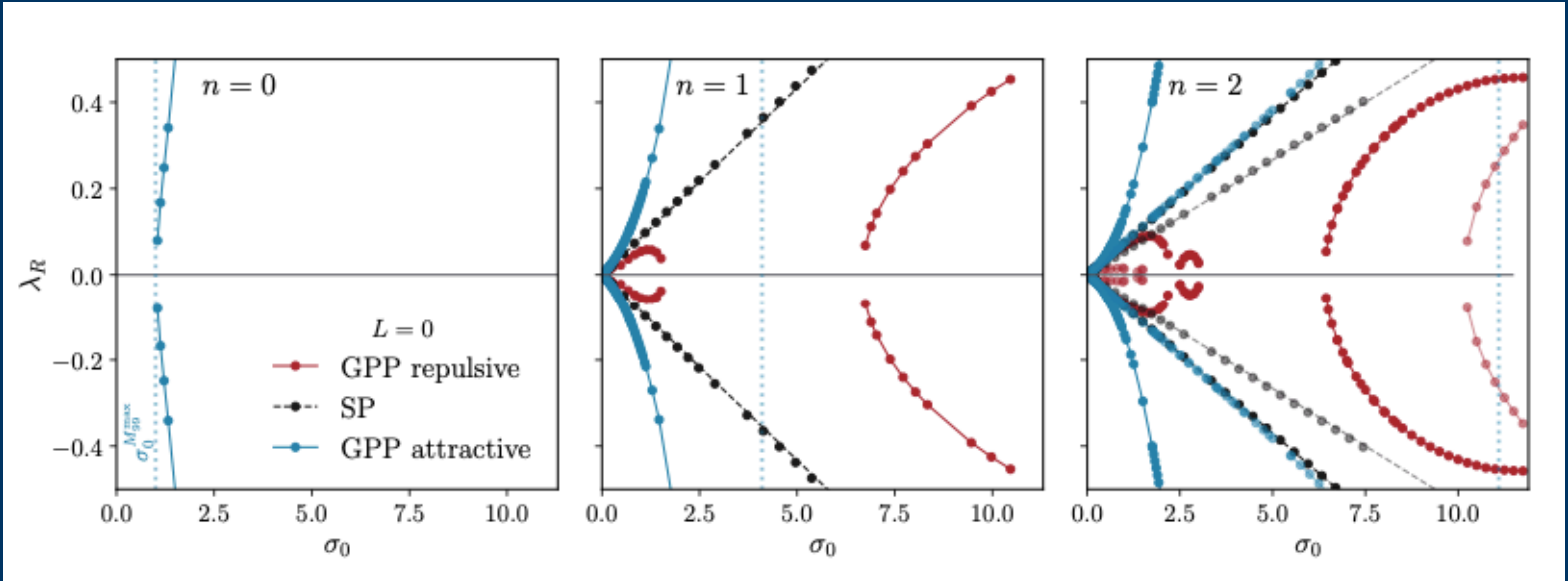
$$\begin{pmatrix} 0 & \tilde{\mathbb{D}}_{\mathsf{N}}^2 \mp \Sigma_0^2 + \mathsf{U}_L^{\mathrm{eff}} \\ \tilde{\mathbb{D}}_{\mathsf{N}}^2 \mp 3\Sigma_0^2 + \mathsf{U}_L^{\mathrm{eff}} - 2\alpha\Sigma_0\left(\tilde{\mathbb{D}}_{\mathsf{N}}^2 - \mathbb{L}\right)^{-1}\Sigma_0 & 0 \end{pmatrix} \begin{pmatrix} \mathsf{a}_L \\ \mathsf{b}_L \end{pmatrix} = -i\lambda \begin{pmatrix} \mathsf{a}_L \\ \mathsf{b}_L \end{pmatrix},$$

$$\begin{aligned} \Sigma_0 &:= \mathbf{diag}\left(\sigma^{(0)}(x_1), \sigma^{(0)}(x_2), \dots, \sigma^{(0)}(x_{\mathsf{N}-1})\right), \\ \mathsf{U}_L^{\mathrm{eff}} &:= \mathbf{diag}\left(U_L^{\mathrm{eff}}(x_1), U_L^{\mathrm{eff}}(x_2), \dots, U_L^{\mathrm{eff}}(x_{\mathsf{N}-1})\right), \\ \mathbb{L} &:= \mathbf{diag}\left(\frac{L(L+1)}{x_1^2}, \frac{L(L+1)}{x_2^2}, \dots, \frac{L(L+1)}{x_{\mathsf{N}-1}^2}\right), \end{aligned}$$

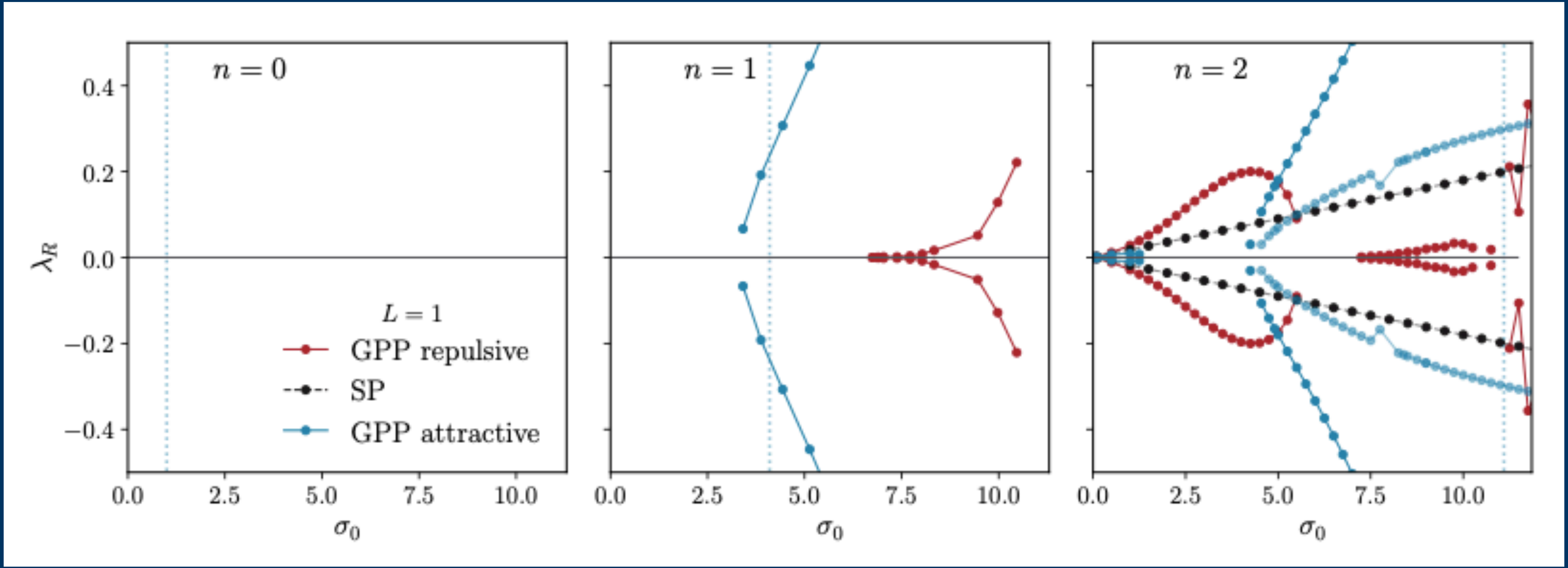
$$\begin{pmatrix} \mathsf{a}_L \\ \mathsf{b}_L \end{pmatrix} := \left(a_L(x_1), \dots, a_L(x_{\mathsf{N}-1}), b_L(x_1), \dots, b_L(x_{\mathsf{N}-1})\right)^T$$

¡¡Un problema de Autovalores!!

L=0 (perturbación radial)



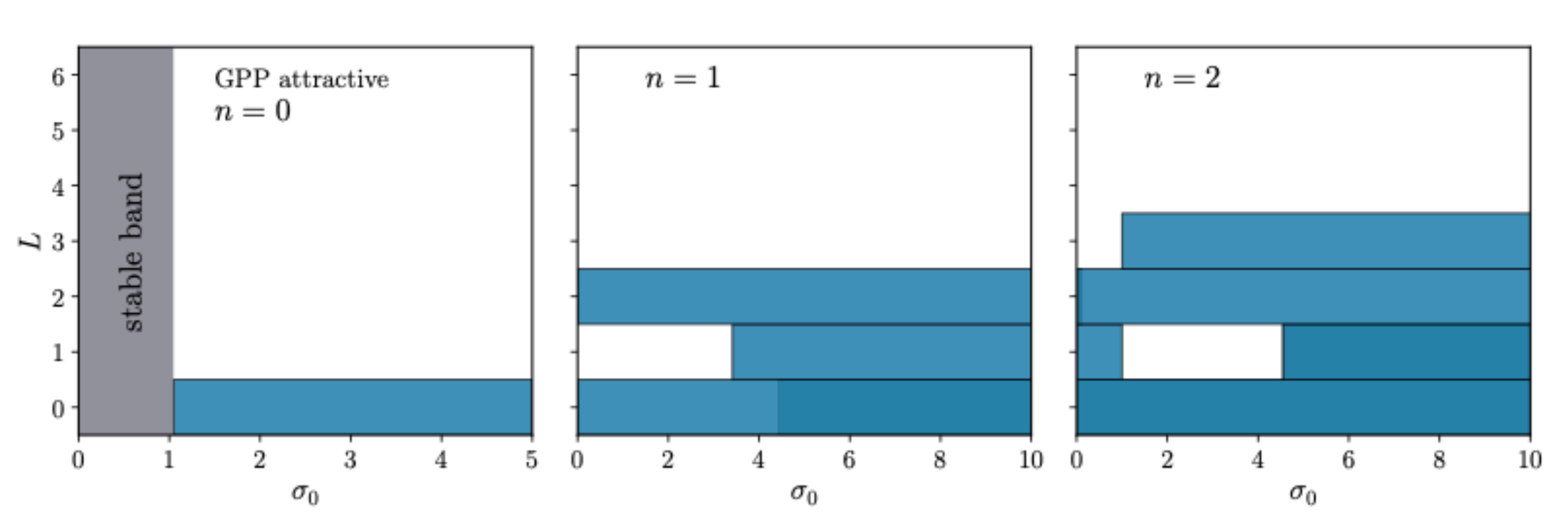
L=1



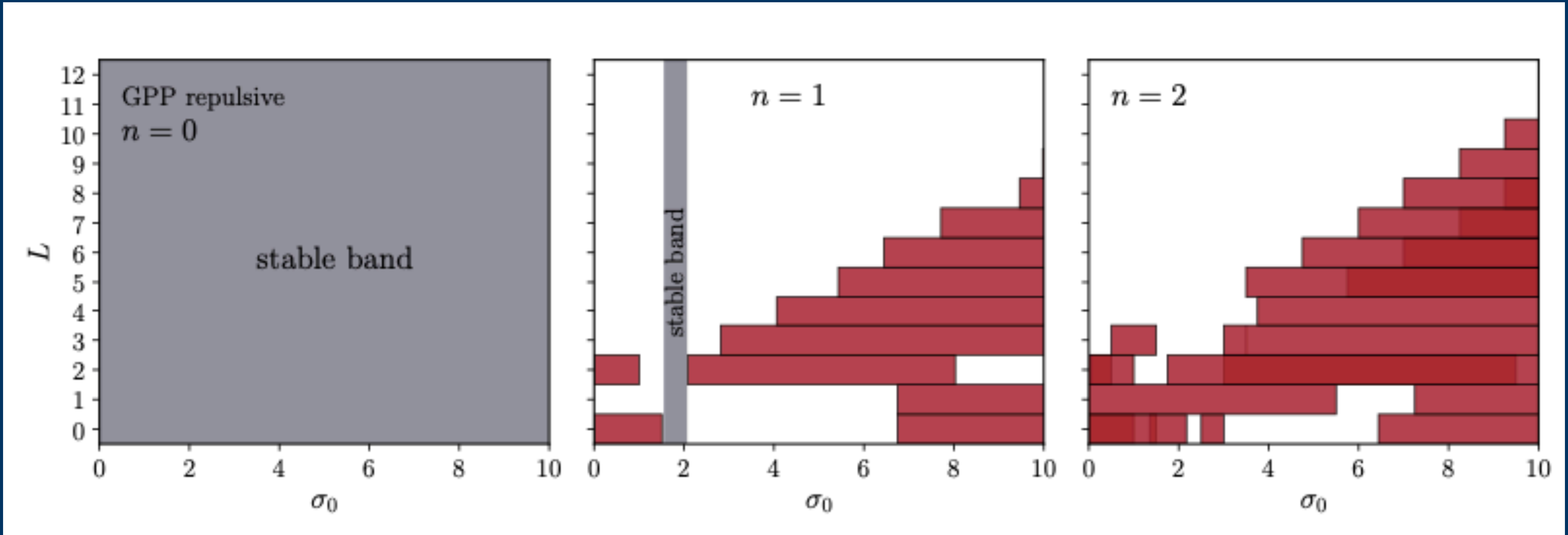
$n = 0$
$n = 1$
$n = 2$
λ_R
0.4
0.2
0.0
−0.2
−0.4
0.0
2.5
5.0
7.5
10.0
σ_0
$L = 1$
GPP repulsive
SP
GPP attractive

Regiones de estabilidad

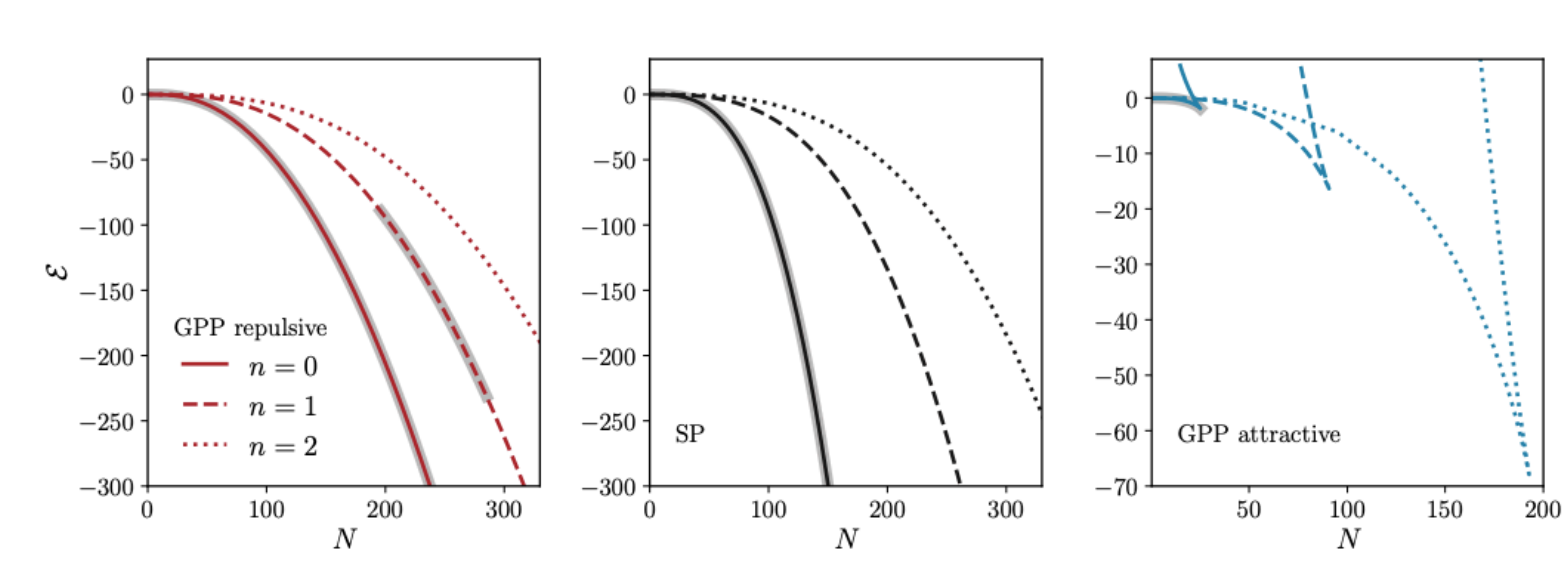
Caso Atractivo



Caso Atractivo

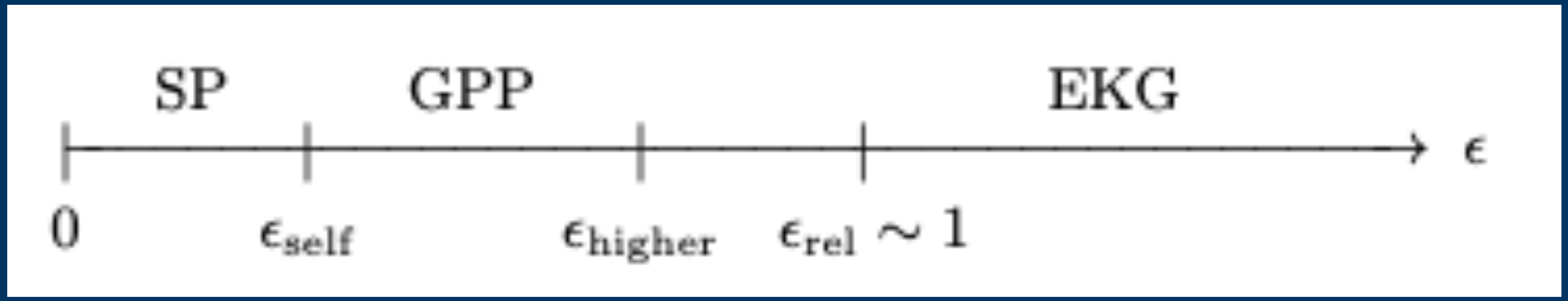


Funcional de energía



Resumen

- **Estados bases:** son estables para el caso repulsivo $\lambda = 1$ (similar al caso sin auto-interacción) existiendo para el caso atractivo $\lambda = -1$ una región estable en el límite $\psi \to 0$ donde la autointeracción es despreciable y el sistema GPP se reduce al sistema SP.



- **Estados exitados:**

Caso atractivo $\lambda = -1$ siempre es Inestables

Caso repulsivo $\lambda = +1$ para el primer estado exitado existe una región finita de estabilidad ante perturbaciones lineales. Para el segundo estado no se encontró una región de estabilidad común.

Comentarios Finales

- Aunque el estudio se realizó para los primeros dos estados excitados, se espera un comportamiento similar para el resto de estados.

- Dada un configuración siempre ocurrirá que a partir de una cierto valor L, solo existen modos estables producto de una perturbación lineal.

- Es lógico pensar que la región estable permanezca una vez que este límite no sea válido.

- Como consecuencia de las variables usadas los resultados son válidos para cualquier valor de la autointeracción.

¡Gracias!