Seminario Teórico

División de Ciencias e Ingenierías. UG

Estabilidad de las Newtonian ℓ-boson stars ante perturbaciones lineales

División de Ciencias e Ingenierías, UGTO, 2021

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Estructura

I. ¿Qué es una ℓ-boson star?

    A. Estabilidad de las ℓ-boson star

II. No relativistas N-boson stars

    A. No relativistas ℓ-boson stars

    B. Perturbaciones (radiales) Lineales

    C. Perturbaciones lineales no esféricas

III. Conclusión

¿Qué es una ℓ-boson star?

$$\mathcal{L} = \frac{R}{16\pi} - \frac{1}{2}\sum_{m=-\ell}^{\ell}\left(\nabla_\mu \Phi_{\ell m}\nabla^\mu \Phi^*_{\ell m} + \mu^2|\Phi_{\ell m}|^2\right)$$

$m = -\ell, \ldots, \ell$ → $N = 2\ell + 1$ of complex scalar

$$\Phi_{\ell m}(t, r, \vartheta, \varphi) = e^{i\omega t}\psi_\ell(r)Y^{\ell m}(\vartheta, \varphi),$$

¿Qué tiene de especial?
¡¡Se mantiene la simetría esférica!!

$$M' = \frac{\kappa_\ell r^2}{2}\left[\frac{\psi_\ell'^2}{\gamma^2} + \left(\mu^2 + \frac{\omega^2}{\alpha^2} + \frac{\ell(\ell+1)}{r^2}\right)\psi_\ell^2\right] = 4\pi r^2\rho,$$

$$\frac{(\alpha\gamma)'}{\alpha\gamma^3} = \kappa_\ell r\left[\frac{\psi_\ell'^2}{\gamma^2} + \frac{\omega^2}{\alpha^2}\psi_\ell^2\right] = 4\pi r(\rho + p_r),$$

Klein-Gordon
$\ell = 0$

$$\frac{1}{r^2\alpha\gamma}\left(\frac{r^2\alpha}{\gamma}\psi_\ell'\right)' = \left(\mu^2 - \frac{\omega^2}{\alpha^2} + \frac{\ell(\ell+1)}{r^2}\right)\psi_\ell,$$

ℓ-Boson stars

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(Dated: August 31, 2018)

Un único campo

Boson stars and their relatives in semiclassical gravity

Miguel Alcubierre,[1] Juan Barranco,[2] Argelia Bernal,[2] Juan Carlos Degollado,[3] Alberto Diez-Tejedor,[2] Miguel Megevand,[4] Darío Núñez,[1] and Olivier Sarbach[5]

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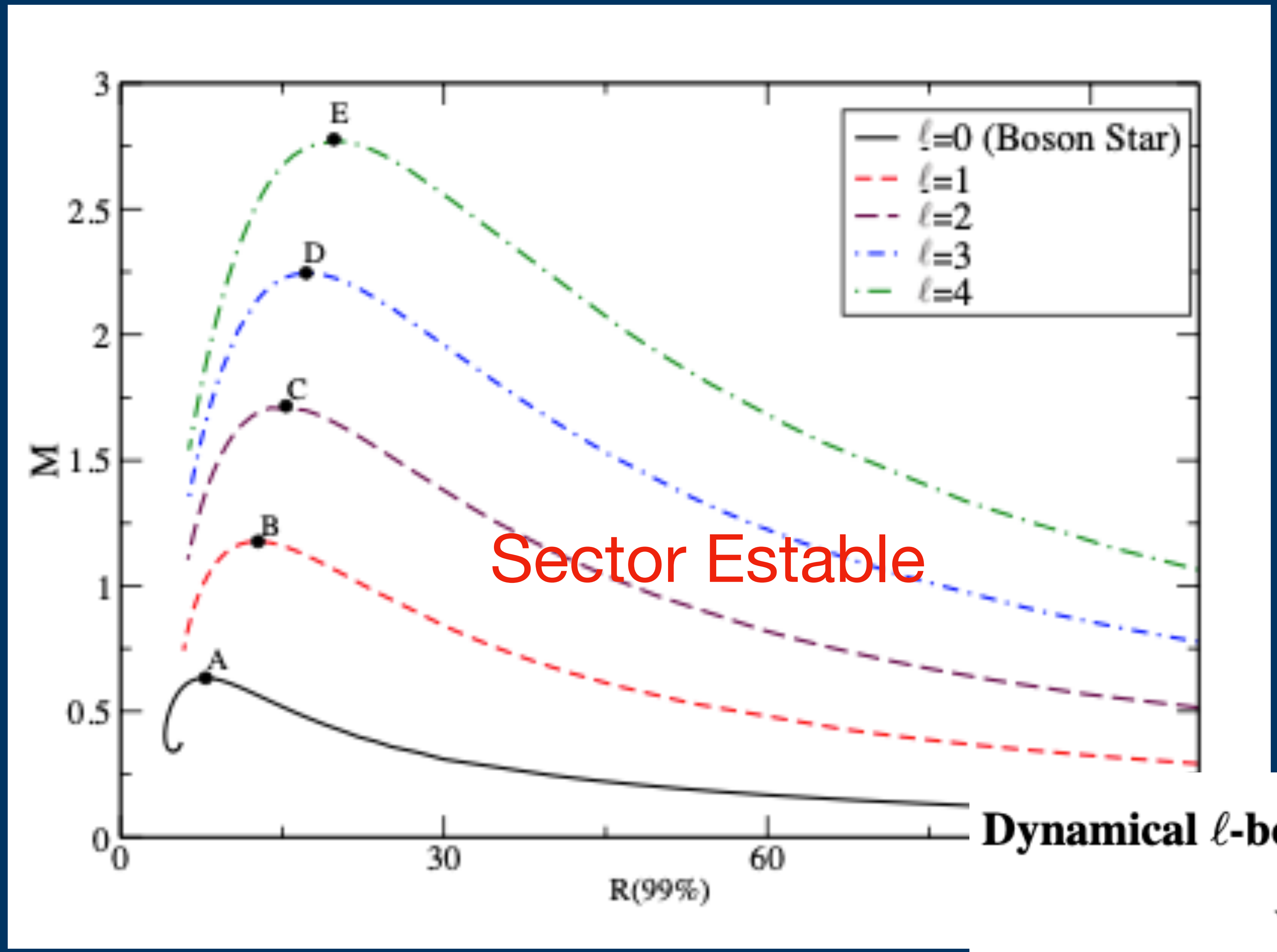
(Dated: February 21, 2023)

Estabilidad de las
ℓ-boson star

arXiv:1805.11488v2



Boson stars driven to the brink of black hole formation

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On the linear stability of ℓ-boson stars with respect to radial perturbations

Miguel Alcubierre,[1] Juan Barranco,[2] Argelia Bernal,[2] Juan Carlos Degollado,[3] Alberto Diez-Tejedor,[2] Miguel Megevand,[4] Darío Núñez,[1] and Olivier Sarbach[5]

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Dynamical evolutions of ℓ-boson stars in spherical symmetry

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Dynamical ℓ-boson stars: generic stability and evidence for non-spherical solutions

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(Dated: April, 2020)

¿Y las demás?

¿Es general?

No relativistas N-boson stars

Radial linear stability of nonrelativistic ℓ-boson stars

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Mecánica Cuántica → No Relativista ← RG

i) N-partículas idénticas de igual masa

ii) Solo interactúan a través del potencial gravitatorio generado por ellas.

i) Partículas idénticas con spin nulo

ii) No correlacionadas y sus funciones de onda son ortornomales

$(\psi_j, \psi_k) = \delta_{jk}$. $\hat{S} = \sum_{\pi \in \sigma(N)} P_\pi / N!$ $\sum_{j=1}^{J} N_j = N$

Caso Estacionario = Hartree

$$\Psi = \sqrt{\frac{N!}{N_1! N_2! \cdots N_J!}} \hat{S}\left(\psi_1^{N_1} \otimes \psi_2^{N_2} \otimes \ldots \otimes \psi_J^{N_J}\right),$$

N-particles Schrödinger-Poisson

$$i\hbar \frac{\partial \Psi(t,X)}{\partial t} = \sum_{i=1}^{N} \left(-\frac{\hbar^2}{2\mu}\nabla^2_{\vec{x}_i} + \mu \mathbf{U}(t,\vec{x}_i)\right)\Psi(t,X),$$

$$\nabla^2 \mathbf{U}(t,\vec{x}) = 4\pi G\mu \sum_{i=1}^{N} \int |\Psi(t,X)|^2 \delta^{(3)}(\vec{x}-\vec{x}_i) d^{3N}X,$$

$$X = (\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_N)$$

Un 3N vector que parametriza el espacio de configuraciones

$$i\hbar \frac{\partial \psi_j(t,\vec{x})}{\partial t} = \left(-\frac{\hbar^2}{2\mu}\nabla^2 + \mu \mathbf{U}(t,\vec{x})\right)\psi_j(t,\vec{x}),$$

$$\nabla^2 \mathbf{U}(t,\vec{x}) = 4\pi G\mu \sum_{j=1}^{J} N_j |\psi_j(t,\vec{x})|^2.$$

ψ_j

No relativistas
ℓ-boson stars

Simetría esférica

$$\psi_j(t,\vec{x}) := f_\ell(t,r) Y^{\ell m}(\vartheta,\varphi).$$

$$j = m + \ell + 1.$$

Sistema estacionario

$$f_\ell(t,r) = e^{\boxed{-iE_\ell t}} \sigma_\ell^{(0)}(r),$$

$$\boxed{(n,\ell,m)}$$

$$N = KJ = K(2\ell+1).$$

$$\hat{\mathcal{H}}_\ell^{(0)} \sigma_\ell^{(0)} = E_\ell \sigma_\ell^{(0)},$$

$$E_\ell^{\text{phys}} = 2\mu v_c^2 (K\Lambda)^2 E_\ell.$$

$$\hat{\mathcal{H}}_\ell^{(0)} := \left[-\nabla_s^2 + \frac{\ell(\ell+1)}{r^2} + \Delta_s^{-1}(|\sigma_\ell^{(0)}|^2)\right]$$

$$\Lambda := \frac{2\ell+1}{\int_0^\infty |k_\ell(t,r)|^2 r^2 dr}.$$

$$\nabla_s^2 := \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\right)$$

$$1 = \int |\psi_j(t,\vec{x})|^2 d^3x = \frac{\Lambda}{2\ell+1}\int_0^\infty |f_\ell(t,r)|^2 r^2 dr.$$

Variables Adimensionales

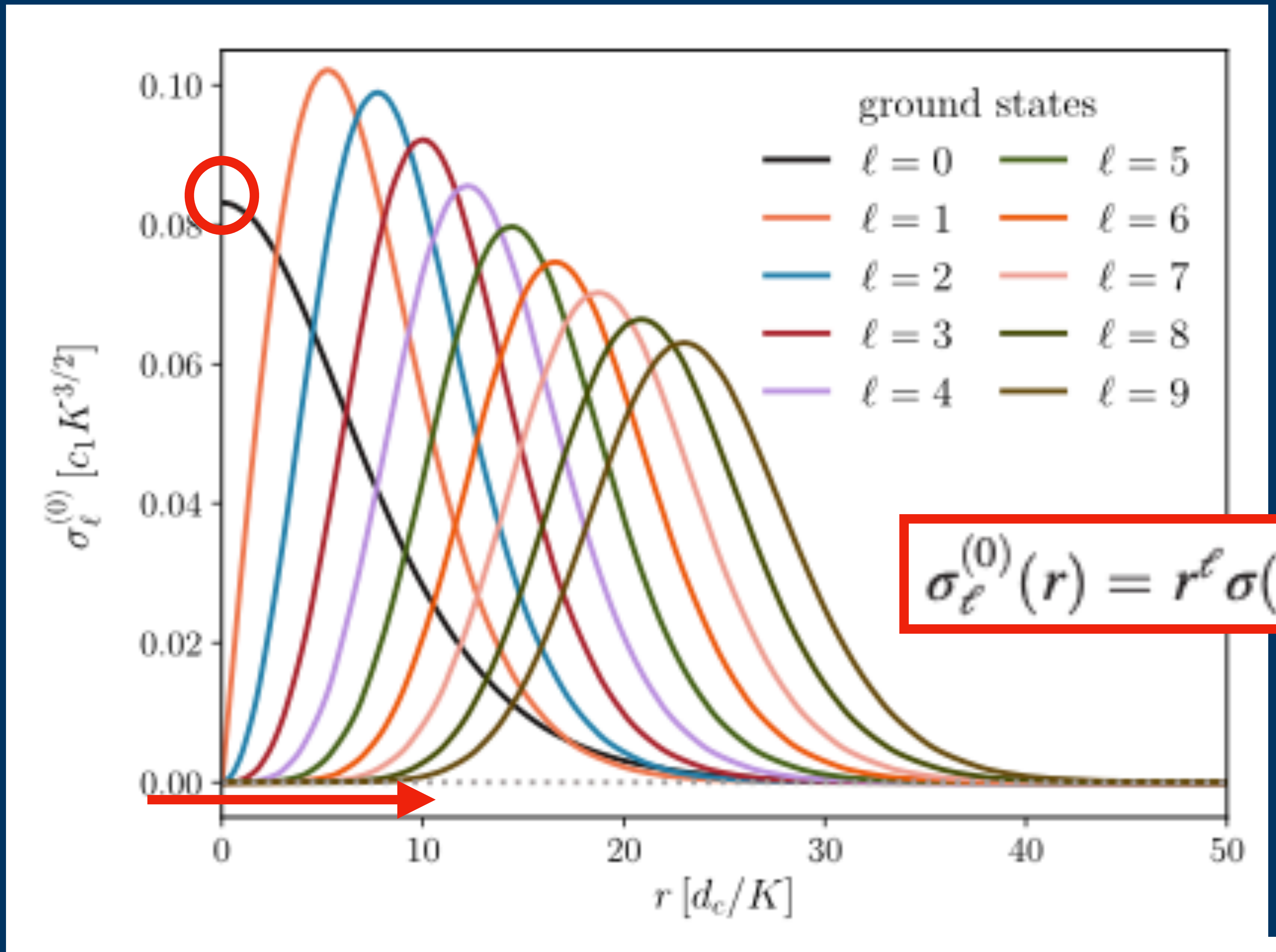
$$d_c := \frac{\hbar^2}{2G\mu^3} \qquad t_c := \frac{\hbar^3}{2G^2\mu^5}$$

$$f_\ell = K^{3/2}\Lambda^2 \bar{f}_\ell / \sqrt{(2\ell+1)d_c^3}, \qquad t = t_c \bar{t}/(K\Lambda)^2,$$

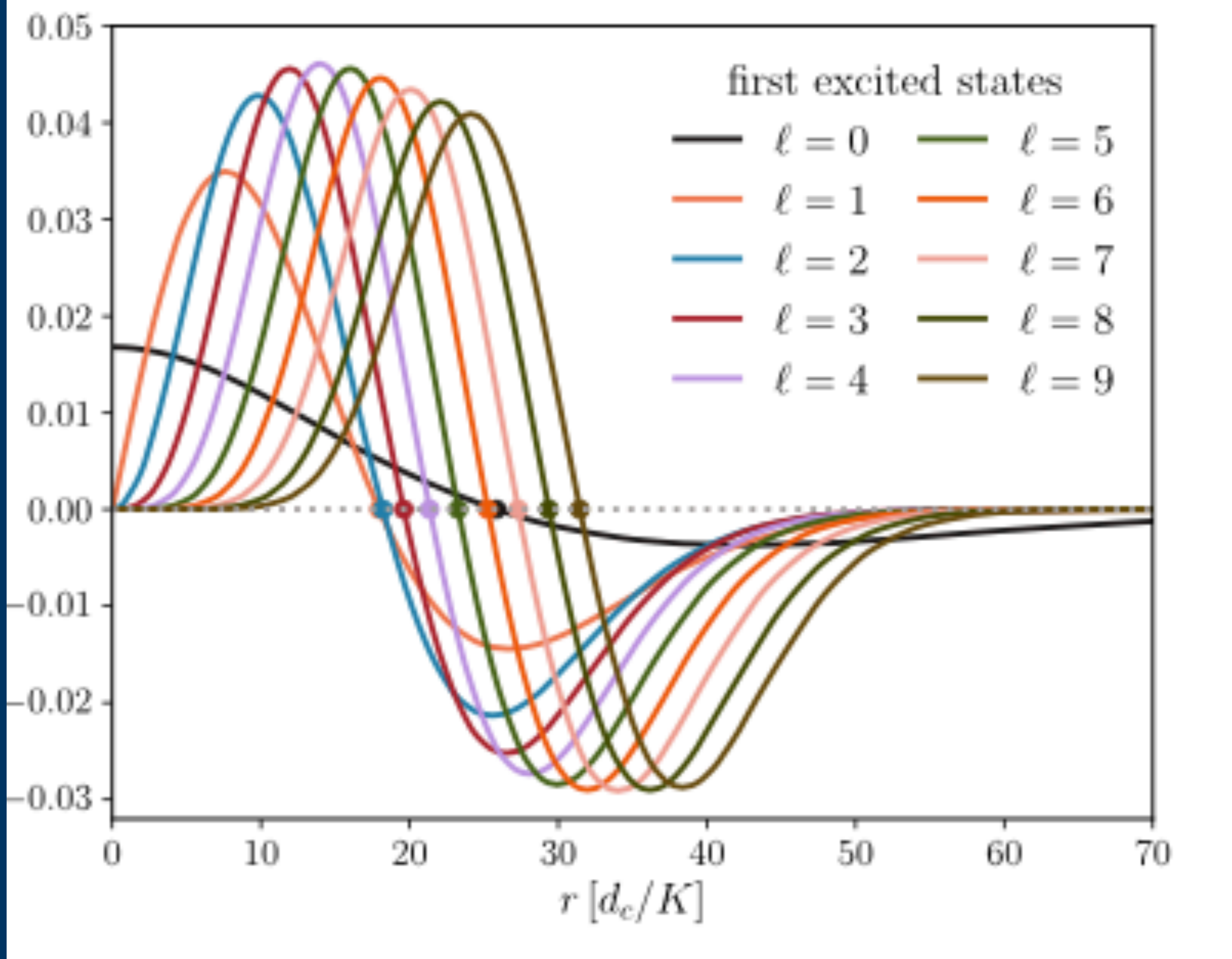
$$r = d_c \bar{r}/(K\Lambda), \qquad \mathrm{U} = 2v_c^2 (K\Lambda)^2 \bar{U},$$

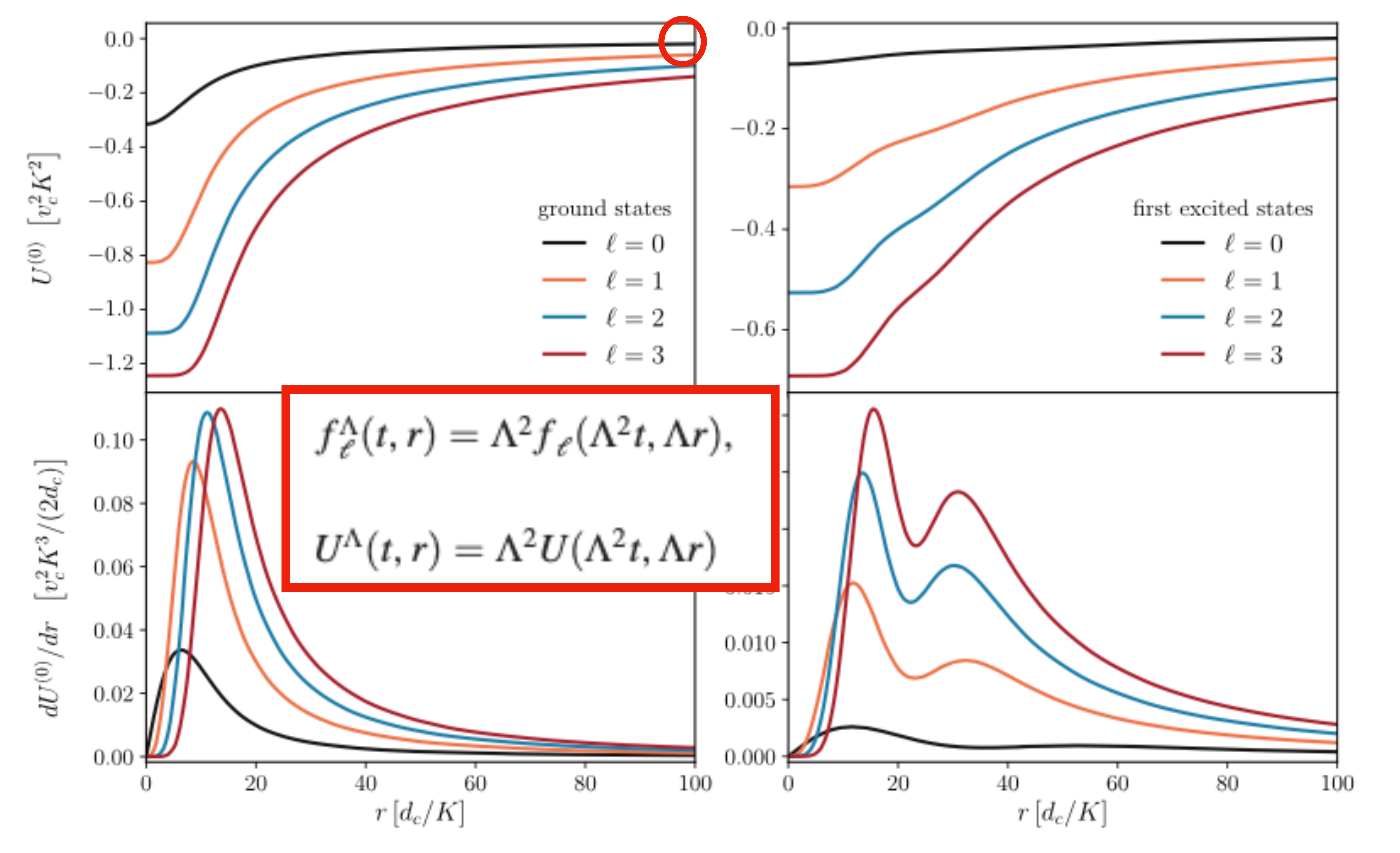
$$\mathcal{E}_\ell[f] = \int_0^\infty \left[|\partial_r f(r)|^2 + \frac{\ell(\ell+1)|f(r)|^2}{r^2}\right] r^2 dr - \frac{1}{2}\int_0^\infty \int_0^\infty \frac{|f(r)|^2 |f(\tilde{r}))|^2}{r_>} r^2 \tilde{r}^2 dr d\tilde{r}.$$

$$\mathcal{E}^{\text{phys}} = K\frac{2\ell+1}{3} E_\ell^{\text{phys}}.$$



$$\sigma_\ell^{(0)}(r) = r^\ell \sigma(r)$$





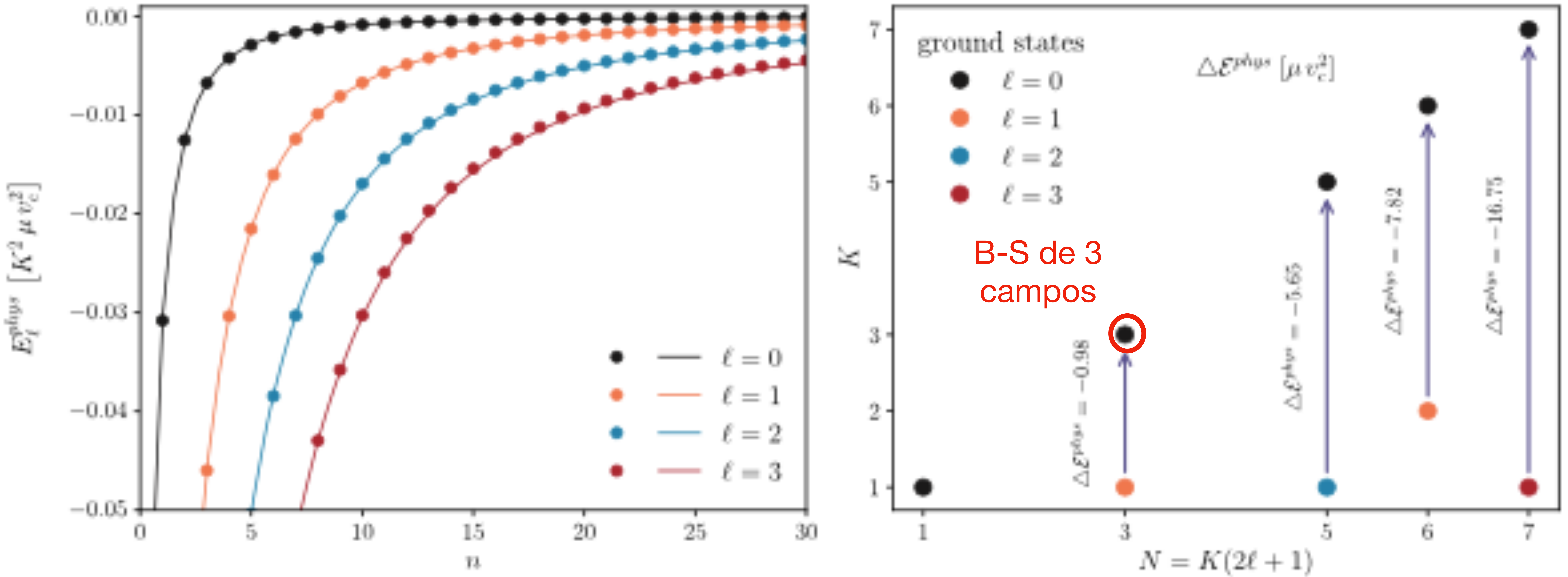
$$f_\ell^\Lambda(t, r) = \Lambda^2 f_\ell(\Lambda^2 t, \Lambda r),$$

$$U^\Lambda(t, r) = \Lambda^2 U(\Lambda^2 t, \Lambda r)$$

Algunos perfiles

$$\sigma(r=0) = \sigma_0, \quad \frac{d\sigma}{dr}(r=0) = 0,$$

$$u^{(0)}(r=0) = u_0, \quad \frac{du^{(0)}}{dr}(r=0) = 0,$$



$$\Delta\mathcal{E}^{\rm phys}_{1\to 2} = \mathcal{E}^{\rm phys}_2 - \mathcal{E}^{\rm phys}_1,$$
$$= \frac{K_1^2 N}{3}\left[\frac{(2\ell_1+1)^2}{(2\ell_2+1)^2}E_2 - E_1\right],$$

$$E_\ell^{\rm phys} = -\frac{\alpha}{(n+\beta)^\gamma}\times[K^2\mu v_c^2],$$

		ℓ-values:					
		0	1	2	3	4	5
Parameters	α	0.0978 ± 0.0001	0.8497 ± 0.0016	2.1737 ± 0.0184	3.8769 ± 0.0637	5.8308 ± 0.1441	7.9516 ± 0.2607
	β	0.7763 ± 0.0005	1.3222 ± 0.0011	1.8116 ± 0.0057	2.2639 ± 0.0128	2.6907 ± 0.0217	3.0987 ± 0.0320
	γ	2.0115 ± 0.0007	1.9935 ± 0.0011	1.9632 ± 0.0040	1.9332 ± 0.0070	1.9058 ± 0.0098	1.8812 ± 0.0123

$\mathbf{U} = -GM/r.$

Balmer

$E_\ell \sim \alpha/n^2$

~ 2 $\quad \beta_U = \ell + 1 \neq \beta_\ell$

Perturbaciones (radiales) Lineales

cantidad compleja

$$\sigma_\ell(t,r) = [A(r)+B(r)]e^{\lambda t} + [A(r)-B(r)]^* e^{\lambda^* t},$$

$$f_\ell(t,r) = e^{-iE_\ell t}[\sigma_\ell^{(0)}(r) + \epsilon\sigma_\ell(t,r) + \mathcal{O}(\epsilon^2)],$$

$$i\frac{\partial f_\ell(t,r)}{\partial t} = \hat{\mathcal{H}}_\ell f_\ell(t,r),$$

Orden cero

$$\hat{\mathcal{H}}_\ell{}^{(0)}\sigma_\ell^{(0)} = E_\ell\sigma_\ell^{(0)},$$

Orden uno

$$i\lambda A = (\hat{\mathcal{H}}_\ell{}^{(0)} - E_\ell)B,$$

$$i\lambda B = (\hat{\mathcal{H}}_\ell{}^{(0)} - E_\ell)A + 2\sigma_\ell^{(0)}\Delta_s^{-1}[\sigma_\ell^{(0)}A].$$

Propiedades

$$(-\lambda, A, -B), \qquad (\lambda^*, A^*, -B^*), \qquad (-\lambda^*, A^*, B^*).$$

$$\lambda^2|(A,B)_{L^2}|^2 \in \mathbb{R},$$

$$i\lambda(A,B)_{L^2} = \frac{1}{2}\delta^2\mathcal{E}_\ell[A_R] + \frac{1}{2}\delta^2\mathcal{E}_\ell[A_I],$$

(i) $\lambda_R = 0, \lambda_I = 0$: This is the zero mode solution we have already discussed above.

(ii) $\lambda_R > 0, \lambda_I = 0$: In this case, λ is real and from Eq. (26) we can assume that $A = A_R$ and $B = iB_I$. It follows from Eq. (39a) that $\delta^2\mathcal{E}_\ell[A_R] = -2\lambda(A_R, B_I)_{L^2}$, such that the sign of the second variation of $\mathcal{E}_\ell$ is opposite to the sign of the product $(A_R, B_I)_{L^2}$.

(iii) $\lambda_R = 0, \lambda_I > 0$: In this case, λ is purely imaginary and we can assume that both A and B are real. It follows from Eq. (39a) that the sign of $\delta^2\mathcal{E}_\ell[A_R]$ is opposite to the sign of $(A_R, B_R)_{L^2}$.

(iv) $\lambda_R > 0, \lambda_I > 0$: In this case, $\lambda^2 \neq \mathbb{R}$, and from the previous points it follows that $(A,B)_{L^2} = 0$. In this case, it follows from Eq. (33) that

$$\delta^2\mathcal{E}_\ell[A_R] + \delta^2\mathcal{E}_\ell[A_I] = 0, \tag{40}$$

such that the condition $\delta^2\mathcal{E}_\ell[A_R] \neq 0$ [which can be verified using Eq. (39a)] implies that the background solution $\sigma_\ell^{(0)}$ corresponds to a critical saddle point of the energy functional.

$$A(r) := a(r)/r,$$

$$B(r) := b(r)/r$$

$$b'' - U_{\text{eff}} b = -i\lambda a,$$

$$a'' - U_{\text{eff}} a - 2\sigma_\ell^{(0)} \left(\frac{d^2}{dr^2}\right)^{-1} [\sigma_\ell^{(0)} a] = -i\lambda b,$$

$$\left(\frac{d^2}{dr^2}\right)^{-1} = r\Delta_s^{-1} r^{-1}$$

$$U_{\text{eff}}(r) := -u^{(0)}(r) + \ell(\ell+1)/r^2$$

Discretizando y utilizando método de Chebyshev $\mathrm{D_C} := [-1, 1],$

$$\begin{pmatrix} 0 & \tilde{\mathbb{D}}_{\mathrm{N}}^2 - U_{\text{eff}} \\ \tilde{\mathbb{D}}_{\mathrm{N}}^2 - \mathrm{U}_{\text{eff}} - 2\Sigma_0 (\tilde{\mathbb{D}}_{\mathrm{N}}^2)^{-1} \Sigma_0 & 0 \end{pmatrix} \begin{pmatrix} \mathrm{a} \\ \mathrm{b} \end{pmatrix} = -i\lambda \begin{pmatrix} \mathrm{a} \\ \mathrm{b} \end{pmatrix},$$

$$x_j = \cos(j\pi/\mathrm{N}),\ j = 0, 1, \ldots, \mathrm{N},$$

$$\mathrm{U}_{\text{eff}} := \mathbf{diag}(U_{\text{eff}}(x_1), U_{\text{eff}}(x_2), \ldots, U_{\text{eff}}(x_{\mathrm{N}-1})),$$

$$\Sigma_0 := \mathbf{diag}(\sigma_\ell^{(0)}(x_1), \sigma_\ell^{(0)}(x_2), \ldots, \sigma_\ell^{(0)}(x_{\mathrm{N}-1}))$$

$$r = r_\star(x+1)/2$$

$$dr/dx = r_\star/2,$$

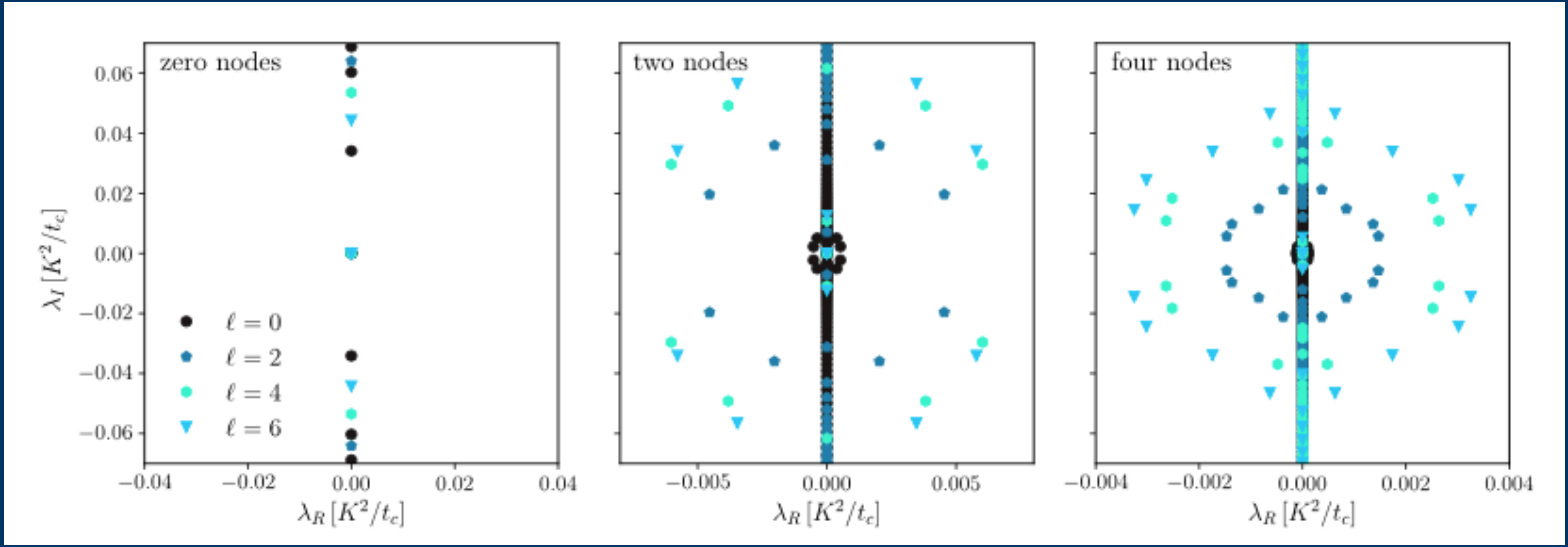
$$\mathrm{N} := 3r_\star/4$$

$$r_\star := 200(n+1)$$

$$\begin{pmatrix} \mathrm{a} \\ \mathrm{b} \end{pmatrix} := (a(x_1), \ldots, a(x_{\mathrm{N}-1}), b(x_1), \ldots, b(x_{\mathrm{N}-1}))^T$$

Inestable/Estable

$$\sigma_\ell(t, r) = 2e^{\lambda_R t} \cos(\lambda_I t)[A_R(r) + iB_I(r)] - 2e^{\lambda_R t} \sin(\lambda_I t)[A_I(r) - iB_R(r)],$$



ℓ-values	n-nodes	$\lambda[K^2/t_c]$	$\frac{1}{2}\delta^2\mathcal{E}_\ell[A_R]$	$\frac{1}{2}\delta^2\mathcal{E}_\ell[A_I]$
0	0	$0 \pm 0.03412558i$	0.00637265	0
	1	$0 \pm 0.00300045i$	0.00039739	0
		$\pm 0.00148347 + 0.00979587i$	0.00011228	−0.00011228
1	0	$0 \pm 0.06385090i$	0.00787292	0
	1	$0 \pm 0.01185117i$	0.00098164	0
		$\pm 0.00619398 + 0.03340186i$	0.00028039	−0.00028043

$$b(r=0) = 0. \quad \lim_{r\to\infty} b(r) = 0.$$

$$a(r=0) = 0, \quad \lim_{r\to\infty} a(r) = 0,$$

Algunos Números

$R_s^{\mathrm{phys}} := 2GM^{\mathrm{phys}}/c^2,$

Validez

$$\boxed{\frac{R_s^{\mathrm{phys}}}{R_{99}^{\mathrm{phys}}} = \frac{(2N)^2}{(2\ell+1)R_{99}}\left(\frac{\mu}{m_{\mathrm{pl}}}\right)^4 \ll 1,}$$

- → Consideremos la configuración: $(N,1,1)$
- → Y su modo más inestable: $\lambda_R = 0.00619398$
- → Y definamos que: $t_{\mathrm{life}} := 1/\lambda_R,$

ultralight mass $N < 10^{99}$

Halo galáctico de materia oscura

$\mu = 10^{-22}\ \mathrm{eV}/c^2 \approx 1.78\times 10^{-58}\ \mathrm{kg}$

$N \approx 10^{98}$ → $M^{\mathrm{phys}} \approx 10^{10} M_\odot$ and $R^{\mathrm{phys}} \approx 1$ Kpc.

$t_{\mathrm{life}} = 10^{13}s \approx 3.17\times 10^5$ yr,

mass $\mu = 10^{-3}\ \mathrm{eV}/c^2 \approx 1.78\times 10^{-39}\ \mathrm{kg}$ $N < 10^{61}$

Tamaño típico de un planeta enano

$N \approx 10^{55}$ → $M^{\mathrm{phys}} \approx 10^{16}$ kg and $R^{\mathrm{phys}} \approx 200$ km.

$t_{\mathrm{life}} \approx 10^4 s \approx 2.8$h.

Perturbaciones lineales no esféricas

Stability analysis of nonrelativistic ℓ-boson stars with respect to non-spherical linear perturbations

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$$i\frac{\partial\Psi(t,\vec{x})}{\partial t} = \left[-\Delta + U(t,\vec{x})\right]\Psi(t,\vec{x}),$$
$$\Delta U(t,\vec{x}) = |\Psi(t,\vec{x})|^2,$$

$$\Psi(t,\vec{x}) = e^{-iEt}[\chi_0(\vec{x}) + \epsilon\chi(t,\vec{x}) + \mathcal{O}(\epsilon^2)],$$

$$\chi(t,\vec{x}) = e^{\lambda t}\left[\mathcal{A}(\vec{x}) + \mathcal{B}(\vec{x})\right] + e^{\lambda^* t}\overline{\left[\mathcal{A}(\vec{x}) - \mathcal{B}(\vec{x})\right]},$$

$$\chi_0(\vec{x}) = \sigma_\ell^{(0)}(r)\mathcal{Y}_\ell(\vartheta,\varphi),$$

$$\mathcal{Y}_\ell := \sqrt{\frac{4\pi}{2\ell+1}}\left(Y^{\ell,-\ell}, Y^{\ell,-\ell+1}, \ldots, Y^{\ell,\ell}\right)^T,$$

$$i\lambda\mathcal{A} = \left(\hat{\mathcal{H}}_0 - E\right)\mathcal{B} \qquad (25a)$$
$$+ \; i\left\{\Delta^{-1}[\chi_0^*(\mathcal{A}+\mathcal{B}) + \chi_0^T(\mathcal{A}-\mathcal{B})]\right\}\mathrm{Im}\{\chi_0\},$$
$$i\lambda\mathcal{B} = \left(\hat{\mathcal{H}}_0 - E\right)\mathcal{A} \qquad (25b)$$
$$+ \; \left\{\Delta^{-1}[\chi_0^*(\mathcal{A}+\mathcal{B}) + \chi_0^T(\mathcal{A}-\mathcal{B})]\right\}\mathrm{Re}\{\chi_0\}.$$

$$\mathcal{A} = \sum_{JLM} A_{JM}{}^L(r) Y^{JM}{}_{L\ell},$$

$$Y^{JM}{}_{L\ell}(\vartheta,\varphi) := \sum_{m,\sigma} C^{JM}{}_{Lm\ell\sigma} Y^{Lm}(\vartheta,\varphi)\chi^{\ell\sigma},$$

$$i\lambda A_{JM}{}^L = \left(\hat{\mathcal{H}}_L^{(0)} - E\right) B_{JM}{}^L,$$
$$i\lambda B_{JM}{}^L = \left(\hat{\mathcal{H}}_L^{(0)} - E\right) A_{JM}{}^L + 2Q_{JM}{}^L,$$

$$L = |J-\ell|, \ldots, J+\ell$$
$$M = -J, \ldots, J$$
$$J = 0, 1, \ldots$$

Propiedades

- Simetría en las soluciones
- Modos estacionario

$$(A_{JM}{}^{L}, B_{JM}{}^{L}) = \gamma_{JM}(0, S_{JM}{}^{L}),$$

$$\chi(t,\vec{x}) = \sigma_\ell^{(0)}(r) \sum_{J=0}^{2\ell} \sum_{M=-J}^{J} \left[\gamma_{JM} Y^{JM}{}_{\ell\ell}(\vartheta,\varphi) - c.c.\right],$$

Posibles soluciones estacionarias fuera de simetría esférica

- Desaparecen los modos inestables cuando J es lo suficientemente grande.

Sistema Discreto

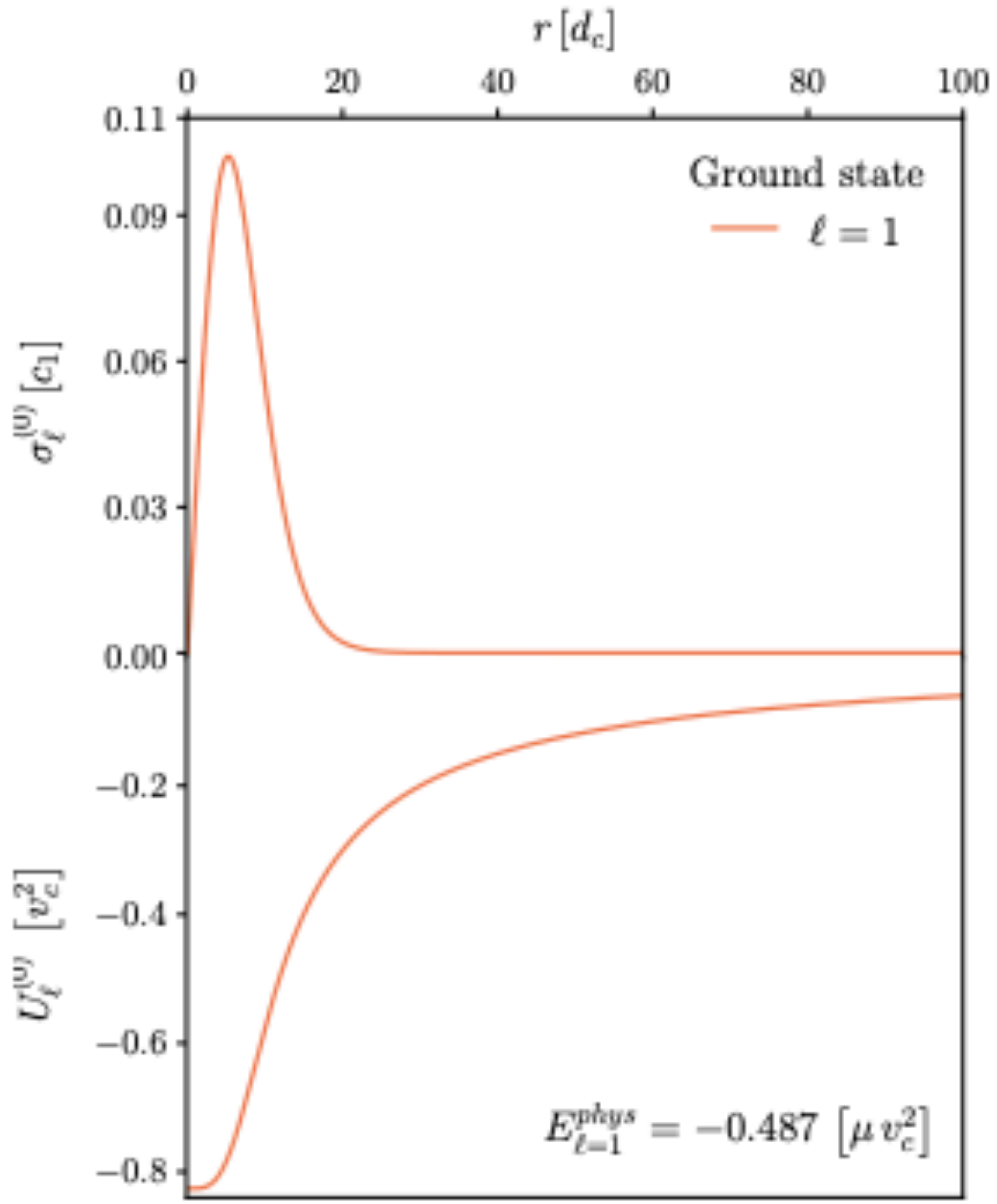
$$\begin{pmatrix} 0 & \mathbb{D}^2 - \mathbb{U}_{\ell J} \\ \mathbb{D}^2 - \mathbb{U}_{\ell J} - 2\Sigma_\ell \mathbb{Z}_{\ell J}(\mathbb{D}^2 - \mathbb{V}_J)^{-1}\Sigma_\ell & 0 \end{pmatrix} \begin{pmatrix} \mathsf{a}_{JM} \\ \mathsf{b}_{JM} \end{pmatrix} = -i\lambda \begin{pmatrix} \mathsf{a}_{JM} \\ \mathsf{b}_{JM} \end{pmatrix},$$

$2c_1(N-1) \times 2c_1(N-1)$

$c_1 = J + 1$ if $J < \ell$

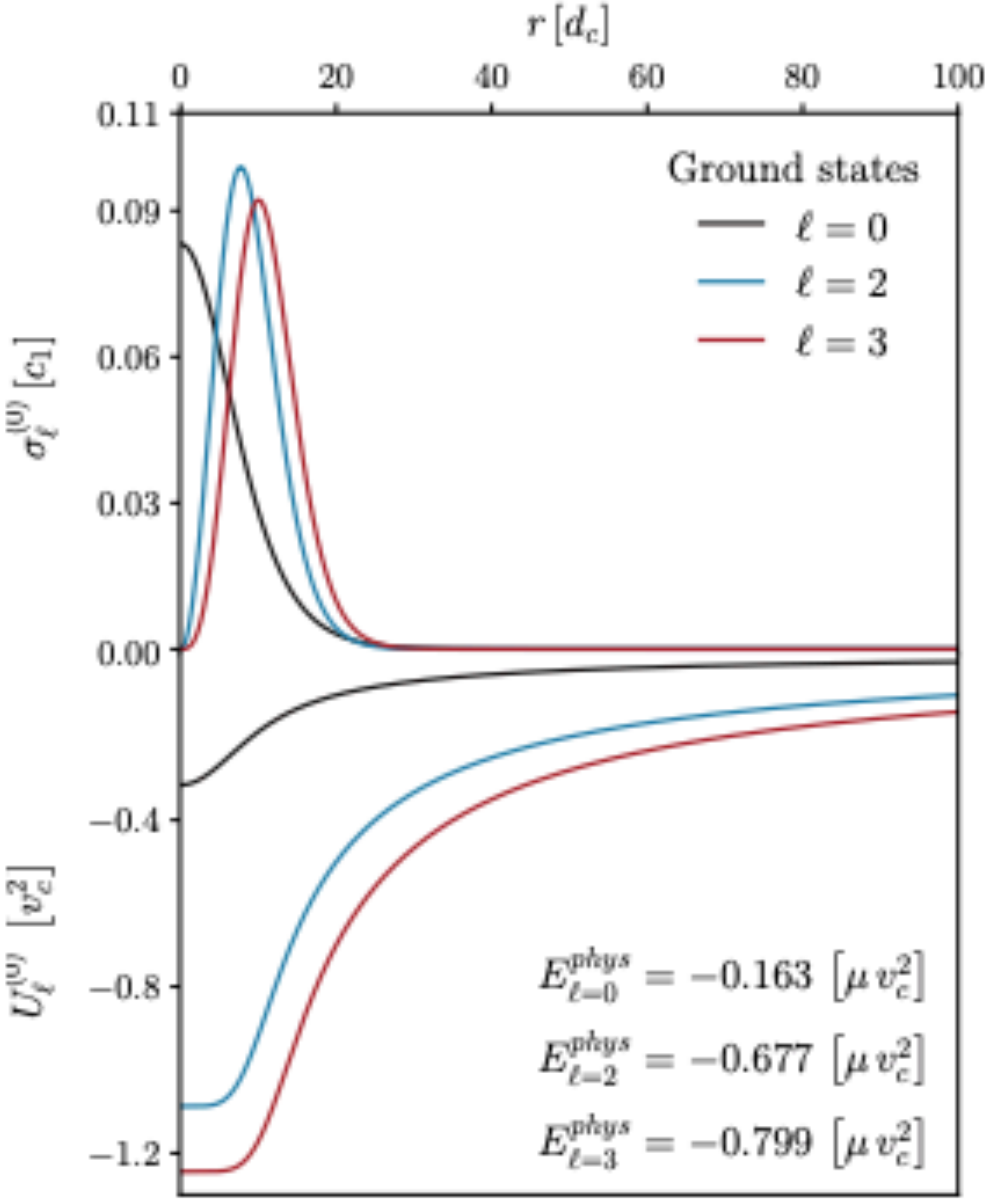
$c_1 = \ell + 1$ if $J \geq \ell$

Sistema $n = 0$, $\ell = 1$



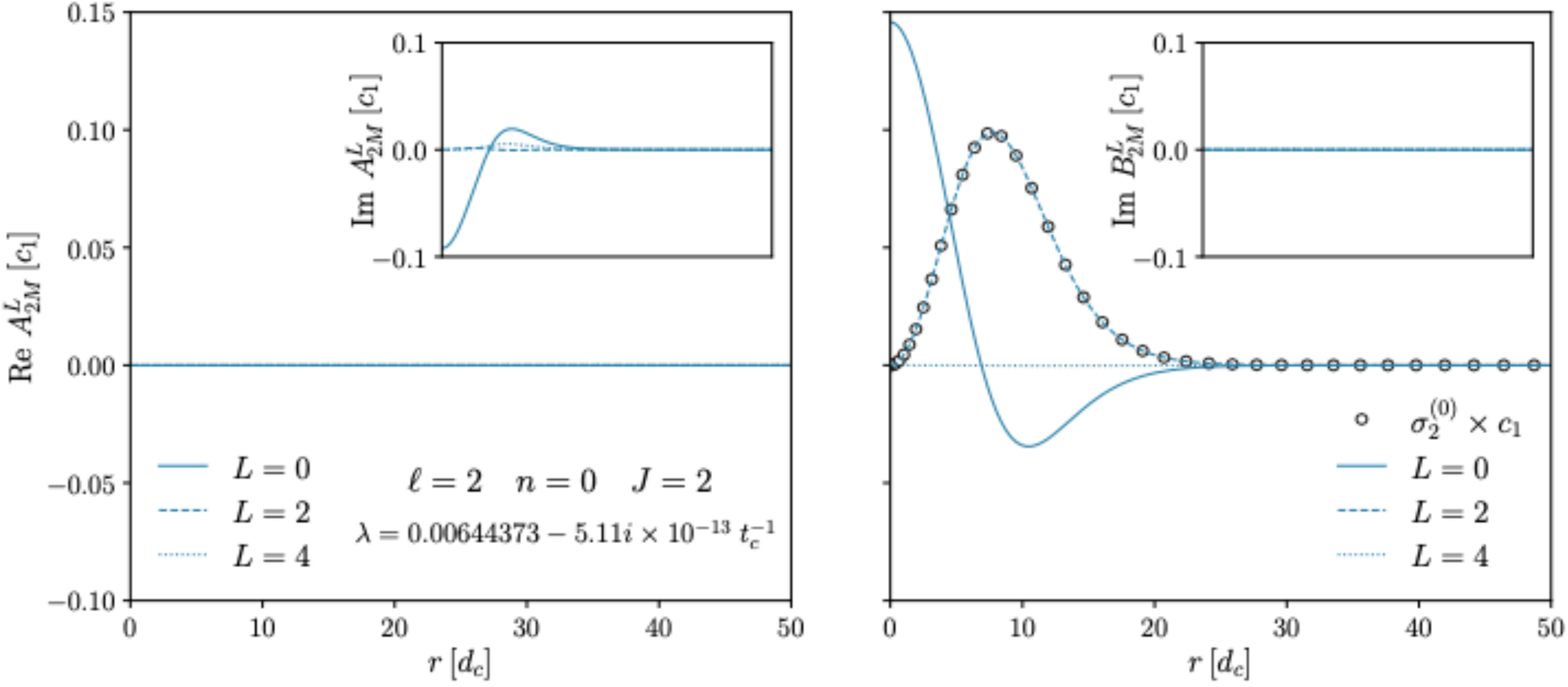
	$\lambda\,[1/t_c]$ Components			$\lambda\,[1/t_c]$ Components	
J	Real	Imaginary	J	Real	Imaginary
0	0	1.05201×10^{-5}	4	0	0.11368481
	0	0.06385090		0	0.16030883
	0	0.14328038		0	0.18097544
1	0	0.00086601	5	0	0.15468802
	0	0.05499307		0	0.18203923
	0	0.08194250		0	0.19756177
2	0	2.11418×10^{-6}	6	0	0.18105790
	0	0.09613376		0	0.19765391
	0	0.10590972		0	0.20832581
3	0	0.05557199	7	0	0.19756669
	0	0.13196275		0	0.20833027
	0	0.15373565		0	0.21570444

Sistema $n = 0,\ \ell \neq 1$

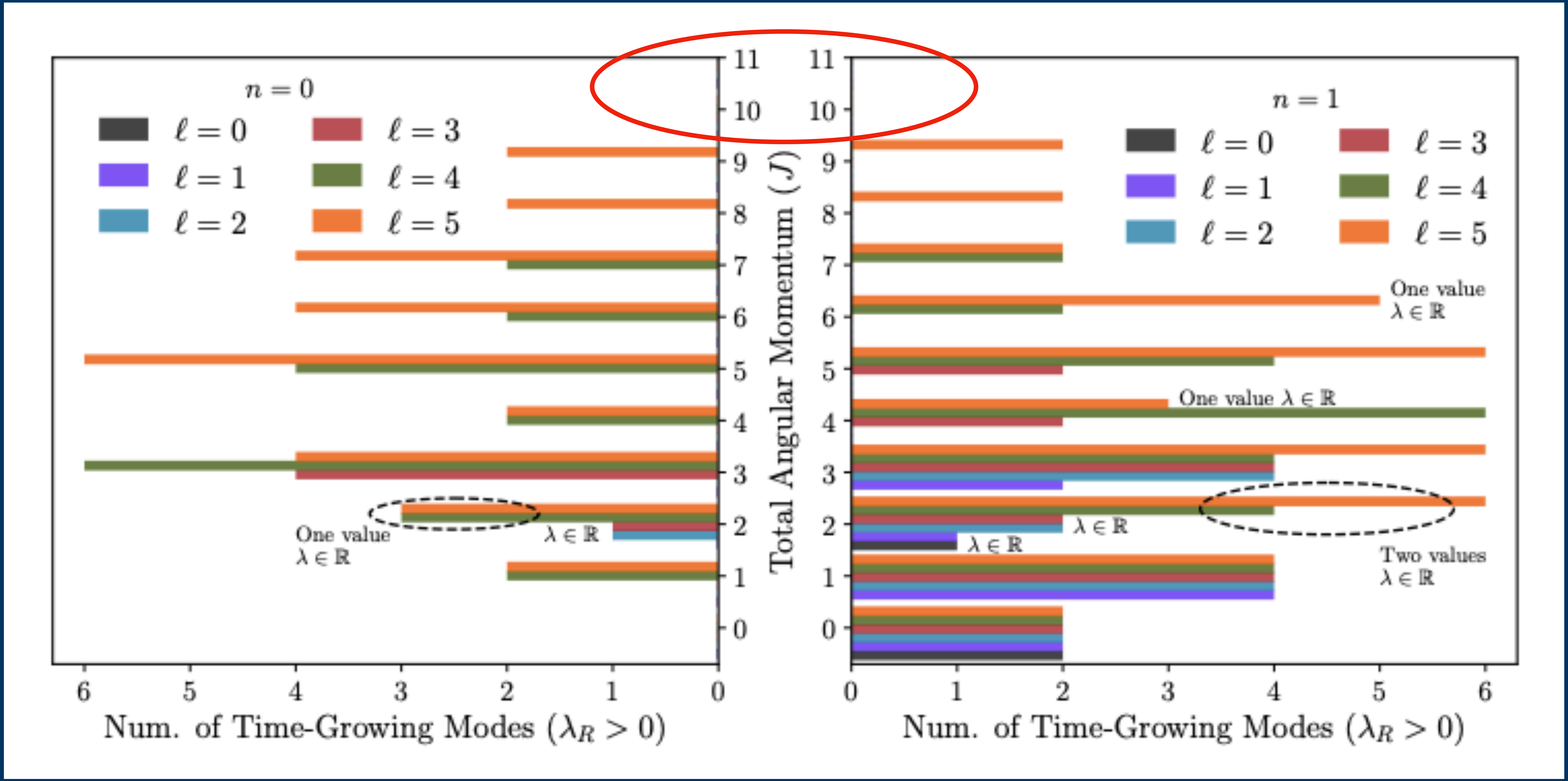


	$\ell = 0,\ \lambda\,[1/t_c]$ Components		$\ell = 2,\ \lambda\,[1/t_c]$ Components		$\ell = 3,\ \lambda\,[1/t_c]$ Components	
J	Real	Imaginary	Real	Imaginary	Real	Imaginary
0	0	4.321×10^{-6}	0	8.766×10^{-6}	0	1.637×10^{-5}
	0	0.03412558	0	0.06408695	0	0.05903587
	0	0.06030198	0	0.16575905	0	0.16827468
1	0	0.00049999	0	0.00111802	0	0.00132286
	0	[illegible]	[illegible]	[illegible]	[illegible]	0.05925506
	0	[illegible]	[illegible]	[illegible]	[illegible]	0.06609698
2	0	0.05285868	4.32×10^{-7}	2.63×10^{-15}	7.99×10^{-7}	1.37×10^{-14}
	0	0.06564270	0.00644373	-5.11×10^{-13}	0.00907756	2.1×10^{-12}
	0	0.07136165	0	0.10237935	0	0.09954769
3	0	0.06571493	0	0.05550210	0.00363560	0.05313153
	0	0.07135829	0	0.06147704	0	0.05910000
	0	0.07442765	0	0.07000546	0.00085739	0.05953874

$$\boxed{\chi(t,\vec{x}) = e^{\lambda t}\left[\mathcal{A}_R(\vec{x}) + i\mathcal{B}_I(\vec{x})\right],}$$



Re A_{2M}^{L} [c_1]
Im A_{2M}^{L} [c_1]
Im B_{2M}^{L} [c_1]
r [d_c]
r [d_c]
$L = 0$
$L = 2$
$L = 4$
$\ell = 2 \quad n = 0 \quad J = 2$
$\lambda = 0.00644373 - 5.11i \times 10^{-13}\ t_c^{-1}$
$\sigma_2^{(0)} \times c_1$
$L = 0$
$L = 2$
$L = 4$



Falta realizar el estudio de los tiempos de vida

Conclusiones

Pareciera que las únicas configuraciones estables son $n = 0$, $\ell = 0{,}1$.

¡Gracias!