EVOLVING EXCITED STATES

We would like to evolve initial data of the form

$$\psi(t=0,\vec{x}) = \sigma^{(0)}(r), \quad \dot{\psi}(t=0,\vec{x}) = -iE\sigma^{(0)}(r), \tag{1}$$

which corresponds to the stationary and spherically symmetric configurations $\psi(t,\vec{x}) = e^{-iEt}\sigma^{(0)}(r)$ that we discuss in our paper [1]. Representative cases of ground ($n=0$) and first excited ($n=1$) states with central amplitude $\bar{\sigma}^{(0)}(\bar{r}=0) = \bar{\sigma}_0 = 1.0, 1.1, 1.2, \ldots, 2.5$ can be found in this repository, with bars denoting our code variables.

We relate physical and code variables through (see Eqs. (8) in Ref. [1])

$$t = \frac{|\lambda_{\text{ours}}|}{4\pi G m_0^3}\bar{t}, \quad x = \left(\frac{|\lambda_{\text{ours}}|}{8\pi G m_0^4}\right)^{1/2}\bar{x}, \quad \mathcal{U} = \frac{4\pi G m_0^2}{|\lambda_{\text{ours}}|}\bar{\mathcal{U}}, \quad \psi = \left(\frac{8\pi G m_0^5}{\lambda_{\text{ours}}^2}\right)^{1/2}\bar{\psi}, \quad E = \frac{4\pi G m_0^3}{|\lambda_{\text{ours}}|}\bar{E}, \tag{2}$$

where we are using natural units with $c = \hbar = 1$ (note that $[\sigma^{(0)}] = [\psi]$ and $[r] = [x]$). At this point it is important to stress that there is a factor $-2/3$ of difference between our definition of the coupling constant λ and that of Ref. [2]. If we compare Eq. (2.4) in [2] with Eq. (5.a) in [1] we obtain[1]

$$\lambda_{\text{yours}} = -\frac{2}{3}\lambda_{\text{ours}}. \tag{4}$$

This implicates that repulsive selfinteractions are characterized by negative (positive) values of λ_{yours} (λ_{ours}). If we now define the characteristic energy scale

$$\mathcal{E} \equiv \frac{4\pi G m_0^3}{|\lambda_{\text{ours}}|} = \frac{8\pi G m_0^3}{3|\lambda_{\text{yours}}|}, \tag{5}$$

the relation between the code and physical variables of Eq. (2) can be also expressed in the form

$$t = \frac{1}{\mathcal{E}}\bar{t}, \quad x = \left(\frac{1}{2m_0\mathcal{E}}\right)^{1/2}\bar{x}, \quad \mathcal{U} = \frac{\mathcal{E}}{m_0}\bar{\mathcal{U}}, \quad \psi = \left(\frac{\mathcal{E}^2}{2\pi G m_0}\right)^{1/2}\bar{\psi}, \quad E = \mathcal{E}\bar{E}. \tag{6}$$

[1] Here we have assumed that $\mathbf{\Psi} = (\psi, 0, 0)$ in order to have just one scalar field ψ, so that $(\mathbf{\Psi}\cdot\mathbf{\Psi})\mathbf{\Psi}^\dagger + (\mathbf{\Psi}^\dagger\cdot\mathbf{\Psi})\mathbf{\Psi} = 3|\psi|^2\psi$ in Eq. (2.4). Note that if we compare instead Eq. (1) in [3] with Eq. (5.a) in [1] the relation is

$$\lambda_{\text{yours}} = -\frac{1}{2}\lambda_{\text{ours}}. \tag{3}$$

We believe these factors can be absorbed in the scale $\mathcal{E}$ and are not very relevant at this point, apart form the relative sign.

Comparing this with Eq. (2.11) in [2] we obtain that our code variables can be related with those of [2] through:

$$\tilde{t} = \bar{t}, \quad \tilde{x} = \frac{1}{\sqrt{2}}\bar{x}, \quad \tilde{\mathcal{U}} = \bar{\mathcal{U}}, \quad \tilde{\psi} = 2\bar{\psi}, \quad \tilde{E} = \bar{E}, \tag{7}$$

with $\tilde{\lambda} = -\frac{1}{12}\lambda$, $\lambda = 0, \pm 1$, in Eq. (2.12).

To make things simpler in this repository we provide you with all the information associated to the configurations $\bar{\sigma}_0 = 1.0, 1.1, 1.2, \ldots, 2.5$ expressed in our code variables [1], i.e. $\bar{x} = x_{\text{ours}}$, and also in yours [2], i.e. $\tilde{x} = x_{\text{yours}}$. In addition, we also provide you with a simple code capable of generating any configuration $\bar{\sigma}_0$, utilizing both types of code variables. If everything works well, all ground state configurations should remain stable under dynamical evolutions. Conversely, for the first excited states, only those configurations with $1.55 \lesssim \bar{\sigma}_0 \lesssim 2.07$ must remain stable (the numerical error should produce the other configurations to collapse or disperse).

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[1] E. C. Nambo, A. Diez-Tejedor, A. A. Roque, and O. Sarbach (2024), 2402.07998.

[2] M. Jain and M. A. Amin, JCAP **04**, 053 (2023), 2211.08433.

[3] M. Jain, W. Wanichwecharungruang, and J. Thomas, Phys. Rev. D **109**, 016002 (2024), 2310.00058.