

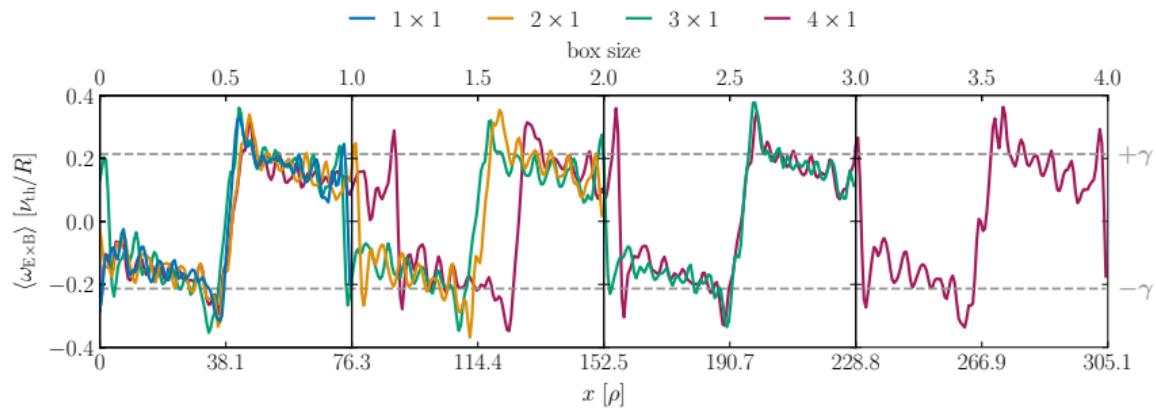


SIZE CONVERGENCE OF THE $E \times B$ STAIRCASE PATTERN IN FLUX TUBE SIMULATIONS OF ION TEMPERATURE GRADIENT DRIVEN TURBULENCE

June 21, 2023

Manuel Lippert

Theoretical Physics V



MOTIVATION

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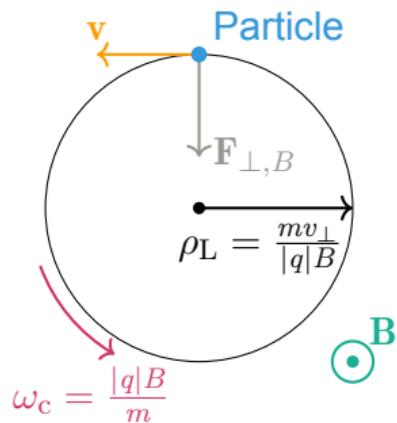
Does the basic pattern size always converges to the box size, or is there a typical mesoscale size inherent to staircase structures also in a local flux-tube description?

CHARGED PARTICLE MOTION

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Lorentz force

$$F_{\perp,B} = |q|v_{\perp}B$$



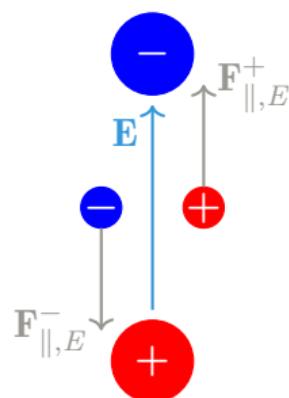
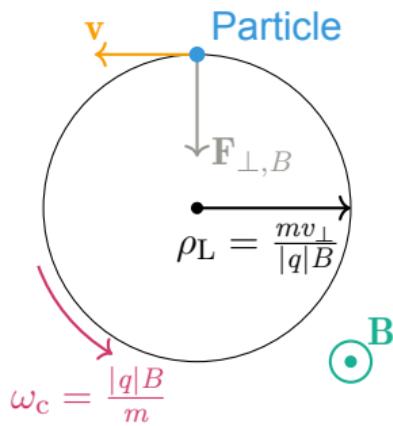
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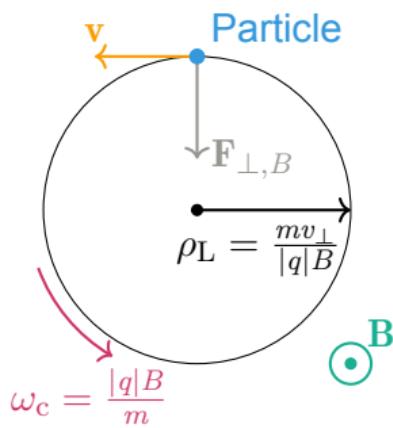
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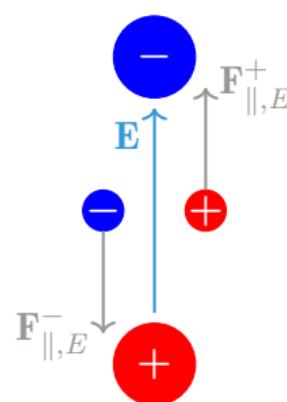
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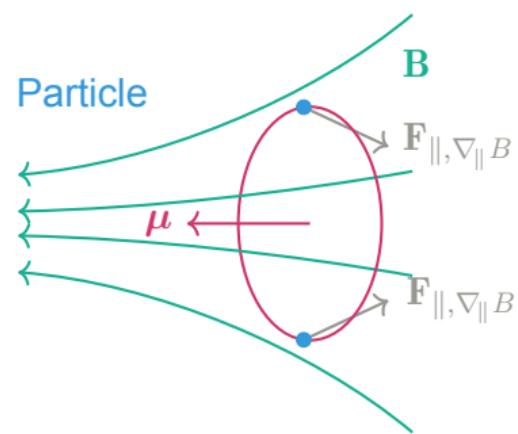
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Inhomogeneous magnetic field

$$F_{\parallel,\nabla_{\parallel}B} = - \underbrace{\frac{mv_{\perp}^2}{2B}}_{\mu} \nabla_{\parallel}B$$

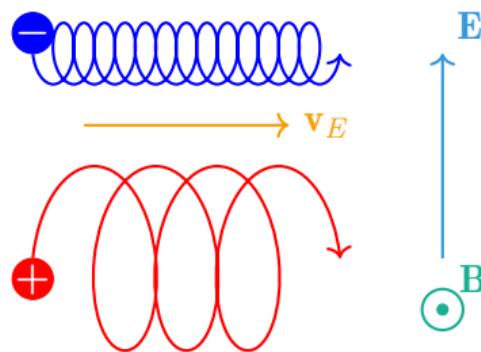


DRIFT IN THE GYROCENTER

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$E \times B$ Drift

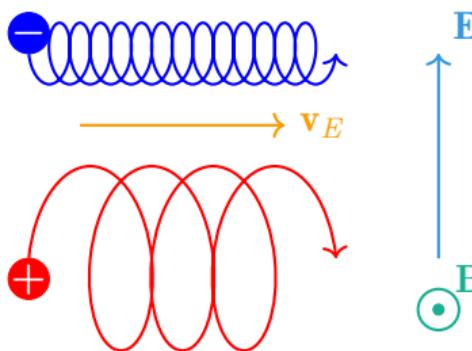
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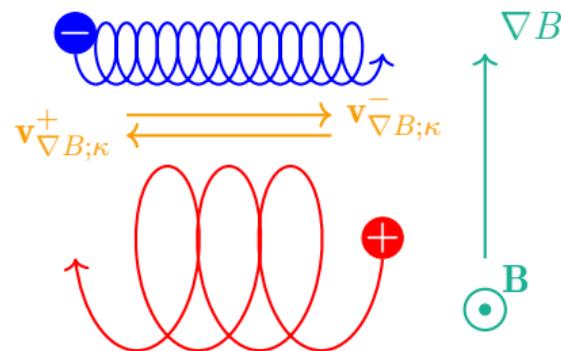


∇B Drift

$$\mathbf{v}_{\nabla B} = \frac{mv_{\perp}^2}{2q} \frac{\mathbf{B} \times \nabla B}{B^3}$$

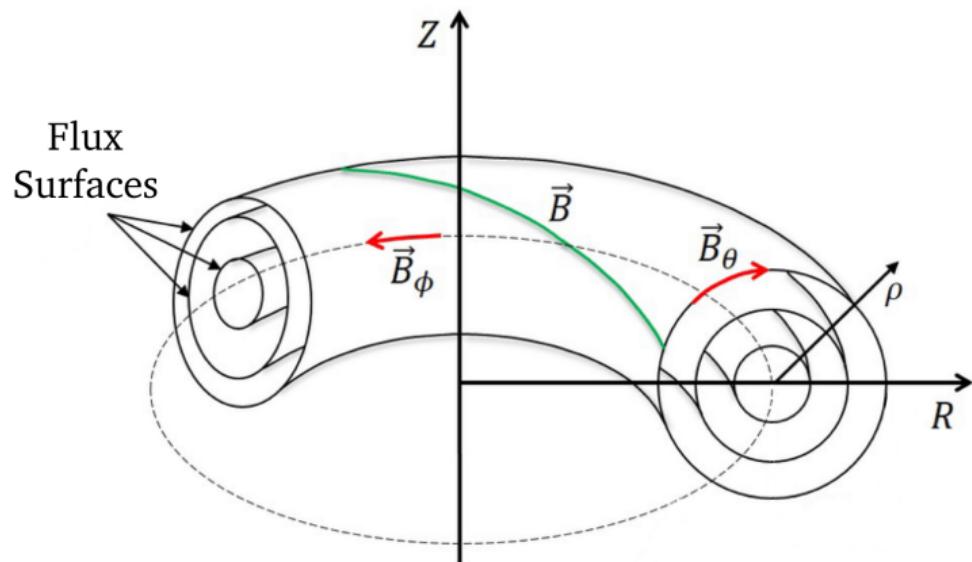
Curvature Drift

$$\mathbf{v}_{\kappa} = \frac{mv_{\parallel}^2}{qB^2} \mathbf{B} \times \underbrace{\kappa}_{\frac{\nabla B}{B}} = \frac{mv_{\parallel}^2}{q} \frac{\mathbf{B} \times \nabla B}{B^3}$$



MAGNETIC CONFINEMENT IN TOKAMAK

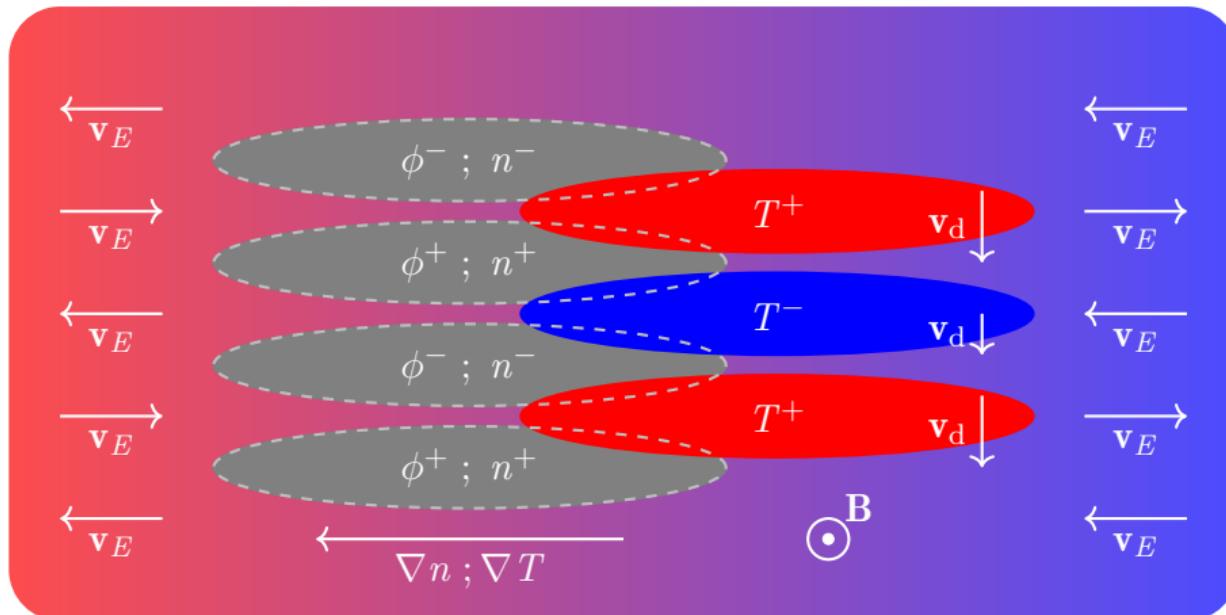
MAGNETIC CONFINEMENT IN TOKAMAK



$$\beta = \frac{nT}{\mu_0 B^2/2}$$

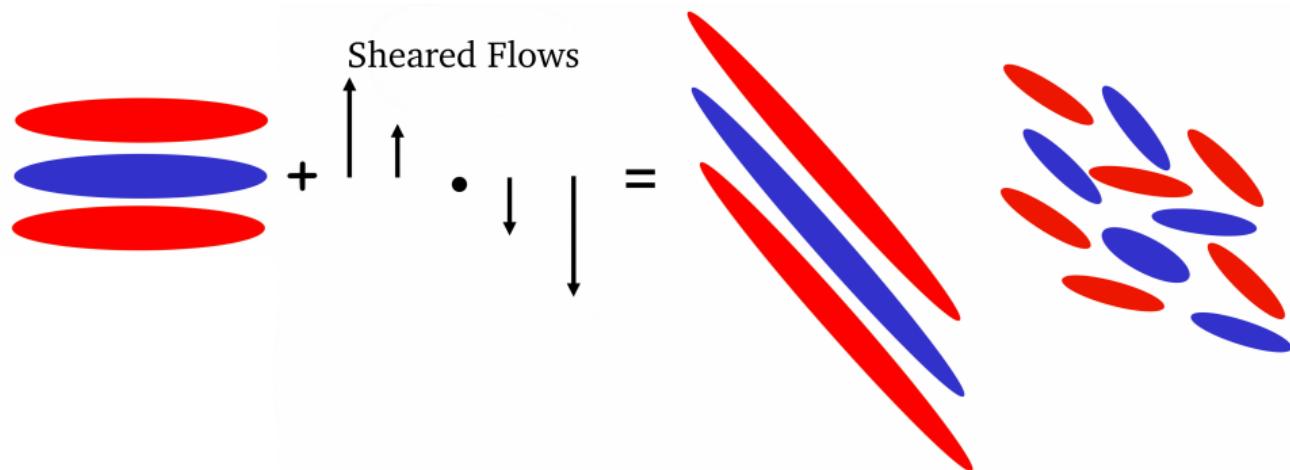
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$$\omega_{E \times B} = \frac{1}{2} \frac{\partial^2 \langle \phi \rangle}{\partial x^2} \quad \langle \phi \rangle = \frac{1}{L_y} \int_0^{L_y} dy \phi(x, y, s=0)$$

GYROKINETICS

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$$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial \mathbf{x}} \cdot \frac{d\mathbf{x}}{dt} + \frac{\partial f}{\partial \mathbf{v}} \cdot \frac{d\mathbf{v}}{dt} = 0 \quad f(\mathbf{x}, \mathbf{v}, t)$$

Vlasov Equation

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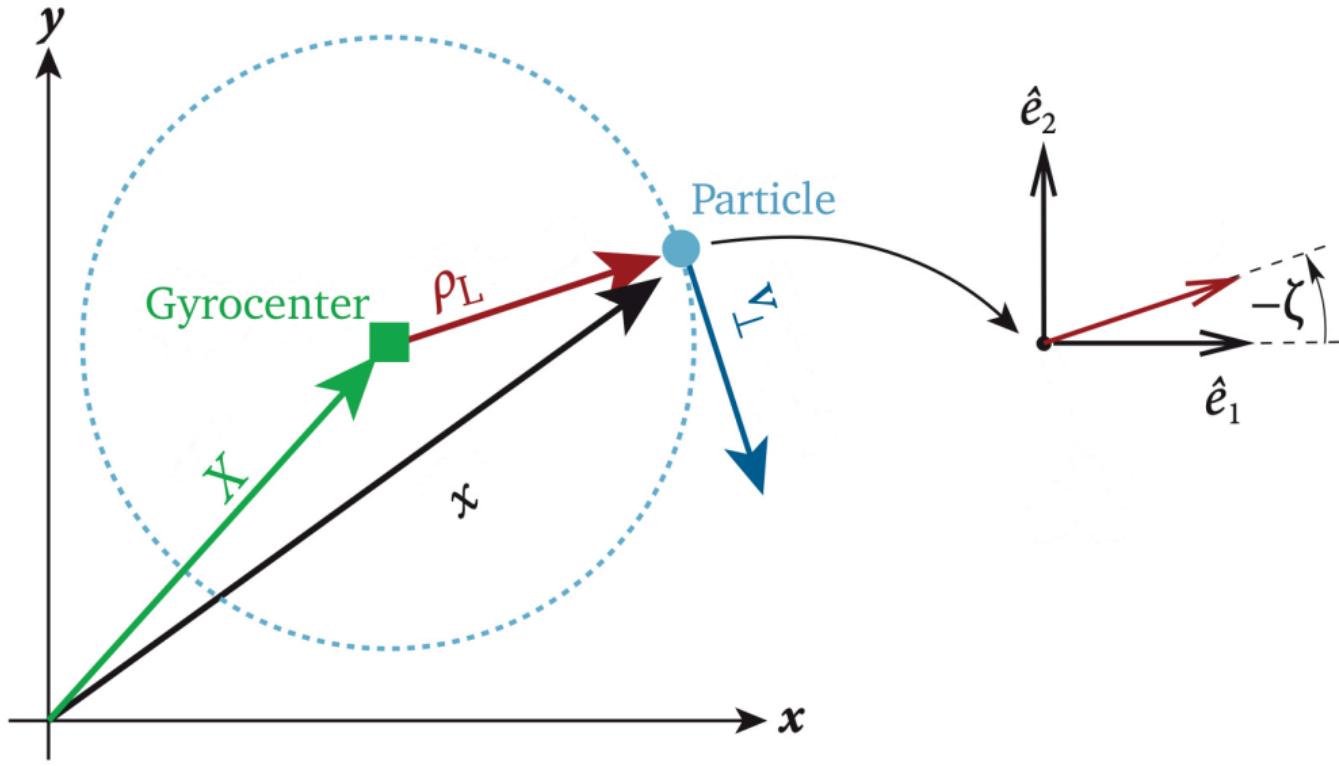
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Gyrokinetic Ordering

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$$\Downarrow \quad f = f_0 + \delta f \quad \delta f \text{ Approx & Local Limit}$$

$$\frac{\partial g}{\partial t} + \mathbf{v}_{\chi} \cdot \nabla g + (v_{\parallel} \mathbf{b} + \mathbf{v}_D) \cdot \nabla (\delta f) - \frac{\mu B}{m} \frac{\mathbf{B} \cdot \nabla B}{B^2} \frac{\partial (\delta f)}{\partial v_{\parallel}} = S \quad \text{Gyrokinetic Equation}$$

$$g \sim (f_0 + \delta f) \quad S \sim (f + f_0) \quad \mathbf{v}_D \sim \nabla B \quad \mathbf{v}_{\chi} \sim \mathbf{E} \times \mathbf{B} \quad f(x, y, s, v_{\parallel}, \mu)$$

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S6	21	83	16	64	9	1	$ \nu_{\parallel} $	0.2	0.1	0.1	6	1.4	2.1

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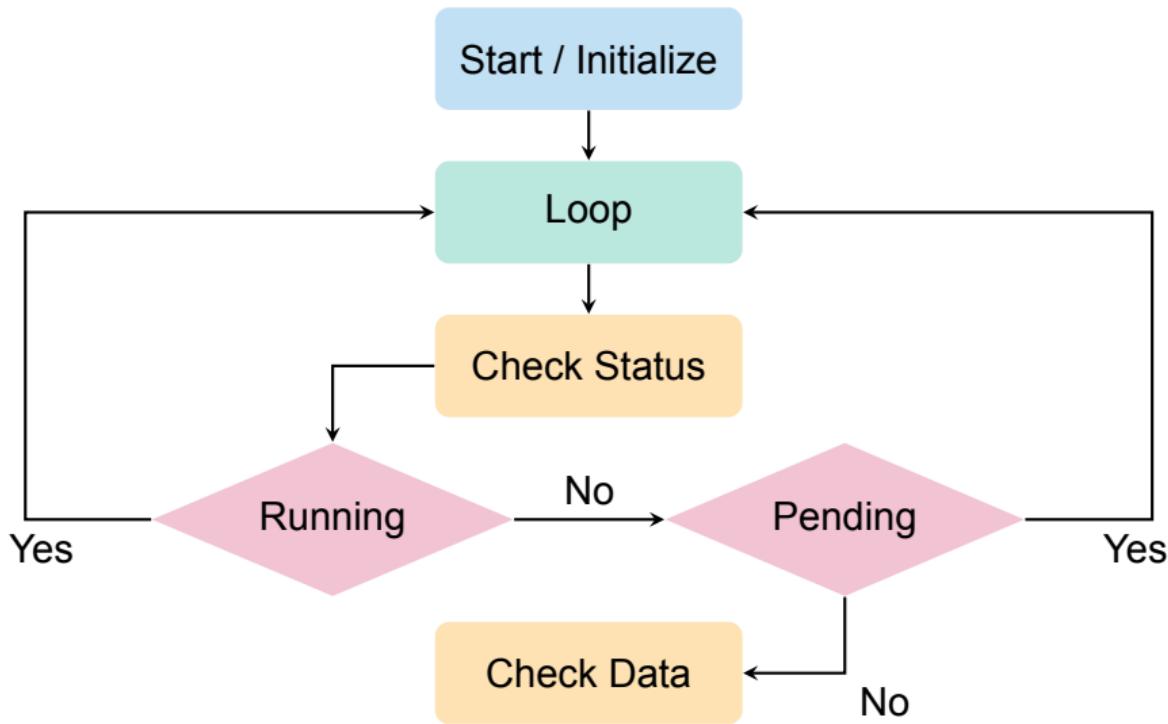
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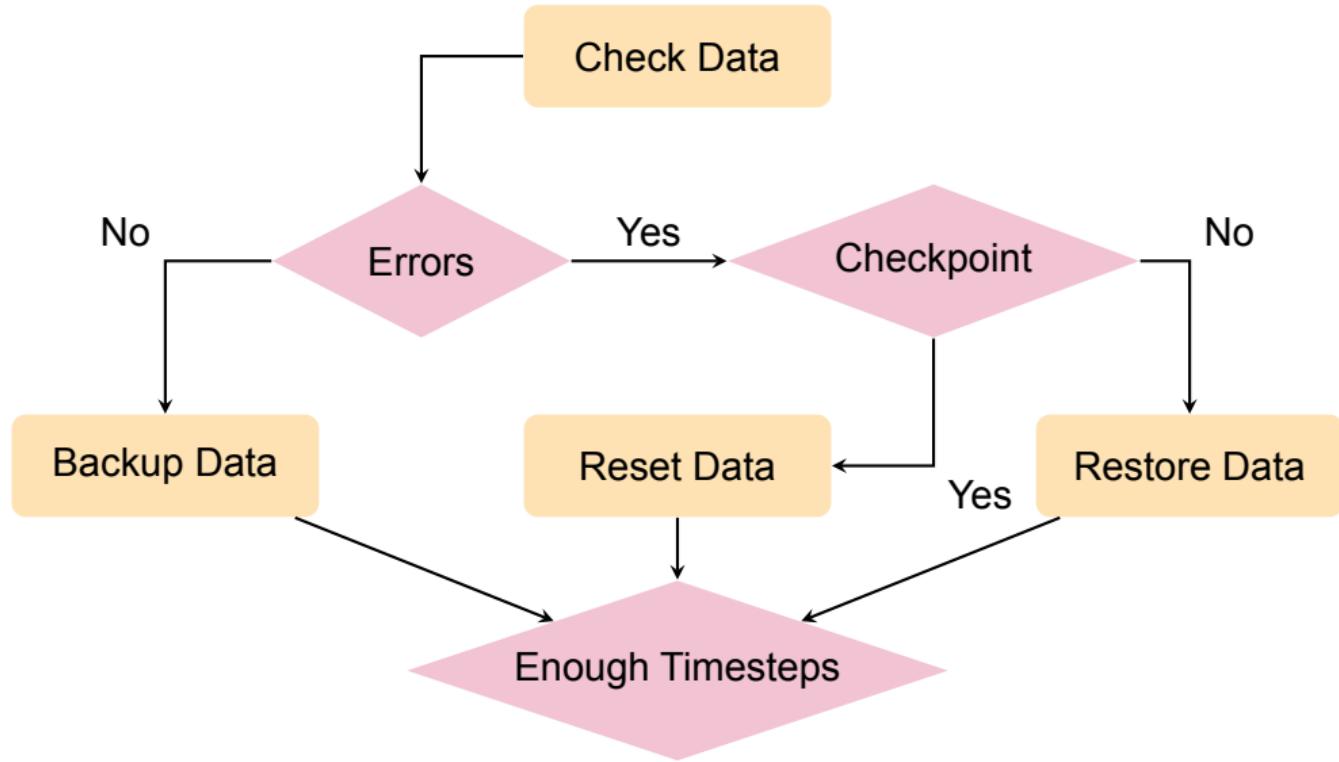
- Zonal flow mode that dominates the $E \times B$ staircase pattern are called **basic mode**
- The basic mode exhibits the maximum amplitude in the spectrum $|\hat{\omega}_{E \times B}|_{n_{ZF}}$

RESTART SCRIPT

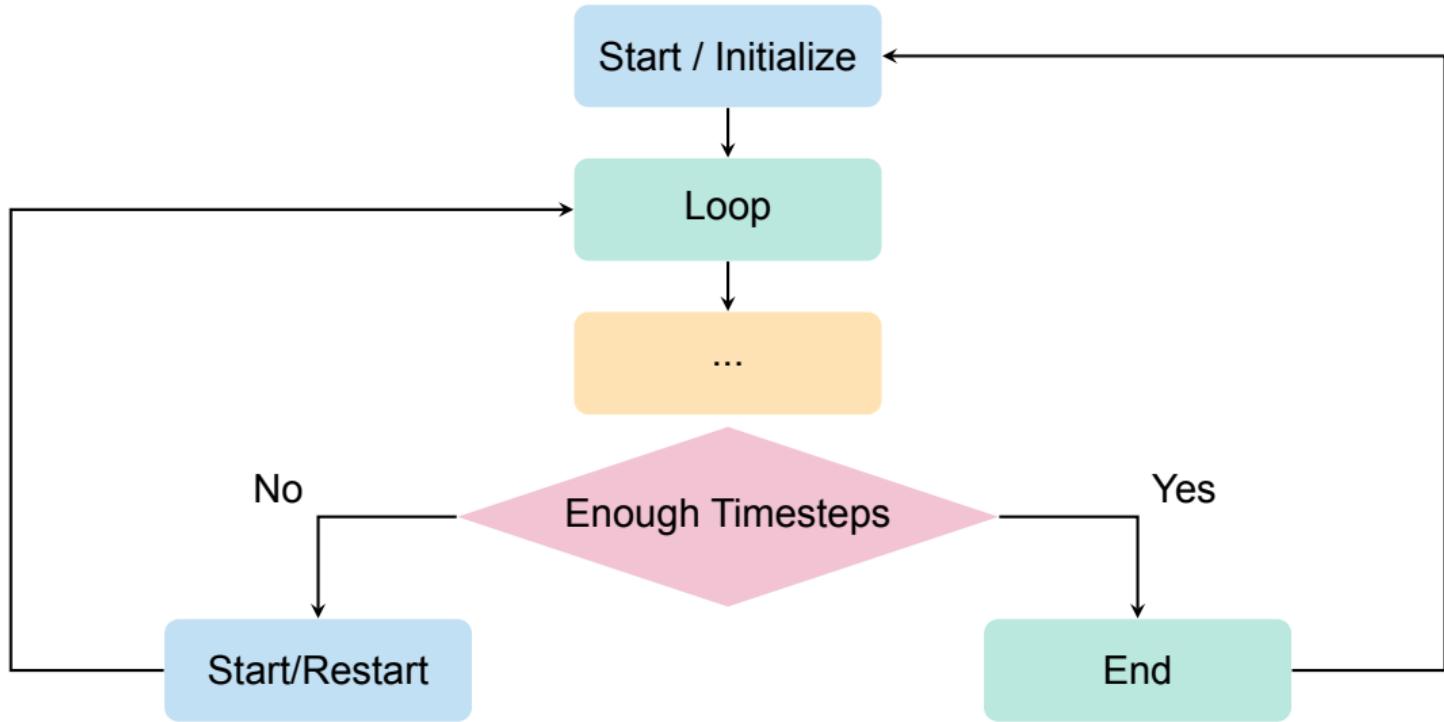
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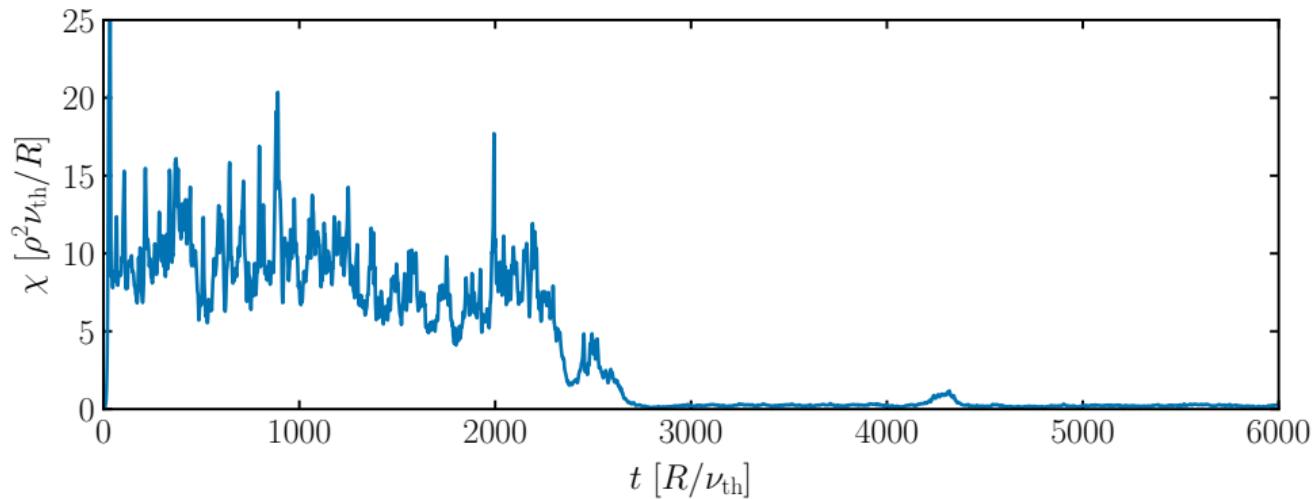
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Verification:

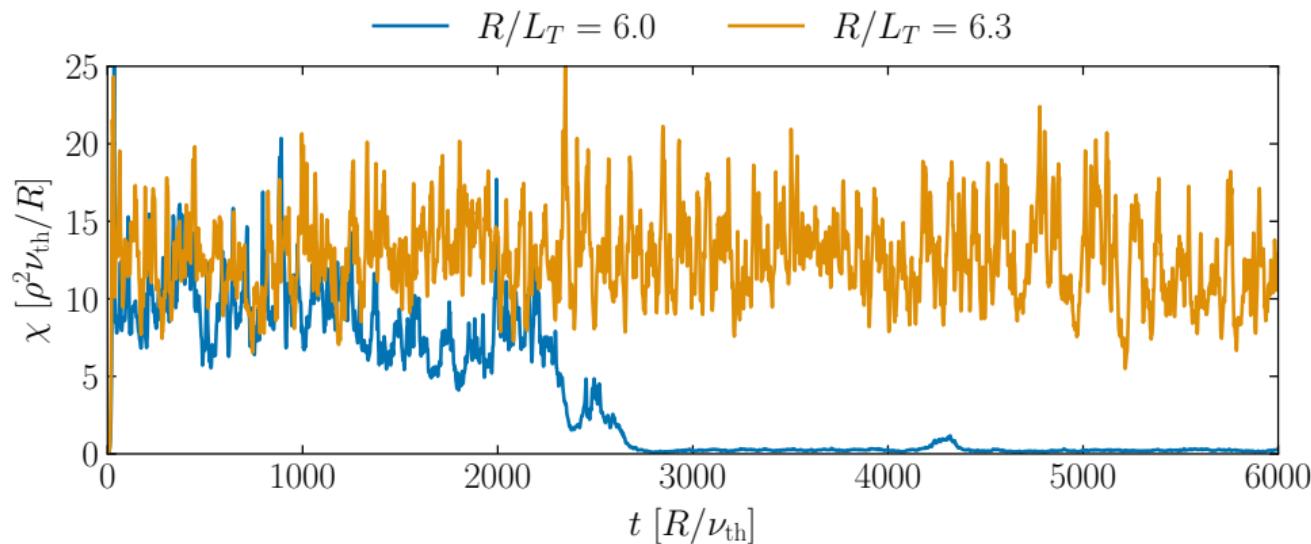
1. Reduce only one number of grid points and look if criterias (1), (2) are satisfied
2. Reduce to known the minimum number of grid points to verify result in general.

BENCHMARK

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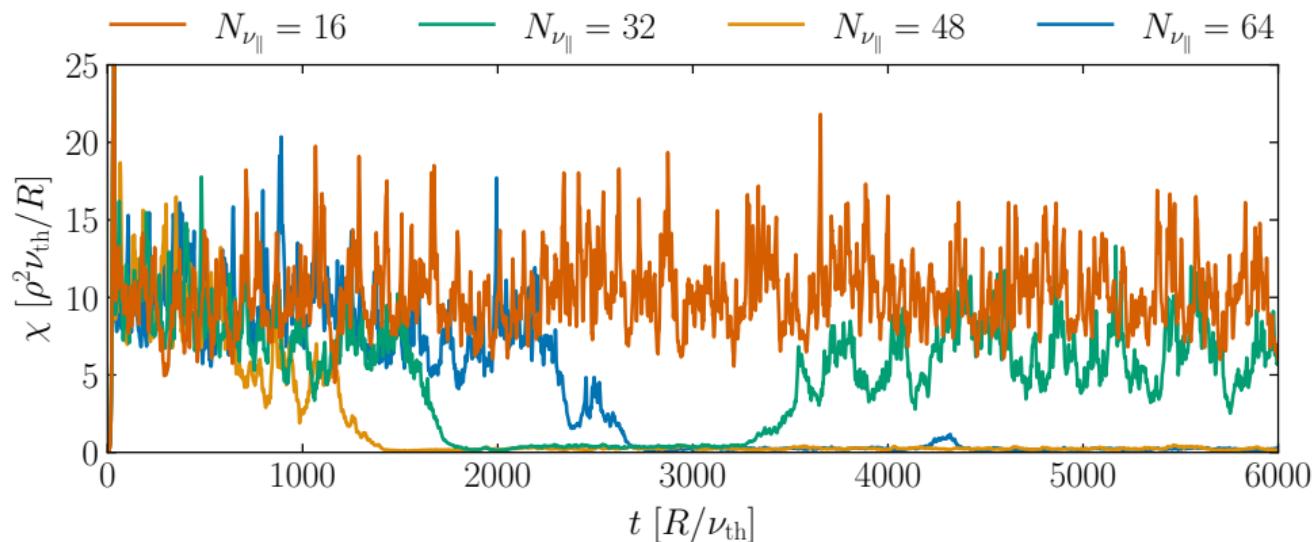


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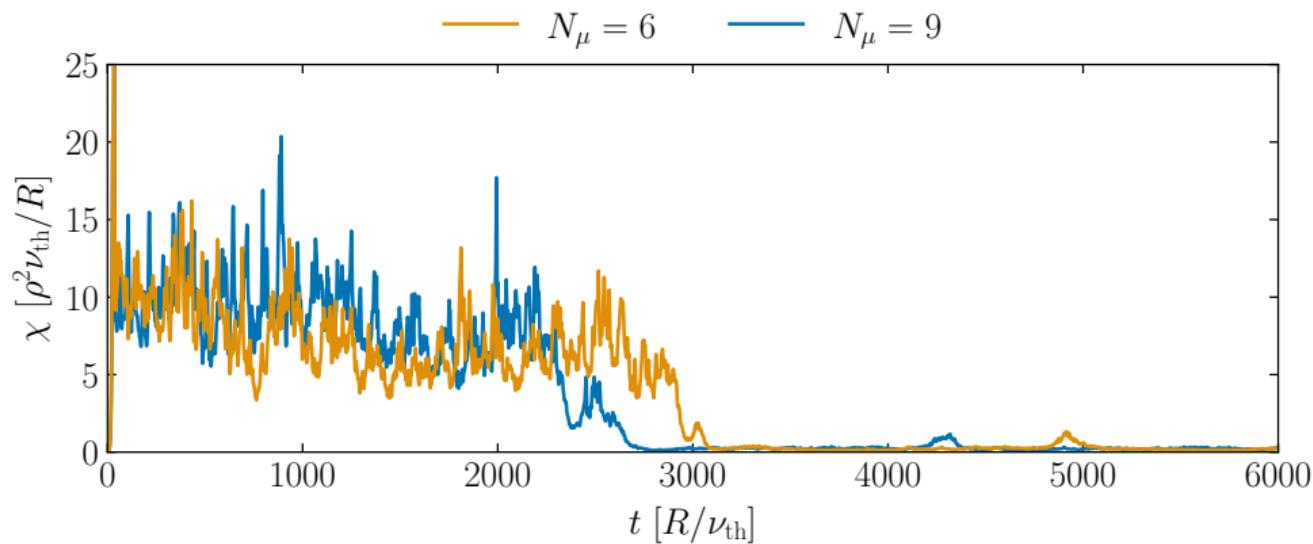


REDUCTION OF GRID POINTS

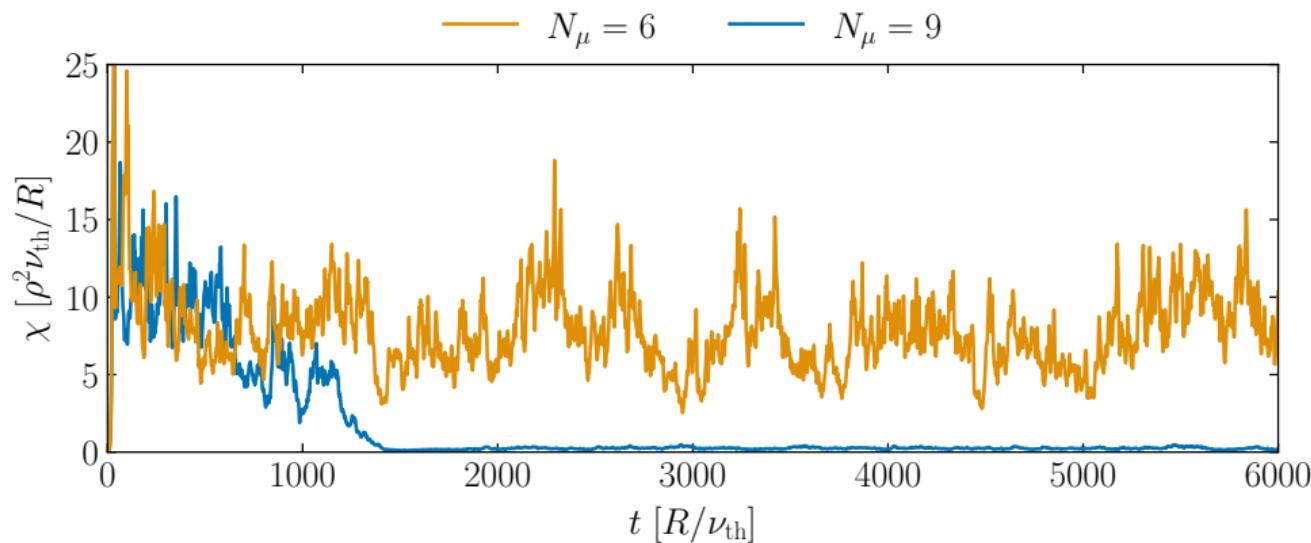
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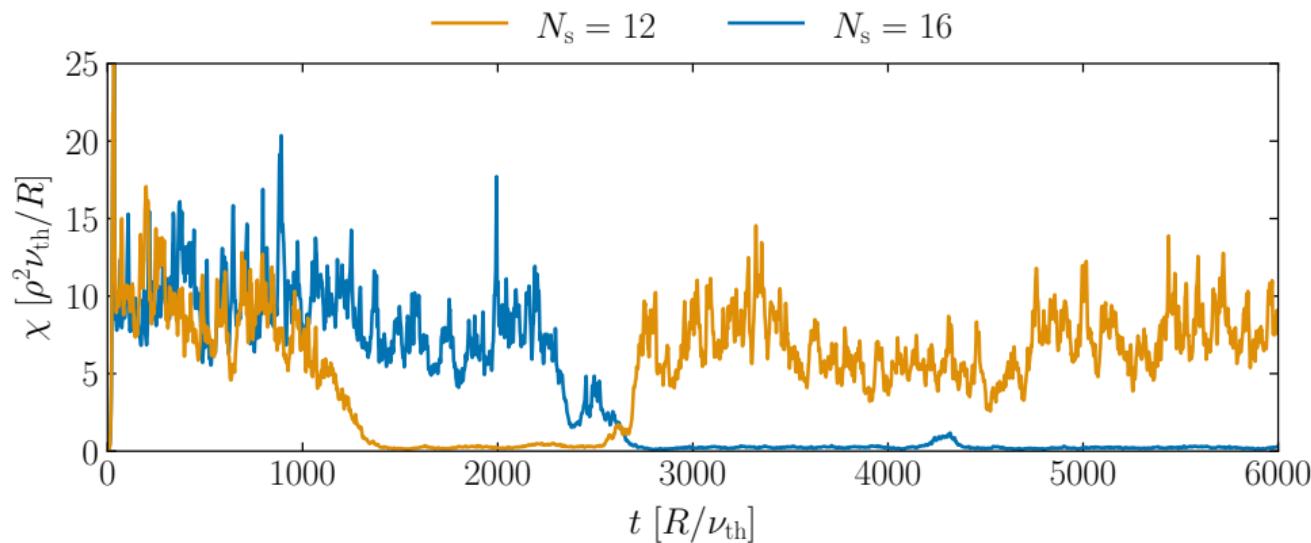
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Final Resolution

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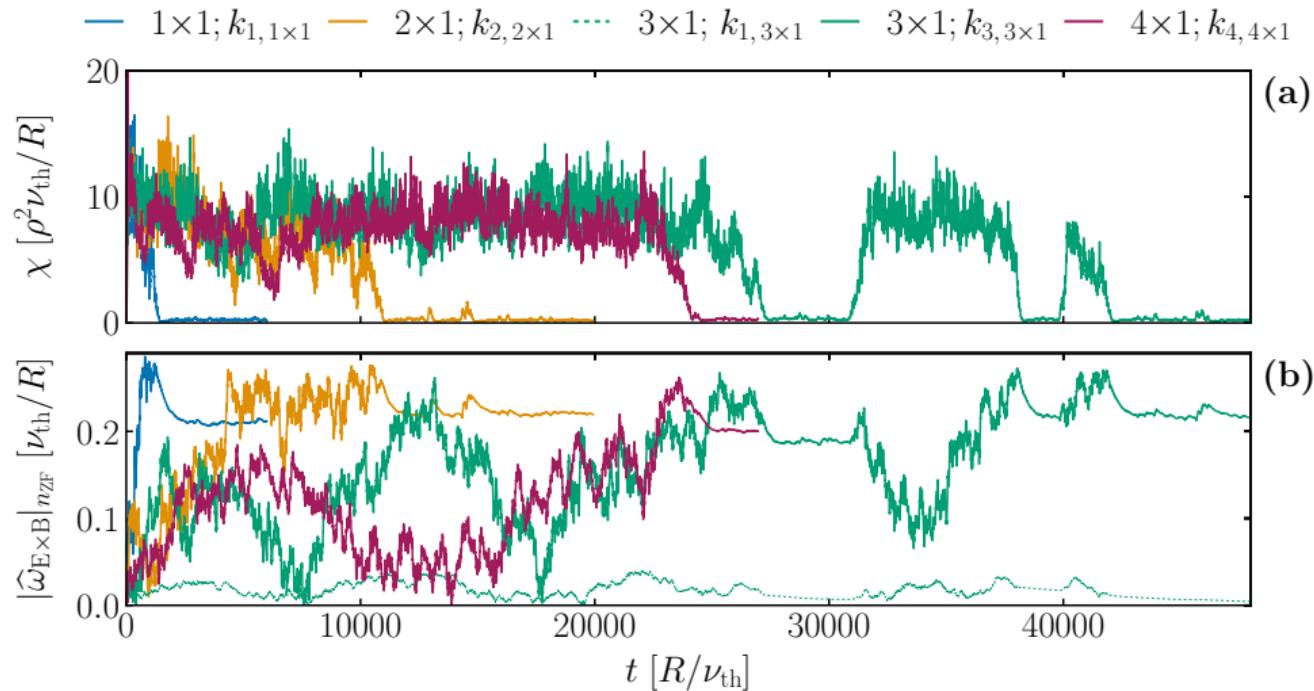
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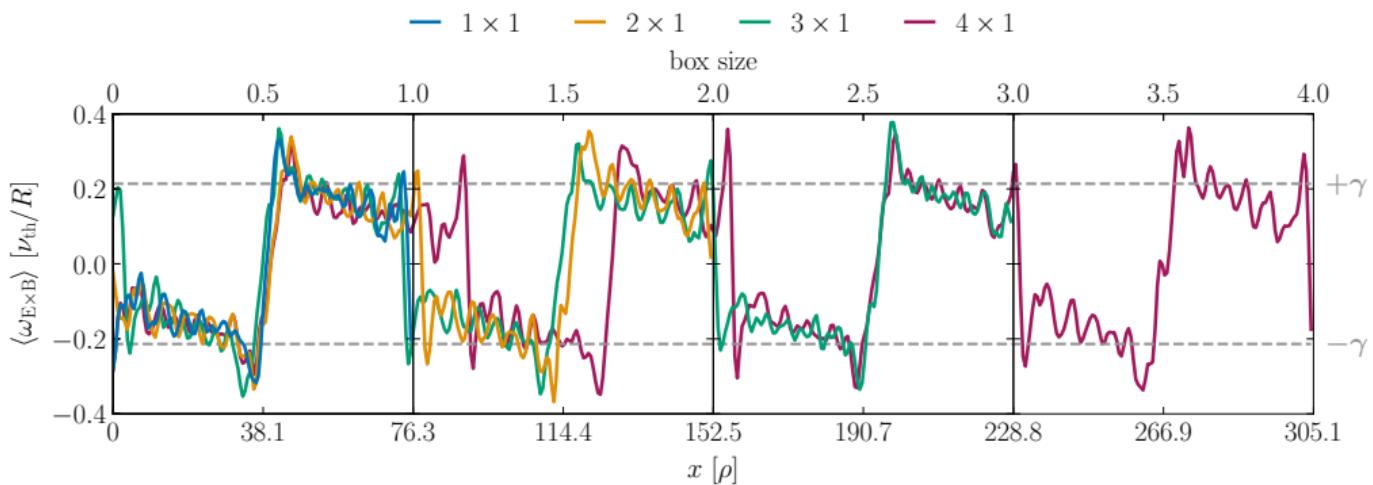
(1) Radial

$$N_R \times N_B \in [1 \times 1, 2 \times 1, 3 \times 1, 4 \times 1]$$

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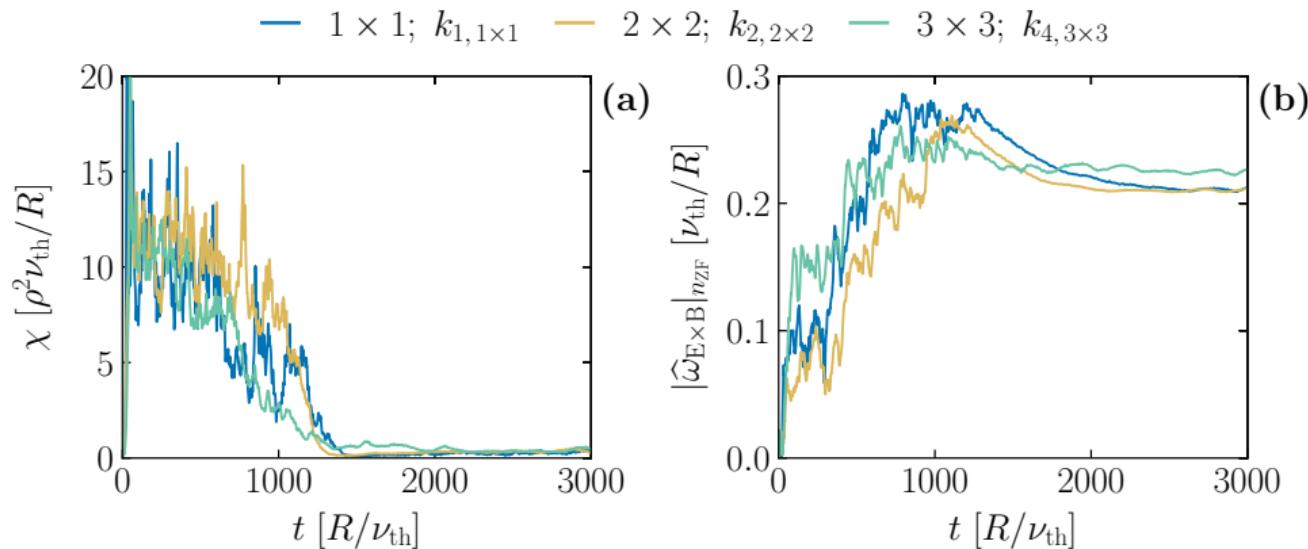


SIZE CONVERGENCE OF E × B STAIRCASE PATTERN

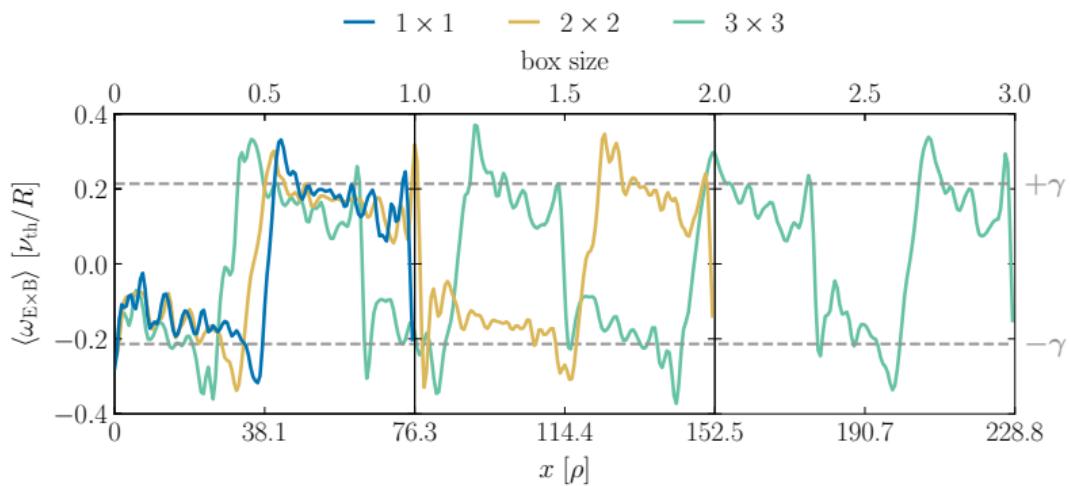
(2) Isotropic

$$N_R \times N_B \in [1 \times 1, 1.5 \times 1.5, 2 \times 2, 2.5 \times 2.5, 3 \times 3]$$

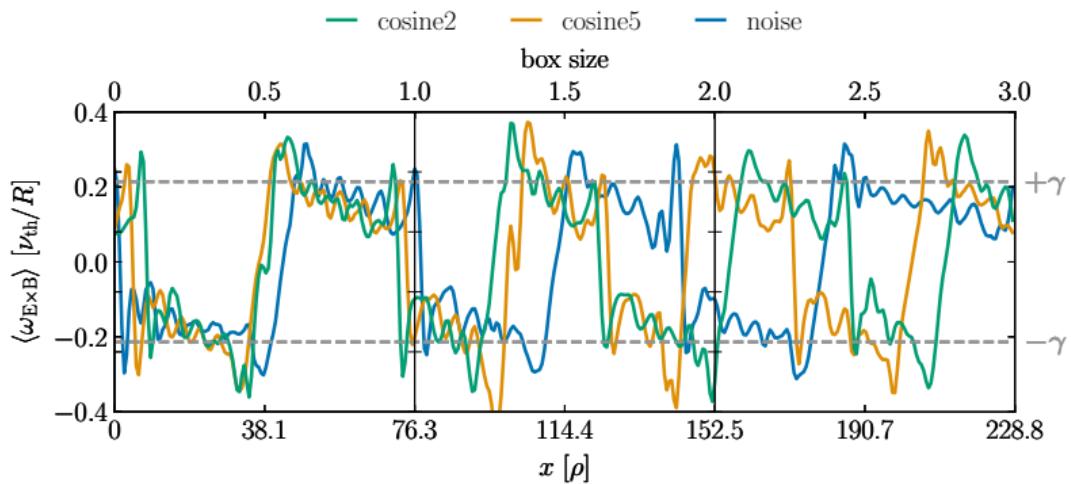
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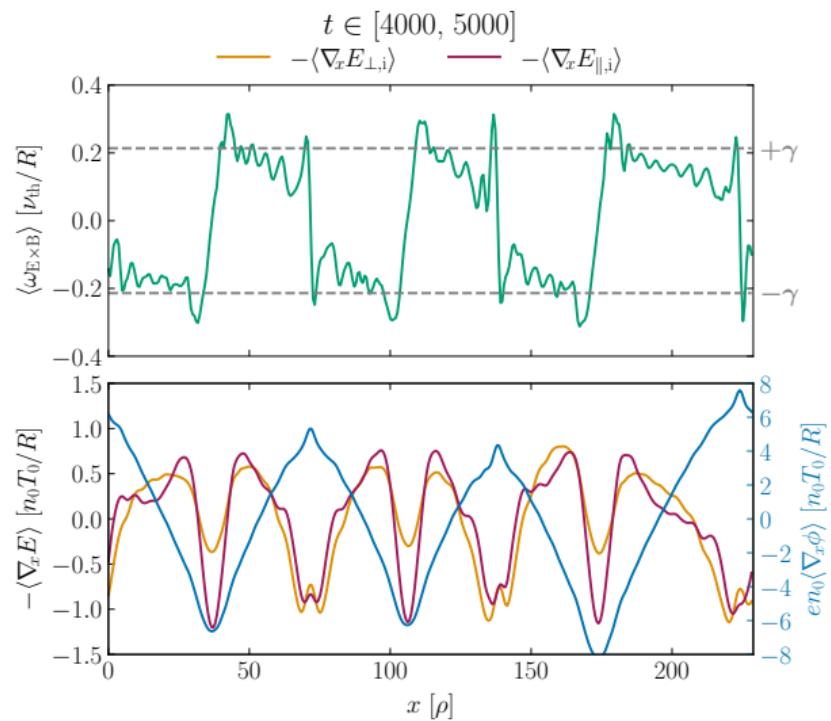
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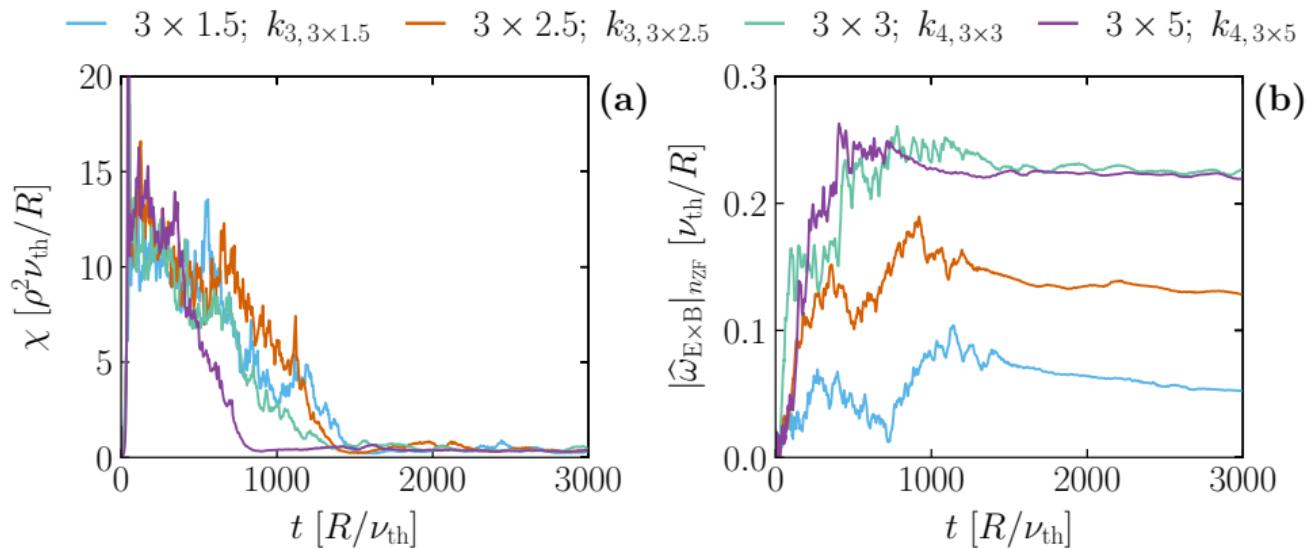


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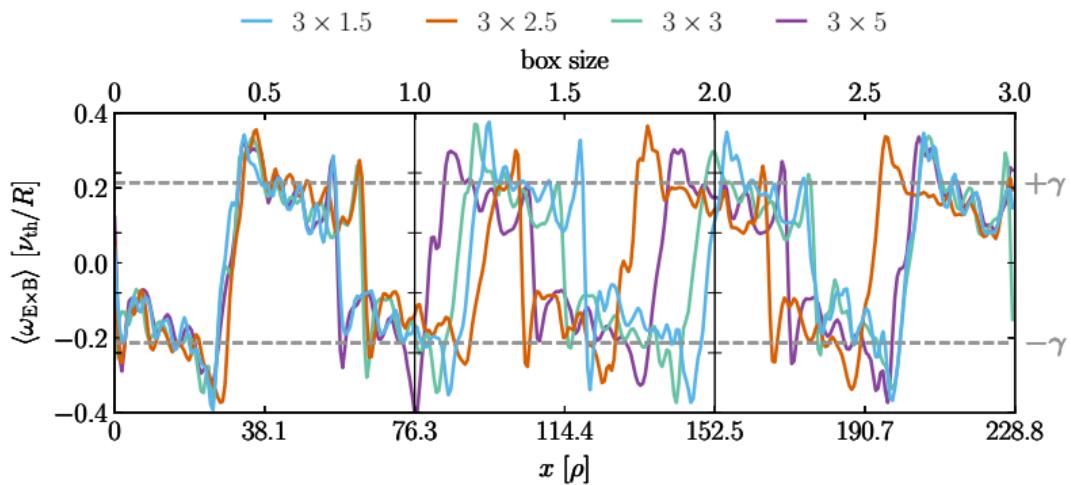
(3) Binormal

$$N_R \times N_B \in [3 \times 1.5, 3 \times 2.5, 3 \times 3, 3 \times 5]$$

SIZE CONVERGENCE OF $E \times B$ STAIRCASE PATTERN



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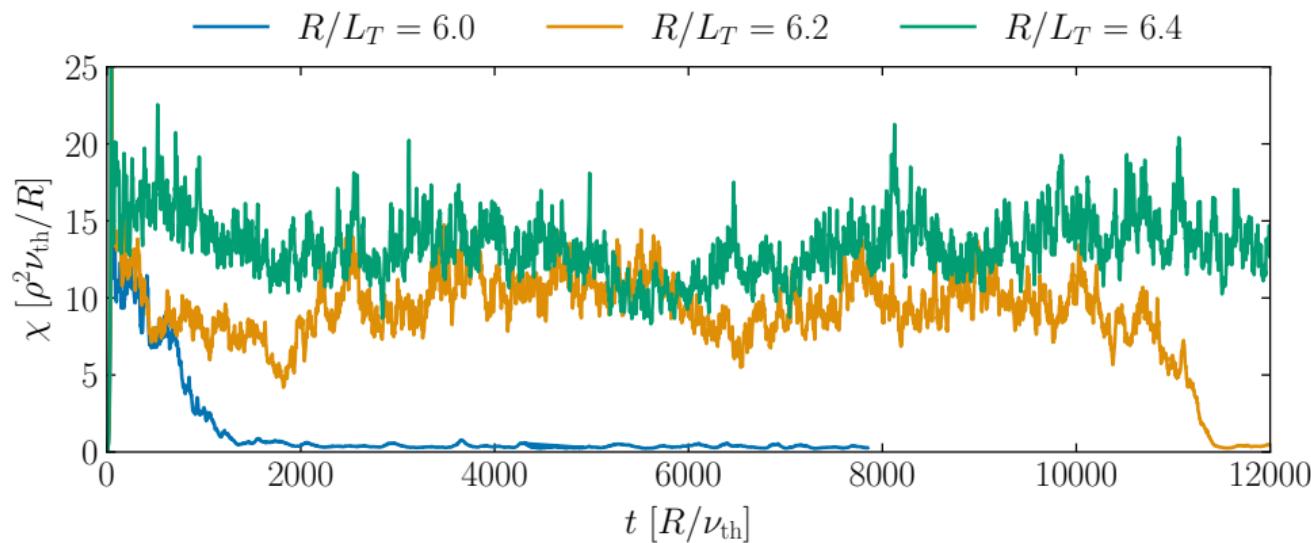
⇒ Mesoscale pattern size of:

$$\sim 57 - 76 \rho$$

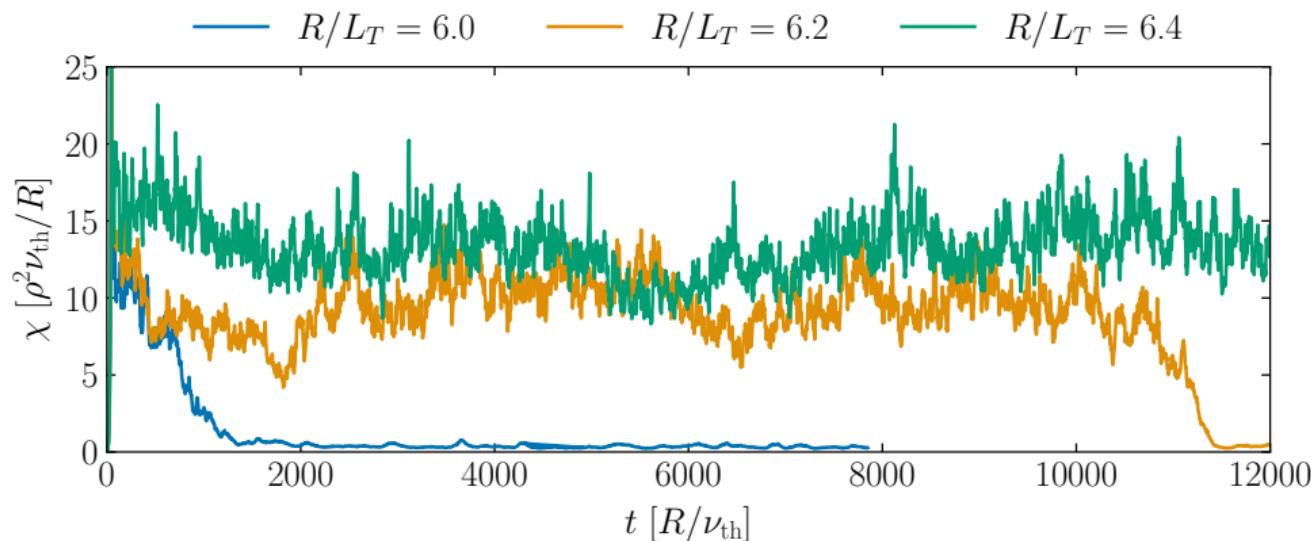
- Non-locality is inherent to ITG-driven turbulence
- Avalanches are spatially organized by the E × B staircase pattern

THE FINITE HEAT FLUX THRESHOLD

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$$\Rightarrow \boxed{R/L_T|_{\text{finite}} = 6.3 \pm 0.1}$$

CONCLUSION

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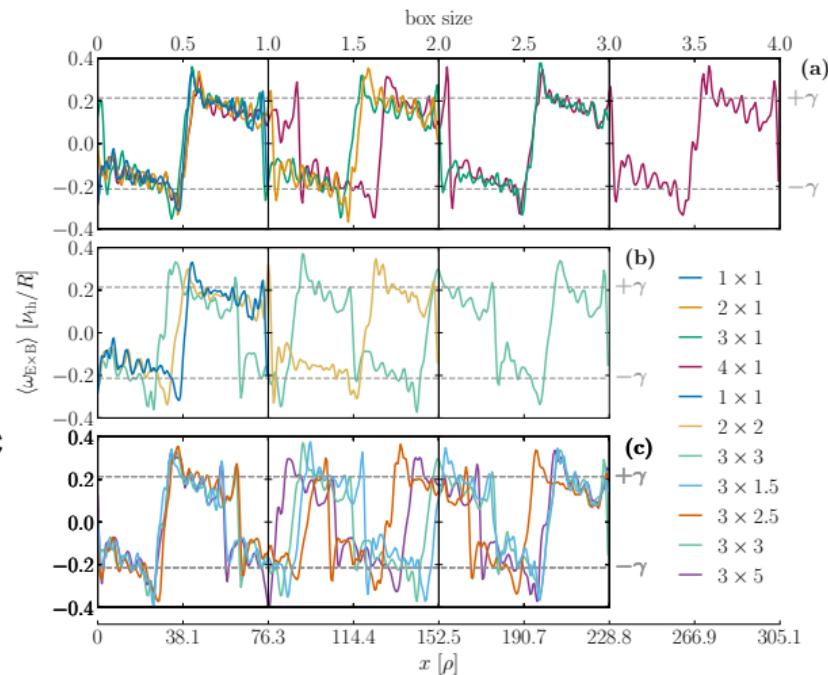
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- Restart Script with python led to further convenience during the task of performing simulations
- Mesoscale pattern size of $\sim 57 - 76 \rho$ is found to be intrinsic to ITG driven turbulence for Cyclone Base Case parameters
- Finite heat flux threshold is located at $R/L_T|_{\text{finite}} = 6.3 \pm 0.1$

