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## AFFILIATIONS

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## ABSTRACT

It is shown that an average number of in-flight fusion reactions catalyzed by one negative muon in the reactor proposed by Iiyoshi *et al.* [AIP Conf. Proc. **2179**, 020010 (2019)] will be less than 120 rather than about 1000, and therefore, the efficiency of the use of this reactor for practical purposes will be much less than expected earlier.

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Recently, two new scenarios of in-flight muon catalyzed fusion (IFMCF) were proposed.<sup>1,2</sup> It is assumed that at least one of them will be highly effective, namely, an average number  $N_{if}$  of in-flight fusion reactions catalyzed by one negative muon will be as high as 1000 and can provide creation of a compact fusion reactor, which “could rely upon existing technologies.”<sup>1</sup> Also, it is assumed that both scenarios will be applicable for the transmutation of long-lived fission products from nuclear power plants.<sup>1,2</sup> Deuteron–triton fusion was considered; the assumptions are based, in particular, on new results of calculations of cross section  $\sigma$  for the in-flight fusion reaction between the muon atom  $t\mu$ , consisting of triton and muon, and deuteron.<sup>1,2</sup> In some range of collision energies, the new values  $\sigma_{new}$  of  $\sigma$  are significantly greater than those obtained earlier.<sup>1</sup> For example, at the collision energy  $\epsilon_{cm}$  in the center-of-mass frame of 1 and about 1.5 keV, the ratio of  $\sigma_{new}$  to  $\sigma$  obtained for the same  $\epsilon_{cm}$  earlier is about 200 and 400, respectively.<sup>1</sup>

The physical importance of IFMCF can be described quantitatively by  $N_{if}$  and/or the ratio  $\eta_{if} = N_{if}/N$ , where  $N$  is the total average number of fusion reactions catalyzed by one muon due to both IFMCF and muon catalyzed fusion reactions involving the formation of muon molecules. According to the results obtained before publication of the paper by Iiyoshi *et al.*,<sup>1</sup> in liquids and gases,  $\eta_{if}$  and, therefore, the physical importance of IFMCF are low because the average distances between nuclei in liquid and gaseous hydrogen are much longer than those in muonic molecules.<sup>3–9</sup> For example, Gershtein *et al.*<sup>7</sup> described the situation when for the deuteron–triton fusion  $\eta_{if} \approx 10^{-7}$ . In plasma, both  $N_{if}$  and  $\eta_{if}$  depend on its density and temperature.<sup>6,10–15</sup>

In an IFMCF reactor proposed by Iiyoshi *et al.*,<sup>1</sup> muons are injected into a gas target heating of which by alpha particles, generated by IFMCF, will result in the formation of plasma with temperature  $T = 10\text{--}100$  eV. It is assumed that the use of such target will resolve the problem of the muon sticking to alpha particle.<sup>1</sup> Below it is shown that even if this assumption is valid, the real value of  $N_{if}$  in the reactor proposed by Iiyoshi *et al.*<sup>1</sup> will be much less than 1000 due to relatively long average distances between nuclei of hydrogen isotopes in the gas target and plasma arising due to heating of this target.

Iiyoshi *et al.*<sup>1</sup> describe quantitatively the gas target of the reactor by the density of  $10^{21}\text{ cm}^{-3}$  without mentioning particles (i.e., molecules or nuclei of hydrogen isotopes) corresponding to this parameter. The information that the range of alpha particle with the initial kinetic energy  $\epsilon_{\alpha 0}$  of 3.5 MeV in the target equals 5 mm<sup>1</sup> and the NIST data<sup>16</sup> on ranges of alpha particles in H<sub>2</sub> allow us to conclude that the parameter under consideration corresponds to the nuclei. Here, the fact that the range of alpha particle with  $\epsilon_{\alpha 0}$  of several MeV is mainly determined by the electron stopping power<sup>16</sup> is taken into account. Assuming that in the gas target, the density  $n_d$  of deuterons coincides with the density  $n_t$  of tritons and transformation of the target into plasma does not result in the significant decreases in  $n_d$  and  $n_t$ , let us consider several parameters related to IFMCF in gas and plasma with

$$n_d = n_t = 5 \times 10^{20} \text{ cm}^{-3}. \quad (1)$$

Iiyoshi *et al.*<sup>1</sup> presented several values of  $\sigma_{new}$  for  $\epsilon_{cm}$  from 0.1 to about 1.5 keV. These values increase with  $\epsilon_{cm}$ . The upper boundary  $\sigma_{new}^u$  of  $\sigma_{new}$ , corresponding to

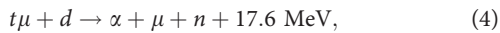
$$\varepsilon_{cm} \approx 1.5 \text{ keV}, \quad (2)$$

is about  $2.2 \times 10^{-21} \text{ cm}^2$ .<sup>1</sup> Below, we will assume that at  $0.1 \text{ keV} \leq \varepsilon_{cm} \leq 1.5 \text{ keV}$ , the real values of  $\sigma$  are close to  $\sigma_{new}$  and increase with  $\varepsilon_{cm}$ . Let us denote the velocities of the muon atom  $t\mu$  and deuteron in the laboratory frame as  $v_{t\mu}$  and  $v_d$ , respectively. In the situations when  $v_{t\mu}$  is much greater than  $v_d$  and much less than the velocity of light, the kinetic energy  $\varepsilon_{t\mu}$  of the muon atom in the laboratory frame is given by

$$\varepsilon_{t\mu} \approx \frac{m_\mu + m_t + m_d}{m_d} \varepsilon_{cm}, \quad (3)$$

where  $m_\mu$ ,  $m_t$ , and  $m_d$  are the masses of muon, triton, and deuteron, respectively.<sup>17,18</sup> Substituting  $m_\mu$ ,  $m_t$ , and  $m_d$  (see, e.g., Refs. 19–21) into Eq. (3), we obtain  $\varepsilon_{t\mu} \approx 2.551 \varepsilon_{cm}$ . Thus, Eq. (2) corresponds to  $\varepsilon_{t\mu} \approx 3.83 \text{ keV}$  and  $v_{t\mu} \approx 4.86 \times 10^7 \text{ cm/s}$ . Below, for the sake of convenience, the velocity of  $4.86 \times 10^7 \text{ cm/s}$  will be denoted as  $v'_{t\mu}$ .

In order to analyze validity of the assumption about attainability of  $N_{if} \approx 1000$  in the reactor proposed by Iiyoshi *et al.*,<sup>1</sup> let us assume that any muon, injected into the gas target of the reactor or plasma arising from it, instantaneously becomes a component of the muon atom  $t\mu$  with  $v_{t\mu} \approx v'_{t\mu}$ , this atom does not leave the target or the plasma and its collisions with other particles result only in the IFMCF reaction



after which the muon instantaneously becomes a component of the new muon atom  $t\mu$  with  $v_{t\mu} \approx v'_{t\mu}$ , etc. In the situation under consideration, when, in particular,  $\varepsilon_{cm}$  cannot exceed 1.5 keV (see below), these assumptions correspond to the upper boundary  $N_{if}^u$  of  $N_{if}$  and yield

$$N_{if}^u \approx \sigma_{new}^u n_d v'_{t\mu} \tau_\mu, \quad (5)$$

where  $\tau_\mu \approx 2.2 \times 10^{-6} \text{ s}$  is the muon lifetime (see, e.g., Ref. 20). Here, the fact that the typical values of  $v_d$  are much less than  $v'_{t\mu}$ , because  $T$  is relatively low,<sup>1</sup> is used. Substituting  $n_d$  from Eq. (1) into Eq. (5), we obtain  $N_{if}^u \approx 120$ . Thus, the assumption about attainability of  $N_{if} \approx 1000$  is not valid.

Similar estimates were published by Zel'dovich<sup>3,4</sup> who obtained  $N_{if}$  of about  $5 \times 10^{-4}$  and of the order of  $10^{-6}$  for deuteron–deuteron and deuteron–proton reactions, respectively, in liquid media within the framework of the models not taking into account resonance effects. The value  $N_{if} \approx 5 \times 10^{-4}$  was obtained, assuming that the product of the cross section for the in-flight fusion reaction between the muon atom, consisting of deuteron and muon, and deuteron on the collision velocity was about  $6 \times 10^{-21} \text{ cm}^3 \text{ s}^{-1}$ .<sup>3,4</sup> This product and density of deuterons in liquid deuterium were substituted into formulae equivalent to Eq. (5).<sup>3,4</sup> Zel'dovich<sup>3,4</sup> also discussed the possibility of an increase in  $N_{if}$  for deuteron–deuteron reaction due to resonance effects and presented estimate according to which such an increase results in  $N_{if} \approx 10^{-2}$ .

It should be emphasized that in the reactor proposed by Iiyoshi *et al.*,<sup>1</sup>  $N_{if}$  will be significantly less than  $N_{if}^u$ . Some of the reasons are the following. The parameter  $N_{if}^u$  is obtained within the framework of the model not taking into account the formation of the muon molecules. At the aforementioned  $T$ , which are much less than 1 keV, not only the typical values of  $v_d$  but also those of  $v_{t\mu}$  will be much less

than  $v'_{t\mu}$ . An average value  $\langle \varepsilon_{\mu 0} \rangle$  of the initial kinetic energy of the muon released in reaction (4) will be about an average value  $\langle \varepsilon_{\mu 0}^{mol} \rangle$  of the initial kinetic energy of muon released in the deuteron–triton fusion reaction accompanying the formation of the muon molecule  $dt\mu$ . If in the situation under consideration, the muon released in reaction (4) instantaneously becomes a component of the new muon atom  $t\mu$  or, in other terms, is immediately bound with triton, an average value  $\langle \varepsilon_{t\mu} \rangle$  of the arising muon atom is given by

$$\langle \varepsilon_{t\mu} \rangle \approx \frac{m_\mu}{m_\mu + m_t} \langle \varepsilon_{\mu 0} \rangle \approx 3.633 \times 10^{-2} \langle \varepsilon_{\mu 0}^{mol} \rangle \quad (6)$$

(see, e.g., Refs. 3, 4, 17, 20, and 21). Substituting  $\langle \varepsilon_{\mu 0}^{mol} \rangle \approx 18 \text{ keV}^9$  into Eq. (6), we obtain  $\langle \varepsilon_{t\mu} \rangle \approx 0.654 \text{ keV}$ . At  $\varepsilon_{t\mu}$  of about this value, corresponding to  $\varepsilon_{cm} \approx 0.26 \text{ keV}$ ,  $\sigma_{new}$  and  $v_{t\mu}$  are much less than  $\sigma_{new}^u$  and  $v'_{t\mu}$ , respectively [see Ref. 1 and Eqs. (3) and (5)]. Moreover, the formation of the majority of the muon  $t\mu$  and  $d\mu$  atoms will probably occur after deceleration of the muons released in reaction (4) (see, e.g., Refs. 3 and 4). Even the model not taking into account the formation of the muon molecules yields that this deceleration will result in the additional decreases in  $N_{if}$  and  $\eta_{if}$  due to the decreases in  $\sigma_{new}$  and  $v_{t\mu}$  (see Refs. 1, 3, and 4).

The assumption of Iiyoshi *et al.*<sup>1</sup> about effective suppression of sticking of muons to alpha particles in the situation under consideration seems to require additional studies.

It is worth also noting that the assumption according to which the muon atom  $t\mu$  does not leave the gas target or the plasma arising from it means that sizes of the target in all dimensions should be of the order of 10 cm or greater. The model used when obtaining  $N_{if}^u$  yields that the average path  $\langle l_{t\mu} \rangle$  of the muon atom  $t\mu$  in the target or the plasma equals  $1/(\sigma_{new}^u n_d) \approx 0.91 \text{ cm}$  [see Eq. (1)]. In turn, the assumption according to which collisions of this atom with other particles result only in the IFMCF reaction means that the trajectory of the atom is straight. Thus, the model yields that the average distance between the point where the muon becomes the component of the muon atom for the first time and the point of this decay is about  $\sqrt{N_{if}^u \langle l_{t\mu} \rangle} \approx 10 \text{ cm}$  (see, e.g., Ref. 22). The losses of muons due to exit of the muon atoms from the target and the plasma will be negligible only if sizes of the target in all dimensions are significantly greater than this value. At least for the transversal sizes of the target proposed by Iiyoshi *et al.*,<sup>1</sup> this condition is not satisfied.

Note that the reactor proposed by Yamamoto *et al.*<sup>2</sup> is similar to that proposed by Iiyoshi *et al.*,<sup>1</sup> but should operate at  $T < 500 \text{ K}$  and density  $> 10^{22} \text{ cm}^{-3}$  (particles corresponding to this parameter are not mentioned) in the region of fusion reaction. Expected value of  $N_{if}$  is not presented;<sup>2</sup> therefore, estimate of upper boundary of  $N_{if}$  in the reactor proposed by Yamamoto *et al.*<sup>2</sup> is outside the scope of this paper. It should be emphasized that relatively low  $T$  is the factor preventing strong suppression of sticking of muon to alpha particle even if the assumption about such suppression in the reactor proposed by Iiyoshi *et al.*<sup>1</sup> is valid.

The exact values of  $N_{if}$  and  $N$  in the reactor proposed by Iiyoshi *et al.*<sup>1</sup> can be obtained only after the detailed studies taking into account IFMCF, muon catalyzed fusion reactions involving the formation of muon molecules, and sticking of muon to alpha particle. However, the results presented above are sufficient to conclude that in such reactor the values of  $N_{if} \approx 1000$  are not achievable and,

therefore, the efficiency of its use for practical purposes would be much less than expected by Iiyoshi *et al.*<sup>1</sup>

It is worth noting that the values  $\sigma_{new}$  obtained by Iiyoshi *et al.*<sup>1</sup> and similar values for higher  $\varepsilon_{cm}$  are useful for analysis of possibility and expedience of the use of muons for heating the deuterium-tritium fuel with the density of the order of 100 g/cm<sup>3</sup> in the scenarios similar to that proposed by Tan.<sup>10</sup> High  $n_d$ ,  $n_t$ , and temperatures in such scenarios, which can be considered as fast ignition scenarios (see, e.g., Ref. 23 and bibliography therein), are probably favorable for realization of high  $N_{if}$ .

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## AUTHOR DECLARATIONS

### Conflict of Interest

The author has no conflicts to disclose.

## DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

## REFERENCES

- <sup>1</sup>A. Iiyoshi, Y. Kino, M. Sato, Y. Tanahashi, N. Yamamoto, S. Nakatani, T. Yamashita, M. Tendler, and O. Motojima, "Muon catalyzed fusion, present and future," *AIP Conf. Proc.* **2179**, 020010 (2019).
- <sup>2</sup>N. Yamamoto, M. Sato, H. Takano, and A. Iiyoshi, "Transmutation of LLFP by irradiation of neutrons on muon catalyzed fusion (MCF) reactor," *Plasma Fusion Res.* **16**, 1405074 (2021).
- <sup>3</sup>Y. B. Zel'dovich, "Reactions induced by  $\mu$ -mesons in hydrogen," *Dokl. Akad. Nauk SSSR* **95**, 493 (1954) (in Russian).
- <sup>4</sup>Y. B. Zel'dovich, "Reactions induced by  $\mu$ -mesons in hydrogen," in *In the Intermissions... Collected Works on Research into the Essentials of Theoretical Physics in Russian Federal Nuclear Center, Arzamas-16*, edited by Y. A. Trutnev (World Scientific Publishing Co. Pte. Ltd, 1998), pp. 104–108.
- <sup>5</sup>Y. B. Zel'dovich and S. S. Gershtein, "Nuclear reactions in cold hydrogen I. Mesonic catalysis," *Usp. Fiz. Nauk* **71**, 581–630 (1960). [*Sov. Phys. Usp.* **3**, 593 (1961)].
- <sup>6</sup>L. I. Men'shikov, "Processes of muon catalysis in dense low-temperature plasma," Preprint No. IAE-4589/2 of I. V. Kurchatov Atomic Energy Institute (1988) (in Russian).
- <sup>7</sup>S. S. Gershtein, Y. V. Petrov, and L. I. Ponomarev, "Muon catalysis and nuclear breeding," *Usp. Fiz. Nauk* **160**(8), 3 (1990) [*Sov. Phys. Usp.* **33**, 591 (1990)].
- <sup>8</sup>L. I. Ponomarev, "Muon catalysis," in *Physical Encyclopedia*, edited by A. M. Prokhorov (Bol'shaya Rossiiskaya Encyclopedia, 1992), Vol. 3, pp. 229–230 (in Russian).
- <sup>9</sup>R. S. Kelly, "Muon catalyzed fusion, an investigation of reactor design," Ph.D. dissertation (Imperial College, London, 2018).
- <sup>10</sup>W. P. S. Tan, "Muon catalysed fusion for pellet ignition," *Nature* **263**, 656 (1976).
- <sup>11</sup>L. I. Men'shikov and L. I. Ponomarev, "Formation of dt  $\mu$  muonic molecules in three-body collisions at high temperatures," *Pis'ma Zh. Eksp. Teor. Fiz.* **46**, 246 (1987) [*JETP Lett.* **46**, 312 (1987)].
- <sup>12</sup>L. I. Men'shikov and L. N. Somov, "Status of muon catalysis of nuclear fusion reactions," *Usp. Fiz. Nauk* **160**(8), 47 (1990) [*Sov. Phys. Usp.* **33**, 616 (1990)].
- <sup>13</sup>H. E. Rafelski, D. Harley, G. R. Shin, and J. Rafelski, "Cold fusion: Muon-catalysed fusion," *J. Phys. B: At. Mol. Opt. Phys.* **24**, 1469 (1991).
- <sup>14</sup>J. Rafelski and D. Harley, "Muon catalyzed fusion at high density," *Part. Accel.* **37–38**, 409 (1992).
- <sup>15</sup>D. Harley, "Regeneration and stopping of ( $\alpha\mu^+$ ) in a degenerate plasma," *Phys. Rev. A* **45**, 8981 (1992).
- <sup>16</sup>M. J. Berger, J. S. Coursey, M. A. Zuker, and J. Chang, see [www.nist.gov/pml/data/star/](http://www.nist.gov/pml/data/star/) for "Stopping-Power and Range Tables for Electrons, Protons and Helium Ions" (2009).
- <sup>17</sup>L. D. Landau and E. M. Lifshitz, *Mechanics*, 2nd ed. (Pergamon Press, 1962).
- <sup>18</sup>H.-S. Bosch and G. M. Hale, "Improved formulas for fusion cross-sections and thermal reactivities," *Nucl. Fusion* **32**, 611 (1992).
- <sup>19</sup>V. M. Kolybasov, "Deuteron," in *Physical Encyclopedia*, edited by A. M. Prokhorov (Sovetskaya Encyclopedia, 1988), Vol. 1, pp. 577–578 (in Russian).
- <sup>20</sup>L. I. Ponomarev, "Muon atom," in *Physical Encyclopedia*, edited by A. M. Prokhorov (Bol'shaya Rossiiskaya Encyclopedia, 1992), Vol. 3, p. 229 (in Russian).
- <sup>21</sup>V. M. Kolybasov, "Triton," in *Physical Encyclopedia*, edited by A. M. Prokhorov (Bol'shaya Rossiiskaya Encyclopedia, 1998), Vol. 5, p. 168 (in Russian).
- <sup>22</sup>R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics* (Addison-Wesley Publishing Company, Inc., 1963), Vol. 1.
- <sup>23</sup>M. Tabak, D. Hinkel, S. Atzeni, E. M. Campbell, and K. Tanaka, "Fast ignition: Overview and background," *Fusion Sci. Technol.* **49**, 254 (2006).