

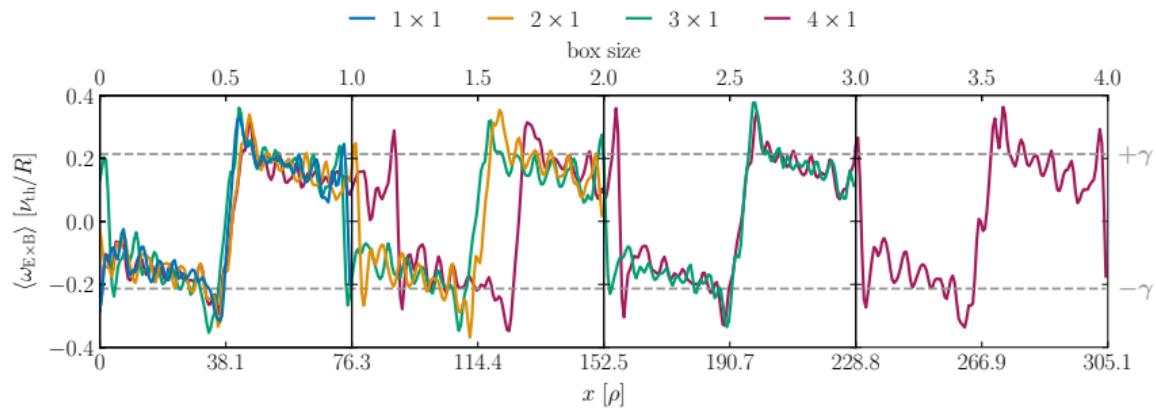


SIZE CONVERGENCE OF THE $E \times B$ STAIRCASE PATTERN IN FLUX TUBE SIMULATIONS OF ION TEMPERATURE GRADIENT-DRIVEN TURBULENCE

June 22, 2023

Manuel Lippert

Theoretical Physics V



MOTIVATION

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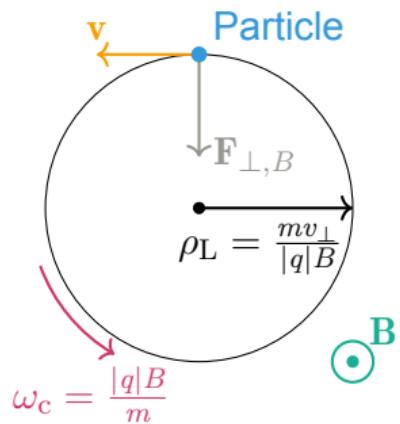
Does the basic pattern size always converges to the box size, or is there a typical mesoscale size inherent to staircase structures also in a local flux-tube description?

CHARGED PARTICLE MOTION

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Lorentz force

$$F_{\perp,B} = |q|v_{\perp}B$$



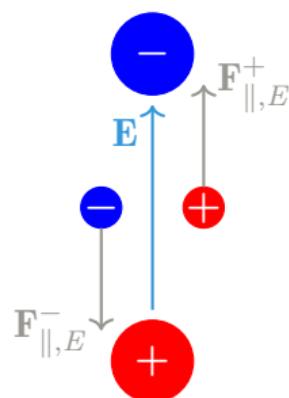
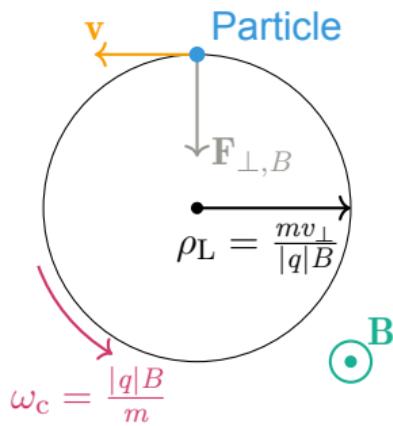
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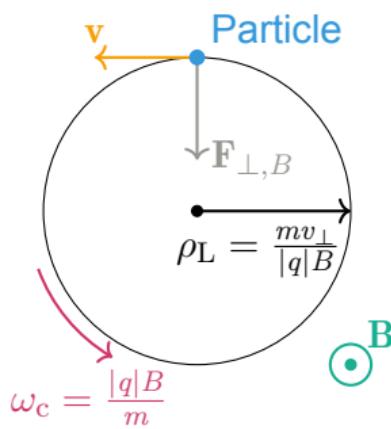
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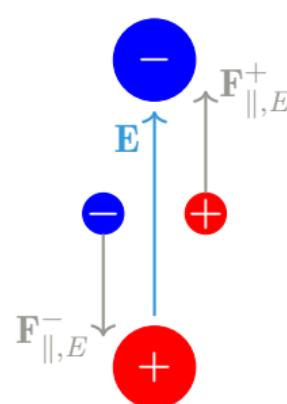
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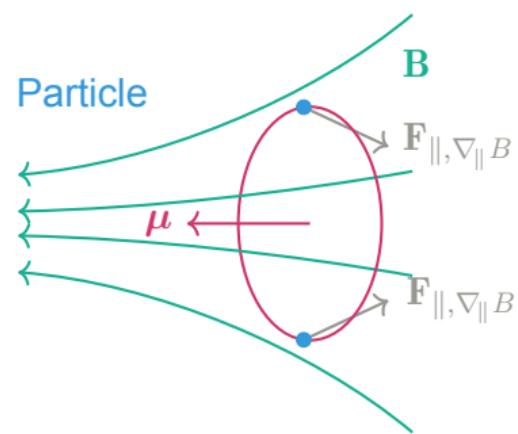
Electric force

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Inhomogeneous magnetic field

$$F_{\parallel,\nabla_{\parallel}B} = - \underbrace{\frac{mv_{\perp}^2}{2B}}_{\mu} \nabla_{\parallel}B$$

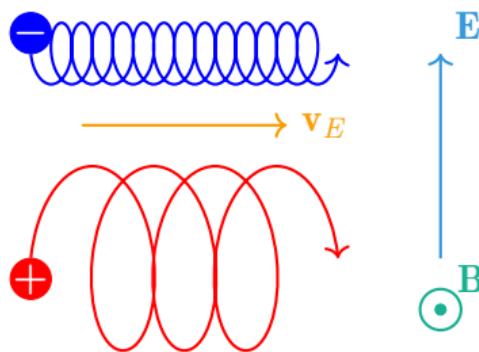


DRIFT IN THE GYROCENTER

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$E \times B$ Drift

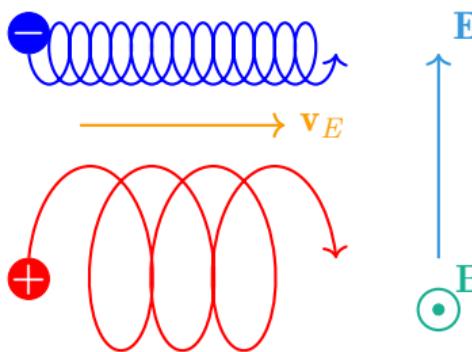
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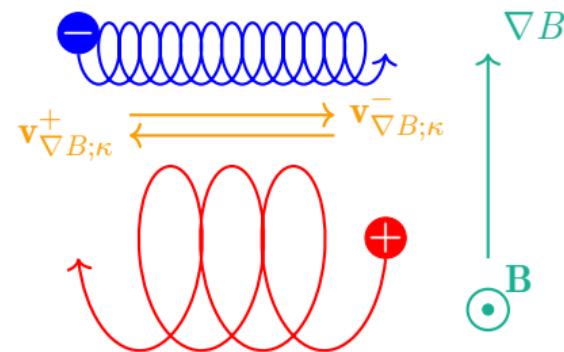


∇B Drift

$$\mathbf{v}_{\nabla B} = \frac{mv_{\perp}^2}{2q} \frac{\mathbf{B} \times \nabla B}{B^3}$$

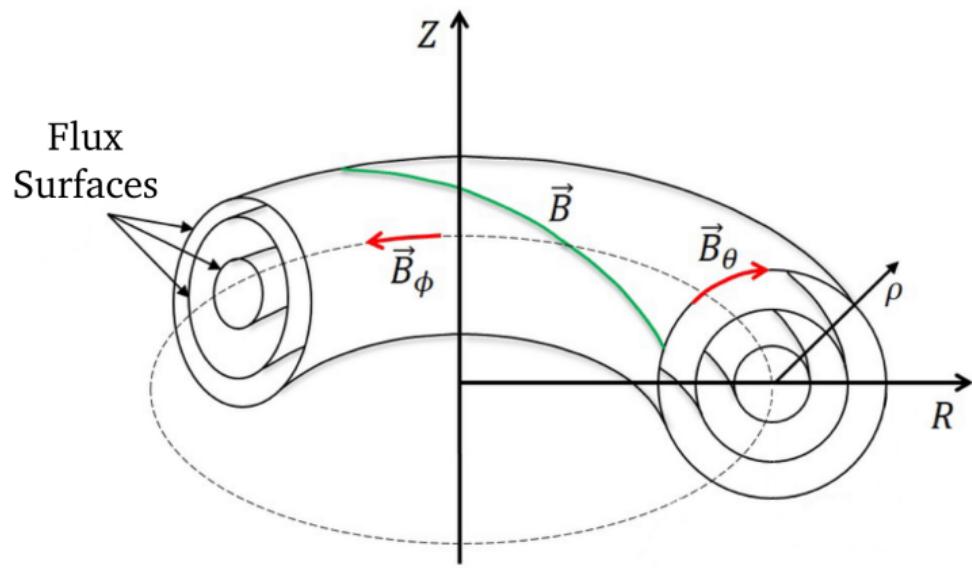
Curvature Drift

$$\mathbf{v}_{\kappa} = \frac{mv_{\parallel}^2}{qB^2} \mathbf{B} \times \underbrace{\kappa}_{\frac{\nabla B}{B}} = \frac{mv_{\parallel}^2}{q} \frac{\mathbf{B} \times \nabla B}{B^3}$$



MAGNETIC CONFINEMENT IN TOKAMAK

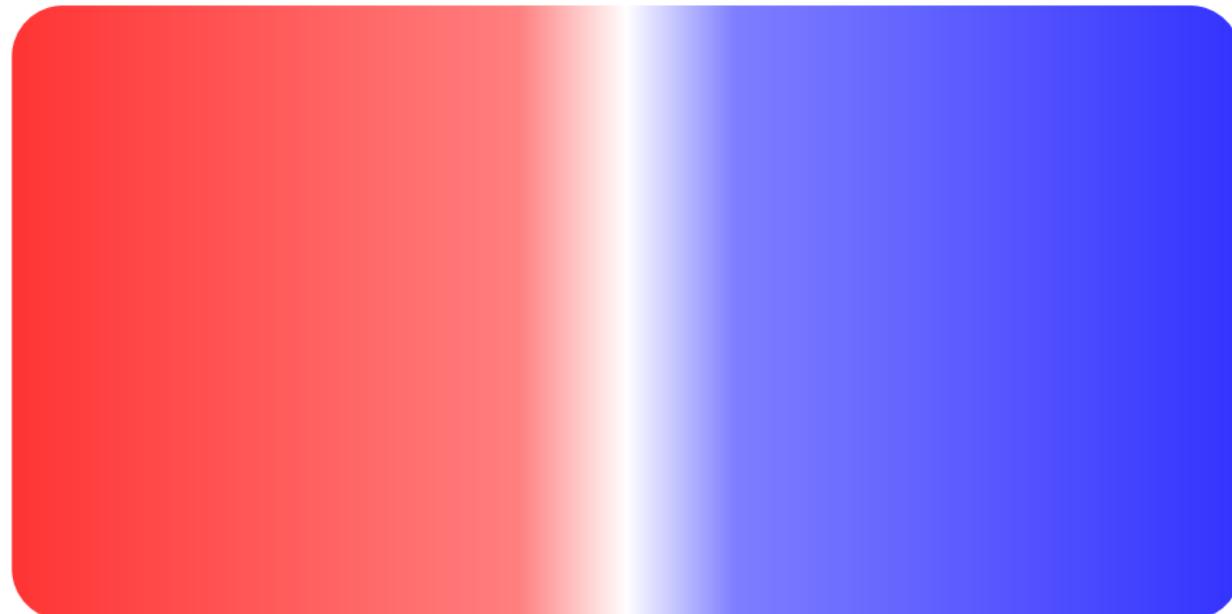
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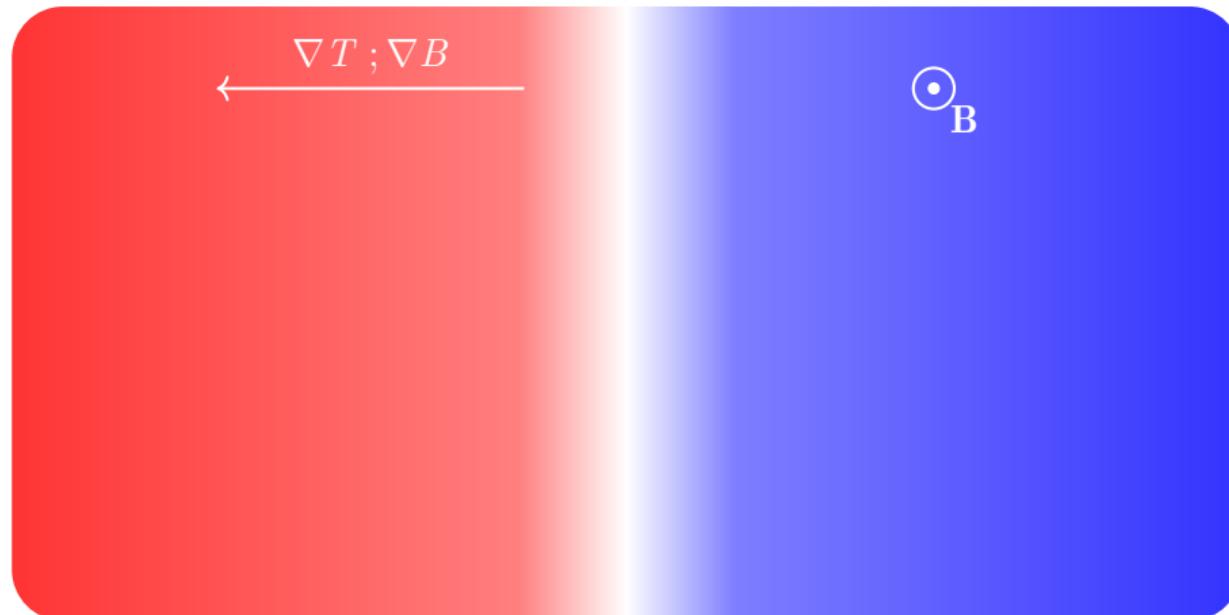
$$\beta = \frac{nT}{\mu_0 B^2/2}$$

ION TEMPERATURE GRADIENT (ITG)-DRIVEN INSTABILITY

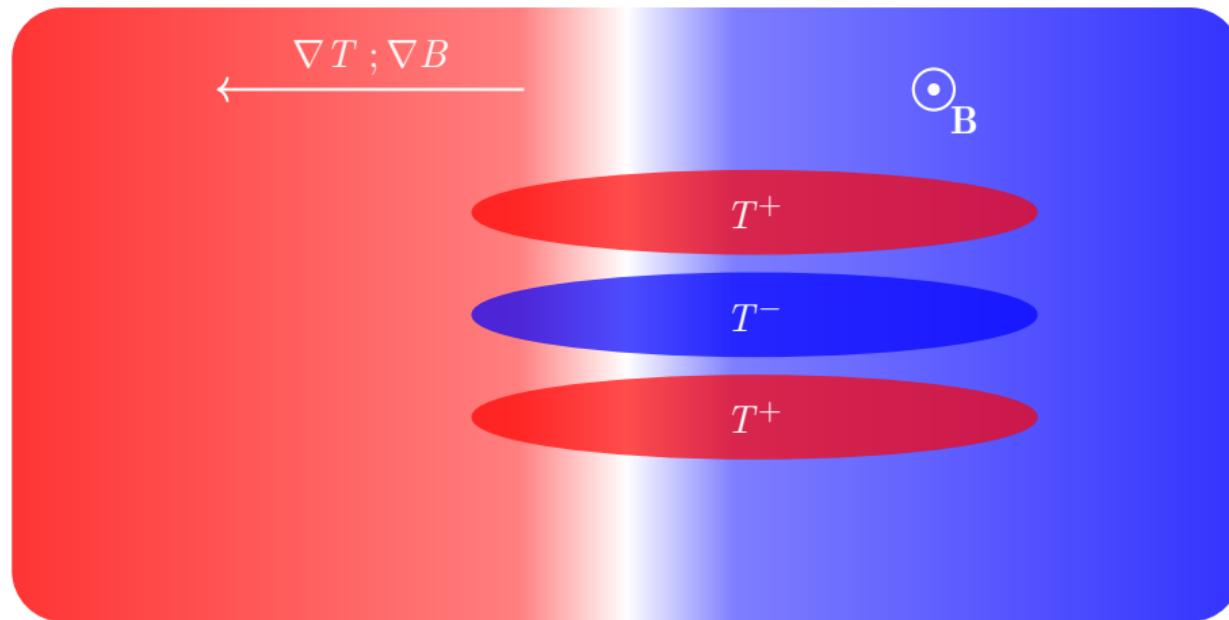
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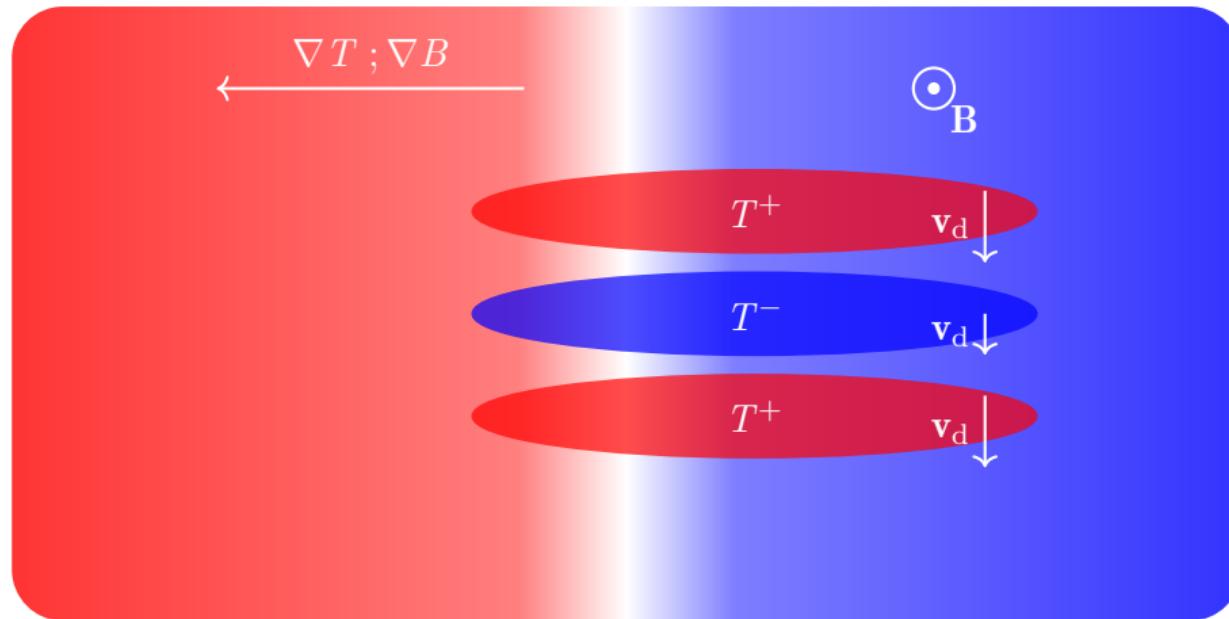
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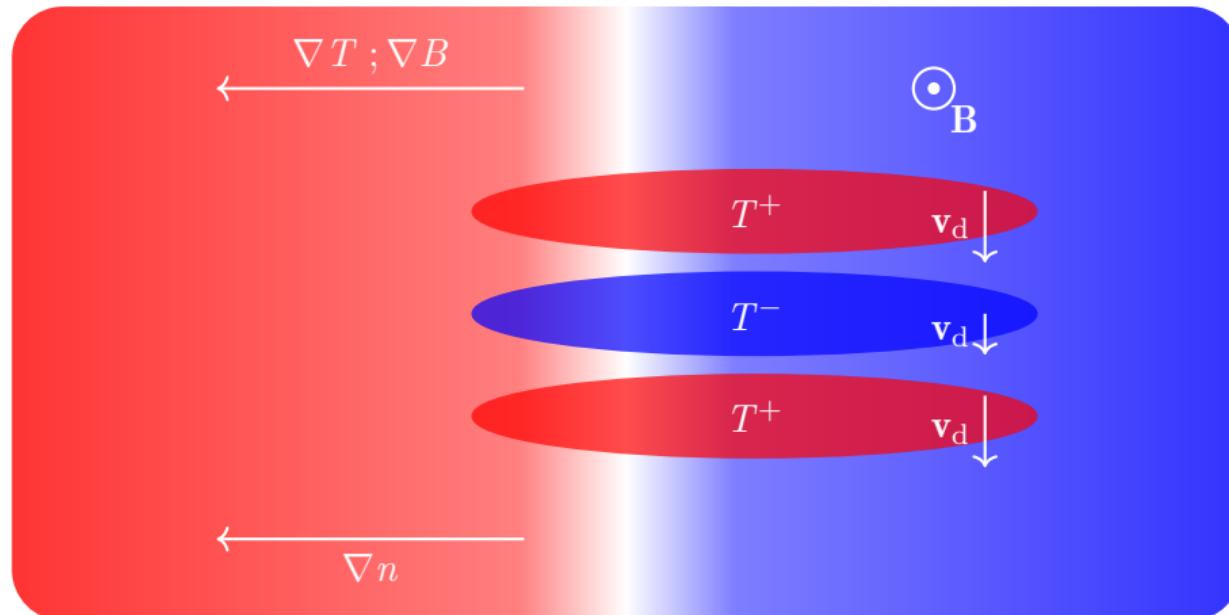
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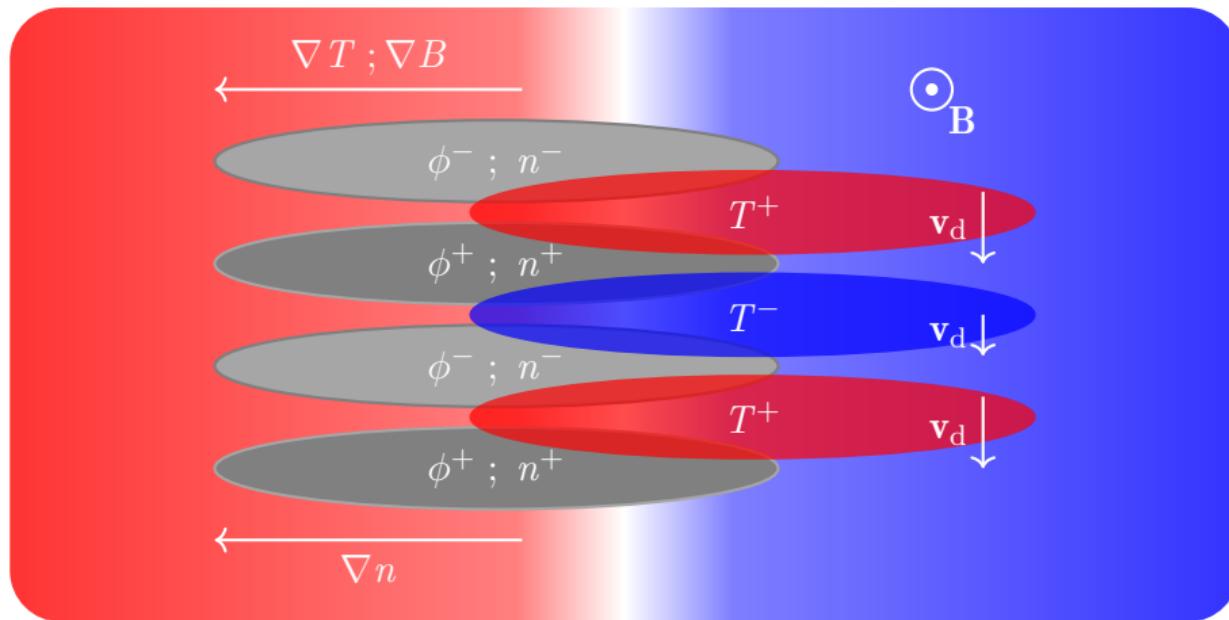
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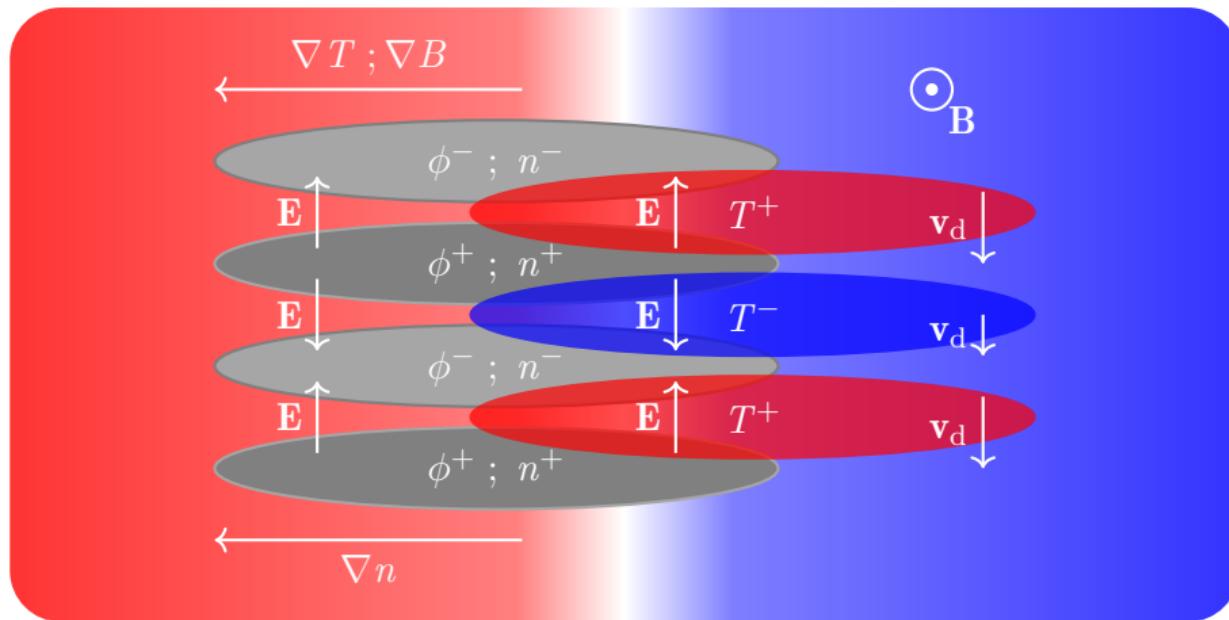
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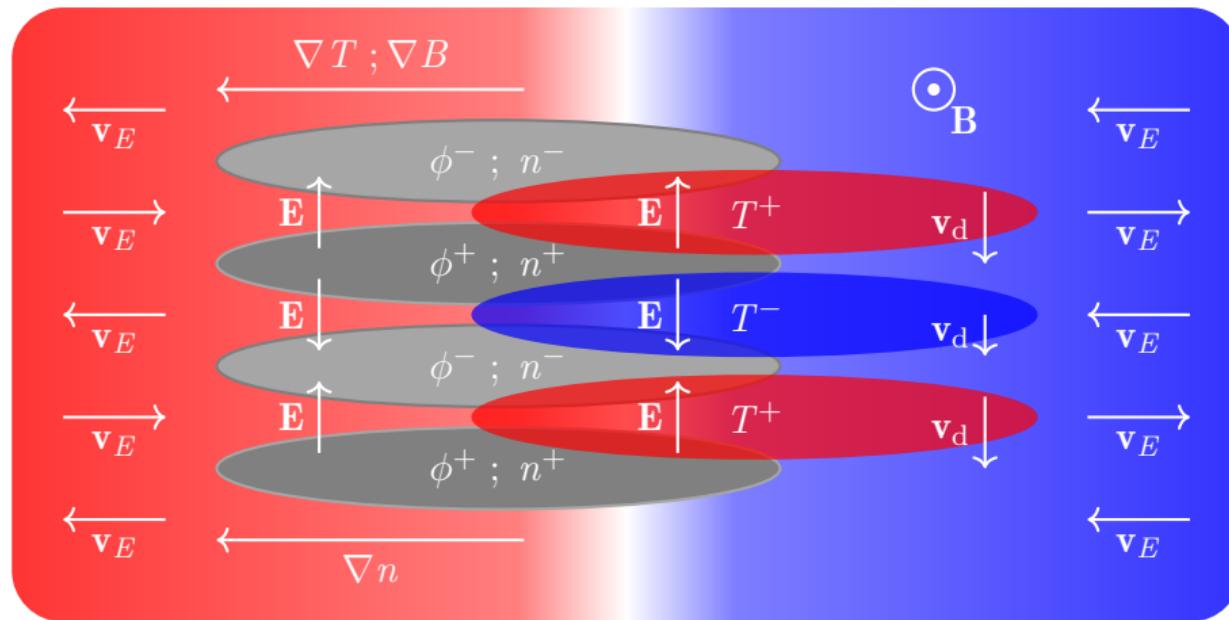
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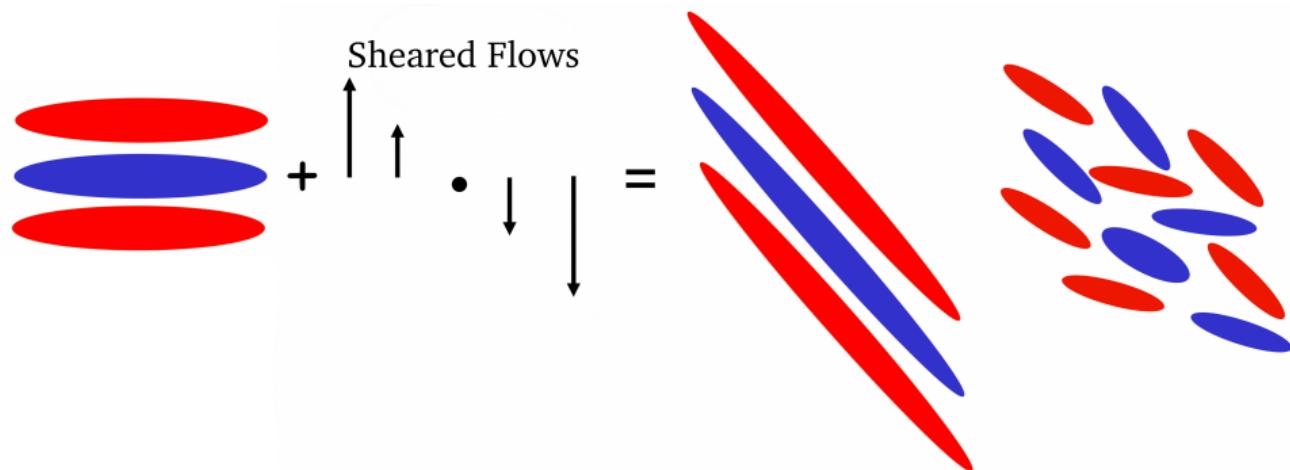


ION TEMPERATURE GRADIENT (ITG)-DRIVEN INSTABILITY



ZONAL FLOWS

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$$\omega_{E \times B} = \frac{1}{2} \frac{\partial^2 \langle \phi \rangle}{\partial x^2} \quad \langle \phi \rangle = \frac{1}{L_y} \int_0^{L_y} dy \phi(x, y, s=0)$$

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$$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial \mathbf{x}} \cdot \frac{d\mathbf{x}}{dt} + \frac{\partial f}{\partial \mathbf{v}} \cdot \frac{d\mathbf{v}}{dt} = 0 \quad f(\mathbf{x}, \mathbf{v}, t)$$

Vlasov Equation

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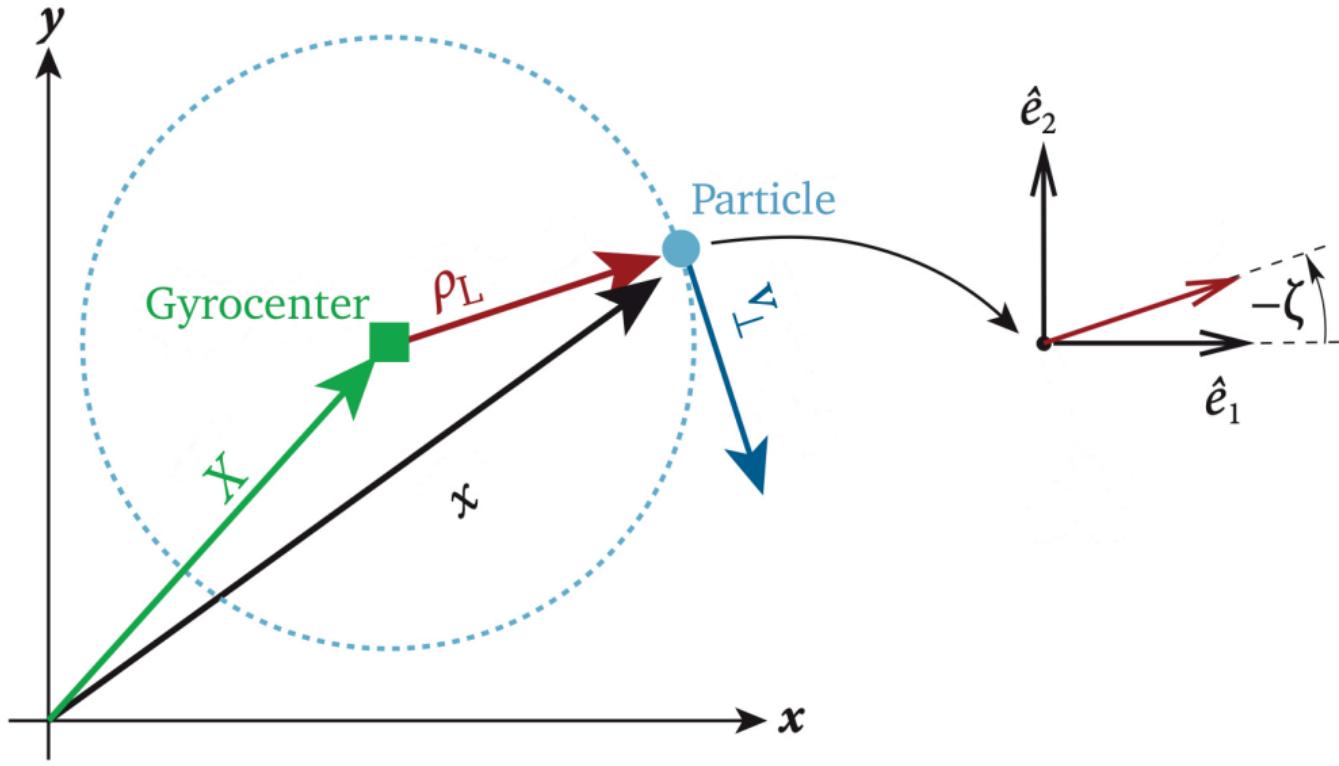
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$$\frac{\partial g}{\partial t} + \mathbf{v}_{\chi} \cdot \nabla g + (v_{\parallel} \mathbf{b} + \mathbf{v}_D) \cdot \nabla (\delta f) - \frac{\mu B}{m} \frac{\mathbf{B} \cdot \nabla B}{B^2} \frac{\partial (\delta f)}{\partial v_{\parallel}} = S \quad \text{Gyrokinetic Equation}$$

$$g \sim (f_0 + \delta f) \quad S \sim (f + f_0) \quad \mathbf{v}_D \sim \nabla B \quad \mathbf{v}_{\chi} \sim \mathbf{E} \times \mathbf{B} \quad f(x, y, s, v_{\parallel}, \mu)$$

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S6	21	83	16	64	9	1	$ \nu_{ } $	0.2	0.1	0.1	6	1.4	2.1

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- Standard box size $(L_x, L_y) = (76.3, 89.8) \rho$

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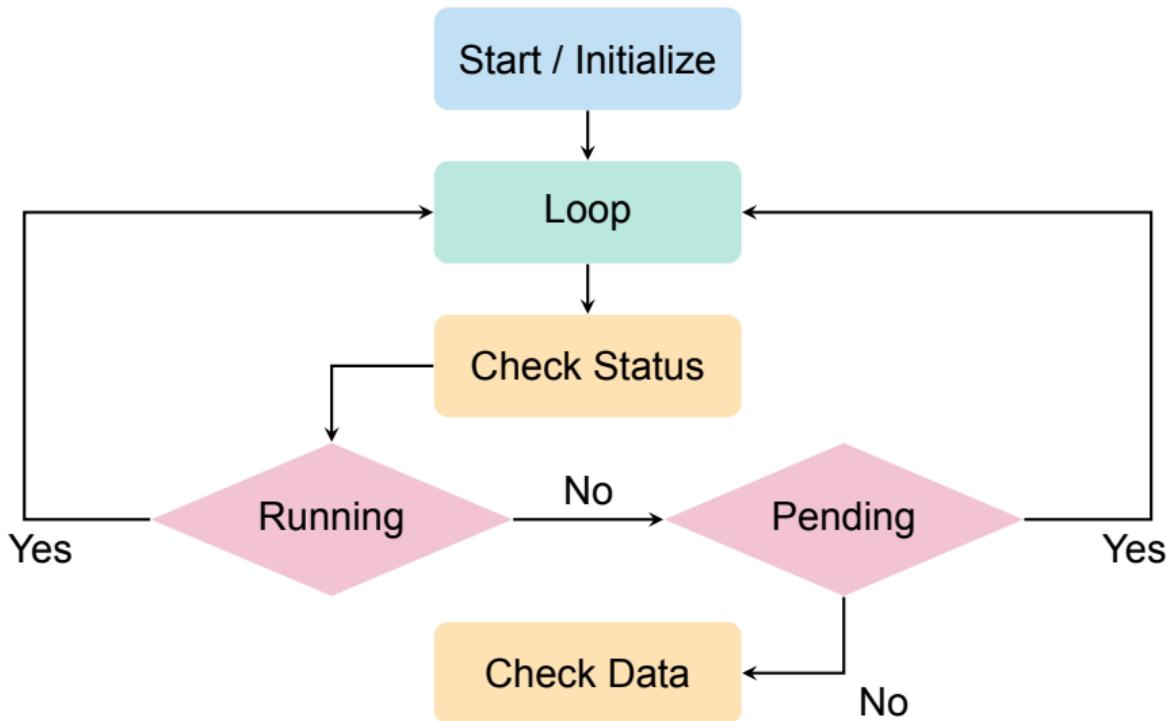
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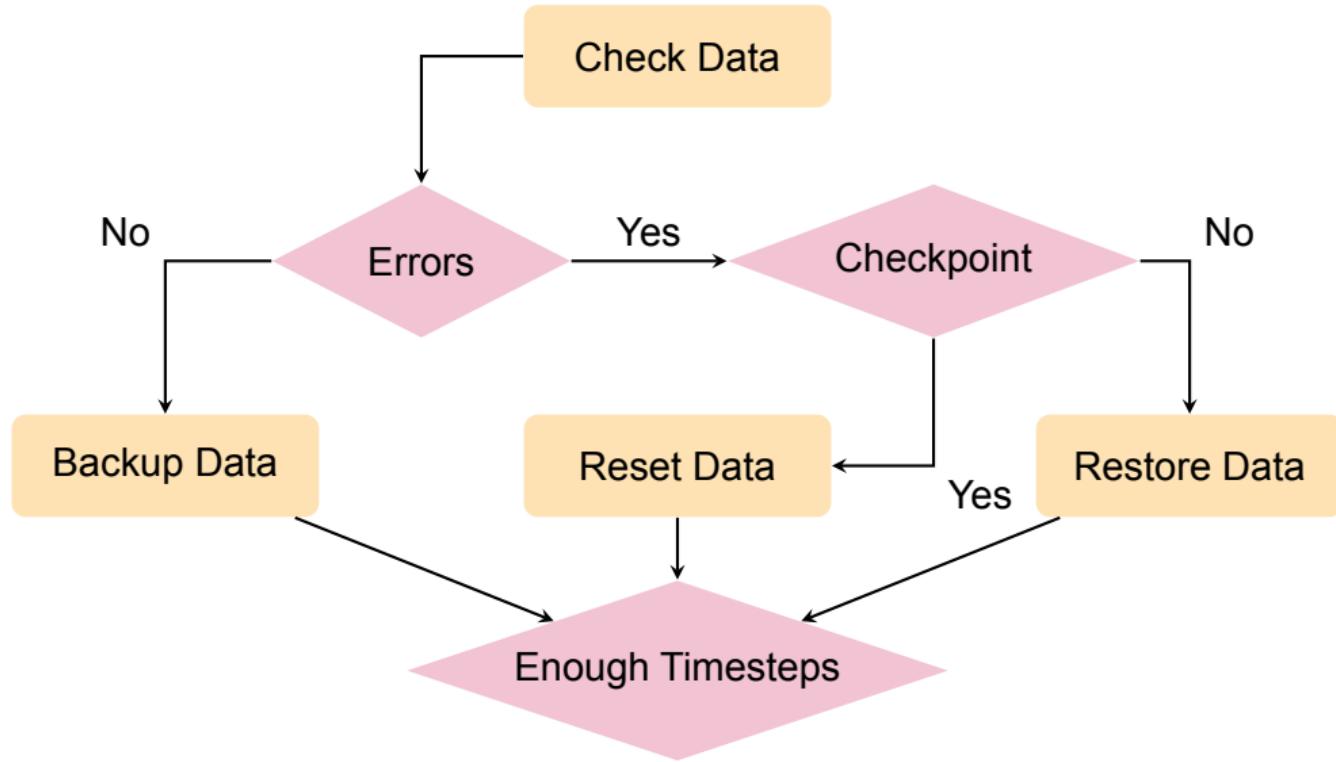
- Zonal flow mode that dominates the $E \times B$ staircase pattern are called **basic mode**
- The basic mode exhibits the maximum amplitude in the spectrum $|\hat{\omega}_{E \times B}|_{n_{ZF}}$

RESTART SCRIPT

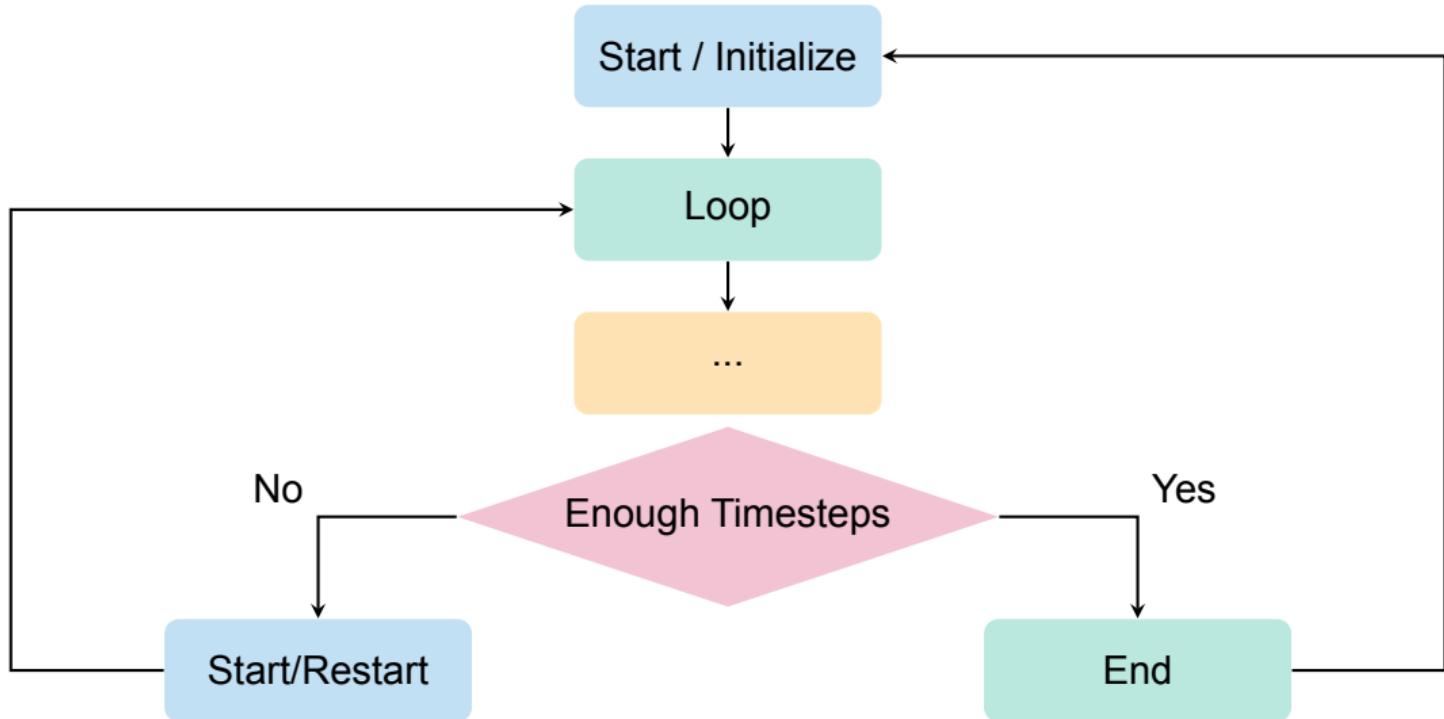
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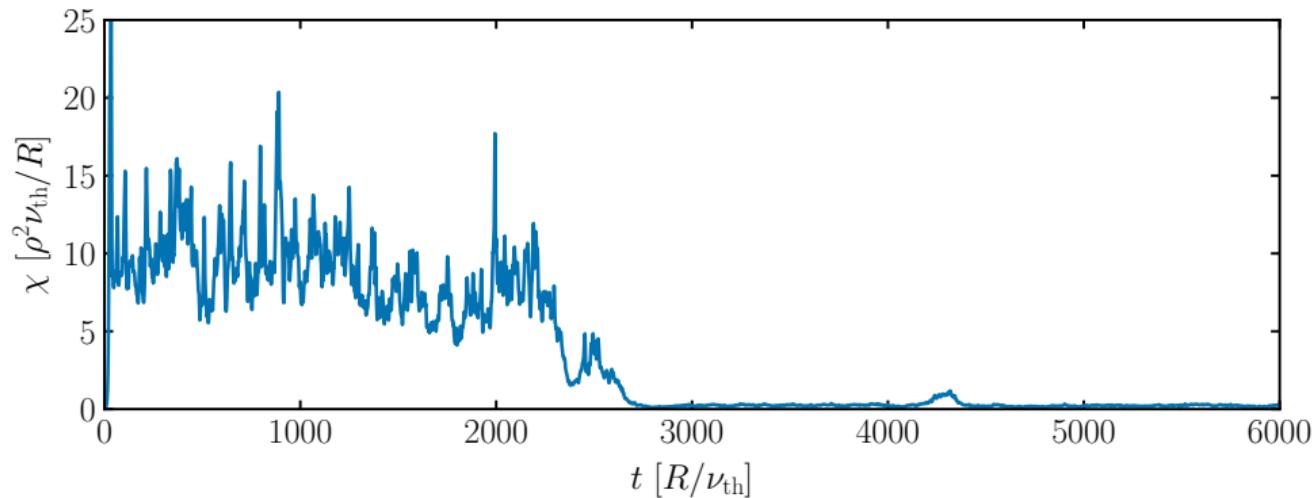
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Verification:

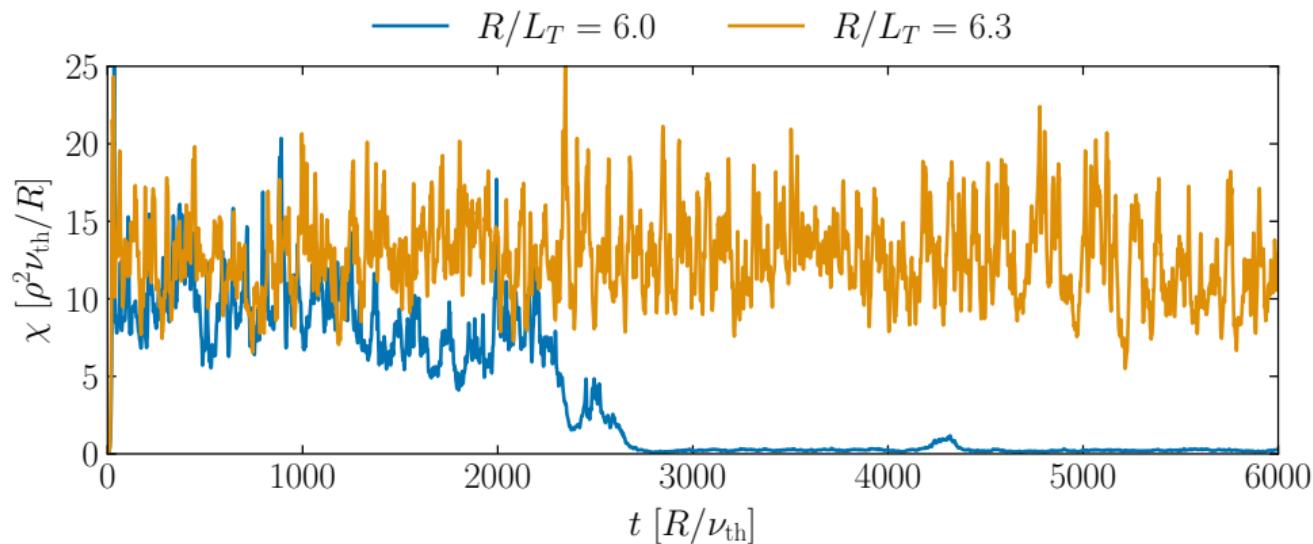
1. Reduce only one number of grid points and look if criterias (1), (2) are satisfied
2. Reduce to known the minimum number of grid points to verify result in general.

BENCHMARK

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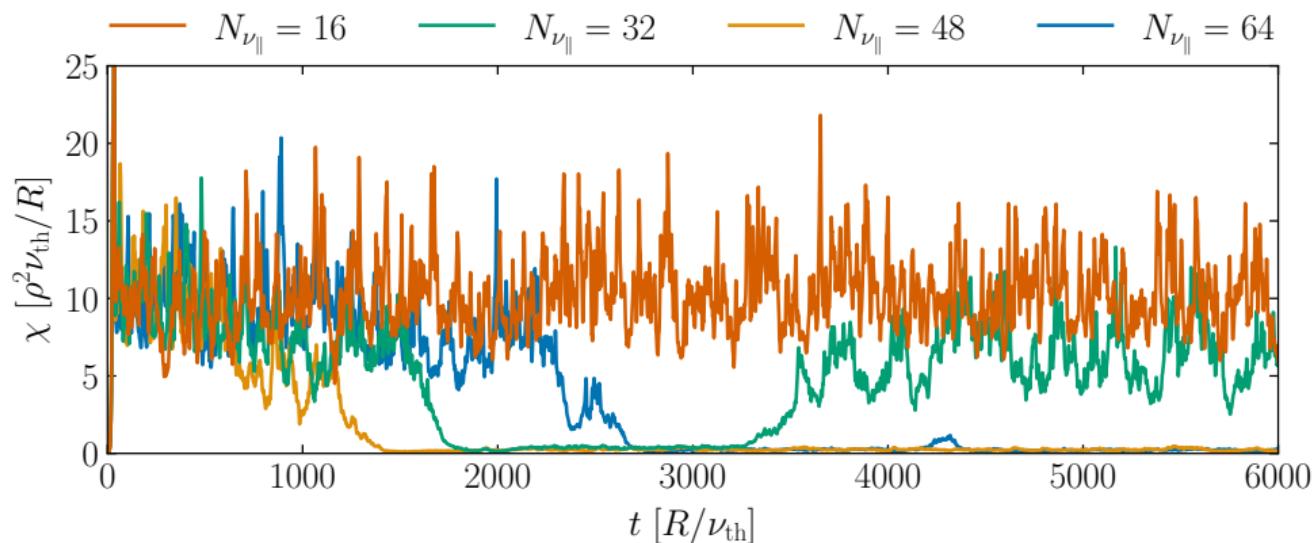


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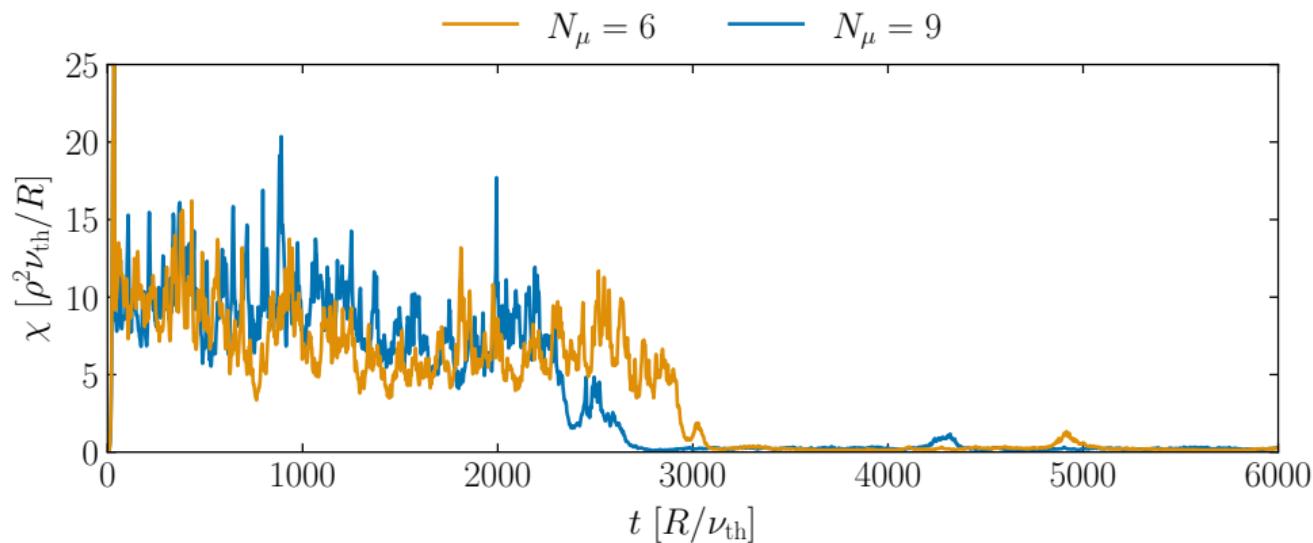


REDUCTION OF GRID POINTS

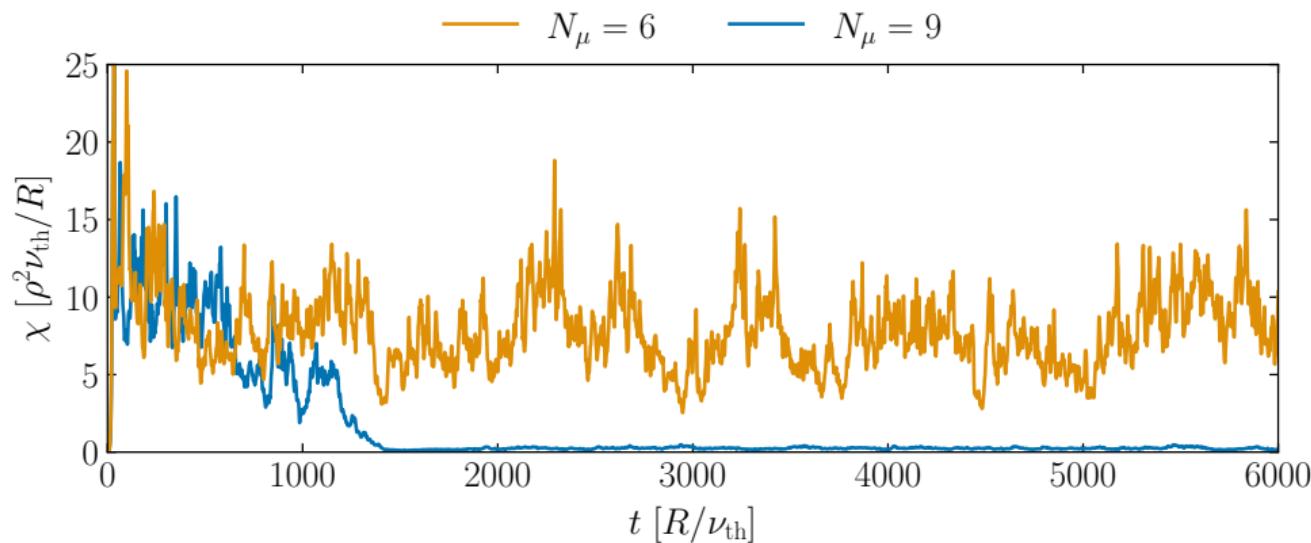
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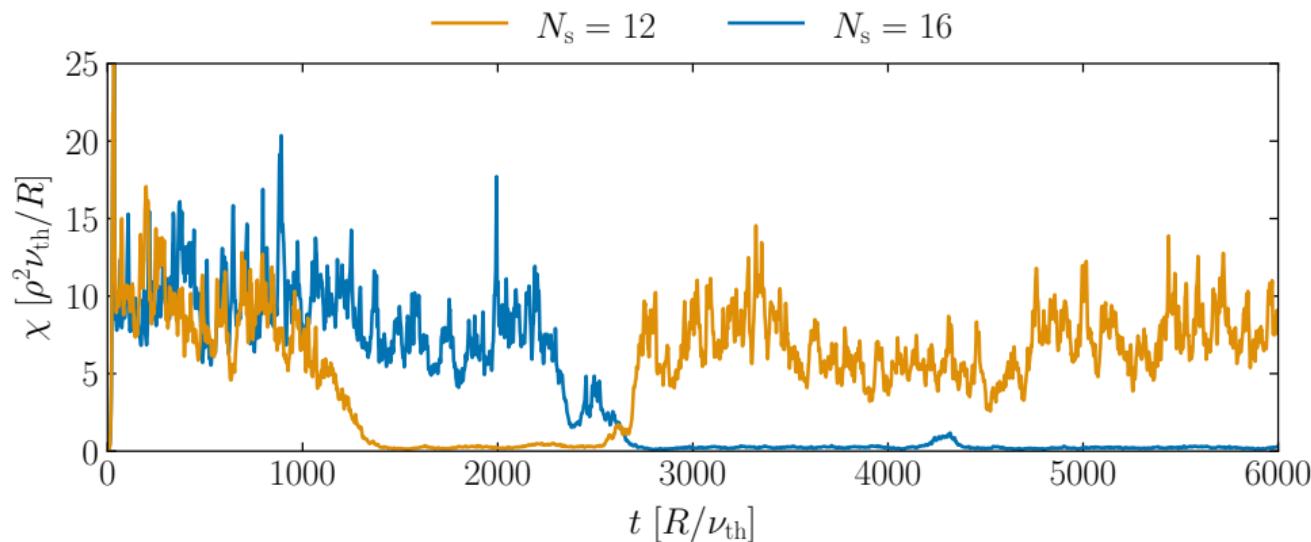
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Final Resolution

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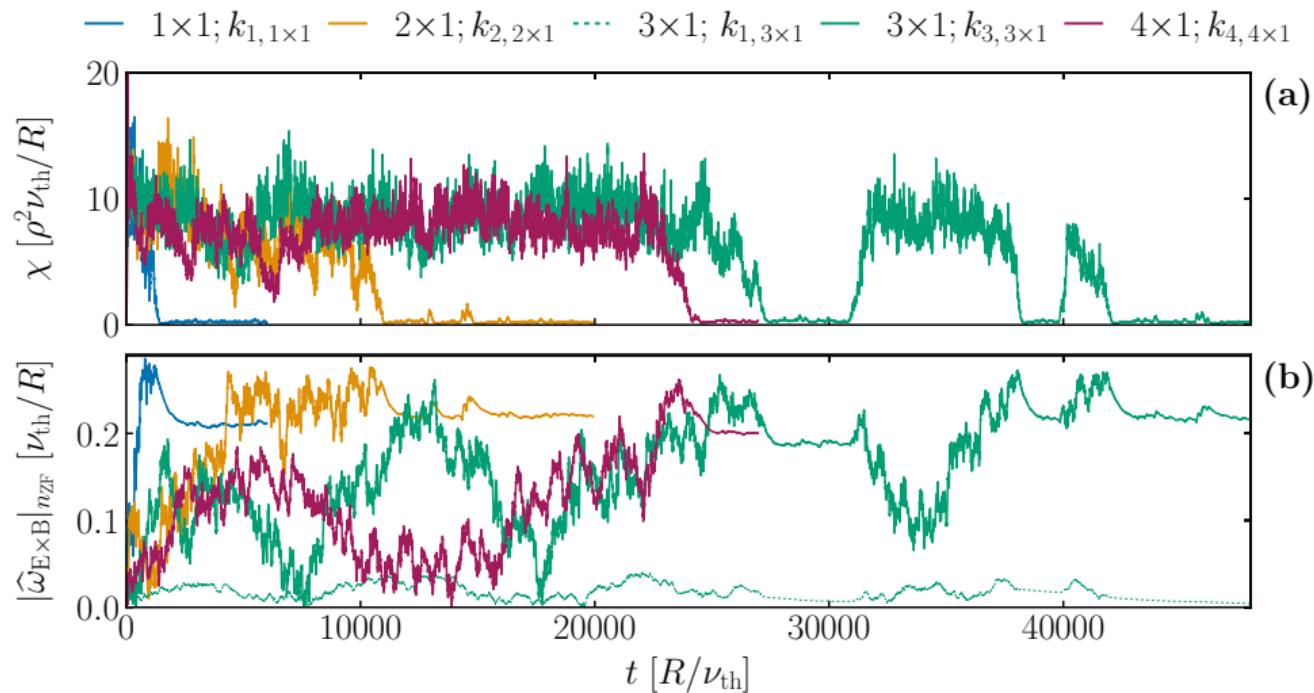
SIZE CONVERGENCE OF E × B STAIRCASE PATTERN

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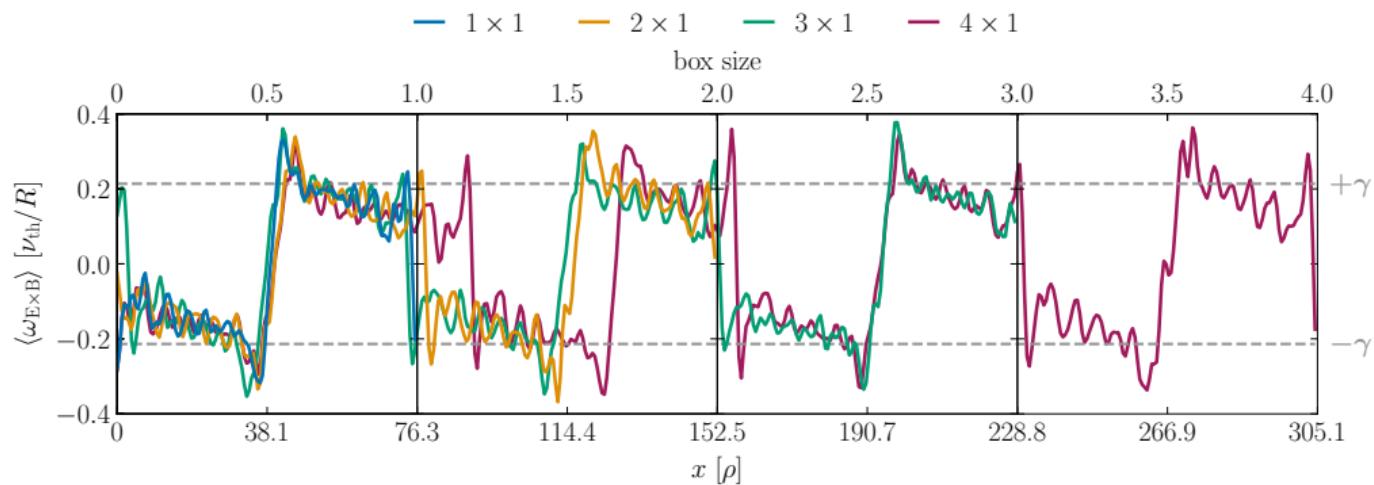
(1) Radial

$$N_R \times N_B \in [1 \times 1, 2 \times 1, 3 \times 1, 4 \times 1]$$

SIZE CONVERGENCE OF $E \times B$ STAIRCASE PATTERN



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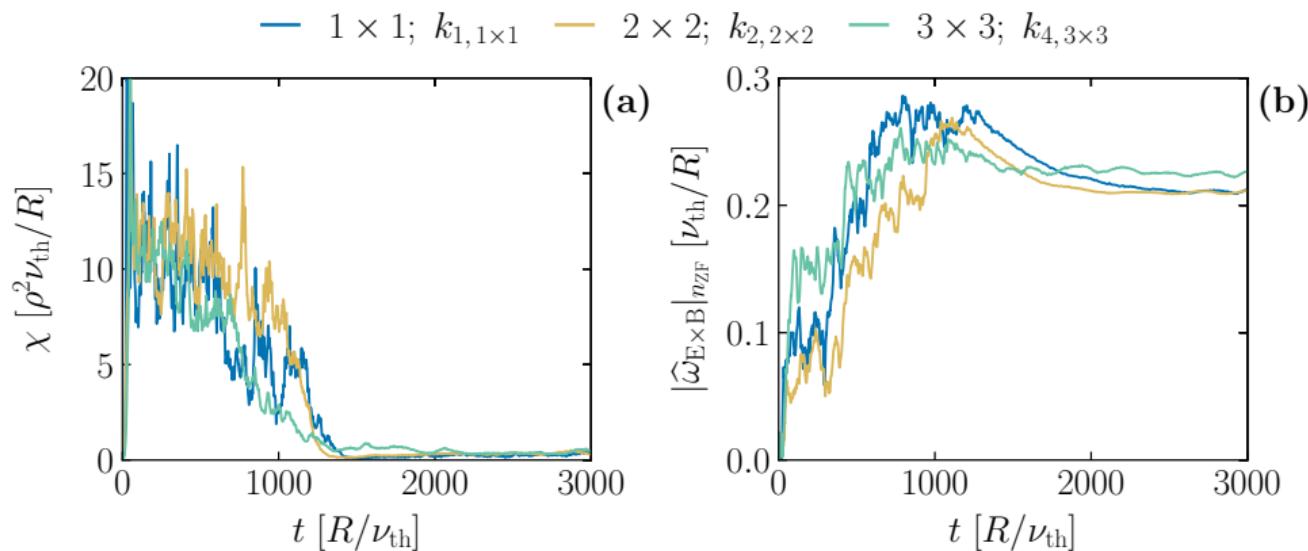


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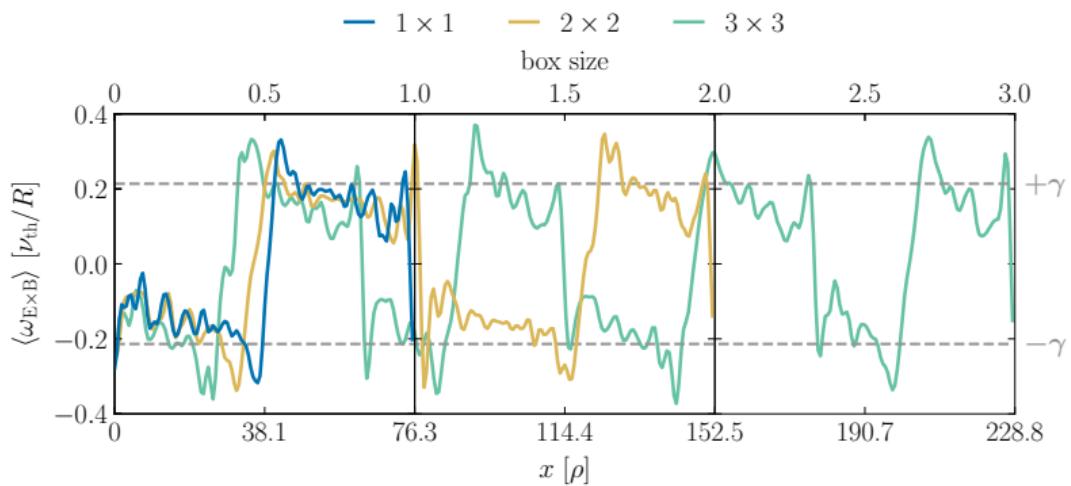
(2) Isotropic

$$N_R \times N_B \in [1 \times 1, 1.5 \times 1.5, 2 \times 2, 2.5 \times 2.5, 3 \times 3]$$

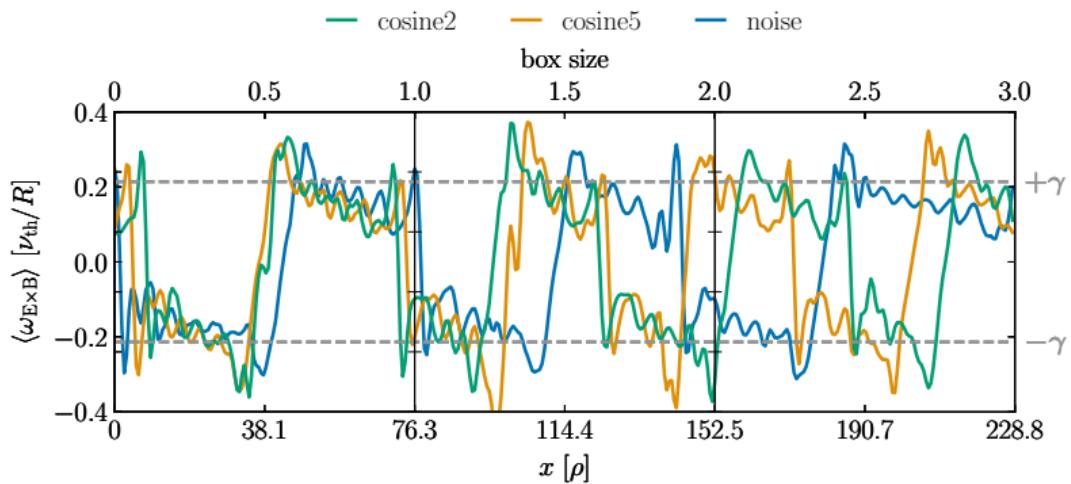
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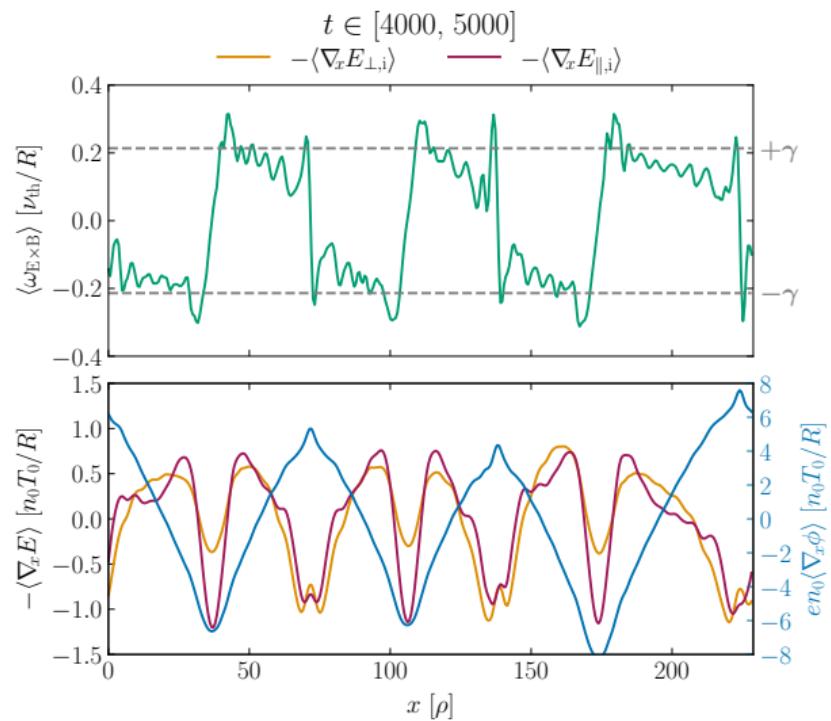
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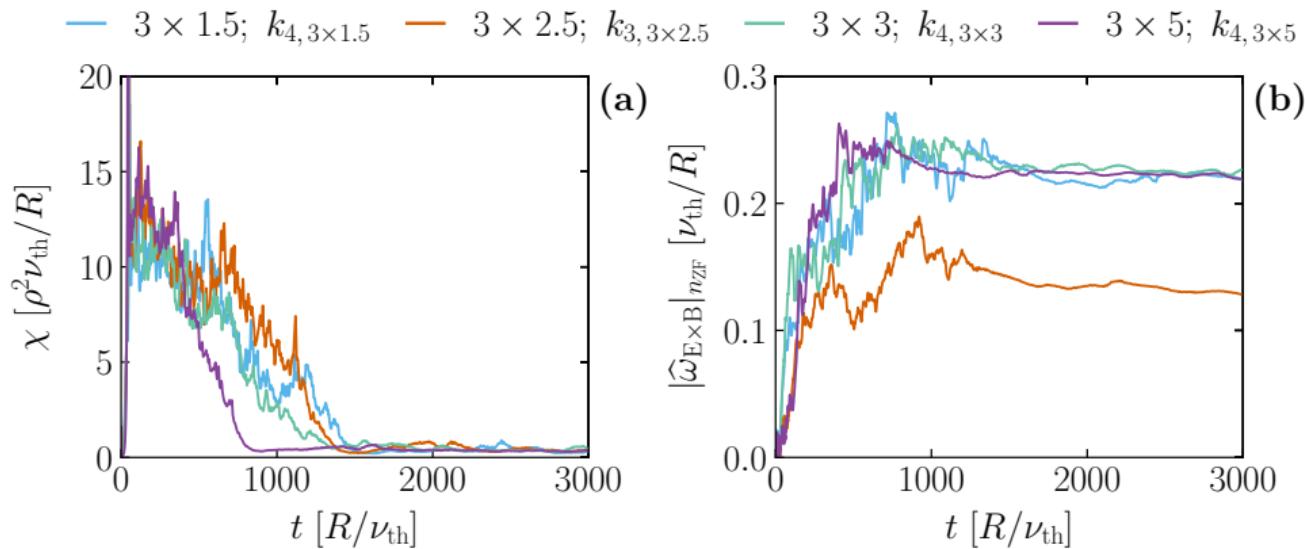


SIZE CONVERGENCE OF E × B STAIRCASE PATTERN

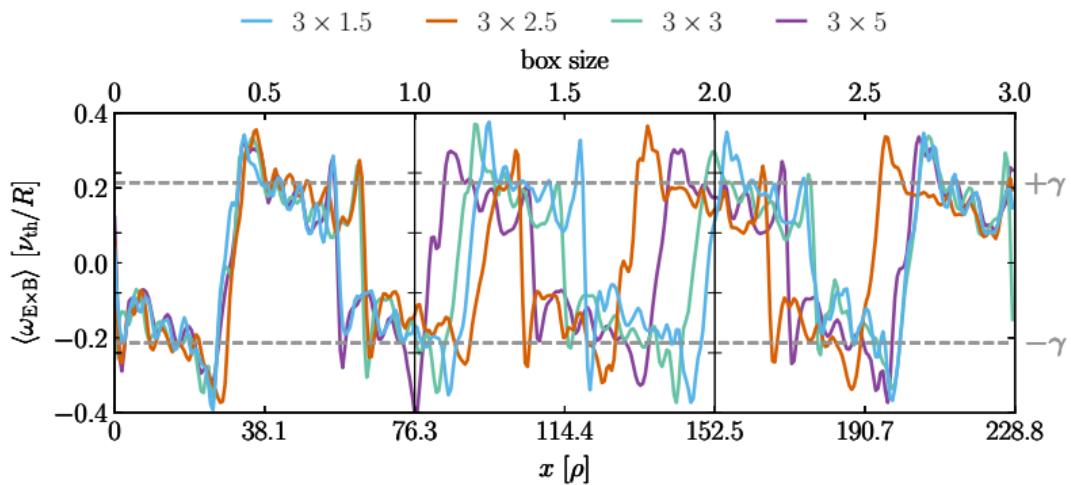
(3) Binormal

$$N_R \times N_B \in [3 \times 1.5, 3 \times 2.5, 3 \times 3, 3 \times 5]$$

SIZE CONVERGENCE OF $E \times B$ STAIRCASE PATTERN



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SIZE CONVERGENCE OF E × B STAIRCASE PATTERN

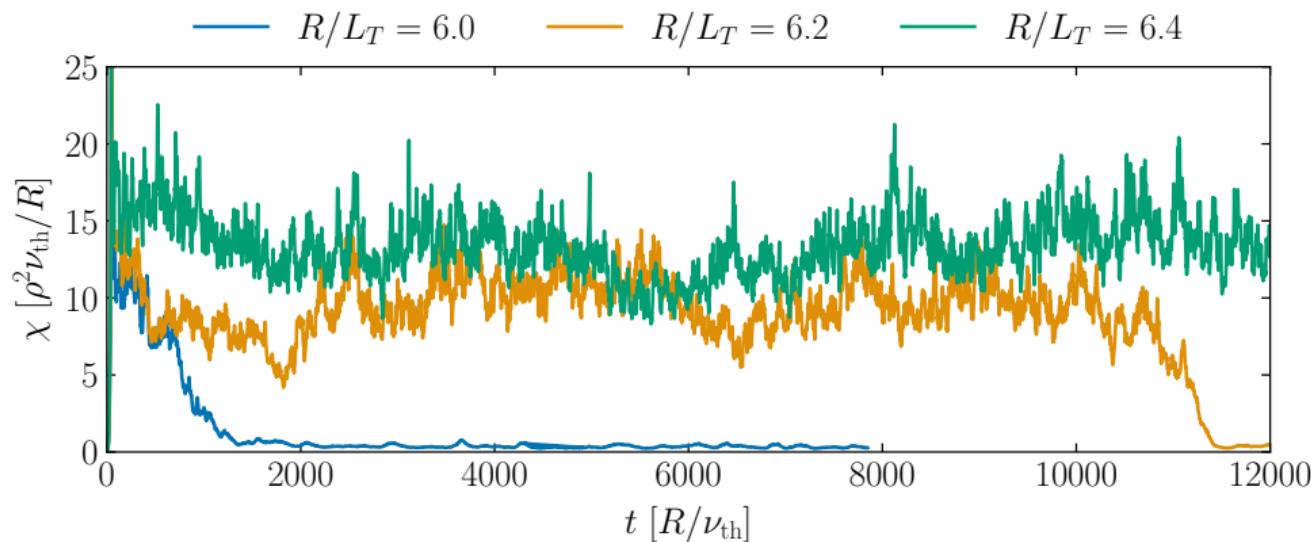
⇒ Mesoscale pattern size of:

$$\sim 57 - 76 \rho$$

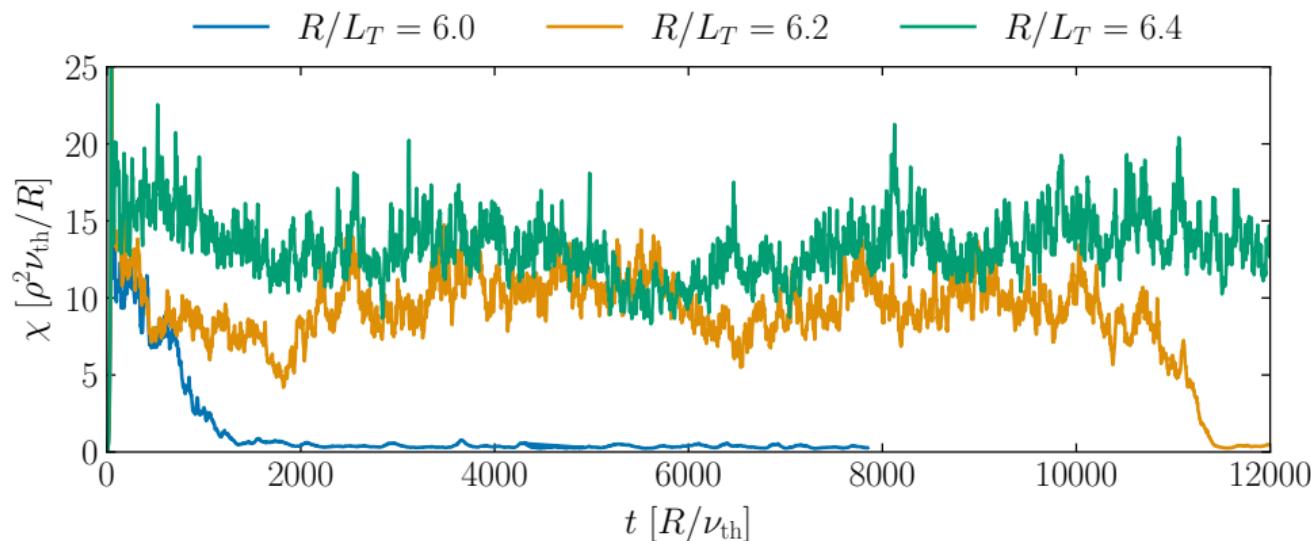
- Non-locality is inherent to ITG-driven turbulence
- Avalanches are spatially organized by the E × B staircase pattern

THE FINITE HEAT FLUX THRESHOLD

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$$\Rightarrow \boxed{R/L_T|_{\text{finite}} = 6.3 \pm 0.1}$$

CONCLUSION

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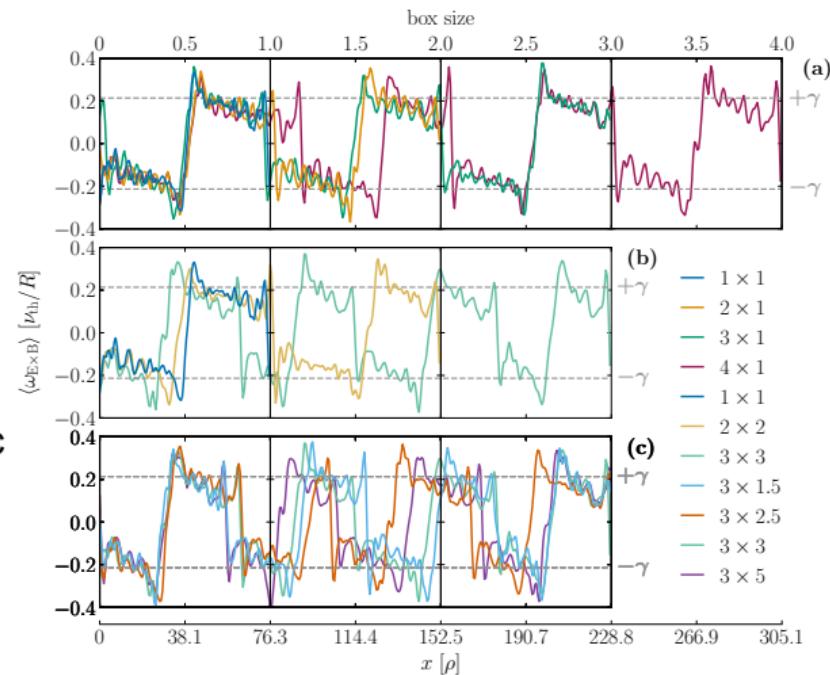
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CONCLUSION

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- Restart Script with python led to further convenience during the task of performing simulations
- Mesoscale pattern size of $\sim 57 - 76 \rho$ is found to be intrinsic to ITG-driven turbulence for Cyclone Base Case parameters
- Finite heat flux threshold is located at $R/L_T|_{\text{finite}} = 6.3 \pm 0.1$

