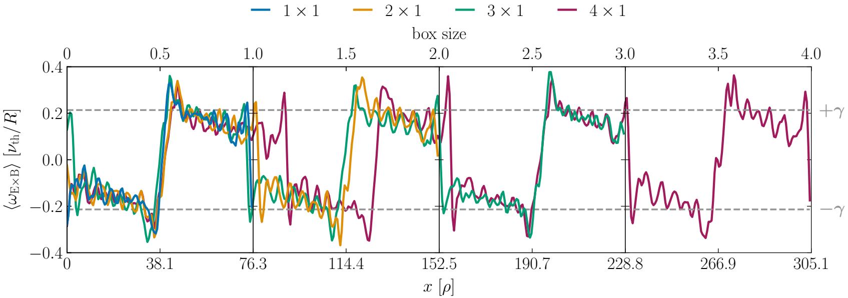
SIZE CONVERGENCE OF THE E×B STAIRCASE PATTERN IN FLUX TUBE SIMULATIONS OF ION TEMPERATURE GRADIENT-DRIVEN TURBULENCE

June 30, 2023



Manuel Lippert

Theoretical Physics V

MOTIVATION

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- **Local studies:** Radial size of $\mathrm{E} \times \mathrm{B}$ staircase structures always converge to the radial box size

Does the basic pattern size always converges to the box size, or is there a typical mesoscale size inherent to staircase structures also in a local flux-tube description?

CHARGED PARTICLE MOTION

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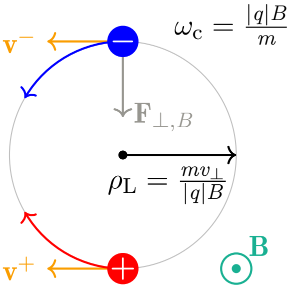
Lorentz force

$$F_{\perp,B} = |q| v_\perp B$$



$$\rho = \frac{m v_{th}}{|q|B} \; ; \; v_{th} = \sqrt{\frac{2T}{m}}$$

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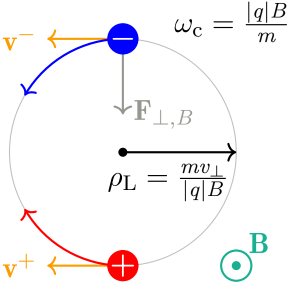

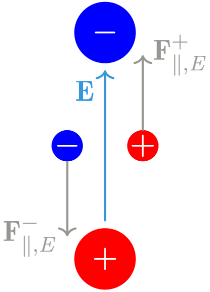

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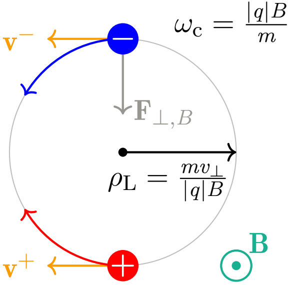
Inhomogeneous magnetic field

$$F_{\|,\nabla_\| B} = -\underbrace{\frac{mv_\perp^2}{2B}}_{\mu} \nabla_\| B$$



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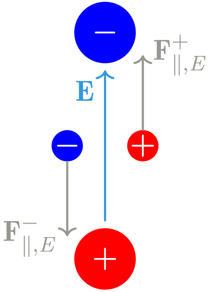

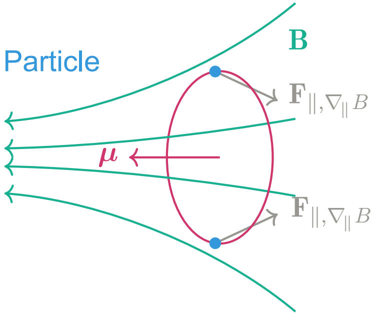

DRIFT IN THE GYROCENTER

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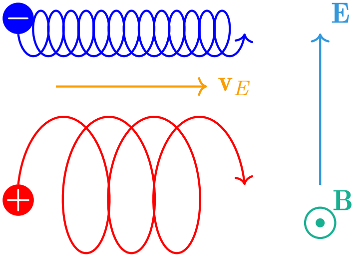
$\mathbf{E} \times \mathbf{B}$ Drift

$$\mathbf{v}_E = \frac{\mathbf{E}\times\mathbf{B}}{B^2}$$



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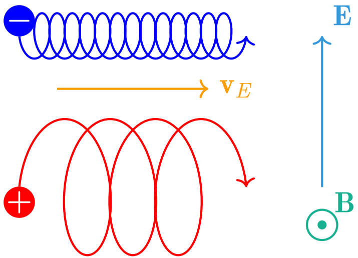
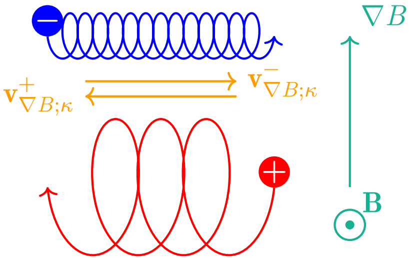
∇B Drift

$$\mathbf{v}_{\nabla B} = \frac{m v_\perp^2}{2q}\frac{\mathbf{B}\times\nabla B}{B^3}$$

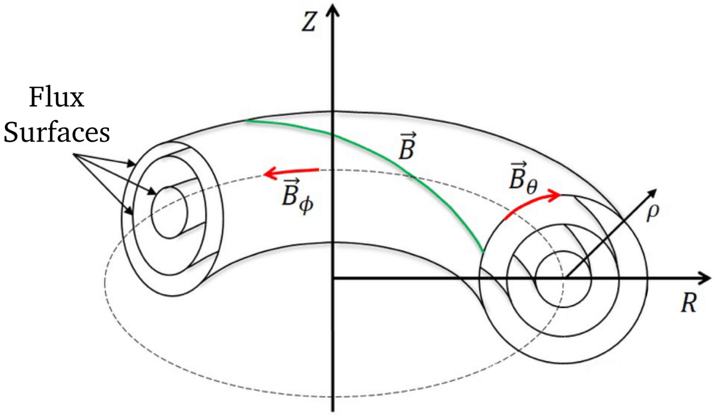
Curvature Drift

$$\mathbf{v}_\kappa = \frac{m v_\parallel^2}{qB^2}\mathbf{B}\times\underbrace{\boldsymbol{\kappa}}_{\frac{\nabla B}{B}} = \frac{m v_\parallel^2}{q}\frac{\mathbf{B}\times\nabla B}{B^3}$$





ION TEMPERATURE GRADIENT (ITG)-DRIVEN INSTABILITY



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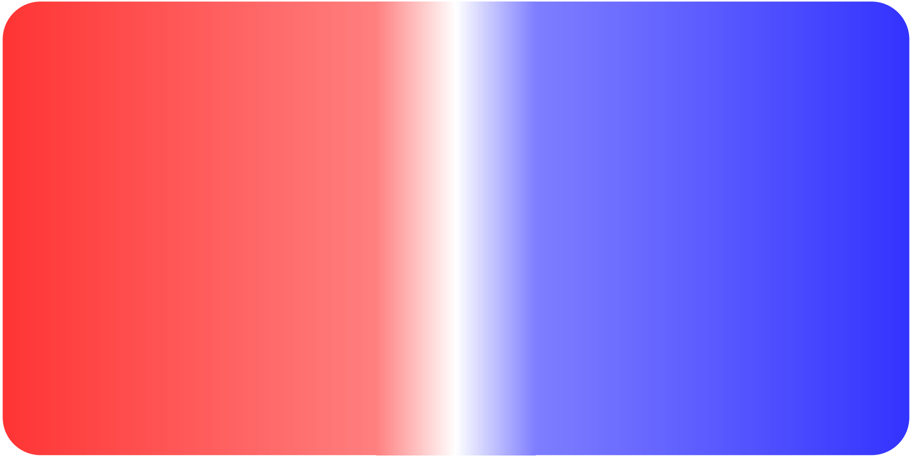

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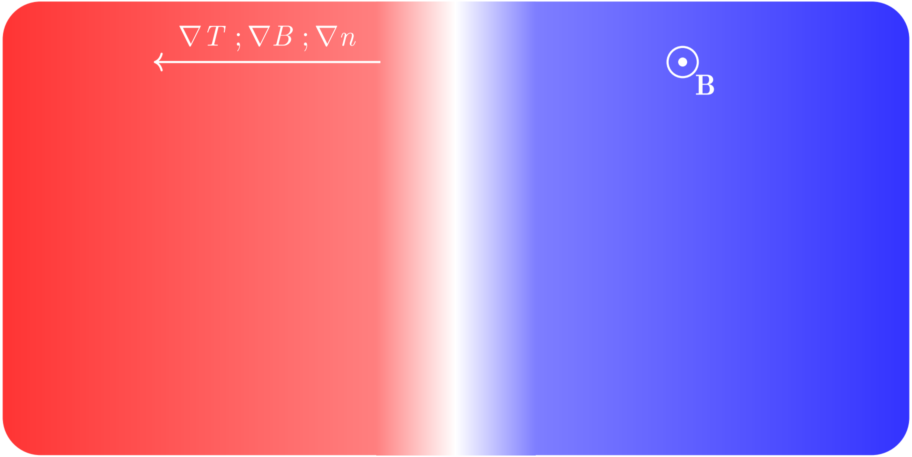

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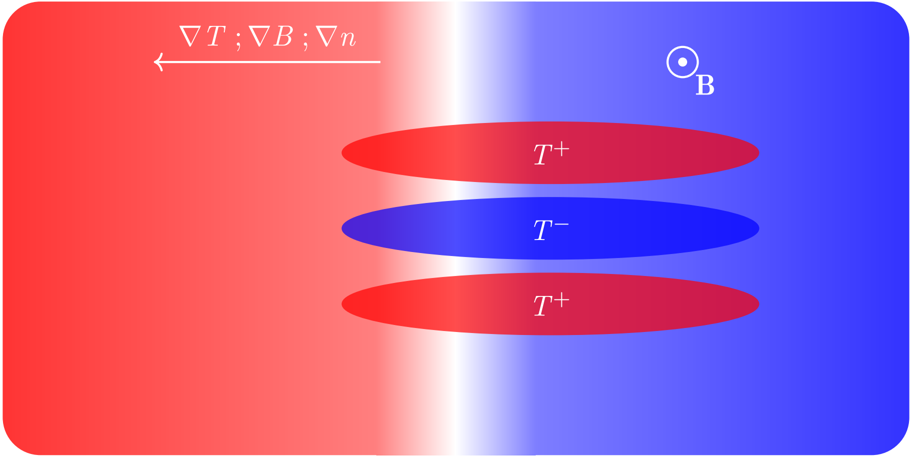

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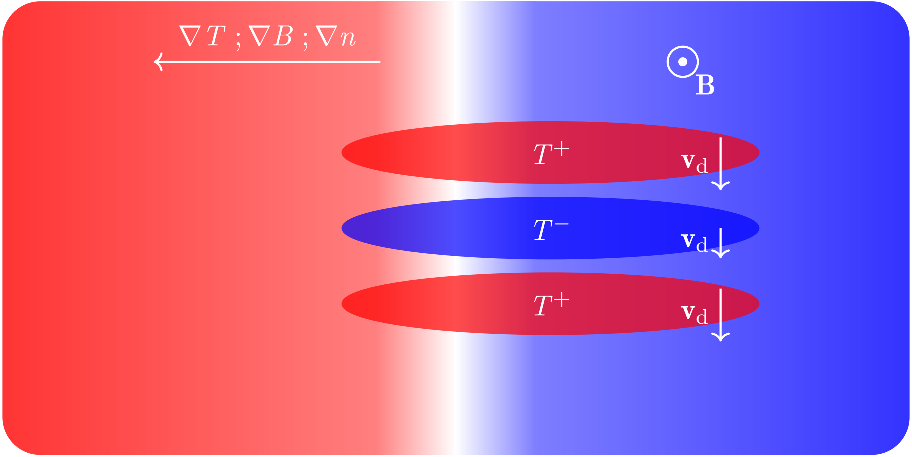

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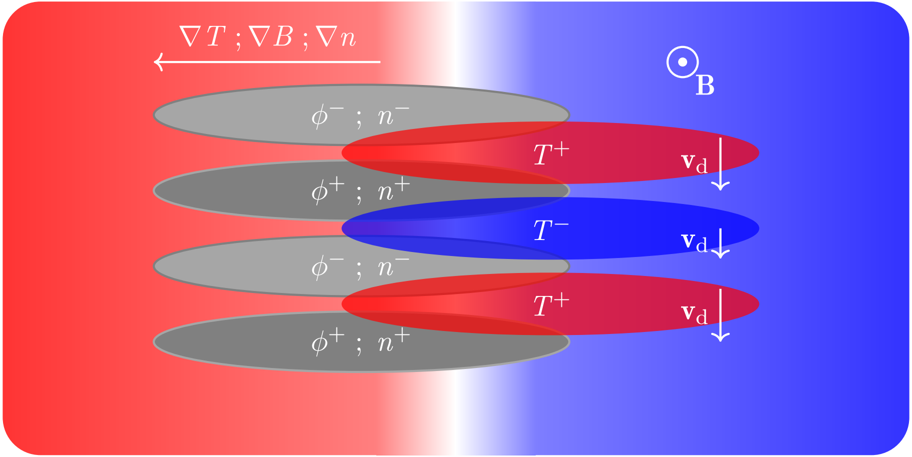

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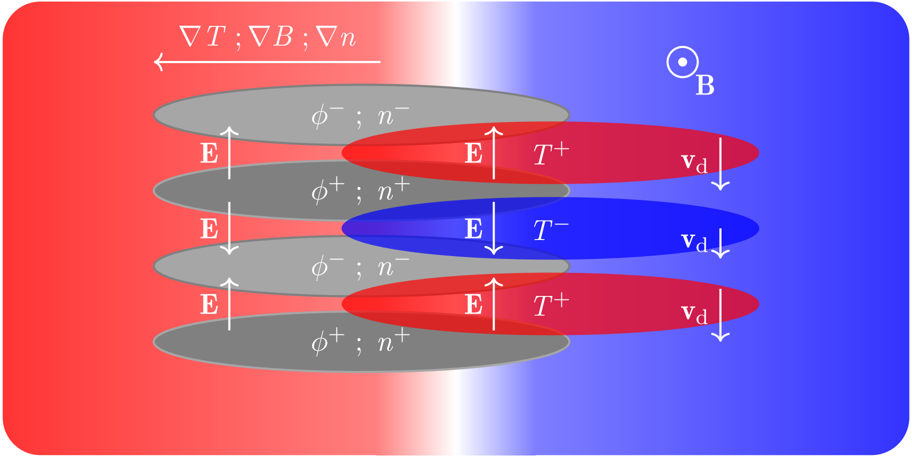

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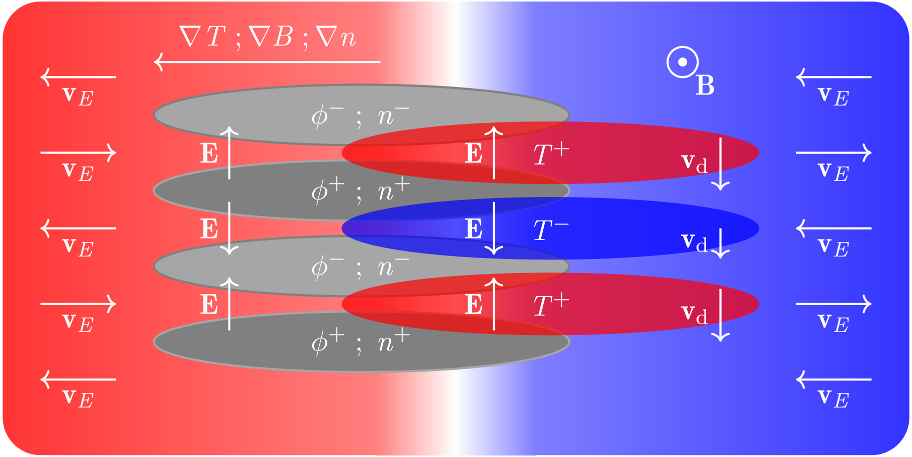

ZONAL FLOWS & SHEARING RATE

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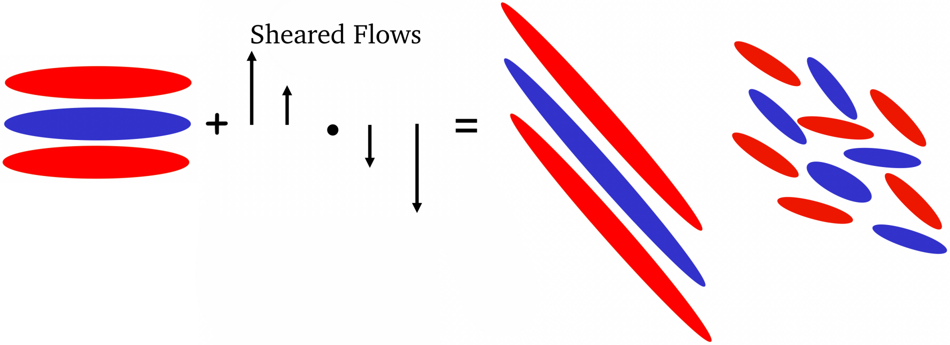

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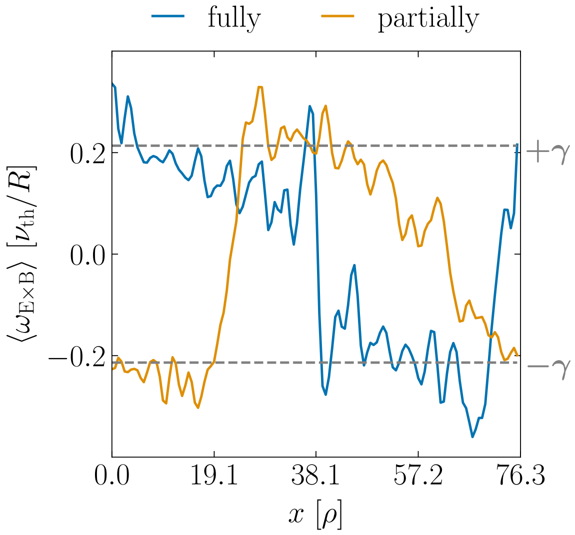

$$\omega_{\mathrm{E\times B}} = \frac{1}{2}\frac{\partial^2 \langle \phi \rangle}{\partial x^2}$$

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	N_{m}	N_x	N_{s}	$N_{\nu_{\parallel}}$	N_μ	D	ν_{d}	$D_{\nu_{\parallel}}$	D_x	D_y	Order	$k_y\rho$	$k_x\rho$
S6	21	83	16	64	9	1	$\lvert\nu_{\parallel}\rvert$	0.2	0.1	0.1	6	1.4	2.1

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$$\omega_{\mathrm{E\times B}} = \sum_{k_{\mathrm{ZF}}} \widehat{\omega}_{\mathrm{E\times B}}(k_{\mathrm{ZF}}, t)\ \exp(\mathrm{i} k_{\mathrm{ZF}} x)$$

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- Waltz criterion $|\omega_{\mathrm{E\times B}}| \approx \gamma$

RESTART SCRIPT

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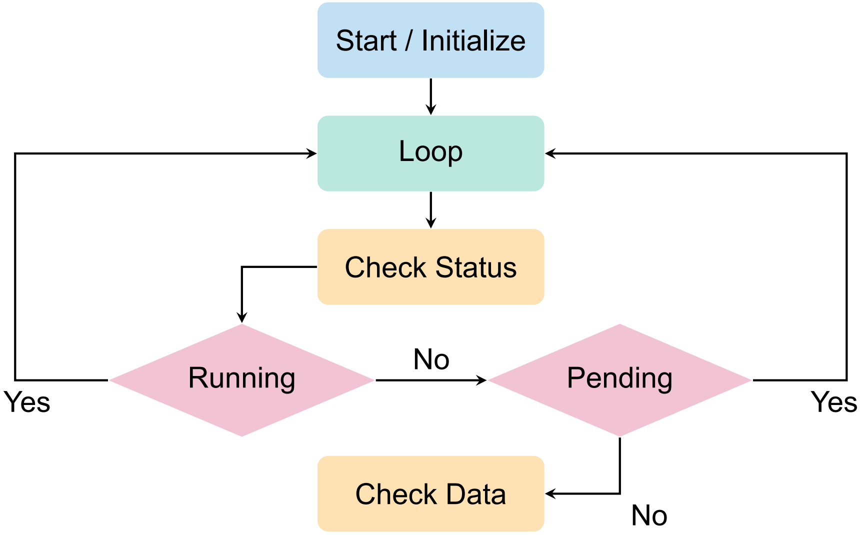

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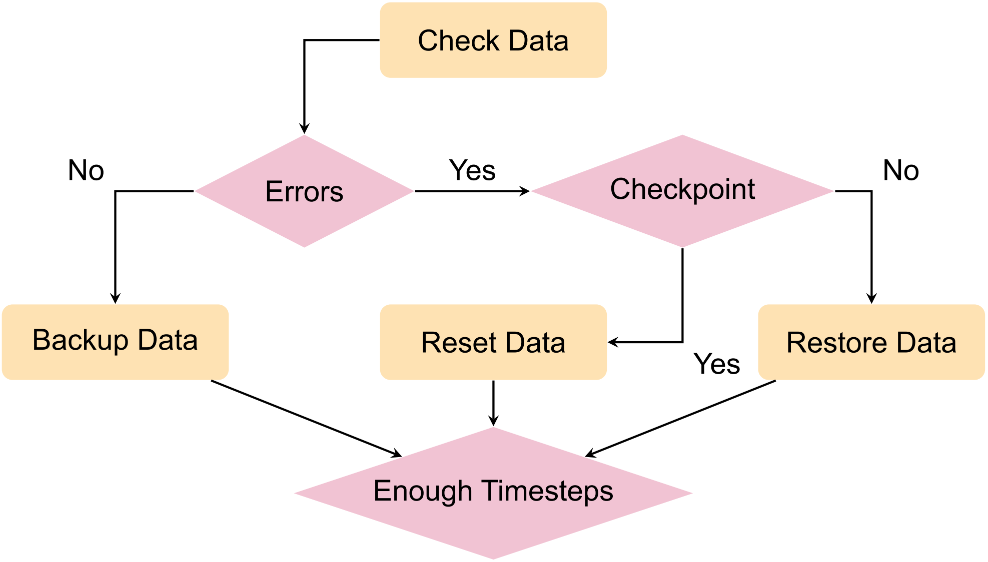

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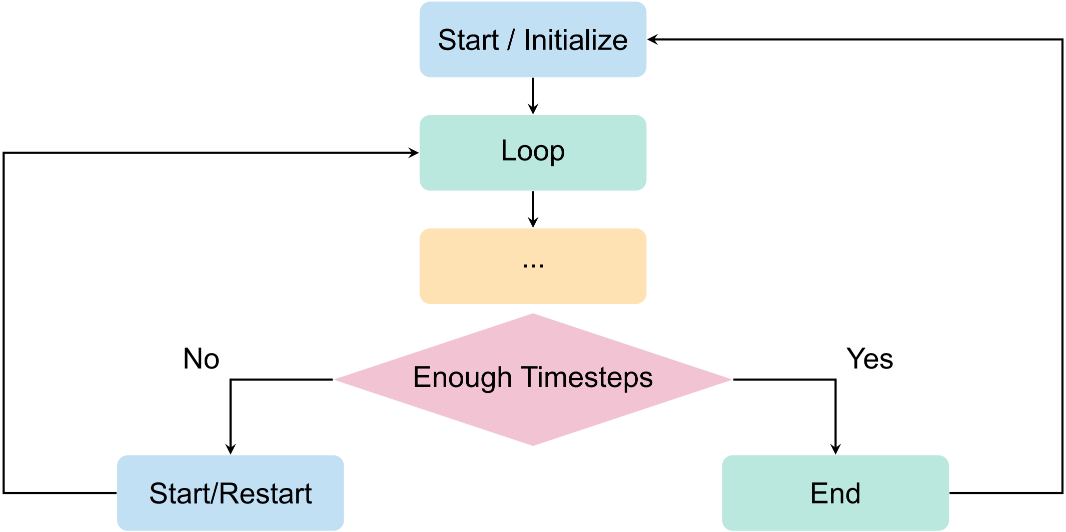

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Verification:

1. Reduce only one number of grid points and look if criteria **(1), (2)** are satisfied,
2. Reduce to the known minimum number of grid points to verify result in general.

BENCHMARK

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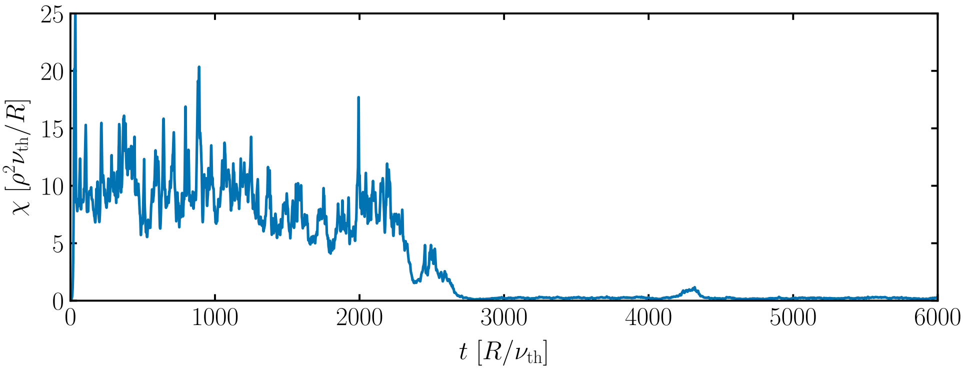
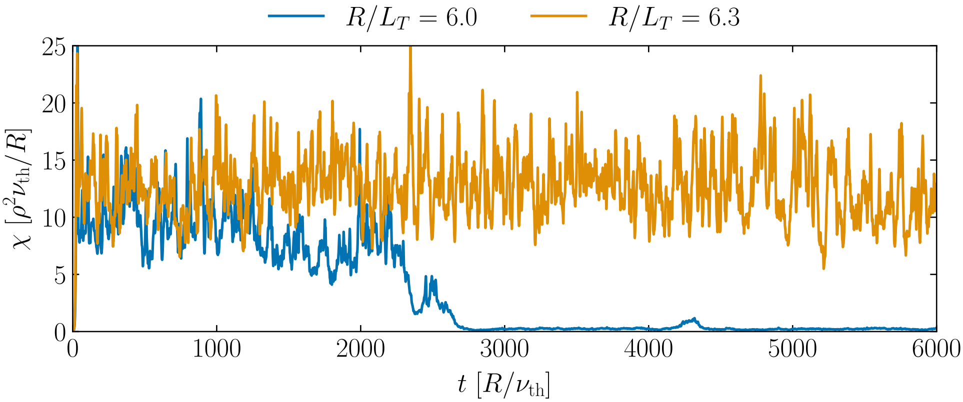

25
20
15
10
5
0
0
1000
2000
3000
4000
5000
6000
$\chi\ [\rho^2 \nu_{\mathrm{th}}/R]$
$t\ [R/\nu_{\mathrm{th}}]$

BENCHMARK


$R/L_T = 6.0$
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0
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REDUCTION OF GRID POINTS

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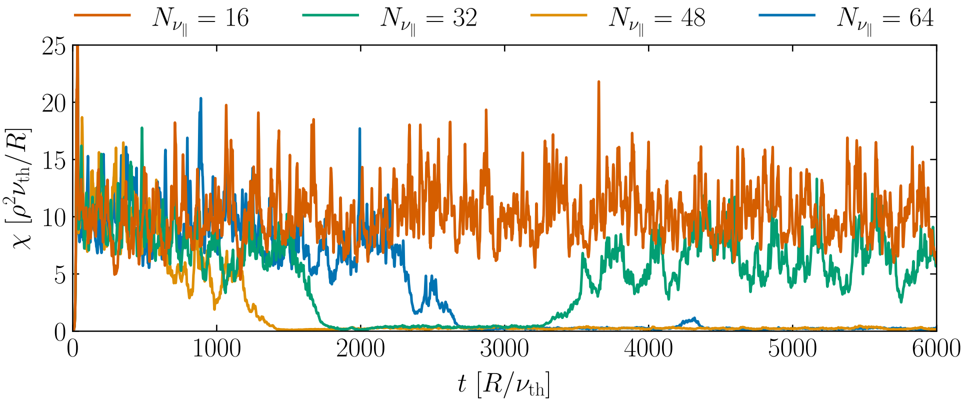

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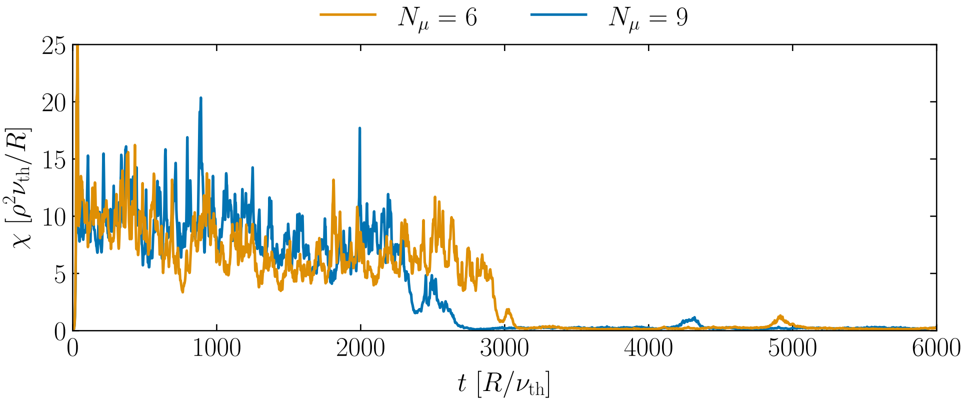

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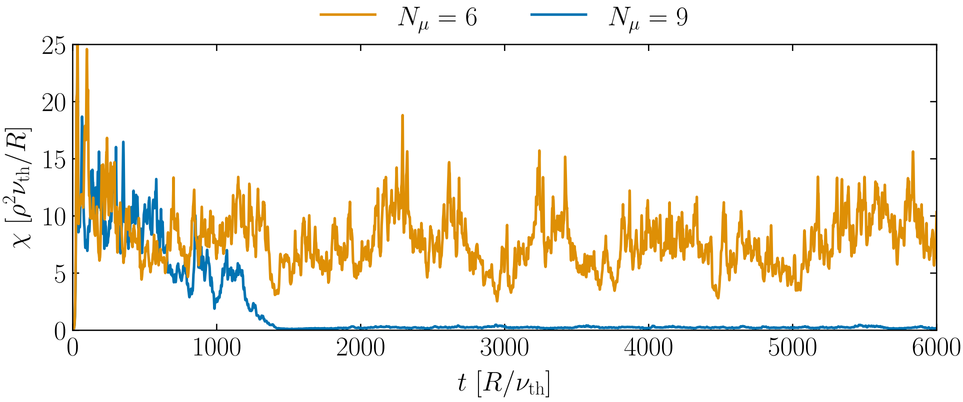

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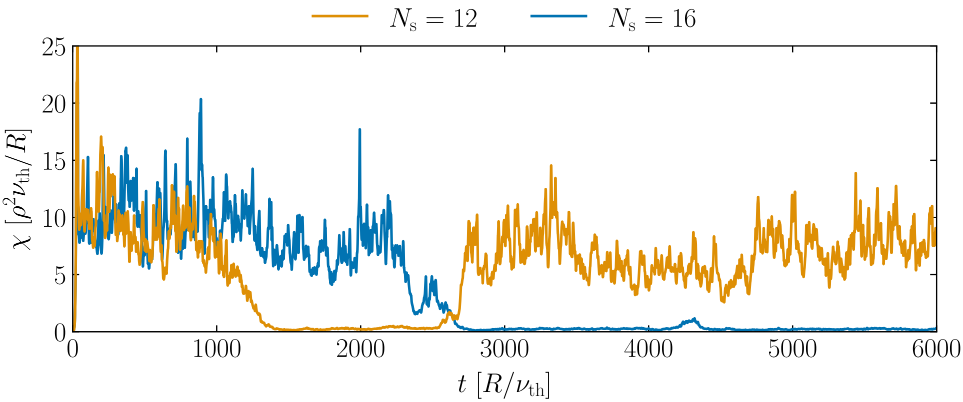

REDUCTION OF GRID POINTS

Final Resolution

	N_m	N_x	N_s	$N_{\nu_\parallel}$	N_μ	D	ν_d	$D_{\nu_\parallel}$	D_x	D_y	Order	$k_y\rho$	$k_x\rho$
S6	21	83	16	48	9	1	$\lvert\nu_\parallel\rvert$	0.2	0.1	0.1	6	1.4	2.1

SIZE CONVERGENCE OF $\mathrm{E} \times \mathrm{B}$ STAIRCASE PATTERN

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(1) Radial

$$N_{\mathrm{R}} \times N_{\mathrm{B}} \in [1 \times 1,\ 2 \times 1,\ 3 \times 1,\ 4 \times 1]$$

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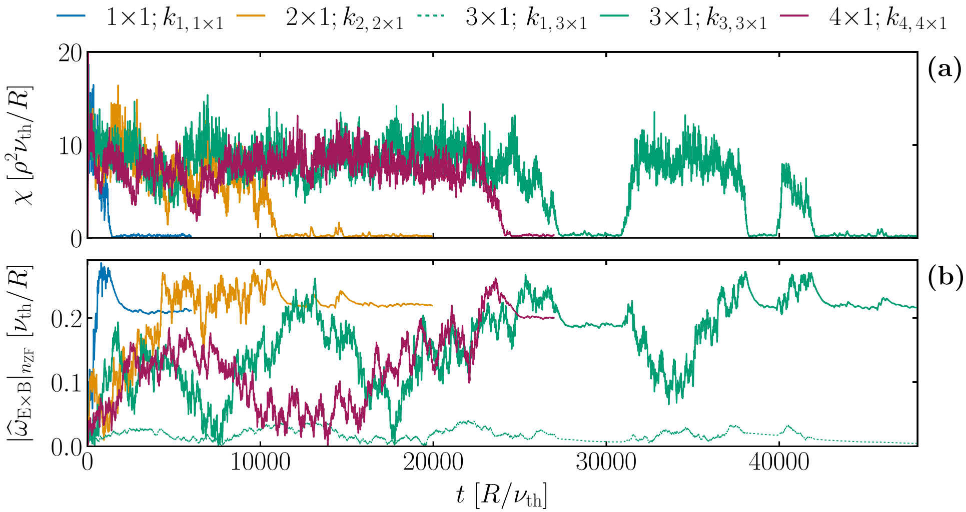

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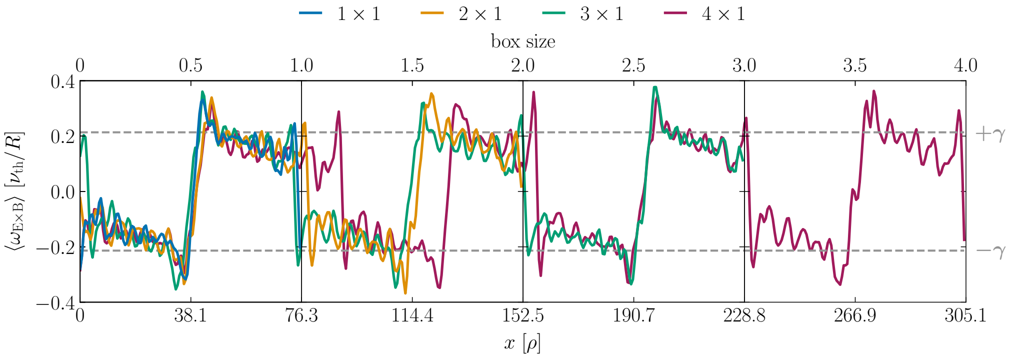

SIZE CONVERGENCE OF $\mathrm{E} \times \mathrm{B}$ STAIRCASE PATTERN

(2) Isotropic

$$N_{\mathrm{R}} \times N_{\mathrm{B}} \in [1 \times 1,\ 1.5 \times 1.5,\ 2 \times 2,\ 2.5 \times 2.5,\ 3 \times 3]$$

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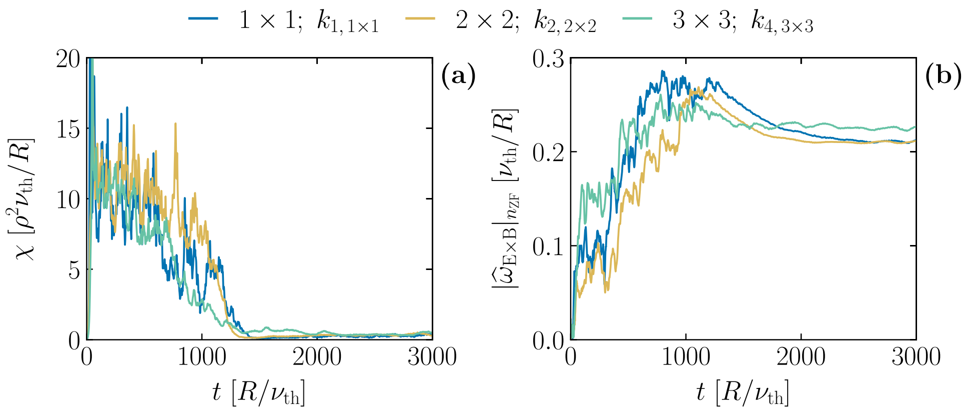

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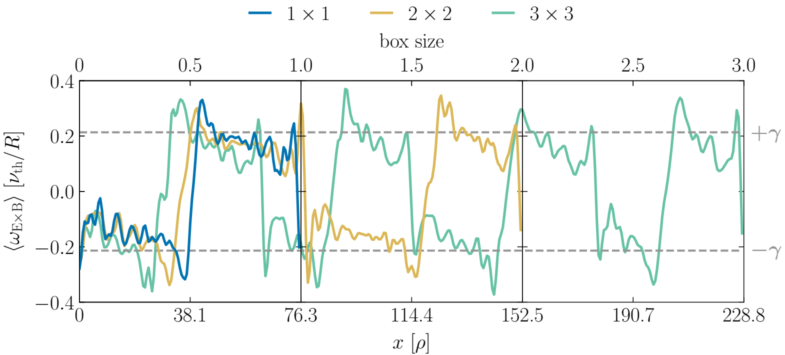

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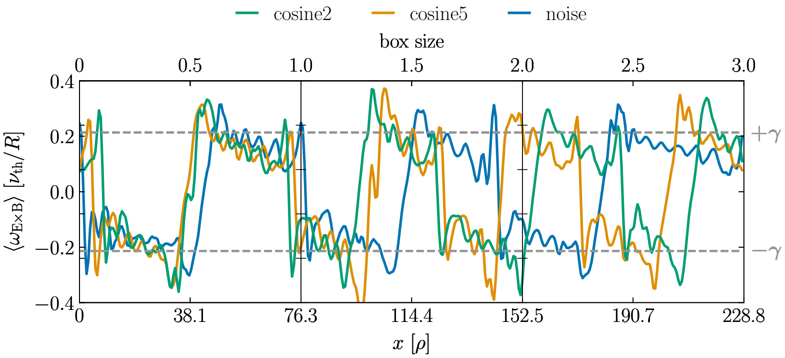

SIZE CONVERGENCE OF $\mathrm{E} \times \mathrm{B}$ STAIRCASE PATTERN

(3) Binormal

$$N_{\mathrm{R}} \times N_{\mathrm{B}} \in [3 \times 1.5,\ 3 \times 2.5,\ 3 \times 3,\ 3 \times 5]$$

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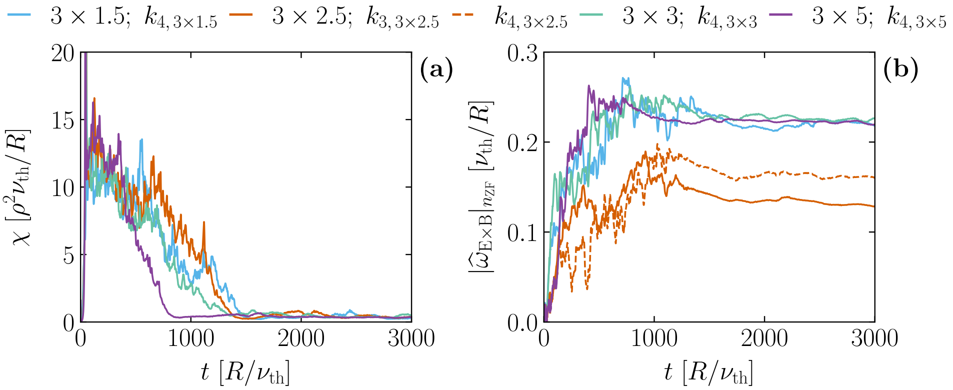

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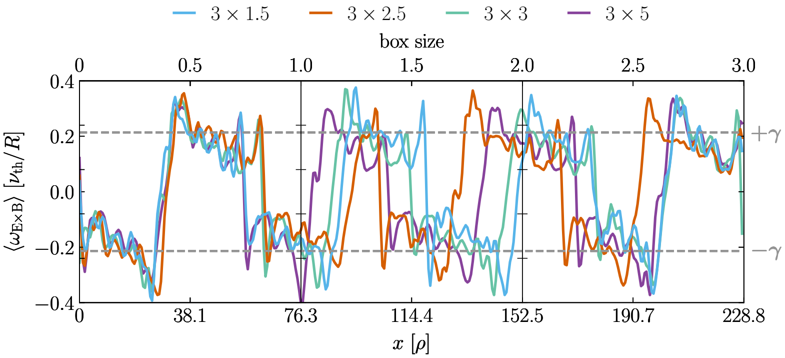

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Does the basic pattern size always converges to the box size, or is there a typical mesoscale size inherent to staircase structures also in a local flux-tube description?

$\Rightarrow$ Mesoscale pattern size of:

$$\boxed{\sim 57 - 76\,\rho}$$

- Non-locality is inherent to ITG-driven turbulence
- Avalanches are spatially organized by the $\mathrm{E} \times \mathrm{B}$ staircase pattern

THE FINITE HEAT FLUX THRESHOLD

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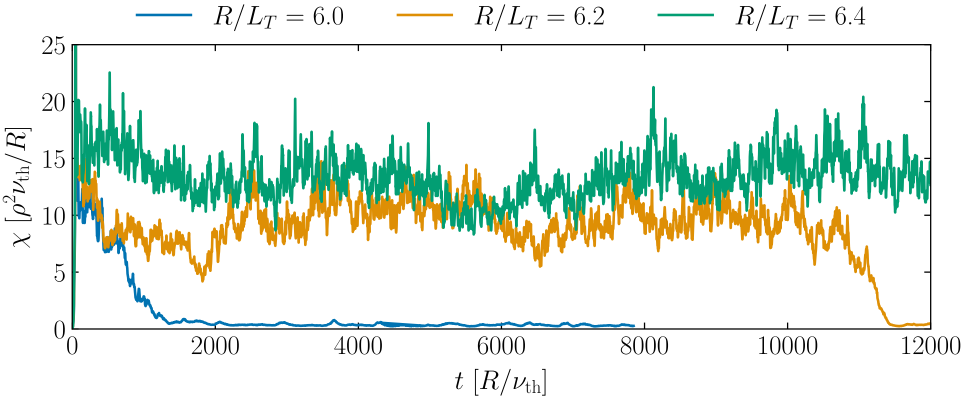
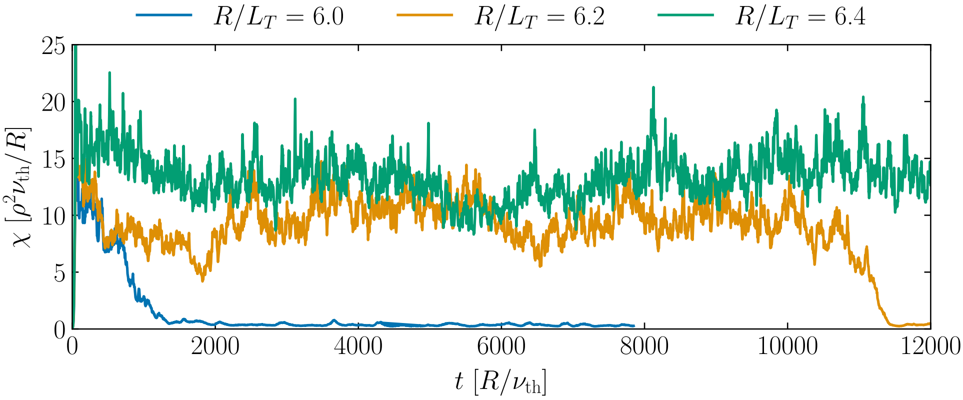


THE FINITE HEAT FLUX THRESHOLD



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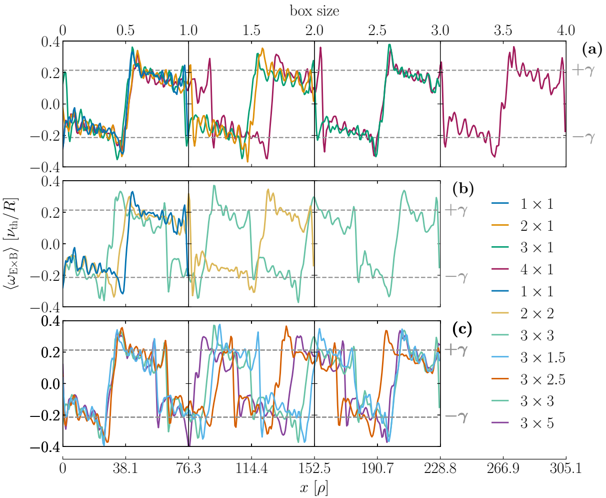

MAGNETIC CONFINEMENT IN TOKAMAK

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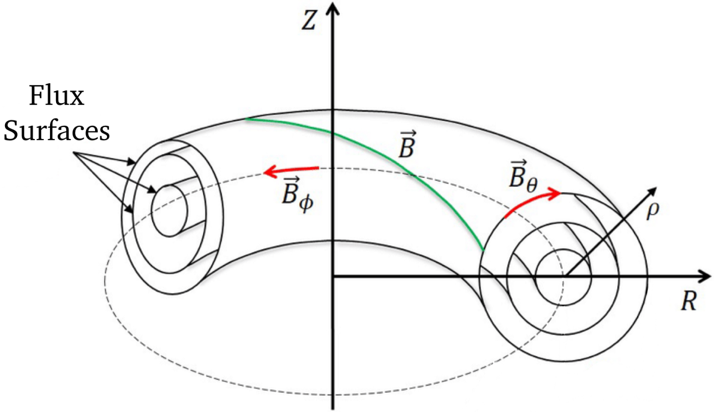

$$\beta = \frac{nT}{\mu_0 B^2/2}$$

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$$\omega_{\,\mathrm{E}\times\mathrm{B}} = \frac{1}{2}\frac{\partial^2 \langle\phi\rangle}{\partial x^2} \qquad \langle\phi\rangle = \frac{1}{L_y}\int_0^{L_y} \mathrm{d}y\; \phi(x, y, s = 0)$$

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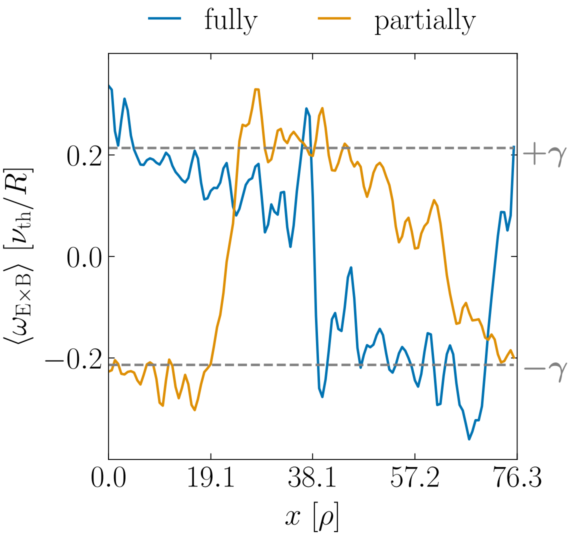

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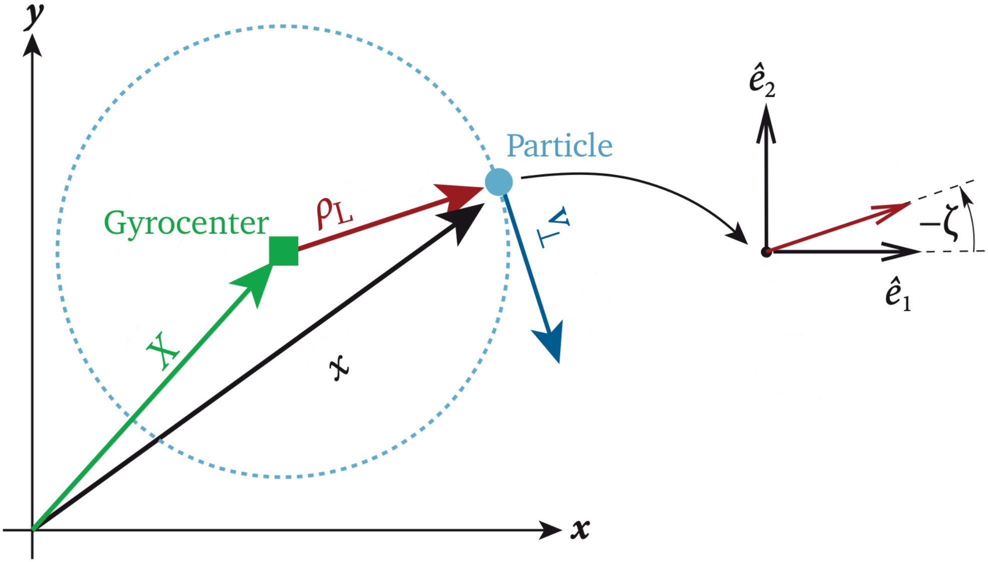

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$$\frac{\partial g}{\partial t} + \mathbf{v}_\chi \cdot \nabla g + (v_\parallel \mathbf{b} + \mathbf{v}_\mathrm{D}) \cdot \nabla(\delta f) - \frac{\mu B}{m} \frac{\mathbf{B}\cdot\nabla B}{B^2} \frac{\partial(\delta f)}{\partial v_\parallel} = S$$

Gyrokinetic Equation

$$g \sim (f_0 + \delta f) \qquad S \sim (f + f_0) \qquad \mathbf{v}_\mathrm{D} \sim \nabla B \qquad \mathbf{v}_\chi \sim \mathrm{E} \times \mathrm{B} \qquad f(x, y, s, v_\parallel, \mu)$$

SIZE CONVERGENCE OF $\mathrm{E} \times \mathrm{B}$ STAIRCASE PATTERN