

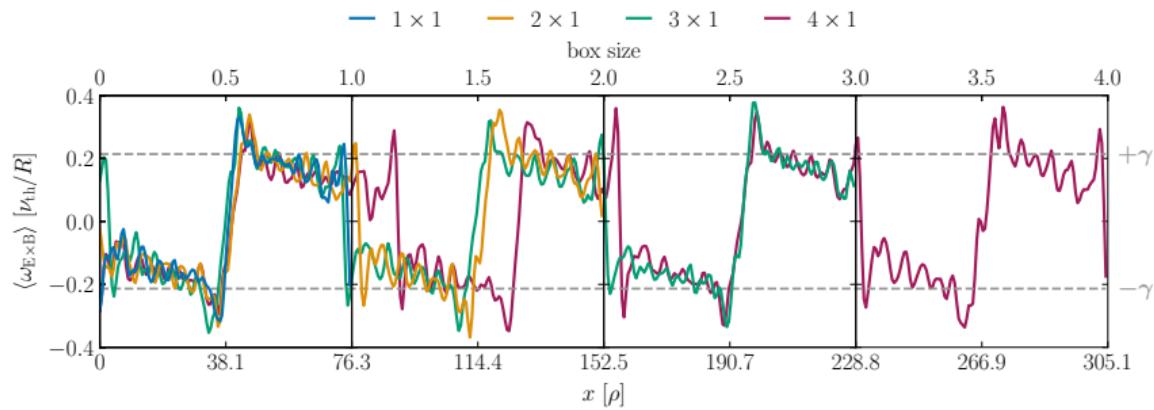


SIZE CONVERGENCE OF THE $E \times B$ STAIRCASE PATTERN IN FLUX TUBE SIMULATIONS OF ION TEMPERATURE GRADIENT-DRIVEN TURBULENCE

June 30, 2023

Manuel Lippert

Theoretical Physics V



MOTIVATION

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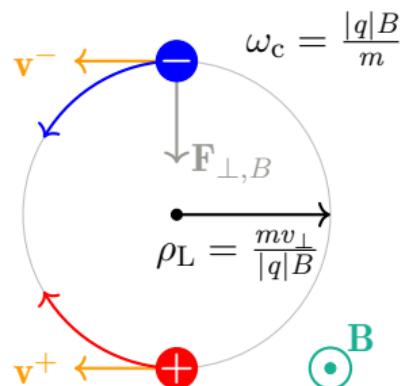
Does the basic pattern size always converges to the box size, or is there a typical mesoscale size inherent to staircase structures also in a local flux-tube description?

CHARGED PARTICLE MOTION

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Lorentz force

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$$\rho = \frac{mv_{\text{th}}}{|q|B} ; v_{\text{th}} = \sqrt{\frac{2T}{m}}$$

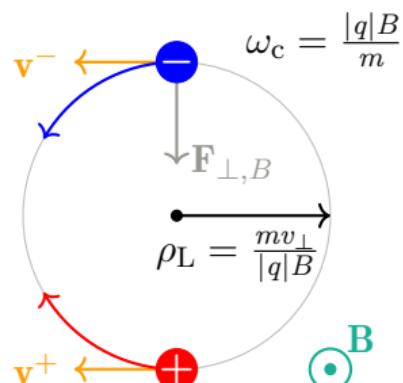
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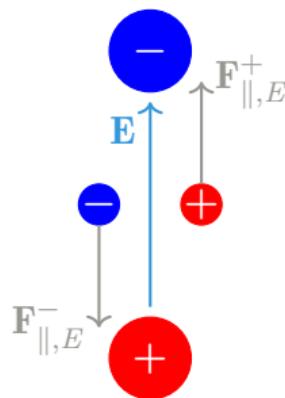
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Electric force

$$F_{\parallel,E} = qE_{\parallel}$$



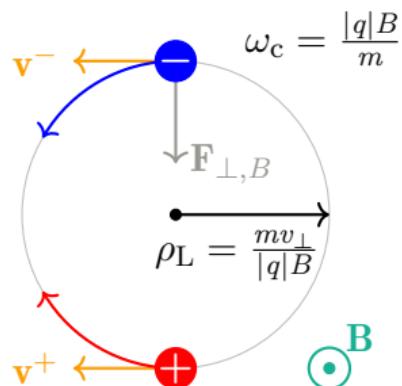
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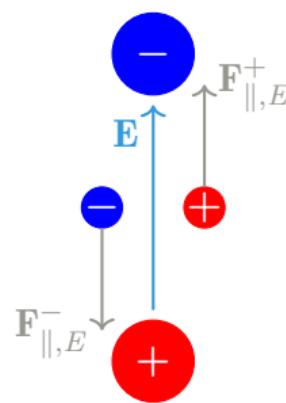
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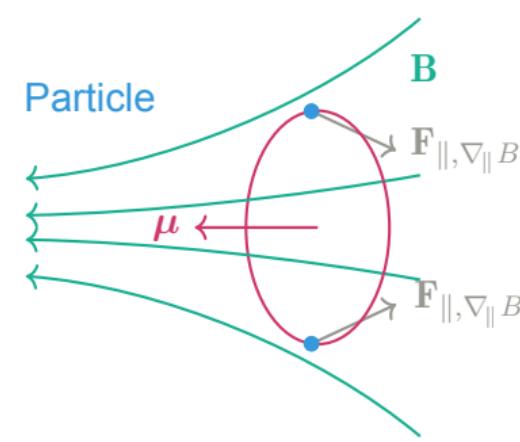
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Inhomogeneous magnetic field

$$F_{\parallel,\nabla_{\parallel}B} = - \underbrace{\frac{mv_{\perp}^2}{2B}}_{\mu} \nabla_{\parallel}B$$

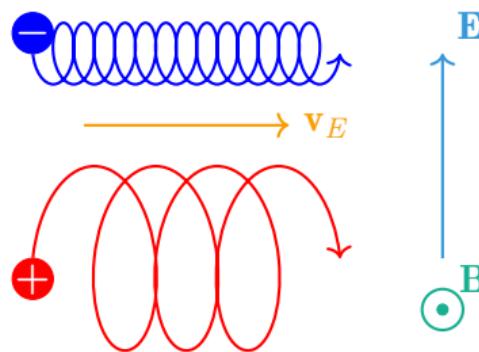


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$E \times B$ Drift

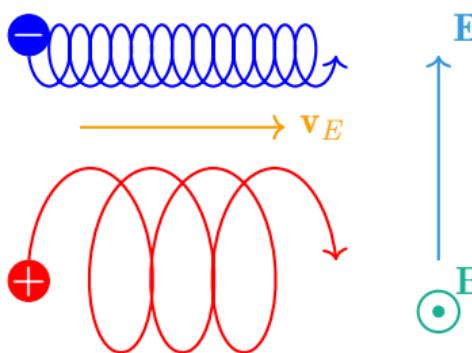
$$\mathbf{v}_E = \frac{\mathbf{E} \times \mathbf{B}}{B^2}$$



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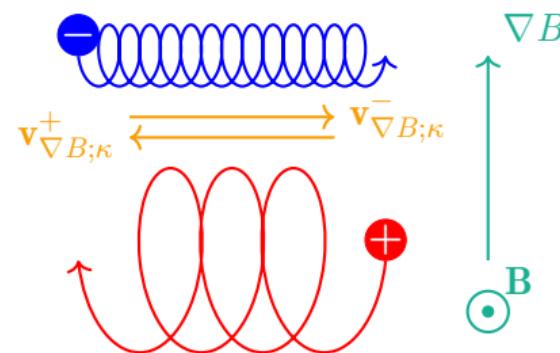


∇B Drift

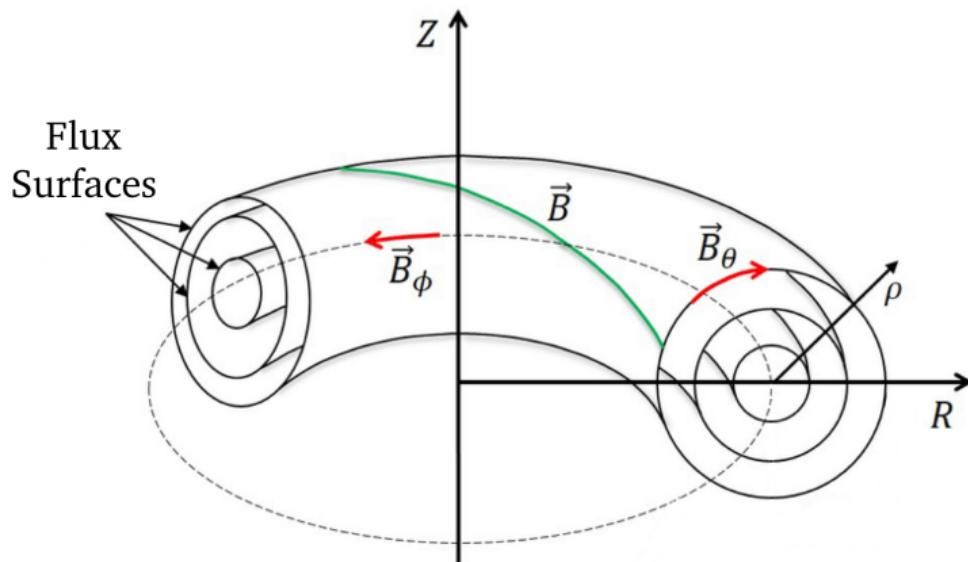
$$\mathbf{v}_{\nabla B} = \frac{mv_{\perp}^2}{2q} \frac{\mathbf{B} \times \nabla B}{B^3}$$

Curvature Drift

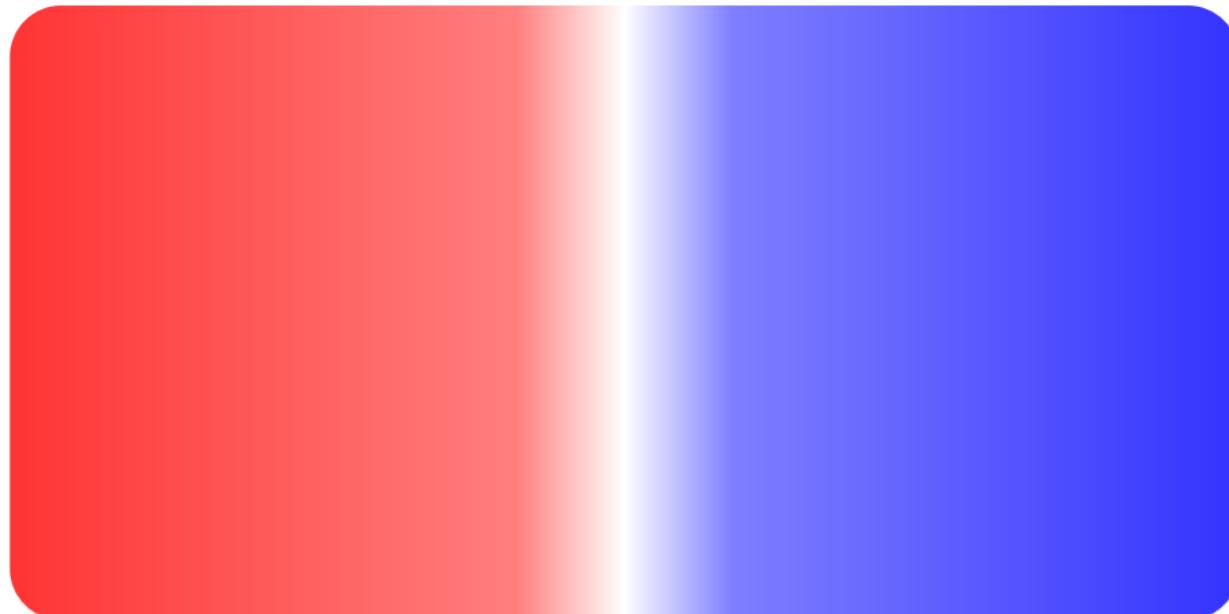
$$\mathbf{v}_{\kappa} = \frac{mv_{\parallel}^2}{qB^2} \mathbf{B} \times \underbrace{\boldsymbol{\kappa}}_{\frac{\nabla B}{B}} = \frac{mv_{\parallel}^2}{q} \frac{\mathbf{B} \times \nabla B}{B^3}$$



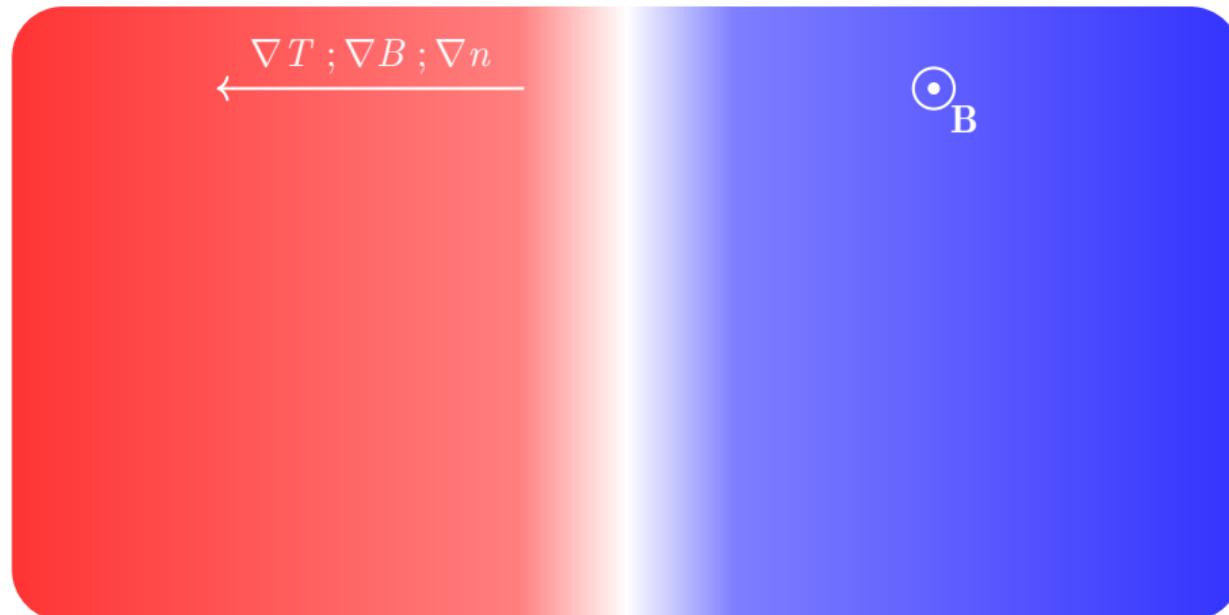
ION TEMPERATURE GRADIENT (ITG)-DRIVEN INSTABILITY



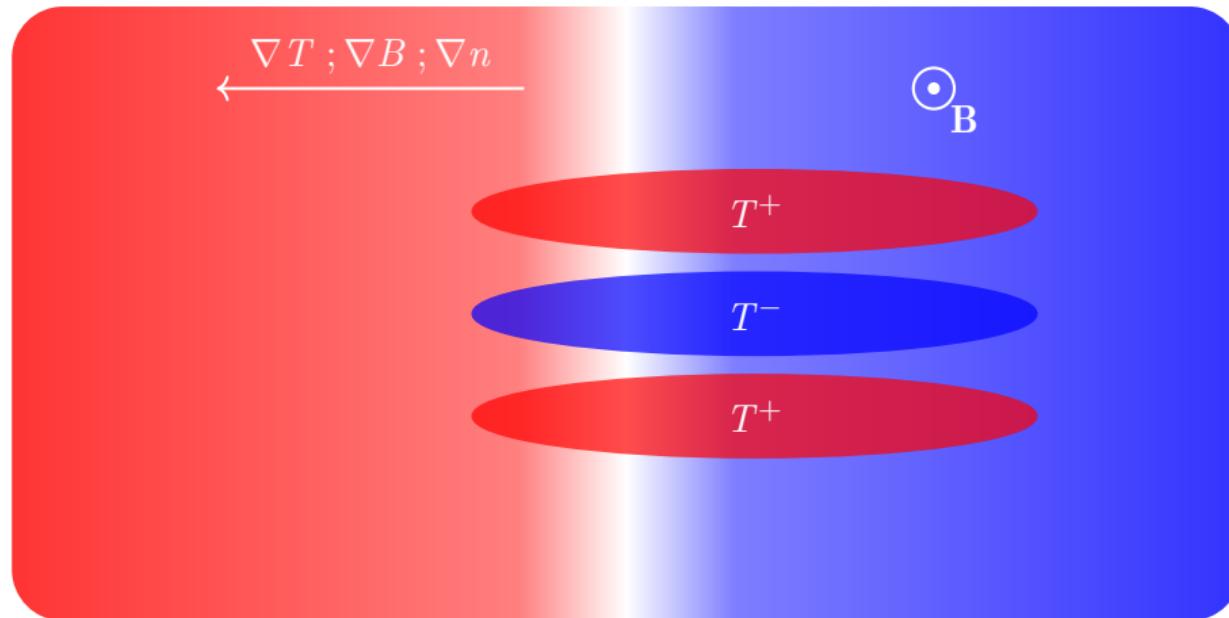
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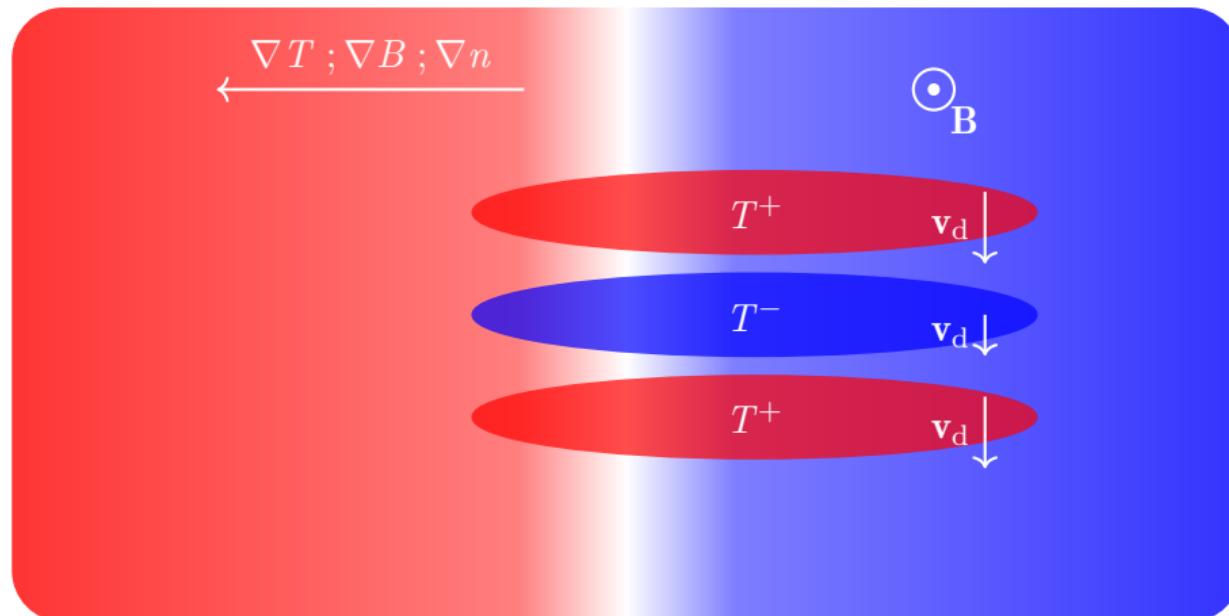
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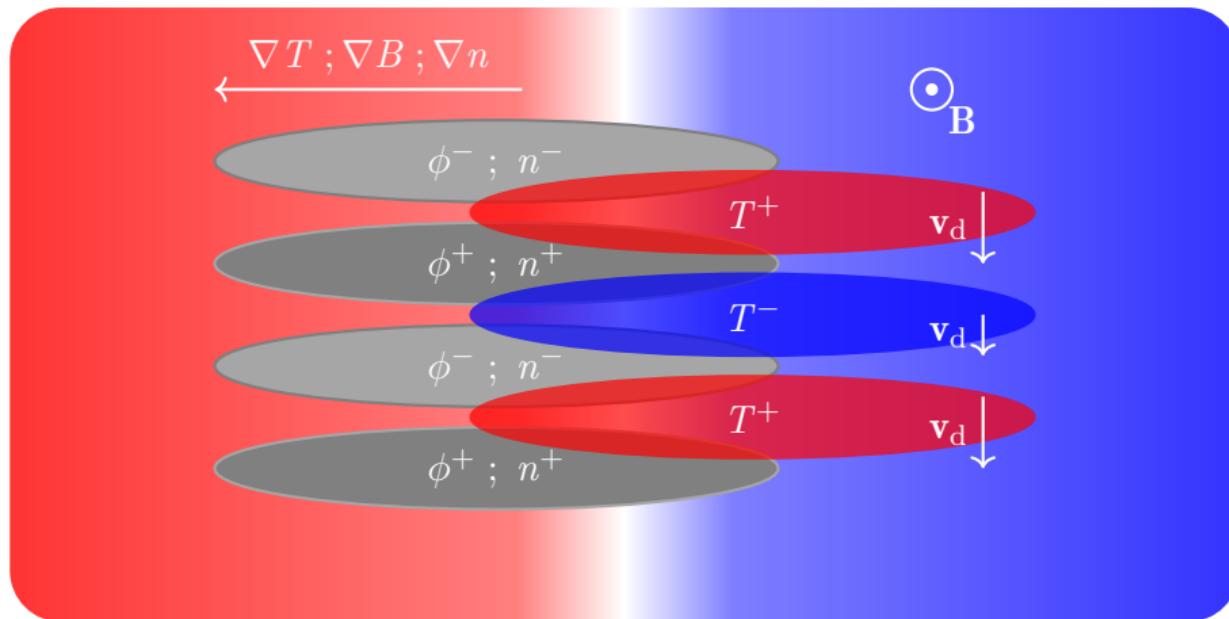
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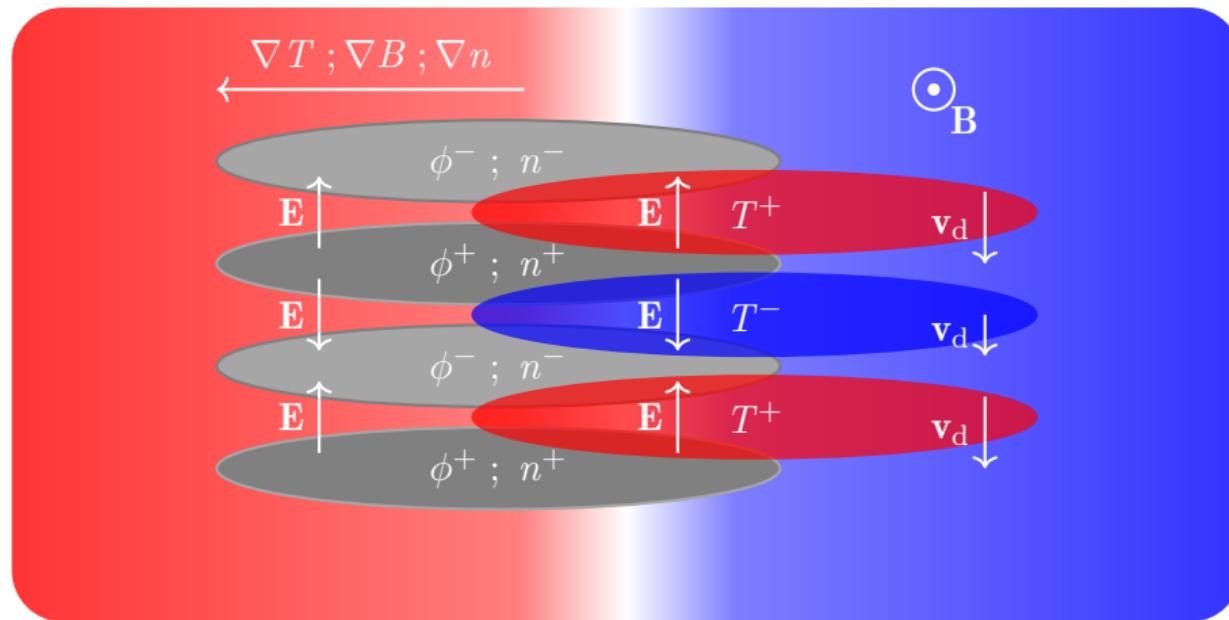
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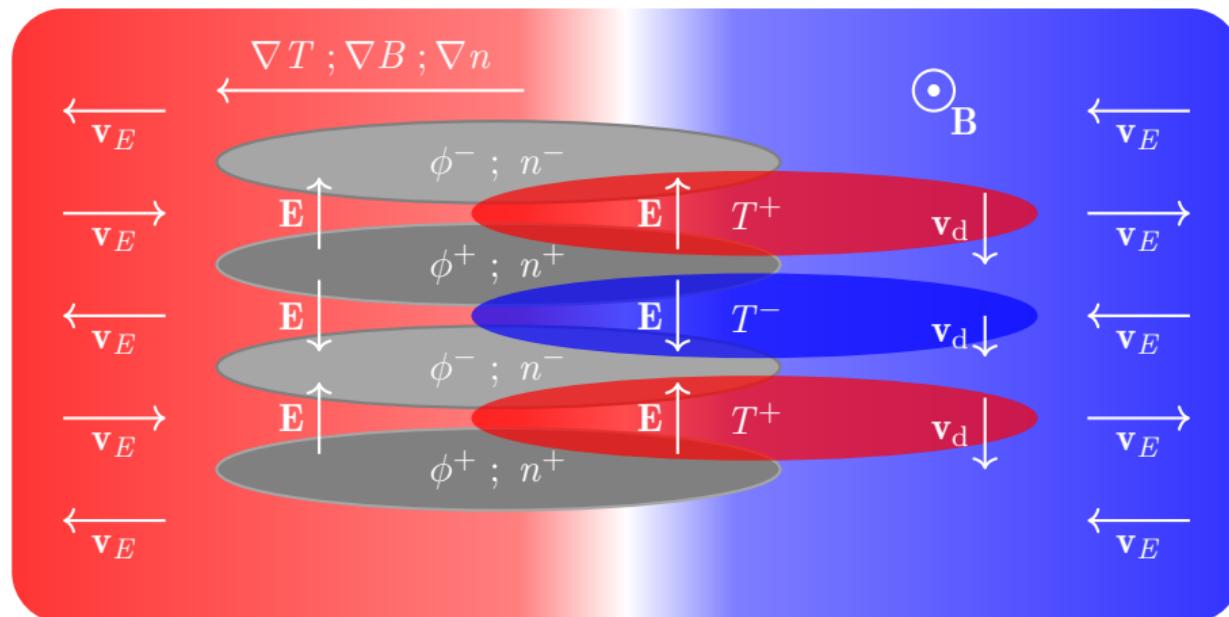
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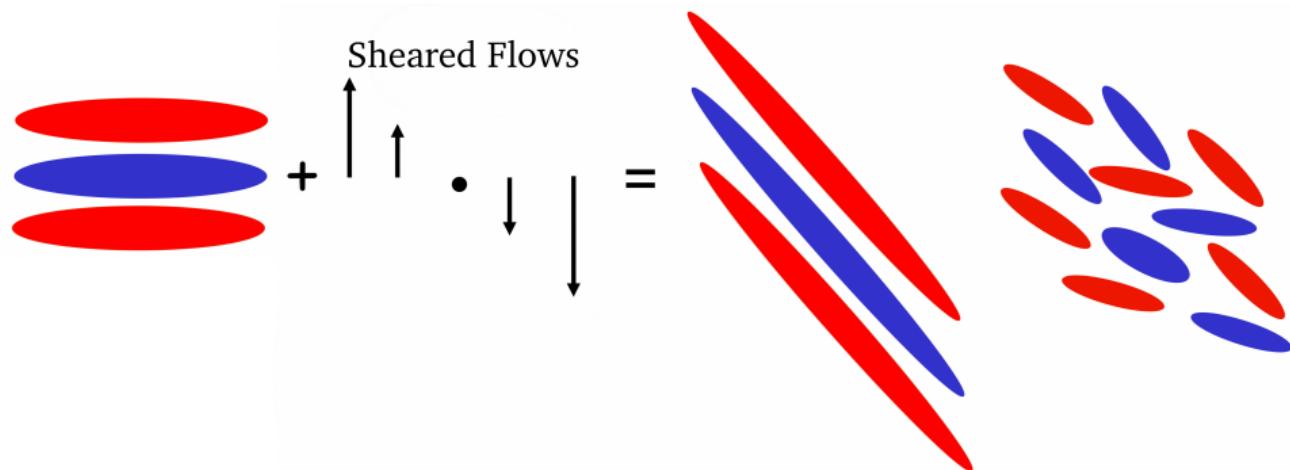


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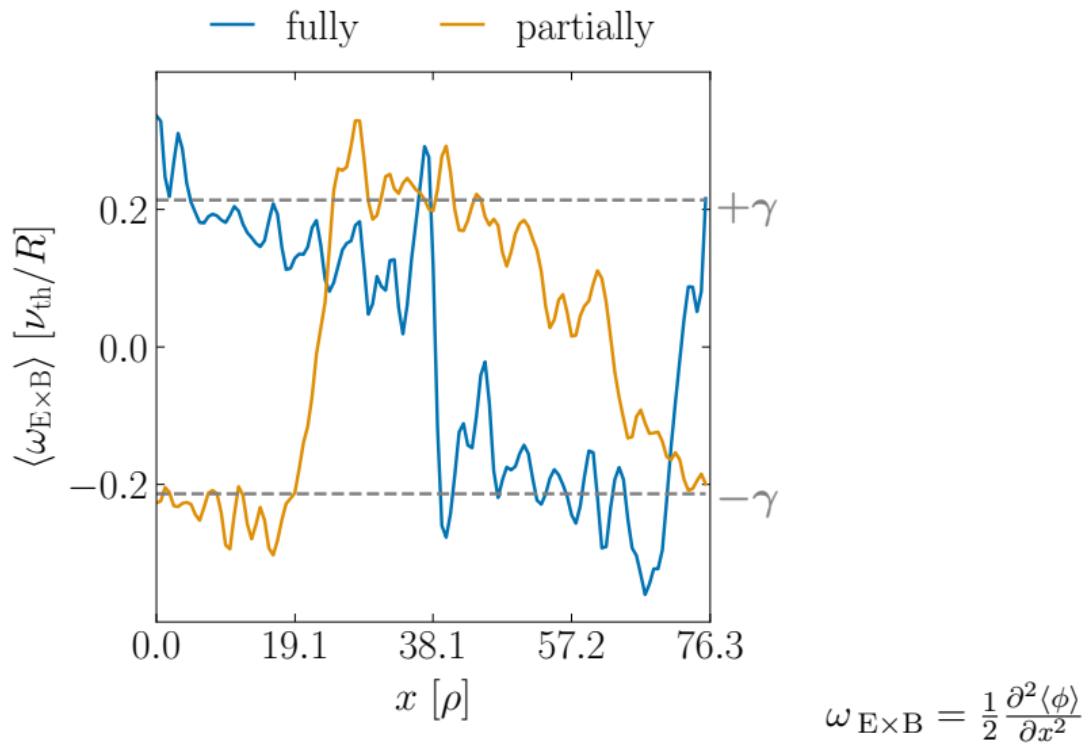


ZONAL FLOWS & SHEARING RATE

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	N_m	N_x	N_s	$N_{\nu_{ }}$	N_μ	D	ν_d	$D_{\nu_{ }}$	D_x	D_y	Order	$k_y \rho$	$k_x \rho$
S6	21	83	16	64	9	1	$ \nu_{ } $	0.2	0.1	0.1	6	1.4	2.1

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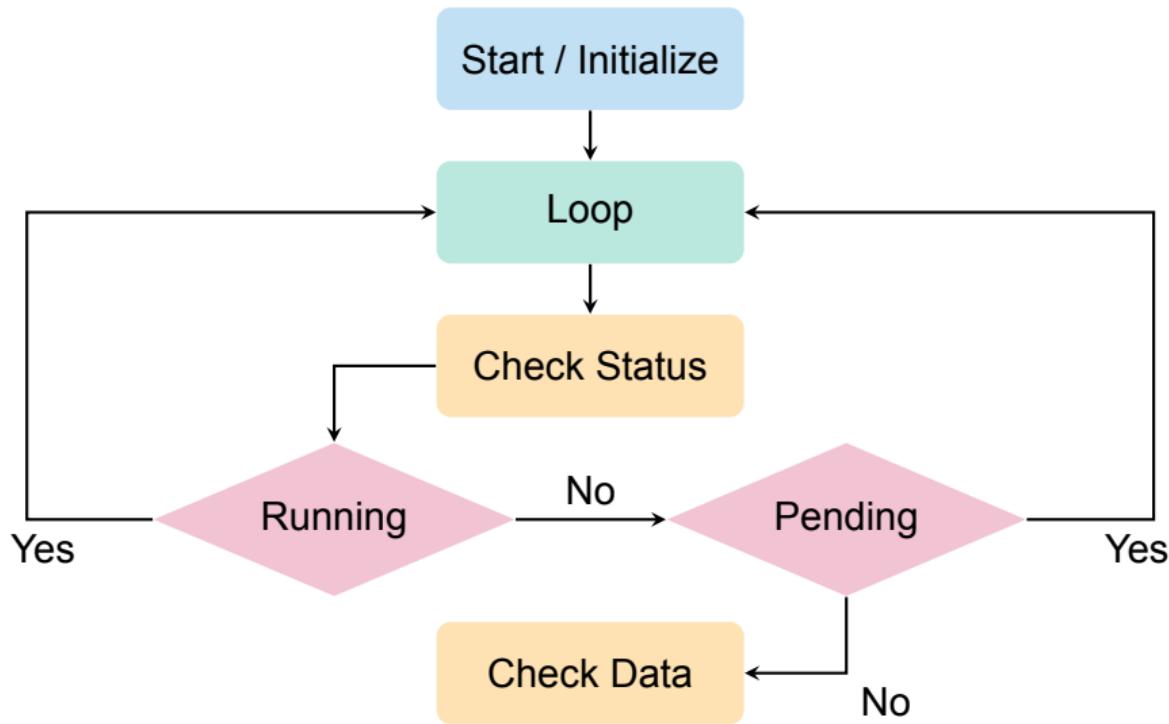
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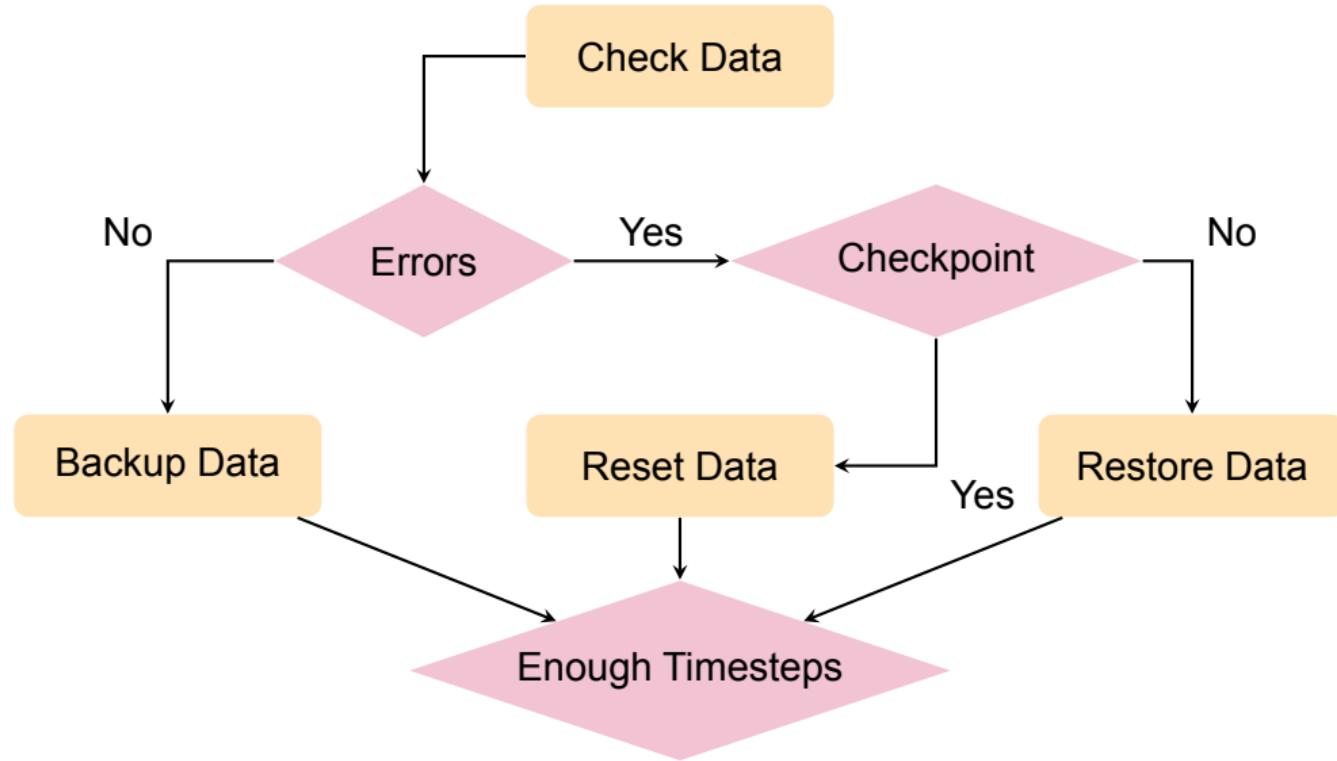
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- Waltz criterion $|\omega_{E \times B}| \approx \gamma$

RESTART SCRIPT

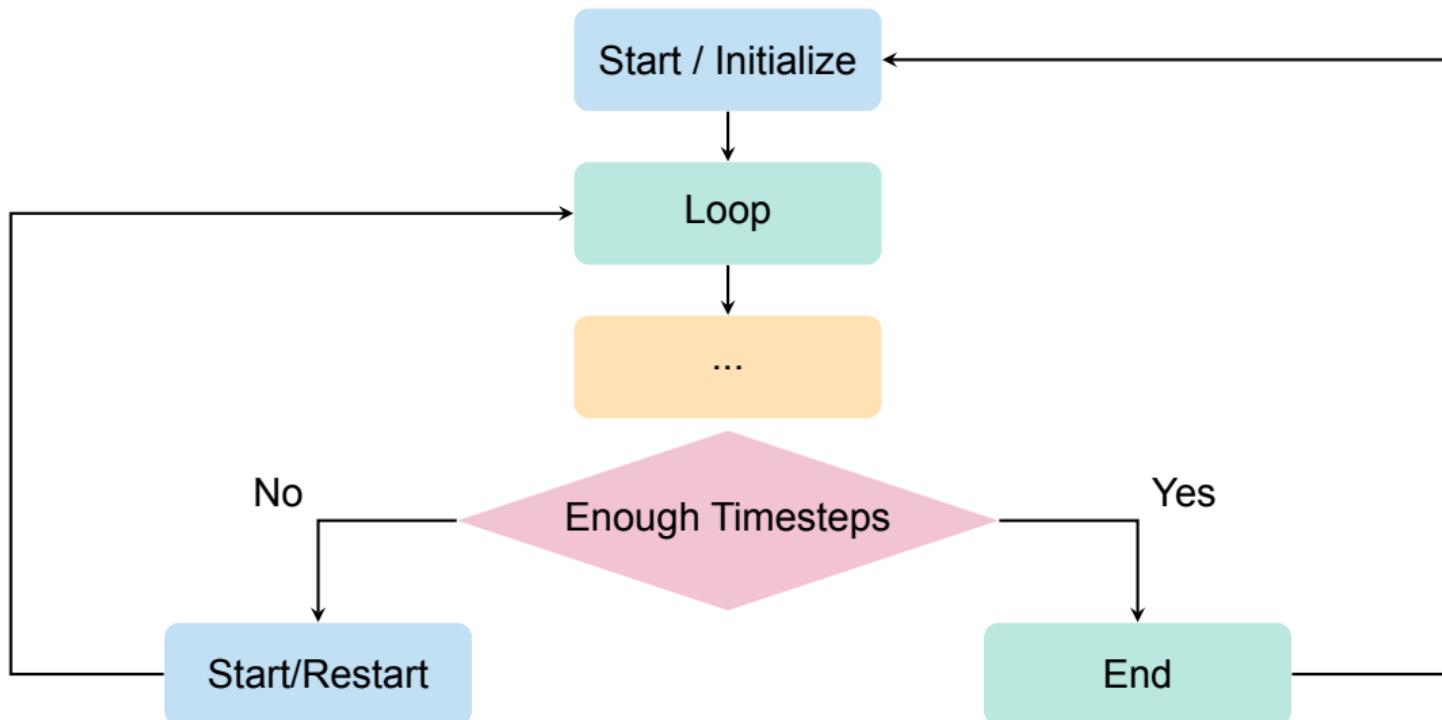
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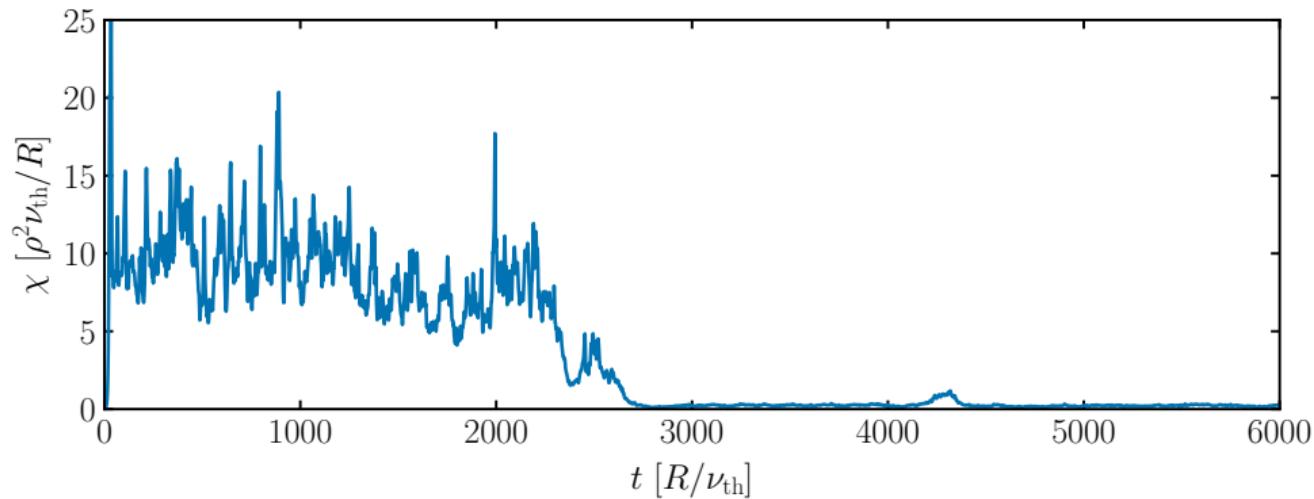
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Verification:

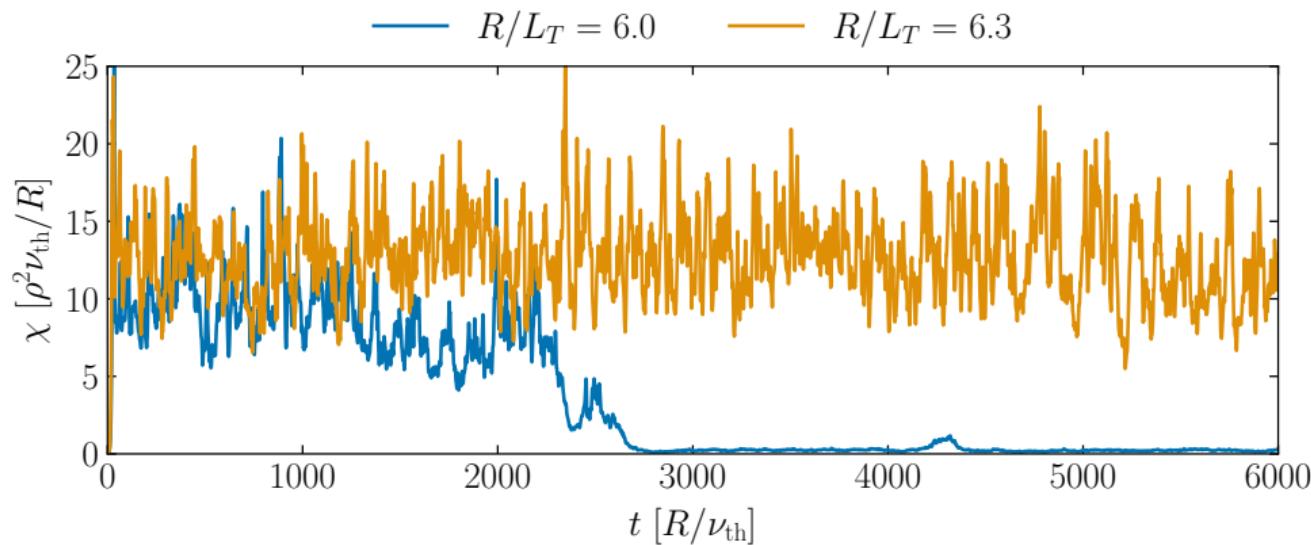
1. Reduce only one number of grid points and look if criteria (1), (2) are satisfied,
2. Reduce to the known minimum number of grid points to verify result in general.

BENCHMARK

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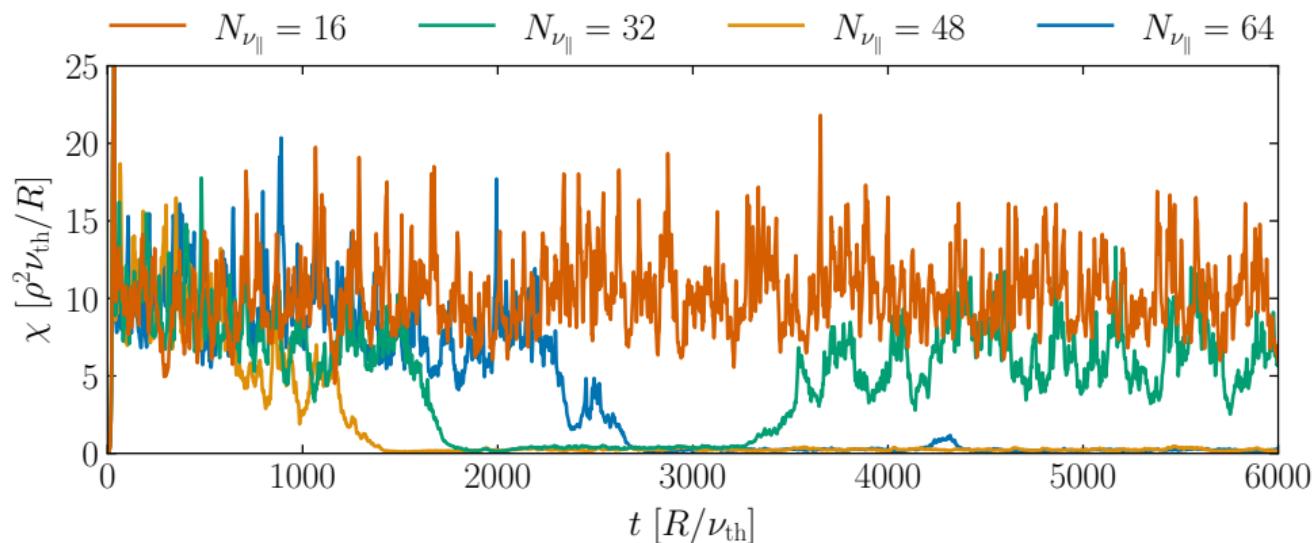


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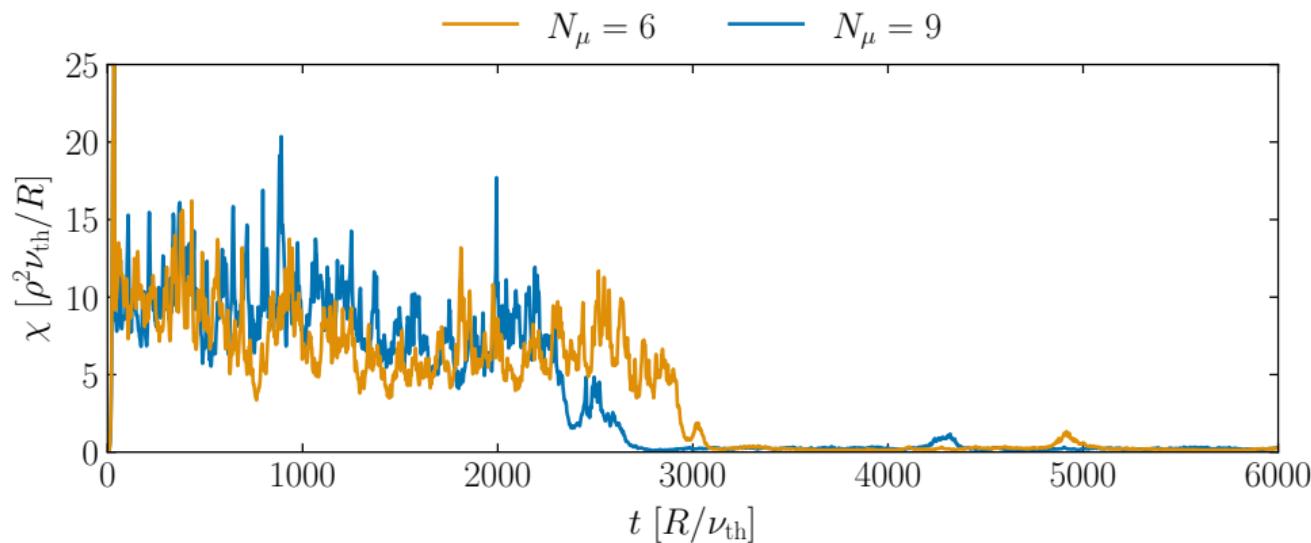


REDUCTION OF GRID POINTS

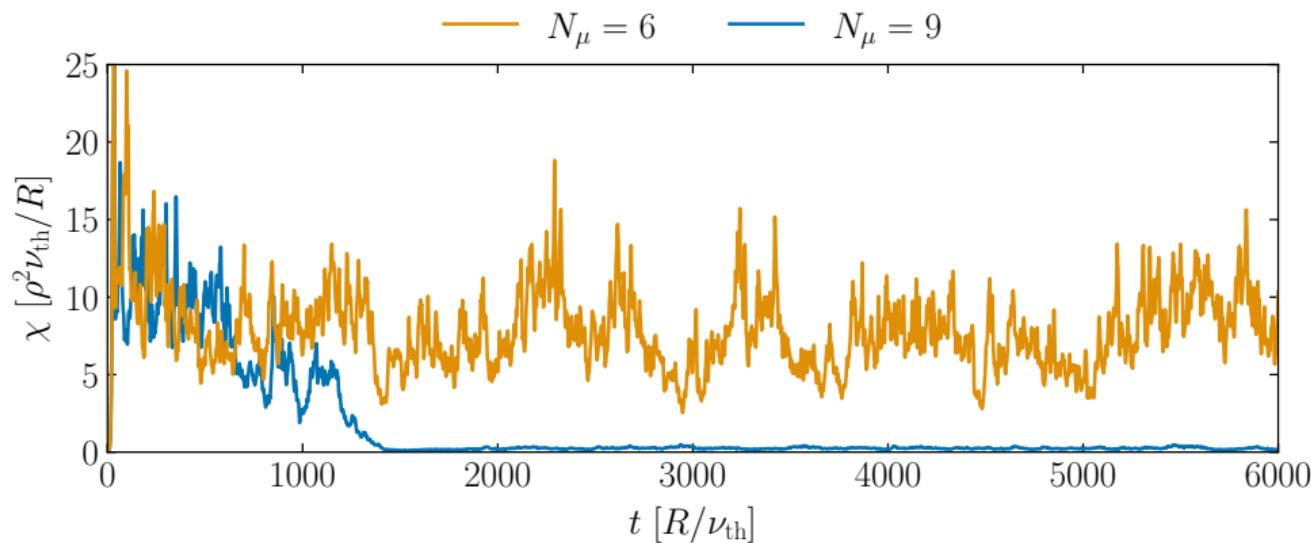
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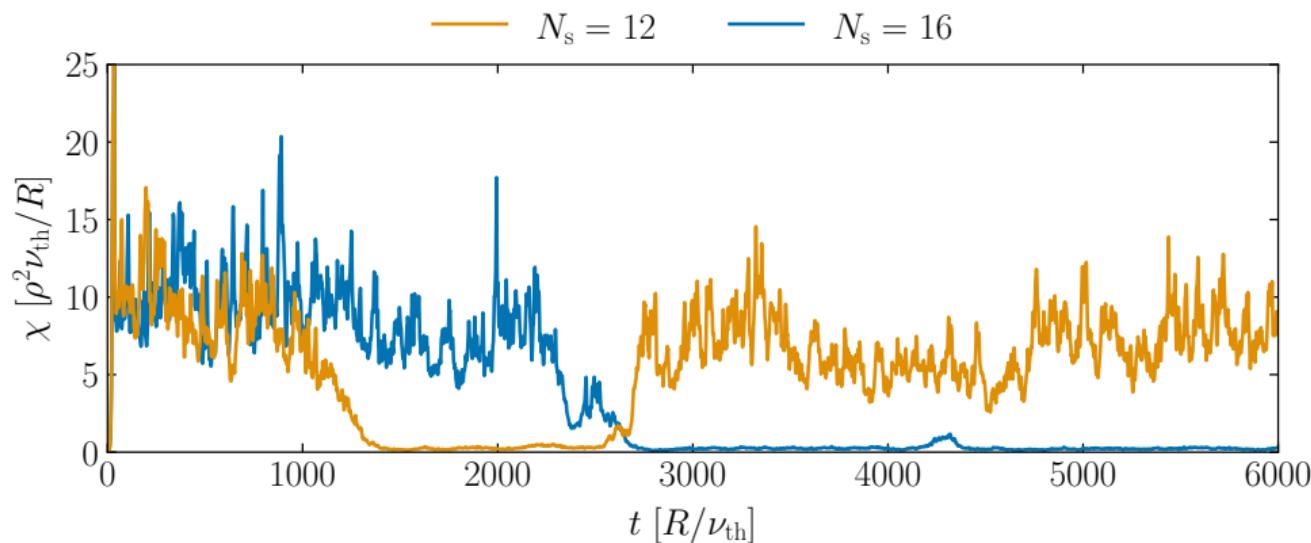
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Final Resolution

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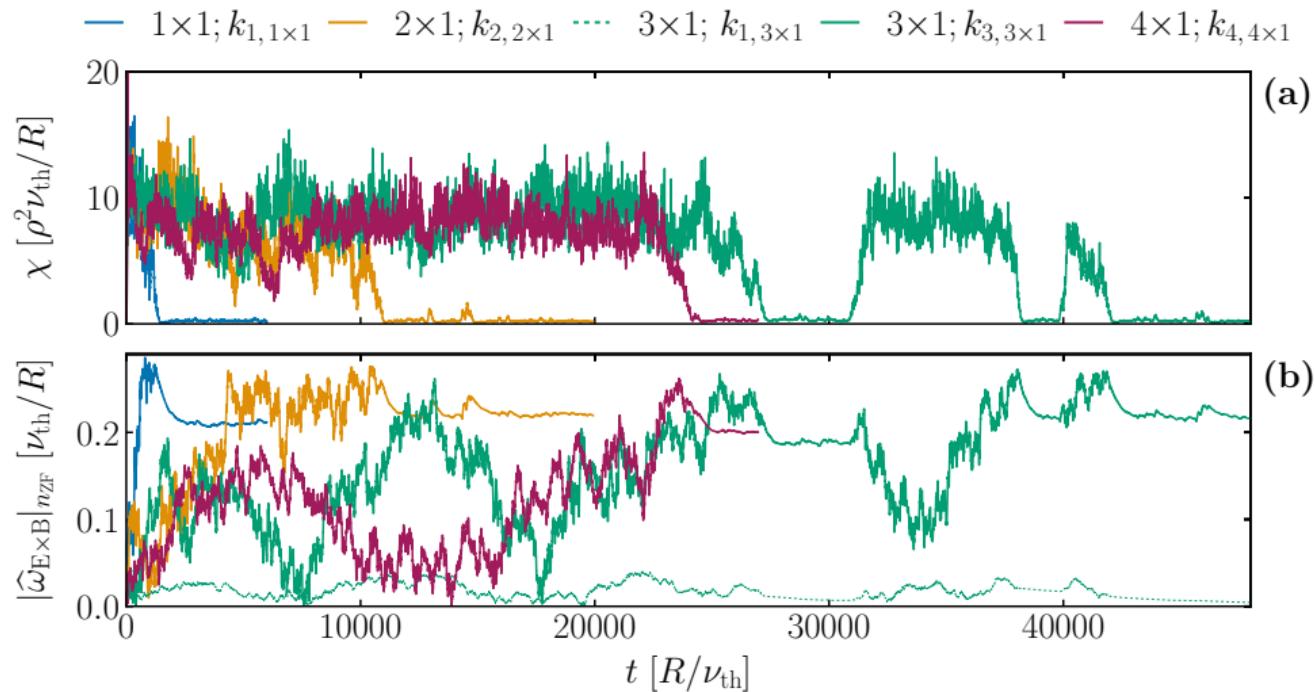
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SIZE CONVERGENCE OF E × B STAIRCASE PATTERN

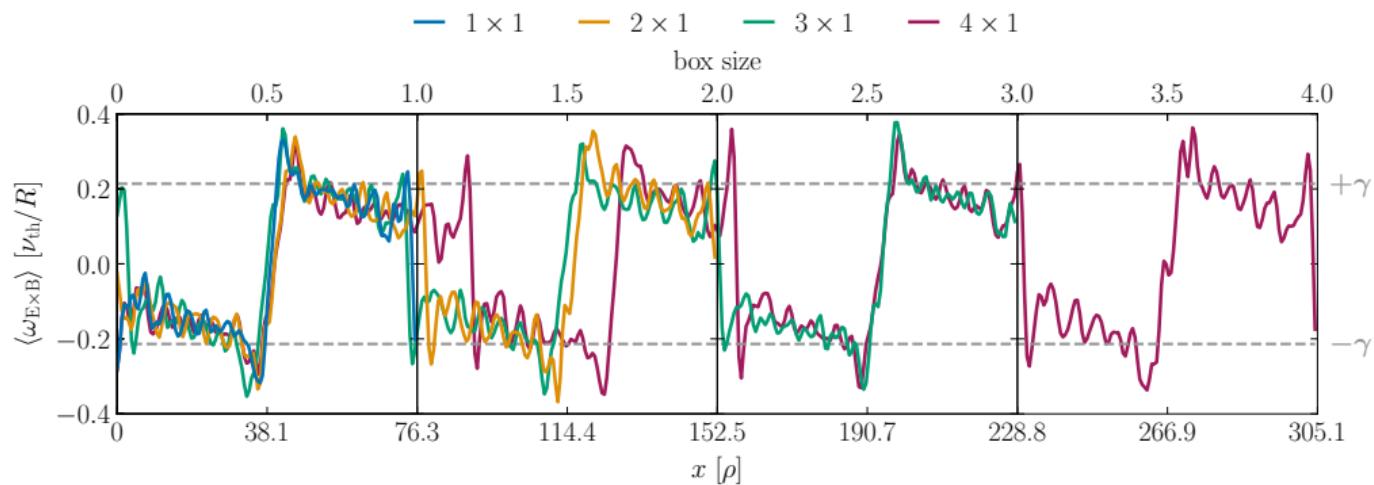
(1) Radial

$$N_R \times N_B \in [1 \times 1, 2 \times 1, 3 \times 1, 4 \times 1]$$

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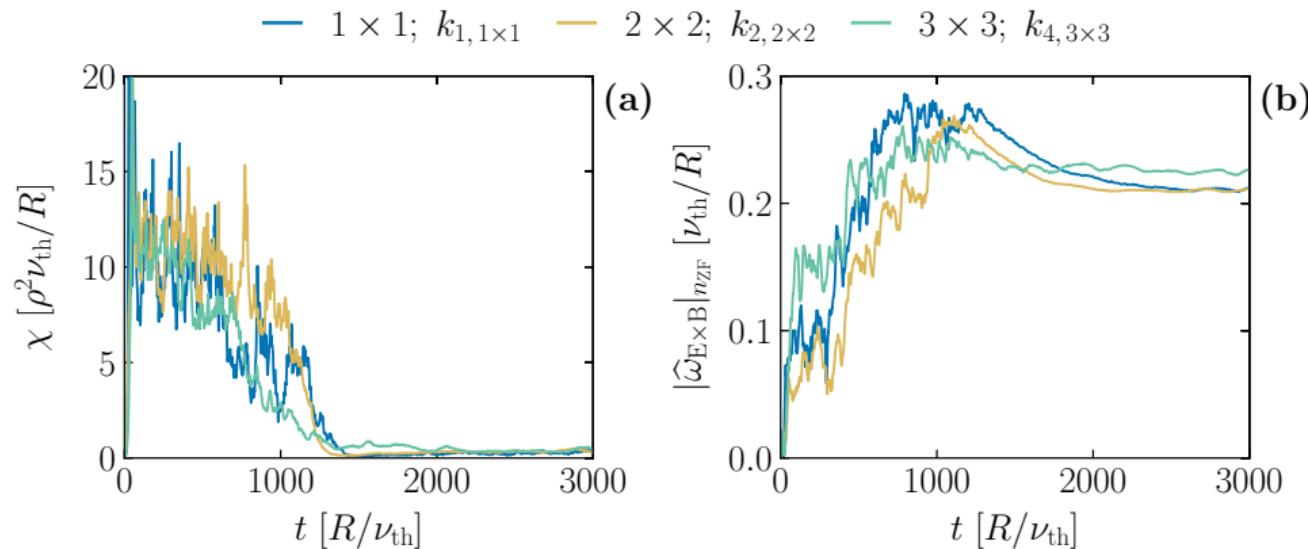


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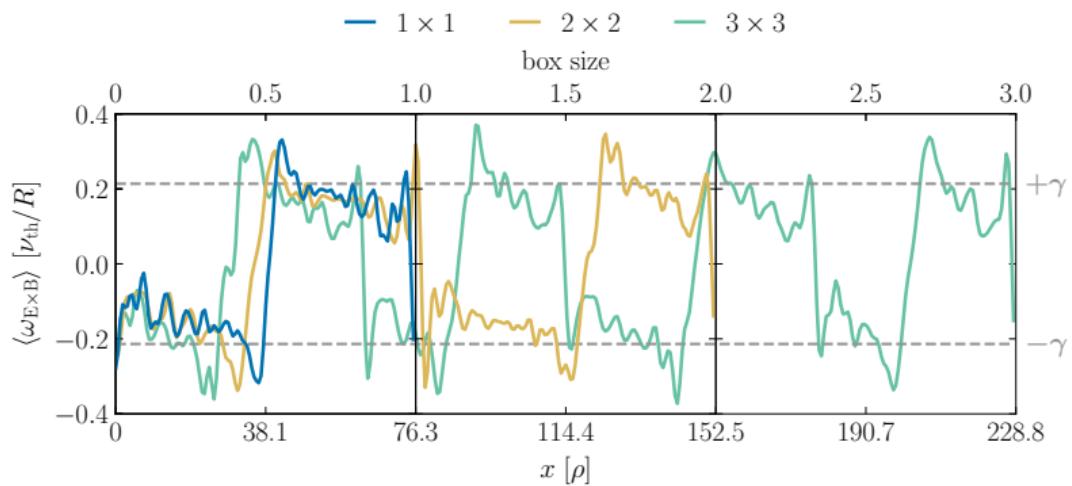
(2) Isotropic

$$N_R \times N_B \in [1 \times 1, 1.5 \times 1.5, 2 \times 2, 2.5 \times 2.5, 3 \times 3]$$

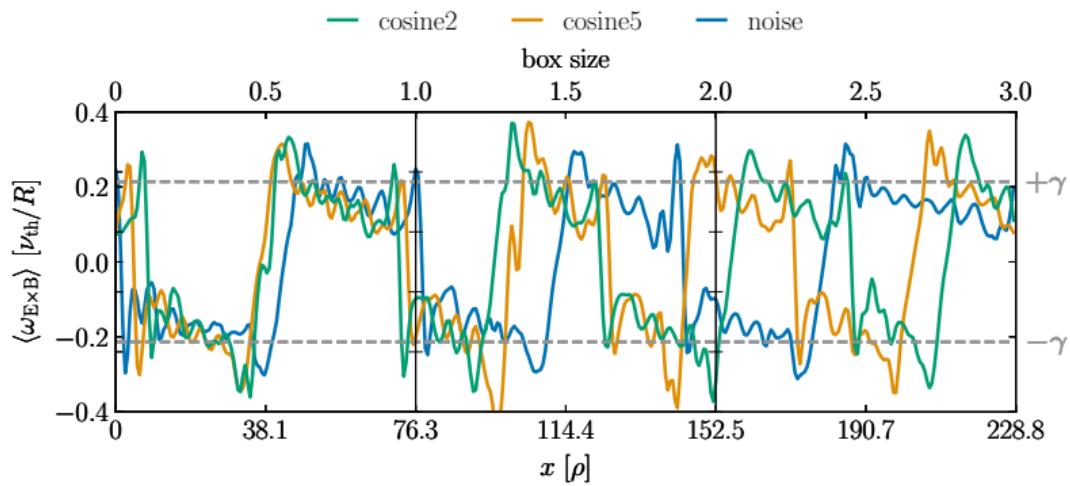
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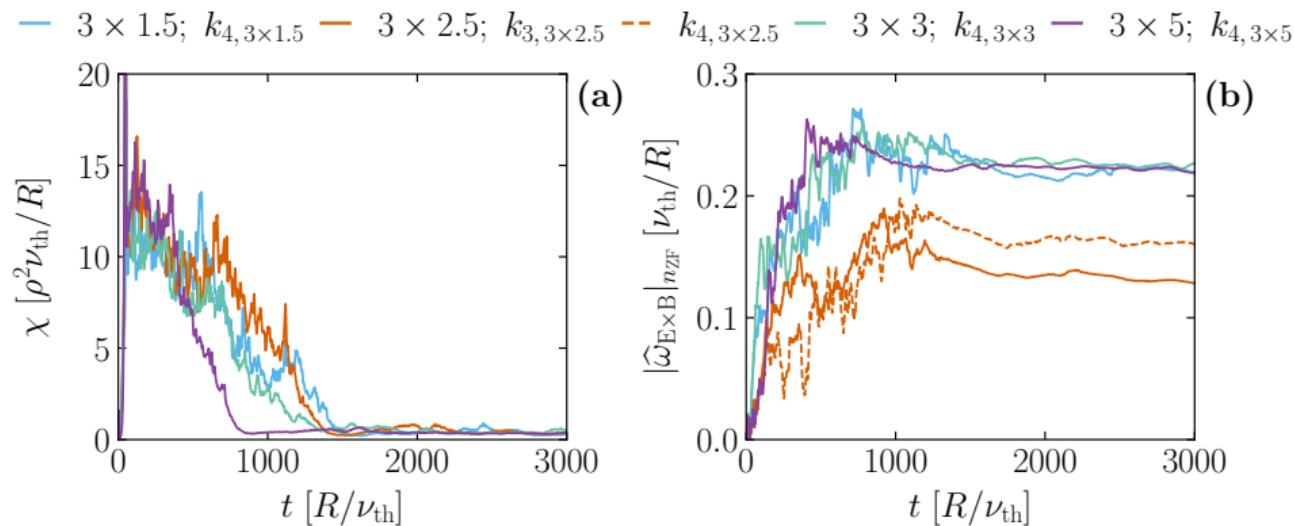


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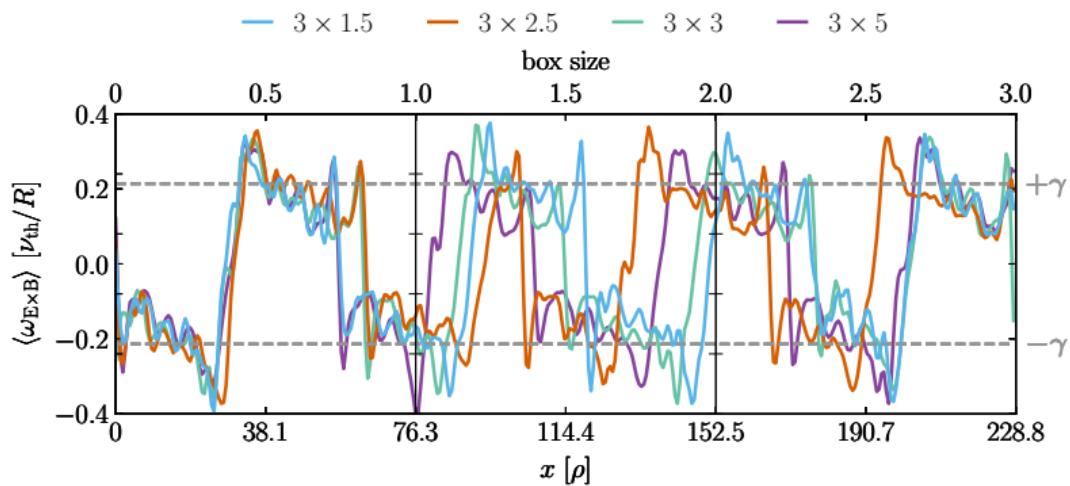
(3) Binormal

$$N_R \times N_B \in [3 \times 1.5, 3 \times 2.5, 3 \times 3, 3 \times 5]$$

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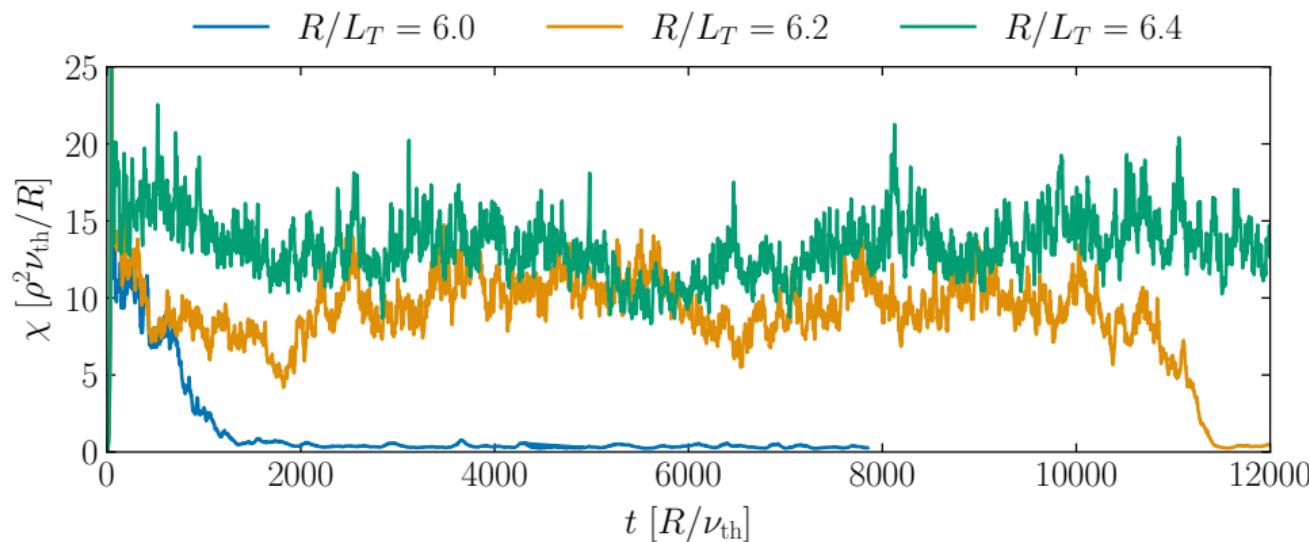
⇒ Mesoscale pattern size of:

$$\sim 57 - 76 \rho$$

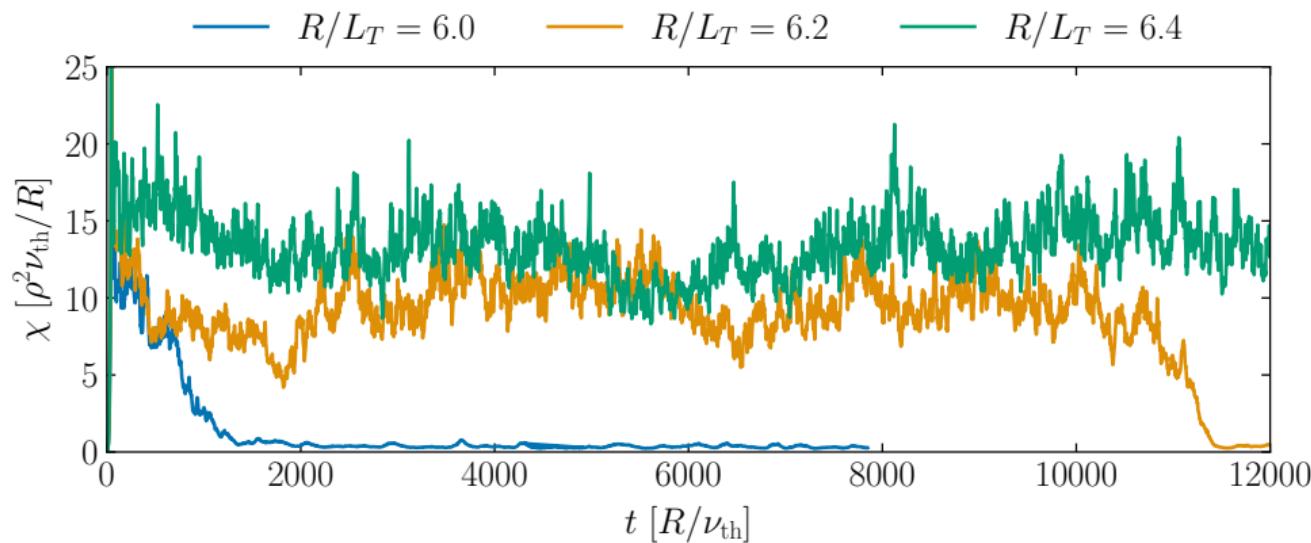
- Non-locality is inherent to ITG-driven turbulence
- Avalanches are spatially organized by the $E \times B$ staircase pattern

THE FINITE HEAT FLUX THRESHOLD

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$$\Rightarrow \boxed{R/L_T|_{\text{finite}} = 6.3 \pm 0.1}$$

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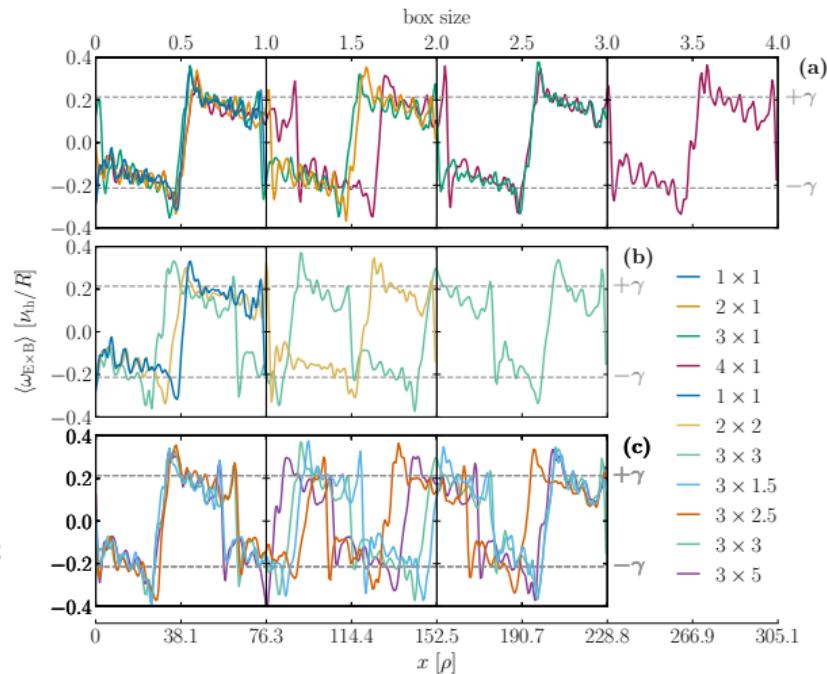
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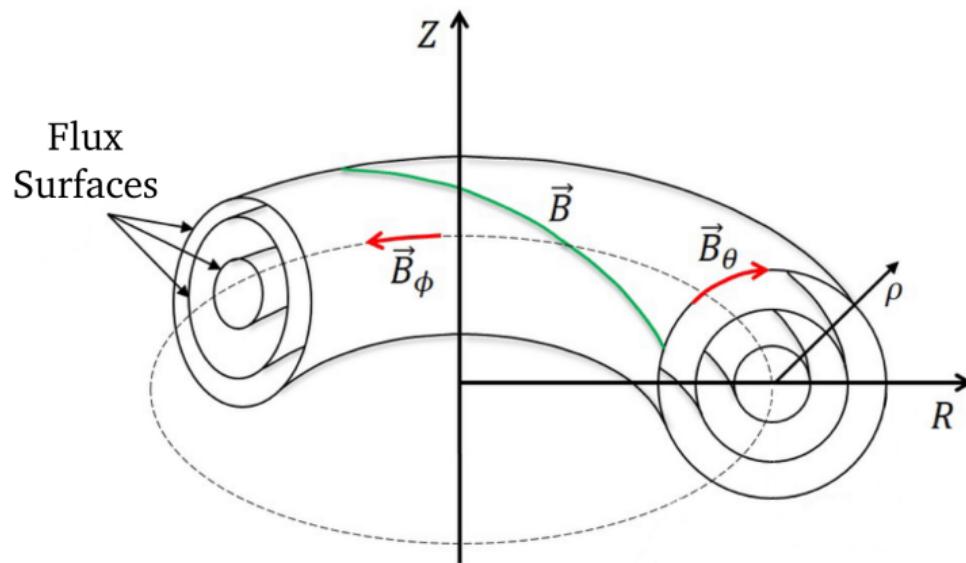
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- Parallel velocity $N_{\nu_{||}}$ could be reduced from 64 to 48, which halved the time until suppression of turbulence
- Mesoscale pattern size of $\sim 57 - 76 \rho$ is found to be intrinsic to ITG-driven turbulence for Cyclone Base Case parameters



MAGNETIC CONFINEMENT IN TOKAMAK

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$$\beta = \frac{nT}{\mu_0 B^2/2}$$

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Pattern formation occurs in the stabilized tokamak plasma the so-called $E \times B$ staircase structure.

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- (4) *Typical amplitude* of the order of $10^{-1} v_{th}/R$ (Waltz criterion $|\omega_{E \times B}| \approx \gamma$)
- (5) Turbulent transport strongly linked to the local $E \times B$ shearing through *avalanches*

SHEARING RATE

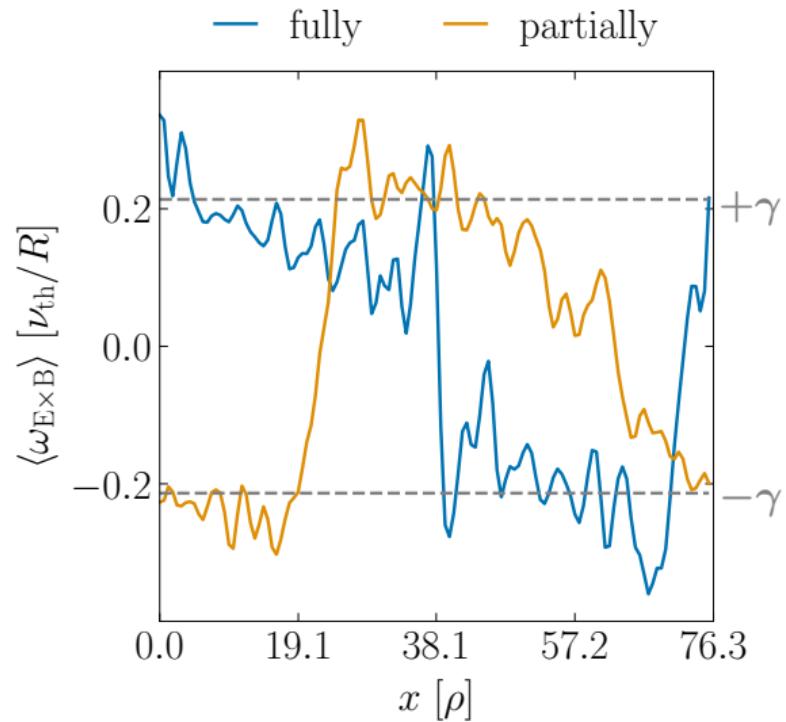
Pattern formation occurs in the stabilized tokamak plasma the so-called $E \times B$ staircase structure.

Properties:

- (1) *Radial mesoscale* of order of $\rho_L < 10 \rho < R$
- (2) *Quasi stationary* in space
- (3) Time scales much longer than the typical turbulence time scales
- (4) *Typical amplitude* of the order of $10^{-1} v_{th}/R$ (Waltz criterion $|\omega_{E \times B}| \approx \gamma$)
- (5) Turbulent transport strongly linked to the local $E \times B$ shearing through *avalanches*

$$\omega_{E \times B} = \frac{1}{2} \frac{\partial^2 \langle \phi \rangle}{\partial x^2} \quad \langle \phi \rangle = \frac{1}{L_y} \int_0^{L_y} dy \phi(x, y, s=0)$$

SHEARING RATE



GYROKINETICS

GYROKINETICS

$$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial \mathbf{x}} \cdot \frac{d\mathbf{x}}{dt} + \frac{\partial f}{\partial \mathbf{v}} \cdot \frac{d\mathbf{v}}{dt} = 0 \quad f(\mathbf{x}, \mathbf{v}, t)$$

Vlasov Equation

GYROKINETICS

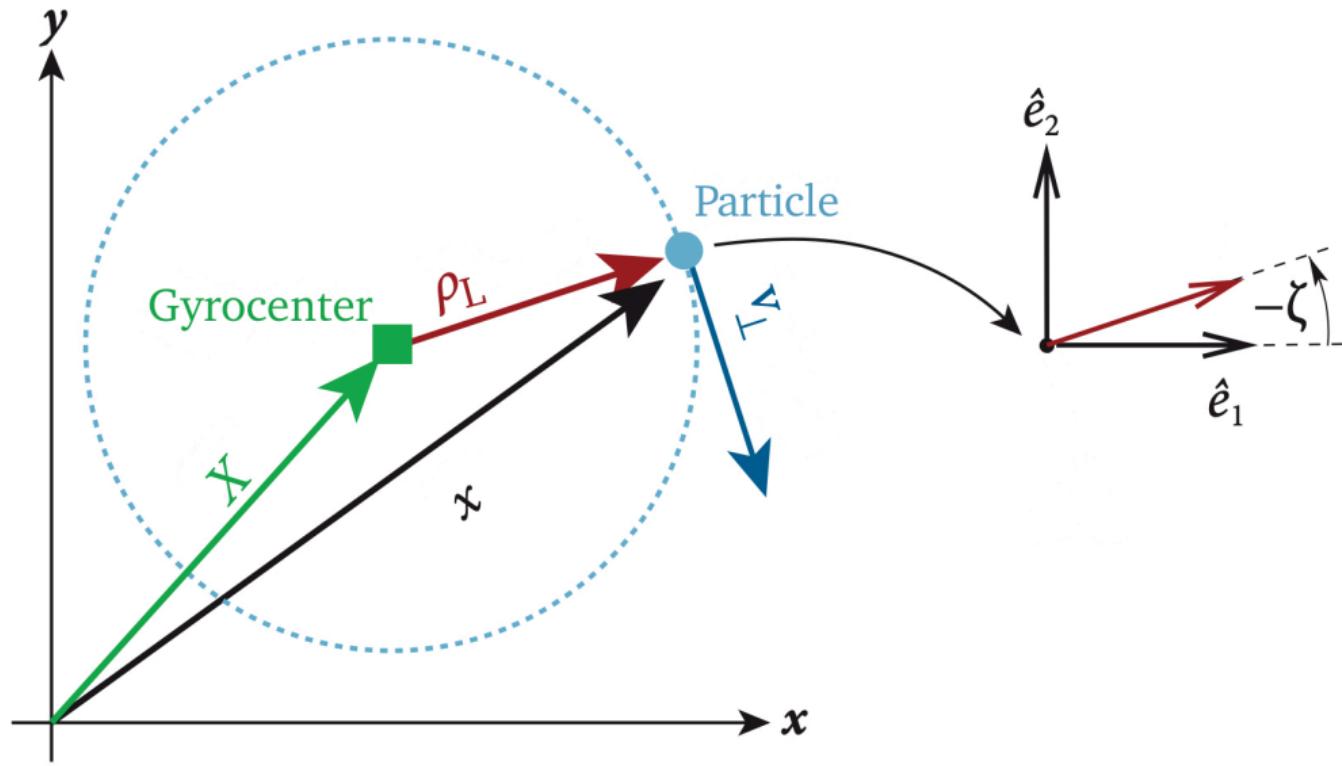
$$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial \mathbf{x}} \cdot \frac{d\mathbf{x}}{dt} + \frac{\partial f}{\partial \mathbf{v}} \cdot \frac{d\mathbf{v}}{dt} = 0 \quad f(\mathbf{x}, \mathbf{v}, t)$$

Vlasov Equation

$$\Downarrow \quad \frac{\omega}{\omega_c} \sim \frac{k_{\parallel}}{k_{\perp}} \sim \frac{\rho}{L_n} \sim \frac{\delta n}{n_0} \sim \frac{\delta B}{B_0} \sim \frac{v_d}{v_{th}} \sim \epsilon_g$$

Gyrokinetic Ordering

GYROKINETICS



GYROKINETICS

$$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial \mathbf{x}} \cdot \frac{d\mathbf{x}}{dt} + \frac{\partial f}{\partial \mathbf{v}} \cdot \frac{d\mathbf{v}}{dt} = 0 \quad f(\mathbf{x}, \mathbf{v}, t) \quad \text{Vlasov Equation}$$

$$\Downarrow \quad \frac{\omega}{\omega_c} \sim \frac{k_{\parallel}}{k_{\perp}} \sim \frac{\rho}{L_n} \sim \frac{\delta n}{n_0} \sim \frac{\delta B}{B_0} \sim \frac{v_d}{v_{th}} \sim \epsilon_g \quad \text{Gyrokinetic Ordering}$$

$$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial \mathbf{X}} \cdot \frac{d\mathbf{X}}{dt} + \frac{\partial f}{\partial v_{\parallel}} \cdot \frac{dv_{\parallel}}{dt} = 0 \quad f(\mathbf{X}, v_{\parallel}, \mu) ; \frac{d\mu}{dt} = 0 \quad \text{Gyrokinetic Formalism}$$

GYROKINETICS

$$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial \mathbf{x}} \cdot \frac{d\mathbf{x}}{dt} + \frac{\partial f}{\partial \mathbf{v}} \cdot \frac{d\mathbf{v}}{dt} = 0 \quad f(\mathbf{x}, \mathbf{v}, t) \quad \text{Vlasov Equation}$$

$$\Downarrow \quad \frac{\omega}{\omega_c} \sim \frac{k_{\parallel}}{k_{\perp}} \sim \frac{\rho}{L_n} \sim \frac{\delta n}{n_0} \sim \frac{\delta B}{B_0} \sim \frac{v_d}{v_{th}} \sim \epsilon_g \quad \text{Gyrokinetic Ordering}$$

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$$\Downarrow \quad f = f_0 + \delta f \quad \delta f \text{ Approx & Local Limit}$$

GYROKINETICS

$$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial \mathbf{x}} \cdot \frac{d\mathbf{x}}{dt} + \frac{\partial f}{\partial \mathbf{v}} \cdot \frac{d\mathbf{v}}{dt} = 0 \quad f(\mathbf{x}, \mathbf{v}, t) \quad \text{Vlasov Equation}$$

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$$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial \mathbf{X}} \cdot \frac{d\mathbf{X}}{dt} + \frac{\partial f}{\partial v_{\parallel}} \cdot \frac{dv_{\parallel}}{dt} = 0 \quad f(\mathbf{X}, v_{\parallel}, \mu); \quad \frac{d\mu}{dt} = 0 \quad \text{Gyrokinetic Formalism}$$

$$\Downarrow \quad f = f_0 + \delta f \quad \delta f \text{ Approx & Local Limit}$$

$$\frac{\partial g}{\partial t} + \mathbf{v}_{\chi} \cdot \nabla g + (v_{\parallel} \mathbf{b} + \mathbf{v}_D) \cdot \nabla(\delta f) - \frac{\mu B}{m} \frac{\mathbf{B} \cdot \nabla B}{B^2} \frac{\partial(\delta f)}{\partial v_{\parallel}} = S \quad \text{Gyrokinetic Equation}$$

$$g \sim (f_0 + \delta f) \quad S \sim (f + f_0) \quad \mathbf{v}_D \sim \nabla B \quad \mathbf{v}_{\chi} \sim \mathbf{E} \times \mathbf{B} \quad f(x, y, s, v_{\parallel}, \mu)$$

SIZE CONVERGENCE OF $E \times B$ STAIRCASE PATTERN

