

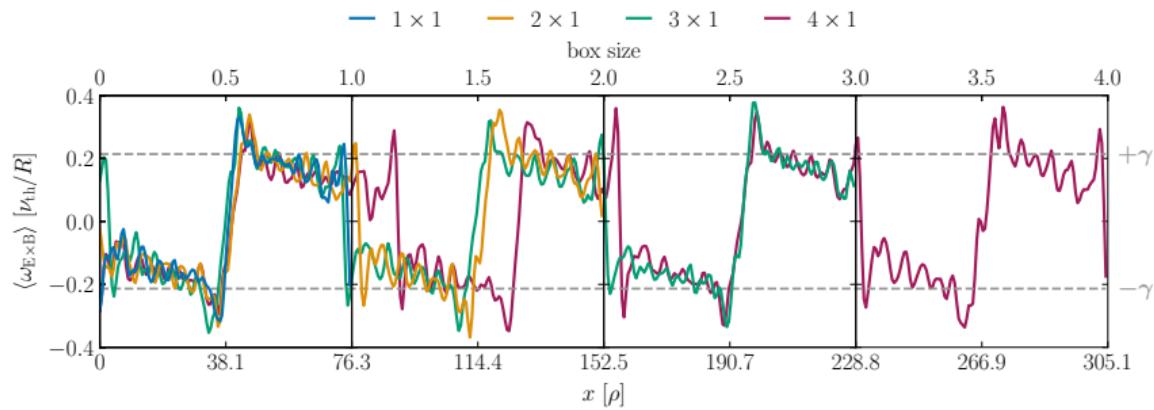


SIZE CONVERGENCE OF THE $E \times B$ STAIRCASE PATTERN IN FLUX TUBE SIMULATIONS OF ION TEMPERATURE GRADIENT-DRIVEN TURBULENCE

June 21, 2023

Manuel Lippert

Theoretical Physics V



MOTIVATION

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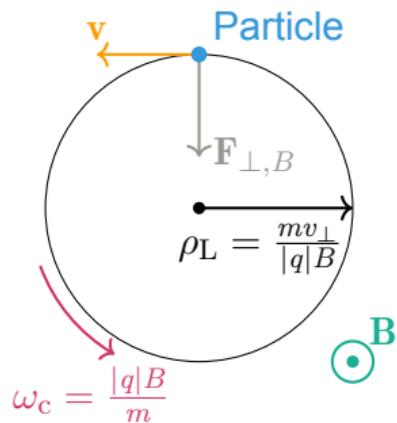
Does the basic pattern size always converges to the box size, or is there a typical mesoscale size inherent to staircase structures also in a local flux-tube description?

CHARGED PARTICLE MOTION

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Lorentz force

$$F_{\perp,B} = |q|v_{\perp}B$$



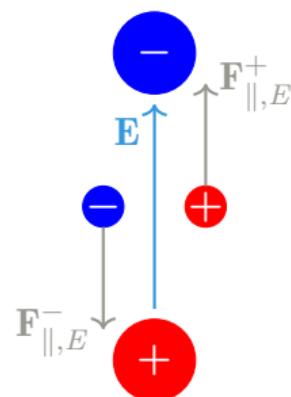
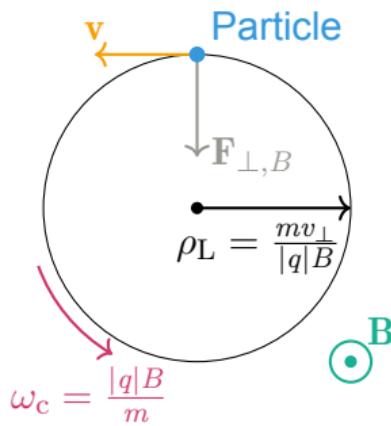
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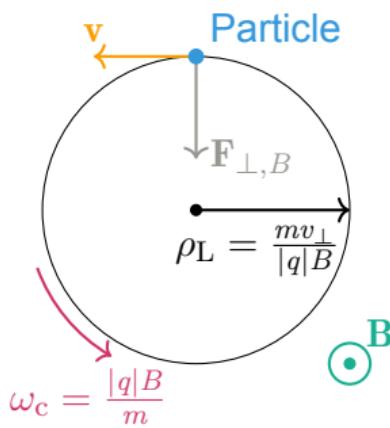
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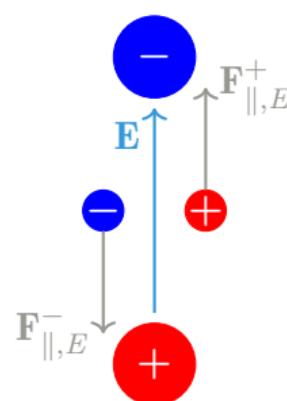
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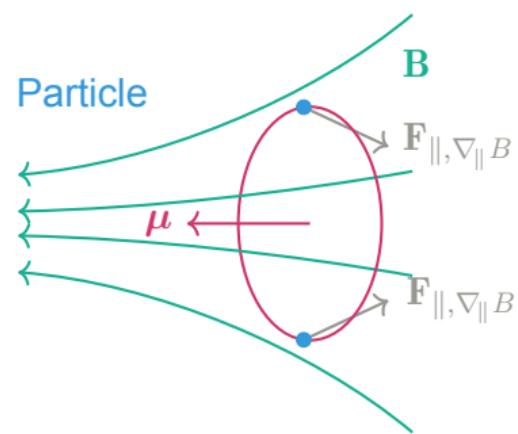
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Inhomogeneous magnetic field

$$F_{\parallel,\nabla_{\parallel}B} = - \underbrace{\frac{mv_{\perp}^2}{2B}}_{\mu} \nabla_{\parallel}B$$

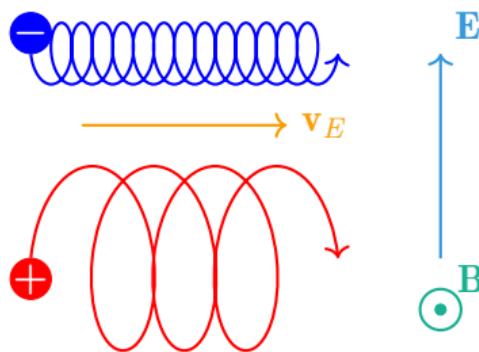


DRIFT IN THE GYROCENTER

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$E \times B$ Drift

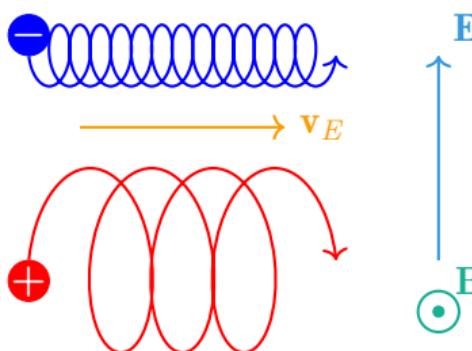
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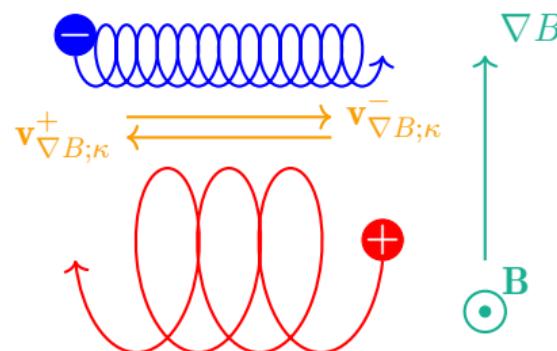


∇B Drift

$$\mathbf{v}_{\nabla B} = \frac{mv_{\perp}^2}{2q} \frac{\mathbf{B} \times \nabla B}{B^3}$$

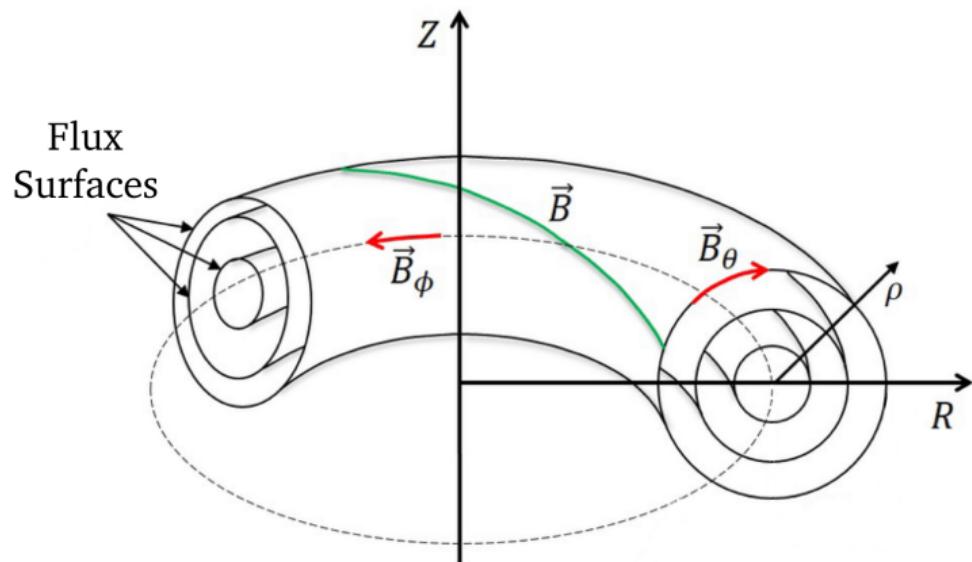
Curvature Drift

$$\mathbf{v}_{\kappa} = \frac{mv_{\parallel}^2}{qB^2} \mathbf{B} \times \underbrace{\kappa}_{\frac{\nabla B}{B}} = \frac{mv_{\parallel}^2}{q} \frac{\mathbf{B} \times \nabla B}{B^3}$$



MAGNETIC CONFINEMENT IN TOKAMAK

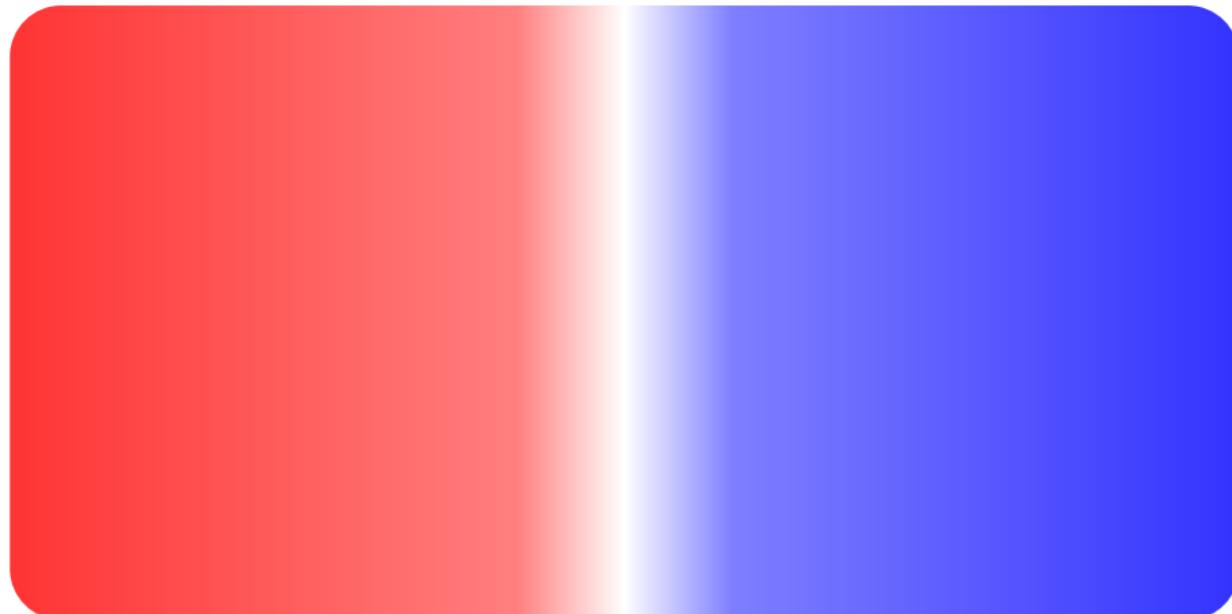
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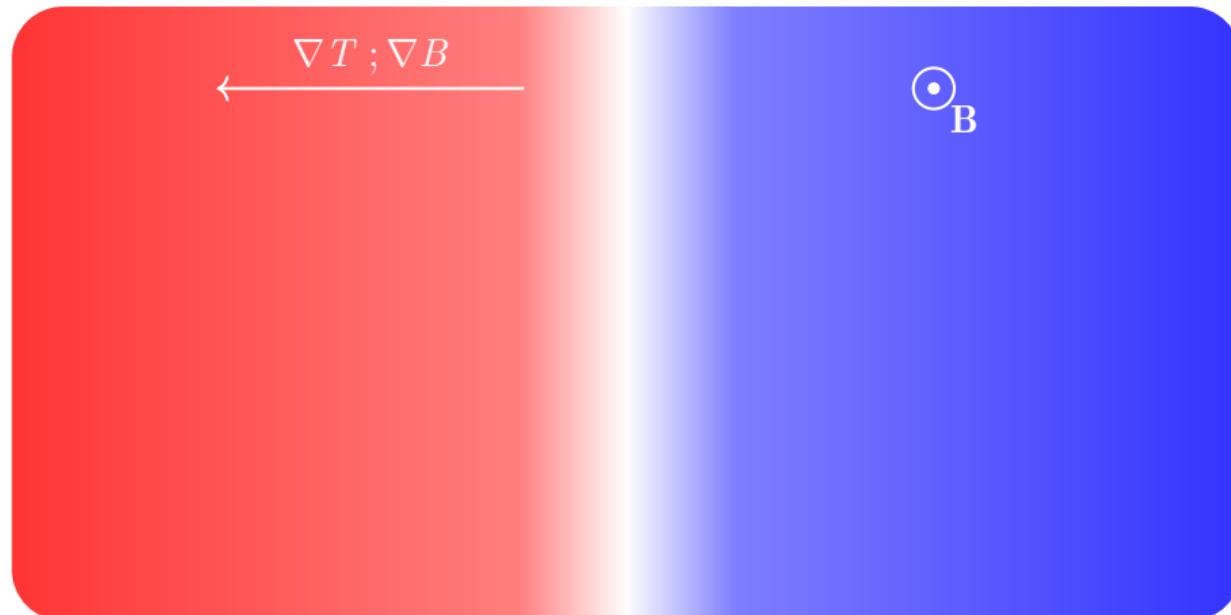
$$\beta = \frac{nT}{\mu_0 B^2/2}$$

ION TEMPERATURE GRADIENT (ITG)-DRIVEN INSTABILITY

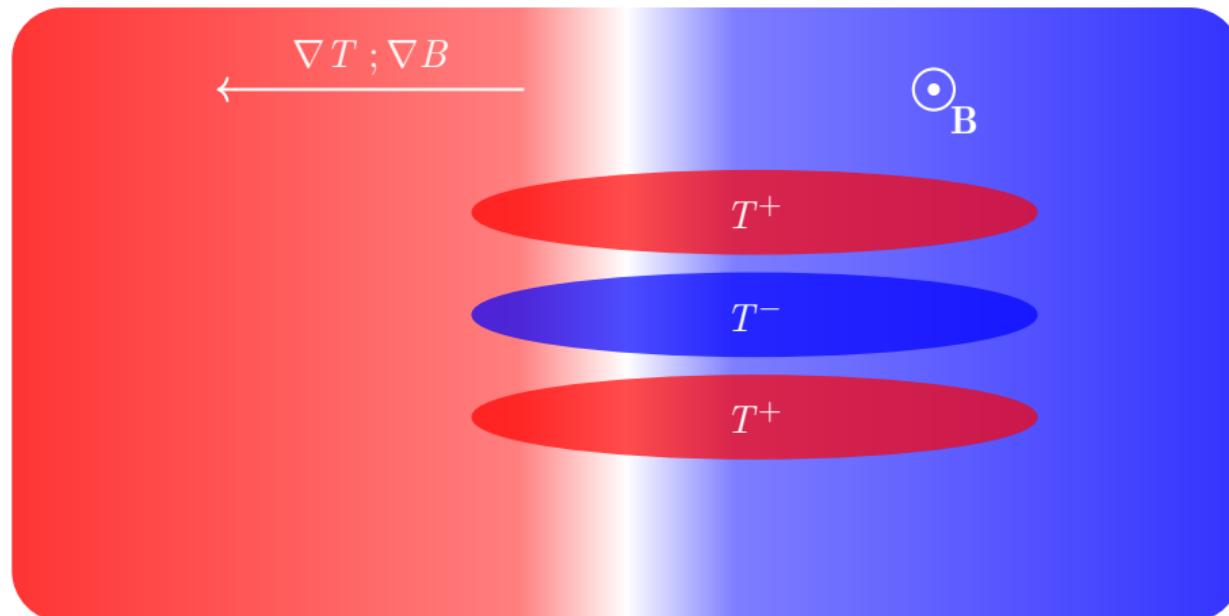
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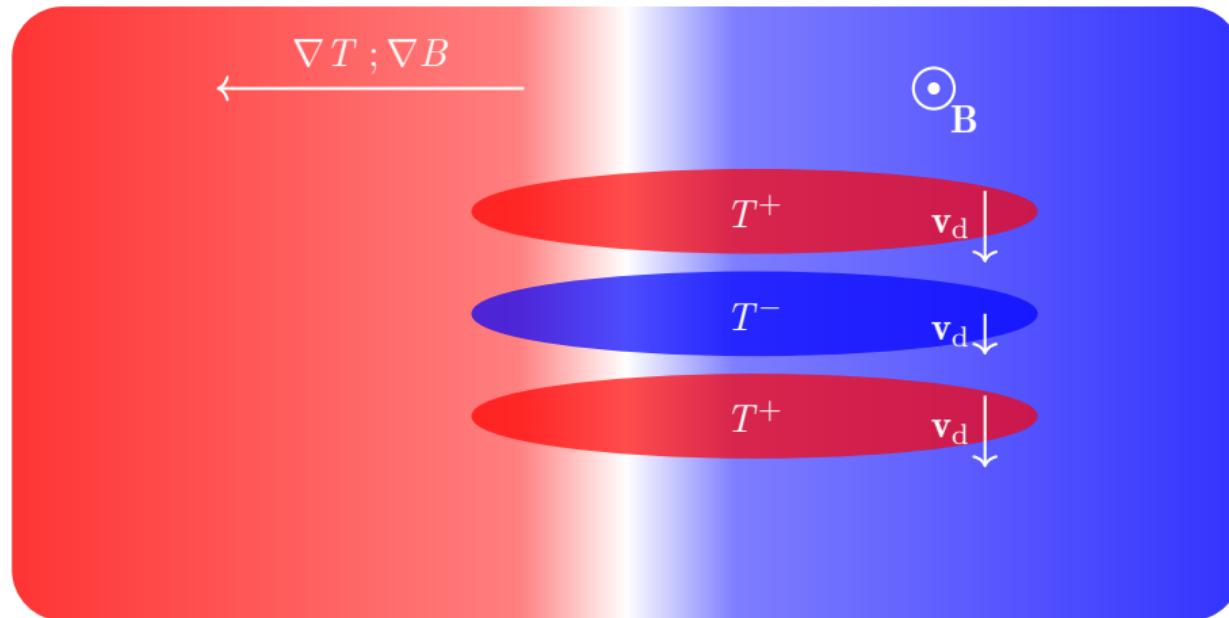
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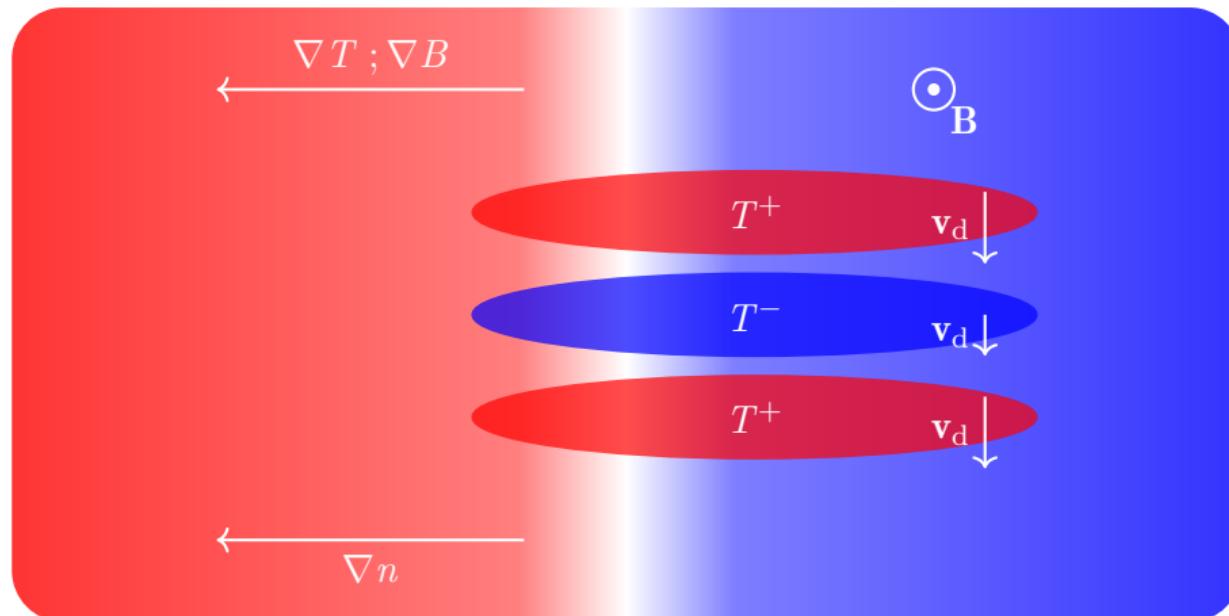
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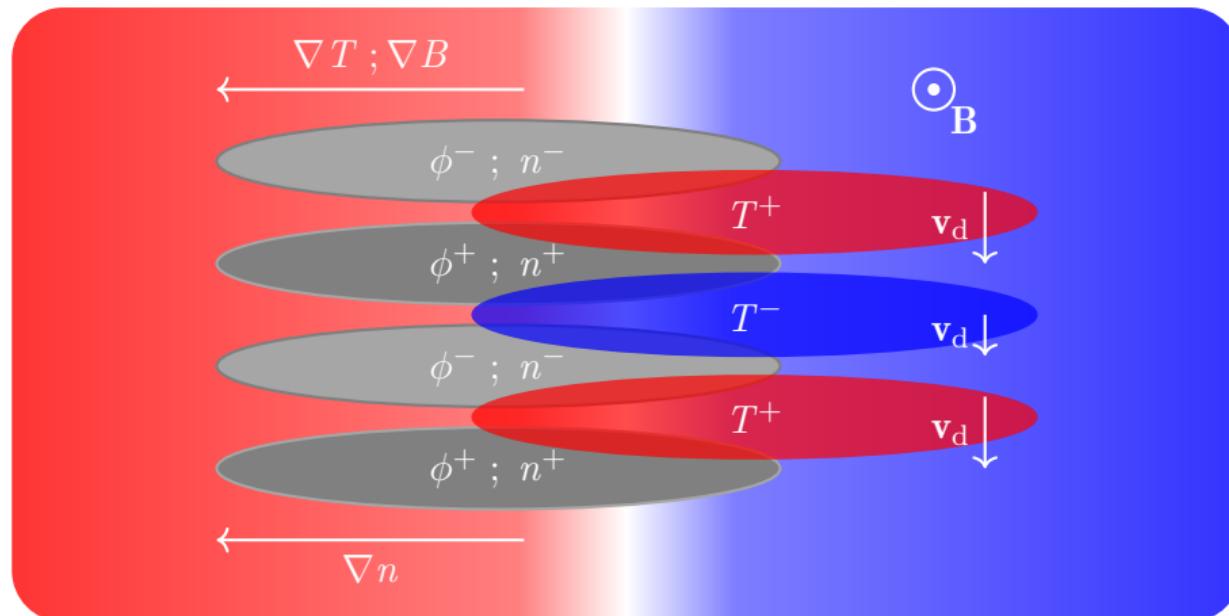
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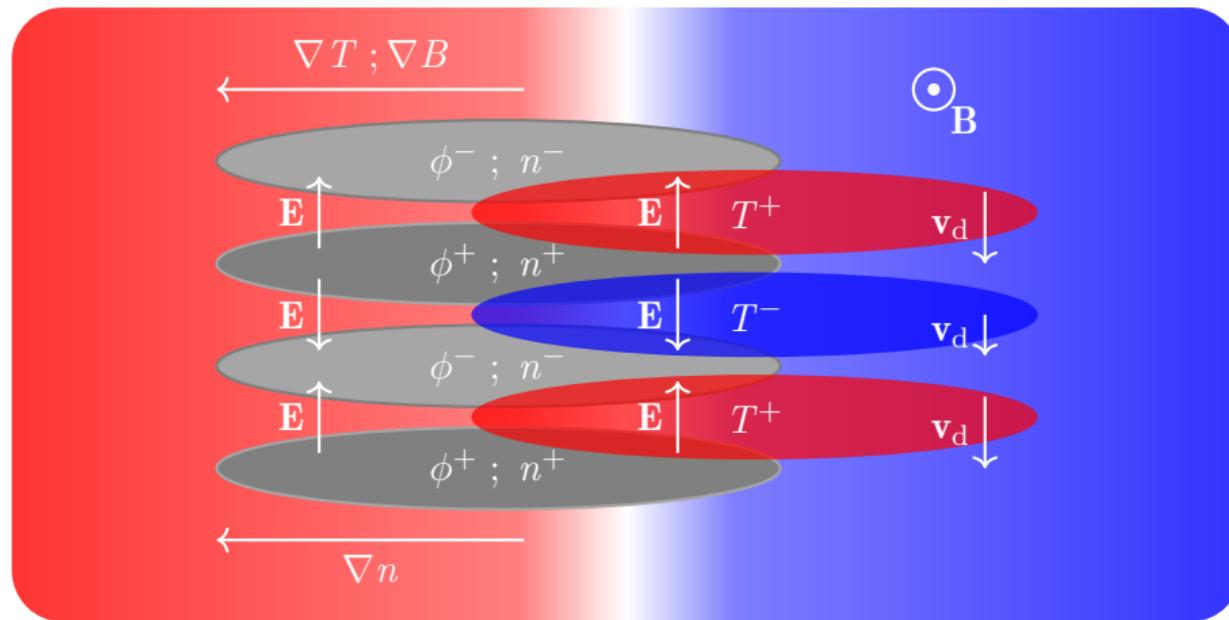
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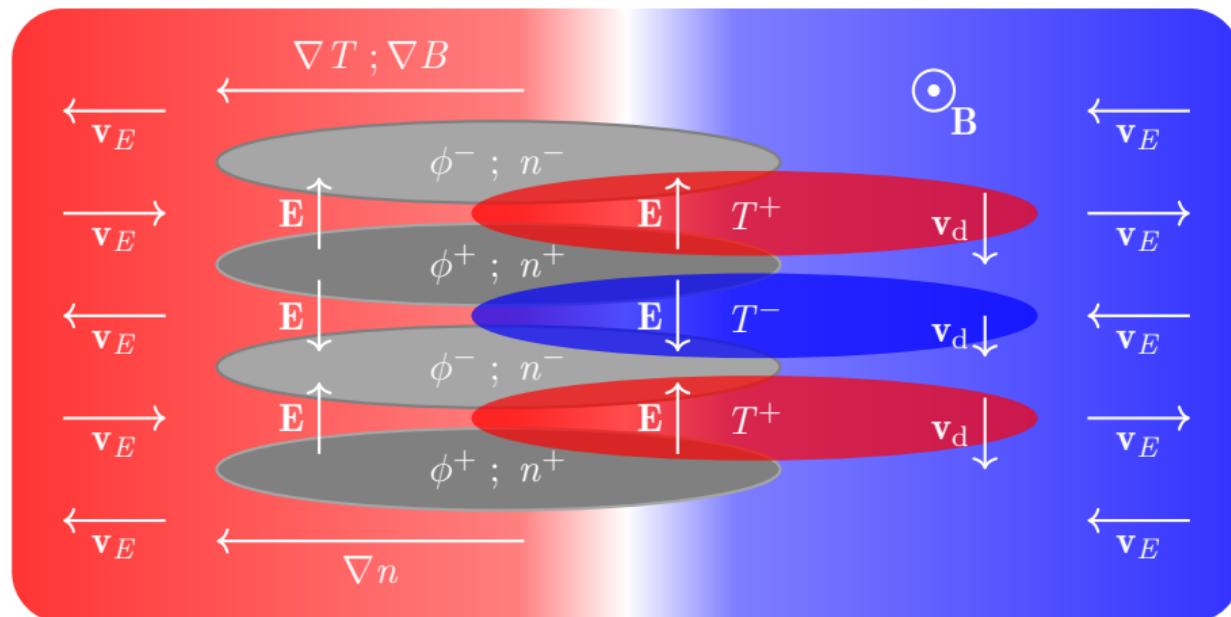
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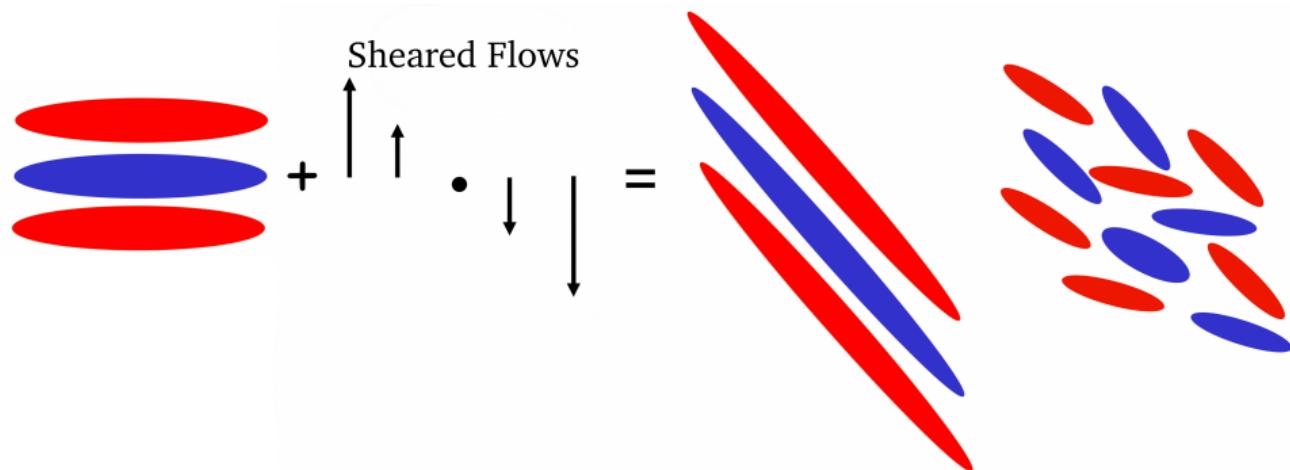


ION TEMPERATURE GRADIENT (ITG)-DRIVEN INSTABILITY



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$$\omega_{E \times B} = \frac{1}{2} \frac{\partial^2 \langle \phi \rangle}{\partial x^2} \quad \langle \phi \rangle = \frac{1}{L_y} \int_0^{L_y} dy \phi(x, y, s=0)$$

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$$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial \mathbf{x}} \cdot \frac{d\mathbf{x}}{dt} + \frac{\partial f}{\partial \mathbf{v}} \cdot \frac{d\mathbf{v}}{dt} = 0 \quad f(\mathbf{x}, \mathbf{v}, t)$$

Vlasov Equation

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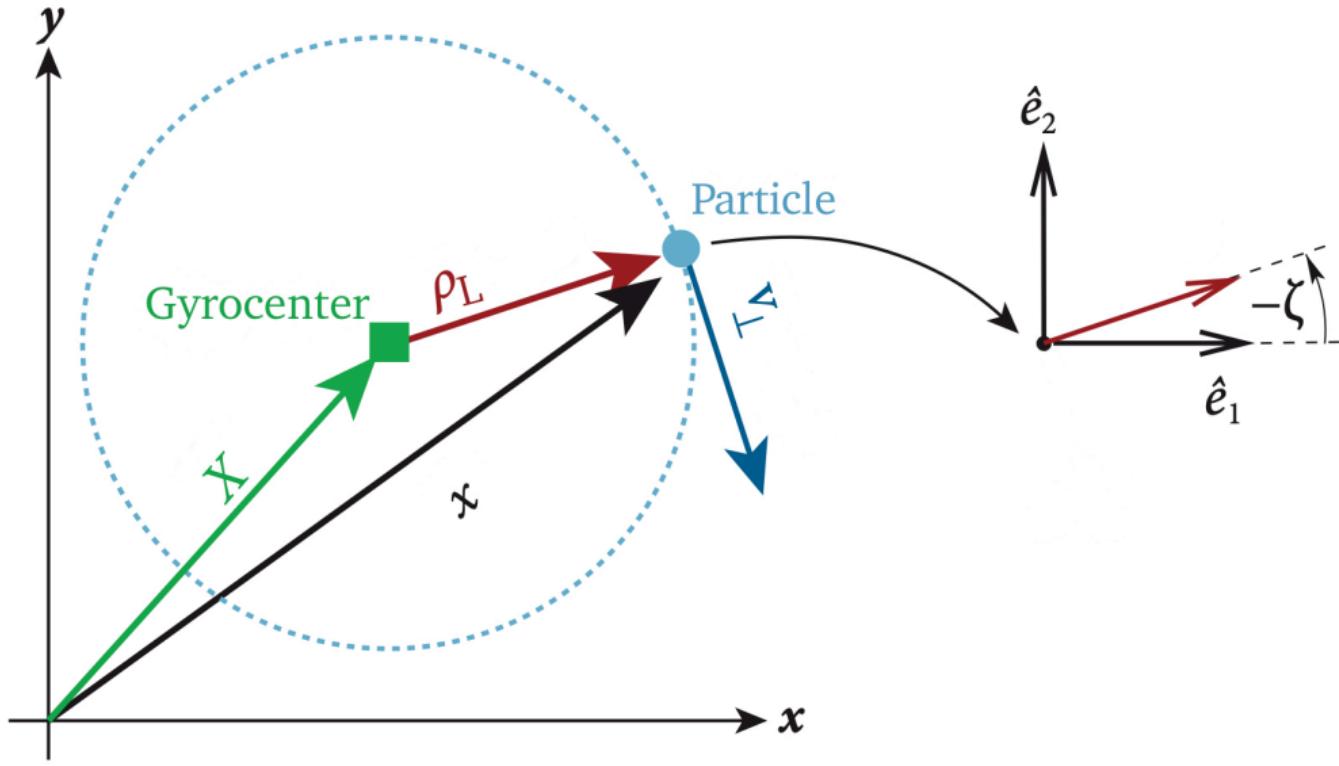
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$$\frac{\partial g}{\partial t} + \mathbf{v}_{\chi} \cdot \nabla g + (v_{\parallel} \mathbf{b} + \mathbf{v}_D) \cdot \nabla (\delta f) - \frac{\mu B}{m} \frac{\mathbf{B} \cdot \nabla B}{B^2} \frac{\partial (\delta f)}{\partial v_{\parallel}} = S \quad \text{Gyrokinetic Equation}$$

$$g \sim (f_0 + \delta f) \quad S \sim (f + f_0) \quad \mathbf{v}_D \sim \nabla B \quad \mathbf{v}_{\chi} \sim \mathbf{E} \times \mathbf{B} \quad f(x, y, s, v_{\parallel}, \mu)$$

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S6	21	83	16	64	9	1	$ \nu_{\parallel} $	0.2	0.1	0.1	6	1.4	2.1

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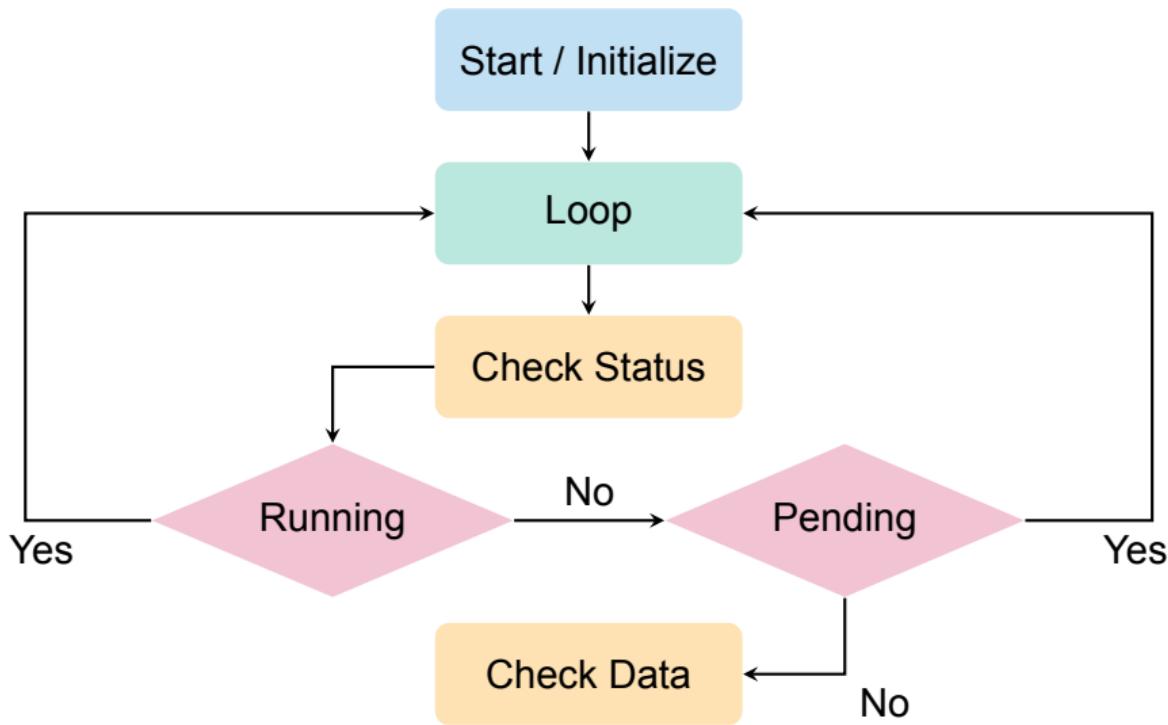
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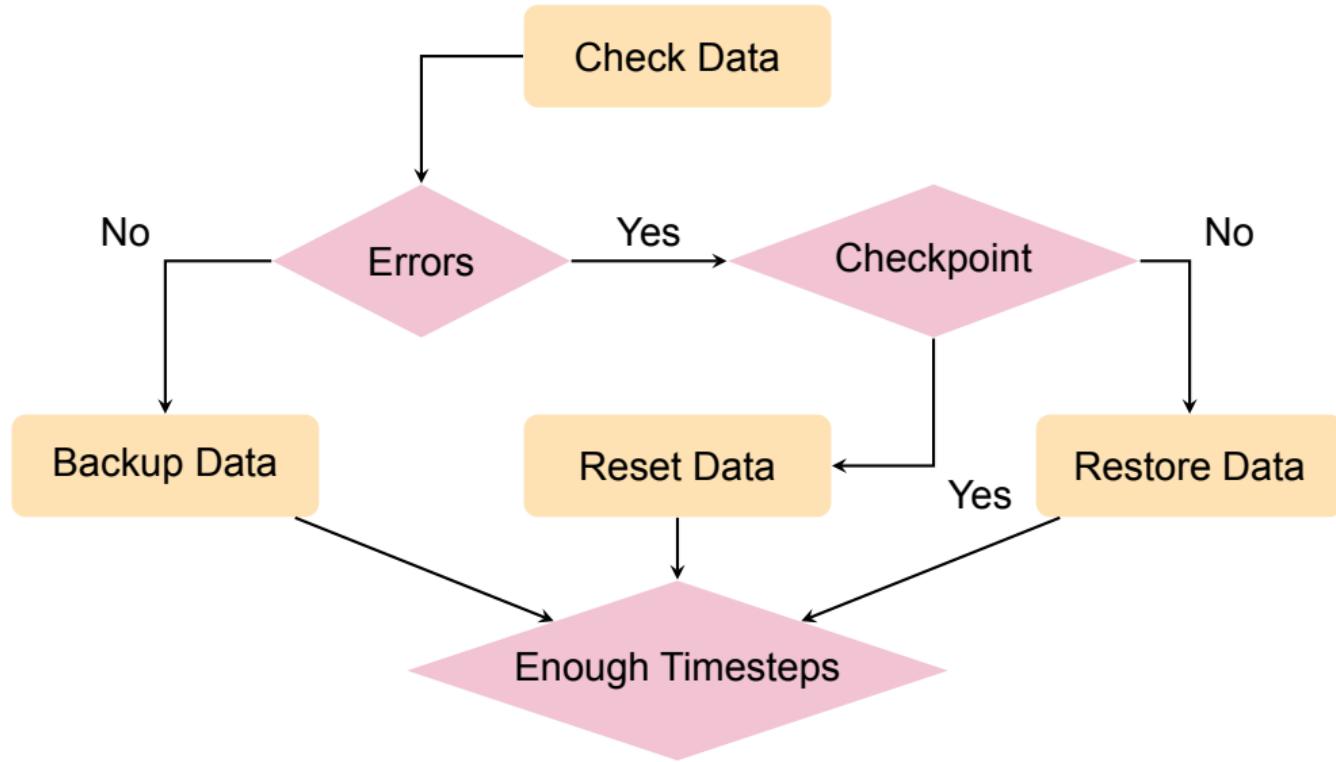
- Zonal flow mode that dominates the $E \times B$ staircase pattern are called **basic mode**
- The basic mode exhibits the maximum amplitude in the spectrum $|\hat{\omega}_{E \times B}|_{n_{ZF}}$

RESTART SCRIPT

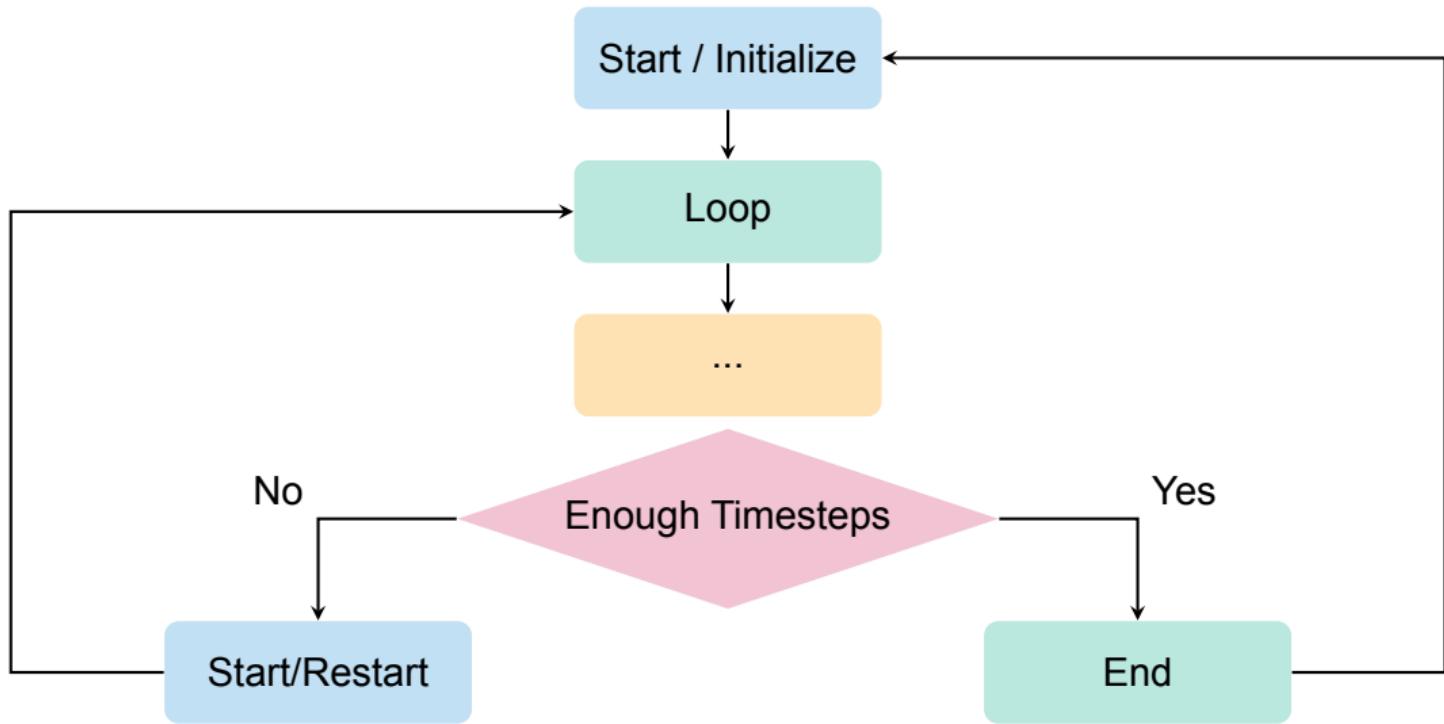
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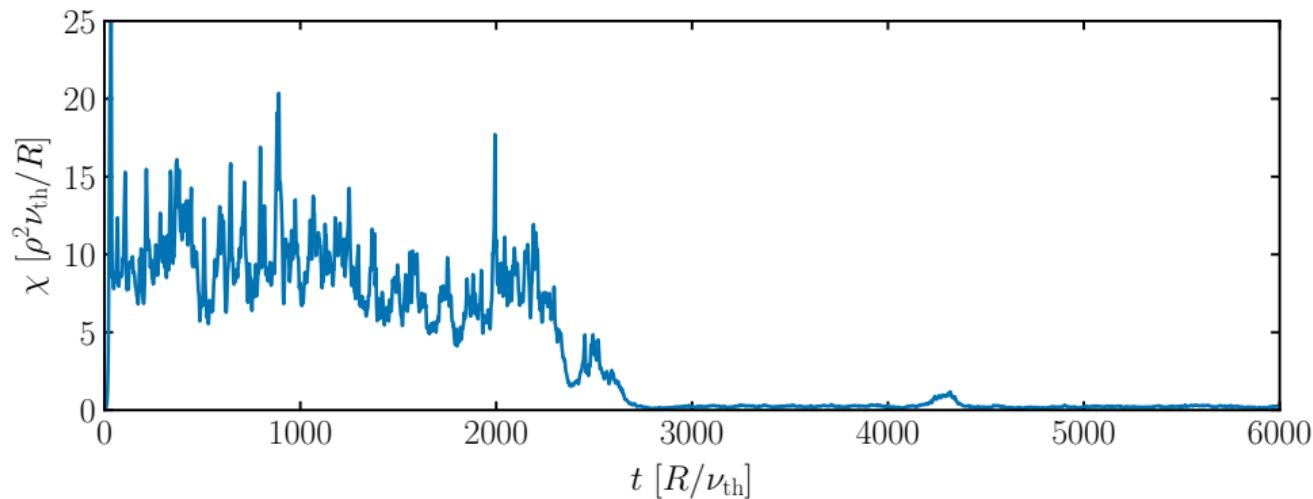
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Verification:

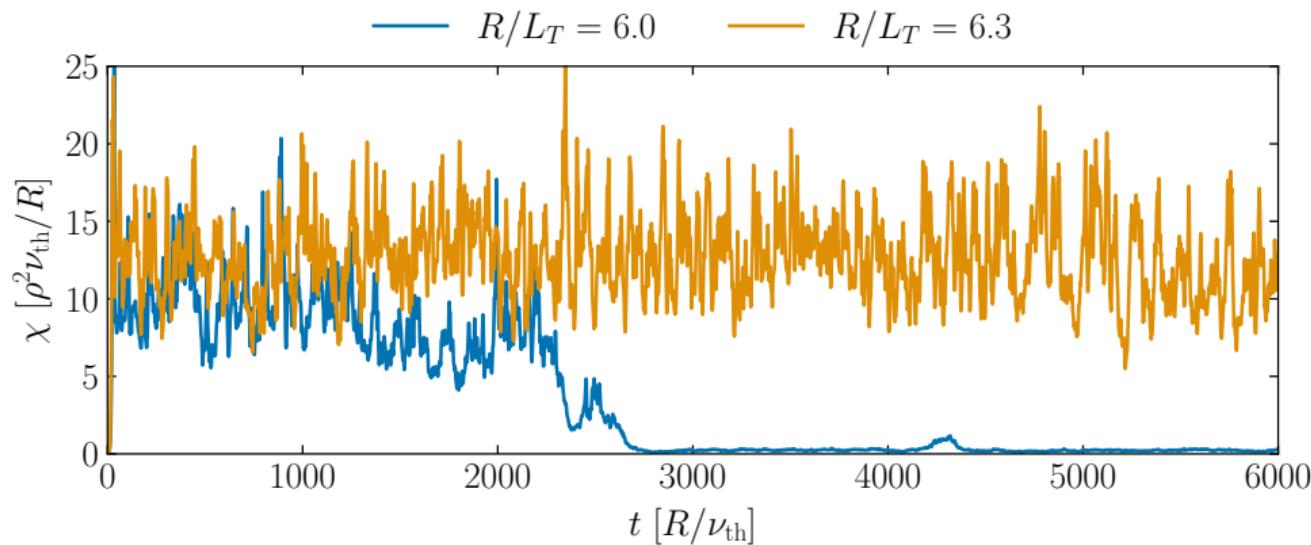
1. Reduce only one number of grid points and look if criterias (1), (2) are satisfied
2. Reduce to known the minimum number of grid points to verify result in general.

BENCHMARK

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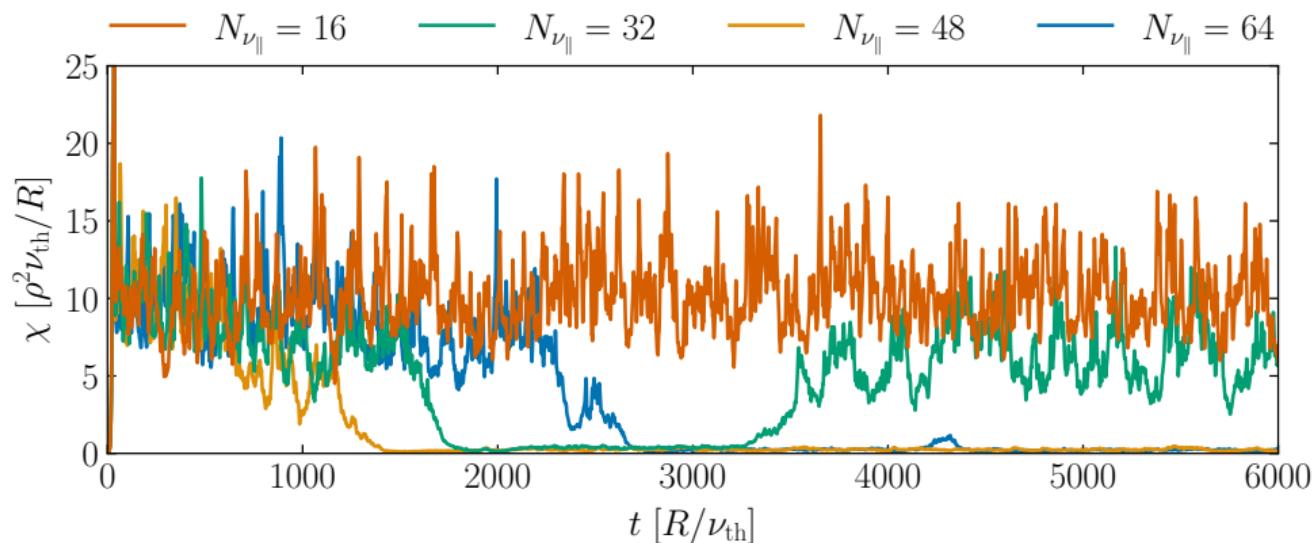


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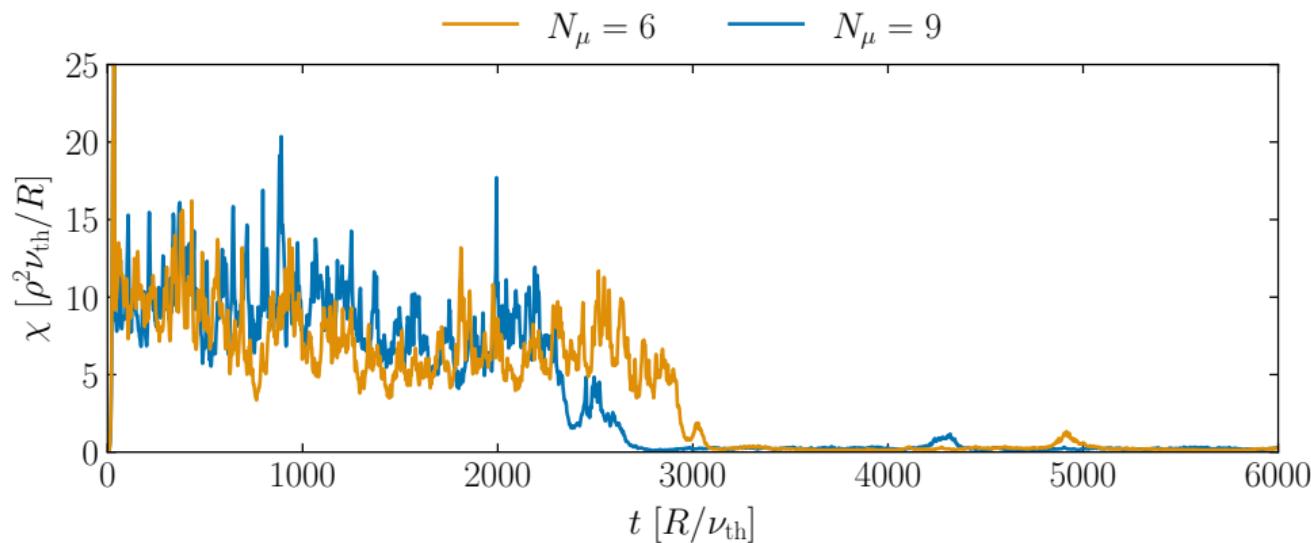


REDUCTION OF GRID POINTS

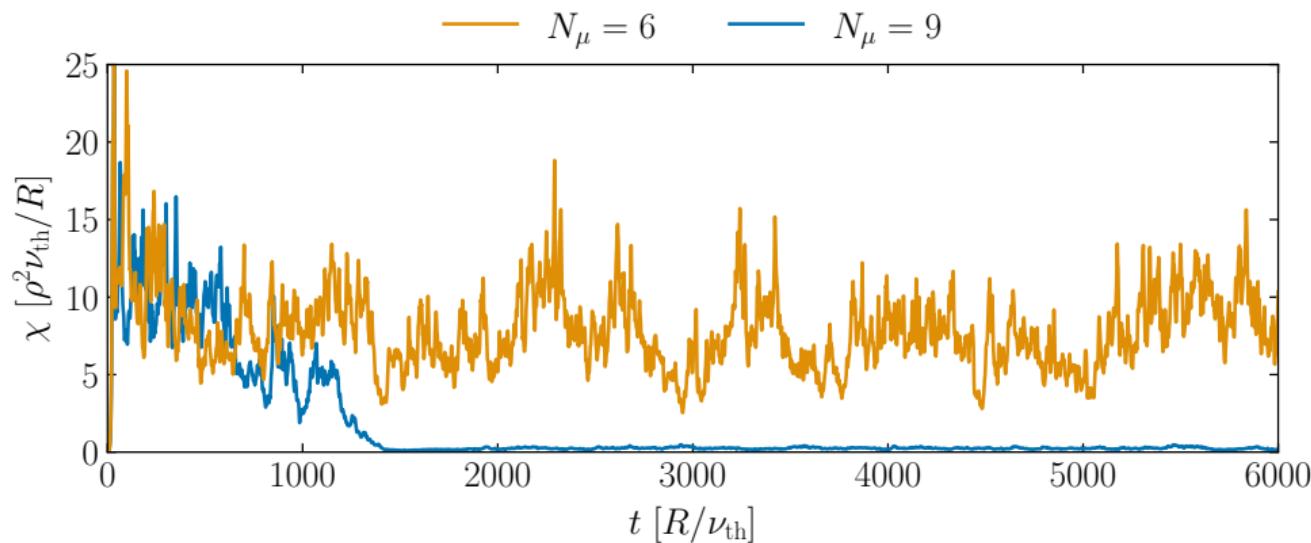
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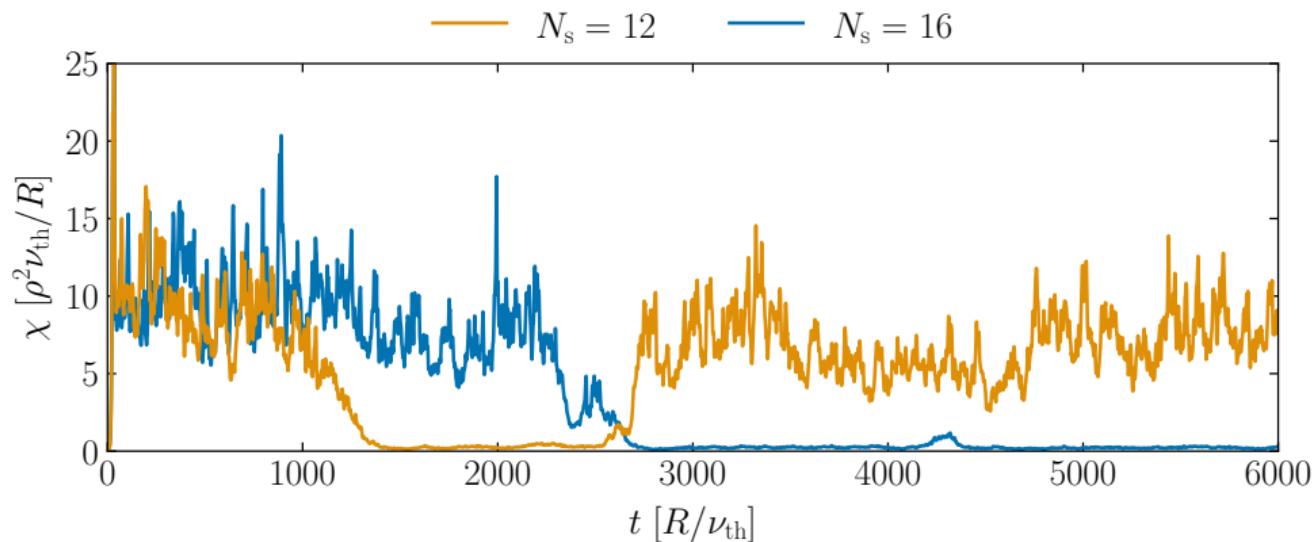
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Final Resolution

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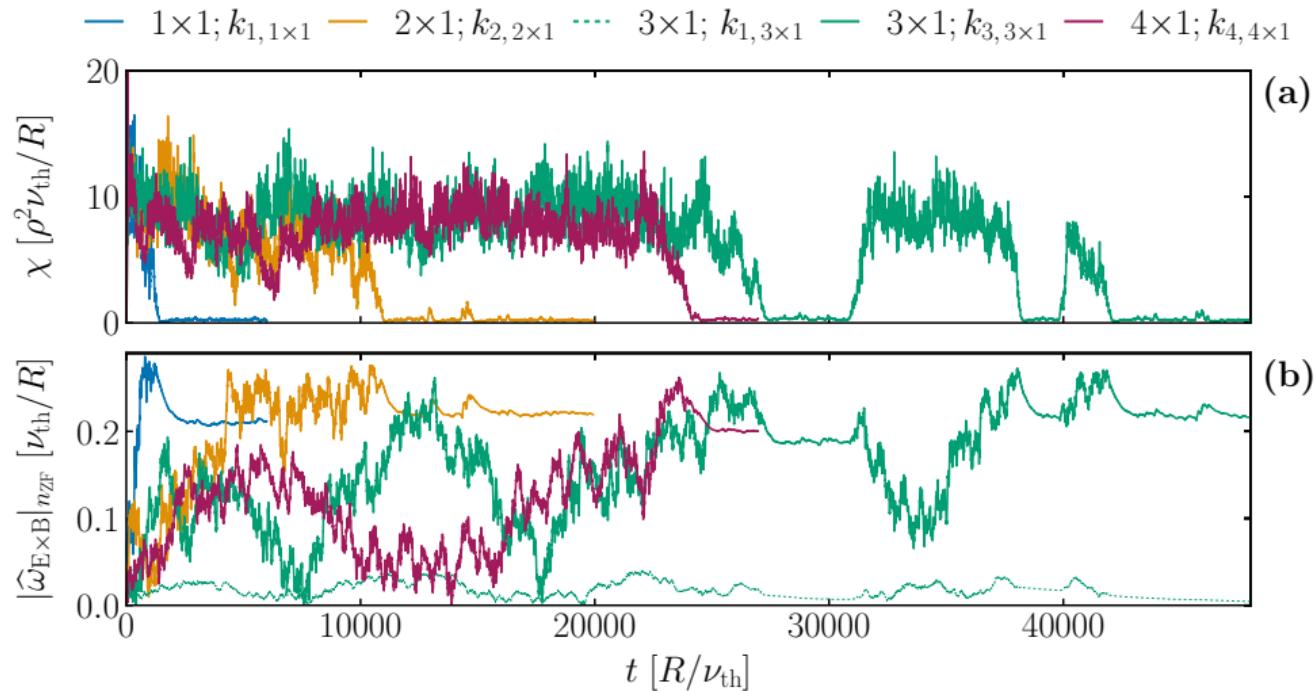
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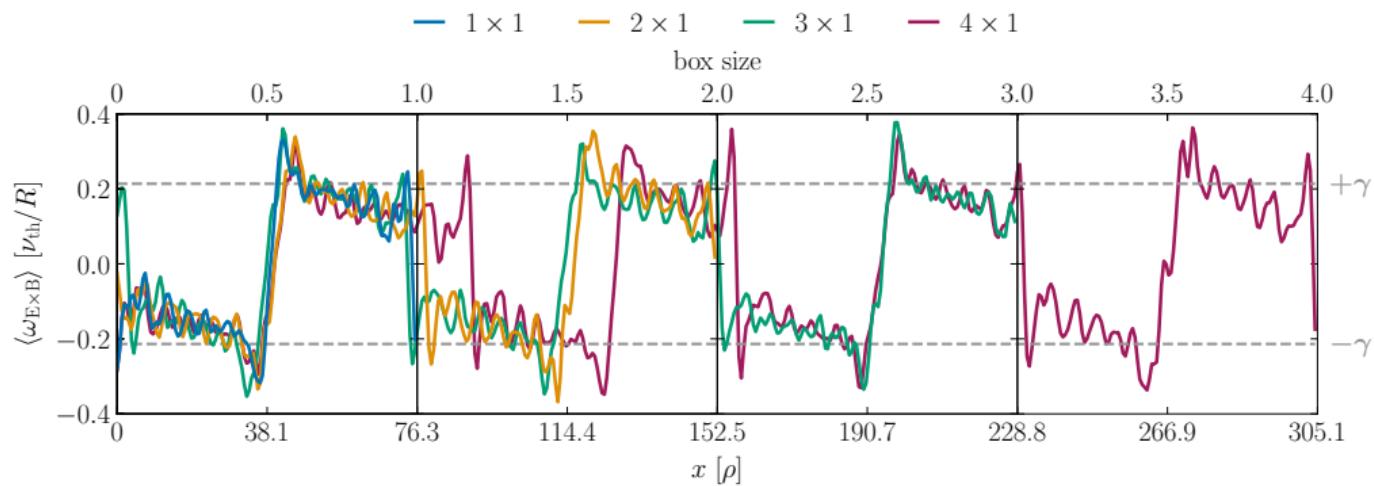
(1) Radial

$$N_R \times N_B \in [1 \times 1, 2 \times 1, 3 \times 1, 4 \times 1]$$

SIZE CONVERGENCE OF $E \times B$ STAIRCASE PATTERN



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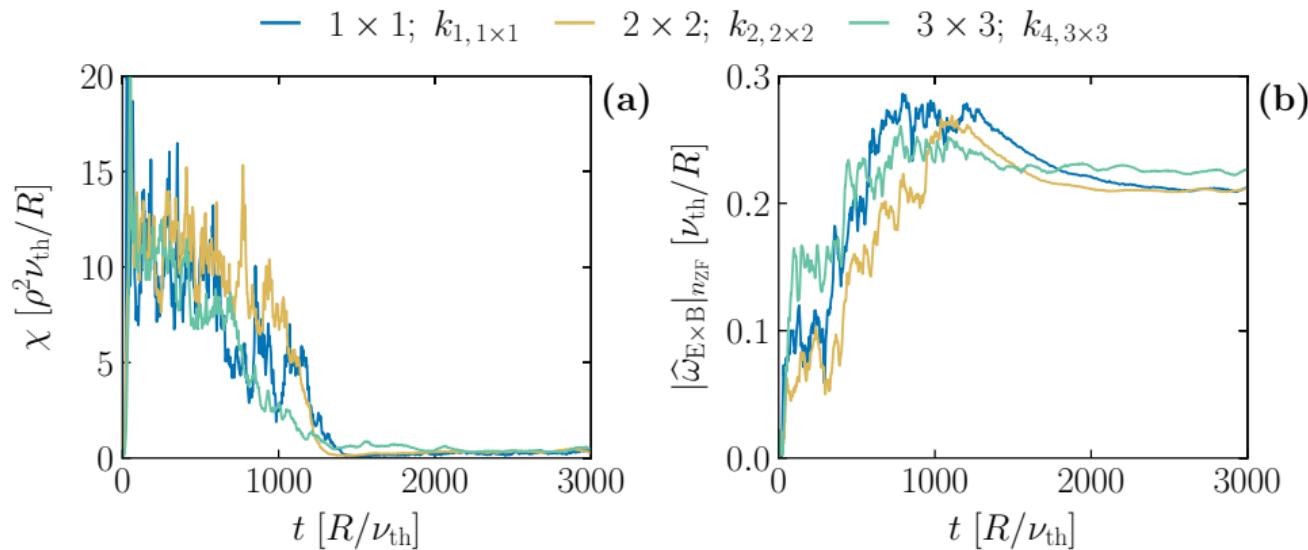


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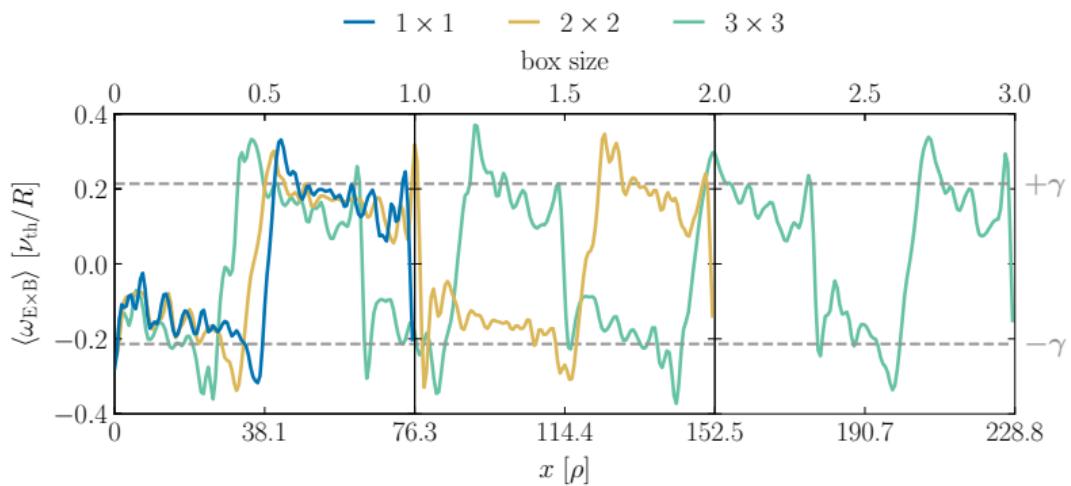
(2) Isotropic

$$N_R \times N_B \in [1 \times 1, 1.5 \times 1.5, 2 \times 2, 2.5 \times 2.5, 3 \times 3]$$

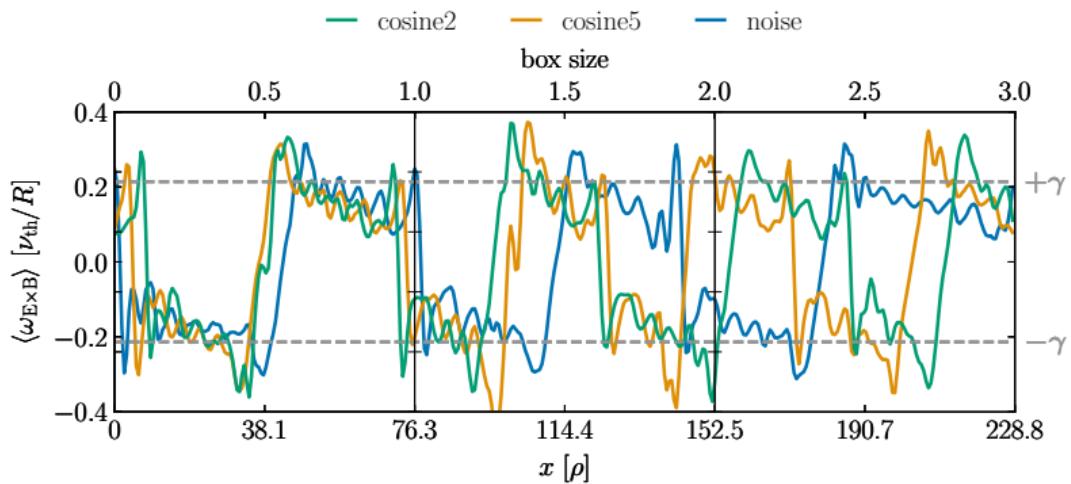
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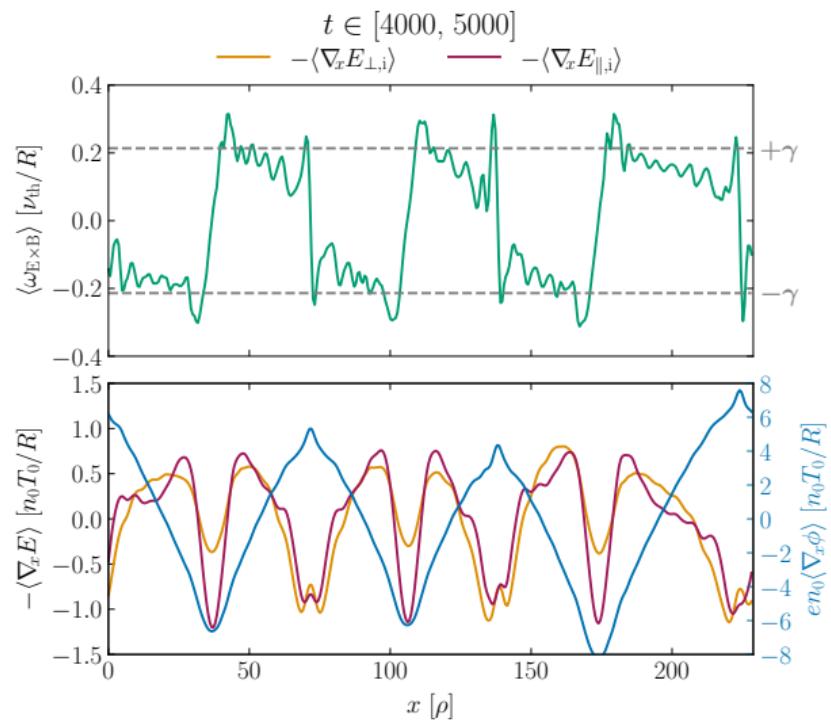
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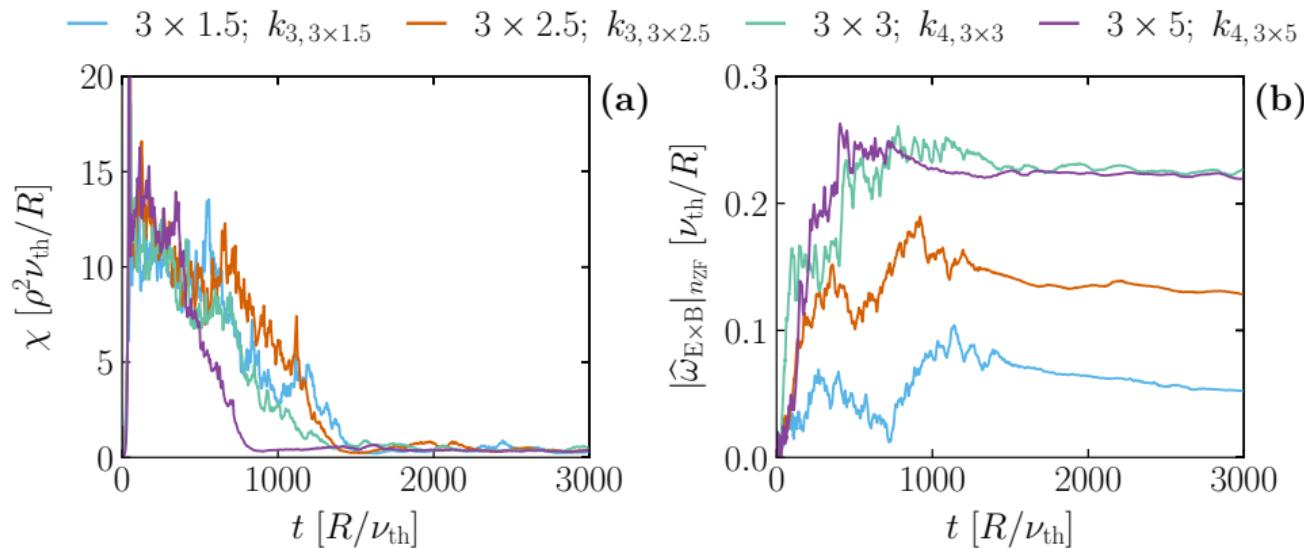


SIZE CONVERGENCE OF E × B STAIRCASE PATTERN

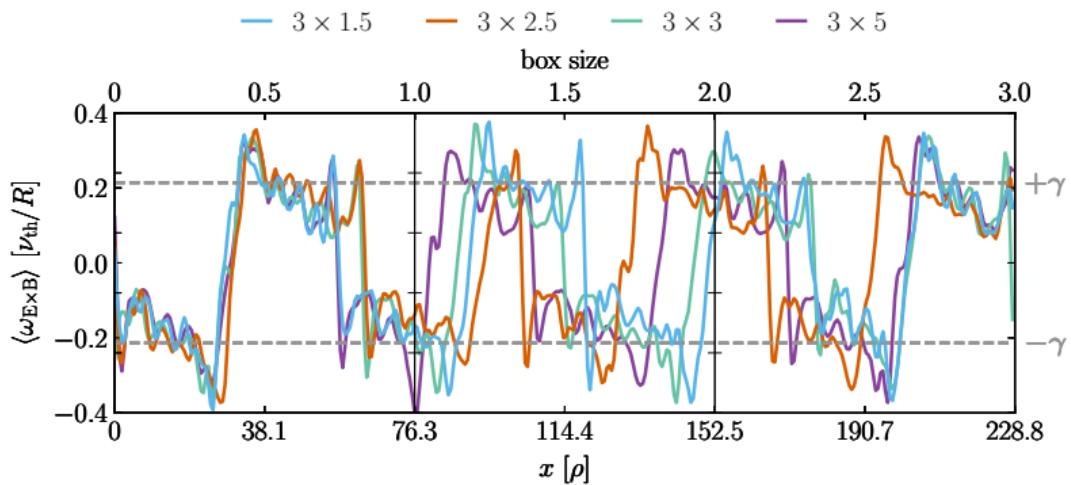
(3) Binormal

$$N_R \times N_B \in [3 \times 1.5, 3 \times 2.5, 3 \times 3, 3 \times 5]$$

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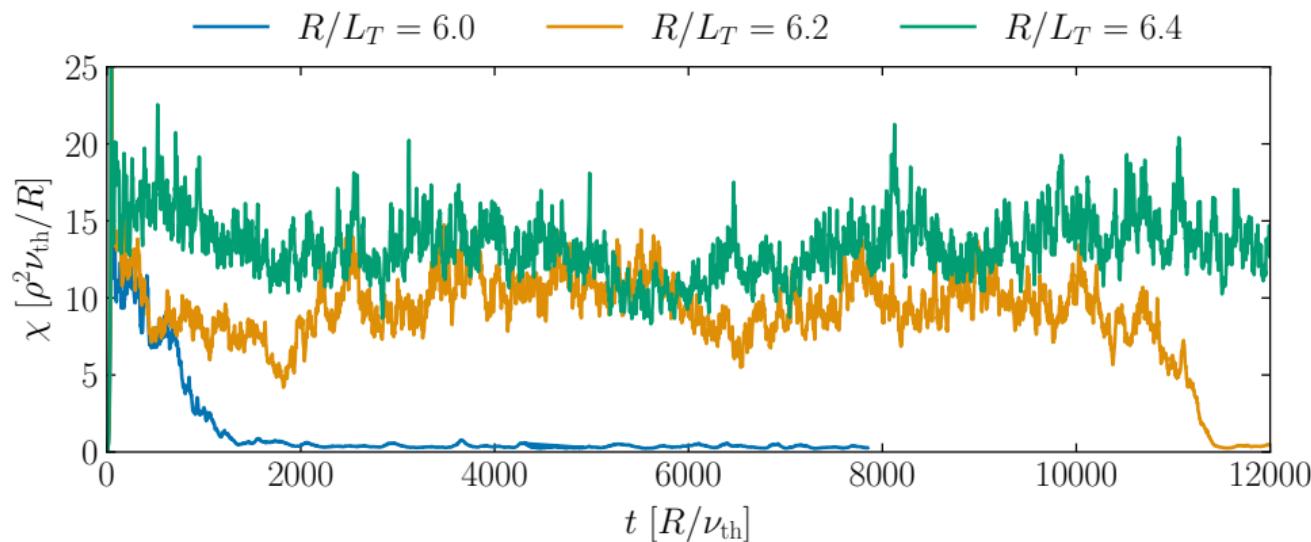
⇒ Mesoscale pattern size of:

$$\sim 57 - 76 \rho$$

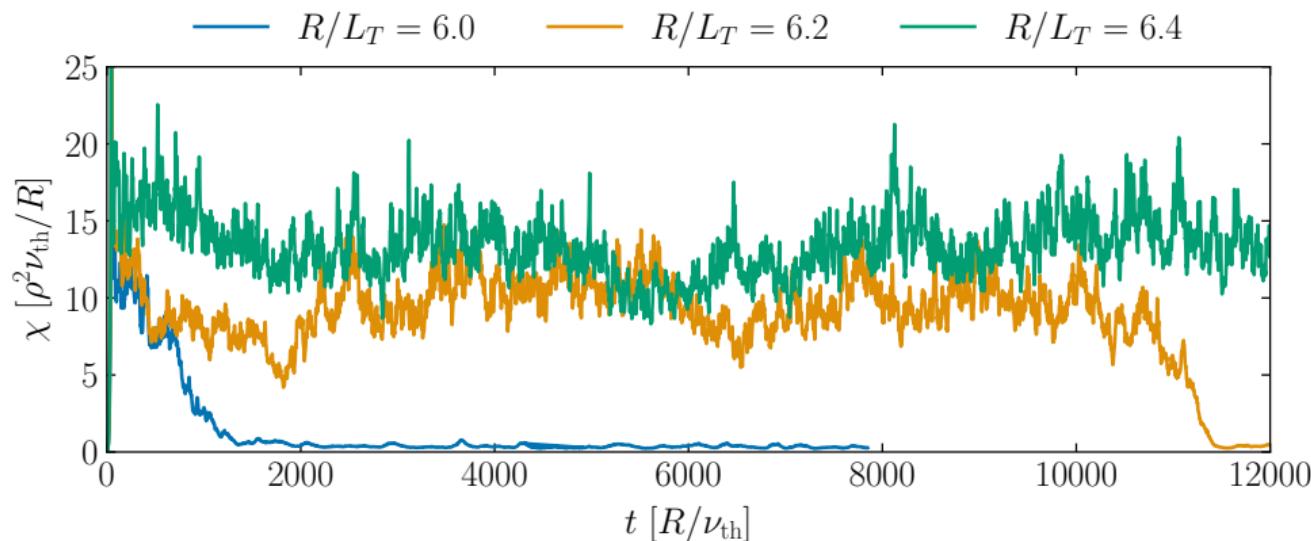
- Non-locality is inherent to ITG-driven turbulence
- Avalanches are spatially organized by the E × B staircase pattern

THE FINITE HEAT FLUX THRESHOLD

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$$\Rightarrow \boxed{R/L_T|_{\text{finite}} = 6.3 \pm 0.1}$$

CONCLUSION

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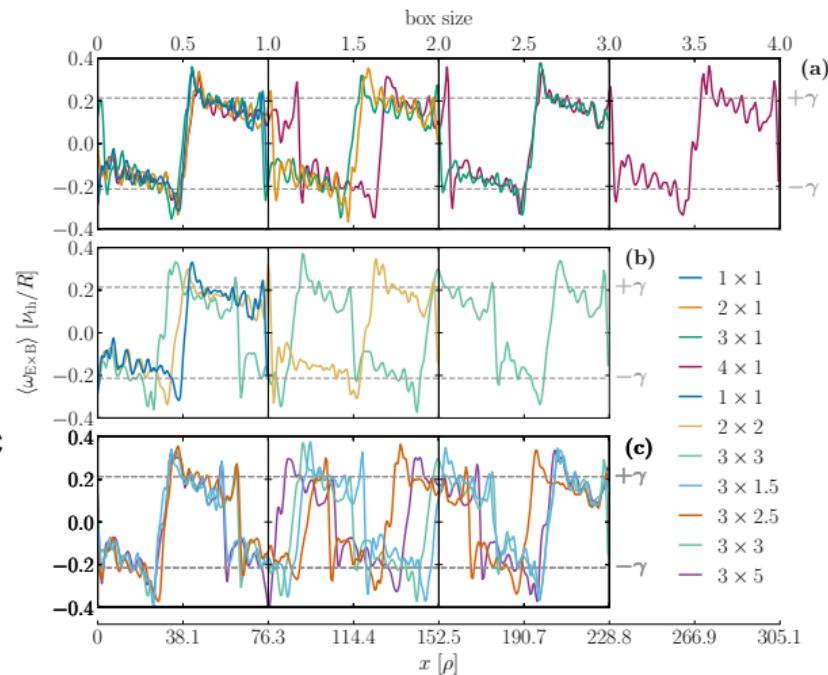
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- Finite heat flux threshold is located at $R/L_T|_{\text{finite}} = 6.3 \pm 0.1$

