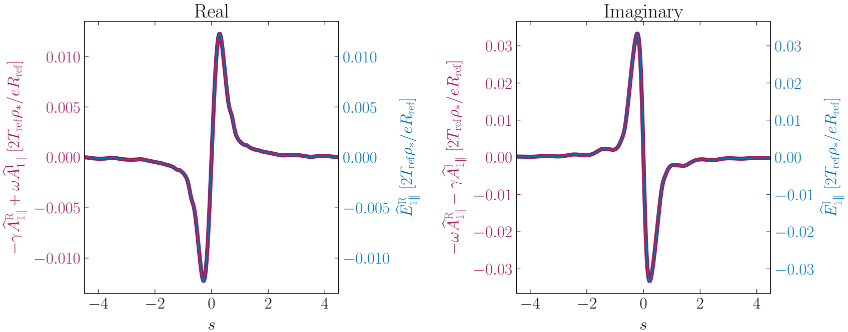
UNIVERSITÄT BAYREUTH

MITIGATION OF THE CANCELLATION PROBLEM IN LOCAL GYROKINETIC SIMULATIONS

October 2, 2024



Manuel Lippert

Theoretical Physics V

MOTIVATION

MOTIVATION

- In plasma physics the plasma beta β is one of the fundamental dimensionless parameters

MOTIVATION

- In plasma physics the plasma beta β is one of the fundamental dimensionless parameters
- Kinetic Models describes plasma in the 6D phase space through the Vlasov equation and Maxwell equations.

MOTIVATION

- In plasma physics the plasma beta β is one of the fundamental dimensionless parameters
- Kinetic Models describes plasma in the 6D phase space through the Vlasov equation and Maxwell equations.
- **Cancellation Problem:** Limits gyrokinetic investigations to low plasma beta high perpendicular wave vectors $k_{\perp}$

MOTIVATION

- In plasma physics the plasma beta β is one of the fundamental dimensionless parameters
- Kinetic Models describes plasma in the 6D phase space through the Vlasov equation and Maxwell equations.
- **Cancellation Problem:** Limits gyrokinetic investigations to low plasma beta high perpendicular wave vectors $k_\perp$
- New technique for global nonlinear simulations found to mitigate the cancellation problem by using an additional field equation

MOTIVATION

- In plasma physics the plasma beta β is one of the fundamental dimensionless parameters
- Kinetic Models describes plasma in the 6D phase space through the Vlasov equation and Maxwell equations.
- **Cancellation Problem:** Limits gyrokinetic investigations to low plasma beta high perpendicular wave vectors $k_\perp$
- New technique for global nonlinear simulations found to mitigate the cancellation problem by using an additional field equation

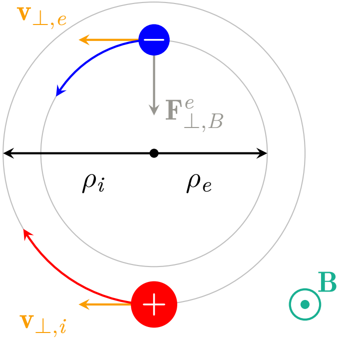
Is it possible to mitigate the cancellation problem in local gyrokinetic simulations with the introduction of a new field equation?

CHARGED PARTICLE MOTION

CHARGED PARTICLE MOTION

Lorentz force

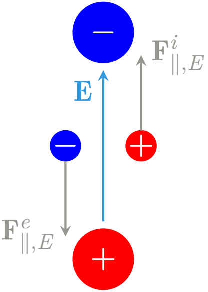
$$F_{\perp,B} = |q| v_\perp B$$



$$\rho = \frac{m\mathbf{v}_\perp}{|q|B} \quad \omega_c = \frac{|q|B}{m}$$

Electric force

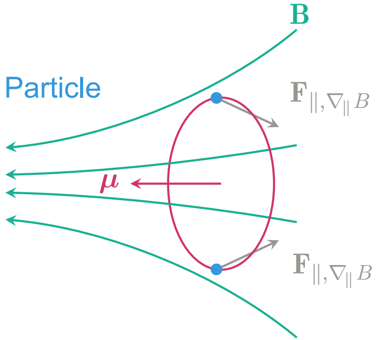
$$F_{\parallel,E} = qE_\parallel$$



$$\rho_{\text{th}} = \frac{m v_{\text{th}}}{|q|B}$$

Inhomogeneous magnetic field

$$F_{\parallel,\nabla_\parallel B} = -\underbrace{\frac{m v_\perp^2}{2B}}_{\mu} \nabla_\parallel B$$



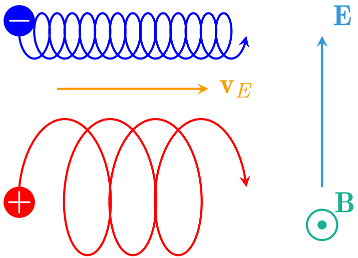
$$v_{\text{th}} = \sqrt{\frac{2T}{m}}$$

DRIFTS IN THE GUIDING CENTER

DRIFTS IN THE GUIDING CENTER

$\mathbf{E} \times \mathbf{B}$ Drift

$$\mathbf{v}_E = \frac{\mathbf{E}\times\mathbf{B}}{B^2}$$



DRIFTS IN THE GUIDING CENTER

$\mathbf{E} \times \mathbf{B}$ Drift

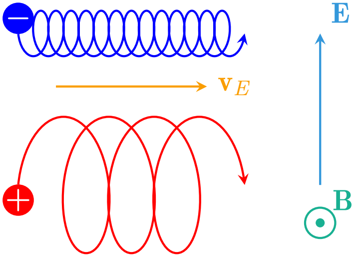
$$\mathbf{v}_E = \frac{\mathbf{E}\times\mathbf{B}}{B^2}$$

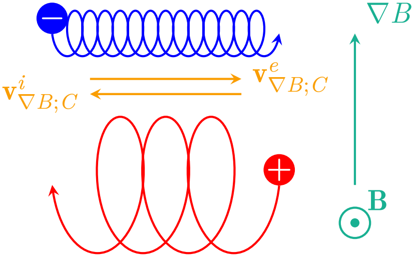
∇B Drift

$$\mathbf{v}_{\nabla B} = \frac{m v_\perp^2}{2q}\frac{\mathbf{B}\times\nabla B}{B^3}$$

Curvature Drift

$$\mathbf{v}_C = \frac{m v_\parallel^2}{qB^2}\mathbf{B} \times \underbrace{\mathbf{C}}_{\frac{\nabla B}{B}} = \frac{m v_\parallel^2}{q}\frac{\mathbf{B}\times\nabla B}{B^3}$$





PLASMA BETA

PLASMA BETA

$$\beta = \frac{\overbrace{nT}^{\text{plasma pressure}}}{\underbrace{B^2/2\mu_0}_{\text{magnetic field pressure}}}$$

PLASMA BETA

$$\beta = \frac{\overbrace{nT}^{\text{plasma pressure}}}{\underbrace{B^2/2\mu_0}_{\text{magnetic field pressure}}}$$

- Characterizes quality of confinement (Best confinement for $\beta < 1$)
- Relevant for fusion rate ($\sim \beta^2$), MHD stability of a fusion device
- Indicator for the relevance of electromagnetic effects
- Electromagentic fields vanish in the limit $\beta \to 0$

GYROKINETIC ORDERING

GYROKINETIC ORDERING

- Low frequency: $\omega/\omega_{\mathrm{c}} \ll 1$
- Anisotropy: $k_{\|}/k_{\perp} \ll 1$
- Strong Magnetization: $\rho/L_n \sim \rho/L_T \sim \rho/L_B \ll 1$ $\qquad L_G = G_0 \left(\frac{\mathrm{d}\,G_0}{\mathrm{dx}}\right)^{-1}$
- Small fluctuations: $F_1/F_0 \ll 1$

GYROKINETIC ORDERING

- Low frequency: $\omega/\omega_{\mathrm{c}} \ll 1$
- Anisotropy: $k_{\|}/k_{\perp} \ll 1$
- Strong Magnetization: $\rho/L_n \sim \rho/L_T \sim \rho/L_B \ll 1$ $\qquad L_G = G_0 \left(\frac{\mathrm{d}G_0}{\mathrm{dx}}\right)^{-1}$
- Small fluctuations: $F_1/F_0 \ll 1$

$$\omega/\omega_{\mathrm{c}} \sim k_{\|}/k_{\perp} \sim \rho/L_n \sim \rho/L_T \sim \rho/L_B \sim F_1/F_0 \sim \epsilon_\delta$$

$$\rho_\star = \frac{\rho_{\mathrm{th,ref}}}{L_B} = \frac{m_{\mathrm{ref}} v_{\mathrm{th,ref}}}{eB_{\mathrm{ref}}} \sim \epsilon_\delta$$

DERIVATION OF GYROKINETIC EQUATION

DERIVATION OF GYROKINETIC EQUATION

- Find fundamental one-form of the gyrocenter phase space

$$\int \mathrm{d}t \; L = \int \gamma$$

DERIVATION OF GYROKINETIC EQUATION

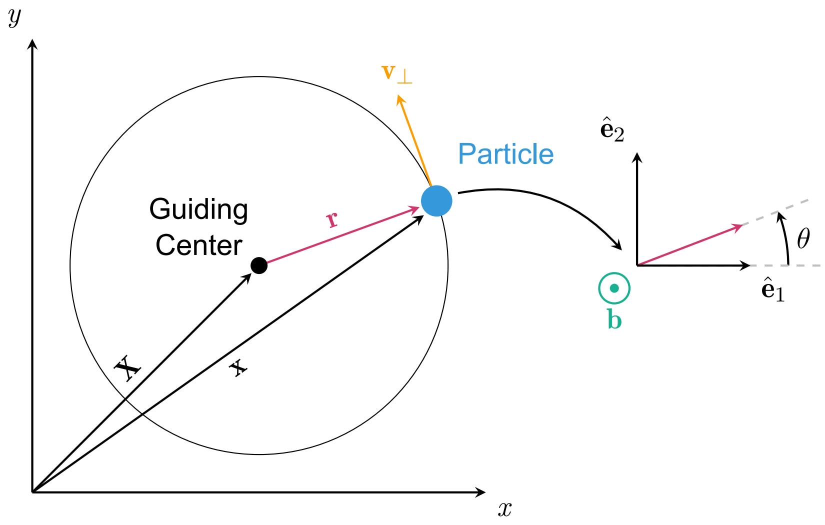
- Find fundamental one-form of the gyrocenter phase space

$$\int \mathrm{d}t\ L = \int \gamma$$

- Particle phase space $\{\mathbf{x}, \mathbf{v}\} \rightarrow$ guiding center $\{\mathbf{X}, v_{\parallel}, \mu, \theta\} \rightarrow$ gyrocenter $\{\bar{\mathbf{X}}, \bar{v}_{\parallel}, \bar{\mu}\}$

$$\Phi = \Phi_0 + \underbrace{\tilde{\Phi}_1 + \bar{\Phi}_1}_{\Phi_1} \qquad \mathbf{A} = \mathbf{A}_0 + \underbrace{\tilde{\mathbf{A}}_1 + \bar{\mathbf{A}}_1}_{\mathbf{A}_1}$$

DERIVATION OF GYROKINETIC EQUATION



DERIVATION OF GYROKINETIC EQUATION

- Find fundamental one-form of the gyrocenter phase space

$$\int \mathrm{d}t\; L = \int \gamma$$

- Particle phase space $\{\mathbf{x}, \mathbf{v}\} \rightarrow$ guiding center $\{\mathbf{X}, v_{\|}, \mu, \theta\} \rightarrow$ gyrocenter $\{\bar{\mathbf{X}}, \bar{v}_{\|}, \bar{\mu}\}$

$$\Phi = \Phi_0 + \underbrace{\tilde{\Phi}_1 + \bar{\Phi}_1}_{\Phi_1} \qquad \mathbf{A} = \mathbf{A}_0 + \underbrace{\tilde{\mathbf{A}}_1 + \bar{\mathbf{A}}_1}_{\mathbf{A}_1}$$

DERIVATION OF GYROKINETIC EQUATION

- Find fundamental one-form of the gyrocenter phase space

$$\int \mathrm{d}t\ L = \int \gamma$$

- Particle phase space $\{\mathbf{x}, \mathbf{v}\} \rightarrow$ guiding center $\{\mathbf{X}, v_{\|}, \mu, \theta\} \rightarrow$ gyrocenter $\{\bar{\mathbf{X}}, \bar{v}_{\|}, \bar{\mu}\}$

$$\Phi = \Phi_0 + \underbrace{\tilde{\Phi}_1 + \bar{\Phi}_1}_{\Phi_1} \qquad \mathbf{A} = \mathbf{A}_0 + \underbrace{\tilde{\mathbf{A}}_1 + \bar{\mathbf{A}}_1}_{\mathbf{A}_1}$$

- Vlasov Equation in gyrocenter phase space without collisions

$$\frac{\partial F}{\partial t} + \dot{\mathbf{X}} \cdot \frac{\partial F}{\partial \mathbf{X}} + \dot{v}_{\|} \cdot \frac{\partial F}{\partial v_{\|}} = 0$$

DERIVATION OF GYROKINETIC EQUATION

- Delta-f approximation $F = F_0 + F_1$

$$\frac{\partial F_1}{\partial t} + \dot{\mathbf{X}} \cdot \nabla F_1 + \dot{v}_{\|} \cdot \frac{\partial F_1}{\partial v_{\|}} = \underbrace{-\dot{\mathbf{X}} \cdot \nabla F_0 - \dot{v}_{\|} \frac{\partial F_0}{\partial v_{\|}}}_{S}$$

DERIVATION OF GYROKINETIC EQUATION

- Delta-f approximation $F = F_0 + F_1$

$$\frac{\partial F_1}{\partial t} + \dot{\mathbf{X}} \cdot \nabla F_1 + \dot{v}_\parallel \cdot \frac{\partial F_1}{\partial v_\parallel} = \underbrace{-\dot{\mathbf{X}} \cdot \nabla F_0 - \dot{v}_\parallel \frac{\partial F_0}{\partial v_\parallel}}_{S}$$

- Maxwellian as equilibrium distribution $F_0 = F_\mathrm{M}$ and $\dot{\mathbf{X}}$ and $\dot{v}_\parallel$ from Lagrangian

$$\frac{\partial F_1}{\partial t} + \dot{\mathbf{X}} \cdot \nabla F_1 - \frac{\mathbf{b}_0}{m} \cdot \left(Ze\nabla\Phi_0 + \mu\nabla B_0 - mR\Omega^2\nabla R\right) \cdot \frac{\partial F_1}{\partial v_\parallel} = S$$

$$\begin{aligned} S = &-(\mathbf{v}_\chi + \mathbf{v}_\mathrm{D}) \cdot \widetilde{\nabla} F_\mathrm{M} - \frac{Zev_\parallel}{T} \partial_t \bar{A}_{1\parallel} F_\mathrm{M} \\ &- \frac{F_\mathrm{M}}{T}(v_\parallel \mathbf{b}_0 + \mathbf{v}_\mathrm{D} + \mathbf{v}_{\bar{B}_{1\perp}}) \cdot (Ze\nabla\bar{\Phi} + \mu\nabla\bar{B}_{1\parallel}) \end{aligned}$$

DERIVATION OF FIELD EQUATIONS

DERIVATION OF FIELD EQUATIONS

- Maxwell's equations

$$\sum_s Z_s e\, n_s = 0 \qquad \nabla \times \mathbf{E}_1 = -\frac{\partial \mathbf{B}_1}{\partial t}$$

$$\nabla \cdot \mathbf{B}_1 = 0 \qquad \nabla \times \mathbf{B}_1 = \mu_0 \sum_s \mathbf{j}_s$$

DERIVATION OF FIELD EQUATIONS

- Maxwell's equations

$$\sum_s Z_s e\, n_s = 0 \qquad \nabla \times \mathbf{E}_1 = -\frac{\partial \mathbf{B}_1}{\partial t}$$

$$\nabla \cdot \mathbf{B}_1 = 0 \qquad \nabla \times \mathbf{B}_1 = \mu_0 \sum_s \mathbf{j}_s$$

- Moments of the distribution f

$$n(\mathbf{x}) = \int \mathrm{d}\mathbf{v}\; f(\mathbf{x}, \mathbf{v})$$

$$j_{||} = Ze \int \mathrm{d}\mathbf{v}\; v_{||} f(\mathbf{x}, \mathbf{v}) \qquad \mathbf{j}_{\perp} = Ze \int \mathrm{d}\mathbf{v}\; \mathbf{v}_{\perp} f(\mathbf{x}, \mathbf{v})$$

DERIVATION OF FIELD EQUATIONS

$$\begin{tikzcd} \text{Moments of } f\text{: } n(\mathbf{x}), \mathbf{j}(\mathbf{x}) \arrow[r] & \text{Guiding Center Phase Space} & \text{Vlasov Equation for } F \arrow[l, "\text{pull}", "\text{back}"'] \end{tikzcd}$$

- Pullback transformation

$$F^{\mathrm{gc}} = \mathcal{P}\left\{F\right\} = F \underbrace{- \frac{F_{\mathrm{M}}}{T}\left(Ze\widetilde{\Phi}_1 - \mu \bar{B}_{1\|}\right)}_{\text{Correction Term}}$$

DERIVATION OF FIELD EQUATIONS

Moments of f: $n(\mathbf{x}), \mathbf{j}(\mathbf{x})$ → Guiding Center Phase Space $\xleftarrow{\text{pull back}}$ Vlasov Equation for F

- Pullback transformation

$$F^{\mathrm{gc}} = \mathcal{P}\left\{F\right\} = F \underbrace{- \frac{F_{\mathrm{M}}}{T}\left(Ze\widetilde{\Phi}_1 - \mu \bar{B}_{1\parallel}\right)}_{\text{Correction Term}}$$

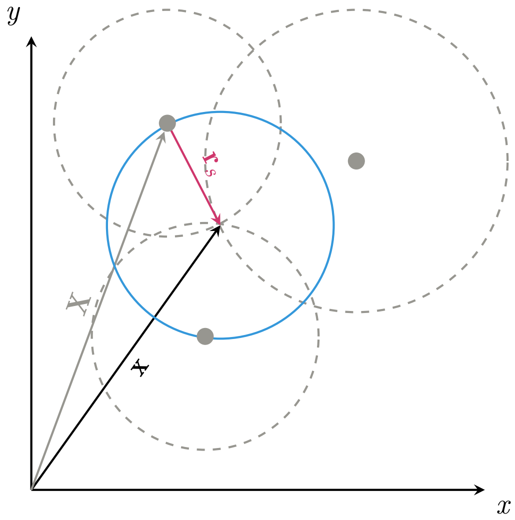
- Moments of the distribution F^{gc}

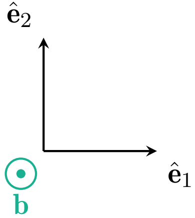
$$n = \int \mathrm{d}\mathbf{v}\, f(\mathbf{x},\mathbf{v}) = \frac{B_0}{m}\int \mathrm{d}\mathbf{X}\mathrm{d}v_\parallel \mathrm{d}\theta \mathrm{d}\mu\, \delta(\mathbf{X}+\mathbf{r}-\mathbf{x})F^{\mathrm{gc}}$$

$$j_\parallel = Ze\int \mathrm{d}\mathbf{v}\, v_\parallel f(\mathbf{x},\mathbf{v}) = \frac{ZeB_0}{m}\int \mathrm{d}\mathbf{X}\mathrm{d}v_\parallel \mathrm{d}\theta \mathrm{d}\mu\, \delta(\mathbf{X}+\mathbf{r}-\mathbf{x})v_\parallel F^{\mathrm{gc}}$$

$$\mathbf{j}_\perp = Ze\int \mathrm{d}\mathbf{v}\, \mathbf{v}_\perp f(\mathbf{x},\mathbf{v}) = \frac{ZeB_0}{m}\int \mathrm{d}\mathbf{X}\mathrm{d}v_\parallel \mathrm{d}\theta \mathrm{d}\mu\, \delta(\mathbf{X}+\mathbf{r}-\mathbf{x})\mathbf{v}_\perp F^{\mathrm{gc}}$$

DERIVATION OF FIELD EQUATIONS





DERIVATION OF FIELD EQUATIONS

- Coulomb's law
  $\rightarrow \sum_s Z_s e\, n_s = 0 \rightarrow \Phi_1$
- Magnetic Compression
  $\rightarrow \nabla^2 \mathbf{A}_{1\perp} = (\nabla \times B_{1\parallel})_\perp = -\mu_0 \mathbf{j}_{1\perp} \rightarrow B_{1\parallel}$
- Ampere's law
  $\rightarrow \nabla^2 A_{1\parallel} = -\mu_0 j_{1\parallel} \rightarrow A_{1\parallel}$

DERIVATION OF FIELD EQUATIONS

- Coulomb's law
  $\rightarrow \sum_s Z_s e\, n_s = 0 \rightarrow \Phi_1$
- Magnetic Compression
  $\rightarrow \nabla^2 \mathbf{A}_{1\perp} = (\nabla \times B_{1\|})_\perp = -\mu_0 \mathbf{j}_{1\perp} \rightarrow B_{1\|}$
- Ampere's law
  $\rightarrow \nabla^2 A_{1\|} = -\mu_0 j_{1\|} \rightarrow A_{1\|}$

$$k_{\perp\mathrm{N}}^2 \widehat{A}_{1\|\mathrm{N}} = 2\pi B_\mathrm{N} \beta_\mathrm{ref} \sum_s Z_s n_{\mathrm{R},s} v_{\mathrm{thR},s} \int \mathrm{d}v_{\|\mathrm{N}} \mathrm{d}\mu_\mathrm{N} \; v_{\|\mathrm{N}} J_0(k_\perp \rho_s) \widehat{F}_{1\mathrm{N},s}$$

THE CANCELLATION PROBLEM

THE CANCELLATION PROBLEM

- Gyrokinetic equation

$$\frac{\partial F_1}{\partial t} + \dot{\mathbf{X}} \cdot \nabla F_1 - \frac{\mathbf{b}_0}{m} \cdot \left(Ze\nabla\Phi_0 + \mu\nabla B_0 - mR\Omega^2\nabla R \right) \cdot \frac{\partial F_1}{\partial v_\parallel} = S$$

$$S = -(\mathbf{v}_\chi + \mathbf{v}_\mathrm{D}) \cdot \widetilde{\nabla} F_\mathrm{M} - \frac{Zev_\parallel}{T} \partial_t \bar{A}_{1\parallel} F_\mathrm{M}$$
$$- \frac{F_\mathrm{M}}{T} (v_\parallel \mathbf{b}_0 + \mathbf{v}_\mathrm{D} + \mathbf{v}_{\bar{B}_{1\perp}}) \cdot (Ze\nabla\bar{\Phi} + \mu\nabla\bar{B}_{1\parallel})$$

THE CANCELLATION PROBLEM

- Gyrokinetic equation

$$\frac{\partial F_1}{\partial t} + \dot{\mathbf{X}} \cdot \nabla F_1 - \frac{\mathbf{b}_0}{m} \cdot \left(Ze\nabla\Phi_0 + \mu\nabla B_0 - mR\Omega^2\nabla R \right) \cdot \frac{\partial F_1}{\partial v_\parallel} = S$$

$$S = -(\mathbf{v}_\chi + \mathbf{v}_\mathrm{D}) \cdot \widetilde{\nabla} F_\mathrm{M} - \frac{Zev_\parallel}{T}\partial_t \bar{A}_{1\parallel} F_\mathrm{M}$$
$$-\frac{F_\mathrm{M}}{T}(v_\parallel \mathbf{b}_0 + \mathbf{v}_\mathrm{D} + \mathbf{v}_{\bar{B}_{1\perp}}) \cdot (Ze\nabla\bar{\Phi} + \mu\nabla\bar{B}_{1\parallel})$$

THE CANCELLATION PROBLEM

- Modified distribution function

$$g = F_1 + \frac{Zev_{\parallel}}{T}\bar{A}_{1\parallel}F_{\mathrm{M}}$$

- Modified gyrokinetic equation

$$\frac{\partial g}{\partial t} + \mathbf{v}_{\chi}\cdot\nabla g \;+ (v_{\parallel}\mathbf{b}_0 + \mathbf{v}_{\mathrm{D}})\cdot\nabla F_1 - \frac{\mathbf{b}_0}{m}\cdot(Ze\nabla\Phi_0 + \mu\nabla B_0 - mR\Omega^2\nabla R)\frac{\partial F_1}{\partial v_{\parallel}} = S$$

$$S = -(\mathbf{v}_{\chi} + \mathbf{v}_{\mathrm{D}})\cdot\tilde{\nabla}F_{\mathrm{M}} - \;\frac{F_{\mathrm{M}}}{T}(v_{\parallel}\mathbf{b}_0 + \mathbf{v}_{\mathrm{D}})\cdot(Ze\nabla\bar{\Phi} + \mu\nabla\bar{B}_{1\parallel})$$

THE CANCELLATION PROBLEM

- Modified distribution function

$$g = F_1 + \frac{Zev_\parallel}{T}\bar{A}_{1\parallel}F_\mathrm{M}$$

- Modified gyrokinetic equation

$$\boxed{\frac{\partial g}{\partial t} + \mathbf{v}_\chi \cdot \nabla g} + (v_\parallel \mathbf{b}_0 + \mathbf{v}_\mathrm{D}) \cdot \nabla F_1 - \frac{\mathbf{b}_0}{m} \cdot (Ze\nabla\Phi_0 + \mu\nabla B_0 - mR\Omega^2\nabla R)\frac{\partial F_1}{\partial v_\parallel} = S$$

$$S = -(\mathbf{v}_\chi + \mathbf{v}_\mathrm{D}) \cdot \tilde{\nabla} F_\mathrm{M} - \boxed{\frac{F_\mathrm{M}}{T}(v_\parallel \mathbf{b}_0 + \mathbf{v}_\mathrm{D}) \cdot (Ze\nabla\bar{\Phi} + \mu\nabla\bar{B}_{1\parallel})}$$

THE CANCELLATION PROBLEM

- Adjusted Ampere's Law

$$\left(k_{\perp\mathrm{N}}^2 + \beta_{\mathrm{ref}} \sum_s \frac{Z_s^2 n_{\mathrm{R},s}}{m_{\mathrm{R},s}} \Gamma_0(b_s) e^{-\mathcal{E}_{\mathrm{N},s}/T_{\mathrm{R},s}}\right) \widehat{A}_{1\|\mathrm{N}} =$$

$$2\pi B_{\mathrm{N}} \beta_{\mathrm{ref}} \sum_s Z_s n_{\mathrm{R},s} v_{\mathrm{thR},s} \int \mathrm{d}v_{\|\mathrm{N}} \mathrm{d}\mu_{\mathrm{N}} \; v_{\|\mathrm{N}} J_0(k_\perp \rho_s) \widehat{g}_{\mathrm{N},s}$$

THE CANCELLATION PROBLEM

- Adjusted Ampere's Law

$$\left(k_{\perp\mathrm{N}}^2 + \boxed{\beta_{\mathrm{ref}} \sum_s \frac{Z_s^2 n_{\mathrm{R},s}}{m_{\mathrm{R},s}} \Gamma_0(b_s) e^{-\mathcal{E}_{\mathrm{N},s}/T_{\mathrm{R},s}}}\right) \widehat{A}_{1\|\mathrm{N}} =$$

$$2\pi B_{\mathrm{N}} \beta_{\mathrm{ref}} \sum_s Z_s n_{\mathrm{R},s} v_{\mathrm{thR},s} \int \mathrm{d}v_{\|\mathrm{N}} \mathrm{d}\mu_{\mathrm{N}}\ v_{\|\mathrm{N}} J_0(k_\perp \rho_s) \widehat{F}_{1\mathrm{N},s}$$

$$+ \boxed{2\pi B_{\mathrm{N}} \beta_{\mathrm{ref}} \sum_s Z_s n_{\mathrm{R},s} v_{\mathrm{thR},s} \int \mathrm{d}v_{\|\mathrm{N}} \mathrm{d}\mu_{\mathrm{N}}\ v_{\|\mathrm{N}}^2 J_0^2(k_\perp \rho_s) \frac{Z e v_{\mathrm{th}}}{T_{\mathrm{ref}} T_{\mathrm{R}}} F_{\mathrm{MN}}} \widehat{A}_{1\|\mathrm{N}}$$

- Error scales with $\sim \beta/k_\perp^2$

MITIGATION OF THE CANCELLATION PROBLEM - FARADAY'S LAW

MITIGATION OF THE CANCELLATION PROBLEM - FARADAY'S LAW

- Faraday's law

$$\rightarrow E_{1\|} = -\frac{\partial A_{1\|}}{\partial t} \rightarrow \nabla^2 \left(-\frac{\partial A_{1\|}}{\partial t} \right) = \mu_0 \frac{\partial j_{1\|}}{\partial t} \rightarrow E_{1\|}$$

$$k_{\perp \mathrm{N}}^2 \underbrace{\left(-\frac{\partial \widehat{A}_{1\|\mathrm{N}}}{\partial t_{\mathrm{N}}} \right)}_{\widehat{E}_{1\|\mathrm{N}}} = -2\pi B_{\mathrm{N}} \beta_{\mathrm{ref}} \sum_s Z_s n_{\mathrm{R},s} v_{\mathrm{thR},s} \int \mathrm{d}v_{\|\mathrm{N}} \mathrm{d}\mu_{\mathrm{N}}\ v_{\|\mathrm{N}} J_0(k_\perp \rho_s) \frac{\partial \widehat{F}_{1\mathrm{N},s}}{\partial t_{\mathrm{N}}}$$

MITIGATION OF THE CANCELLATION PROBLEM - FARADAY'S LAW

- Faraday's law

$$\rightarrow E_{1\|} = -\frac{\partial A_{1\|}}{\partial t} \rightarrow \nabla^2 \left(-\frac{\partial A_{1\|}}{\partial t} \right) = \mu_0 \frac{\partial j_{1\|}}{\partial t} \rightarrow E_{1\|}$$

$$k_{\perp\mathrm{N}}^2 \underbrace{\left(-\frac{\partial \widehat{A}_{1\|\mathrm{N}}}{\partial t_\mathrm{N}} \right)}_{\widehat{E}_{1\|\mathrm{N}}} = -2\pi B_\mathrm{N} \beta_\mathrm{ref} \sum_s Z_s n_{\mathrm{R},s} v_{\mathrm{thR},s} \int \mathrm{d}v_{\|\mathrm{N}} \mathrm{d}\mu_\mathrm{N} \; v_{\|\mathrm{N}} J_0(k_\perp \rho_s) \frac{\partial \widehat{F}_{1\mathrm{N},s}}{\partial t_\mathrm{N}}$$

- Replacing the time derivative of the gyrocenter distribution function

$$\frac{\partial F_1}{\partial t} = \mathcal{V} - \frac{Zev_\|}{T} \frac{\partial \bar{A}_{1\|}}{\partial t} F_\mathrm{M} = \mathcal{V} + \frac{Zev_\|}{T} \bar{E}_{1\|} F_\mathrm{M}$$

MITIGATION OF THE CANCELLATION PROBLEM - FARADAY'S LAW

- Field Equation for plasma induction $E_{1\|}$

$$\left(k_{\perp\mathrm{N}}^2 + \beta_{\mathrm{ref}} \sum_s \frac{Z_s^2 n_{\mathrm{R},s}}{m_{\mathrm{R},s}} \Gamma_0(b_s) e^{-\mathcal{E}_{\mathrm{N},s}/T_{\mathrm{R},s}}\right) \widehat{E}_{1\|\mathrm{N}} =$$
$$- 2\pi B_{\mathrm{N}} \beta_{\mathrm{ref}} \sum_s Z_s n_{\mathrm{R},s} v_{\mathrm{thR},s} \int \mathrm{d}v_{\|\mathrm{N}} \mathrm{d}\mu_{\mathrm{N}} \; v_{\|\mathrm{N}} J_0(k_\perp \rho_s) \widehat{\mathcal{V}}_{\mathrm{N},s}$$

MITIGATION OF THE CANCELLATION PROBLEM - FARADAY'S LAW

- Field Equation for plasma induction $E_{1\|}$

$$\left(k_{\perp\mathrm{N}}^2 + \beta_{\mathrm{ref}} \sum_s \frac{Z_s^2 n_{\mathrm{R},s}}{m_{\mathrm{R},s}} \Gamma_0(b_s) e^{-\mathcal{E}_{\mathrm{N},s}/T_{\mathrm{R},s}}\right) \widehat{E}_{1\|\mathrm{N}} = - 2\pi B_{\mathrm{N}} \beta_{\mathrm{ref}} \sum_s Z_s n_{\mathrm{R},s} v_{\mathrm{thR},s} \int \mathrm{d}v_{\|\mathrm{N}} \mathrm{d}\mu_{\mathrm{N}}\; v_{\|\mathrm{N}} J_0(k_\perp \rho_s) \widehat{\mathcal{V}}_{\mathrm{N},s}$$

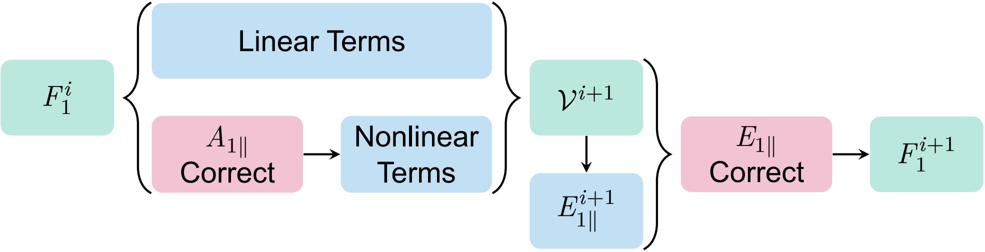
- New source term in gyrokinetic equation

$$\frac{\partial F_1}{\partial t} + \dot{\mathbf{X}} \cdot \nabla F_1 - \frac{\mathbf{b}_0}{m} \cdot \left(Ze\nabla\Phi_0 + \mu\nabla B_0 - mR\Omega^2\nabla R\right) \cdot \frac{\partial F_1}{\partial v_\|} = S$$

$$S = -(\mathbf{v}_\chi + \mathbf{v}_{\mathrm{D}}) \cdot \widetilde{\nabla} F_{\mathrm{M}} - \boxed{\frac{Zev_\|}{T} \bar{E}_{1\|} F_{\mathrm{M}}} - \frac{F_{\mathrm{M}}}{T} (v_\| \mathbf{b}_0 + \mathbf{v}_{\mathrm{D}} + \mathbf{v}_{\bar{B}_{1\perp}}) \cdot (Ze\nabla\bar{\Phi} + \mu\nabla\bar{B}_{1\|})$$

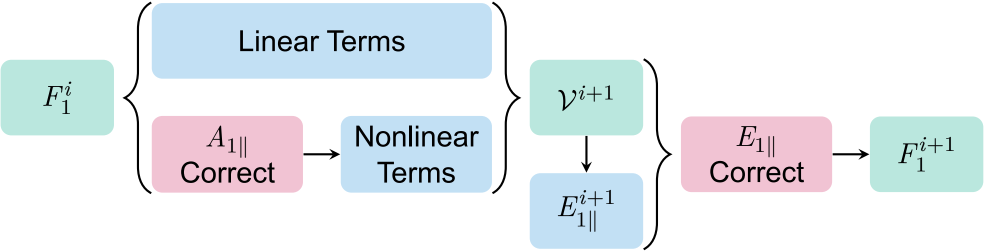
IMPLEMENTATION OF FARADAY'S LAW

IMPLEMENTATION OF FARADAY'S LAW



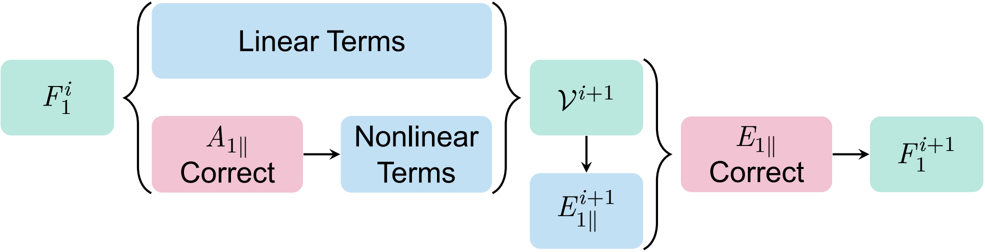
- GKW calculates the modified distribution g (g-version)
  $\rightarrow$ Switch to the calculation of the gyrocenter distribution F_1 (f-version)

IMPLEMENTATION OF FARADAY'S LAW



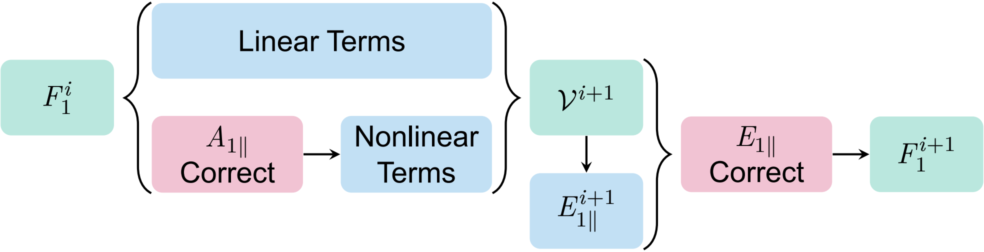
- GKW calculates the modified distribution g (g-version)
  $\rightarrow$ Switch to the calculation of the gyrocenter distribution F_1 (f-version)
- Input switch `nlepar`

IMPLEMENTATION OF FARADAY'S LAW



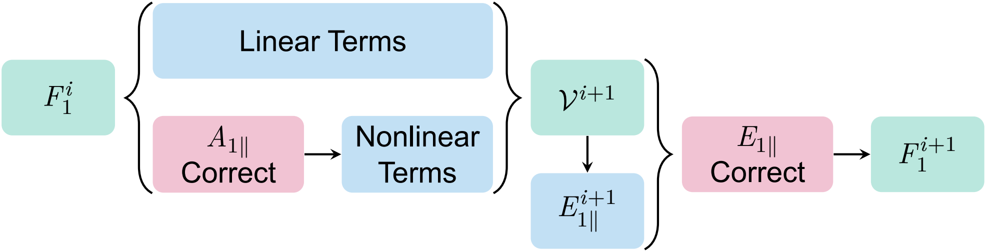
- GKW calculates the modified distribution g (g-version)
  $\rightarrow$ Switch to the calculation of the gyrocenter distribution F_1 (f-version)
- Input switch `nlepar`
- Apply $A_{1\|}$ correction on F_1 for the calculation of the nonlinear terms

IMPLEMENTATION OF FARADAY'S LAW



- GKW calculates the modified distribution g (g-version)
  $\rightarrow$ Switch to the calculation of the gyrocenter distribution F_1 (f-version)
- Input switch `nlepar`
- Apply $A_{1\|}$ correction on F_1 for the calculation of the nonlinear terms
- Calculation of Faraday's law in an additional subroutine with $\mathcal{V}$

IMPLEMENTATION OF FARADAY'S LAW



- GKW calculates the modified distribution g (g-version)
  $\rightarrow$ Switch to the calculation of the gyrocenter distribution F_1 (f-version)
- Input switch `nlepar`
- Apply $A_{1\|}$ correction on F_1 for the calculation of the nonlinear terms
- Calculation of Faraday's law in an additional subroutine with $\mathcal{V}$
- Apply $E_{1\|}$ correction on $\mathcal{V}$ for the calculation of F_1

BENCHMARK OF THE F-VERSION

BENCHMARK OF THE F-VERSION

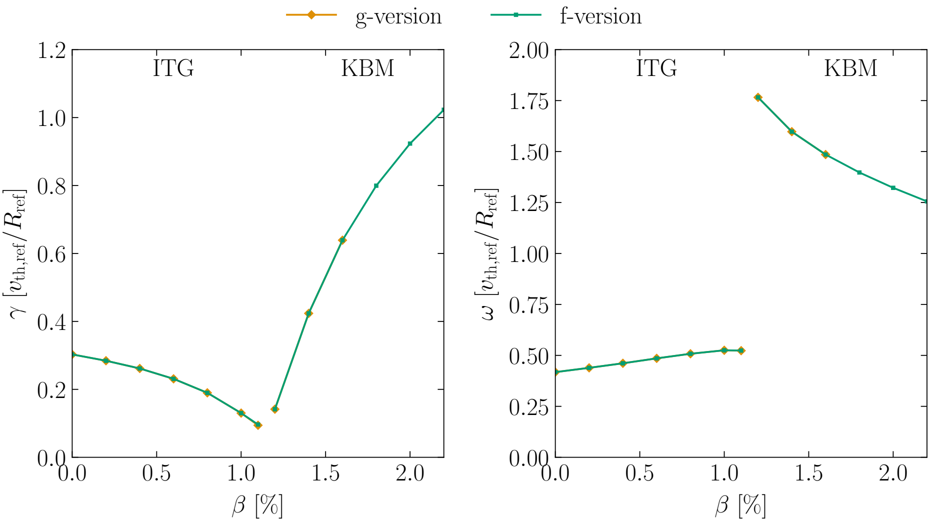
- Flux-tube version of GKW with field aligned Hamada coordinates $\{s, \psi, \zeta, v_\parallel, \mu\}$
- Cyclone base case (CBC) beta scan for

$$\beta \in [0.0,\ 0.2,\ 0.4,\ 0.6,\ 0.8,\ 1.0,\ 1.1,\ 1.2,\ 1.4,\ 1.6,\ 1.8,\ 2.0,\ 2.2]\,\%$$

- Kinetic electrons (`adiabatic_electrons = .false.`)
- $k_\zeta \rho = 0.3$ (Maximum of the nonlinear transport spectrum)

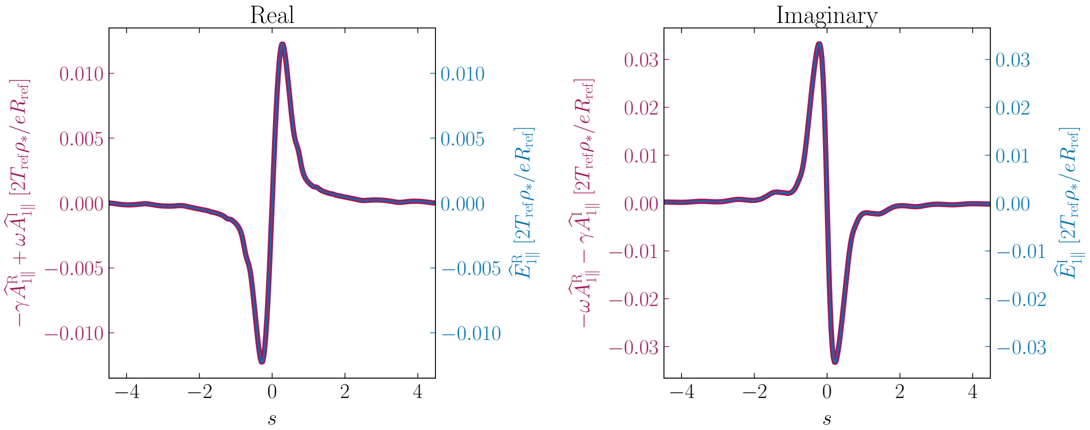
DTIM	NTIME	NAVERAGE	N_{mod}	N_x	N_s	$N_{v_\parallel}$	N_μ	N_{sp}	nperiod
0.01	2000	100	1	1	288	64	16	2	5

BENCHMARK OF THE F-VERSION



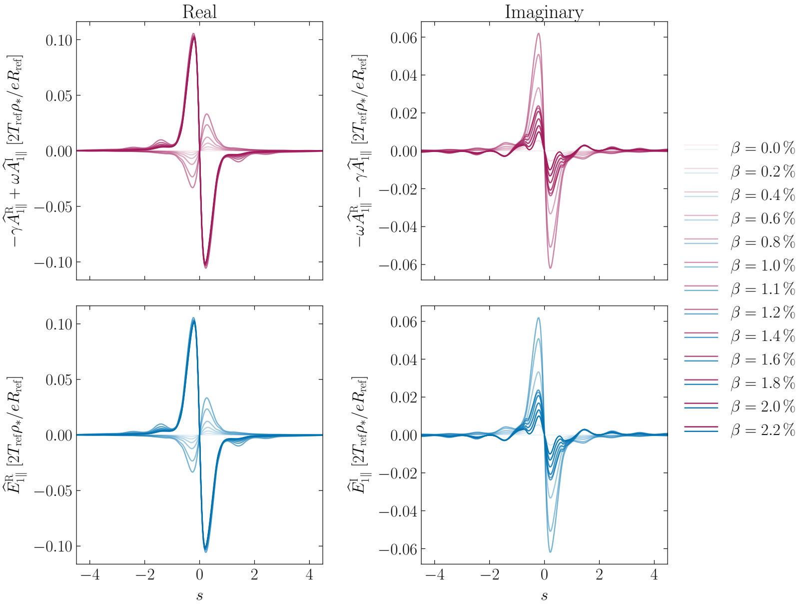
$10\,\%$ more runtime for the f-version

BENCHMARK OF THE F-VERSION



$$\beta = 0.8\,\%$$

BENCHMARK OF THE F-VERSION



MITIGATION IN LOCAL SIMULATIONS

MITIGATION IN LOCAL SIMULATIONS

- Mitigation could only be tested in the local linear simulations
  $\rightarrow$ Local nonlinear simulations not benchmarked yet

MITIGATION IN LOCAL SIMULATIONS

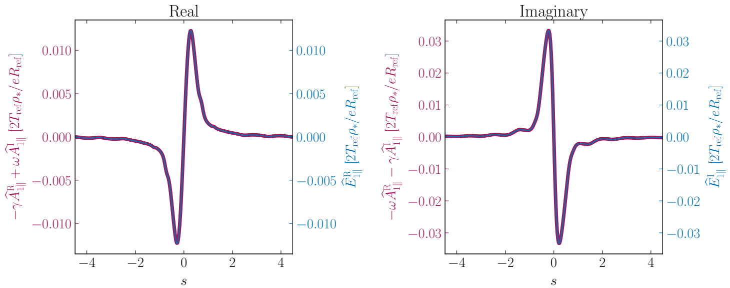
- Mitigation could only be tested in the local linear simulations
  $\rightarrow$ Local nonlinear simulations not benchmarked yet
- CBC and AUG (ASDEX-Upgrade) test cases fails
  for the same value of $k_\perp$ in both versions

$$\left(k_{\perp \mathrm{N}}^2 + \beta_{\mathrm{ref}} \sum_s \frac{Z_s^2 n_{\mathrm{R},s}}{m_{\mathrm{R},s}} \Gamma_0(b_s) e^{-\mathcal{E}_{\mathrm{N},s}/T_{\mathrm{R},s}} \right) \widehat{E}_{1\|\mathrm{N}} = \\ - 2\pi B_{\mathrm{N}} \beta_{\mathrm{ref}} \sum_s Z_s n_{\mathrm{R},s} v_{\mathrm{thR},s} \int \mathrm{d}v_{\|\mathrm{N}} \mathrm{d}\mu_{\mathrm{N}} \ v_{\|\mathrm{N}} J_0(k_\perp \rho_s) \widehat{\mathcal{V}}_{\mathrm{N},s}$$

$$\left(k_{\perp \mathrm{N}}^2 + \beta_{\mathrm{ref}} \sum_s \frac{Z_s^2 n_{\mathrm{R},s}}{m_{\mathrm{R},s}} \Gamma_0(b_s) e^{-\mathcal{E}_{\mathrm{N},s}/T_{\mathrm{R},s}} \right) \widehat{A}_{1\|\mathrm{N}} = \\ 2\pi B_{\mathrm{N}} \beta_{\mathrm{ref}} \sum_s Z_s n_{\mathrm{R},s} v_{\mathrm{thR},s} \int \mathrm{d}v_{\|\mathrm{N}} \mathrm{d}\mu_{\mathrm{N}} \ v_{\|\mathrm{N}} J_0(k_\perp \rho_s) \widehat{g}_{\mathrm{N},s}$$

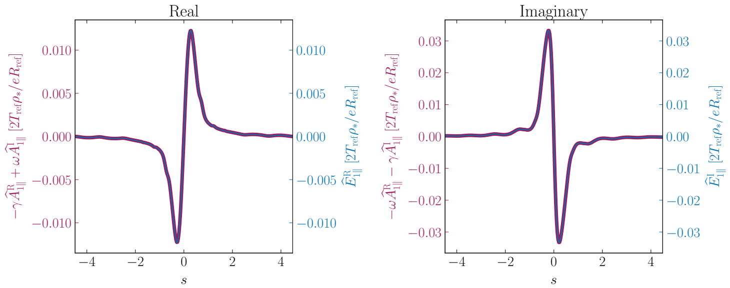
CONCLUSION

CONCLUSION



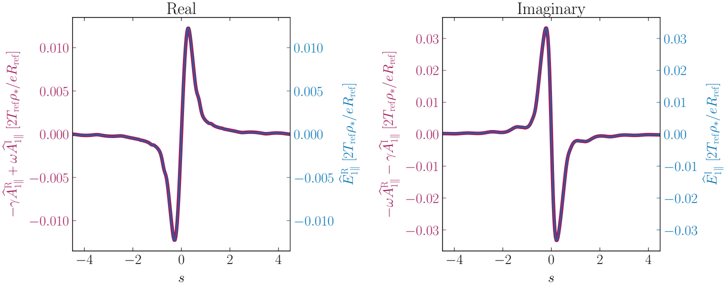
- Local linear f-Version successfully implemented
  $\rightarrow 10\,\%$ more runtime

CONCLUSION



- Local linear f-Version successfully implemented
  $\rightarrow 10\,\%$ more runtime
- Nonlinear benchmark not completed yet

CONCLUSION



- Local linear f-Version successfully implemented
  $\rightarrow 10\,\%$ more runtime
- Nonlinear benchmark not completed yet
- Groundwork for global f-version of GKW done