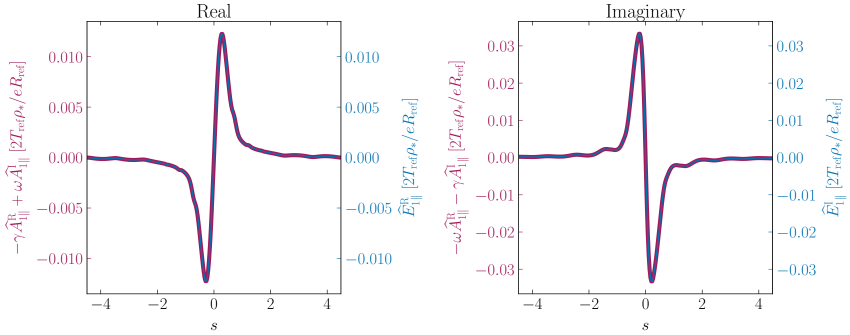
UNIVERSITÄT BAYREUTH

MITIGATION OF THE CANCELLATION PROBLEM IN LOCAL GYROKINETIC SIMULATIONS

September 23, 2024



Manuel Lippert

Theoretical Physics V

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Is it possible to mitigate the cancellation problem in local gyrokinetic simulations with the introduction of a new field equation?

CHARGED PARTICLE MOTION

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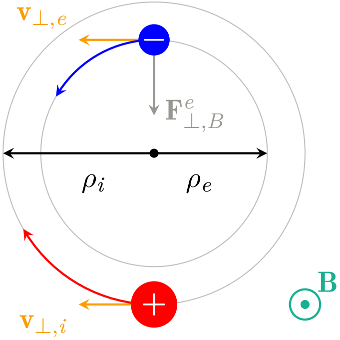
Lorentz force

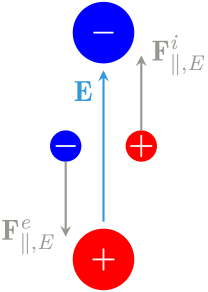
$$F_{\perp,B} = |q| v_\perp B$$



$$\rho = \frac{m \mathbf{v}_\perp}{|q| B} \quad \omega_c = \frac{|q| B}{m}$$

Electric force

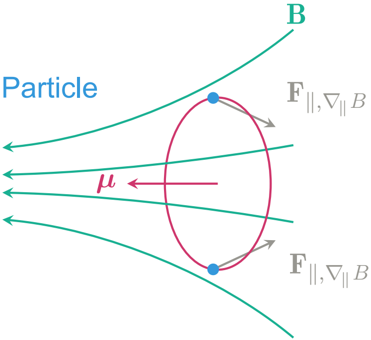
$$F_{\parallel,E} = q E_\parallel$$



$$\rho_{\mathrm{th}} = \frac{m v_{\mathrm{th}}}{|q| B}$$

Inhomogeneous magnetic field

$$F_{\parallel,\nabla_\parallel B} = -\underbrace{\frac{m v_\perp^2}{2B}}_{\mu} \nabla_\parallel B$$



$$v_{\mathrm{th}} = \sqrt{\frac{2T}{m}}$$

DRIFTS IN THE GUIDING CENTER

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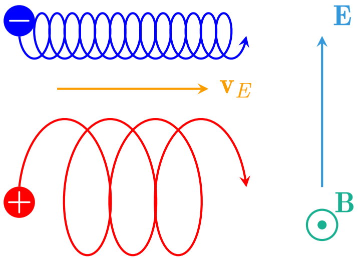
$\mathbf{E} \times \mathbf{B}$ Drift

$$\mathbf{v}_E = \frac{\mathbf{E}\times\mathbf{B}}{B^2}$$



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$\mathbf{E} \times \mathbf{B}$ Drift

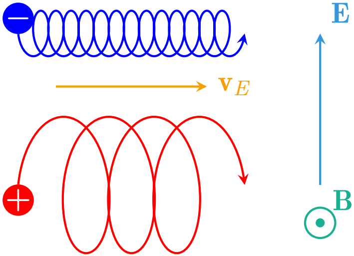
$$\mathbf{v}_E = \frac{\mathbf{E}\times\mathbf{B}}{B^2}$$

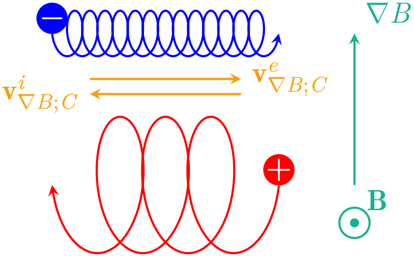
∇B Drift

$$\mathbf{v}_{\nabla B} = \frac{m v_\perp^2}{2q}\frac{\mathbf{B}\times\nabla B}{B^3}$$

Curvature Drift

$$\mathbf{v}_C = \frac{m v_\parallel^2}{qB^2}\mathbf{B}\times\underbrace{\mathbf{C}}_{\frac{\nabla B}{B}} = \frac{m v_\parallel^2}{q}\frac{\mathbf{B}\times\nabla B}{B^3}$$





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- Characterizes quality of confinement (Best confinement for $\beta < 1$)
- Relevant for fusion rate ($\sim \beta^2$), MHD stability of a fusion device
- Indicator for the relevance of electromagnetic effects
- Electromagentic fields vanish in the limit $\beta \to 0$

GYROKINETIC ORDERING

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- Low frequency: $\omega / \omega_{\mathrm{c}} \ll 1$
- Anisotropy: $k_{\|} / k_{\perp} \ll 1$
- Strong Magnetization: $\rho / L_n \sim \rho / L_T \sim \rho / L_B \ll 1$ $\quad L_G = G_0 \left(\frac{\mathrm{d}\, G_0}{\mathrm{dx}} \right)^{-1}$
- Small fluctuations: $F_1 / F_0 \ll 1$

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$$\omega/\omega_{\mathrm{c}} \sim k_{\|}/k_{\perp} \sim \rho/L_n \sim \rho/L_T \sim \rho/L_B \sim F_1/F_0 \sim \epsilon_\delta$$

$$\rho_\star = \frac{\rho_{\mathrm{th,ref}}}{L_B} = \frac{m_{\mathrm{ref}} v_{\mathrm{th,ref}}}{eB_{\mathrm{ref}}} \sim \epsilon_\delta$$

DERIVATION OF GYROKINETIC EQUATION

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- Particle phase space $\{\mathbf{x}, \mathbf{v}\} \rightarrow$ guiding center $\{\mathbf{X}, v_{\|}, \mu, \theta\} \rightarrow$ gyrocenter $\{\bar{\mathbf{X}}, \bar{v}_{\|}, \bar{\mu}\}$

$$\Phi = \Phi_0 + \underbrace{\tilde{\Phi}_1 + \bar{\Phi}_1}_{\Phi_1} \qquad \mathbf{A} = \mathbf{A}_0 + \underbrace{\tilde{\mathbf{A}}_1 + \bar{\mathbf{A}}_1}_{\mathbf{A}_1}$$

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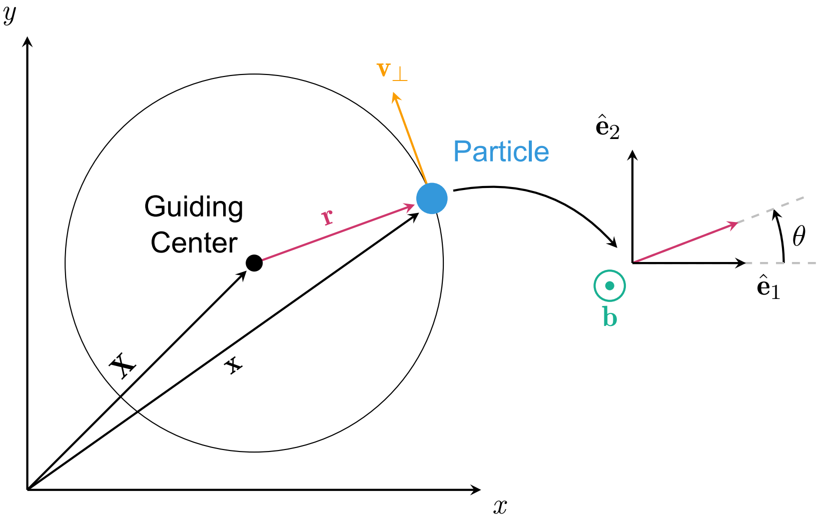

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- Vlasov Equation in gyrocenter phase space without collisions

$$\frac{\partial F}{\partial t} + \dot{\mathbf{X}} \cdot \frac{\partial F}{\partial \mathbf{X}} + \dot{v}_{\|} \cdot \frac{\partial F}{\partial v_{\|}} = 0$$

DERIVATION OF GYROKINETIC EQUATION

- Delta-f approximation $F = F_0 + F_1$

$$\frac{\partial F_1}{\partial t} + \dot{\mathbf{X}} \cdot \nabla F_1 + \dot{v}_{\parallel} \cdot \frac{\partial F_1}{\partial v_{\parallel}} = \underbrace{-\dot{\mathbf{X}} \cdot \nabla F_0 - \dot{v}_{\parallel} \frac{\partial F_0}{\partial v_{\parallel}}}_{S}$$

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- Maxwellian as equilibrium distribution $F_0 = F_\mathrm{M}$ and $\dot{\mathbf{X}}$ and $\dot{v}_\|$ from Lagrangian

$$\frac{\partial F_1}{\partial t} + \dot{\mathbf{X}} \cdot \nabla F_1 - \frac{\mathbf{b}_0}{m} \cdot \left(Ze\nabla\Phi_0 + \mu\nabla B_0 - mR\Omega^2\nabla R\right) \cdot \frac{\partial F_1}{\partial v_\|} = S$$

$$\begin{aligned} S = &-(\mathbf{v}_\chi + \mathbf{v}_\mathrm{D}) \cdot \widetilde{\nabla} F_\mathrm{M} - \frac{Zev_\|}{T}\partial_t \bar{A}_{1\|} F_\mathrm{M} \\ &-\frac{F_\mathrm{M}}{T}\left(v_\|\mathbf{b}_0 + \mathbf{v}_\mathrm{D} + \mathbf{v}_{\bar{B}_{1\perp}}\right) \cdot \left(Ze\nabla\bar{\Phi} + \mu\nabla\bar{B}_{1\|}\right) \end{aligned}$$

DERIVATION OF FIELD EQUATIONS

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- Maxwell's equations

$$\sum_s Z_s e\, n_s = 0 \qquad \nabla \times \mathbf{E}_1 = -\frac{\partial \mathbf{B}_1}{\partial t}$$

$$\nabla \cdot \mathbf{B}_1 = 0 \qquad \nabla \times \mathbf{B}_1 = \mu_0 \sum_s \mathbf{j}_s$$

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- Moments of the distribution f

$$n(\mathbf{x}) = \int \mathrm{d}\mathbf{v}\; f(\mathbf{x}, \mathbf{v})$$

$$j_{||} = Ze \int \mathrm{d}\mathbf{v}\; v_{||} f(\mathbf{x}, \mathbf{v}) \qquad \mathbf{j}_{\perp} = Ze \int \mathrm{d}\mathbf{v}\; \mathbf{v}_{\perp} f(\mathbf{x}, \mathbf{v})$$

DERIVATION OF FIELD EQUATIONS

Moments of f: $n(\mathbf{x}), \mathbf{j}(\mathbf{x})$ → Guiding Center Phase Space $\xleftarrow[\text{back}]{\text{pull}}$ Vlasov Equation for F

- Pullback transformation

$$F^{\mathrm{gc}} = \mathcal{P}\left\{F\right\} = F \underbrace{- \frac{F_{\mathrm{M}}}{T}\left(Ze\widetilde{\Phi}_1 - \mu \bar{B}_{1\|}\right)}_{\text{Correction Term}}$$

DERIVATION OF FIELD EQUATIONS

$$\boxed{\begin{array}{c}\text{Moments of } f\text{:}\\ n(\mathbf{x}), \mathbf{j}(\mathbf{x})\end{array}} \longrightarrow \boxed{\begin{array}{c}\text{Guiding Center}\\ \text{Phase Space}\end{array}} \xleftarrow[\text{back}]{\text{pull}} \boxed{\begin{array}{c}\text{Vlasov Equation}\\ \text{for } F\end{array}}$$

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- Moments of the distribution F^{gc}

$$n = \int \mathrm{d}\mathbf{v}\, f(\mathbf{x},\mathbf{v}) = \frac{B_0}{m}\int \mathrm{d}\mathbf{X}\mathrm{d}v_{\|}\mathrm{d}\theta\mathrm{d}\mu\, \delta(\mathbf{X}+\mathbf{r}-\mathbf{x})F^{\mathrm{gc}}$$

$$j_{\|} = Ze\int \mathrm{d}\mathbf{v}\, v_{\|} f(\mathbf{x},\mathbf{v}) = \frac{ZeB_0}{m}\int \mathrm{d}\mathbf{X}\mathrm{d}v_{\|}\mathrm{d}\theta\mathrm{d}\mu\, \delta(\mathbf{X}+\mathbf{r}-\mathbf{x})v_{\|}F^{\mathrm{gc}}$$

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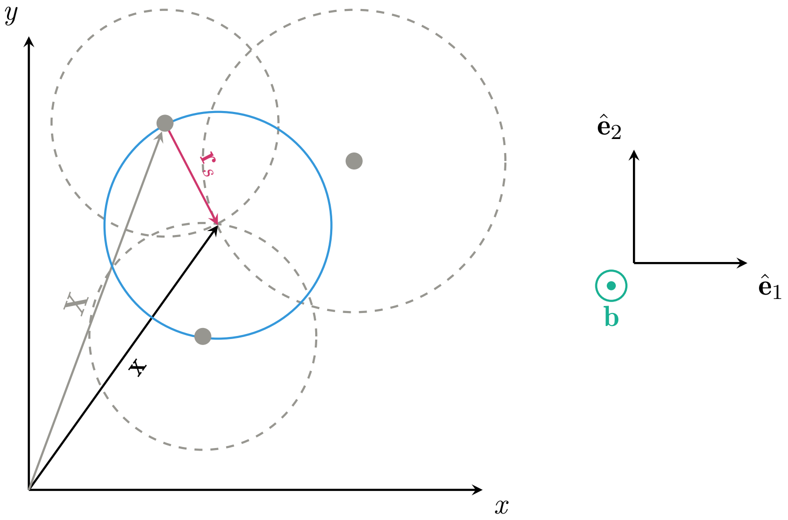

DERIVATION OF FIELD EQUATIONS

- Coulomb's law
  $\rightarrow \sum_s Z_s e\, n_s = 0 \rightarrow \Phi_1$
- Magnetic Compression
  $\rightarrow \nabla^2 \mathbf{A}_{1\perp} = (\nabla \times B_{1\|})_\perp = -\mu_0 \mathbf{j}_{1\perp} \rightarrow B_{1\|}$
- Ampere's law
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$$k_{\perp\mathrm{N}}^2 \widehat{A}_{1\|\mathrm{N}} = 2\pi B_\mathrm{N} \beta_\mathrm{ref} \sum_s Z_s n_{\mathrm{R},s} v_{\mathrm{thR},s} \int \mathrm{d}v_{\|\mathrm{N}} \mathrm{d}\mu_\mathrm{N}\ v_{\|\mathrm{N}} J_0(k_\perp \rho_s) \widehat{F}_{1\mathrm{N},s}$$

THE CANCELLATION PROBLEM

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- Gyrokinetic equation

$$\frac{\partial F_1}{\partial t} + \dot{\mathbf{X}} \cdot \nabla F_1 - \frac{\mathbf{b}_0}{m} \cdot \left(Ze\nabla\Phi_0 + \mu\nabla B_0 - mR\Omega^2\nabla R \right) \cdot \frac{\partial F_1}{\partial v_\parallel} = S$$

$$S = -(\mathbf{v}_\chi + \mathbf{v}_\mathrm{D}) \cdot \widetilde{\nabla} F_\mathrm{M} - \frac{Zev_\parallel}{T} \partial_t \bar{A}_{1\parallel} F_\mathrm{M}$$
$$- \frac{F_\mathrm{M}}{T} (v_\parallel \mathbf{b}_0 + \mathbf{v}_\mathrm{D} + \mathbf{v}_{\bar{B}_{1\perp}}) \cdot (Ze\nabla\bar{\Phi} + \mu\nabla\bar{B}_{1\parallel})$$

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THE CANCELLATION PROBLEM

- Modified distribution function

$$g = F_1 + \frac{Zev_\parallel}{T}\bar{A}_{1\parallel}F_\mathrm{M}$$

- Modified gyrokinetic equation

$$\frac{\partial g}{\partial t} + \mathbf{v}_\chi \cdot \nabla g \ + (v_\parallel \mathbf{b}_0 + \mathbf{v}_\mathrm{D}) \cdot \nabla F_1 - \frac{\mathbf{b}_0}{m} \cdot (Ze\nabla\Phi_0 + \mu\nabla B_0 - mR\Omega^2\nabla R)\frac{\partial F_1}{\partial v_\parallel} = S$$

$$S = -(\mathbf{v}_\chi + \mathbf{v}_\mathrm{D}) \cdot \tilde{\nabla} F_\mathrm{M} - \ \frac{F_\mathrm{M}}{T}(v_\parallel \mathbf{b}_0 + \mathbf{v}_\mathrm{D}) \cdot (Ze\nabla\bar{\Phi} + \mu\nabla\bar{B}_{1\parallel})$$

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THE CANCELLATION PROBLEM

- Adjusted Ampere's Law

$$\left(k_{\perp\mathrm{N}}^2 + \beta_{\mathrm{ref}} \sum_s \frac{Z_s^2 n_{\mathrm{R},s}}{m_{\mathrm{R},s}} \Gamma_0(b_s) e^{-\mathcal{E}_{\mathrm{N},s}/T_{\mathrm{R},s}}\right) \widehat{A}_{1\|\mathrm{N}} =$$

$$2\pi B_{\mathrm{N}} \beta_{\mathrm{ref}} \sum_s Z_s n_{\mathrm{R},s} v_{\mathrm{thR},s} \int \mathrm{d}v_{\|\mathrm{N}} \mathrm{d}\mu_{\mathrm{N}}\; v_{\|\mathrm{N}} J_0(k_\perp \rho_s) \widehat{g}_{\mathrm{N},s}$$

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$$+ \boxed{2\pi B_{\mathrm{N}} \beta_{\mathrm{ref}} \sum_s Z_s n_{\mathrm{R},s} v_{\mathrm{thR},s} \int \mathrm{d}v_{\|\mathrm{N}} \mathrm{d}\mu_{\mathrm{N}} \; v_{\|\mathrm{N}}^2 J_0^2(k_\perp \rho_s) \frac{Zev_{\mathrm{th}}}{T_{\mathrm{ref}} T_{\mathrm{R}}} F_{\mathrm{MN}}} \widehat{A}_{1\|\mathrm{N}}$$

- Error scales with $\sim \beta / k_\perp^2$

MITIGATION OF THE CANCELLATION PROBLEM - FARADAY'S LAW

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- Faraday's law

$$\rightarrow E_{1\|} = -\frac{\partial A_{1\|}}{\partial t} \rightarrow \nabla^2 \left(-\frac{\partial A_{1\|}}{\partial t} \right) = \mu_0 \frac{\partial j_{1\|}}{\partial t} \rightarrow E_{1\|}$$

$$k_{\perp \mathrm{N}}^2 \underbrace{\left(-\frac{\partial \widehat{A}_{1\|\mathrm{N}}}{\partial t_{\mathrm{N}}} \right)}_{\widehat{E}_{1\|\mathrm{N}}} = -2\pi B_{\mathrm{N}} \beta_{\mathrm{ref}} \sum_s Z_s n_{\mathrm{R},s} v_{\mathrm{thR},s} \int \mathrm{d}v_{\|\mathrm{N}} \mathrm{d}\mu_{\mathrm{N}} \; v_{\|\mathrm{N}} J_0(k_\perp \rho_s) \frac{\partial \widehat{F}_{1\mathrm{N},s}}{\partial t_{\mathrm{N}}}$$

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- Replacing the time derivative of the gyrocenter distribution function

$$\frac{\partial F_1}{\partial t} = \mathcal{V} - \frac{Zev_\|}{T} \frac{\partial \bar{A}_{1\|}}{\partial t} F_\mathrm{M} = \mathcal{V} + \frac{Zev_\|}{T} \bar{E}_{1\|} F_\mathrm{M}$$

MITIGATION OF THE CANCELLATION PROBLEM - FARADAY'S LAW

- Field Equation for induced electric field $E_{1\|}$

$$\left(k_{\perp \mathrm{N}}^2 + \beta_{\mathrm{ref}} \sum_s \frac{Z_s^2 n_{\mathrm{R},s}}{m_{\mathrm{R},s}} \Gamma_0(b_s) e^{-\mathcal{E}_{\mathrm{N},s}/T_{\mathrm{R},s}} \right) \widehat{E}_{1\|\mathrm{N}} = \\ - 2\pi B_{\mathrm{N}} \beta_{\mathrm{ref}} \sum_s Z_s n_{\mathrm{R},s} v_{\mathrm{thR},s} \int \mathrm{d}v_{\|\mathrm{N}} \mathrm{d}\mu_{\mathrm{N}} \; v_{\|\mathrm{N}} J_0(k_\perp \rho_s) \widehat{\mathcal{V}}_{\mathrm{N},s}$$

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- New source term in gyrokinetic equation

$$\frac{\partial F_1}{\partial t} + \dot{\mathbf{X}} \cdot \nabla F_1 - \frac{\mathbf{b}_0}{m} \cdot \left(Ze\nabla\Phi_0 + \mu\nabla B_0 - mR\Omega^2\nabla R\right) \cdot \frac{\partial F_1}{\partial v_\|} = S$$

$$S = -(\mathbf{v}_\chi + \mathbf{v}_{\mathrm{D}}) \cdot \widetilde{\nabla} F_{\mathrm{M}} - \boxed{\frac{Zev_\|}{T} \bar{E}_{1\|} F_{\mathrm{M}}}$$
$$-\frac{F_{\mathrm{M}}}{T} (v_\| \mathbf{b}_0 + \mathbf{v}_{\mathrm{D}} + \mathbf{v}_{\bar{B}_{1\perp}}) \cdot (Ze\nabla\bar{\Phi} + \mu\nabla\bar{B}_{1\|})$$

IMPLEMENTATION OF FARADAY'S LAW

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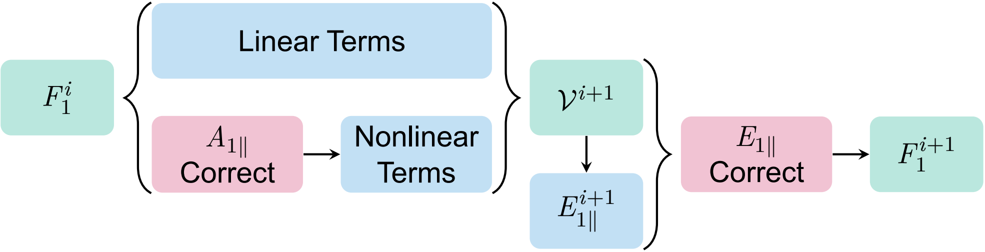
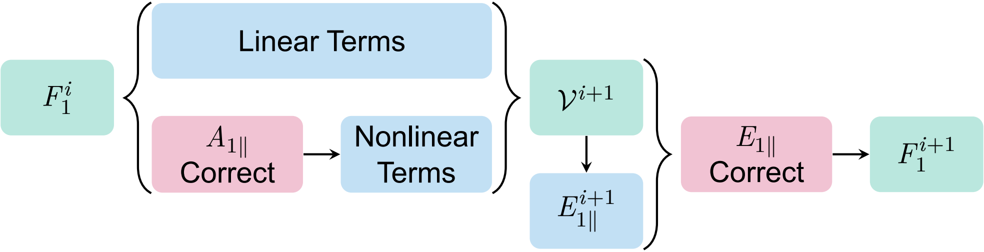


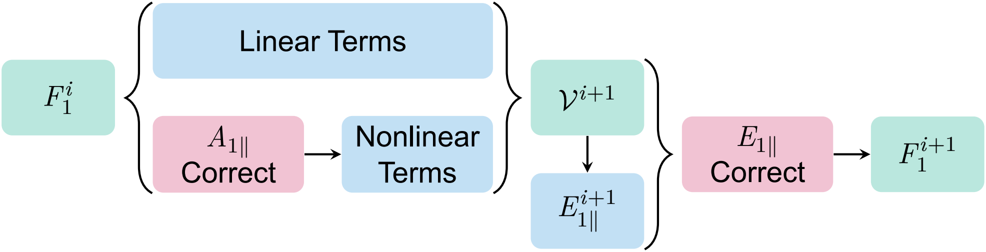
- GKW calculates the modified distribution g (g-version)
  $\rightarrow$ Switch to the calculation of the gyrocenter distribution F_1 (f-version)

IMPLEMENTATION OF FARADAY'S LAW



- GKW calculates the modified distribution g (g-version)
  $\rightarrow$ Switch to the calculation of the gyrocenter distribution F_1 (f-version)
- Input switch `nlepar`

IMPLEMENTATION OF FARADAY'S LAW



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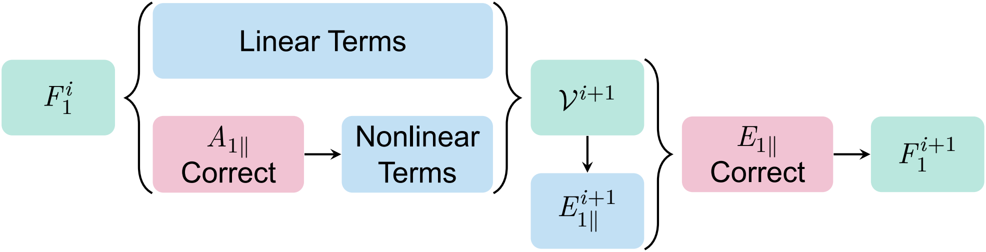

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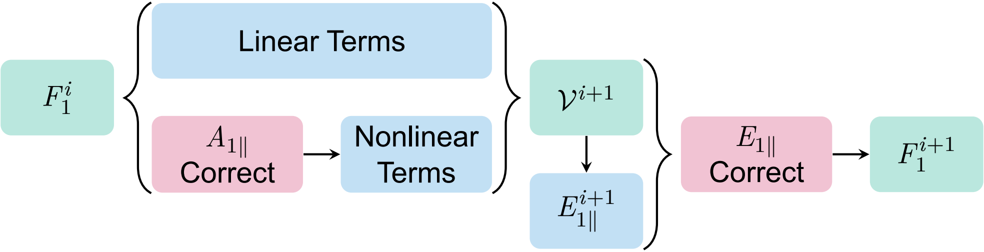
IMPLEMENTATION OF FARADAY'S LAW



- GKW calculates the modified distribution g (g-version)
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- Input switch `nlepar`
- Apply $A_{1\|}$ correction on F_1 for the calculation of the nonlinear terms
- Calculation of Faraday's law in an additional subroutine with $\mathcal{V}$
- Apply $E_{1\|}$ correction on $\mathcal{V}$ for the calculation of F_1

BENCHMARK OF THE F-VERSION

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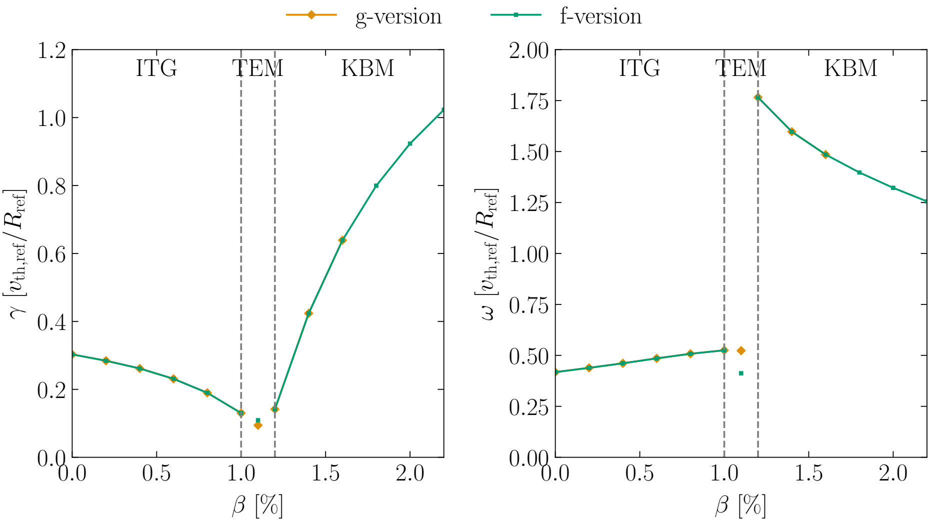
- Flux-tube version of GKW with field aligned Hamada coordinates $\{s, \psi, \zeta, v_\parallel, \mu\}$
- Cyclone base case (CBC) beta scan for

$$\beta \in [0.0,\ 0.2,\ 0.4,\ 0.6,\ 0.8,\ 1.0,\ 1.1,\ 1.2,\ 1.4,\ 1.6,\ 1.8,\ 2.0,\ 2.2]\,\%$$

- Kinetic electrons (`adiabatic_electrons = .false.`)
- $k_\zeta \rho = 0.3$ (Maximum of the nonlinear transport spectrum)

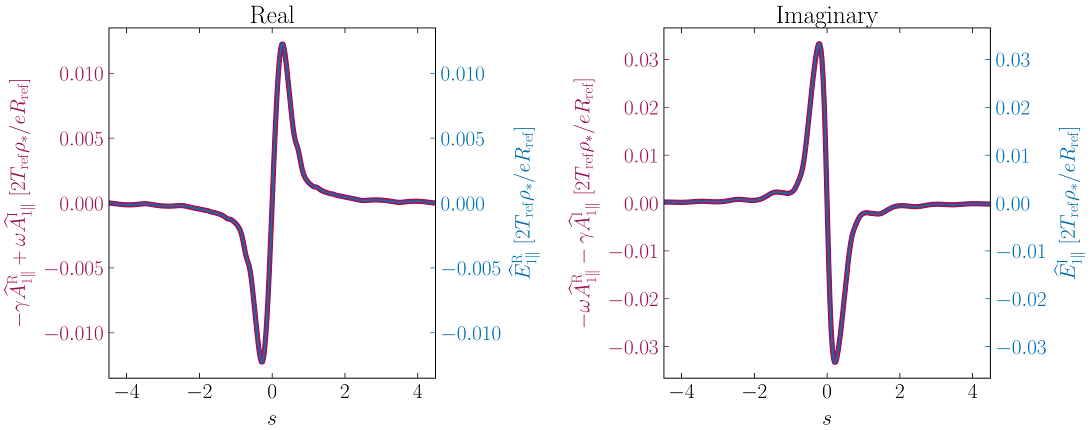
DTIM	NTIME	NAVERAGE	N_{mod}	N_x	N_s	$N_{v_\parallel}$	N_μ	N_{sp}	nperiod
0.01	2000	100	1	1	288	64	16	2	5

BENCHMARK OF THE F-VERSION



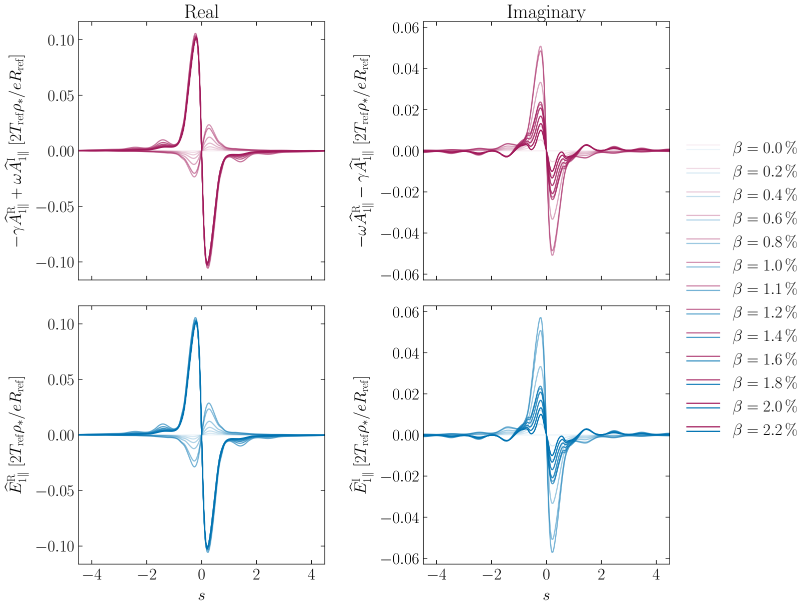
$10\,\%$ more runtime for the f-version

BENCHMARK OF THE F-VERSION



$$\beta = 0.8\,\%$$

BENCHMARK OF THE F-VERSION



MITIGATION IN LOCAL SIMULATIONS

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- Mitigation could only be tested in the local linear simulations
  $\rightarrow$ Local nonlinear simulations not benchmarked yet

MITIGATION IN LOCAL SIMULATIONS

- Mitigation could only be tested in the local linear simulations
  $\rightarrow$ Local nonlinear simulations not benchmarked yet
- CBC and AUG (ASDEX-Upgrade) test cases fails for the same value of $k_\perp$ in both versions

$$\left(k_{\perp\mathrm{N}}^2 + \beta_\mathrm{ref}\sum_s \frac{Z_s^2 n_{\mathrm{R},s}}{m_{\mathrm{R},s}}\Gamma_0(b_s)\, e^{-\mathcal{E}_{\mathrm{N},s}/T_{\mathrm{R},s}}\right)\widehat{E}_{1\|\mathrm{N}} =$$

$$-2\pi B_\mathrm{N}\beta_\mathrm{ref}\sum_s Z_s n_{\mathrm{R},s} v_{\mathrm{thR},s}\int \mathrm{d}v_{\|\mathrm{N}}\mathrm{d}\mu_\mathrm{N}\ v_{\|\mathrm{N}} J_0(k_\perp\rho_s)\widehat{\mathcal{V}}_{\mathrm{N},s}$$

$$\left(k_{\perp\mathrm{N}}^2 + \beta_\mathrm{ref}\sum_s \frac{Z_s^2 n_{\mathrm{R},s}}{m_{\mathrm{R},s}}\Gamma_0(b_s)\, e^{-\mathcal{E}_{\mathrm{N},s}/T_{\mathrm{R},s}}\right)\widehat{A}_{1\|\mathrm{N}} =$$

$$2\pi B_\mathrm{N}\beta_\mathrm{ref}\sum_s Z_s n_{\mathrm{R},s} v_{\mathrm{thR},s}\int \mathrm{d}v_{\|\mathrm{N}}\mathrm{d}\mu_\mathrm{N}\ v_{\|\mathrm{N}} J_0(k_\perp\rho_s)\widehat{g}_{\mathrm{N},s}$$

CONCLUSION

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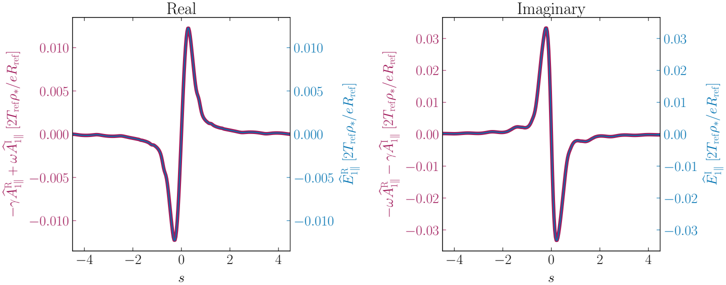
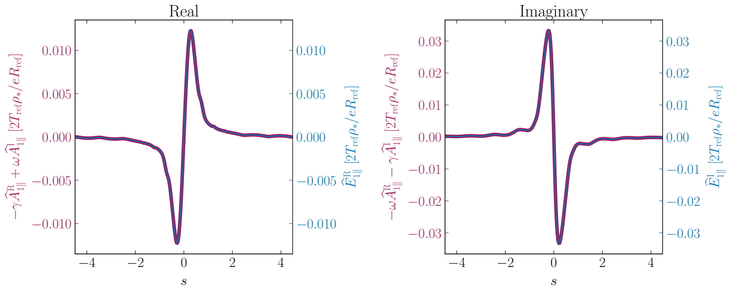


- Local linear f-Version successfully implemented
  $\rightarrow 10\,\%$ more runtime

CONCLUSION



- Local linear f-Version successfully implemented
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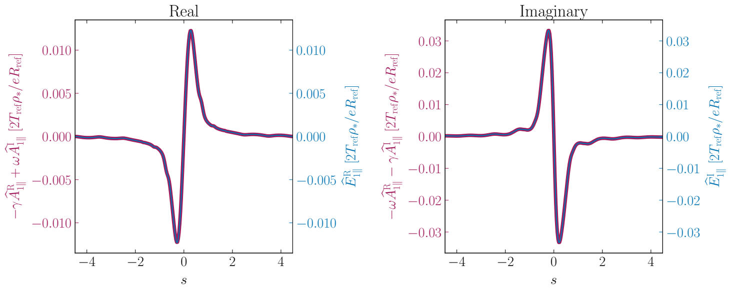
- Nonlinear benchmark not completed yet

CONCLUSION



- Local linear f-Version successfully implemented
  $\rightarrow 10\,\%$ more runtime
- Nonlinear benchmark not completed yet
- Groundwork for global f-version of GKW done