



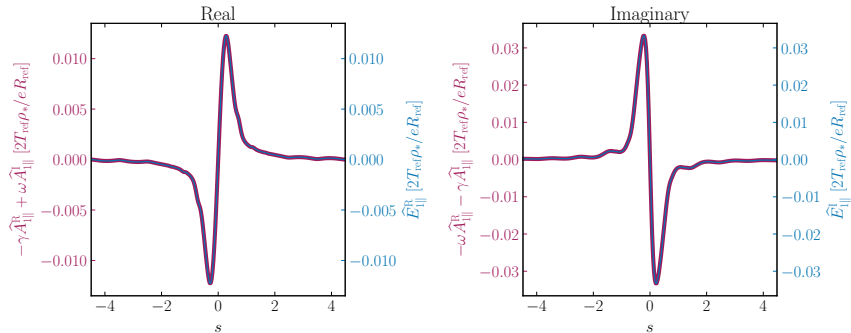
UNIVERSITÄT
BAYREUTH

MITIGATION OF THE CANCELLATION PROBLEM IN LOCAL GYROKINETIC SIMULATIONS

September 26, 2024

Manuel Lippert

Theoretical Physics V



MOTIVATION



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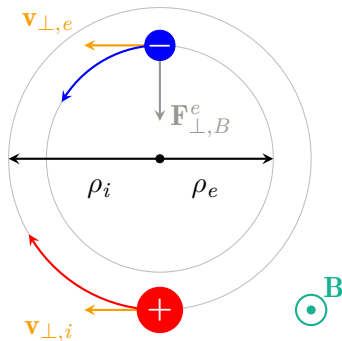
Is it possible to mitigate the cancellation problem in local gyrokinetic simulations with the introduction of a new field equation?

CHARGED PARTICLE MOTION

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Lorentz force

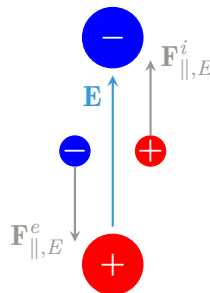
$$F_{\perp,B} = |q|v_{\perp}B$$



$$\rho = \frac{m\mathbf{v}_{\perp}}{|q|B} \quad \omega_c = \frac{|q|B}{m}$$

Electric force

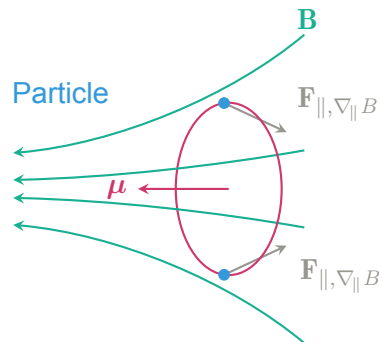
$$F_{\parallel,E} = qE_{\parallel}$$



$$\rho_{th} = \frac{mv_{th}}{|q|B}$$

Inhomogeneous magnetic field

$$F_{\parallel,\nabla_{\parallel}B} = - \underbrace{\frac{mv_{\perp}^2}{2B}}_{\mu} \nabla_{\parallel}B$$



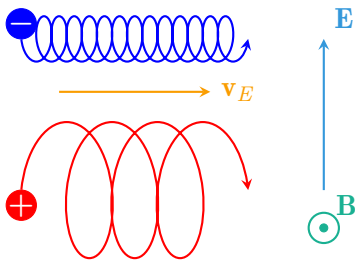
$$v_{th} = \sqrt{\frac{2T}{m}}$$

DRIFTS IN THE GUIDING CENTER

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$\mathbf{E} \times \mathbf{B}$ Drift

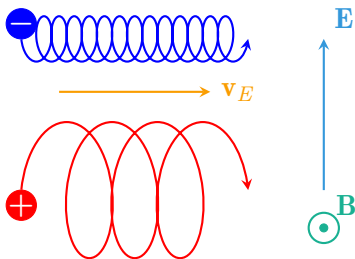
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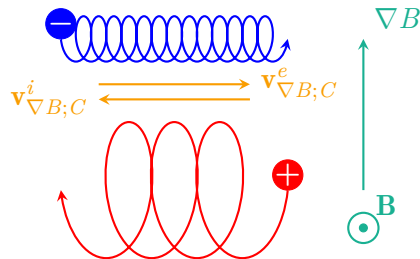


∇B Drift

$$\mathbf{v}_{\nabla B} = \frac{mv_{\perp}^2}{2q} \frac{\mathbf{B} \times \nabla B}{B^3}$$

Curvature Drift

$$\mathbf{v}_C = \frac{mv_{\parallel}^2}{qB^2} \mathbf{B} \times \underbrace{\mathbf{C}}_{\frac{\nabla B}{B}} = \frac{mv_{\parallel}^2}{q} \frac{\mathbf{B} \times \nabla B}{B^3}$$



PLASMA BETA

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- Characterizes quality of confinement (Best confinement for $\beta < 1$)
- Relevant for fusion rate ($\sim \beta^2$), MHD stability of a fusion device
- Indicator for the relevance of electromagnetic effects
- Electromagnetic fields vanish in the limit $\beta \rightarrow 0$

GYROKINETIC ORDERING

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- Low frequency: $\omega/\omega_c \ll 1$
- Anisotropy: $k_{\parallel}/k_{\perp} \ll 1$
- Strong Magnetization: $\rho/L_n \sim \rho/L_T \sim \rho/L_B \ll 1$ $L_G = G_0 \left(\frac{dG_0}{dx} \right)^{-1}$
- Small fluctuations: $F_1/F_0 \ll 1$

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$$\omega/\omega_c \sim k_{\parallel}/k_{\perp} \sim \rho/L_n \sim \rho/L_T \sim \rho/L_B \sim F_1/F_0 \sim \epsilon_{\delta}$$
$$\rho_{\star} = \frac{\rho_{\text{th,ref}}}{L_B} = \frac{m_{\text{ref}} v_{\text{th,ref}}}{e B_{\text{ref}}} \sim \epsilon_{\delta}$$

DERIVATION OF GYROKINETIC EQUATION

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- Find fundamental one-form of the gyrocenter phase space

$$\int dt L = \int \gamma$$

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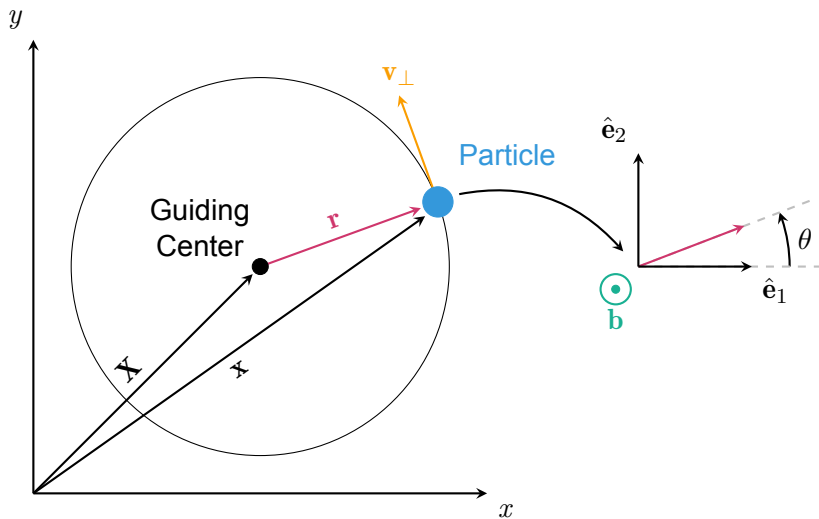
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- Particle phase space $\{\mathbf{x}, \mathbf{v}\} \rightarrow$ guiding center $\{\mathbf{X}, v_{\parallel}, \mu, \theta\} \rightarrow$ gyrocenter $\{\bar{\mathbf{X}}, \bar{v}_{\parallel}, \bar{\mu}\}$

$$\Phi = \Phi_0 + \underbrace{\tilde{\Phi}_1 + \bar{\Phi}_1}_{\Phi_1} \quad \mathbf{A} = \mathbf{A}_0 + \underbrace{\tilde{\mathbf{A}}_1 + \bar{\mathbf{A}}_1}_{\mathbf{A}_1}$$

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- Vlasov Equation in gyrocenter phase space without collisions

$$\frac{\partial F}{\partial t} + \dot{\mathbf{X}} \cdot \frac{\partial F}{\partial \mathbf{X}} + \dot{v}_{\parallel} \cdot \frac{\partial F}{\partial v_{\parallel}} = 0$$

DERIVATION OF GYROKINETIC EQUATION

- Delta- f approximation $F = F_0 + F_1$

$$\frac{\partial F_1}{\partial t} + \dot{\mathbf{X}} \cdot \nabla F_1 + \dot{v}_{\parallel} \cdot \frac{\partial F_1}{\partial v_{\parallel}} = \underbrace{-\dot{\mathbf{X}} \cdot \nabla F_0 - \dot{v}_{\parallel} \frac{\partial F_0}{\partial v_{\parallel}}}_S$$

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- Maxwellian as equilibrium distribution $F_0 = F_M$ and $\dot{\mathbf{X}}$ and \dot{v}_{\parallel} from Lagrangian

$$\frac{\partial F_1}{\partial t} + \dot{\mathbf{X}} \cdot \nabla F_1 - \frac{\mathbf{b}_0}{m} \cdot (Ze \nabla \Phi_0 + \mu \nabla B_0 - m R \Omega^2 \nabla R) \cdot \frac{\partial F_1}{\partial v_{\parallel}} = S$$

$$S = -(\mathbf{v}_{\chi} + \mathbf{v}_D) \cdot \tilde{\nabla} F_M - \frac{Z e v_{\parallel}}{T} \partial_t \bar{A}_{1\parallel} F_M \\ - \frac{F_M}{T} (v_{\parallel} \mathbf{b}_0 + \mathbf{v}_D + \mathbf{v}_{\bar{B}_{1\perp}}) \cdot (Ze \nabla \bar{\Phi} + \mu \nabla \bar{B}_{1\parallel})$$

DERIVATION OF FIELD EQUATIONS

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- Maxwell's equations

$$\sum_s Z_s e n_s = 0$$

$$\nabla \cdot \mathbf{B}_1 = 0$$

$$\nabla \times \mathbf{E}_1 = -\frac{\partial \mathbf{B}_1}{\partial t}$$

$$\nabla \times \mathbf{B}_1 = \mu_0 \sum_s \mathbf{j}_s$$

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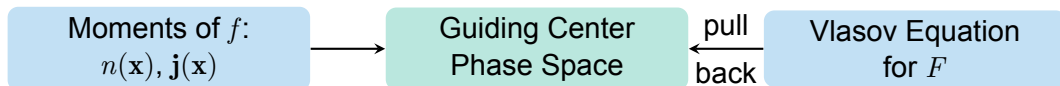
$$\nabla \times \mathbf{B}_1 = \mu_0 \sum_s \mathbf{j}_s$$

- Moments of the distribution f

$$n(\mathbf{x}) = \int d\mathbf{v} f(\mathbf{x}, \mathbf{v})$$

$$j_{\parallel} = Ze \int d\mathbf{v} v_{\parallel} f(\mathbf{x}, \mathbf{v}) \quad \mathbf{j}_{\perp} = Ze \int d\mathbf{v} \mathbf{v}_{\perp} f(\mathbf{x}, \mathbf{v})$$

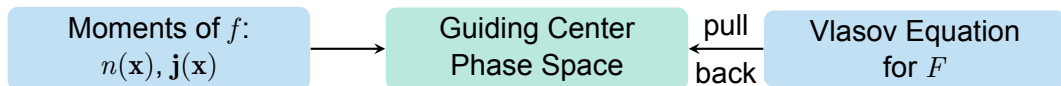
DERIVATION OF FIELD EQUATIONS



- Pullback transformation

$$F^{\text{gc}} = \mathcal{P}\{F\} = F - \underbrace{\frac{F_{\text{M}}}{T} \left(Ze\tilde{\Phi}_1 - \mu\bar{B}_{1\parallel} \right)}_{\text{Correction Term}}$$

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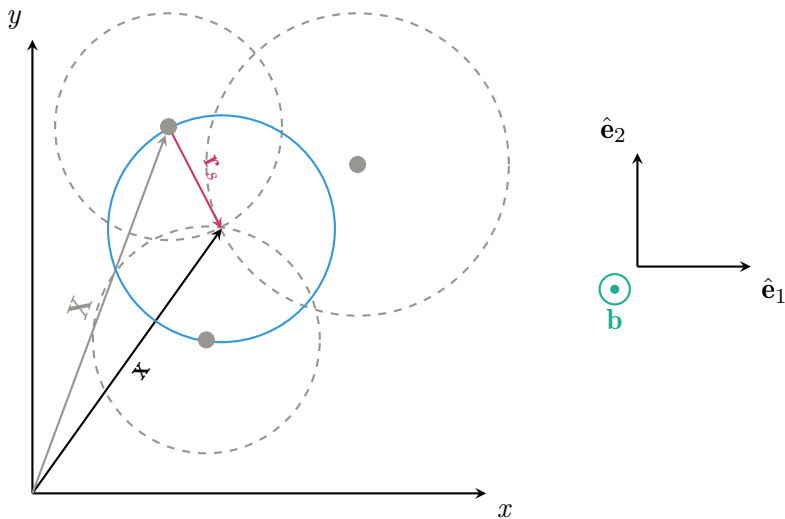
- Moments of the distribution F^{gc}

$$n = \int d\mathbf{v} f(\mathbf{x}, \mathbf{v}) = \frac{B_0}{m} \int d\mathbf{X} dv_{\parallel} d\theta d\mu \delta(\mathbf{X} + \mathbf{r} - \mathbf{x}) F^{\text{gc}}$$

$$j_{\parallel} = Ze \int d\mathbf{v} v_{\parallel} f(\mathbf{x}, \mathbf{v}) = \frac{ZeB_0}{m} \int d\mathbf{X} dv_{\parallel} d\theta d\mu \delta(\mathbf{X} + \mathbf{r} - \mathbf{x}) v_{\parallel} F^{\text{gc}}$$

$$\mathbf{j}_{\perp} = Ze \int d\mathbf{v} \mathbf{v}_{\perp} f(\mathbf{x}, \mathbf{v}) = \frac{ZeB_0}{m} \int d\mathbf{X} dv_{\parallel} d\theta d\mu \delta(\mathbf{X} + \mathbf{r} - \mathbf{x}) \mathbf{v}_{\perp} F^{\text{gc}}$$

DERIVATION OF FIELD EQUATIONS



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- Coulomb's law
 $\rightarrow \sum_s Z_s e n_s = 0 \rightarrow \Phi_1$
- Magnetic Compression
 $\rightarrow \nabla^2 \mathbf{A}_{1\perp} = (\nabla \times B_{1\parallel})_{\perp} = -\mu_0 \mathbf{j}_{1\perp} \rightarrow B_{1\parallel}$
- Ampere's law
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$$k_{\perp N}^2 \hat{A}_{1\parallel N} = 2\pi B_N \beta_{\text{ref}} \sum_s Z_s n_{R,s} v_{\text{th}R,s} \int dv_{\parallel N} d\mu_N v_{\parallel N} J_0(k_{\perp} \rho_s) \hat{F}_{1N,s}$$

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THE CANCELLATION PROBLEM

- Modified distribution function

$$g = F_1 + \frac{Zev_{\parallel}}{T} \bar{A}_{1\parallel} F_M$$

- Modified gyrokinetic equation

$$\frac{\partial g}{\partial t} + \mathbf{v}_{\chi} \cdot \nabla g + (v_{\parallel} \mathbf{b}_0 + \mathbf{v}_D) \cdot \nabla F_1 - \frac{\mathbf{b}_0}{m} \cdot (Ze \nabla \Phi_0 + \mu \nabla B_0 - mR\Omega^2 \nabla R) \frac{\partial F_1}{\partial v_{\parallel}} = S$$

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THE CANCELLATION PROBLEM

- Adjusted Ampere's Law

$$\left(k_{\perp N}^2 + \beta_{\text{ref}} \sum_s \frac{Z_s^2 n_{R,s}}{m_{R,s}} \Gamma_0(b_s) e^{-\mathcal{E}_{N,s}/T_{R,s}} \right) \hat{A}_{1\parallel N} =$$

$$2\pi B_N \beta_{\text{ref}} \sum_s Z_s n_{R,s} v_{\text{th}R,s} \int dv_{\parallel N} d\mu_N v_{\parallel N} J_0(k_{\perp} \rho_s) \hat{g}_{N,s}$$

THE CANCELLATION PROBLEM

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$$\begin{aligned}
 & \left(k_{\perp N}^2 + \beta_{\text{ref}} \sum_s \frac{Z_s^2 n_{R,s}}{m_{R,s}} \Gamma_0(b_s) e^{-\mathcal{E}_{N,s}/T_{R,s}} \right) \hat{A}_{1\parallel N} = \\
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 & + 2\pi B_N \beta_{\text{ref}} \sum_s Z_s n_{R,s} v_{\text{th}R,s} \int dv_{\parallel N} d\mu_N v_{\parallel N}^2 J_0^2(k_{\perp} \rho_s) \frac{Z e v_{\text{th}}}{T_{\text{ref}} T_R} F_{MN} \hat{A}_{1\parallel N}
 \end{aligned}$$

- Error scales with $\sim \beta/k_{\perp}^2$

MITIGATION OF THE CANCELLATION PROBLEM - FARADAY'S LAW

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- Faraday's law

$$\rightarrow E_{1\parallel} = -\frac{\partial A_{1\parallel}}{\partial t} \rightarrow \nabla^2 \left(-\frac{\partial A_{1\parallel}}{\partial t} \right) = \mu_0 \frac{\partial j_{1\parallel}}{\partial t} \rightarrow E_{1\parallel}$$

$$\underbrace{k_{\perp N}^2 \left(-\frac{\partial \hat{A}_{1\parallel N}}{\partial t_N} \right)}_{\hat{E}_{1\parallel N}} = -2\pi B_N \beta_{\text{ref}} \sum_s Z_s n_{R,s} v_{\text{th}R,s} \int dv_{\parallel N} d\mu_N v_{\parallel N} J_0(k_{\perp} \rho_s) \frac{\partial \hat{F}_{1N,s}}{\partial t_N}$$

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- Replacing the time derivative of the gyrocenter distribution function

$$\frac{\partial F_1}{\partial t} = \mathcal{V} - \frac{Zev_{\parallel}}{T} \frac{\partial \bar{A}_{1\parallel}}{\partial t} F_M = \mathcal{V} + \frac{Zev_{\parallel}}{T} \bar{E}_{1\parallel} F_M$$

MITIGATION OF THE CANCELLATION PROBLEM - FARADAY'S LAW

- Field Equation for plasma induction $E_{1\parallel}$

$$\left(k_{\perp N}^2 + \beta_{\text{ref}} \sum_s \frac{Z_s^2 n_{R,s}}{m_{R,s}} \Gamma_0(b_s) e^{-\mathcal{E}_{N,s}/T_{R,s}} \right) \widehat{E}_{1\parallel N} =$$

$$- 2\pi B_N \beta_{\text{ref}} \sum_s Z_s n_{R,s} v_{\text{th}R,s} \int dv_{\parallel N} d\mu_N v_{\parallel N} J_0(k_{\perp} \rho_s) \widehat{\mathcal{V}}_{N,s}$$

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- New source term in gyrokinetic equation

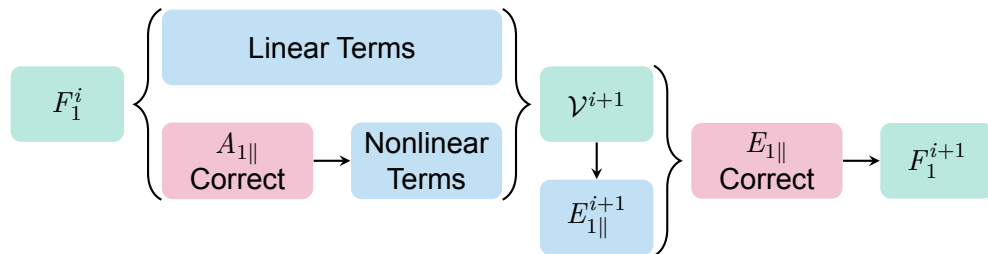
$$\frac{\partial F_1}{\partial t} + \dot{\mathbf{X}} \cdot \nabla F_1 - \frac{\mathbf{b}_0}{m} \cdot (Ze \nabla \Phi_0 + \mu \nabla B_0 - m R \Omega^2 \nabla R) \cdot \frac{\partial F_1}{\partial v_{\parallel}} = S$$

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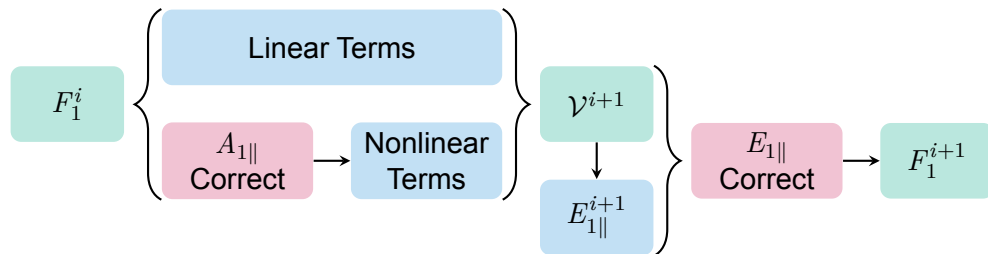
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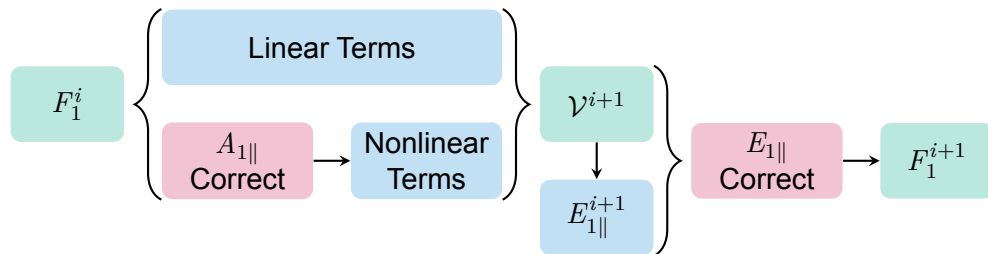
- GKW calculates the modified distribution g (g-version)
→ Switch to the calculation of the gyrocenter distribution F_1 (f-version)

IMPLEMENTATION OF FARADAY'S LAW



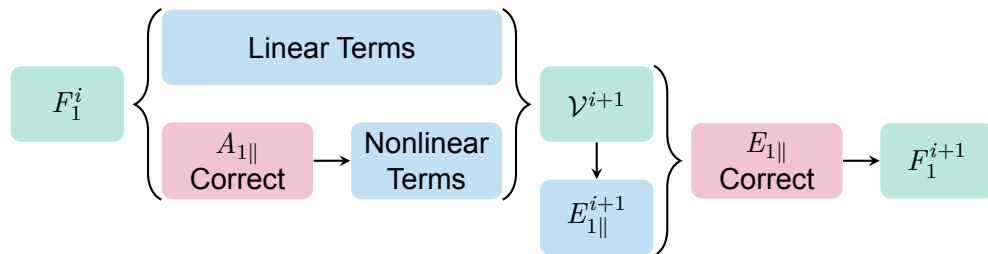
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IMPLEMENTATION OF FARADAY'S LAW



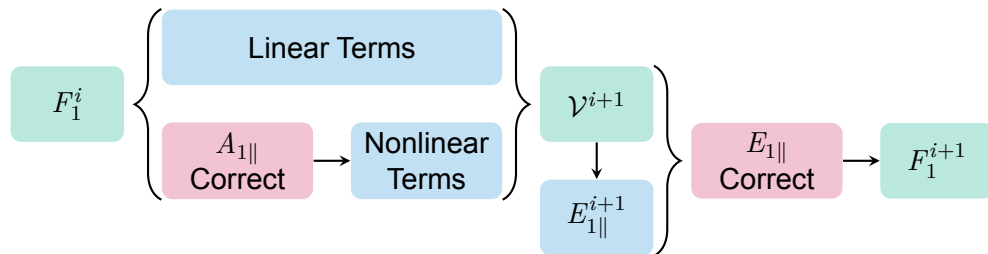
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- Apply $A_{1\parallel}$ correction on F_1 for the calculation of the nonlinear terms
- Calculation of Faraday's law in an additional subroutine with \mathcal{V}
- Apply $E_{1\parallel}$ correction on \mathcal{V} for the calculation of F_1

BENCHMARK OF THE F-VERSION

BENCHMARK OF THE F-VERSION

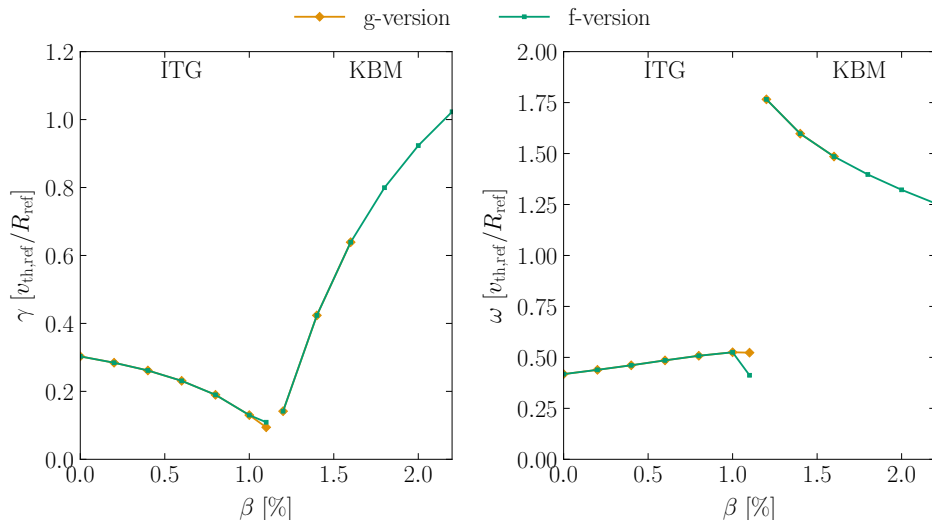
- Flux-tube version of GKW with field aligned Hamada coordinates $\{s, \psi, \zeta, v_{\parallel}, \mu\}$
- Cyclone base case (CBC) beta scan for

$$\beta \in [0.0, 0.2, 0.4, 0.6, 0.8, 1.0, 1.1, 1.2, 1.4, 1.6, 1.8, 2.0, 2.2] \%$$

- Kinetic electrons (`adiabatic_electrons = .false.`)
- $k_{\zeta} \rho = 0.3$ (Maximum of the nonlinear transport spectrum)

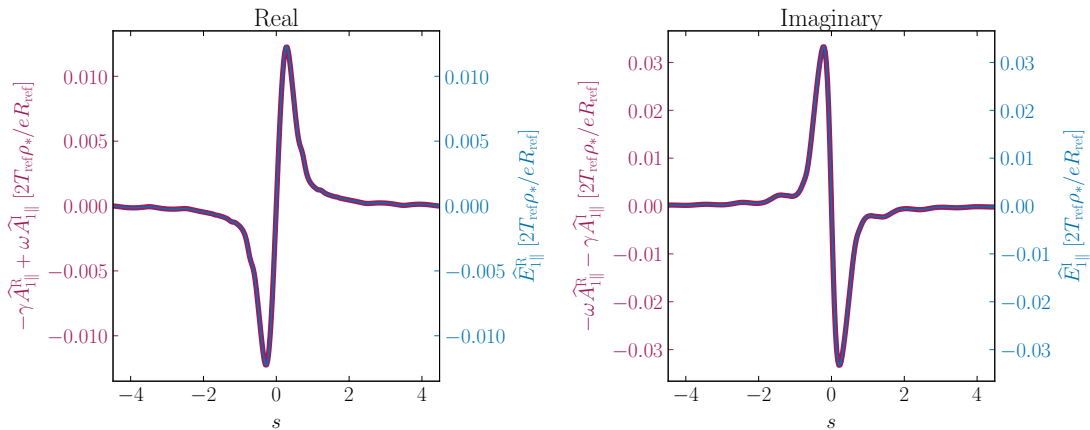
| DTIM | NTIME | NAVERAGE | N_{mod} | N_x | N_s | $N_{v_{\parallel}}$ | N_{μ} | N_{sp} | nperiod |
|------|-------|----------|------------------|-------|-------|---------------------|-----------|-----------------|---------|
| 0.01 | 2000 | 100 | 1 | 1 | 288 | 64 | 16 | 2 | 5 |

BENCHMARK OF THE F-VERSION



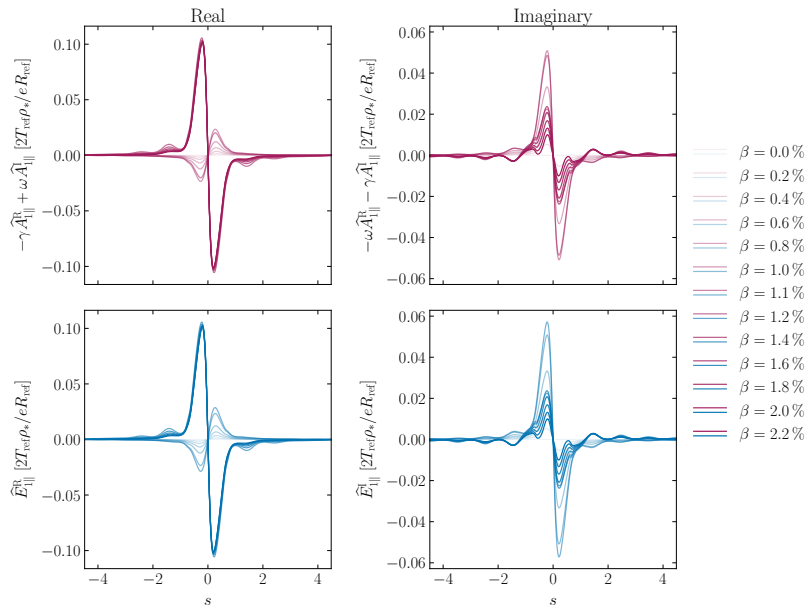
10 % more runtime for the f-version

BENCHMARK OF THE F-VERSION



$$\beta = 0.8 \%$$

BENCHMARK OF THE F-VERSION



MITIGATION IN LOCAL SIMULATIONS

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- Mitigation could only be tested in the local linear simulations
→ Local nonlinear simulations not benchmarked yet

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- Mitigation could only be tested in the local linear simulations
→ Local nonlinear simulations not benchmarked yet
- CBC and AUG (ASDEX-Upgrade) test cases fails
for the same value of k_{\perp} in both versions

$$\left(k_{\perp N}^2 + \beta_{\text{ref}} \sum_s \frac{Z_s^2 n_{R,s}}{m_{R,s}} \Gamma_0(b_s) e^{-\mathcal{E}_{N,s}/T_{R,s}} \right) \widehat{E}_{1\parallel N} =$$

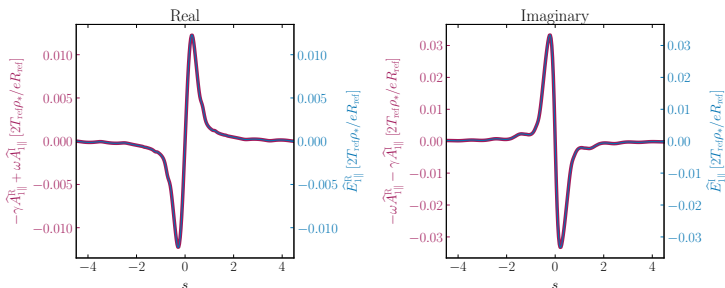
$$- 2\pi B_N \beta_{\text{ref}} \sum_s Z_s n_{R,s} v_{\text{th}R,s} \int dv_{\parallel N} d\mu_N v_{\parallel N} J_0(k_{\perp} \rho_s) \widehat{\mathcal{V}}_{N,s}$$

$$\left(k_{\perp N}^2 + \beta_{\text{ref}} \sum_s \frac{Z_s^2 n_{R,s}}{m_{R,s}} \Gamma_0(b_s) e^{-\mathcal{E}_{N,s}/T_{R,s}} \right) \widehat{A}_{1\parallel N} =$$

$$2\pi B_N \beta_{\text{ref}} \sum_s Z_s n_{R,s} v_{\text{th}R,s} \int dv_{\parallel N} d\mu_N v_{\parallel N} J_0(k_{\perp} \rho_s) \widehat{\mathcal{G}}_{N,s}$$

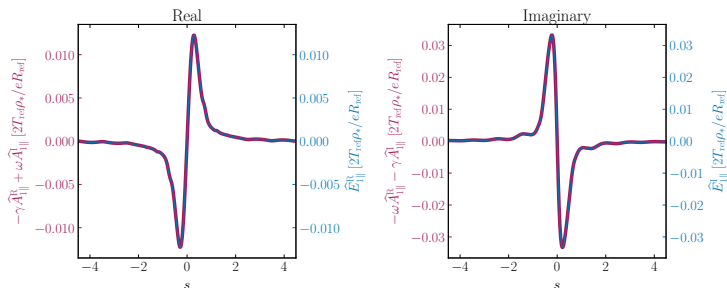
CONCLUSION

CONCLUSION



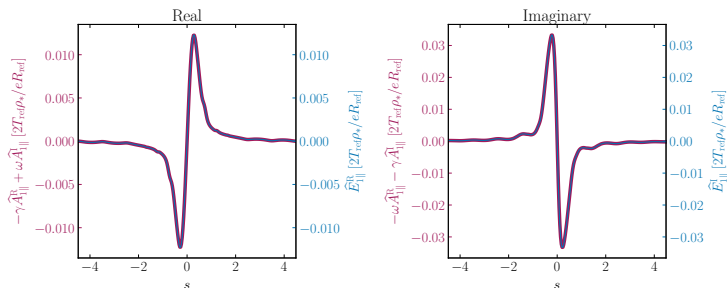
- Local linear f-Version successfully implemented
→ 10 % more runtime

CONCLUSION



- Local linear f-Version successfully implemented
→ 10 % more runtime
- Nonlinear benchmark not completed yet

CONCLUSION



- Local linear f-Version successfully implemented
→ 10 % more runtime
- Nonlinear benchmark not completed yet
- Groundwork for global f-version of GWK done