| KEC303 | Network Analysis and Synthesis | 3L:0T:0P | 3 Credits |
|---|---|---|---|

| Unit | Topics | Lectures |
|---|---|---|
| I | Node and mesh analysis, matrix approach of network containing voltage & current sources and reactances, source transformation and duality. | 8 |
| II | Network theorems: Superposition, reciprocity, Thevenin's, Norton's, Maximum power transfer, compensation and Tallegen's theorem as applied to A.C. circuits. | 8 |
| III | Trigonometric and exponential Fourier series: Discrete spectra and symmetry of waveform, steady state response of a network to non-sinusoidal periodic inputs, power factor, effective values, Fourier transform and continuous spectra, three phase unbalanced circuit and power calculation. | 8 |
| IV | Laplace transforms and properties: Partial fractions, singularity functions, waveform synthesis, analysis of RC, RL, and RLC networks with and without initial conditions with Laplace transforms evaluation of initial conditions. | 8 |
| V | Transient behaviour, concept of complex frequency, driving points and transfer functions poles and zeros of immittance function, their properties, sinusoidal response from pole-zero locations, convolution theorem and two four port network and interconnections, behaviour of series and parallel resonant circuits, introduction to band pass, low pass, high pass and band reject filters. | 8 |

**Text/Reference Books**


1. Franklin F. Kuo, "Network Analysis and Synthesis," Wiley India Education, 2nd Ed., 2006.
2. Van, Valkenburg, "Network analysis," Pearson, 2019.
3. Sudhakar, A., Shyammohan, S. P., "Circuits and Network," Tata McGraw-Hill New Delhi, 1994.
4. A William Hayt, "Engineering Circuit Analysis," 8th Edition, McGraw-Hill Education.
5. A. Anand Kumar, "Network Analysis and Synthesis," PHI publication, 2019.


**Course Outcomes:**

At the end of this course students will demonstrate the ability to:

1. Understand basics electrical circuits with nodal and mesh analysis.
2. Appreciate electrical network theorems.
3. Apply Laplace transform for steady state and transient analysis.
4. Determine different network functions.
5. Appreciate the frequency domain techniques.

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# CONTENTS

## KEE 403 : NETWORK ANALYSIS & SYNTHESIS

# 1 UNIT

# Graph Theory

## CONTENTS







PART-1

*Pre-Requisites : Basic Circuital Law, Mesh and Nodal Analysis.*

**Questions-Answers**

**Long Answer Type and Medium Answer Type Questions**

**Que 1.1. Explain the basic circuital law.**

**Answer**

There are two basic circuital law :

A. **Kirchhoff 's voltage law (KVL) :** This law states that the algebraic sum of voltage around any closed path is zero.

$$\sum_{n=1}^{N} V_n = 0$$

where, $V_n$ is the voltage in $n^{\text{th}}$ element of a closed loop having $n$ element.

B. **Kirchhoff 's current law (KCL) :** This law states that the algebraic sum of current entering the node is equal to zero.

*i.e.,* $$\sum_{n=1}^{N} i_n = 0$$

where, $i_n$ is the current in $n^{th}$ branch.

**Que 1.2. Discuss the mesh and nodal analysis with the help of example.**

**Answer**

A. **Nodal analysis :** In this analysis independent nodes are considered and voltages are assumed at these nodes with respect to one node called datum node. The equations are then framed according to KCL.

**Example :**

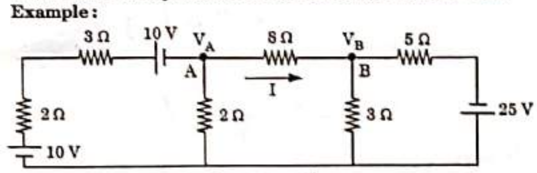



**Fig. 1.2.1.**

**Answer**

1. At node $A$, $\dfrac{V_A + 10 - 10}{5} + \dfrac{V_A - 0}{2} + \dfrac{V_A - V_B}{8} = 0$

$$8V_A + 20V_A + 5V_A - 5V_B = 0$$
$$33\,V_A = 5V_B$$
$$\therefore \quad V_A = \frac{5V_B}{33} \quad \text{...(1.2.1)}$$

2. At node $B$, $\dfrac{V_B - V_A}{8} + \dfrac{V_B - 0}{3} + \dfrac{V_B + 25}{5} = 0$

$$15V_B - 15V_A + 40V_B + 24V_B + 600 = 0$$
$$79V_B - 15V_A = -600 \quad \text{...(1.2.2)}$$

3. Putting value of $V_A$ in eq. (1.2.2), we get

$$79V_B - \frac{75}{33}V_B = -600$$
$$\therefore \quad V_B = -7.82\text{ V}$$

4. Putting value of $V_B$ in eq. (1.2.1),

$$V_A = \frac{5}{33} \times -7.82 = -1.18\text{ V}$$

5. Current through 8 Ω resistor

$$I = \frac{V_A - V_B}{8} = \frac{-1.18 + 7.82}{8} = 0.83\text{ A}$$

**B. Mesh analysis :**

1. A mesh (or loop) is defined as a closed path around a circuit that does not contain any other closed path within it.
2. The method of mesh analysis is based on assigning the mesh currents and then applying KVL around each mesh in the network. Usually, the mesh current directions are taken clockwise.

**Example :**

1. The equivalent circuit is :

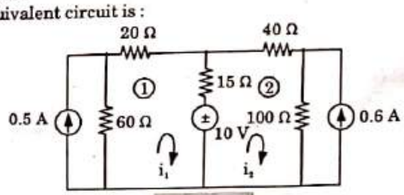



**Fig. 1.2.2.**

2. **In mesh 1 :**

$$20i_1 + 15(i_1 - i_2) - 10 + 60(i_1 - 0.5) = 0$$
$$95i_1 - 15i_2 = 40$$



$$19i_1 - 3i_2 = 8 \quad \text{...(1.2.1)}$$

3. **In mesh 2 :**

$$40i_2 + 100(i_2 + 0.6) + 10 + 15(i_2 - i_1) = 0$$
$$155i_2 - 15i_1 = -70$$
$$31i_2 - 3i_1 = -14 \quad \text{...(1.2.2)}$$

4. From eq. (1.2.1) and (1.2.2),

$$i_1 = \frac{103}{290}\text{ A},\ i_2 = -\frac{121}{290}\text{ A}$$

5. Now, the current flowing through 20 Ω is $I_1$, hence

$$I_1 = i_1 = \frac{103}{290} = 0.355\text{ A}$$

6. Current flowing through 40 Ω is $I_2$, hence

$$I_2 = i_2 = -\frac{121}{290} = -0.4127\text{ A}$$

and current flowing through 15 Ω is $I_3$

$$I_3 = (i_1 - i_2) = \left(\frac{103-(-121)}{290}\right) = \frac{224}{290} = 0.772\text{ A}$$

## PART-2

*Importance of Graph Theory in Network analysis, Graph of a Network, Definitions, Planar and Non Planar Graphs, Isomorphism, Tree, Co-tree, Link, Basic Loop and Basic Cutset, Incidence Matrix.*

**Questions-Answers**

**Long Answer Type and Medium Answer Type Questions**

**Que 1.3.** **What is the importance of graph theory in network analysis ?**

**Answer**

1. It is used to topological analysis of linear, passive and transformer less networks.
2. It is used to topological synthesis of pure resistive network.

**Que 1.4.** **What do you mean by "the graph of a network" ? Also mention its significance and limitations.**

**Answer**

**A. Graph of a network :**

1. A linear graph is defined as a collection of points called nodes, and line segments called branches, the nodes being joined together by the branches.
2. The graph of the network of Fig. 1.4.1(a) is shown in Fig. 1.4.1(b). The graph of a planar network is drawn by keeping all the points of intersection of two or more branches, known as nodes and representing the network elements by lines, voltage and current sources by their internal impedances.
3. The internal impedance of an ideal voltage source is zero and to be replaced by short-circuit, and that of an ideal current sources is infinite and hence to be replaced by an open circuit.

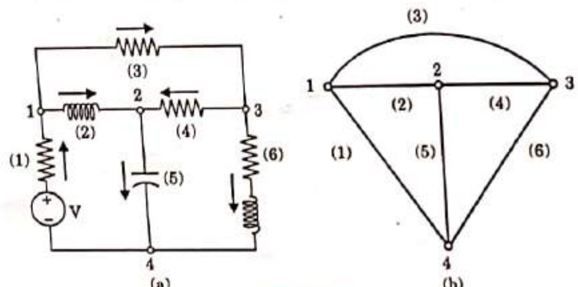



**Fig. 1.4.1.**

**B. Significance :** The graph of a network is significant because it makes the circuit analysis easier as only the geometrical pattern of a network is considered and no distinction is made between different types of physical elements of which it is composed.

**C. Limitations :** The graph of a network can only be used to find out the KVL and KCL equations and not the exact magnitude of voltages and currents.

**Que 1.5.** **Define the following terms related to a network graph.**

| | |
|---|---|
| **i. Network graph** | **ii. Oriented graph** |
| **iii. Subgraph** | **iv. Branch** |
| **v. Node** | **vi Degree of node** |
| **vii. Path** | **viii. Loop** |
| **ix. Tree** | **x. Twigs** |
| **xi. Co-tree** | **xii. Link** |
| **xiii. f-cut-set** | |



**Answer**

**i. Network graph :**

1. A network graph is defined as the set of nodes and branches with the condition that each branch terminates at each end into a node.
2. In a network graph no distinction is made among the different types of physical elements such as resistor, inductor and capacitor.

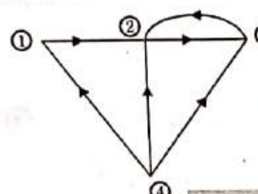
**ii. Oriented graph :** A graph is said to be oriented or directed when all its nodes and branches are numbered and directions are assigned to the branches by arrows.

**iii. Subgraph :** A subgraph is a subset of the branches and node of a graph. A subgraph is said to be "proper subgraph" if it does not contain all the branches and nodes of the graph.

**iv. Branch :** A branch is a line segment which represents a network element or a combination of elements connected between two points. Sometimes it is called an edge. The line segments do not say anything about the type of elements.

**v. Node :** A node is an end point of a branch or an isolated point. It is also called a vertex or a junction.

**vi. Degree of node :** The degree of a node is the number of branches associated to it.




Degree of node (1) is = 2
Degree of node (2) is = 4
Degree of node (4) is = 3

**Fig. 1.5.1.**

A node of degree two is called simple node and a node of degree zero is called an isolated node.

**vii. Path :** An ordered sequence of branches traversing from one node to another is called path.

**viii. Loop :** A loop is a collection of branches in a graph which form a closed path with the restriction that exactly two branches of the path are connected to each of the nodes.

**ix. Trees :** A tree of a connected graph is defined as a set of branches which connect all the nodes of graph without forming any loop.

**x. Twigs :** The branches of tree are called twigs.

**xi. Co-tree :** The set of all branches of the graph which are not present in the tree is called co-tree.

**xii. Link :** The branches of a co-tree are called links.

**xiii. f-cut-set (Basic cut-set) :**

1. Cut-set matrix with respect to a tree is formed by one and only one twig and a set of links. In a graph for each twig of chosen tree there is a cut-set matrix.
2. A graph having $N$ nodes will have $(N - 1)$ fundamental cut-set. *i.e.*, equal to number of twigs.
3. The orientation of a cut-set coincides with the orientation of twig.

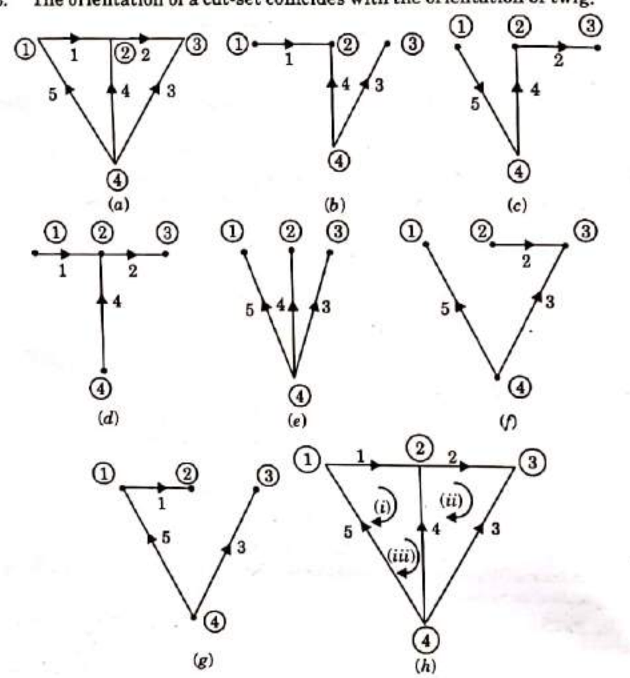



**Fig. 1.5.2.**

4. In Fig. 1.5.2, various subgraph are shown in which ($a$) is the graph while ($b$), ($c$), ($d$), ($e$), ($f$) and ($g$) are the tree of graph ($a$).
5. In Fig. 1.5.2($a$), 1, 2, 3, 4, 5 are called branches. In Fig. 1.5.2($b$), 1, 4, 3 and Fig. 1.5.2($c$), 5, 4, 3 (and similarly for others) are called twigs.
6. In Fig. 1.5.2($b$), 5, 2 and Fig. 1.5.2($c$), 1, 3 (and similarly for others) are called links. By links co-tree is made.



**Que 1.6.** **Describe the planar and non-planar graphs.**

**Answer**

**A. Planar graph :**

1. A graph is said to be planar if it can be drawn in a plane so that no edge cross.
2. **Region of a planar graph :** A region is defined to be an area of the plane that is bounded by edges and cannot be further subdivided. A planar graph divides the plans into one or more regions.

a. **Finite region :** If the area of the region is finite, then that region is called a finite region.

b. **Infinite region :** If the area of the region is infinite, that region is called a infinite region. A planar graph has only one infinite region.

3. **Example :** Consider the graph shown in Fig. 1.6.1. Determine the number of regions, finite regions and an infinite region.

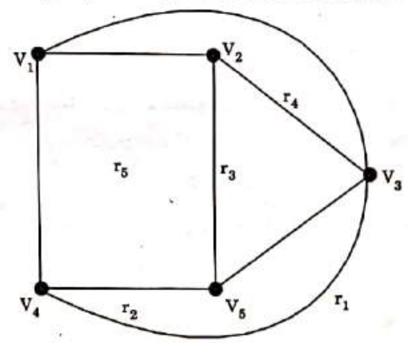



**Fig. 1.6.1.**

**Solution :**

a. There are five regions in the above graph, *i.e.* $r_1, r_2, r_3, r_4, r_5$.

b. There are four finite regions in the graph, *i.e.*, $r_2, r_3, r_4, r_5$.

c. There is only one infinite region, *i.e.*, $r_1$

4. **Properties of planar graphs :**

i. If a connected planar graph $G$ has $e$ edges and $r$ regions, then $r \le 2/3\, e$.

ii. If a connected planar graph $G$ has $e$ edges, $v$ vertices, and $r$ regions, then $v - e + r = 2$.

iii. If a connected planar graph $G$ has $e$ edges and $v$ vertices, then $3v - e \ge 6$.

iv. A complete graph $K_n$ is a planar if and only if $n < 5$.

**B. Non-planar graph :**

1. A graph is said to be non planar if it cannot be drawn in a plane so that no edge cross.
2. **Example :** The graphs shown in Fig. 1.6.2 are non planar graphs.


**Fig. 1.6.2.**

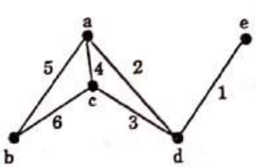
These graphs cannot be drawn in a plane so that no edges cross hence they are non-planar graphs.

3. **Properties of non-planar graphs :** A graph is non-planar if and only if it contains a subgraph homeomorphic to $K_5$.

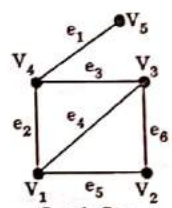
**Que 1.7.** **Explain the isomorphism with the help of example.**

**Answer**

1. Two graphs $G$ and $G'$ are said to be isomorphic to each other if there is a one-to-one correspondence between their vertices and between their edges such that the incidence relationship is preserved.
2. **Example :**




Graph G :




Graph G' :

| Correspondence of vertices | Correspondence of edges |
|---|---|
| $f(a) = V_1$ | $f(1) = e_1$ |
| $f(b) = V_2$ | $f(2) = e_2$ |
| $f(c) = V_3$ | $f(3) = e_3$ |
| $f(d) = V_4$ | $f(4) = e_4$ |
| $f(e) = V_5$ | $f(5) = e_5$ |

Adjacency also preserved. Therefore $G$ and $G'$ are said to be isomorpic.



**Que 1.8.** **Define fundamental loop (Basic loop). Explain with an example.**

**Answer**

1. Fundamental loop in any graph is a loop formed by the branches such that it contains one link and remainders are tree branches.
2. Therefore, the number of fundamental loops will be equal to the number of links. Let us explain it with an example.
3. Consider the graph and the tree as shown in Fig. 1.8.1(*a*) and Fig. 1.8.1(*b*) respectively.

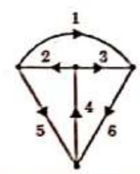



**Fig. 1.8.1.** (*a*) Graph and tree.

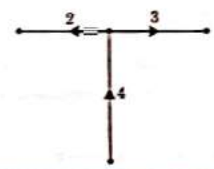



**Fig. 1.8.1.** (*b*) Tree of the graph.

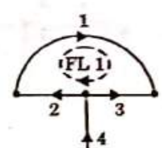



**Fig. 1.8.2.** (*a*) Loop-1.

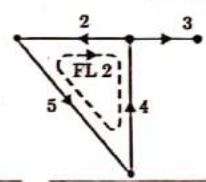



**Fig. 1.8.2.** (*b*) Loop-2.

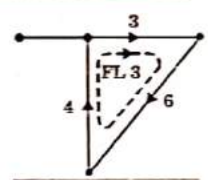



**Fig. 1.8.2.** (*c*) Loop-3.

4. This selected tree will result in three fundamental loops as we connect each link.

**Fundamental loop 1 :** Connecting link 1 to the tree.
**Fundamental loop 2 :** Connecting link 5 to the tree.
**Fundamental loop 3 :** Connecting link 6 to the tree.

**Que 1.9.** **Explain following terms with reference to network topology**

**i. Tree** **ii. Co-tree**
**iii. Oriented graph** **v. Twig and link**
**v. Incidence matrix.**

**AKTU 2017-18, Marks 07**

**Answer**

i. **Tree :** Refer Q. 1.5, Page 1–5C, Unit-1.

ii. **Co-tree :** Refer Q. 1.5, Page 1–5C, Unit-1.

iii. **Oriented graph :** Refer Q. 1.5, Page 1–5C, Unit-1.

iv. **Twig link :** Refer Q. 1.5, Page 1–5C, Unit-1.

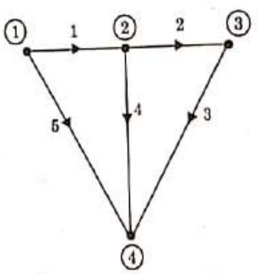
v. **Incidence matrix :**

1. When a graph has $N$ nodes and $B$ branches, the complete incidence matrix $[A_i]$ is $[a_{ij}]$ and is $N \times B$ rectangular matrix whose elements are defined as :

a. If branch $j$ is incident at node $i$ and oriented away from the node, $a_{ij} = 1$

b. If branch $j$ enters node $i$, *i.e.*, oriented towards node $i$, $a_{ij} = -1$

c. If branch $j$ is not associated with node $i$, $a_{ij} = 0$




(*a*) Tree

Nodes ↓ Branches ⟶

$$\begin{array}{c} \\ ① \\ ② \\ ③ \\ ④ \end{array}\begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ -1 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 & -1 \end{bmatrix} \end{array}$$

(*b*) Incidence matrix

**Fig. 1.9.1.**

**Que 1.10.** **What is incidence matrix ? Give its properties.**

**Answer**

A. **Incidence matrix :** Refer Q. 1.9, Page 1–10C, Unit-1.

B. **Properties :**

a. Algebraic sum of the column entries of an incidence matrix is zero.

b. Determinant of the incidence matrix of a closed loop is zero.

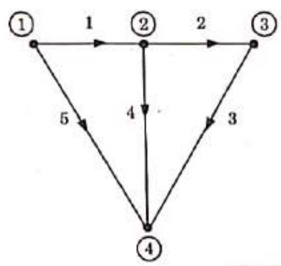
c. The rank of a connected graph is $(n - 1)$, where $n$ = number of nodes.

**Que 1.11.** **Explain the reduced incidence matrix with the help of example.**



**Answer**

1. When one row is deleted from the complete incidence matrix $A_i$, the remaining matrix is called reduced incidence matrix.
2. In other words, suppression of the reference node from incidence matrix results in reduced incidence matrix.
3. **Example :**




$$[A_i] = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ -1 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 & -1 \end{bmatrix}$$

**Fig. 1.11.1.**

$$\text{Reduced incidence matrix } [A] = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ -1 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 \end{bmatrix}$$

**Que 1.12.** **For given reduced incidence matrix, draw the graph and hence obtain the f-cutset matrix**

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 & 1 & 1 \\ -1 & 0 & 1 & 0 & 0 & -1 & 0 \end{bmatrix}$$

**AKTU 2017-18, Marks 10**

**Answer**

1. From reduced incident matrix, we get the complete incidence matrix as

$$A_i = \begin{array}{c} \\ \text{A} \\ \text{B} \\ \text{C} \\ \text{D} \end{array}\begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \leftarrow \text{Branch} \\ \begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 & 1 & 1 \\ -1 & 0 & -1 & 0 & 0 & -1 & 0 \\ 1 & -1 & 0 & -1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

↑
Node

2. Graph

Fig. 1.12.1.

3. For the tree selected, the fundamental cut-sets are

$$C_3 = \{1, 3, 6\}$$
$$C_2 = \{2, 5, 6, 7\}$$
$$C_1 = \{3, 4, 5, 7\}$$

Fig. 1.12.2.

4. Hence, f-cut set matrix

$$= \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ \uparrow \\ \text{Cut-set} \end{array} \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \leftarrow \text{Branch} \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 & 1 & -1 \\ -1 & 0 & -1 & 0 & 0 & -1 & 0 \end{bmatrix} \\ \\ \\ \end{array}$$

**Que 1.13.** **For the network shown in Fig. 1.13.1 draw the directed graph. And also find number of possible trees.**



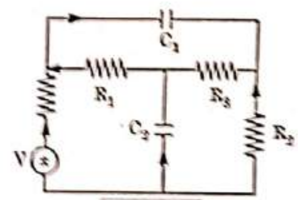


Fig. 1.13.1.

**AKTU 2017-18, Marks 10**

**Answer**

1. Graph :

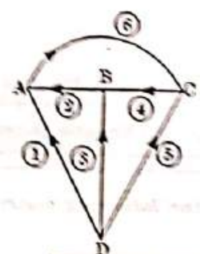


Fig. 1.13.2.

2. The incidence matrix A is

$$= \begin{array}{c} \\ A \\ B \\ C \\ D \\ \uparrow \\ \text{Nodes} \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \leftarrow \text{Branch} \end{array} \\ \begin{bmatrix} -1 & -1 & 0 & 0 & 0 & 1 \\ 0 & 1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & -1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \\ \\ \\ \end{array}$$

3. Reduced incidence matrix $[A_r]$ is

$$= \begin{bmatrix} -1 & -1 & 0 & 0 & 0 & 1 \\ 0 & 1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & -1 \end{bmatrix}$$

4. Number of possible trees = det$([A_r] \times [A_r^T])$

$$= \det\left\{\begin{bmatrix} -1 & -1 & 0 & 0 & 0 & 1 \\ 0 & 1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & -1 \end{bmatrix} \times \begin{bmatrix} -1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}\right\}$$

$$= \begin{vmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{vmatrix}$$

$$= 3(9-1) + 1(-3-1) - 1(1+3) = 16$$

## PART-3

*Cut-set Matrix, Tie-set Matrix, Duality, Loop and Nodal Method of Analysis.*

### Questions-Answers

### Long Answer Type and Medium Answer Type Questions

**Que 1.14.** **Explain the following matrices taking a suitable example :**

**i. Reduced incidence matrix**

**ii. Basic cut-set matrix**

**iii. Basic tie-set matrix**

**AKTU 2014-15, Marks 10**

**OR**

**Define with suitable example :**

**i. Incidence matrix.**

**ii. Cut-set matrix.**

**AKTU 2018-19, Marks 07**

**Answer**

**i. Reduced incidence matrix :** Refer Q. 1.11, Page 1–11C, Unit-1.

**ii. Basic cut-set matrix or cut-set matrix :**

1. A cut-set is a minimum set of branches of a connected graph such that when removed these branches from the graph, then the graph get separated into 2 distinct parts called sub-graphs.
2. And the cut set matrix is the matrix which is obtained by row-wise taking one cut-set at a time. The cutset matrix is denoted by symbol $[Q_f]$.



**Example of cut-sets matrix of a circuit :**

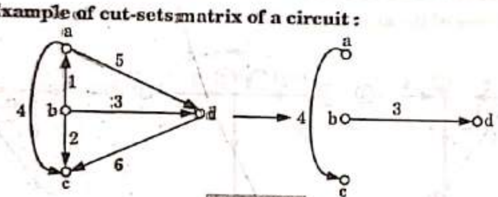



**Fig. 1.14.1.**

1. Two sub-graphs are obtained from a graph by selecting cut-sets consisting of branches [1, 2, 5, 6].
2. Thus, in other words we can say that fundamental cut set of a given graph with reference to a tree is a cut-set formed with one twig and remaining links. Twigs are the branches of tree and links are the branches of co-tree.
3. Thus, the number of cutset is equal to the number of twigs.
   Number of twigs = $N - 1$
   where, $N$ is the number of nodes of the given graph or drawn tree.
4. The orientation of cut-set is the same as that of twig and that is taken positive.

**iii. Basic tie-set matrix :**

1. A tie-set is a set of branches contained in a loop such that each loop contains one link or chord and the remainder are tree branches.
2. For a given graph having '$N$' nodes and '$B$' branches, tie-set matrix is a rectangular matrix with '$b$' columns and as many rows as there are loops.
3. Its elements have the following values :

$$B_{ij} = 1, \text{ if branch } j \text{ is in loop } i \text{ and their orientations coincides;}$$
$$= -1, \text{ if branch } j \text{ is in loop } i \text{ and their orientations do not coincide;}$$
$$= 0, \text{ if branch } j \text{ is not in loop } i.$$

4. Number of $f$-tie-sets is equal to the number of links.
5. Let us consider a graph and a tree having twigs [2, 4, 5] as shown in Fig. 1.14.2.

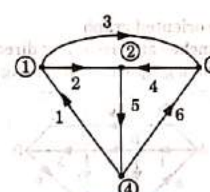



**Fig. 1.14.2.**

**Trees of the graph :**

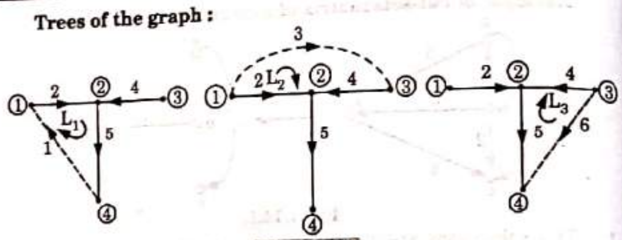


Fig. 1.14.3.

$$\begin{array}{r} \\ L_1 \\ \text{Tie-set matrix is} : L_2 \\ L_3 \end{array}\begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & 1 \end{bmatrix} \end{array}$$

iv. **Incidence matrix :** Refer Q. 1.9, Page 1–10C, Unit-1.

**Que 1.15.** **Draw a graph of resistive network in Fig. 1.15.1. Select a suitable tree and obtain the tie-set matrix.**

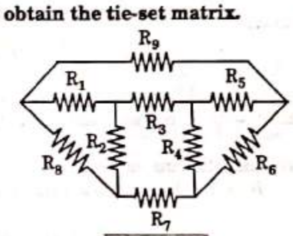



Fig. 1.15.1.

**AKTU 2018-19, Marks 07**

**Answer**

1. **Step-1 :** To draw the oriented graph.
Mark the nodes, branches and assuming directions of branch current arbitrarily. The oriented graph is shown in Fig. 1.15.2.
**Graph theory :**

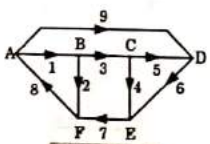


Fig. 1.15.2.



2. **Step-2 :** To select any one tree.
The selected tree is shown in Fig. 1.15.3.

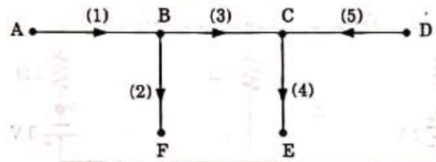


Fig. 1.15.3.

3. **Step-3 :** To obtain tie-sets and tie-set matrix.
The no. of $f$-tiesets = No. of links = $b - n + 1$
$= 9 - 6 + 1$
$= 4$

4. The $f$-tiesets are :
$f$-tieset – 6 : [4, 5, 6]
$f$-tieset – 7 : [2, 3, 4, 7]
$f$-tieset – 8 : [1, 2, 8]
$f$-tieset – 9 : [1, 3, 5, 9]

5. The $f$-tiesets are shown in Fig. 1.15.4.

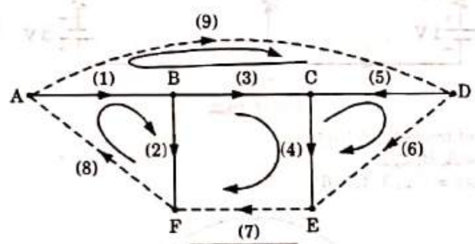


Fig. 1.15.4.

The orientations of $f$-tieset are the orientation of the corresponding links.

6. The $f$-tieset schedule is given by

$B_f =$

| $f$-tiesets ↓ \ Branches → | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|---|
| 6 | 0 | 0 | 0 | –1 | –1 | +1 | 0 | 0 | 0 |
| 7 | 0 | –1 | +0 | +1 | 0 | +1 | +1 | 0 | 0 |
| 8 | +1 | +1 | 0 | 0 | 0 | 0 | 0 | +1 | 0 |
| 9 | –1 | 0 | –1 | 0 | –1 | 0 | 0 | 0 | +1 |

**Que 1.16.** **Obtain cut-set matrix for following electrical network.**

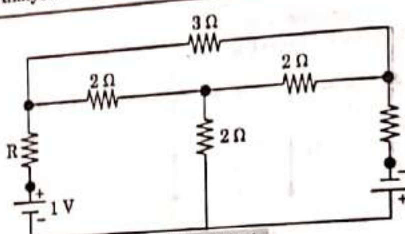



**Fig. 1.16.1.**

**AKTU 2016-17, Marks 10**

**Answer**

Assuming value of resistance $R$ is $1\ \Omega$.

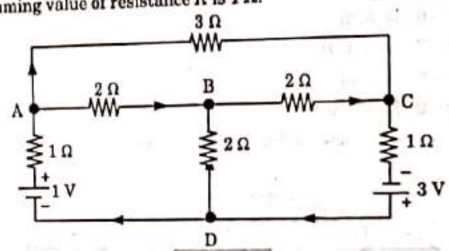



**Fig. 1.16.2.**

1. Graph of network is drawn as :
   Nodes = A, B, C, D
   Branches = 1, 2, 3, 4, 5, 6

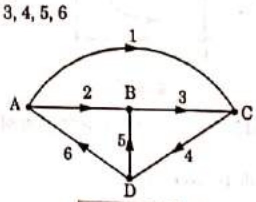



**Fig. 1.16.3.**

2. Selected tree is


**Fig. 1.16.4.**



3. **Fundamental cut-set :**
   Cut-set – 1 : [1, 2, 6]
   Cut-set – 2 : [3, 1, 4]
   Cut-set – 3 : [6, 5, 4]

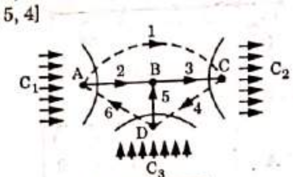



**Fig. 1.16.5.**

Cut-sets ↓ Branches ⟶

$$Q = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \end{array}\begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 \end{bmatrix} \end{array}$$

**Que 1.17.** **The reduced incidence matrix is,**

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ -1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

**Do the following :**

i. **Draw the graph.**
ii. **How many trees are possible ?**
iii. **Write tie-set and cut-set matrix.**

**AKTU 2015-16, Marks 10**

**Answer**

Given,
Reduced incidence matrix :

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ -1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

Complete incidence matrix :

$$A_i = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \leftarrow \text{Branch} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ -1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

↑
node

i.

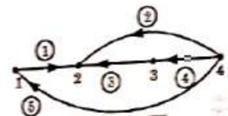


Fig. 1.17.1.

ii. Number of trees = det $[A.A^T]$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ -1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & -1 & 0 \\ 0 & -1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix}$$

$$AA^T = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ -1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 0 & -1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} (1+1) & -1 & 0 \\ -1 & \begin{pmatrix} 1+1 \\ +1 \end{pmatrix} & -1 \\ 0 & -1 & (1+1) \end{bmatrix} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

$$\text{Number of trees} = \det \begin{bmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

$$= 2(6-1) + 1(-2+0) = (2 \times 5) - 2 = 10 - 2 = 8$$

Tree [1, 2, 3, 4] is selected.

iii. **For tie-set and cut-set matrix :**

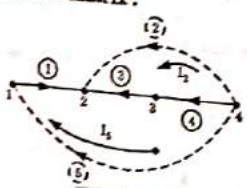


Fig. 1.17.2.



Tree [1, 2, 3, 4] is selected.

$$\text{Tie-set matrix} = \begin{array}{c} \\ I_2 \\ I_5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \leftarrow \text{Branch} \\ \begin{bmatrix} 0 & 1 & -1 & -1 & 0 \\ 1 & 0 & -1 & -1 & 1 \end{bmatrix} \end{array}$$

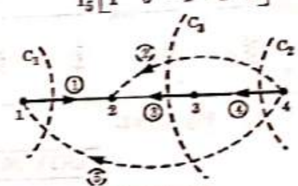


Fig. 1.17.3.

$$\text{Cut-set matrix} = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ \uparrow \\ \text{cut-set} \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \leftarrow \text{Branch} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \end{bmatrix} \\ \\ \\ \end{array}$$

**Que 1.18. What do you understand by the term 'duality' ? Discuss the procedure to find out the dual of a given network having both voltage and current sources.**

**Answer**

A. **Duality :**

1. Duality is a transformation in which currents and voltages are interchanged. Two phenomena are said to be dual if they are described by equations of the same mathematical form.
2. There are a number of similarities and analogies between the two circuit analysis techniques based on loop-current method and node voltage method.
3. The principle quantities and concepts involved in these two methods based on KVL and KCL are dual of each other with voltage variables substituted by current variables, independent loop by independent node-pair, etc. This similarity is termed as 'principle of duality'.

B. **Dual quantities and concepts :**

| S. No. | Quantities | Dual |
|---|---|---|
| 1. | Current | Voltage |
| 2. | Resistance | Conductance |
| 3. | Inductance | Capacitance |
| 4. | Impedance | Admittance |
| 5. | Reactance | Susceptance |

**Que 1.19.** **Explain the concept of duality. Find the dual of the network shown in Fig. 1.19.1.**

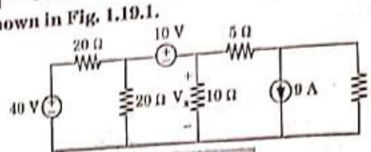



**Fig. 1.19.1.**

**AKTU 2014-15, Marks 10**

**Answer**

A. **Duality :** Refer Q. 1.18, Page 1–22C, Unit-1.

B. **Numerical :**

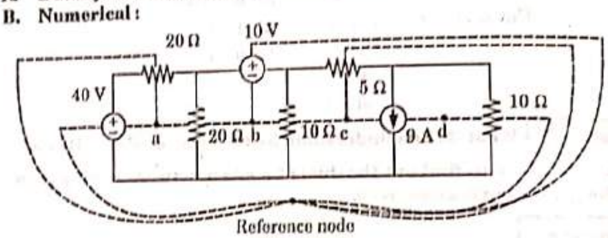



**Fig. 1.19.2.**

1. **Step-1 :** A node is placed inside each loop. In given Fig. 1.19.2, there are four loops.
2. **Step-2 :** An extra node is placed outside the network.
3. **Step-3 :** All the nodes are joined through element of original network, transversing only one element at a time.
4. **Step-4 :** For each element transversed in original network, dual element is connected.
5. **Dual network is as follows :**

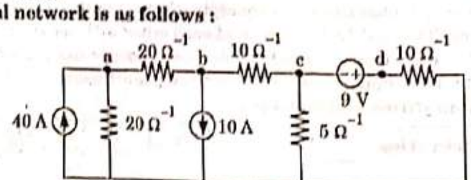



**Fig. 1.19.3.**

**Que 1.20.** **Obtain the dual network of the network shown in Fig. 1.20.1.**



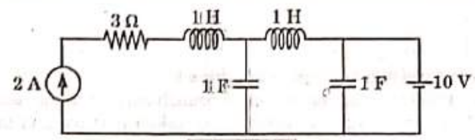



**Fig. 1.20.1.**

**AKTU 2018-19, Marks 07**

**Answer**

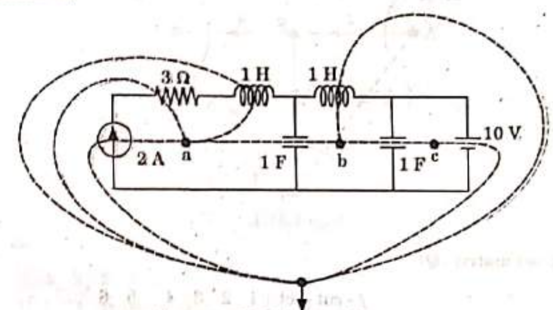



**Fig. 1.20.2.**

A. **Procedure**

**Step-1 :** A node is placed inside each loop. In given Fig. 1.20.2, there are three loops.

**Step-2 :** An extra node is placed outside the network.

**Step-3 :** All the nodes are joined through element of original network, transversing only one element at a time.

**Step-4 :** For each element transversed in original network, dual element is connected.

B. **Dual network is as follows :**

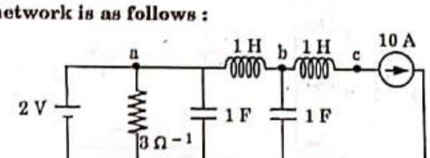



**Fig. 1.20.3.**

**Que 1.21.** **Derive the KCL and KVL using network graph variables.**

**AKTU 2014-15, Marks 10**

**Answer**

**A. KCL using network graph variables :**

1. This law states that algebraic sum of branch current at any node must be equal to zero, outgoing currents are taken positive and incoming currents are taken negative.
2. Consider the tree (Fig. 1.21.1) corresponding to twigs [2, 3, 4]. The number of fundamental cut-set = $n - 1 = 4 - 1 = 3$ of a graph :

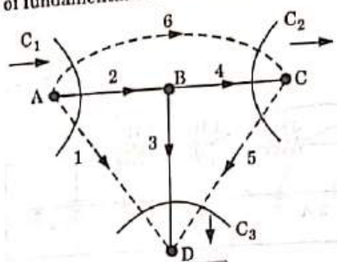



**Fig. 1.21.1.**

3. Cut-set matrix ($Q$) :

| $f$-cut-set | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 0 | 0 | 0 | 1 |
| 2 | 0 | 0 | 0 | 1 | –1 | 1 |
| 3 | 1 | 0 | 1 | 0 | 1 | 0 |

Branches

Node $A$ : $i_{b1} + i_{b2} + i_{b6} = 0$

Node $C$ : $i_{b4} - i_{b5} + i_{b6} = 0$

Node $D$ : $i_{b1} + i_{b3} + i_{b5} = 0$

4. In matrix form they can be written as

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_{b1} \\ i_{b2} \\ i_{b3} \\ i_{b4} \\ i_{b5} \\ i_{b6} \end{bmatrix} = 0$$

or $\quad QI_b = 0$

**B. KVL using network graph variables :**

1. It states that the algebraic sum of voltage drops in any set of branches forming a closed circuit or loop must be equal to zero.



2. While we move along the closed path along the current in any branch, this results a voltage drop and for this we use positive sign.
3. If we move along the closed path in opposite direction of current, this results a voltage rise and to assume voltage drop we use negative sign.

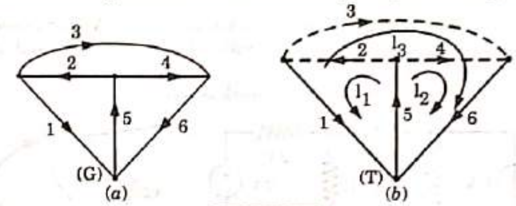



**Fig. 1.21.2.**

3. Let $T$ be the tree of graph $G$ as shown in Fig. 1.21.2($b$). The number of fundamental loop = $(b - n + 1) = 3$.
4. Apply KVL for loop $l_1, l_2, l_3$

$l_1$ : $\quad V_1 + V_2 + V_5 = 0$

$l_2$ : $\quad V_4 + V_5 + V_6 = 0$

$l_3$ : $\quad -V_1 + V_3 + V_6 = 0$

5. In matrix form

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ -1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \end{array} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$B_f \cdot V_b = 0$$

where $\quad B_f$ = fundamental loop matrix.

**Que 1.22.** **Determine the currents in all the branches of the network shown in Fig. 1.22.1 using node analysis method of the graph theory.**

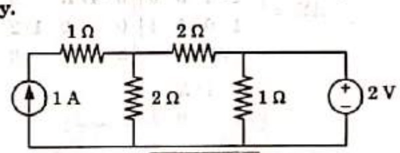



**Fig. 1.22.1.**

**AKTU 2017-18, Marks 07**

**Answer**

1. Here, the 1 Ω resistance in parallel with 2 V voltage source can be ignored.
2. Also there is no passive element in parallel with 1 A current source. We assume resistance $R$ in parallel with 1 A current source and finally let $R \to \infty$.
3. Therefore, the graph of the network is shown in Fig. 1.22.2.

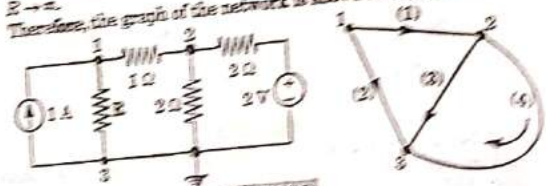



Fig. 1.22.2.

4. The complete incidence matrix is

$$A_a = \begin{bmatrix} 1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 1 \\ 0 & -1 & -1 & -1 \end{bmatrix}$$

5. Reduced incidence matrix is

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 1 \end{bmatrix}$$

6. Branch admittance matrix is

$$Y_b = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/R & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1/2 \end{bmatrix}$$

$$\therefore \quad AY_b = \begin{bmatrix} 1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/R & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1/2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1/R & 0 & 0 \\ -1 & 0 & 1/2 & 1/2 \end{bmatrix}$$

$$\therefore \quad AY_bA^T = \begin{bmatrix} 1 & 1/R & 0 & 0 \\ -1 & 0 & 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$$



$$= \begin{bmatrix} (1+1/R) & -1 \\ -1 & 2 \end{bmatrix}$$

7. Now,

$$AY_bV_s - AI_s = \begin{bmatrix} 1 & 1/R & 0 & 0 \\ -1 & 0 & 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 2 \end{bmatrix} - \begin{bmatrix} 1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 1 \end{bmatrix} - \begin{bmatrix} -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

8. The node equations are

$$\begin{bmatrix} (1+1/R) & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad ...(1.22.1)$$

9. With $R \to \infty$, the eq. (1.22.1) become

$$V_1 - V_2 = 1 \quad ...(1.22.2)$$

$$-V_1 + 2V_2 = 1 \quad ...(1.22.3)$$

10. Solving eq. (1.22.2) and (1.22.3), we get

$$V_1 = 3\text{ V}, V_2 = 2\text{ V}$$

11. Hence, the current in different branches are shown in Fig. 1.22.3.

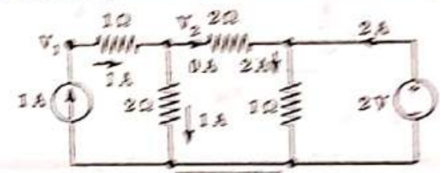



Fig. 1.22.3.

**VERY IMPORTANT QUESTIONS**

***Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.***

**Q. 1. Define the following terms related to a network graph.**

i. Network graph
ii. Oriented graph
iii. Subgraph
iv. Branch
v. Node
vi. Degree of node

vii. Path
viii. Loop
ix. Tree
x. Twigs
xi. Co-tree
xii. Link
xiii. f-cut-set

**Ans.** Refer Q. 1.5.

**Q. 2. Describe the planar and non-planar graphs.**

**Ans.** Refer Q. 1.6.

**Q. 3. For given reduced incidence matrix, draw the graph and hence obtain the f-cutset matrix**

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 & 1 & 1 \\ -1 & 0 & 1 & 0 & 0 & -1 & 0 \end{bmatrix}$$

**Ans.** Refer Q. 1.12.

**Q. 4. For the network shown in Fig. 1 draw the directed graph. And also find number of possible tree.**

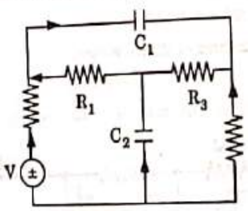



**Fig. 1.**

**Ans.** Refer Q. 1.13.

**Q. 5. Explain the following matrices taking a suitable example:**

i. **Reduced incidence matrix**
ii. **Basic cut-set matrix**
iii. **Basic tie-set matrix**

**Ans.** Refer Q. 1.14.

**Q. 6. Draw a graph of resistive network in Fig. 2. Select a suitable tree and obtain the tie-set matrix.**



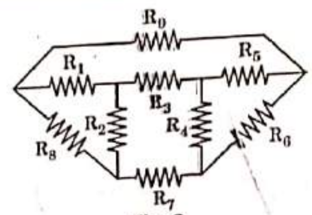



**Fig. 2.**

**Ans.** Refer Q. 1.15.

**Q. 7. Obtain cut-set matrix for following electrical network.**

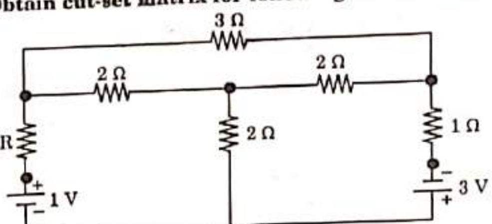



**Fig. 3.**

**Ans.** Refer Q. 1.16.

**Q. 8. The reduced incidence matrix is,**

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ -1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

**Do the following :**

i. **Draw the graph.**
ii. **How many trees are possible ?**
iii. **Write tie-set and cut-set matrix.**

**Ans.** Refer Q. 1.17.

**Q. 9. What do you understand by the term 'duality' ? Discuss the procedure to find out the dual of a given network having both voltage and current sources.**

**Ans.** Refer Q. 1.18.

☺☺☺

# 2 UNIT

# AC Network Theorem

## CONTENTS







## PART-1

*Pre-Requisites : Concepts of DC Network Theorems, Electrical Sources.*

### Questions-Answers

### Long Answer Type and Medium Answer Type Questions

**Que 2.1.** **Explain superposition theorem.**

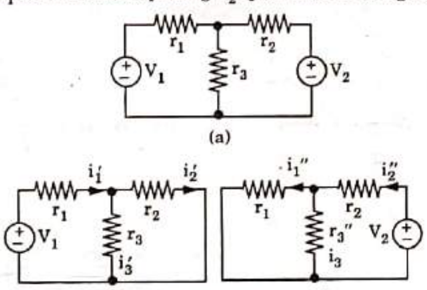
**Answer**

**A. Statement :**

If a number of voltage or current sources are acting simultaneously in a linear network, the resultant current in any branch is the algebraic sum of the current that would be produced in it, when each source acts alone replacing all other independent sources by their internal resistances.

**B. Explanation :**

1. In Fig. 2.1.1(*a*), to apply superposition theorem, let us first take the source $V_1$ alone at first replacing $V_2$ by short circuit [Fig. 2.1.1(*b*)].




(b) **Fig. 2.1.1.** (c)

2. Here,
$$i_1' = \frac{V_1}{\dfrac{r_2 r_3}{r_2 + r_3} + r_1}$$

$$i_2' = i_1' \frac{r_3}{r_2 + r_3} \text{ and } i_3' = i_1' - i_2'$$

3. Next, replacing $V_1$ by short circuit. Let the circuit be energized by $V_2$ only [Fig. 2.1.1(c)]

Here,
$$i_2'' = \frac{V_2}{\frac{r_1 r_3}{r_1 + r_3} + r_2} \text{ and } i_1'' = i_2'' \frac{r_3}{r_1 + r_3}$$

Also,
$$i_3'' = i_2'' - i_1''$$

4. As per superposition theorem,
$$i_3 = i_3' + i_3''$$
$$i_2 = i_2' - i_2''$$
$$i_1 = i_1' - i_1''$$

**Que 2.2.** **Discuss Thevenin's theorem.**

**Answer**

A. **Statement :**

1. A linear active bilateral network can be replaced at any two of its terminals by an equivalent voltage source (Thevenin's voltage source), $V_{oc}$, in series with an equivalent impedance (Thevenin's impedance), $R_{TH}$.
2. Here, $V_{oc}$ is the open circuit voltage between the two terminals under the action of all sources and initial conditions, and $R_{TH}$ is the impedance obtained across the terminals with all sources removed by their internal impedance and initial conditions reduced to zero.

B. **Explanation :**

1. Let us consider a simple DC circuit as shown in Fig. 2.2.1($a$). We are to find $I_L$ by Thevenin's theorem.

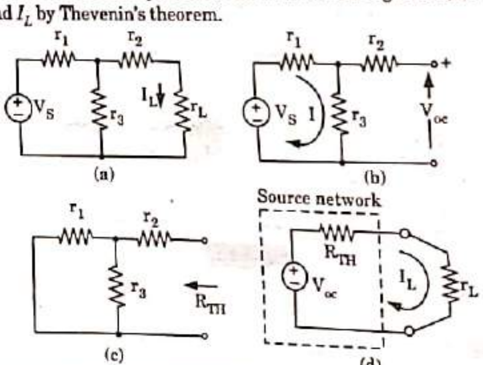



**Fig. 2.2.1.** (a) A simple DC circuit, (b) Finding of $V_{oc}$ (c) Finding of $R_{TH}$ (d) Finding of $I_L$, forming Thevenin's equivalent circuit.



2. In order to find the equivalent voltage source, $r_L$ is removed [Fig. 2.2.1($b$)] and $V_{oc}$ is calculated
$$V_{oc} = I\,r_3 = \frac{V_S}{r_1 + r_3}\, r_3$$
3. Now, to find the internal resistance of the network (Thevenin's resistance or equipment resistance) in series with $V_{oc}$, the voltage source is removed by a short circuit as shown in Fig, 2.2.1($c$)
$$R_{TH} = r_2 + \frac{r_1 r_3}{r_1 + r_3}$$
4. As per Thevenin's theorem, the equivalent circuit is shown in Fig. 2.2.1($d$).
$$I_L = \frac{V_{oc}}{R_{TH} + r_L}$$

**Que 2.3.** **Explain Norton's theorem.**

**Answer**

A. **Statement :**

1. A linear active bilateral network can be replaced at any two of its terminals, by an equivalent current source (Norton's current source), $i_{sc}$, in parallel with a resistance (or internal resistance), $R_{int}$.
2. Here, $i_{sc}$ is the short-circuit current flowing from one terminal to the other under the action of all sources and initial conditions, and $R_{int}$ obtained across the terminals with all sources removed by their internal impedance and initial conditions reduced to zero.

B. **Explanation :**

1. In order to find the current through $r_L$, (Fig. 2.3.1), replace $r_L$ by short circuit [Fig. 2.3.2].

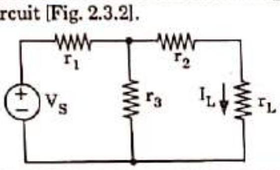



**Fig. 2.3.1.** A simple DC network.

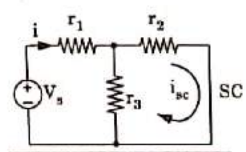


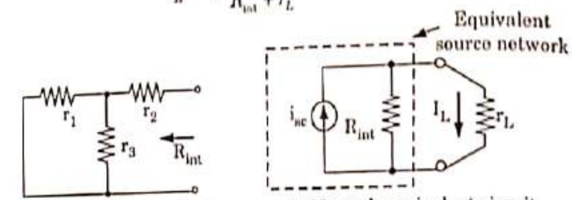

**Fig. 2.3.2.** Finding of $i_{sc}$.

$$i = \frac{V_S}{r_1 + \frac{r_2 r_3}{r_2 + r_3}} \text{ and } i_{sc} = i\,\frac{r_3}{r_3 + r_2}$$

2. Now, the short circuit is removed and the independent source is deactivated [Fig. 2.3.3($a$)].
$$R_{int} = r_2 + \frac{r_1 r_3}{r_1 + r_3}.$$

3. As per Norton's theorem, the equivalent circuit is drawn in Fig. 2.3.3(*b*).

$$I_L = i_{sc}\,\frac{R_{int}}{R_{int} + r_L}$$




(a) Finding of $R_{TH}$ (or $R_{int}$). (b) Norton's equivalent circuit.

**Fig. 2.3.3.**

**Que 2.4.** **State and prove maximum power transfer theorem.**

**Answer**

A. **Statement :** It states that maximum power is transferred from the source to the load when load resistance ($R_L$) equals the internal resistance ($R_{th}$) of the network.
For maximum power $R_L = R_{th}$

B. **Proof :**

1. Consider a linear network whose Thevenin's equivalent is given as shown in Fig. 2.4.1 :

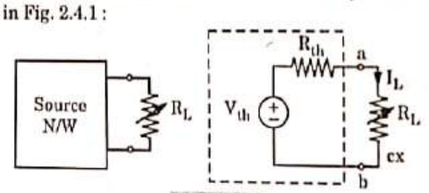



**Fig. 2.4.1.**

2. Current in resistance $R_L$ is given by :

$$I_L = \frac{V_{th}}{R_{th} + R_L}$$

3. Now the power drawn by $R_L$ will be :

$$P = I_L^2 R_L = \frac{V_{th}^2}{(R_{th} + R_L)^2}\,.\,R_L$$

4. Condition for maximum power can be found out by equating,

$$\frac{dP}{dR_L} = 0$$

$$\therefore \quad \frac{d}{dR_L}\left(\frac{V_{th}^2}{(R_{th} + R_L)^2}\,.\,R_L\right) = 0$$



$$\frac{(R_{th} + R_L)^2\,.\,V_{th}^2 - V_{th}^2 R_L\,.\,2(R_{th} + R_L)}{(R_{th} + R_L)^4} = 0$$

$$(R_{th} + R_L)\,V_{th}^2 - 2V_{th}^2 R_L = 0$$

$$(R_{th} + R_L) - 2R_L = 0$$

$$\therefore \quad R_{th} = R_L$$

5. Expression for maximum efficiency :

$$P = \frac{V_{th}^2}{(R_{th} + R_L)^2}\,.\,R_L \qquad \text{...(2.4.1)}$$

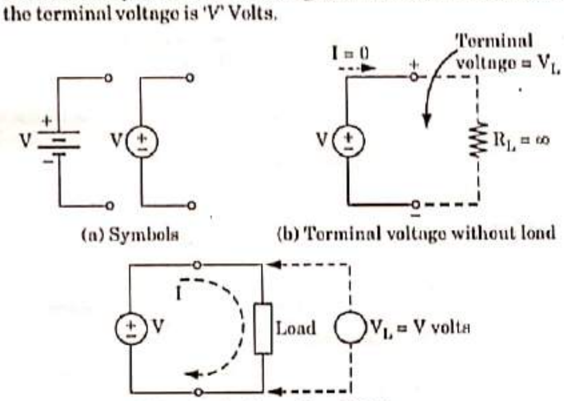
Substituting $R_{th} = R_L$ in the eq. (2.4.1)

$$P_{max} = \frac{V_{th}^2}{(2R_L)^2}\,.\,R_L = \frac{V_{th}^2}{4R_L}$$

**Que 2.5.** **Discuss the ideal voltage source.**

**Answer**

1. An ideal voltage source is defined as a voltage source that maintains a constant voltage across the load irrespective of the load current.
2. The circuit symbol for an ideal voltage source is as shown in Fig. 2.5.1(a). Note that the polarities of the voltage are indicated inside the circle and the terminal voltage is 'V' Volts.




(c) Terminal voltage $V_L$ = V Volts

**Fig. 2.5.1.**

3. The source resistance of an ideal voltage source is zero. Therefore the terminal voltage remains constant equal to 'V' Volts without load ($R_L = \infty$) or with load as shown in Fig. 2.5.1(b) and (c).

4. Thus for an ideal voltage source the terminal voltage is independ load. This is as shown in Fig. 2.5.2.
5. Without load, the current is zero as shown in Fig. 2.5.1(b), but whe load is connected, the load current I will flow from the positive end source, through load back to the negative end as shown in Fig. 2.5. This is the conventional current flow.

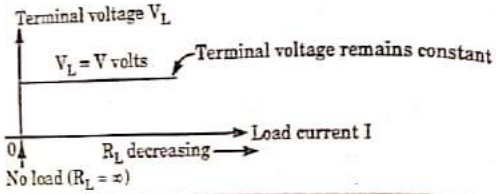



Fig. 2.5.2. Terminal voltage of an ideal voltage source.

**Que 2.6.** **Explain the ideal current source.**

**Answer**

1. An ideal current source is defined as the current source which abs or delivers a constant current irrespective of the load.
2. The symbol of an ideal current source is shown in Fig. 2.6.1(a).
3. The arrow indicates the direction of conventional current flow and internal shunt resistance is infinite for the ideal current sou ($R_{sh} = \infty$).
4. When the load resistance is connected between the output termin shown in Fig. 2.6.1(b), a constant current $I$ flows through the lo $I_L = I$ and the voltage appears across the load, $V_L = I R_L$.

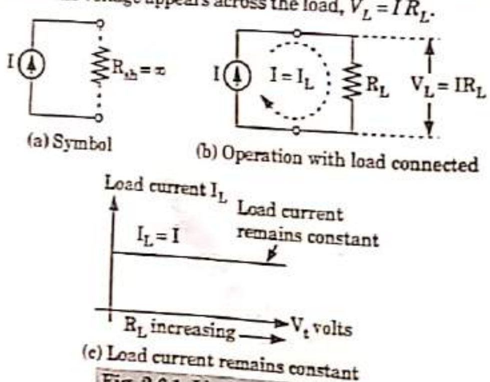



Fig. 2.6.1. Ideal current source.



5. Fig. 2.6.1(c) suggests that irrespective of change in the load, the current remains constant but the terminal voltage will change.
6. The current sources are not available directly but they are derived from the voltage sources. Such a current source contains a voltage source along with some other components such as inductance.

**Que 2.7.** **Find the current $I_o$ using source transformation in Fig. 2.7.1.**

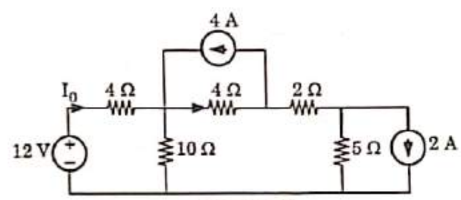



Fig. 2.7.1.

**AKTU 2014-15, Marks 05**

**Answer**

Using source transformation :

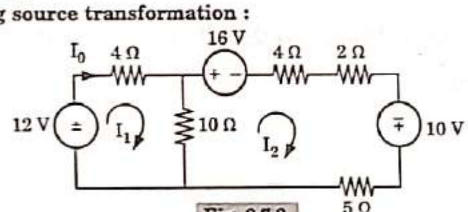



Fig. 2.7.2.

1. Using KVL in loop (1)

$$-12 + 4I_1 + 10I_1 - 10I_2 = 0$$

$$\therefore \quad 14I_1 - 10I_2 = 12 \quad \text{....(2.7.1)}$$

2. Again, using KVL in loop (2)

$$16 + 6I_2 - 10 + 5I_2 + 10I_2 - 10I_1 = 0$$

$$10I_1 - 21I_2 = 6 \quad \text{...(2.7.2)}$$

3. From eq. (2.7.1) and (2.7.2), we get $I_1 = I_o = 0.9896$ A

**PART-2**

*Basic Circuital Law.*

## Questions-Answers

### Long Answer Type and Medium Answer Type Questions

**Que 2.8.** **Explain the basic circuital law with the help of example.**

**OR**

**Find the voltage $V_o$ in Fig. 2.8.1.**

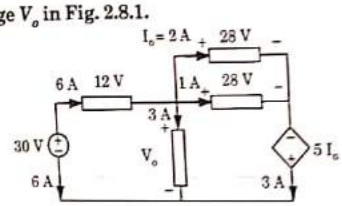



**Fig. 2.8.1.**

**AKTU 2014-15, Marks 05**

**Answer**

**A. Basic circuital law :** Refer Q. 1.1, Page 1–2C, Unit-1.

**B. Numerical :**

**Using KVL :**

$$-30 + 12 + V_0 = 0$$
$$V_0 = 18 \text{ V}$$

## PART-3

*Super Position Theorem, Thevenin's Norton's Theorem, Maximum Power Transfer Theorem.*

## Questions-Answers

### Long Answer Type and Medium Answer Type Questions

**Que 2.9.** **State and explain superposition theorem and also give the advantage and limitation.**

**Answer**

**A. State and proof :** Refer Q. 2.1, Page 2–2C, Unit-2.



**B. Advantages :**

1. This theorem is valid for all types of linear circuits having time varying or time-invariant elements.
2. This theorem is used to find the current or voltage in a branch when the circuit has a large number of independent sources.

**C. Limitations :**

1. Not applicable for the network containing non-linear elements or unilateral elements.
2. Not applicable for the non-linear parameters such as power.

**Que 2.10.** **Using superposition theorem, find the voltage across $(4 + j3)\ \Omega$ in the network shown in Fig. 2.10.1.**

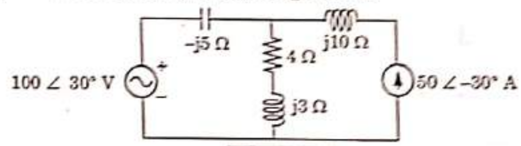



**Fig. 2.10.1.**

**AKTU 2018-19, Marks 07**

**Answer**

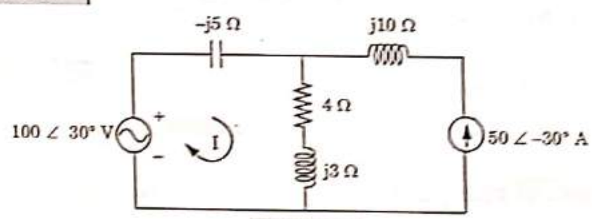



**Fig. 2.10.2.**

**Step 1 :** When the $100 \angle 30°$ V source is acting alone in Fig. 2.10.2.

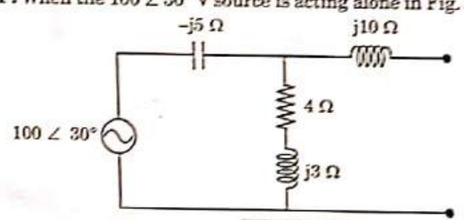



**Fig. 2.10.3.**

$$I = \frac{100 \angle 30^\circ}{-j5 + 4 + j3}$$

$$= \frac{100 \angle 30^\circ}{4 - j2} = \frac{100 \angle 30^\circ}{4.47 \angle -26^\circ} = 22.37 \angle 56^\circ \text{ Amp}$$

Voltage across $(4 + j3)\,\Omega$

$$V' = (4 + j3) \times (22.37 \angle 56^\circ)$$
$$= 5 \angle 36.86^\circ \times 22.37 \angle 56^\circ$$
$$= 111.35 \angle 92.86^\circ \text{ V}$$

**Step 2 :** When the $50 \angle -30^\circ$ A current source is acting alone, Fig. 2.10.3.

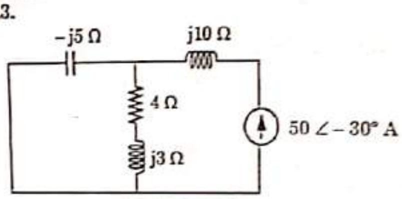



**Fig. 2.10.4.**

By current division rule

$$I = 50 \angle -30^\circ \times \frac{(-j5)}{(4 - j2)}$$
$$= 50 \angle -30^\circ \times 1.11 \angle -63.43^\circ$$
$$= 55.5 \angle -93.43^\circ \text{ Amp}$$

Voltage across $(4 + j3)\,\Omega$

$$V'' = (4 + j3) \times (55.5 \angle -93.43^\circ)$$
$$= 5 \angle 36.86^\circ \times 55.5 \angle -93.43^\circ = 277.5 \angle -56.57^\circ$$

**Step 3 :** By superposition theorem

$$V = V' + V''$$
$$= 111.35 \angle 92.86^\circ + 277.5 \angle -56.57^\circ$$
$$= 190.25 \angle -39.25^\circ \text{ V}$$

**Que 2.11.** **Find $i_o$ in the circuit in Fig. 2.11.1 using superposition theorem.**

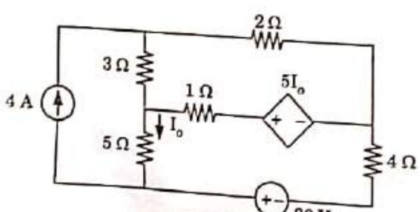



**Fig. 2.11.1.**

**AKTU 2016-17, Marks 10**



**Answer**

1. Current source *i.e.*, 4 A is open circuited.

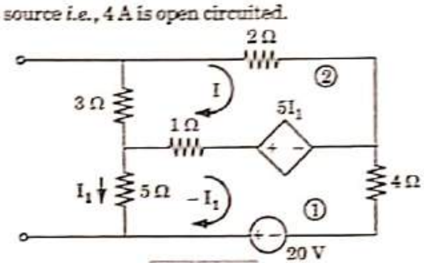



**Fig. 2.11.2.**

2. Using KVL in loop 1, $-20 - 5I_1 - I_1 + 5I_1 - 4I_1 - I = 0$
$$-5I_1 - I = 20$$
$$\therefore \quad 5I_1 + I = -20 \quad \text{...(2.11.1)}$$
3. Using KVL in loop 2, $6I + I_1 - 5I_1 = 0$
$$6I = 4I_1$$
$$I = \frac{4}{6} I_1 = \frac{2}{3} I_1 \quad \text{...(2.11.2)}$$
4. Putting value $I$ in eq. (2.11.1), we get
$$5I_1 + \frac{2}{3} I_1 = -20$$
$$\frac{17 I_1}{3} = -20$$
$$\therefore \quad I_1 = -\frac{60}{17} \text{ A}$$
5. Voltage source *i.e.*, 20 V is short-circuited.
6. Using KCL at node A,
$$\frac{V_A - V_C}{2} + \frac{V_A - V_B}{3} - 4 = 0$$
$$\frac{5}{6} V_A - \frac{1}{2} V_C - \frac{V_B}{3} = 4 \quad \text{...(2.11.3)}$$

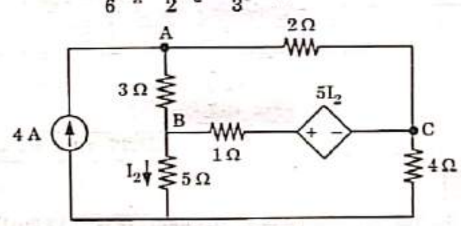



**Fig. 2.11.3.**

7. Using KCL at node $B$,

$$\frac{V_B - V_A}{3} + \frac{V_B - 5I_2 - V_C}{1} + \frac{V_B}{5} = 0$$

$$V_B\left(\frac{1}{3}+1+\frac{1}{5}\right) - 5\times\frac{V_B}{5} - V_C - \frac{V_A}{3} = 0$$

$$V_B\left(\frac{5+15+3-15}{15}\right) - V_C - \frac{V_A}{3} = 0$$

$$-\frac{V_A}{3} + \frac{8}{15}V_B - V_C = 0 \quad ...(2.11.4)$$

8. Using KCL at node $C$,

$$\frac{V_C - V_A}{2} + \frac{V_C + 5I_2 - V_B}{1} + \frac{V_C}{4} = 0$$

$$-\frac{V_A}{2} + 0\,V_B + \frac{7}{4}V_C = 0 \quad ...(2.11.5)$$

9. Solving eq. (2.11.3), (2.11.4) and (2.11.5), we get

$$V_A = 13.18\text{ V};\ V_B = 15.30\text{ V};\ V_C = 3.76\text{ V}$$

10. $$I_2 = \frac{V_B}{5} = \frac{15.30}{5} = 3.06\text{ A}$$

11. Using superposition theorem,

$$I_o = I_1 + I_2 = -\frac{60}{17} + 3.06 = -0.469\text{ A}$$

**Que 2.12.** **Determine the current in capacitor $C$ by the principle of superposition of the network shown in Fig. 2.12.1.**

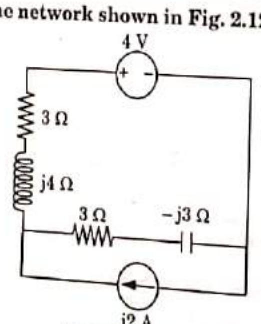



**Fig. 2.12.1.**

**AKTU 2017-18, Marks 07**

**Answer**

1. **When the voltage source is acting alone [Fig. 2.12.2(*a*)] :**
Here, the current in the capacitor branch is



$$I' = \frac{4\angle 0^\circ}{(3+j4)+(3-j4)} = \frac{2}{3}\angle 0^\circ\text{ A}$$

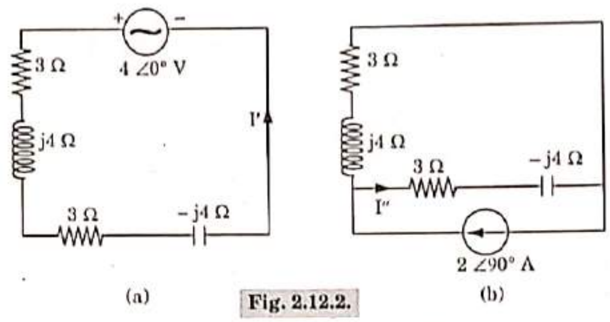



**Fig. 2.12.2.**

2. **When the current source is acting alone [Fig. 2.12.2(*b*)] :**
Here, the current in the capacitor branch is

$$I'' = 2\angle 90^\circ \times \frac{(3+j4)}{(3+j4)+(3-j4)} = \left(-\frac{4}{3}+j1\right)\text{ A}$$

3. Total current when both the sources are acting simultaneously, is

$$I = (I' + I'') = \left(\frac{2}{3} - \frac{4}{3} + j1\right) = \left(-\frac{2}{3} + j1\right)$$

$$= 1.2\angle 123.7^\circ\text{ A}$$

**Que 2.13.** **State and prove Thevenin's theorem also gives the advantage and disadvantage.**

**Answer**

A. **State and proof :** Refer Q. 2.2, Page 2–3C, Unit-2.

B. **Advantage :**
1. Thevenin's theorem useful to analyze the large distributed networks by making it into a voltage source in series with internal impedance.
2. This theorem is applicable to both AC and DC networks.
3. We can easily calculate the maximum power transferred to load.

C. **Disadvantage :** It is applicable only for linear and bilateral networks.

**Que 2.14.** **Find current through 5 Ω resistor using Thevenin's theorem.**

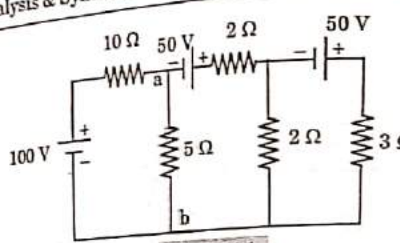



Fig. 2.14.1.

**AKTU 2017-18, Marks 10**

**Answer**

1. **Calculation of $R_{TH}$ :** (Voltage source is short circuited and 5 Ω resistor is open circuited).

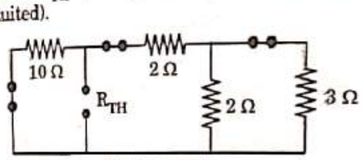



Fig. 2.14.2.

$$R_{TH} = [(2 \parallel 3) + 2] \parallel 10 = (1.2 + 2) \parallel 10$$
$$= 2.424\ \Omega$$

2. **Calculation of $V_{TH}$ :** (5 Ω resistor is open circuited).

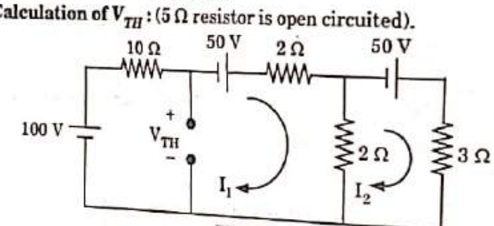



Fig. 2.14.3.

i. **Apply KVL in loop 1 :**

$$100 + 50 = (10 + 2 + 2)I_1 - 2I_2$$
$$150 = 14\,I_1 - 2I_2 \quad \text{...(2.14.1)}$$

ii. **Apply KVL in loop 2 :**

$$50 = (3 + 2)I_2 - 2I_1 = -2I_1 + 5I_2 \quad \text{...(2.14.2)}$$

iii. From eq. (2.14.1) and (2.14.2),

$$I_1 = 12.88 \text{ A and } I_2 = 15.15 \text{ A}$$

iv.

$$V_{TH} = 100 - 10\,I_1 = 100 - 10 \times 12.88 = -28.8 \text{ V}$$

3.

$$I = \frac{V_{TH}}{R_L + R_{TH}} = \frac{28.8}{5 + 2.42}$$
$$= 3.88 \text{ A (From } b \text{ to } a)$$



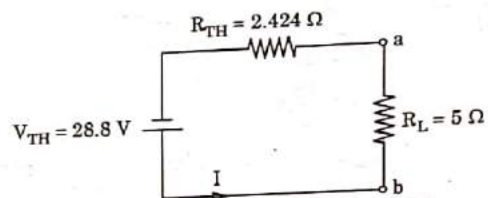



Fig. 2.14.4. Thevenin's circuit.

**Que 2.15.** **Find the current $I$ and voltage $V_{ab}$ in Fig. 2.15.1.**

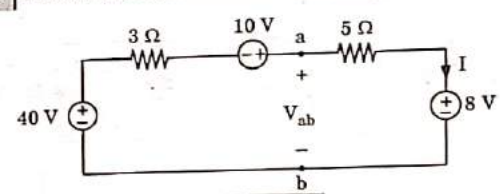



Fig. 2.15.1.

**AKTU 2014-15, Marks 05**

**Answer**

**Using Thevenin's theorem :**

**Calculation for $R_{th}$ :**

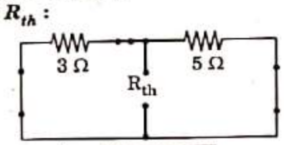



Fig. 2.15.2.

$$R_{th} = (3 \parallel 5) = \frac{3 \times 5}{3 + 5} = \frac{15}{8}\ \Omega$$

**Calculation for $V_{th}$ :**

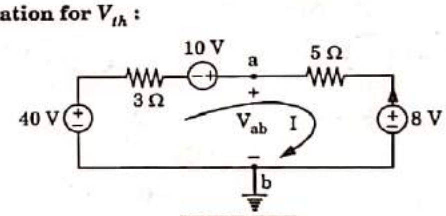



Fig. 2.15.3.

1. Using KVL in Fig. 2.15.2,
$$40 - 3I + 10 - 5I - 8 = 0$$
$$8I = 42$$
$$I = \frac{42}{8} \text{ A}$$
2. Now,
$$V_{ab} = 40 - 3I + 10 = 50 - \frac{3 \times 42}{8} = 34.25 \text{ V}$$
3. $$I = 5.25 \text{ A and } V_{ab} = 34.25 \text{ V}$$

**Que 2.16.** **Find the current $I_x$ through the 5 ohm resistor using Thevenin's theorem in Fig. 2.16.1.**

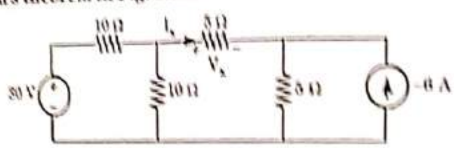


**Fig. 2.16.1.**

**AKTU 2014-15, Marks 05**

**Answer**

Using Thevenin's theorem,

**Calculation for $R_{th}$ :**

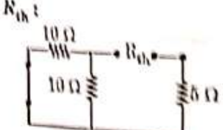


**Fig. 2.16.2.**

$$R_{th} = (10 \| 10) + 5 = \left(\frac{10 \times 10}{10 + 10}\right) + 5 = 5 + 5 = 10 \ \Omega$$

**Calculation for $V_{th}$ :**

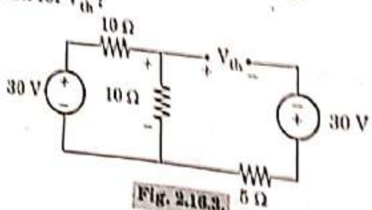


**Fig. 2.16.3.**

Voltage across 10 Ω = $30 \times \frac{10}{10 + 10} = 15$ V

Hence, $V_{th} = 30 + 15 = 45$ V



3. The equivalent Thevenin's circuit is

4. Current
$$I_x = \frac{45}{10 + 5} = 3 \text{ A}$$

**Fig. 2.16.4.**

**Que 2.17.** **Find Thevenin's equivalent circuit across *a-b* and find current through 10 Ω resistor.**

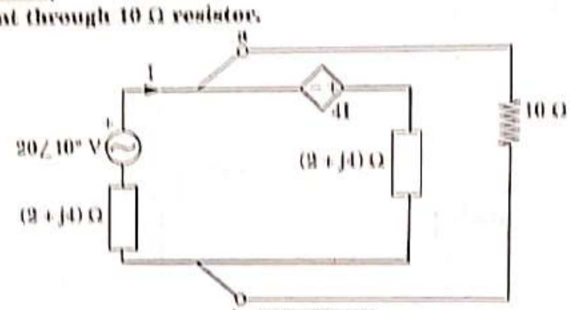


**Fig. 2.17.1.**

**AKTU 2015-16, Marks 10**

**Answer**

A. **Calculation for open circuit voltage ($V_{oc}$) :**

1. Load terminal is opened.

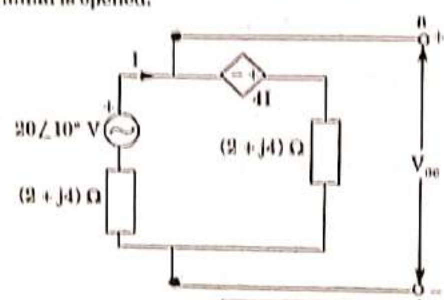


**Fig. 2.17.2.**

2. Using KVL in Fig. 2.17.2,

$$-20\angle 10^\circ - 4I + (2+j4)I + (2+j4)I = 0$$

$$I(-4+4+j8) = 20\angle 10^\circ$$

$$I = \frac{20\angle 10^\circ}{j8} = \frac{20\angle 10^\circ}{8\angle 90^\circ}\text{ A}$$

3.
$$V_{oc} = 20\angle 10^\circ - (2+j4)I$$

$$V_{oc} = 20\angle 10^\circ - (2+j4)\,\frac{20\angle 10^\circ}{8\angle 90^\circ}$$

$$V_{oc} = 20\angle 10^\circ - (2+j4)\,(2.5\angle -80^\circ)$$

$$= (19.7 + j3.47) - (2+j4)\,(0.43 - j2.462)$$

$$\therefore \quad V_{oc} = 9 + j6.67\text{ V}$$

**B. Calculation for $I_{sc}$ :**

1. Load terminal is short circuited.

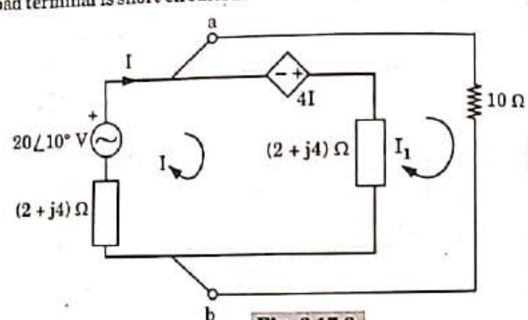



**Fig. 2.17.3.**

2. Using KVL in both the loops in Fig. 2.17.3,

$$-20\angle 10^\circ - 4I + (4+j8)I - (2+j4)$$

$$I_1 = \frac{20\angle 10^\circ + (j8)I}{(2+j4)}$$

$$\therefore \quad I_1 = \frac{20\angle 10^\circ + (j8)I}{(2+j4)}\text{ A} \qquad ...(2$$

and $-20\angle 10^\circ + (2+j4)I = 0$

$$\therefore \quad I = \frac{20\angle 10^\circ}{2+j4} = \frac{19.7 + j3.47}{2+j4} = 2.65 - j3.6\text{ A}$$

3. Putting the value of $I$ in eq. (2.17.1),

$$I_1 = \frac{(19.7 + j3.47) + (j8)(2.65 - j3.6)}{(2+j4)}$$

$$\therefore \quad I_1 = 9.78 - j7.2$$

$$\therefore \quad I_{sc} = I_1 = (9.78 - j7.2)\text{ A}$$

4.
$$Z_{th} = \frac{V_{oc}}{I_{sc}} = \frac{(9.0 + j6.67)}{(9.78 - j7.2)} = (0.27 + j0.88)\ \Omega$$



5. Thevenin's equivalent circuit is,

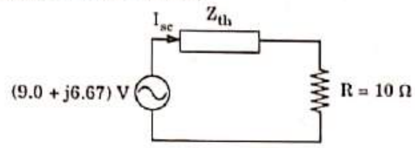



**Fig. 2.17.4.**

6.
$$I_1 = \frac{(9.0 + j6.67)}{(10.27 + j0.88)} = 0.93 + j0.57 = 1.09\angle 31.5^\circ\text{ A}$$

**Que 2.18.** **State and prove Norton's theorem also gives the limitation.**

**Answer**

**A. State and proof :** Refer Q. 2.3, Page 2–4C, Unit-2.

**B. Limitation :** Norton's theorem is not applicable to the circuits consists of unilateral elements or non-linear elements.

**Que 2.19.** **Find the Norton's equivalent of network shown in Fig. 2.19.1.**

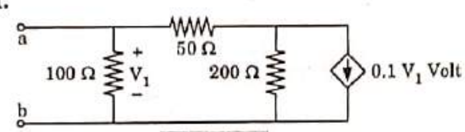



**Fig. 2.19.1.**

**AKTU 2018-19, Marks 07**

**Answer**

Circuit has only one dependent source hence $I_{SC} = I_N = 0$ amp

**For $R_N$ :** In this case a voltage source of V voltage is applied

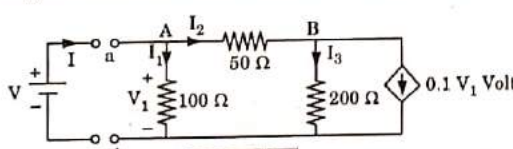



**Fig. 2.19.2.**

Applying nodal analysis at $A$ :

$$I = I_1 + I_2 = \frac{V_1}{100} + \frac{V_A - V_B}{50} \qquad ...(2.19.1)$$

Applying nodal analysis at $B$:

$$I_2 = I_3 + 0.1\, V_1$$

$$\frac{V_A - V_B}{50} = \frac{V_B}{200} + 0.1\, V_1 \quad ...(2.1$$

We know, $\quad V = V_1 = V_A \quad ...(2.1$

From (2.19.1) and (2.19.3):

$$I = \frac{V_1}{100} + \frac{V_1 - V_B}{50} \quad ...(2.1$$

From eq. (2.19.2) and (2.19.3):

$$\frac{V_1 - V_B}{50} = \frac{V_B}{200} + 0.1\, V_1 \quad ...(2.1$$

$$V_B = -\frac{16}{50} V_1$$

Put the value of $V_B$ in eq. (2.19.4)

$$I = \frac{V_1}{100} + \frac{V_1}{50} - \frac{V_B}{50} = \frac{V_1}{100} + \frac{V_1}{50} + \frac{16}{2500} V_1$$

$$I = \frac{91}{2500} V_1 \quad ...(2.1$$

Now, from eq. (2.19.6),

$$\frac{V_1}{I} = \frac{V}{I} = \frac{2500}{91} = R_N$$

$$R_N = 27.47 \text{ ohm}$$

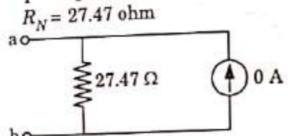



Fig. 2.19.3.

**Que 2.20.** **Obtain Norton's equivalent network at terminal a-b given Fig. 2.20.1.**

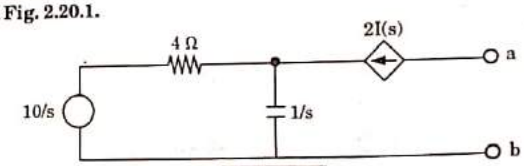



Fig. 2.20.1.

**Answer**

1. Let us apply short circuit across $a$-$b$. Let $V_x(s)$ be the voltage at node in the circuit. Then by nodal analysis,



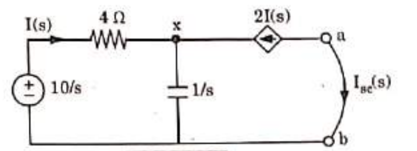



Fig. 2.20.2.

$$\frac{V_x(s)}{1/s} = 2I(s) + I(s) = 3I(s)$$

$$sV_x(s) = 3I(s)$$

$$V_x(s) = \frac{3I(s)}{s} \quad ...(2.20.1)$$

2. Also, $\quad 4 \times I(s) = \frac{10}{s} - V_x(s) = \frac{10}{s} - \frac{3I(s)}{s}$

or $\quad I(s)\left[4 + \frac{3}{s}\right] = \frac{10}{s}$

or $\quad I(s) = \frac{10/s}{4 + \frac{3}{s}} = \frac{10}{4s+3} \quad ...(2.20.2)$

3. Again inspection reveals that,

$$-2I(s) = I_{sc}(s)$$

$$I_{sc}(s) = -2\frac{10}{4s+3} = -\frac{20}{4s+3} \quad ...(2.20.3)$$

4. This $I_{sc}(s)$ is the Norton's equivalent current. Let us now remove the short circuit across $a$-$b$ as shown in Fig. 2.20.3 and apply a voltage source $V_0(s)$ so that input current is $I_0(s)$. (10/$s$) source is deactivated.

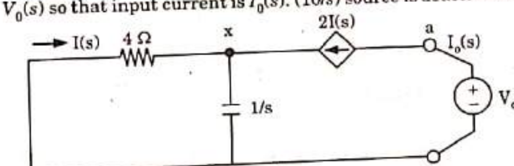



Fig. 2.20.3.

5. At node $x$, nodal analysis yields

$$\frac{V_x(s)}{1/s} = I(s) + 2I(s) = 3I(s) \quad ...(2.20.4)$$

or $\quad sV_x(s) = 3I(s) \quad ...(2.20.5)$

6. From the left most loop, $I(s) = -V_x(s)/4$

Using eq. (2.20.5) in (2.20.4)

$$I(s) = 0$$

Thus $I_0(s)$ becomes zero $Z_{int}(s) = \frac{V_o(s)}{I_o(s)} = \infty$

7. Fig. 2.20.4 represents Norton's equivalent circuit where $Z_{int} = \infty$

$$I_{sc}(s) = -\frac{20}{4s+3}$$

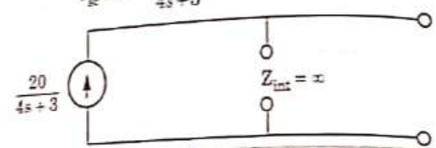



Fig. 2.20.4. Norton's equivalent circuit.

**Que 2.21.** **Derive the maximum power transfer theorem for the case when the source impedance is complex and the load is variable with its power factor being unity.**

**AKTU 2014-15, Marks 05**

**OR**

**State and prove the maximum power transfer theorem applied to the AC circuits.**

**AKTU 2017-18, Marks 07**

**OR**

**Give statement and prove maximum power transfer theorem.**

**AKTU 2016-17, Marks 10**

**Answer**

1. Maximum power will be delivered by a network, to an impedance $Z_R$, if the impedance of $Z_R$ in the complex conjugate of the impedance $Z$ of the network, measured looking back into the terminals of the network.
2. From Fig. 2.21.1,

$$I = \frac{E}{Z + Z_R} = \frac{E}{(R_A + R) + j(X_R + X)} \quad ...(2.21.1)$$

3. Power delivered to the load is

$$P = \frac{(E)^2 R_R}{(R_R + R)^2 + j(X_R + X)^2} \quad ...(2.21.2)$$

where, $Z = R + jX$

$Z_R = R_R + jX_R$

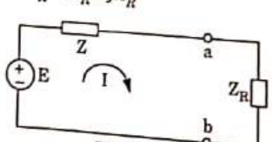



Fig. 2.21.1.

4. For maximum power, $\partial P/\partial X_R$ must be zero.



Now,

$$\frac{\partial P}{\partial X_R} = \frac{-2(E)^2 R_R (X_R + X)}{[(R_R + R)^2 + (X_R + X)^2]^2} = 0 \quad ...(2.21.3)$$

from which $X_R + X = 0$ or $X_R = -X$

*i.e.*, the reactance of the load impedance is of opposite sign to the reactance of the source impedance.

Putting $X_R = -X$ in eq. (2.21.2),

$$P = \frac{(E)^2 R_R}{(R_R + R)^2} \quad ...(2.21.4)$$

5. For maximum power, $\partial P/\partial R_R = 0$

$$\frac{\partial P}{\partial R_R} = \frac{(E)^2 (R_R + R)^2 - 2(E)^2 R_R (R_R + R)}{(R_R + R)^4} = 0$$

or $E^2 (R_R + R) - 2(E)^2 R_R = 0$ or $R_R = R$ ...(2.21.5)

6. Therefore, make $X_R = -X$ and $R_R = R$. Then, maximum power will be transferred from source to load. For maximum power transfer, load impedance $Z_R$ should be complex conjugate of the internal impedance $Z$ of the source, *i.e.*, $Z_R = Z^*$.
7. The maximum power transferred will be $P = (E)^2/4R_R$

**Que 2.22.** **State maximum power transfer theorem also determine the maximum power transfer to the load $R_L$ for the following circuit.**

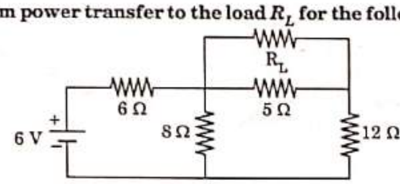



Fig. 2.22.1.

**AKTU 2018-19, Marks 07**

**Answer**

A. **State :** Refer Q. 2.21, Page 2–23C, Unit-2.

B. **Numerical :**

1. **To find $R_{th}$ or $R_{AB}$ or $R_L$ :**

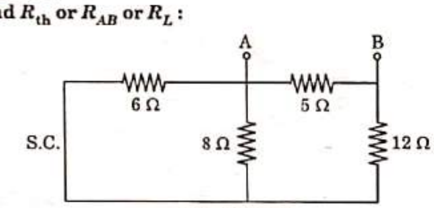



Fig. 2.22.2.

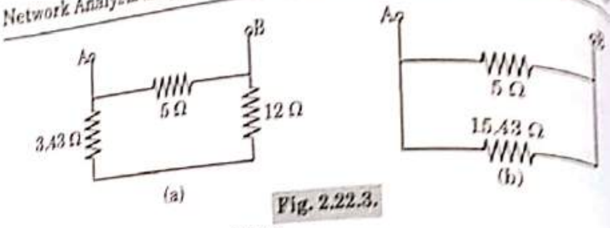



Fig. 2.22.3.

$$R_{th} = 3.77\ \Omega$$

2. To find $V_{th}$ or $V_{OC}$ or $V_{AB}$ :

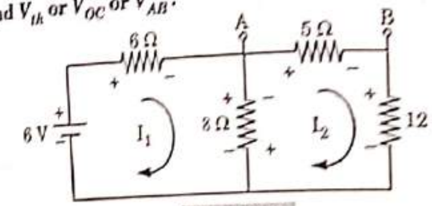



Fig. 2.22.4.

3. Applying KVL in loop 1 :
$$-6 + 6I_1 + 8(I_1 - I_2) = 0$$
$$14I_1 - 8I_2 = 6 \quad ...(2.2$$
4. Applying KVL in loop 2 :
$$8(I_2 - I_1) + 5I_2 + 12I_2 = 0$$
$$25I_2 - 8I_1 = 0 \quad ...(2.2$$
6. By solving eq. (2.22.1) and (2.22.2), we get
$$I_1 = 0.524\ \text{A}$$
$$I_2 = 0.167\ \text{A}$$
7. Now, from the circuit in Fig. 6, $V_{OC}$ or $V_{AB}$ is
$$V_A - 5I_2 - V_B = 0$$
$$V_{OC} \text{ or } V_{AB} = 5I_2 = 5 \times 0.167 = 0.835\ \text{V}$$
8. Hence, the maximum power through the load resistance ($R_L$) is;
$$P_{max} = V_{OC}^2 / 4R_L = (0.835 \times 0.835)/ 4 \times 3.77 = 0.046\ \text{Watt}$$

**Que 2.23.** **What should be the value of $R_L$ so the maximum pow can be transferred from the source to $R_L$ for the given Fig. 2.23.1**

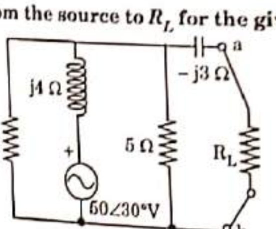



Fig. 2.23.1.

**AKTU 2015-16, Marks 1**



**Answer**

To satisfy maximum power transfer, we have to find internal impedance of the circuit.

1. **Finding internal impedance :**
   i. Voltage source is removed by short circuit since it does not have any internal resistance.
   ii. Load terminal is opened.
2. **Reduced network :**

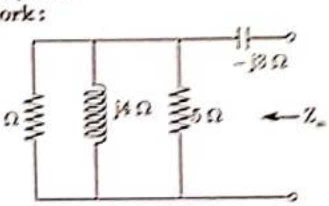



Fig. 2.23.2.

Component 3 Ω, $j4$ Ω and 5 Ω are parallel to each other.
Let equivalent of these = $Z_1$

$$\frac{1}{Z_1} = \frac{1}{3} + \frac{1}{j4} + \frac{1}{5} = \frac{8}{15} + \frac{1}{j4} = \frac{j32 + 15}{j60}$$

$$\therefore \quad Z_1 = \frac{j60}{j32 + 15} = \frac{j60 \times (15 - j32)}{15^2 + 32^2}$$
$$= 1.54 + j0.72\ \Omega$$

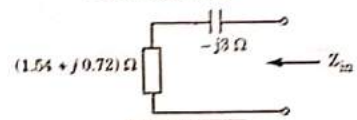



Fig. 2.23.3.

$$Z_{in} = Z_1 - j3 = 1.54 + j0.72 - j3 = (1.54 + j2.28)\ \Omega$$

Then,
$$R_1 = \text{Real part of } Z_{in}$$
$$= 1.54\ \Omega.$$

**PART-4**

*Reciprocity Theorem, Millman's Theorem, Compensation Theorem, Tellegen's Theorem.*

**Questions-Answers**

**Long Answer Type and Medium Answer Type Questions**

**Que 2.24.** State reciprocity theorem in AC network.

**Answer**

1. For a linear, time invariant, bilateral network, the ratio of input to the output remains constant in a reciprocal network with respect to an interchange between the points of application of input and measurement of output.

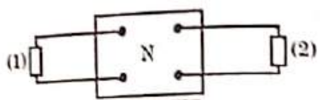


Fig. 2.24.1.

2. Consider a reciprocal network $N$, as shown in Fig. 2.24.1. Now, apply a voltage source of emf $E_1$ in branch 1 which produces current $I_2$ in branch 2 as shown in Fig. 2.24.2.

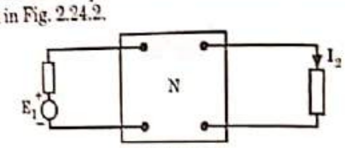


Fig. 2.24.2.

3. If the $E_2$ emf voltage source applied in branch 2 of network then the current $I_1$ is produced in branch 1 as shown in Fig. 2.24.3.

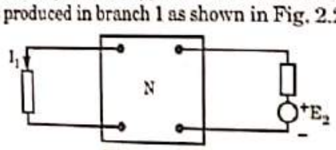


Fig. 2.24.3.

4. According to reciprocity theorem, $E_1/I_2 = E_2/I_1$

**Que 2.25.** State and prove Millman's theorem.

**Answer**

**A. For voltage source :** If there are '$n$' number of voltage source having magnitude $V_1, V_2, V_3, ...., V_n$ having internal impedances $Z_1, Z_2, ....., Z_n$ respectively, then these sources may be replaced by a single voltage source $V_m$ having equivalent series internal impedances $Z_m$ given by

$$V_m = \frac{V_1Y_1 + V_2Y_2 + V_3Y_3 + .... + V_nY_n}{Y_1 + Y_2 + ... + Y_n}$$

where $Y_1, Y_2, ...., Y_n$ are the admittances corresponding to $Z_1, Z_2, ....., Z_n$

where $$Z_m = \frac{1}{\sum_{i=1}^{n} Y_i} = \frac{1}{Y_1 + Y_2 + .... + Y_n}$$



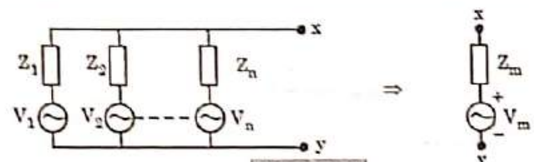


Fig. 2.25.1.

**B. For current source :**

1. To prove, each Thevenin's voltage equivalent is transferred into Norton's current equivalent as shown in Fig. 2.25.2.

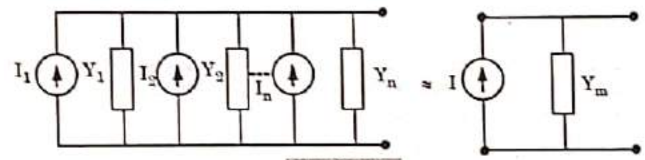


Fig. 2.25.2.

$$I_i = \frac{V_i}{Z_i} = V_i Y_i \; ; i = 1, 2, ...., n \text{ and } Y_i = \frac{1}{Z_i} ; i = 1, 2, ...., n$$

2. By KCL, all current sources are added,

$$I = \sum_{i=1}^{n} I_i = \sum_{i=1}^{n} V_i Y_i = V_1Y_1 + V_2Y_2 + .... + V_nY_n$$

and $$Y_m = \sum_{i=1}^{n} Y_i = Y_1 + Y_2 + .... + Y_n$$

3. Transforming back to voltage source $V_m$ in series with impedance $Z_m$ as shown in Fig. 2.25.1, we get

$$V_m = \frac{\sum_{i=1}^{n} V_i Y_i}{\sum_{i=1}^{n} Y_i} \text{ and } Z_m = \frac{1}{Y_m} = \frac{1}{\sum_{i=1}^{n} Y_i}$$

**Que 2.26.** Give the statement of compensation theorem. Also prove it for linear network.

**Answer**

**A. Statement :** "In a linear time variant network when the resistance ($R$) of an uncoupled branch, carrying a current ($I$), is changed by ($\Delta R$), the current in all the branches would change and can be obtained by assuming that an ideal voltage source of ($V_c$) has been connected in series with ($R + \Delta R$) when all other sources in the network are replaced by their internal resistance.

**B. Proof :**

1. Let us assume a load $R_L$ be connected to a DC source network whose Thevenin's equivalent gives $V_0$ as the Thevenin's voltage and $R_{th}$ as Thevenin's resistance as evident from Fig. 2.26.1.

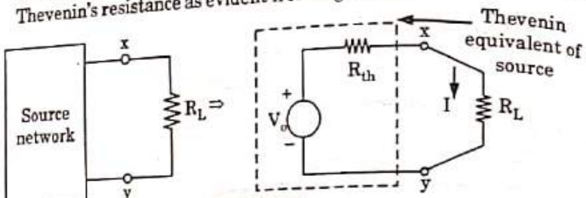



Fig. 2.26.1.

2. Let the load resistance $R_L$ be changed to $(R_L + \Delta R_L)$. Since the rest of the circuit remains unchanged, the Thevenin's equivalent network remains the same as in Fig. 2.26.2.

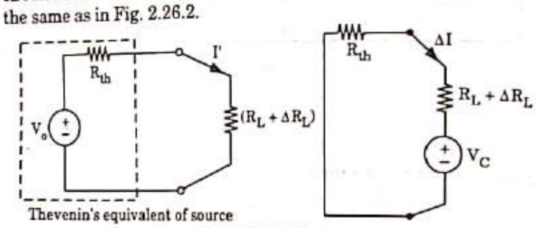



Fig. 2.26.2.

Here $\qquad I' = \dfrac{V_o}{R_{th} + (R_L + \Delta R_L)}$

3. The change of current being termed as $\Delta I$.

We find, $\Delta I = I' - I$

$$= \frac{V_o}{R_{th} + (R_L + \Delta R_L)} - \frac{V_o}{R_{th} + R_L} = -\left(\frac{V_o}{R_{th} + R_L}\right)\frac{\Delta R_L}{R_{th} + R_L + \Delta R_L}$$

$$= -\frac{I\Delta R_L}{R_{th} + R_L + \Delta R_L} = \frac{(-V_c)}{R_{th} + R_L + \Delta R_L}$$

4. Thus it has been proved that change of branch resistance, branch current is changed and the change is equivalent to an ideal compensating voltage source in series with the branch opposing the original current.

**Que 2.27.** **State and prove the Tellegen's theorem.**

**AKTU 2014-15, Marks 05**

**Answer**

**A. Tellegen's theorem :**

1. "For any given time, the sum of power delivered to each branch of any electric network is zero".



2. Thus for $K^{th}$ branch, this theorem states that $\sum_{K=1}^{n} V_K i_K = 0$; $n$ being the number of branches, $V_K$ the drop in the branch and $i_K$ the through current.
3. Tellegen's theorem is applicable to any network made up of lumped two terminal elements.

**B. Proof :**

1. Let $i_{pq} (= i_k) = K_{th}$ branch through current.
   $V_K$ = Voltage drop in branch $K = V_p - V_q$, where $V_p$ and $V_q$ are the respective node voltages at $p$ and $q$ nodes.
   We have, $\qquad V_K\, i_{pq} = (V_p - V_q)\, i_{pq} = V_K\, i_K$
   [also $V_K\, i_K = (V_q - V_p)\, i_{qp}$, obviously, $i_{qp} = -i_{pq}$]
2. Summing these two relations

$$2\, V_K\, i_K = (V_p - V_q)\, i_{pq} + (V_q - V_p)\, i_{qp}$$

or $\qquad V_K\, i_K = \dfrac{1}{2}[(V_p - V_q)\, i_{pq} + (V_q - V_p)\, i_{qp}]$

$$\sum_{k=1}^{n} V_k i_k = \frac{1}{2}\sum_{p=1}^{n}\sum_{q=1}^{n}(V_p - V_q) i_{pq} = \frac{1}{2}\sum_{p=1}^{n} V_p\left(\sum_{q=1}^{n} i_{pq}\right) - \frac{1}{2}\sum_{q=1}^{n} V_q\left(\sum_{q=1}^{n} i_{pq}\right)$$

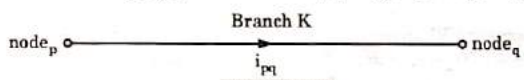



Fig. 2.27.1.

3. Following Kirchhoff's current law, the algebraic sum of current at each node is equal to zero.

*i.e.* $\qquad \sum_{p=1}^{n} i_{pq} = 0$ and $\sum_{q=1}^{n} i_{qp} = 0$

With this finally we get $\sum_{k=1}^{n} V_k i_k = 0$

4. Thus, it has been observed that the sum of power delivered to a closed network is zero.

**Que 2.28.** **Consider the network shown in Fig: 2.28.1.**

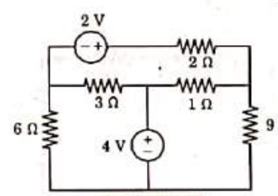



Fig. 2.28.1.

**Verify the Tellegen's theorem for the network shown in Fig. 2.28.1.**

**Answer**

**Numerical :**

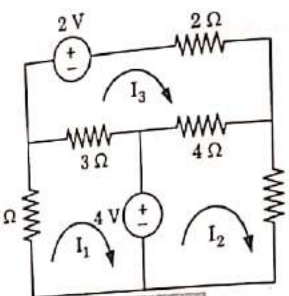



Fig. 2.28.2.

1. Applying KVL in loop 1
$$6I_1 + 3(I_1 - I_3) + 4 = 0$$
$$9I_1 - 3I_3 = -4 \quad ...(2.2$$
2. Applying KVL in loop 2
$$-4 + 4(I_2 - I_3) + 9I_2 = 0$$
$$13I_2 - 4I_3 = 4 \quad ...(2.2$$
3. Applying KVL in loop 3
$$0 = -2 + 2I_3 + 4(I_3 - I_2) + 3(I_3 - I_1)$$
$$9I_3 - 4I_2 - 3I_1 = 2 \quad ...(2.2$$
Solving eq. (2.28.1), (2.28.2) and (2.28.3)
$$I_1 = -0.351 \text{ A and } I_2 = 0.394 \text{ A and } I_3 = 0.28 \text{ A}$$
4. For Tellegen's theorem, $\Sigma v_k i_k = 0$
$$V_1 = 2 \text{ V}, i_1 = -0.351 \text{ A}$$
$$V_2 = 2 \times (-0.351) = -0.702 \text{ V}, i_2 = -0.351 \text{ A}$$
$$V_3 = 3 \times (I_1 - I_3) = 3 \times (-0.631) = -1.893 \text{ V}$$

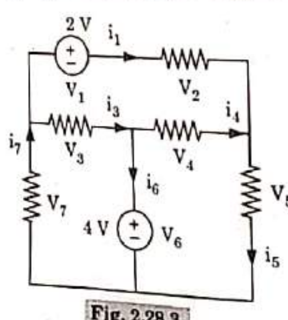



Fig. 2.28.3.



$$i_3 = I_1 - I_3 = -0.631 \text{ A}$$
$$V_4 = 4(I_3 - I_2) = 4 \times (-0.114) = -0.456 \text{ V}$$
$$I_4 = -0.114 \text{ A}$$
$$V_5 = 9 \times I_2 = 3.546 \text{ V}$$
$$i_5 = 0.394 \text{ A}$$
$$V_6 = 4 \text{ V}$$
$$i_6 = I_1 - I_2 = -0.745 \text{ A}$$
$$V_7 = 6 \times I_1 = -2.106 \text{ V}$$
$$I_7 = I_1 = -0.351 \text{ A}$$
$$\Sigma v_k i_k = v_1 i_1 + v_2 i_2 + v_3 i_3 + v_4 i_4 + v_5 i_5 + v_6 i_6 + v_7 i_7$$
$$= 2(-0.351) + (-0.702)(-0.351) + (-1.893)(-0.631)$$
$$+ (-0.456)(-0.114) + (3.546)(0.394) + 4(-0.745)$$
$$+ (-2.106)\, 0(-0.351) = 0$$

Hence Tellegen's theorem is verified.

**Que 2.29.** **Using Millman's Theorem, find the current through $(4 + j3)\ \Omega$ in the network shown in Fig. 2.29.1.**

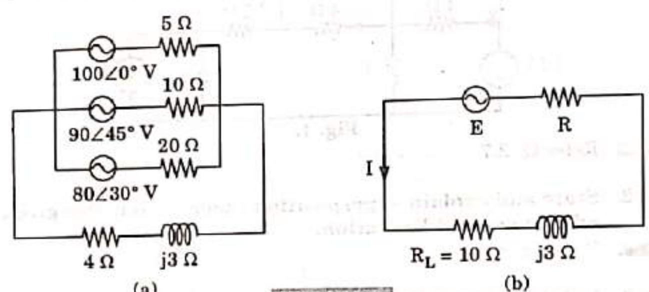



Fig. 2.29.1.

**Answer**

Using Millman's Theorem,

$$E = \frac{E_1G_1 + E_2G_2 + E_3G_3}{G_1 + G_2 + G_3}$$

$$= \frac{\dfrac{100\angle 0^\circ}{5} + \dfrac{90\angle 45^\circ}{10} - \dfrac{80\angle 30^\circ}{20}}{\dfrac{1}{5} + \dfrac{1}{10} + \dfrac{1}{20}}$$

$$= 66.6 \angle 10.78^\circ \text{ V}$$

$$R = \frac{1}{G} = \frac{1}{G_1 + G_2 + G_3} = \frac{1}{\dfrac{1}{5} + \dfrac{1}{10} + \dfrac{1}{20}} = 2.86\ \Omega$$

$$\therefore \quad I = \frac{E}{R + (4 + j3)} + \frac{66.6\angle 10.78°}{6.86 + j3}$$

$$= 8.89 \angle -12.84° \text{ A}$$

## VERY IMPORTANT QUESTIONS

*Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.*

**Q. 1. Find the current $I_o$ using source transformation in Fig. 1.**

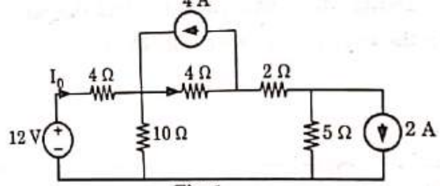



Fig. 1.

**Ans.** Refer Q. 2.7.

**Q. 2. State and explain superposition theorem and also give the advantage and limitation.**

**Ans.** Refer Q. 2.9.

**Q. 3. Using superposition theorem, find the voltage across (4 + j3) Ω in the network shown in Fig. 2.**

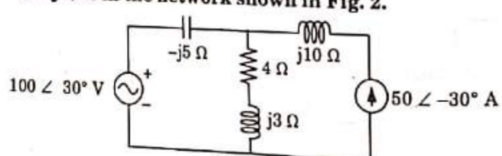



Fig. 2.

**Ans.** Refer Q. 2.10.

**Q. 4. Find $i_o$ in the circuit in Fig. 3 using superposition theorem.**



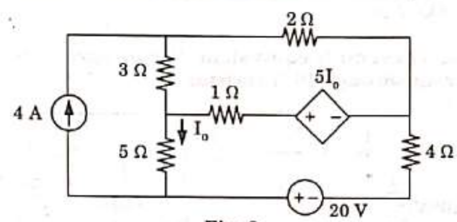



Fig. 3.

**Ans.** Refer Q. 2.11.

**Q. 5. Find current through 5 Ω resistor using Thevenin's theorem.**

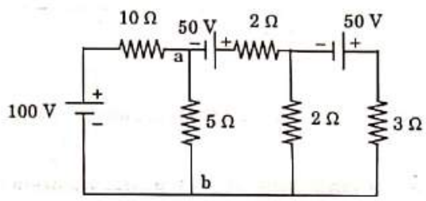



Fig. 4.

**Ans.** Refer Q. 2.14.

**Q. 6. Find the current $I$ and voltage $V_{ab}$ in Fig. 5.**

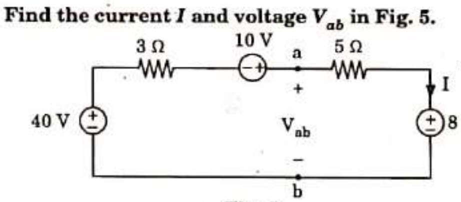



Fig. 5.

**Ans.** Refer Q. 2.15.

**Q. 7. Find the current $I_x$ through the 5 ohm resistor using Thevenin's theorem in Fig. 6.**

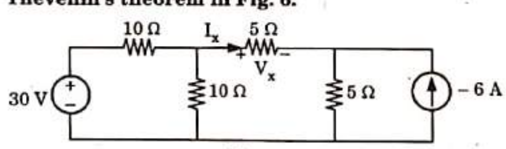



Fig. 6.

**Ans.** Refer Q. 2.16.

**Q. 8. Find Thevenin's equivalent circuit across *a-b* and current through 10 Ω resistor.**

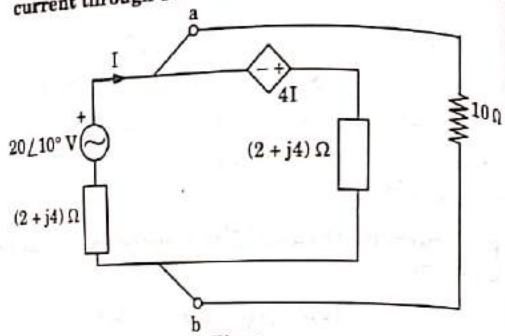



**Fig. 7.**

**Ans.** Refer Q. 2.17.

**Q. 9. State and prove Norton's theorem also gives the limit**

**Ans.** Refer Q. 2.18.

**Q. 10. Derive the maximum power transfer theorem for the when the source impedance is complex and the l variable with its power factor being unity.**

**Ans.** Refer Q. 2.21.

**Q. 11. State and prove the Tellegen's theorem.**

**Ans.** Refer Q. 2.27.

☺☺☺

**3**

**UNIT**

# Transient Circuit Analysis

## CONTENTS

PART-1

*Pre-Requisites : Laplace Transform and Concept of Initial Conditions.*

**Questions-Answers**

**Long Answer Type and Medium Answer Type Questions**

**Que 3.1.** What do you understand by "Laplace Transform" ?

**Answer**

**Laplace transform :**

1. Laplace transform of a time function $f(t)$ is define as :

$$F(s) = L[f(t)] = \int_{-\infty}^{\infty} f(t)e^{-st}dt \quad ...(3.1$$

where, $s$ is a complex variable and is equal to $\sigma + j\omega$.

2. The Laplace Transform as define in eq. (3.1.1) with $-\infty$ as the lower lim for the integral is called the 'two-sided' or 'bilateral' Laplace transfor

3. If the lower limit is changed to '0', we get the 'one-sided' or 'unilater Laplace transform.

4. Hence, we define the one-sided Laplace transform as :

$$F(s) = L[f(t)] = \int_{0}^{\infty} f(t)e^{-st}dt$$

The two-sided Laplace transform is useful to get a better insight into th system.

**Que 3.2.** State and prove initial value theorem of Laplac transform.

**Answer**

i. **Initial value theorem :** The initial value theorem states that if $F(s)$ i the Laplace transform of $f(t)$ then,

$$f(0^+) = \lim_{t \to 0^+} f(t) = \lim_{s \to \infty} s\,F(s)$$

**Proof :**

1. Consider the Laplace transform of the real differentiation,

$$L\left[\frac{df(t)}{dt}\right] = s\,F(s) - f(0^-)$$



2. Taking limit as $s \to \infty$ on both the sides

$$\lim_{s \to \infty} L\left[\frac{df(t)}{dt}\right] = \lim_{s \to \infty}\left[sF(s) - F(0^-)\right] \quad ...(3.3.1)$$

3. Consider the left hand side of eq. (3.3.1)

$$\lim_{s \to \infty} L\left[\frac{df(t)}{dt}\right] = \lim_{s \to \infty} \int_0^{\infty} e^{-st}\left[\frac{df(t)}{dt}\right]dt \quad \text{(by definition of Laplace)}$$

$$\therefore \quad \lim_{s \to \infty} L\left[\frac{df(t)}{dt}\right] = 0 \quad \text{as } \lim_{s \to \infty} e^{-st} \text{ is zero} \quad ...(3.3.2)$$

4. Putting eq. (3.3.2) in eq. (3.3.1), we get

$$0 = \lim_{s \to \infty} [s\,F(s) - f(0^-)]$$

*i.e.* $f(0^-) = \lim_{s \to \infty} s\,F(s)$

But as the function $f(t)$ is continuous, $f(0^-) = f(0^+)$ *i.e.*, the initial value of $f(t)$.

$$\therefore \quad f(0^+) = \lim_{t \to 0^+} f(t) = \lim_{s \to \infty} sF(s)$$

PART-2

*Natural Response and Forced Response, Transient Response and Steady State Response for Arbitrary Inputs (DC and AC), Evaluation of Time Response both through Classical and Laplace Methods.*

**Questions-Answers**

**Long Answer Type and Medium Answer Type Questions**

**Que 3.3.** What do you mean by natural response and forced response of electrical circuit ?

**Answer**

A. **Natural response :**

1. The analysis of circuits containing inductors and/or capacitors is dependent upon the formulation and solution of the integro-differential equation that characterize the circuits.
2. The solution of differential equation represents a response of the circuit, and it is known as natural response since it depends on the nature of circuit element.
3. However any storage element cannot store energy forever so the resistance of the circuit converts all the energy into heat and the response must eventually die-out. For this reason it is also called transient response.

B. **Forced response :** We consider the independent sources acting on the circuit and response is dependent upon the nature of source. This part of response is known as forced response or steady-state response.

C. The complete response is the combination of both natural response and forced response.

**Que 3.4.** **Discuss time constant in series *RL* circuit.**

**Answer**

1. It is the time taken for the current to reach 63 % of its final value. Thus, it is a measure of the rapidity with which the steady state is reached.
2. Also, at $t = 5\tau$, $i = 0.993i_p$, the transient is therefore, said to be practically disappeared in five time constant.
3. The tangent to the equation $i = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)$ at $t = 0$, intersects the straight line, $i = \frac{V}{R}$ at $t = \tau = \frac{L}{R}$. Thus, time-constant is the time in which steady state would be reached if the current increases at the initial rate.
4. Physically, time-constant represents the speed of the response of a circuit. A low value of time-constant represents a fast response and a high value of time-constant represents a sluggish response.
5. Voltage across the resistor,
$$V_R = Ri(t) = V\left(1 - e^{-\frac{R}{L}t}\right)$$
6. Voltage across the inductor,
$$V_L = L\frac{di(t)}{dt} = L\frac{d}{dt}\left[\frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)\right] = Ve^{-\frac{R}{L}t}$$

**Que 3.5.** **Discuss time constant in series *RC* circuit.**

**Answer**

1. It is the time in which current decays to 37 % of its initial value.
2. Also, at $t = 5\tau$, $i = 0.07\frac{V}{R}$; the transient is therefore, said to be practically disappeared in five time constant.
3. The tangent to the equation $i = \frac{V}{R}e^{-t/RC}$ at $t = 0$, intersects the time axis at $t = \tau = RC$. Thus, time-constant is the time in which the current would reach the steady state zero value if the current decays at the initial rate.



4. Physically, time-constant represents the speed of the response of a circuit. A low value of time-constant represents a fast response and a high value of time-constant represents a sluggish response.
5. Voltage across the resistor,
$$V_R = Ri(t) = Ve^{-t/RC}$$
6. Voltage across the capacitor,
$$V_C = \frac{1}{C}\int_0^t i(t)\,dt = \frac{1}{C}\int_0^t \frac{V}{R}e^{-t/RC}\,dt = V\left(1 - e^{-t/RC}\right)$$

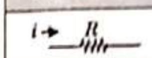
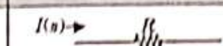
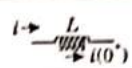
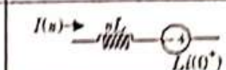
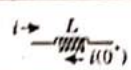
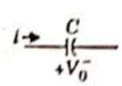
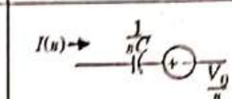
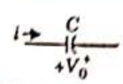
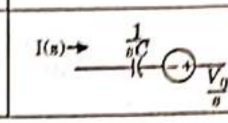
**Que 3.6.** **What are the steps for circuit analysis using Laplace method ?**

**Answer**

A. **Steps :**
1. All circuit elements are transformed from time domain to Laplace domain with initial conditions.
2. Excitation function is transformed into Laplace domain.
3. The circuit is solved using different analysis techniques, such as mesh analysis, node analysis, etc.
4. Time domain solution is obtained by taking inverse Laplace transform of the solution.

B. **Behaviour of circuit elements in *s*-domain :**

Table 3.6.1.

| Time domain | *s*-domain | *s*-domain voltage term |
|---|---|---|
|  |  | $RI(s)$ |
|  |  | $sLI(s) + Li(0^+)$ |
|  |  | $sLI(s) + Li(0^+)$ |
|  |  | $\frac{I(s)}{sC} + \frac{V_0}{s}$ |
|  |  | $\frac{I(s)}{sC} - \frac{V_0}{s}$ |

**Que 3.7.** In the circuit shown $v(t) = 2u(t)$ and $i_L(0^-) = 2$ amps. Find and sketch $i_2(t)$.

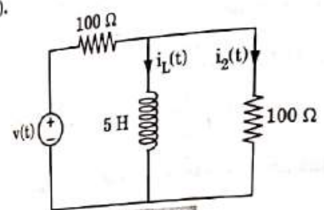


**Fig. 3.7.1.**

**AKTU 2017-18, Marks 07**

**Answer**

1. The transformed network,

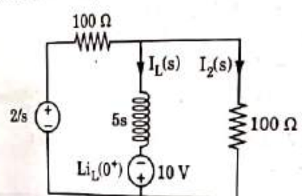


**Fig. 3.7.2.**

Applying KVL,

$$\frac{2}{s} + 10 = 100[I_L(s) + I_2(s)] + 5s\, I_L(s)$$

$$\frac{2+10s}{s} = (5s+100)\, I_L(s) + 100\, I_2(s) \quad ...(3.7.1)$$

3. And,

$$\frac{2}{s} = 100[I_L(s) + I_2(s)] + 100\, I_2(s)$$

$$100\, I_L(s) = \frac{2}{s} - 200\, I_2(s)$$

$$I_L(s) = \frac{1}{50s} - 2I_2(s) \quad ...(3.7.2)$$

4. From eq. (3.7.1) and (3.7.2),

$$\frac{2+10s}{s} = (5s+100)\left[\frac{1}{50s} - 2I_2(s)\right] + 100 I_2(s)$$

$$9.9 = -(100+10s) I_2$$



$$I_2(s) = \frac{-9.9}{10s+100}$$

$$I_2(s) = \frac{-0.99}{(s+10)}$$

Negative sign shows that direction taken is opposite to original.

5. Taking inverse Laplace transform of $I_2(s)$,

$$i_2(t) = 0.99\, e^{-10t}$$

$$i_2(0) = 0.99$$

$$i_2(\infty) = 0$$

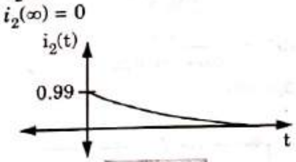


**Fig. 3.7.3**

**Que 3.8.** **In the circuit shown in Fig. 3.8.1, the switch is kept closed for a long time and is then opened at $t = 0$.**

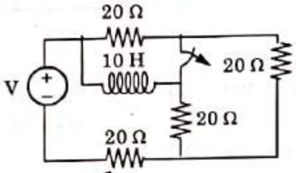


**Fig. 3.8.1.**

**Find the values of current $i$ just before opening the switch ($t = 0^-$) and just after opening the switch ($t = 0^+$).** **AKTU 2014-15, Marks 05**

**Answer**

1. Since switch is kept closed for a long time, hence, the steady state current flows.
2. At $t < 0$, switch is closed.

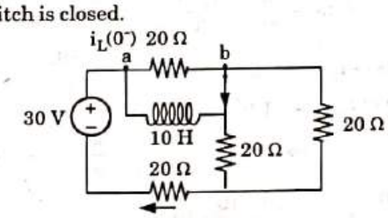


**Fig. 3.8.2.**

3. At steady state, inductor behaves as short circuit. Hence, no current flows through $ab$ branch.

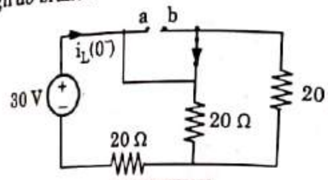



Fig. 3.8.3.

4. $$i_L(0^-) = \frac{30}{[20+(20 \parallel 20)]} = \frac{30}{(20+10)} = 1 \text{ A}$$

5. At $t > 0$, switch is opened.

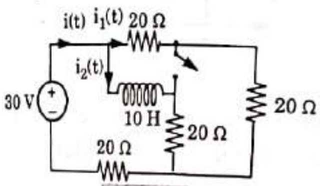



Fig. 3.8.4.

6. Applying KVL in Fig. 3.8.4,

$$-30 + 20\, i_1(t) + 20\, i_1(t) + 20\, i_1(t) + 20\, i_2(t) = 0$$

$$60\, i_1(t) + 20\, i_2(t) = 30 \quad \text{...(3.8.1)}$$

7. Again apply KVL in Fig. 3.8.4,

$$-30 + 10\frac{di_2(t)}{dt} + 20i_2(t) + 20i_2(t) + 20i_1(t) = 0$$

$$\therefore \; 10\frac{di_2(t)}{dt} + 40i_2(t) + 20i_1(t) = 30 \quad \text{...(3.8.2)}$$

8. Taking Laplace on both side of eq. (3.8.1) and eq. (3.8.2),

$$60\, I_1(s) + 20\, I_2(s) = \frac{30}{s}$$

9. $$I_1(s) = \frac{30}{60s} - \frac{I_2(s)}{3} \quad \text{...(3.8.3)}$$

and, $$10[s\, I_2(s) - i_L(0^-)] + 40I_2(s) + 20\, I_1(s) = \frac{30}{s} \quad \text{...(3.8.4)}$$

10. Putting value of $I_1(s)$ in eq. (3.8.4), we get

$$10[sI_2(s) - 1] + 40\, I_2(s) + 20\left[\frac{1}{2s} - \frac{I_2(s)}{3}\right] = \frac{30}{s} \quad (\because \; i_L(0^-) = 1 \text{ A})$$

$$\left(10s + 40 - \frac{20}{3}\right) I_2(s) = \frac{30}{s} - \frac{10}{s} + 10$$



$$\left(\frac{30s + 120 - 20}{3}\right) I_2(s) = \frac{20 + 10s}{s}$$

$$\left(\frac{3s + 10}{3}\right) I_2(s) = \frac{s+2}{s}$$

$$I_2(s) = \frac{3(s+2)}{s(3s+10)}$$

11. Using partial fraction,

$$I_2(s) = \frac{6}{10s} + \frac{12}{10(3s+10)}$$

12. Taking inverse Laplace transform,

$$i_2(t) = \frac{3 + 2e^{\frac{-10}{3}t}}{5}$$

13. $$i_1(t) = \frac{1}{60}\left[30 - 20\left(\frac{3 + 2e^{\frac{-10}{3}t}}{5}\right)\right]$$

$$= \frac{1}{60}\left[18 - 8e^{\frac{-10}{3}t}\right]$$

14. $$i(t) = i_1(t) + i_2(t)$$

$$= \frac{1}{60}\left[18 - 8e^{\frac{-10}{3}t}\right] + \left[\frac{3 + 2e^{\frac{-10}{3}t}}{5}\right]$$

15. $$i(t) = \frac{9}{10} + \frac{4}{15}e^{\frac{-10}{3}t} \text{ A}$$

**Que 3.9.** **In the Fig. 3.9.1 shown, the ideal switch has been open for a long time. If it is closed at $t = 0$, then find the magnitude of current (in mA) through the 4 kΩ resistor at $t = 0^+$?**

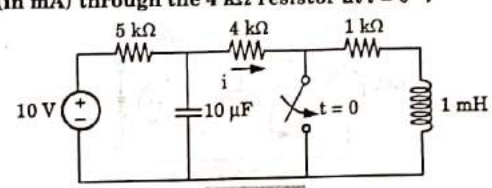



Fig. 3.9.1.

**AKTU 2014-15, Marks 05**

**Answer**

1. At $t < 0$.
   i. Circuit achieves steady state.
   ii. Capacitor behaves as open-circuit.
   iii. Inductor behaves as short-circuit.
   iv. Switch is kept open.

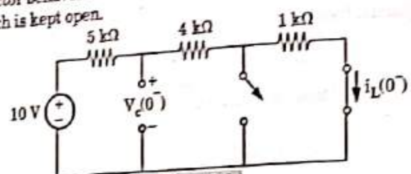



**Fig. 3.9.2.**

$$i_L(0^-) = \frac{10}{10} = 1\text{ A}$$

and,
$$V_c(0^-) = 10 - (5 \times 1) = 5\text{ V}$$

2. At $t > 0$, switch is closed,

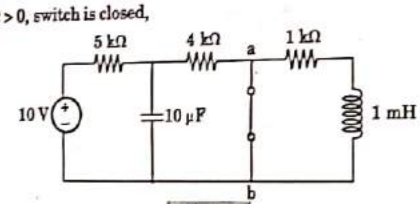



**Fig. 3.9.3.**

3. Since, $ab$ branch is short-circuited. Hence, no current flows through 1 kΩ and 1 mH.

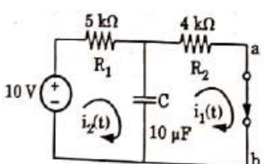



**Fig. 3.9.4.**

4. Apply KVL in Fig. 3.9.4,

$$-10 + R_1 i_2(t) + \frac{1}{C}\int (i_2(t) - i_1(t))dt + V_c(0^-) = 0$$

$$-10 + R_1 i_2(t) + \frac{1}{C}\int (i_2(t) - i_1(t))dt + 5 = 0$$



$$R_1 i_2(t) + \frac{1}{C} i_2(t) - i_1(t) = 5 - 1 \qquad ...(3.9.1)$$

5. Again,

$$0 = \frac{1}{C}\int (i_1(t) - i_2(t))dt + R_2 i_1(t) + [-V_c(0^-)]$$

$$0 = \frac{1}{C}\int (i_1(t) - i_2(t))dt + R_2 i_1(t) - 5$$

$$\therefore \quad \frac{1}{C}\int (i_1(t) - i_2(t))dt + R_2 i_1(t) = 5 \qquad ...(3.9.2)$$

6. Taking Laplace transform of eq. (3.9.1) and eq. (3.9.2),

$$R_1 I_2(s) + \frac{1}{Cs}[I_2(s) - I_1(s)] = \frac{5}{s}$$

$$R_1 I_2(s) + \frac{I_2(s)}{Cs} - \frac{5}{s} = \frac{I_1(s)}{Cs}$$

$$\frac{(R_1 Cs + 1 - 5C) I_2(s)}{Cs} = \frac{I_1(s)}{Cs}$$

$$\therefore \quad I_2(s) = \frac{I_1(s)}{(R_1 Cs - 5C + 1)}$$

7. Again,

$$\frac{1}{Cs}[I_1(s) - I_2(s)] + R_2 I_1(s) = \frac{5}{s}$$

8. Putting value of $I_2(s)$,

$$\frac{1}{Cs} I_1(s) - \frac{I_1(s)}{Cs(R_1 Cs - 5C + 1)} + R_2 I_1(s) = \frac{5}{s}$$

$$I_1(s)\left[\frac{1}{Cs} - \frac{1}{Cs(R_1 Cs - 5C + 1)} + R_2\right] = \frac{5}{s}$$

$$I_1(s)\left[\frac{R_1 Cs - 5C + 1 - 1 + R_2 R_1 C^2 s^2 - 5C^2 R_2 s + R_2 Cs}{Cs(R_1 Cs - 5C + 1)}\right] = \frac{5}{s}$$

$$I_1(s)\left[\frac{R_1 R_2 C^2 s^2 + (R_1 C - 5C^2 R_2 + R_2 C)s + (-5C)}{C(R_1 Cs - 5C + 1)}\right] = 5$$

9. Putting

$$R_1 = 5 \times 10^3\ \Omega$$
$$R_2 = 4 \times 10^3\ \Omega$$
$$C = 10\ \mu\text{F} = 10^{-5}\text{ F}$$

10.
$$I_1(s)\left[\frac{200 s^2 + (5 \times 10^3 - 5 \times 10^{-5} \times 4 \times 10^3)s - 5}{(5 \times 10^3 \times 10^{-5} - 5 \times 10^{-5} + 1)}\right] = 5$$

$$= \frac{5(0.05 s + 1)}{200 (s + 25)(s + 20)}$$

11. Using partial fraction,

$$I_1(s) = \frac{A}{s+25} + \frac{B}{s+20}$$

$$\therefore \quad I_1(s) = \frac{1}{200}\left[\frac{0.25}{s+25}\right].$$

12. Taking inverse Laplace transforms,

$$i_1(t) = \frac{0.25}{200} e^{-25t} u(t)$$

$$= 1.25 e^{-25t} u(t)$$

**Que 3.10.** **Find the expression of the current $i(t)$ in the circuit of Fig. 3.10.1 assuming that the switch is opened at $t = 0$, the steady state having already reached before that.**

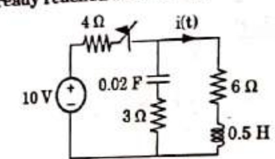



Fig. 3.10.1.

**AKTU 2014-15, Marks 10**

**Answer**

1. At $t < 0$, switch is closed and circuit reaches steady state. At that time capacitor behaves as open circuit and inductor behaves as short circuit.

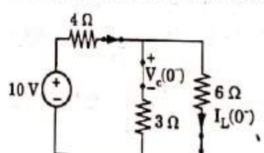



Fig. 3.10.2.

2. $$I_L(0^-) = \frac{10}{10} = 1\text{ A}$$

$$V_c(0^-) = 10 - (4 \times 1) = 6\text{ V}$$

3. At $t > 0$, switch is opened.



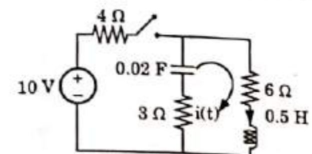



Fig. 3.10.3.

4. Using KVL in Fig. 3.10.3,

$$\frac{1}{0.02}\int i(t)dt - V_c(0^-) + 9i(t) + 0.5\frac{di(t)}{dt} = 0$$

5. Taking Laplace transform,

$$\frac{1}{0.02s}I(s) - \frac{V_c(0^-)}{s} + 9I(s) + 0.5[sI(s) - I_L(0^-)] = 0$$

$$\frac{1}{0.02s}I(s) - \frac{6}{s} + 9I(s) + 0.5sI(s) - 0.5 = 0$$

$$I(s) = \frac{(0.010s + 0.12)}{(0.010s^2 + 0.18s + 1)} = \frac{(0.010s + 0.12)}{0.01(s+0.9)^2}$$

$$= \left[\frac{0.010}{(s+0.9)} + \frac{0.03}{(s+0.9)^2}\right] \times \frac{1}{0.010}$$

6. $$i(t) = e^{-0.9t} + 3te^{-0.9t}\text{ A}$$

**Que 3.11.** **Find the voltage $V_o$ in the circuit of Fig. 3.11.1. The switch was open for a long time before it was closed at $t = 0$.**

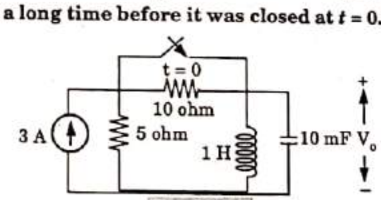



Fig. 3.11.1.

**AKTU 2014-15, Marks 10**

**Answer**

1. At $t > 0$, switch opened for long time, hence steady state reached.
2. At steady state, capacitor behaves as open circuit and inductor behaves as short-circuit.

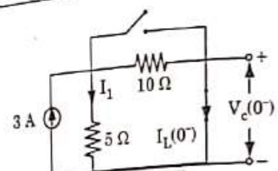



**Fig. 3.11.2.**

3. Using current division rule,

$$I_L(0^-) = \frac{5}{(5+10)} \times 3 = \frac{5}{15} \times 3$$
$$= 1\text{ A}$$
$$I_1 = 2\text{ A}$$

4. $$V_c(0^-) = 5 \times 2 - (10 \times 1)$$
$$= 0$$

5. If $t > 0$, switch is closed.

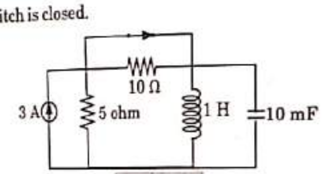



**Fig. 3.11.3.**

6. Hence, no current flows through 10 Ω resistor.
7. Current source is converted into voltage source.

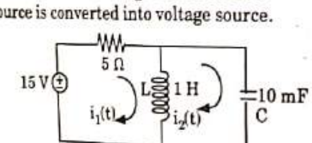



**Fig. 3.11.4.**

Apply KVL in Fig. 3.11.4,

$$-15 + 5\, i_1(t) + \frac{d[i_1(t) - i_2(t)]}{dt} = 0 \quad \text{...(3.11.1)}$$

Again

$$\frac{d(i_2(t) - i_1(t))}{dt} + \frac{1}{C}\int i_2(t)dt + V_c(0^-) = 0 \quad \text{...(3.11.2)}$$

8. Taking Laplace transform of eq. (3.11.1) and (3.11.2)

$$\frac{-15}{s} + 5\,I_1(s) + [s\,(I_1(s) - I_2(s)) - 1] = 0$$

$$(5+s)\,I_1(s) - sI_2(s) = \frac{s+15}{s}$$



$$\therefore \quad I_1(s) = \frac{s+15}{s(5+s)} + \frac{sI_2(s)}{(5+s)} \quad \text{...(3.11.3)}$$

9. $$[s(I_2(s) - I_1(s)) + 1] + \frac{1}{Cs}I_2(s) + 0 = 0$$

$$sI_2(s) - s\left[\frac{s+15}{s(s+5)} + \frac{sI_2(s)}{s+5}\right] + 1 + \frac{1}{Cs}I_2(s) = 0$$

$$I_2(s)\left[s - \frac{s^2}{s+5} + \frac{1}{Cs}\right] = -1 + \frac{s(s+15)}{s(s+5)}$$

$$I_2(s)\left[\frac{Cs^2(s+5) - Cs^3 + s + 5}{Cs(s+5)}\right] = \frac{-s-5+s+15}{s+5}$$

$$I_2(s)\left[\frac{Cs^3 + 5\,Cs^2 - Cs^3 + s + 5}{Cs}\right] = \frac{10}{1}$$

$$I_2(s)\left[\frac{5Cs^2 + s + 5}{Cs}\right] = \frac{10}{1}$$

10. Putting $\quad C = 10^{-2}$ F,

$$I_2(s)\left[\frac{5\,s^2 + 100\,s + 500}{s}\right] = \frac{10}{1}$$

$$I_2 = \frac{10s}{5(s+10)^2} = \frac{2s}{(s+10)^2}$$

11. $$V_C(t) = V_C(0^-) + \frac{1}{C}\int i_2(t)\,dt$$

12. $$V_C(t) = \frac{1}{Cs}I_2(s)$$

$$= \frac{2s}{0.01s(s+10)^2} = \frac{200s}{s(s+10)^2}$$

13. $$V_o(t) = V_1(t) = 200t\,e^{-10t}$$

**Que 3.12.** **In the circuit shown in Fig. 3.12.1, the switch is moved from $A$ to $B$ at $t = 0$. Find $v(t)$ for $t > 0$.**

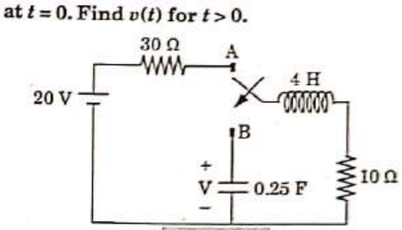



**Fig. 3.12.1.**

**AKTU 2018-19, Marks 07**

**Answer**

**Step 1 :** First find all the initial conditions and then transform into the s-domain since the capacitor is not connected to a circuit, we do not know its initial condition so we can assume it is zero $[V(0) = 0]$. We can find $i_L(0)$ by letting the inductor be a short

$$i_L(0) = \frac{20}{40} = 0.5 \text{ Amp}$$

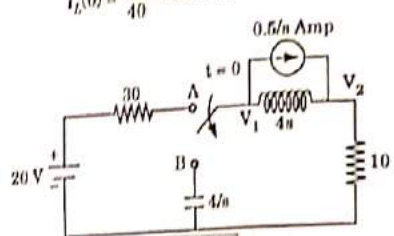



**Fig. 3.12.2.**

$$\frac{V_1 - 0}{4/s} + \frac{V_1 - V_2}{4s} + \frac{0.5}{s} = 0 \quad ...(3.12.1)$$

$$\frac{V_1 - V_2}{4s} - \frac{0.5}{s} + \frac{V_2 - 0}{10} = 0 \quad ...(3.12.2)$$

Adding eq. (3.12.1) and eq. (3.12.2), we get

$$\frac{sV_1}{4} + \frac{V_2}{10} = 0$$

$$V_2 = -2.5\,sV_1$$

**Step 2 :**

$$\left(\frac{s}{4} + \frac{1}{4s} + \frac{2.5s}{4s}\right) V_1 = -\frac{0.5}{s}$$

$$\left(\frac{s^2 + 2.5s + 1}{4s}\right) V_1 = \frac{0.5}{s}$$

$$V_1 = \frac{-0.5 \times 4}{s^2 + 2.5s + 1}$$

$$V_1 = \frac{-2}{s^2 + 2.5s + 1}$$

$$V_1 \text{ or } V(t) = \frac{-2}{(s + 0.5)(s + 2)}$$

$$= \frac{-1.333}{(s + 0.5)} + \frac{1.333}{s + 2}$$

$$V(t) = [-1.333\,e^{-t/2} + 1.333e^{-2t}]\,u(t) \text{ Volts}$$

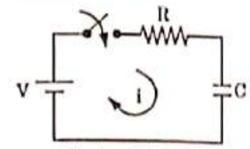


**Que 3.13.** **For RC series circuit switch is closed at $t = 0$, find out current expression also draw its transient curve.**

**AKTU 2018-19, Marks 07**

**Answer**

1. Consider a series RC circuit shown in Fig. 3.13.1. The capacitor in the circuit is initially uncharged. When the switch is closed at $t = 0$, we can determine the complete solution for the current.




Series RC circuit

**Fig. 3.13.1.**

2. Applying KVL to the circuit for $t > 0$, we have

$$V = Ri + \frac{1}{C}\int i\,dt \quad ...(3.13.1)$$

3. By differentiating the eq. (3.13.1), we get

$$0 = R\frac{di}{dt} + \frac{1}{C}i$$

or

$$\frac{di}{dt} + \frac{1}{RC}i = 0$$

4. The general solution for a linear differential equation $\frac{di}{dt} + Pi = 0$ is :

$$i = Ke^{-Pt}$$

Here,

$$P = \frac{1}{RC}$$

$\therefore$

$$i = Ke^{-\frac{1}{RC}t} \quad ...(3.13.2)$$

5. To find the value of $K$, we use the initial conditions. At $t = 0^-$, the switch is open. So $i(0^-) = 0$ and $V_C(0^-) = 0$. Since the voltage across the capacitor cannot change instantaneously, the capacitor acts as a short circuit at $t = 0^+$, *i.e.*,

$$V_C(0^+) = 0$$

$\therefore$ At $t = 0$, the current $i = \frac{V}{R}$

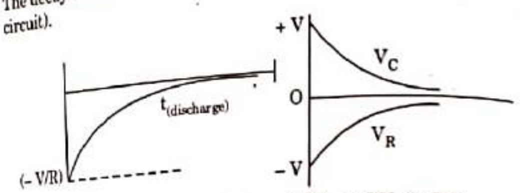
$\therefore$

$$i(0) = \frac{V}{R} = Ke^{-\frac{1}{RC}(0)} = K \text{ or } K = \frac{V}{R}$$

6. The current expression in eq. (3.13.2) becomes

$$i = \frac{V}{R}e^{-t/RC}$$

7. The decay transient is plotted in Fig. 3.13.2(a) (natural response of the circuit).




(a) Current decay transient In RC discharging circuit.

(b) $V_R$ and $V_C$ in RC discharging circuit.

**Fig. 3.13.2.**

8. The corresponding transient voltages are given by
$V_R$(Voltage drop across $R$) = $iR = -Ve^{-t/RC}$

and $V_C$ (Voltage drop across $C$) = $\frac{1}{C}\int i\,dt = Ve^{-t/RC}$

Obviously $V_R + V_C = 0$

9. Fig. 3.13.2(b) represents the profiles of $V_R$ and $V_C$ with $t$.

**Que 3.14.** **In Fig. 3.14.1 the initial voltage in the capacitor is 1 V with the polarity as shown, find the voltage appearing across the capacitor using Laplace method with application of step voltage &**

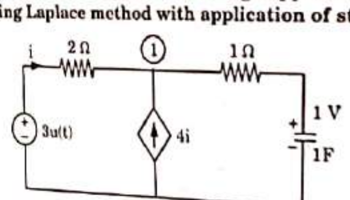



**Fig. 3.14.1.**

**AKTU 2015-16, Marks 10**

**Answer**

1. Given network is, $V_c(0) = 1$ V
2. By KVL in Fig. 3.14.2,

$$-3 + 2i + 5i + \frac{5}{1}\int_0^t i\,dt + V_c(0) = 0$$



$$7i + 5\int_0^t i\,dt + V_c(0) = 3$$

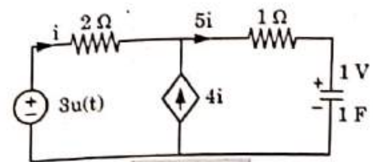



**Fig. 3.14.2.**

3. Taking Laplace transform,

$$7I(s) + \frac{5I(s)}{s} + \frac{V_c(0)}{s} = \frac{3}{s}$$

$$I(s)\left[7 + \frac{5}{s}\right] = \left(\frac{3}{s} - \frac{1}{s}\right)$$

$$I(s)\left(\frac{5+7s}{s}\right) = \frac{2}{s}$$

$$\therefore \quad I(s) = \frac{2}{7s+5}$$

4. Now,

$$V_c(s) = \frac{I_{(s)}}{s} + \frac{V_c(0)}{s}$$

$$V_c(s) = \frac{2}{s(7s+5)} + \frac{1}{s}$$

$$V_c(s) = \frac{2}{7}\left(\frac{1}{s(s+5/7)}\right) + \frac{1}{s}$$

$$V_c(s) = \frac{2}{7}\left(\frac{1}{s} - \frac{1}{(s+5/7)}\right) \times \frac{1}{(5/7)} + \frac{1}{s}$$

$$V_c(s) = \frac{2}{5}\left(\frac{1}{s} - \frac{1}{(s+5/7)}\right) + \frac{1}{s}$$

5. Taking inverse Laplace transform,

$$V_c(t) = \left(\frac{2}{5} + 1\right) - \frac{2}{5}e^{-5/7t}$$

$$\therefore \quad V_c(t) = \frac{7}{5} - \frac{2}{5}e^{-5/7t}$$

**Que 3.15.** **A network has been shown in Fig. 3.15.1. Switch $K$ is closed at $t = 0$. Find the current in $R_L$ using Thevenin's theorem. Assume steady state condition before switching. Use the following state condition before switching. Use the following values : ($r_1 = r_2 = r_3 = 10\ \Omega$; $L_1 = L_2 = 1$ H; $V = 10$ V)**

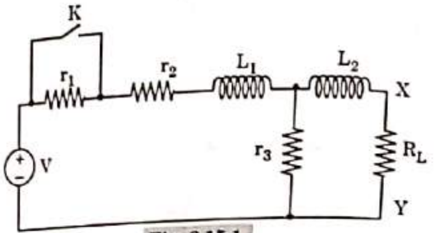



Fig. 3.15.1.

**AKTU 2015-16, Marks 10**

**Answer**

1. Since, switch $K$ is closed at $t = 0$, then effect of resistance $r_1$ would be zero, *i.e.*, no current will flow through this. Hence, it is removed.

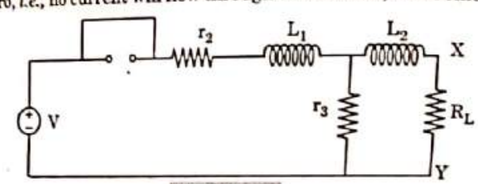



Fig. 3.15.2.

2. Calculation for $V_{th}$ : Load terminal is opened.

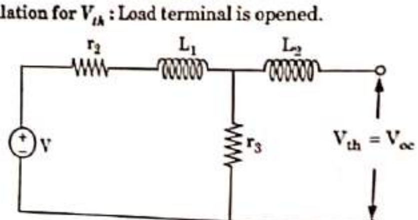



Fig. 3.15.3.

$$V_{th} = \frac{V}{r_2 + r_3 + j\omega L_1} \times r_3$$

$$= \frac{10}{10 + 10 + j\omega L_1} \times 10$$

$$= \frac{100}{20 + j\omega L_1} \text{ V}$$



3. **Calculation for $R_{th}$ :**

i. Voltage source is short circuited.

ii. Load terminal is opened.

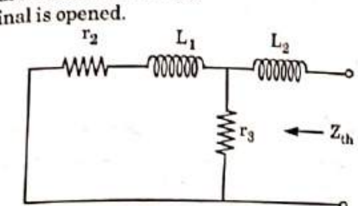



Fig. 3.15.4.

$r_2 + L_1$ in series and parallel with $r_3$.

Then,
$$\frac{1}{Z_1} = \frac{1}{r_2 + j\omega_1 L_1} + \frac{1}{r_3}$$

$$\frac{1}{Z_1} = \frac{1}{10 + j\omega L_1} + \frac{1}{10} = \frac{20 + j\omega L_1}{10(10 + j\omega L_1)}$$

$$\therefore \quad Z_1 = \frac{100 + j10\omega L_1}{20 + j\omega L_1}$$

4. Then,
$$Z_{th} = Z_1 + j\omega L_2$$

$$= \frac{100 + j10\omega L_1}{20 + j\omega L_1} + j\omega L_2$$

$$= \frac{100 + j10\omega L_1 + j20\omega L_2 - \omega^2 L_1 L_2}{20 + j\omega L_1}$$

$$\therefore \quad Z_{th} = \frac{(100 - \omega^2 L_1 L_2) + j(10\omega L_1 + 20\omega L_2)}{20 + j\omega L_1}$$

5. **Thevenin's network is :**

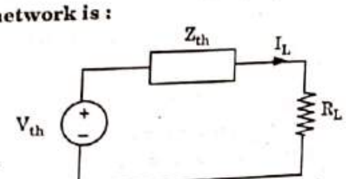



Fig. 3.15.5.

$$I_L = \frac{V_{th}}{Z_{th} + R_L}$$

or,
$$I_L = \frac{100}{20 + j\omega} \times \frac{1}{\dfrac{(100 - \omega^2) + j30\omega}{20 + j\omega} + R_L} \quad [\because \; L_1 = L_2 = 1 \text{ H}]$$

$$I_L = \frac{100}{(100-\omega^2+j30\omega)+R_L(20+j\omega)}$$

$\therefore$ at $\omega = 0$,

$$I_L = \frac{100}{100+20R_L}\text{ A}$$

**Que 3.16.** In Fig. 3.16.1 with switch open, steady state is reached with $v = 100 \sin 314t$ volts. The switch is closed at $t = 0$. The circuit is allowed to come to steady state again. Determine the steady state current and complete solution of transient current.

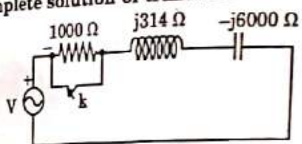



**Fig. 3.16.1.**

**AKTU 2015-16, Marks 10**

**Answer**

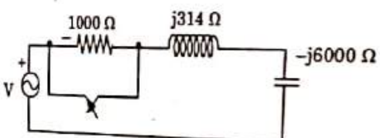



**Fig. 3.16.2.**

1. Before the switch is operated *i.e.*, at $t < 0$,

$$Z = 1000 + j(314 - 6000) = 5773.27 \angle(-1.39 \text{ rad})\ \Omega$$

$\therefore$ $$|I_{max}| = \frac{100}{5773.24} = 0.0173 \text{ A}$$

at an angle of 79.64°

$\therefore$ $$I_{max} = 0.0173\angle 79.64° = (0.003 + j0.017) \text{ A}$$

This is steady state peak current before switching.

2. After $K$ closed, resistance is by passed and hence,

$$I'_{max} = \frac{100}{j(314-6000)} = -0.0176 \text{ at } 90°$$

$\therefore$ $|I'_{max}| = 0.0176$ and 90° (lag)

*i.e.*, $I_{s.state} = 0.0176 \sin(314t - 90°)\ (t > 0)$

$i(0^-) = i(0^+) = 0.017$ A

Now, root of $RLC$ circuit,



$$P^2 + \frac{R}{L}P + \frac{1}{LC} = 0$$

$$P_1, P_2 = \frac{-\frac{R}{L} \pm \sqrt{\frac{R^2}{L^2} - \frac{4}{LC}}}{2}$$

In this problem at $t = 0^-$, $R = 0$

$$P_1, P_2 = \pm\sqrt{\frac{-4}{LC}} = \pm j1.37 \times 10^3$$

$$\left[\because L = \frac{X_L}{2\pi f} = \frac{314}{314} = 1\text{ H}, C = \frac{1}{2\pi f X_c} = 5.3 \times 10^{-7}\text{ F}\right]$$

$\therefore$ $i_c = C_1 e^{P_1 t} + C_2 e^{P_2 t} = C_1 \cos(1.37 \times 10^3)t + C_2 \sin(1.37 \times 10^3)t$

$i_p$ being equivalent of steady state current.

$$i = i_c + i_p$$
$$= C_1 \cos(1.37 \times 10^3)t + C_2 \sin(1.37 \times 10^3)t + 0.0176 \cos 314t$$

4. When steady state current was flowing,

$$V_c(0^-) = 0.0173 \times 6000\angle(79.64° - 90°) = (102 - j18.66)\text{ V}$$

5. Across the capacitor, the reactive part of voltage will remain same at $t = 0^-$ and $t = 0^+$.

then, $$V_c(0^-) = V_c(0^+) = -18.66\text{ V}$$

6. Initial conditions are,

$$i(0^+) = 0.017\text{ A}$$
$$V_c(0^+) = -18.66\text{ V}$$

Also, $$i = 0.0176 \cos 314t + C_1 \cos(1.37 \times 10^3)t + C_2 \sin(1.37 \times 10^3)t$$

and $$\frac{di}{dt} = -0.0176 \times 314 \sin 314t - C_1 \times 1.37 \times 10^3 \sin(1.37 \times 10^3 t) + C_2 \times 1.37 \times 10^3 \cos(1.37 \times 10^3)t$$

7. With initial condition,

$$0.017 = 0.0176 + C_1$$

and $$18.66 = C_2 \times 1.37 \times 10^3$$

$\therefore$ $$C_1 = -6 \times 10^{-4}$$
$$C_2 = 13.7 \times 10^{-3}$$

Therefore, $$i = 0.0176 \cos 314t + (-6 \times 10^{-4} \cos 1.37 \times 10^3 t) + (13.7 \times 10^{-3}) \sin(1.37 \times 10^3)t$$

$\therefore$ $$i = (17.6 \cos 314t - 0.6 \cos(1.37 \times 10^3 t) + 13.7 \sin(1.37 \times 10^3 t)\text{ mA}$$

## VERY IMPORTANT QUESTIONS

*Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.*

**Q. 1.** In the circuit shown $v(t) = 2u(t)$ and $i_L(0^-) = 2$ amps. Find and sketch $i_2(t)$.

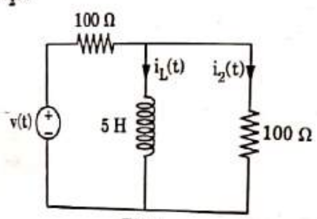



Fig. 1.

**Ans.** Refer Q. 3.7.

**Q. 2.** In the circuit shown in Fig. 2, the switch is kept closed for a long time and is then opened at $t = 0$.

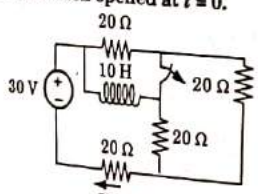



Fig. 2.

Find the values of current $i$ just before opening the switch ($t = 0^-$) and just after opening the switch ($t = 0^+$).

**Ans.** Refer Q. 3.8.

**Q. 3.** In the Fig. 3 shown, the ideal switch has been open for a long time. If it is closed at $t = 0$, then find the magnitude of current (in mA) through the 4 kΩ resistor at $t = 0^+$ ?

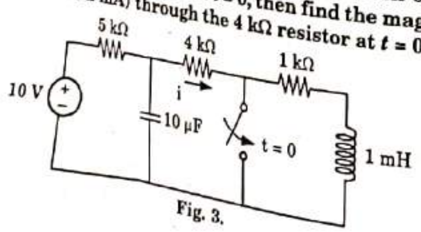



Fig. 3.



**Ans.** Refer Q. 3.9.

**Q. 4.** Find the voltage $V_o$ in the circuit of Fig. 4. The switch was open for a long time before it was closed at $t = 0$.

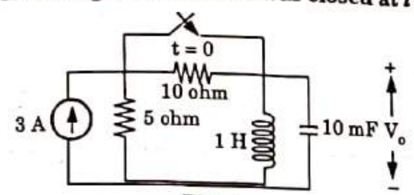



Fig. 4.

**Ans.** Refer Q. 3.11.

**Q. 5.** For RC series circuit switch is closed at $t = 0$, find out current expression also draw its transient curve.

**Ans.** Refer Q. 3.13.

**Q. 6.** In Fig. 5 the initial voltage in the capacitor is 1 V with the polarity as shown, find the voltage appearing across the capacitor using Laplace method with application of step voltage 8.

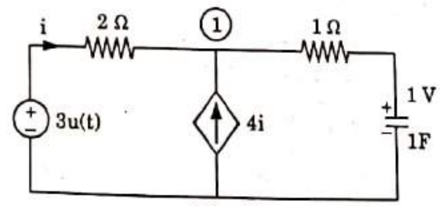



Fig. 5.

**Ans.** Refer Q. 3.14.

☺☺☺

4
UNIT

# Network Functions

## CONTENTS







## PART-1

*Pre-Requisites : Concepts of Basic Circuital Law, Parallel, Series Circuits*

### Questions-Answers

### Long Answer Type and Medium Answer Type Questions

**Que 4.1.** **Discuss the concept of basic circuital law.**

**Answer**

Refer Q. 1.1, Page 1–2C, Unit-1.

**Que 4.2.** **Explain the series and parallel circuits.**

**Answer**

i. **Resistances in series :**

1. When some conductors having resistances $R_1$, $R_2$, $R_3$ and $R_4$ etc. are joined end-to-end as shown in Fig. 4.2.1, they are said to be connected in series combination.

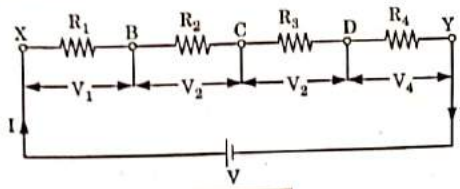



**Fig. 4.2.1.**

2. The equivalent resistance between $X$ and $Y$ is equal to the sum of the four individual resistances $R_1$, $R_2$, $R_3$ and $R_4$.
3. In a series circuit, magnitude of current is same through all four resistances.
4. However, the voltage drop across each resistance differs due to different magnitudes of resistances.
5. The supply voltage is equal to the sum of individual drop across the resistances in a series network and the current through each of the resistances is same.

5. Thus, $V = V_1 + V_2 + V_3 + V_4 = IR_1 + IR_2 + IR_3 + IR_4$ ...(4.2.1)
6. But, $V = IR$ (By Ohm's law) ...(4.2.2)

where $R$ is the equivalent resistance between $X$ and $Y$ of the series resistance combination.

7. From eq. (4.2.1) and eq. (4.2.2)

$$IR = IR_1 + IR_2 + IR_3 + IR_4$$

or

$$R = R_1 + R_2 + R_3 + R_4 \quad ...(4.2.3)$$

Also,

$$\frac{1}{G} = \frac{1}{G_1} + \frac{1}{G_2} + \frac{1}{G_3} + \frac{1}{G_4} \quad ...(4.2.4)$$

$G$ is the conductance of the path and is reciprocal of resistance.

ii. **Resistance in parallel :**

1. When more than one resistance is joined as shown in Fig. 4.2.2, the combination is said to be a parallel combination.

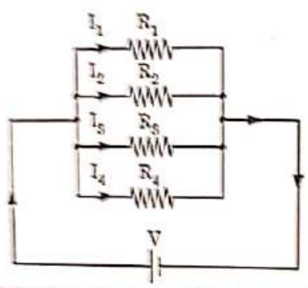



Fig. 4.2.2. Parallel combination of resistance.

2. In a parallel combination, voltage drop across each resistance is same, current in each branch is different and the total circuit current is the summation of all separate branch currents.

$$I = I_1 + I_2 + I_3 + I_4 \quad ...(4.2.1)$$

or

$$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} + \frac{V}{R_4} \quad ...(4.2.2)$$

But,

$$I = \frac{V}{R} \quad \text{(By Ohm's)} \quad ...(4.2.3)$$

where $R$ is the equivalent resistance of the parallel combination circuit.

3. From eq. (4.2.2) and (4.2.3), we have

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} + \frac{V}{R_4}$$



or

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \quad ...(4.2.4)$$

Also,

$$G = G_1 + G_1 + G_1 + G_1 \quad ...(4.2.5)$$

## PART-2

***Concept of Complex Frequency, Transform Impedances Network Functions of One Port and Two Port Networks, Concept of Poles and Zeros, Properties of Driving Point and Transfer Functions.***

### Questions-Answers

### Long Answer Type and Medium Answer Type Questions

**Que 4.3.** **Explain the concept of complex frequency.**

**Answer**

1. It is a generalised frequency whose real part $\sigma$ describes growth or decay of the amplitudes of signals and whose imaginary part $j\omega$ is the angular frequency.
2. This complex frequency is applicable for cisoidal signals where

$$r(t) = Ae^{j\omega t}$$

3. The angular frequency $\omega$ can be taken as a velocity at the end of the phasor $r(t)$ [since the velocity is also at right angles to the phasor].
4. Next, let us consider a general case when the velocity (symbolised as 's') is inclined with an angle $\theta$ as shown in Fig. 4.3.1(*b*).

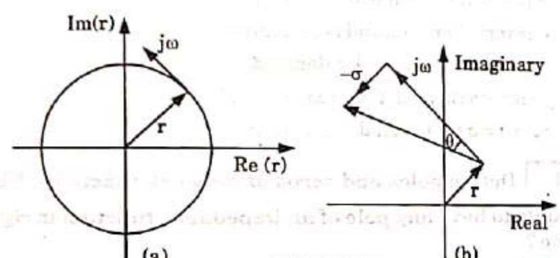



**Fig. 4.3.1.**

5. Here $s = -\sigma + j\omega$

[s is composed of a component $\omega$ at right angles to $r$ and another component $-\sigma$ parallel to $r$. $-\sigma$ component reduces magnitude of $r$ as it rotates counter clockwise towards origin]

6. $\therefore$ $Re\,[r(t)] = Ae^{-\sigma t}\cos \omega t$

$Im\,[r(t)] = Ae^{-\sigma t}\sin \omega t$ [as shown in Fig. 4.3.2(a) and (b)]

7. On the other hand, if $s = \sigma + j\omega$ as shown in Fig. 4.3.2(c), the phasor increases exponentially in magnitude.

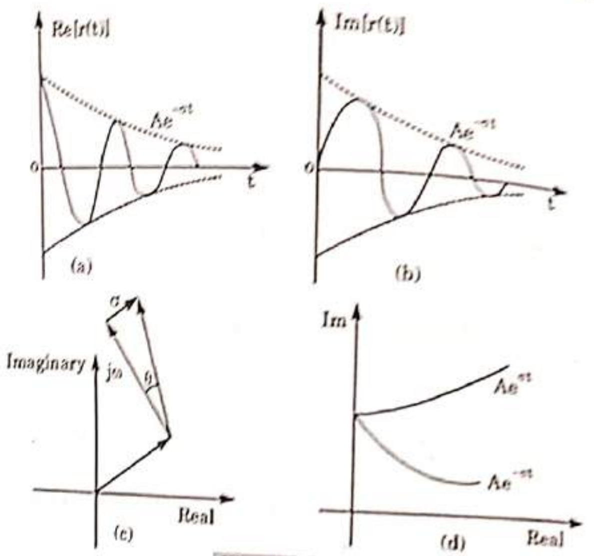



Fig. 4.3.2.

8. Then, in general, for a cisoidal signal,
$r(t) = Ae^{st} = Ae^{(\sigma + j\omega)t}$

9. If $\sigma$ is +ve then signal amplitude increases,
$\sigma$ is zero then sinusoid is undamped,
$\sigma$ is –ve then sinusoid is damped,
$j\omega$ is zero then signal is exponential,
$j\omega = 0 = \sigma$ then signal is constant.

**Que 4.4.** **Define poles and zeros of network functions. Why it is not possible to have any pole of an impedance function in right half of s-plane ?**

**Answer**

1. Any network function may be expressed in the form of a quotient of polynomial in $s$.



$$N(s) = \frac{P(s)}{Q(s)} = \frac{a_0 s^n + a_1 s^{n-1} + \ldots a_{n-1}s + a_n}{b_0 s^m + b_1 s^{m-1} + \ldots + b_{m-1}s + b_m} \quad ...(4.5.1)$$

where coefficients $a_0$ to $a_n$ and $b_0$ to $b_m$ are real and positive for a network containing passive elements only and containing no controlled sources.

$$N(s) = H\frac{(s - Z_1)(s - Z_2)\ldots(s - Z_n)}{(s - P_1)(s - P_2)\ldots(s - P_m)} \quad ...(4.5.2)$$

$H$ is a constant and called scale factor.

2. When the variable $s$ has value equal to any of roots $Z_1, Z_2 \ldots Z_n$, the network function $N(s)$ becomes zero. Hence these complex frequencies $Z_1, Z_2 \ldots Z_n$, are called the zeros of the network function.
3. When the variable $s$ has any of the values $P_1, P_2, \ldots P_m$, the network function $N(s)$ becomes infinite. Hence these complex frequencies $P_1, P_2 \ldots P_m$, are called the poles of the network function.
4. Partial fraction of eq. (4.5.2) is

$$N(s) = \frac{K_1}{s - P_1} + \frac{K_2}{s - P_2} + \ldots \frac{K_m}{s - P_m}$$

5. For the stability of the network, the poles should lie only in left-half and not in right-half of $s$-plane.

**Que 4.5.** **Explain properties of driving point function.**

**Answer**

1. The coefficients in the polynomials $P(s)$ and $Q(s)$ of network functions $N(s) = \dfrac{P(s)}{Q(s)}$ must be real and positive.
2. Poles and zeros must be conjugate; if imaginary or complex.
3. The real part of all poles and zeros must be negative or zero. If the real part is zero then the pole or zero must be simple.
4. The polynomial $P(s)$ and $Q(s)$ may not have missing terms between that of highest and lowest degree, unless all even or all odd terms are missing.
5. The degrees of $P(s)$ and $Q(s)$ may differ by either zero or one only.
6. The terms of lowest degree in $P(s)$ and $Q(s)$ may differ in degree by one at most.

**Que 4.6.** **Write the necessary conditions for the existence of transfer functions giving a suitable example.**

**AKTU 2014-15, Marks 05**

**Answer**

1. The coefficients in the polynomials $N(s)$ and $D(s)$ of $T = N/D$ must be real and those for $D(s)$ must be positive.
2. Poles and zeroes must be conjugate if imaginary or complex.
3. The real part of poles must be negative or zero, if the real part is zero, then that pole must be simple. This includes the origin.
4. The polynomial $D(s)$ may not have any missing term between that of highest and lowest degrees, unless all even all odd terms are missing.
5. The polynomial $N(s)$ may have terms missing, and some of the coefficients may be negative.
6. The degree of $N(s)$ may be as small as zero independent of the degree of $D(s)$.
7.
   a. **For $G$ and $\alpha$ :** The maximum degree of $N(s)$ is equal to the degree of $D(s)$.
   b. **For $Z$ and $Y$ :** The maximum degree of $N(s)$ is equal to the degree of $D(s)$ plus one.
8. **Example :** $G_{21}(s) = \dfrac{2s^2+5}{3s^2+9s+1}$

   Yes ; all conditions are satisfied.

**Que 4.7.** **The output voltage $V(s)$ of a network is given by :**

$$V(s) = \frac{4s}{(s+2)(s^2+2s+2)}$$

**Plot its pole-zero configuration and hence obtain $V(t)$.**

**Answer**

1. $$V(s) = \frac{4s}{(s+2)(s^2+2s+2)}$$

   $$\therefore \quad V(s) = \frac{4s}{(s+2)(s+1-j1)(s+1+j1)}$$

2. The poles are located at $s = -2, -1+j1, -1-j1$

   and zero at $s = 0$

3. Now $$V(s) = \frac{K_1}{s+2} + \frac{K_2}{(s+1-j1)} + \frac{K_3}{(s+1+j1)}$$
4. $$V(t) = K_1 e^{-2t} + K_2 e^{-(1-j1)t} + K_3 e^{-(1+j1)t}$$

   $$K_1 = H\left[\frac{M_{0-P}\angle\phi_{0P}}{M_{P_1-P}\angle\phi_{P_1P} \times M_{P_2-P}\angle\phi_{P_2P}}\right]$$



$$= 4\left[\frac{2\angle 180^\circ}{\sqrt{2}\angle -45^\circ \times \sqrt{2}\angle 45^\circ}\right] = 4\angle 180^\circ = -4$$

$$K_2 = H\left[\frac{M_{0-P_1}\angle\phi_{0P1}}{M_{P-P_1}\angle\phi_{PP1} \times M_{P_2-P_1}\angle\phi_{P_2P_1}}\right]$$

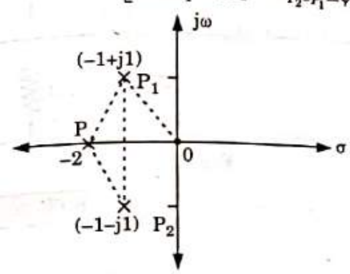



**Fig. 4.7.1.**

$$= 4\left[\frac{\sqrt{2}\angle 135^\circ}{\sqrt{2}\angle 45^\circ \times 2\angle 90^\circ}\right] = 2\angle 0 = 2$$

$$K_3 = K_2^* = 2$$

$$\therefore \quad V(t) = -4e^{-2t} + 2e^{-(1-j1)t} + 2e^{-(1+j1)t}$$

**Que 4.8.** **For the given network function, draw the pole zero diagram and hence obtain the time response $i(t)$**

$$I(s) = \frac{5s}{(s+1)(s^2+4s+8)}$$

**AKTU 2017-18, Marks 07**

**Answer**

The procedure is same as Q. 4.7, Page 4–7C, Unit-4.

(Ans. $i(t) = -0.625\, e^{-t} + 5e^{-(2-j2)t} + 5e^{-(2+j2)t}$)

**Pole-zero configuration :**

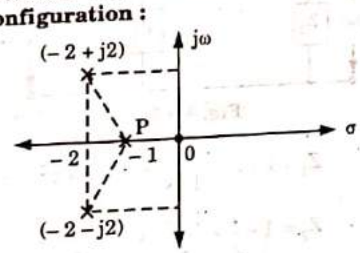



**Fig. 4.8.1.**

**Que 4.9.** **Obtain $V_2/V_1$ of the network shown in Fig. 4.9.1. Also find pole zero configuration.**

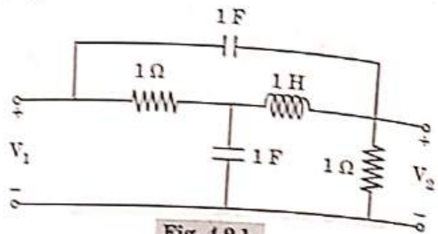



Fig. 4.9.1.

**AKTU 2018-19, Marks 07**

**Answer**

1. The s-domain circuit is shown in Fig. 4.9.1.

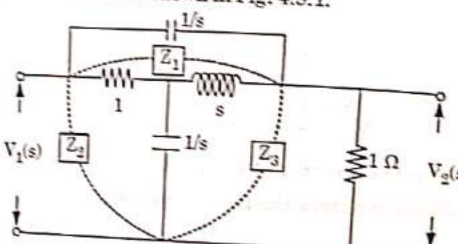



Fig. 4.9.2.

2. Now changing star to delta form :

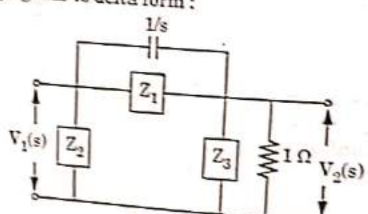



Fig. 4.9.3.

$$Z_1 = 1 + s + \frac{s}{1/s} = 1 + s + s^2$$

$$Z_2 = 1 + \frac{1}{s} + \frac{1/s}{s} = \frac{s^2 + 1 + s}{s^2}$$

$$Z_3 = s + \frac{1}{s} + \frac{1}{1} = \frac{s^2 + 1 + s}{s}$$



3. Now on solving the parallel combinations of $(Z_1 \,||\, (1/s))$ and $(Z_3 \,||\, 1)$ of Fig. 4.9.4.

Transfer function,

where,
$$Z_3 \,||\, 1 = \frac{(s^2 + s + 1)/s}{\left(\frac{s^2 + s + 1}{s}\right) + 1} = \frac{s^2 + s + 1}{s^2 + 2s + 1} \quad ...(4.9.1)$$

$$Z_1 \,||\, (1/s) = \frac{(s^2 + s + 1)/s}{s^2 + s + 1 + \frac{1}{s}} = \frac{s^2 + s + 1}{s(s^2 + s + 1) + 1} \quad ...(4.9.2)$$

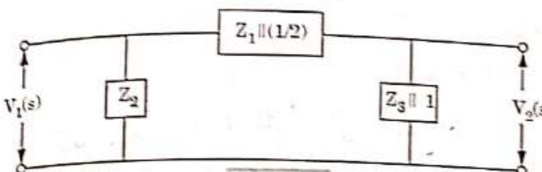



Fig. 4.9.4.

$$\frac{V_2(s)}{V_1(s)} = \frac{(Z_3 \,||\, 1)}{(Z_3 \,||\, 1) + (Z_1 \,||\, (1/s))}$$

4. Substituting values calculated in eq. (4.9.1) and (4.9.2), we get

$$\therefore \quad \frac{V_2(s)}{V_1(s)} = \frac{\frac{s^2 + s + 1}{s^2 + 2s + 1}}{\frac{s^2 + s + 1}{s^2 + 2s + 1} + \frac{s^2 + s + 1}{s(s^2 + s + 1) + 1}}$$

$$= \frac{(s^3 + s^2 + s + 1)}{(s^3 + s^2 + s + 1) + (s^2 + 2s + 1)}$$

$$= \frac{s^3 + s^2 + s + 1}{s^3 + 2s^2 + 3s + 2}$$

**Que 4.10.** **Consider the network shown in Fig. 4.10.1. Determine the transfer function.**

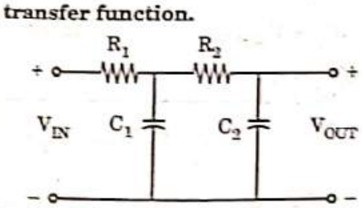



Fig. 4.10.1.

**Answer**

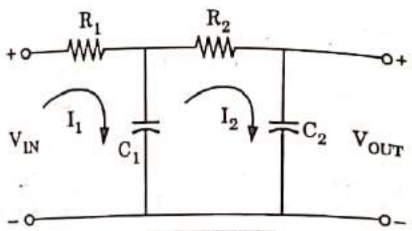



**Fig. 4.10.2.**

1. In 1[st] loop

$$V_{IN} = I_1(s)R_1 + (I_1(s) - I_2(s))\frac{1}{sC_1}$$

$$= I_1(s)R_1 + \frac{I_1(s)}{sC_1} - \frac{I_2(s)}{sC_1} = I_1\left(R_1 + \frac{1}{sC_1}\right) - \frac{I_2}{sC_1} \quad ...(4.10.1)$$

2. In 2[nd] loop

$$\frac{1}{sC_1}(I_2(s) - I_1(s)) + \left(R_2 + \frac{1}{sC_2}\right)I_2 = 0$$

$$\frac{I_2}{sC_1} - \frac{I_1}{sC_1} + R_2I_2 + \frac{I_2}{sC_2} = 0 \quad ...(4.10.2)$$

3. In 3[rd] loop $\quad V_{OUT} = \dfrac{I_2}{sC_2}$

4. From eq. (4.10.2) $\quad I_1 = sC_1I_2\left(\dfrac{1}{sC_1} + R_2 + \dfrac{1}{sC_2}\right) = I_2\left(1 + sC_1R_2 + \dfrac{C_1}{C_2}\right)$

5. Substituting in eq. (4.10.1)

$$I_2\left(1 + sC_1R_2 + \frac{C_1}{C_2}\right)\left(R_1 + \frac{1}{sC_1}\right) - \frac{I_2}{sC_1} = V_{IN}$$

$$V_{IN} = I_2\left(R_1 + sC_1R_1R_2 + R_2 + \frac{C_1R_1}{C_2} + \frac{1}{sC_2}\right)$$

6. But $\quad I_2 = V_{OUT}\,sC_2$

$$V_{IN} = V_{OUT}\,sC_2\left(R_1 + sC_1R_1R_2 + R_2 + \frac{C_1R_1}{C_2} + \frac{1}{sC_2}\right)$$

7.

$$\frac{V_{OUT}}{V_{IN}} = \frac{1}{sC_2\left(R_1 + sC_1R_1R_2 + R_2 + \dfrac{C_1R_1}{C_2} + \dfrac{1}{sC_2}\right)}$$

$$= \frac{1}{s^2C_1C_2R_1R_2 + s(R_1C_2 + R_2C_2 + R_1C_1) + 1}$$



**Que 4.11.** **A two terminal network consisting of a coil having inductance $L$ and resistance $R$ shunted by a capacitor $C$. The poles and zeros of the driving point impedance function $Z(s)$ of this network are shown in Fig. 4.11.1. Find the values of $L$, $R$ and $C$.**

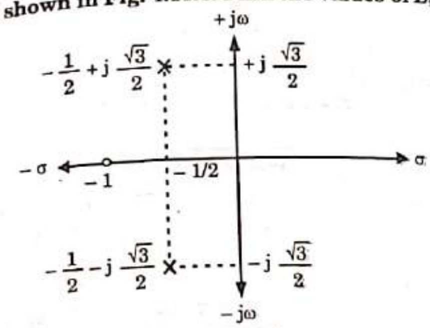



**Fig. 4.11.1.**

**Answer**

1.

$$Z(s) = \frac{(sL + R) \times \dfrac{1}{sC}}{sL + R + \dfrac{1}{sC}} = \frac{\dfrac{sL + R}{sC}}{\dfrac{s^2LC + RsC + 1}{sC}}$$

$$= \frac{sL + R}{s^2LC + RCs + 1} \quad ...(4.11.1)$$

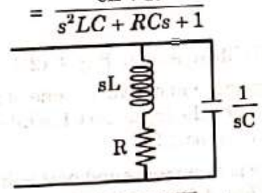



**Fig. 4.11.2.**

Given zeros are at –1

Poles at $\dfrac{-1}{2} + j\dfrac{\sqrt{3}}{2}$ and $\dfrac{-1}{2} - j\dfrac{\sqrt{3}}{2}$

2. Transfer function

$$= \frac{s - (-1)}{\left[s - \left(\dfrac{-1}{2} + j\dfrac{\sqrt{3}}{2}\right)\right]\left[s - \left(\dfrac{-1}{2} - j\dfrac{\sqrt{3}}{2}\right)\right]}$$

$$= \frac{s+1}{\left(s+\frac{1}{2}-j\frac{\sqrt{3}}{2}\right)\left(s+\frac{1}{2}+j\frac{\sqrt{3}}{2}\right)}$$

$$= \frac{s+1}{s^2+\frac{1}{4}+s+\frac{3}{4}} = \frac{s+1}{s^2+s+1} \quad ...(4.11.2)$$

3. Comparing eq. (4.11.1) and (4.11.2), we get
$$R = 1\,\Omega, L = 1\text{ H and } C = 1\text{ F}$$

## PART-3

*Two Port Networks-Characterization of LTI Two Port Networks, Z, Y, ABCD, A'B'C'D, g and h Parameters, Reciprocity and Symmetry, Inter-relationships Between the Parameters, Inter-Connections of Two Port Networks, Ladder and Lattice Networks : T & π Representation, Terminated two Port Networks, Image Impedance.*

**Questions-Answers**

**Long Answer Type and Medium Answer Type Questions**

**Que 4.12.** **Discuss the characterisation of linear time invariant two-port network.**

**Answer**

1. A two-port network is illustrated in Fig. 4.12.1.
2. By analogy with transmission networks, one of the ports (normally the port labelled 1-1') is called the input port 1, while the other (labelled port 2-2') is termed the output port 2.
3. The port variables are port currents and port voltages, with the standard references shown in Fig. 4.12.1.
4. External networks that may be connected at the input and output ports are called terminations.

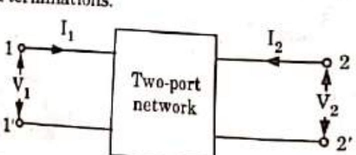



**Fig. 4.12.1.** Two-port network with standard reference directions for voltages and currents.



5. In order to describe the relationships among the port voltages and currents of a linear multi-port, as many linear equations are required as there are ports.
6. Thus, for a two-port, two linear equations are required among the four variables.
7. However, the choice of two 'independent' and two 'dependent' variables is a matter of convenience in a given application.

**Que 4.13.** **Define [z], [g], [h], [y], parameter regarding with two-port network.**

**OR**

**Explain Z-impedance parameter in detail.**

**AKTU 2016-17, Marks 10**

**OR**

**Explain admittance parameters in detail.**

**AKTU 2016-17, Marks 10**

**Answer**

A. **Z-impedance parameter :**

1. In Z-impedance parameter the expression of two-port voltages in terms of two-port currents, *i.e.*,
$$(V_1, V_2) = f(I_1, I_2)$$
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$
Or
$$[V] = [Z]\,[I]$$
2. Where [Z] is the open circuit impedance matrix of the two port network and impedances $Z_{ij}$ are the open circuit impedance (Z) parameters.
$$V_1 = Z_{11}I_1 + Z_{12}I_2 \quad ...(4.13.1)$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2 \quad ...(4.13.2)$$

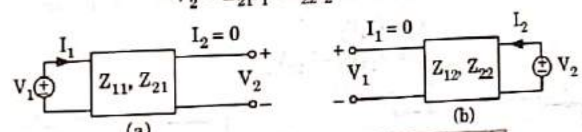



**Fig. 4.13.1.** Determination of Z-parameters.

3. In order to determine the Z-parameters, open the output port and applied some voltage $V_1$ to input port as shown in Fig. 4.13.1(a). We determine $I_1$ and $V_2$ to obtain $Z_1 L$ and $Z_{21}$.
4. Then, the input port is open circuited and the output port is excited with the voltage $V_2$ as shown in Fig. 4.13.1(b). The circuit is analysed to determine $I_2$ and $V_1$, so as to obtain $Z_{12}$ and $Z_{22}$. Mathematically;

5. **Case I :** $V_1 = V_1, I_1 = ?, V_2 = ?, I_2 = 0$ [Output port open circuit as shown in Fig. 4.13.1(a)].

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$$

(Input driving point impedance with the output port open circuited)

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

(Forward transfer impedance with the output port open circuited)

6. **Case II :** $V_2 = V_2, I_2 = ?, V_1 = ?, I_1 = 0$ [Input port open circuited as shown in Fig. 4.13.1(b)]

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

(Reverse transfer impedance with the input port open circuited)

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

(Output driving point impedance with the input port open circuited)

**B. Admittance parameters [Y] :**

1. In admittance parameter the expression of two-port current in terms of two-port voltages, *i.e.*,

$$(I_1, I_2) = f(V_1, V_2)$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

or $\quad [I] = [Y]\,[V]$

2. Where [Y] is the short circuit admittance matrix of the two port network and admittance $Y_{ij}$ are the short circuit admittance (Y) parameters.

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \quad \text{...(4.13.1)}$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 \quad \text{...(4.13.2)}$$

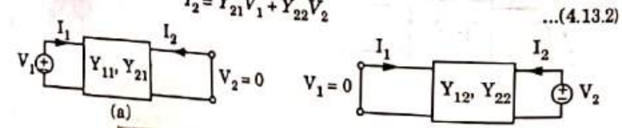



**Fig. 4.13.2. Determination of Y-parameters.**

3. In order to determine the Y-parameters, short the output port and applied some voltage $V_1$ to input port as shown in Fig. 4.13.1(a). We determine $I_1$ and $I_2$ to obtain $Y_{11}$ and $Y_{21}$.



4. Then, the input port is short circuited and the output port is excited with the voltage $V_2$ as shown in Fig. 4.13.1(b). The circuit is analysed to determine $I_2$ and $I_1$, so as to obtain $Y_{12}$ and $Y_{22}$. Mathematically;

5. **Case I :** $V_1 = V_1, I_1 = ?, I_2 = ?, V_2 = 0$ [Output port short circuited as shown in Fig. 4.13.1(a)].

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$

(Input driving point admittance with the output port short circuited)

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

(Forward transfer admittance with the output short circuit)

6. **Case II :** $V_2 = V_2, I_2 = ?, I_1 = ?, V_1 = 0$ [Input port short circuited as shown in Fig. 4.13.1(b)]

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

(Reverse transfer admittance with the input port short circuited)

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

(Output driving point admittance with the input port short circuited)

**C. Inverse hybrid parameter (g) :**

1. The inverse hybrid parameter represents a mixed or hybrid relation between the voltages and the currents in the two-port network.
2. The inverse hybrid parameter matrix may be written as

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

or

$$I_1 = g_{11}V_1 + g_{12}I_2$$

$$V_2 = g_{21}V_1 + g_{22}I_2$$

where,

$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0} \quad \text{(Open-circuit input admittance)}$$

$$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0} \quad \text{(Short-circuit reverse current gain)}$$

$$g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0} \quad \text{(Open-circuit voltage gain)}$$

$$g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0} \quad \text{(Short-circuit output impedance)}$$

**D. Hybrid parameters (*h*) :**

1. The hybrid parameters represent a mixed or hybrid relation between the voltages and the currents in the two-port network.
2. The hybrid parameter matrix may be written as

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

or,

$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \quad \text{(Short-circuit input impedance)}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} \quad \text{(Open-circuit reverse voltage gain)}$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} \quad \text{(Short-circuit current gain)}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} \quad \text{(Open-circuit output admittance).}$$

**Que 4.14.** **Define transmission and inverse transmission parameters.**

**Answer**

**A. Transmission parameters :**

1. The transmission parameters or chain parameters or *ABCD* parameter equation of the two port network can be written as

$$V_1 = AV_2 + B(-I_2)$$
$$I_1 = CV_2 + D(-I_2)$$

which in matrix form is,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

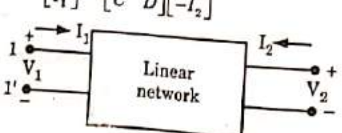



Fig. 4.14.1.



2. Transmission parameters are used in the analysis of power transmission line, where they are also known as "general circuit parameters".
3. The input port is called the sending end and the output port the receiving end.
4. Note that the variable used is $-I_2$ instead of $I_2$. The negative sign in this case arises from the fact that $I_2$ is considered outward.
5. Hence, the negative sign indicates that the current $(-I_2)$ is leading port 2. For cascade connection, transmission parameter representation is very useful.
6. The transmission parameters are :

a. The reverse voltage ratio with the receiving end open-circuited,

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$$

b. The transfer admittance with the receiving end open-circuited,

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$

c. The transfer impedance with the receiving end open-circuited,

$$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0}$$

d. The reverse current ratio with the receiving end short-circuited,

$$D = \left.\frac{I_1}{-I_2}\right|_{V_2=0}$$

**B. Inverse transmission parameters :**

1. The transmission parameter and inverse transmission parameter are duals of each other.
2. The inverse transmission parameters of the two-port network having directions of voltages and currents as shown in Fig. 4.14.2, are given by

$$V_2 = A'V_1 + B'(-I_1)$$

and

$$I_2 = C'V_1 + D'(-I_1)$$

which, in matrix form,

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix}$$

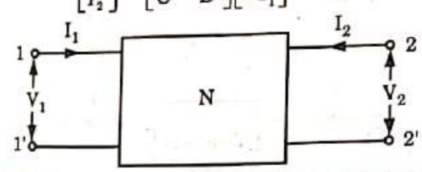



**Fig. 4.14.2. Two-port network for consideration of inverse transmission parameters.**

3. The inverse transmission parameters can be defined as :

a. Forward voltage ratio with sending end open-circuited,

$$A' = \left.\frac{V_2}{V_1}\right|_{I_1=0}$$

b. Transfer admittance with sending end open-circuited,

$$C' = \left.\frac{I_2}{V_1}\right|_{I_1=0}$$

c. Transfer impedance with the sending end short-circuited,

$$B' = \left.\frac{V_2}{-I_1}\right|_{V_1=0}$$

d. Forward current ratio with the sending end short-circuited,

$$D' = \left.\frac{I_2}{-I_1}\right|_{V_1=0}$$

**Que 4.15.** **What is meant by reciprocity and symmetry in two-port network ? Determine the conditions for reciprocity and symmetry in terms of all two-port parameters.**

**OR**

**What is meant by reciprocal and symmetric networks ? Explain with the help of an example.** **AKTU 2014-15, Marks 05**

**OR**

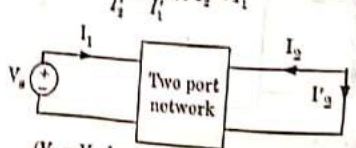
**Derive the condition of reciprocity and symmetry for *h*-parameters.** **AKTU 2014-15, Marks 05**

**Answer**

A. **Reciprocity :**

1. A two port network is said to be reciprocal, if the ratio of the excitation to response is invariant to an interchange of the position of the excitation and response in the network.
2. Networks containing resistors, inductors, and capacitors are generally reciprocal.
3. Networks that additionally have dependent sources are generally non-reciprocal. Mathematically, we can say from Fig. 4.15.1.

$$\frac{V_s}{I_2'} = \frac{V_s}{I_1'} \text{ or } I_2' = I_1'$$



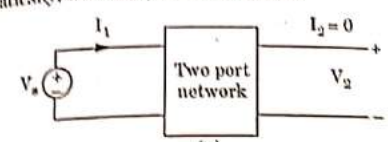

$(V_1 = V_s, I_1 = I_1, V_2 = 0, I_2 = -I'_2)$
(a)




$(V_2 = V_s, I_2 = I_2, V_1 = 0, I_1 = -I'_1)$
(b)

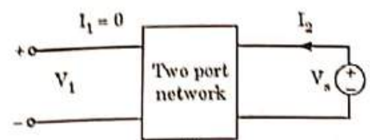
**Fig. 4.15.1.**

B. **Symmetric :** A two port network is said to be symmetrical if the ports can be interchanged without changing the port voltages and currents. Mathematically, we can say from Fig. 4.15.2.




(a)

$(V_1 = V_s, I_1 = I_1, I_2 = 0, V_2 = V_2)$




(b)

$(V_2 = V_s, I_2 = I_2, I_1 = 0, V_1 = V_1)$

**Fig. 4.15.2.**

$$\left.\frac{V_s}{I_1}\right|_{I_2=0} = \left.\frac{V_s}{I_2}\right|_{I_1=0}$$

I. **Condition in terms of Z-parameters :**

a. **Condition for reciprocity :**

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \quad ...(4.15.1)$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2 \quad ...(4.15.2)$$

$$V_1 = V_s, \quad I_1 = I_1, \quad V_2 = 0, \quad I_2 = -I'_2 \quad ...(4.15.3)$$

Put the value of eq. (4.15.3) in (4.15.1) and (4.15.2) then we get,

$$V_s = Z_{11}I_1 - Z_{12}I'_2 \quad ...(4.15.4)$$

$$0 = Z_{21}I_1 - Z_{22}I'_2 \quad ...(4.15.5)$$

Solving eq. (4.15.4) and (4.15.5) then we get,

$$I_2' = \frac{V_s Z_{21}}{Z_{11}Z_{22} - Z_{21}Z_{12}}$$

$$V_2 = V_s,\quad I_2 = I_2,\quad V_1 = 0,\quad I_1 = -I'_1 \qquad ...(4.15.6)$$

Put the value of eq. (4.15.6) in eq. (4.15.1) and (4.15.2) then we get

$$0 = -Z_{11} I_1' + Z_{12} I_2 \qquad ...(4.15.7)$$

$$V_s = -Z_{21} I_1' + Z_{22} I_2 \qquad ...(4.15.8)$$

Solving eq. (4.15.7) and (4.15.8) then we get

Hence $$I_1' = \frac{V_s Z_{12}}{Z_{11}Z_{22} - Z_{21}Z_{12}}$$

Comparing $I_2'$ and $I_1'$, we get

$$Z_{12} = Z_{21}$$

This is the condition of reciprocity in terms of Z-parameters.

b. **Condition for symmetry :**

From Fig. 4.15.2, $V_1 = V_s, I_1 = I_1, I_2 = 0, V_2 = V_2$.

$$V_s = Z_{11} I_1$$

$$\left.\frac{V_s}{I_1}\right|_{I_2=0} = Z_{11}$$

From Fig 4.15.2, $V_2 = V_s, I_2 = I_2, I_1 = 0, V_1 = V_1$.

$$V_s = Z_{22} I_2$$

$$\left.\frac{V_s}{I_2}\right|_{I_1=0} = Z_{22}$$

From the definition of symmetry, $\left.\frac{V_s}{I_1}\right|_{I_2=0} = \left.\frac{V_s}{I_2}\right|_{I_1=0}$ leads to

$$Z_{11} = Z_{22}$$

II. **Conditions in terms of Y-parameters :**

a. **Condition for reciprocity :**

1. From Fig. 4.15.3(a), writing the Y-parameter equations

$$I_1 = Y_{11} V_s$$

$$-I'_2 = Y_{21} V_s \qquad ...(4.15.5)$$

$$\Rightarrow \quad -\frac{I_2'}{V_s} = Y_{21}$$

2. From Fig. 4.15.3(b), writing the Y-parameter equations

$$-I'_1 = Y_{12} V_s$$

$$I_2 = Y_{22} V_s \qquad ...(4.15.6)$$



$$\Rightarrow \quad -\frac{I_1'}{V_s} = Y_{12}$$

From the principle of reciprocity, the condition for reciprocity is

$$Y_{12} = Y_{21}$$

b. **Condition for symmetry :** As already stated, a two-port network is said to be symmetric if the ports can be interchanged without changing the port voltages and currents and thus, the condition of symmetry becomes

$$Y_{11} = Y_{22}$$

III. **Conditions in terms of ABCD-parameters :**

a. **Condition for reciprocity :**

1. From Fig. 4.15.3(a), writing the ABCD-parameter equations

$$V_s = A, 0 - B(-I'_2) = BI'_2$$

$$I_1 = C, 0 - D(-I'_2) = DI'_2$$

$$\Rightarrow \quad \frac{I_2'}{V_s} = \frac{1}{B} \qquad ...(4.15.7)$$

2. From Fig. 4.15.1(b), writing the ABCD-parameter equations

$$0 = AV_s - BI_2$$

$$-I'_1 = CV_s - DI_2$$

$$\Rightarrow \quad \frac{I_1'}{V_s} = \frac{AD - BC}{B} \qquad ...(4.15.8)$$

3. From the principle of reciprocity, the condition for reciprocity is

$$\frac{1}{B} = \frac{(AD - BC)}{B}$$

$$(AD - BC) = 1$$

b. **Condition for symmetry :**

1. From eq. (4.15.7),

$$I_1 = DI'_2 = D\,\frac{V_s}{B} \qquad ...(4.15.9)$$

2. From eq. (4.15.8),

$$I_1 = \frac{I_1' + CV_s}{D} = \frac{1}{D}\left\{V_s\left(\frac{AD - BC}{B}\right) + CV_s\right\} = V_s\,\frac{A}{B} \qquad ...(4.15.10)$$

3. From eq. (4.15.9) and (4.15.10), we have the condition for symmetry as

$$A = D$$

IV. **Conditions in terms of h-parameters :**

a. **Condition for reciprocity :**

1. From Fig. 4.15.3(a), writing the h-parameter equations

$$V_s = h_{11} I_1 + h_{12} \cdot 0 = h_{11} I_1$$
$$-I'_2 = h_{21} I_1 + h_{22} \cdot 0 = h_{21} I_1$$

$$\Rightarrow \quad \frac{I'_2}{V_s} = -\frac{h_{21}}{h_{11}} \quad ...(4.15.11)$$

2. From Fig. 4.15.2(b), writing the *h*-parameter equations

$$0 = h_{11} I'_2 + h_{12} V_s$$

$$\Rightarrow \quad \frac{I'_1}{V_s} = \frac{h_{12}}{h_{11}} \quad ...(4.15.12)$$

3. From the principle of reciprocity, the condition for reciprocity is

$$h_{12} = -h_{21}$$

b. **Condition for symmetry :**

1. From eq. (4.15.11),

$$I_1 = \frac{V_s}{h_{11}} \quad ...(4.15.13)$$

2. From eq. (4.15.12),

$$I_2 = -h_{21}\left(\frac{h_{12}}{h_{11}} V_s\right) + h_{22} V_s = V_s \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{11}} \quad ...(4.15.14)$$

3. From eq. (4.15.13) and (4.15.14), we have the condition for symmetry as

$$(h_{11}h_{22} - h_{12}h_{21}) = 1$$

v. **Conditions in terms of inverse *T*-parameters :**

a. **Condition for reciprocity :**

1. From Fig. 4.15.1(*a*), writing the *T*-parameter equations

$$0 = A'V_s - B'I_1$$
$$-I'_2 = -C'V_s - DI_1 \quad ...(4.15.15)$$

$$\Rightarrow \quad \frac{I'_2}{V_s} = \frac{A'D' - B'C'}{B'}$$

2. From Fig. 4.15.1(*b*), writing the *T*-parameter equation

$$V_s = 0 - B'(-I'_1) = B'I'_1$$

$$\Rightarrow \quad \frac{I'_2}{V_s} = \frac{1}{B'} \quad ...(4.15.16)$$

$$I_2 = 0 - D'(-I'_1) = D'I'_1$$

3. From the principle of reciprocity, the condition for reciprocity is

$$(A'D' - B'C') = 1$$



b. **Condition for symmetry :**

1. From eq. (4.15.15),

$$0 = A'V_s - B'I_1$$
$$-I'_2 = C'V_s - D'I_1 \quad ...(4.15.17)$$

2. From eq. (4.15.16),

$$I_2 = \frac{D'}{B'} V_s \quad ...(4.15.18)$$

3. From eq. (4.15.17) and (4.15.18), we have the condition for symmetry as,

$$A' = D'$$

vi. **Conditions in terms of inverse hybrid (*g*)-parameters :**

a. **Condition for Reciprocity :**

1. From Fig. 4.15.1(*a*), writing the *g*-parameter equations

$$I_1 = g_{11} V_s - g_{12} I'_2$$
$$0 = g_{21} V_s - g_{22} I'_2 \quad ...(4.15.19)$$

$$\Rightarrow \quad \frac{I'_2}{V_s} = \frac{g_{21}}{g_{22}}$$

2. From Fig. 4.15.1(*b*), writing the *g*-parameter equations

$$-I'_1 = g_{11}(0) + g_{12} I_2 = g_{12} I_2$$
$$V_s = g_{21}(0) + g_{22} I_2 = g_{22} I_2 \quad ...(4.15.20)$$

$$\Rightarrow \quad \frac{I'_1}{V_s} = \frac{g_{12}}{g_{22}}$$

3. From the principle of reciprocity, the condition for reciprocity is

$$g_{12} = -g_{21}$$

b. **Condition for symmetry :**

1. From eq. (4.15.19), $I_1 = \left(\frac{g_{11}g_{22} - g_{12}g_{21}}{g_{22}}\right) V_s$ ...(4.15.21)

2. From eq. (4.15.20), $I_2 = \frac{1}{g_{22}} V_s$ ...(4.15.22)

3. From eq. (4.15.21) and (4.15.22), we have the condition for symmetry as,

$$(g_{11}g_{22} - g_{12}g_{21}) = 1$$

**Que 4.16.** **Explain in detail with figure (without proofs) the interconnection of all two-port networks.**

**Answer**

The two-port networks can be interconnected in various configurations :

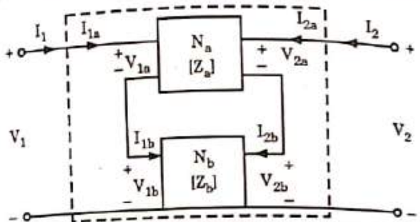



**Fig. 4.16.1.**

i. **Series connection :**
Series connection of two port network $N_a$ and $N_b$ with open circuit parameters $Z_a$ and $Z_b$.

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

where,
$$Z_{11} = Z_{11a} + Z_{11b}$$
$$Z_{12} = Z_{12a} + Z_{12b}$$
$$Z_{21} = Z_{21a} + Z_{21b}$$
$$Z_{22} = Z_{22a} + Z_{22b}$$

Z-parameter matrix of a series two-port network is the sum of the Z-parameter matrices of each individual two port network connected in series.

ii. **Parallel connection :**

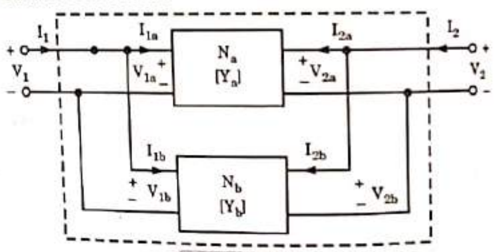



**Fig. 4.16.2.**

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

where,
$$Y_{11} = Y_{11a} + Y_{11b}$$
$$Y_{12} = Y_{12a} + Y_{12b}$$



$$Y_{21} = Y_{21a} + Y_{21b}$$
$$Y_{22} = Y_{22a} + Y_{22b}$$

Y-parameters matrix of parallel connected two-port network is the sum of the Y-parameter matrices of each individual two-port network.

iii. **Cascade connection :**

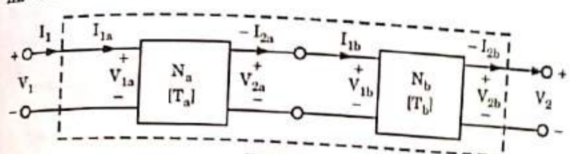



**Fig. 4.16.3.**

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

where,
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix}$$

$$A = A_a A_b + B_a C_b$$
$$B = A_a B_b + B_a D_b$$
$$C = C_a A_b + D_a C_b$$
$$D = C_a B_b + D_a D_b$$

T matrix for cascade connected two-port network is the matrix product of T-parameter of each individual two-port network.

iv. **Series-parallel connection :**

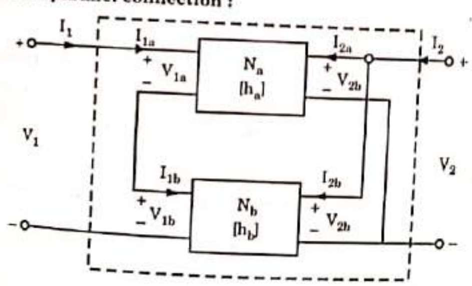



**Fig. 4.16.4.**

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

where,
$$h_{11} = h_{11a} + h_{11b}$$
$$h_{12} = h_{12a} + h_{12b}$$
$$h_{21} = h_{21a} + h_{21b}$$
$$h_{22} = h_{22a} + h_{22b}$$

The $h$-parameters matrix for series-parallel connected two-port network is the sum of individual $h$-parameter of each two-port network connected.

v. **Parallel-series connection :**

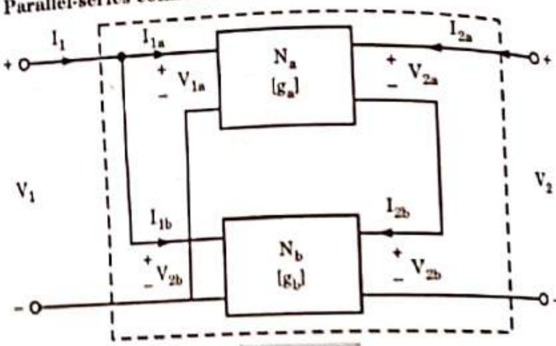



**Fig. 4.16.5.**

$$g_{11} = g_{11a} + g_{11b}$$
$$g_{12} = g_{12a} + g_{12b}$$
$$g_{21} = g_{21a} + g_{21b}$$
$$g_{22} = g_{22a} + g_{22b}$$

The $g$-parameter matrix for parallel-series connected two-port network is sum of $g$-parameter matrices of each individual two-port network connected.

**Que 4.17.** **Prove that the overall Z parameters of series connected two port networks are the sum of corresponding Z parameters of the two network.** **AKTU 2018-19, Marks 07**

**Answer**

**Series connection of two port networks :** Consider two 2-port networks $a$ and $b$ connected in series.

1. Fig. 4.17.1, shown a series connection of two 2-port networks with open circuit Z-parameters $Z_a$ and $Z_b$.
2. For network $N_a$

$$\begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix} = \begin{bmatrix} Z_{11a} & Z_{12a} \\ Z_{21a} & Z_{22a} \end{bmatrix} \begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix} \quad ...(4.17.1)$$



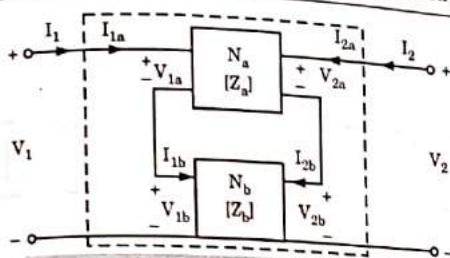



**Fig. 4.17.1.** Series connection of two-port network.

3. Similarly for network $N_b$

$$\begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix} = \begin{bmatrix} Z_{11b} & Z_{12b} \\ Z_{21b} & Z_{22b} \end{bmatrix} \begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix} \quad ...(4.17.2)$$

4. According to Fig. 4.17.1, at port 1

$$I_{1a} = I_{1b} = I_1, V_{1a} + V_{1b} = V_2$$

5. At port 2 :

$$I_{2a} = I_{2b} = I_2, V_{2a} + V_{2b} = V_2$$

6. So from eq. (4.17.1) and eq. (4.17.2)

$$V_{1a} = Z_{11a} I_{1a} + Z_{12a} I_{2a} \quad ...(4.17.3)$$
$$V_{1b} = Z_{11b} I_{1b} + Z_{12b} I_{2b} \quad ...(4.17.4)$$

7. Adding eq. (4.17.3) and eq. (4.17.4),

$$V_{1a} + V_{1b} = (Z_{11a} + Z_{11b}) I_1 + (Z_{12a} + Z_{12b}) I_2$$
$$V_1 = (Z_{11a} + Z_{11b}) I_1 + (Z_{12a} + Z_{12b}) I_2$$
$$V_{2a} = Z_{21a} I_{1a} + Z_{22a} I_{2a}$$
$$V_{2b} = Z_{21b} I_{1b} + Z_{22b} I_{2b}$$
$$V_2 = (Z_{21a} + Z_{21b}) I_1 + (Z_{22a} + Z_{22b}) I_2$$

8. In matrix form

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11a} + Z_{11b} & Z_{12a} + Z_{12b} \\ Z_{21a} + Z_{21b} & Z_{22a} + Z_{22b} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

9. In matrix form, Z parameters of equivalent network :

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

10. Hence
$$Z_{11} = Z_{11a} + Z_{11b}$$
$$Z_{12} = Z_{12a} + Z_{12b}$$
$$Z_{21} = Z_{21a} + Z_{21b}$$
$$Z_{22} = Z_{22a} + Z_{22b}$$

or
$$[Z] = [Z_a] + [Z_b]$$

**Que 4.18.** Derive the expression for ladder network shown in Fig 4.18.1.

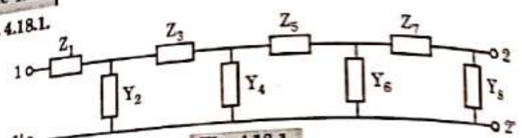



Fig. 4.18.1.

**Answer**

1. This is a general Ladder network. Each $Z$ or $Y$ may contain a number of resistance, inductance and capacitance in series or parallel or both.
2. The driving point impedance at port 1, with port 2 open, can be verified to be

$$Z_{11} = Z_1 + \cfrac{1}{Y_2 + \cfrac{1}{Z_3 + \cfrac{1}{Y_4 + \cfrac{1}{Z_5 + \cfrac{1}{Y_6 + ...}}}}}$$

Note : This type of equation is referred to as a continuous fraction.

**Que 4.19.** Explain lattice network in terms of Z-parameters.

**Answer**

1. Lattice network form the basis of design of most four-terminal network like attenuators, filters, etc.

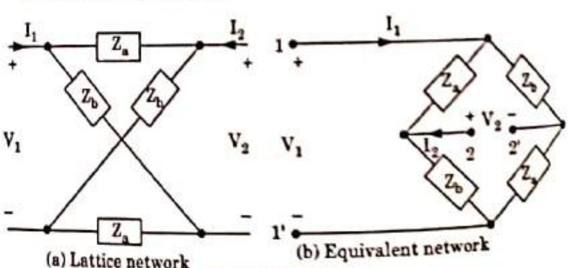



Fig. 4.19.1.

2. Here, $Z_a$ are the series arms and $Z_b$ are the diagonal or shunt arms as shown in Fig. 4.19.1(a).
3. To find the Z-parameters, we redraw the network as shown in Fig 4.19.1(b).



4. Assuming $I_2 = 0$, the current $I_1$ enters the bridge at point $A$ and divides equally between the two arms.

$$\therefore \quad \frac{I_1}{2} Z_a + V_2 = \frac{I_1}{2} Z_b \Rightarrow V_2 = \left(\frac{Z_b - Z_a}{2}\right)$$

$$\therefore \quad Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \left(\frac{Z_b - Z_a}{2}\right)$$

5. Also,
$$V_1 = \frac{I_1}{2}(Z_a + Z_b) \Rightarrow Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \left(\frac{Z_b + Z_a}{2}\right)$$

6. As the network is reciprocal and symmetrical,

$$\therefore \quad Z_{21} = Z_{12} = \left(\frac{Z_b - Z_a}{2}\right) \text{ and } Z_{11} = Z_{22} = \left(\frac{Z_b + Z_a}{2}\right)$$

$$\therefore \quad Z_a = (Z_{11} - Z_{12}) \text{ and } Z_b = (Z_{11} + Z_{12})$$

**Que 4.20.** Find the Y-parameter of the following π-circuit and draw the Y-parameter model.

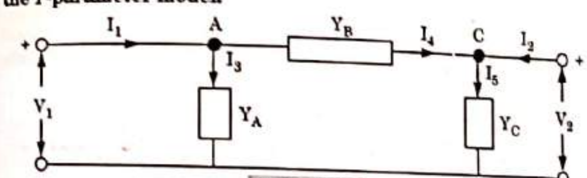



Fig. 4.20.1.

**Answer**

1. Using KCL at node $A$

$$I_1 = I_3 + I_4$$
$$I_1 = V_1 Y_A + (V_1 - V_2) Y_B$$
$$I_1 = V_1 (Y_A + Y_B) + (-Y_B) V_2 \quad ...(4.20.1)$$

2. Application of KCL at node $C$ results :

$$I_2 = I_5 - I_4$$
$$I_2 = V_2 Y_C - (V_1 - V_2) Y_B$$
$$I_2 = (-Y_B) V_1 + (Y_C + Y_B) V_2 \quad ...(4.20.2)$$

3. Comparing these two eq. (4.20.1) and (4.20.2)

$$Y_{11} = (Y_A + Y_B)$$
$$Y_{12} = -Y_B$$
$$Y_{21} = -Y_B$$
$$Y_{22} = Y_C + Y_B$$

4. Y-parameter model :

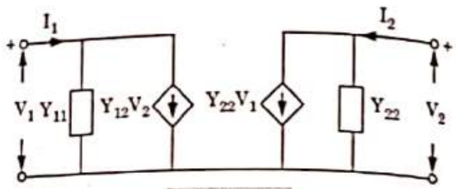



**Fig. 4.20.2.**

**Que 4.21.** **Prove that if two 2-port networks are connected in cascade, the transmission parameter matrix of the composite two port network is the product of the two individual transmission parameter matrices.**

**AKTU 2014-15, Marks 05**

**Answer**

1. The simplest possible interconnection of two-port networks is cascade or tandem-connection. Two two-port network are said to be connected in cascade if the output port of the first becomes the input port of the second as shown in Fig. 4.21.1.

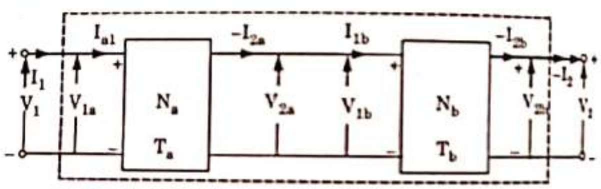



**Fig. 4.21.1.**

2. It the $T_a$ and $T_b$ are the $T$-parameters of the network $N_a$ and $N_b$ respectively. Then, for network $N_a$,

$$\begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_{2a} \\ -I_{2a} \end{bmatrix}$$

3. Similarly, for the network $N_b$,

$$\begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix} = \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix}$$

4. Then their cascade connection requires that

$$I_1 = I_{1a}, -I_{2a} = I_{1b}, I_{2b} = I_2$$
$$V_1 = V_{1a}, V_{2a} = V_{1b}, V_{2b} = V_2$$



5. Now,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_{2a} \\ -I_{2a} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix}$$

$$= \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix}$$

$$= \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

6. So, in matrix form the T-parameters of the cascade connected combined network can be written as

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Where $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix}$

7. Or in the equation form

$$A = A_a A_b + B_a C_b$$
$$B = A_a B_b + B_a D_b$$
$$C = C_a A_b + D_a C_b$$
$$D = C_a B_b + D_a D_b$$

8. Or in the matrix form

$$[T] = [T_a]\ [T_b]$$

**Que 4.22.** **What do you understand by image impedances ?**

**Answer**

**Image impedance :**

1. Consider a two-port network.
2. Let, $Z_{i1}$ = driving-point impedance at port 1 with impedance $Z_{i2}$ connected across port 2, and
   $Z_{i2}$ = driving-point impedance at port 2 with impedance $Z_{i1}$ connected across port 1.

   Then the impedance $Z_{i1}$ and $Z_{i2}$ are known as image impedances of the two-port network.
3. From Fig. 4.22.1(a), we get input impedance

$$Z_{i1} = \frac{AV_2 - BI_2}{CV_2 - DI_2}$$

But $V_2 = -I_2 Z_{i2}$

$$\therefore \quad Z_{i1} = \frac{AZ_{i2} + B}{CZ_{i2} + D} \quad ...(4.22.1)$$

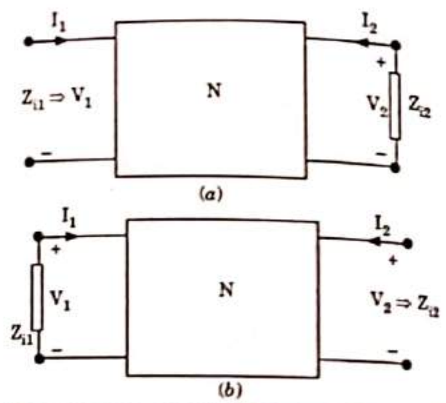



**Fig. 4.22.1.** Image impedance of two-port network.

4. Similarly, from Fig. 4.22.2(b), we get input impedance

$$Z_{i2} = \frac{DZ_{i1} + B}{CZ_{i1} + D} \quad ...(4.22.2)$$

5. These two expressions represent the image impedance in terms of ABCD parameters but they do not completely define a two-port network; a third parameter, called image transfer parameter is needed.
6. It is obtained as follows :

$$V_1 = AV_2 - BI_2 = \left[A + \frac{B}{Z_{i2}}\right] V_2 \; [\because V_2 = -I_2 Z_{i2}] \quad ...(4.22.3)$$

$$I_1 = CV_2 - DI_2 = -[CZ_{i2} + D] I_2 \quad ...(4.22.4)$$

7. From eq. (4.22.3)

$$\frac{V_1}{V_2} = \left(A + \frac{B}{Z_{i2}}\right) = \left(A + B\sqrt{\frac{AC}{BD}}\right)$$

$$= \left(A + \frac{\sqrt{ABCD}}{D}\right) \qquad \left[\because Z_{i2} = \sqrt{\frac{BD}{AC}}\right] \quad ...(4.22.5)$$

8. From eq. (4.22.4)

$$\frac{-I_1}{I_2} = (D + CZ_{i2}) = \left(D + C\sqrt{\frac{BD}{AC}}\right) = \left(D + \frac{\sqrt{ABCD}}{A}\right) \quad ...(4.22.6)$$



9. Multiplying eq. (4.22.5) and (4.22.6)

$$-\frac{V_1}{V_2} \times \frac{I_1}{I_2} = \frac{(AD + \sqrt{ABCD}\,)^2}{AD} = (\sqrt{AD} + \sqrt{BC})^2$$

$$\sqrt{\frac{-V_1}{V_2} \times \frac{I_1}{I_2}} = \sqrt{AD} + \sqrt{BC} = \sqrt{AD} + \sqrt{AD - 1}$$

$$(\because \; AD - BC = 1)$$

10. Let,

$$\sqrt{AD} = \cos h\theta, \; \sqrt{AD-1} = \sin h\theta$$

$$\sqrt{\frac{-V_1 I_1}{V_2 I_2}} = \cos h\theta + \sin h\theta = e^{\theta}$$

$$\theta = \log_e \sqrt{-\frac{V_1 I_1}{V_2 I_2}}$$

where, θ is called image transfer parameter.

**Que 4.23.** **Find the $Y$ and $h$ parameters of the network shown in Fig. 4.23.1.**

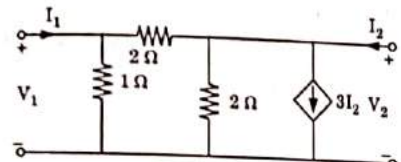



**Fig. 4.23.1.**

**AKTU 2014-15, Marks 05**

**Answer**

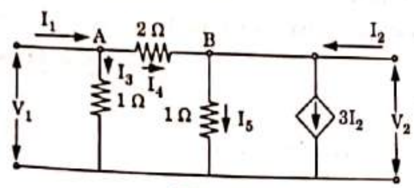



**Fig. 4.23.2.**

1. At node A,

$$I_1 = I_3 + I_4$$

$$I_1 = \frac{V_1}{1} + \frac{V_1 - V_2}{2}$$

$$2I_1 = 3V_1 - V_2$$

$$I_1 = 1.5V_1 - 0.5V_2 \quad ...(4.23.1)$$

2. At node B, $I_2 + I_4 = 3I_2 + I_5$

$$I_2 + \frac{V_1 - V_2}{2} = 3I_2 + \frac{V_2}{2}$$

$$2I_2 = \frac{V_1}{2} - V_2$$

$$I_2 = 0.25V_1 - 0.5V_2$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 1.5 & -0.5 \\ 0.25 & -0.5 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad ...(4.23.2)$$

3. So,
$$[Y] = \begin{bmatrix} 1.5 & -0.5 \\ 0.25 & -0.5 \end{bmatrix}$$

4. Now considering eq. (4.23.1)

$$1.5V_1 = I_1 + 0.5V_2$$

$$V_1 = \frac{2}{3}I_1 + \frac{1}{3}V_2 \quad ...(4.23.3)$$

and from eq. (4.23.2)

$$I_2 = 0.25\,V_1 - 0.5\,V_2 = 0.25\left[\frac{2}{3}I_1 + \frac{1}{3}V_2\right] - 0.5V_2$$

$$\therefore \quad I_2 = \frac{1}{6}I_1 - \frac{0.5}{3}V_2 \quad ...(4.23.4)$$

5.
$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 2/3 & 1/3 \\ 1/6 & -0.5/3 \end{bmatrix}\begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

6.
$$[h] = \begin{bmatrix} 2/3 & 1/3 \\ 1/6 & -0.5/3 \end{bmatrix}$$

**Que 4.24.** **Prove that the star-delta conversion does not bring any change in the Z-parameter matrix for the case of a resistive network.**

**AKTU 2014-15, Marks 05**

**Answer**

1. Let star and delta network are as follows :

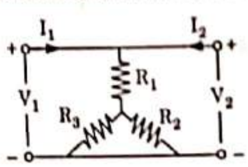



**Fig. 4.24.1.** Star network.

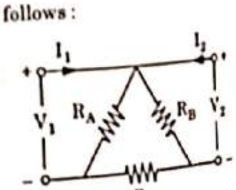



**Fig. 4.24.2.** Delta network.



2. For star network

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = R_1 + R_3 \quad ; \quad Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = R_1$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = R_1 \quad ; \quad Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = R_1 + R_2$$

$$[Z]_Y = \begin{bmatrix} R_1 + R_3 & R_1 \\ R_1 & R_1 + R_2 \end{bmatrix} \quad ...(4.24.1)$$

3. In case of delta network,

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = R_A \cdot \frac{(R_B + R_C)}{R_A + R_B + R_C}$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{R_A \cdot R_B}{R_A + R_B + R_C}$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{R_B \cdot (R_A + R_C)}{R_A + R_B + R_C}$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{R_B \cdot R_A}{R_A + R_B + R_C}$$

$$\therefore \quad [Z]_\Delta = \begin{bmatrix} \dfrac{R_A \cdot (R_B + R_C)}{R_A + R_B + R_C} & \dfrac{R_A \cdot R_B}{R_A + R_B + R_C} \\ \dfrac{R_B \cdot R_A}{R_A + R_B + R_C} & \dfrac{R_B \cdot (R_A + R_C)}{R_A + R_B + R_C} \end{bmatrix} \quad ...(4.24.2)$$

4. Using star-delta conversion,

$$\frac{R_A \cdot (R_B + R_C)}{R_A + R_B + R_C} = R_1 + R_3 \quad ...(4.24.3)$$

$$\frac{R_A \cdot R_B}{R_A + R_B + R_C} = R_1 \quad ...(4.24.4)$$

$$\frac{R_B \cdot (R_A + R_C)}{R_A + R_B + R_C} = R_1 + R_2 \quad ...(4.24.5)$$

5. From eq. (4.24.1), (4.24.2), (4.24.3) and (4.24.4), we have

$$[Z]_Y = [Z]_\Delta$$

**Que 4.25.** **Determine Y parameters for the network shown in Fig. 4.25.1.**

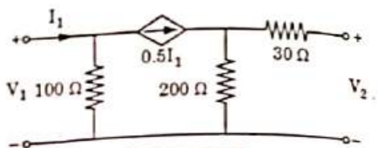



**Fig. 4.25.1.**

AKTU 2018-19, Marks 10

**Answer**

**Numerical :**

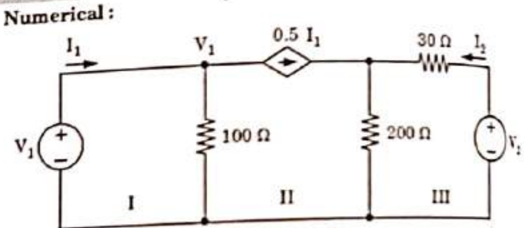



**Fig. 4.25.2.**

1. Applying KCL in loop I and II,

$$I_1 = 0.5\,I_1 + \frac{V_1}{100}$$

$$\Rightarrow \quad 0.5\,I_1 = \frac{V_1}{100}$$

$$\Rightarrow \quad V_1 = 50\,I_1$$

$$\Rightarrow \quad I_1 = \frac{V_1}{50} \quad \text{...(4.25.1)}$$

2. Applying KVL in loop III

$$V_2 = 30\,I_2 + (0.5\,I_1 + I_2)\,200$$

$$\Rightarrow \quad V_2 = 100\,I_1 + 230\,I_2 \quad \text{...(4.25.2)}$$

3. Now putting the value of $I_1$ in eq. (4.25.2)

$$V_2 = \frac{100}{50}V_1 + 230\,I_2$$

$$\Rightarrow \quad I_2 = \frac{-100\,V_1}{50 \times 230} + \frac{V_2}{230}$$

$$\Rightarrow \quad I_2 = -\frac{2}{230}V_1 + \frac{V_2}{230} \quad \text{...(4.25.3)}$$



4. Comparing eq. (4.25.2) and (4.25.3) with standard equation, we get

$$Y_{11} = \frac{1}{50}\,℧,\; Y_{21} = \frac{2}{230}\,℧$$

$$Y_{12} = 0\,℧,\; Y_{22} = \frac{2}{320}\,℧$$

**Que 4.26.** **Find $Y$ and $Z$ parameters of the network.**

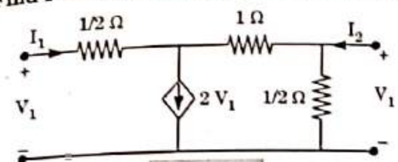



**Fig. 4.26.1.**

AKTU 2017-18, Marks 10

**Answer**

1. When $I_2 = 0$,

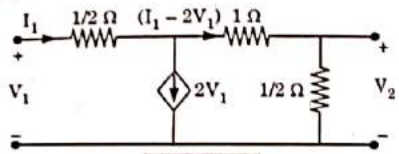



**Fig. 4.26.2.**

$$V_1 = \frac{1}{2}I_1 + (I_1 - 2V_1) + \frac{1}{2}(I_1 - 2V_1)$$

$$V_1 = 2I_1 - 3V_1$$

$$2V_1 = I_1$$

Now, $$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{1}{2}\,\Omega$$

Also, $$V_2 = (I_1 - 2V_1) \times \frac{1}{2}$$

$$V_2 = \left(I_1 - 2 \times \frac{I_1}{2}\right) \times \frac{1}{2}$$

$$V_2 = 0$$

Now, $$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = 0\,\Omega$$

2. When $I_1 = 0$,

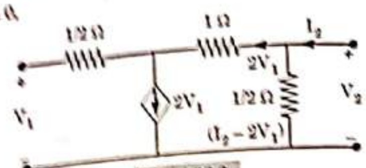



Fig. 4.26.3.

$$V_1 = -2V_1(1) + \frac{1}{2}(I_2 - 2V_1)$$

$$V_1 = -3V_1 + \frac{1}{2}I_2$$

$$8V_1 = I_2$$

Now, $$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{1}{8}\ \Omega$$

Also, $$V_2 = \frac{1}{2}(I_2 - 2V_1)$$

$$2V_2 = I_2 - 2\left(\frac{I_2}{8}\right)$$

$$8V_2 = 3I_2$$

Now, $$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{3}{8}\ \Omega$$

$\therefore$ $$Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 1/2 & 1/8 \\ 0 & 3/8 \end{bmatrix}\Omega$$

3. For $Y$ parameters

$$\Delta Z = Z_{11}Z_{22} - Z_{12}Z_{21}$$

$$= \frac{1}{2} \times \frac{3}{8} - \frac{1}{8} \times 0 = \frac{3}{16}$$

$$Y_{11} = \frac{Z_{22}}{\Delta Z} = \frac{3/8}{3/16} = 2\ \Omega^{-1}$$

$$Y_{12} = \frac{-Z_{12}}{\Delta Z} = \frac{-1/8}{3/16} = \frac{-2}{3}\ \Omega^{-1}$$

$$Y_{21} = \frac{-Z_{21}}{\Delta Z} = \frac{0}{3/16} = 0\ \Omega^{-1}$$

$$Y_{22} = \frac{Z_{11}}{\Delta Z} = \frac{1/2}{3/16} = \frac{8}{3}\ \Omega^{-1}$$



$$Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} 2 & -2/3 \\ 0 & 8/3 \end{bmatrix}\Omega^{-1}$$

**Que 4.27.** **Determine the $Z$-parameters of Fig. 4.27.1.**

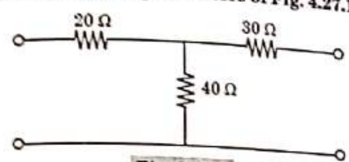



Fig. 4.27.1.

**AKTU 2016-17, Marks 10**

**Answer**

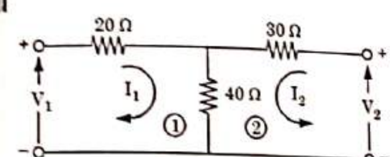



Fig. 4.27.2.

1. Using KVL in loop 1,

$$V_1 = 20\,I_1 + 40\,(I_1 + I_2) = 60\,I_1 + 40\,I_2 \qquad ...(4.27.1)$$

2. Using KVL in loop 2,

$$V_2 = 30\,I_2 + 40(I_1 + I_2) = 40\,I_1 + 70\,I_2 \qquad ...(4.27.2)$$

3. Converting eq. (4.27.1) and (4.27.2) in matrix form,

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 60 & 40 \\ 40 & 70 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

4. Hence, $Z$-parameter is

$$[Z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 60 & 40 \\ 40 & 70 \end{bmatrix}\Omega$$

**Que 4.28.** **On short circuit tests, the currents and voltages were determined experimentally for an unknown two port network as :**

| | |
|---|---|
| **at $V_2 = 0$** | **at $V_1 = 0$** |
| **$I_1 = 1$ mA ; $I_2 = 0.5$ mA ;** | **$I_1 = -1$ mA ; $I_2 = -10$ mA ;** |
| **$V_1 = 25$ V** | **$V_2 = 50$ V** |

**Determine the $Y$-parameters and draw the $Y$-parameter model.**

**AKTU 2015-16, Marks 10**

**Answer**

1. Equation of $Y$-Parameter :

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \quad \text{...(4.28.1)}$$
$$I_2 = Y_{21} V_1 + Y_{22} V_2 \quad \text{...(4.28.2)}$$

2. At $V_2 = 0$,

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$
$$= \frac{1\times10^{-3}}{25} = 0.04\times10^{-3}\ \Omega^{-1}$$
$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = \frac{0.5\times10^{-3}}{25} = 2\times10^{-5}\ \Omega^{-1}$$

3. At $V_1 = 0$

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{10\times10^{-3}}{50} = 2\times10^{-4}\ \Omega^{-1}$$
$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = \frac{1\times10^{-3}}{50} = 2\times10^{-5}\ \Omega^{-1}$$

4.
$$Y = \begin{bmatrix} 4\times10^{-5} & 2\times10^{-5} \\ 2\times10^{-5} & 2\times10^{-4} \end{bmatrix}$$

$Y$-parameter model,

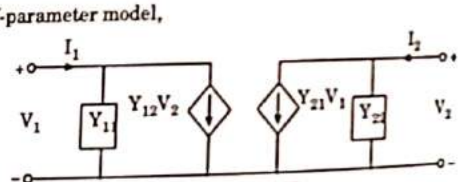



**Fig. 4.28.1.**

**Que 4.29.** **The current $I_1$ and $I_2$ at input and output port respectively of a two-port network can be expressed as : $I_1 = 5V_1 - V_2$; $I_2 = V_1 + V_2$.**

**i. Find the equivalent $\pi$-network.**

**ii. Find the input impedance when a load of $(3 + j5)\Omega$ is connected across the output port.**

**AKTU 2015-16, Marks $7\frac{1}{2}$**



**Answer**

**i. Equivalent $\pi$-network :**

1. Given equation is :

$$I_1 = 5V_1 - V_2 \quad \text{...(4.29.1)}$$
$$I_2 = V_1 + V_2 \quad \text{...(4.29.2)}$$

Let $\pi$ network is :

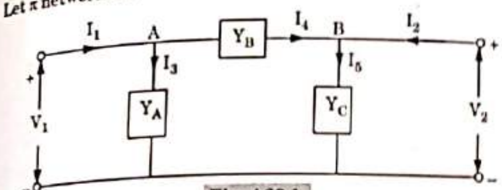



**Fig. 4.29.1.**

2. Using KCL at node $A$ in Fig. 4.29.1,

$$I_1 = I_3 + I_4$$
$$I_1 = V_1Y_A + (V_1 - V_2)Y_B$$
$$\therefore \quad I_1 = (Y_A + Y_B)V_1 - Y_BV_2 \quad \text{...(4.29.3)}$$

3. Using KCL at node $B$ in Fig. 4.29.1,

$$I_2 = I_5 - I_4$$
$$I_2 = I_5 - (V_1 - V_2)Y_B$$
$$I_2 = Y_CV_2 - V_1Y_B + Y_BV_2$$
$$I_2 = -V_1Y_B + (Y_B + Y_C)V_2 \quad \text{...(4.29.4)}$$

4. Comparing eq. (4.29.3) and (4.29.4) with eq. (4.29.1) and (4.29.2),

$$Y_B = 1$$
$$Y_B + Y_C = 1$$
$$\therefore \quad Y_C = 0$$
$$Y_A = 4$$

**ii. Input impedance**

1. Load $(3 + j5)\ \Omega$ is connected, then

$$V_2 = -I_2Z_1$$
$$= -I_2(3 + j5)$$

2. Putting in eq. (4.29.1) and (4.29.2),

$$I_1 = 5V_1 + I_2(3 + j5) \quad \text{...(4.29.5)}$$
$$I_2 = V_1 - I_2(3 + j5)$$
$$I_2 = V_1 - I_2(3 + j5)$$

$$I_2(4+j5) = V_1$$

$$\therefore \quad I_2 = \frac{V_1}{4+j5}$$

3. Putting value of $I_2$ in eq. (4.29.5)

$$I_1 = 5V_1 + \frac{V_1(3+j5)}{4+j5}$$

$$I_1 = V_1\left[\frac{20+j25+3+j5}{4+j5}\right]$$

$$\frac{V_1}{I_1} = \frac{4+j5}{23+j30} = \frac{6.40\angle 51°}{37.80\angle 52.52°}$$

4. Input impedance = $0.169 \angle -1.52°$

**Que 4.30.** **A network has two input terminals *a*, *b* and output terminals *c*, *d*. The input impedance with *c*-*d* open circuited is (250 + *j*100) Ω and with *c*-*d* short circuited is (400 + *j*3000) Ω. The impedance across *c*-*d* with *a*-*b* open circuited is 200 Ω. Determine equivalent *T*-network parameters.** **AKTU 2015-16, Marks $7\frac{1}{2}$**

**Answer**

1. Let $T$ network is :

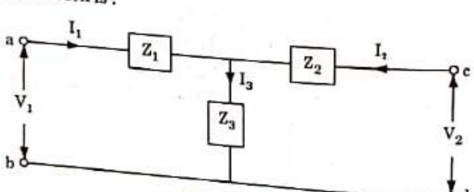


Fig. 4.30.1.

2. $c-d$ terminal is open-circuited

*i.e.,* $I_2 = 0$

$$I_1 = I_3 = \frac{V_1}{Z_1 + Z_3}$$

$$Z_1 + Z_3 = \frac{V_1}{I_1}$$

$$\therefore \quad Z_1 + Z_3 = 250 + j100\ \Omega \qquad ...(4.30.1)$$

3. Again $c-d$ is short circuited,



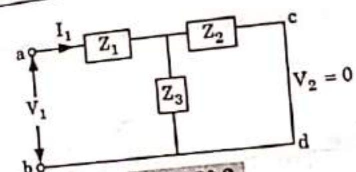


Fig. 4.30.2.

$$Z_1 + \frac{Z_2 Z_3}{Z_2 + Z_3} = \frac{V_1}{I_1} \qquad ...(4.30.2)$$

$$\therefore \quad Z_1 + \frac{Z_2 Z_3}{Z_2 + Z_3} = 400 + j3000$$

4. Subtracting eq. (4.30.1) from (4.30.2),

$$\frac{Z_2 Z_3}{Z_2 + Z_3} - Z_3 = 150 + j2900 \qquad ...(4.30.3)$$

$$\frac{Z_2 Z_3 - Z_2 Z_3 - Z_3^2}{Z_2 + Z_3} = 150 + j2900$$

5. $a-b$ terminal is open circuited.

Then, $I_1 = 0$

$$Z_2 + Z_3 = \frac{V_2}{I_2} \qquad ...(4.30.4)$$

$$\therefore \quad Z_2 + Z_3 = 200$$

6. Putting value of $(Z_2 + Z_3)$ in eq. (4.30.3)

$$\frac{-Z_3^2}{200} = 150 + j2900$$

$$-Z_3^2 = 200(150 + j2900)$$

$$-Z_3^2 = (75 + j1450) \times 400$$

$$Z_3^2 = (75 + j1450) \times 400$$

$$Z_3 = \sqrt{(-1)(20)^2(75 + j1450)}$$

$$= 20 \times j\sqrt{1452\angle 87°}$$

$$= j(760\angle 43.5)$$

$$= (-523.15 + j551.284)\ \Omega$$

7. **Since value of $Z_3$ comes negative which does not exists. Hence, values given are incorrect.**

**Que 4.31.** Determine *h*-parameters for the network shown in Fig. 4.31.1.

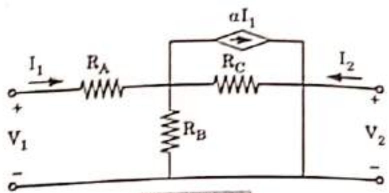



Fig. 4.31.1.

**AKTU 2018-19, Marks 07**

**Answer**

1. Short circuiting the output port as shown in Fig. 4.31.2.

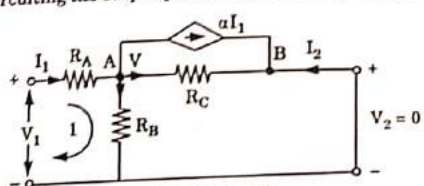



Fig. 4.31.2.

2. Applying KCL at node A

$$I_1 = \frac{V}{R_C} + \frac{V}{R_B} + \alpha I_1$$

$$V = \frac{(1-\alpha) R_B R_C I_1}{R_B + R_C} \quad \text{...(4.31.1)}$$

3. In loop 1 :

$\because$ $$V_1 = I_1 R_A + V$$

Putting value of $V$ from eq. (4.31.1),

$$V_1 = I_1 R_A + \left[\frac{(1-\alpha) R_B R_C I_1}{R_B + R_C}\right]$$

$$= I_1\left[R_A + \frac{(1-\alpha) R_B R_C}{R_B + R_C}\right]$$

$\therefore$ $$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = R_A + \frac{(1-\alpha) R_B R_C}{R_B + R_C}$$

4. Applying KCL at node B,



$$\frac{V}{R_C} + \alpha I_1 + I_2 = 0$$

$$I_2 = -\alpha I_1 - \frac{1}{R_C}\left[\frac{(1-\alpha) I_1 R_B R_C}{R_B + R_C}\right]$$

$$= I_1\left[-\alpha - \frac{(1-\alpha) R_B}{R_B + R_C}\right]$$

$$= -I_1\left[\frac{\alpha R_C + R_B}{R_B + R_C}\right]$$

$\therefore$ $$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = -\left[\frac{\alpha R_C + R_B}{R_B + R_C}\right]$$

5. For finding $h_{12}$ and $h_{22}$, open circuiting the input port as shown in Fig. 4.31.3.

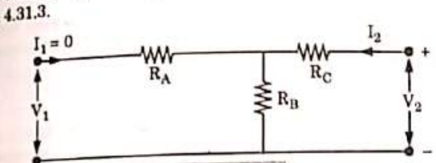



Fig. 4.31.3.

$$V_2 = I_2(R_B + R_C)$$

$$V_1 = I_2 R_B$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \frac{R_B}{R_B + R_C}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \frac{1}{R_B + R_C}$$

**VERY IMPORTANT QUESTIONS**

***Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.***

**Q. 1. Write the necessary conditions for the existence of transfer functions giving a suitable example.**

**Ans.** Refer Q. 4.6.

**Q. 2. For the given network function, draw the pole zero diagram and hence obtain the time response $i(t)$**

$$I(s) = \frac{5s}{(s+1)(s^2+4s+8)}$$

**Ans.** Refer Q. 4.8.

**Q. 3. Define [z], [g], [h], [y], parameter regarding with two-port network.**

**Ans.** Refer Q. 4.13.

**Q. 4. What is meant by reciprocity and symmetry in two-port network ? Determine the conditions for reciprocity and symmetry in terms of all two-port parameters.**

**Ans.** Refer Q. 4.15.

**Q. 5. Prove that the overall Z parameters of series connected two port networks are the sum of corresponding Z parameters of the two network.**

**Ans.** Refer Q. 4.17.

**Q. 6. Prove that if two 2-port networks are connected in cascade, the transmission parameter matrix of the composite two port network is the product of the two individual transmission parameter matrices.**

**Ans.** Refer Q. 4.21.

**Q. 7. Find the Y and h parameters of the network shown in Fig. 1.**


Fig. 2.

**Ans.** Refer Q. 4.23.

**Q. 8. Determine Y parameters for the network shown in Fig. 2.**

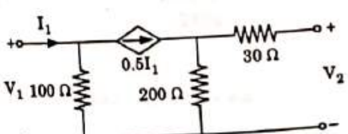



Fig. 2.



**Ans.** Refer Q. 4.25.

**Q. 9. Find Y and Z parameters of the network.**

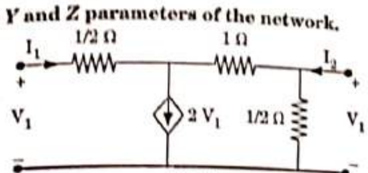



Fig. 3.

**Ans.** Refer Q. 4.26.

**Q. 10. Determine the Z-parameters of Fig. 4.**

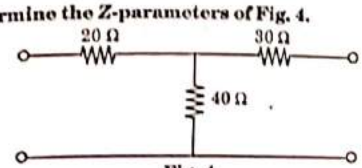



Fig. 4.

**Ans.** Refer Q. 4.27.

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# 5 UNIT

# Network Synthesis and Filters

## CONTENTS







## PART-1

*Pre-Requisites : Laplace Transform, Concept of Immittance Functions.*

### Questions-Answers

**Long Answer Type and Medium Answer Type Questions**

**Que 5.1.** **What do you understand by Laplace transform ? Find the Laplace transform of $te^{-at}$.**

**Answer**

A. **Laplace Transform :** Refer Q. 3.1, Page 3–2C, Unit-3.

B. **Numerical :** $f(t) = te^{-at}$

$$L(t) = \frac{1}{s^2}$$

$$L[te^{-at}] = \frac{1}{(s+a)^2}$$

**Que 5.2.** **Discuss the concept of immittance function.**

**Answer**

1. Driving point immittance function $[Z_{LC}(s)]$ or $[Y_{LC}(s)]$ is the ratio of even to odd or odd to even polynomials.
2. Since, both $E_i(s)$ and $O_i(s)$ are Hurwitz, they have only imaginary roots. Thus, the poles and zeros of the immittance function are on the imaginary ($j\omega$) axis.
3. The poles and zeros of the immittance function alternate on the imaginary axis.
4. The highest as well as the lowest power of numerator and denominator must differ by unity.
5. There must be either a zero or pole at origin ($s = 0$) and infinity ($s = \infty$).

## PART-2

*Positive Real Function, Definition and Properties, Properties of LC, RC and RL Driving Point Functions, Synthesis of LC, RC and RL Point Immittance Functions using Foster and Cauer First and Second Forms.*

Questions-Answers

Long Answer Type and Medium Answer Type Questions

**Que 5.3.** **Write five necessary conditions (or properties) for positive real function. Test whether given polynomial is positive real function or not $Z(s) = \frac{s^3 + 2s + 26}{s + 4}$.**

**AKTU 2018-19, Marks 07**

**Answer**

i. **Conditions :** The necessary and sufficient conditions to test positive real function are as follows :

1. All poles of the function should lie on the left half of the $s$-plane, they cannot lie in the right half of the $s$-plane.
2. Only simple poles with positive real residues can exist on the imaginary ($j\omega$) axis.
3. The poles and zeros of a positive real function are real or occur in conjugate pairs.
4. The highest power of the numerator and denominator polynomials can at most differ by one.
5. The lowest powers of the numerator and denominator polynomials can at most differ by one.

ii. **Numerical :**

$$Z(s) = \frac{N(s)}{D(s)} = \frac{s^3 + 2s + 26}{s + 4}$$

Now, $N(s) = s^3 + 2s + 26$

It can be seen that the term of $s^2$ is absent and the entire polynomial is not odd. Thus it does not satisfy the necessary condition of Hurwitz polynomial. So $N(s)$ is not Hurwitz.

Hence the given function is not positive real.

**Que 5.4.** **Explain the properties of positive real function, LC functions and RL functions.**

**AKTU 2014-15, Marks 10**



**Answer**

A. **Properties of positive real function (p.r.f.) :** Refer Q. 5.3, Page 5–3C, Unit-5.

B. **Properties of LC function :**

i. $Z_{LC}(s)$ and hence $Y_{LC}(s)$ are the ratio of even to odd or odd to even polynomials. (This property is also called as "Foster's Reactance Theorem").

ii. Since both $M_i(s)$ and $N_j(s)$ are Hurwitz, they have only imaginary roots, and it follow that the poles and zeros of $Z_{LC}(s)$ or $Y_{LC}(s)$ are on the imaginary axis (including origin).

iii. The poles and zeros interlace (or alternate) on the $j\omega$-axis.

iv. The highest powers of numerator and denominator must differ by unity.

v. The lowest powers of numerator and denominator must also differ by unity.

vi. There must be either a zero or a pole at the origin and infinity.

C. **Properties of RL Function :**

i. Poles and zeros lie on the negative real axis, and they are alternate.

ii. The residues of the poles must be real and positive.

iii. The critical frequencies nearest to the origin or at the origin must be a zero, whereas, the critical frequency nearest to infinity or at infinity must be a pole.

**Que 5.5.** **State the properties of RL driving point impedance function. Also realize the given network impedance function using foster form I**

$$Z(s) = (s + 1)(s + 3)/(s + 2)(s + 4)$$

**AKTU 2017-18, Marks 07**

**Answer**

A. **Properties of RL driving point impedance functions :** Refer Q. 5.4, Page 5–3C, Unit-5.

B. **Numerical :**

1. As zero is nearest to origin so it will be RL impedance function.

2. $$Z(s) = \frac{(s + 1)(s + 3)}{(s + 2)(s + 4)}$$

3. And, $$\frac{Z(s)}{s} = \frac{(s + 1)(s + 3)}{s(s + 2)(s + 4)}$$

4. Using partial fraction,

$$\frac{Z(s)}{s} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+4}$$

$$A = \left.\frac{(s+1)(s+3)}{(s+2)(s+4)}\right|_{s=0} = \frac{3}{8}$$

$$B = \left.\frac{(s+1)(s+3)}{s(s+4)}\right|_{s=-2} = \frac{-1}{-4} = \frac{1}{4}$$

$$C = \left.\frac{(s+1)(s+3)}{s(s+2)}\right|_{s=-4} = \frac{3}{8}$$

5. Therefore,

$$Z(s) = \frac{3}{8} + \frac{s/4}{s+2} + \frac{3s/8}{s+4} \qquad ...(5.5.1)$$

6. On comparing eq. (5.5.1) with

$$Z(s) = R_1 + \frac{R_2 s}{s + \frac{R_2}{L_2}} + \frac{R_3}{s + \frac{R_3}{L_3}}$$

$$R_1 = \frac{3}{8}\,\Omega;\ R_2 = \frac{1}{4}\,\Omega$$

$$R_3 = \frac{3}{8}\,\Omega;\ L_2 = \frac{1}{8}\,\text{H}$$

$$L_3 = \frac{3}{32}\,\text{H}$$

7. Realization of network is shown in Fig. 5.5.1.

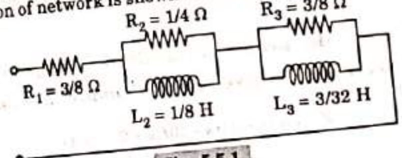



**Fig. 5.5.1.**

**Que 5.6.** **Give the method to find Foster and Cauer first and second forms for *LC* network.**

**Answer**

**A. Foster form-I :**

1. The *LC* impedance function can be written as

$$Z(s) = \frac{K_0}{s} + \frac{2K_2 s}{s^2 + \omega_2^2} + \frac{2K_4 s}{s^2 + \omega_4^2} + ... + K_\infty s$$

2. Foster form-I of *LC* impedance function is drawn as



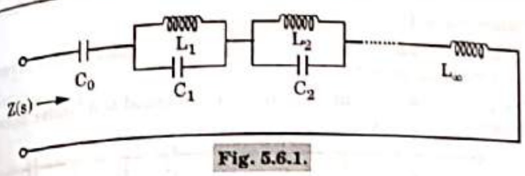



**Fig. 5.6.1.**

3. Here,

$$C_0 = \frac{1}{K_0};\ C_1 = \frac{1}{2K_2},\ L_1 = \frac{2K_2}{\omega_2^2}$$

$$L_\infty = K_\infty;\ C_2 = \frac{1}{4K_4},\ L_2 = \frac{2K_4}{\omega_4^2}$$

**B. Foster form-II :**

1. The *LC* admittance function can be written as

$$Y(s) = \frac{K_0}{s} + \frac{2K_2 s}{s^2 + \omega_2^2} + \frac{2K_4 s}{s^2 + \omega_4^2} + ... + K_\infty s$$

2. Foster form-II of *LC* admittance function can be drawn as

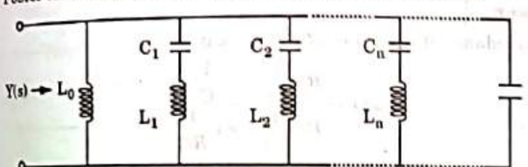



**Fig. 5.6.2.**

3. Here

$$L_0 = \frac{1}{K_0};\ L_1 = \frac{1}{2K_2};\ C_1 = \frac{2K_2}{\omega_2^2}$$

$$L_2 = \frac{1}{2K_4};\ C_2 = \frac{2K_4}{\omega_4^2};\ C_\infty = K_\infty$$

**C. Cauer form-I :**

1. In this, the numerator and denominator polynomials are arranged in the descending power of *s*.

2. The final network synthesized by this method is a ladder network as shown in Fig. 5.6.3.

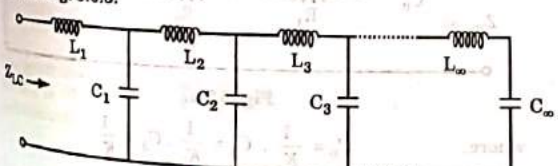



**Fig. 5.6.3.**

**B. Cauer form-II :**

1. In this the numerator and denominator polynomials are arranged in the ascending power of $s$.
2. The final network synthesised by this method is a ladder network as shown in Fig. 5.6.4.

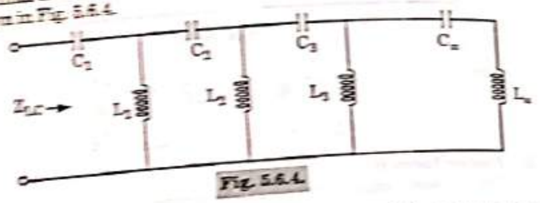



Fig. 5.6.4.

**Que 5.7.** Give the method to find Foster and Cauer first and second forms for *RC* and *RL* network.

**Answer**

1. Impedance of a parallel *RC* network is

$$Z_{RC}(s) = \frac{R \times \frac{1}{Cs}}{R + \frac{1}{Cs}} = \frac{\frac{1}{C}}{s + \frac{1}{RC}}$$

2. Admittance of a series *RL* network is

$$Y_{RL}(s) = \frac{1}{R + sL} = \frac{\frac{1}{L}}{s + \frac{R}{L}}$$

3. The Foster form-I of *RC* impedance function $Z_{RC}(s)$ is drawn as

$$Z_{RC}(s) = \frac{K_0}{s} + \frac{K_1}{s+\sigma_1} + \frac{K_2}{s+\sigma_2} + \ldots + K_\infty$$

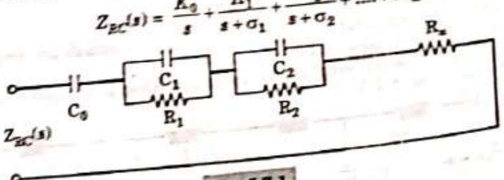



Fig. 5.7.1.

where,
$$C_0 = \frac{1}{K_0},\ C_1 = \frac{1}{K_1},\ C_2 = \frac{1}{K_2}$$



$$R_1 = \frac{K_1}{\sigma_1},\ R_2 = \frac{K_2}{\sigma_2},\ R_\infty = K_\infty$$

4. If $Y_{RC}$ is an *RC* admittance function, then Foster form-II of *RC* admittance function will be drawn as

$$Y_{RC}(s) = \frac{K_0}{s} + \frac{K_1}{s+\sigma_1} + \frac{K_2}{s+\sigma_2} + \ldots + K_\infty$$

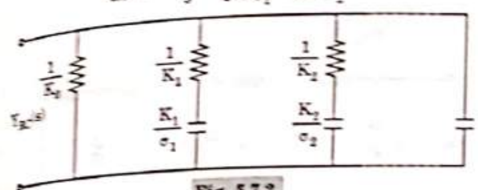



Fig. 5.7.2.

5. If $Z_{RL}(s)$ is *RL* impedance function, then Foster form-I will be drawn as

$$Z_{RL}(s) = \frac{K_0}{s} + \frac{K_1}{s+\sigma_1} + \frac{K_2}{s+\sigma_2} + \ldots + K_\infty$$

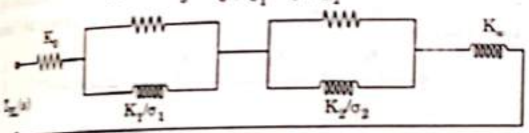



Fig. 5.7.3.

6. The Foster form-II of *RL* admittance function $Y_{RL}(s)$ is drawn as:

$$Y_{RL}(s) = 1 + \ldots + K_\infty$$

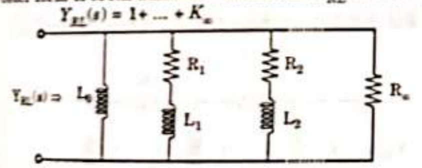



Fig. 5.7.4.

where
$$L_0 = \frac{1}{K_0},\ L_1 = \frac{1}{K_1},\ L_2 = \frac{1}{K_2}$$

$$R_1 = \frac{\sigma_1}{K_1},\ R_2 = \frac{\sigma_2}{K_2},\ R_\infty = K_\infty$$

7. The synthesis of *RC* impedance (or *RL* admittance) functions by Cauer form-I is obtained by continued fraction expansion method. The synthesized network is a ladder network.

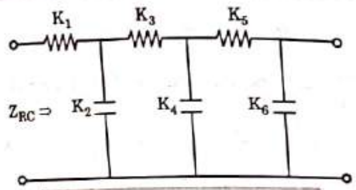



**Fig. 5.7.5.** Cauer form-I of $Z_{RC}(s)$.

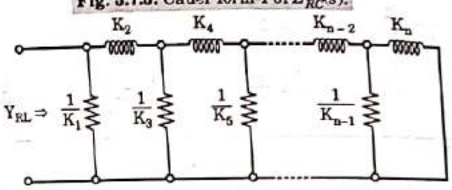



**Fig. 5.7.6.** Cauer form-I of *RL* admittance function.

where $K_1, K_2, K_3 \ldots K_n$ are quotients of continued fraction expansion.

8. The synthesis of *RC* impedance (or *RL* admittance) functions by Cauer-II is drawn as :

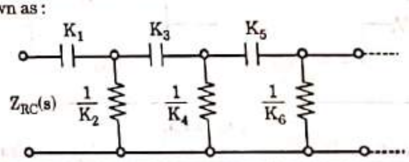



**Fig. 5.7.7.** Cauer form-II of *RC* impedance function.

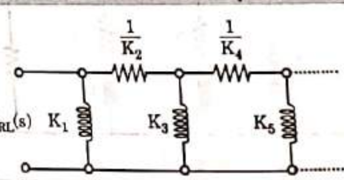



**Fig. 5.7.8.** Cauer form-II of *RL* admittance function.

**Que 5.8.** **Find the first order and second order Foster form of the driving point impedance function.**

$$Z(s) = \frac{2(s^2+1)(s^2+9)}{s(s^2+4)}$$

**AKTU 2015-16, Marks 15**



**Answer**

A. **Foster-I form :**

1. Since the order of the numerator polynomial exceeds that of the denominator polynomial, hence there is a simple pole at $\omega = \infty$.
2. On the other hand, the lowest order term of the numerator is of the order lower than that of the denominator.
3. Hence there is a simple pole at $\omega = 0$. Thus, there would be first and last elements present in the first form of Foster network.
4. Thus, the partial expansion gives,

$$Z(s) = \frac{A_0}{s} + \frac{A_2}{s+j2} + \frac{A_2^*}{s-j2} + Hs$$

where,
$$A_0 = \left.\frac{2(s^2+1)(s^2+9)}{s^2+4}\right|_{s=0} = \frac{2\times 9}{4} = 4.5$$

$$A_2 = \left.\frac{2(s^2+1)(s^2+9)}{s(s-j2)}\right|_{s=-j2} = \frac{2(-4+1)(-4+9)}{(-j2)\times(-j4)}$$

$$= \frac{-3\times 2\times 5}{-8} = 3.75$$

5. By inspection, $H = 2$
6. In the first form of Foster network,

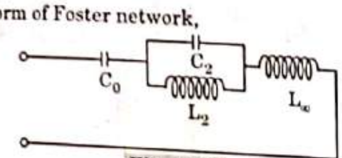



**Fig. 5.8.1.**

$$C_0 = \frac{1}{A_0} = \frac{1}{4.5}\text{ F}$$

$$L_\infty = H = 2\text{ H}$$

$$C_2 = \frac{1}{2A_2} = \frac{1}{2\times 3.75} = \frac{1}{7.5}\text{ F}$$

$$L_2 = \frac{2A_2}{\omega_2^2} = \frac{2\times 3.75}{2^2} = 1.875\text{ H}$$

B. **Foster-II form :**

1. In order to find the second Foster form, we will represent the given function into admittance form.

$$Y(s) = \frac{s(s^2+4)}{2(s^2+1)(s^2+9)}$$

2. Since there is one *s* term in the numerator and an excess term in the denominator than in the numerator, two zeros exist at $\omega = 0$ and at $\omega = \infty$.

3. Thus, the given network is simply a $LC$ parallel combination.
4. The presence of zero indicates that there would be no end elements in the 2nd form of network.
5. The partial fraction expansion is then $Y(s) = \frac{2B_1 s}{s^2+1} + \frac{2B_2 s}{s^2+9}$

$$\text{where } B_1 = \frac{1}{2} \cdot \frac{s(s^2+4)}{(s-j1)(s^2+9)}\bigg|_{s=-j1} = \frac{1}{2}\frac{(-j1)(-1+4)}{(-j2)(-1+9)} = \frac{3}{32}$$

$$B_2 = \frac{1}{2}\frac{s(s^2+4)}{(s^2+1)(s-j3)}\bigg|_{s=-j3} = \frac{1}{2}\frac{(-j3)(-9+4)}{(-9+1)(-j6)}$$

$$= \frac{15}{48 \times 2} = \frac{5}{32}.$$

6. The values of elements in the second form are thus

$$L_1 = \frac{1}{2B_1} = \frac{16}{3} \text{ H}$$

$$L_2 = \frac{1}{2B_2} = 3.2 \text{ H}$$

$$C_1 = \frac{2B_1}{\omega_1^2} = \frac{2 \times 3/32}{1} = \frac{3}{16} \text{ F}$$

$$C_2 = \frac{2B_2}{\omega_2^2} = \frac{2 \times 5/32}{5^2} = \frac{2 \times 5}{32 \times 5^2} = \frac{1}{80} \text{ F}$$

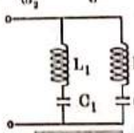



Fig. 5.8.2.

**Que 5.9.** **Synthesize the following network function in Cauer-2 form :**

$$Z(s) = \frac{8s^3+10s}{5+6s^2+s^4}$$

**AKTU 2015-16, Marks 10**

**Answer**

1. For using $Z(s)$ in continued fraction form, it is arranged in descending order polynomial for both the numerator and denominator.

$$Z(s) = \frac{10s+8s^3}{5+6s^2+s^4} = \frac{1}{\dfrac{5+6s^2+s^4}{10s+8s^3}}$$



$$10s + 8s^3 \,\big)\, 5 + 6s^2 + s^4 \,\big(\, \frac{1}{2s}$$
$$\underline{5 + 4s^2}$$
$$2s^2 + s^4 \,\big)\, 10s + 8s^3 \,\big(\, \frac{5}{s}$$
$$\underline{10s + 5s^3}$$
$$3s^3 \,\big)\, 2s^2 + s^4 \,\big(\, \frac{2}{3s}$$
$$\underline{2s^2}$$
$$s^4 \,\big)\, 3s^3 \,\big(\, \frac{3}{s}$$
$$\underline{3s^3}$$
$$\times$$

2. With $s \to 0$, $Y(s) \to \infty$, *i.e.*, $Z(s) \to 0$ then first element is absent.

$$Y(s) = \cfrac{1}{Z_1(s) + \cfrac{1}{Y_2(s) + \cfrac{1}{Z_3(s) + \cfrac{1}{Y_4(s) + \cfrac{1}{Z_5(s) + \cfrac{1}{Y_6(s)}}}}}}$$

3. Thus

$$Y_2(s) = \frac{1}{2s} = \frac{1}{L_2 s} \quad \therefore L_2 = 2 \text{ H}$$

$$Z_3(s) = \frac{5}{s} = \frac{1}{\frac{1s}{5}} = \frac{1}{C_3 s} \quad \therefore C_3 = \frac{1}{5} \text{ F}$$

$$Y_4(s) = \frac{2}{3s} = \frac{1}{L_4 s} \quad \therefore L_4 = \frac{3}{2} \text{ H}$$

$$Z_5(s) = \frac{3}{s} = \frac{1}{\frac{1s}{3}} = \frac{1}{C_5 s} \quad \therefore C_5 = \frac{1}{3} \text{ F}$$

4. Required network is

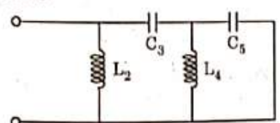



Fig. 5.9.1.

**Que 5.10.** With example, explain first Foster form realization of *LC* networks.

**AKTU 2016-17, Marks 15**

**Answer**

A. **Foster form-I realization :** Refer Q. 5.6, Page 5–5C, Unit-5.

B. **Example :**

1. Performing partial fraction, we get

$$Z(s) = \frac{A}{s+1} + \frac{B}{s+6} + \frac{C}{s+8}$$

$$s^2 + 12s + 3s = A(s+6)(s+8) + B(s+1)(s+8) + C(s+1)(s+6)$$

2. Solving, we get $A = \frac{24}{35}, B = \frac{1}{10}$ and $C = \frac{3}{14}$

$$Z(s) = \frac{\frac{24}{35}}{s+1} + \frac{\frac{1}{10}}{s+6} + \frac{\frac{3}{14}}{s+8}$$

$$C_1 = \frac{35}{24}, C_2 = 10, C_3 = \frac{14}{3}$$

$$R_1 = \frac{24}{35}, R_2 = \frac{1}{60}, R_3 = \frac{3}{112}$$

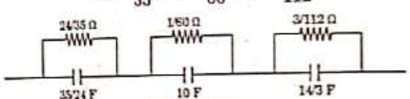



**Fig. 5.10.1.**

**Que 5.11.** Obtain Cauer form realization of following and obtain network.

$$Z(s) = \frac{(s+1)}{s(s+2)}; \quad Z(s) = \frac{(s+1)(s+3)}{s(s+2)}$$

**AKTU 2016-17, Marks 15**

**Answer**

A. $$Z(s) = \frac{s+1}{s(s+2)}$$

a. **Cauer form-I :**

1. The polynomial order of numerator is less than denominator. Also, with $s \to \infty$, $Z(s) \to 0$.

2. Hence, continued fraction expansion is performed as



$$s+1 \overline{)\, s^2 + 2s \,(}\; s; C_2 = 1\text{ F}$$
$$\underline{s^2 + s}$$
$$s \overline{)\, s + 1 \,(}\; 1; R_2 = 1\,\Omega$$
$$\underline{s}$$
$$1 \overline{)\, s \,(}\; s; C_3 = 1\text{ F}$$

3. Cauer form - I network :

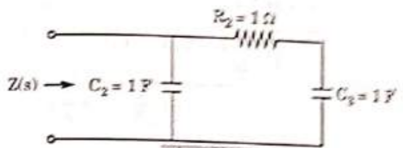



**Fig. 5.11.1.**

b. **Cauer form-II :**

$$Z(s) = \frac{1+s}{2s+s^2}$$

1. Continued fraction :

$$2s + s^2 \overline{)\, 1 + s \,(}\; \frac{1}{2s}; C_1 = 2\text{ F}$$
$$\underline{1 + \frac{1}{2}s}$$
$$\frac{1}{2}s \overline{)\, 2s + s^2 \,(}\; 4; R_2 = \frac{1}{4}\,\Omega$$
$$\underline{2s}$$
$$s^2 \overline{)\, \frac{1}{2}s \,(}\; \frac{1}{2s}; C_2 = 2\text{ F}$$
$$\underline{\frac{1}{2}s}$$
$$\underline{\times}$$

2. Cauer form-II network :

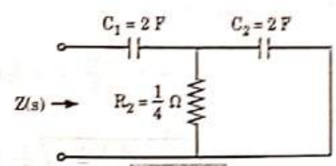



**Fig. 5.11.2.**

B. $$Z(s) = \frac{(s+1)(s+3)}{s(s+2)}$$

$$Z(s) = \frac{s^2 + 4s + 3}{s^2 + 2s}$$

a. *Cauer form-I :*

1. *Continued fraction :*

$$s^2 + 2s \,\big)\, s^2 + 4s + 3 \,\big(\, 1 \;;\; R_1 = 1\,\Omega$$
$$\underline{s^2 + 2s}$$
$$2s + 3 \,\big)\, s^2 + 4s \,\big(\, \frac{s}{2} \;;\; C_2 = \frac{1}{2}\,\text{F}$$
$$\underline{s^2 + \frac{3}{2}s}$$
$$\frac{5}{2}s \,\big)\, 2s + 3 \,\big(\, \frac{4}{5} \;;\; R_2 = \frac{5}{4}\,\Omega$$

$$\frac{1}{2}s \,\big)\, 2s + 3 \,\big(\, 4 \;;\; R_2 = 4\,\Omega$$
$$\underline{2s}$$
$$3 \,\big)\, \frac{1}{2}s \,\big(\, \frac{s}{6} \;;\; C_3 = \frac{1}{6}\,\text{F}$$
$$\underline{\frac{1}{2}s}$$
$$\times$$

2. *Cauer form-I network :*

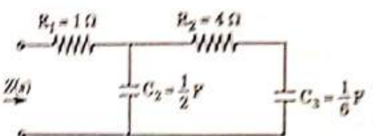


Fig. 5.11.3.

b. **Cauer form-II :**

1. **Continued fraction :**

$$2s + s^2 \,\big)\, 3 + 4s + s^2 \,\big(\, \frac{3}{2s} \;;\; C_1 = \frac{2}{3}\,\text{F}$$
$$\underline{3 + \frac{3}{2}s}$$
$$\frac{5}{2}s + s^2 \,\big)\, 2s + s^2 \,\big(\, \frac{4}{5} \;;\; R_2 = \frac{5}{4}\,\Omega$$
$$\underline{2s + \frac{4}{5}s^2}$$
$$\frac{1}{5}s^2 \,\big)\, \frac{5}{2}s + s^2 \,\big(\, \frac{25}{2s} \;;\; C_2 = \frac{2}{25}\,\text{F}$$
$$\underline{\frac{5}{2}s}$$
$$s^2 \,\big)\, \frac{1}{5}s^2 \,\big(\, \frac{1}{5} \;;\; R_3 = 5\,\Omega$$
$$\underline{\frac{1}{5}s^2}$$
$$\times$$



2. Cauer form-II network :

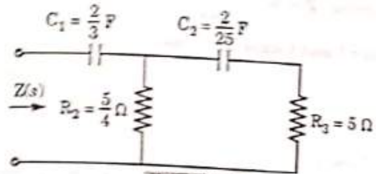


Fig. 5.11.4.

**Que 5.12.** **Test whether the polynomial $P(s)$ is Hurwitz or not :**

i. $s^5 + 3s^3 + 2s$  ii. $s^4 + 5s^3 + 5s^2 + 4s + 10$.

**AKTU 2017-18, Marks 10**

**Answer**

i.
$$P(s) = s(s^4 + 3s^2 + 2)$$

The polynomial $P(s)$ is not Hurwitz because a root of $P(s)$ lies at origin, which is not permitted according to Hurwitz polynomial.

ii.

1.
$$P(s) = s^4 + 5s^3 + 5s^2 + 4s + 10$$

2. Here, odd part is, $o(s) = 5s^3 + 4s$
   and even part is, $e(s) = s^4 + 5s^2 + 10$

3. Continued fraction expansion is, $C(s) = \dfrac{e(s)}{o(s)}$

$$5s^3 + 4s \,\big)\, s^4 + 5s^2 + 10 \,\big(\, s/5$$
$$\underline{s^4 + \frac{4s^2}{5}}$$
$$\frac{21}{5}s^2 + 10 \,\big)\, 5s^3 + 4s \,\big(\, \frac{25}{21}s$$
$$\underline{5s^3 + \frac{250}{21}s}$$
$$-\frac{166}{21}s \,\big)\, \frac{25}{5}s^2 + 10 \,\big(\, -\frac{441}{830}s$$
$$\underline{\frac{25}{5}s^2}$$
$$10$$

4. **The given polynomial is not Hurwitz because of presence of negative quotient term in the continued fraction expression.**

**Que 5.13.** Given $Z(s) = \dfrac{10(s^2+4)(s^2+6)}{s(s^2+5)}$

**Find the Foster I and Cauer II forms of network.**

**AKTU 2014-15, Marks 10**

**Answer**

**A. Foster-I form :**

1. Since order of numerator polynomial exceeds the denominator polynomial. Hence, there is a simple pole at $\omega = \infty$.
2. The lowest order term of the numerator is of the order lower than that of the denominator. There is a simple pole at $\omega = \infty$.

$$Z(s) = \frac{A_0}{s} + \frac{A_1}{(s+j\sqrt{5})} + \frac{A_2}{(s-j\sqrt{5})} + 10s$$

$$A_0 = \left.\frac{10(s^2+4)(s^2+6)}{(s^2+5)}\right|_{s=0} = 48$$

$$A_1 = \left.\frac{10(s^2+4)(s^2+6)}{s(s+j\sqrt{5})}\right|_{s=-j\sqrt{5}} = 2$$

$$C_0 = \frac{1}{A_0} = \frac{1}{48}\text{ F}$$

$$L_\infty = 10\text{ H}$$

$$C_1 = \frac{1}{2A_1} = \frac{1}{2\times 2} = \frac{1}{4}\text{ F}$$

$$L_1 = \frac{2A_1}{\omega_1^2} = \frac{2\times 2}{(\sqrt{5})^2} = \frac{4}{5}\text{ H}$$

3. The required network

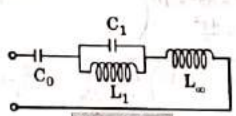



**Fig. 5.13.1.**

**B. Cauer-II form :**

1. $$Z(s) = \frac{10(s^2+4)(s^2+6)}{s(s^2+5)}$$

2. $$Z(s) = \frac{10(s^4+10s^2+24)}{s(s^2+5)} = \frac{240+100s^2+10s^4}{5s+s^3}$$



3. $$\begin{array}{l}
5s+s^3\,\big)\,240+100s^2+105s^4\,\big(\,\dfrac{48}{s} \\
\qquad\quad\; \underline{240+48s^2} \\
\qquad\quad\; 52s^2+10s^4\,\big)\,5s+s^3\,\big(\,\dfrac{5}{52s} \\
\qquad\qquad\qquad\qquad\;\; \underline{5s+\dfrac{25}{26}s^3} \\
\qquad\qquad\qquad\qquad\;\; \dfrac{s^3}{26}\,\big)\,52s^2+10s^4\,\big(\,\dfrac{1352}{s} \\
\qquad\qquad\qquad\qquad\qquad\qquad \underline{52s^2} \\
\qquad\qquad\qquad\qquad\qquad\qquad 10s^4\,\big)\,\dfrac{s^3}{26}\,\big(\,\dfrac{1}{260s} \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \underline{\dfrac{1}{26}s^3} \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \times
\end{array}$$

4. So, $$Z(s) = \frac{48}{s} + \cfrac{1}{\cfrac{5}{52s} + \cfrac{1}{\cfrac{1352}{s} + \cfrac{1}{\cfrac{1}{260s}}}}$$

5. Hence,

$$Z_1(s) = \frac{48}{s} = \frac{1}{C_1 s} \Rightarrow C_1 = \frac{1}{48}\text{ F}$$

$$Y_2(s) = \frac{5}{52s} = \frac{1}{L_2 s} \Rightarrow L_2 = \frac{52}{5}\text{ H}$$

$$Z_3(s) = \frac{1352}{s} = \frac{1}{C_3 s} \Rightarrow C_3 = \frac{1}{1352}\text{ F}$$

$$Y_4(s) = \frac{1}{2s} = \frac{1}{L_4 s} \Rightarrow L_4 = 2\text{ H}$$

6. The required network

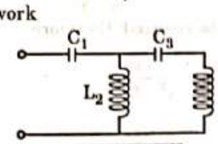



**Fig. 5.13.2.**

**Que 5.14.** **Synthesize :**

i. $Z(s) = (s+1)(s+3)/[(s+6)s]$ **in Cauer-I form.**
ii. $Z(s) = (s+5)/[(s+1)(s+6)]$ **in Foster's-II form.**

**AKTU 2018-19, Marks 07**

**Answer**

i. $$Z(s) = \frac{(s+1)(s+3)}{s(s+6)} \text{ in Cauer-}I\text{ form}$$

$$Z(s) = \frac{s^2+4s+3}{s^2+6s}$$

$$\begin{array}{l} s^2+6s\,\big)\,s^2+4s+3\,\big(\,1 \leftrightarrow Z_1 \\ \qquad\quad \underline{s^2+6s} \\ \qquad\quad -2s+3\,\big)\,s^2+6s\,\big(\,\frac{s}{2} \\ \qquad\qquad\qquad\quad -s^2+\frac{3s}{2} \end{array}$$

Representation is not possible.

Since a negative quotient has appeared in expression of continued fraction, hence.

ii. Given, $$Z(s) = \frac{(s+5)}{(s+1)(s+6)}$$

$$Y(s) = \frac{(s+1)(s+6)}{(s+5)}$$

$$\begin{array}{l} s+5\,\big)\,s^2+7s+6\,\big(\,1 \\ \qquad\quad \underline{s^2+5s} \\ \qquad\qquad 2s+6 \\ \qquad\qquad \underline{2s+10} \\ \qquad\qquad\quad -4 \end{array}$$

$$Y(s) = (s+2) + \frac{-4}{(s+5)}$$

This function can not be realized, therefore

$$\frac{Y(s)}{s} = \frac{s^2+7s+6}{s^2+5s}$$

$$\begin{array}{l} s^2+5s\,\big)\,s^2+7s+6\,\big(\,1 \\ \qquad\quad \underline{s^2+5s} \\ \qquad\qquad 2s+6 \end{array}$$

$$\frac{Y(s)}{s} = 1 + \frac{2s+6}{s(s+5)}$$



Using partial fraction

$$\frac{Y(s)}{s} = 1 + \frac{\frac{6}{5}}{s} + \frac{\frac{4}{5}}{s+5}$$

$$Y(s) = s + \frac{\frac{6}{5}s}{s} + \frac{\frac{4}{5}s}{s+5} = s + \frac{6}{5} + \frac{\frac{4}{5}s}{s+5}$$

The synthesized network,

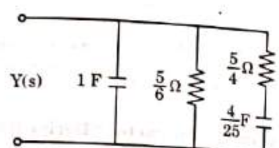



**Fig. 5.14.1**

## PART-3

*Pre-Requisites : Concept of Passive and Active Elements.*

**Questions-Answers**

**Long Answer Type and Medium Answer Type Questions**

**Que 5.15.** **Discuss the active and passive elements.**

**Answer**

A. **Active elements :**

1. The elements which supply energy to the network are known as active elements.
2. The voltage sources like batteries, DC generator, AC generator and current sources like photoelectric cells, metadyne generators fall under the category of active elements.

B. **Passive elements :** The components which dissipate or store energy are known as passive components. Resistors, inductors and capacitors fall under the category of passive elements.

## PART-4

*Image Parameters and Characteristics Impedance, Passive and Active Filter Fundamentals, Low Pass Filters, High Pass (Constant K Type) Filters, Introduction to Active Filters.*

### Questions-Answers

**Long Answer Type and Medium Answer Type Questions**

**Que 5.16.** **What is image parameter ? Derive its expression. What are active and passive filters ? Explain the advantages of active filters.**

**AKTU 2014-15, Marks 10**

**OR**

**Define active, passive filters. List advantages of active filter over passive filter.**

**AKTU 2018-19, Marks 07**

**OR**

**Explain the advantages of active filter in comparison to passive filter in detail.**

**AKTU 2017-18, Marks 07**

**Answer**

**A. Image parameter and its expression :**

1. We consider a two-port network. Let,

   $Z_{i1}$ = Driving point impedance at Port 1 with impedance $Z_{i2}$ connected across Port 2.

   $Z_{i2}$ = Driving point impedance at Port 2 with impedance $Z_{i1}$ connected across Port 1.

2. Then the impedances $Z_{i1}$ and $Z_{i2}$ are known as the image impedances of the two-port network.

3. From Fig. 5.16.1(a), we get the input impedance, $Z_{i1} = \dfrac{AV_2 - BI_2}{CV_2 - DI_2}$

   But, $V_2 = -I_2 Z_{i2}$

   $$\therefore \quad Z_{i1} = \frac{AZ_{i2} + B}{CZ_{i2} + D} \quad \text{...(5.16.1)}$$

4. Similarly, from Fig. 5.16.1(b), we get the input impedance



$$Z_{i2} = \frac{DZ_{i1} + B}{CZ_{i1} + D} \quad \text{...(5.16.2)}$$

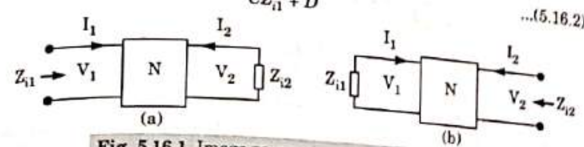



**Fig. 5.16.1.** Image parameters of a two-port network.

5. From eq. (5.16.1) and (5.16.2), we get

$$Z_{i1} = \sqrt{\frac{AB}{CD}}$$

$$Z_{i2} = \sqrt{\frac{BD}{AC}}$$

6. These two expressions represent the image impedances in terms of the *ABCD* parameters. However, these two image impedances do not completely define a two-port network; a third parameter, called image transfer parameter, is needed.

7. It is obtained as :

i. From Fig. 5.16.1(a), $V_1 = AV_2 - BI_2 = \left[A + \dfrac{B}{Z_{i2}}\right]V_2$ ...(5.16.3)

$$I_1 = CV_2 - DI_2 = -[CZ_{i2} + D]I_2 \quad \text{...(5.16.4)}$$

ii. From eq. (5.16.3), $\dfrac{V_1}{V_2} = \left(A + \dfrac{B}{Z_{i2}}\right)$

$$= \left(A + B\sqrt{\frac{AC}{BD}}\right) = \left(A + \frac{\sqrt{ABCD}}{D}\right) \quad \text{...(5.16.5)}$$

iii. From eq. (5.16.4), $-\dfrac{I_1}{I_2} = (D + CZ_{i2})$

$$= \left(D + C\sqrt{\frac{BD}{AC}}\right) = \left(D + \frac{\sqrt{ABCD}}{A}\right) \quad \text{...(5.16.6)}$$

iv. Multiplying eq. (5.16.5) and (5.16.6),

$$-\frac{V_1}{V_2} \times \frac{I_1}{I_2} = \frac{\left(AD + \sqrt{ABCD}\right)^2}{AD} = \left(\sqrt{AD} + \sqrt{BC}\right)^2$$

$$\sqrt{-\frac{V_1}{V_2} \times \frac{I_1}{I_2}} = \sqrt{AD} + \sqrt{BC}$$

$$= \sqrt{AD} + \sqrt{AD - 1} \quad (\because \; AD - BC = 1)$$

v. Let, $\sqrt{AD} = \cosh\theta$, $\sqrt{AD - 1} = \sinh\theta$

$$\therefore \sqrt{\frac{V_1 I_1}{V_2 I_2}} = \cosh \theta + \sinh \theta = e^{\theta}$$

$$\therefore \quad \theta = \log_e \sqrt{\frac{V_1 I_1}{V_2 I_2}}$$

where, $\theta$ is called the image transfer parameter.

**B. Active filters :** Active filters contain active components (such as operation amplifiers that introduce some gain into the signal) suitable *RC* feedback circuits are also combined to give them the desired frequency response characteristics.

**C. Passive filters :** Passive filters consist of impedances arranged in series and in parallel ($L - C$).

**D. Advantages of active filters :**

i. They eliminate the need for inductors which are large, heavy and costly, particularly in audio frequency range.

ii. They provide gain and isolation properties *i.e.*, high input impedance and low output impedance.

iii. Active filters are readily compatible with ICs due to absence of inductors.

iv. High range of $Q$-factor is possible.

v. Due to excellent isolation property, active filters can be designed and tuned independently with minimum interaction.

**Que 5.17.** **Give classification of filters.**

**AKTU 2016-17, Marks 10**

**Answer**

**Classification :** The filters are classified into four common types :

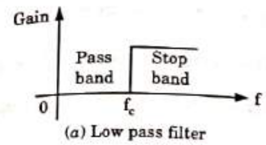
a. **Low pass filter :** These filters reject all frequencies above a specified value called the cut-off frequency. The attenuation characteristic of an ideal low pass filter is shown in Fig. 5.17.1(a).

b. **High pass filter :** These filters reject all frequencies below the cut-off frequency. The attenuation characteristic of a high pass filter is shown in Fig. 5.17.1(b).

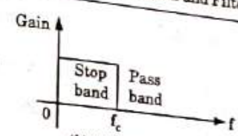
c. **Band pass filter :** A band pass filter passes or allows transmission of a band of frequencies and reject all frequencies beyond this band. As shown in Fig. 5.17.1(c).

d. **Band stop filter :** A band stop filter rejects or disallows transmission of a limited band of frequencies but allows transmission of all other frequencies and is shown in Fig. 5.17.1(d).

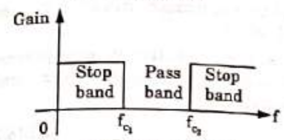




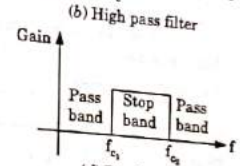

(*a*) Low pass filter




(*b*) High pass filter




(*c*) Band pass filter




(*d*) Band stop filter

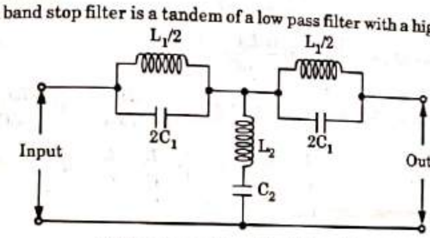
**Fig. 5.17.1.**

**Que 5.18.** **Explain in detail band stop filter with proof.**

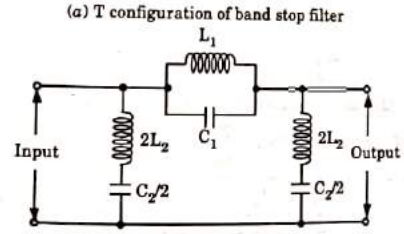
**AKTU 2016-17, Marks 10**

**Answer**

1. A band stop filter is a tandem of a low pass filter with a high pass filter.




(*a*) T configuration of band stop filter




(*b*) $\pi$ configuration of band stop filter

**Fig. 5.18.1.**

2. Here, the cut off frequency of high pass filter is higher than the cut-off frequency of the low pass filter.
3. The overlapping attenuation band of the two filters constitutes the stop band of the band stop filter.
4. In practice, the actions of LPF and HPF are combined into a single band stop filter.
5. $Z_1$ becomes the series impedance of a parallel tuned circuit and $Z_2$ is the shunt impedance of a series tuned circuit.
6. Operation is initiated when resonance occurs so that shunt impedance ($Z_2$) becomes minimum and series impedance ($Z_1$) becomes maximum.
7. Thus incident signal is blocked at this particular condition.

   Thus at resonance, $\omega_0^2 L_1 C_1 = 1$ ...(5.18.1)

   and $\omega_0^2 L_2 C_2 = 1$ ...(5.18.2)

8. From eq. (5.18.1) and (5.18.2), $L_1 C_1 = L_2 C_2 = \dfrac{1}{\omega_0^2}$ ...(5.18.3)

$$Z_1 = 2\left[\frac{j\dfrac{\omega L_1}{2}\left(\dfrac{1}{2j\,\omega C_1}\right)}{j\dfrac{\omega L_1}{2} + \dfrac{1}{2j\,\omega C_1}}\right] = j\frac{\omega L_1}{1-\omega^2 L_1 C_1} \quad \text{...(5.18.4)}$$

And, $Z_2 = j\,\omega L_2 + \dfrac{1}{j\,\omega C_2} = j\left[\dfrac{\omega^2 L_2 C_2 - 1}{\omega C_2}\right]$ ...(5.18.5)

9. Multiplying eq. (5.18.4) and (5.18.5), $Z_1 Z_2 = j\dfrac{\omega L_1}{1-\omega^2 L_1 C_1} \times j\left[\dfrac{\omega^2 L_2 C_2 - 1}{\omega C_2}\right]$

$$= \frac{L_1}{C_2} = \frac{L_2}{C_1} = R_0^2 \quad \left[\because \frac{L_1}{C_2} = \frac{L_2}{C_1} = R_0^2\right] \quad \text{...(5.18.6)}$$

10. The characteristic impedance is given by

$$Z_{0T}^2 = \frac{Z_1^2}{4} + Z_1 Z_2$$

$$\frac{Z_{0T}^2}{Z_1 Z_2} = \frac{Z_1}{4Z_2} + 1 \quad \text{...(5.18.7)}$$

11. For critical condition, $Z_{0T} = 0$

    Putting $Z_{0T} = 0$ in eq. (5.18.7), $\dfrac{Z_1}{4Z_2} = -1$

12. Multiplying both sides by $Z_1$,

$$Z_1 = -4Z_2$$

$$Z_1^2 = -4Z_1 Z_2 = -4R_0^2$$

Hence $Z_1 = \pm j2R_0$



13. $Z_1$ will be positive with $f_1$ and $Z_2$ will be negative with system frequency $f_2$. Considering $Z_1$ to be positive, $Z_1 = j\,2R_0$

$$j\frac{\omega_1 L_1}{1-\omega_1^2 L_1 C_1} = j\,2R_0$$

$$\omega_1 L_1 = 2R_0\,(1-\omega_1^2 L_1 C_1)$$

$$2\omega_1^2 L_1 C_1 R_0 + \omega_1 L_1 - 2R_0 = 0$$

$$\therefore \quad \omega_1 = \frac{-L_1 \pm \sqrt{L_1^2 + 16 L_1 C_1 R_0^2}}{4L_1 C_1 R_0}$$

$$f_1 = \frac{-L_1 \pm \sqrt{L_1^2 + 16 L_1 C_1 R_0^2}}{8\pi L_1 C_1 R_0}$$

$$= \frac{-1 \pm \sqrt{1 + 16\dfrac{C_1}{L_1}R_0^2}}{8\pi C_1 R_0}\ \text{Hz} \quad \text{...(5.18.8)}$$

14. Eq. (5.18.8) provides the lower cut-off frequency of the band stop filter. Considering the negative impedance of $Z_1$ for $f_2$,

$$Z_1 = -j2R_0$$

$$j\frac{\omega_2 L_1}{1-\omega_2^2 L_1 C_1} = -j2R_0$$

[$\omega_2$ being the frequency at which $Z_1 = -j2R_0$]

$$\frac{\omega_2 L_1}{\omega_2^2 L_1 C_1 - 1} = 2R_0$$

$$\omega_2 L_1 = 2R_0\,(\omega_2^2 L_1 C_1 - 1)$$

$$2R_0 \omega_2^2 L_1 C_1 - \omega_2 L_1 - 2R_0 = 0$$

$$\omega_2 = \frac{L_1 \pm \sqrt{L_1^2 + 16R_0^2 L_1 C_1}}{4R_0 L_1 C_1} = \frac{1 \pm \sqrt{1 + 16R_0^2\dfrac{C_1}{L_1}}}{4R_0 C_1}$$

$$f_2 = \frac{1 \pm \sqrt{1 + 16R_0^2\dfrac{C_1}{L_1}}}{8\pi R_0 C_1}\ \text{Hz} \quad \text{...(5.18.9)}$$

15. $f_2$ gives the higher cut-off frequency and hence eq. (5.18.8) and (5.18.9) gives the two cut off frequencies of the band stop filter.

**Que 5.19.** **Design a constant *k*-low pass filter having cut-off frequency 2.5 kHz, design resistance $R_0$ = 700 Ω. Also find the frequency at which the filter produces attenuation of 19.1 dB. Find the characteristics impedances and phase constant at pass band and stop or attenuation band.** **AKTU 2015-16, Marks 15**

**Answer**

1. Given, $f_c = 2.5$ kHz
   $R_o = 700\ \Omega$
2. The design elements of LPF are $L$ and $C$ given by the relations,
$$L = \frac{R_o}{\pi f_c}$$
$$= \frac{700}{3.14 \times 2.5 \times 10^3}$$
$$= 89.17 \times 10^{-3} = 0.089 \text{ H}$$
$$C = \frac{1}{\pi f_c R_o} = \frac{1}{3.14 \times 2.5 \times 10^3 \times 700}$$
$$= 1.82 \times 10^{-7}$$
$$= 0.182\ \mu\text{F}$$
3. Attenuation $\alpha = 19.1$ dB
$$= \frac{19.1}{8.686} = 2.199 \text{ Nepers}$$
4. Attenuation in attenuation band of a LPF is given by the relation
$$\alpha = 2\cosh^{-1}\left(\frac{f}{f_c}\right)$$
$$\frac{f}{f_c} = \cosh\left(\frac{2.19}{2}\right)$$
5. $\therefore$ $$f = f_c \cosh\left(\frac{2.19}{2}\right)$$
$$= 2.5 \times \cosh(1.095)$$
$$= 4.154 \text{ Hz}$$
6. Characteristics impedance of a $T$-type and $\pi$-type LPF are given by the relations,

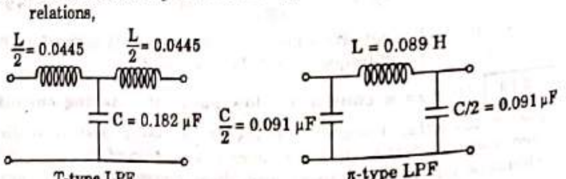



**Fig. 5.19.1.**



$$Z_{0T} = R_o\sqrt{1 - \left(\frac{f}{f_c}\right)^2}$$
$$= 700\sqrt{1 - \left(\frac{4.154}{2.5}\right)^2} = j928.898\ \Omega$$
$$Z_{0\pi} = \frac{R_o}{\sqrt{1 - \left(\frac{f}{f_c}\right)^2}} = \frac{R_o^2}{Z_{0T}} = \frac{(700)^2}{Z_{0T}} = -j527.52\ \Omega$$

7. In attenuation band the phase constant is given by,
$$\beta = \pi$$

**Que 5.20.** **Determine what type of filter is shown in Fig. 5.20.1? Calculate the corner or cut off frequency. Take $R = 2$ k and $L = 2$ H and $C = 2\ \mu$F.**

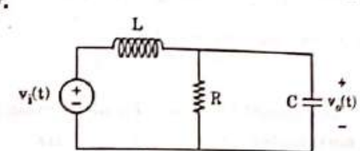



**Fig. 5.20.1.**

**AKTU 2016-17, Marks 15**

**Answer**

Assuming $R = 2$ kΩ.

1. The transfer function is
$$H(s) = \frac{V_o}{V_i} = \frac{R \| 1/sC}{sL + R \| 1/sC} \quad ...(5.20.1)$$
But
$$R \| \frac{1}{sC} = \frac{R/sC}{R + 1/sC} = \frac{R}{1 + sRC}$$
2. Substituting this into eq. (5.20.1) gives
$$H(s) = \frac{R/(1+sRC)}{sL + R/(1+sRC)} = \frac{R}{s^2RLC + sL + R}$$
Putting $s = j\omega$, $$H(\omega) = \frac{R}{-\omega^2RLC + j\omega L + R} \quad ...(5.20.2)$$
3. Here $H(0) = 1$ and $H(\infty) = 0$. Hence, the circuit in Fig. 5.20.1 is a second order low pass filter.
4. The magnitude of $H$ is $$H = \frac{R}{\sqrt{(R - \omega^2RLC)^2 + \omega^2L^2}} \quad ...(5.20.3)$$

5. The corner frequency is the same as the half-power frequency, *i.e.*, where $H$ is reduced by a factor of $\frac{1}{\sqrt{2}}$.

6. Since the DC value of $H(\omega)$ is 1, at the corner frequency, eq. (3) becomes after squaring,

$$H^2 = \frac{1}{2} = \frac{R^2}{(R - \omega_c^2 RLC)^2 + \omega_c^2 L^2}$$

$$2 = (1 - \omega_c^2 LC) + \left(\frac{\omega_c L}{R}\right)^2$$

7. Substituting the values of $R$, $L$, and $C$, we obtain

$$2 = (1 - \omega_c^2 4 \times 10^{-6})^2 + (\omega_c 10^{-3})^2$$

Assuming that $\omega_c$ is in krad/s,

$$2 = (1 - 4\omega_c)^2 + \omega_c^2 \quad \text{or} \quad 16\omega_c^4 - 7\omega_c^2 - 1 = 0$$

8. Solving the quadratic equation, we get

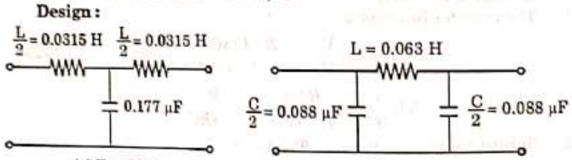
$$\omega_c = 0.742 \text{ krad/s} = 742 \text{ rad/s}$$

**Que 5.21.** **Design constant $K$ low pass $T$ and $\pi$ section filters to be terminated in 600 Ω having cut-off frequency 3 kHz.**

**AKTU 2017-18, Marks 07**

**Answer**

The procedure is same as Q. 5.20, Page 5–28C, Unit-5.
(Ans. $L$ = 0.063 H and $C$ = 0.177 μF)
**Design :**




(a) T-section (b) π-section

**Fig. 5.21.1.**



## VERY IMPORTANT QUESTIONS

*Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.*

**Q. 1. Write five necessary conditions (or properties) for positive real function. Test whether given polynomial is positive real function or not $Z(s) = \frac{s^3 + 2s + 26}{s + 4}$.**

**Ans.** Refer Q. 5.3.

**Q. 2. Explain the properties of positive real function, $LC$ functions and $RL$ functions.**

**Ans.** Refer Q. 5.4.

**Q. 3. State the properties of RL driving point impedance function. Also realize the given network impedance function using foster form I**

$$Z(s) = (s + 1)(s + 3)/(s + 2)(s + 4)$$

**Ans.** Refer Q. 5.5.

**Q. 4. Find the first order and second order Foster form of the driving point impedance function.**

$$Z(s) = \frac{2(s^2 + 1)(s^2 + 9)}{s(s^2 + 4)}$$

**Ans.** Refer Q. 5.8.

**Q. 5. Synthesize the following network function in Cauer-2 form :**

$$Z(s) = \frac{8s^3 + 10s}{5 + 6s^2 + s^4}$$

**Ans.** Refer Q. 5.9.

**Q. 6. With example, explain first Foster form realization of $LC$ networks.**

**Ans.** Refer Q. 5.10.

**Q. 7. Obtain Cauer form realization of following and obtain network.**

$$Z(s) = \frac{(s + 1)}{s(s + 2)}; \quad Z(s) = \frac{(s + 1)(s + 3)}{s(s + 2)}$$

**Ans.** Refer Q. 5.11.

**Q. 8. Synthesize :**

**i. $Z(s) = (s + 1)(s + 3) / [(s + 6)s]$ in Cauer-I form.**

**ii. $Z(s) = (s + 5)/[(s + 1)(s + 6)]$ in Foster's-II form.**

**Ans.** Refer Q. 5.14.

**Q. 9. Define active, passive filters. List advantages of active filter over passive filter.**

**Ans.** Refer Q. 5.16.

**Q. 10. Design a constant *k*-low pass filter having cut-off frequency 2.5 kHz, design resistance $R_0 = 700\ \Omega$. Also find the frequency at which the filter produces attenuation of 19.1 dB. Find the characteristics impedances and phase constant at pass band and stop or attenuation band.**

**Ans.** Refer Q. 5.19.

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# 1 UNIT

# Graph Theory (2 Marks Questions)

**1.1. What do you mean by graph of a network ?**

**Ans.** It is a geometrical pattern of an electrical network, in which no distinction is made between the different types of physical elements of which it is composed. It facilitates network analysis.

**1.2. Write two properties of complete incidence matrix.**

AKTU 2017-18, Marks 02

**Ans.**

1. Algebraic sum of the column entries of an incidence matrix is zero.
2. Determinant of the incidence matrix of a closed loop is zero.

**1.3. Describe the following : Tree, Co-Tree, Twig, Link, Cut-set and Tie-set.**

AKTU 2017-18, Marks 02

**OR**

**Define tree, twig, links.**

AKTU 2018-19, Marks 02

**OR**

**Define tree in graph theory.**

AKTU 2016-17, Marks 02

**OR**

**Define twig and link.**

AKTU 2016-17, Marks 02

**Ans.**

1. **Trees :** A tree of a connected graph is defined as a set of branches which connect all the nodes of graph without forming any loop.
2. **Co-tree :** The set of all branches of the graph which are not present in the tree is called co-tree.
3. **Twigs :** The branches of tree are called twigs.
4. **Links :** The branches of a co-tree are called links.
5. **Tie-set :** It is a set of branches contained in a loop such that each loop contains one link and remainder are tree branches.
6. **Cut-set :**

   i. Cut-set matrix with respect to a tree is formed by only one twig and a set of links. In a graph for each twig of chosen tree there is a cut-set matrix.

ii. A graph having $N$ nodes will have $(N-1)$ fundamental cut-set, *i.e.*, equal to number of twigs.
iii. The orientation of a cut-set coincides with the orientation of twig.

**1.4. Write down all the properties of loop impedance matrix.**

**AKTU 2016-17, Marks 02**

**Ans.**
1. If network has no coupling elements, the branch impedance matrix $Z_b$ is diagonal and loop impedance matrix is symmetric.
2. If the network is resistive and if all its resistance are positive, then $\det(Z) > 0$.

**1.5. How can you say that a network is stable ? Give definition.**

**AKTU 2016-17, Marks 02**

**Ans.**
1. For a linear system to be stable, all of its poles must have negative real parts, *i.e.*, they must lie within the left-half of the $s$-plane.
2. A system having one or more poles lying on the imaginary axis of the $s$-plane and having non-decaying oscillatory components in its natural response is defined as a marginally stable system.

**1.6. Write the relation between twigs and links.**

**AKTU 2015-16, Marks 02**

**Ans.** Links = $B$ − Twigs

where, $B$ = Number of branches

**1.7. List out the properties of a tree in a graph.**

**AKTU 2015-16, Marks 02**

**Ans.**
1. In a tree, there exists only one path between any pair of nodes.
2. Every connected graph has at least one tree.
3. If a tree contains $n$ nodes, then it has $(n-1)$ branches.
4. The rank of a tree is $(n-1)$.

**1.8. Define principle of duality.** **AKTU 2018-19, Marks 02**

**Ans.** Duality is a transformation in which currents and voltages are interchanged. Two phenomena are said to be dual if they are described by equations of the same mathematical form.

**1.9. What do you understand by degree of a node ? Give an example.**

**Ans.** The degree of a node is the number of branches associated to it.



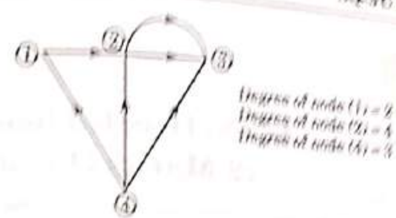


Fig. 1.

**1.10. What are the properties of cut-set ?**

**Ans.** **Properties :**
1. A cut-set divides the set of nodes into two subsets.
2. Each fundamental cut-set contains one tree branch, the remaining elements being links.
3. Each branch of the cut-set has one of its terminals incident at a node in one subset and its other terminal at a node in the other subset.

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# 2 UNIT

# AC Network Theorems (2 Marks Questions)

**2.1. Give statement of superposition theorem.**

**AKTU 2016-17, Marks 02**

**Ans.** If a number of voltage or current sources are acting simultaneously in a linear network, the resultant current in any branch is the algebraic sum of the currents that would be produced in it, when each source acts alone replacing all other independent sources by their internal resistances.

**2.2. What is the condition for maximum power transfer in network ? Also mention any two applications of maximum power transfer theorem.**

**AKTU 2015-16, Marks 02**

**Ans.**

A. **Condition for maximum power transfer :** Maximum power will be transferred when the load resistance is equal to the source resistance *i.e.*,

$$R_s = R_L.$$

B. **Applications of maximum power transfer theorem :**

i. In communication system.

ii. In electric power transmission system.

**2.3. Compare dependent and independent sources.**

**AKTU 2018-19, Marks 02**

**Ans.**

| S. No. | Dependent source | Independent source |
|---|---|---|
| 1. | Dependent source are the sources, whose output value depends upon the voltage or current in some other part of the circuit. | Independent source are the sources, whose output value does not depend upon the circuit parameters like voltage or current. |

**2.4. State Tellegen's theorem.**

**AKTU 2015-16, Marks 02**



OR

**Give statement of Tellegen's theorem.**

**AKTU 2018-19, Marks 02**

**Ans.**

1. For any given time, the sum of power delivered to each branch of any electric network is zero.
2. Thus for $K^{th}$ branch, this theorem states that $\sum_{K=1}^{n} v_K i_K = 0$; $n$ being the number of branches, $v_K$ the drop in the branch and $i_K$ the through current.

**2.5. Give the advantages and limitations of superposition theorem.**

**Ans.** **Advantages :**

1. This theorem is valid for all types of linear circuits having time varying or time-invariant elements.
2. This theorem is used to find the current or voltage in a branch when the circuit has a large number of independent sources.

**Limitations :**

1. Not applicable for the network containing non-linear elements or unilateral elements.
2. Not applicable for the non-linear parameters such as power.

**2.6. State Thevenin's theorem.**

**Ans.**

1. A linear active bilateral network can be replaced at any two of its terminals by an equivalent voltage source (Thevenin's voltage source), $V_{oc}$, in series with an equivalent impedance (Thevenin's impedance), $Z_{th}$.
2. Here, $V_{oc}$ is the open circuit voltage between the two terminals under the action of all sources and initial conditions, and $Z_{th}$ is the impedance obtained across the terminals with all sources removed by their internal impedance and initial conditions reduced to zero.

**2.7. Give the important points of Thevenin's theorem.**

**Ans.**

1. This theorem is very useful for replacement of a large portion of a network with a small equivalent circuit. This is useful for calculating the load resistance in impedance-matching problems.
2. This theorem is applicable to any linear, bilateral, active network.

**2.8. Write the statement of Norton's theorem.**

**Ans.**

1. A linear active bilateral network can be replaced at any two of its terminals, by an equivalent current source (Norton's current

source), $I_{sc}$, in parallel with an equivalent admittance (Norton's admittance), $Y_N$.

2. Here, $I_{sc}$ is the short-circuit current flowing from one terminal to the other under the action of all sources and initial conditions, and $Y_N$ is the admittance obtained across the terminals with all sources removed by their internal impedance and initial conditions reduced to zero.

**2.9. Mention the important points of Norton's theorem.**

**Ans.**
1. This theorem is very useful for replacement of a large portion of a network with a small equivalent circuit. This is useful for calculating the load resistance in impedance-matching problems.
2. This theorem is applicable to any linear, bilateral, active network.
3. To apply this theorem, the load branch should not be magnetically coupled to any other branch in the circuit and the load should not contain any dependent source.
4. This theorem is inapplicable to non-linear and unilateral networks.
5. This theorem is inapplicable for active load.

**2.10. What is reciprocity theorem and give its advantages ?**

**Ans.** It states that in any linear time-varying network, the ratio of response to excitation remains same for an interchange of the position of excitation and response in the network.

**Advantages :**
1. This theorem is applicable to the network comprising of linear, time-invariant, bilateral, passive elements, such as ordinary resistors, inductors, capacitors and transformers.
2. To apply this theorem, we have to consider only the zero-state response by taking all the initial condition to be zero.

**2.11. What do you understand by Millman's theorem ?**

**Ans.** It states that if several ideal voltage sources ($V_1$, $V_2$, ...) in series with impedances ($Z_1$, $Z_2$, ...) are connected in parallel, then the circuit may be replaced by a single ideal voltage ($V$) in series with an impedance ($Z$).

$$V = \frac{\sum_{i=1}^{n} V_i Y_i}{\sum_{i=1}^{n} Y_i} \text{ and } Z = \frac{1}{\sum_{i=1}^{n} Y_i}$$

☺☺☺



# 3 UNIT

# Transient Circuit Analysis (2 Marks Questions)

**3.1. What is transient and steady state response ?**

AKTU 2018-19, Marks 02

**Ans.**
1. In a network containing energy storage elements, the currents and voltages change from one state to other state. The behaviour of the voltage or current when it is changed from one state to another is called the transient state.
2. A circuit having constant sources is said to be in steady state if the currents and voltages do not change with time and corresponding response of the circuit is called steady state response.

**3.2. Write the time constants of *RC* and *RL* networks.**

AKTU 2015-16, Marks 02

**Ans.**

**A. Time constant in RL circuit :**

Current equation, $i = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)$

Time constant, $\tau = \frac{L}{R}$.

**B. Time constant in RC circuit :**

Current equation, $i = \frac{V}{R} e^{-t/RC}$

Time constant, $\tau = RC$.

**3.3. Write a definition of convolution.**

AKTU 2016-17, Marks 02

**Ans.**
1. This is a process by which one signal is time reversed, shifted, multiplied with another signal and then its integral is calculated to generate a third signal. This operation is useful in describing output of a system by convolving input signal and its unit impulse response.

2. Mathematically, convolution is represented as

$$w(t) = v(t) \otimes h(t) = \int_{-\infty}^{\infty} v(\tau)\, h(t-\tau)\, d\tau$$

where, $w(t)$ = Convolved output
$v(t)$, $h(t)$ = Signals being convolved
$\otimes$ = Convolution operator

**3.4. What do you mean by natural response and forced response ?**

**Ans.**
1. When we consider a circuit containing storage elements which are independent of the sources, the response depends upon the nature of the circuit and is called the natural response.
2. When we consider a source acting on a circuit, the response depends upon the nature of source and is called forced response.

**3.5. Discuss the advantages of analysing the circuits using frequency domain rather than the time domain.**

**Ans.** Following are some advantages of analysing an electrical network in $s$-domain rather than in $t$-domain :

i. Each element can easily be replaced by transform impedance.
ii. No integration or differentiation is involved in the transform equations.
iii. The response obtained after solution is a complete response *i.e.*, both the steady state and transient response are obtained.

**3.6. A network series $R$-$L$ circuit as shown in Fig. 1 and switch $S$ is closed at time $t = 0$. Evaluate graph response of $i(t)$.**

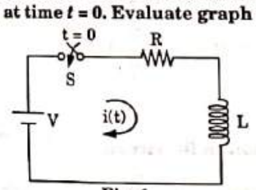


**Fig. 1.**

**Ans.**

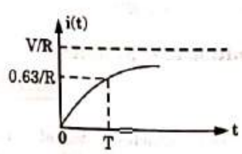


**Fig. 2.**



**3.7. In the circuit shown in Fig. 3, switch $S$ is closed at $t = 0$. Find the time constant of the circuit.**

**Ans.**

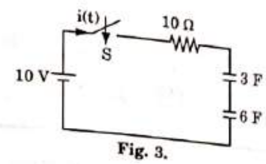


**Fig. 3.**

$$C = \frac{3 \times 6}{3+6} = 2 \text{ F}$$

Time constant $RC$ = 20 sec

**3.8. A circuit is shown in Fig. 4. Find the voltage $V_L(t)$.**

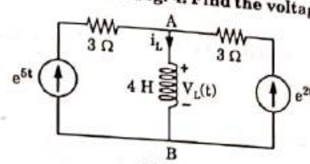


**Fig. 4.**

**Ans.** From Fig. 6, $i_L(t) = e^{5t} + e^{2t}$

We know, $$V_L(t) = L\frac{di_L(t)}{dt}$$

$$V_L = 4\frac{d}{dt}(e^{5t} + e^{2t})$$

$$V_L = 4\,(5e^{5t} + 2e^{2t})$$

$$V_L = 20e^{5t} + 8e^{2t}$$

☺☺☺

# 4 UNIT

# Network Functions (2 Marks Questions)

**4.1. What do you mean by transfer function ?**

**AKTU 2016-17, Marks 02**

**Ans.** It is defined as the ratio of an output transform to an input transform, with zero initial condition and with no internal energy source except the controlled sources.

**4.2. Define transfer admittance and impedance of two port network.**

**AKTU 2015-16, Marks 02**

**Ans.**

1. Transfer admittance function

$$Y_{12}(s) = \frac{I_1(s)}{V_2(s)},\ Y_{21}(s) = \frac{I_2(s)}{V_1(s)}$$

2. Transfer impedance function

$$Z_{12}(s) = \frac{V_1(s)}{I_2(s)},\ Z_{21}(s) = \frac{V_2(s)}{I_1(s)}$$

**4.3. What are the advantages of transfer function ?**

**Ans.**

1. It gives simple mathematical algebraic equation.
2. It gives poles and zeros of the system directly.
3. Stability of the system can be determined easily.
4. The output of the system for any input can be determined easily.

**4.4. What are the disadvantages of transfer function ?**

**Ans.**

1. It is applicable for LTI system.
2. It does not take initial condition into account.
3. The internal status of the system cannot be determined.
4. Applicable only for single input and single output.
5. Controllability and observability cannot be determined.

**4.5. Define a two port network.**

**AKTU 2016-17, Marks 02**



**Ans.**

1. Two port network is defined as any pair of terminals into which energy is supplied, or from which energy is withdrawn, or where the network variables may be measured.
2. Two ports containing no sources in their branches are called passive ports and two ports containing sources in their branches are called active ports.

**4.6. Write hybrid parameters in terms of Z parameters.**

**AKTU 2017-18, Marks 02**

**Ans.**

$$h_{11} = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{22}},\ h_{12} = \frac{Z_{12}}{Z_{22}}$$

$$h_{21} = -\frac{Z_{21}}{Z_{22}} \text{ and } h_{22} = \frac{1}{Z_{22}}$$

**4.7. Describe complex frequency in brief.**

**AKTU 2017-18, Marks 02**

**Ans.** A type of frequency, which depends on two parameters ; one is of which controls the magnitude of signal and other is of which controls the rotation of signal, is known as complex frequency.

$$s = \sigma + j\omega$$

**4.8. Find out $Z_{11}$, $Z_{21}$ for the following network.**

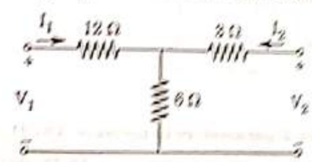


Fig. 1.

**AKTU 2018-19, Marks 02**

**Ans.**

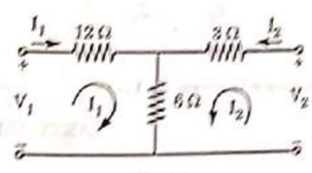


Fig. 2.

1. By writing KVL equation for two meshes

$$12I_1 + 6(I_1 + I_2) - V_1 = 0$$
$$18I_1 + 6I_2 = V_1 \quad ...(1)$$
$$3I_2 + 6(I_1 + I_2) - V_2 = 0$$
$$6I_1 + 9I_2 = V_2 \quad ...(2)$$

2. $$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = 18\,\Omega$$

3. $$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = 6\,\Omega$$

**4.9. Give the reciprocity conditions of various parameters.**

**OR**

**Find reciprocity condition in Y-parameter.**

**AKTU 2018-19, Marks 02**

**Ans.** Reciprocity condition of various parameters,

1. Z parameters, $Z_{12} = Z_{21}$
2. Y parameters, $Y_{12} = Y_{21}$
3. ABCD (transmission) parameters,
   $$AD - BC = 1$$
4. A'B'C'D' (inverse transmission) parameters,
   $$A'D' - B'C' = 1$$
5. Hybrid parameters, $h_{12} = h_{21}$
6. Inverse hybrid parameters,
   $$g_{12} = -g_{21}$$

**4.10. Write the Z-parameters in terms of ABCD parameters.**

**AKTU 2015-16, Marks 02**

**Ans.**
$$Z_{11} = \frac{A}{C},\ Z_{12} = \frac{AD - BC}{C}$$
$$Z_{21} = \frac{1}{C},\ Z_{22} = \frac{D}{C}$$

**4.11. An admittance is given by $Y(s) = \dfrac{1}{s+2}$. Find the pole-zero plot.**

**AKTU 2015-16, Marks 02**



**Ans.** Given,

$$Y(s) = 1/(s+2)$$

For pole, $s + 2 = 0$

$\therefore$ $s = -2$

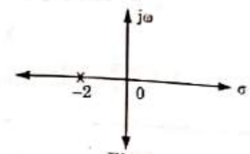


Fig. 1.

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# 5 UNIT
# Network Synthesis and Filters (2 Marks Questions)

**5.1. Define network synthesis.** AKTU 2016-17, Marks 02

**Ans.** Network synthesis is a method of designing signal processing filters.

**5.2. What do you mean by filters ?** AKTU 2016-17, Marks 02

**Ans.** An electric filter is a four-terminal frequency-selective network designed generally with reactive elements to transmit freely a specified band of frequencies and block or attenuate signals of frequency outside this band.

**5.3. Draw the reactance frequency characteristics of low pass filter.** AKTU 2015-16, Marks 02

**Ans.**

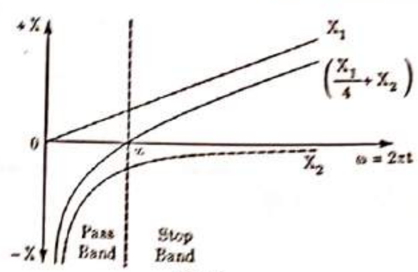


Fig. 1.

**5.4. List out the characteristics of filter.** AKTU 2015-16, Marks 02

**Ans.**

1. Low-pass filters pass low frequencies and attenuate high frequencies.
2. High-pass filters pass high frequencies and attenuate low frequencies.



3. Band-pass filters pass a certain band of frequencies.
4. Band-stop filters attenuate a certain band of frequencies.

**5.5. State two properties of the R-L driving point impedance function.** AKTU 2017-18, Marks 02

**Ans.**

a. Poles and zeros lie on the negative real axis, and they are alternate.
b. The residues of the poles must be real and positive.

**5.6. What are the properties of positive real function ?**

**Ans.**

1. If $F(s)$ is PRF, then $F\left(\frac{1}{s}\right)$ is also a PRF.
2. If $F(s)$ is PRF, then $\frac{1}{F(s)}$ is also a PRF.
3. If $F(s) = \frac{N(s)}{D(s)}$ is PRF, then $F_1(s) = N(s) + D(s)$ is also a PRF.
4. If $F_1(s)$ and $F_2(s)$ are PRF, then $F_3(s) = \frac{F_1(s)F_2(s)}{F_1(s)+F_2(s)}$ is also a PRF.

**5.7. Define cut-off frequency.**

**Ans.** The frequency which separates the pass-band and the stop-band is defined as the cutoff frequency of the filter.

**5.8. Differentiate passive and active filters.**

**Ans.**

| S. No. | Active filters | Passive filters |
|---|---|---|
| 1. | Made up of resistances, capacitances and operational amplifiers. | Mainly uses inductors. |
| 2. | Readily compatible with ICs. | Not compatible with ICs. |
| 3. | Consume less power. | Consume more power. |
| 4. | Can provide gain. | Cannot provide gain. |

**5.9. State and describe the properties of RL and RC DPI network.** AKTU 2017-18, Marks 02

**Ans.**

1. **Properties of RL DPI network :**
   - i. The poles and zeros interlace each other on the negative real axis.
   - ii. The poles and zeros are the critical frequencies. The critical frequency nearest to the origin is always a zero, which may be located at the origin.
   - iii. The critical frequency at a greatest distance away from the origin is always a pole, which may be located at infinity also.
2. **Properties of RC DPI network :**
   - i. The poles and zeros are simple. There are no multiple poles and zeros.
   - ii. The poles and zeros are located on negative real axis.
   - iii. The poles and zeros interlace each other on the negative real axis.

**5.10. Give the advantages of active filters.**

**Ans.** **Advantages of active filters :**

1. They eliminate the need for inductors which are large, heavy and costly, particularly in audio frequency range.
2. They provide gain and isolation properties *i.e.*, high input impedance and low output impedance.
3. Active filters are readily compatible with ICs due to absence of inductors.
4. High range of *Q*-factor is possible.

**5.11. What are the necessary and sufficient conditions of a network function for a stable network ?**

**Ans.**

1. The function should not have any pole in the right half of the s-plane.
2. The poles on the imaginary axis should be simple (not repeated).
3. The difference in the degree of the numerator and denominator polynomial can at most be unity.

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# B. Tech.
# (SEM. IV) EVEN SEMESTER THEORY EXAMINATION, 2014-15
# NETWORK ANALYSIS & SYNTHESIS

**Time : 3 Hours** **Max. Marks : 100**

**Note :** Attempt all question.

**1.** Attempt any **two** parts of the following : (10 × 2 = 20)

**a. Explain the following matrices taking a suitable example :**

**i. Reduced incidence matrix**

**ii. Basic cut-set matrix**

**iii. Basic tie-set matrix**

**Ans.** Refer Q. 1.14, Page 1–15C, Unit-1.

**b. Derive the KCL and KVL using network graph variables.**

**Ans.** Refer Q. 1.21, Page 1–24C, Unit-1.

**c. Explain the concept of duality. Find the dual of the network shown in Fig. 1.**

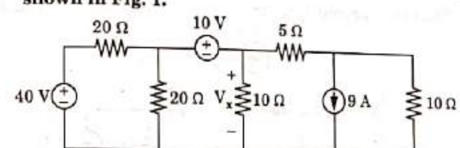


**Fig. 1.**

**Ans.** Refer Q. 1.19, Page 1–23C, Unit-1.

**2.** Attempt any **four** parts of the following. (5 × 4 = 20)

**a. Find the current *I* and voltage $V_{ab}$ in Fig. 2.**

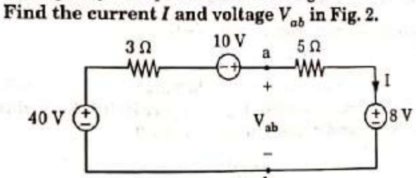


**Fig. 2.**

**Ans.** Refer Q. 2.15, Page 2–16C, Unit-2.

**b. Derive the maximum power transfer theorem for the case when the source impedance is complex and the load is variable with its power factor being unity.**

**Ans.** Refer Q. 2.21, Page 2–23C, Unit-2.

c. **Find the voltage $V_o$ in Fig. 13.**

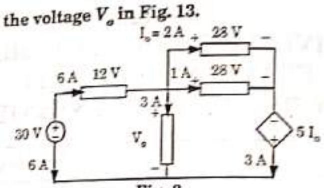


Fig. 3.

**Ans.** Refer Q. 2.8, Page 2–9C, Unit-2.

d. **Find the current $I_x$ through the 5 ohm resistor using Thevenin's theorem in Fig. 4.**

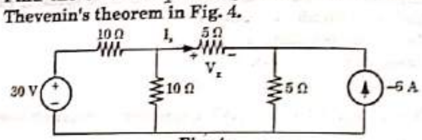


Fig. 4.

**Ans.** Refer Q. 2.16, Page 2–17C, Unit-2.

e. **Find the current $I_o$ using source transformation in Fig. 5.**

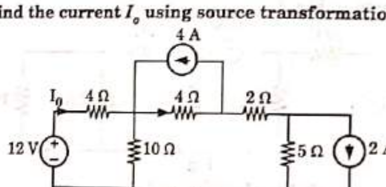


Fig. 5.

**Ans.** Refer Q. 2.7, Page 2–8C, Unit-2.

f. **State and prove the Tellegen's theorem.**

**Ans.** Refer Q. 2.27, Page 2–29C, Unit-2.

3. Attempt any two parts of the following : **(10 × 2 = 20)**

a. i. **In the circuit shown in Fig. 6, the switch is kept closed for a long time and is then opened at $t = 0$.**

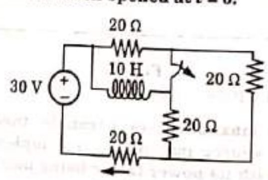


Fig. 6.



**Find the values of current $i$ just before opening the switch ($t = 0^-$) and just after opening the switch ($t = 0^+$).**

**Ans.** Refer Q. 3.8, Page 3–7C, Unit-3.

ii. **In the Fig. 7 shown, the ideal switch has been open for a long time. If it is closed at $t = 0$, then find the magnitude of current (in mA) through the 4 kΩ resistor at $t = 0^+$ ?**

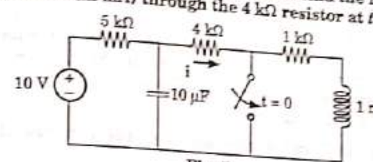


Fig. 7.

**Ans.** Refer Q. 3.9, Page 3–9C, Unit-3.

b. **Find the expression of the current $i(t)$ in the circuit of Fig. 8 assuming that the switch is opened at $t = 0$, the steady state having already reached before that.**

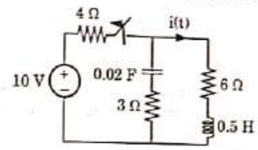


Fig. 8.

**Ans.** Refer Q. 3.10, Page 3–12C, Unit-3.

c. **Find the voltage $V_o$ in the circuit of Fig. 9. The switch was open for a long time before it was closed at $t = 0$.**

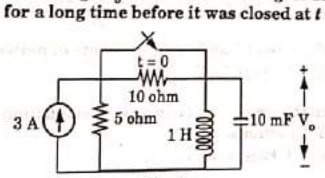


Fig. 9.

**Ans.** Refer Q. 3.11, Page 3–13C, Unit-3.

4. Attempt any four parts of the following : **(5 × 4 = 20)**

a. **Write the necessary conditions for the existence of transfer functions giving a suitable example.**

**Ans.** Refer Q. 4.6, Page 4–6C, Unit-4.

b. **What is meant by reciprocal and symmetric networks ? Explain with the help of an example.**

**Ans:** Refer Q. 4.15, Page 4–19C, Unit-4.

c. **Derive the condition of reciprocity and symmetry for *h*-parameters.**

**Ans:** Refer Q. 4.15, Page 4–19C, Unit-4.

d. **Prove that if two 2-port networks are connected in cascade, the transmission parameter matrix of the composite two port network is the product of the two individual transmission parameter matrices.**

**Ans:** Refer Q. 4.21, Page 4–31C, Unit-4.

e. **Find the *Y* and *h* parameters of the network shown in Fig. 10.**

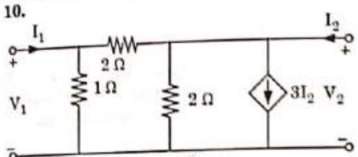


**Fig. 10.**

**Ans:** Refer Q. 4.23, Page 4–34C, Unit-4.

f. **Prove that the star-delta conversion does not bring any change in the Z-parameter matrix for the case of a resistive network.**

**Ans:** Refer Q. 4.24, Page 4–35C, Unit-4.

5. Attempt any two parts of the following. **(10 × 2 = 20)**

a. **Given** $$Z(s) = \frac{10(s^2+4)(s^2+6)}{s(s^2+5)}$$

**Find the Foster I and Cauer II forms of network.**

**Ans:** Refer Q. 5.13, Page 5–17C, Unit-5.

b. **Explain the properties of positive real function, *LC* functions and *RL* functions.**

**Ans:** Refer Q. 5.4, Page 5–3C, Unit-5.

c. **What is image parameter ? Derive its expression. What are active and passive filters ? Explain the advantages of active filters.**

**Ans:** Refer Q. 5.16, Page 5–21C, Unit-5.

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# B. Tech.
# (SEM. IV) EVEN SEMESTER THEORY EXAMINATION, 2015-16
# NETWORK ANALYSIS & SYNTHESIS

**Time : 3 Hours** **Max. Marks : 100**

## SECTION - A

1. **Attempt all parts. All parts carry equal marks. Write answer of each part in short.** **(2 × 10 = 20)**

a. **Write the relation between twigs and links.**

**Ans:** Refer Q. 1.6, Page SQ–2C, Unit-1, 2 Marks Questions.

b. **List out the properties of a tree in a graph.**

**Ans:** Refer Q. 1.7, Page SQ–2C, Unit-1, 2 Marks Questions.

c. **State Tellegen's theorem.**

**Ans:** Refer Q. 2.4, Page SQ–4C, Unit-2, 2 Marks Questions.

d. **What is the condition for maximum power transfer in network ? Also mention any two applications of maximum power transfer theorem.**

**Ans:** Refer Q. 2.2, Page SQ–4C, Unit-2, 2 Marks Questions.

e. **Write the time constants of *RC* and *RL* networks.**

**Ans:** Refer Q. 3.2, Page SQ–7C, Unit-3, 2 Marks Questions.

f. **An admittance is given by** $Y(s) = \frac{1}{s+2}$. **Find the pole-zero plot.**

**Ans:** Refer Q. 4.11, Page SQ–12C, Unit-4, 2 Marks Questions.

g. **Define transfer admittance and impedance of two port network.**

**Ans:** Refer Q. 4.2, Page SQ–10C, Unit-4, 2 Marks Questions.

h. **Write the Z-parameters in terms of *ABCD* parameters.**

**Ans:** Refer Q. 4.10, Page SQ–12C, Unit-4, 2 Marks Questions.

i. **Draw the reactance frequency characteristics of low pass filter.**

**Ans:** Refer Q. 5.3, Page SQ–14C, Unit-5, 2 Marks Questions.

j. List out the characteristics of filter.

**Ans.** Refer Q. 5.4, Page SQ-14C, Unit-5, 2 Marks Questions.

## SECTION-B

2. Attempt any five questions from this section. (10 × 5 = 50)

a. Find Thevenin's equivalent circuit across *a-b* and find current through 10 Ω resistor.

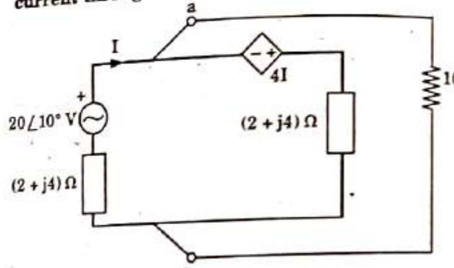


Fig. 1.

**Ans.** Refer Q. 2.17, Page 2-18C, Unit-2.

b. What should be the value of $R_L$ so the maximum power can be transferred from the source to $R_L$ for the given Fig. 2?

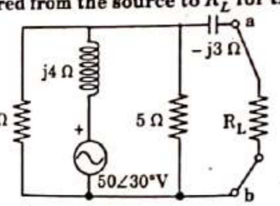


Fig. 2.

**Ans.** Refer Q. 2.23, Page 2-25C, Unit-2.

c. The reduced incidence matrix is,

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ -1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

Do the following :

i. Draw the graph.

ii. How many trees are possible ?

iii. Write tie-set and cut-set matrix.

**Ans.** Refer Q. 1.17, Page 1-20C, Unit-1.



d. In Fig. 3 the initial voltage in the capacitor is 1 V with the polarity as shown, find the voltage appearing across the capacitor using Laplace method with application of step voltage 8.

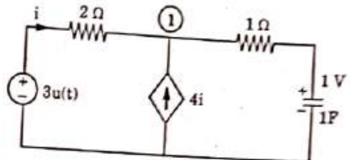


Fig. 3.

**Ans.** Refer Q. 3.14, Page 3-18C, Unit-3.

e. A network has been shown in Fig. 4. Switch *K* is closed at $t = 0$. Find the current in $R_L$ using Thevenin's theorem. Assume steady state condition before switching. Use the following state condition before switching. Use the following values : ($r_1 = r_2 = r_3 = 10\ \Omega$; $L_1 = L_2 = 1$ H; $V = 10$ V).

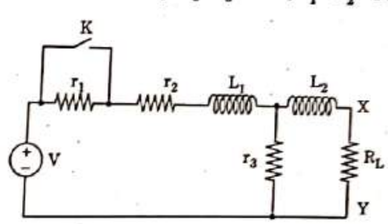


Fig. 4.

**Ans.** Refer Q. 3.15, Page 3-19C, Unit-3.

f. In Fig. 5 with switch open, steady state is reached with $v = 100 \sin 314t$ volts. The switch is closed at $t = 0$. The circuit is allowed to come to steady state again. Determine the steady state current and complete solution of transient current.




Fig. 5.

**Ans.** Refer Q. 3.16, Page 3-22C, Unit-3.

**g. On short circuit tests, the currents and voltages were determined experimentally for an unknown two port network as :**

at $V_2 = 0$: $I_1 = 1$ mA ; $I_2 = 0.5$ mA ; $V_1 = 25$ V

at $V_1 = 0$: $I_1 = -1$ mA ; $I_2 = -10$ mA ; $V_2 = 50$ V

**Determine the Y-parameters and draw the Y-parameter model.**

**Ans.** Refer Q. 4.28, Page 4–40C, Unit-4.

**h. Synthesize the following network function in Cauer-2 form :**

$$Z(s) = \frac{8s^3 + 10s}{5 + 6s^2 + s^4}$$

**Ans.** Refer Q. 5.9, Page 5–11C, Unit-5.

## SECTION-C

**Note : Attempt any two questions from this section. (15 × 2 = 30)**

**3. a. The current $I_1$ and $I_2$ at input and output port respectively of a two-port network can be expressed as : $I_1 = 5V_1 - V_2$; $I_2 = V_1 + V_2$.**

**i. Find the equivalent π-network.**

**ii. Find the input impedance when a load of $(3 + j5)\Omega$ is connected across the output port.**

**Ans.** Refer Q. 4.29, Page 4–41C, Unit-4.

**b. A network has two input terminals $a$, $b$ and output terminals $c$, $d$. The input impedance with $c$-$d$ open circuited is $(250 + j100)\ \Omega$ and with $c$-$d$ short circuited is $(400 + j300)\ \Omega$. The impedance across $c$-$d$ with $a$-$b$ open circuited is $200\ \Omega$. Determine equivalent T-network parameters.**

**Ans.** Refer Q. 4.30, Page 4–43C, Unit-4.

**4. Find the first order and second order Foster form of the driving point impedance function.**

$$Z(s) = \frac{2(s^2+1)(s^2+9)}{s(s^2+4)}$$

**Ans.** Refer Q. 5.8, Page 5–9C, Unit-5.

**5. Design a constant $k$-low pass filter having cut-off frequency 2.5 kHz, design resistance $R_0 = 700\ \Omega$. Also find the frequency at which the filter produces attenuation of 19.1 dB. Find the characteristics impedances and phase constant at pass band and stop or attenuation band.**

**Ans.** Refer Q. 5.19, Page 5–26C, Unit-5.

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# B.Tech.

# (SEM. IV) EVEN SEMESTER THEORY EXAMINATION, 2016-17

# NETWORK ANALYSIS AND SYNTHESIS

**Time : 3 Hours** **Max. Marks : 100**

**Note :** Be precise in your answer. In case of numerical problem assume data wherever not provided.

## SECTION-A

**1. Attempt all of the following questions : (2 × 10 = 20)**

**a. Define a two port network.**

**Ans.** Refer Q. 4.5, Page SQ–10C, Unit-4, 2 Marks Questions.

**b. Define network synthesis.**

**Ans.** Refer Q. 5.1, Page SQ–14C, Unit-5, 2 Marks Questions.

**c. What do you mean by transfer function ?**

**Ans.** Refer Q. 4.1, Page SQ–10C, Unit-4, 2 Marks Questions.

**d. Define twig and link.**

**Ans.** Refer Q. 1.3, Page SQ–1C, Unit-1, 2 Marks Questions.

**e. Write a definition of convolution.**

**Ans.** Refer Q. 3.3, Page SQ–7C, Unit-3, 2 Marks Questions.

**f. How can you say that a network is stable ? Give definition.**

**Ans.** Refer Q. 1.5, Page SQ–2C, Unit-1, 2 Marks Questions.

**g. What do you mean by filters ?**

**Ans.** Refer Q. 5.2, Page SQ–14C, Unit-5, 2 Marks Questions.

**h. Give statement of superposition theorem.**

**Ans.** Refer Q. 2.1, Page SQ–4C, Unit-2, 2 Marks Questions.

**i. Write down all the properties of loop impedance matrix.**

**Ans.** Refer Q. 1.4, Page SQ–2C, Unit-1, 2 Marks Questions.

**j. Define tree in graph theory.**

**Ans.** Refer Q. 1.3, Page SQ–1C, Unit-1, 2 Marks Questions.

## SECTION-B

2. Attempt any five of the following questions : **(10 × 5 = 50)**

a. **Explain Z-impedance parameter in detail.**

**Ans.** Refer Q. 4.13, Page 4–14C, Unit-4.

b. **Give classification of filters.**

**Ans.** Refer Q. 5.17, Page 5–23C, Unit-5.

c. **Obtain cut-set matrix for following electrical network.**

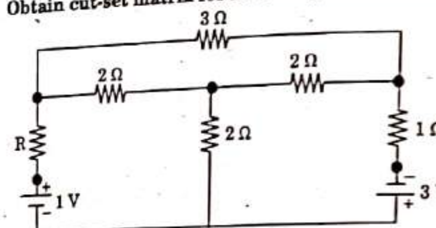


**Fig. 1.**

**Ans.** Refer Q. 1.16, Page 1–18C, Unit-1.

d. **Determine the Z-parameters of Fig. 2.**

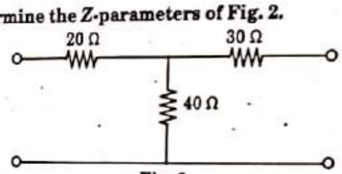


**Fig. 2.**

**Ans.** Refer Q. 4.27, Page 4–40C, Unit-4.

e. **Find $i_o$ in the circuit in Fig. 3 using superposition theorem.**

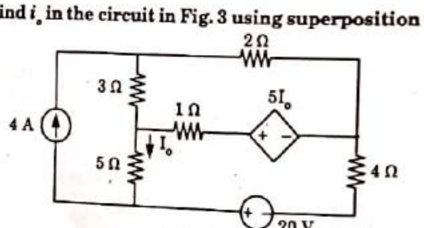


**Fig. 3.**

**Ans.** Refer Q. 2.11, Page 2–11C, Unit-2.



f. **Explain admittance parameters in detail.**

**Ans.** Refer Q. 4.13, Page 4–14C, Unit-4.

g. **Explain in detail band stop filter, with proof.**

**Ans.** Refer Q. 5.18, Page 5–24C, Unit-5.

h. **Give statement and prove maximum power transfer theorem.**

**Ans.** Refer Q. 2.21, Page 2–23C, Unit-2.

## SECTION-C

Attempt any two of the following questions : **(15 × 2 = 30)**

3. **With example, explain first Foster form realization of LC networks.**

**Ans.** Refer Q. 5.10, Page 5–13C, Unit-5.

4. **Determine what type of filter is shown in Fig. 4. Calculate the corner or cut off frequency. Take $R = 2$ k and $L = 2$ H and $C = 2\ \mu$F.**

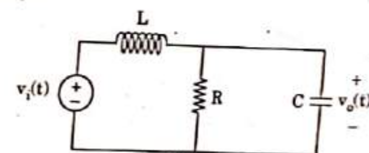


**Fig. 4.**

**Ans.** Refer Q. 5.20, Page 5–28C, Unit-5.

5. **Obtain Cauer form realization of following and obtain network.**

$$Z(s) = \frac{(s+1)}{s(s+2)}; \quad Z(s) = \frac{(s+1)(s+3)}{s(s+2)}$$

**Ans.** Refer Q. 5.11, Page 5–13C, Unit-5.

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# B.Tech.

## (SEM. IV) EVEN SEMESTER THEORY EXAMINATION, 2017-18

## NETWORK ANALYSIS AND SYNTHESIS

Time : 3 Hours Max. Marks : 70

Note : 1. Attempt all sections.

### SECTION-A

1. Attempt all questions in brief : $(2 \times 7 = 14)$

a. **Write two properties of complete incidence matrix.**

**Ans.** Refer Q. 1.2, Page SQ–1C, Unit-1, 2 Marks Questions.

b. **Write hybrid parameters in terms of Z parameters.**

**Ans.** Refer Q. 4.6, Page SQ–11C, Unit-4, 2 Marks Questions.

c. **State two properties of the R-L driving point impedance function.**

**Ans.** Refer Q. 5.5, Page SQ–15C, Unit-5, 2 Marks Questions.

d. **Describe the following : Tree, Co-Tree, Twig, Link, Cut-set and Tie-set.**

**Ans.** Refer Q. 1.3, Page SQ–1C, Unit-1, 2 Marks Questions.

e. **State and describe the properties of RL and RC DPI network.**

**Ans.** Refer Q. 5.9, Page SQ–15C, Unit-5, 2 Marks Questions.

f. **State and describe Thevenin's theorem with suitable example.**

**Ans.** i. **State and explanation :** Refer Q. 2.2, Page 2–3C, Unit-2.
ii. **Example :** Refer Q. 2.14, Page 2–14C, Unit-2.

g. **Describe complex frequency in brief.**

**Ans.** Refer Q. 4.7, Page SQ–11C, Unit-4, 2 Marks Questions.

### SECTION-B

2. Attempt any five of the following questions : $(10 \times 5 = 50)$

a. **For given reduced incidence matrix, draw the graph and hence obtain the f-cutset matrix**

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 & 1 & 1 \\ -1 & 0 & 1 & 0 & 0 & -1 & 0 \end{bmatrix}$$

**Ans.** Refer Q. 1.12, Page 1–12C, Unit-1.



b. **For the network shown in Fig. 1 draw the directed graph. And also find number of possible tree.**

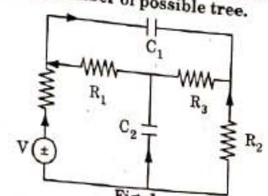

Fig. 1.

**Ans.** Refer Q. 1.13, Page 1–13C, Unit-1.

c. **Find current through 5 Ω resistor using Thevenin's theorem.**

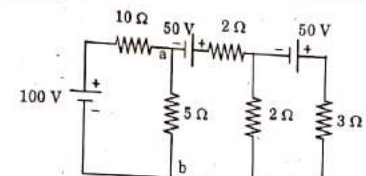

Fig. 2.

**Ans.** Refer Q. 2.14, Page 2–14C, Unit-2.

d. **Test whether the polynomial $P(s)$ is Hurwitz or not :**
i. $s^5 + 3s^2 + 2s$ ii. $s^4 + 5s^3 + 5s^2 + 4s + 10$.

**Ans.** Refer Q. 5.12, Page 5–16C, Unit-5.

e. **Find $Y$ and $Z$ parameters of the network.**

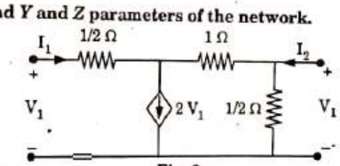

Fig. 3.

**Ans.** Refer Q. 4.26, Page 4–38C, Unit-4.

### SECTION-C

3. Attempt any one/two part of the following : $(7 \times 1 = 7)$

a. **State the properties of RL driving point impedance function. Also realize the given network impedance function using foster form I**

$$Z(s) = (s + 1)(s + 3)/(s + 2)(s + 4)$$

**Ans.** Refer Q. 5.5, Page 5–4C, Unit-5.

b. **Explain the advantages of active filter in comparison to passive filter in detail.**

**Ans.** Refer Q. 5.16, Page 5–21C, Unit-5.

4. Attempt any one/two part of the following : **(7 × 1 = 7)**

a. **For the given network function, draw the pole zero diagram and hence obtain the time response $i(t)$**

$$I(s) = \frac{5s}{(s+1)(s^2+4s+8)}$$

**Ans.** Refer Q. 4.8, Page 4–8C, Unit-4.

b. **Design constant $K$ low pass $T$ and $\pi$ section filters to be terminated in 600 Ω having cut-off frequency 3 kHz.**

**Ans.** Refer Q. 5.21, Page 5–29C, Unit-5.

5. Attempt any one/two part of the following : **(7 × 1 = 7)**

a. **Determine the currents in all the branches of the network shown in Fig. 4 using node analysis method of the graph theory.**

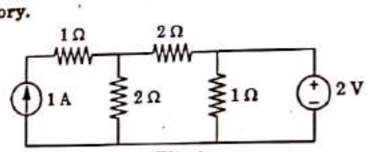


**Fig. 4.**

**Ans.** Refer Q. 1.22, Page 1–26C, Unit-1.

b. **Explain following terms with reference to network topology**

i. **Tree** ii. **Co-tree**
iii. **Incidence matrix** iv. **Oriented graph**
v. **Twig and link.**

**Ans.** Refer Q. 1.9, Page 1–10C, Unit-1.

6. Attempt any one/two part of the following : **(7 × 1 = 7)**

a. **Sketch the following signals :**

i. $t^2[u(t-1)-u(t-3)]$

ii. $(t-4)[u(t-1)-u(t-4)]$

**Ans.**

i.

1. Graph of $t^2$ :



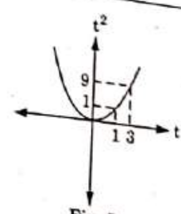


**Fig. 5.**

2. Graph of $[u(t-1)-u(t-3)]$ :

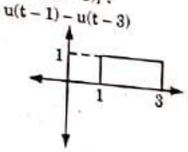


**Fig. 6.**

3. Now the graph of $t^2[u(t-1)-u(t-3)]$ :

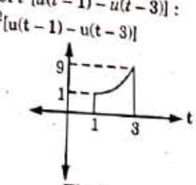


**Fig. 7.**

ii.

1. Graph of $(t-4)$ :

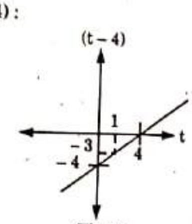


**Fig. 8.**

2. Graph of $[u(t-1)-u(t-4)]$ :

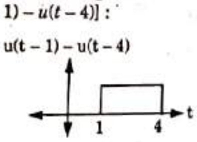


**Fig. 9.**

3. Now, the graph of $(t-4)[u(t-1)-u(t-4)]$ :

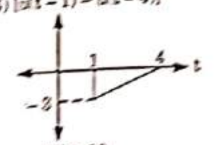


Fig. 10.

b. In the circuit shown $v(t) = 2u(t)$ and $i_L(0^-) = 2$ amps. Find and sketch $i_2(t)$.

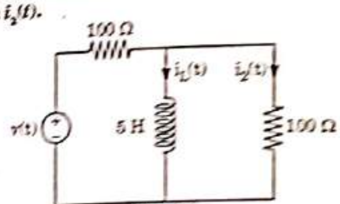


Fig. 11.

**Ans.** Refer Q. 3.7, Page 3–6C, Unit-3.

7. Attempt any one/two part of the following : (7 × 1 = 7)

a. **State and prove the maximum power transfer theorem applied to the AC circuits.**

**Ans.** Refer Q. 2.21, Page 2–23C, Unit-2.

b. **Determine the current in capacitor *C* by the principle of superposition of the network shown in Fig. 12.**

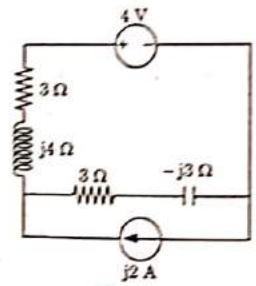


Fig. 12.

**Ans.** Refer Q. 2.12, Page 2–13C, Unit-2.

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# B.Tech.

# (SEM. IV) EVEN SEMESTER THEORY EXAMINATION, 2018-19

# NETWORK ANALYSIS AND SYNTHESIS

**Time : 3 Hours** **Max. Marks : 70**

**Note :** 1. Attempt all sections. If require any missing data; then choose suitably.

## SECTION-A

1. Attempt all questions in brief : (2 × 7 = 14)

a. **Compare dependent and independent sources.**

**Ans.** Refer Q. 2.3, Page SQ–4C, Unit-2, 2 Marks Questions.

b. **Give statement of Tellegen's theorem.**

**Ans.** Refer Q. 2.4, Page SQ–4C, Unit-2, 2 Marks Questions.

c. **Define tree, twig, links.**

**Ans.** Refer Q. 1.3, Page SQ–1C, Unit-1, 2 Marks Questions.

d. **Define principle of duality.**

**Ans.** Refer Q. 1.8, Page SQ–2C, Unit-1, 2 Marks Questions.

e. **What is transient and steady state response ?**

**Ans.** Refer Q. 3.1, Page SQ–7C, Unit-3, 2 Marks Questions.

f. **Find out $Z_{11}$, $Z_{21}$ for the following network.**

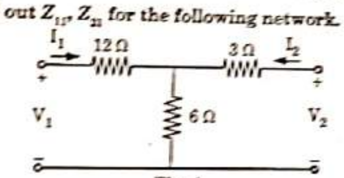


Fig. 1.

**Ans.** Refer Q. 4.8, Page SQ–11C, Unit-4, 2 Marks Questions.

g. **Find reciprocity condition in *Y*-parameter.**

**Ans.** Refer Q. 4.9, Page SQ–12C, Unit-4, 2 Marks Questions.

## SECTION-B

2. Attempt any three of the following : (7 × 3 = 21)

a. **State maximum power transfer theorem also determine the maximum power transfer to the load $R_L$ for the following circuit.**

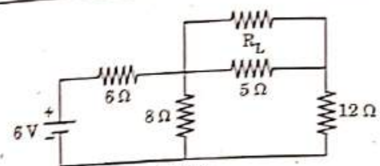


Fig. 2.

**Ans.** Refer Q. 2.22, Page 2–24C, Page-2.

**b. Define with suitable example :**

**i. Incidence matrix**

**ii. Cut-set matrix.**

**Ans.** Refer Q. 1.14, Page 1–15C, Page-1.

**c. Determine Y parameters for the network shown in Fig. 3.**

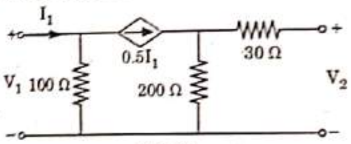


Fig. 3.

**Ans.** Refer Q. 4.25, Page 4–36C, Page-4.

**d. In the circuit shown in Fig. 4, the switch is moved from $A$ to $B$ at $t = 0$. Find $v(t)$ for $t > 0$.**

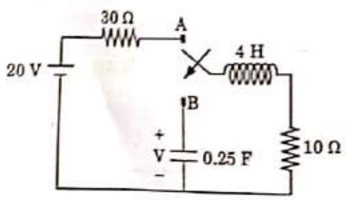


Fig. 4.

**Ans.** Refer Q. 3.12, Page 3–15C, Page-3.

**e. Synthesize :**

**i. $Z(s) = (s + 1)(s + 3) / [(s + 6)s]$ in Cauer-I form.**

**ii. $Z(s) = (s + 5)/[(s + 1)(s + 6)]$ in Foster's-II form.**

**Ans.** Refer Q. 5.14, Page 5–18C, Page-5.

## SECTION-C

3. Attempt any **one** part of the following : **(7 × 1 = 7)**

**a. Find the Norton's equivalent of network shown in Fig. 5.**



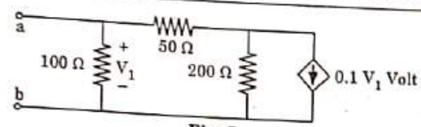


Fig. 5.

**Ans.** Refer Q. 2.19, Page 2–20C, Page-2.

**b. Using superposition theorem, find the voltage across $(4 + j3)$ Ω in the network shown in Fig. 6.**

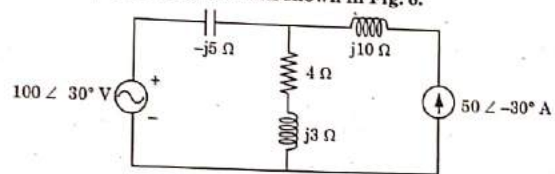


Fig. 6.

**Ans.** Refer Q. 2.10, Page 2–10C, Page-2.

4. Attempt any **one** part of the following : **(7 × 1 = 7)**

**a. Obtain $V_2/V_1$ of the network shown in Fig. 7. Also find pole zero configuration.**

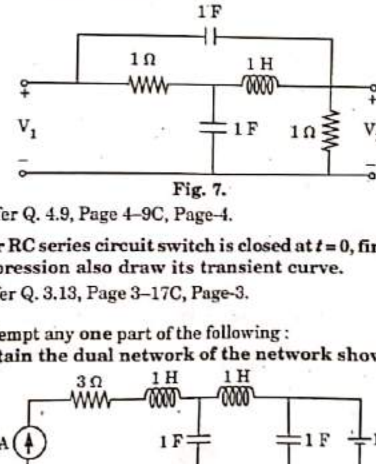


Fig. 7.

**Ans.** Refer Q. 4.9, Page 4–9C, Page-4.

**b. For RC series circuit switch is closed at $t = 0$, find out current expression also draw its transient curve.**

**Ans.** Refer Q. 3.13, Page 3–17C, Page-3.

5. Attempt any **one** part of the following : **(7 × 1 = 7)**

**a. Obtain the dual network of the network shown in Fig. 8.**




Fig. 8.

**Ans.** Refer Q. 1.20, Page 1–23C, Unit-1.

b. **Draw a graph of resistive network in Fig. 9. Select a suitable tree and obtain the tie-set matrix.**

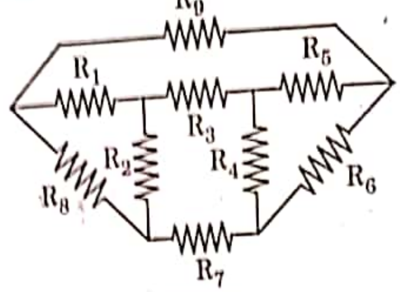


**Fig. 9.**

**Ans:** Refer Q. 1.15, Page 1–17C, Unit-1.

6. Attempt any one part of the following : **(7 × 1 = 7)**

a. **Prove that the overall Z parameters of series connected two port networks are the sum of corresponding Z parameters of the two network.**

**Ans:** Refer Q. 4.17, Page 4–27C, Unit-4.

b. **Determine *h*-parameters for the network shown in Fig. 10.**

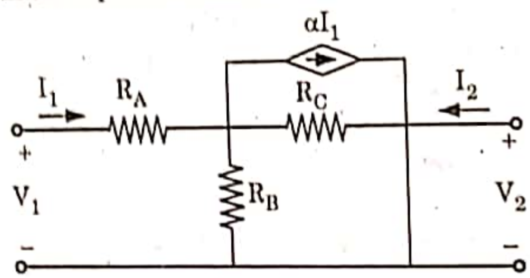


**Fig. 10.**

**Ans:** Refer Q. 4.31, Page 4–45C, Unit-4.

7. Attempt any one part of the following : **(7 × 1 = 7)**

a. **Define active, passive filters. List advantages of active filter over passive filter.**

**Ans:** Refer Q. 5.16, Page 5–21C, Unit-5.

b. **Write five necessary conditions for positive real function. Test whether given polynomial is positive real function or not $Z(s) = \dfrac{s^3 + 2s + 26}{s + 4}$.**

**Ans:** Refer Q. 5.3, Page 5–3C, Unit-5.

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