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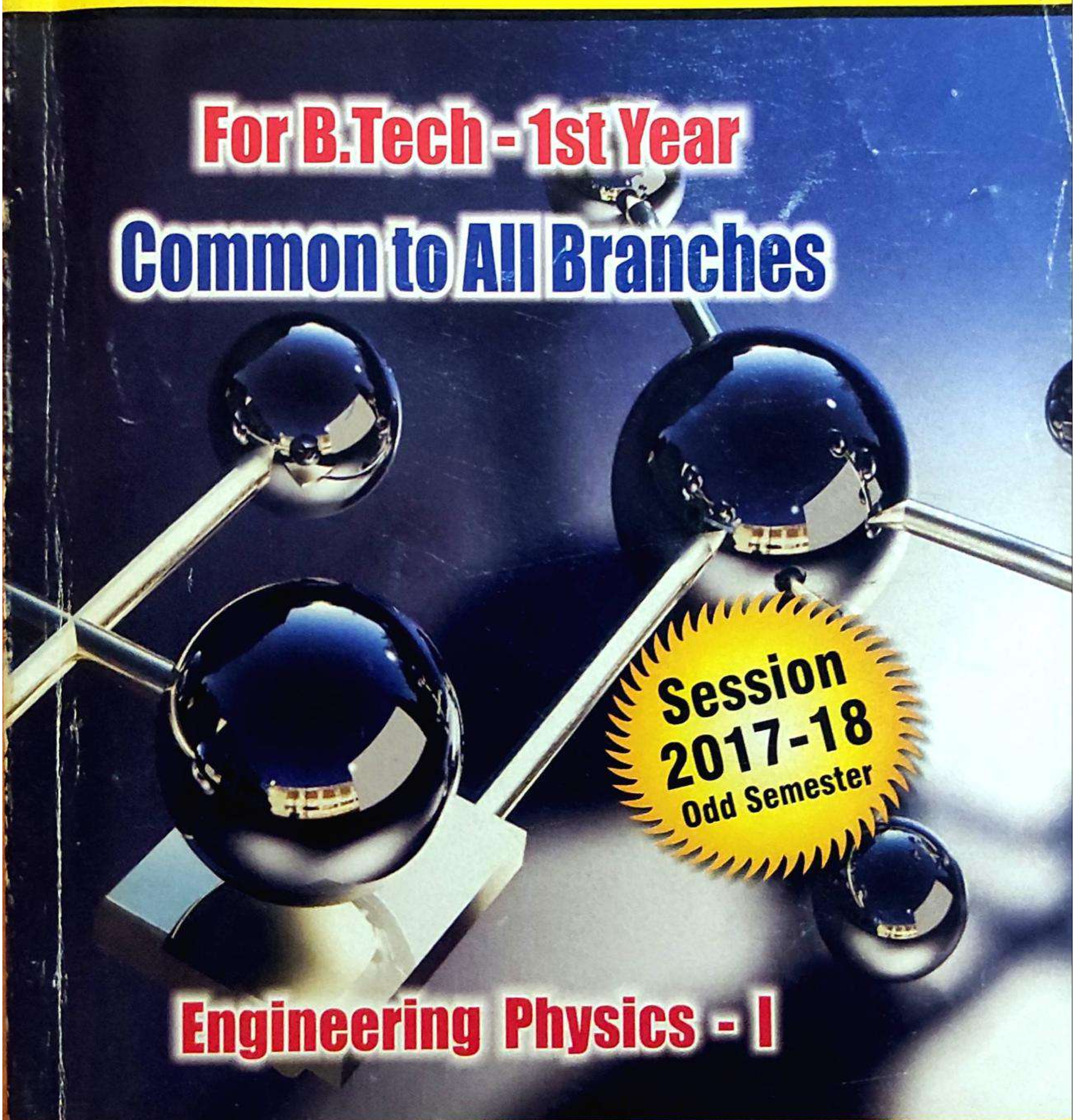

Session 2017-18
Odd Semester

Engineering Physics - I

Also Include Previous Years Solved University Question Papers

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CONTENTS

RAS 101 : ENGINEERING PHYSICS - I

1 UNIT

Relativistic Mechanics

PART-1

Inertial and non-inertial frames, Galilean transformation, Michelson-Morley experiment, Einstein's postulates.

CONCEPT OUTLINE : PART-1

Frame of Reference : It is that coordinate system which is used to identify the position or motion of an object.

Types of Frame of Reference :

a. Inertial frame of reference, and
b. Non-inertial frame of reference.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.1. Show that the frame of reference moving with constant velocity v is an inertial frame of reference.

Answer

1. Let us s is a frame of reference which is in rest to an observer and s' is another frame of reference moving with constant velocity v in the positive x direction with respect to the same observer.
2. o and o' are origin of frame s and s' respectively.

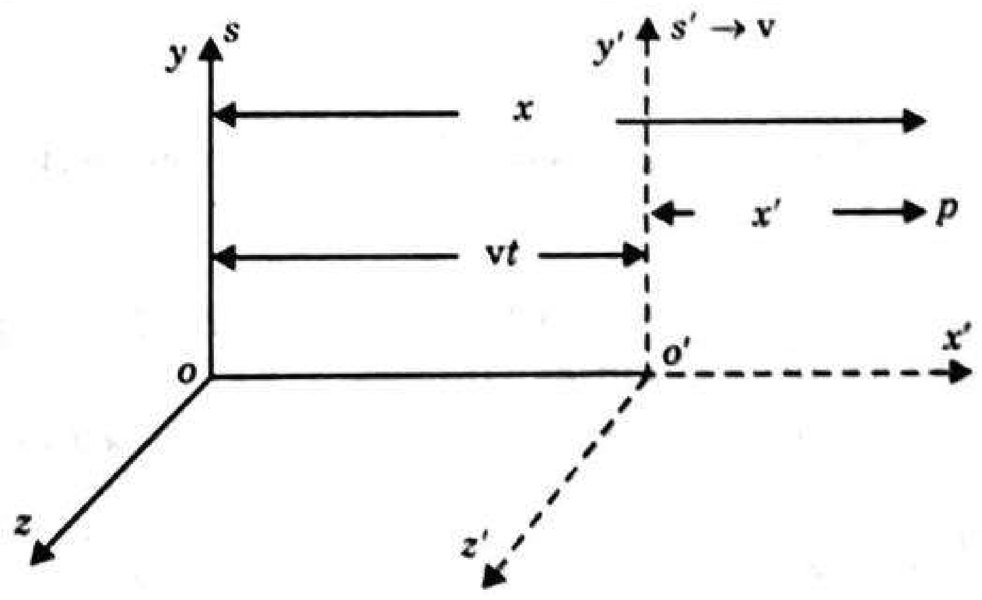

Fig. 1.1.1.

3. Initially o and o' coincide with each other at time $t = t' = 0$, where t and t' = time measured in s and s' frames respectively.

4. Suppose P be a point in the space.
5. Now from Fig. 1.1.1,

$$x = x' + vt \quad ...(1.1.1)$$
$$y = y' \quad ...(1.1.2)$$
$$z = z' \quad \text{No relative motion} \quad ...(1.1.3)$$
$$t = t' \quad ...(1.1.4)$$

6. Equation (1.1.1) to equation (1.1.4) are position and time transformation equations in s and s' frame.
7. Differentiating equation (1.1.1) w.r.t. t on both sides,

$$\frac{dx}{dt} = \frac{dx'}{dt} + \frac{v\,dt'}{dt} \quad ...(1.1.5)$$
$$\frac{dx}{dt} = \frac{dx'}{dt'} + \frac{v\,dt'}{dt'} \quad (\because \; t = t' \;\; \therefore \;\; dt = dt')$$
$$\Rightarrow \quad u_x = u'_x + v \quad ...(1.1.6)$$

8. Differentiating equation (1.1.2) w.r.t. t,

$$\frac{dy}{dt} = \frac{dy'}{dt}$$
$$\frac{dy}{dt} = \frac{dy'}{dt'} \quad (\because \; t = t') \quad ...(1.1.7)$$
$$\Rightarrow \quad u_y = u'_y$$

9. Similarly, $u_z = u'_z$ ...(1.1.8)
10. Now again differentiate equation (1.1.6) w.r.t. t, we get

$$\frac{du_x}{dt} = \frac{du'_x}{dt} + \frac{dv}{dt}$$
$$\frac{du_x}{dt} = \frac{du'_x}{dt} \quad (\because \; v = \text{constant})$$
$$\frac{du_x}{dt} = \frac{du'_x}{dt'} \quad (\because \; t = t') \quad ...(1.1.9)$$
$$\Rightarrow \quad a_x = a_x'$$

11. Similarly differentiating equation (1.1.8) and equation (1.1.9),

$$a_y = a'_y \quad ...(1.1.10)$$
$$a_z = a'_z \quad ...(1.1.11)$$

12. Equation (1.1.9), equation (1.1.10) and equation (1.1.11) shows that the acceleration is invariant in both frames.
13. So a frame of reference moving with constant velocity is an inertial frame.

Que 1.2. **Derive the Galilean transformation equations and show that its acceleration components are invariant.**

UPTU 2015-16, Marks 15

Answer

1. Suppose we are in an inertial frame of reference s and the coordinates of some event that occurs at the time t are x, y, z as shown in Fig. 1.2.1.
2. An observer located in a different inertial frame s' which is moving with respect to s at the constant velocity $\vec{v}$, will find that the same event occurs at time t' and has the position coordinates x', y' and z'.

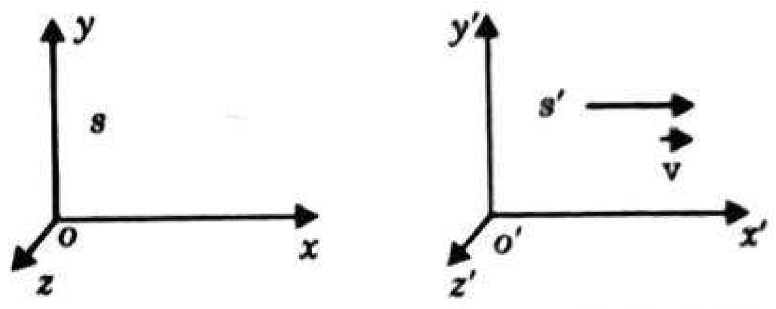

Fig. 1.2.1. Galilean transformation.

3. Assume that $\vec{v}$ is in positive x direction.
4. When origins of s and s' coincide, measurements in the x direction made in s is greater than those of s' by $\vec{v}t$ (distance).
5. Hence,

$$x' = x - vt \quad ...(1.2.1)$$
$$y' = y \quad ...(1.2.2)$$
$$z' = z \quad \text{no relative motion} \quad ...(1.2.3)$$
$$t' = t \quad ...(1.2.4)$$

6. These set of equations are known as Galilean transformations.
7. Differentiating equation (1.2.1) with respect to t, we get

$$\frac{dx'}{dt} = \frac{dx}{dt} - v\frac{dt}{dt}$$
$$\frac{dx'}{dt'} = \frac{dx}{dt} - v \quad (\because t = t' \;\; \therefore \;\; dt' = dt)$$

8. Similarly $\dfrac{dy'}{dt'} = \dfrac{dy}{dt}$

and $\dfrac{dz'}{dt'} = \dfrac{dz}{dt}$

9. Since, $dx'/dt' = u_x'$, the x-component of the velocity measured in s', and $dx/dt = u_x$, etc., then,

$$u_x' = u_x - v \quad ...(1.2.5)$$
$$u_y' = u_y \quad ...(1.2.6)$$
$$u_z' = u_z \quad ...(1.2.7)$$

10. The equation (1.2.5), equation (1.2.6) and equation (1.2.7) can be written collectively in the vector form as

$$\vec{u}' = \vec{u} - \vec{v} \quad ...(1.2.8)$$

11. To obtain the acceleration transformation, we differentiate the equation (1.2.5), equation (1.2.6) and equation (1.2.7) with respect to time such that

$$\frac{du_x'}{dt'} = \frac{d}{dt}(u_x - v) = \frac{du_x}{dt}$$

Similarly,
$$\frac{du_y'}{dt'} = \frac{du_y}{dt} \text{ and } \frac{du_z'}{dt'} = \frac{du_z}{dt}$$

12. Since,
$$\frac{du_x'}{dt'} = a_x' \,;\, \frac{du_y'}{dt'} = a_y' \,;\, \frac{du_z'}{dt'} = a_z'$$
$$\frac{du_x}{dt} = a_x \,;\, \frac{du_y}{dt} = a_y \,;\, \frac{du_z}{dt} = a_z$$

13. Then we get
$$a_x' = a_x \quad \text{...(1.2.9)}$$
$$a_y' = a_y \quad \text{...(1.2.10)}$$
$$a_z' = a_z \quad \text{...(1.2.11)}$$

or writing these equations collectively,

$$\vec{a}' = \vec{a}$$

14. The measured components of acceleration of a particle are independent of the uniform relative velocity of the reference frames.
15. In other words, acceleration remains invariant when passing from one inertial frame to another that is in uniform relative translational motion.

Que 1.3. **Show that the distance between points is invariant under Galilean transformations.**

Answer

1. Let (x_1, y_1, z_1) and (x_2, y_2, z_2) be the coordinate of two points P and Q in rest frame s. Then the distance between them will be
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$
2. Now the distance between them measured in moving frame s' is
$$d' = \sqrt{(x_2' - x_1')^2 + (y_2' - y_1')^2 + (z_2' - z_1')^2}$$
3. Using Galilean transformation
$$x_2' = x_2 - v_x t,\ y_2' = y_2 - v_y t \text{ and } z_2' = z_2 - v_z t$$
$$x_1' = x_1 - v_x t,\ y_1' = y_1 - v_y t \text{ and } z_1' = z_1 - v_z t$$
4. Hence
$$d' = \sqrt{[(x_2 - v_x t) - (x_1 - v_x t)]^2 + [(y_2 - v_y t) - (y_1 - v_y t)]^2 + [(z_2 - v_z t) - (z_1 - v_z t)]^2}$$
$$d' = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$
$$d' = d$$

Que 1.4. **Discuss the objective and outcome of Michelson-Morley experiment.** **[UPTU 2012-13, Marks 05]**

Answer

A. Objective of Michelson-Morley Experiment :

1. According to Huygens the atmosphere is surrounded by an imaginary medium known as 'ether'.
2. The relative motion of bodies is affected by the presence of this imaginary medium.
3. To explain the Huygens concept of 'ether drag' Michelson-Morley did an experiment.
4. According to Morley if there exist some imaginary medium like 'ether' in the earth atmosphere, there should be some time difference between relative motion of body with respect to earth and against the motion of earth.

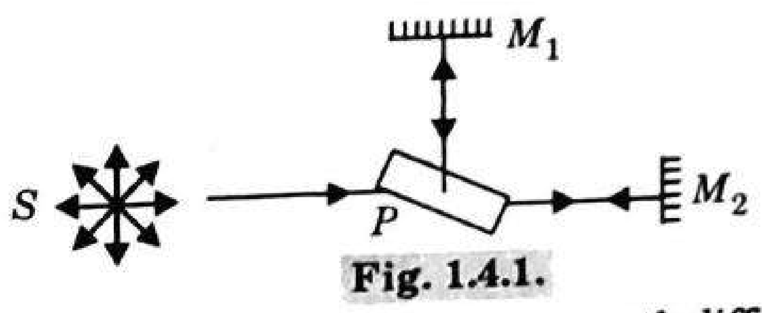

Fig. 1.4.1.

5. Due to this time difference there exist some path difference and if such path difference occurs, Huygens concept is correct and if it does not occur then Huygens concept is wrong.

B. Michelson-Morley Experiment :

1. In Michelson-Morley experiment there is a semi-silvered glass-plate P and two plane mirrors M_1 and M_2 which are mutually perpendicular and equidistant from plate P.
2. There is a monochromatic light source in front of glass plate P.
3. The whole arrangement is fixed on a wooden stand and that wooden stand is dipped in a mercury pond. So it becomes easy to rotate.
4. Let v be the speed of imaginary medium (ether) w.r.t. earth and c is the velocity of light, so time taken to move the light ray from plate P to M_2 and reflected back,
$$T_1 = \frac{d}{c+v} + \frac{d}{c-v} = d\left[\frac{2c}{(c^2 - v^2)}\right]$$
$$T_1 = \frac{2dc}{c^2\left[1 - \frac{v^2}{c^2}\right]} = \frac{2d}{c\left[1 - \frac{v^2}{c^2}\right]}$$
5. Expanding binomially,

$$T_1 = \frac{2d}{c}\left[1+\frac{v^2}{c^2}\right] \qquad ...(1.4.1)$$

[neglecting higher power term]

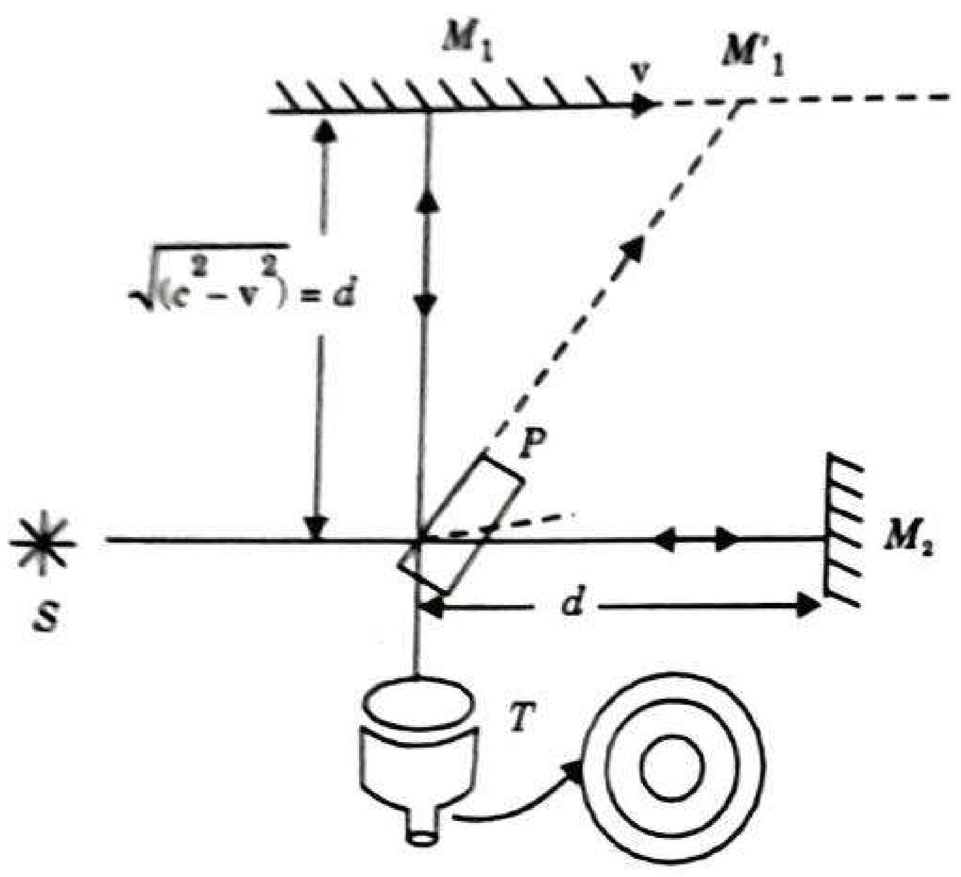

Fig. 1.4.2.

6. Time taken to move a light ray from plate P to M_1 and to reflect back,

$$T_2 = \frac{2d}{\sqrt{c^2 - v^2}} = \frac{2d}{c\sqrt{1-\frac{v^2}{c^2}}}$$

$$T_2 = \frac{2d}{c}\left[1-\frac{v^2}{c^2}\right]^{\frac{-1}{2}}$$

7. Expanding binomially,

$$T_2 = \frac{2d}{c}\left[1+\frac{v^2}{2c^2}\right] \qquad ...(1.4.2)$$

8. So, time difference

$$\Delta t = T_1 - T_2$$

$$= \frac{2d}{c}\left[1+\frac{v^2}{c^2}\right] - \frac{2d}{c}\left[1+\frac{v^2}{2c^2}\right]$$

$$= \frac{2d}{c}\left[1+\frac{v^2}{c^2}-1-\frac{v^2}{2c^2}\right] = \frac{2d}{c}\left[\frac{v^2}{2c^2}\right] = \frac{dv^2}{c^3} \qquad ...(1.4.3)$$

9. Now, the apparatus is rotated by 90° so that the position of mirror M_1 and M_2 gets interchange. So time taken from P to M_1 is

$$T_1' = \frac{2d}{c}\left[1+\frac{v^2}{2c^2}\right]$$

and time taken from P to M_2 is

$$T_2' = \frac{2d}{c}\left[1+\frac{v^2}{c^2}\right]$$

10. Time difference

$$\Delta t' = T_1' - T_2'$$

$$= \frac{2d}{c}\left[1+\frac{v^2}{2c^2}-\frac{v^2}{c^2}-1\right] = -\frac{2dv^2}{2c^3} = -\frac{dv^2}{c^3} \qquad ...(1.4.4)$$

11. So, total time difference

$$\Delta T = \Delta t - \Delta t'$$

$$= \frac{dv^2}{c^3} - \left(-\frac{dv^2}{c^3}\right) = \frac{2dv^2}{c^3}$$

12. Since, path difference = speed of light × ΔT

$$\Delta = c \times \Delta T$$

$$\Delta = c \times \frac{2dv^2}{c^3}$$

$$\Delta = \frac{2dv^2}{c^2} \qquad ...(1.4.5)$$

13. Let in this path difference there are n waves of wavelength λ each are showing interference

So, $\Delta = n\lambda$

14. Equating equation (1.4.5) and equation (1.4.6),

$$n\lambda = \frac{2dv^2}{c^2}$$

$$n = \frac{2dv^2}{c^2\lambda}$$

n = fringe shift

15. Taking $d = 11$ m

Velocity of earth = 3×10^4 m/s

Velocity of light = 3×10^8 m/s

λ = 6000 Å [1 Å = 10^{-10} m]

$$n = \frac{2 \times 11 \times 9 \times 10^8}{9 \times 10^{16} \times 6000 \times 10^{-10}} = \frac{22 \times 10^2}{6000} = \frac{22}{60} = \frac{11}{30}$$

$$n = 0.36$$

16. If such type of medium like 'ether' exists in the atmosphere, there must be a fringe shift of 0.36.

17. Michelson–Morley performed that experiment several times in different situations, in different weather conditions but no fringe shift was obtained

hence Huygens concept of 'ether drag' is wrong. This is known as 'Negative Result'.

Que 1.5. **What will be the expected fringe shift on the basis of stationary ether hypothesis in Michelson-Morley experiment ? If the effective length of each part is 8 m and wavelength used is 8000 Å ?**

Answer

1. **Given :**

$$d = 8 \text{ m}$$
$$v = 3 \times 10^4 \text{ m/s}$$
$$\lambda = 8000 \times 10^{-10} \text{ m}$$
$$c = 3 \times 10^8 \text{ m/s}$$

2. We know that fringe shift is given by

$$\Delta = \frac{2d\,v^2}{c^2 \lambda}$$

3. On putting the given values, we have

$$\Delta = \frac{2 \times 8 \times (3 \times 10^4)^2}{(3 \times 10^8)^2 \times 8 \times 10^{-7}} = 0.2$$

Que 1.6. **State Einstein's postulates of special theory of relativity. Explain why Galilean relativity failed to explain actual results of Michelson-Morley experiment.**

UPTU 2013-14, Marks 05

Answer

1. There are two postulates of the special theory of relativity proposed by Einstein.

Postulate 1 :

The laws of physics are the same in all inertial frames of reference moving with a constant velocity with respect to one another.

Explanation :

1. If the laws of physics had different forms for observers in different frames in relative motion, one could determine from these differences which objects are stationary in space and which are moving.
2. As there is no universal frame of reference, therefore this distinction between objects cannot be made. Hence universal frame of reference is absent.

Postulate 2 :

The speed of light in free space has the same value in all inertial frames of reference. This speed is 2.998×10^8 m/s.

Explanation :

1. This postulate is directly followed from the result of Michelson-Morley experiment.

A. Galilean transformation :

1. In Galilean transformations the speed of light was not taken to be constant in all inertial frames.
2. These equations were based on absolute time and absolute space.
3. The above two assumptions contradict the Einstein postulates.
4. So the Galilean transformation failed to explain the actual results of Michelson-Morley experiment.

PART-2

Lorentz transformation equations, Length contraction and Time dilation, Relativistic addition of velocities.

CONCEPT OUTLINE : PART-2

Lorentz Transformation : The equation relating the coordinates of a particle in the two inertial frame on the basis of special theory of relativity are called Lorentz transformations.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.7. **What are Lorentz transformation equations ? Derive the expression for it.**

OR

State the fundamental postulates of the special theory of relativity. Deduce the Lorentz transformation equations.

UPTU 2009-10, Marks 05

Answer

A. Fundamental Postulates of the Special Theory of Relativity :
Refer Q. 1.6, Page 12B, Unit-1.

B. Lorentz Transformation Equations :

1. The equations in special theory of relativity, which relate to the space and time coordinates of an event in two inertial frames of reference moving with a uniform velocity relative to one another, are called Lorentz transformation.
2. Let us consider two frames of reference s and s' in which frame s' is moving with velocity v along x-axis. The coordinates of frame s are (x, y, z, t) while the coordinates of frame s' are (x', y', z', t').

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3. According to first postulates of special theory of relativity in frame s',

$$x' \propto (x - vt)$$

$$x' = k(x - vt) \quad \text{...(1.7.1)}$$

4. In frame s, $x \propto (x' + vt')$

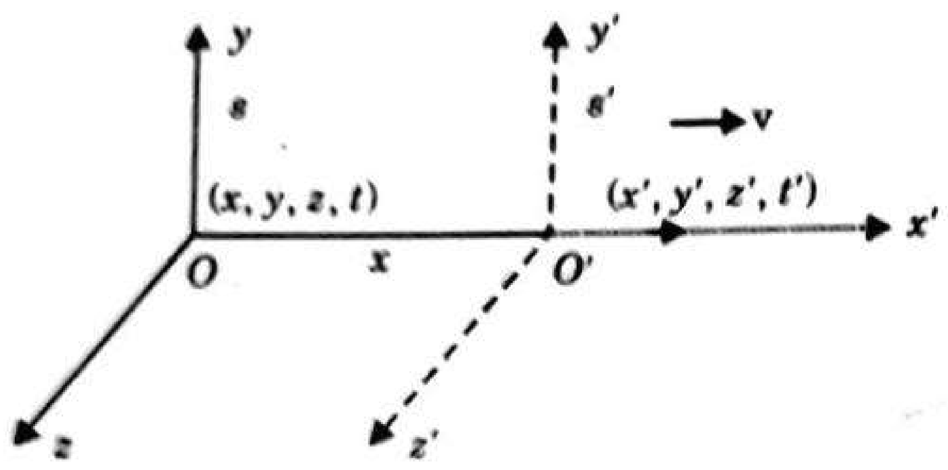


Fig. 1.7.1.

$$x = k(x' + vt') \quad \text{...(1.7.2)}$$

5. Putting x' in equation (1.7.2),

$$x = k[(k(x - vt) + vt')] = [k^2(x - vt) + v\,kt']$$

$$x = k^2x - k^2vt + kvt'$$

$$kvt' = (1 - k^2)x + k^2vt$$

$$t' = \frac{(1-k^2)x + k^2vt}{kv}$$

$$t' = \frac{(1-k^2)x}{kv} + kt \quad \text{...(1.7.3)}$$

6. According to second postulate of special theory of relativity, speed of light is a constant quantity.

In frame s $\quad x = ct \quad$ ...(1.7.4)

In frame s' $\quad x' = ct' \quad$ ...(1.7.5)

7. Putting the value of x' and t' from equation (1.7.1) and equation (1.7.3) in equation (1.7.5),

$$k(x - vt) = \frac{cx}{vk}(1-k^2) + ckt$$

$$x\left[k - \frac{c}{vk}(1-k^2)\right] = [ck + vk]\,t$$

$$x = \frac{[ck + vk]t}{\left[k - \frac{c}{vk}(1-k^2)\right]} \quad \text{...(1.7.6.)}$$

9. On comparing equation (1.7.6) with equation (1.7.4),

$$c = \frac{(ck + vk)}{\left[k - \frac{c}{vk}(1-k^2)\right]}$$

$$ck + vk = ck - \frac{c^2}{vk}(1-k^2)$$

$$vk = -\frac{c^2}{vk}(1-k^2)$$

$$v^2k^2 = -c^2 + c^2k^2$$

$$k^2(v^2 - c^2) + c^2 = 0$$

$$k^2(c^2 - v^2) = c^2$$

$$\frac{k^2}{c^2}[c^2 - v^2] = 1$$

$$k^2\left[1 - \frac{v^2}{c^2}\right] = 1$$

$$k^2 = \frac{1}{1 - \frac{v^2}{c^2}}$$

or

$$k = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

10. Putting the value of k in equation (1.7.1),

$$x' = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}[x - vt]$$

(First Lorentz transformation equation)

$y' = y$ (Second Lorentz transformation equation)

$z' = z$ (Third Lorentz transformation equation)

11. Now,

$$t' = \frac{x(1-k^2)}{kv} + kt = \frac{x}{v}\left[\frac{1}{k} - k\right] + kt$$

$$= \frac{x}{v}\left[\frac{1}{\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}} - \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}\right] + t\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{x}{v}\left[\sqrt{1 - \frac{v^2}{c^2}} - \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}\right] + \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{x}{v}\left[\frac{1 - \frac{v^2}{c^2} - 1}{\sqrt{1 - \frac{v^2}{c^2}}}\right] + \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{x}{v}\left[\frac{-\frac{v^2}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}\right] + \frac{t}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$= -\frac{x}{c^2}\left[\frac{v}{\sqrt{1-\frac{v^2}{c^2}}}\right] + \frac{t}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{-vx + c^2 t}{c^2\sqrt{1-\frac{v^2}{c^2}}}$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

12. Similarly putting the value of k in equation (1.7.2) inverse Lorentz transformation equations,

$$x = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}[x' + vt']$$

$$y = y'$$

$$z = z'$$

$$t = \frac{t' + \frac{vx'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

Que 1.8. **Deduce the Lorentz transformation equations from Einstein's postulates. Also show that at low velocities, the Lorentz transformations reduce to Galilean transformations.**

UPTU 2014-15, Marks 05

Answer

A. Lorentz Transformation Equations from Einstein's Postulates : Refer Q. 1.7, Page 13B, Unit-1.

B. Galilean Transformations :

1. Lorentz transformation equation,

$$x = \frac{x' + vt'}{\sqrt{1-\frac{v^2}{c^2}}} \quad \text{...(1.8.1)}$$

2. At low velocities means $v <<< c$

Thus $\quad 1 - \frac{v^2}{c^2} = 1$

3. From equation (1.8.1)

$$x = x' + vt' \quad \text{...(1.8.2)}$$

4. Equation (1.8.2) is a Galilean transformation equation.
5. It means at low velocities, the Lorentz transformation reduces to Galilean transformation.

Que 1.9. **Show from Lorentz transformation that two simultaneous events ($t'_1 = t'_2$) at different position ($x'_1 \neq x'_2$) in a frame s' are not in general simultaneous in another frame s.**

OR

Show from Lorentz transformation that two events simultaneous ($t_1 = t_2$) at different positions ($x_1 \neq x_2$) in a reference frame s are not in general simultaneous in another reference frame s'.

UPTU 2008-09, Marks 05

Answer

1. Suppose s and s' are two frames of reference. Frame s' moving with constant velocity v w.r.t. s.
2. Let two events at P and Q occurring at time $t'_1 = t'_2$ at x'_1 and x'_2 in frame s'.

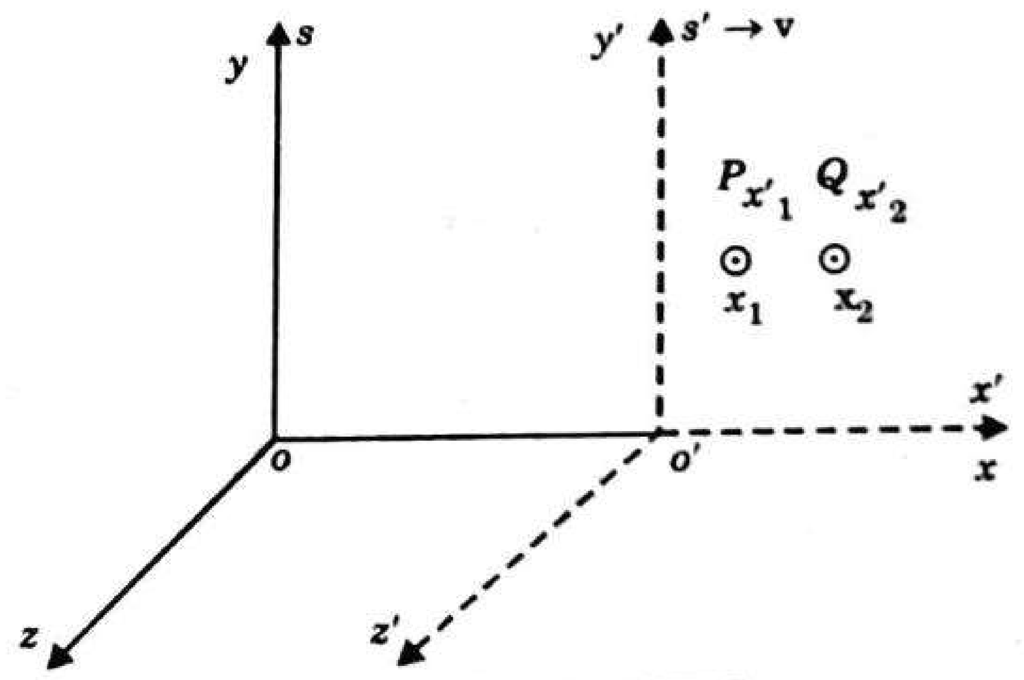


Fig. 1.9.1.

3. If $x_2' \neq x_1'$ and $t_2' = t_1'$
4. Then

$$t_2 - t_1 = \frac{t_2' + \frac{v}{c^2}x_2'}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{t_1' + \frac{v}{c^2}x_1'}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{t_2' + \frac{v}{c^2}x_2' - t_1' - \frac{v}{c^2}x_1'}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$= \frac{t_2' - t_1' + \frac{v}{c^2}(x_2' - x_1')}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{\frac{v}{c^2}(x_2' - x_1')}{\sqrt{1-\frac{v^2}{c^2}}} \quad (\because t_2' = t_1')$$

5. So,
$$t_2 - t_1 = \frac{\frac{v}{c^2}(x_2' - x_1')}{\sqrt{1-\frac{v^2}{c^2}}} \neq 0$$

as $x_2' \neq x_1'$ and $v \neq 0$

Que 1.10. **Show that $x^2 + y^2 + z^2 = c^2t^2$ is invariant under Lorentz transformation.**

OR

Show that the space time interval $x^2 + y^2 + z^2 - c^2t^2 = 0$ is invariant under Lorentz transformation.

OR

Show that space-time interval between two events remains invariant under Lorentz transformation.

UPTU 2011-12, Marks 05

Answer

1. The inverse Lorentz transformation are
$$x = \frac{x' + vt'}{\sqrt{1-\frac{v^2}{c^2}}}, y = y', z = z' \text{ and } t = \frac{t' + \frac{vx'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

2. Using above equation in equation
$$x^2 + y^2 + z^2 = c^2t^2$$
$$\Rightarrow \left[\frac{x'+vt'}{\sqrt{1-\frac{v^2}{c^2}}}\right]^2 + y'^2 + z'^2 = c^2\left[\frac{t'+\frac{vx'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}\right]^2$$
$$\Rightarrow y'^2 + z'^2 = \frac{1}{1-\frac{v^2}{c^2}}\left[c^2t'^2 + \frac{v^2x'^2}{c^2} - x'^2 - v^2t'^2\right]$$
$$\Rightarrow y'^2 + z'^2 = \frac{1}{1-\frac{v^2}{c^2}}\left[(c^2t'^2 - x'^2)\left(1-\frac{v^2}{c^2}\right)\right]$$
$$\Rightarrow x'^2\left(1-\frac{v^2}{c^2}\right) + y'^2\left(1-\frac{v^2}{c^2}\right) + z'^2\left(1-\frac{v^2}{c^2}\right) = c^2t'^2\left(1-\frac{v^2}{c^2}\right)$$
$$\Rightarrow x'^2 + y'^2 + z'^2 = c^2t'^2$$

3. Hence $x^2 + y^2 + z^2 = c^2t^2$ is invariant under Lorentz transformation.

Que 1.11. **As measured by O a bulb goes off at $x = 100$ km, $y = 10$ km, $z = 1$ km and $t = 5 \times 10^{-4}$ sec. What are the coordinates x', y', z' and t' of this event as determined by a second observer O' moving relative to O at $-0.8\,c$ along the common x-x' axis ?**

Answer

1. Given :
$x = 100$ km
$v = -0.8 \times 3 \times 10^5$ km/s
$t = 5 \times 10^{-4}$ s
and $c = 3 \times 10^8$ m/s $= 3 \times 10^5$ km/s

2. Since,
$$x' = \frac{x - vt}{\sqrt{1-\frac{v^2}{c^2}}}$$

3. So
$$x' = \frac{100 - (-0.8 \times 3 \times 10^5) \times 5 \times 10^{-4}}{\sqrt{1-(0.8)^2}} = 367 \text{ km}$$

4.
$$t' = \frac{t - \frac{v}{c^2}x}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{5 \times 10^{-4} - \frac{(-0.8 \times 3 \times 10^5) \times 100}{(3 \times 10^5)^2}}{\sqrt{1-(0.8)^2}}$$
$$= 12.8 \times 10^{-4}$$

5.
$y' = y = 10$ km
$z' = z = 1$ km

Que 1.12. **Show that a moving circle will appeared to be an ellipse if it is seen from a frame which is at rest.**

Answer

1. The equation of circle is $x^2 + y^2 = a^2$.
2. Putting the values from Lorentz transformation,
$$\left[\frac{x'+vt'}{\sqrt{1-\frac{v^2}{c^2}}}\right]^2 + y'^2 = a^2$$
$$\frac{x'^2 + v^2t'^2 + 2x'.vt'}{\left(1-\frac{v^2}{c^2}\right)} + y'^2 = a^2$$
$$c^2x'^2 + c^2v^2t'^2 + 2x'\,vt'\,c^2 + y'^2(c^2 - v^2) = a^2(c^2 - v^2)$$
$$x'^2 + v^2t'^2 + 2x'\,vt' + y'^2 - y'^2\,\frac{v^2}{c^2} = a^2 - \frac{a^2v^2}{c^2}$$

$$x'^2 + y'^2\left(1-\frac{v^2}{c^2}\right) = a^2\left(1-\frac{v^2}{c^2}\right) - v^2t'^2 - 2x'vt'$$

$$\frac{x'^2}{\left(1-\frac{v^2}{c^2}\right)} + \frac{y'^2}{1} = a^2 - \frac{v^2t'^2 + 2x'vt'}{\left(1-\frac{v^2}{c^2}\right)}$$

3. Let $P^2 = 1-\frac{v^2}{c^2}$

and $Q^2 = a^2 - \frac{v^2t'^2 + 2x'vt'}{\left(1-\frac{v^2}{c^2}\right)}$

4. $$\frac{x'^2}{P^2} + \frac{y'^2}{1^2} = Q^2$$

$$\frac{x'^2}{(QP)^2} + \frac{y'^2}{Q^2} = 1 \quad ...(1.12.1)$$

5. Equation (1.12.1) represents an equation of ellipse.

Que 1.13. **What is length contraction ? Find out its equation using Lorentz transformation.**

Answer

1. The length of a moving rod will appeared to be contracted if it is seen from a frame of reference which is at rest. This decrease in length in the direction of motion is called length contraction.
2. Let us consider two frame of reference s and s' in which frame s' is moving with velocity v along x-axis.
3. A rod of length 'L_0' is moving horizontally in frames.

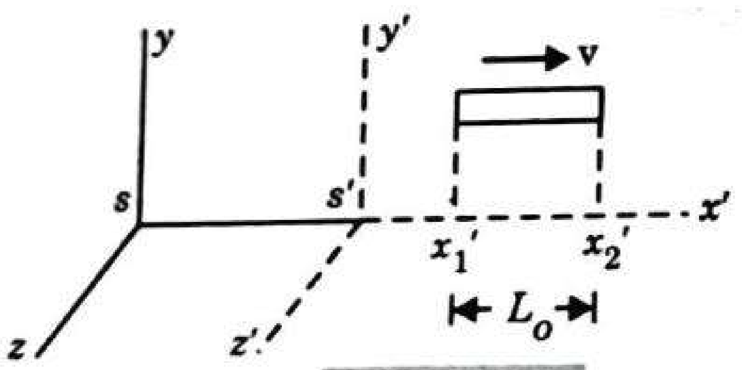


Fig. 1.13.1.

3. According to Lorentz transformation,

$$x_1' = \frac{x_1 - vt}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$x_2' = \frac{x_2 - vt}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$x_2' - x_1' = \frac{x_2 - x_1}{\sqrt{1-\frac{v^2}{c^2}}} \quad \text{(apparent length)}$$

4. $x_2 - x_1 = L$

$x_2' - x_1' = L_0$

$$L_0 = \frac{L}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$L = L_0\sqrt{1-\frac{v^2}{c^2}}$$

If v = c then $L = 0$

Que 1.14. **What do you mean by proper length ? Derive the expression for relativistic length. Calculate the percentage contraction of a rod moving with a velocity of 0.6 c in a direction inclined at 30° to its own length.**

UPTU 2015-16, Marks 10

Answer

A. Proper Length :

1. Suppose a body is at rest with respect to an observer, its length is determined by measuring the difference between the spatial coordinates of the end points of the body.
2. Since the body is not moving, these measurements may be made at any time and the length so determined is called rest length or proper length of the body.

B. Numerical :

1. Suppose the proper length of rod is L_0.
2. Observed length, along the direction of motion

$$L_x = L_0 \cos 30° \sqrt{1-(0.6)^2} = 0.7L_0$$

3. Observed length, perpendicular to the direction of motion,

$$L_y = L_0 \sin 30°$$

$$L_y = \frac{L_0}{2}$$

4. ∴ Length of moving rod,

$$L = \sqrt{L_x^2 + L_y^2}$$

$$L = \left[(0.7L_0)^2 + \left(\frac{L_0}{2}\right)^2\right]^{1/2} = 0.86L_0$$

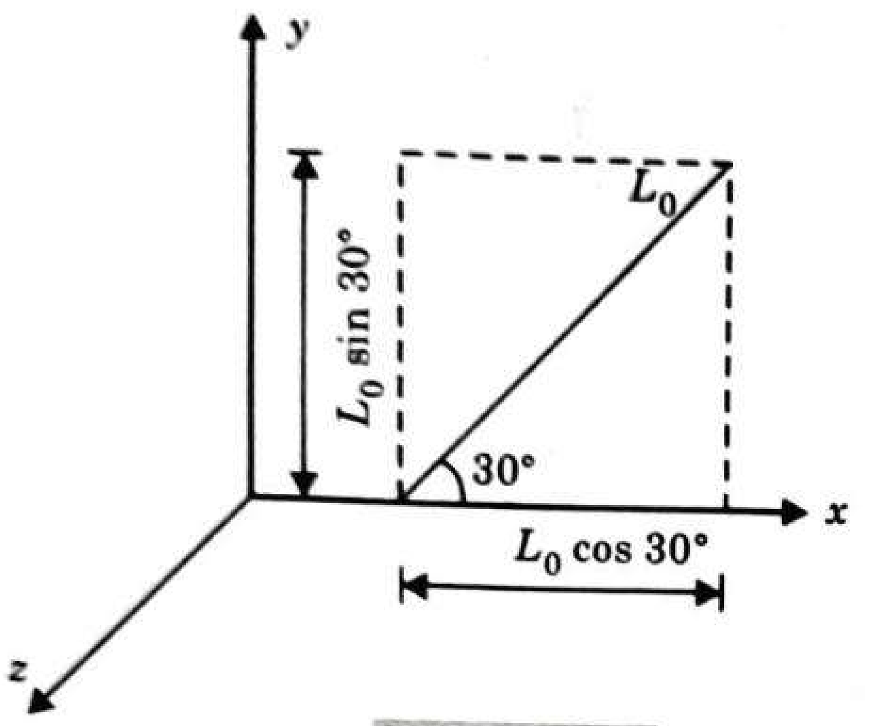


Fig. 1.14.1.

5. Percentage contraction in length $= \frac{L_0 - L}{L_0} \times 100$

$$= \frac{L_0 - 0.86L_0}{L_0} \times 100$$

$$= 14\ \%$$

Que 1.15. **What will be the apparent length of rod of length 5 m and inclined at an angle 60° to horizontal. This rod is moving with a speed of 3×10^7 m/s.**

Answer

1. Since, frame of reference is moving along x-direction. So length of rod appears to change in x-direction only.
2. So, new length in x-direction,

$$L = L_0\sqrt{1 - \frac{v^2}{c^2}}$$

$$= 5\cos 60^\circ \sqrt{1 - \left(\frac{3 \times 10^7}{3 \times 10^8}\right)^2}$$

$$L_x = 2.487 \text{ m}$$

3. New apparent length of rod

$$= \sqrt{L_x^2 + (5\sin 60^\circ)^2}$$

$$= \sqrt{(2.487)^2 + (5\sin 60^\circ)^2}$$

$$= 4.99 \text{ m}$$

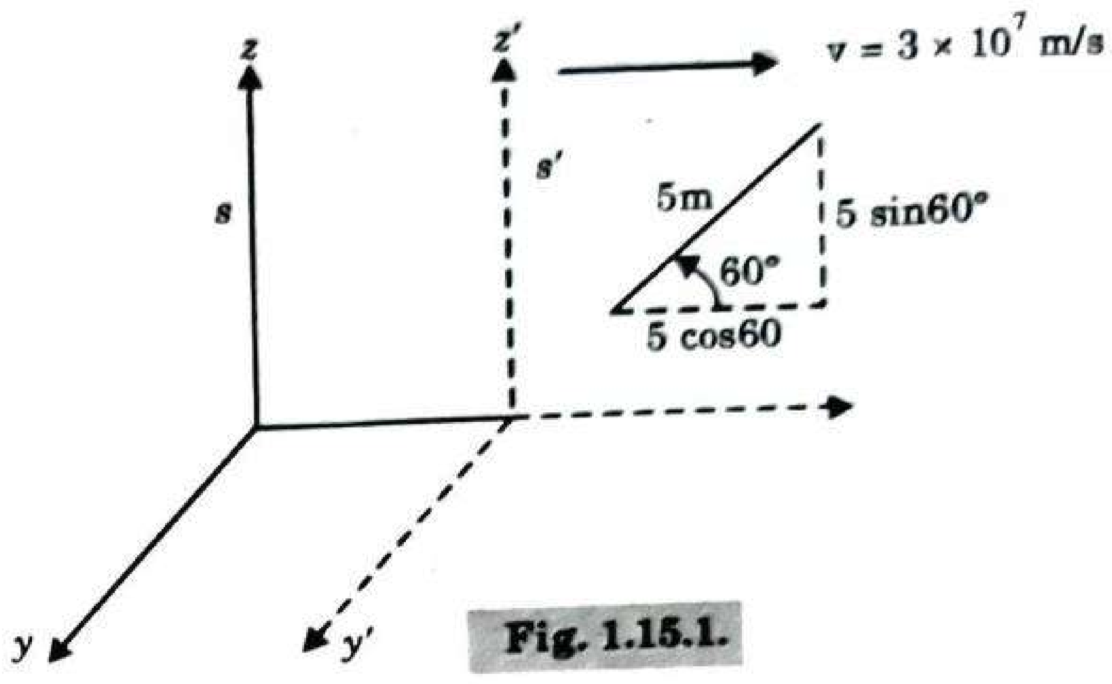


Fig. 1.15.1.

Que 1.16. **How much time does a metre stick moving at 0.1 c relative to an observer take to pass the observer ? The metre stick is parallel to its motion.**

Answer

1. Given : $L_0 = 1$ m
   $v = 0.1\,c$
2. Since $L = L_0\sqrt{1 - \frac{v^2}{c^2}}$
3. So $L = L_0\sqrt{1 - 0.01} = \sqrt{0.99} = 0.994$ m
4. $\text{Time} = \frac{L}{v} = \frac{0.994}{0.1 \times 3 \times 10^8}$
   $\text{Time} = 3.31 \times 10^{-8}$ sec

Que 1.17. **What is time dilation ? Find out its equation using Lorentz transformation and give an example to show that time dilation is a real effect.**

OR

Derive the expression of time dilation. Show that time dilation is real effect. **UPTU 2012-13, Marks 05**

Answer

NO A. **Time Dilation :**

1. Suppose s and s' are two frame of references. Frame s' is moving with constant velocity v in the positive x direction w.r.t. frame s.
2. Let an event occurred at point P in t'_1 time. Second event occurs at the same point P in t'_2 time as measured by observer o'.

3. Let point P be at rest w.r.t. o'.

4. So, ΔT_o = proper time $= t'_2 - t'_1$

$$\Delta T = t_2 - t_1$$

5. Using Lorentz transformation equation,

$$\Delta T = \frac{t'_2 + \frac{vx'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{t'_1 + \frac{vx'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Delta T = \frac{t'_2 + \frac{vx'}{c^2} - t'_1 - \frac{vx'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{t'_2 - t'_1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Delta T = \frac{\Delta T_o}{\sqrt{1-\frac{v^2}{c^2}}}$$

6. $\therefore \Delta T > \Delta T_o$

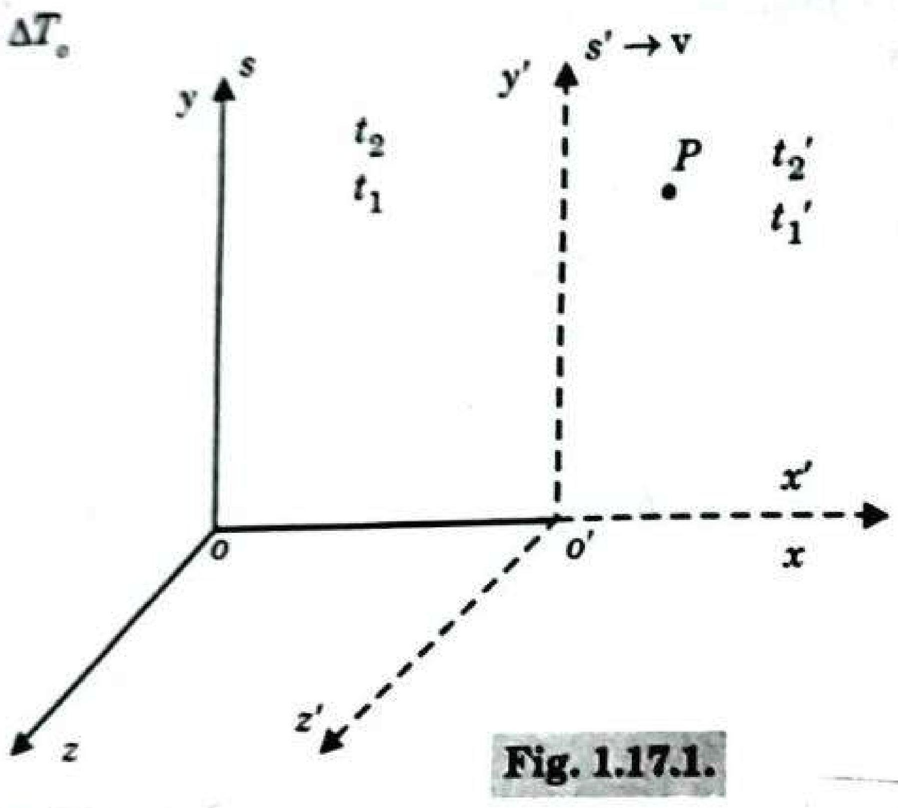


Fig. 1.17.1.

7. So the relativistic interval of time is more than proper interval of time.

8. Therefore a moving clock appears to go slow (*i.e.*, take more time to complete a rotation compared to a rest clock).

Physical significance →

9. If $v = c$ then $\Delta T = \infty$

10. If v is very less than c then $\Delta T = \Delta T_o$

B. Time Dilation is a Real Effect :

1. The cosmic ray particles called μ-meson are created at high altitude about 10 km above earth's atmosphere and are projected towards earth surface with velocity 2.994×10^8 m/s.

2. It decays into e^+ (Positron) with an average life time of about 2×10^{-6} sec.

3. Therefore in its life time μ-meson can travel a distance

$$d = vt = 2.994 \times 10^8 \times 2 \times 10^{-6}$$
$$= 600 \text{ m}$$

4. But it is found at earth surface also. It is possible because of time dilation effect.

$$t_o = 2 \times 10^{-6} \text{ sec.}$$

$$t = \frac{t_o}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{2 \times 10^{-6}}{\sqrt{1-\left(\frac{2.994 \times 10^8}{3 \times 10^8}\right)^2}}$$

$$= 3.17 \times 10^{-5} \text{ sec}$$

5. In this time a μ-meson can travel a distance

$$d = 2.994 \times 10^8 \times 3.17 \times 10^{-5} = 10 \text{ km}$$

6. This shows that time dilation is a real effect.

Que 1.18. **The proper mean life time of + μ meson is 2.5×10^{-8} sec.**

Calculate :

1. **Mean life time of + μ meson travelling with the velocity 2.4×10^8 m/s.**
2. **The distance travelled by this + μ meson during one mean life time.**
3. **The distance travelled without relativistic effect.**

Answer

1. Mean life time $$t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{2.5 \times 10^{-8}}{\sqrt{1-\left(\frac{2.4 \times 10^8}{3 \times 10^8}\right)^2}} = \frac{2.5 \times 10^{-8}}{\sqrt{1-(0.8)^2}}$$

$$= \frac{2.5 \times 10^{-8}}{0.6} = 4.17 \times 10^{-8} \text{ sec}$$

2. The distance travelled

$$= (2.4 \times 10^8) \times (4.17 \times 10^{-8}) = 10 \text{ m}$$

3. The distance travelled without relativistic effect

$$= (2.4 \times 10^8) \times (2.5 \times 10^{-8}) = 6 \text{ m.}$$

Que 1.19. **The half life of a particular particle as measured in the laboratory comes out to be 4.0×10^{-8} sec, when its speed is 0.8 *c* and 3.0×10^{-8} sec, when its speed is 0.6 *c*. Explain this.**

Answer

1. The time interval in motion is given by

$$t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}} \quad \text{(where, } t_0 = \text{proper time interval)}$$

2. The proper half life of the given particle is

$$t_0 = t\sqrt{1-\frac{v^2}{c^2}}$$

3. In the first case $t = 4.0 \times 10^{-8}$ sec and $v = 0.8\,c$

$$t_0 = 4.0 \times 10^{-8}\sqrt{1-\left(\frac{0.8\,c}{c}\right)^2} = 2.4 \times 10^{-8} \text{ sec}$$

4. As proper half life is independent of velocity, therefore half life of the particle when speed is 0.6 c must be given by

$$t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{2.4 \times 10^{-8}}{\sqrt{1-\left(\frac{0.6\,c}{c}\right)^2}} = \frac{2.4 \times 10^{-8}}{0.8}$$

$$= 3 \times 10^{-8} \text{ sec}$$

which is actual observation.

5. Thus the variation of half life of given particle is due to relativistic time dilation.

Que 1.20. **At what speed should a clock be moved so that it may appear to lose 1 minute in each hour ?**

Answer

1. ∵ Rest clock takes 60 minutes for T time interval.
   ∴ Rest clock takes 1 minute for $T/60$ time interval.
2. Now, moving clock takes 59 minutes for same T time interval.
   ∴ Moving clock takes 1 minute for $T/59$ time interval.
3. Here $t_0 = \frac{T}{60}$ and $t = \frac{T}{59}$
4. From time dilation formula,

$$t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

5. $$\frac{T}{59} = \frac{\frac{T}{60}}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow \sqrt{1-\frac{v^2}{c^2}} = \frac{59}{60}$$

$\Rightarrow$ $v = 5.45 \times 10^7$ m/s

Que 1.21. **Derive the relativistic velocity addition theorem. Show that it is consistent with Einstein's second postulate.**

UPTU 2012-13, 2014-15; Marks 05

Answer

A. Relativistic Velocity Addition Theorem :

1. Let s and s' be two frame of references in which s' is moving with a constant velocity v in the x direction w.r.t. frame s.
2. Let P be a point having coordinate (x, y, z, t) and (x', y', z', t') in frame s and s' at any instant of time.
3. In these two frames the components of the velocities of that particle along x, y and z axis will be given by

$$u_x = \frac{dx}{dt} \text{ and } u_x' = \frac{dx'}{dt'}$$

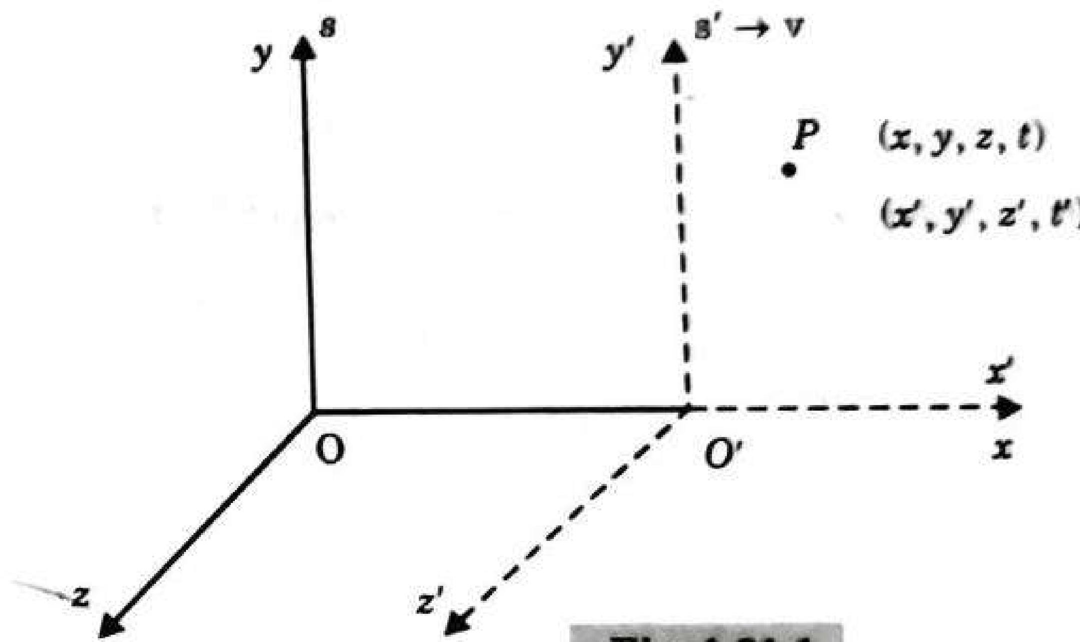


Fig. 1.21.1.

$$u_y = \frac{dy}{dt} \text{ and } u_y' = \frac{dy'}{dt'}$$

$$u_z = \frac{dz}{dt} \text{ and } u_z' = \frac{dz'}{dt'}$$

4. Now using the Lorentz transformation equation,

$$x = \frac{x' + vt'}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$y = y'$$

$$z = z' \text{ and } t = \frac{t' + \frac{v}{c^2}x'}{\sqrt{1-\frac{v^2}{c^2}}}$$

5. Differentiating above equations,

$$dx = \frac{dx' + v\,dt'}{\sqrt{1-\frac{v^2}{c^2}}} \quad \text{...(1.21.1)}$$

$$dy = dy' \quad ...(1.21.2)$$

$$dz = dz' \quad ...(1.21.3)$$

$$dt = \frac{dt' + \frac{v}{c^2}dx'}{\sqrt{1-\frac{v^2}{c^2}}} \quad ...(1.21.4)$$

6. Dividing equation (1.21.1) by equation (1.21.4),

$$\frac{dx}{dt} = \frac{dx' + v\,dt'}{dt' + \frac{v}{c^2}dx'}$$

7. Multiply and divide by $\frac{1}{dt'}$ on R.H.S.

$$\frac{dx}{dt} = \frac{\frac{dx'}{dt'} + v}{\frac{dt'}{dt'} + \frac{v}{c^2}\frac{dx'}{dt'}} \Rightarrow u_x = \frac{u_x' + v}{1 + \frac{v}{c^2}u_x'} \quad ...(1.21.5)$$

8. Similarly dividing equation (1.21.2) by equation (1.21.4),

$$\frac{dy}{dt} = \frac{dy'\sqrt{1-\frac{v^2}{c^2}}}{dt' + \frac{v}{c^2}dx'} \times \frac{\frac{1}{dt'}}{\frac{1}{dt'}}$$

$$u_y = \frac{u_y'\sqrt{1-\frac{v^2}{c^2}}}{1 + \frac{v}{c^2}u_x'} \quad ...(1.21.6)$$

9. Similarly $$u_z = \frac{u_z'\sqrt{1-\frac{v^2}{c^2}}}{1 + \frac{v}{c^2}u_x'} \quad ...(1.21.7)$$

10. If motion of object is only in x direction then $u = u_x$ and equation becomes

$$u = \frac{u' + v}{1 + \frac{vu'}{c^2}}$$

B. Consistency with Einstein's Second Postulate :

1. **Case–1 :** If $u' = c$ then

$$u = \frac{c + v}{1 + \frac{v}{c^2}c} \Rightarrow u = \frac{(c+v).c}{(c+v)} = c$$

2. **Case–2 :** If $v = c$ then

$$u = \frac{u' + c}{1 + \frac{cu'}{c^2}} = c$$

3. **Case–3 :** If $v = c$ and $u' = c$ then

$$u = \frac{c + c}{1 + \frac{c.c}{c^2}} = c$$

4. So the velocity of any object cannot be greater than 'c', whatever be the velocity of moving frame or velocity of object in that frame.

Que 1.22. **Rocket A travels towards the right and rocket B travels towards the left with velocity 0.8 c and 0.6 c respectively relative to the earth. What is the velocity of rocket :**

a. ***A*, measured from *B*, and**

b. ***B*, measured from *A* ?**

Answer

1. The velocity addition formula,

$$u = \frac{u' + v}{1 + \frac{u'v}{c^2}}$$

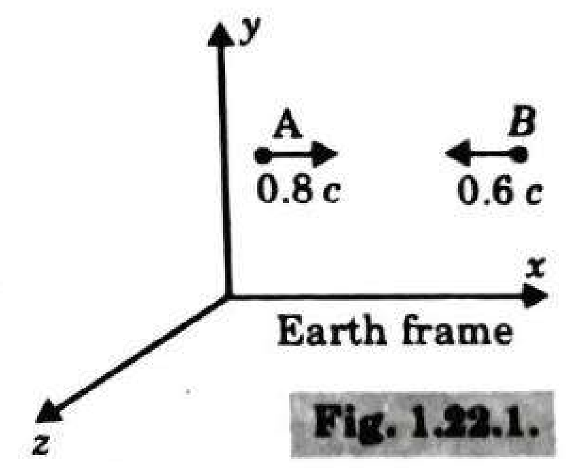


Fig. 1.22.1.

2. Velocity of earth w.r.t. B,

$$v = 0.6\,c$$

3. Velocity of A w.r.t. earth,

$$u' = 0.8\,c$$

4. Velocity of A w.r.t. B,

$$u = \frac{u' + v}{1 + \frac{u'v}{c^2}} = \frac{0.8c + 0.6c}{1 + \frac{(0.8c)(0.6c)}{c^2}}$$

$$\Rightarrow \quad u = \frac{1.4c}{1.48} = 0.946c$$

5. Velocity of earth w.r.t. A,

$$v = -0.8\,c$$

6. Velocity of B w.r.t. earth,

$$u' = -0.6\,c$$

7. Velocity of B w.r.t. A,

$$u = \frac{u' + v}{1 + \frac{u'v}{c^2}} = \frac{(-0.6c) + (-0.8c)}{1 + \frac{(-0.8)(-0.6)c^2}{c^2}}$$

$$u = -0.946\,c$$

8. Negative sign indicates that velocity of B w.r.t. A is towards left.

Que 1.23. **Show that no signal can travel faster than velocity of light.**

UPTU 2010-11, Marks 05

OR

Show that no particle can attain a velocity larger than velocity of light.

UPTU 2011-12, Marks 05

Answer

1. Given $v_x' = c$ and $v = c$
2. Since, $v_x = \dfrac{v_x' + v}{1 + \dfrac{v_x' v}{c^2}}$
3. Therefore, $v_x = \dfrac{c + c}{1 + \dfrac{c^2}{c^2}} = \dfrac{2c}{2} = c$

PART-3

Variation of mass with velocity, Mass energy equivalence, Concept of rest mass of photon.

CONCEPT OUTLINE : PART-3

Variation of Mass with Velocity : Mass is a function of the velocity of the body. It increases with increasing velocity represented by the relation :

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Mass Energy Equivalence : The variation of mass with velocity has modified the idea of energy, so that, a relationship can be established between mass and energy.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.24. **Deduce expression for variation of mass with velocity.**

UPTU 2012-13, 2013-14, 2015-16 ; Marks 05

Answer

1. Suppose s and s' are two frame of references in which s' is moving with a constant velocity 'v' w.r.t. observer O.
2. Two identical bodies A and B having same mass m are moving with velocity u' but in opposite direction in s' frame.
3. After some time both collides and stick together and momentarily comes to rest in s' frame.
4. Now from velocity addition theorem

$$u_1 = \frac{u' + v}{1 + \frac{u'v}{c^2}} \quad ...(1.24.1)$$

$$u_2 = \frac{-u' + v}{1 - \frac{u'v}{c^2}} \quad ...(1.24.2)$$

Where u_1 and u_2 are velocity of A and B in s frame before collision and m_1 and m_2 are their masses in s frame.

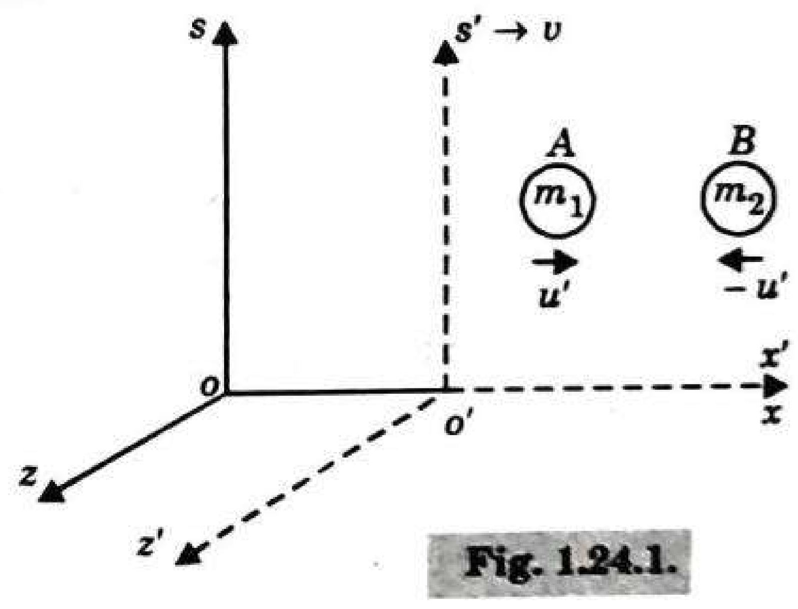


Fig. 1.24.1.

5. From the law of conservation of linear momentum,

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2)\, v \quad ...(1.24.3)$$

6. Putting the value of u_1 and u_2 from equation (1.24.1) and equation (1.24.2)

$$\Rightarrow \quad m_1\left(\frac{u' + v}{1 + \frac{u'v}{c^2}}\right) + m_2\left(\frac{-u' + v}{1 - \frac{u'v}{c^2}}\right) = (m_1 + m_2)v$$

$$\Rightarrow \quad m_1\left(\frac{u' + v}{1 + \frac{u'v}{c^2}} - v\right) = m_2\left(v - \frac{-u' + v}{1 - \frac{u'v}{c^2}}\right)$$

$$\Rightarrow \quad m_1\left[\frac{u'+v-v-\frac{u'v^2}{c^2}}{1+\frac{u'v}{c^2}}\right] = m_2\left[\frac{v-\frac{u'v^2}{c^2}+u'-v}{1-\frac{u'v}{c^2}}\right]$$

$$\Rightarrow \quad m_1 u'\frac{\left[1-\frac{v^2}{c^2}\right]}{1+\frac{u'v}{c^2}} = m_2 u'\frac{\left[1-\frac{v^2}{c^2}\right]}{1-\frac{u'v}{c^2}}$$

$$\Rightarrow \quad \frac{m_1}{m_2} = \frac{1+\frac{u'v}{c^2}}{1-\frac{u'v}{c^2}} \qquad ...(1.24.4)$$

7. Now calculating

$$1-\frac{u_1^2}{c^2} = 1-\frac{1}{c^2}\left(\frac{u'+v}{1+\frac{u'v}{c^2}}\right)^2 = \frac{\left(1+\frac{u'v}{c^2}\right)^2-\frac{(u'+v)^2}{c^2}}{\left(1+\frac{u'v}{c^2}\right)^2}$$

$$= \frac{1+\frac{u'^2v^2}{c^4}+\frac{2u'v}{c^2}-\left(\frac{u'^2}{c^2}+\frac{v^2}{c^2}+\frac{2u'v}{c^2}\right)}{\left(1+\frac{u'v}{c^2}\right)^2}$$

$$1-\frac{u_1^2}{c^2} = \frac{\left(1-\frac{v^2}{c^2}\right)-\frac{u'^2}{c^2}\left(1-\frac{v^2}{c^2}\right)}{\left(1+\frac{u'v}{c^2}\right)^2}$$

$$1+\frac{u'v}{c^2} = \sqrt{\frac{\left(1-\frac{v^2}{c^2}\right)\left(1-\frac{u'^2}{c^2}\right)}{1-\frac{u_1^2}{c^2}}} \qquad ...(1.24.5)$$

8. Similarly we can take equation (1.24.2) and proceed in the same manner

$$1-\frac{u'v}{c^2} = \sqrt{\frac{\left(1-\frac{v^2}{c^2}\right)\left(1-\frac{u'^2}{c^2}\right)}{1-\frac{u_2^2}{c^2}}} \qquad ...(1.24.6)$$

9. Putting equation (1.24.5) and equation (1.24.6) in equation (1.24.4),

$$\frac{m_1}{m_2} = \sqrt{\frac{1-\frac{u_2^2}{c^2}}{1-\frac{u_1^2}{c^2}}}$$

$$m_1\sqrt{1-\frac{u_1^2}{c^2}} = m_2\sqrt{1-\frac{u_2^2}{c^2}} = m_o = \text{constant}$$

10. If body B is at rest in stationary frame s that is $u_2 = 0$ before collision and $m_2 = m_0$ in frame s.
11. As bodies A and B are identical and have same mass in s'. So, $m_1 = m$ (relativistic mass) for $u_1 = v$.

Therefore, $$m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}$$

Que 1.25. Derive Einstein mass energy relation $E = mc^2$ and discuss it. Give some evidence showing its validity.

Answer

1. Suppose a force 'F' is acting on a body of mass 'm' in the same direction as its velocity 'v'.
2. The gain in K.E. in the body is in the form of work done.

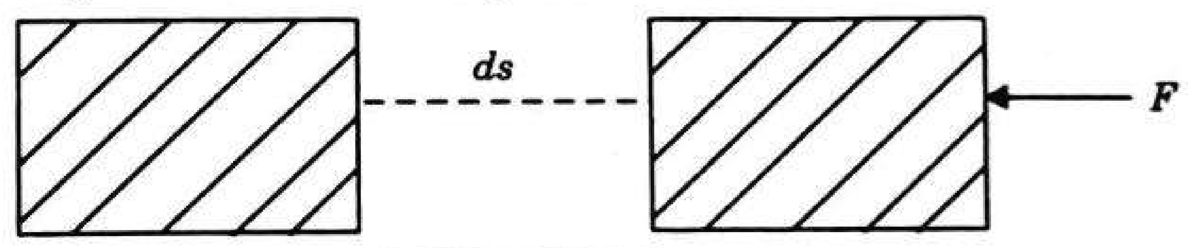


Fig. 1.25.1.

3. If a force 'F' displaces the particle through a small distance 'ds', then work done,

$$dW = dK = F.\,ds \qquad ...(1.25.1)$$

4. According to Newton's Law of motion,

$$F = \frac{dp}{dt} = \frac{d(mv)}{dt}$$

$$\Rightarrow \quad F = \frac{m\,dv}{dt} + v\frac{dm}{dt}$$

5. Multiplying 'ds' on both sides, we get

$$\Rightarrow \quad F.ds = m\,\frac{ds}{dt}.dv + v\frac{ds}{dt}.dm$$

6. From equation (1.25.1)

$$dK = m\,v\,dv + v^2\,dm \qquad ...(1.25.2) \quad \left(\because \frac{ds}{dt} = v\right)$$

7. But we know that

$$m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (m_0 = \text{rest mass of particle})$$

8. On differentiating, we get

$$dm = m_0\left(-\frac{1}{2}\right)\left(1-\frac{v^2}{c^2}\right)^{-\frac{3}{2}}\left(\frac{-2v}{c^2}\right)dv$$

$$dm = \frac{m_0 v\,dv}{c^2\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}}}$$

$$dm = \frac{mv\,dv}{c^2\left(1-\frac{v^2}{c^2}\right)} \qquad \left(\because \quad m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}\right)$$

$$dm\,(c^2 - v^2) = mv\,dv$$

9. Now putting this value in equation (1.25.2),

$$dK = (c^2 - v^2)\,dm + v^2\,dm$$

$$dK = c^2\,dm$$

10. If the change in kinetic energy of a particle be K when it's mass changes from rest mass m_0 to relativistic mass m, then

$$\int_0^K dK = \int_{m_0}^{m} c^2\,dm$$

$$K = c^2\,(m - m_0) = c^2\,(\Delta m)$$

11. Total energy of particle,

$$E = \text{Relativistic K.E.} + \text{Rest mass energy}$$

$$E = (m - m_0)\,c^2 + m_0c^2$$

$$E = mc^2$$

12. This is Einstein's mass energy-relation, which states mass energy equivalence.

A. **Evidence of its Validity :**

(1) In nuclear reaction such as fission and fusion.

(2) In process of annihilation of matter, an electron and a positron give up all its mass into two photons. The entire mass is converted into energy. This verifies mass-energy relation.

Que 1.26. **Derive the relation**

a. $E^2 = P^2c^2 + m_0^2c^4$, **and**

b. $P = \sqrt{\frac{K^2}{c^2} + 2m_0K}$ **where, K is kinetic energy.**

Answer

1. Total energy of a particle is

$$E = mc^2 \qquad ...(1.26.1)$$

2. The relativistic mass

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \qquad ...(1.26.2)$$

3. Put the value of equation (1.26.2) in equation (1.26.1),

$$E = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{m_0c^2}{\sqrt{1-\frac{m^2v^2}{m^2c^2}}} = \frac{m_0c^2}{\sqrt{1-\frac{m^2v^2c^2}{m^2c^4}}}$$

$$\Rightarrow \quad E = \frac{m_0c^2}{\sqrt{1-\frac{P^2c^2}{m^2c^4}}}$$

$$\Rightarrow \quad E^2 = \frac{m_0^2c^4}{1-\frac{P^2c^2}{m^2c^4}} = \frac{m_0^2c^4}{1-\frac{P^2c^2}{E^2}} \qquad [\because \; E = mc^2]$$

$$\Rightarrow \quad E^2\left(1-\frac{P^2c^2}{E^2}\right) = m_0^2c^4$$

$$\Rightarrow \quad E^2 - P^2c^2 = m_0^2c^4$$

$$\Rightarrow \quad E^2 = P^2c^2 + m_0^2c^4$$

4. Total energy, E = relativistic kinetic energy + rest mass energy

$$\Rightarrow \quad E = \text{K.E.} + m_0c^2$$

$$\Rightarrow \quad \text{K.E.} = E - m_0c^2$$

Here $\quad E = \sqrt{m_0^2c^4 + P^2c^2}$

$$\Rightarrow \quad K = \sqrt{m_0^2c^4 + P^2c^2} - m_0c^2$$

$$\Rightarrow \quad K + m_0c^2 = \sqrt{m_0^2c^4 + P^2c^2}$$

5. On squaring both side,

$$\Rightarrow \quad K^2 + m_0^2c_4 + 2Km_0c^2 = m_0^2c^4 + P^2c^2$$

$$\Rightarrow \quad K^2 + 2Km_0c^2 = P^2c^2$$

$$\Rightarrow \quad P^2 = 2Km_0 + \frac{K^2}{c^2}$$

$$\Rightarrow \quad P = \sqrt{\frac{K^2}{c^2} + 2m_0K}$$

Que 1.27. **Show that the relativistic form of Newton's second law, when $\bar{F}$ is parallel to $\bar{v}$ is**

$$\bar{F} = m_0\frac{d\bar{v}}{dt}\left(1-\frac{v^2}{c^2}\right)^{-3/2}$$

Answer

1. Newton's second law,

$$\bar{F} = \frac{d\bar{P}}{dt} = \frac{d}{dt}(m\bar{v})$$

2. But
$$m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}$$

3. So,
$$\bar{F} = \frac{d}{dt}\left[\frac{m_o\bar{v}}{\sqrt{1-\frac{v^2}{c^2}}}\right]$$

$$\bar{F} = m_o\left[\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\frac{d\bar{v}}{dt} + \frac{\bar{v}\left(-\frac{1}{2}\right)\left(1-\frac{v^2}{c^2}\right)^{-1/2}\left(\frac{-2\bar{v}}{c^2}\right)}{\left(1-\frac{v^2}{c^2}\right)}\frac{d\bar{v}}{dt}\right]$$

$$= m_o\left[\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\frac{d\bar{v}}{dt} + \frac{\frac{v^2}{c^2}}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}\frac{d\bar{v}}{dt}\right]$$

$$\bar{F} = m_o\frac{d\bar{v}}{dt}\left(1-\frac{v^2}{c^2}\right)^{-3/2}\left[\left(1-\frac{v^2}{c^2}\right)+\frac{v^2}{c^2}\right]$$

$$\bar{F} = m_o\frac{d\bar{v}}{dt}\left(1-\frac{v^2}{c^2}\right)^{-3/2}$$

Que 1.28. **The mass of a moving electron is 11 times its rest mass. Find its kinetic energy and momentum.** **UPTU 2011-12, Marks 05**

Answer

1. Given : $m = 11\,m_0$

2. Since, $m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}$

3. So $11\,m_o = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}$

$$\sqrt{1-\frac{v^2}{c^2}} = \frac{1}{11} \Rightarrow 1-\frac{v^2}{c^2} = \frac{1}{121}$$

$$\frac{v}{c} = 0.995 \Rightarrow v = 0.995 \times 3 \times 10^8 = 2.985 \times 10^8 \text{ m/s}$$

4. Total energy $E = \text{K.E.} + m_0c^2$

$$\text{K.E.} = (m - m_0)\,c^2$$
$$= (11\,m_0 - m_0)\,c^2 = 10\,m_0c^2$$
$$= 10 \times 9.1 \times 10^{-31} \times (3 \times 10^8)^2 = 8.19 \times 10^{-13}\text{ J}$$

5. $P = mv = 11\,m_0v$

$$= 11 \times 9.1 \times 10^{-31} \times 2.985 \times 10^8$$
$$P = 2.987 \times 10^{-21}\text{ N-s}$$

Que 1.29. **The total energy of a moving meson is exactly twice its rest energy. Find the speed of meson.** **UPTU 2012-13, Marks 05**

Answer

1. Given : $E = 2E_o$

$$mc^2 = 2m_oc^2$$

or $m = 2m_o$

2. Since
$$m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow 2m_o = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}$$

3. Therefore
$$2 = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow v = 0.866c$$
$$= 0.866 \times 3 \times 10^8$$
$$= 2.59 \times 10^8 \text{ m/s}$$

Que 1.30. **Show that the relativistic K.E. will convert classical K.E. if $v << c$.**

Answer

1. The expression for relativistic K.E. is

$$K = (m - m_0)\,c^2 = \left(\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} - m_0\right)c^2 = m_0c^2\left[\left(1-\frac{v^2}{c^2}\right)^{-1/2} - 1\right]$$

2. Expanding using binomial theorem,

$$K = m_0c^2\left[1+\frac{1}{2}\frac{v^2}{c^2}+\frac{3}{8}\frac{v^4}{c^4}+\ldots-1\right]$$

3. Since, v<< c i.e., v/c << 1, so, higher terms may be neglected.

4. Thus, $K = m_0c^2\left[1+\frac{1}{2}\frac{v^2}{c^2}-1\right] \approx \frac{1}{2}m_0 v^2$ .... (classical K.E.)

5. Therefore, if v<< c then relativistic K.E. will convert into classical K.E.

Que 1.31. **Calculate the amount of work to be done to increase the speed of an electron from 0.6 c to 0.8 c. Given that the rest energy of an electron is 0.5 MeV.** **UPTU 2014-15, Marks 05**

Answer

1. K.E. of electron

$$K = m - m_oc^2 = \left[\frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}} - m_o\right]c^2$$

$$K = m_oc^2\left[\left\{1-\left(\frac{v}{c}\right)^2\right\}^{-\frac{1}{2}}-1\right] \quad ...(1.31.1)$$

2. Rest energy of electron, $m_oc^2 = 0.5$ MeV
3. Now initial kinetic energy,

$$K_1 = m_oc^2\left[\left\{1-\left(\frac{0.6c}{c}\right)^2\right\}^{-\frac{1}{2}}-1\right] = 0.25\ m_oc^2$$

$$\Rightarrow \quad K_1 = 0.25 \times 0.5 \times 10^6 \text{ eV} = 1.25 \times 10^5 \text{ eV}$$

4. Final K.E.,

$$K_2 = m_oc^2\left[\left\{1-\left(\frac{0.8c}{c}\right)^2\right\}^{-\frac{1}{2}}-1\right]$$

$$K_2 = 3.33 \times 10^5 \text{ eV}$$

5. Amount of work $= K_2 - K_1$

$$= 3.33 \times 10^5 - 1.25 \times 10^5 = 2.08 \times 10^5 \text{ eV}$$
$$= 2.08 \times 10^5 \times 1.6 \times 10^{-19} = 3.328 \times 10^{-14} \text{ J}$$

Que 1.32. **A charged particle shows an acceleration of 4.2×10^{12} cm/s² under an electric field at low speed. Compute the acceleration of the particle under the same field when the speed has reached a value 2.88×10^{10} cm/s. The speed of light is 3×10^{10} cm/s.**

Answer

1. The force acting on a charged particle $F = qE$

2. At low speed v << c, the effective mass $m = m_o$
3. Acceleration at low speed,

$$a_0 = \frac{F}{m_o} = \frac{qE}{m_o} = 4.2 \times 10^{12} \text{ cm/s}^2 \quad ...(1.32.1)$$

4. Now,

$$m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{m_o}{\sqrt{1-\left(\frac{2.88\times10^{10}}{3\times10^{10}}\right)^2}} = \frac{m_o}{0.28}$$

5. Now, acceleration at high speed,

$$a = \frac{F}{m} = \frac{F}{m_o/0.28} = \frac{0.28F}{m_o} \quad ...(1.32.2)$$

6. Using equation (1.32.1) in equation (1.32.2)

$$a = 0.28 \times 4.2 \times 10^{12} = 1.176 \times 10^{12} \text{ cm/s}^2$$

Que 1.33. **If the kinetic energy of a body is twice its rest mass energy, find its velocity.**

Answer

1. **Given :** Kinetic energy, $K = 2 \times$ rest mass energy

$$\frac{1}{2}mv^2 = 2 \times m_0c^2$$

2. We know that

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

3.

$$\frac{1}{2}\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}.v^2 = 2 \times m_0c^2$$

$$v^2 = 4c^2\sqrt{1-\frac{v^2}{c^2}}$$

$$v^4 = 16\,c^4\left(1-\frac{v^2}{c^2}\right)$$

$$v^4 = 16\,c^4 - 16\,c^2v^2$$

4. $v^4 + 16\,c^2v^2 - 16\,c^4 = 0$

$$v^2 = \frac{-16\,c^2 \pm \sqrt{256\,c^4 + 4\times16\,c^4}}{2}$$

$$v^2 = \frac{-16\,c^2 \pm 17.89\,c^2}{2}$$

$$v = 0.971\,c$$

Que 1.34. **Show that the rest mass of photon is zero.**

Answer

1. A photon travels with the velocity of light. Its momentum is given by,

$$P = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}$$

2. But the momentum of a photon of radiation of wavelength λ is

$$P = \frac{h}{\lambda} \text{ (h is Planck's constant)}$$

$$\therefore \quad \frac{h}{\lambda} = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}$$

or $$m_0 = \frac{h\sqrt{1 - \frac{v^2}{c^2}}}{v\lambda}$$

3. But for photon $v = c$,

So, $m_0 = 0$

4. Therefore, the rest mass of a photon is zero.

☺☺☺

2 UNIT Modern Physics

PART-1

Black body radiation spectrum, Wien's Law and Rayleigh-Jean's law, Wave particle duality, Bohr's quantization rule, Davisson-Germer experiment.

CONCEPT OUTLINE : PART-1

Wave Particle Duality :
According to Einstein, the energy of light is concentrated in small bundles called photon. Hence, light behaves as a wave on one hand and as a particle on the other hand. This nature of light is known as dual nature, while this property of light is known as wave particle duality.

de-Broglie Matter Waves :
de-Broglie wavelength is given by

$$\lambda = \frac{h}{p}$$

de-Broglie wavelength in terms of temperature is given by

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{3mkT}}$$

Group Velocity : The velocity with which a wave packet moves forward in the medium is called group velocity.

$$v_g = \frac{\Delta\omega}{\Delta k}$$

Phase Velocity : The average velocity of the individual monochromatic wave in the medium with which the wave packet is constructed is called wave velocity or phase velocity.

$$v_p = \frac{\omega}{k}$$

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.1. **Explain black body radiation and discuss energy distribution in the spectrum of a black body radiation.**

Answer

A. Black Body :

1. A body which absorbs completely all the radiations incident upon it, reflecting none and transmitting none, is called a black body.
2. Absorptivity of a black body is unity for all wavelengths.
3. It appears black whatever the wavelength of incident radiation be.
4. When a black body is heated to a suitable high temperature it emits total radiation which is known as black body radiations.
5. From the energy point of view, black body radiation is equivalent to the radiation of an infinitely large number of non interacting harmonic oscillations, the so called radiation oscillations.
6. No actual body is a perfect black body, it is only an ideal conception.
7. Lamp black is the nearest approach to black body which absorbs nearly 99 % of the incident radiation.
8. Platinum black is another example of a black body.
9. An ideal model of a perfectly black surface is obtained if a small hole is made in the opaque walls (heat insulated walls) of a closed hollow cavity.

B. Energy Distribution :

1. Results of the studies of black body radiation spectra are shown in Fig. 2.1.1 in which variation of intensity with wavelength for various temperatures are shown.

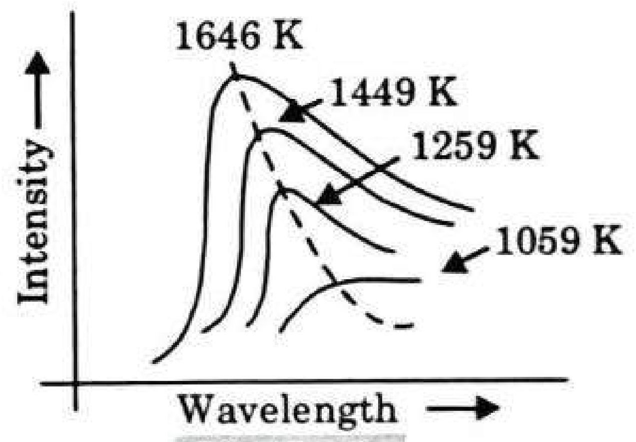


Fig. 2.1.1.

a. The energy distribution in the radiation spectrum of black body is not uniform. As the temperature of the body rises, the intensity of radiation for each wavelength increases.
b. At a given temperature, the intensity of radiation increases with increase in wavelength and becomes maximum at a particular wavelength. With further increase in wavelength, the intensity of radiation decreases.
c. The point of maximum energy shifts towards the shorter wavelengths as the temperature increases.
d. For a given temperature the total energy of radiation is represented by the area between the curve and the horizontal axis and the area

increases with increase of temperature, being directly proportional to the fourth power of absolute temperatures.

Que 2.2. **Explain Wien's laws of energy distribution.**

Answer

A. Fifth Power Law :

1. The total amount of energy emitted by a black body per unit volume at an absolute temperature T and contained in the spectral region included within the wavelength λ and $\lambda + d\lambda$ is given as,

$$u_\lambda \, d\lambda = \frac{A}{\lambda^5} f(\lambda T) \, d\lambda \quad ...(2.2.1)$$

where A is a constant and $f(\lambda T)$ is a function of the product λT and is given as

$$f(\lambda T) = e^{-ch/\lambda kT} = e^{-a/\lambda T} \quad \text{where, } a = ch/k$$

$$\therefore \quad u_\lambda \, d\lambda = A\lambda^{-5} e^{-a/\lambda T} d\lambda \quad ...(2.2.2)$$

2. For $\lambda = \infty$, $u_\lambda = 0$ and for $\lambda = 0$, $u_\lambda = 0$.
3. Thus equation (2.2.2) shows that no energy is emitted by a wave of infinite wavelength as well as by a wave of zero wavelength.
4. For $T = \infty$, equation (2.2.2) reduces to : $u_\lambda \, d\lambda = A\lambda^{-5}$,
   which is finite quantity and is in open contradiction with the Stefan's fourth power law (σT^4).
5. Wien's law of energy distribution explains the energy distribution at short wavelengths at higher temperature and fails for long wavelengths.

B. Displacement Law :

1. As the temperature of the body is raised the maximum energy tends to be associated with shorter wavelength, *i.e.*,

$$\lambda_m T = \text{constant} \quad ...(2.2.3)$$

where, λ_m = wavelength at which the energy is maximum

T = absolute temperature.

2. Thus, if radiation of a particular wavelength at a certain temperature is adiabatically altered to another wavelengths then temperature changes in the inverse ratio.

Que 2.3. **Discuss Rayleigh-Jean's Law.**

Answer

1. The total amount of energy emitted by a black body per unit volume at an absolute temperature T in the wavelength range λ and $\lambda + d\lambda$ is given as,

$$u_\lambda \, d\lambda = \frac{8\pi kT}{\lambda^4} d\lambda$$

where, k = Boltzmann's constant = 1.381×10^{-23} J/K.

2. The energy radiated in a given wavelength range λ and $\lambda + d\lambda$ increases rapidly as λ decreases and approaches infinity for very short wavelengths which however can't be true.
3. This law explains the energy distribution at longer wavelengths at all temperatures and fails totally for shorter wavelengths.
4. The energy distribution curves of black body show a peak while going towards the ultra-violet wavelength (shorter λ) and then fall while Rayleigh-Jean's law indicate continuous rise only.
5. This is the failure of classical physics and this failure was often called as the "Ultra-violet catastrophe" of classical physics

Que 2.4. **What are the various assumptions of quantum theory of radiation ?**

OR

Explain the development of quantum theory of radiation.

Answer

1. In 1900, Planck made the assumption that energy was made of individual units, or quanta.
2. In 1905, Albert Einstein theorized that not just the energy, but the radiation itself was quantized in the same manner.
3. In 1924, Louis de-Broglie proposed that there is no fundamental difference in the makeup and behavior of energy and matter; on the atomic and subatomic level either may behave as if made of either particles or waves.
4. In 1927, Werner Heisenberg proposed that precise, simultaneous measurement of two complementary values such as the position and momentum of a subatomic particle is impossible.

Que 2.5. **Derive Planck's radiation law and show how this law successfully explained observed spectrum of black body radiation.**

Answer

1. According to Planck's quantum hypothesis the exchange of energy by radiations with matters do not take place continuously but discontinuously and discretely as an integral multiple of an elementary quantum of energy represented by the relation

$$E = h\nu$$

where, ν = is the frequency of radiations, and

h = Planck's constant.

2. The resonators can oscillate only with integral energy values $h\nu$, $2h\nu$, $3h\nu$, ... , $nh\nu$ or in general $E_n = n\,h\nu$ (n = 1, 2, 3,...).

3. Hence emission and absorption of energy by the particles of a radiating body interchanging energy with the radiation oscillation occur discretely, not in a continuous sequence.
4. In relation $E_n = nh\nu$, n is called a quantum number and the energies of the radiators are said to be quantised and allowed energy states are called quantum states.
5. On the basis of his assumptions Planck derived a relation for energy density (u_ν) of resonators emitting radiation of frequency lying between ν and $\nu + d\nu$ which is given as follows :

$$u_\nu \, d\nu = \frac{8\pi h\nu^3}{c^3} \frac{d\nu}{e^{h\nu/kT} - 1} \quad ...(2.5.1)$$

or

$$u_\lambda \, d\lambda = \frac{8\pi hc}{\lambda^5} \frac{d\lambda}{e^{hc/\lambda kT} - 1} \quad ...(2.5.2)$$

6. The equation (2.5.1) and equation (2.5.2) are known as Planck's radiation law.

A. Experimental Verification of Planck's Radiation Law and Comparison with Wien's Law and Rayleigh-Jean's Law :

1. According to Planck's radiation law expression for energy density is given as

$$u_\lambda \, d\lambda = \frac{8\pi hc}{\lambda^5} \frac{d\lambda}{e^{hc/\lambda kT} - 1}$$

a. Wien's Law from Planck's Radiation Law :

1. For shorter wavelengths λT will be small and hence $e^{hc/\lambda kT} >> 1$
2. Hence, for small values of λT Planck's formula reduces to

$$u_\lambda \, d\lambda = \frac{8\pi hc}{\lambda^5} \frac{d\lambda}{e^{hc/\lambda kT}} = 8\pi hc \, \lambda^{-5} \, e^{-hc/\lambda kT} d\lambda$$

or

$$u_\lambda \, d\lambda = A\lambda^{-5} e^{-a/\lambda T} d\lambda \quad ...(2.5.3)$$

where A ($= 8\pi hc$) and a ($= hc/k$) are constants.

3. Equation (2.5.3) is Wien's law.
4. This result shows that at shorter wavelengths Planck's law approaches Wien's law and hence at shorter wavelengths Planck's law and Wien's law agrees (Fig. 2.5.1).

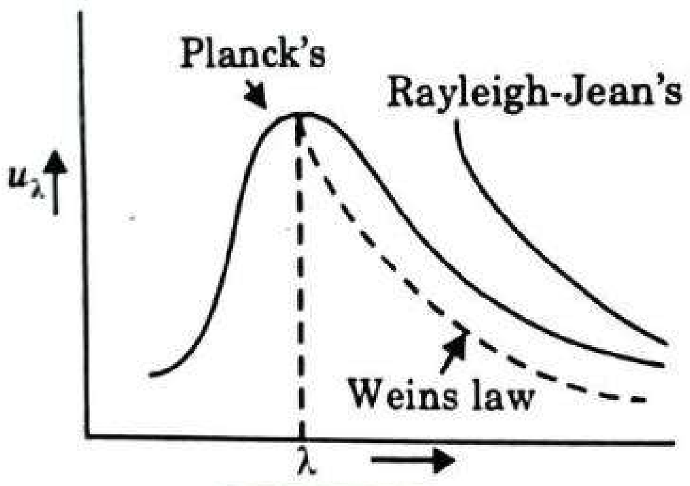


Fig. 2.5.1.

b. Rayleigh-Jean's Law :

1. For longer wavelengths $e^{hc/\lambda kT}$ is small and can be expanded as follows :

$$e^{hc/\lambda kT} = 1 + \frac{hc}{\lambda \, kT} \approx \frac{hc}{\lambda \, kT}$$

2. Hence, for longer wavelengths Planck's formula reduces to

$$u_\lambda d\lambda = \frac{8\pi \, hc}{\lambda^5} \frac{\lambda kT}{hc} \, d\lambda$$

or

$$u_\lambda d\lambda = \frac{8\pi \, kT}{\lambda^4} \, d\lambda \quad ...(2.5.4)$$

3. Equation (2.5.4) shows that for longer wavelengths Planck's law approaches to Rayleigh-Jean's law and thus at longer wavelengths Planck's law and Rayleigh-Jean's law agree as shown in Fig. 2.5.1.
4. Thus, it is concluded that the Planck's radiation law successfully explained the entire shape of the curves giving the energy distribution in black body radiation.

Que 2.6. **What are de-Broglie's matter waves ?**

UPTU 2005-06, 2006-07; Marks 05

Answer

1. In 1923-1924, Louis de-Broglie proposed that material particles such as an electron, proton, atoms etc., may have dual nature just like light, because matter and light both are forms of energy and each of them can transform energy from one place to other place.
2. According to de-Broglie's concept, each material particle in motion behaves as waves, having wavelength 'λ' associated with moving particle of momentum p.
3. $$\lambda = \frac{h}{p} \Rightarrow \lambda \propto \frac{1}{p}$$

$$\text{Wave nature} \propto \frac{1}{\text{Particle nature}}$$

4. Such waves associated with the moving particle are called matter waves or de-Broglie's waves.

Que 2.7. **Deduce expression for wavelength of de-Broglie wave.**

Answer

1. Let a photon having energy,

$$E = h\nu = \frac{hc}{\lambda} \quad ...(2.7.1)$$

2. If a photon possesses mass, it is converted into energy.
3. Now according to Einstein's law,
$$E = mc^2 \quad ...(2.7.2)$$
4. From equation (2.7.1) and equation (2.7.2),
$$mc^2 = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{mc^2}$$
$$\lambda = \frac{h}{mc} \Rightarrow \lambda = \frac{h}{p} \quad [\text{Since } mc = p]$$
5. In place of photon, we take material particle having mass 'm' moving with velocity 'v'. The momentum,
$$p = mv$$
6. The wavelength of wave associated with particle is,
$$\lambda = \frac{h}{mv} = \frac{h}{p}$$
This is de-Broglie's wavelength.
7. If E_k is kinetic energy of material particle of mass 'm' moving with velocity 'v' then
$$E_k = \frac{1}{2} mv^2$$
$$\Rightarrow \quad E_k = \frac{m^2 v^2}{2m}$$
$$\text{or} \quad E_k = \frac{(mv)^2}{2m} = \frac{p^2}{2m}$$
$$\text{or} \quad p = \sqrt{2mE_k}$$
8. The de-Broglie's wavelength, $\lambda = \frac{h}{\sqrt{2mE_k}}$
9. According to kinetic theory of gases, the average kinetic energy (E_k) of the material particle is given as
$$E_k = \frac{3}{2}KT$$
10. The de-Broglie's wavelength,
$$\lambda = \frac{h}{\sqrt{2m \times \frac{3KT}{2}}} = \frac{h}{\sqrt{3mKT}}$$
where, $K = 1.38 \times 10^{-23}$ J/K
T = temperature (K).
11. Suppose material particle is accelerated by potential difference of V volt then
$$E_k = qV \text{ where, } q = \text{charge of particle.}$$

12. The de-Broglie's wavelength,
$$\lambda = \frac{h}{\sqrt{2mqV}}$$

Que 2.8. **The kinetic energy of an electron is 4.55×10^{-25} J. Calculate the velocity, momentum and wavelength of the electron.**

UPTU 2008-09, Marks 05

Answer

1. **Given :** $E_k = 4.55 \times 10^{-25}$ J
2. If m_o is the rest mass, v is the velocity of the electron, then its kinetic energy (E_k) is given by
$$E_k = \frac{1}{2} m_o v^2$$
$$v = \sqrt{\frac{2E_k}{m_o}} = \sqrt{\frac{2 \times 4.55 \times 10^{-25}}{9.1 \times 10^{-31}}} = 1 \times 10^3 \text{ m/s}$$
3. Momentum of electron,
$$p = m_o v = 9.1 \times 10^{-31} \times 10^3 = 9.1 \times 10^{-28} \text{ kg m/s}$$
4. Wavelength of electron,
$$\lambda = h/p = (6.63 \times 10^{-34})/(9.1 \times 10^{-28}) = 7.29 \times 10^{-7} \text{ m}$$

Que 2.9. **Find the de-Broglie wavelength of neutron of energy 12.8 MeV (given that $h = 6.625 \times 10^{-34}$J–s, mass of neutron $(m_n) = 1.675 \times 10^{-27}$ kg and 1 eV = 1.6×10^{-19} Joule).**

UPTU 2006-07, Marks 05

Answer

1. Rest mass energy of neutron is given as,
$$m_o c^2 = 1.675 \times 10^{-27} \times (3 \times 10^8)^2$$
$$= 1.5075 \times 10^{-10} \text{ J}$$
$$= \frac{1.507 \times 10^{-10}}{1.6 \times 10^{-19}} = 942.18 \text{ MeV}$$
2. The given energy 12.8 MeV is very less compared to the rest mass energy of neutron, therefore relativistic consideration in this case is not applicable.
3. Now de-Broglie wavelength of the neutron is given as
$$\lambda = \frac{h}{\sqrt{2mE_k}}$$
$$E_k = 12.8 \times 10^6 \times (1.6 \times 10^{-19}) \text{ J}$$

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 1.675 \times 10^{-27} \times 12.8 \times 10^{6} \times 1.6 \times 10^{-19}}}$$

$$= \frac{6.63 \times 10^{-34}}{8.28 \times 10^{-20}}$$

$$= 8 \times 10^{-15} \text{ m}$$

$$= 8 \times 10^{-5} \text{ Å}$$

Que 2.10. **Calculate the de-Broglie's wavelength associated with a proton moving with a velocity equal to $\frac{1}{20}$ th of light velocity.**

UPTU 2007-08, Marks 05

Answer

1. **Given :** $v = \frac{1}{20} \times \text{light velocity} = \frac{1}{20} \times 3 \times 10^{8}$ m/s

$$= 1.5 \times 10^{7} \text{ m/s}$$

$$m_p = 1.67 \times 10^{-27} \text{ kg}$$

2. Formula for de-Broglie's wavelength :

$$\lambda = \frac{h}{mv}$$

$$\lambda = \frac{6.63 \times 10^{-34}}{1.67 \times 10^{-27} \times 1.5 \times 10^{7}} = 2.646 \times 10^{-14} \text{ m}$$

$$= 2.646 \times 10^{-4} \text{ Å}$$

Que 2.11. **Distinguish between group velocity (v_g) and phase velocity (v_p) of a wave packet and show that $v_p.v_g = c^2$.**

UPTU 2008-09, Marks 05

Answer

1. A wave packet consists of a group of waves slightly differing in their wavelength, velocities, phase and amplitude as shown in Fig. 2.11.1.

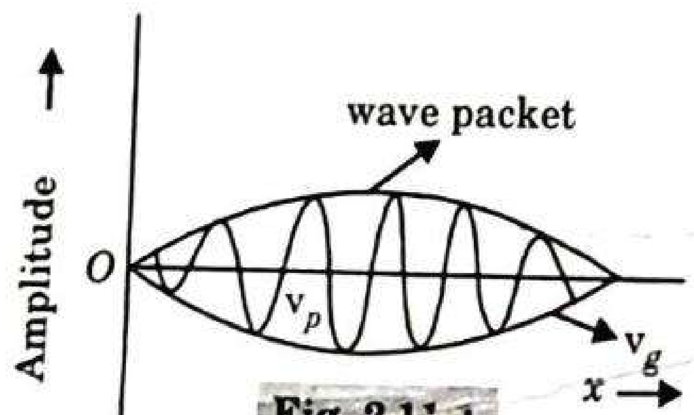


Fig. 2.11.1.

2. Such a wave packet moves with its own velocity called group velocity (v_g) or particle velocity and velocity of individual waves forming the wave packet is called phase velocity (v_p).
3. We know that

$$v_p = \nu\lambda \quad \text{...(2.11.1)}$$

and the de-Broglie's wavelength

$$\lambda = \frac{h}{mv}$$

and $E = h\nu = mc^2 \Rightarrow \nu = mc^2/h$

4. Equation (2.11.1) becomes,

$$v_p = \left(\frac{mc^2}{h}\right)\left(\frac{h}{mv}\right) = \frac{c^2}{v}$$

5. Since group velocity (v_g) is equal to particle velocity (v) *i.e.*, $v_g = v$

Then $v_p = \frac{c^2}{v_g}$ or $v_p . v_g = c^2$

Que 2.12. **Deduce expression for wave velocity and group velocity.**

Answer

1. Consider a group of two waves having equal amplitude 'a' and slightly unequal angular frequencies ω_1 and ω_2 and propagation constant k_1 and k_2.
2. The displacement equation is expressed as :

$$\psi_1 = a \sin(\omega_1 t - k_1 x) \quad \text{...(2.12.1)}$$

$$\psi_2 = a \sin(\omega_2 t - k_2 x) \quad \text{...(2.12.2)}$$

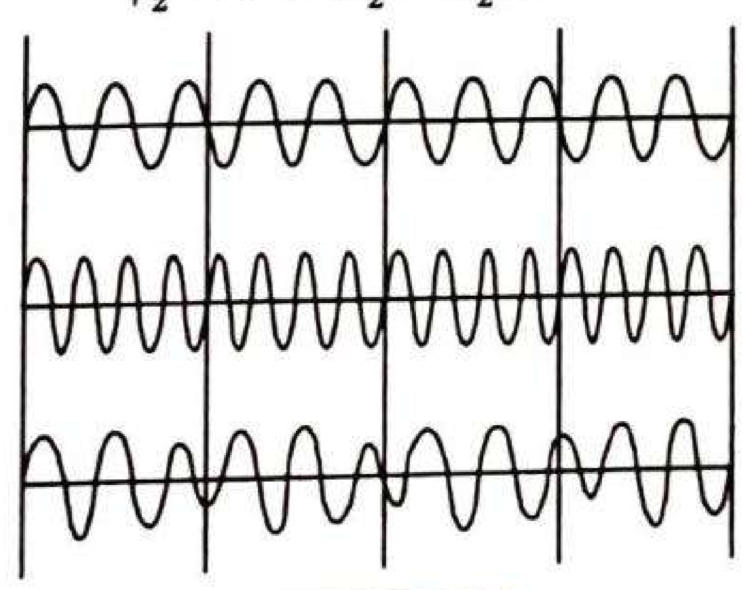


Fig. 2.12.1.

3. According to superposition theorem the resultant displacement

$$\psi = \psi_1 + \psi_2$$

$$= a[\sin(\omega_1 t - k_1 x) + \sin(\omega_2 t - k_2 x)]$$

$$\psi = 2a\sin\left[\left(\frac{\omega_1+\omega_2}{2}\right)t-\left(\frac{k_1+k_2}{2}\right)x\right]\cos\left[\left(\frac{\omega_1-\omega_2}{2}\right)t-\left(\frac{k_1-k_2}{2}\right)x\right] \quad ...(2.12.3)$$

4. Let $\omega = \dfrac{\omega_1+\omega_2}{2}$, $k = \dfrac{k_1+k_2}{2}$

   and $\Delta\omega = \omega_1 - \omega_2$, $\Delta k = k_1 - k_2$

5. Thus equation (2.12.3) becomes,

$$\psi = 2a\sin(\omega t - kx)\cos\left(\frac{\Delta\omega t}{2}-\frac{\Delta kx}{2}\right) \quad ...(2.12.4)$$

   This is modulated wave equation.

6. This effect produce successive wave group as shown in Fig. 2.12.1.
7. The phase velocity or wave velocity v_p is given by

$$v_p = \nu\lambda = \frac{2\pi\nu\lambda}{2\pi}$$

   or

$$v_p = \frac{2\pi\nu}{\frac{2\pi}{\lambda}} \qquad \left[\text{Since } \omega = 2\pi\nu \text{ and } k = \frac{2\pi}{\lambda}\right]$$

$$\therefore \quad v_p = \frac{\omega}{k}$$

8. And group velocity v_g is given as

$$v_g = \frac{\Delta\omega}{\Delta k}$$

Que 2.13. **Prove that group velocity is equal to particle wave velocity.**

OR

Explain group velocity. Establish a relation between group velocity and phase velocity and show that these velocities are equal in non-dispersive medium.

UPTU 2014-15, Marks 05

Answer

Group Velocity : The velocity with which a wave packet moves forward in the medium is called group velocity.

Relation between Group Velocity and Phase Velocity : Refer Q. 2.11, Page 50B, Unit-2.

Velocities in Non-Dispersive Medium :

1. The angular frequency ω of de-Broglie's waves associated with particle of rest mass m_o and moving with velocity v is given by,

$$\omega = 2\pi\nu \text{ and } E = h\nu$$

$$\Rightarrow \quad \nu = \frac{E}{h} \text{ and } E = mc^2$$

$$\omega = \frac{2\pi E}{h} = \frac{2\pi mc^2}{h} = \frac{2\pi m_o c^2}{h\times\sqrt{1-\frac{v^2}{c^2}}} \qquad \left[\because m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}\right]$$

$$\omega = \frac{2\pi m_o c^2}{h}\left(1-\frac{v^2}{c^2}\right)^{\frac{-1}{2}}$$

2. Differentiate both sides of above equation w.r.t. v, we get

$$\frac{d\omega}{dv} = \frac{2\pi m_o c^2}{h}\left(\frac{-1}{2}\right)\left(1-\frac{v^2}{c^2}\right)^{\frac{-3}{2}}\left(-\frac{2v}{c^2}\right)$$

$$\frac{d\omega}{dv} = \frac{2\pi m_o v}{h\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}}} \quad ...(2.13.1)$$

3. Also, the wave number k of de-Broglie's wave is

$$k = \frac{2\pi}{\lambda} = \frac{2\pi mv}{h} = \frac{2\pi m_o v}{h\times\sqrt{1-\frac{v^2}{c^2}}}$$

4. Differentiate both sides of above equation w.r.t. v, we get

$$\frac{dk}{dv} = \frac{2\pi m_o}{h\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}}} \quad ...(2.13.2)$$

5. We know that group velocity,

$$v_g = \frac{d\omega}{dk}$$

6. From equation (2.13.1) and equation (2.13.2), we get

$$v_g = \frac{2\pi m_o v}{h\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}}}\cdot\frac{h\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}}}{2\pi m_o}$$

$$v_g = v$$

Que 2.14. **Deduce a relation between phase velocity and group velocity in a medium where wave velocity is frequency dependent. What happens if the phase velocity is independent of frequency ?**

UPTU 2013-14, Marks 05

Answer

1. As phase velocity $v_p = \frac{dx}{dt} = \frac{\omega}{K}$
2. For the amplitude of wave packet to be constant
$$\frac{\Delta\omega t}{2} - \frac{\Delta K}{2}x = \text{Constant}$$
3. Hence, group velocity
$$v_g = \frac{dx}{dt} = \frac{\frac{\Delta\omega}{2}}{\frac{\Delta K}{2}} = \frac{\Delta\omega}{\Delta K}$$
$$v_g = \lim_{\Delta K \to 0} \frac{\Delta\omega}{\Delta K} = \frac{d\omega}{dK}$$
4. Since $\frac{\omega}{K} = v_p$ and $\omega = Kv_p$
5. Group velocity, $v_g = \frac{d}{dK}(Kv_p) = v_p + K\frac{dv_p}{dK}$
$$v_g = v_p + \left(\frac{2\pi}{\lambda}\right)\frac{dv_p}{d\left(\frac{2\pi}{\lambda}\right)}$$
$$v_g = v_p - \lambda\frac{dv_p}{d\lambda}$$
6. This is a relation between group velocity v_g and wave velocity v_p in a dispersive medium in which wave velocity is frequency dependent.
7. If $\frac{dv_p}{d\lambda} = 0$, then the phase velocity does not depends on frequency and become independent
$$v_p = v_g$$

Que 2.15. **Describe Davisson-Germer experiment to demonstrate the wave-character of electrons.**

Answer

Construction :

1. A collimated beam of electrons is produced using an electron gun. This beam is incident on a target of nickel crystal.
2. The electrons are scattered in all directions by the atoms of the target.
3. The intensity of the scattered electrons in a given direction is measured by allowing it to enter in a collector, which can be moved along a circular scale.

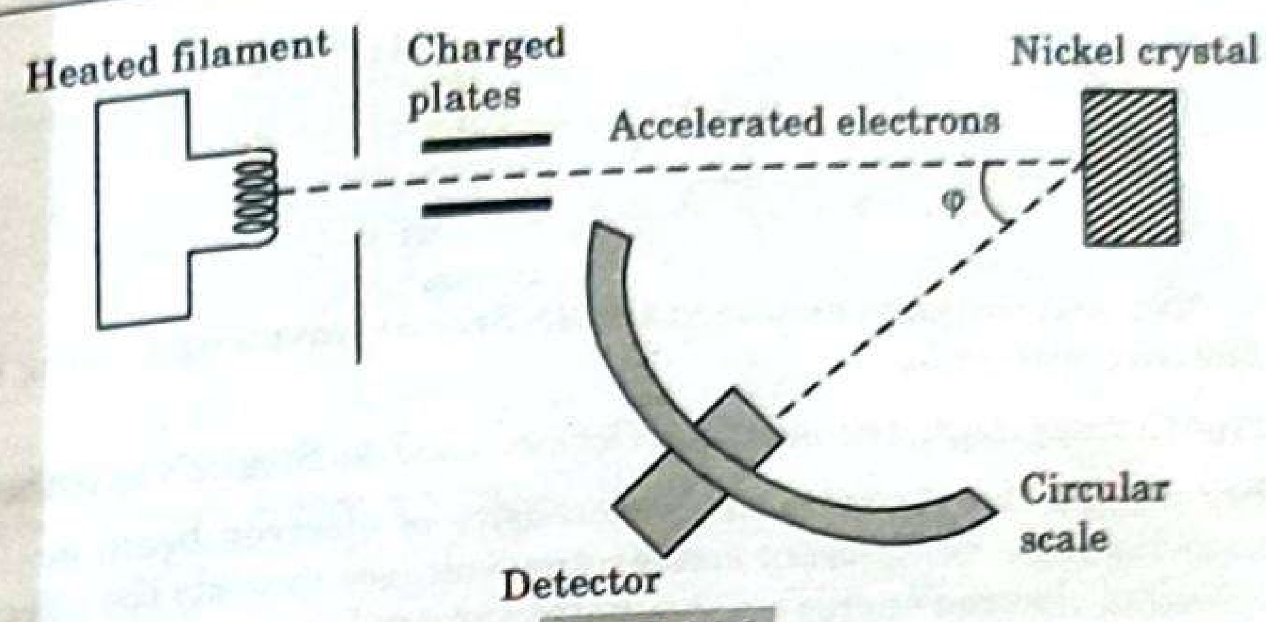


Fig. 2.15.1.

Principle :

1. If the material particles have a wave character, they are expected to show the interference and diffraction phenomena.
2. Davisson and Germer experimentally demonstrated the diffraction of electron beam.

Working :

1. Let an electron of rest mass m_0 be accelerated by potential V, then its kinetic energy is given by
$$K = \frac{1}{2}m_0 v^2 = eV \qquad ...(2.15.1)$$
where v is the velocity of the accelerated electron.
2. From equation (2.15.1),
$$v = \sqrt{\frac{2eV}{m_0}} \qquad ...(2.15.2)$$
3. The wavelength of the de-Broglie wave associated with this electron is expressed as
$$\lambda = \frac{h}{m_0 v} \qquad ...(2.15.3)$$
4. Using equation (2.15.2) in equation (2.15.3),
$$\lambda = \frac{h}{m_0\sqrt{\frac{2eV}{m_0}}}$$
i.e.,
$$\lambda = \frac{h}{\sqrt{2em_0V}} \qquad ...(2.15.4)$$
5. Substituting $m_0 = 9.1 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-16}$ C and $h = 6.62 \times 10^{-34}$ J-s in equation (2.15.4)

$$\lambda = \frac{h}{\sqrt{2em_0V}} = \frac{6.62\times10^{-34}}{\sqrt{2\times1.6\times10^{-19}\times9.1\times10^{-31}\times V}}$$

or
$$\lambda = \frac{12.24}{\sqrt{V}}\,\text{Å}$$

6. Davisson and Germer calculated the de-Broglie wavelength using two different approaches.
7. In the first approach, Davisson and Germer used de-Broglie's hypothesis.
8. They plotted the variation in the intensity of electron beam against scattering angle for different accelerating voltages to study the effect of increasing electron energy on the scattering angle φ.
9. They found that a bump begins to appear in the curve for $V = 44$ volts.
10. With increasing potential, the bump moves upward, and becomes more prominent in the curve for $V = 54$ volts at $\varphi = 50°$, thereby indicating the maximum suffering in electron beam for $V = 54$ volts as shown in Fig. 2.15.2.

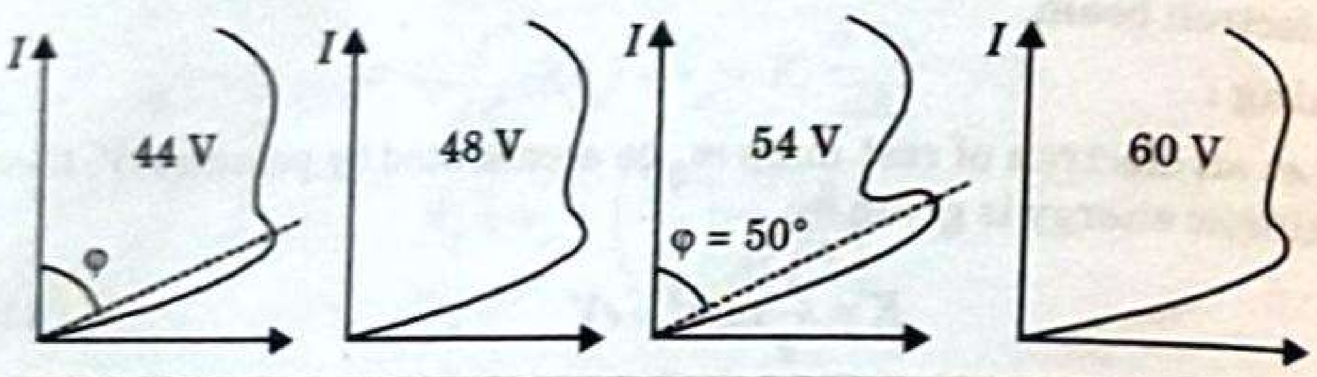


Fig. 2.15.2. Plots of intensity of electron beam against scattering angle for different values of accelerating voltage.

11. Thus, for $V = 54$ V, the de-Broglie wavelength of the electrons is
$$\lambda = \frac{12.24}{\sqrt{V}} = \frac{12.24}{\sqrt{54}} = 1.66\ \text{Å} \quad \dots (2.15.5)$$
12. In the second approach, Davisson and Germer calculated the de-Broglie wavelength by treating the electron beam as a wave.
13. They used Bragg's equation, $n\lambda = 2d \sin\theta$.
14. For nickel crystal, $d = 0.91$ Å. Also, $\theta = 65°$.
15. Hence for the first order ($n = 1$) reflection, we have
$$\lambda = 2d \sin\theta = 2 \times 0.91 \times \sin 65° = 1.65\ \text{Å} \dots (2.15.6)$$
16. Equation (2.15.3) and equation (2.15.4) show an excellent agreement between the two approaches.
17. Thus, the Davisson–Germer experiment provides a direct verification of wave nature of electrons and hence it also verifies the de Broglie's hypothesis.

PART-2

Heisenberg's Uncertainty Principle and its Applications, Wave Function and its Significance, Schrodinger's wave equation, Eigen values and Eigen function.

CONCEPT OUTLINE : PART-2

Heisenberg's Uncertainty Principle :
It is impossible to measure the exact position and momentum of a moving particle simultaneously. This law can be stated as the product of change in position (Δx) and momentum (Δp) of a particle is at least equal to $\hbar$ *i.e.,*
$$\Delta x\ \Delta p \ge \hbar \quad \text{where, } \hbar = h/2\pi$$
Wave Function and its Significance :
The wave function ψ is described as mathematical function whose variation builds up matter waves. $|\psi|^2$ defines the probability density of finding the particle within the given confined limits. ψ is defined as probability amplitude and $|\psi|^2$ is defined as probability density.
Schrodinger's Wave Equation :
This wave equation is a fundamental equation in quantum mechanics and describes the variation of wave function ψ in space and time.
Two forms of Schrodinger's Wave Equation are :

a. Time-independent wave equation,
$$\nabla^2\psi + \frac{2m}{\hbar^2}(E - V)\psi = 0$$
where, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

b. Time-dependent wave esquation,
$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\psi = i\hbar\frac{\partial\psi}{\partial t} \quad \text{where, } \hbar = \frac{h}{2\pi}$$

Particle in a Box : For a particle confined in a one dimensional box, the energy eigen values are quantized. The quantized eigen values are represented as
$$E_n = \frac{n^2\hbar^2\pi^2}{2mL^2}$$
The corresponding wave function is given as
$$\psi_n = \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}$$

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.16. What is uncertainty principle ?

UPTU 2009-10, Marks 02

Answer

1. According to this principle, "It is impossible to determine the exact position and momentum of a particle simultaneously".
2. If Δx and Δp are the uncertain position and momentum of particle then according to this principle

$$\Delta x\, \Delta p \geq \frac{h}{2\pi}$$

or

$$\Delta x\, \Delta p \geq \hbar$$

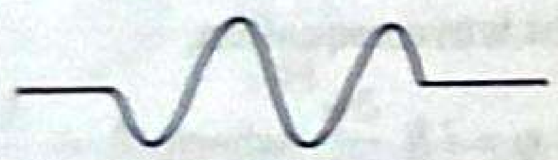
The product of uncertainty position and uncertainty momentum of particle is greater than or equal to $h/2\pi$.

3. Relation between uncertainty energy ΔE and uncertainty time Δt is

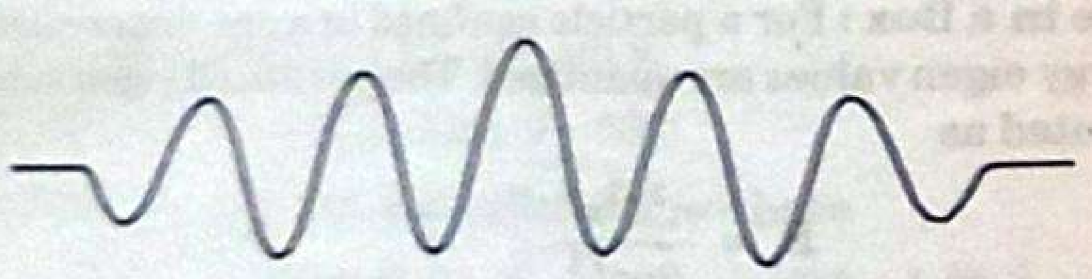
$$\Delta E\, \Delta t \geq \frac{h}{2\pi}$$

4. If $\Delta\theta$ and ΔJ are uncertainty angular position and angular momentum then

$$\Delta\theta\, \Delta J \geq \frac{h}{2\pi}$$



(a) Δp large, Δx small



(b) Δx large, Δp small

Fig. 2.16.1. Wave-packet : (a) Narrow, and (b) Wide.

Que 2.17. Explain Heisenberg's uncertainty principle ? Describe Heisenberg's gamma ray microscope. **UPTU 2014-15, Marks 05**

Answer

A. **Heisenberg's Uncertainty Principle :** Refer Q. 2.16, Page 58B, Unit-2.

B. **Heisenberg's Gamma-ray Microscope :**

1. Let us try to measure the position and linear momentum of an electron using an imaginary microscope with a very high resolving power as shown in Fig. 2.17.1.
2. The electron can be observed if at least one photon is scattered by it into the microscope lens.
3. The limit of resolution of the microscope is given by the relation.

$$d = \frac{\lambda}{2 \sin\theta}$$

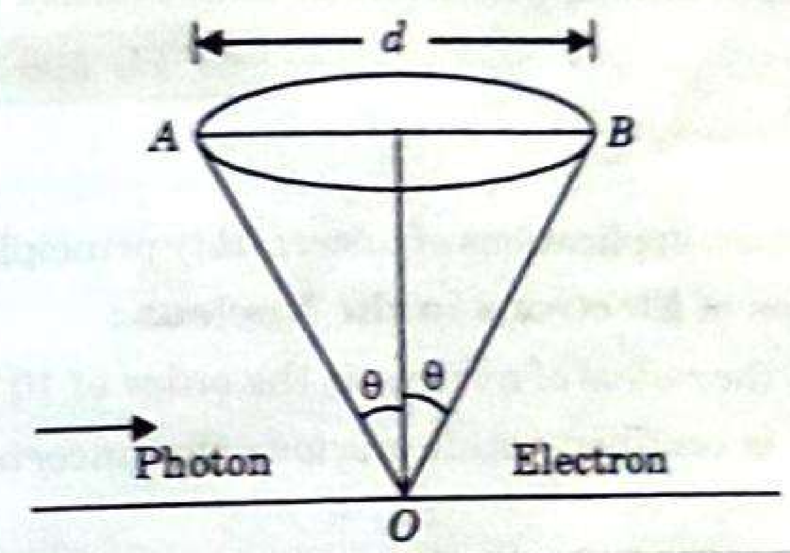


Fig. 2.17.1. Measurement of position and linear momentum of an electron.

4. Here d represents the distance between the two points which can be just resolved by the microscope.
5. This is the range in which the electron would be visible when disturbed by the photon.
6. Therefore, the uncertainty in the measurement of the position of the electron is

$$\Delta x = d = \frac{\lambda}{2 \sin\theta} \quad \text{...(2.17.1)}$$

7. However, the incoming photon will interact with the electron through the Compton effect.
8. To see this electron, the scattered photon should enter the microscope within the angle 2θ.
9. The momentum imparted by the photon to the electron during the impact is of the order of h/λ.
10. The component of this momentum along OA is $(-\,h/\lambda \sin\theta)$ and that along OB is $(h/\lambda \sin\theta)$.
11. Hence the uncertainty in the measurement of the momentum of the electron is

$$\Delta p = \left(\frac{h}{\lambda}\sin\theta\right) - \left(-\frac{h}{\lambda}\sin\theta\right) = 2\frac{h}{\lambda}\sin\theta \quad ...(2.17.2)$$

12. Multiplying equation (2.17.1) by equation (2.17.2), we obtain

$$\Delta x \Delta p = \frac{\lambda}{2\sin\theta} 2\frac{h}{\lambda}\sin\theta$$

$$\Delta x \Delta p = h$$

13. A more sophisticated approach will show that $\Delta x \Delta p \geq \hbar$.

Que 2.18. **Apply uncertainty principle to calculate the radius of the Bohr's first orbit.**

UPTU 2009-10, Marks 03

OR

Discuss some important application of uncertainty principle.

UPTU 2007-08, Marks 05

Answer

1. Followings are the applications of uncertainty principle :

a. Non-existence of Electrons in the Nucleus :

1. We know that the radius of nucleus is the order of 10^{-14} m.
2. If an electron is confined within nucleus the uncertainty position of electron is

$$\Delta x = 2 \times 10^{-14} \text{ m}$$

3. Now according to uncertainty principle,

$$\Delta x\, \Delta p \geq \frac{h}{2\pi}$$

and
$$\Delta p = \frac{h}{2\pi\Delta x} = \frac{6.63 \times 10^{-34}}{2 \times \pi \times 2 \times 10^{-14}} = 5.276 \times 10^{-21} \text{ kg m/s}$$

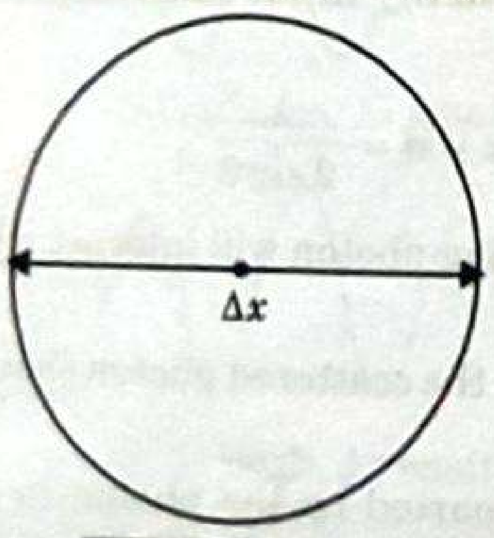


Fig. 2.18.1.

4. Using relativistic formula for the energy of the electron

$$E^2 = p^2c^2 + m_o^2c^4$$

As the rest energy m_oc^2 of an electron is of the order of 0.511 MeV, which is much smaller than the value of first term.

5. Hence the second term is neglected therefore,

$$E^2 = p^2c^2$$

$$E = pc = (5.276 \times 10^{-21}) \times (3 \times 10^8) \text{ J}$$

$$E = \frac{5.276 \times 10^{-21} \times 3 \times 10^8}{1.6 \times 10^{-19}} \text{ eV} \approx 97 \text{ MeV}$$

5. Thus, if an electron exists inside the nucleus then its energy should be of the order of 97 MeV. But the experiment shows that no electron in the atom possesses kinetic energy greater than 4 MeV.
6. Hence, no electron can exist inside the nucleus.

b. Radius of Bohr's First Orbit :

1. The energy of electron in a hydrogen atom is given by

$$E = KE + PE = \frac{p^2}{2m_0} + \frac{(-e^2)}{4\pi\varepsilon_0 x} \quad ...(2.18.1)$$

where x is the distance between the electron and the centre of the nucleus.

2. Equation (2.18.1) in terms of uncertainty can be expressed as

$$\Delta E = \frac{(\Delta p)^2}{2m_0} - \frac{e^2}{4\pi\varepsilon_0\Delta x} \quad ...(2.18.2)$$

3. Using the Heisenberg's uncertainty principle

$$\Delta p \Delta x \approx \hbar \quad ...(2.18.3)$$

4. Putting equation (2.18.3) in equation (2.18.2)

$$\Delta E = \frac{(\hbar)^2}{2m_0(\Delta x)^2} - \frac{e^2}{4\pi\varepsilon_0\Delta x} \quad ...(2.18.4)$$

5. For minimum energy (*i.e.*, for ground state) of electron,

$$\frac{\partial(\Delta E)}{\partial(\Delta x)} = 0$$

$$\frac{\partial}{\partial(\Delta x)}\left[\frac{\hbar^2}{2m_0(\Delta x)^2} - \frac{e^2}{4\pi\varepsilon_0\Delta x}\right] = 0$$

$$\frac{\partial}{\partial(\Delta x)}\left[\frac{\hbar^2}{2m_0}(\Delta x)^{-2} - \frac{e^2}{4\pi\varepsilon_0}(\Delta x)^{-1}\right] = 0$$

$$-2\frac{\hbar^2}{2m_0}(\Delta x)^{-3} + \frac{e^2}{4\pi\varepsilon_0}(\Delta x)^{-2} = 0$$

$$-\frac{\hbar^2}{m_0(\Delta x)^3} + \frac{e^2}{4\pi\varepsilon_0(\Delta x)^2} = 0$$

$$\frac{\hbar^2}{m_0(\Delta x)^3} = \frac{e^2}{4\pi\varepsilon_0(\Delta x)^2}$$

$$\Delta x = \frac{4\pi\varepsilon_0\hbar^2}{m_0e^2}$$

6. For this value of Δx,

$$\frac{\partial^2(\Delta E)}{\partial(\Delta x)^2} > 0$$

7. Hence for the given value of Δx the value of ΔE will give the minimum or ground state energy of an electron, *i.e.*,

$$E_{\min} = [\Delta E]_{\Delta x = \frac{4\pi\varepsilon_0\hbar^2}{m_0e^2}} = \left[\frac{(\hbar)^2}{2m_0(\Delta x)^2} - \frac{e^2}{4\pi\varepsilon_0\Delta x}\right]_{\Delta x = \frac{4\pi\varepsilon_0\hbar^2}{m_0e^2}}$$

i.e.,
$$E_{\min} = \left[\frac{(\hbar)^2}{2m_0\left(\frac{4\pi\varepsilon_0\hbar^2}{m_0e^2}\right)^2} - \frac{e^2}{4\pi\varepsilon_0\left(\frac{4\pi\varepsilon_0\hbar^2}{m_0e^2}\right)}\right]$$

$$E_{\min} = \left[\frac{m_0e^4}{32\pi^2\varepsilon_0^2\hbar^2} - \frac{m_0e^4}{16\pi^2\varepsilon_0^2\hbar^2}\right]$$

$$E_{\min} = \frac{-m_0e^4}{32\pi^2\varepsilon_0^2\hbar^2}$$

8. This is the required expression for the minimum or ground state energy of an electron in the hydrogen atom.
9. Also, the value of Δx for which the ground state energy of an electron is obtained gives the value of radius for the first Bohr's orbit.
10. This value is known as Bohr's radius and it is denoted by r_0.
11. Thus the Bohr's radius is given by

$$r_0 = \Delta x = \frac{4\pi\varepsilon_0\hbar^2}{m_0e^2}$$

12. Using the values of m, e and $\hbar$, we get

$$r_0 = \frac{4\times3.14\times(8.85\times10^{-12})\times(1.054\times10^{-14})^2}{(9.1\times10^{-31})\times(1.6\times10^{-19})^2}$$

$$= 0.53\ \text{Å}$$

c. **Binding Energy of an Electron in Atom :**

1. The uncertainty in position Δx of an electron is of order of $2R$, where R is radius of orbit.

2. The corresponding uncertainty in its momentum is

$$\Delta p \ge \frac{h}{2\pi.2R}$$

$$R = 10^{-10}\ \text{m}$$

then
$$\Delta p \approx 0.527\times10^{-24}\ \text{kg-m/s}$$

3. Kinetic energy of electron is

$$E_k = \frac{p^2}{2m_0} = \left(\frac{h}{4\pi R}\right)^2\frac{1}{2m_o} = \frac{h^2}{32\pi^2m_oR^2}$$

4. Potential energy of electron in electrostatic field of the nucleus is

$$V = \frac{-Ze^2}{4\pi\in_o R}$$

5. So the total energy of its orbit will be

$$E = E_k + V$$

$$= \frac{h^2}{32\pi^2m_oR^2} - \frac{Ze^2}{4\pi\in_o R}$$

$$= \frac{(6.63\times10^{-34})^2}{32\times(3.14)^2\times9.1\times10^{-31}\times R^2} - \frac{Z(1.6\times10^{-19})^2}{4\times3.14\times8.85\times10^{-12}R}$$

$$E = \frac{10^{-20}}{R^2} - \frac{15\times10^{-10}Z}{R}\ \text{eV}$$

Taking $R = 10^{-10}$ m

$$E = (1-15Z)\ \text{eV}$$

6. Now the binding energy of outermost electron in H is -13.6 eV
7. For H atom

$$E = (1-15) = -14\ \text{eV} \qquad (\because \text{for } H \text{ atom}, Z = 1)$$

It is very near to -13.6 eV.

8. Hence, binding energy of an electron can be calculated.

Que 2.19. A nucleon is confined to a nucleus of diameter 5×10^{-14} m. Calculate minimum uncertainty in the momentum of the nucleon. Also calculate the minimum kinetic energy of the nucleon.

UPTU 2009-10, Marks 05

Answer

1. **Given :** Diameter $= 5\times10^{-14}$ m or $\Delta x = 5\times10^{-14}$ m
2. According to uncertainty principle

$$\Delta x\,\Delta p = \frac{h}{2\pi}$$

So,
$$\Delta p = \frac{h}{2\pi\Delta x} = \frac{6.63\times10^{-34}}{2\times3.14\times5\times10^{-14}}$$

$$= 2.112 \times 10^{-21} \text{ kg-m/s}$$

3. The minimum kinetic energy of the nucleon is given by

$$E_{\min} = \frac{p^2}{2m_0} = \frac{2.112 \times 10^{-21} \times 2.112 \times 10^{-21}}{2 \times 1.67 \times 10^{-27}}$$

$$= 13.36 \times 10^{-16} \text{ J}$$

$$E_{\min} = \frac{13.36 \times 10^{-16}}{1.6 \times 10^{-19}} \text{ eV} = 8.348 \times 10^3 \text{ eV}$$

Que 2.20. **The speed of an electron is measured to be 5.0×10^3 m/s to an accuracy of 0.003 %. Find the uncertainty in determining the position of this electron (mass of electron is 9.1×10^{-31} kg and Planck's constant is 6.62×10^{-34} Js).**

UPTU 2013-14, Marks 05

Answer

1. **Given :** $m_e = 9.1 \times 10^{-31}$ kg
$h = 6.62 \times 10^{-34}$ J s

$$\text{Uncertainty in velocity} = \frac{0.003}{100} \times 5.0 \times 10^3 = 0.15 \text{ m/s}$$

2. $$\Delta x \, \Delta p = \frac{h}{2\pi} \Rightarrow \Delta x = \frac{h}{2\pi \times \Delta p}$$

3. $$\Delta x = \frac{h}{2\pi m(\Delta v)} = \frac{6.62 \times 10^{-34}}{2 \times 3.14 \times 9.1 \times 10^{-31} \times 0.15}$$

$$\Delta x = 7.72 \times 10^{-4} \text{ m.}$$

Que 2.21. **Calculate the uncertainty in the position of a dust particle with mass equal to 1 mg if uncertainty in its velocity is 5.5×10^{-20} m/s.**

UPTU 2008-09, Marks 05

Answer

1. **Given :** $m = 1$ mg $= 10^{-6}$ kg
2. From the uncertainty principle, we have

$$\Delta x . \Delta p \geq \frac{h}{2\pi}$$

$$\Delta v = 5.5 \times 10^{-20} \text{ m/s}$$

3. So,

$$\Delta x = \frac{h}{2\pi \times m\Delta v} = \frac{6.63 \times 10^{-34}}{2 \times 3.14 \times 10^{-6} \times 5.5 \times 10^{-20}}$$

$$= \frac{6.63 \times 10^{-34}}{34.54 \times 10^{-26}}$$

$$\Delta x = 19.2 \text{ Å}$$

Que 2.22. **What is physical significance of wave function ?**

UPTU 2009-10, Marks 03

OR

What is wave function ? Write its physical significance.

Answer

1. The quantity in quantum mechanics undergoes periodic changes and gives information about the particle within the wave packet. It is called wave function ψ.
2. The wave function ψ itself has no physical significance but the square of its absolute magnitude $|\psi|^2$ gives the probability of finding the particle at that time.

a. Normalization of Wave Function :

1. If the wave function ψ of any system is such that it gives the value of given integral a finite quantity say 'N'.

$$\int_{-\infty}^{\infty} \psi \psi^* dx = \int_{-\infty}^{\infty} |\psi|^2 dx$$

$$= N \text{ (Integral)} \quad (\psi^* \text{ is complex conjugate.})$$

then ψ is called normalization of wave function.

b. Orthogonal Wave Function :

1. When the value of the integral is equal to zero ($N = 0$), the wave function ψ is known as orthogonal wave function.

$$\int_{-\infty}^{\infty} |\psi|^2 dx = 0$$

Que 2.23. **The wave function of a particle confined to a box of length L is**

$$\psi(x) = \sqrt{\frac{2}{L}} \sin \frac{\pi x}{L} \quad 0 < x < L$$

and $\psi(x) = 0$ everywhere else.
Calculate probability of finding the particle in region $0 < x < \frac{L}{2}$.

Answer

1. The probability of finding the particle in interval dx at distance x is

$$p(x)dx = |\psi|^2 dx$$

$$= \frac{2}{L} \sin^2\left(\frac{\pi x}{L}\right) dx$$

2. The probability in region $0 < x < \frac{L}{2}$ is

$$P = \int_0^{L/2} p(x)dx = \int_0^{L/2} \frac{2}{L}\sin^2\left(\frac{\pi x}{L}\right)dx$$

$$= \frac{1}{L}\int_0^{L/2}\left(1-\cos\frac{2\pi x}{L}\right)dx$$

$$= \frac{1}{L}\cdot\frac{L}{2} = \frac{1}{2}$$

Que 2.24. **Normalize the eigen function $\phi(x) = e^{icx}$ within the region $-a \le x \le a$.**

UPTU 2008-09, Marks 05

Answer

1. **Given :** $\phi(x) = e^{icx}$
2. The wave function $\phi(x)$ can be normalized by multiplying it by k. Therefore
$$\phi(x) = ke^{icx}$$
3. Now, by applying normalized condition, we get
$$\int_{-a}^{a} \phi(x)\phi^*(x)dx = 1$$
Here $\phi(x) = ke^{icx}$ and $\phi^*(x) = ke^{-icx}$
4. Therefore, $k^2 \int_{-a}^{a} e^{icx} e^{-icx} dx = 1$
$$k^2 \int_{-a}^{a} dx = 1$$
$$k^2 [x]_{-a}^{a} = 1$$
$$k^2 [a + a] = 1$$
$$k^2 . 2a = 1$$
$$k = \frac{1}{\sqrt{2a}}$$
5. Thus normalized wave function can be given as
$$\phi(x) = \frac{1}{\sqrt{2a}} e^{icx}$$

Que 2.25. **Derive time independent Schrodinger wave equation.**

UPTU 2009-10, Marks 03

OR

Derive time dependent Schrodinger wave equation.

UPTU 2009-10, Marks 05

OR

What is Schrodinger wave equation ? Derive time independent and time dependent Schrodinger wave equations.

Answer

1. Schrodinger's equation which is the fundamental equation of quantum mechanics is a wave equation in the variable ψ.

A. Time Independent Schrodinger Wave Equation :

1. Consider a system of stationary wave to be associated with particle and the position coordinate of the particle (x, y, z) and ψ is the periodic displacement of any instant time 't'.
2. The general wave equation in 3-D in differential form is :
$$\nabla^2\psi = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} \quad ...(2.25.1)$$
where, v = velocity of wave, and
$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} = \text{Laplacian operator.}$$
3. The wave function may be written as
$$\psi = \psi_o e^{-i\omega t} \quad ...(2.25.2)$$
4. Differentiate equation (2.25.2) w.r.t. time, we get
$$\frac{\partial\psi}{\partial t} = -i\,\omega\,\psi_o e^{-i\omega t} \quad ...(2.25.3)$$
5. Again differentiating equation (2.25.3)
$$\frac{\partial^2\psi}{\partial t^2} = +i^2\omega^2\psi_o e^{-i\omega t}$$
$$\frac{\partial^2\psi}{\partial t^2} = -\omega^2\psi \quad ...(2.25.4)$$
6. Putting these value in equation (2.25.1),
$$\nabla^2\psi = \frac{-\omega^2}{v^2}\psi \quad ...(2.25.5)$$
7. But $\omega = 2\pi v = \frac{2\pi v}{\lambda} \Rightarrow \frac{\omega}{v} = \frac{2\pi}{\lambda}$
8. Equation (2.25.5) becomes
$$\nabla^2\psi = -\frac{4\pi^2}{\lambda^2}\psi \quad ...(2.25.6)$$
9. From de-Broglie's wavelength, $\lambda = \frac{h}{mv}$

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then $$\nabla^2 \psi = \frac{-4\pi^2 m^2 v^2}{h^2} \psi \quad \text{...(2.25.7)}$$

10. If E and V are the total and potential energies of a particle and E_k is kinetic energy, then
$$E_k = E - V \text{ or } \frac{1}{2} mv^2 = E - V \quad \text{or} \quad m^2v^2 = 2m(E - V)$$

11. Now equation (2.25.7) becomes
$$\nabla^2\psi = \frac{-4\pi^2 2m[E - V]\psi}{h^2} \qquad \left[\text{Since } \hbar = \frac{h}{2\pi}\right]$$
$$\therefore \quad \nabla^2\psi + \frac{2m[E - V]\psi}{\hbar^2} = 0 \quad \text{...(2.25.8)}$$
This is required time-independent Schrodinger wave equation.

12. For free particle ($V = 0$)
$$\therefore \quad \nabla^2 \psi + \frac{2m}{\hbar^2} E\psi = 0$$

B. Time Dependent Schrodinger Wave Equation :

1. We know that wave function is $\psi = \psi_o e^{-i\omega t}$ ...(2.25.9)
2. On differentiating w.r.t. time, we get
$$\frac{\partial\psi}{\partial t} = -i\omega\, \psi_o e^{-i\omega t}$$
or $$\frac{\partial\psi}{\partial t} = -i\,(2\pi v)\,\psi \quad \text{...(2.25.10)}$$
3. But $$E = hv \;\Rightarrow\; v = \frac{E}{h}$$
4. Equation (2.25.10) becomes
$$\frac{\partial\psi}{\partial t} = -i2\pi\left(\frac{E}{h}\right)\psi \qquad \left[\text{Since } \hbar = \frac{h}{2\pi}\right]$$
5. $$\frac{\partial\psi}{\partial t} = -\frac{i}{\hbar} E\psi$$
and $$E\psi = -\frac{\hbar}{i}\frac{\partial\psi}{\partial t}$$
or $$E\psi = i\hbar\frac{\partial\psi}{\partial t} \quad \text{...(2.25.11)}$$
6. Now time independent Schrodinger wave equation is
$$\nabla^2 \psi + \frac{2m}{\hbar^2}(E - V)\psi = 0$$
or $$\nabla^2 \psi + \frac{2m}{\hbar^2}[E\psi - V\psi] = 0$$

7. Using equation (2.25.11), we get
$$\nabla^2 \psi + \frac{2m}{\hbar^2}\left[i\hbar\frac{\partial\psi}{\partial t} - V\psi\right] = 0$$
$$\nabla^2\psi - \frac{2m}{\hbar^2}V\psi = -\frac{2m}{\hbar^2}i\hbar\,.\,\frac{\partial\psi}{\partial t}$$
$$\left(\nabla^2 - \frac{2m}{\hbar^2}V\right)\psi = -\frac{2m}{\hbar^2}i\hbar\frac{\partial\psi}{\partial t}$$
or $$\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\psi = i\hbar\frac{\partial\psi}{\partial t}$$
This is required time dependent Schrodinger wave equation.
8. $$-\frac{\hbar^2}{2m}\nabla^2 + V = H \rightarrow \text{is known as Hamiltonian operator.}$$
$$i\hbar\frac{\partial\psi}{\partial t} = E \rightarrow \text{energy operator.}$$
Then, $$H\psi = E\psi$$

Que 2.26. **What do you mean by eigen values and eigen functions. Also, give the eigen value equation.**

Answer

1. The values of energy E_n for which Schrodinger's steady-state equation can be solved are called eigen values and the corresponding wave functions Ψ_n are called eigen functions.
2. The discrete energy levels of the hydrogen atom are an example of a set of eigen values.
3. Eigen value equation
$$\hat{G}\,\Psi_n = G_n\Psi_n \quad \text{...(2.33.1)}$$
where $\hat{G}$ is the operator that corresponds to G and each G_n is a real number.
4. When equation (2.33.1) holds for the wave function of a system, it is a fundamental postulate of quantum mechanics that any measurement of G can only yield one of the values G_n.
5. If measurements of G are made on a number of identical systems all in states described by the particular eigen function Ψ_k, each measurement will yield the single value G_k.

Que 2.27. **Write the Schrodinger wave equation for the particle in a box and solve it to obtain the eigen value and eigen function.**

Answer

1. Let a particle is confined in one-dimensional box of length 'L'. The particle is free *i.e.*, no external force, so potential energy inside box is zero ($V = 0$).

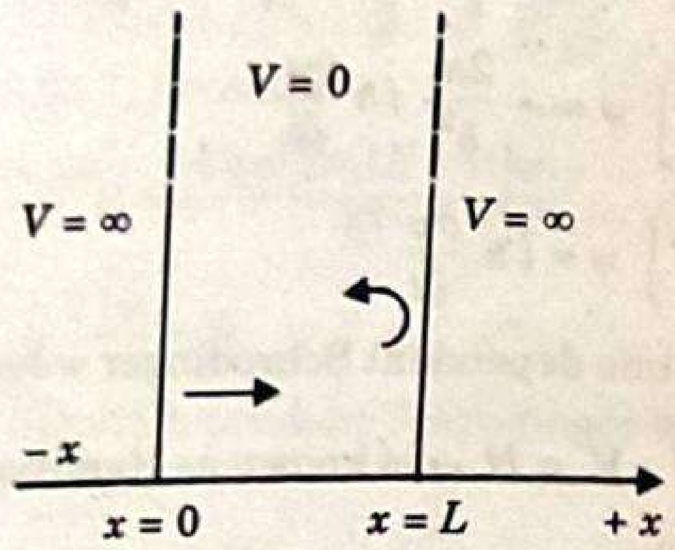

Fig. 2.26.1.

$V = 0$ for $0 < x < L$

$V = \infty$ for $x \le 0$ and $x \ge L$

i.e., outside the box the potential energy is infinite.

2. The particle cannot exist outside the box.

$\psi = 0$ for $x \ge L$ and $x \le 0$

3. Schrodinger time independent equation for free particle ($V = 0$),

$$\frac{\partial^2\psi}{\partial x^2} + \frac{2m}{\hbar^2} E\psi = 0 \qquad ...(2.26.1)$$

or $$\frac{\partial^2\psi}{\partial x^2} + k^2\psi = 0 \left[\text{where } k^2 = \frac{2mE}{\hbar^2}\right] \qquad ...(2.26.2)$$

4. Solution of equation (2.26.2) is

$$\psi(x) = A \sin kx + B \cos kx \qquad ...(2.26.3)$$

5. Using boundary condition, $\psi = 0$ at $x = 0$

$0 = A \sin 0 + B \cos 0$

$\Rightarrow$ $B = 0$

and $\psi = 0$ at $x = L$

$0 = A \sin kL + B \cos kL \; (B = 0)$

or $A \sin kL = 0$ or $\sin kL = \sin n\pi$

$kL = n\pi \quad n = 1, 2, 3,...$ But $n \ne 0$

$$k = \frac{n\pi}{L}$$

6. Now equation (2.26.3) becomes

$$\psi_n(x) = A \sin \frac{n\pi x}{L} \qquad \text{[Eigen function]}$$

and $$\frac{n^2\pi^2}{L^2} = \frac{2mE}{\hbar^2} \Rightarrow E = \frac{n^2\pi^2\hbar^2}{2mL^2} \qquad \left[\hbar = \frac{h}{2\pi}\right]$$

7. Then $$E_n = \frac{n^2h^2}{8mL^2} \qquad \text{[Eigen energy]}$$

8. For $n = 1$, $E_1 = \dfrac{h^2}{8mL^2}$, it is ground state energy of particle.

Que 2.28. **A particle is in motion along a line between $x = 0$ and $x = L$ with zero potential energy. At points for which $x < 0$ and $x > L$, the potential energy is infinite. The wave function for the particle in n^{th} state is given by :**

$$\psi_n = A \sin \frac{n\pi x}{L}$$

Find the expression for the normalized wave function.

UPTU 2009-10, Marks 05

OR

Derive normalization wave function.

Answer

1. The eigen function is

$$\psi_n(x) = A \sin \frac{n\pi x}{L} \qquad ...(2.27.1)$$

2. Now applying normalization condition.

$$\int_0^L |\psi_n(x)|^2 \, dx = 1$$

$$\int_0^L A^2 \sin^2\left(\frac{n\pi x}{L}\right) dx = 1$$

$$\frac{A^2}{2}\int_0^L \left(1 - \cos\frac{2n\pi}{L}x\right) dx = 1$$

$$\frac{A^2}{2}\left[x - \frac{\sin\frac{2n\pi x}{L}}{\frac{2n\pi}{L}}\right]_0^L = 1$$

$$\frac{A^2}{2} L = 1$$

$$A = \sqrt{\frac{2}{L}}$$

3. Equation (2.27.1) becomes

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$$

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this is normalization function.

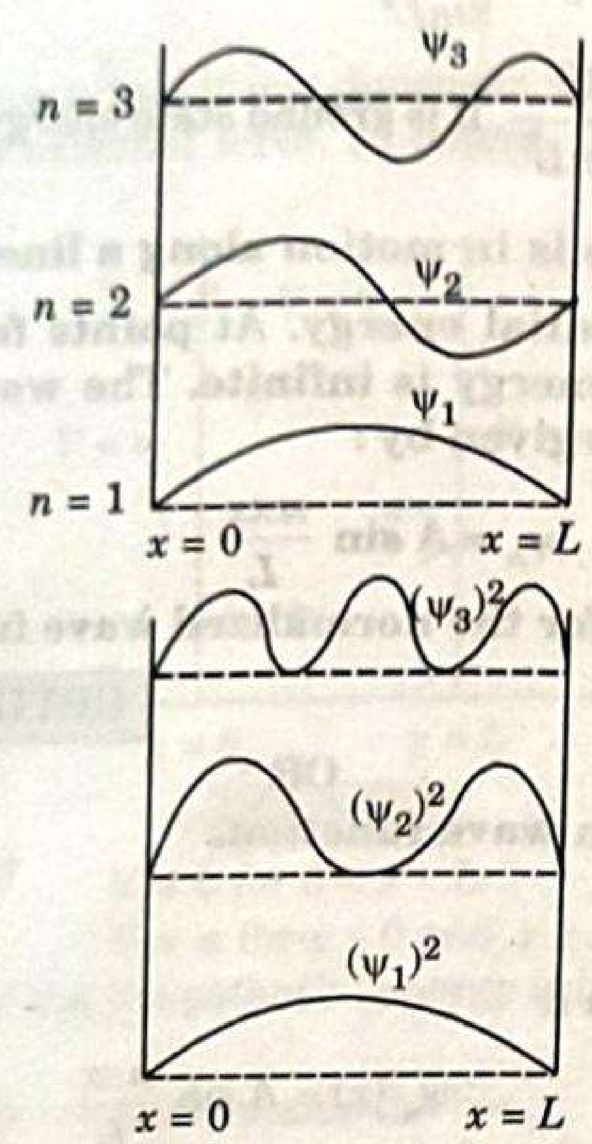

Fig. 2.27.1.

Que 2.29. **An electron is bound in one dimensional potential box which has width 2.5×10^{-10} m. Assuming the height of the box to be infinite, calculate the lowest two permitted energy values of the electron.**

UPTU 2014-15, Marks 05

Answer

1. **Given :** $L = 2.5 \times 10^{-10}$ m
2. We know that,

$$E_n = \frac{n^2 h^2}{8mL^2}$$

$$E_n = \frac{n^2 (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2.5 \times 10^{-10})^2}$$

$$= 9.66 \times 10^{-19} n^2 \text{ J}$$

$$= 6.037 n^2 \text{ eV}$$

3. For $n = 1$, $E_1 = 6.037$ eV

$n = 2$, $E_2 = 24.15$ eV

Que 2.30. **Find the energy of an electron in one dimension in an infinitely high potential box of width 1 Å. Given : $m_e = 9.1 \times 10^{-31}$ kg, $h = 6.63 \times 10^{-34}$ J-s.**

Answer

1. **Given :** $L = 1 \text{ Å} = 1 \times 10^{-10}$ m
2. We know $E_n = \dfrac{n^2 h^2}{8mL^2}$

$$E_n = \frac{n^2 (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (1 \times 10^{-10})^2}$$

$$= 0.6038 \times 10^{-17} n^2 \text{ J}$$

$$E_n = 37.7 n^2 \text{ eV}$$

3. For $n = 1$, $E_1 = 37.7$ eV

$n = 2$, $E_2 = 150.8$ eV

Que 2.31. **Compute the energy difference between the ground state and first excited state of an electron in one-dimensional box of length 10^{-8} m.**

Answer

1. **Given :** $L = 10^{-8}$m, $m = 9.1 \times 10^{-31}$ kg, $h = 6.63 \times 10^{-34}$ Js
2. We know that eigen energy, $E_n = \dfrac{n^2 h^2}{8mL^2}$

$$E_n = n^2 \frac{(6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (10^{-8})^2} = 0.60 \times 10^{-21} n^2 \text{ J}$$

$$= \frac{0.6 \times 10^{-21} n^2}{1.6 \times 10^{-19}} \text{ eV}$$

$$E_n = 3.75 \times 10^{-3} n^2 \text{ eV}$$

4. For ground state ($n = 1$), $E_1 = 3.75 \times 10^{-3}$ eV
5. First excited state ($n = 2$), $E_2 = 0.015$ eV
6. Difference between first excited and ground state,

$$E_2 - E_1 = (15 - 3.75)\, 10^{-3} \text{ eV} = 11.25 \text{ meV}$$

Que 2.32. **A particle confined to move along x-axis has the wave function $\psi = ax$ between $x = 0$ and $x = 1.0$, and $\psi = 0$ elsewhere. Find the probability that the particle can be found between $x = 0.35$ to $x = 0.45$. Also, find the expectation value $\langle x \rangle$ of particle's position.**

Answer

1. **Given :** $\psi_n = ax$ between $x = 0$ and $x = 1$
2. The probability of finding the particle between x_1 and x_2 when it is in n^{th} state is

$$P = \int_{x_1}^{x_2} |\psi_n|^2 \, dx$$

3. Here $\quad x_1 = 0.35$ and $x_2 = 0.45$

Therefore
$$P = \int_{0.35}^{0.45} (ax)^2 dx$$

$$P = a^2 \int_{0.35}^{0.45} x^2 dx$$

$$P = \frac{a^2}{3}\left[x^3\right]_{0.35}^{0.45} = \frac{a^2}{3}\,[(0.45)^3 - (0.35)^3]$$

$$= \frac{a^2}{3}\,[0.091125 - 0.042875]$$

$$= 0.0161\, a^2$$

4. The expectation value of the position of particle is given by

$$< x > = \int_{-\infty}^{\infty} x|\psi_n(x)|^2 \, dx$$

5. Since the particle is confined in a box having its limit $x = 0$ to $x = 1$ then

$$< x > = \int_0^1 ax.\, x.\, ax\, dx \;= a^2 \int_0^1 x^3 dx$$

$$< x > = \frac{a^2}{4} = 0.25\, a^2$$

Que 2.33. **Determine the probabilities of finding a particle trapped in a box of length L in the region from 0.45 L to 0.55 L for the ground state.**

Answer

1. The eigen function of particle trapped in a box of length L are

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$$

2. And probability $\quad P = \int_{x_1}^{x_2} |\psi_n(x)|^2 \, dx = \frac{2}{L}\int_{x_1}^{x_2} \sin^2 \frac{n\pi x}{L}\, dx$

$$P = \frac{2}{L}\int_{x_1}^{x_2} \frac{1}{2}\left(1 - \cos\frac{2\pi n x}{L}\right) dx$$

$$= \frac{1}{L}\left[x - \frac{L}{2\pi n}\cdot \sin\frac{2n\pi x}{L}\right]_{x_1}^{x_2}$$

3. Since, $x_1 = 0.45\, L$ and $x_2 = 0.55\, L$ for ground state, $n = 1$

$$P = \frac{1}{L}\left[x - \frac{L}{2\pi}\sin\frac{2\pi x}{L}\right]_{0.45L}^{0.55L}$$

$$= \frac{1}{L}\left[\left(0.55L - \frac{L}{2\pi}\sin 1.1\pi\right) - \left(0.45L - \frac{L}{2\pi}\sin 0.9\pi\right)\right]$$

$$= \left[\left(0.55 - \frac{1}{2\pi}\sin 198°\right) - \left(0.45 - \frac{1}{2\pi}\sin 162°\right)\right]$$

$$P = [0.59918 - 0.400818]$$

$$P = 0.198362$$

$$P = 19.8\ \%$$

☺☺☺

3 UNIT

Wave Optics

PART-1

Coherent Sources, Interference of thin films, Newton's rings.

CONCEPT OUTLINE : PART-1

Interference : The non-uniform distribution of the light intensity due to the superposition of two waves is called interference.

Necessary Conditions for Interference :

1. Light sources must be coherent in nature.
2. Light waves should be of same frequency.
3. The sources of light must be very close to each other.
4. Light sources should be monochromatic in nature.
5. The light waves must propagate along the same direction.

Types of Interference :

1. **Constructive Interference :** At certain points the resultant intensity (I) is greater than the sum of individual intensity of two waves. The interference produced at this point is known as constructive interference, it results into bright fringe. At constructive interference

$$I > I_1 + I_2$$

2. **Destructive Interference :** At certain points the resultant intensity (I) is less than the sum of individual intensity of two waves. The interference produced at this point is known as destructive interference and it results into dark fringe. At destructive interference

$$I < I_1 + I_2$$

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.1. What do you understand by coherent sources ? How are these obtained in practice ? **UPTU 2012-13, Marks 05**

Answer

A. **Coherent Sources :**

1. Two sources are said to be coherent if they emit continuous light waves of the same frequency or wavelength, nearly the same amplitude and

having sharply defined phase difference that remains constant with time.

B. Production of Coherent Sources :

1. If two sources are derived from a single source by some device, then any phase-change in one is simultaneously accompanied by the same phase-change in the other.
2. Thus the phase difference between the two sources remains constant.
3. Following are the devices for creating coherent sources of light :

a. Young's Double Slit :

1. In this device, two narrow slits S_1 and S_2 receive light from the same narrow slit S.
2. Hence S_1 and S_2 act as coherent sources, as shown in Fig. 3.1.1.

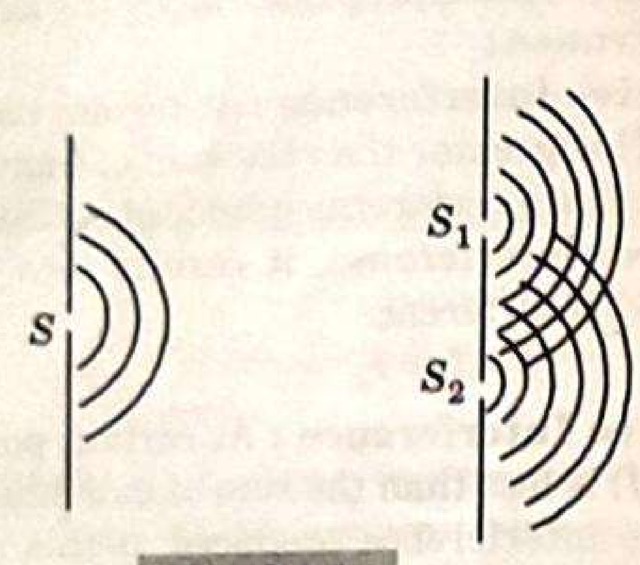

Fig. 3.1.1.

b. Lloyd's Mirror :

1. In this device, a slit S and its virtual image S' formed by reflection at a mirror are the coherent sources, as shown in Fig. 3.1.2.

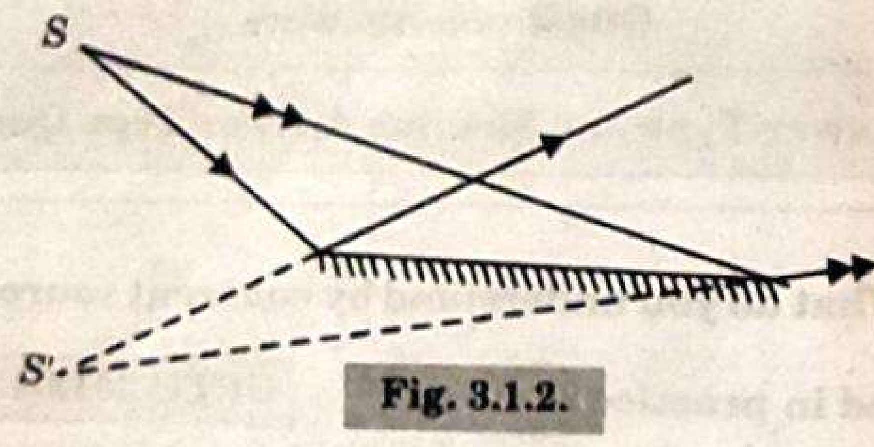

Fig. 3.1.2.

c. Fresnel's Double Mirror :

1. In this device two virtual images S_1 and S_2 of a single slit S, formed by reflection at two plane mirrors M_1 and M_2 inclined at a small angle to each other, are the coherent sources as shown in Fig. 3.1.3.

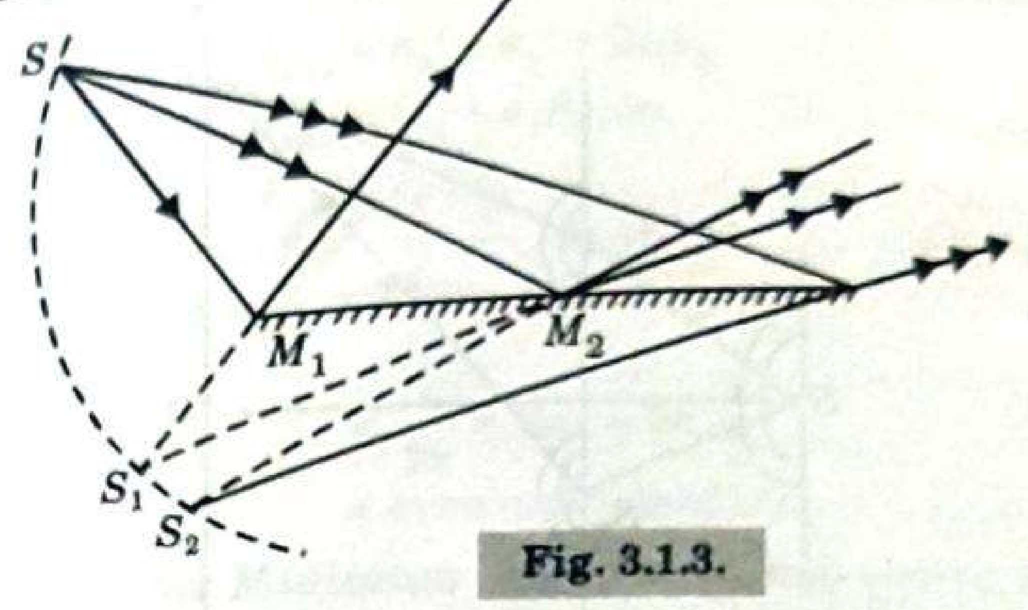

Fig. 3.1.3.

d. Fresnel's Biprism :

1. In this device, S_1 and S_2, which are the images of a slit S formed by refraction through a biprism, act as coherent sources, as shown in Fig. 3.1.4.

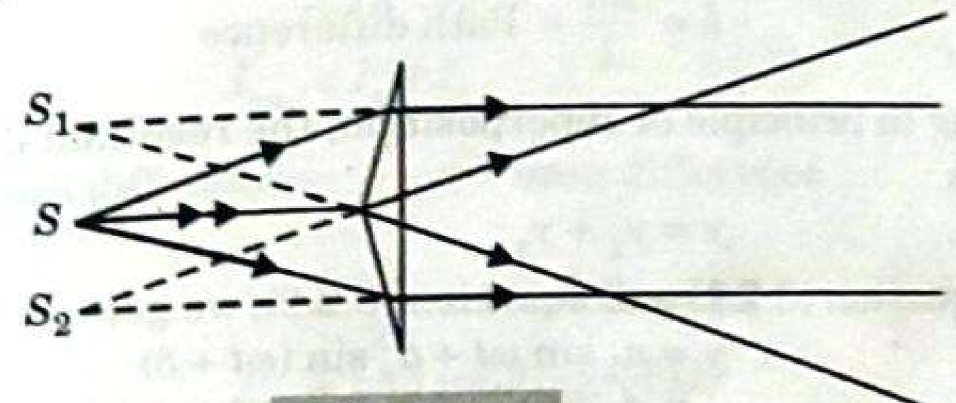

Fig. 3.1.4.

e. Michelson's Interferometer :

1. In this device, a single beam is broken into two light waves perpendicular to each other, one by reflection and the other by refraction.
2. The two beams, when reunite produce interference fringes. Here these two beams act as coherent sources.

Que 3.2. **Explain theory of interference by two waves.**

OR

Explain Young's double slit experiment.

Answer

1. Let us consider two superimposed waves travelling with same frequency $\left(\frac{\omega}{2\pi}\right)$ and having constant phase difference in the same region.
2. If a_1 and a_2 are amplitude of two waves, the displacement of two waves at any instant t is given by

$$y_1 = a_1 \sin \omega t \quad ...(3.2.1)$$
$$y_2 = a_2 \sin (\omega t + \delta) \quad ...(3.2.2)$$

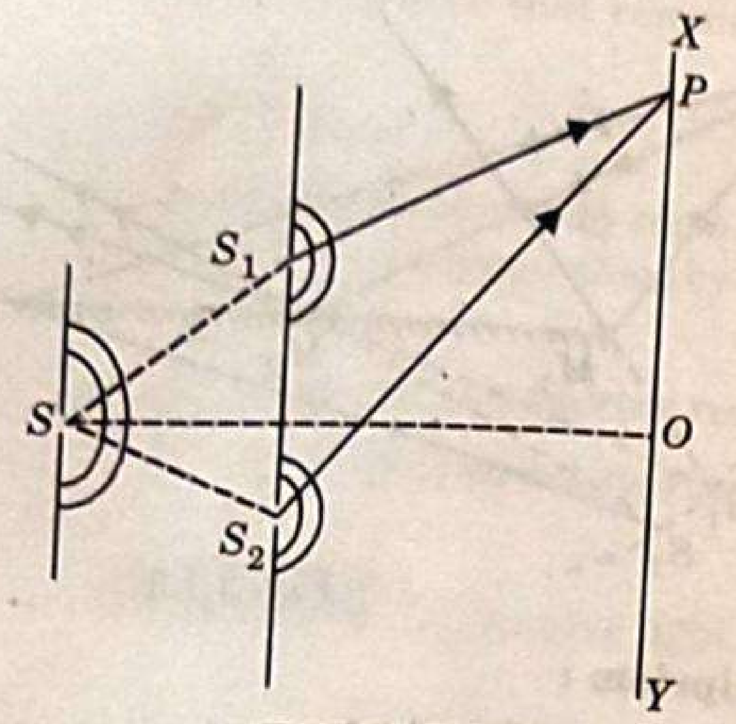

Fig. 3.2.1.

(where $\delta \rightarrow$ initial phase difference)

or, $\delta = \frac{2\pi}{\lambda} \times$ Path difference

3. According to principle of superposition, the resultant displacement at point P is

$$y = y_1 + y_2 \quad ...(3.2.3)$$

4. Using equation (3.2.1) and equation (3.2.2), we get

$$y = a_1 \sin \omega t + a_2 \sin (\omega t + \delta)$$
$$= a_1 \sin \omega t + a_2 [\sin \omega t \cos \delta + \cos \omega t \sin \delta]$$

or, $y = \sin \omega t (a_1 + a_2 \cos \delta) + (a_2 \sin \delta) \cos \omega t \quad ...(3.2.4)$

5. Let us take

$$A \cos \phi = a_1 + a_2 \cos \delta \quad ...(3.2.5)$$
$$A \sin \phi = a_2 \sin \delta \quad ...(3.2.6)$$

6. Equation (3.2.4) becomes

$$y = A \cos \phi \sin \omega t + A \sin \phi \cos \omega t$$

or, $y = A \sin (\omega t + \phi)$

7. Since, this is the resultant wave equation, squaring equation (3.2.5) and equation (3.2.6) and adding

$$A^2 = a_1^2 + a_2^2 \cos^2 \delta + 2a_1 a_2 \cos \delta + a_2^2 \sin^2 \delta$$
$$A^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos \delta$$

8. By the definition, intensity is directly proportional to the square of amplitude

i.e., $I \propto A^2$

or, $I = KA^2$ (K = 1, in arbitrary unit)

$\therefore$ $I = A^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos \delta$

A. Condition for Maximum Intensity (Constructive Interference, I_{max}) :

1. If $\cos \delta = 1$ *i.e.*, $\delta = 2n\pi$

where, $n = 0, 1, 2, 3,...$

2. $I_{max} = a_1^2 + a_2^2 + 2a_1 a_2$

$I_{max} = (a_1 + a_2)^2$

3. So, $I_{max} > I_1 + I_2$

4. Path difference $= \frac{\lambda}{2\pi} \times$ Phase difference

$$= \frac{\lambda}{2\pi} \times 2n\pi = 2n \frac{\lambda}{2}$$

= even multiple of $\lambda/2$.

B. Condition for Minimum Intensity (Destructive Interference, I_{min}) :

1. If $\cos \delta = -1$ *i.e.*, $\delta = (2n + 1)\pi$

where, $n = 0, 1, 2, 3...$

2. $I_{min} = a_1^2 + a_2^2 - 2a_1 a_2$

$= (a_1 - a_2)^2$

3. Hence, $I_{min} < I_1 + I_2$

4. Path difference $= \frac{\lambda}{2\pi} \times$ Phase difference

$$= \frac{\lambda}{2\pi} \times (2n + 1)\pi = (2n + 1) \frac{\lambda}{2}$$

= odd multiple of $\lambda/2$.

Que 3.3. **Discuss the interference in thin film due to reflected light. What happens when film is excess thin ?**

UPTU 2013-14, Marks 05

OR

Explain the phenomenon of interference in thin films due to reflected light.

UPTU 2011-12, 2015-16 Marks 05

Answer

1. Thin film like soap bubbles or a drop of oil spread on the surface of water show different colours when exposed to sunlight. In this case the interference pattern is produced by division of amplitude.
2. Let a ray SA from monochromatic light source S be incident at A as shown in Fig. 3.3.1.
3. A part of it is reflected along AC and remaining refracted along AE.
4. Again E is partially reflected along EB and partially refracted along EG parallel to SA.
5. We get two sets of rays first AC, BD, etc., reflected from the film and second EG, FH, etc transmitted through the film.

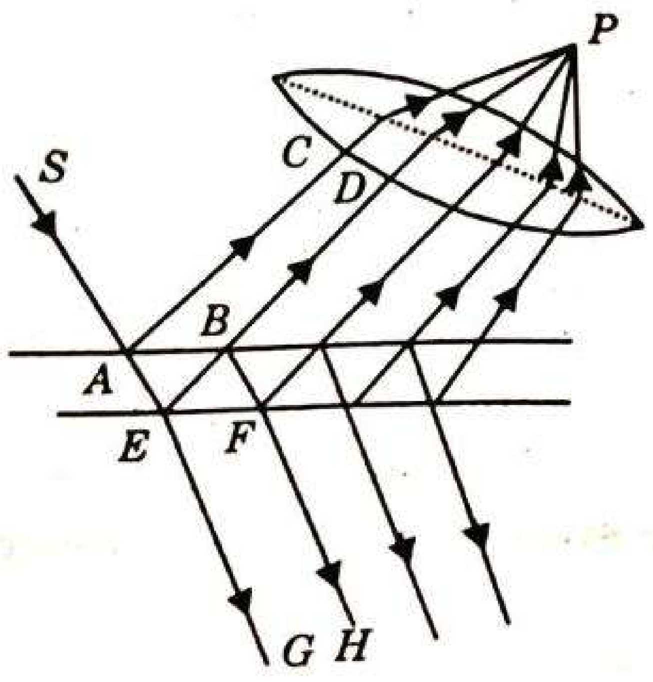

Fig. 3.3.1.

A. Parallel Sided Thin Film :

1. Consider a parallel sided transparent thin film of thickness t and refractive index $\mu > 1$.
2. Let SA a monochromatic light of wavelength λ be incident on the upper surface of the film at an angle i. This ray gets partially reflected along AB and partially refracted along AC direction.
3. Now at point C it again gets reflected along CD and transmitted along DE.

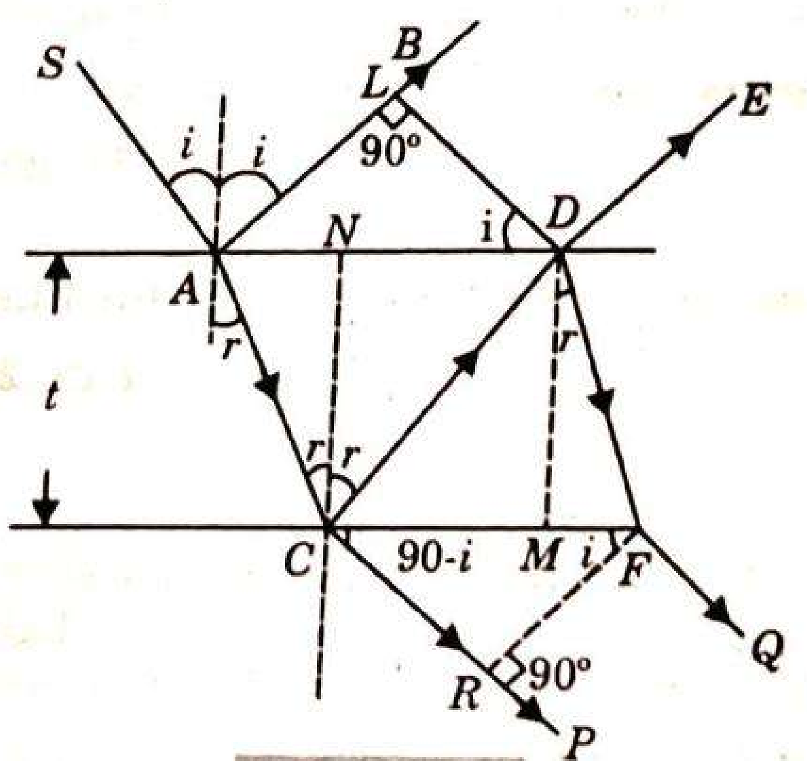

Fig. 3.3.2.

B. Reflected System :

1. According to Fig. 3.3.2, the path difference between AB and DE rays,

$$\Delta = \text{path } ACD \text{ in film} - \text{path } AL \text{ in air}$$

$$\Delta = \mu (AC + CD) - AL \quad ...(3.3.1)$$

2. Now in ΔANC,

$$\cos r = \frac{CN}{AC} \Rightarrow AC = \frac{CN}{\cos r} = \frac{t}{\cos r}$$

3. In ΔNCD,

$$\cos r = \frac{t}{CD} \Rightarrow CD = \frac{t}{\cos r}$$

4. Now in ΔALD,

$$\sin i = \frac{AL}{AD} \Rightarrow AL = AD \sin i$$

$$AL = (AN + ND) \sin i$$

5. But from ΔANC and ΔNCD,

$$AN = t \tan r \text{ and } ND = t \tan r$$

$$AL = 2t \tan r \,.\, \sin i$$

6. Putting the values of AC, CD, and AL in equation (3.3.1),

$$\Delta = \mu \left(\frac{2t}{\cos r}\right) - 2t \tan r . \sin i$$

$$= \frac{2\mu t}{\cos r} - 2t \frac{\sin r}{\cos r} . \sin i \qquad \left[\because \mu = \frac{\sin i}{\sin r}\right]$$

$$\therefore \quad \Delta = \frac{2\mu t}{\cos r}[1 - \sin^2 r]$$

$$\Delta = 2\mu t \cos r$$

7. Since, the ray AB is reflected at the surface of a denser medium therefore, it undergoes a phase change of π or path difference of $\frac{\lambda}{2}$.
8. The effective path difference between AB and DE is

$$\Delta = 2\mu t \cos r + \frac{\lambda}{2} \quad ...(3.3.2)$$

a. Condition for Maxima :

1. If $\quad \Delta = 2n \frac{\lambda}{2}$

where, $\quad n = 0, 1, 2, 3...$

2. Then, $\quad 2\mu t \cos r + \frac{\lambda}{2} = 2n \frac{\lambda}{2}$

$$2\mu t \cos r = (2n - 1) \frac{\lambda}{2} \quad ...(3.3.3)$$

b. Condition for Minima :

1. $$\Delta = (2n + 1) \frac{\lambda}{2}$$

where, $\quad n = 0, 1, 2, 3...$

2. Then, $2\mu t \cos r + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$

$$2\mu t \cos r = n\lambda \quad ...(3.3.4)$$

C. Transmitted System :

1. From Fig. 3.3.2, the path difference between two transmitted rays, CP and FQ

$$\Delta = \text{Path } CDF \text{ in film} - CR \text{ in air}$$
$$\Delta = \mu (CD + DF) - CR \quad ...(3.3.5)$$

2. Now in ΔCDM,

$$\cos r = \frac{DM}{CD} \Rightarrow CD = \frac{t}{\cos r} \text{ and } CM = t \tan r$$

3. In ΔDMF

$$\cos r = \frac{DM}{DF} \Rightarrow DF = \frac{t}{\cos r} \text{ and } MF = t \tan r$$

4. And in ΔCRF

$$\sin i = \frac{CR}{CF} \Rightarrow CR = CF \sin i$$

or, $CR = (CM + MF) \sin i$

$CR = 2t \tan r \sin i$

5. On putting the value of CD, DF and CR in equation (3.3.5),

$$\Delta = \frac{2\mu t}{\cos r} - 2t \tan r \sin i$$

$$= \frac{2\mu t}{\cos r} - 2t\frac{\sin r}{\cos r}.\mu \sin r \quad \left[\because \mu = \frac{\sin i}{\sin r}\right]$$

$$\Delta = \frac{2\mu t}{\cos r}[1 - \sin^2 r]$$

$$\Delta = 2\mu t \cos r \quad ...(3.3.6)$$

a. Condition for Maxima :

1. If $\Delta = 2n\frac{\lambda}{2}$ $(n = 0, 1, 2, 3...)$

2. Then $2\mu t \cos r = 2n\frac{\lambda}{2}$

or, $2\mu t \cos r = n\lambda$ ...(3.3.7)

b. Condition for Minima :

1. If $\Delta = (2n+1)\frac{\lambda}{2}$ $(n = 0, 1, 2, 3....)$

2. Then, $2\mu t \cos r = (2n+1)\frac{\lambda}{2}$ ...(3.3.8)

3. Hence, we can say that the reflected and transmitted interference patterns are complementary to each other.

D. Condition for Excess Thin Film :

1. When the film is excessively thin such that its thickness t is very small as compared to the wavelength of light, then $2\mu t \cos r$ is almost zero.
2. Hence effective path difference becomes $\frac{\lambda}{2}$.
3. Thus every wavelength will be absent and film will appear black in reflected light.

Que 3.4. **White light falls normally on a film of soapy water whose thickness is 1.5×10^{-5} cm and refractive index is 1.33. Which wavelength in the visible region will be reflected most strongly ?**

UPTU 2011-12, Marks 05

Answer

1. Given : $\mu = 1.33$, and $t = 1.5 \times 10^{-5}$ cm
$\alpha = 0°$, white light on the film falls normally.
So, $\cos \alpha = 1$

2. Since,
$$2\mu t \cos \alpha = \frac{(2n-1)\lambda}{2}$$

3. Then, $2 \times 1.33 \times 1.5 \times 10^{-5} = (2n-1)\lambda/2$

$$\lambda = \frac{4 \times 1.33 \times 1.5 \times 10^{-5}}{2n-1} = \frac{7.98 \times 10^{-5}}{2n-1} \text{ cm}$$

$$\lambda = \frac{7.98 \times 10^{-7}}{(2n-1)} \text{ m}$$

4. For $n = 1$, $\lambda_1 = 7980 \times 10^{-10} = 7980$ Å (visible region).

5. For $n = 2$, $\lambda_2 = \frac{7.98 \times 10^{-7}}{(2n-1)} = \frac{7.98 \times 10^{-7}}{3} = 2660 \times 10^{-10}$

$= 2660$ Å (not in visible region).

Que 3.5. **A man whose eyes are 150 cm above the oil film on water surface observes greenish colour at a distance of 100 cm from his feet. Find the thickness of the film.**

($\mu_{oil} = 1.4, \mu_{water} = 1.33, \lambda_{green} = 5000$ Å)

Answer

1. The condition for maxima,

$$2\mu t \cos r = (2n-1)\frac{\lambda}{2} \quad (n = 1, 2, 3....)$$

or $$t = \frac{(2n-1)\lambda}{4\mu\cos r}$$

2. From Fig. 3.5.1,
$$\tan i = \frac{100}{150} = \frac{2}{3}$$
$$\sin i = \frac{2}{\sqrt{13}}$$

3. Since, $$\mu = \frac{\sin i}{\sin r}$$
$\therefore$ $$\sin r = \frac{\sin i}{\mu} = \frac{\frac{2}{\sqrt{13}}}{1.4} = 0.3962$$
and $$\cos r = \sqrt{1-\sin^2 r} = \sqrt{[1-(0.3962)^2]} = 0.9182$$

4. Therefore, $$t = \frac{(2n-1)\lambda}{4\mu\cos r} = \frac{(2n-1)5\times10^{-7}}{4\times1.4\times0.9182}$$
$$= (2n-1)\times 9.725\times10^{-8}\text{ m}$$

5. Putting $n = 1, 2, 3......$ value of t is calculated.

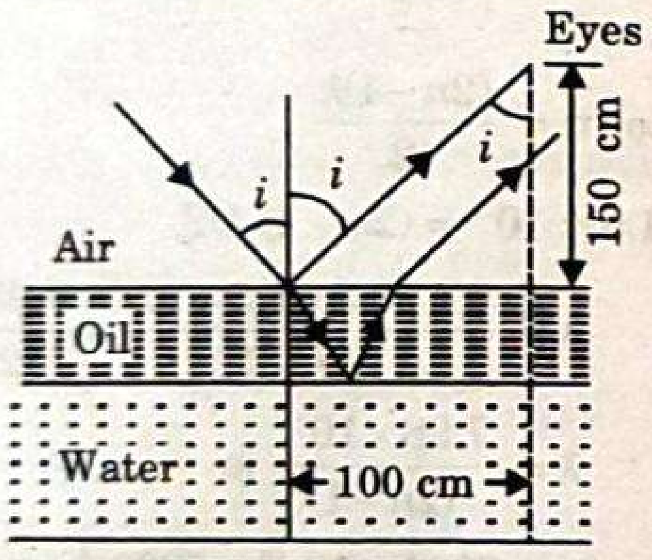

Fig. 3.5.1.

Que 3.6. **Light of wavelength 5893 Å is reflected at nearly normal incidence from a soap film of $\mu = 1.42$. What is the least thickness of this film that will appear :**
a. dark
b. bright ?

Answer

Given : $\lambda = 5893$ Å, $\mu = 1.42$

A. Since, the condition for the dark film in reflected system is
$$2\mu t\cos r = n\lambda$$

1. For normal incidence, $r = 0$ and $\cos r = 1$
$\therefore$ $$2\mu t = n\lambda \text{ or } t = n\lambda/2\mu$$

2. For least thickness of the film, $n = 1$

$$t = \frac{\lambda}{2\mu}$$
$$t = \frac{5893\times10^{-8}}{2\times1.42} = 2.075\times10^{-5}\text{ cm}$$

B. The condition for bright film,
$$2\mu t\cos r = (2n-1)\frac{\lambda}{2}$$

1. For normal incidence, $r = 0$ and $\cos r = 1$
$\therefore$ $$2\mu t = (2n-1)\frac{\lambda}{2}$$

2. For least thickness, $n = 1$
$$2\mu t = (2\times1-1)\frac{\lambda}{2} \text{ or } 2\mu t = \frac{\lambda}{2}$$
and $$t = \frac{\lambda}{4\mu} = \frac{5893\times10^{-8}}{4\times1.42} = 1.0375\times10^{-5}\text{ cm}$$

Que 3.7. **Explain interference in a wedge-shaped thin film.**

Answer

1. Consider two planes OP and OQ make an angle θ and encloses a wedge shaped film in between them, the thickness increases from O to P and have refractive index μ for the film.

2. When a ray of monochromatic light falls on the upper surface of the film, interference occurs between reflected ray from upper surface along AB and other obtained from internal reflection on lower surface and transmitted along DE, we will calculate path difference between AB and DE rays.

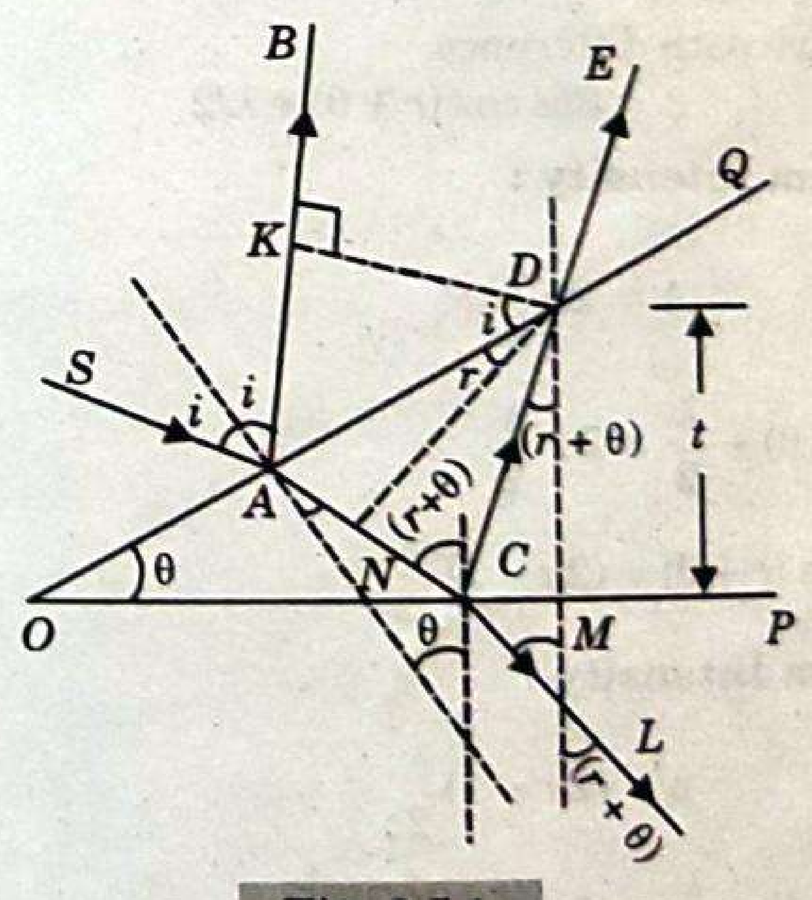

Fig. 3.7.1.

3. ∴ The path difference, Δ = path ACD in film $- AK$ in air

$$\Delta = \mu\,(AC + CD) - AK$$

$$\Delta = \mu\,[AN + NC + CD] - AK \qquad ...(3.7.1)$$

4. But, $\mu = \dfrac{\sin i}{\sin r}$

5. Now in ΔAKD and ΔAND

$$\sin i = \frac{AK}{AD} \text{ and } \sin r = \frac{AN}{AD}$$

$$\therefore \quad \mu = \frac{AK / AD}{AN / AD} \Rightarrow AK = \mu AN \qquad ...(3.7.2)$$

6. From equation (3.7.1) and equation (3.7.2),

$$\Delta = \mu\,(AN + NC + CD) - \mu AN$$

or $$\Delta = \mu\,(NC + CD) \qquad ...(3.7.3)$$

7. Now in ΔDCM and ΔCML

$CM = CM$ (common), $\angle L = \angle D = r + \theta$,

$\angle LMC = \angle CMD = 90°$

∴ $\Delta DCM \cong \Delta CML$, hence $CL = CD$ and $DM = ML = t$

8. Equation (3.7.3) becomes

$$\Delta = \mu(NC + CL) = \mu NL \qquad ...(3.7.4)$$

9. In ΔDNL,

$$\cos(r + \theta) = \frac{NL}{DL} \Rightarrow NL = DL \cos(r + \theta)$$

$$NL = 2t \cos(r + \theta)$$

10. Putting the value of NL in equation (3.7.4), we get

$$\Delta = 2\mu t \cos(r + \theta)$$

11. Since, AB ray is reflected from denser medium therefore, it covers extra path difference $\lambda/2$.

12. Hence, effective path difference,

$$\Delta = 2\mu t \cos(r + \theta) + \lambda/2$$

a. For Maximum Intensity :

1. If $$\Delta = 2n\frac{\lambda}{2} \qquad (n = 0, 1, 2, 3...)$$

2. $$2\mu t.\cos(r + \theta) + \frac{\lambda}{2} = 2n\,\frac{\lambda}{2}$$

or, $$2\mu t \cos(r + \theta) = (2n - 1)\,\frac{\lambda}{2}$$

b. For Minimum Intensity :

1. If $$\Delta = (2n + 1)\,\frac{\lambda}{2} \qquad (n = 0, 1, 2, ...)$$

2. $$2\mu t \cos(r + \theta) + \frac{\lambda}{2} = (2n + 1)\,\frac{\lambda}{2}$$

$$2\mu t \cos(r + \theta) = n\lambda.$$

c. Fringe Width :

1. Let distance of n^{th} maxima from O is x_n and thickness of film is t.

$$\tan\theta = \frac{t}{x_n} \Rightarrow t = x_n \tan\theta$$

2. For n^{th} maxima,

$$2\mu t \cos(r + \theta) = (2n - 1)\,\frac{\lambda}{2}$$

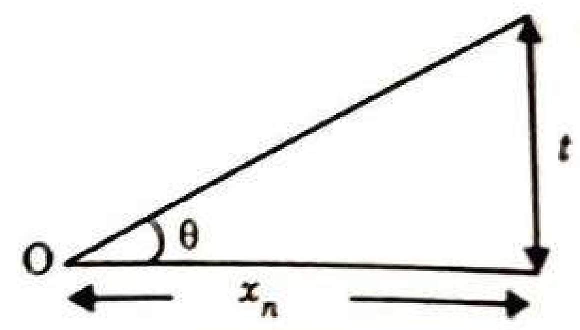

Fig. 3.7.2.

3. For normal incidence, $r = 0$ and for air film, $\mu = 1$.

$$2t \cos\theta = (2n - 1)\,\frac{\lambda}{2},$$

(On putting the value of t)

$$2x_n \tan\theta.\cos\theta = (2n - 1)\,\frac{\lambda}{2} \text{ or } 2x_n \sin\theta = (2n - 1)\,\frac{\lambda}{2} \qquad ...(3.7.1)$$

4. If x_{n+1} is the distance of $(n + 1)^{th}$ maxima,

Then, $$2x_{n+1} \sin\theta = (2n + 1)\,\frac{\lambda}{2} \qquad ...(3.7.2)$$

5. Now subtracting equation (3.7.1) and equation (3.7.2), we get

$$2\sin\theta\,(x_{n+1} - x_n) = \frac{2\lambda}{2}$$

$$(x_{n+1} - x_n) = \beta = \frac{\lambda}{2\sin\theta}$$

6. For small θ, $$\beta = \frac{\lambda}{2\theta}$$

7. If $\mu \to$ material $\Rightarrow$ $\beta = \dfrac{\lambda}{2\mu\theta}$

Que 3.8. **White light is incident on a soap film at an angle $\sin^{-1}\frac{4}{5}$ and the reflected light is observed with spectroscope. It is found that two consecutive dark bands correspond to wavelength 6.1×10^{-5} cm and 6.0×10^{-5} cm. If the μ of the film be 4/3, calculate its thickness.**

UPTU 2006-07, 2008-09, 2012-13 ; Marks 05

Answer

1. **Given :** $\lambda_1 = 6.1 \times 10^{-5}$ cm, $\lambda_2 = 6.0 \times 10^{-5}$ cm, $\mu = \dfrac{4}{3}$
2. Since, the condition for dark band is
$$2\mu t \cos r = n\lambda \quad ...(3.8.1)$$
3. If n and $(n + 1)$ are the orders for dark bands for wavelengths λ_1 and λ_2 respectively, then
$$2\mu t \cos r = n\lambda_1$$
and $$2\mu t \cos r = (n + 1)\lambda_2 \quad ...(3.8.2)$$
or, $$2\mu t \cos r = n\lambda_1 = (n + 1)\lambda_2 \quad ...(3.8.3)$$
or, $$n = \frac{\lambda_2}{\lambda_1 - \lambda_2}$$
4. On putting the value of n in equation (3.8.2),
$$2\mu t \cos r = \frac{\lambda_1\lambda_2}{\lambda_1 - \lambda_2} \text{ or } t = \frac{\lambda_1\lambda_2}{(\lambda_1 - \lambda_2)2\mu \cos r}$$
5. But $$\cos r = \sqrt{1 - \sin^2 r} = \sqrt{1 - \left(\frac{\sin i}{\mu}\right)^2} \qquad \left(\because \mu = \frac{\sin i}{\sin r}\right)$$
$$\cos r = \sqrt{1 - \left(\frac{4/5}{4/3}\right)^2} = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$$
6. Now, $$t = \frac{6.1 \times 10^{-5} \times 6.0 \times 10^{-5}}{(6.1 \times 10^{-5} - 6.0 \times 10^{-5}) \times 2 \times \frac{4}{3} \times \frac{4}{5}}$$
$$= 0.0017 \text{ cm.}$$

Que 3.9. **Two plane glass surfaces in contact along one edge are separated at the opposite edge by a thin wire. If 20 interference fringes are observed between these edges, in sodium light of wavelength, λ = 5890 Å of normal incidence, find the diameter of the wire.**

UPTU 2012-13, Marks 05

Answer

1. **Given :** $N = 20$, $\lambda = 5890$ Å
2. Let the diameter of the wire be 'd' and the length of the wedge be 'l'.
3. The wedge angle is given as,
$$\tan \theta = \frac{d}{l}$$
$$\tan \theta \approx \theta \text{ (As } \theta \text{ is small)}$$
$$\therefore \quad \theta = \frac{d}{l}$$

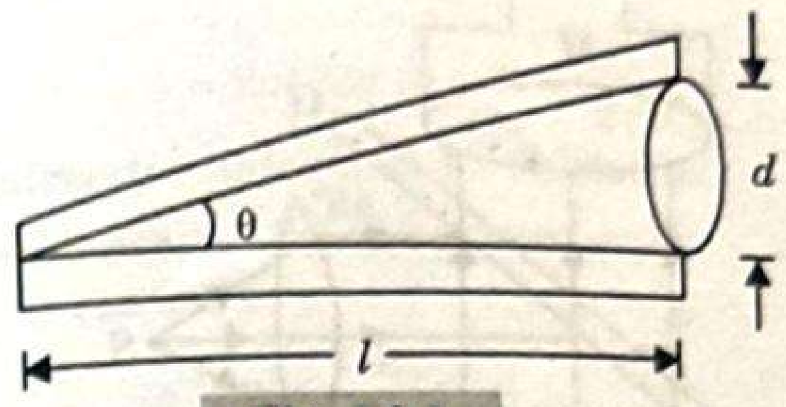

Fig. 3.9.1.

4. Now, fringe-width in air wedge is
$$\bar{X} = \frac{\lambda}{2\lambda\theta} = \frac{1}{2\theta} \qquad (\because \lambda = 1 \text{ for air wedge})$$
or, $$\bar{X} = \frac{l}{2d} \qquad \left(\because \theta = \frac{d}{l}\right)$$
5. If N fringes are seen,
$$l = N\bar{X}$$
6. Thus, $$\bar{X} = \frac{\lambda N\bar{X}}{2d}$$
$$d = \frac{N\lambda}{2}$$
7. $$d = \frac{20 \times 5890}{2}$$
$$d = 58900 \text{ Å}$$
$$d = 5.89 \times 10^{-4} \text{ cm}$$

Que 3.10. **What are Newton's rings ? Explain with diagram.**

Answer

1. When a plano-convex lens of large radius of curvature is placed on a plane glass plate with convex surface in contact, a thin film between the lower surface of the lens and the upper surface of glass plate is formed.
2. The thickness of this film is very small (or zero) at the contact point and gradually increases from contact point to outward.
3. When a monochromatic light falls on the film, we get dark and bright concentric circular fringes having uniform thickness.
4. These rings are first investigated by Newton and are called Newton's rings.

A. Newton's Rings by Reflected Light :

1. According to Fig. 3.10.1, S is a monochromatic source of light placed at the focus of lens L_1.
2. A horizontal beam of light fall on the glass plate G placed at 45° to the incident beam.

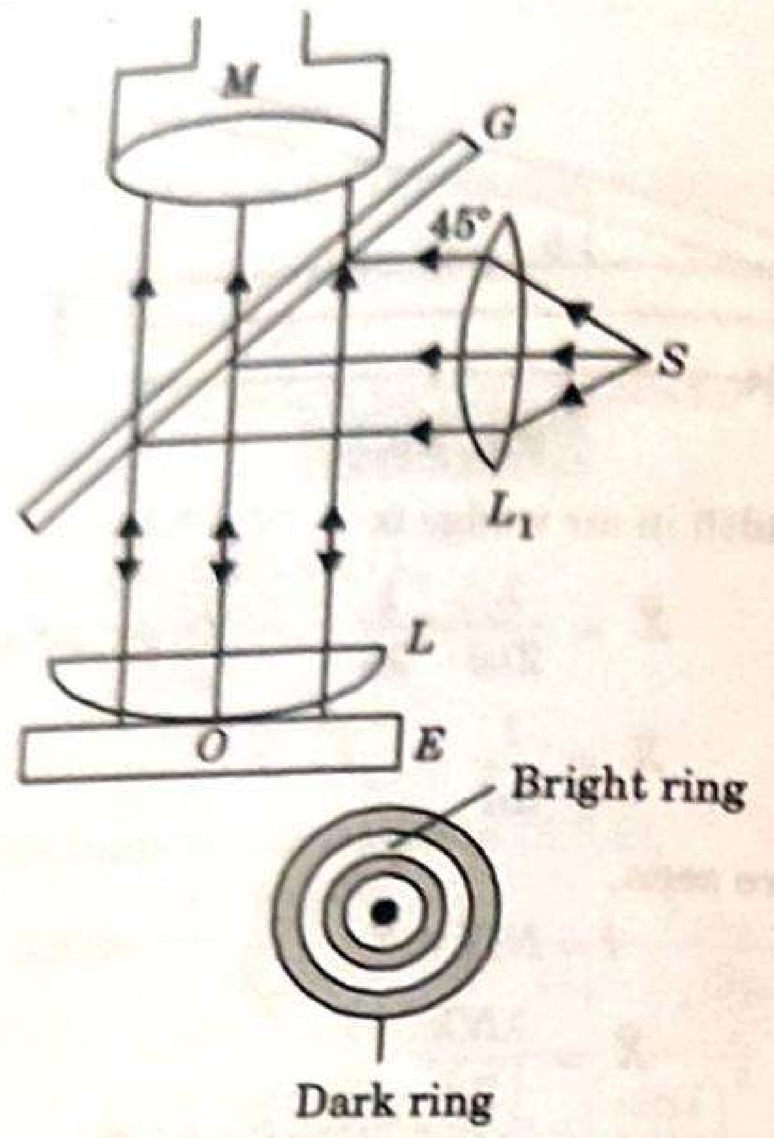

Fig. 3.10.1.

3. This beam is partly reflected from glass plate G.
4. This reflected beam fall normally on the lens L, placed on glass plate E.
5. Hence, the interference occurs between the rays reflected from the upper and lower surface of the film.
6. Interference rings are seen with the help of lower power microscope M.
7. The fringes are circular because the air film is symmetrical about the point of contact with lens and glass plate.

B. Explanation :

1. According to Fig. 3.10.2, rays (1) and (2) are reflected interfering rays corresponding to incident ray SP.

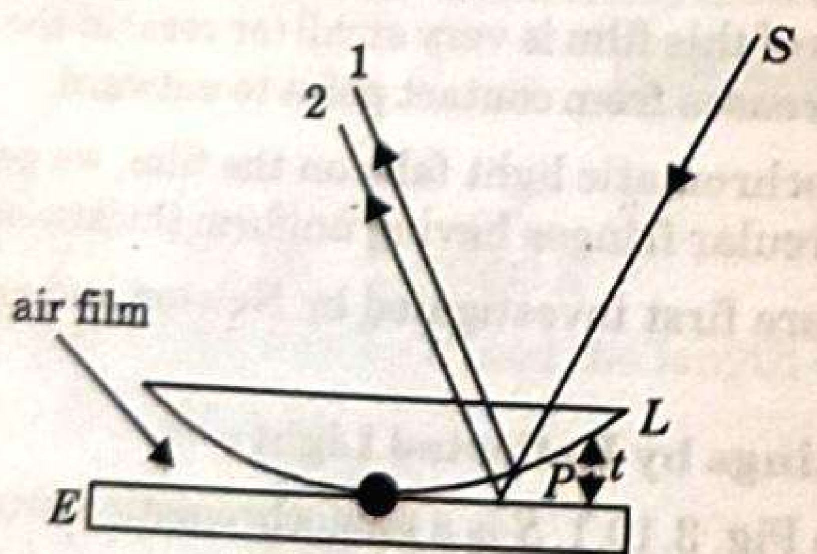

Fig. 3.10.2.

2. Now the effective path difference between (1) and (2) rays is given as :

$$\Delta = 2\mu t \cos (r + \theta) + \frac{\lambda}{2}$$

where, μ = Refractive index and t = thickness

3. For normal incidence, $r = 0$ and θ is very small.
$$\cos \theta = 1$$
4. Hence,
$$\Delta = 2\mu t + \frac{\lambda}{2}$$
5. At point of contact (O) of the lens,
$$t = 0$$
$$\Rightarrow \quad \Delta = \frac{\lambda}{2}$$
6. This is condition of minimum intensity hence the central spot of the ring is dark.

a. Condition for Maximum Intensity (Bright Rings) :

1. If path difference, $\Delta = 2n \frac{\lambda}{2}$ (where, $n = 0, 1, 2, 3...$)
2. Hence, $2\mu t + \frac{\lambda}{2} = 2n \frac{\lambda}{2}$ or $2\mu t = (2n - 1) \frac{\lambda}{2}$
3. For air, $\mu = 1 \Rightarrow 2t = (2n - 1) \frac{\lambda}{2}$

b. Condition for Minimum Intensity (Dark Rings) :

1. If path difference, $\Delta = (2n + 1) \frac{\lambda}{2}$ (where, $n = 0, 1, 2, 3...$)
2. Hence, $2\mu t + \frac{\lambda}{2} = (2n + 1) \frac{\lambda}{2}$
$$2\mu t = n\lambda$$
3. For air $\mu = 1 \Rightarrow 2t = n\lambda$

Que 3.11. Derive the expression for diameter of the bright and dark ring of n order in Newton's ring.

OR

Describe and explain the formation of Newton's rings in reflected monochromatic light. Prove that in reflected light diameters of the dark rings are proportional to the square root of natural numbers.

UPTU 2014-15, Marks 05

OR

What are Newton's rings ? Prove that in reflected light diameters of the bright rings are proportional to the square root of odd natural number.

UPTU 2009-10, Marks 05

OR

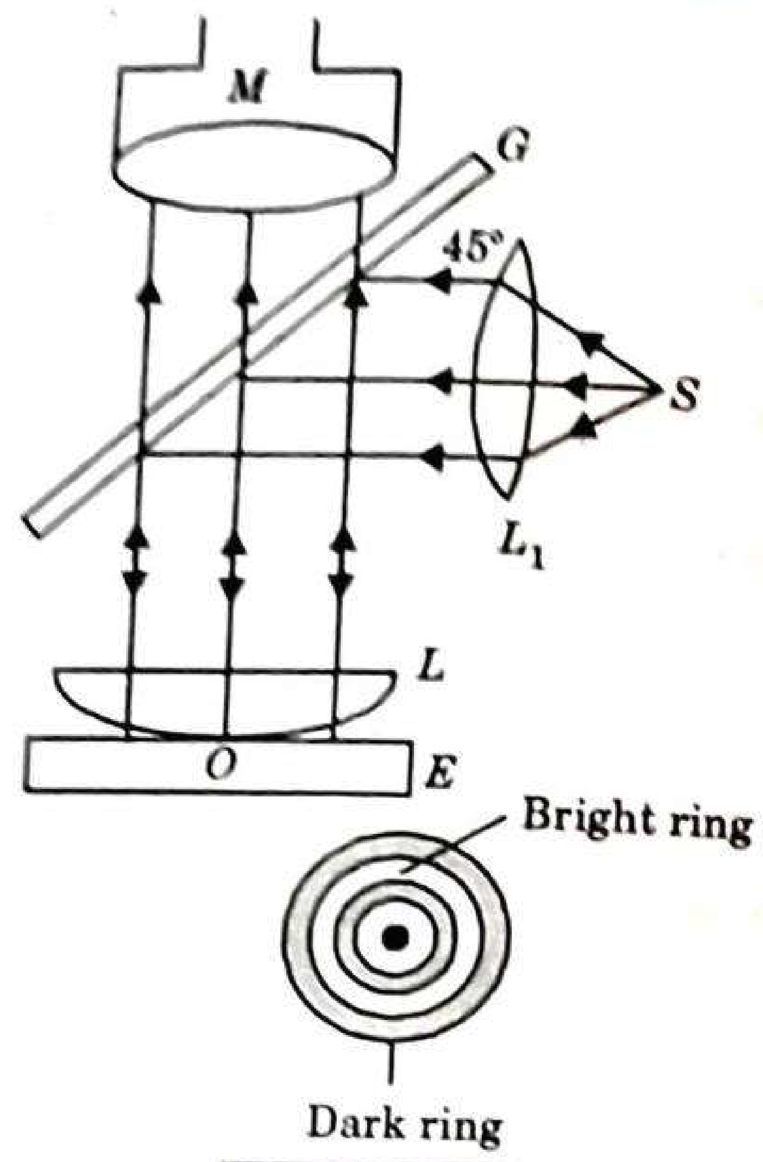

Fig. 3.10.1.

3. This beam is partly reflected from glass plate G.
4. This reflected beam fall normally on the lens L, placed on glass plate E.
5. Hence, the interference occurs between the rays reflected from the upper and lower surface of the film.
6. Interference rings are seen with the help of lower power microscope M.
7. The fringes are circular because the air film is symmetrical about the point of contact with lens and glass plate.

B. Explanation :

1. According to Fig. 3.10.2, rays (1) and (2) are reflected interfering rays corresponding to incident ray SP.

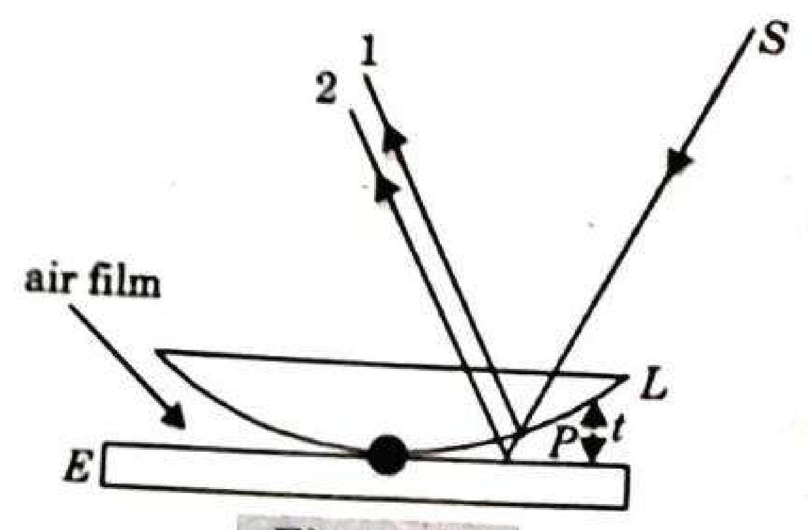

Fig. 3.10.2.

2. Now the effective path difference between (1) and (2) rays is given as :

$$\Delta = 2\mu t \cos(r + \theta) + \frac{\lambda}{2}$$

where, μ = Refractive index and t = thickness

3. For normal incidence, $r = 0$ and θ is very small.

$$\cos\theta = 1$$

4. Hence, $$\Delta = 2\mu t + \frac{\lambda}{2}$$

5. At point of contact (O) of the lens,

$$t = 0$$

$$\Rightarrow \quad \Delta = \frac{\lambda}{2}$$

6. This is condition of minimum intensity hence the central spot of the ring is dark.

a. Condition for Maximum Intensity (Bright Rings) :

1. If path difference, $\Delta = 2n\dfrac{\lambda}{2}$ (where, n = 0, 1, 2, 3...)
2. Hence, $$2\mu t + \frac{\lambda}{2} = 2n\frac{\lambda}{2} \text{ or } 2\mu t = (2n-1)\frac{\lambda}{2}$$
3. For air, $$\mu = 1 \Rightarrow 2t = (2n-1)\frac{\lambda}{2}$$

b. Condition for Minimum Intensity (Dark Rings) :

1. If path difference, $\Delta = (2n+1)\dfrac{\lambda}{2}$ (where, n = 0, 1, 2, 3...)
2. Hence, $$2\mu t + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$ $$2\mu t = n\lambda$$
3. For air $$\mu = 1 \Rightarrow 2t = n\lambda$$

Que 3.11. **Derive the expression for diameter of the bright and dark ring of *n* order in Newton's ring.**

OR

Describe and explain the formation of Newton's rings in reflected monochromatic light. Prove that in reflected light diameters of the dark rings are proportional to the square root of natural numbers.

UPTU 2014-15, Marks 05

OR

What are Newton's rings ? Prove that in reflected light diameters of the bright rings are proportional to the square root of odd natural number.

UPTU 2009-10, Marks 05

OR

Find the expression of diameter of Newton's ring due to curve surfaces in case of reflected light.
UPTU 2012-13, Marks 05

Answer

A. **Newton's Rings :** Refer Q. 3.10, Page 91B, Unit-3.

B. **Diameter of Rings :**

1. Let R is radius of curvature of lens 'L' and 't' is thickness of air film at point 'P'.
2. From the geometrical properties of circle as shown in Fig. 3.11.1,

$$AP \times AB = AO \times AF$$
$$r \times r = t \times (2R - t)$$
$$r^2 = 2Rt - t^2$$

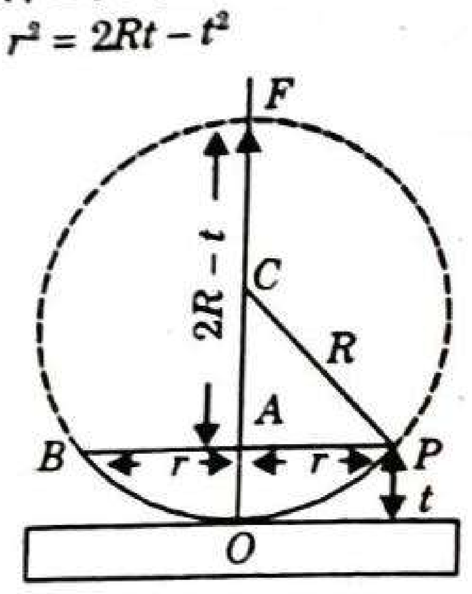

Fig. 3.11.1.

3. In actual, R is quite large and t is very small. So, t^2 is neglected.
4. Hence, $r^2 = 2Rt$

$$t = \frac{r^2}{2R} \quad \text{...(3.11.1)}$$

a. **For Bright Rings :**

1. Since we know that

$$2t = (2n-1)\frac{\lambda}{2}$$

2. On putting the value of t from equation (3.11.1)

$$2\frac{r^2}{2R} = (2n-1)\frac{\lambda}{2} \text{ or } r^2 = (2n-1)\frac{\lambda R}{2}$$

3. If radius of n^{th} bright ring is r_n,

Then, $$r_n^2 = \frac{(2n-1)\lambda R}{2} \quad \text{...(3.11.2)}$$

4. And D_n is diameter of n^{th} bright ring,

$$r_n^2 = \left(\frac{D_n}{2}\right)^2$$

5. Now, from equation (3.11.2),

$$\left(\frac{D_n}{2}\right)^2 = \frac{(2n-1)\lambda R}{2}$$
$$D_n^2 = 2(2n-1)\lambda R$$

or, $$D_n = \sqrt{2(2n-1)\lambda R}$$

or, $$K = \sqrt{2\lambda R}$$

6. Let $$D_n = K\sqrt{2n-1} \quad \text{(where, } n = 0, 1, 2, 3\text{...)}$$
$$D_n \propto \sqrt{2n-1}$$

7. Diameter of bright ring is proportional to the square root of odd natural numbers.

b. **For Dark Rings :**

1. Since, we know $2t = n\lambda$
2. The value of 't' from equation (3.11.1),

$$2\frac{r^2}{2R} = n\lambda \text{ or } r^2 = n\lambda R$$

3. If radius of n^{th} dark ring is r_n.
Then, $r_n^2 = n\lambda R$
and D_n is diameter of n^{th} dark ring,

$$r_n = \frac{D_n}{2}$$

4. $$\left(\frac{D_n}{2}\right)^2 = n\lambda R \text{ or } D_n^2 = 4n\lambda R$$
$$D_n = \sqrt{4n\lambda R}$$

5. Let $K = \sqrt{4\lambda R}$

Therefore, $D_n = K\sqrt{n}$ or $D_n \propto \sqrt{n}$

6. Diameter of dark ring is proportional to the square root of natural number.

Que 3.12. Explain the formation of Newton's ring ? If in a Newton's ring experiment, the air in the interspaces is replaced by a liquid of refractive index 1.33, in what proportion would the diameter of the rings changed ?
UPTU 2015-16, Marks 10

Answer

A. **Formation of Newton's Ring :** Refer Q. 3.10, Page 91B, Unit-3.

B. Numerical :

1. **Given :** $\mu = 1.33$ (refractive index of liquid)

2. Since, $$\frac{\text{diameter of a ring in liquid film}}{\text{diameter of the same ring in air film}} = \frac{1}{\sqrt{\mu}}$$
$$= \frac{1}{\sqrt{1.33}} = 0.867$$

3. So, the diameter of rings decreased by the portion of 0.867 of natural diameter.

Que 3.13. **Show that the diameter D_n of the n^{th} Newton's ring, when two surface of radius R_1 and R_2 are placed in contact is given by the relation : $\frac{1}{R_1} \pm \frac{1}{R_2} = \frac{4n\lambda}{D_n^2}$.**

Answer

A. Newton's Rings formed by two Curved Surfaces :

a. Case I :

1. When a planoconvex lens of radius of curvature R_1 is placed on the planoconcave lens of radius R_2.
2. Let at point A the thickness of air film is 't' and n^{th} dark ring is passing through A and its radius is r_n.
3. According to Fig. 3.13.1.

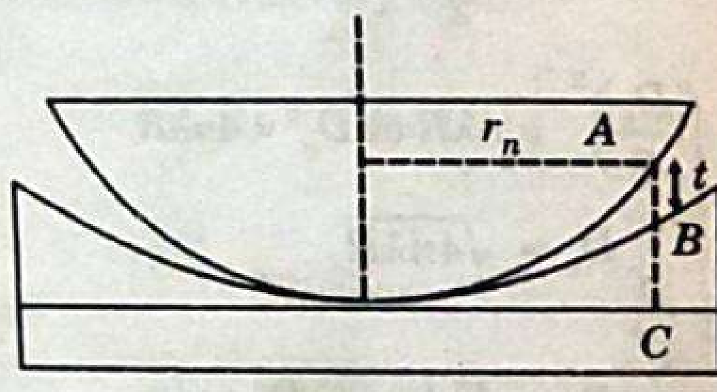

Fig. 3.13.1.

$$t = t_1 - t_2 \qquad (AC = t_1 \text{ and } BC = t_2)$$
$$= \frac{r_n^2}{2R_1} - \frac{r_n^2}{2R_2} \text{ or } t = \frac{r_n^2}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

or $$2t = r_n^2\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad ...(3.13.1)$$

4. For dark ring,
$$2\mu t = n\lambda$$
$$2t = n\lambda \qquad (\mu = 1 \text{ for air}) \qquad ...(3.13.2)$$

5. From equation (3.13.1) and equation (3.13.2),

$$r_n^2\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = n\lambda$$

or, $$\frac{1}{R_1} - \frac{1}{R_2} = \frac{n\lambda}{r_n^2}$$

6. But, $$D_n = 2r_n$$

$\therefore$ $$\frac{1}{R_1} - \frac{1}{R_2} = \frac{4n\lambda}{D_n^2}$$

b. Case II :

1. Let both the lenses are planoconvex and their curved surface is in contact.
2. Let 't' is thickness of air film at point A and R_1 and R_2 are radius of curvature of lenses respectively as shown in Fig 3.13.2.

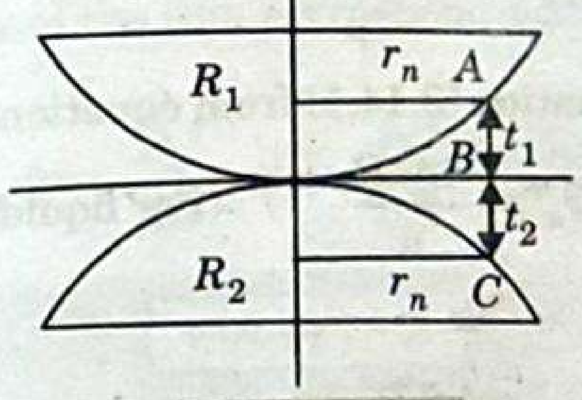

Fig. 3.13.2.

3. Since, $$t = t_1 + t_2$$
$$= \frac{r_n^2}{2R_1} + \frac{r_n^2}{2R_2} \Rightarrow \frac{r_n^2}{2}\left[\frac{1}{R_1} + \frac{1}{R_2}\right]$$

or $$2t = r_n^2\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

4. For dark ring, $$2t = n\lambda$$

$\therefore$ $$r_n^2\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = n\lambda$$
$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{n\lambda}{r_n^2},$$

5. But $r_n = D_n/2$
$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{4n\lambda}{D_n^2}$$

6. Hence $$\frac{1}{R_1} \pm \frac{1}{R_2} = \frac{4n\lambda}{D_n^2}$$

Que 3.14. **Describe how Newton's ring experiment can be used to determine the refractive index of a liquid ?**

UPTU 2011-12, Marks 05

Answer

1. The transparent liquid whose refractive index is to be determined is introduced between lens and glass plate.
2. Since diameter of n^{th} dark ring is given by

$$D_n^2 = \frac{4n\lambda R}{\mu} \quad ...(3.14.1)$$

3. Similarly, for $(n+p)^{th}$ dark ring,

$$D_{n+p}^2 = \frac{4(n+p)\lambda R}{\mu} \quad ...(3.14.2)$$

4. On subtracting equation (3.14.1) from equation (3.14.2),

$$D_{n+p}^2 - D_n^2 = \frac{4p\lambda R}{\mu} \quad \text{(For liquid)} \quad ...(3.14.3)$$

5. For air, $\mu = 1$

$$D_{n+p}^2 - D_n^2 = 4p\lambda R \quad ...(3.14.4)$$

6. On dividing equation (3.13.4) by equation (3.14.3),

$$\mu = \frac{(D_{n+p}^2 - D_n^2)_{air}}{(D_{n+p}^2 - D_n^2)_{liquid}}$$

Que 3.15. **Newton's rings are observed by keeping a spherical surface of 100 cm radius on a plane glass plate. If the diameter of the 15^{th} bright ring is 0.590 cm and the diameter of the 5^{th} ring is 0.336 cm, what is the wavelength of light used ?**

UPTU 2014-15, Marks 05

Answer

1. **Given :** $D_{15} = 0.590$ cm, $D_5 = 0.336$ cm
   $p = 15 - 5 = 10$ and $R = 100$ cm
2. If D_{n+p} and D_n be the diameter of $(n+p)^{th}$ and n^{th} bright ring, then

$$\lambda = \frac{D_{n+p}^2 - D_n^2}{4\,pR}$$

3. $\therefore$

$$\lambda = \frac{(0.590)^2 - (0.336)^2}{4 \times 10 \times 100} = 5.88 \times 10^{-5} \text{ cm} = 5880 \text{ Å}$$

Que 3.16. **Newton's rings are observed normally in reflected light of wavelength 6000 Å. The diameter of the 10^{th} dark ring is 0.50 cm. Find the radius of curvature of lens and thickness of the film.**

UPTU 2013-14, Marks 05

Answer

1. Given : $D_n = 0.50$ cm, $\lambda = 6000$ Å $= 6.0 \times 10^{-5}$ cm and $n = 10$
2. The diameter of n^{th} dark ring is given by

$$D_n^2 = 4n\lambda R \text{ or } R = \frac{D_n^2}{4n\lambda}$$

$\therefore$

$$R = \frac{0.50 \times 0.50}{4 \times 10 \times 6.0 \times 10^{-5}} = 104.17 \text{ cm}$$

3. If t is the thickness of the film corresponding to a ring of D_n diameter, then

$$2t = \frac{D_n^2}{4R} \text{ or } t = \frac{D_n^2}{8R} = \frac{0.50 \times 0.50}{8 \times 104.17} = 2.99 \times 10^{-4} \text{ cm}$$

PART-2

Single, double and N-slit diffraction, Diffraction grating, Grating spectra, Dispersive power, Rayleigh's criterion and Resolving power.

CONCEPT OUTLINE : PART-2

Diffraction : Diffraction of light is a phenomenon of bending of light and spreading out towards the geometrical shadow when passed through an obstruction.

Rayleigh's Criteria : The spectral lines of equal intensity are said to be resolved, if the position of the principal maxima of one spectral line coincide with first minima of the other spectral line.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.17. **What is meant by diffraction of light ? Write name of the two classes of diffraction and explain it.**

Answer

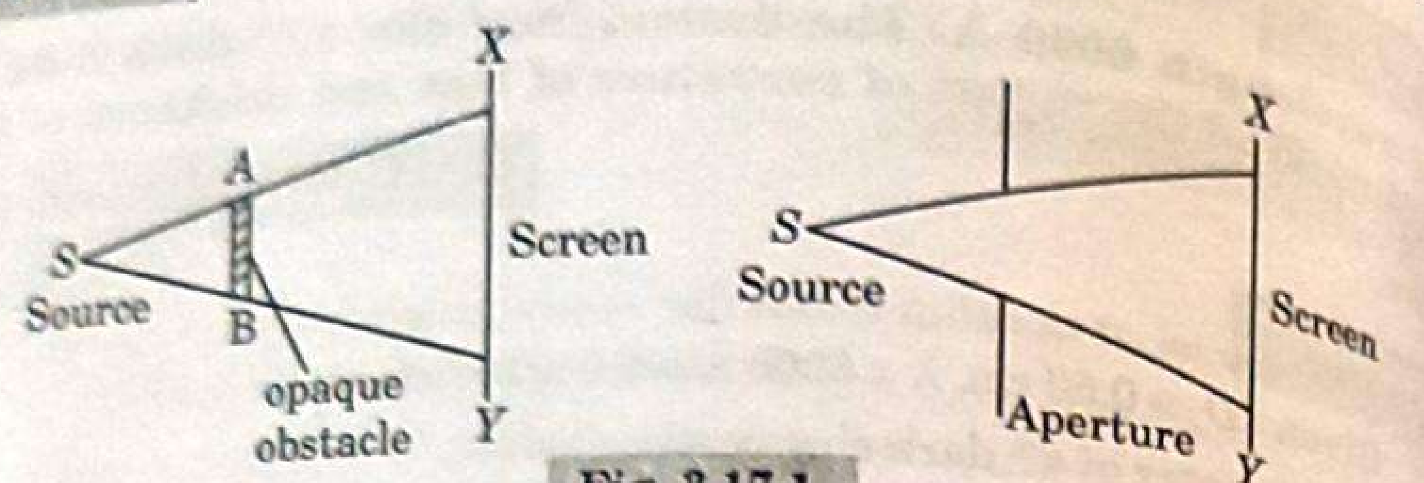

Fig. 3.17.1.

1. When light passes through small apertures or by the side of small obstacles it does not follow rectilinear path strictly, but bends at round corners of the obstacles.
2. The departure of light path from true rectilinear path or the bending of light around corners of an obstacle is called diffraction of light.
3. Diffraction phenomena can be divided into two types :

a. **Fresnel Diffraction :**

1. In Fresnel class of diffraction, the source of light and the screen, both are placed at finite distance from the diffraction element (obstacle or apertures) in which incident wavefront is either spherical or cylindrical and no lens are used.

b. **Fraunhofer Diffraction :**

1. In Fraunhofer diffraction, source of light and the screen both are placed at infinite distance from diffraction element in which incident wavefront is often plane.
2. Here convex lenses are used for focusing diffracted light.

Que 3.18. Describe Fraunhofer diffraction due to a single slit and show that relative intensities of successive maxima are nearly

$1 : \frac{1}{22} : \frac{1}{61} : \frac{1}{121}$ .....

OR

Discuss the phenomenon of Fraunhofer diffraction at a single slit and show that the relative intensities of successive maxima are nearly

$1 : \frac{4}{9\pi^2} : \frac{4}{25\pi^2} : \frac{4}{49\pi^2}$ ......

UPTU 2014-15, 2015-16; Marks 05

OR

Discuss Fraunhofer diffraction pattern due to a single slit. Find the expression for the width of central maxima.

UPTU 2012-13, Marks 05

OR

Explain the diffraction pattern obtained with diffraction at single slit. By what fraction the intensity of second maximum reduced from principal maximum ?

UPTU 2013-14, Marks 05

Answer

A. **Fraunhofer Diffraction due to a Single Slit :**

1. The light from a monochromatic source S is converted into parallel beam of light by convex lens L_1.
2. Now this beam is incident normally on a slit AB of width 'e'.
3. Now according to Huygens wave theory, every point in AB sends out secondary waves which are superimposed to give diffraction pattern on screen XY.
4. In this diffraction pattern, a central bright band is obtained because the rays from AB reach at C in same phase and here the intensity is maximum.
5. The rays which are directed through an angle θ are focused at point 'P'.
6. To find intensity at P let us draw normal AK.
7. Path difference of rays meeting at P is

$$BK = e \sin \theta$$

and phase difference $= \left(\frac{2\pi}{\lambda}\right) e \sin \theta$

8. Let AB be divided into large number of equal parts. The secondary waves originating from these parts will be of equal amplitude 'a' (say).
9. Then phase difference between two successive waves will be

$$\delta = \frac{1}{n}\left(\frac{2\pi}{\lambda}\right) e \sin \theta$$

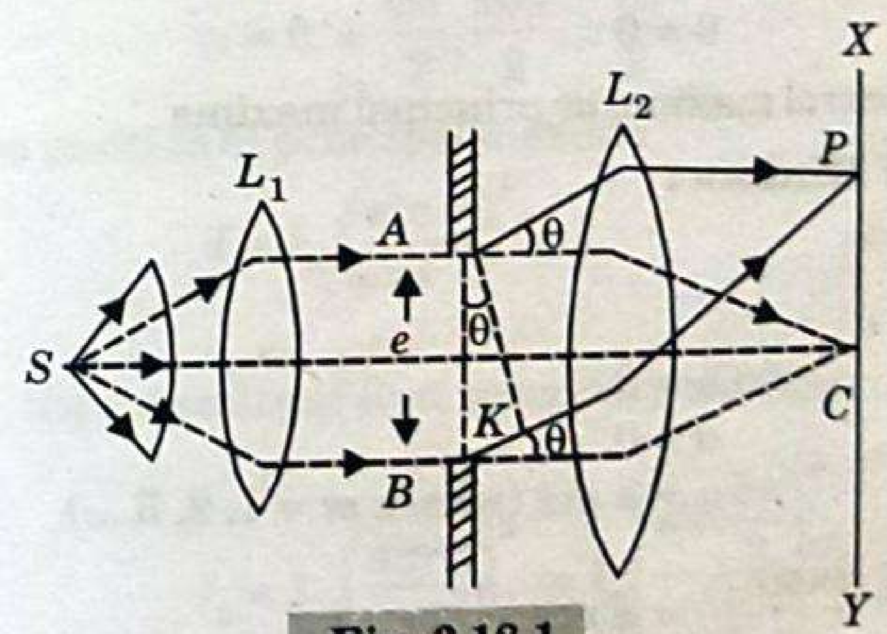

Fig. 3.18.1.

10. Now, according to n simple Harmonic waves,

$$R = \frac{a\sin\frac{n\delta}{2}}{\sin\delta/2} = \frac{a\sin\left(\frac{\pi e\sin\theta}{\lambda}\right)}{\sin\left(\frac{\pi e\sin\theta}{n}\right)}$$

11. Let $n\delta = 2\alpha = \frac{2\pi}{\lambda}t$

and $\alpha = \frac{\pi}{\lambda}e\sin\theta$

12. Then $R = \frac{a\sin\alpha}{\sin\left(\frac{\alpha}{n}\right)}$ $\left(\frac{\alpha}{n}\text{ is very small. So, }\sin\frac{\alpha}{n} = \frac{\alpha}{n}\right)$

or, $R = na\frac{\sin\alpha}{\alpha} \Rightarrow R = \frac{A_o\sin\alpha}{\alpha}$ $[na = A_o]$

13. Intensity at point '*P*'

$$I = R^2 = \frac{A_o^2\sin^2\alpha}{\alpha^2} \quad ...(3.18.1)$$

a. Position of Maxima :

1. When $\frac{\sin\alpha}{\alpha} = 1$ and when $\alpha \to 0$

$$\lim_{\alpha\to 0}\frac{\sin\alpha}{\alpha} = \lim_{\alpha\to 0}\frac{1}{\alpha}\left(\alpha - \frac{\alpha^3}{\lfloor 3} + \frac{\alpha^5}{\lfloor 5} - ...\right) = 1$$

2. $I = I_o(1)^2 \Rightarrow I = I_o$

3. And $\alpha = \frac{\pi}{\lambda}e\sin\theta$

$$\frac{\pi}{\lambda}e\sin\theta = 0 \Rightarrow \sin\theta = 0$$

$$\theta = 0$$

4. Point *C* is central maxima or principal maxima.

b. Position of Minima :

1. If $\frac{\sin\alpha}{\alpha} = 0$

$\Rightarrow$ $\sin\alpha = 0$

$\alpha \neq 0$

$\alpha = \pm m\pi$ (where $m = 1, 2, 3....$)

2. Hence, $\frac{e\pi\sin\theta}{\lambda} = \pm m\pi$

$e\sin\theta = \pm(m\lambda)$ [where $m = 1, 2, 3.....$] ...(3.18.2)

3. Equation (3.18.2) gives the direction of first, second, third .... minima and this equation is called diffraction equation.

c. Secondary Maxima :

1. The condition of secondary maxima may be obtained by differentiating equation (3.18.1) w.r.t. α and equating it to zero.

$$\frac{dI}{d\alpha} = \frac{d}{d\alpha}\left[A_o^2\left(\frac{\sin\alpha}{\alpha}\right)^2\right] = A_o^2 2\frac{\sin\alpha}{\alpha}\left[\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2}\right]$$

2. Either $\frac{\sin\alpha}{\alpha} = 0 \Rightarrow \sin\alpha = 0$

or $\alpha\cos\alpha - \sin\alpha = 0$

$\alpha = \tan\alpha$

3. $\sin\alpha = 0$, gives position of principal minima and position of secondary maxima is given by

$$\alpha = \tan\alpha \quad ...(3.18.3)$$

4. Equation (3.18.3) can be solved graphically by plotting the curves $y = \alpha$ and $y = \tan\alpha$

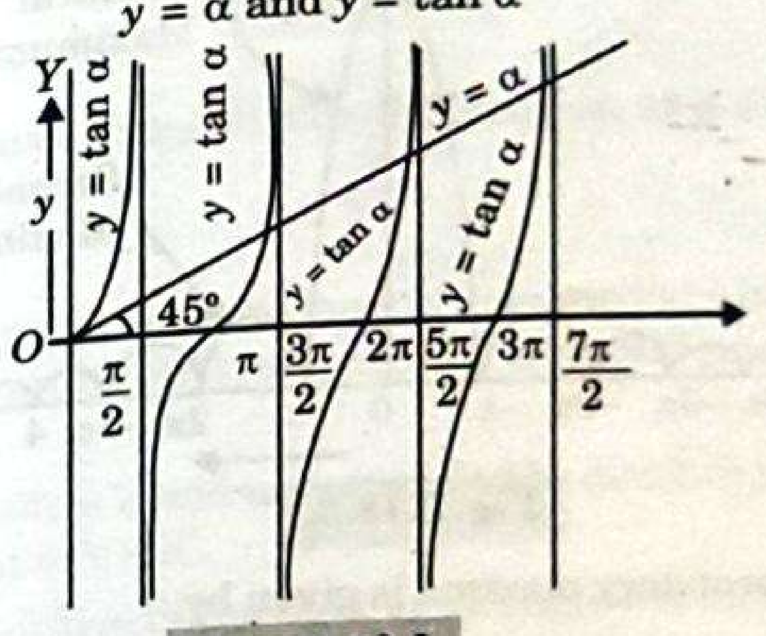

Fig. 3.18.2.

5. According to curves, the point of intersection of these two curves gives the value of α satisfying the equation $\alpha = \tan\alpha$. These points correspond to the value of

$$\alpha = 0, \pm\frac{3\pi}{2}, \frac{\pm 5\pi}{2}, \pm\frac{7\pi}{2}....$$

6. At $\alpha = 0$, the position of principal maxima

$$I = I_o\left(\frac{\sin\alpha}{\alpha}\right)^2 = I_o$$

7. At $\alpha = \frac{3\pi}{2}$, the intensity of first secondary maxima

$$I_1 = I_o\left(\frac{\sin\alpha}{\alpha}\right)^2 \Rightarrow I_o\left(\frac{\sin\frac{3\pi}{2}}{\frac{3\pi}{2}}\right)^2 = \frac{4I_o}{9\pi^2}$$

8. At $\alpha = \frac{5\pi}{2}$, the intensity of second secondary maxima,

$$I_2 = I_o\left(\frac{\sin\frac{5\pi}{2}}{\frac{5\pi}{2}}\right)^2 = \frac{4I_o}{25\pi^2}$$

9. At $\alpha = \frac{7\pi}{2}$, $\quad I_3 = \frac{4I_o}{49\pi^2}$

10. The ratio of relative intensity $1:\frac{4}{9\pi^2}:\frac{4}{25\pi^2}:\frac{4}{49\pi^2}:.....$

or, $1:\frac{1}{22}:\frac{1}{61}:\frac{1}{121}:.....$

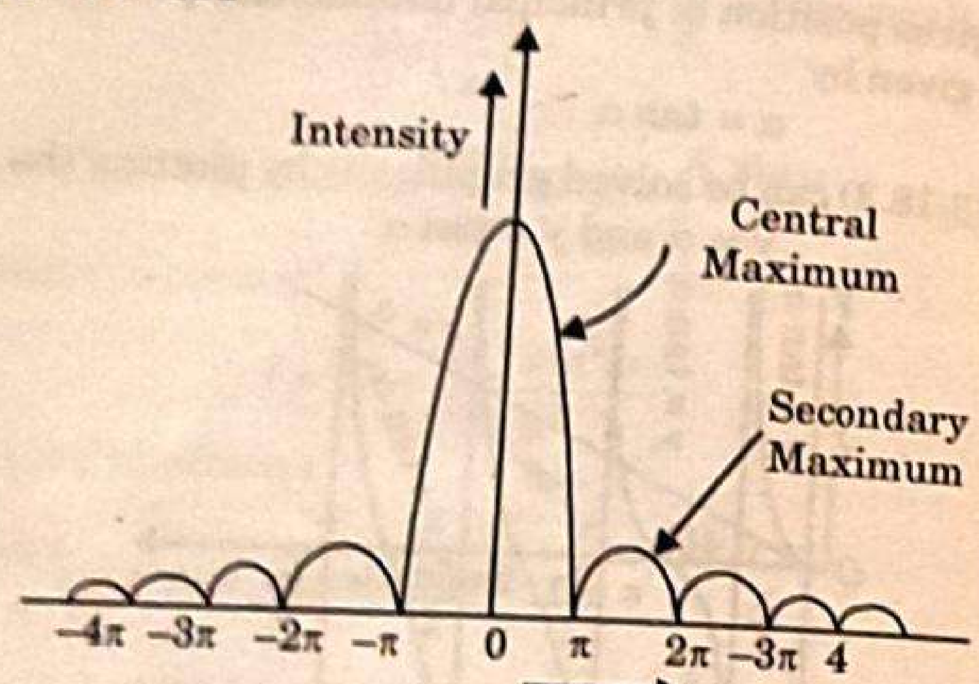

Fig. 3.18.3.

11. Direction of secondary maxima is given by :

$$e\sin\theta = \pm(2m+1)\frac{\lambda}{2}$$

12. But $\quad \alpha = \frac{\pi e}{\lambda}\sin\theta \quad$ or $\quad \alpha = \frac{\pi}{\lambda}.(2m+1)\frac{\lambda}{2}$

$$\alpha = (2m+1)\frac{\pi}{2} \qquad m = 1, 2, 3, 4.....$$

13. Hence, $\quad \alpha = \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}$

Que 3.19. **A light of wavelength 6000Å falls normally on a straight slit of width 0.10 mm. Calculate the total angular width of the central maxima and also the linear width as observed on a screen placed 1 meter away.**

Answer

1. **Given :** $\lambda = 6.0 \times 10^{-5}$ cm, $a = 0.01$ cm
2. Since, $a\sin\theta = n\lambda$

3. For $n = 1$ and for small θ,

$$\theta = \frac{\lambda}{a} = \frac{6\times10^{-5}}{0.01} = 6\times10^{-3} \text{ rad.}$$

4. Total angular width, $2\theta = 1.2 \times 10^{-2}$ rad or $0.688°$
5. Linear half width $= \theta.D = 6 \times 10^{-3} \times 100 = 0.6$ cm
6. Total linear width $= 2\theta.D = 1.2$ cm

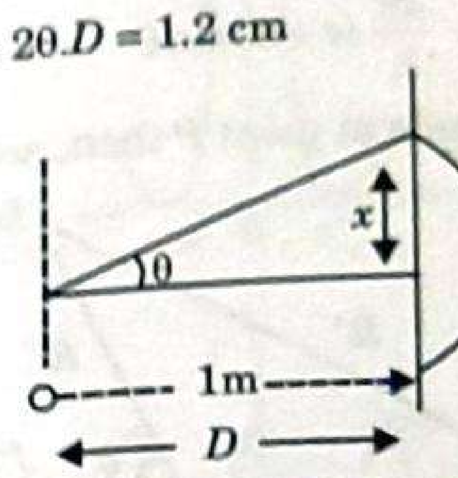

Fig. 3.19.1.

Que 3.20. **Discuss Fraunhofer diffraction at a double slit.**

Answer

1. Consider a parallel beam of monochromatic light having wavelength λ incident normally on two parallel slits AB and CD as shown in Fig. 3.20.1.
2. Width of each slit is 'e' and are separated by distance 'd'. Distance between S_1 and S_2 point is '$e + d$'.
3. Now each slit diffracts the light at an angle θ to incident direction.

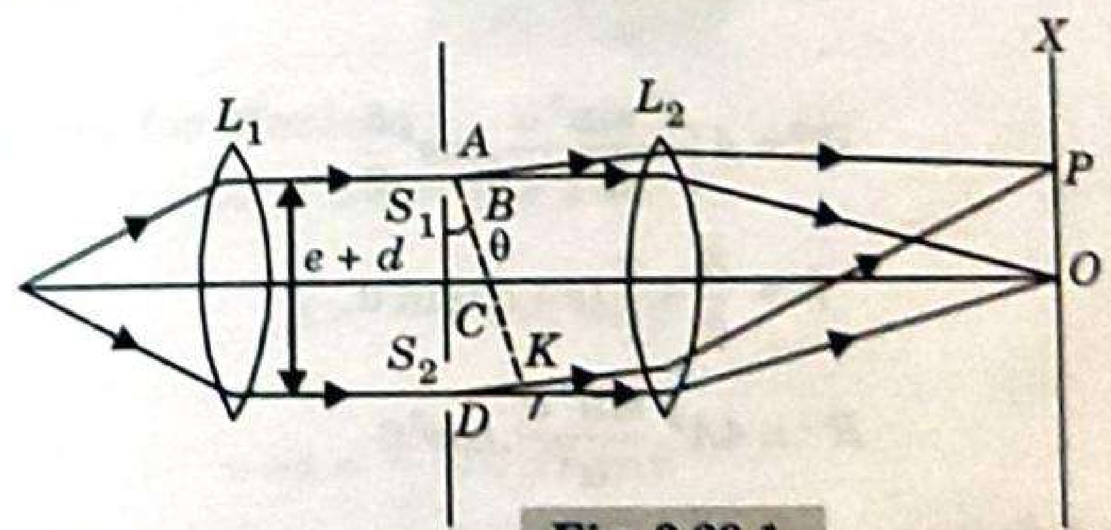

Fig. 3.20.1.

5. From the theory of diffraction due to single slit we know that, resultant amplitude is

$$R = A\frac{\sin\alpha}{\alpha} \text{ and } \alpha = \frac{\pi}{\lambda}e\sin\theta$$

6. Let S_1 and S_2 are two coherent sources, each sending wavelets of amplitude $A\frac{\sin\alpha}{\alpha}$ in the direction of θ.

7. Therefore, the resultant amplitude due to interference of these two waves at point P can be calculated as :

a. Draw perpendiculars S_1K on S_2K.

b. Path difference, $S_2K = (e + d) \sin \theta$ and phase difference,

$$\delta = \frac{2\pi}{\lambda}.(e+d)\sin\theta \quad ...(3.20.1)$$

c. If R' is resultant amplitude at point P then, according to Fig. 3.20.2. ...(3.21.2)

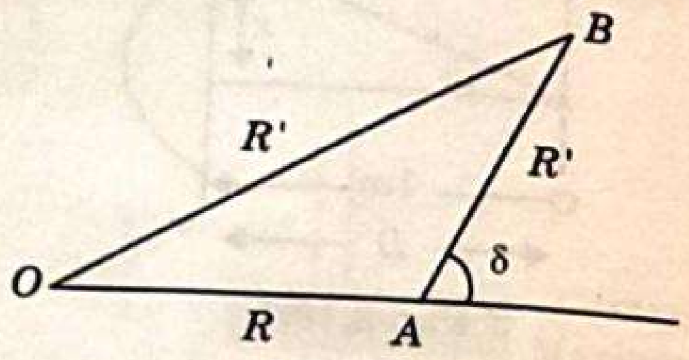

Fig. 3.20.2.

$$OB^2 = OA^2 + AB^2 + 2AB.OA \cos \delta$$
$$R'^2 = R^2 + R^2 + 2RR \cos \delta$$
$$= 2R^2 + 2R^2 \cos \delta$$
$$R'^2 = 2R^2(1 + \cos \delta) = 2R^2 \left(2\cos^2\frac{\delta}{2}\right)$$
$$R'^2 = 4R^2 \cos^2 \frac{\delta}{2}$$

d. But $$R = A \frac{\sin\alpha}{\alpha}$$

e. Then $$R'^2 = 4\frac{A^2\sin^2\alpha}{\alpha^2}.\cos^2\frac{\delta}{2}$$

f. Let $$\beta = \frac{\delta}{2} = \frac{\pi}{\lambda}(e+d)\sin\theta$$

then, $$R'^2 = 4A^2\frac{\sin^2\alpha}{\alpha^2}.\cos^2\beta$$

8. Now by definition

$$I = R'^2 = \frac{4A^2\sin^2\alpha.\cos^2\beta}{\alpha^2}$$

9. Hence, the resultant intensity depends upon following two factors :

a. $\frac{\sin^2\alpha}{\alpha^2}$ due to diffraction, and

b. $\cos^2\beta$ due to interference.

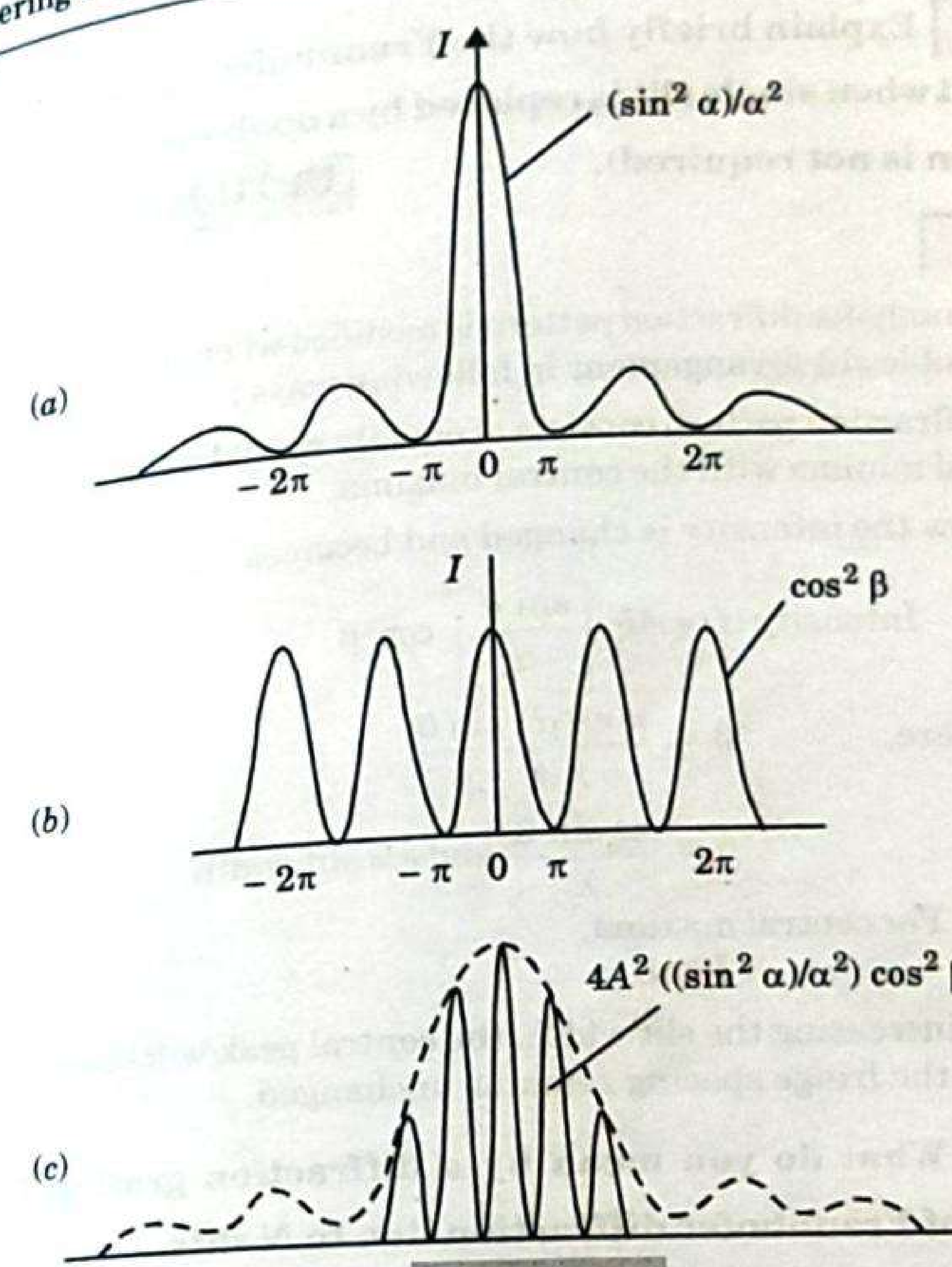

Fig. 3.20.3.

A. **Conditions for Maxima :**

1. If $\cos^2 \beta = 1$ or $\beta = \pm n\pi$ $\quad n = 0, 1, 2, 3,...$

2. But $$\beta = \frac{\pi}{\lambda}(e+d)\sin\theta$$

3. $$\pm n\pi = \frac{\pi}{\lambda}(e+d)\sin\theta$$
$$(e + d) \sin \theta = \pm n\lambda \quad n = 0, 1, 2, 3,....$$

B. **Conditions for Minima :**

1. If $\cos^2 \beta = 0$ or $\cos \beta = 0$
$$\beta = \pm (2n + 1)\frac{\pi}{2} \quad n = 0, 1, 2, 3,...$$

2. Then, $(e + d) \sin \theta = \pm (2n + 1)\frac{\lambda}{2}$

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Que 3.21. **Explain briefly how the Fraunhofer diffraction pattern is modified when single slit is replaced by a double slit arrangement? (Derivation is not required).**

UPTU 2010-11, Marks 05

Answer

1. The Fraunhofer diffraction pattern is modified when single slit is replaced by a double slit arrangement in following ways :
   a. Diffraction pattern consists of equally spaced interference maxima and minima with the central maxima.
   b. Now the intensity is changed and becomes

$$\text{Intensity } (I) = 4I_o \left[\frac{\sin \alpha}{\alpha}\right]^2 \cos^2 \beta$$

where,
$$\beta = \frac{\pi(e+d)\sin\theta}{\lambda}$$
$$\alpha = \frac{\pi e \sin\theta}{\lambda} \text{ and } e = \text{slit width}$$

$\therefore$ For central maxima,
$$I = 4I_o$$

   c. On increasing the slit width, the central peak will become sharper, but the fringe spacing remains unchanged.

Que 3.22. **What do you mean by a diffraction grating ? Derive expression of Fraunhofer diffraction due to N slits.**

OR

Give the theory of plane transmission grating and show how would you use it to determine the wavelength of light ?

UPTU 2009-10, Marks 05

Answer

1. The diffraction grating consists of a large number (N) of parallel slits having equal width and separated by an equal opaque space.
2. It is constructed by rolling a large number of parallel and equidistant lines on a glass plate with a diamond point.

A. Explanation :

1. Let a parallel beam of monochromatic light of wavelength 'λ' be incident on 'N' slits.
2. This light diffracted at an angle θ is focused at point P on the screen by lens L_2 having same amplitude.

$$R = A\,\frac{\sin\alpha}{\alpha}$$

3. Let e be the width of each slit and 'd' be the opaque space between two slits, then $(e + d)$ is called grating element.
4. The path difference is $(e + d)\sin\theta$ and phase difference is

$$2\beta = \frac{2\pi}{\lambda}(e+d)\sin\theta.$$

5. Therefore, as we pass from one vibration to another, the path goes on increasing by an amount $(e + d)\sin\theta$ and phase goes on increasing by an amount $\frac{2\pi}{\lambda}(e+d)\sin\theta$. Thus, phase increases in arithmetic progression.
6. Now, the resultant amplitude and intensity at point due to N slits can be obtained by vector polygon method.

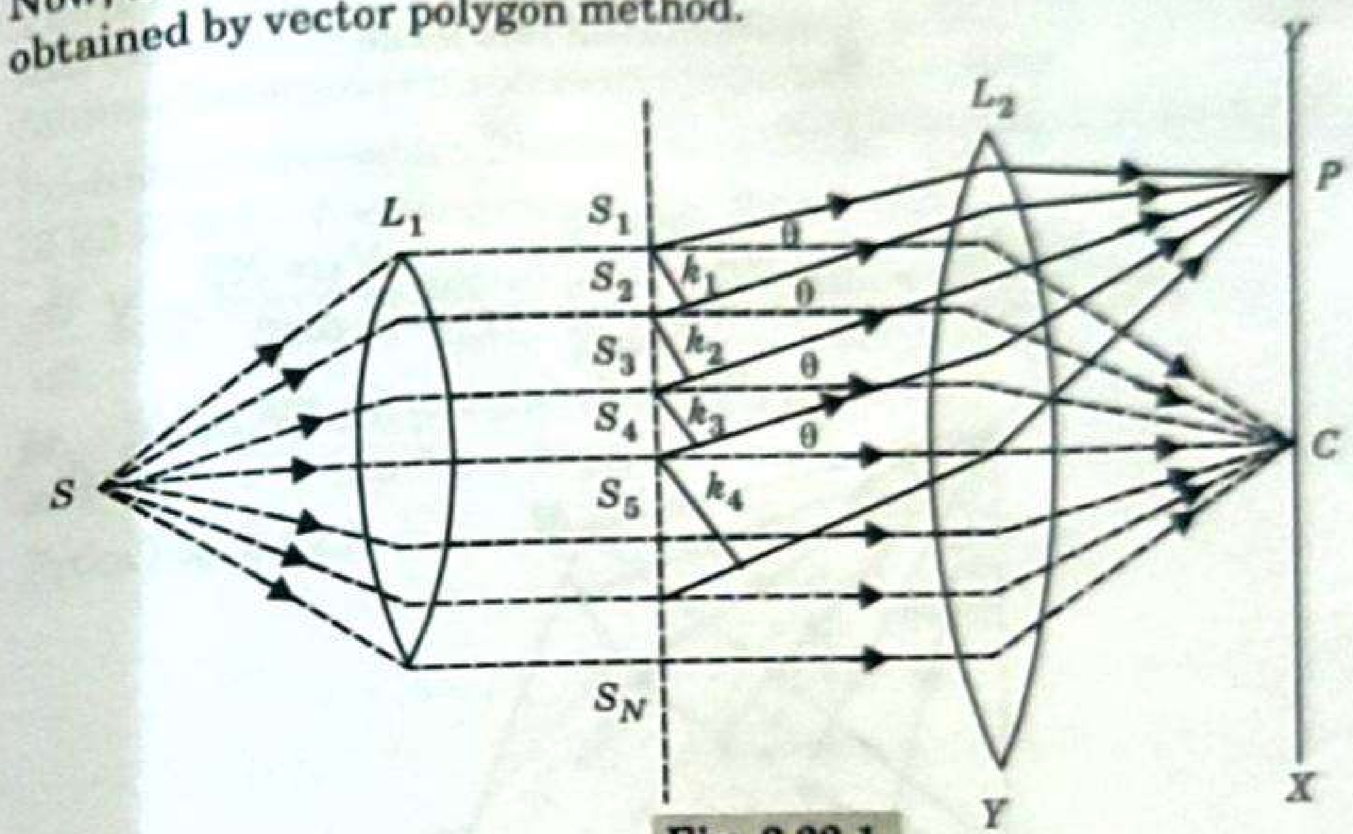


Fig. 3.22.1.

where,
$$R' = R\,\frac{\sin\frac{N\delta}{2}}{\sin\frac{\delta}{2}}$$

7. But
$$\delta = 2\beta$$

8. Hence,
$$R' = \frac{R\sin\frac{2N\beta}{2}}{\sin\frac{2\beta}{2}}$$

or,
$$R' = R\,\frac{\sin N\beta}{\sin\beta} \quad \text{...(3.22.1)}$$

where,
$$R = A\,\frac{\sin\alpha}{\alpha} \text{ (due to single slit)}$$

9. Intensity,
$$I = R'^2 = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2 \cdot \left(\frac{\sin N\beta}{\sin\beta}\right)^2 \quad \text{...(3.22.2)}$$

10. Hence, intensity distributed is product of two terms I[st] term $A_o^2\left(\frac{\sin\alpha}{\alpha}\right)^2$ represents diffraction pattern due to single slit and II[nd] term $\left(\frac{\sin N\beta}{\sin\beta}\right)^2$ represents interference pattern due to N slits.

B. Principal Maxima :

1. When $\sin\beta = 0$
   For $\quad \beta = \pm n\pi \qquad n = 0, 1, 2, 3...$
2. Then, $\quad \sin N\beta = 0$
   Hence, $\quad \frac{\sin N\beta}{\sin\beta} = \frac{0}{0}$ is indeterminate form.
3. It is solved by L-Hospital rule.

$$\lim_{\beta\to\pm n\pi}\frac{\sin N\beta}{\sin\beta} = \lim_{\beta\to\pm n\pi}\frac{\frac{d}{d\beta}(\sin N\beta)}{\frac{d}{d\beta}(\sin\beta)} = \lim_{\beta\to\pm n\pi}\frac{N\cos N\beta}{\cos\beta} = N$$

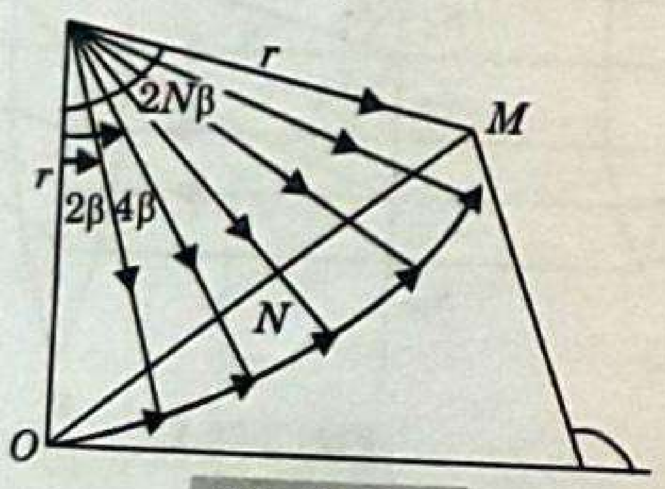


Fig. 3.22.2.

$$I' = A^2\,\frac{\sin^2\alpha}{\alpha^2}.N^2 \qquad ...(3.22.3)$$

4. Putting the value of $\frac{\sin N\beta}{\sin\beta} = N$ in equation (3.22.2), we get

a. Condition for Principal Maxima :

1. $$\sin\beta = 0$$
   $$\beta = \pm n\pi$$
2. But $$\beta = \frac{\pi}{\lambda}(e+d)\sin\theta$$
   So, $$\pm n\pi = \frac{\pi}{\lambda}(e+d)\sin\theta \Rightarrow (e+d)\sin\theta = \pm n\lambda \qquad ...(3.22.4)$$
3. For $n = 0$ we get $\theta = 0$ and get zero order principal maxima. $n = 1, 2, 3, \ldots\ldots\ldots$ correspond to I[st], II[nd], III[rd] order principal maxima.

b. Minima :

1. $\sin N\beta = 0.$
2. But $\sin\beta \neq 0$
   $$N\beta = \pm m\pi \text{ or } \beta = \pm\frac{m\pi}{N}$$
3. $$\text{or,}\quad \frac{N\pi}{\lambda}(e+d)\sin\theta = \pm m\pi \text{ or } N(e+d)\sin\theta = \pm m\lambda \qquad ...(3.22.5)$$
   where m can take all integral values except $0, N, 2N, 3N \ldots\ldots nN$; $m = 0$ gives maxima and $m = 1, 2, 3, \ldots\ldots (N-1)$ give maxima.

C. Secondary Maxima :

1. There are $(N-1)$ minima between two consecutive principal maxima's therefore, there are $(N-2)$ other maxima coming alternatively with the minima between two successive principal maxima.
2. Position of secondary maxima is obtained by differentiating equation (3.22.2) w.r.t. β and equating it to zero.
   $$\frac{dI}{d\beta} = A^2\frac{\sin^2\alpha}{\alpha^2}.2\left[\frac{\sin N\beta}{\sin\beta}\right]\left[\frac{N\cos N\beta\sin\beta - \sin N\beta\cos\beta}{\sin^2\beta}\right] = 0$$
   or, $\quad N\cos N\beta\sin\beta - \sin N\beta\cos\beta = 0 \qquad ...(3.22.6)$
   and $\quad \tan N\beta = N\tan\beta$
3. From Fig. 3.22.3,

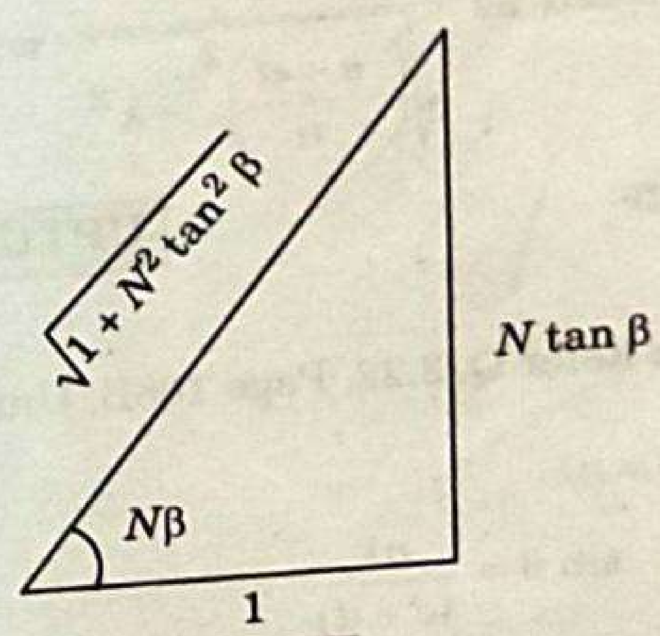


Fig. 3.22.3.

$$\sin N\beta = \frac{N\tan\beta}{\sqrt{1+N^2\tan^2\beta}}$$

4. Squaring both sides and dividing by $\sin^2\beta$
   $$\frac{\sin^2 N\beta}{\sin^2\beta} = \frac{N^2\tan^2\beta}{[(1+N^2\tan^2\beta)\sin^2\beta]} = \frac{N^2}{[1+(N^2-1).\sin^2\beta]}$$
5. Putting the value of $\frac{\sin^2 N\beta}{\sin^2\beta}$ in equation (3.22.2),

$$I' = A_o^2 \left(\frac{\sin\alpha}{\alpha}\right)^2 \frac{N^2}{[1+(N^2-1)\sin^2\beta]} \quad ...(3.22.7)$$

6. $$\frac{\text{Intensity of secondary maxima}}{\text{Intensity of principal maxima}} = \frac{I'}{I} = \frac{1}{[1+(N^2-1)\sin^2\beta]}$$

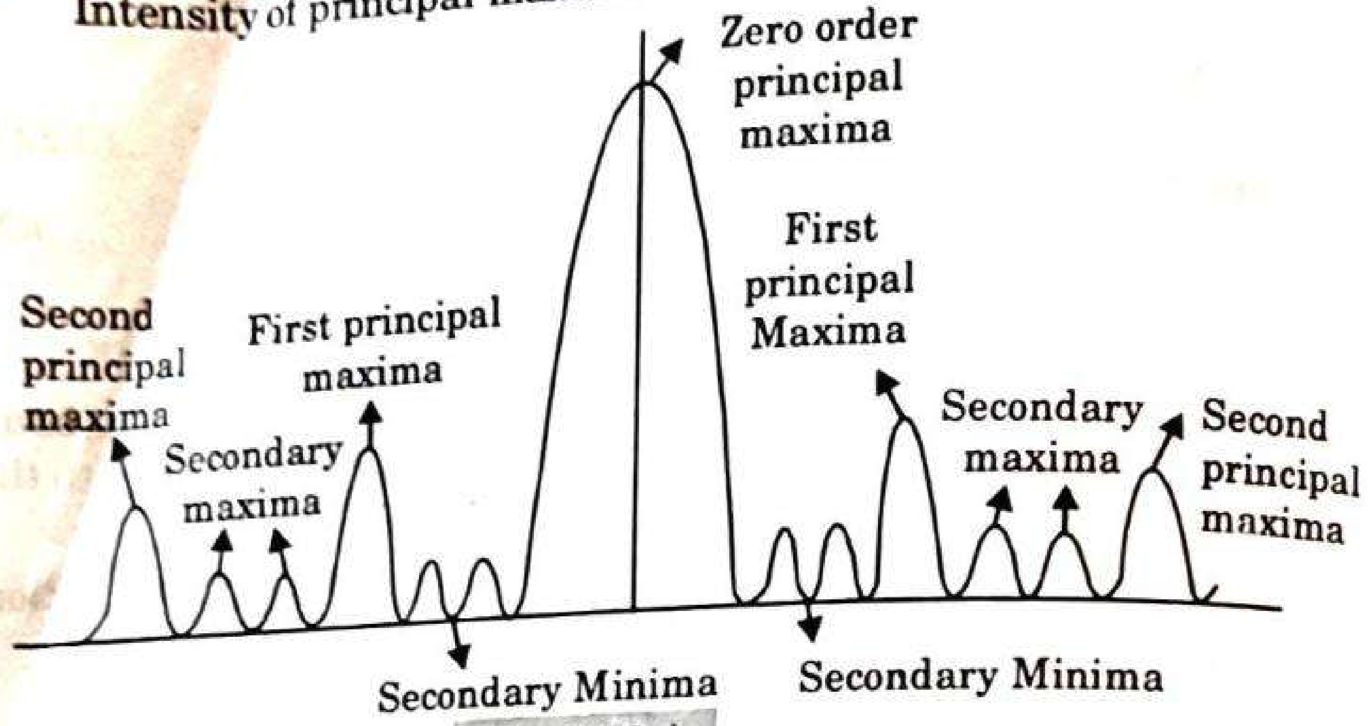


Fig. 3.22.4.

Que 3.23. **What is diffraction grating ? Show that its dispersive power can be expressed as $\dfrac{1}{\sqrt{\left(\dfrac{e+d}{n}\right)^2 - \lambda^2}}$ where all terms have their usual meanings.** **UPTU 2013-14, Marks 05**

Answer

Diffraction Grating : Refer Q. 3.22, Page 108B, Unit-3.

Proof :

1. Since, $(e + d) \sin\theta = n\lambda$

$$\sin\theta = \frac{n\lambda}{(e+d)} \quad ...(3.23.1)$$

2. Differentiate with respect to λ, we get

$$(e+d)\cos\theta \frac{d\theta}{d\lambda} = n$$

$$d\theta = \frac{n d\lambda}{(e+d)\cos\theta} = \frac{n d\lambda}{(e+d)\sqrt{1-\sin^2\theta}} \quad ...(3.23.2)$$

3. Putting the valve of $\sin\theta$ in equation (3.23.2)

$$d\theta = \frac{n d\lambda}{(e+d)\sqrt{1-\dfrac{n^2\lambda^2}{(e+d)^2}}} = \frac{n d\lambda}{\sqrt{(e+d)^2 - n^2\lambda^2}} = \frac{d\lambda}{\sqrt{\left(\dfrac{e+d}{n}\right)^2 - \lambda^2}}$$

Que 3.24. **What do you understand by missing order spectrum ? What particular spectra would be absent if the width of transparencies and opacities of the grating are equal ? Show that only first order spectra is possible if the width of the grating element is more than wavelength of light and less than twice the wavelength of light.**

OR

What do you understand by missing orders ? Which order will be missing if opacities are twice the transparencies ?

UPTU 2012-13, Marks 05

Answer

1. Sometime for a particular angle of diffraction 'θ' satisfying the relation $(e + d) \sin\theta = n\lambda$, there is no visible spectrum obtained. This phenomenon is known as missing order spectrum.
2. We know that condition for a minima in a single slit is given by
$$e \sin\theta = m\lambda \quad ...(3.24.1)$$
3. And the condition for the principal maxima in the n^{th} order spectrum is given by
$$(e + d) \sin\theta = n\lambda \quad ...(3.24.2)$$
4. If both conditions are simultaneously satisfied, the diffracted rays from all transparencies are superimposed upon each other but the resultant intensity is zero, *i.e.* the spectrum is absent.
5. From equation (3.24.1) and equation (3.24.2), we have
$$\frac{e+d}{e} = \frac{n}{m}$$
6. If $e = d$ then, $n = 2m$
So that 2^{nd}, 4^{th}, 6^{th}, ... order of the spectra will be missing corresponding to $m = 1, 2, 3, ...$
7. The maximum number of spectra available with a diffraction grating in the visible region can be evaluated by using the grating equation for normal incidence as
$$(e + d)\sin\theta_n = n\lambda \text{ or } n = \frac{(e+d)\sin\theta_n}{\lambda}$$
8. The maximum possible value is 90^0.
$$n_{max} = \frac{(e+d)\sin 90}{\lambda} = \frac{(e+d)}{\lambda}$$
9. If the grating element $(e + d)$ lies between λ and 2λ or grating element
$$(e + d) < 2\lambda \text{ then } n_{max} < \frac{2\lambda}{\lambda} < 2.$$

10. Therefore, for normal incidence only first order will be obtained.
11. Hence if the width of grating element is less than twice the wavelength of light, then only first order is possible.

Que 3.25. **What are the conditions of absent spectra in the grating ?**

OR

Derive the condition of missing spectra.

UPTU 2012-13, Marks 05

Answer

1. The direction of principal maxima in diffraction grating is

$$(e + d) \sin \theta = \pm n\lambda \quad (n = 0, 1, 2, ...) \quad ...(3.25.1)$$

n = order of maxima

2. Direction of minima due to single slit is

$$e \sin \theta = m\lambda \quad m = 1, 2, 3, ... \quad ...(3.25.2)$$

3. The resultant intensity in grating is given by

$$I = A^2 \frac{\sin^2 \alpha \sin^2 N\beta}{\alpha^2 \sin^2 \beta} \text{ when } \alpha = \frac{\pi}{\lambda} e \sin\theta$$

$$\beta = \frac{\pi}{\lambda}(e+d)\sin\theta$$

4. In any direction $\sin^2 N\beta/\sin^2 \beta$ is maxima and $\frac{A^2 \sin^2 \alpha}{\alpha^2}$ is zero. The principal maxima will not be present in this direction.
5. Thus, equation (3.25.1) and equation (3.25.2) are simultaneously satisfied for some value of e and θ. Here particular maxima order is absent in grating pattern.
6. Now dividing equation (3.25.1) by equation (3.25.2), we get

$$\frac{e+d}{e} = \frac{n}{m}$$; This is condition for absent spectra.

7. When $d = e$ then $n = 2m$

$m = 1, 2, 3.....$ $n = 2, 4, 6, ...$

Hence 2nd, 4th, 6th, ... spectra will be absent.

8. When $d = 2e$ then $n = 3m$

$(m = 1, 2, 3,...)$

$n = 3, 6, 9, ...$

Hence 3rd, 6th, 9th,....... spectra will be absent.

Que 3.26. **What is dispersive power of grating and resolving power of an optical instrument ? Explain Rayleigh's criterion of resolution.**

OR

What do you understand by the resolving power of an optical instrument ? Explain the Rayleigh criterion of resolution.

UPTU 2011-12, Marks 05

OR

What is diffraction grating ? Derive an expression for dispersive power of grating and explain it.

UPTU 2011-12, Marks 05

Answer

A. **Diffraction Grating :** Refer Q. 3.22, Page 108B, Unit-3.

B. **Dispersive Power of a Diffraction Grating :**

1. The dispersive power of a diffraction grating is defined as the rate of change of the diffraction angle with the wavelength.
2. It is expressed as $\frac{d\theta}{d\lambda}$.
3. For a grating, $(e + d) \sin \theta = n\lambda$

Differentiating w.r.t. λ, we get

$$(e + d) \cos \theta. \frac{d\theta}{d\lambda} = n$$

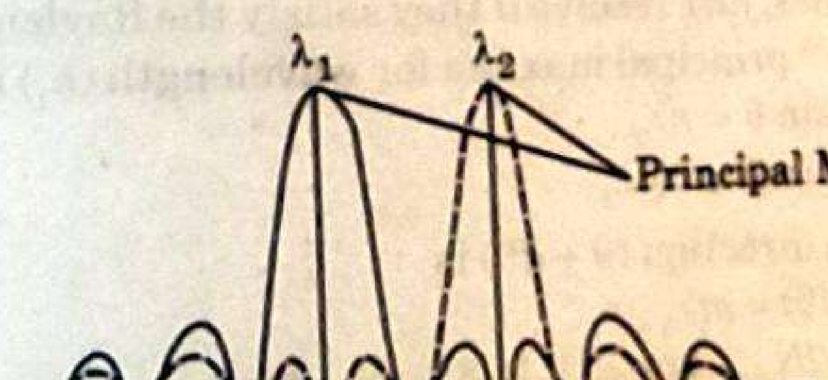
or, $$\frac{d\theta}{d\lambda} = \frac{n}{(e+d)\cos\theta}$$

C. **Resolving Power of an Optical Instrument :**

1. The ability of an optical instrument to separate images of two objects placed very close to each other is known as resolving power.

D. **Rayleigh's Criteria of Resolution :**

1. According to Rayleigh's criterion, the spectral lines of equal intensity are said to be resolved, if the position of the principal maxima of one spectral line coincide with first minima of the other spectral line.



(a) Easily Resolvable

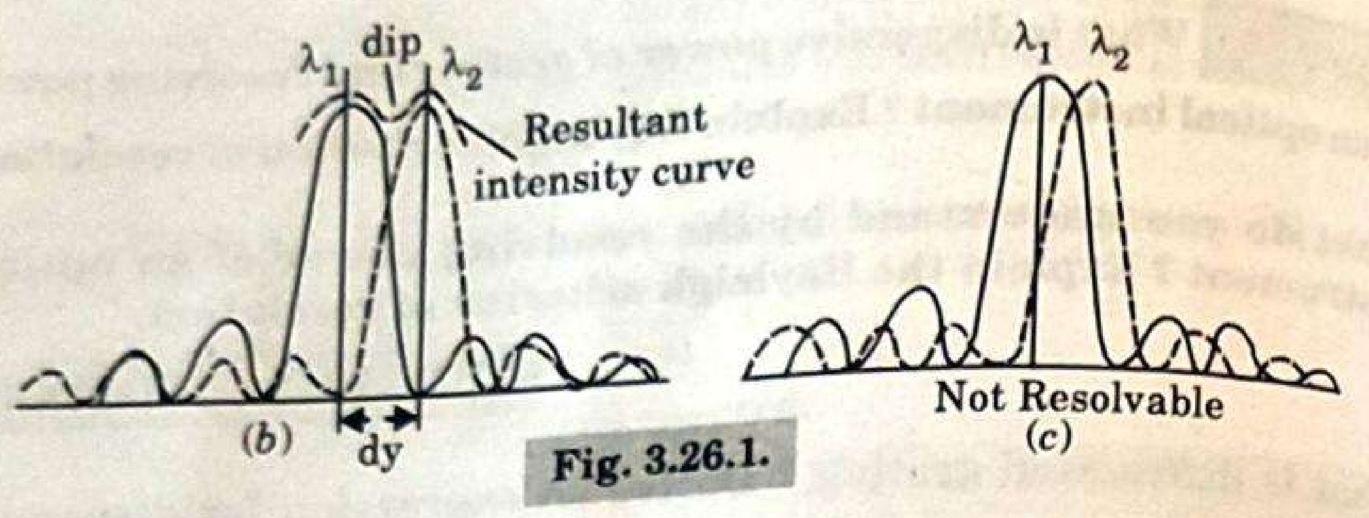


Fig. 3.26.1.

Que 3.27. **What do you mean by resolving power of grating? Derive the necessary expression.**

Answer

A. Resolving Power of Grating :

1. It is defined as the ratio of wavelength (λ), of any spectral line to the smallest difference of two wavelengths ($d\lambda$), for which the spectral lines can be resolved at the wavelength λ.

i.e., Resolving power of grating $= \dfrac{\lambda}{d\lambda}$

B. Expression :

1. Let a light consisting of two wavelengths λ_1 and λ_2 is incident normally on a grating element $(e + d)$ and the spectral lines corresponding to λ_1 and λ_2 are formed on screen P_1 and P_2.

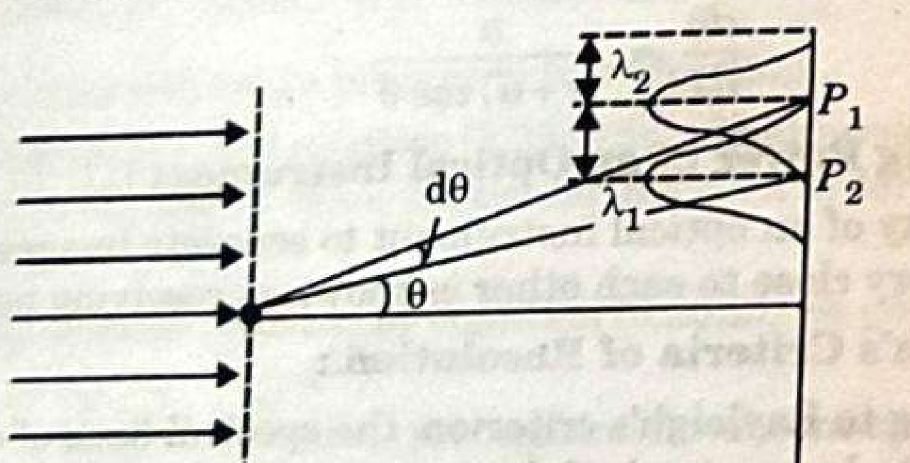


Fig. 3.27.1.

2. These spectral lines, just resolve if they satisfy the Rayleigh's criterion, the direction of n^{th} principal maxima for wavelength (λ_1) is given by,

$$(e + d) \sin \theta = n\lambda_1$$
$$N(e + d) \sin \theta = Nn\lambda_1 \quad ...(3.27.1)$$

3. And Ist minima in direction $(\theta + d\theta)$ is

$$N(e + d) \sin (\theta + d\theta) = m\lambda_1 \quad ...(3.27.2)$$

except $(m = 0, N, 2N .....)$ or $(1, 2, 3..., N - 1)$

4. When, $m = (nN + 1)$, equation (3.27.2) becomes

$$N(e + d) \sin (\theta + d\theta) = (nN + 1)\lambda_1 \quad ...(3.27.3)$$

5. The principal maxima due to wavelength λ_2 in direction $\theta + d\theta$ is

$$(e + d) \sin (\theta + d\theta) = n\lambda_2$$
$$N(e + d) \sin (\theta + d\theta) = Nn\lambda_2 \quad ...(3.27.4)$$

6. Comparing equation (3.27.3) and equation (3.27.4), we get

$$(nN + 1)\lambda_1 = Nn\lambda_2 \quad ...(3.27.5)$$

7. If $\lambda_1 = \lambda$, $\lambda_2 = \lambda + d\lambda$, $\quad d\lambda = \lambda_2 - \lambda_1$ equation (3.27.5) becomes

$$(nN + 1)\lambda = Nn(\lambda + d\lambda)$$
$$\lambda = Nnd\lambda \text{ or } \frac{\lambda}{d\lambda} = nN$$

or

$$(e + d) \sin \theta = n\lambda \text{ or } n = \frac{(e+d)\sin\theta}{\lambda}$$
$$\frac{\lambda}{d\lambda} = \frac{N(e+d)\sin\theta}{\lambda}$$

Que 3.28. **Find the angular separation of 5048 Å and 5016 Å wavelengths in second order spectrum obtained by a plane diffraction grating having 150000 lines per cm.**

UPTU 2010-11, 2012-13; Marks 05

Answer

1. **Given :**

$$n = 2$$
$$\lambda_1 = 5048 \text{ Å} = 5048 \times 10^{-10} \text{ m}$$
$$\lambda_2 = 5016 \text{ Å} = 5016 \times 10^{-10} \text{ m}$$
$$(e + d) = \frac{1}{150000} = 6.6 \times 10^{-6} \text{ cm}$$
$$= 6.6 \times 10^{-8} \text{ m}$$

2. Since,

$$d\lambda = \lambda_1 - \lambda_2$$
$$= 5048 - 5016$$
$$= 32 \text{ Å} = 32 \times 10^{-10} \text{ m}$$

3. And

$$\lambda = \frac{\lambda_1 + \lambda_2}{2} = 5032 \times 10^{-10} \text{ m}$$

4. Now angular separation is given by,

$$d\theta = \frac{d\lambda}{\sqrt{\left[\left(\frac{e+d}{n}\right)^2 - \lambda^2\right]}}$$
$$= \frac{32 \times 10^{-10}}{\sqrt{\left[\left(\frac{6.6 \times 10^{-8}}{2}\right)^2 - (5032 \times 10^{-10})^2\right]}}$$

$$= \frac{32 \times 10^{-10}}{\sqrt{10.89 \times 10^{-16} - 25.32 \times 10^{-14}}}$$

$$= \frac{32 \times 10^{-10}}{\sqrt{10^{-16}(10.89 - 0.2532)}}$$

$$= \frac{32 \times 10^{-10}}{3.26 \times 10^{-8}}$$

$$= 9.81 \times 10^{-2} \text{ rad} = 0.098 \text{ rad}$$

Que 3.29. A diffraction grating used at normal incidence gives a yellow line ($\lambda = 6000$ Å) in a certain spectral order superimposed on a blue line ($\lambda = 4800$ Å) of next higher order. If the angle of diffraction is $\sin^{-1}$ (3/4), calculate the grating element.

UPTU 2015-16, Marks 05

Answer

1. **Given :** $\lambda_1 = 6000$ Å $= 6000 \times 10^{-8}$ cm
$\lambda_2 = 4800$ Å $= 4800 \times 10^{-8}$ cm

Angle of diffraction $= \sin^{-1}\left(\frac{3}{4}\right) = 48.6°$

2. $\lambda_1 - \lambda_2 = (6000 - 4800) \times 10^{-8}$
$= 1200 \times 10^{-8}$ cm

3. Since grating element,

$$a + b = \frac{\lambda_1\lambda_2}{(\lambda_1 - \lambda_2)\sin\theta}$$

$$a + b = \frac{6000 \times 10^{-8} \times 4800 \times 10^{-8}}{1200 \times 10^{-8} \times 0.75}$$

$$= 3.2 \times 10^{-4}$$

Que 3.30. A diffraction grating used at normal incidence gives a yellow line ($\lambda = 6000$ Å) in a certain spectral order superimposed on a blue line ($\lambda = 4800$ Å) of next higher order. If the angle of diffraction is 60°, calculate the grating element. **UPTU 2013-14, Marks 05**

Answer

Same as Q. 3.29, Page 118B, Unit-3. (Ans. 2.77×10^{-4} cm)

Que 3.31. Find out if a diffraction grating will resolve the lines 8037.20 Å and 8037.50 Å in the second order given that the grating is just able to resolve two lines of wavelengths 5140.34 Å and 5140.85 Å in the first order. **UPTU 2014-15, Marks 05**

Answer

1. The resolving power of a grating is given by $\frac{\lambda}{d\lambda} = nN$

Therefore, $N = \frac{1}{n}\left(\frac{\lambda}{d\lambda}\right)$

2. where $d\lambda$ is the smallest wavelength difference at the mean wavelength λ, N the number of the lines on grating and n is the order of the spectrum.

$$\lambda = \frac{5140.34 + 5140.85}{2} = 5140.595 \text{ Å}$$

$$d\lambda = 5140.85 - 5140.34 = 0.51 \text{ Å and } n = 1$$

$$\therefore \quad n = \frac{1}{1}\left(\frac{5140.595}{0.51}\right) = 10080$$

3. Hence the resolving power ($\lambda/\lambda d$) of a grating in second order
$= nN = 2 \times 10080 = 20160.$

4. The resolving power required to resolve the lines 8037.20 Å and 8037.50 Å in the second order is $\lambda/d\lambda$

In this case $\lambda = \frac{8037.20 + 8037.50}{2} = 8037.35$ Å

$d\lambda = 8037.50 - 8037.20 = 0.30$

5. Resolving power $= \frac{8037.35}{0.30} = 26791.17$

6. Thus, the grating will not be able to resolve the lines 8037.20 Å and 8037.50 Å in the second order because the required resolving power (26791.17) is greater than the actual resolving power (20160).

Que 3.32. A diffraction grating used at normal incidence gives a green line (5400 Å) in a certain order n superimposed on the violet line (4050 Å) of the next higher order. If the angle of diffraction is 30°, find the value of n, also find how many lines per cm there in the grating.

Answer

1. **Given :** $\lambda_1 = 5400$ Å $= 5400 \times 10^{-8}$ cm, $\lambda_2 = 4050 \times 10^{-8}$ cm, $\theta = 30°$ and
$\lambda_1 - \lambda_2 = 1350 \times 10^{-8}$ cm

2. The direction of principal maxima for normal incidence is given by

$$(a + b)\sin\theta = n\lambda \quad ...(3.32.1)$$

3. Let n^{th} order maxima of λ_1, coincide with $(n + 1)^{th}$ order maxima of λ_2, we have

$$(a + b)\sin\theta = n\lambda_1 = (n + 1)\lambda_2$$

or $\quad n\lambda_1 = (n + 1)\lambda_2$ or $n = \frac{\lambda_2}{\lambda_1 - \lambda_2}$

4. Now, $(a+b)\sin\theta = \dfrac{\lambda_1\lambda_2}{\lambda_1-\lambda_2}$ or $a+b = \dfrac{\lambda_1\lambda_2}{(\lambda_1-\lambda_2)\sin\theta}$

5. $\therefore$ $$n = \frac{4050\times10^{-8}}{1350\times10^{-8}} = 3$$

and $$a+b = \frac{5400\times10^{-8}\times4050\times10^{-8}}{1350\times10^{-8}\times\sin 30^\circ}$$

6. Number of lines per cm

$$N = \frac{1}{a+b} = \frac{1350\times10^{8}}{5400\times4050\times2} = 3086$$

☺☺☺

4 UNIT Polarization and Laser

PART-1

Double refraction, Nicol prism, Production and analysis of plane, Circular and polarized light, Retardation plate.

CONCEPT OUTLINE : PART-1

Polarized Light : The light having vibrations only along a single line perpendicular to the direction of propagation of light is said to be polarized light.

Unpolarized Light : The light having vibrations along all possible straight lines perpendicular to the direction of propagation of light is said to be unpolarized.

Double Refraction : If an object is viewed through a calcite crystal, two images of the object are observed, this phenomenon is called double refraction.

Nicol Prism : It is an optical device made from a calcite crystal for producing and analyzing plane polarized light

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.1. What are unpolarized and polarized light ? Explain it.

Answer

A. **Unpolarized Light :**

1. The light having vibration along all possible straight lines perpendicular to the direction of propagation of light is said to be unpolarized.
2. For any position of the crystal, there will be one vibration parallel to the crystal axis, if a beam of unpolarized light is passed through a tourmaline crystal.
3. Hence no change in the intensity of emergent light is observed.
4. The unpolarized light is represented as

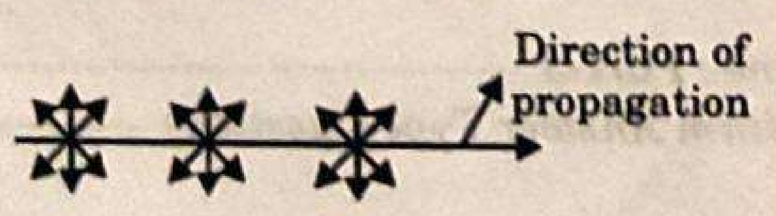


Fig. 4.1.1.

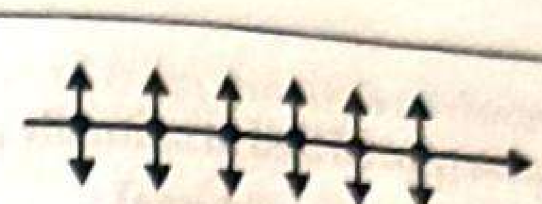
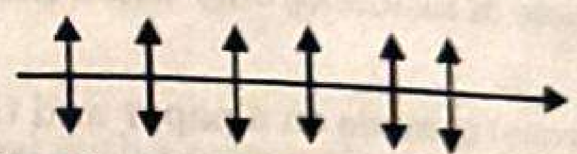


Fig. 4.1.2.

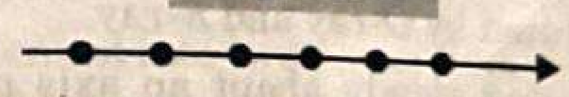
B. **Polarized Light :**

1. The light having vibrations only along a single line perpendicular to the direction of propagation of light is said to be polarized light.
2. A change in the intensity of emergent light is observed when a plane polarized light is passed through tourmaline crystal.
3. A light wave is said to be plane or linearly polarized if the electric vector ($\vec{E}$) or magnetic vector ($\vec{H}$) at any point goes on vibrating along the same line perpendicular to the direction of propagation of light wave.
4. The plane polarized light is represented as :



Vibrations parallel to plane of paper.

Fig. 4.1.3.



Vibrations perpendicular to plane of paper.

Fig. 4.1.4.

Que 4.2. What do you mean by double refraction ? Describe briefly the double refraction in a calcite crystal.

OR

Differentiate between the ordinary and extra-ordinary rays.

Answer

1. When a beam of unpolarized light is incident on the surface of an anisotropic crystal such as calcite or quartz, it is found that it will separate into two rays that travel in different directions.
2. This phenomenon is called birefringence or double refraction.

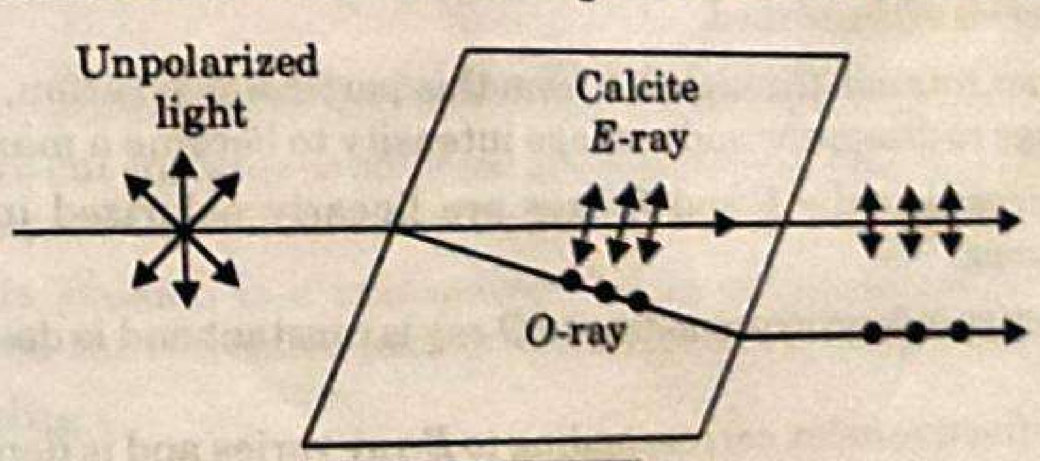


Fig. 4.2.1.

3. The two rays are known as ordinary ray (*O*-ray) and extraordinary ray (*E*-ray), which are linearly polarized in mutually perpendicular directions.

A. Double Refraction in Calcite Crystal :

1. A ray of light is incident on the face *AB* of the crystal and it travels along the principal section as shown in Fig. 4.2.2(*a*).
2. The ray is split into two rays, namely *O*-ray (fast ray) and *E*-ray (slow ray).
3. The *O*-ray travels through the crystal without deviation while the *E*-ray refracted at some angle.
4. As the opposite faces of the crystal are parallel, the rays emerge out parallel to the incident ray.
5. Within the crystal the *O*-ray always lies in the plane of incidence whereas *E*-ray does not lie in the plane of incidence.
6. *E*-ray lies in the plane of incidence only when the plane of incidence is a principal section.
7. If a mark (dot or cross) is made on a paper and then the calcite crystal (*AB* face) is placed on it, two images are seen through the crystal, as shown in Fig. 4.2.2(*b*).
8. The images are produced by *O*-ray and *E*-ray.
9. If the crystal is rotated slowly about an axis passing through the *o*-image, the *e*-image moves round in a circle while the *o*-image remains stationary.
10. It shows that the velocity of propagation of *O*-ray is the same in all directions, while that of *E*-ray changes with direction.
11. The *E*-ray and *O*-ray are linearly polarized.
12. The *E*-ray has its vibrations (*i.e.*, the optical vector) parallel to the principal section whereas the vibrations in *O*-ray are perpendicular to the principal section.
13. The vibration directions can be established by examining the rays through a polarizer.
14. As the polarizer is held in the path of the rays and rotated slowly, in one position, the intensity of the *o*-image will be a maximum while the *e*-image is extinguished.
15. Further rotation through 90° from this particular position, causes the *o*-image to disappear and *e*-image intensity to become a maximum.
16. It proves that the *E*-and *O*-rays are linearly polarized in mutually directions.
17. Refractive index corresponding to *O*-ray is constant and is designated by μ_o.
18. The refractive index corresponding to *E*-ray varies and is denoted by μ_e.

19. The difference between the refractive indices is known as the amount of double refraction or birefringence.

$$\Delta\mu = \mu_e - \mu_o$$

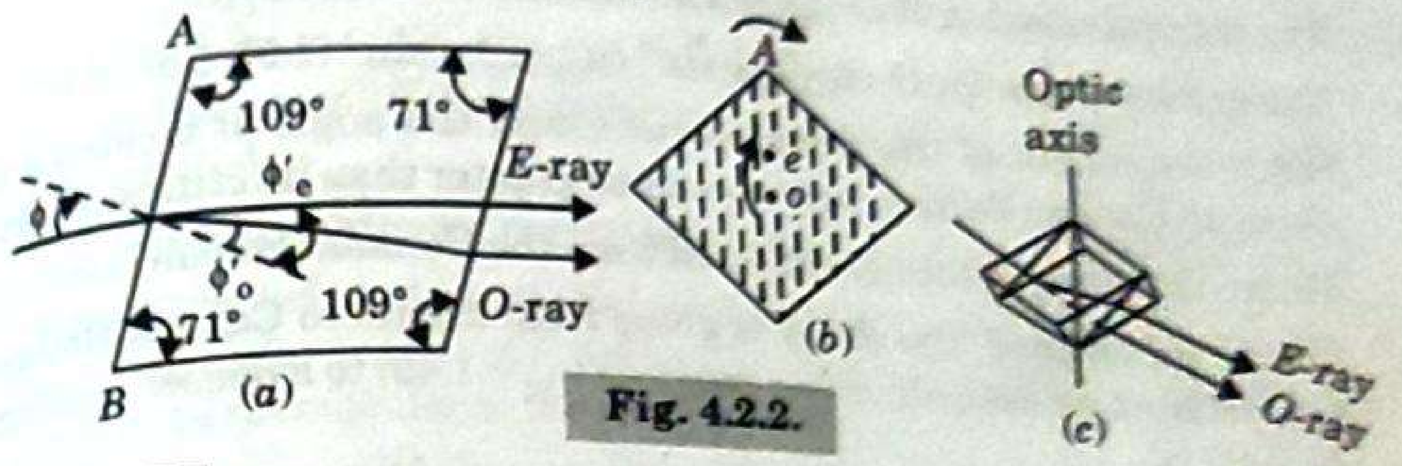


Fig. 4.2.2.

Que 4.3. **Describe the construction, working and application of Nicol prism.**

UPTU 2014-15, 2015-16 ; Marks 05

Answer

A. Construction :

1. It is a calcite crystal with a principal section *ABCD* whose length is three times its breadth as shown in Fig. 4.3.1.
2. In this situation, the new blunt corners will *A'* and *C'*.
3. The crystal is then cut into two pieces from one blunt corner *A'* to other blunt corner *C'* along a plane *A'C'* perpendicular to the principal section *ABCD* and perpendicular to both the faces *A'D'* and *BC'*.

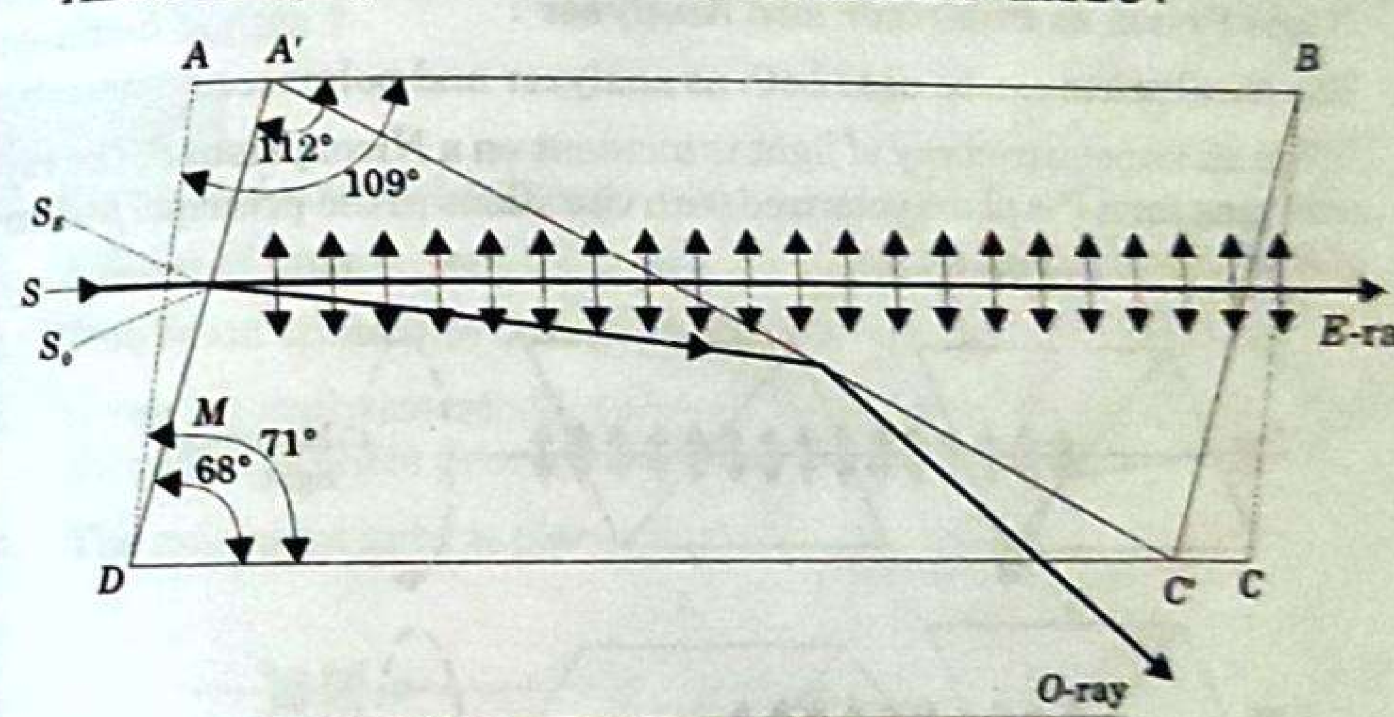


Fig. 4.3.1. Construction of a calcite crystal.

4. The two cut surfaces so obtained are polished optically flat and joined by a transparent medium called Canada balsam.
5. Canada Balsam is a transparent liquid whose refractive index lies between the refractive indices of calcite for the *O*-ray and *E*-ray.

B. Working :

1. When an unpolarized light ray *SM* parallel to the face *DC'* is incident on the face *A' D*, it splits into *O*-ray and *E*-ray.

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2. The *O*-ray is going from calcite to Canada Balsam travels from optically denser medium ($\mu_o = 1.66$) to a rarer medium ($\mu_{CB} = 1.55$).
3. The refractive index of *O*-ray w.r.t. Canada Balsam = μ_{CB}/μ_o
4. The critical angle for *O*-ray, $\theta_c = \sin^{-1}(\mu_{CB}/\mu_O) = \sin^{-1}(1.55/1.66) = 69°$
5. Since the length of the prism is sufficient, the angle of incidence of *O*-ray at Canada Balsam layer becomes greater than its critical angle.
6. Hence the *O*-ray is totally reflected from the Canada Balsam layer.
7. On the other hand, the *E*-ray is going from calcite to Canada Balsam travels from an optically rarer medium ($\mu_E = 1.49$) to a denser medium ($\mu_{CB} = 1.55$).
8. In this way, the light emerging from the Nicol prism is plane polarized with vibrations parallel to the principal section.

C. Application :

1. Use in microscopy and polarimetry.
2. Used for observing the sample placed between orthogonally oriented polarizers.

Que 4.4. **How Nicol prism can be used as polarizer and as an analyzer ?**

Answer

A. Nicol Prism as Polarizer and Analyzer :

1. The Nicol prism can be used both as analyzer and polarizer.
2. When an unpolarized ray of light is incident on a Nicol prism *P*, the ray emerging from *P* is plane polarized with vibrations in the principal section of *P* as shown in Fig. 4.4.1.

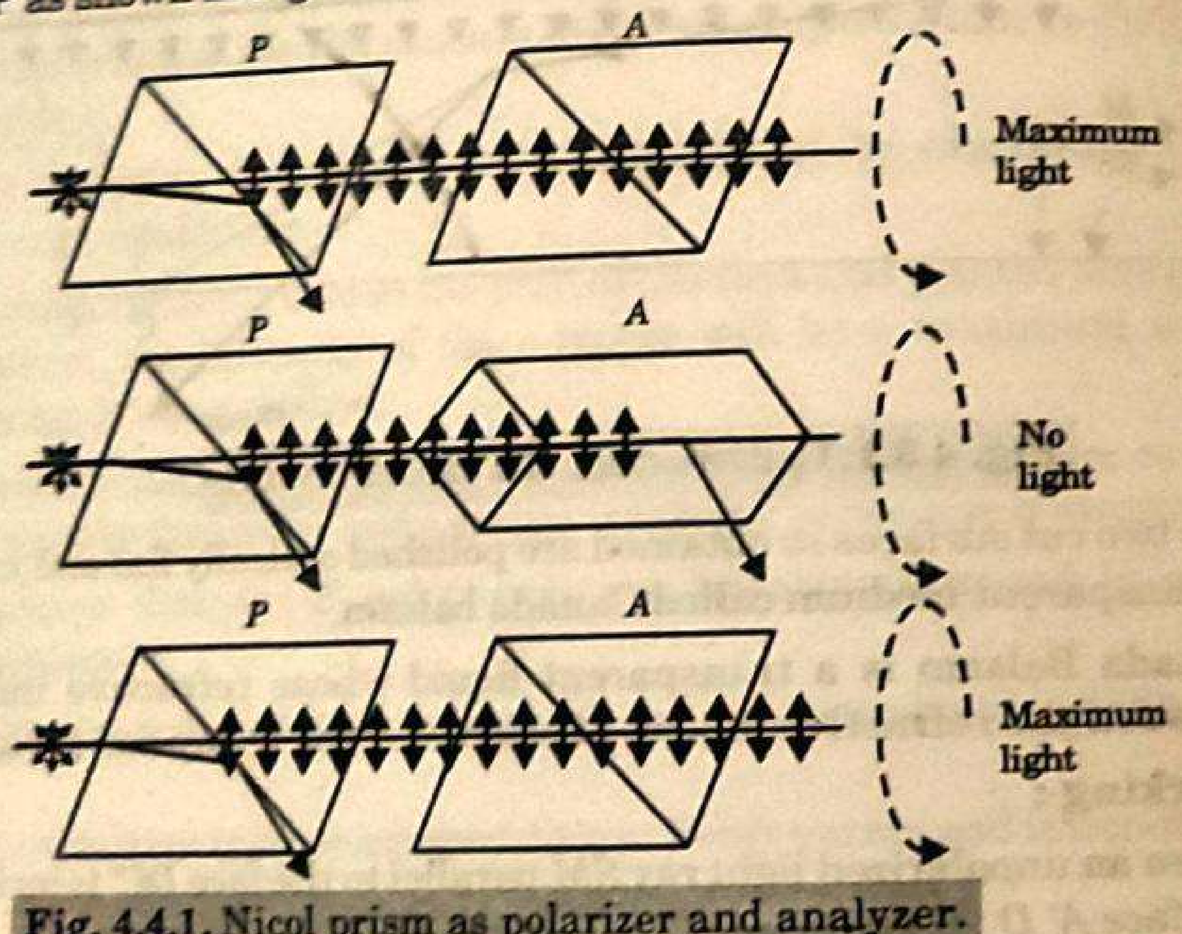

Fig. 4.4.1. Nicol prism as polarizer and analyzer.

3. This prism is known as polarizer.
4. If the emerging ray falls on a second Nicol prism *A*, whose principal section is parallel to that of *P*, its vibrations will be parallel to the principal section of *A*.
5. Hence the ray will behave as *E*-ray in the prism *A* and will be completely transmitted.
6. The intensity of the emergent light will be maximum in this case.
7. Now if the prism *A* is rotated such that its principal section becomes perpendicular to that of *P*, then the vibrations of the ray incident on *A* will be perpendicular to the principal section of *A*.
8. Hence the ray will behave as *O*-ray inside the prism *A* and will be lost by total internal-reflection at the Canada Balsam layer.
9. Now, when the Nicol prism *A* is further rotated about its axis, the intensity of light emerging through it increases and becomes maximum for the position in which the principal sections of both prisms are parallel to each other.
10. Hence the Nicol prisms *P* and *A* act as polarizer and analyzer respectively.

Que 4.5. **Describe how plane, circular and elliptically polarized light is produced.**

OR

How will you produce and detect plane, elliptically and circularly polarized lights ?

Answer

A. Plane Polarized Light :

1. A beam of unpolarized light passed through a Nicol prism.
2. This beam splits into *O*-ray and *E*-ray.
3. *O*-ray is totally internally reflected and is absorbed, while *E*-ray passes through the Nicol prism.
4. The emergent light is plane polarized.

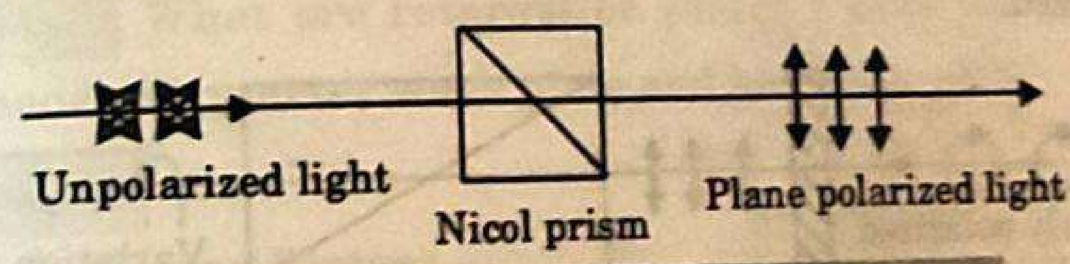

Fig. 4.5.1. Production of plane polarized light.

a. Detection :

1. For the detection of plane polarized light it is passed through another rotating Nicol.
2. If the intensity of emerging light from rotating Nicol varies with zero minimum, the light is plane polarized.

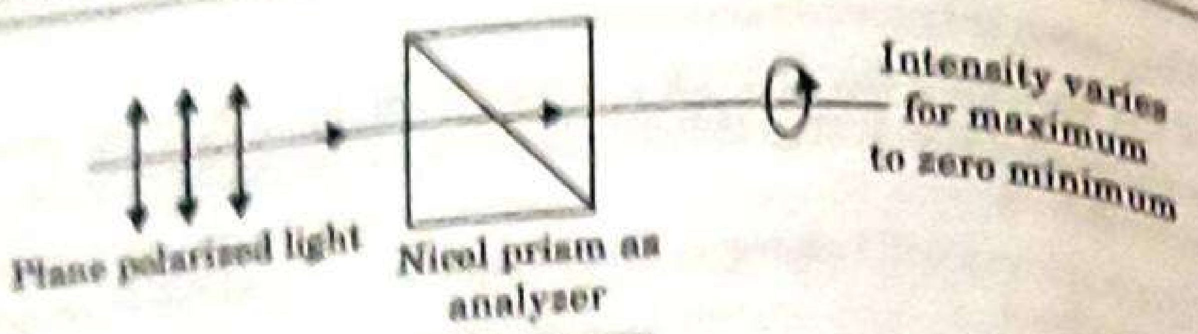

Fig. 4.5.2.

B. Circularly Polarized Light :

1. Ordinary monochromatic light is passed through a Nicol prism and the emerging light is plane polarized.
2. This emergent light is allowed to fall normally on a quarter wave plate such that the vibrations of light make an angle of 45° with the optic axis of the plane.
3. After entering in quarter wave plate, it splits into E-components and O-components having equal amplitude and period.
4. Emerging light from quarter wave plate (QWP) is circularly polarized.

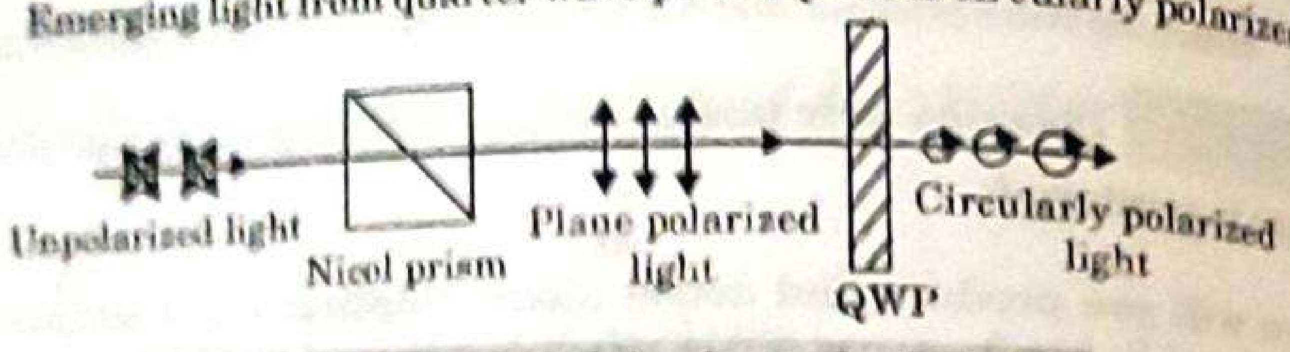

Fig. 4.5.3. Production of circulary polarized light.

a. Detection :

1. When circularly polarized light observed by rotating Nicol, it shows no variation.
2. We get the same result for unpolarized light.
3. For detection of circularly polarized light, firstly it is passed through a quarter wave plate, which converts it into plane polarized light.
4. The emergent light is viewed through a rotating Nicol, it shows a variation in intensity with zero minimum. Then the light is circularly polarized.

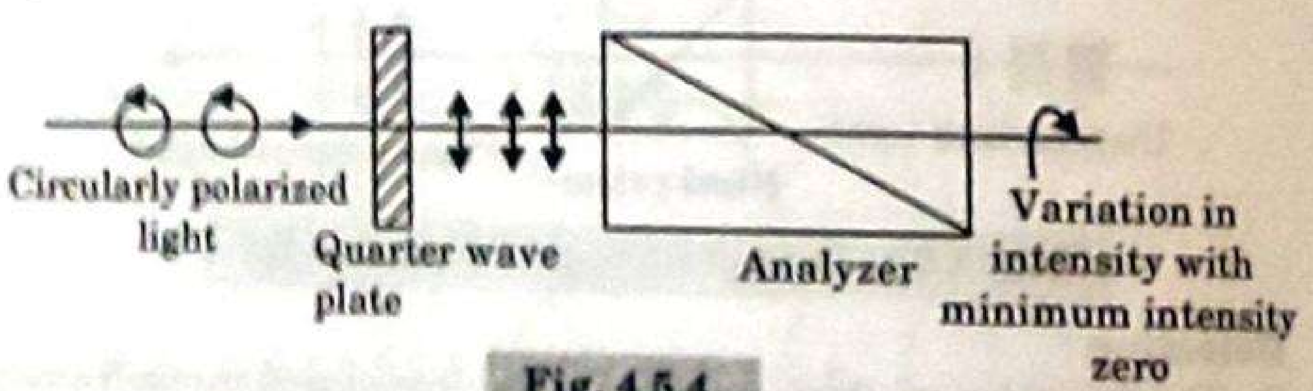

Fig. 4.5.4.

C. Elliptically Polarized Light :

1. Ordinary monochromatic light is passed through a Nicol prism and the emerging light is plane polarized.

2. This emergent light is allowed to fall normally on a quarter wave plate (QWP) such that the vibrations of light makes an angle θ (θ ≠ 0°, 45°, 90°) with the optic axis of plate.
3. After entering into quarter wave plate, it splits into O-component and E-component having unequal amplitudes and equal periods.
4. Emerging light from quarter wave plate is elliptically polarized.

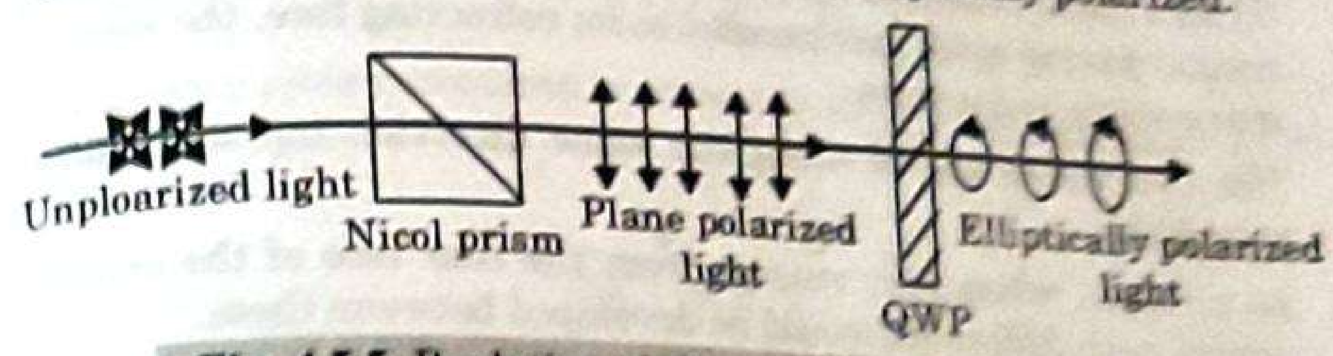

Fig. 4.5.5. Prodution of elliptically polarized light.

a. Detection :

1. When elliptically polarized light observed by rotating Nicol, it shows variation in intensity but minimum intensity is not zero.
2. We get the same result for partially plane-polarized light.
3. For detection of elliptically polarized light, firstly it is passed through a quarter wave plate, which converts it into plane polarised light.
4. The emergent light is viewed through a rotating Nicol, if variation of intensity is from a maximum to zero minimum. Then the light is elliptically polarized.

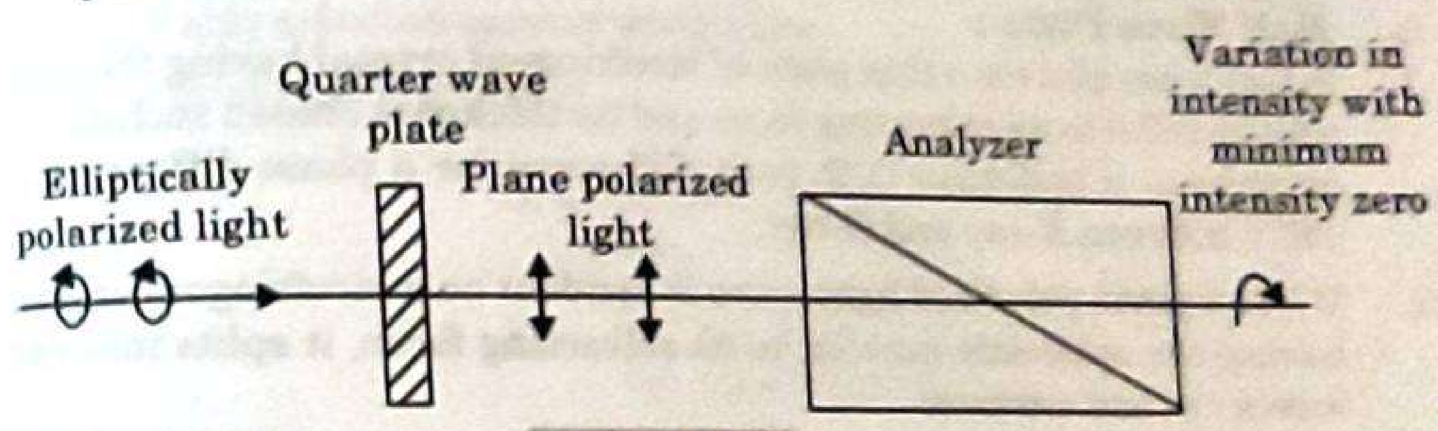

Fig. 4.5.6.

Que 4.6. What are retardation plates ? Give the construction and theory of quarter wave plate and half wave plate.

Answer

A. Retardation Plates :

1. A plate of uniform thickness cut from a double refracting crystal, with its faces parallel to optic axis of crystal, so as to introduce a desired value of path difference between O-ray and E-ray is called a retardation plate.
2. These are of two types :

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a. Quarter Wave Plate :

1. It is a thin plate of birefringent crystal having the optic axis parallel to its refracting faces and its thickness adjusted such that it introduces a quarter-wave ($\lambda/4$) path difference (or a phase difference of 90°) between the E-ray and O-ray propagating through it.
2. When a plane polarized light wave is incident on a negative birefringent crystal having optic axis parallel to its refracting face, the wave splits into e-wave and o-wave.
3. The two waves travel along the same direction but with different velocities.
4. As a result, when they emerge from the rear face of the crystal, an optical path difference would be developed between them.
5. Thus,

$$\therefore \quad (\mu_0 - \mu_e)d = \frac{\lambda}{4} \quad \text{...(4.6.1)}$$

$$d = \frac{\lambda}{4(\mu_0 - \mu_e)} \quad \text{...(4.6.2)}$$

6. A quarter wave plate introduces a phase difference δ, between E-ray and O-ray given by $\delta = (2\pi/\lambda)\Delta = \pi/2 = 90°$.
7. A quarter-wave plate is used for producing elliptically or circularly polarized light.

b. Half Wave Plate :

1. A half wave plate is a thin plate of birefringent crystal having the optic axis parallel to its refracting faces and its thickness chosen such that it introduces a half-wave ($\lambda/2$) path difference (or a phase difference of 180°) between E-ray and O-ray.
2. When a plane polarized light wave is incident on a birefringent crystal having the optic axis parallel to its refracting faces, it splits into two waves : o- and e-waves.
3. The two waves travel along the same direction inside the crystal but with different velocities.
4. As a result, when they emerge from the rear face of the crystal, an optical path difference would be developed between them.
5. $\therefore$

$$(\mu_0 - \mu_e)d = \frac{\lambda}{2} \quad \text{...(4.6.3)}$$

$$\therefore \quad d = \frac{\lambda}{2(\mu_0 - \mu_e)} \quad \text{...(4.6.4)}$$

6. A half wave plate introduces a phase differences δ, between E-ray and O-ray given by $\delta = (2\pi/\lambda)\Delta = \pi = 180°$.

Que 4.7. **A half-wave plate is fabricated for a wavelength of 3800 Å. For what wave-length does it work as quarter-wave plate ?**

Answer

1. Thickness of a half-wave plates is $d = \dfrac{\lambda_1}{2(\mu_e - \mu_0)}$.
2. The same plate is required to act as a quarter-wave plate. Therefore, we can write that

$$d = \frac{\lambda_2}{4(\mu_e - \mu_0)}.$$

$$\therefore \quad d = \frac{\lambda_1}{2(\mu_e - \mu_0)} = \frac{\lambda_2}{4(\mu_e - \mu_0)}$$

$$\therefore \quad \lambda_2 = 2\lambda_1 = 2 \times 3800 \text{Å} = 7600 \text{Å}.$$

Que 4.8. **A beam of plane polarized light is changed into circularly polarized light by passing it through a slice of crystal 0.003 cm thick. Calculate the birefringence of the crystal assuming to be the minimum thickness that will produce the effect, ($\lambda = 6 \times 10^{-5}$cm).**

Answer

1. Plane polarized light is converted into circularly polarized light by a suitably oriented quarter wave plate.
2. Its thickness is given by

$$d = \frac{\lambda_2}{4(\mu_e - \mu_0)} = \frac{\lambda}{4\Delta\mu}$$

$$\therefore \quad \Delta\mu = \frac{\lambda}{4d} = \frac{6 \times 10^{-5} \text{ cm}}{4 \times 0.003 \text{ cm}} = 0.005.$$

Que 4.9. **The value of μ_e and μ_o of quartz are 1.5508 and 1.5418 respectively. Calculate the phase retardation for λ = 5000 Å when the plate thickness is 0.032 mm.** **UPTU 2013-14, Marks 05**

Answer

1. **Given :** $\mu_e = 1.5508$, $\mu_o = 1.5418$, $t = 0.032$ mm $= 0.0032$ cm, $\lambda = 5000$ Å $= 5000 \times 10^{-8}$ cm
2. The phase retardation $= \dfrac{2\pi}{\lambda} \cdot \Delta = \dfrac{2\pi}{\lambda}(\mu_e - \mu_o)t$
3. Phase retardation $= \dfrac{2 \times 3.14(1.5508 - 1.5418)}{5000 \times 10^{-8}} \times 0.0032$

$= 3.617$ rad.

PART-2

Optical activity, Fresnel's theory, Specific rotation.

CONCEPT OUTLINE : PART-2

Optical Activity : The substances which rotate the plane of polarization are said to be optically active substances and this property is known as optical activity.

Specific Rotation : It is defined as the rotation produced by 1 decimeter length of the substances in solution when its concentration is 1 g/cm³.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.10. **What is optical activity ?**

Answer

1. When plane-polarized light passes through certain substances the plane of polarization (or plane of vibration) of light is rotated about the direction of propagation of light through a certain angle.
2. This property is known as optical activity and these substances are said to be optically active and this phenomenon is called optical rotation.
3. There are two types of optically active substances :
   - A. **Right handed or Dextrorotatory :** The substances which rotate the plane of polarization in the clockwise direction are called dextrorotatory.
   - B. **Left handed or Laevorotatory :** The substances which rotate the plane of polarization in the anticlockwise direction are called laevorotatory.

   Example : Fruit sugar.
4. According to Biot's laws the angle of rotation depends upon the following :
   - a. $\theta \propto l$ (length of optically active substances).
   - b. $\theta \propto C$ (concentration of the solution).
   - c. $\theta \propto \frac{1}{\lambda}$ (wavelength of light).

   - d. The amount of rotation also depends on the nature and temperature of the substance.

Que 4.11. **How can you detect optical rotation in an optical active crystal ?**

Answer

1. When a beam of plane polarized light propagates through a quartz crystal along the optic axis, the plane of polarization steadily turns about the direction of the beam.
2. The optical rotation can be detected as follows :
   - a. If two polaroid sheets or Nicol prisms are held in crossed configuration and if a beam of unpolarized light is viewed through them, the field of view appears to be completely dark.
   - b. Now let a quartz crystal, cut with its faces perpendicular to the optic axis, be inserted between the polarizers such that light is incident normally on the crystal.
   - c. The field of view now appears lit up indicating that the light is not cut off by the analyzer.
   - d. In order to cut off the transmitted light, we find that the analyzer is to be rotated through a certain angle.
   - e. The experiment establishes that the plane polarized light produced by the polarizer remains plane polarized while passing through the quartz crystal but the plane of polarization is rotated through an angle.
   - f. This angle is the angle through which the analyzer is rotated in order to cutoff the light totally.
   - g. The optical rotation, *i.e.*, the plane of polarized light is shown in Fig. 4.11.1.

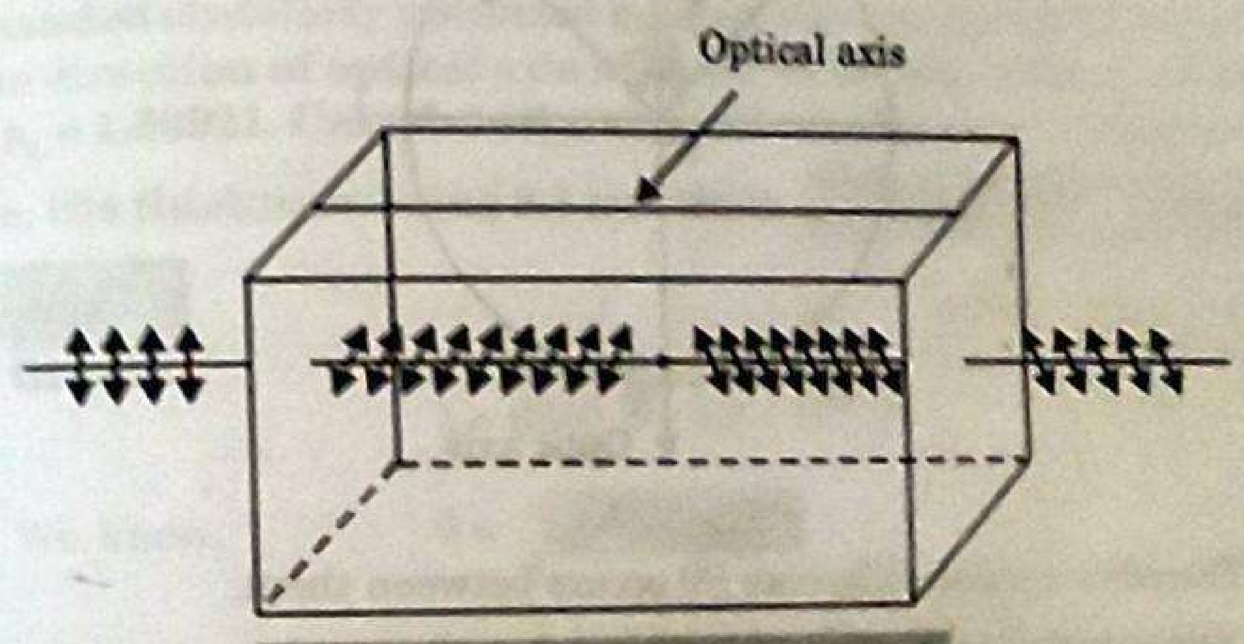


Fig. 4.11.1. Optically active crystal.

Que 4.12. **Explain Fresnel's theory and show that $\theta = \frac{\pi}{\lambda}(\mu_L - \mu_R)t$.**

Answer

A. Assumptions of Fresnel's Explanation for Optical Rotation :

1. When beam of plane-polarized light incident on an optically active substance along its optic axis it splits into two oppositely directed circular motion, one is in clockwise direction, while another is in anticlockwise direction.
2. The velocities of two circularly polarized beams is different for optically active substance, and same for optically inactive substance.
3. Due to different velocities the phase difference occurs between them.
4. In dextrorotatory substance, the velocity of right handed component greater than left handed component.

$$v_R > v_L$$

5. In laevorotatory substance, the velocity of $v_L > v_R$
6. On emergence from the substance, these two circular motions recombine to produce a plane polarized light.

B. Mathematical Explanation :

1. Let a plane polarized beam be incident normally on a doubly refracting crystal like quartz plate with its faces perpendicular to the optic axis.
2. These beam divided into two parts, clockwise and anticlockwise directions, in circular motion.
3. The circular motions travelling along the optic axis have different velocities.

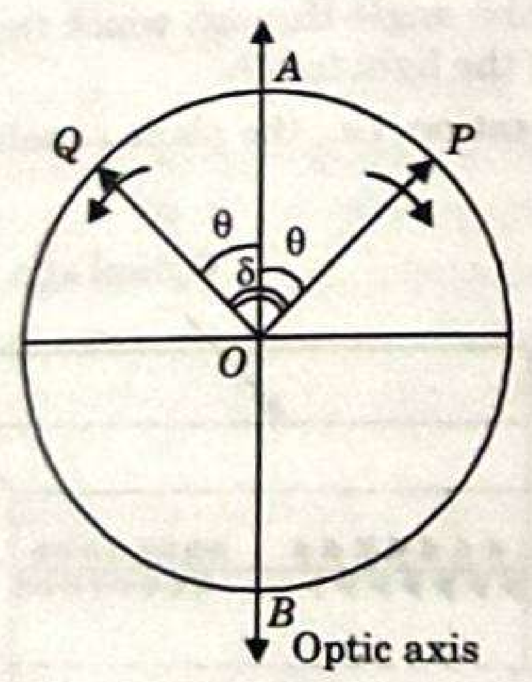


Fig. 4.12.1.

4. Therefore a phase difference (δ) occurs between them.
5. Now considering the case of dextrorotatory ($v_R > v_L$).
   AB – Optic axis.
   OP and *OQ* – Two circular motions rotating in opposite directions.
6. At the time of emergence these vibration are represented as

$x_1 = a\cos(\omega t + \delta)$, $y_1 = a\sin(\omega t + \delta)$ clockwise.
$x_2 = -a\cos(\omega t)$, $y_2 = a\sin(\omega t)$ anticlockwise.

7. By superposition theorem the resultant of x and y component are

$$X = x_1 + x_2 = a\cos(\omega t + \delta) - a\cos\omega t$$

$$X = 2a\sin\frac{\delta}{2}\cdot\sin\left(\omega t + \frac{\delta}{2}\right) \quad ...(1.12.1)$$

and

$$Y = y_1 + y_2 = a\sin(\omega t + \delta) + a\sin\omega t$$

$$Y = 2a\cos\frac{\delta}{2}\cdot\sin\left(\omega t + \frac{\delta}{2}\right) \quad ...(1.12.2)$$

8. The resulting vibrations along *X*-and *Y*-axis are in same period and phase.
9. Now dividing equation (1.12.1) by (1.12.2), we get

$$\frac{X}{Y} = \tan\frac{\delta}{2} \text{ or } X = Y\tan\frac{\delta}{2}$$

10. This equation is of a straight line having slope tan δ/2 with *Y*-axis.
11. If μ_R and μ_L are refractive indices of right and left handed light and t is thickness of crystal,

$$\text{path difference} = (\mu_L - \mu_R)t$$

and phase difference = $\delta = \frac{2\pi}{\lambda}(\mu_L - \mu_R)t$

and angle of rotation $2\theta = \delta$

$$2\theta = \frac{2\pi}{\lambda}(\mu_L - \mu_R)t$$

$$\theta = \frac{\pi}{\lambda}(\mu_L - \mu_R)t$$

Que 4.13. The indices of refraction of quartz for right handed and left handed circularly polarized lights of wavelength 6300 Å travelling in the direction of optical axis have the following values μ_R = 1.53915 and μ_L = 1.53921. Calculate the angle of rotation produced by quartz plate, the thickness being 0.5 mm thick. **UPTU 2011-12, Marks 05**

Answer

1. **Given :** d = 0.5 mm = 0.05 cm
   λ = 6300 Å = 6.3×10^{-5} cm

2. We know,

$$\theta = \frac{\pi}{\lambda}(\mu_L - \mu_R)d$$

$$\theta = \frac{3.14 \times 0.05(1.53921 - 1.53915)}{6.3 \times 10^{-5}} = 8.57°$$

Que 4.14. **The rotation in the plane of polarization in a certain substance is 10°/cm. Calculate the difference between refractive indices for right and left circularly polarized light in the substance. Given $\lambda = 5893$ Å.**

Answer

1. Given : $\frac{\theta}{d} = 10° = \frac{10 \times \pi}{360°} = \frac{\pi}{36}$ radian/cm and $\lambda = 5893$ Å $= 5893 \times 10^{-8}$ cm.
2. $$\theta = \frac{\pi}{\lambda}[\mu_L - \mu_R]d$$
3. $\therefore$ $$\mu_L - \mu_R = \frac{\pi}{36} \cdot \frac{5893 \times 10^{-8}\ \text{cm}}{\pi} = 1.6 \times 10^{-6}$$

Que 4.15. **The indices of refraction of quartz for right-handed and left-handed circularly polarized waves of wavelength 7620 Å travelling in the direction of optic axis have the following values. $\mu_R = 1.53914$ and $\mu_L = 1.53920$**
Calculate the rotation of the plane of polarization of light in degrees produced by a plate of 0.5 mm thick.

Answer

1. Given : $\lambda = 7620$ Å, $\mu_R = 1.53914$, $\mu_L = 1.53920$, $d = 0.5$ mm
2. We know, $$\theta = \frac{\pi}{\lambda}[\mu_L - \mu_R]d$$
$$= \frac{3.14 \times 0.5 \times 10^{-3}}{7620 \times 10^{-10}} \times (1.53920 - 1.53914)$$
$$= 0.1236 \text{ radian}$$
$$= \frac{0.1236 \times 180°}{\pi} = \frac{0.1236 \times 180°}{3.14} = 7°5'.$$

Que 4.16. **What is specific rotation ? Explain with formula.**

Answer

1. The specific rotation of an optically active substance at a given temperature for a given wavelength of light is defined as the rotation (in degrees) produced by a path of one decimeter length in a substance of unit density.
2. The optical activity of a substance is measured by its specific rotation.
3. θ depends upon the following :

a. The thickness of optically active substance.
b. The concentration of the solution.
c. Wavelength of light.
d. Temperature.

4. $\theta \propto l$ where, $\theta \rightarrow$ Rotation angle (degree)
$\theta \propto C$ $l \rightarrow$ Length of the substance (dm)
$C \rightarrow$ Concentration in (gm/cc)
5. Hence, $\theta \propto lC$
or, $\theta = SlC$
or, $S = \frac{\theta}{lC}$ $S \rightarrow$ Specific rotation
6. If l is in centimeter, then $$S = \frac{10\theta}{lC}$$
7. If $C = \frac{m}{V}$ then $S = \frac{10\theta V}{lm}$
8. If $l = 1$ dm
$C = 1$ gm/cc
Then $\theta = S$
9. Hence, specific rotation is defined as the rotation provided by 1 dm length of the solution containing 1 gm of the optically active substance per cc of the solution.
10. Specific rotation by graph
$$\tan\phi = \frac{\theta}{\frac{m}{V}} = \frac{\theta V}{m}$$
or, $$\theta = \frac{m \tan\phi}{V}$$
11. Here, $$S = \frac{10 \tan\phi}{l}$$

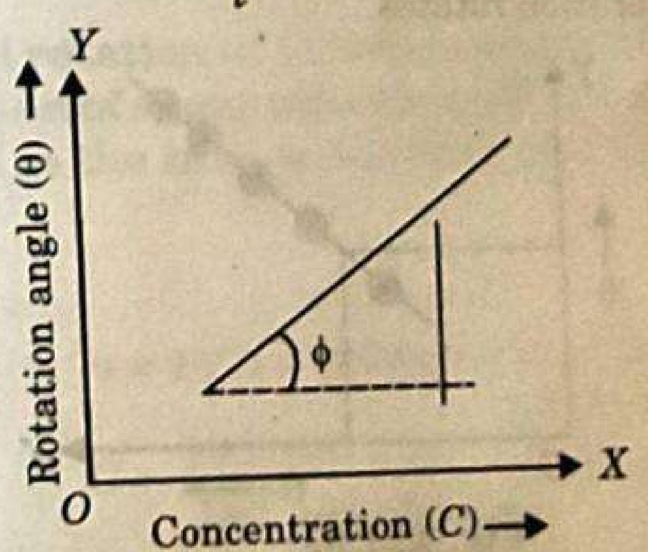


Fig. 4.16.1.

Que 4.17. **Outline the principal of a half-shade polarimeter and explain how will you use it to determine the specific rotation of a substance ?**

OR

Define specific rotation. Describe the construction and working of Laurent's half-shade polarimeter.

Answer

A. **Specific Rotation :** Refer Q. 4.16, Page 136B, Unit-4.

B. **Laurent's Half-Shade Polarimeter :**

1. Monochromatic light from source S is allowed to fall on Nicol N_1.
2. This light is rendered plane polarized with its vibrations in principal plane of Nicol N_1. Thus Nicol N_1 acts as a polarizer.
3. The emerging light is passed through a half-shade device and then through the solution and then after passing through Nicol N_2 is viewed by telescope T.

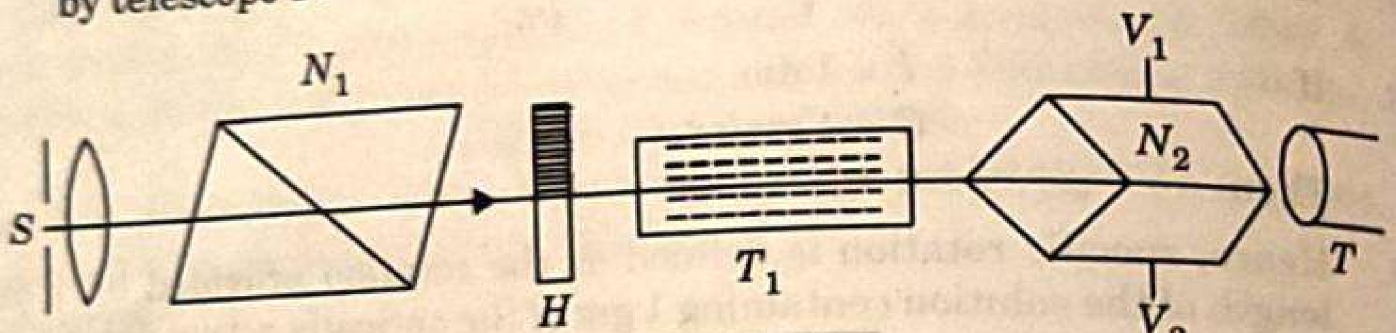


Fig. 4.17.1.

C. **Working :**

1. Firstly tube T_1 is kept empty.
2. Position of Nicol N_2 is adjusted in condition (1), so that field of view is equally bright (or equally dark) on two halves. Readings of verniers V_1 and V_2 are noted.
3. Tube is filled with a solution containing known concentration of an optically active substance.
4. The plane of vibration rotates.

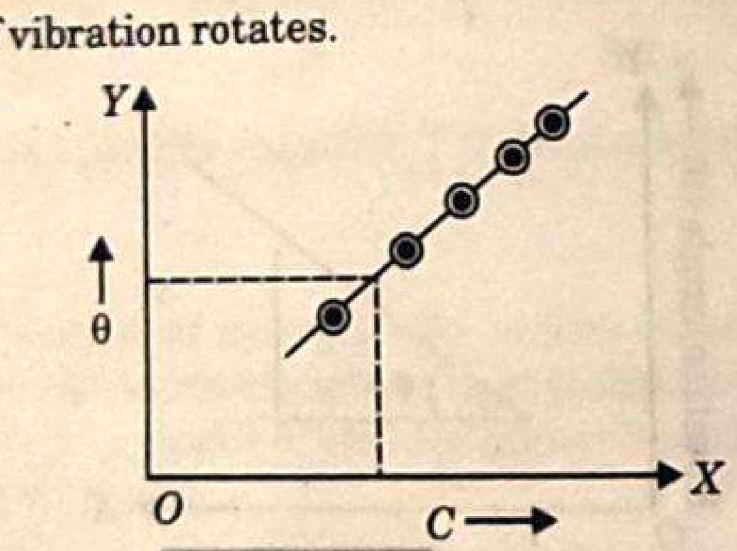


Fig. 4.17.2.

5. Nicol N_2 is rotated and set in condition (2), again note the readings of Vernier scales V_1 and V_2.
6. The difference in two readings gives angle through which the plane of vibration of incident plane polarized beam is rotated.
7. Experiment readings are taken for various concentration of solution.
8. The graph between concentration and angle of rotation is a straight line.

Specific rotations, $S = \dfrac{10\theta}{lC}$

Que 4.18. **A certain length of 5 % solution causes the optical rotation 20°. How much length of 10 % solution of the same substance will cause 35° rotation ?** **UPTU 2012-13, Marks 05**

Answer

1. **Given :** $\theta_1 = 20°$ and $\theta_2 = 35°$
2. Let length of 5 % solution is $= l_1$

   Length of 10 % solution is $= l_2$
3. Since, substance is same.

$\therefore$ Specific rotation $S = \dfrac{\theta_1}{l_1 C_1} = \dfrac{\theta_2}{l_2 C_2}$

or $\dfrac{20°}{l_1 \times 5\,\%} = \dfrac{35°}{l_2 \times 10\,\%}$

or $l_2 = \dfrac{7}{8} l_1$

Que 4.19. **A 200 mm long tube containing 48 cm³ of sugar solution produces an optical rotation of 11° when placed in a saccharimeter. If the specific rotation of sugar solution is 66°, calculate the quantity of sugar contained in the tube in the form of a solution.**

Answer

1. **Given :** $\theta = 11°$, $l = 200$ mm $= 20$ cm, $S = 66°$ and $V = 48$ cm³.
2. $C = \dfrac{10\,\theta}{lS} = \dfrac{10 \times 11°}{20 \times 66°} = 0.0833$ g/cm³
3. Mass of sugar in solution, $M = CV = 0.083$ g/cm³ $\times$ 48 cm³ $= 4$ grams.

PART-3

Spontaneous and stimulated emission, Population inversion, Einstein's coefficient, Types of lasers.

CONCEPT OUTLINE : PART-3

Laser : It is an acronym for Light Amplification by Stimulated Emission of Radiation.

Spontaneous Emission : It takes place when excited atoms make transition to lower energy level without any external stimulation.

Stimulated Emission : It takes place when a photon of energy ($h\nu = E_2 - E_1$) stimulates an excited atom to make transition to lower energy level.

Einstein's Coefficients : $\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3}$

Population Inversion : The process of making $N_2 > N_1$ is called population inversion.

Types of Laser :

a. Ruby laser, and
b. He-Ne laser.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.20. Explain LASER and different types of emission of radiations.

OR

Explain the spontaneous and stimulated emission of radiation. Why is spontaneous radiation incoherent ?

Answer

1. LASER stands for "Light Amplification by Stimulated Emission of Radiation".
2. It is a device use to produce a strong, monochromatic, collimated and highly coherent beam of light and it depends on the phenomenon of "stimulated emission".

A. Absorption of Radiation :

1. When an atom is in its ground state and a photon energy $h\nu$ is incident over it, it comes to its excited state after absorbing that photon.

2. This process is known as absorption of radiation.

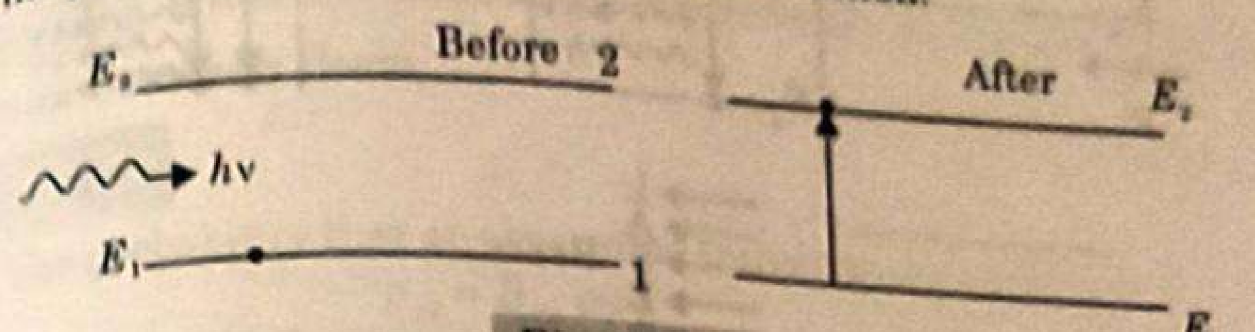


Fig. 4.20.1.

3. The probability of absorption of radiation is given by :

$$P_{12} = A_{12}\, u(\nu)$$

where A_{12} is Einstein's coefficient of absorption of radiation, $u(\upsilon)$ is energy density.

B. Spontaneous Emission of Radiation :

1. When an atom is in its excited state, it can remain there only for 10^{-8} sec.
2. After that it comes to its ground state and releases a photon of energy $h\nu$.
3. This process is called spontaneous emission of radiation.

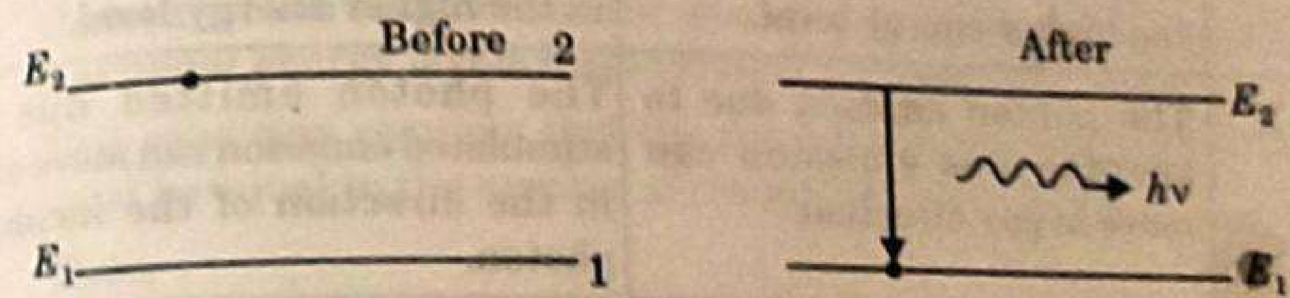


Fig. 4.20.2.

4. The probability of spontaneous emission of radiation is given by :

$$P_{21} = A_{21}$$

where A_{21} is Einstein's coefficient of spontaneous emission of radiation.

C. Stimulated Emission (Induced Emission) of Radiation :

1. When an atom is in its excited state and a photon of energy $h\nu$ is incident over it, atom comes to its ground state.
2. But now instead of one, two photons of energy $h\nu$ each are released.
3. When these two photons of energy $h\nu$ each are incident on another two excited state atoms, four photons of energy $h\nu$ each are released.
4. This process goes on continuously and as a result, a monochromatic, unidirectional beam of photon is released, which is known as stimulated emission of radiation.
5. The probability of stimulated emission of radiation is given by :

$$P'_{21} = A_{21} + B_{21}u(\nu)$$

where B_{21} is Einstein's coefficient of stimulated emission of radiation and $u(\upsilon)$ is energy density.

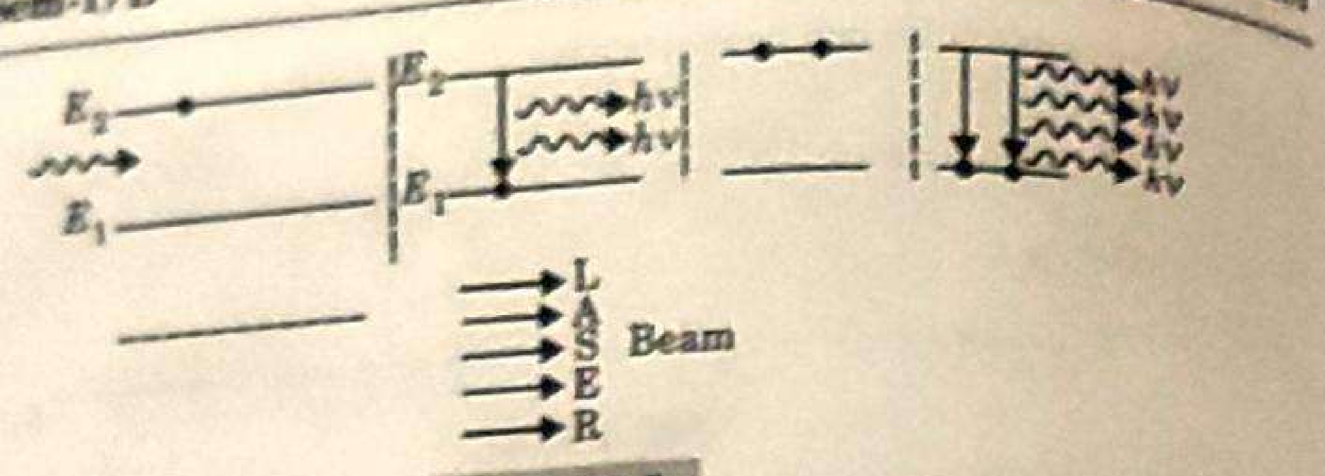


Fig. 4.20.3.

Que 4.21. **Differentiate between spontaneous emission and stimulated emission.**

Answer

S. No.	Spontaneous Emission	Stimulated Emission
1.	It is a natural transition in which an atom is de-excited after the end of its life-time in the higher energy level.	It is an artificial transition which occurs due to de-excitation of an atom before the end of its life-time in the higher energy level.
2.	The photon emitted due to spontaneous emission can move in any direction.	The photon emitted due to stimulated emission can move only in the direction of the incident photon.
3.	The probability of spontaneous emission depends only on the properties of the two energy levels between which the transition occurs.	The probability of stimulated emission depends on the properties of the two energy levels involved in the transition as well as on the energy density of incident radiation.

Que 4.22. **Discuss necessary condition to achieve laser action.**

OR

What are the necessary conditions of laser ?

Answer

a. There are three conditions to achieve laser action as follows :

1. The number of atoms in higher energy state must be greater than that in lower energy state so that the rate of emission becomes greater than the rate of absorption.
2. The radiation must be coherent so the probability of spontaneous emission should be negligible in comparison to the probability of stimulated emission.
3. The coherent beam of light must be sufficiently amplified.

Que 4.23. **What are Einstein's coefficients A and B ? Establish a relation between them.**

Answer

1. The probability that an absorption transition occurs is given by

$$P_{12} = B_{12}\,\rho(\nu)$$

where B_{12} is the constant of proportionality known as the Einstein's coefficient for induced absorption.

2. The probability that a spontaneous transition occurs is given by

$$(p_{21})_{\text{stimulated}} = A_{21} \qquad ...(4.23.1)$$

where A_{21} is a constant known as the Einstein's coefficient for spontaneous emission.

3. The probability that a stimulated transition occurs is given by

$$(p_{21})_{\text{stimulated}} = B_{21}\rho(\nu) \qquad ...(4.23.2)$$

where B_{21} is the constant of proportionality known as the Einstein's coefficient for stimulated emission.

A. **Relation Between Einstein's Coefficients *A* and *B* :**

1. Under thermal equilibrium, the mean population N_1 and N_2 in the lower and upper energy levels respectively must remain constant.
2. This condition requires that the number of transitions from E_2 to E_1 must be equal to the number of transitions from E_1 to E_2.
3. Thus,

$$\begin{pmatrix}\text{The number of atoms absorbing}\\ \text{photons per second per unit volume}\end{pmatrix} = \begin{pmatrix}\text{The number of atoms emitting}\\ \text{photons per unit volume}\end{pmatrix}$$

4. The number of atoms absorbing photons per second per unit volume $= B_{12}\rho(\nu)N_1$
5. The number of atoms emitting photons per second per unit volume $= A_{21}N_2 + B_{21}\rho(\nu)N_2$
6. As the number of transitions from E_1 to E_2 must equal the number of transitions from E_2 to E_1, we have

$$B_{12}\rho(\nu)N_1 = A_{21}N_2 + B_{21}\rho(\nu)N_2 \qquad ...(4.23.3)$$

$$\rho(\nu)\,[B_{12}N_1 - B_{21}N_2] = A_{21}N_2$$

$$\therefore \qquad \rho(\nu) = \frac{A_{21}N_2}{[B_{12}N_1 - B_{21}N_2]} \qquad ...(4.23.4)$$

7. By dividing both the numerator and denominator on the right hand side of the above equation with $B_{12}N_2$, we obtain,

$$\rho(\nu) = \frac{A_{21}/B_{12}}{\left[\dfrac{N_1}{N_2} - \dfrac{B_{21}}{B_{12}}\right]} \qquad ...(4.23.5)$$

8. But $$\frac{N_2}{N_1} = e^{-(E_2 - E_1)/kT}$$

As $$E_2 - E_1 = h\nu,$$

$$\frac{N_2}{N_1} = e^{-h\nu/kT} \quad \text{or} \quad \frac{N_1}{N_2} = e^{h\nu/kT}$$

$\therefore$ $$\rho(\nu) = \frac{A_{21}}{B_{12}}\left[\frac{1}{e^{h\nu/kT} - B_{21}/B_{12}}\right] \quad ...(4.23.6)$$

9. To maintain thermal equilibrium, the system must release energy in the form of electromagnetic radiation.
10. It is required that the radiation be identical with black body radiation and be consistent with Planck's radiation law for any value of T.
11. According to Planck's law

$$\rho(\nu) = \left(\frac{8\pi h\nu^3\mu^3}{c^3}\right)\left[\frac{1}{e^{h\nu/kT} - 1}\right] \quad ...(4.23.7)$$

where μ is the refractive index of the medium and c is the velocity of light in free space.

12. Energy density $\rho(\nu)$ given by equation (4.23.6) will be consistent with Planck's law (4.23.7), only if

$$\frac{A_{21}}{B_{12}} = \frac{8\pi h\nu^3\mu^3}{c^3} \quad ...(4.23.8)$$

and $$\frac{A_{21}}{B_{12}} = 1 \quad \text{or} \quad B_{12} = B_{21} \quad ...(4.23.9)$$

13. The above equations are known as the Einstein's relations. The coefficients B_{12}, B_{21} and A_{21} are known as Einstein's coefficients.
14. It follows that the coefficients are related through

$$B_{12} = B_{21} = \frac{c^3}{8\pi h\nu^3\mu^3} A_{21} \quad ...(4.23.10)$$

15. The relation (4.23.10) shows that the ratio of coefficients of spontaneous versus stimulated emission is proportional to the third power of frequency of the radiation. This is why it is difficult to achieve laser action in higher frequency ranges such as X-rays.

Que 4.24. **What is population inversion ?**

Answer

1. According to Boltzmann's equation, if N_1 and N_2 are the number of atoms in the ground and excited states, then

$$\frac{N_2}{N_1} = e^{(E_2 - E_1)/kT}$$

or $$\frac{N_2}{N_1} = e^{\Delta E/kT}$$

where ΔE is energy difference between the ground state and excited state, k is Boltzmann's constant and 'T' is absolute temperature.

2. But for atomic radiation ΔE is much greater than kT.
3. Therefore in thermal equilibrium the population of higher state is very much smaller than the ground state. *i.e.*, $N_1 > N_2$.
4. As a result the numbers of stimulated emissions are very little as compared to absorption.
5. Therefore laser action will not take place.
6. If somehow the number of atoms in excited state are made larger than in the ground state *i.e.*, $N_2 > N_1$, the process of stimulated emission dominates and the laser action can be achieved.

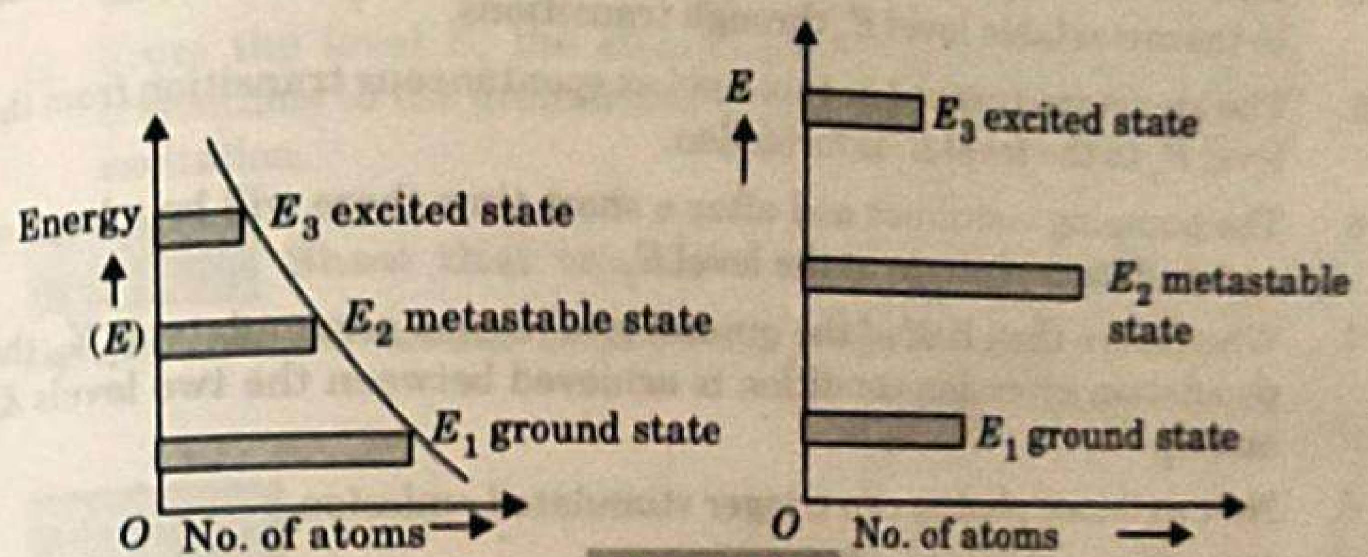


Fig. 4.24.1.

7. "The state in which the population of atoms in a higher state is larger than that in the ground state is known as population inversion *i.e.*, $(N_2 > N_1)$".

Que 4.25. **Explain the concept of 3 and 4 level laser.**

OR

Discuss the principal pumping schemes.

Answer

A. **Three Level Pumping Scheme :**

1. A typical three level pumping scheme is shown in Fig. 4.25.1.
2. The state E_1 is the ground level; E_3 is the pump level and E_2 is the metastable upper lasing level.

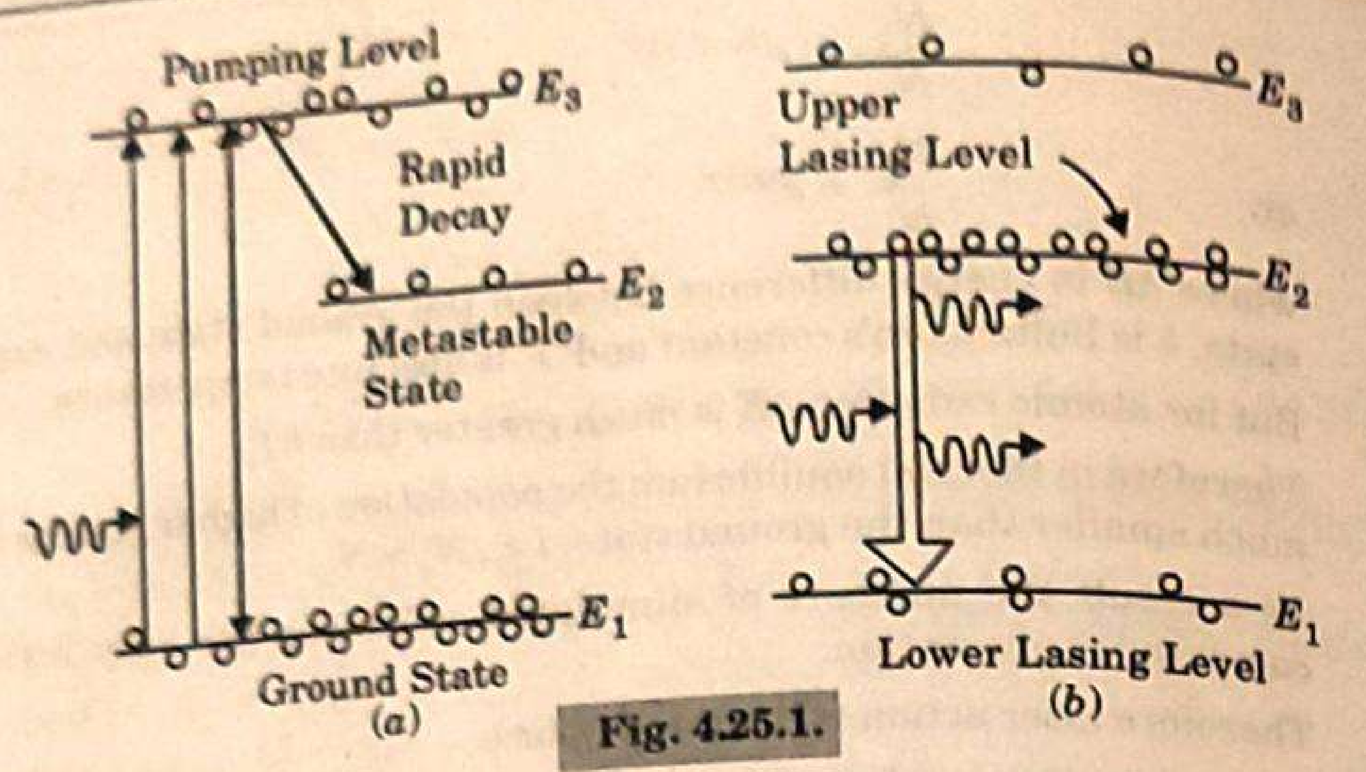


Fig. 4.25.1.

3. When the medium is exposed to pump frequency radiation, a large number of atoms will be excited to E_3 level.
4. They do not stay at that level but rapidly undergo downward transitions to the metastable level E_2 through transitions.
5. The atoms are trapped at this level as spontaneous transition from the level E_2 to the level E_1 is forbidden.
6. The pumping continues and after a short time there will be a large of accumulation of atoms at the level E_2.
7. When more than half of the ground level atoms accumulate at E_2, the population inversion condition is achieved between the two levels E_1 and E_2.
8. Now a chance photon can trigger stimulated emission.

B. Four Level Pumping Scheme :

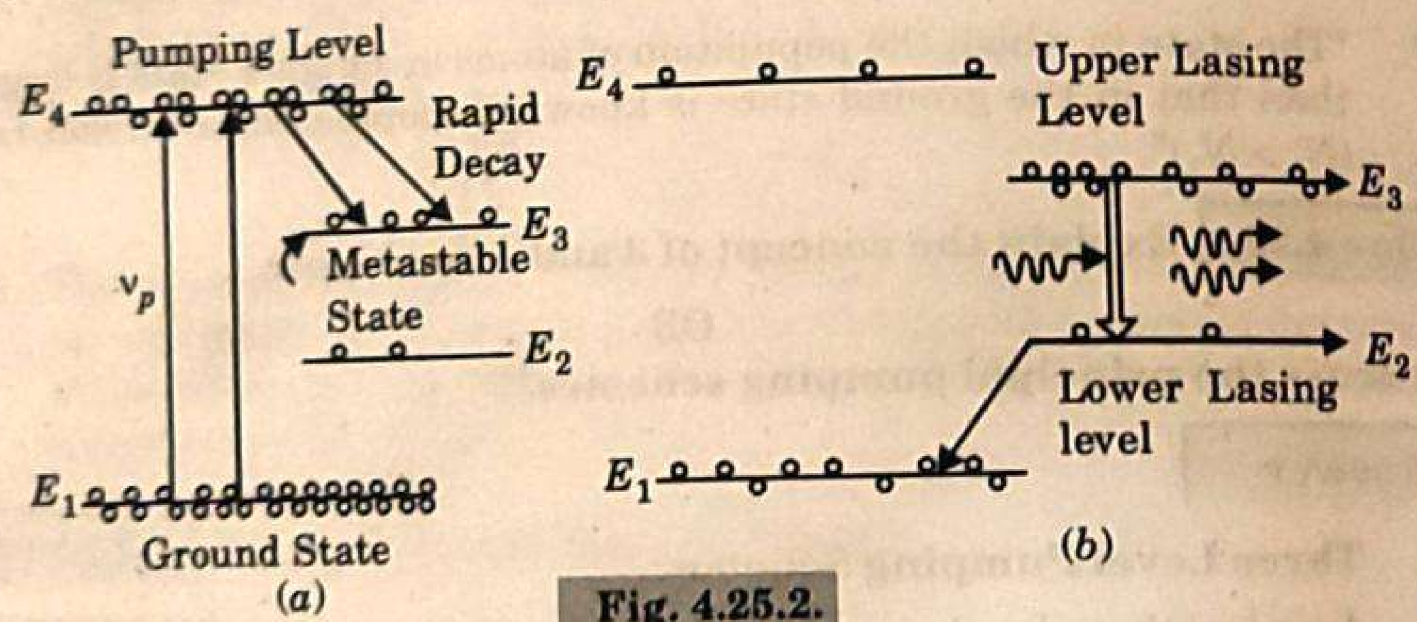


Fig. 4.25.2.

1. A typical four-level pumping scheme is shown in Fig. 4.25.2.
2. The level E_1 is the ground level, E_4 is the pumping level, E_3 is the metastable upper lasing level and E_2 is the lower lasing level.
3. E_2, E_3 and E_4 are the excited levels.

4. When light of pump frequency ν_p is incident on the lasing medium, the active centers are readily excited from the ground level to the pumping level E_4.
5. The atoms stay at the E_4 level for only about 10^{-8} sec, and quickly drop to the metastable level E_3.
6. As spontaneous transitions from the level E_3 to level E_2 cannot take place, the atoms get trapped at the level E_3.
7. The population at the level E_3 grows rapidly.
8. The level E_2 is well above the ground level such that $(E_2 - E_1) > kT$.
9. Therefore, at normal temperature atoms cannot jump to level E_2 on the strength of thermal energy.
10. As a result, the level E_2 is virtually empty.
11. Therefore, population inversion is attained between the levels E_3 and E_2.
12. A chance photon of energy $h\nu = (E_3 - E_2)$ emitted spontaneously can start a chain of stimulated emissions, bringing the atoms to the lower laser level E_2.
13. From the level E_2 the atoms subsequently undergo non-radiative transitions to the ground level E_1 and will be once again available for excitation.

Que 4.26. **Show that two level laser system has no practical significance for lasing. Explain the principle of three level lasers.**

UPTU 2013-14, Marks 05

Answer

1. The two level laser system has no practical significance because in two level system pumping is not suitable for obtaining population inversion.
2. The time span Δt, for which atoms have to stay at the upper level E_2, must be longer for achieving population inversion condition.
3. Hence, in two level system condition of population inversion will not achieve because $(N_1 = N_2)$.
4. Thus stimulated emission will not take place and laser amplification will not occur.

Principle of Three Level Laser : Refer Q. 4.25, Page 145B, Unit-4.

Que 4.27. **Explain Ruby laser with its construction and working.**

Answer

1. Ruby laser was built by T.H. Maiman in 1960.

2. Ruby is basically Al_2O_3 (silica) crystal containing about 0.05 % (by weight) of chromium atoms.
3. The Al^{3+} ions in the crystal lattice are substituted by Cr^{3+} ions.
4. Cr^{3+} ions constitute the active centres whereas the aluminium and oxygen atoms are inert.
5. The chromium ions give the transparent Al_2O_3 crystal a pink or red colour depending upon its concentration.

A. Construction :

1. The construction for generating Ruby laser is shown in Fig. 4.27.1.
2. Active material is a small cylinder of pink synthetic ruby, about 0.5 cm in diameter and few centimetres long.
3. Two parallel mirrors are used, one mirror M_1 is fully silvered and the other mirror M_2 is partly silvered so as to enable the coherent light radiation to be emitted through that end.
4. The mirrors must be separated by a distance that is an exact number of half wavelengths apart.

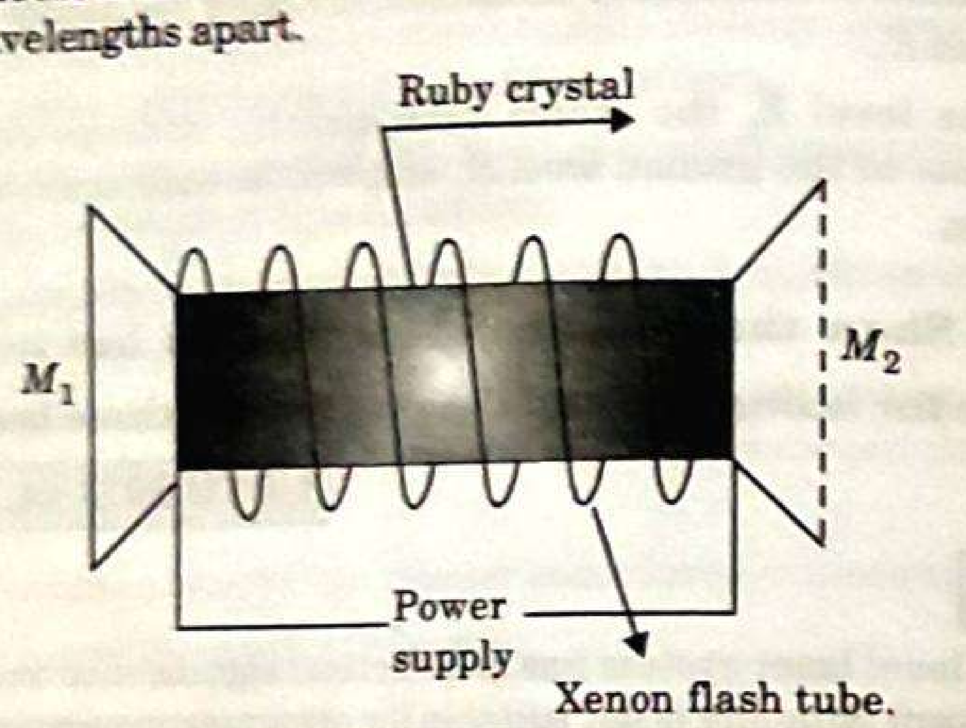


Fig. 4.27.1. Construction of Ruby laser.

5. Cooling is used to keep the ruby at a constant temperature.
6. Since quite a lot of the energy pumped into it is dissipated into heat, pumping process is carried with the help of a xenon flash tube.

B. Working :

1. Chromium ions are excited by the optical pumping, which is achieved by the xenon flash tube and raised to higher energy states *H*.
2. The excited atoms return to the lower state *L* from higher state *H* in two steps as shown in Fig. 4.27.2.
3. First they return to meta-stable state *M*.
4. This transition is radiationless transition and energy of this transition is passed to the crystal lattice as heat loss due to collisions.

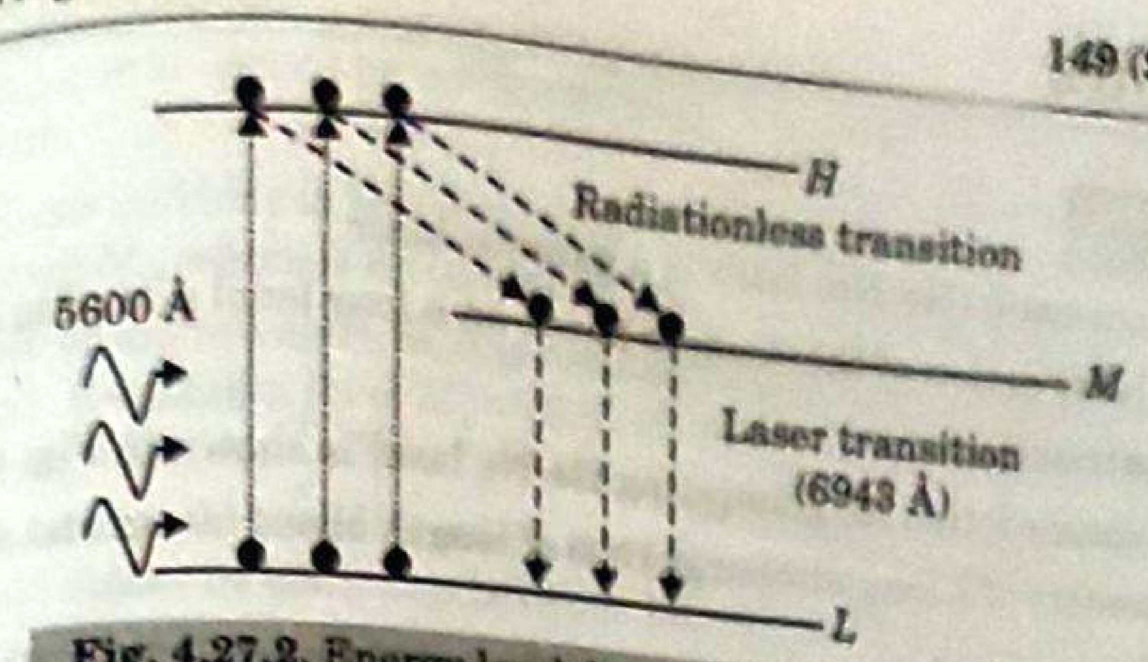


Fig. 4.27.2. Energy level diagram of Ruby laser.

5. The chromium ions that returned to *M* level can remain in this state for several milli-seconds.
6. Thus, the accumulation of the coming excited atoms at *M* level from *H* level increases its population.
7. When a chromium ion falls to the lower level *L* from the meta-stable level *M* by spontaneous emission, it emits a photon of 6943 Å.
8. This photon travels along the axis of ruby rod and is reflected back and forth by the silvered mirrors as shown in Fig. 4.27.3.
9. The photon travelling parallel to the axis of the tube will start photon multiplication by stimulated emission of other chromium ions of *M* level.
10. When the photon beam becomes sufficiently intense, it emerges through the partially silvered end of the ruby rod in the form of laser pulses.
11. The laser beam is red in colour and corresponds to a wavelength of (6943 Å).

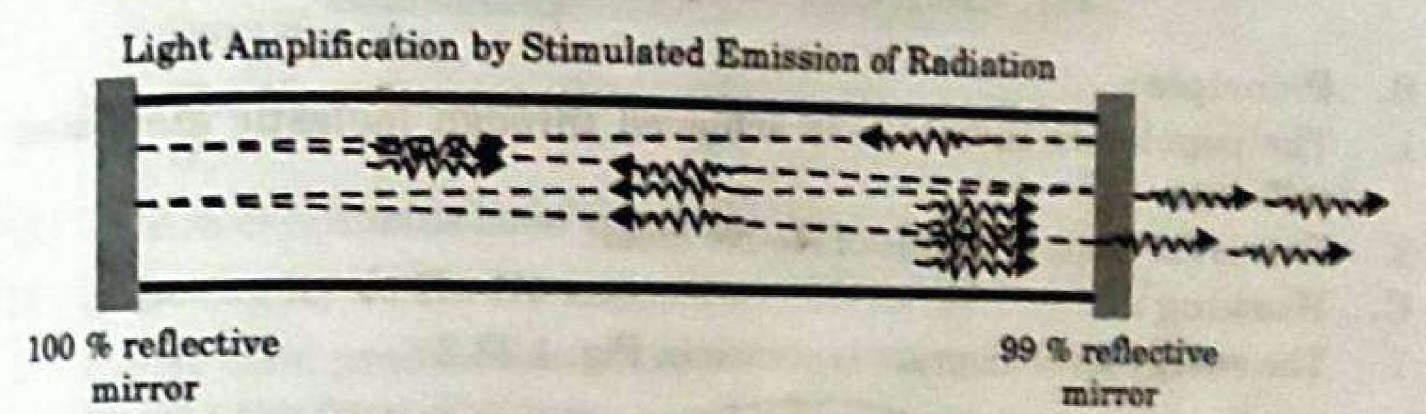


Fig. 4.27.3. Photon multiplication in Ruby laser.

C. Drawbacks :

1. The output of the laser is not continuous but occurs in the form of pulses of microseconds duration.
2. The efficiency of ruby lasers is very less.
3. It requires greater excitation in order to achieve population inversion.

Que 4.28. **Discuss the He-Ne laser with necessary diagrams. Give its superiority over ruby laser.** **UPTU 2014-15, 2015-16; Marks 05**

Answer

1. Helium-neon (He-Ne) laser is a gaseous laser.
2. The laser action of this laser is based on a four level pumping scheme.

A. Construction :

1. The construction for generating He-Ne laser is shown in Fig. 4.28.1.
2. It consists of a long discharge tube of length about 50 cm and diameter 1 cm.
3. The tube is filled with a mixture of He and Ne gases in the ratio 80 : 20.
4. Helium is the pumping medium and Neon is the lasing medium.
5. Electrodes are provided to produce a discharge in the gas and they are connected to a high voltage power supply.
6. On the axis of the tube, two reflectors M_1 (fully silvered) and M_2 (partially silvered) are fixed.
7. The distance between the mirrors is adjusted such that it equals $m\lambda/2$.

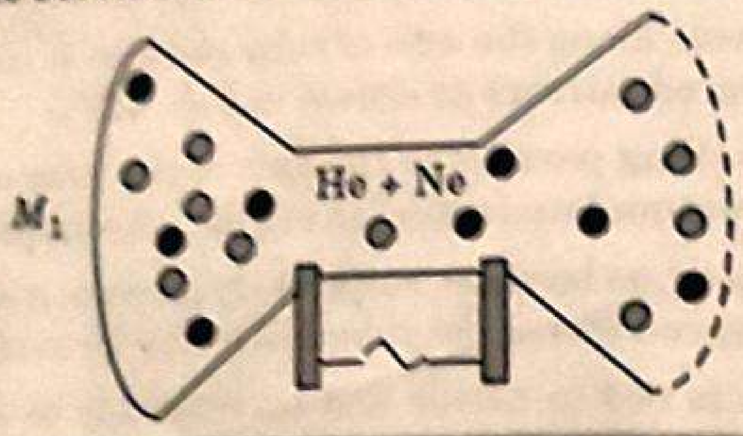


Fig. 4.28.1. Construction of He-Ne laser.

B. Principle :

1. The population inversion is achieved through inelastic atom-atom collisions.
2. This is the basic principle of He-Ne laser.

C. Working :

1. The energy level diagram is shown in Fig. 4.28.2.

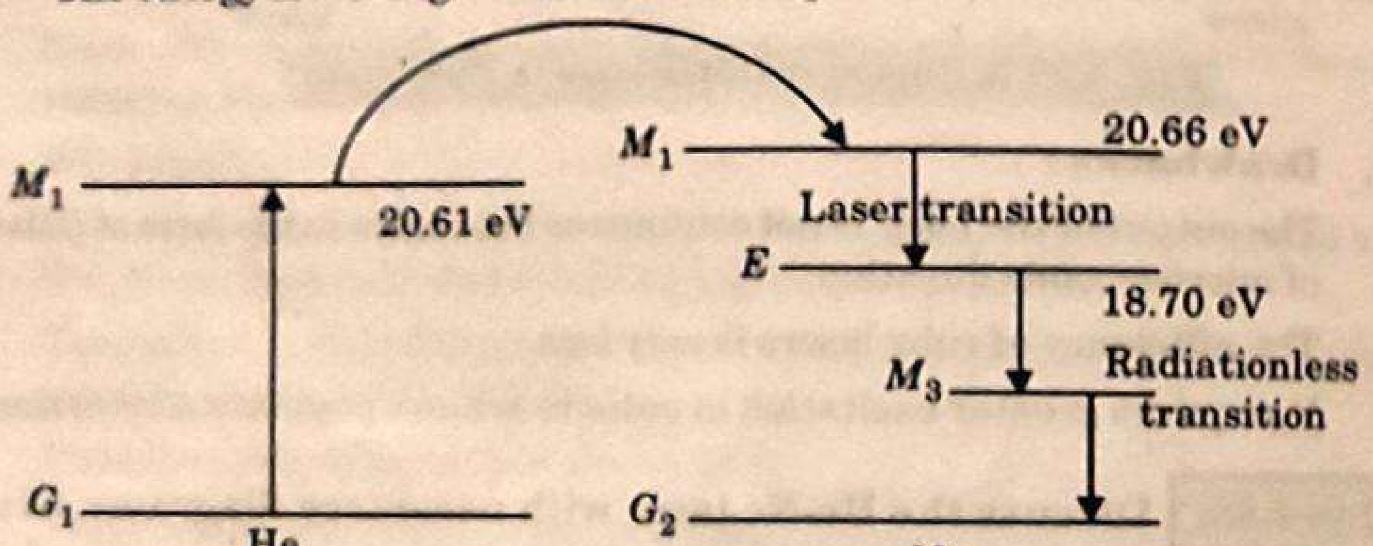


Fig. 4.28.2. Energy level diagram of He-Ne laser.

2. Pumping is achieved by using electrical discharge in the helium-neon mixture.
3. Electrons and ions in this discharge collide with He atoms raising them to a level M_1, which is meta-stable.
4. The He atoms are more readily excitable than Ne atom because they are lighter. Excitation level M_1 = 20.61 eV of He is very close to excitation level M_2 = 20.66 eV of Ne.
5. Some of the excited He atoms transfer their energy to Ne atoms of ground states by collisions between helium and neon atoms.
6. Thus, the excited He atoms return to ground state by transferring their energy to Ne atoms through collisions.
7. The kinetic energy of helium atoms provides the additional 0.05 % for exciting the neon atom.
8. This is the main pumping scheme of He-Ne system.
9. Thus, neon atoms are active centres.
10. The helium gas in the laser tube provides the pumping medium to attain the necessary population inversion for laser action.
11. In this way, M_2 state of Ne can become more highly populated than level E.
12. The laser transition occurs when Ne atoms fall from level M_2 to E through stimulated emission.
13. A red laser light of wavelength 6328 Å is generated in He-Ne laser.
14. This population inversion is maintained because :
    a. The meta-stability of level E_3 ensures a ready supply of Ne atoms in level E_2.
    b. The Ne atoms from level E decay rapidly to the neon ground state G_2.
15. Thus, a continuous laser beam is obtained in He-Ne laser.

D. Superiority of He-Ne laser over Ruby laser :

1. He-Ne laser produces continuous laser beam while ruby laser produces light in the form of pulses.
2. He-Ne laser employs a four level pumping scheme while ruby laser employs a three level pumping scheme.

Que 4.29. What are various applications of LASER beam ?

Answer

1. The laser beam is used for drilling, welding and melting of hard materials like diamonds, iron, steel, etc.
2. It is used in heat treatments for hardening or annealing in metallurgy.

3. The laser beam is used in delicate surgery like cornea grafting and in the treatment of kidney stone, cancer and tumor.
4. Laser is used in holography, fibre optics and nonlinear optics.
5. During war-time, lasers are used to detect and destroy enemy missiles. Now, laser-pistols, laser-rifles and laser bombs are also being made, which can be aimed at the enemy in the night.
6. Laser is very useful in science and research areas.
7. Laser is used for communications and measuring large distances.
8. Semiconductor laser is used for recording and erasing of data on compact disks.
9. Semiconductor lasers and helium-neon lasers are used to scan the universal barcodes to identify products in supermarket scanners.

Que 4.30. **In a Ruby laser, total number of Cr^{+3} is 2.8×10^{19}. If the laser emits radiation of wavelength 7000 Å, calculate the energy of the laser pulse.**

Physical constants :

Mass of electron	$m_0 = 9.1 \times 10^{-31}$ kg
Mass of proton	$m_p = 1.67 \times 10^{-27}$ kg
Speed of light	$c = 3 \times 10^8$ m/s
Planck's constant	$h = 6.63 \times 10^{-34}$ J/s
Charge of electron	$e = 1.67 \times 10^{-19}$ C
Boltzmann's constant	$k = 1.38 \times 10^{-23}$ m^2 kg s^{-2} k^{-1}

UPTU 2015-16, Marks 05

Answer

1. **Given :** Total number of Cr^{+3}, $n = 2.8 \times 10^{19}$
   Wavelength, $\lambda = 7000$ Å $= 7000 \times 10^{-10}$ m

   $$c = 3 \times 10^8 \text{ m/s}, \; h = 6.63 \times 10^{-34} \text{ J/s}$$

2. We know that,

$$\text{Energy} = \frac{nhc}{\lambda}$$

$$\text{Energy} = \frac{2.8 \times 10^{19} \times 6.63 \times 10^{-34} \times 3 \times 10^8}{7000 \times 10^{-10}}$$

$$= 7.956 \text{ J}$$

$$= \frac{7.956}{1.67 \times 10^{-19}}$$

$$= 4.764 \times 10^{19} \text{ eV}$$

☺☺☺

5 UNIT

Fiber Optics and Holography

PART-1

Fundamental ideas about optical fibre, Propagation mechanism, Acceptance angle and cone, Numerical aperture, Single and Multi mode fibres, Dispersion and attenuation.

CONCEPT OUTLINE : PART-1

Optical Fibre : Optical fibre consists of a core surrounded by a cladding and a sheath. It is a thin, transparent and flexible strand. It made up of glass or plastic. It works on the principle of total internal reflection.

Acceptance Angle : It is defined as the maximum angle that a light ray can have relative to the axis of the fibre and propagates down the fibre.

Numerical Aperture : It is a dimensionless number that characterizes the range of angles over which the fibre can accept or emit light.

Dispersion : The amplitude of the optical signal propagating in an optical fibre attenuates due to losses in fibres as well as it spreads during its propagation. Thus, the output signal received at the end becomes wider compared to the input signal. This type of distortion arises due to dispersion effect in optical fibres.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.1. **What is optical fibre ?**

Answer

1. Optical fibre is a long, thin transparent dielectric material made up of glass or plastic, which carries electromagnetic waves of optical frequencies (visible to infrared) from one end of the fibre to the other by means of multiple total internal reflections.
2. Optical fibres work as wave guides in optical communication systems.
3. An optical fibre consists of an inner cylindrical material made up of glass or plastic called core.
4. The core is surrounded by a cylindrical shell of glass or plastic called the cladding.
5. The refractive index of core (n_1) is slightly larger than the refractive index of cladding (n_2), (*i.e.*, $n_1 > n_2$).

6. The cladding is enclosed in a polyurethane jacket as shown in Fig. 5.1.1.
7. This layer protects the fibre from the surrounding atmosphere.
8. Many fibres are grouped to form a cable. A cable may contain one to several hundred such fibres.

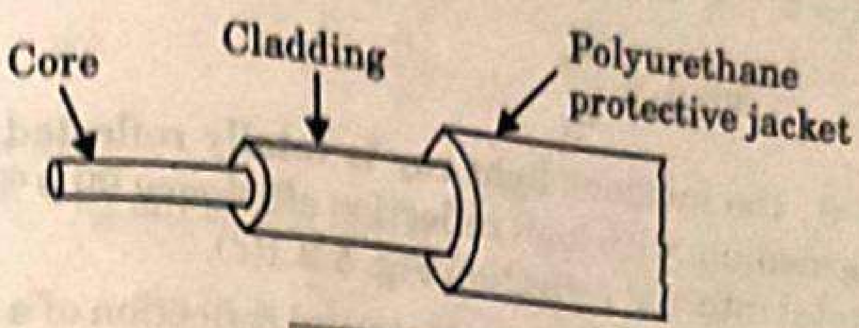


Fig. 5.1.1.

Que 5.2. **Explain the principle of optical fibre.**

Answer

1. The working of optical fibre is based on the principle of total internal reflection.
2. Total internal reflection is the phenomenon in which a light ray reflects completely in the first medium, when it is incident on the boundary of two different mediums.
3. When a light ray is incident on a high to low refractive index interface, then from Snell's law Fig. 5.2.1(*a*).

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1} \qquad ...(5.2.1)$$

where n_1 and n_2 are the refractive indices of denser and rarer mediums respectively.

5. Since $n_1 > n_2$, so from equation (5.2.1), we have

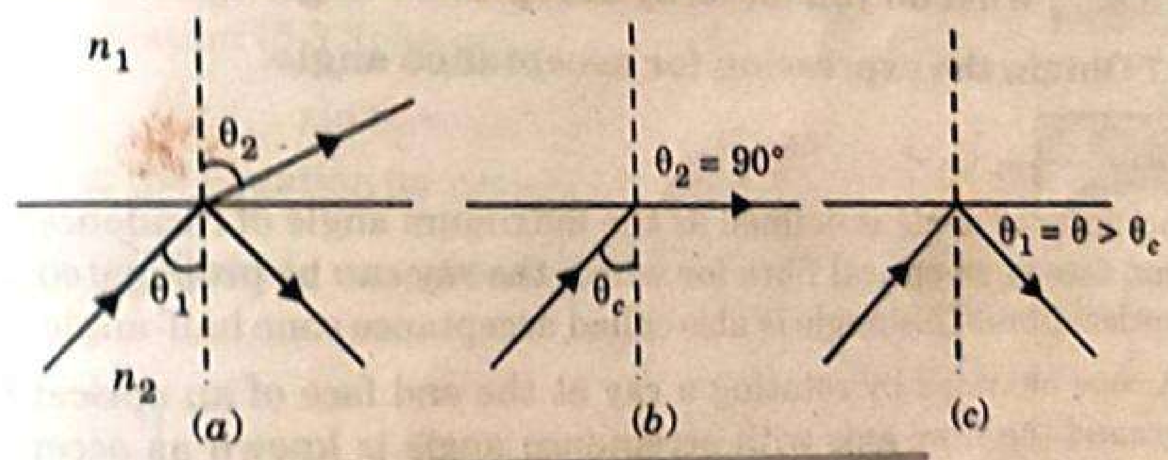


Fig. 5.2.1. Principle of optical fibre.

$$\frac{\sin\theta_1}{\sin\theta_2} < 1$$

i.e., $\sin\theta_1 < \sin\theta_2$ ...(5.2.2)

i.e., $\theta_1 < \theta_2$

6. When the refracted light ray emerges along the interface, *i.e.*, when the angle of refraction becomes 90°, *i.e.* $\theta_2 = 90°$, the corresponding value of the angle of incidence is called the critical angle and denoted by θ_c Fig. 5.2.1(*b*).
7. Thus for $\theta_1 = \theta_c$, $\theta_2 = 90°$, equation (5.2.1) becomes

$$\sin\theta_c = \frac{n_2}{n_1} \qquad ...(5.2.3)$$

8. If $\theta_1 = \theta > \theta_c$, the incident light ray is totally reflected back into the originating medium with high reflection efficiency 99.9 %. This event is known as total internal reflection Fig. 5.2.1(*c*).
9. The necessary conditions for total internal reflection of a light ray in an optical fibre are therefore as follows :
   a. The refractive index of fibre core should be higher than that of the cladding.
   b. The light ray should be incident between the core-cladding interface and the normal to the core-cladding interface at an angle greater than the critical angle.
   c. The respective refractive indices n_1 and n_2 of core and cladding materials of the fibre should be related to the critical angle by the relation given in equation (5.2.2).

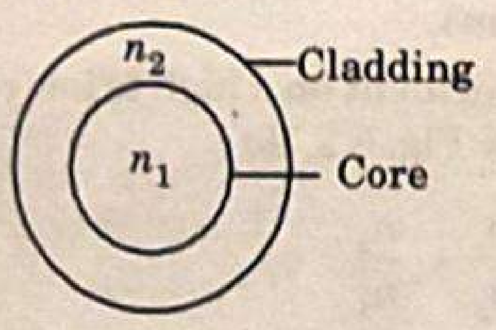


Fig. 5.2.2.

Que 5.3. **What do you mean by acceptance angle and acceptance cone ? Obtain the expression for acceptance angle.**

Answer

1. Acceptance angle is defined as the maximum angle of incidence at the end face of an optical fibre for which the ray can be propagated in the optical fibre. This angle is also called acceptance cone half-angle.
2. A cone obtained by rotating a ray at the end face of an optical fibre, around the fibre axis with acceptance angle is known as acceptance cone.

A. Expression for Acceptance Angle :

1. Applying Snell's law at points *B* and 0°.

$$n_1 \sin(90° - \theta_1) = n_2 \sin 90°$$
$$n_1 \cos\theta_1 = n_2$$

$$\cos\theta_1 = \frac{n_2}{n_1}$$

or
$$\sin\theta_1 = \sqrt{1-\cos^2\theta_1}$$
$$= \sqrt{1-\frac{n_2^2}{n_1^2}} \qquad ...(5.3.1)$$

2. Snell's law at *O*,

$$n_0 \sin\theta_0 = n_1 \sin\theta_1$$

or
$$\sin\theta_0 = \frac{n_1}{n_0}\sin\theta_1 \qquad ...(5.3.2)$$

3. On substituting equation (5.3.1) in equation (5.3.2),

$$\sin\theta_0 = \frac{n_1}{n_0}\sqrt{1-\frac{n_2^2}{n_1^2}} = \frac{\sqrt{n_1^2 - n_2^2}}{n_0} \qquad ...(5.3.3)$$

4. As the fibre is in air.

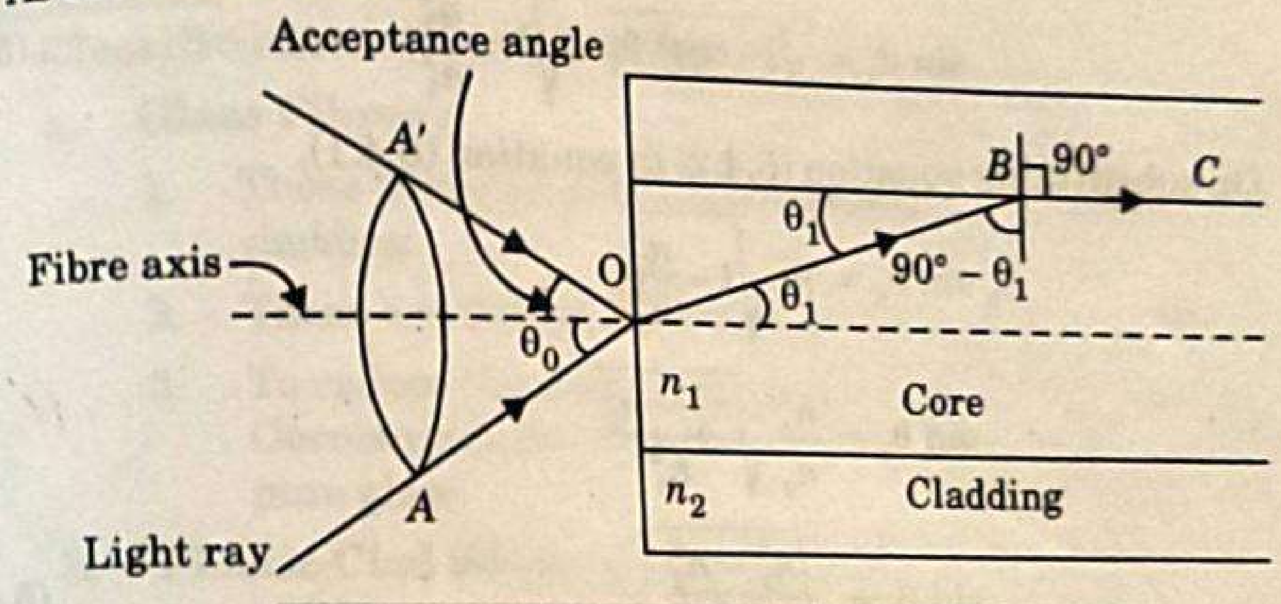


Fig. 5.3.1. Light propagation in an optical fibre.

So, the refractive index $n_0 = 1$

5. The equation (5.3.3) becomes,

$$\sin\theta_0 = \sqrt{n_1^2 - n_2^2} \qquad ...(5.3.4)$$

This is the equation for acceptance angle.

Que 5.4. **What is numerical aperture ? Derive the expression for it.**

Answer

A. Numerical Aperture (*NA*) :

1. Numerical aperture represents the light-gathering capacity of an optical fibre.
2. Light-gathering capacity is proportional to the acceptance angle, θ_0.
3. So, numerical aperture can be the sine of acceptance angle of the fibre *i.e.*, $\sin\theta_0$.

B. Expression for Numerical Aperture (*NA*) :

1. Expression for numerical aperture can be obtained by applying Snell's law at *O* and *B* in Fig. 5.3.1.
2. Let n_1, n_2 and n_0 be the refractive indices of core, cladding and the surrounding medium (air), respectively.
3. Applying Snell's law at the point of entry of the ray [*i.e.*, at *O*],

We have,

$$n_0 \sin \theta_0 = n_1 \sin \theta_1 \quad \text{...(5.4.1)}$$

4. At point *B* on the core-cladding interface,
The angle of incidence = $90° - \theta_1$.
Applying Snell's law at *B*, we have

$$n_1 \sin (90° - \theta_1) = n_2 \sin 90°$$
$$n_1 \cos \theta_1 = n_2$$
$$\cos \theta_1 = \frac{n_2}{n_1}$$
$$\sin \theta_1 = \sqrt{1 - \cos^2 \theta_1} = \sqrt{1 - \frac{n_2^2}{n_1^2}} \quad \text{...(5.4.2)}$$

5. On substituting equation (5.4.2) in equation (5.4.1),

$$n_0 \sin \theta_0 = n_1 \sqrt{1 - \frac{n_2^2}{n_1^2}}$$
$$\sin \theta_0 = \frac{n_1}{n_0} \sqrt{\frac{n_1^2 - n_2^2}{n_1^2}}$$
$$\sin \theta_0 = \sqrt{\frac{n_1^2 - n_2^2}{n_0^2}} \quad \text{...(5.4.3)}$$

6. If the surrounding medium of the fibre is air, then $n_0 = 1$.

So, $\sin \theta_0 = \sqrt{n_1^2 - n_2^2}$

7. According to the definition for numerical aperture (NA),

$$NA = \sin \theta_0 = \sqrt{n_1^2 - n_2^2} \quad \text{...(5.4.4)}$$

8. Let the fractional change in the refractive index (Δ) be the ratio between the difference in refractive indices of core and cladding to the refractive index of core.

i.e., $$\Delta = \frac{n_1 - n_2}{n_1} \quad \text{...(5.4.5)}$$

or $$n_1 - n_2 = \Delta n_1 \quad \text{...(5.4.6)}$$

9. Equation (5.4.4) can be written as :

$$NA = \sqrt{n_1^2 - n_2^2} = \sqrt{(n_1 - n_2)(n_1 + n_2)} \quad \text{...(5.4.7)}$$

10. Substituting equation (5.4.6) in equation (5.4.7), we get

$$NA = \sqrt{\Delta n_1 (n_1 + n_2)}$$

11. Since $n_1 = n_2$; so, $n_1 + n_2 \approx 2n_1$

$$\therefore \quad NA = \sqrt{2\Delta n_1^2} = n_1 \sqrt{2\Delta} \quad \text{...(5.4.8)}$$

12. Numerical aperture can be increased by increasing 'Δ' and thus enhances the light-gathering capacity of the fibre.
13. We cannot increase Δ to a very large value because it leads to intermodal dispersion, which cause signal distortion.

Que 5.5. **Write the classification of optical fibres.**

OR

Describe various types of optical fibre based on modes and core refractive index.

UPTU 2011-12, Marks 05

Answer

A. Classification of Optical Fibre Depending on Material used :

a. Glass Fibres :

1. These fibres consist of glass as the core and also glass as the cladding.
2. These are the most widely used fibres.
3. To reduce the refractive index of cladding impurities such as Germanium, Boron, Phosphorous or Fluoride are added to the pure glass.

b. Plastic Clad Silica or P.C.S. Fibres :

1. By replacing the cladding with a plastic coating of the refractive index lower than that of core, a plastic clad fibre is achieved.
2. Its advantage is only that the replacement of the glass cladding with plastic offers the saving in cost.
3. The limitations are :
   i. Losses are more than the glass fibres.
   ii. Refractive index varies with temperature.
   iii. Fibre life is small, mainly in humid environment.

c. Plastic Fibres :

1. These fibres consist of both core and cladding of the plastic material.
2. These fibres are cheaper in comparison to the above fibres.
3. But these fibres have high losses and low bandwidth.

4. Also life of these fibres is small and refractive index varies with temperature.
5. These fibres don't need protective coating and they are more flexible.
6. Attenuation of plastic fibres is more than glass or silica fibres but even then they are frequently used for short distance computer applications.

B. Classification of Optical Fibres Depending on Number of Modes :

a. Monomode or Single Mode Fibre :

1. In this, fibre is capable of transmitting only one mode.
2. Suppose we make the core of the fibre for any small ray of order of 2 to 8 μm, then only one ray of light can enter the core and get guided by total internal reflection.
3. Major advantage of single mode fibre is that it exhibits minimum dispersion loss and hence, the highest transmission bandwidth.
4. Only high-quality laser sources that produce a very focused beam of nearly monochromatic light can be used for single-mode operation.
5. Because of the superior transmission characteristics, such fibres are extensively used for long-distance applications.

b. Multimode Fibres :

1. In this, the fibre is capable of transmitting more than one mode, so the name multimode fibre.
2. The multimode fibre has the core diameter of the order of 50 μm *i.e.*, larger than the monomode fibre.
3. As the core radius is large enough, it accommodates many different ray of light or modes, each entering the core at different angles.
4. Since the different mode have different groups velocities, there exists considerable broadening of transmitted light pulses.
5. Hence, dispersion losses are more and bandwidth length product is small of order of 1 GHz-km.
6. These fibres are useful for moderate distances.
7. The loss of information capacity, however is compensated by certain benefits of multimode fibres over monomode fibre such as :
   i. Incoherent optical source can be used in multimode fibre due to large core diameter and large acceptance angle.
   ii. Ease of splicing or joining.
   iii. Lower tolerance requirements on fibre connectors.

C. Classification of Optical Fibres Depending on the Index Profile :

a. Multimode Step Index Fibre (MMSI) :

1. It consists of a core material surrounded by a concentric layer of cladding material with a uniform index of refraction n_2 that is only slightly less than that of core of refractive index n_1.
2. If the refractive index is plotted against the radial distance from the core, the refractive index abruptly changes at the core-cladding surface creating a STEP, hence the name step index.

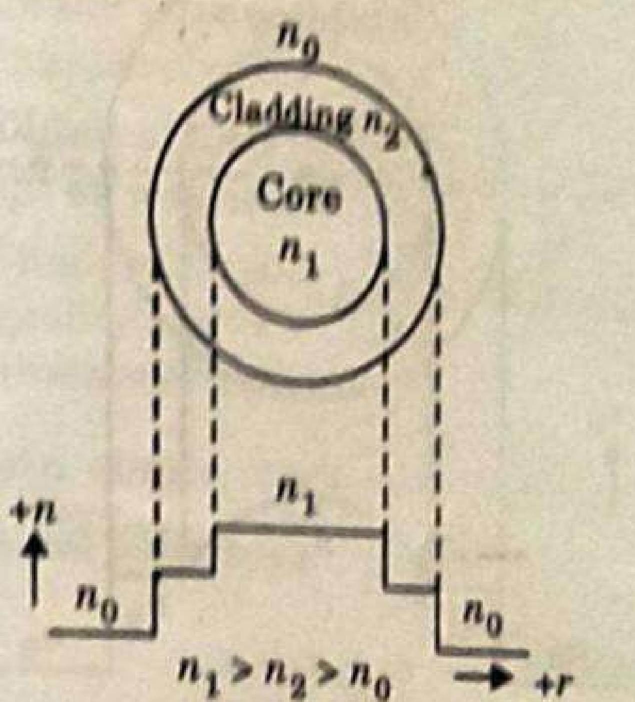


Fig. 5.5.1.

3. The name step index is due to this index profile and the term multimode is due to its feature of propagating a number of modes.
4. Its manufacturing is such that its core radius is large enough to accommodate many different rays of light or mode each entering the core at different angles.

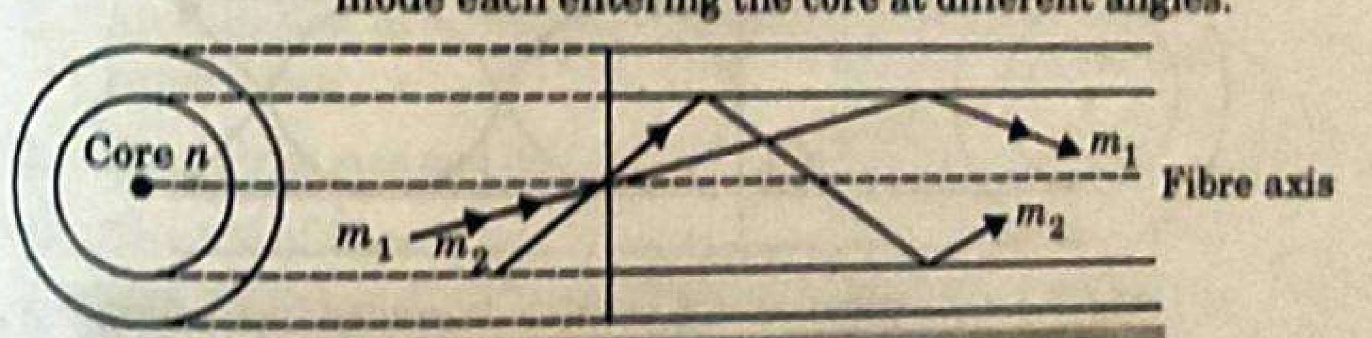


Fig. 5.5.2. Propagation in multimode step index fibre.

b. Multimode Graded Index Fibre (MMGIF) :

1. In this, the material in the core is modified so that the refractive index profile does not exhibit step index change but a parabolic refractive index profile which is maximum at the fibre axis.
2. In this fibre, index of refraction has a maximum value n_1 at the axis and lesser values falling off gradually and hence the name graded index is given to this fibre.

3. Since the light travels faster in a medium with lower refractive index, the light ray, which is farther from the fibre axis travels faster than the ray which is nearer to the axis.
4. As the refractive index is continuously changing across the fibre axis, the light ray is bent towards the fibre axis in almost sinusoidal fashion.
5. Light rays are curved towards the fibre axis by refraction.

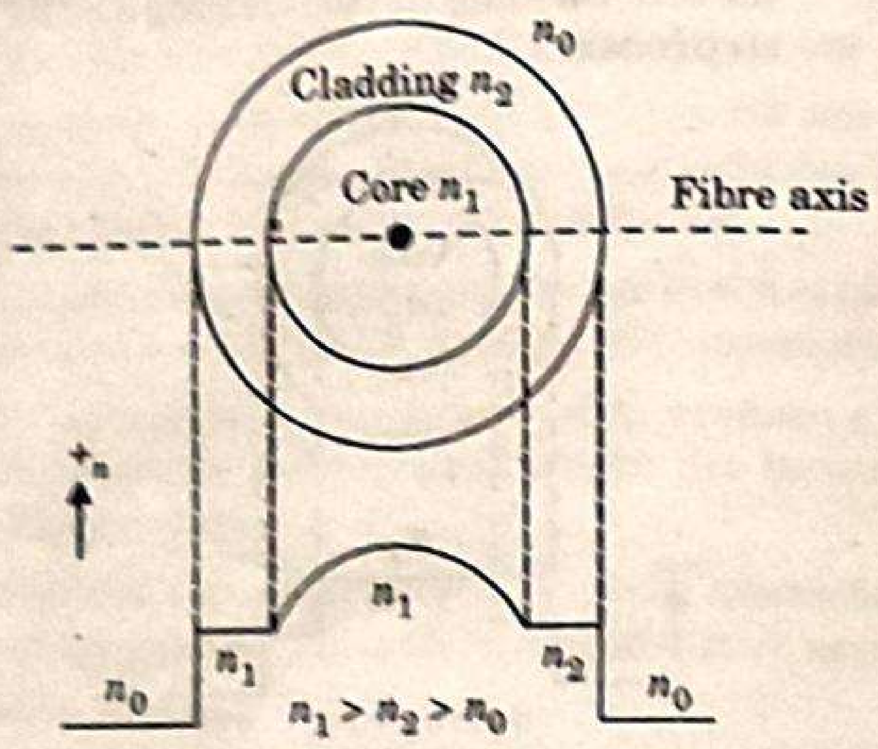


Fig. 5.5.3.

6. Light rays periodically diverge and converge along the length of the fibre.

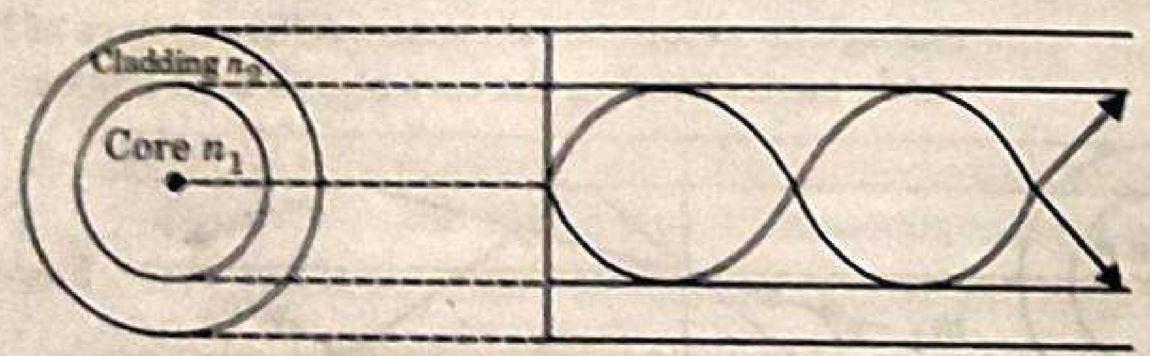


Fig. 5.5.4. Propagation in a multimode graded index fibre.

c. **Single Mode Step Index Fibre (SMSIF) :**

1. In this fibre, the core of a fibre is made so small that only one ray of light can enter the core and get guided by the total internal reflection hence the name single mode.
2. This will be the only ray of light or mode that can enter the core at such a shallow angle.

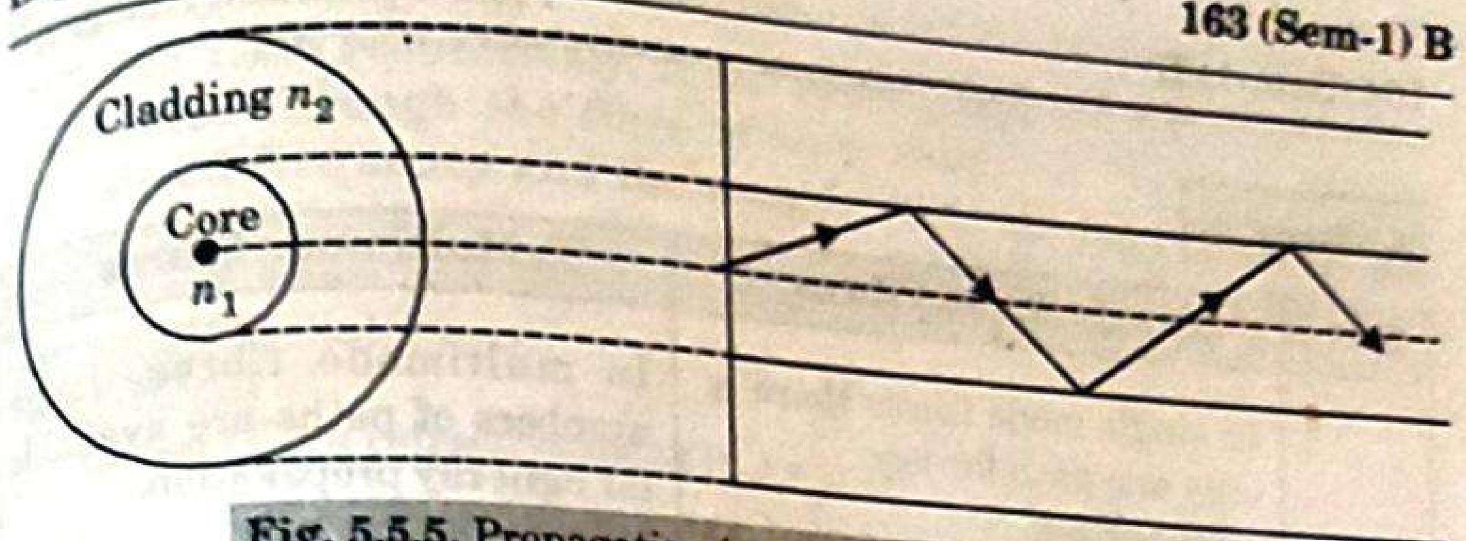


Fig. 5.5.5. Propagation in a single mode step index fibre.

3. Major advantage of this fibre is that modal dispersion is totally eliminated and because of this, such fibres are extensively used for long distance communication.
4. Different fibre designs have a specific wavelength called cut off wavelength above which it carries only one mode.
5. Single mode step index fibre has a superior transmission quality over other fibre types of the above because of the absence of modal dispersion.

Que 5.6. **Explain single mode and multimode fibre. What are advantages of optical fibre over copper wire ?**

UPTU 2009-10, Marks 05

Answer

A. **Single Mode and Multimode Fibre :** Refer Q. 5.5, Page 159B, Unit-5.

B. **Advantages of Optical Fibre over Copper Wire :**

1. The information carrying capacity of a fibre is much greater than the microwave radio system.
2. Attenuation in optical fibre is much lower than that of coaxial cable or twisted pair.
3. Smaller in size and lighter in weight.
4. The life of fibre is longer than corresponding copper wire.
5. Fibre communication system is more reliable as it can better withstand environmental conditions.
6. The cost per channel is lower than that of metal counterpart.
7. Handling and installation cost of optical fibre system is very nominal.

Que 5.7. **Differentiate between single mode fibres and multimode fibres.**

Answer

S. No.	Single Mode Fibres	Multimode Fibres
1.	In single mode fibres there is only one path for ray propagation.	In multimode fibres, large numbers of paths are available for light ray propagation.
2.	Single mode step index fibres have less core diameter (<10 μm) and the difference between the refractive indices of core and cladding is very small.	Multimode step index fibres have larger core diameter (50 to 200 μm) and the difference between the refractive indices of core and cladding is large.
3.	In single mode fibres, there is no dispersion.	There is signal distortion and dispersion takes place in multimode fibres.
4.	Signal transmission capacity is less but the single mode fibres are suitable for long distance communication.	Signal transmission capacity is more in multimode fibres. Because of large dispersion and attenuation, they are less suitable for long distance transmission.
5.	Launching of light into single mode fibres is difficult.	Launching of light into multi mode fibres is easy.
6.	Fabrication cost is very high.	Fabrication cost is less.

Que 5.8. **If refractive indices of core and cladding of an optical fibre are 1.50 and 1.45 respectively determine the values of numerical aperture, acceptance angle and critical angle of the fibre.**

UPTU 2014-15, Marks 05

Answer

1. Numerical aperture,

$$NA = n_1\sqrt{(2\Delta)}$$

where $$\Delta = \frac{n_1 - n_2}{n_1} = \frac{1.50 - 1.45}{1.50} = 0.033$$

2. So $$NA = 1.50\sqrt{(2 \times 0.033)} = 1.50 \times 0.257 = 0.385$$

3. Acceptance angle, $\alpha = \sin^{-1}(NA) = \sin^{-1}(0.385) = 22.64°$

4. According to Snell's law

$$\sin\theta_c = \frac{n_2}{n_1} \text{ or } \theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}\left(\frac{1.45}{1.50}\right)$$

or $$\theta_c = \sin^{-1}(0.967) = 75.3°$$

Que 5.9. **A step index fibre has core and cladding refractive indices 1.466 and 1.460 respectively. If the wavelength of light 0.85 μm is propagated through the fibre of core diameter 50 μm, find the normalized frequency and the number of mode supported by the fibre.**

UPTU 2012-13, Marks 05

Answer

1. **Given :** Core refractive index $(n_1) = 1.466$

Cladding refractive index $(n_2) = 1.460$

Wavelength $(\lambda) = 0.85\ \mu m$

$$\text{Core radius, } a = \frac{d}{2} = \frac{50}{2} = 25\ \mu m$$

2. Normalized frequency is given by,

$$v = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2}$$

$$= \frac{2 \times \pi \times 25}{0.85}\sqrt{(1.466)^2 - (1.460)^2} = 24.48 \text{ Hz}$$

3. Number of guided modes :

$$N = \frac{v^2}{2} = \frac{(24.48)^2}{2} = 299.635 \approx 300$$

Que 5.10. **Describe the basic principle of communication of wave in optical fibre. A step index fibre has core refractive index 1.468, cladding refractive index 1.462. Compute the maximum radius allowed for a fibre, if it supported only one mode at a wavelength 1300 nm.**

UPTU 2015-16, Marks 05

Answer

A. **Basic Principle of Communication :** Refer Q. 5.2, Page 155B, Unit-5.

B. **Numerical :**

1. **Given :** Fibre supported only one mode.

So, $N_{max} = 1$

Wavelength, $\lambda = 1300$ nm

Core refractive index, $n_1 = 1.468$

Cladding refractive index, $n_2 = 1.462$

2. Number of modes supported, $N = \frac{v^2}{2}$

$$1 = \frac{v^2}{2}$$

$$\nu = 1.414$$

3. Let a is radius allowed for a fibre.

$$\nu = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2}$$

$$1.414 = \frac{2 \times 3.14 \times a}{1300 \times 10^{-9}}\sqrt{(1.468)^2 - (1.462)^2}$$

$$292.7 \times 10^{-9} = a\sqrt{(1.468)^2 - (1.462)^2}$$

$$a = 2.2 \times 10^{-6}$$

$$a = 2.2\ \mu m$$

Que 5.11. **Explain attenuation in optical fibre.**

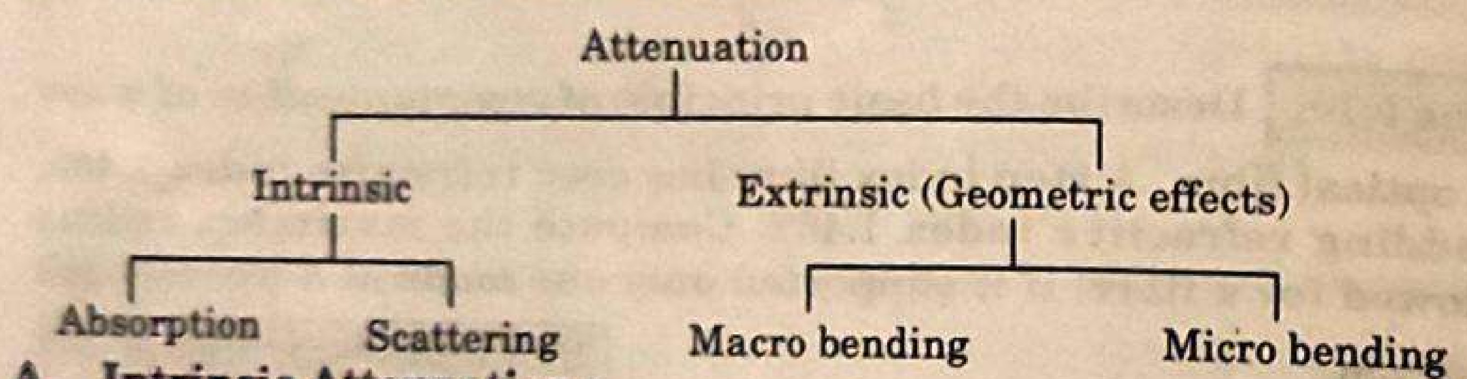
OR

What do you understand by attenuation in optical fibre ? Discuss the important factors responsible for the loss of power in optical fibre.

UPTU 2010-11, Marks 05

Answer

1. Attenuation is defined as the loss in the strength of signal due to propagation of wave in optical fibres.
2. Attenuation can be classified as follows :



A. **Intrinsic Attenuation :**

a. **Intrinsic Absorption :**

1. It is a natural property of glass itself.
2. It is of two types :

i. **Ultraviolet Absorption :**

1. It is due to electronic and molecular transition band.
2. Ultraviolet absorption takes place because for pure fused silica, valence electrons can be ionized into conduction by the light and the energy for this ionization is drawn from the light fields being propagated and constitutes a transmission loss.
3. The ultraviolet absorption loss does not occur at fixed wavelength.
4. This loss occurs over a broadband extending up into the visible part of spectrum.

5. The loss decreases with increasing wavelength and becomes negligible in the 1.2 to 1.3 μm band.
6. Absorption loss increases by introduction of impurity.

ii. **Infrared Absorption :**

1. It is due to the vibration of chemical bonds and it takes place because photons of the light energy are absorbed by the atoms within the glass molecules and converted to the random mechanical vibrations to typical of heating.
2. This loss prohibits the use of silica fibres beyond 1.6 μm.

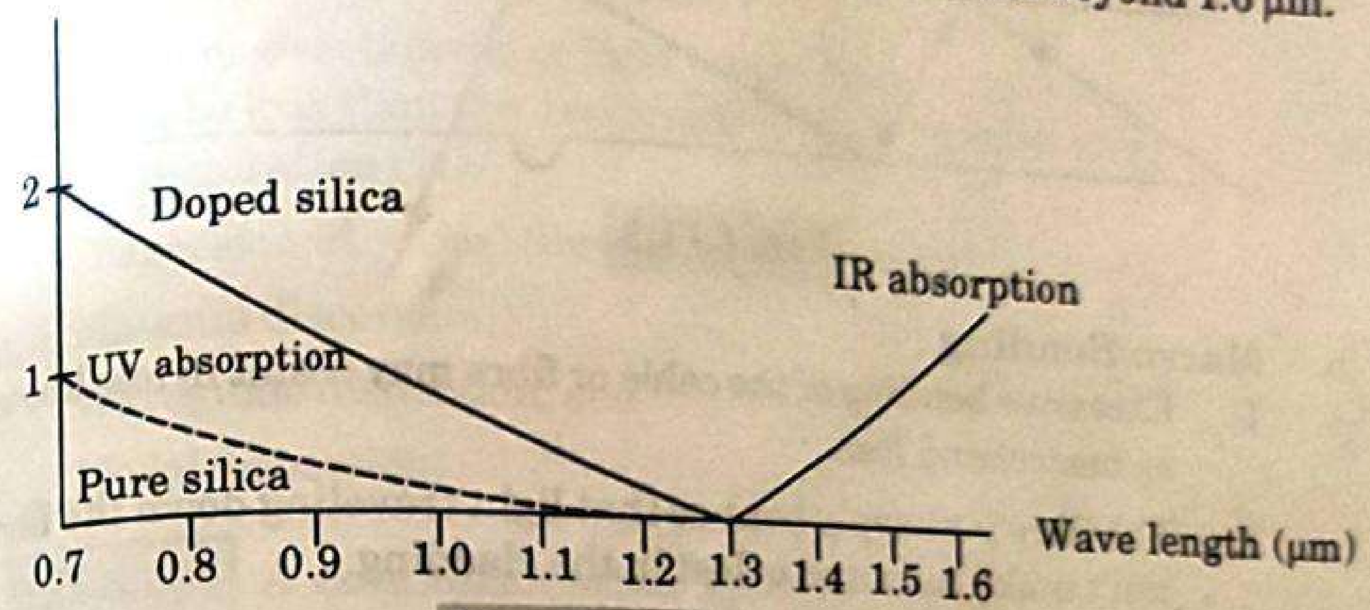


Fig. 5.11.1.

b. **Scattering :**

1. It is the loss of optical energy due to imperfections in the fibre.
2. Due to this phenomenon, the light is scattered in all directions which causes the loss of the optical power in the forward direction.
3. This loss is known as Rayleigh scattering loss.
4. Rayleigh scattering loss is found to be inversely proportional to the fourth power of the light wavelength.

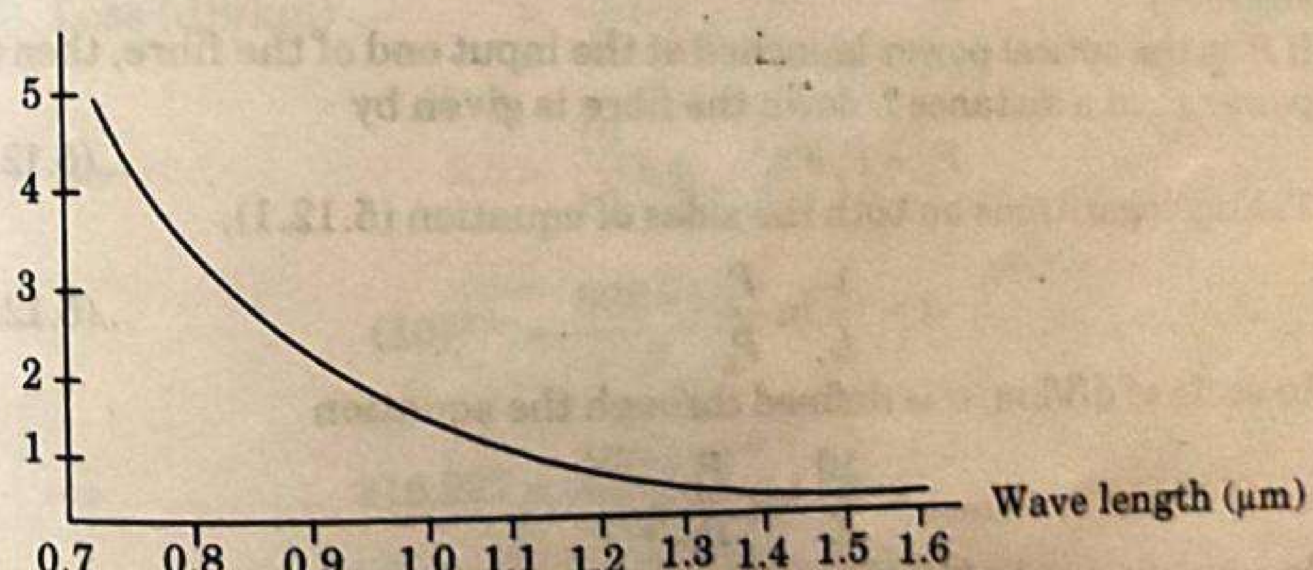


Fig. 5.11.2.

B. **Extrinsic Attenuation :**

1. It is the loss of optical power to external sources.

2. Extrinsic attenuation occurs mainly due to geometric effects which are caused by the bending of the fibre.

a. **Micro Bending :**

1. Micro bending losses are caused either during the manufacturing or during the cabling process.
2. Microbends may not be visible with the naked eyes.
3. During the manufacturing the microscopic bending of the core of the fibre occurs due to thermal contraction between the core and cladding.

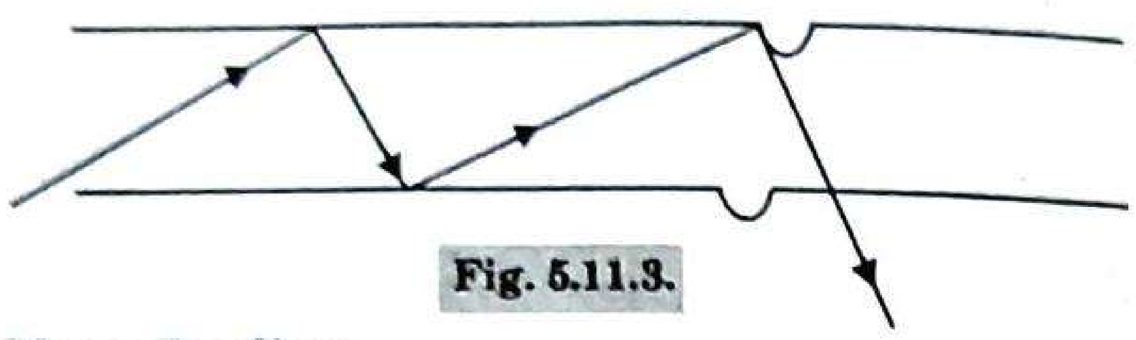


Fig. 5.11.3.

b. **Macro Bending :**

1. Excessive bending of the cable or fibre may result in loss known as macrobend loss.
2. The fibre is sharply bent so that light traveling down the fibre can't make turn and is lost in the cladding.

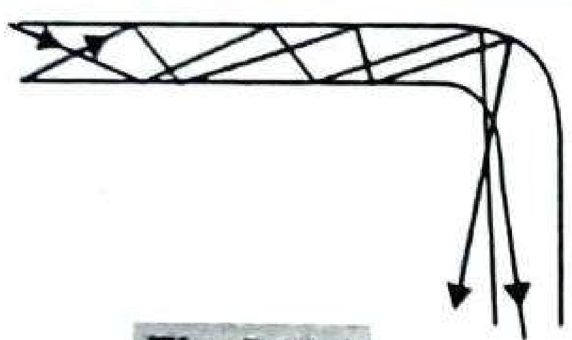


Fig. 5.11.4.

Que 5.12. **Explain attenuation constant.**

Answer

1. If P_i is the optical power launched at the input end of the fibre, then the power P_0 at a distance L down the fibre is given by
$$P_0 = P_i e^{-\alpha L} \quad ...(5.12.1.)$$
2. Taking logarithms on both the sides of equation (5.12.1),
$$\alpha = \frac{1}{L} \ln \frac{P_i}{P_0} \quad ...(5.12.2.)$$
3. In units of dB/km, α is defined through the equation
$$\alpha_{dB/km} = \frac{10}{L} \log \frac{P_i}{P_0}$$
4. In case of an ideal fibre, $P_0 = P_i$ and the attenuation would be zero.

Que 5.13. **Write a short note on dispersion.**

Answer

1. Dispersion is the time distortion of an optical signal that results from the time of flight differences of different component of that signal, typically resulting in the pulse broadening.
2. In digital transmission, dispersion limits the maximum data rate, the maximum distance, or the information carrying capacity of a single-mode fibre link.
3. In analog transmission, dispersion can cause a waveform to become significantly distorted and can result in unacceptable levels of composite second order distortion (CSO).
4. When no overlapping of light pulses takes place, the digital bit rate BT must be less than the reciprocal of the broadened (through dispersion) pulse duration (2τ).
5. Dispersion in optical fibres can be classified into three main types :

a. Material dispersion,

b. Waveguide dispersion, and

c. Modal dispersion.

Que 5.14. **A communication system uses a 10 km fibre having a loss of 2.5 dB/km. Compute the output power if the input power is 500 μW.**

UPTU 2011-12, Marks 05

Answer

1. **Given :** $L = 10$ km, loss = 2.5 dB/km
$$P_{in} = 500 \times 10^{-6} \text{ W}$$
2. We know that loss in fibre,
$$\text{Loss (dB/km)} = \frac{10}{L} \log_{10} \frac{P_{in}}{P_{out}}$$
3. So
$$2.5 = \frac{10}{10} \log_{10} \frac{500}{P_{out}} \times 10^{-6}$$
$$(10)^{(2.5)} = \frac{500 \times 10^{-6}}{P_{out}}$$
$$\Rightarrow \quad 316.227 = \frac{500 \times 10^{-6}}{P_{out}}$$
4.
$$P_{out} = \frac{500 \times 10^{-6}}{316.227} = 1.58 \ \mu\text{W}$$

PART-2

Basic principle of Holography, Construction and reconstruction of image on hologram and Applications of holography.

CONCEPT OUTLINE : PART-2

Holography : It is a method of producing a three dimensional image of an object employing the coherence properties of a laser beam.

Types of Holograms :

1. **Transmission Hologram :** This hologram is made by the light from object and reference beam approaching the film from same side. It is visible with laser light.
2. **Reflection Hologram :** It is made by two light rays approaching the holographic film from opposite side. It is visible with white light (like spot light, flash light etc.).
3. **Multiplex Hologram :** A large number of flat pictures of an object viewed from different angles are combined into a single 3-D image of the object.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.15. **What is the basic principle of holography ? Explain.**

Answer

1. An object is illuminated with a beam of coherent light. Then every point on the surface of the object acts as a source of secondary waves.
2. These secondary waves spread in all directions. Some of these waves are allowed to fall on a holographic plate.
3. Simultaneously, another beam of same coherent light is allowed to fall on this holographic plate.
4. In the holographic plate, both the beams combine and interference pattern will be formed.
5. This interference pattern is recorded on the holographic plate.
6. The 3-D image of the object can be seen by exposing the recorded holographic plate to coherent light.

Que 5.16. **Discuss the construction and reconstruction of image in a hologram.**

Answer

A. **Construction :**

1. The monochromatic light from a laser has been passed through a 50 % beam splitter so that the amplitude division of the incident beam into two beams takes place.
2. One beam falls on mirror M_1 and the light reflect from M_1 falls on the object. This beam is known as an object beam.
3. The object scatters this beam in all directions, so that a part of the scattered beam falls on the holographic plate.
4. The other beam is reflected by mirror M_2 and falls on the holographic plate. This beam is known as reference beam.
5. Superposition of the scattered rays from the object and the reference beam takes place on the plane of the holographic plate, so that interference pattern is formed on the plate and it is recorded.
6. The recorded interference pattern contains all the information of the scattered rays *i.e.*, the phases and intensities of the scattered rays.
7. For proper recording, the holographic plate has to be exposed to the interference pattern for a few seconds.
8. After exposing, the holographic plate is to be developed and fixed as like in the case of ordinary photograph.
9. The recorded holographic plate is known as hologram.
10. The hologram does not contain a distinct image of the object. It contains information in the form of interference pattern.
11. Fig. 5.16.1 shows the method of recording an image on a holographic plate.

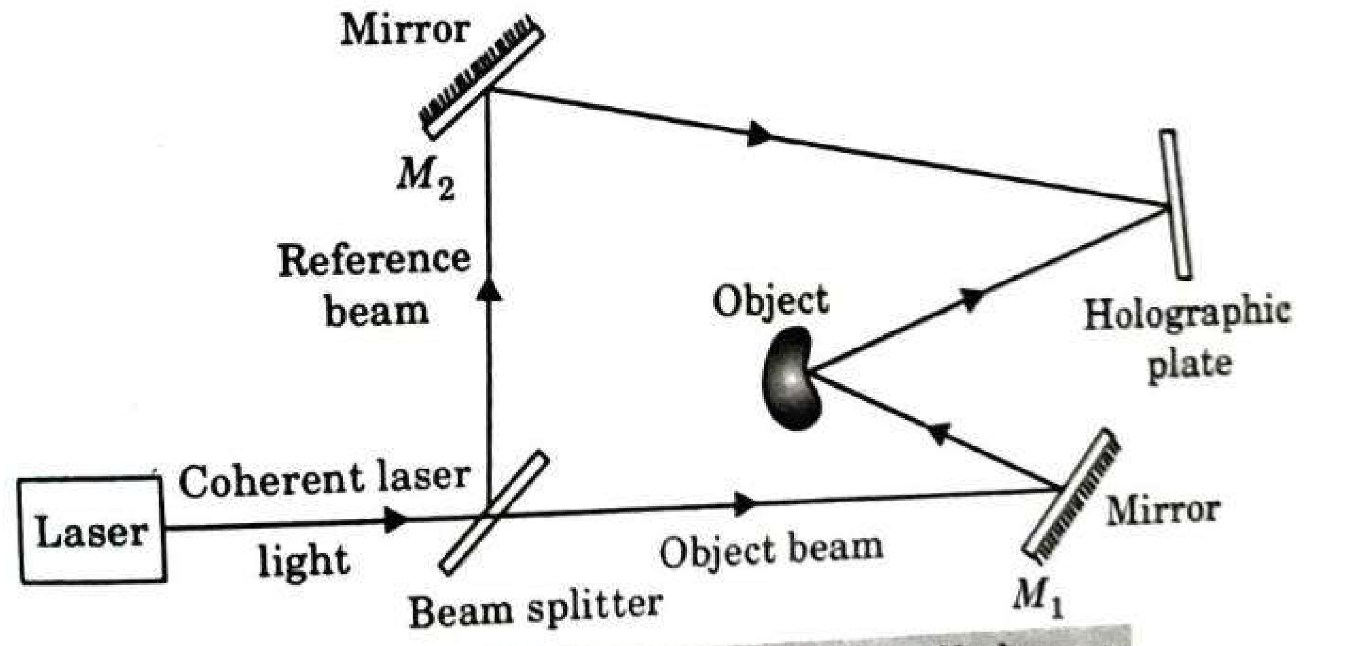


Fig. 5.16.1. Recording of hologram.

B. Reconstruction :

1. As shown in Fig. 5.16.2, the hologram is exposed to the laser beam from one side and it can be viewed from the other side.

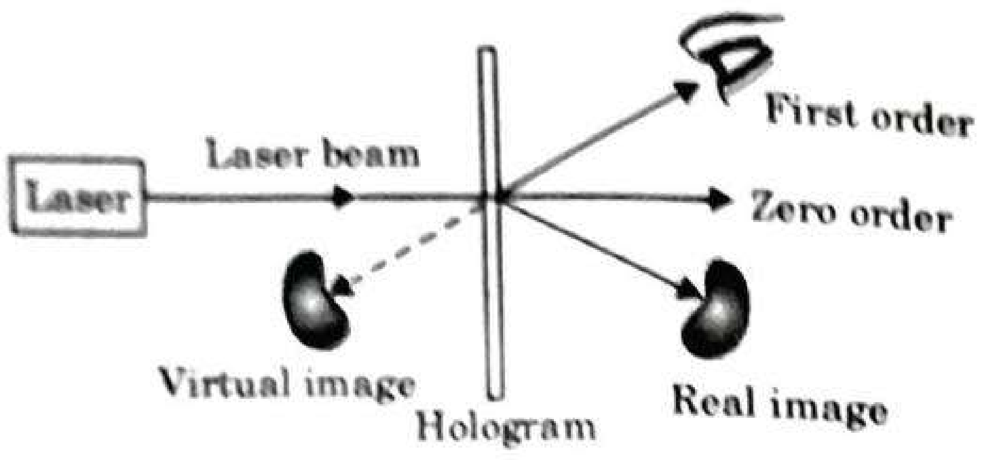


Fig. 5.16.2. Image reconstruction.

2. This beam is known as reconstruction beam.
3. The reconstruction beam illuminates the hologram at the same angle as the reference beam.
4. The hologram acts as a diffraction grating, so constructive interference takes place in some directions and destructive interference takes place in other direction.
5. A real image is formed in front of the hologram and a virtual image is formed behind the hologram.
6. It is identical to the object and hence it appears as if the object is present. The 3-D effect in the image can be seen by moving the head of the observer.
7. During recording, the secondary waves from every point of the object reach complete plate.
8. So, each bit of the plate contains complete information of the object.
9. Hence, image can be constructed using a small piece of hologram.

Que 5.17. **Write short notes on :**

a. **Electron holography.**
b. **Acoustic holography.**
c. **Atomic holography.**

Answer

a. Electron Holography :

1. Electron holography is the application of holography techniques to electron wave rather than light waves.
2. Electron holography was invented by Dennis Gabor to improve the resolution and avoid the aberrations of the transmission in electron microscope.

3. Today it is commonly used to study electric and magnetic fields in thin films, as magnetic and electric fields can shift the phase of the interfering wave passing through the sample.
4. The principle of electron holography can also be applied to interference lithography.

b. Acoustic Holography :

1. Acoustic holography is a method used to estimate the sound field near a source by measuring acoustic parameters away from the source via an array of pressure and / or particle velocity transducers.
2. Measuring techniques included within acoustic holography are becoming increasingly popular in various fields, most notably those of transportation, vehicle and aircraft design.
3. The general idea of acoustic holography has led to different versions such as near-field acoustic holography (NAH) and statistically optimal near-field acoustic holography (SONAH).

c. Atomic Holography :

1. Atomic holography has evolved out the development of the basic elements of atom optics.
2. With the Fresnel diffraction lens and atomic mirrors, atomic holography follows a natural step in the development of the physics (and applications) of atomic beams.
3. Recent developments including atomic mirrors and specially ridged mirrors have provided the tools necessary for the creation of atomic holograms, although such holograms have not yet been commercialized.

Que 5.18. **What are the applications of holography ?**

Answer

1. The 3-D images produced by holograms have been used in various fields, such as technical, educational also in advertising, artistic display etc.
2. Hologram is a reliable object for data storage, because even a small broken piece of hologram contains complete data or information about the object with reduced clarity.
3. In hospitals, holography can be used to view the working of inner organs three dimensionally. *i.e.*, the beating of the heart, the fetus of the pregnant lady and flowing blood based on motion holography.
4. Holographic interferometry is used in non-destructive testing of materials to find flaws in structural parts and minute distortions due to stress or vibrations, etc. in the objects.
5. Holography is used in information coding.
6. Many museums have made holograms of valuable articles in their collections.

☺☺☺

1 UNIT

Relativistic Mechanics (2 Marks Questions)

Memory Based Questions

1.1. What is frame of reference ?

Ans. A coordinate system with respect to which we measure the position of a point object of an event is called a frame of reference.

1.2. Define inertial frame of reference.

Ans. Inertial frame of reference is defined as the frame in which a body is at rest or moving with uniform velocity and is not under any force.

1.3. What are non-inertial frames ?

Ans. The frames of reference with respect to which an unaccelerated body appears accelerated are called non-inertial frames.

1.4. What was the aim of Michelson-Morley experiment ?

Ans. The aim of this experiment was to prove the existence of the ether and to test whether the ether is fully or partially dragged with bodies moving in it.

negative result

1.5. What are the conclusions of Michelson-Morley experiment ?

Ans.
a. There is no existence of hypothetical medium ether.
b. The velocity of light in all inertial frames of reference remains constant.

1.6. What are the Einstein postulates of special theory of relativity ?

Ans. **Postulate I :** The principle of equivalence.
Postulate II : The principle of constancy of the speed of light.

1.7. What do you understand by variant and invariant under the Galilean transformation ?

[UPTU 2011-12, Marks 02]

Ans. Variant means the physical quantities which change from one frame of reference to another frame of reference. E.g. velocity. Invariant means the physical quantities which do not change from one frame of reference to another frame of reference. Example : distance between two points is invariant in true inertial frames.

1.8. What do you mean by Lorentz transformation ?

Ans. The equations in special theory of relativity, which relate to the space and time coordinates of an event in two inertial frames of reference moving with a uniform velocity relative to one-another, are called Lorentz transformations.

1.9. What is the conclusion of Lorentz transformation ?

Ans. The conclusion of Lorentz transformation is that it limits the maximum velocity of the material bodies.

1.10. Write down the inverse Lorentz transformation equations.

Ans. Lorentz inverse transformation equations are :

$$x = \frac{x' + vt'}{\sqrt{1 - v^2/c^2}}, \; y = y', \; z = z' \text{ and } t = \frac{t' + vx'/c^2}{\sqrt{1 - v^2/c^2}}$$

1.11. Define length contraction.

Ans. The length of a moving rod will appear to be contracted if it is seen from a frame of reference which is at rest. This decrease in length in the direction of motion is called length contraction.

1.12. What do you understand by time dilation ?

[UPTU 2013-14, Marks 02]

Ans. In the special theory of relativity, the moving clock is found to run slower than a clock at rest does. This effect is known as time dilation.

1.13. The proper length of each edge of a cube is l_0. When it is moving with a velocity v along one of its edges, what will be the volume of a cube ?

Ans. $l_0^3\sqrt{1 - \frac{v^2}{c^2}}$

1.14. Give the Einstein's mass-energy relation.

Ans.
$$E = mc^2$$
This relation is known as Einstein's mass-energy relation.

1.15. Give some examples of mass-energy equivalence.

Ans. Some important examples of the mass-energy equivalence are as follows :
a. Pair production phenomenon
b. Annihilation phenomenon (production of γ-rays photon)

c. Nuclear fusion
d. Nuclear fission

1.16. What are massless particles ?

[UPTU 2012-13, 2013-14; Marks 02]

Ans. A particle which has zero rest mass (m_0) is called a massless particle. In relativistic mechanics, a particle with zero rest mass can exist.
For massless particle, $m_0 = 0$
As the relativistic total energy E of a particle of rest mass (m_0) in terms of its momentum p may be expressed as

$$E = \sqrt{m_0^2 c^4 + p^2 c^2}$$

$$\because \quad m_0 = 0$$

$$\therefore \quad E = pc \qquad \text{or} \qquad p = E/c$$

As $p = \dfrac{E\text{v}}{c^2}$, therefore, $\text{v} = c$, *i.e.*, the velocity of the massless particle is same as that of light in free space. Thus, every massless particle has energy pc and momentum E/c, and moves with the velocity of light.

1.17. A particle of rest mass m_0 moves with speed $\dfrac{c}{\sqrt{2}}$. Calculate its mass.

Ans. $$m = \frac{m_0}{\sqrt{1-\dfrac{\text{v}^2}{c^2}}} = \frac{m_0}{\sqrt{1-\dfrac{(c/\sqrt{2})^2}{c^2}}} = \sqrt{2}m_0 = 1.42m_0$$

1.18. At what speed will the mass of a body be 2.25 times its rest mass.

Ans. The relativistic mass is given by

$$m = \frac{m_o}{\sqrt{1-\dfrac{\text{v}^2}{c^2}}}$$

Given $\quad m = 2.25\, m_o$

$$\therefore \quad 2.25\, m_o = \frac{m_o}{\sqrt{1-\dfrac{\text{v}^2}{c^2}}}$$

$$1-\frac{\text{v}^2}{c^2} = \frac{1}{(2.25)^2} = \frac{1}{5.0625}$$

$$\frac{\text{v}^2}{c^2} = 0.8024$$

$$\text{v} = 0.895c = 0.895 \times 3 \times 10^8 = 2.68 \times 10^8 \text{ m/s}$$

1.19. Which frames are known as accelerated of frames ?

Ans. Non inertial frames are known as accelerated frames.

1.20. Find relativistic relation between energy and momentum.

[UPTU 2015-16, Marks 02]

Ans. $E = PC$

Evaluation Based Questions

1.21. Why are Galilean transformations used ?

Ans. Galilean transformations are used to convert the laws of mechanics from one frame of reference to another frame, moving with constant velocity with respect to the first frame

1.22. Is earth an inertial frame of reference or not ?

Ans.

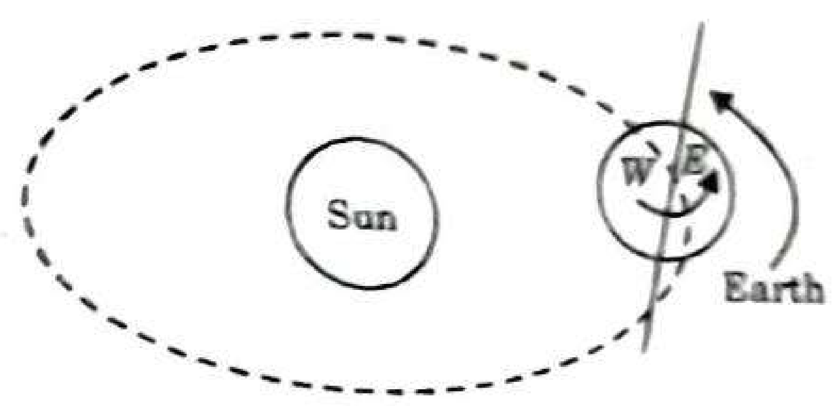


Fig. 1.22.1.

According to Newton's assumption earth is an inertial frame because for the study of any particle or body on earth, we can take earth as inertial reference frame.
But on other hand earth rotates about its axis as well as revolves around the sun in its orbit so it can also be treated as non-inertial frame of reference.

1.23. How the negative results of Michelson-Morley experiment interpreted ? **[UPTU 2015-16, Marks 02]**

Ans. Refer Q. 1.5, 2 Marks Questions, Unit-1.

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2 UNIT

Modern Physics (2 Marks Questions)

Memory Based Questions

2.1. What is a black body ?

Ans. A body which absorbs completely all the radiations incident upon it, reflecting none and transmitting none, is called a black body.

2.2. Define black body radiations.

Ans. When a black body is heated to a suitable high temperature it emits total radiations which are known as black body radiations.

2.3. Which body is assumed to be perfectly black ?

Ans. Lamp black is the nearest approach to black body which absorbs nearly 99 % of the incident radiation.

2.4. Define Wien's law.

Ans. Wien showed that the maximum energy, E_m of the emitted radiation from black body is proportional to fifth power of absolute temperature (T^5)

$$E_m \propto T^5$$

or $$E_m = \text{constant} \times T^5$$

2.5. What is Rayleigh-Jean's law ?

Ans. Rayleigh-Jean's law states that the total amount of energy emitted by a black body per unit volume at an absolute temperature T in the wavelength range λ and $\lambda + d\lambda$ is given as

$$u_\lambda d\lambda = \frac{8\pi kT}{\lambda^4} d\lambda$$

2.6. Define Planck's law ?

Ans. Planck's derived an equation for the energy per unit volume of black body in the entire spectrum of black body radiation. It is given by

$$u_\lambda d\lambda = \frac{8\pi hc}{\lambda^5}\left[\frac{1}{e^{hc/\lambda k_B T}} - 1\right] d\lambda$$

This is Planck's law.

2.7. What do you mean by wave particle duality ?

Ans. According to Einstein, the energy of light is concentrated in small bundles called photons. Hence, light behave as a wave on one hand and as a particle on the other hand. This nature of light is known as dual nature, while this property of light is known as wave particle duality.

2.8. What are the properties of matter waves ?

Ans. Following are the properties of matter waves :

a. Each wave of the group travel with a velocity known as phase velocity.

b. These waves cannot be observed.

c. The wavelength of these waves, $\lambda = \frac{h}{p}$

2.9. What is matter or de-Broglie waves ?

Ans. According to de-Broglie, a particle of mass m, moving with velocity v is associated with a wave called matter wave or de-Broglie wave.

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

2.10. Define group velocity.

Ans. The velocity with which the wave packet obtained by superposition of wave travelling in group is called group velocity

$$v_g = \frac{\Delta\omega}{\Delta K}$$

2.11. What is phase velocity ?

Ans. The average velocity of the individual monochromatic wave in the medium with which the wave packet is constructed is called wave velocity or phase velocity.

$$v_p = \frac{\omega}{k}$$

2.12. What was the main aim of Davisson – Germer experiment ?

Ans. The main aim of this experiment was to find the intensity of scattered electrons by a metal target in different directions.

2.13. Given the expression for phase velocity and group velocity and how they are related ?

Ans. Group velocity, $v_g = \frac{\Delta\omega}{\Delta K}$

Phase velocity, $v_p = \frac{\omega}{k}$

Relation between v_p and v_g :

$$v_p = \frac{c^2}{v_g}$$

2.14. Give the relation between v_g and v_p for non-dispersive medium.

Ans. $v_g = v_p$

The group velocity of a wave packet is equal to wave velocity or phase velocity.

1.15. Define Heisenberg's uncertainty principle.

Ans. According to Heisenberg's principle, it is impossible to measure the exact position and momentum of a moving particle simultaneously.

2.16. If uncertainty in the position of a particle is equal to de-Broglie wavelength, what will be uncertainty in the measurement of velocity ? **[UPTU 2015-16, Marks 02]**

Ans. **Given :** Uncertainty in the position of a particle = de-Broglie wavelength

$$\Delta x = \lambda = \frac{h}{p} \quad ...(2.16.1)$$

where, h = Planck's constant, and
p = momentum of particle.

According to Heisenberg's uncertainty principle,

$$\Delta x \, . \, \Delta p \geq \frac{h}{2\pi} \quad ...(2.16.2)$$

Let Δv be the uncertainty in velocity,

$$\Delta p = m.\Delta v \quad ...(2.16.3)$$

Putting equation (2.16.1) and equation (2.16.3) in equation (2.16.2)

$$\frac{h}{p} \cdot m \cdot \Delta v = \frac{h}{2\pi} \quad [\because p = mv]$$

So, uncertainty in velocity,

$$\Delta v = \frac{v}{2\pi}$$

2.17. Define wave function.

Ans. The quantity in quantum mechanics undergoes periodic changes and gives information about the particle within the wave packet. It is called wave function.

2.18. What do you mean by normalization of wave function ?

Ans. If the wave function Ψ of any system is such that it gives value of integral a finite quantity say 'N'

$$\int_{-\infty}^{\infty} \Psi \Psi^* dx = \int_{-\infty}^{\infty} |\Psi|^2 dx = N \text{ (integral)}$$

when Ψ^* = complex conjugate

Then, Ψ is called normalization of wave function.

2.19. Define orthogonal wave function.

Ans. When the value of the integral is equal to zero ($N = 0$), the wave function Ψ is known as orthogonal wave function.

2.20. What are eigen values and eigen functions ?

Ans. The values of energy for which steady state equation can be solved are called eigen values and the corresponding wave functions are called eigen functions.

2.21. Write the characteristics of wave function.

UPTU 2015-16, Marks 02

Ans.
a. The wave function Ψ contains all the measurable information about the particle.
b. It can interfere with itself. This property explains the phenomenon of electron diffraction.
c. The wave function Ψ permits the calculation of most probable value of a given variable.

Application Based Questions

2.22. What are the applications of Heisenberg's uncertainty principle ?

Ans. Following are the applications of Heisenberg's uncertainty principle :
a. Non-existence of electrons in the nucleus.
b. The zero point energy.
c. The radius of the Bohr's first orbit.
d. Finite width of spectral lines.
e. Binding energy of an electron in an atom.

Evaluation Based Questions

2.23. How can we obtain a perfect black surface is obtained ?

Ans. An ideal model of a perfectly black surface is obtained if a small hole is made in the opaque walls of a closed hollow cavity.

2.24. Why uncertainty principle has no significance in case of macroscopic objects ?

Ans. Uncertainty principle has no significance in case of macroscopic objects because the value of h (= 6.63×10^{-34} J-s) is extremely small.

2.25. What is the physical significance of wave function ?

Ans. The wave function Ψ itself has no physical significance but the square of its absolute magnitude $|\Psi|^2$ gives the probability of finding the particle at that time.

☺☺☺

3 UNIT

Wave Optics (2 Marks Questions)

Memory Based Questions

3.1. Define interference.

Ans. The modification in the intensity of light resulting from the superposition of two (or more) waves of light is called interference.

3.2. What do you mean by coherent sources ?

Ans. Two sources are said to be coherent if they emit continuous light waves of the same frequency or wavelength, nearly the same amplitude and having sharply defined phase difference that remains constant with time.

3.3. What are the different methods of obtaining coherent sources ?

Ans. Following are the different methods of obtaining coherent sources :

a. Young's double-slit experiment,
b. Fresnel's biprism experiment,
c. Lloyd's mirror, and
d. Thin film.

3.4. Explain interference in thin films.

Ans. When a thin film of transparent material like oil drop spread over the surface of water is exposed to an extended source of light, it appears coloured. This phenomenon can be explained as interference of thin films.

3.5. Define Newton's rings.

Ans. When a monochromatic light falls on the film, we get dark and bright concentric fringes having uniform thickness these rings are called Newton's rings.

3.6. Write down the conditions for bright and dark rings.

Ans. **Condition for Bright Rings :**

$$2t = (2m - 1)\lambda/2$$

Condition for Dark Rings :

$$2t = m\lambda$$

3.7. On which factor the condition of brightness or darkness depends ?

Ans. The condition of brightness or darkness depends on the path difference between the two reflected rays.

3.8. Define diffraction.

Ans. Diffraction of light is a phenomenon of bending of light and spreading out towards the geometrical shadow when passed through an obstruction.

3.9. What are the types of diffraction ?

Ans. There are two types of diffraction :

(a) Fresnel diffraction

(b) Fraunhofer diffraction

3.10. If first secondary maxima of wavelength 4600 Å fall on the first minima of some wavelength λ in single slit diffraction pattern. What is the value of λ ?

Ans. Secondary maxima,

$$e \sin \theta = \frac{(2m+1)}{2}\lambda$$

First secondary maxima,

$$e \sin \theta = \frac{3}{2} \times 4600$$

First minima,

$$e \sin \theta = \lambda$$

Since, both first secondary maxima and first minima coincide.

Thus, $\frac{3}{2} \times 4600 = \lambda$

$\therefore$ $\lambda = 6900$ Å

3.11. Differentiate between Fresnel and Fraunhofer diffractions.

Ans.

S.No.	Fresnel diffraction	Fraunhofer diffraction
a.	Lateral distances are important.	The angular inclinations are important.
b.	Observed pattern is a projection of a diffracting element.	Observed pattern is an image of the source.
c.	The centre of diffraction pattern may be bright or dark depending upon the number of Fresnel zones.	The centre of the diffraction pattern is always bright for all paths parallel to the axis of the lens.

3.12. Define diffraction grating.

Ans. Diffraction grating is an arrangement consisting of a large number of close parallel, straight, transparent and equidistant slits, each of equal width a, with neighbouring slits being separated by an opaque region of width b.

3.13. What do you mean by dispersive power of a plane diffraction grating ? [UPTU 2012-13, Marks 02]

Ans. Dispersive power of a grating is defined as the ratio of the difference in the angle of diffraction of any two neighbouring spectral lines to the difference in wavelength between the two spectral lines.

3.14. How the dispersive power related to order of the spectrum ?

Ans. The dispersive power is directly proportional to the order of spectrum, *i.e.*, higher is the order greater is the dispersive power.

3.15. What is Rayleigh's criterion of resolution ? [UPTU 2015-16, Marks 02]

Ans. According to Rayleigh, the two point sources or two equally intense spectral lines are just resolved by an optical instrument when the central maximum of the diffraction pattern due to one source falls exactly on the first minimum of the diffraction pattern of the other and vice-versa.

3.16. Define resolving power of a diffraction grating.

Ans. The resolving power of a grating is defined as its ability to show the two neighbouring spectral lines in a spectrum as separate.

3.17. Differentiate between the dispersive power and resolving power of grating.

Ans.

S.No.	Dispersive power	Resolving power
a.	It is defined as the rate of change of angle of diffraction with the wavelength used.	It is defined as the ratio of the wavelength of any spectral line to the smallest wavelength difference between neighbouring lines for which the spectral lines can be just resolved.
b.	Dispersive power is given by, $\frac{d\theta}{d\lambda} = \frac{n}{(e+d)\cos\theta}$	The resolving power of a grating is given by $\frac{\lambda}{d\lambda} = nN$
c.	Dispersive power depends upon the grating element	Resolving power is independent of grating element.

3.18. Differentiate the interference and diffraction.

Ans.

S.No.	Interference	Diffraction
a.	In this, the interference occurs between the two separate wavefronts emanating from two coherent sources.	In this, the interference occurs between the innumerable secondary wavelets produced by the unobstructed portion of the same wavefront.
b.	The interference fringes are usually equally spaced.	The diffraction fringes are never equally spaced.
c.	In an interference pattern all the bright fringes are of equal intensity.	In diffraction pattern the intensity of central maximum is maximum and goes on decreasing as the order of maxima increases on either side of the central maxima.

3.19. Why the centre of Newton's ring is dark.

UPTU 2015-16, Marks 02

Ans. At the point of contact of lens and glass plate, the path difference is zero and phase change 'π' takes place due to reflection on glass. Hence dark spot will be formed at the centre of ring system.

Application Based Questions

3.20. What are the applications of thin film interference ?

Ans. Applications of thin film interference are as follows :

a. Measurement of small displacements,

b. Testing of surface finish,
c. Testing of a lens surface, and
d. Thickness of a thin film coating.

Evaluation Based Questions

3.21. Two independent sources could not produce interference. Why ?

[UPTU 2011-12, 2012-13; Marks 02]

Ans. Two independent sources could not produce interference because there will be phase difference development between the two waves and hence sustained interference will not develop.

3.22. What will be the effect on the intensity of principal maxima of diffraction pattern when single slit is replaced by double slit ?

[UPTU 2011-12, Marks 02]

Ans. In single slit diffraction, intensity of principal maxima

$$I = R^2 = \frac{A^2\sin^2\alpha}{\alpha^2}$$

By replacing single slit by double slit, the resultant intensity at any point on the screen is given by :

$$I = \frac{4A^2\sin^2\alpha}{\alpha^2}\cos^2\beta$$

So, the intensity of principal maxima becomes 4 times.

☺☺☺

4 UNIT Polarization and Laser (2 Marks Questions)

Memory Based Questions

4.1. Define polarization of light.

Ans. Polarization of light is the phenomenon of limiting the vibrations of their electric vector in only one particular direction in a plane perpendicular to the direction of propagation of light.

4.2. What is unpolarized light ?

Ans. The light in which the vibrations of electric vector are distributed in all directions in a plane perpendicular to the direction of propagation of light waves are known as unpolarized light.

4.3. What is polarized light ?

Ans. The light in which the vibrations of electric vector are limited in one particular direction in a plane perpendicular to the direction of propagation of light is known as polarized light.

4.4. Define plane of polarization and plane of vibration.

UPTU 2015-16, Marks 02

Ans. **Plane of Polarization :** The plane containing the direction of propagation of the light, but containing no vibrations is called the plane of polarization.

Plane of Vibration : The plane containing the direction of vibration and direction of propagation of light is called the plane of vibration.

4.5. Define polarizer.

Ans. It is a device, which limits the vibrations of electric vector in natural light in only one direction in a plane perpendicular to the direction of propagation of light.

4.6. Define analyzer.

Ans. It is a device, which detects whether any given light is polarized light or unpolarized light.

4.7. What are the different types of polarization ?

Ans. Following are the different types of polarization :

(1) Plane polarized light,
(2) Circularly polarized light, and
(3) Elliptically polarized light.

4.8. Define double refraction.

Ans. The phenomenon in which we get two refracting plane polarized rays corresponding to one incident polarized light ray is called double refraction.

4.9. What are doubly refracting crystals ?

Ans. Doubly refracting crystals are those which split the unpolarized light into two refracted rays, which are plane polarized.

4.10. What are the different types of doubly refracting crystals ?

Ans. Doubly refracting crystals are of following two types :

(*a*) Uniaxial crystals, and
(*b*) Biaxial crystals

4.11. Define optic axis of doubly refracting crystal.

UPTU 2015-16, Marks 02

Ans. A certain direction in a doubly refracting crystal along which the speed of light of two refracted light rays remains the same, is known as optic axis of that doubly refracting crystal.

4.12. Give examples of uniaxial crystals and biaxial crystals.

Ans. (*a*) Calcite, tourmaline and quartz are the examples of uniaxial crystals.

(*b*) Topaz and aragonite are the examples of biaxial crystal.

4.13. Define optical activity.

Ans. The ability to rotate the plane of polarization of plane polarized light by certain substance is called optical activity.

4.14. Give full form of LASER.

Ans. LASER is the acronym for Light Amplification by Stimulated Emission of Radiation.

4.15. Differentiate between the ordinary beam and laser beam.

Ans.

S. No.	Ordinary beam	Laser beam
1.	It is not monochromatic.	It is monochromatic.
2.	Ordinary light is produced by spontaneous emission.	Laser beam is produced by stimulated beam.
3.	It is incoherent.	It is coherent.

4.16. Define spontaneous emission and stimulated emission.

OR

What is stimulated emission of radiation ?

[UPTU 2012-13, 2014-15; Marks 02]

Ans. **Spontaneous Emission :** The process in which photon emission occurs without any interaction with external radiation is called spontaneous emission.

Stimulated Emission : The phenomena of forced emission of photons are called induced emission or stimulated emission.

4.17. What do you mean by population inversion ?

[UPTU 2013-14, Marks 02]

Ans. The phenomenon in which the number of atoms in the higher energy state becomes comparatively greater than the number of atoms in the lower energy state is known as population inversion.

4.18. Define pumping.

Ans. The process of supplying energy to the medium to transfer it into the state of population inversion is known as pumping.

4.19. Define metastable state. **UPTU 2015-16, Marks 02**

Ans. Metastable state is particular excited state of an atom, nucleus, or other system that has a longer lifetime than ordinary excited states and that generally has a shorter lifetime than the lowest energy state, called the ground state.

Application Based Questions

4.20. Write down the use of Nicol prism.

Ans. Nicol prism is used to separate the *E*-ray and *O*-ray.

4.21. Write down the use of half wave plate.

Ans. A half wave plate is used in Laurent's half shade polarimeter for rotating the plane of polarization of the plane polarized light.

4.22. Write down some applications of laser.

Ans. Following are the applications of laser :

(1) In surgery,
(2) In holography,
(3) In communications,
(4) In computer industry, and
(5) Laser printing.

Evaluation Based Questions

4.23. Why does a ray of light propagating along optic axis not suffer double refraction ?

Ans. A ray of light propagating along optic axis does not suffer double refraction because the structure of the crystal is symmetric about that direction.

4.24. Why are polaroid sheets used ?

Ans. Polaroid sheets are used for the production and detection of linearly polarized light.

4.25. How can we produce circularly or elliptically polarized light ?

Ans. Retarders are used to produce circularly or elliptically polarized light.

4.26. Why does ruby laser emits red or pink colour ?

Ans. Ruby laser emits red or pink colour due to the presence of chromium ions depending upon its concentration.

4.27. How is population inversion achieved in He-Ne laser ?

Ans. In He-Ne laser, the population inversion is achieved through inelastic atom-atom collisions.

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5
UNIT

Fibre Optics and Holography (2 Marks Questions)

Memory Based Questions

5.1. Define fibre optics.

Ans. Fibre optics is a technology in which signals are converted from electrical into optical signals, transmitted through a thin glass fibre and reconverted into electrical signals.

5.2. What is optical fibre ?

Ans. An optical fibre is a cylindrical wave guide made of transparent dielectric, which guides light waves along its length by total internal reflection.

5.3. What do you understand by total internal reflection ?

Ans. When the angle of incidence exceeds the critical angle (*i.e.*, when $\phi_1 > \phi_c$), there is no refracted ray and we have what is known as total internal reflection.

5.4. Write down the advantages of optical fibres.

Ans. The advantages of optical fibres include :

a. High data transmission rates and bandwidth,
b. Low losses,
c. Small cable size and weight, and
d. Data security.

5.5. What are the functions of cladding ?

Ans. The cladding performs the following important functions :

a. Protects the fibre from physical damage and absorbing surface contaminants.
b. Prevents leakage of light energy from the fibre through evanescent waves.

5.6. What is critical ray ?

Ans. The ray incident, at the core-cladding boundary, at the critical angle is called a critical ray

5.7. Define acceptance angle.

Ans. Acceptance angle is the maximum angle that a light ray can have relative to the axis of the fibre and propagate down the fibre.

5.8. Define modes.

Ans. The light ray paths along which the waves are in phase inside the fibre are known as modes.

5.9. What is attenuation ?

Ans. The attenuation of optical signal is defined as the ratio of the optical output power from a fibre of length to the input optical power.

5.10. Define acceptance cone.

Ans. A cone obtained by rotating a ray at the end face of an optical fibre, around the fibre axis with acceptance angle is known as acceptance cone.

5.11. What is dispersion ?

Ans. The amount by which a pulse broadens as it passes through a multimode fibre is commonly known as dispersion.

5.12. Define holography.

Ans. Holography is the science of producing holograms; it is an advanced form of photography that allows an image to be recorded in three dimensions.

Application Based Questions

5.13. Give few important applications of optical fibre ?

[UPTU 2015-16, Marks 02]

Ans. Applications :

a. In communication,
b. In optical sensors,
c. In illumination applications, and
d. In imaging optics.

5.14. Where are multimode fibre used ?

Ans. Multimode fibres are used for short distance (shorter than 200 meters), communication links or for application where high power must be transmitted.

5.15. Which fibres are generally used under sea water ?

Ans. Single mode fibres are frequently used under sea water.

5.16. Write down the applications of holography.

Ans. Applications :

a. Used for data storage.

b. Used in medical imaging such as CT scan, X-ray, and

c. Used for security purpose in many industries.

Evaluation Based Questions

5.17. Why are light pulses broaden and spread into wider time interval ?

Ans. The light pulses are broadened and spread into wider time interval because of different time taken by different rays propagating through the fibre.

5.18. How can be modal dispersion minimized ?

Ans. Modal dispersion can be minimized by using single mode step or monomode step index fibres or graded index multi mode fibres.

5.19. What is the condition for stable and recordable interference pattern in hologram ?

Ans. For the stable and recordable interference pattern, the maximum path difference between the object wave and the reference wave should not exceed the coherence length.

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