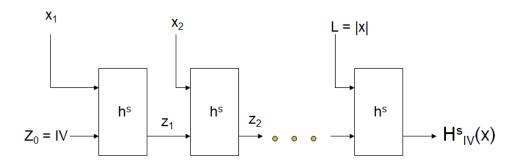
## Construction of variable length hash function from fixed length hash function Merkle Damgard Transform

Theorem:

If (Gen, h) is a fixed length collision resistant hash function, then (Gen, H) is a collision resistant hash function



Let (Gen, h) be a fixed-length collision-resistant hash function for inputs of length 2l(n) and output length l(n). Construct a variable-length hash function (Gen, H) as flows:

- Gen: remains unchanged
- H: on input a key s and a string  $x \in \{0.1\}^*$  of length  $L < 2^{l(n)}$ , do the following (set l = l(n) in what follows:
  - 1. Set  $B \coloneqq \left[\frac{l}{l}\right]$  (i.e., the number of blocks in x). Pad x with zeroes so its length is a multiple of l. Parse the padded result as the sequence of l bit blocks  $x_1, x_2, \dots, x_B$ . Set  $x_{B+1} \coloneqq L$ , where L is encoded using exactly l bits.
  - 2. Set  $z_0 := 0^l$ .
  - 3. For i = 1, ..., B + 1, compute  $z_i := h^s(z_{i-1}||x_i)$ .
  - 4. Output  $z_{B+1}$ .

## References

- [1] J. K. a. Y. Lindell, Introduction to Modern Cryptography.
- [2] B. Micali, "Hardcord bits," [Online]. Available: <a href="https://crypto.stanford.edu/pbc/notes/crypto/hardcore.html">https://crypto.stanford.edu/pbc/notes/crypto/hardcore.html</a>.
- [3] Lecture Slides