MATH*2000 Assignment #1, Due Friday September 24th

The following assignment is to be submitted through courselink before midnight on Friday September 24^{th}. Please ensure that your work is legible before submitting. Any work which is illegible to the point where it is next to impossible to read will be discarded. The student will be properly informed of this happenning and asked to resubmit their assignment subject to the normal late penalites of this course.

1. **(15 marks)**

   For each of the following implicative statements: write both their converse and their contrapositive. For each converse and contrapositive written, indicate whether they are either **True** or **False**.

   (a) If $2 + 2 = 4$ then the area of a circle is equal to π multiplied by its radius squared.

   (b) If $\sqrt{2}$ is a rational number then $10 \times 10 \neq 89$.

   (c) Consider two rectangles, A and B, with side lengths that are elements of the set $\mathbb{N}$.
   If A and B have the same perimeter and the same area, then they are the same shape up to being a rotation of one another.

   (d) $\forall$ pairs of functions, $f(x)$ and $g(x)$, that are both continuous and differentiable,
   if $f'(x) = g'(x)$, then $f(x) = g(x)$.

   (e) $\forall\ a, b \in \mathbb{Z}$, If $a^2 > b^2$, then $a > b$.

2. **(10 marks)**

   Verify whether the following two logic statements are logically equivalent to one another. For this question, your answer may not rely upon the use of truth tables and must instead be constructed using the algebraic identities of logical statements. Be sure to show all your steps in your answer.

   $$B \vee (\sim (A \vee \sim A) \wedge B) \vee \sim (A \wedge \sim B) \equiv \sim A \vee B$$

3. **(10 marks)**

   Write each of the following symbolic mathematical expressions in plain english.

   (a) $\forall x, (x \in \mathbb{R} \wedge x > 0) \rightarrow (\exists y \ni (y > 0 \wedge y^2 = x))$

   (b) $\sim \exists x, x \in \mathbb{R} \wedge (1 < x^2 < x)$

4. **(10 marks)**

   Consider all of the different logical statements that one can make using at most two atomic statements, A and B, along with any number of elementary connectives. When one considers setting up a truth table for these logical statements, they would reason that A and B may both either be true or false and that there exist four different situations where a logical statement may either be true or false. *(i.e. every column of the truth table must have four rows).*

   Whether the statement *is* true or false depends wholly on the contents of the logical statement, so this means that there can only be at most sixteen distinct logical statements that can be made from at most two atomic statements. All other logical statements would be logically equivalent to one of these sixteen statements.

   For this question, write out your own set of sixteen logical statements such that each one has its own distinct column of true and false values. Verify clearly that each statement has its own distinct conditions for truth and falsehood. You will find that a truth table is the best way to do this.

(Hint: remember as you are working that you don't have to use both A and B for every statement in your set; some combinations of true and false may come more easily when using only one of them. Also don't forget how a negation affects the truth values of a logical statement; this may save you a lot of time.)

5. **(5 marks)**

   Consider a logical statement that is written as $\exists y \forall x, F(x, y)$. Consider another logical statement that is written as $\forall x \exists y, F(x, y)$, where $F(x, y)$ is the same arbitrary statement. Can these two statements always be thought of as semantically the same up to a logical equivalence? Briefly discuss why or why not.