Online Homework System

Assignment Worksheet
11/27/20 - 1:20:32 PM EST

Name: ______________________________ **Class:** Calculus 1 - MATH*1200 - F20
Class #: ______________________________ **Section #:** ______________________________
Instructor: Mihai Nica **Assignment:** Term Test #5

Question 1: (1 point)

Fill in your name on the academic integrity pledge:

"As a member of the University of Guelph, I pledge to uphold the highest standards of ethics and academic integrity. This means that I will only use my notes, a calculator, and will NOT use *any other outside assistance* (no *internet* or other *people* including my peers). I understand that there are *serious consequences*, including getting expelled from the course or the university, for violating academic integrity."

Write the phrase "I, --insert name here--, agree to the academic integrity pledge" on your first page.

Reminder:

-Email math1200@uoguelph.ca if you have any kind of issue during the test. Do not wait until after the test.

-Dont forget to save a copy of your questions (e.g. by hitting print -> save as pdf, or by taking a screenshot) to submit to Crowdmark

-It is helpful to your grader to copy out any equations in your question in your answer

-Don't forget to hit "Submit Assignment" on Mobius BEFORE time runs out to submit your answer to the True/False questions in Q1 (If you press "Save+Exit" without hitting submit assignment, your answers are not submitted). Also record your answers for Q1 on paper to submit to Crowdmark as a backup. -Q2,Q3,Q4 are all submitted on Crowdmark only

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Q1 Note that order of True and False is randomized in each question; the order of True and False do not give any information about the correct answer.

Q1a) [1pt] Let g be a continuous function whose domain is all numbers.

True or False: If g has a local maximum at $x = 7$, then it must be that $g'(7) \neq 8$.

(a) True

(b) False

Q1b) [1pt] Let g be a continuous and differentiable function whose domain is all numbers. This question involves the definition of "strictly decreasing" which is a concept we went over in lecture. Here is a reminder: " g is strictly decreasing on the interval $x \in (16, 31)$" means that $g(x_1) > g(x_2)$ for every x_1, x_2 that satisfy $16 < x_1 < x_2 < 31$.

True or False: If $g(x)$ is strictly decreasing on the interval $x \in (16, 31)$, then $g'(x) < 0$ for all $x \in (16, 31)$.

(a) True

(b) False

Q2a) [1pt] Frank says: "I don't understand what's the point of this approximation stuff. Why can't we just type in $(8.99)^{\frac{1}{2}}$ on our calculator? There is no reason to learn this.". Explain to Frank the point of learning about linear approximation.

Q2b) [3pt] Use linear approximation to estimate $(8.99)^{\frac{1}{2}}$. There is no need to simplify your final answer (e.g. leaving your answer as $5+\frac{2}{3\times 7}$ is perfectly fine). Clearly show all your steps and explain your work carefully. (Hint: It might be helpful to know that $9=3^2$.)

Q3 [8pt] This problem is about the function $q(x)$ which is given below along with its first and second derivative.

$$q(x)=\frac{x^{100}}{e^x}$$

$$q'(x)=\frac{x^{99}(100-x)}{e^x}$$

$$q''(x)=\frac{x^{98}(90\times 110-200x+x^2)}{e^x}$$

(Hint #1: Note that in the second derivative I left the giant number $90\times 110=9,900$ as a product of two smaller numbers. This might be helpful to you. The quadratic equation $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ might also be helpful if you wish to use it.)

(Hint #2: Remember that x^p is always positive when p is an <u>even</u> number. When p is an <u>odd</u> number, x^p is positive if and only if x is positive. The function e^x is always positive.)

a) Do NOT try to find the x- and y- intercepts. I did it for you: the <u>only</u> x- and y- intercepts are the point $(x,y)=(0,0)$.

b) Do NOT try to find the vertical or horizontal asymptotes of $q(x)$. I did it for you: It has no V.A's, no left-hand H.A., and the right-hand H.A. is $\lim_{x\to\infty} q(x)=0$

c) Do a first-derivative analysis. (Find intervals where the function is <u>increasing/decreasing</u> and any <u>minimums/maximums</u>.)

d) Do a second-derivative analysis. (Find intervals where the function is <u>concave up/down</u>, find any <u>points of inflection</u>.)

e) Draw a sketch of this function using an appropriate scale. Label some key points that you calculated in parts (a) - (d). Do <u>NOT</u> attempt to find the y-values of any of these points: these will be very large and may explode your calculator. Only label the <u>x-values</u> of the interesting points.

Q4 [6pt]: This question has two parts. In the first part you setup an optimization problem but <u>DO NOT</u> solve it. In the second part, you solve an optimization problem where the function is already given. This is so that you can get full marks in part b) even if your setup in part a) has mistakes in it.

Q4a) [3pt]: (Setup only but do NOT solve)

Two telelphone poles stand 120 m apart from one another. One pole is 3 m tall and the other pole is 18 m tall. A telephone wire is going to run from the top of the 3 m pole, then diagonally down to the ground at some point between the two poles, and then diagonally up to the top of the 18 m pole. The engineering team wants you to minimize the total length of wire to be used for this. Carefully setup the optimization problem. To get full marks, you must clearly explain what is the <u>variable and its domain</u>, what is the <u>function to be minimized</u>, and what <u>other functions</u> that appear in the problem. Draw a picture to help explain your work. DO NOT try to solve the problem; only set it up. (Hint: You will need to use Pythagoras at some point).

Q4b) [3pt]: (Optimize a given function) Find the global minimum of the following function on the domain of <u>positive</u> numbers $x\in(0,\infty)$:

$$g(x)=52x+\frac{1}{x}$$

BONUS: [2pt] Solve the optimization question in Q4a. (Warning: This is a hard problem! Also, since this a bonus, you will not get any part marks here for the easy steps. Do not attempt this unless you are sure you got all the other questions.)