University of Guelph
CIS 2910 Fall 2016 – FINAL (Dec. 6)

Instructor: Joe Sawada

First Name: _____

Last Name: _____

Student Number: _____

Problem 1: (6 marks)		Problem 6: (4 marks)	
Problem 2: (6 marks)		Problem 7: (5 marks)	
Problem 3: (6 marks)		Problem 8: (6 marks)	
Problem 4: (4 marks)		Problem 9: (5 marks)	
Problem 5: (5 marks)			
		Total (47 marks)	

This test is closed book and lasts 120 minutes.
You may not use any electronic/mechanical computation devices.
There are 11 pages including the cover page.

Problem 1: [6 marks]

(a) TRUE or FALSE:
If a simple graph G can be colored with 4 colors, then G is planar.

(b) TRUE or FALSE:
A local ultimate frisbee league has 12 teams. If there is a total of 97 players in the league then one team must have exactly 9 players.

(c) TRUE or FALSE:
If events E and F are dependent then $p(E \cap F) \neq p(E)p(F)$.

(d) TRUE or FALSE:
If a simple graph G has unique edge weights then it has a unique minimum spanning tree.

(e) TRUE or FALSE:
The number of ways to put 8 identical balls into 3 labeled boxes is $\binom{10}{8}$.

(f) TRUE or FALSE:
If a simple graph G has exactly 2 vertices with odd degree, then the graph has an Euler path.

Problem 2: [6 marks] Multiple Choice.

(a) Which of the following most accurately defines a *directed graph*?

(a) A graph $G = (V, E)$ consisting of a non-empty set of vertices V and an edge set E of unordered pairs of distinct elements from V.

(b) A graph $G = (V, E)$ consisting of a non-empty set of vertices V and an edge set E of ordered pairs of elements from V.

(c) A graph $G = (V, E)$ consisting of a non-empty set of vertices V and an edge set E of ordered pairs of distinct elements from V.

(d) A graph $G = (V, E)$ consisting of vertices V and an edge set E of directed edges.

(e) none of the above

(b) Which of the following corresponds to the *Generalized Pigeonhole Principle*?

(a) If $k + 1$ objects are placed into k boxes, then there a box containing two objects.

(b) If more than k objects are placed into k boxes, then then at least one box contains two objects.

(c) If N objects are placed into k boxes, then there is exactly one box containing at least $\lceil N/k \rceil$ objects.

(d) If N objects are placed into k boxes, then there is at least one box containing at least $\lceil N/k \rceil$ objects.

(e) none of the above

(c) Which of the following most accurately defines the *chromatic number* of a simple graph G?

(a) The number of colors that can be used to color the vertices of G.

(b) The size of the largest clique (complete subgraph) in G.

(c) The maximum number of colors required to color the vertices of G.

(d) The minimum number of vertices that require the same color.

(e) none of the above

(d) How many binary strings of length 10 have either exactly 4 ones, or begin and end with the same bit?

(a) $\binom{10}{4} + 2^8$
(b) $\binom{10}{4} + 2^9$
(c) $\binom{10}{4} + 2^9 - \binom{8}{2}$
(d) $\binom{10}{4} + 2^9 - \binom{8}{4} - \binom{8}{2}$
(e) $\binom{10}{2} + \binom{8}{4}$
(f) none of the above

(e) Consider the Monty Hall problem on 4 doors: there are 4 doors and behind one of the doors is a new car. You are asked to choose a door, after which Monty will open one of the other 3 doors that does not contain the car. What is the probability of winning the car if you now switch doors?

(a) 1/2
(b) 1/3
(c) 1/4
(d) 2/3
(e) 3/8
(f) none of the above

(f) Consider a 6 by 3 grid graph. How many paths are there starting from the bottom left corner and ending at the top right if only moves up and to the right are allowed?

(a) 18
(b) 20
(c) 36
(d) 56
(e) 84
(f) none of the above

Problem 3: [6 marks]

(a) **[2 marks]** Given positive integers n, k with $n \geq k$, state Pascal's Identity.

(b) **[2 marks]** Fill out the first 7 rows of Pascal's triangle with the proper integer values. The first row starts with a single 1.

(c) **[2 marks]** Consider a set S with 25 distinct elements. How many ways can you select a subset of S that has at least 3 elements?

Problem 4: [4 marks]

(a) **[1 mark]** Let $T(n) = 2T(n-1) + T(n-2)$ when $n > 2$ and let $T(1) = T(2) = 2$. What is $T(3)$ and $T(4)$?

(b) **[1 mark]** Suppose event E happens with probability $1/2$. Suppose event F happens with probability $1/3$. Suppose both E and F happen with probability $1/5$. What is the probability that event E happens given that F happens?

(c) **[2 marks]** A *ternary* string is a word over the alphabet $\{0, 1, 2\}$. Describe a simple **recursive** algorithm to generate all ternary words of length n in lexicographic order. For $n = 2$ it should list: 00, 01, 02, 10, 11,12, 20, 21, 22.

Problem 5: [5 marks]

Canada currently has 335 members of parliament (MPs), 247 are male and 88 are female. By party affiliation, there are:

182 Liberal, 97 Conservative, 44 NDP, 10 Bloc, 1 Green, 1 Independent.

(a) **[1 mark]** How many ways can the Prime Minister choose 30 MPs to attend a ribbon cutting ceremony?

(b) **[2 marks]** Suppose 30 members are attending a special conference on the environment. Of this group exactly 20 are Liberal and 8 are Conservative. The other two members are from the other parties. How many ways can the MPs be seated in a row of 30 seats so that no two Conservatives sit next to each other?

(c) **[2 marks]** How many ways can the Prime Minister assign seats numbered 1 to 335 to the 335 MPs so that no female is sitting in a seat with an odd number?

Problem 6: [4 marks]

(a) **[2 marks]** Prove or disprove: Every simple graph with integer edge weights has a unique minimum spanning tree.

(b) **[2 marks]** Prove or disprove: In every simple graph with more than one vertex there exists two vertices with the same degree.

Problem 7: [5 marks]

Answer the following questions using the **smallest** number of vertices possible.

(a) [**1 mark**] Draw a simple graph that is not a planar representation.

(b) [**2 marks**] Draw simple graph that has an Euler path with 2 pendant vertices and also contains C_4 as an induced subgraph.

(c) [**2 marks**] Draw a simple graph with chromatic number 3 that does not contain either a C_3 or a C_5 as an induced subgraph

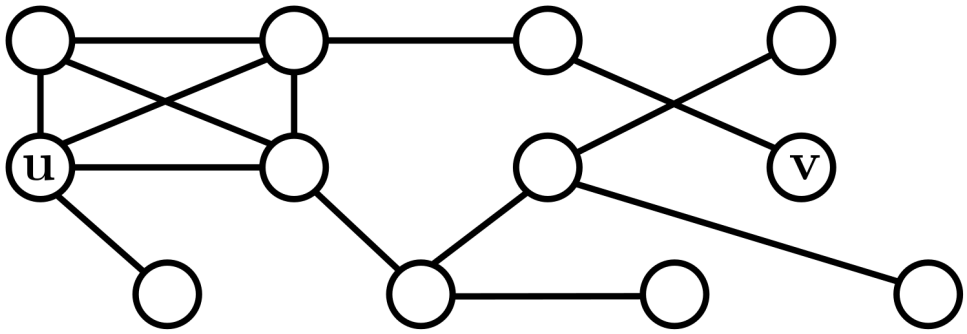
Problem 8: [6 marks]

(a) **[2 marks]** State Euler's Formula.

(b) **[2 marks]** Use Euler's formula to prove the following statement: If a connected planar simple graph has $n \geq 3$ vertices and m edges and no cycles of length three, then $m \leq 2n - 4$.

(c) **[2 marks]** Use the result from (b) to prove that $K_{3,3}$ is not planar.

Problem 9: [5 marks] Answer the following questions **in the corresponding boxes** about the following simple graph. Each question is worth 0.5 marks.



(a) Is it a planar ?

(b) Is this graph bipartite?

(c) Does it have an Euler path?

(d) Does it have a Hamilton path?

(e) How many pendant vertices are there?

(f) How many isolated vertices are there?

(g) What is the degree of u?

(h) Does it contain a cut vertex?

(i) Is the vertex u adjacent to v?

(j) What is the chromatic number of this graph?

– END OF FINAL –