Assignment #2 Math*2000 Maneesh Wijewardhana

1a) $P(A \cup B) = P(A) \cup P(B) \rightarrow$ Disprove with counter example

Lets say $A = \{1, 2\}$ and $B = \{2, 3\}$

$A \cup B = \{1, 2, 3\}$

$P(A \cup B) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

now... $P(A) = \{\emptyset, \{1\}, \{2\}, \{1,2\}\}$

$P(B) = \{\emptyset, \{2\}, \{3\}, \{2,3\}\}$

$P(A) \cup P(B) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}\}$

Since these two are not equal, the set equivalence is False.

1b) $P(A \cap B) = P(A) \cap P(B)$

Lets prove LHS $\subset$ RHS

Let $x \in P(A \cap B)$

$\rightarrow x \subset (A \cap B)$

$\rightarrow x \subset A \wedge x \subset B$

$\therefore x \subset A$, then $x \in P(A)$, so $x \in P(A) \cap P(B)$

$\therefore$ LHS $\subset$ RHS

Lets prove RHS $\subset$ LHS

Let $x \in P(A) \cap P(B)$

$\rightarrow x \subset A \wedge x \subset B$

$\rightarrow x \subset A$, then $x \in P(A)$, so $x \in P(A \cap B)$

$\therefore$ RHS $\subset$ LHS

Since LHS $\subset$ RHS and RHS $\subset$ LHS, this set equivalence is True.

1c) $P(A \setminus B) = P(A) \setminus P(B)$ $\rightarrow$ Disprove with counter example

$A = \{0, 1\}$, $B = \{1, 2\}$

$A \setminus B = \{0\}$

$P(A \setminus B) = \{\emptyset, 0\}$

$P(A) = \{\emptyset, \{0\}, \{1\}, \{0, 1\}\}$ Since there ...

1b) $P(A \cap B) = P(A) \cap P(B)$

Lets prove $LHS \subset RHS$

Let $x \in P(A \cap B)$

$\rightarrow x \subset (A \cap B)$

$\rightarrow x \subset A \wedge x \subset B$

$\therefore x \subset A$, then $x \in P(A)$, so $x \in P(A) \cap P(B)$

$\therefore LHS \subset RHS$

Lets prove $RHS \subset LHS$

Let $x \in P(A) \cap P(B)$

$\rightarrow x \subset A \wedge x \subset B$

$\rightarrow x \subset A$, then $x \in P(A)$, so $x \in P(A \cap B)$

$\therefore RHS \subset LHS$

Since $LHS \subset RHS$ and $RHS \subset LHS$, this set equivalence is True.

1c) $P(A \setminus B) = P(A) \setminus P(B)$ → Disprove with counter example

$A = \{0, 1\}$, $B = \{1, 2\}$

$A \setminus B = \{0\}$

$P(A \setminus B) = \{\emptyset, 0\}$

$P(A) = \{\emptyset, \{0\}, \{1\}, \{0,1\}\}$

$P(B) = \{\emptyset, \{1\}, \{2\}, \{1,2\}\}$

$P(A) \setminus P(B) = \{\{0\}, \{0,1\}\}$

Since these two are not equal we say that this statement is false.

2a) There exists A_i, where i is an element of the natural numbers such that i does not equal j if and only if A_i does not equal A_j; then the intersection of all A_i from i equals 1 ~~to~~ i = infinity is A' such that the cardinality of A' is more than 0 but less than infinity.

This means $A_1 = \{1\}$, $A_2 = \{1,2\}$, $A_3 = \{1,2,3\}$ ... etc

We know that $A_1 \neq A_2$ so consequently $i \neq j$ so

$i \neq j \Leftrightarrow A_i \neq A_j$ is True.

$\bigcap_{i=1}^{\infty} A_i = A' \ni 0 < |A'| < \infty$, the intersection from i=1 to infinity will result in $A' = \{1\}$ which means the cardinality of $A' = 1$ so $\bigcap_{i=1}^{\infty} A_i = A' \ni 0 < |A'| < \infty$ is True

$T \rightarrow T = T$

2b) There exists A_i where i is an element of the natural numbers such that i does not equal j if and only if A_i does not equal A_j, then the union of all A_i from i equals 1 to $i =$ infinity is A' such that the cardinality of A' is more than 0 but less than infinity.

Lets assume this is true. This would mean that A' is the union of infinite sets with finite cardinality. A' can only have a finite cardinality if the sets were equal to each other and have a finite number of elements but looking at the statement, the two sets <u>cannot</u> be equal thus resulting in a contradiction.

3a) $(a,b),(a,b)$, $a+b=a+b$ so this is reflexive
$(a,b),(a',b')$; $a+b'=a'+b$ and $(a',b'),(a,b)$, $a'+b=a+b'$ so the relation is symmetric
$(a,b),(a',b')$, $a+b'=a'+b$, $(b,c),(b',c') \rightarrow b+c'=b'+c'$ and $(a,c),(a',c') \rightarrow a+c'=a'+c$ so this relation is transitive

Representative class: $(2,3),(9,10)$
$(2,3),(2,3)$, $2+3=2+3$
$(2,3),(9,10)$, $2+10=3+9$ and $(9,10),(2,3)$, $9+3=10+2$
$(2,3),(9,10)$, $2+10=3+9$ and $(3,4),(10,11)$, $3+11=4+10$ and so $(2,4),(9,11)$, $2+11=4+9$
Since this relation is reflexive, symmetric, and transitive, it is an equivalence relation.

3b) This relation is reflexive because everyone has a blood relation to themselves.
This relation is symmetric because If A is blood related to B, then [illegible] blood related to A.
[illegible] relat[illegible] NOT transitive because A can be related to B and [illegible] to C, but this does not mean A is related [illegible] blood related [illegible]

$(a,b),(a',b'), a+b' = a'+b, (b,c),(b',c') \rightarrow b+c' = b'+c'$ and
$(a,c),(a',c') \rightarrow a+c' = a'+c$ so this relation is transitive

Representable class: $(2,3),(9,10)$
$(2,3),(2,3), 2+3 = 2+3$
$(2,3),(9,10), 2+10 = 3+9$ and $(9,10),(2,3), 9+3 = 10+2$
$(2,3),(9,10), 2+10 = 3+9$ and $(3,4),(10,11), 3+11 = 4+10$ and so
$(2,4),(9,11), 2+11 = 4+9$
Since this relation is reflexive, symmetric, and transitive, it is an equivalence relation.

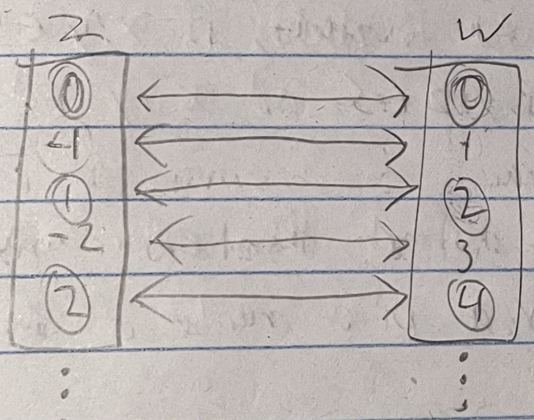
3b) This relation is reflexive because everyone has a blood relation to themselves.
This relation is symmetric because If A is blood related to B, then B is blood related to A.
This relation is NOT transitive because A can be related to B and B can be related to C, but this does not mean A is related to C. For ex, A and B are cousins which means blood related and B and C are cousins so they are blood related. This does not mean A and C have any blood relation as the relationship with B could be through different sides of the family.

3c) This relation is reflexive because for example, Toronto is related to Toronto as they are both in Ontario.
This relation is symmetric because for example, Toronto is related to Guelph as they are in the same province and Guelph is related to Toronto for the same reason.
This is relation is transitive because for example, if Toronto is related to Guelph, and Guelph is related to Cambridge, then Toronto is related to Cambridge as they would all have to be in the same Province.
Since this relation is reflexive, symmetric, and transitive, it is an equivalence relation.

4. A Partial order on a set S is a type of relation on S which is reflexive, anti-symmetric, and transitive. A total order carries the same properties as a partial order except for an additional property that for all $x, y \in S$, either $(x,y) \in P$ or $(y,x) \in P$.

An example of a partial order would be the points on a plane given as ordered pairs (x,y). We can give them a partial order by saying $(x_1, y_1) > (x_2, y_2)$ when both $x_1 > x_2$ and $y_1 > y_2$. This is not a total order because if we use the example $(9,3)$ and $(7,4)$, we see that it is not comparible as one has a bigger x and the other has a bigger y.

5. Z is the set of integers $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$ and
W is the set of whole numbers $\{0, 1, 2, 3, 4, \ldots\}$
lets try to map Z onto W



As you can see, we can map all of the even numbers to positive numbers and the odd numbers to negative numbers.

we know even numbers $= 2k$ where k is some integer and
and odd is $2k+1$ where k is some integer

$$\therefore f(x) \begin{cases} 2k & \text{if } x \geq 0 \\ -2k-1 & \text{if } x < 0 \end{cases}$$