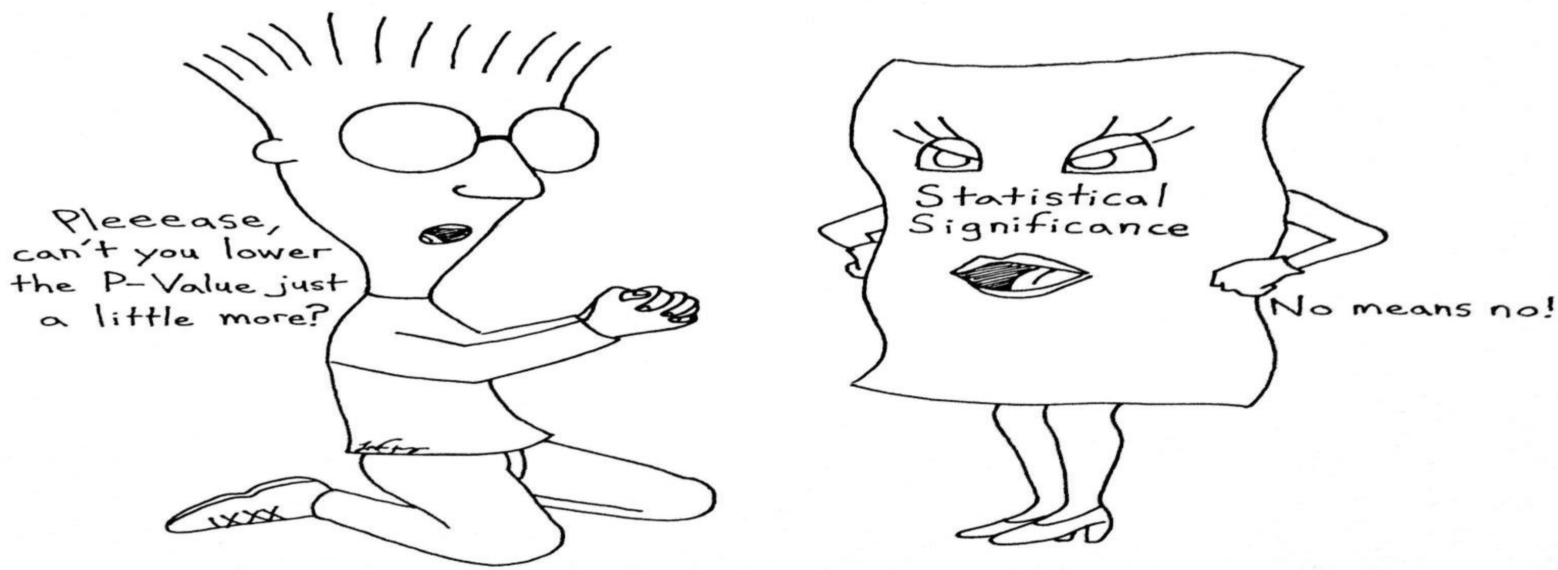
Introductory Statistics: A Problem-Solving Approach

by Stephen Kokoska

Chapter 10

Confidence Intervals and Hypothesis Tests Based on Two Samples or Treatments



Introduction

- To conduct hypothesis tests to compare two population parameters, we will simply modify the single-sample procedures.

- To compare two population means μ_1 and μ_2, we consider the difference $\mu_1 - \mu_2$.

- The hypothesized difference between two means may be nonzero. For example, H_0: $\mu_1 = \mu_2 + 5$, corresponds to a test of H_0: $\mu_1 - \mu_2 = 5$.

- Two samples are **independent** if the process of selecting individuals or objects in sample 1 has no effect on the selection of individuals or objects in sample 2. If the samples are not independent, they are **dependent**.

- A **paired** data set is the result of matching each individual or object in sample 1 with a similar individual or object in sample 2. A common experiment in which paired data are obtained involves a *before* and *after* measurement on each individual or object. Each *before* observation is matched, or paired, with an *after* observation.

Two Independent Samples, Population Variances Known

Suppose

1. $\overline{X}_1$ is the mean of a random sample of size n_1 from a population with mean μ_1 and variance σ_1^2.

2. $\overline{X}_2$ is the mean of a random sample of size n_2 from a population with mean μ_2 and variance σ_2^2.

3. The samples are independent.

If the distributions of both populations are normal, then the random variable $\overline{X}_1 - \overline{X}_2$ has the following properties.

1. $\mathrm{E}(\overline{X}_1 - \overline{X}_2) = \mu_{\overline{X}_1 - \overline{X}_2} = \mu_1 - \mu_2$

   $\overline{X}_1 - \overline{X}_2$ is an unbiased estimator of the parameter $\mu_1 - \mu_2$. The distribution is centered at $\mu_1 - \mu_2$.

2. $\mathrm{Var}(\overline{X}_1 - \overline{X}_2) = \sigma^2_{\overline{X}_1 - \overline{X}_2} = \dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}$ and the standard deviation is

$$\sigma_{\overline{X}_1 - \overline{X}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}.$$

3. The distribution of $\overline{X}_1 - \overline{X}_2$ is normal.

If the underlying distributions are not known, but both n_1 and n_2 are large, then $\overline{X}_1 - \overline{X}_2$ is approximately normal (by the Central Limit Theorem).

Hypothesis Test Concerning Two Population Means When Population Variances Are Known

Given two independent random samples of sizes n_1 and n_2, with population means of μ_1 and μ_2, respectively, assume that

1. The underlying populations are normal and/or both sample sizes are large
2. The population variances are known.

A hypothesis test concerning two population means, in terms of $\mu_1 - \mu_2$, with significance level α, has the form

H_0: $\mu_1 - \mu_2 = \Delta_0$

H_a: $\mu_1 - \mu_2 > \Delta_0$, $\mu_1 - \mu_2 < \Delta_0$, or $\mu_1 - \mu_2 \neq \Delta_0$

$$\text{TS: } z = \frac{(\overline{x}_1 - \overline{x}_2) - \Delta_0}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$$

RR: $z \geq z_\alpha$, $z \leq z_\alpha$, or $|z| \geq z_{\alpha/2}$

Hypothesis Test Concerning Two Population Means When Population Variances Are Known

Δ_0 is the fixed, hypothesized difference in means. Usually, $\Delta_0 = 0$

- H_0: $\mu_1 - \mu_2 = 0$ corresponds to a test of H_0: $\mu_1 = \mu_2$.
- Δ_0 may be some nonzero value. For example, H_0: $\mu_1 - \mu_2 = 12$ would assume the means differ by 12.

Example: Low-Carb Ice Cream

Low-carbohydrate foods are very popular as many Americans try to avoid the sugar and starch combination that many believe causes weight gain. An advertisement for a low-carb ice cream claims that the product has 16 fewer g of carbohydrates per serving than the leading store brand. To check this claim, independent random samples of each type of ice cream were obtained, and the amount of carbohydrates in each serving was measured.

The store brand had a sample mean of 20.6570 g with $n_1 = 38$, and the low-carb brand had a mean of 3.8486 g with $n_2 = 35$. The variance in carbohydrates per serving is known to be 8.5 for the store brand and 0.253 for the low-carb brand. Is there any evidence to suggest that the difference in population means of carbohydrates per serving is different from 16 g? Use $\alpha = 0.01$.

$H_0: \mu_1 - \mu_2 = 16$

$H_a: \mu_1 - \mu_2 \neq 16$

The samples are random and independent, with both sample sizes larger than 30, and both population variances are known.

Example: Low-Carb Ice Cream

Store brand	Low-carb brand
$n_1 = 38$	$n_2 = 35$
$\overline{x}_1 = 20.6570$	$\overline{x}_2 = 3.8486$
$\sigma_1^2 = 8.5$	$\sigma_2^2 = 0.253$

$\alpha = 0.01$

$$z = \frac{(\overline{x}_1 - \overline{x}_2) - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

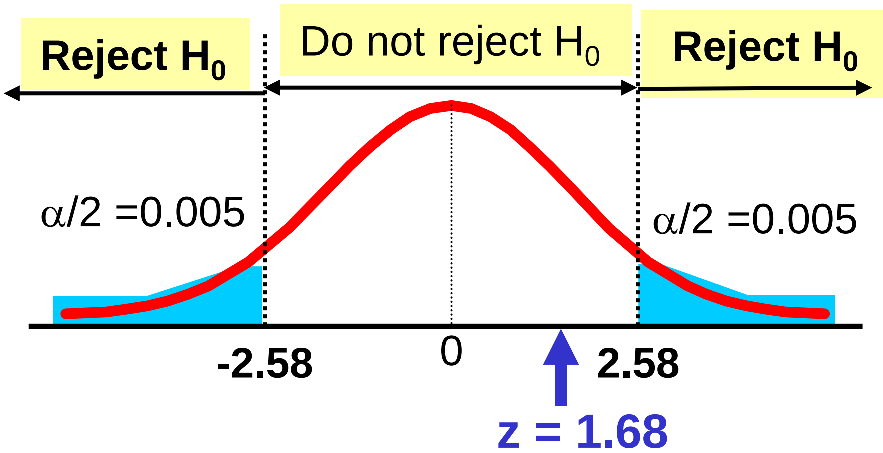
$$= \frac{(20.6579 - 3.8486) - 16}{\sqrt{\frac{8.5}{38} + \frac{0.253}{35}}} = 1.6842$$

RR: $|Z| \geq z_{\alpha/2} = z_{0.005} = \mathbf{2.58}$

Example: Low-Carb Ice Cream

1st. approach using z-critical value	2nd. approach using *p*-value

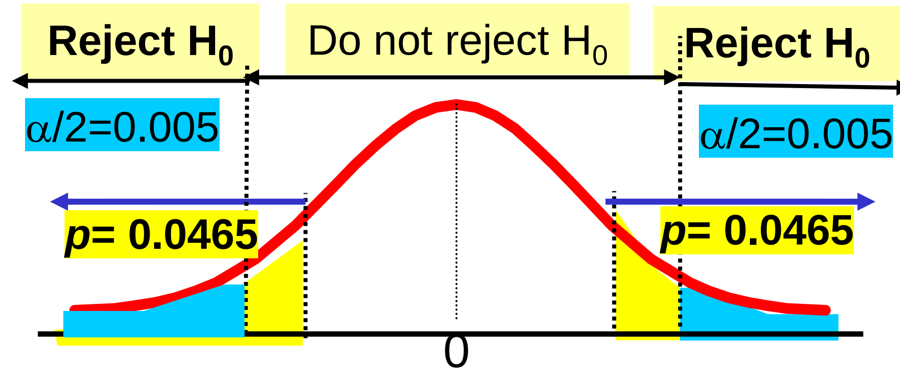
1st. approach using z-critical value



qnorm(0.005,0,1)
-2.575829

Since the z-test value = 1.68 is < the critical value =2.58, we do not reject H_0. TS does not lie in the rejection region.

2nd. approach using *p*-value



pnorm(-1.68,0,1)
0.04647866
p-value = 2(0.0465) = 0.093

The H_0 would be not rejected because the *p*-value = 9.3% is > α = 1%

Conclusion: There is no evidence to suggest that the difference in mean carbohydrates is different from 16 g at the α = 0.01 significance level.

CI for $\mu_1 - \mu_2$ When Population Variances Are Known

How to Find a 100(1 − α)% CI for $\mu_1 - \mu_2$ When Variances Are Known?

Given the two-sample Z test assumptions, a 100(1 − α)% confidence interval for $\mu_1 - \mu_2$ has the following values as endpoints:

$$(\overline{x}_1 - \overline{x}_2) \pm z_{a/2} \cdot \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

Example: Pizza stones designed for home use help cooks produce baked goods with brick oven qualities—for example, a crusty loaf of bread or crispy-crust pizza. However, pizza stones can be very heavy and can also take up a lot of space in a traditional residential oven. Independent random samples of two similar types of round pizza stones were obtained, and the weight (in pounds) of each was recorded. The summary of statistics and known variances are given in the table:

Find a 95% confidence interval for the difference in population mean pizza-stone weights.

Pizza stone	Sample size	Sample mean	Population variance
Kitchen Depot (1)	$n_1 = 35$	$\overline{x}_1 = 6.21$	$\sigma_1^2 = 2.1$
Head Chef (2)	$n_2 = 31$	$\overline{x}_2 = 7.08$	$\sigma_2^2 = 3.5$

CI for $\mu_1 - \mu_2$ When Population Variances Are Known

Sample sizes, sample means, and known variances are given. The underlying weight distributions are unknown, but the sample sizes are both large (≥ 30).

$1 - \alpha = 0.95 \Rightarrow \alpha = 0.05 \Rightarrow \alpha/2 = 0.025$

$z_{\alpha/2} = z_{0.025} = 1.96$

Pizza stone	Sample size	Sample mean	Population variance
Kitchen Depot (1)	$n_1 = 35$	$\bar{x}_1 = 6.21$	$\sigma_1^2 = 2.1$
Head Chef (2)	$n_2 = 31$	$\bar{x}_2 = 7.08$	$\sigma_2^2 = 3.5$

$$(\bar{x}_1 - \bar{x}_2) \pm z_{a/2} \cdot \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$= (6.21 - 7.08) \pm 1.96 \cdot \sqrt{\frac{2.1}{35} + \frac{3.5}{31}}$$

$$= -0.87 \pm 0.8150 = (-1.6850, \; -0.0550)$$

A 95% confidence interval for the difference in population mean weights of the pizza stones is (–1.6850, –0.0550) lb.

Comparing Two Population Means Using Independent Samples from Normal Populations

The two-sample Z test is valid only if both population variances are known, but it is unrealistic to assume that the population variances are always known.

Additional assumptions regarding the population variances are necessary to construct a similar two-sample t test.

Suppose that

1) $\bar{X}_1$ is the mean of a random sample of size n_1 from a normal population with mean μ_1.
2) $\bar{X}_2$ is the mean of a random sample of size n_2 from a normal population with mean μ_2.
3) The samples are independent.
4) The population variances are **unknown** but equal. The common variance is denoted

$$\sigma^2 \left(= \sigma_1^2 = \sigma_2^2\right)$$

Properties of $\bar{X}_1 - \bar{X}_2$

If the two-sample *t* test assumptions are true, then the estimator $\overline{X}_1 - \overline{X}_2$ has the following properties.

1. $\mathrm{E}(\overline{X}_1 - \overline{X}_2) = \mu_{\overline{X}_1 - \overline{X}_2} = \mu_1 - \mu_2$

$\overline{X}_1 - \overline{X}_2$ is still an unbiased estimator of the parameter $\mu_1 - \mu_2$.

2. $\mathrm{Var}(\overline{X}_1 - \overline{X}_2) = \sigma^2_{\overline{X}_1 - \overline{X}_2} = \dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2} = \dfrac{\sigma^2}{n_1} + \dfrac{\sigma^2}{n_2} = \sigma^2\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)$

and the standard deviation is

$$\sigma_{\overline{X}_1 - \overline{X}_2} = \sqrt{\sigma^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

Properties of $\bar{X}_1 - \bar{X}_2$

- S_1^2 and S_2^2 are separate estimators for the common variance, but using only one of them means ignoring additional, useful information.
- Because σ^2 is the variance for both underlying populations, an estimator for this common variance should depend on both samples. Yet, it also seems reasonable for the estimator to rely more on the larger sample.
- Therefore, an estimate of the common variance uses both S_1^2 and S_2^2 in a weighted average.

The **pooled estimator** of the common variance σ^2, denoted S_p^2, is

$$\begin{aligned} S_p^2 &= \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2} \\ &= \left(\frac{n_1 - 1}{n_1 + n_2 - 2}\right) S_1^2 + \left(\frac{n_2 - 1}{n_1 + n_2 - 2}\right) S_2^2 \end{aligned}$$

The pooled estimator of the common standard deviation σ is $S_p = \sqrt{S_p^2}$.

Hypothesis Tests Concerning Two Population Means When Variances Are Unknown But Equal

Given the two-sample *t*-test assumptions, a hypothesis test concerning two population means in terms of the difference in means $\mu_1 - \mu_2$, with significance level α, has the following form:

H_0: $\mu_1 - \mu_2 = \Delta_0$

H_a: $\mu_1 - \mu_2 > \Delta_0$,

$\mu_1 - \mu_2 < \Delta_0$,

or $\mu_1 - \mu_2 \neq \Delta_0$

$$\text{TS:} \quad t = \frac{(\overline{x}_1 - \overline{x}_2) - \Delta_0}{\sqrt{S_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$\text{RR: } t \geq t_{\alpha, n_1+n_2-2}, \; t \leq t_{\alpha, n_1+n_2-2}, \text{ or } |t| \geq t_{\alpha/2, n_1+n_2-2}$$

Example: Weight of Aluminum Cans

Aluminum cans are made from huge solid ingots that are pressed under high-pressure rollers and are cut like cookies from thin sheets. A company claims that a new manufacturing process decreases the amount of aluminum needed to make a can and, therefore, decreases the weight. Independent random samples of aluminum cans made by the old and new processes were obtained, and the statistics summary of each is given in the table. Is there any evidence that the new-process aluminum cans have a smaller population mean weight? Assume the populations are normal, with equal variances, and use $\alpha = 0.01$.

Process	Sample size	Sample mean	Sample variance
Old Process (1)	$n_1 = 21$	$\bar{x}_1 = 0.5048$	$s_1^2 = 0.0003762$
New Process (2)	$n_2 = 21$	$\bar{x}_2 = 0.4886$	$s_2^2 = 0.0004429$

H_0: $\mu_1 = \mu_2 \Rightarrow \mu_1 - \mu_2 = 0$

H_a: $\mu_1 > \mu_2 \Rightarrow \mu_1 - \mu_2 > 0$

Example: Weight of Aluminum Cans

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

$$= \frac{(20)(0.0003762) + (20)(0.0004429)}{40} = 0.0004095$$

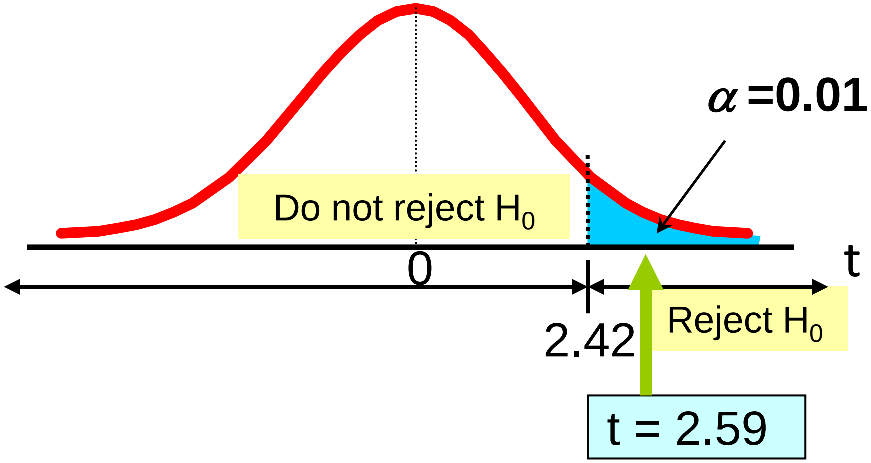
$$t = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$= \frac{0.5048 - 0.4886}{\sqrt{(0.0004095)\left(\frac{1}{21} + \frac{1}{21}\right)}} = \mathbf{2.5925}$$

$$\text{RR: } t \geq t_{\alpha, n_1 + n_2 - 2} = t_{0.01,40} = \mathbf{2.4233}$$

Example: Weight of Aluminum Cans

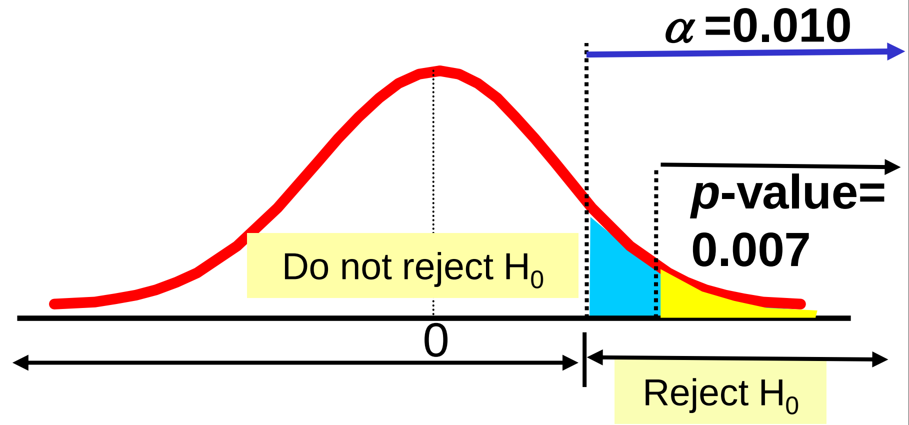
1st. approach using z-critical value



qt(0.99,40)
2.423257

Since the t-test value of 2.59 is > the critical value of 2.42, we reject the H_0

2nd. approach using *p*-value



pt(- 2.59,40)
0.006662288 ≈ 0.007
pt(2.59,40,lower.tail=FALSE)=0.0066623
pt(2.59,40)
0.9933377 → 1-0.9933377=0.0066623

The H_0 would be rejected because the *p*-value = 0.7% is < α = 1%

Conclusion: There is evidence to suggest that the new-process cans have a smaller mean weight.

CI for Two Population Means When Variances Are Equal

Given the two-sample t test assumptions, a $100(1 - \alpha)\%$ confidence interval for $\mu_1 - \mu_2$ has the following values as endpoints:

$$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2, n_1+n_2-2} \cdot \sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

Example10.6: Iron is an essential mineral. It is used by the body to carry oxygen, and even a slight deficiency can cause fatigue, weakness, and even anemia. Certain kinds of mollusks are very high in iron content—for example, clams, mussels, and oysters. Independent random samples of 3-oz servings of clams and oysters were obtained, and the iron content (in milligrams) was measured in each. The summary statistics are in the table. Assume the populations are normal and the variances are equal. Find a 99% confidence interval for the difference in population mean iron content.

Mollusk	Sample size	Sample mean	Sample variance
Clams (1)	12	6.94	2.25
Oysters (2)	15	7.82	1.86

Example: Iron Man

$$1-\alpha = 0.99 \Rightarrow \alpha = 0.01 \Rightarrow \alpha/2 = 0.005$$

$$t_{\alpha/2,n_1+n_2-2} = t_{0.005,25} = \mathbf{2.7874}$$

Mollusk	Sample size	Sample mean	Sample variance
Clams (1)	12	6.94	2.25
Oysters (2)	15	7.82	1.86

$$s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n-2}$$

$$= \frac{(11)(2.25)+(14)(1.86)}{25} = 2.0316$$

$$(\overline{x}_1 - \overline{x}_2) \pm t_{a/2,n_1+n_2-2} \cdot \sqrt{s_p^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$$

$$= (6.94-7.82) \pm (2.7874) \cdot \sqrt{(2.0316)\left(\tfrac{1}{12}+\tfrac{1}{15}\right)}$$

$$= -0.88 \pm 1.5387$$

$$= (-2.4188,\ 0.6588)$$

Because 0 is included in this interval, there is no evidence to suggest the mean iron content is different in 3-oz servings of clams and oysters.

Hypothesis Test for Two Population Means When Variances Are Unknown and Unequal

Given the modified Welch two-sample t-test assumptions (population variances unknown and assumed unequal), an approximate hypothesis test concerning two population means, in terms of $\mu_1 - \mu_2$, with significance level α, has the following form:

H_0: $\mu_1 - \mu_2 = \Delta_0$

H_a: $\mu_1 - \mu_2 > \Delta_0$, $\quad \mu_1 - \mu_2 < \Delta_0$, $\qquad$ or $\mu_1 - \mu_2 \neq \Delta_0$

$$\text{TS: } t' = \frac{(\overline{x}_1 - \overline{x}_2) - \Delta_0}{\sqrt{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)}}$$

$$\text{RR: } t' \geq t_{\alpha,\nu}, \quad t' \leq -t_{\alpha,\nu}, \quad \text{or} \quad |t'| \geq t_{\alpha/2,\nu}$$

$$\text{where } \nu \approx \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(s_1^2/n_1\right)^2}{n_1 - 1} + \frac{\left(s_2^2/n_2\right)^2}{n_2 - 1}}$$

It is likely that the value of v will not be an integer. To be conservative, always round down (to the nearest integer).

Confidence Intervals for Two Population Means When Variances Are Unknown and Unequal

An approximate $100(1 - \alpha)\%$ confidence interval for $\mu_1 - \mu_2$ has the following values as endpoints:

$$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2,\nu} \cdot \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$$

EXAMPLE 10.7: The clay-composite poker chips used in Las Vegas and Atlantic City casinos weigh between 8.5 and 10 g each, and last between 3 and 6 years. In a recent study of poker-chip weights, a casino obtained independent random samples of \$100 and \$500 chips. Is there any evidence to suggest that the population mean weight of \$100 chips is different from that of \$500 chips? Assume both populations are normal and use $\alpha = 0.05$. The statistics summary of each is given in the table.

A detailed solution is given in the textbook

Process	Sample size	Sample mean	Sample variance
\$100 chips (1)	$n_1 = 14$	$\bar{x}_1 = 9.2043$	$s_1^2 = 0.008934$
\$500 chips (2)	$n_2 = 9$	$\bar{x}_2 = 9.4322$	$s_2^2 = 0.1785$

Hypothesis Test for Two Population Means When Data Are Paired

- In the previous two sections, the assumption was that the samples were obtained independently.
- When comparing two population means, the samples may have been obtained independently. However, many experiments involve only n individuals, where two observations are made of each individual.
- When using paired data, we still want to consider the difference of the population means. However, the sample means are not independent because the paired observations are dependent.
- Two-Sample Paired t Test Assumptions:
  1. There are n pairs of individuals or objects, that are related in an important way or share a common characteristic.
  2. There are two observations of each individual. The populations of first and second observations are normal or approximately normal.

Hypothesis Test for Two Population Means When Data Are Paired

- Notation:
  - X_1 represents a randomly selected first observation and X_2 represents the corresponding second observation on the same individual.
  - Consider the random variable $D = X_1 - X_2$, which is the difference in the observations. There are n observed differences: $d_i = x_{1i} - x_{2i}$, $i = 1, 2, \ldots, n$
  - Because X_1 and X_2 are both normal, D is also normal. Thus, the differences are independent and normally distributed.
  - A hypothesis test concerning $\mu_1 - \mu_2$ is based on the sample mean of the differences, $\bar{D}$.
- This random variable, $\bar{D}$ has the following properties:
  - $\mathrm{E}\left(\bar{D}\right) = D$ (it is an unbiased estimator of the difference in means, $\mu_1 - \mu_2$)
  - $Var(\bar{D})$ is unknown, but it can be estimated using the sample variance of the differences, S_D^2.
  - Both underlying populations are normal, so D is normal and, therefore, $\bar{D}$ is also normal.

Hypothesis Test for Two Population Means When Data Are Paired

Given the two-sample paired t test assumptions, a hypothesis test concerning the two population means in terms of the difference $\mu_D = \mu_1 - \mu_2$, with significance level α, has the following form:

H_0: $\mu_D = \Delta_0$

H_a: $\mu_D > \Delta_0$, $\mu_D < \Delta_0$, or $\mu_D \neq \Delta_0$

Usually $\Delta_0 = 0$: The null hypothesis is that the two-population means are equal.

$$\text{TS: } t = \frac{\bar{d} - 0}{s_D / \sqrt{n}}$$

RR: $t \geq t_{\alpha,\, n-1}$, $t \leq -t_{\alpha,\, n-1}$, or $|t| \geq t_{\alpha/2, n-1}$

A paired t test is valid even if the underlying population variances are unequal.

Confidence Interval for μ_D

A $100(1 - \alpha)\%$ confidence interval for μ_D has the following values as endpoints:

$$\bar{d} \pm t_{a/2, n-1} \cdot \frac{s_D}{\sqrt{n}}$$

EXAMPLE 10.8 Relaxing Music

There is no direct scientific measure of stress, but some physical properties of the body that are believed to be related to stress include pulse rate, blood pressure, breathing rate, brain waves, muscle tension, skin resistance, and body temperature. Some researchers claim that music can be relaxing and, therefore, can reduce stress. Twelve patients who claim to be suffering from job-related stress were selected at random. An initial resting pulse rate (in beats per minute, bpm) was obtained, and each person participated in a month-long music-listening, relaxation-therapy program. A final resting pulse rate was taken at the end of the experiment.

a. Is there any evidence to suggest that the music-listening, relaxation-therapy program reduced the mean pulse rate and, therefore, the stress level? Assume the underlying distributions of initial and final pulse rate are normal, and use $\alpha = 0.05$.

b. Find a 95% confidence interval for the true difference in mean pulse rate.

H_0: $\mu_D = 0$

H_a: $\mu_D > 0$

RR: $t_{\alpha,n-1} = t_{0.05,11} = 1.7959$

The summary of statistics are:

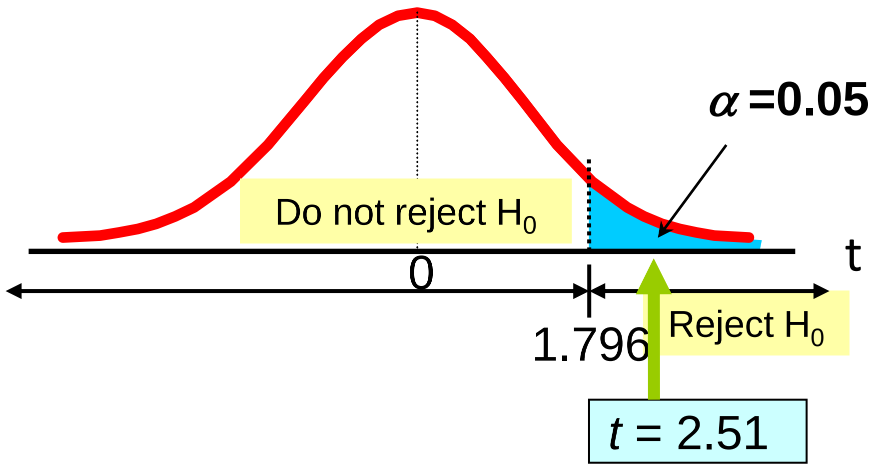
$$\bar{d} \approx 6.3333$$

$$s_D \approx 8.7317$$

EXAMPLE 10.8 Relaxing Music

$$t = \frac{\bar{d} - 0}{s_D/\sqrt{n}} = \frac{6.3333}{8.7317 / \sqrt{12}} \approx 2.51$$

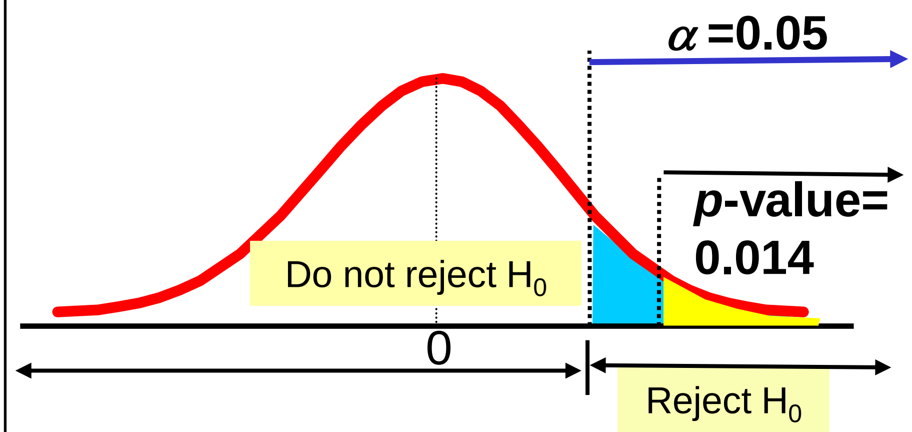
1st. approach using z-critical value



qt(0.95,11)
1.795885

Since the t-test value of 2.51 is > the critical value of 1.796, we reject the H_0

2nd. approach using *p*-value



pt(- 2.51,11)
0.01449323 ≈ 0.014

The H_0 would be rejected because the *p*-value = 1.4% is < α = 5%

Conclusion: There is evidence to suggest that the music-listening, relaxation-therapy program does reduce a person's resting pulse rate (and therefore the stress level).

EXAMPLE 10.8 Relaxing Music

b. Find a 95% confidence interval for the true difference in mean pulse rate.

$$1-\alpha = 0.95 \Rightarrow \alpha = 0.05 \Rightarrow \alpha/2 = 0.025$$

$$t_{\alpha/2,n-1} = t_{0.025,11} = 2.2010$$

$$\bar{d} \pm t_{a/2,n-1} \cdot \frac{s_D}{\sqrt{n}} = 6.33 \pm (2.2010) \cdot \frac{8.73}{\sqrt{12}}$$

$$= 6.33 \pm 5.5468$$

$$= (0.7832,\ 11.8768)$$

Because 0 is not included in this interval, there is an evidence to suggest that μ_D is > 0, and there is an evidence to suggest that the music-listening, relaxation-therapy program does reduce a person's resting pulse rate.

Summary

Hypothesis test for $\mu_1 - \mu_2$

Ddependent Samples

$$t = \frac{\bar{d} - 0}{s_D / \sqrt{n}}$$

Independent Samples

Population Variances Known

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

Population Variances Unknown

Population Variances are Equal

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{S_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

Population Variances are Unequal

$$t' = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)}}$$