University of Guelph

MATH*2000 - F21 : Term Test #2

Name: ________________

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1. *Ordered Relations* **(15 Marks)**

   Let $\mathcal{R}$ be a relation on the set $\mathbb{N} \times \mathbb{N}$. Two elements $(n_1, m_1), (n_2, m_2) \in \mathbb{N} \times \mathbb{N}$ are said to be related, $\big((n_1, m_1), (n_2, m_2)\big) \in \mathcal{R}$, (if $n_1 \leq n_2$) $\vee$ (if $m_1 \leq m_2$).

   (a) Aside from reflexivity, list the three conditions required in order for $\mathcal{R}$ to be a total order relation. **(3 Marks)**

   (b) Prove whether or not each of the conditions listed in part a) are true for $\mathcal{R}$. **(6 Marks)**

   (c) Suppose that the $\vee$ of this question was changed to $\wedge$. How does this change each of our answers to part (b). Be sure to prove all of your claims. **(6 marks)**

2. *Functions on Sets* **(12 Marks)**

For each of following functions listed below, **prove** whether they are *i)* injective, *ii)* surjective, or *iii)* bijective; *iv)* **state** whether the functions have an inverse, and if they do: *v)* **list** their elements using a set notation.

(a) $f : \mathbb{R} \to \mathbb{R},\ \ g(x) = \begin{cases} x^2 - 2x + 4 & x \leq 1 \\ -(x-1)^2 + 3 & x > 1 \end{cases}$ **(6 Marks)**

(b) $A = \{00, 01, 11\},\ B = \{0, 1, 2, 3\},\ \ g : A \to B,\ \ g = \{(00, 0), (01, 1), (11, 2)\}$ **(6 Marks)**

3. *Counting Arguments* **(16 Marks)**

Suppose that we were trying to make a travel plan for a vacation in Timmins, Ontario. Suppose that there are L different locations we could go to and that we wish to visit v of them (where $L \geq v$). Answer the following questions about how many different trips we could plan. For each part, be sure to justify your answer with a formula and a sentence that offers clarification.

(a) How many trips could we make if we wanted to visit v locations without visiting the same place twice, but we cared about the order in which we visited them? **(3 Marks)**

(b) How many trips could we schedule if we didn't want to visit the same place twice, but we didn't care about the order in which we visited our planned locations? **(2 Marks)**

(c) How many trips could we schedule if we were allowed to visit the same place multiple times, but we still cared about the order in which we visited the locations? (visiting the same location twice in a row would be considered possible by quickly pulling in and out of the parking lot). **(2 Marks)**

(d) How many trips could we schedule if we were allowed to visit the same locations multiple times, but we didn't care about the order in which we visited them? **(3 Marks)**

(e) Suppose we wanted to visit all L of the locations during our trip. We will be there for d days and we must visit each location exactly once during our stay. How many different ways can we schedule our trip assuming that we care about the order in which we visit each location and we also care about which day we visit a given location? **(4 Marks)**

(f) How does our answer in part (e) change if we relax our conditions and worry not about which order we visit the locations on a given day? Rather than provide an exact formula, simply discuss the consequences. Similarly, how would our answer change if we made sure never to schedule an empty day that was filled with no visits? **(2 Marks)**

4. *Uncountable Infinities* **(6 Marks)**

Let S be a set that is a proper subset of the real numbers $\mathbb{R}$. Give an example of a surjective function $g : S \to \mathbb{R}$. You must explicitly write your subset S and prove that it is a proper subset of $\mathbb{R}$. You must also prove that your example function is surjective. Discuss briefly what implications arise due to this function mapping existing.