CIS*4720 Image Processing and Vision

Winter 2023, Assignment 2 Part 1/2

Your submission must include the statement below, followed by your signature:
"I have read and understood the Academic Misconduct section in the course outline.
I assert this work is my own."

All answers must be justified in a clear, concise and complete manner. If two answers require the same or very similar explanations, you may justify your first answer only, and refer the reader to that justification for the second answer.

M, N, m and n are given positive integers, with m and n odd.

An *infinite image* is a total function from $\mathbb{Z}^2$ to $\mathbb{R}$.
A *finite image* is a total function from $0..M-1 \times 0..N-1$ to $\mathbb{R}$.
A *neighbourhood* is a total function from $0..m-1 \times 0..n-1$ to $\mathbb{R}$.

Consider a finite image g. The infinite images generated from g through zero padding, circular indexing and reflected indexing are denoted by g^Z, g^C and g^R, respectively.

Consider a neighbourhood f and an infinite image G.
The set $\{(u,v) \in 0..m-1 \times 0..n-1 \mid f(u,v) \neq 0\}$ is denoted by S_f.
The finite images $T_f(G)$, $MIN_f(G)$, $MEDIAN_f(G)$ and $MAX_f(G)$ are defined as follows:

$$\forall (x,y) \in 0..M-1 \times 0..N-1,\ T_f(G)(x,y) = \sum_{u=0}^{m-1} \sum_{v=0}^{n-1} f(u,v)\, G\left(x+u-\frac{m-1}{2}, y+v-\frac{n-1}{2}\right)$$

$$\forall (x,y) \in 0..M-1 \times 0..N-1,\ MIN_f(G)(x,y) = \min_{(u,v)\in S_f} G\left(x+u-\frac{m-1}{2}, y+v-\frac{n-1}{2}\right)$$

$$\forall (x,y) \in 0..M-1 \times 0..N-1,\ MED_f(G)(x,y) = \operatorname{median}_{(u,v)\in S_f} G\left(x+u-\frac{m-1}{2}, y+v-\frac{n-1}{2}\right)$$

$$\forall (x,y) \in 0..M-1 \times 0..N-1,\ MAX_f(G)(x,y) = \max_{(u,v)\in S_f} G\left(x+u-\frac{m-1}{2}, y+v-\frac{n-1}{2}\right)$$

1. Prove or disprove that for any neighbourhood f and any infinite images G_1 and G_2:

a) $T_f(G_1+G_2) = T_f(G_1)+T_f(G_2)$
b) $MIN_f(G_1+G_2) = MIN_f(G_1)+MIN_f(G_2)$
c) $MED_f(G_1+G_2) = MED_f(G_1)+MED_f(G_2)$
d) $MAX_f(G_1+G_2) = MAX_f(G_1)+MAX_f(G_2)$

2. Prove or disprove that for any neighbourhood f, any infinite image G and any real number k:

a) $T_f(k.G) = k.T_f(G)$
b) $MIN_f(k.G) = k.MIN_f(G)$
c) $MED_f(k.G) = k.MED_f(G)$
d) $MAX_f(k.G) = k.MAX_f(G)$