University of Guelph
CIS 2910 F16 – Midterm (Oct. 13)
Instructor: Joe Sawada

First Name: ______________________________

Last Name: ______________________________

Student Number: ______________________________

Problem 1: (5 marks)		Problem 6: (4 marks)	
Problem 2: (6 marks)		Problem 7: (4 marks)	
Problem 3: (4 marks)			
Problem 4: (6 marks)			
Problem 5: (4 marks)			
		Total (33 marks)	

This test is closed book and lasts 75 minutes.
You may not use any electronic/mechanical computation devices.
There are 8 pages including the cover page.

Problem 1: [5 marks]

(a) TRUE or FALSE: In a class of 30 students, at least 3 must be born in December.

(b) TRUE or FALSE: In a class of 30 students, there exists at least one day of the week on which exactly 5 were born.

(c) TRUE or FALSE: $\binom{10}{0} \cdot 0! = 1$

(d) TRUE or FALSE: Let A be a set with 30 elements and let B be a set with 60 elements. Then the number of different subsets of A is greater than the number of subsets of B with exactly 30 elements.

(e) TRUE or FALSE: The number of ways to rearrange the letters in the word RAPTORS is the same as the number of ways to rearrange the letters ABCDEFG such that A comes before B.

Problem 2: [6 marks]
A local bakery wants to display 15 pies in a row in its front window. Because it is near Thanksgiving, they can choose from a selection of Pumpkin, Apple, and Cherry pies. They have an unlimited number of each. How many ways can they set up their display of 15 pies if:

(a) [**1 mark**] There are no cherry pies?

(b) [**1 mark**] The first and last pie is pumpkin?

(c) [**1 mark**] There are exactly 3 pumpkin pies?

(d) [**1 mark**] There are at least 2 pumpkin pies?

(e) [**1 mark**] Each pie appears at least 4 times?

(f) [**1 mark**] Either the first pie is apple, or the middle pie is apple.

Problem 3: [4 marks]

A local market is selling the following 8 different items: **A**pples, **B**ananas, **C**herries, **D**onuts, **E**ggs, **F**ries, **G**um, and **H**otdogs. It's a great day to go shopping. For this question, when you pack your (infinitely large) knapsack with items, the order of items does not matter.

(a) [**1 mark**] How many different ways can you fill your knapsack so that you have at most one of every item?

(b) [**1 mark**] How many different ways can you fill your knapsack so that it has at least 3 items, but no item is repeated?

(c) [**1 mark**] How many different ways can you fill your knapsack with a total of 20 items, and exactly 2 different items? For example, it may contain 18 eggs and 2 apples.

(d) [**1 mark**] How many different ways can you fill your knapsack with 100 items so it contains exactly 1 apple?

Problem 4: [6 marks]

(a) [**1 mark**] Express $\binom{20}{17}$ as a decimal number.

(b)[**1 mark**] How many ways are there to order the 7 letters TORONTO?

(c)[**1 mark**] What is the probability of rolling a 4 with two regular six-sided dice?

(d)[**1 mark**] It is a beautiful Sunday at the track. In the final race, there are 8 horses competing, and the race ends in a three-way tie for first. There are no other ties. Given these constraints, how many different ways can the horses finish?

(e)[**1 mark**] How many binary strings of length $n > 100$ are there such that the first 4 bits are 0's and the last 90 bits are 1's?

(f)[**1 mark**] How many permutations of the 10 letters ABCDEFGHIJ have the A next to J?

Problem 5: [4 marks]

(a) **[2 marks]** State the Pigeon Hole Principle.

(b) **[2 marks]** State Pascal's identity.

Problem 6: [4 marks]

(a) [**1 mark**] Given an example of a sequence that is a geometric progression.

(b) [**1 mark**] Given an example of a sequence that is an arithmetic progression.

(c) [**2 marks**] Simplify the following sum, where $n > 3$:

$$\sum_{i=3}^{n-1} 2^i$$

Problem 7: [4 marks]

Prove the following identity by induction for all $n \geq 2$:

$$\sum_{i=0}^{n} 2^{n-i} = 2^{n+1} - 1$$

– END OF MIDTERM –