CIS*4720 Image Processing and Vision
Winter 2023, Assignment 2 Part 2/2

Your submission must include the statement below, followed by your signature:
"I have read and understood the Academic Misconduct section in the course outline.
I assert this work is my own."

All answers must be justified in a clear, concise and complete manner. If two answers require the same or very similar explanations, you may justify your first answer only, and refer the reader to that justification for the second answer.

Here, an *infinite image* is a total function from $\mathbb{Z}^2$ to $\mathbb{Z}$, where $\mathbb{Z}$ is the set of all integers. Consider two infinite images F and G. We say that F is *related* to G, and we write F $\mathfrak{R}$ G, if and only if : $\exists(a,b)\in\mathbb{Z}^2, \forall(x,y)\in\mathbb{Z}^2, F(x+a,y+b)=G(x,y)$. It is easy to show that $\mathfrak{R}$ is an equivalence relation.

3.
a) Let F be the infinite image defined by F(x,y)=0, for all x and y.
How many infinite images are related to F?
b) Let G be the infinite image defined by G(x,y)=1 if x=y=0 and G(x,y)=0 otherwise.
How many infinite images are related to G?
c) Give an example of an infinite image H that is related to exactly two infinite images.

Here, a *finite image* is a total function from some set $0..M-1 \times 0..N-1$ to some set $0..L-1$, where M, N and L are positive integers. The image is then of size M×N, and it can be represented by the matrix

$$\begin{bmatrix} f(0,0) & f(0,1) & \dots & f(0,N-1) \\ f(1,0) & f(1,1) & \dots & f(1,N-1) \\ \dots & \dots & \dots & \dots \\ f(M-1,0) & f(M-1,1) & \dots & f(M-1,N-1) \end{bmatrix}.$$

Let f be a finite image of size M×N. The symbol f^Z (resp. f^C, f^R) denotes its extension to an infinite image through Zero padding (resp. Circular indexing, Reflected indexing).

4.
f^Z can be formally defined as follows: for any integers x and y, we have $f^Z(x,y)=f(x,y)$ if $x\geq 0$ and $x<M$ and $y\geq 0$ and $y<N$, and we have $f^Z(x,y)=0$ otherwise.
a) Give a formal definition of f^C.
b) Give a formal definition of f^R.

Let F be an infinite image. F is a Z-*image* (resp. C-*image*, R-*image*) if there exists a finite image f such that $f^Z \Re F$ (resp. $f^C \Re F$, $f^R \Re F$).
f is then called a Z-*generator* (resp. C-*generator*, R-*generator*) of F.
For example,

$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$ and $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 \end{bmatrix}$ are two Z-generators of $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}^Z$;

$\begin{bmatrix} 1 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$ are two C-generators of $\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^C$.

5.
a) How many Z-images are also C-images?
b) How many Z-images are also R-images?
c) Explain why R-images are also C-images.
d) Show that a C-image is not necessarily an R-image.

Let F be an infinite image. In the following definition, the term "generator" implicitly refers to a Z-, C- or R-generator of F: If f is a generator of size $M\times N$ and there is no generator g of size $m\times n$ with $m<M$ or $n<N$ then f is a *minimum generator*.
For example,

$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$ is a minimum Z-generator of $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}^Z$, but $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 \end{bmatrix}$ is not;

$\begin{bmatrix} 1 \end{bmatrix}$ is a minimum C-generator of $\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^C$, but $\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$ is not.

6.
a) Show that there exist infinite images with no minimum generator.
b) Show that there exist infinite images with exactly one minimum generator.
c) Show that there exist infinite images with exactly two minimum generators.