

MATH*2000: Practice Term Test #1

Name: _____

E-mail: _____

ID:_____

1. (10 Marks)

Let P, Q , and R be atomic statements that may either be *true* or *false*. Prove whether or not the following two statements are logically equivalent to one another.

$$\neg(P \wedge Q) \leftrightarrow (\neg Q \vee R) \equiv (\neg P \vee R) \leftrightarrow \neg Q$$

2. (15 Marks)

Prove whether or not the following statement is true. If it is *true*, then your proof must use the method of proof by contradiction. If it is *false*, provide a counter-example that contradicts it.

Prove that $\forall n \in \mathbb{N}$ if n is a perfect square ($\exists x \in \mathbb{N} \ni x^2 = n$), then the value $n + 1$ cannot be a perfect square.

3. (15 Marks)

Verify the following mathematical identity for all $n \in \mathbb{N}$ using a proof by induction technique.

$$\sum_{i=0}^n r^i = \frac{1 - r^{n+1}}{1 - r}$$

4. (10 Marks)

Let A , B , and C be sets. Prove that the following sets are equal.

$$A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$$