

CIS*3490 The Analysis and Design of Algorithms

Winter 2022

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Assignment 1

1. Question 3 on page 59 in the textbook. (10%)
2. Question 5 on page 60 in the textbook. (10%)
3. Question 1 on page 67 in the textbook. (10%)
4. Question 2 on page 67 in the textbook. (10%)
5. Question 3 on page 67 in the textbook. (10%)
6. Solve the following recurrences and find the efficiency class for each of them. (10%)
 1. $A(n) = 4A(n - 1)$ for $n > 1$, $A(1) = 5$
 2. $A(n) = A(n - 1) + 3$ for $n > 1$, $A(1) = 0$
 3. $A(n) = A(n - 1) + 2n$ for $n > 0$, $A(0) = 0$
 4. $A(n) = A(n/5) + 1$ for $n > 1$, $A(1) = 1$ (solve for $n = 5^k$)
 5. $A(n) = 2A(n/2) + \log_2 n$ for $n > 1$, $A(1) = 1$ (solve for $n = 2^k$)
7. Consider the following algorithm and answer the questions. (10%)

```
ALGORITHM S( $A[0..n - 1]$ )  
  // Input:  $A$  is an array containing  $n \geq 1$  real numbers  
  for  $i \leftarrow 0$  to  $n - 2$  do  
     $m \leftarrow i$   
    for  $j \leftarrow i + 1$  to  $n - 1$  do  
      if  $A[j] < A[m]$   
         $m \leftarrow j$   
    swap  $A[j]$  and  $A[m]$ 
```

1. What does this algorithm compute?

2. What is the input size?
 3. What is the basic operation?
 4. How many times is the basic operation executed? (Set up a sum, and simplify the sum, to find the function expressing the number of repetitions of the basic operation.)
 5. What is the efficiency class of this algorithm?
8. Consider the following algorithm and answer the questions. (10%)

```

ALGORITHM T( $A[0..n-1]$ )
  // Input:  $A$  contains  $n \geq 1$  real numbers
  for  $i \leftarrow 1$  to  $n-1$  do
     $v \leftarrow A[i]$ 
     $j \leftarrow i-1$ 
    while  $j \geq 0$  and  $A[j] > v$  do
       $A[j+1] \leftarrow A[j]$ 
       $j \leftarrow j-1$ 
     $A[j+1] \leftarrow v$ 

```

1. What does this algorithm compute?
 2. What is the input size?
 3. What is the basic operation?
 4. How many times is the basic operation executed? (Set up a sum, and simplify the sum, to find the function expressing the number of repetitions of the basic operation.)
 5. What is the efficiency class of this algorithm?
9. Consider the following algorithm and answer the questions. (10%)

```

ALGORITHM U( $A[1..n]$ )
  // Input:  $A$  is an array containing  $n \geq 1$  integers
  if  $n = 1$  return 1
  else if  $A[n] < A[n-1]$  return -1
  else return U( $A[1..n-1]$ )

```

1. What does this algorithm compute?
2. What is the input size?

3. What is the basic operation?
 4. Set up a recurrence with an initial condition, and solve the recurrence, to find the number of times the basic operation is executed.
 5. What is the efficiency class of this algorithm?
10. Consider the following algorithm and answer the questions. (10%)

ALGORITHM $W(A, l, r, K)$
 // Input: A is an array of sorted integers,
 // l and r are indexes of elements in A , K is an integer
if $l > r$ **return** -1
else
 $m \leftarrow \lfloor (l + r)/2 \rfloor$
 if $K = A[m]$ **return** m
 else if $K < A[m]$ **return** $W(A, l, m - 1, K)$
 else return $W(A, m + 1, r, K)$

1. What does this algorithm compute?
2. How is the input size n expressed in terms of the parameters?
3. What is the basic operation?
4. Set up a recurrence with an initial condition for the worst case of this algorithm. Solve the recurrence for $n = 2^k$, and determine the Θ efficiency class.
5. What is the Θ efficiency class when $n \neq 2^k$? Why?

Due time: 08:00am, Monday, Jan 31, 2022. Please submit an e-copy to Moodle.