Assignment-5
Monte-Carlo Simulation

Problem-1

In a nuclear chain reaction, a neutron is bombarded to hit a nucleus of an atom (Uranium-235 or Plutonium-239). When an atom undergoes nuclear fission, a few neutrons (the expected number depends on several factors, usually between 2.5 and 3.0) are ejected from the reaction. These free neutrons will then interact with the surrounding material, and if more fissile fuel is present, some may be absorbed and cause more fissions. Thus, the cycle repeats to give a reaction that is self-sustaining.

Suppose p_i is the probability that a neutron will result in a fission that produces i new neutrons. Your task is to calculate the probability distribution of the number of neutrons produced in the nth generation of a chain reaction.

In this problem, a fissile neutron can only produce a maximum of 3 new neutrons, while some may produce zero new ones. Here $p_i = (0.2126)(0.5893)^{i-1}$ for $i = 1, 2, 3$.

For 10 generations, simulate the production of new neutrons for each generation 10,000 times. Then calculate the probabilities of having $j = 0, 1, 2, 3, 4$ number of neutrons in the nth generation; $n \in [1, 10]$.

You can find the sample probabilities calculated for each n, j here.

Problem-2

In this task, you have to simulate the Secretary Problem/Optimal Stopping Marriage Problem. The rules for this problem are as follows:

(1) There is a population of potential candidates (size *n*) from which one has to be selected.
(2) You may meet or interview by picking one candidate from the population. In this way, you can meet the entire population one at a time.
(3) After each interview, you have to decide if this candidate is the one to be selected. If 'yes', the process terminates (success or failure can be determined at this point). If 'no', the process continues until the entire population is exhausted (remember that if the population is exhausted, then the last candidate is the one who will be selected, according to rule 1).
(4) Once a candidate is rejected after the interview, you can not go back for a review and the rejection is final.

A strategy may be followed where a sample group of size *m* is to be interviewed at first only for the purpose of setting a standard which is the best from this sample group. Nobody can be selected from this sample group while interviewing them one after the other. After the standard is set, anyone who is better than the standard will be selected.

For this task, you have to take an input of population size *n*, and the success criteria s. Here, s can be 1, 3, 5, or 10 which means the strategy is successful if the best, or anyone from top 3, or top 5, or top 10 was selected.

For a certain value of *n*, assume that each candidate has a unique rank from 1 to *n* and the sample size *m* can be from 0 to n-1. For a fixed value of *n* and *s*, the output is the probability of success of a sample size *m*. From the output, we can know the probability

distribution of success of a strategy (m), given the value of population size n and the success defined by s.

For n = 100 candidates with each success criteria s, plot the Success Rate vs. sample size (m) graphs to show the optimal strategy from where to choose the candidates.

You can find the sample plots here.