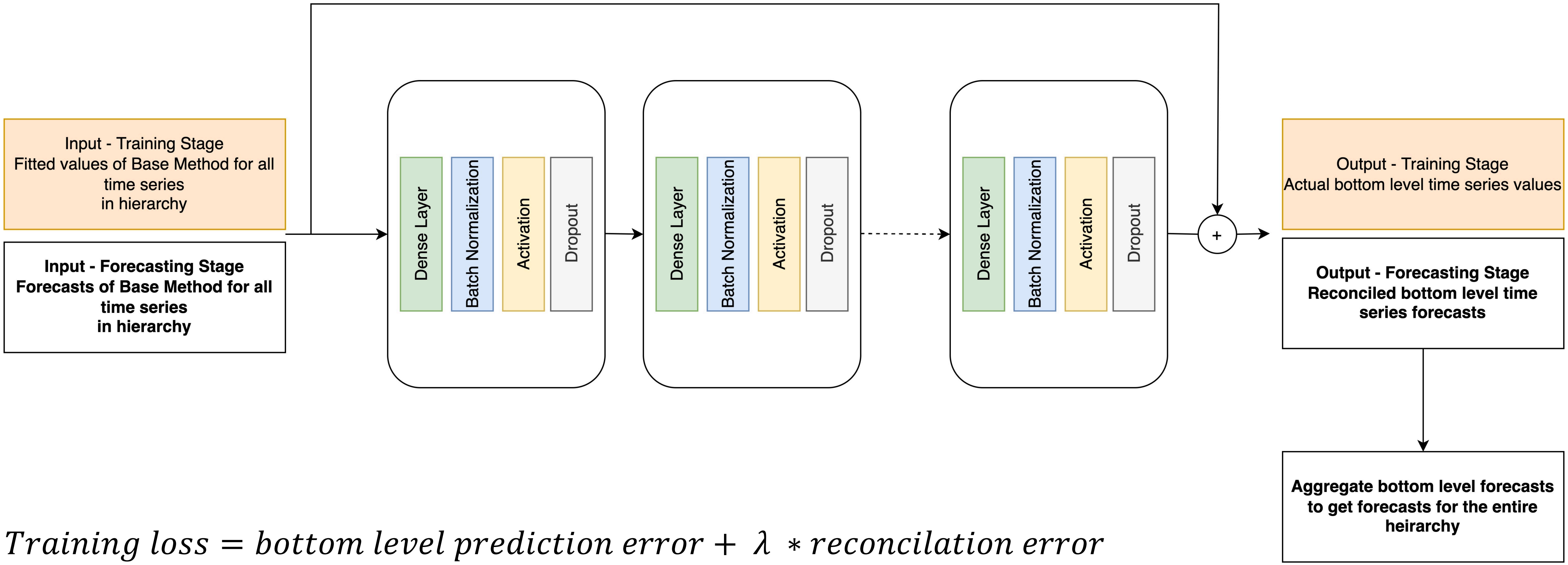
A Machine Learning based Approach to Forecast Hierarchical Time Series using Non-linear Mappings

- We propose a non-linear forecast reconciliation approach using machine learning methods
- We introduce a novel loss function incorporating non-linear mappings to obtain a set of coherent forecasts from the individual base forecasts.
- To obtain the weights of the non-linear mapping between the base forecasts, we train a feed-forward neural network with the proposed loss function using fitted values of the base models as inputs to the network.

Proposed Reconciliation Network

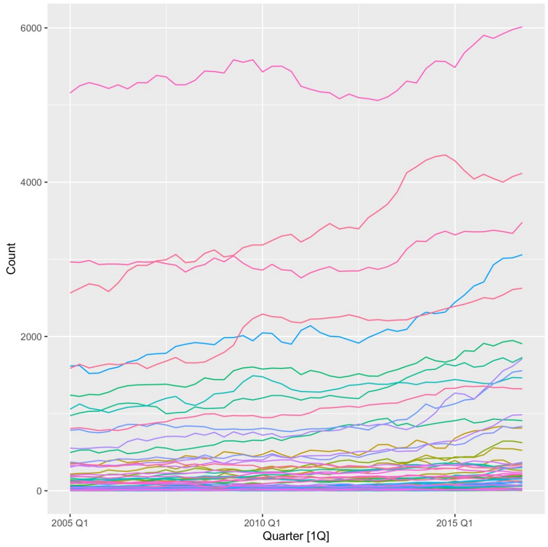


$$Training\ loss = bottom\ level\ prediction\ error + \lambda * reconcilation\ error$$

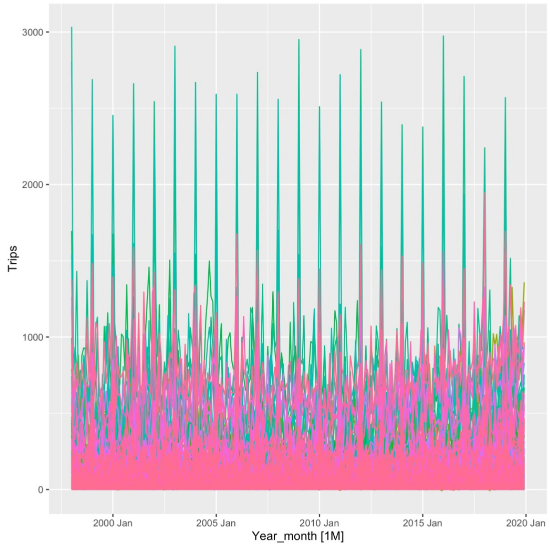
Data

Dataset	Frequency	Number of levels	Number of total time series	Horizon	Short Horizon
Prison	4 (quarterly)	5	121	8	4
Tourism	12 (monthly)	3	85	12	6
Wikipedia	7 (weekly)	6	1095	7	3
Labour	4 (quarterly)	4	57	12	6

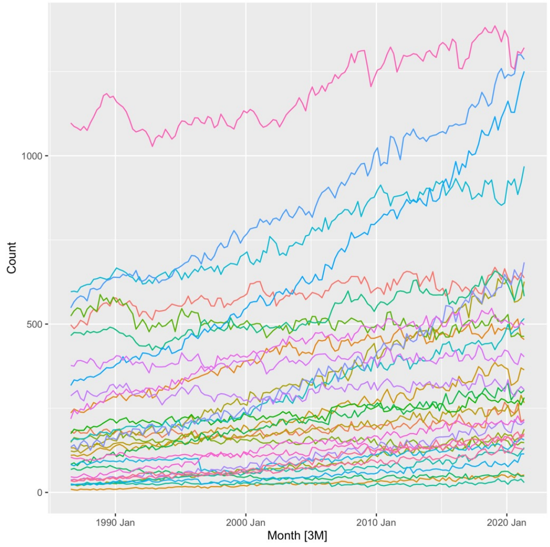
Data



Prison



Tourism



Labour

Wikipedia

Experimental Setup and Evaluation

- Base forecasting methods:
  - ARIMA
  - ETS
  - DeepAR
  - WaveNet

Univariate Model

Global Models

- The proposed ML Reconciliation mehtod is trained five times with different seeds and the average across these are taken as the final reconciled bottom level forecast

- Benchmark Reconciliation Methods:
  - Bottom Up
  - OLS
  - WLS
  - ERM
  - MinTShrink
  - MinTSample

- DeepAR and WaveNet methods are trained as Global Models

- To develop the Global Models – all time series in the hierarchy are clustered using K-mean algorithms and a DeepAR/ WaveNet model is built for each cluster

- We conduct an expanding window evaluation for all datasets. Based on the length of the time series in the datasets the following number of windows are created per dataset

Dataset	**Number of windows**	**Minimum training window length**
Prison	3	24
Tourism	10	144
Wikipedia	10	324
Labour	5	68

Experimental Setup and Evaluation

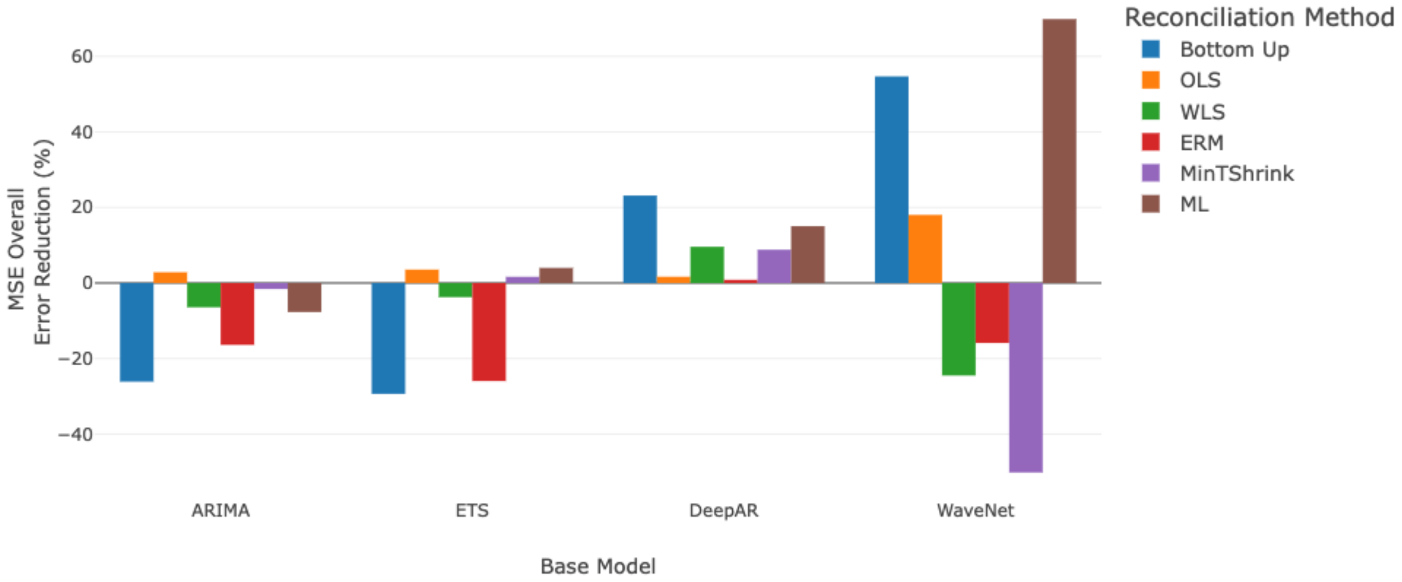
- Error calculation:
  1. MSE is calculated for all time series in the hierarchy
  2. The overall error (considering all levels) is calculated by taking the mean across the errors in step 1
  3. This is repeated for all windows of the dataset
  4. The mean across overall errors of the windows are calculated
  5. The percentage improvement of the overall error is calculated as follow

$$MSE\ error\ reduction\ percentage\ (\%) = \frac{MSE_{baseMethod} - MSE_{reconciled}}{MSE_{baseMethod}} * 100\%$$

- Hyper-parameters of the ML reconciliation method are tuned with HyperOpt – Bayesian Optimization

Hyper-parameter	**Minimum**	**Maximum**
Number of Layers	1	5
Dropout Rate	0	0.5
Learning Rate	0.0001	0.1
Number of Epochs	10	200
Batch Size	1	Size of input data
Max Norm	0	10
Lambada (Reconciliation Loss)	0.01	5

Results – Prison Dataset



ARIMA	ETS	DeepAR	WaveNet
OLS	ML Reconciliation	Bottom UP	ML Reconciliation

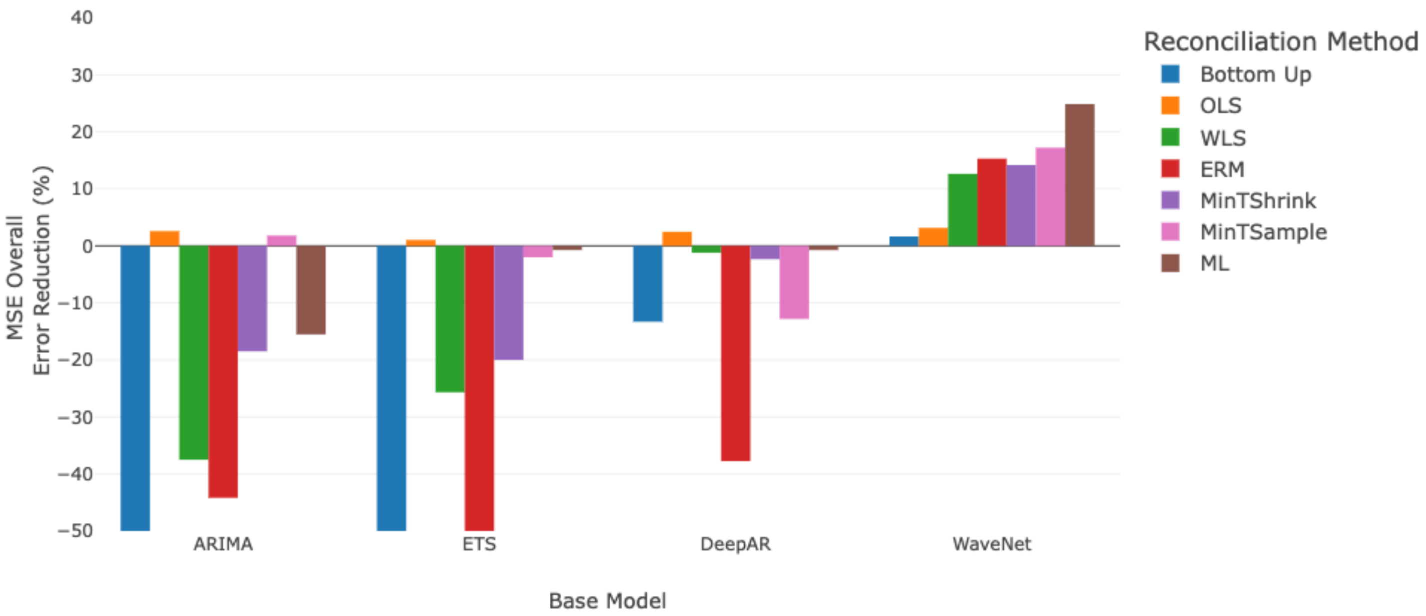
Results – Prison Dataset (Short Horizon)

ARIMA	ETS	DeepAR	WaveNet
ML Reconciliation	ML Reconciliation	Bottom UP	ML Reconciliation

Results – Prison Dataset (One step Horizon)

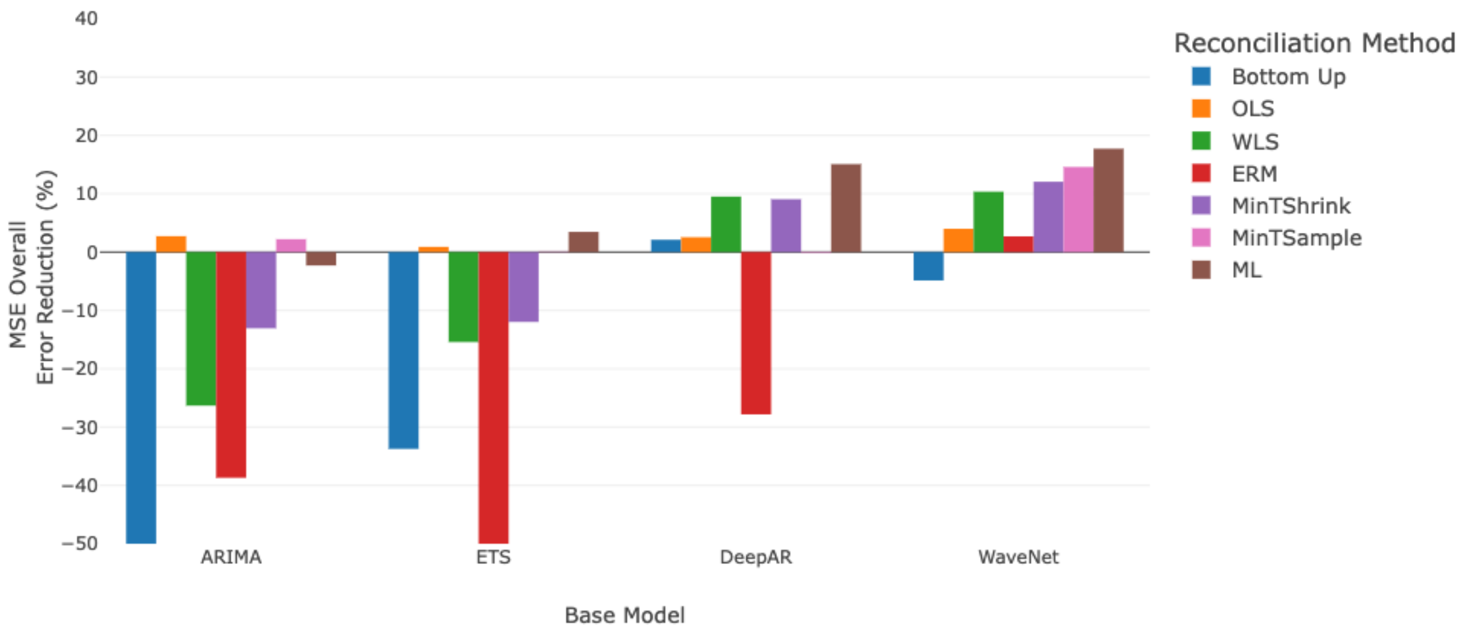
ARIMA	ETS	DeepAR	WaveNet
ML Reconciliation	ML Reconciliation	Bottom UP	ML Reconciliation

Results - Tourism



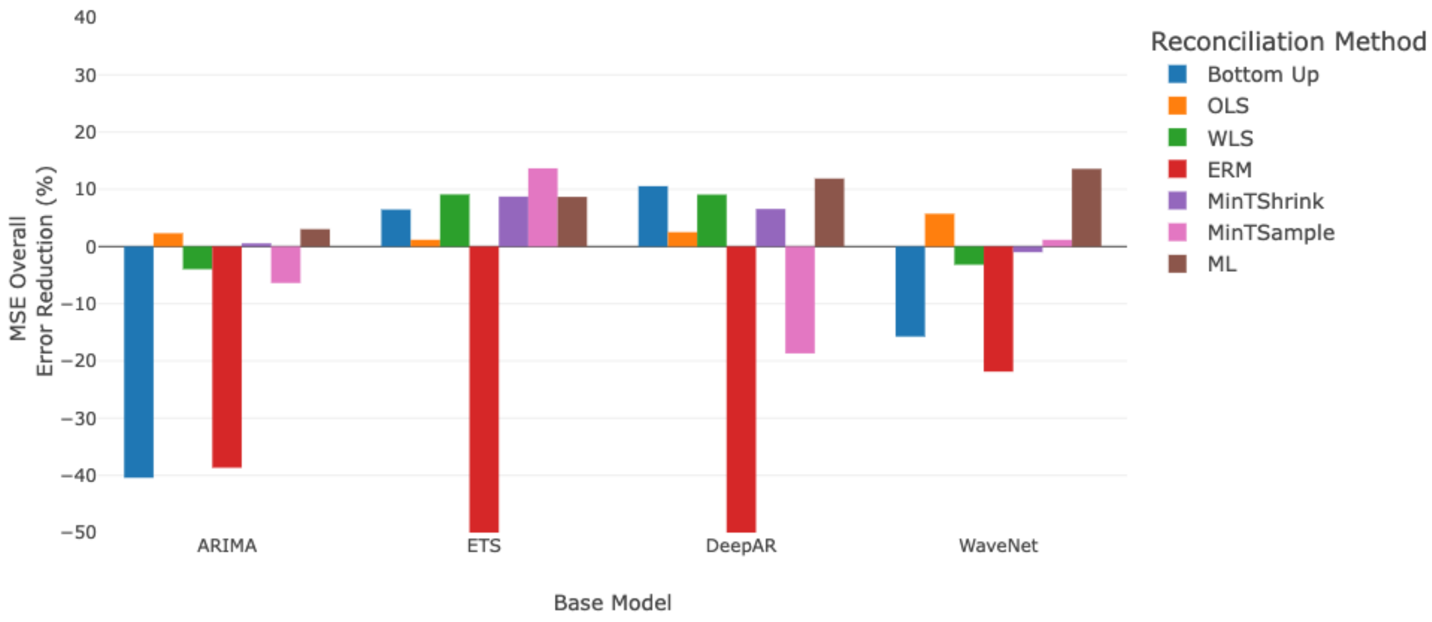
ARIMA	ETS	DeepAR	WaveNet
OLS	OLS	OLS	ML Reconciliation

Results – Tourism (Short Horizon)



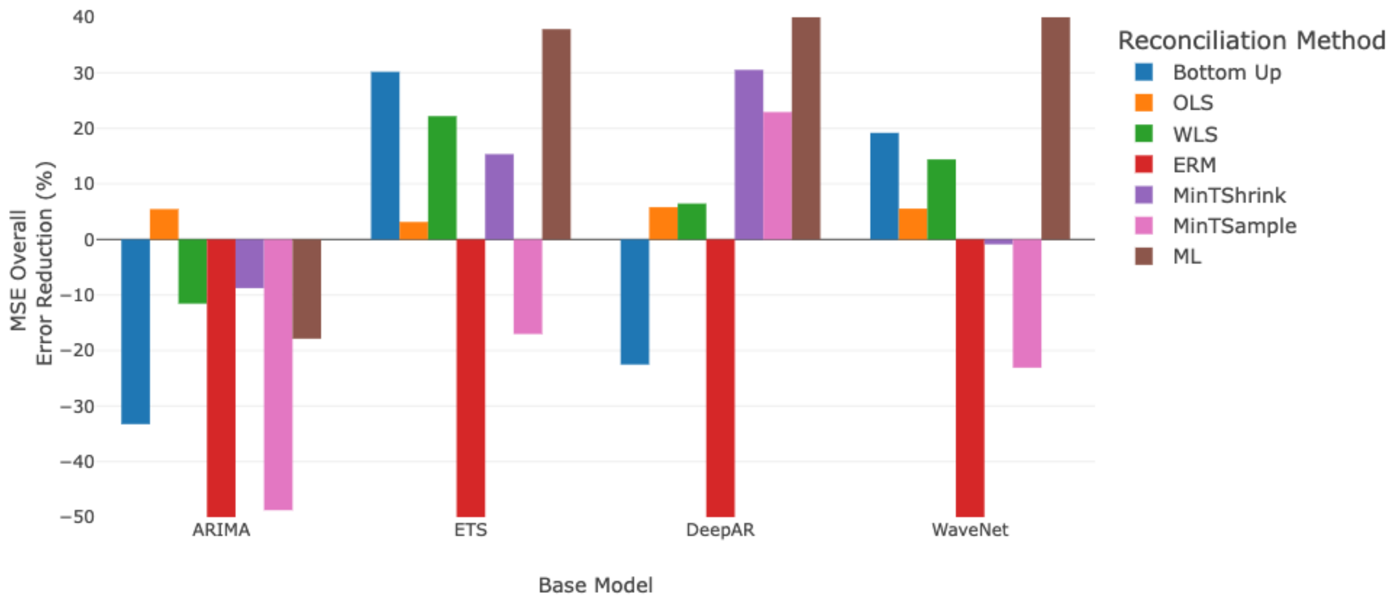
ARIMA	ETS	DeepAR	WaveNet
OLS	ML Reconciliation	ML Reconciliation	ML Reconciliation

Results – Tourism (One step Horizon)



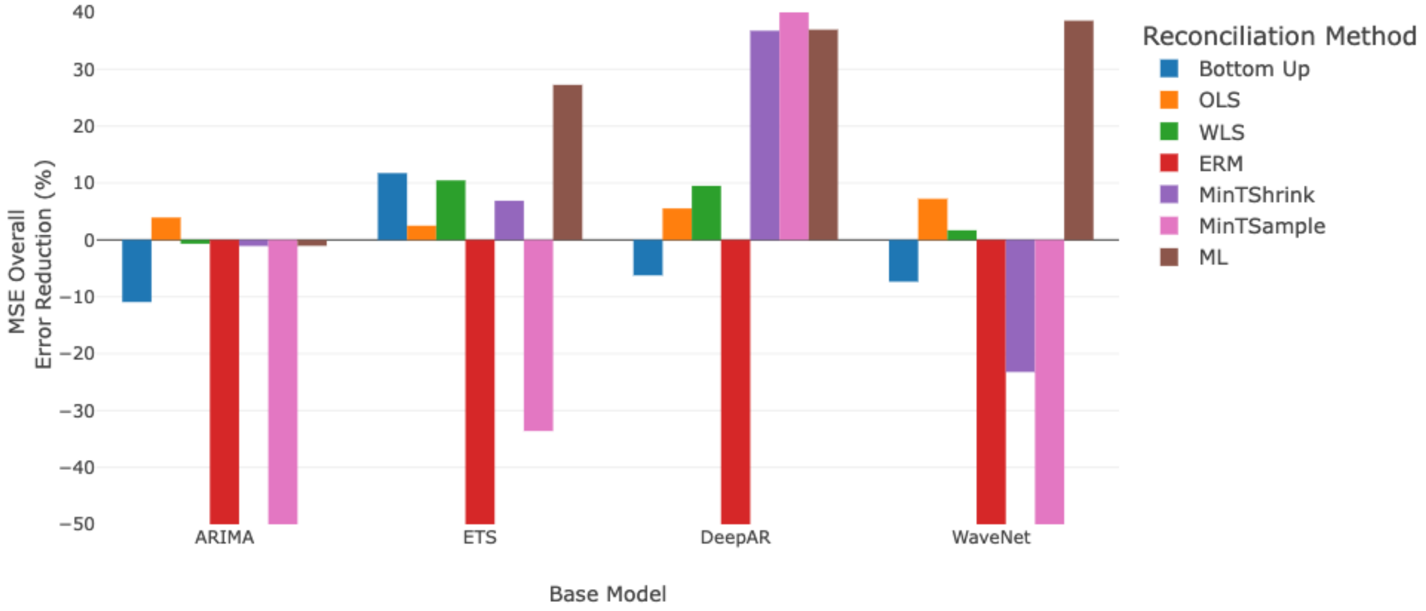
ARIMA	ETS	DeepAR	WaveNet
ML Reconciliation	MinT Sample	ML Reconciliation	ML Reconciliation

Results - Labour



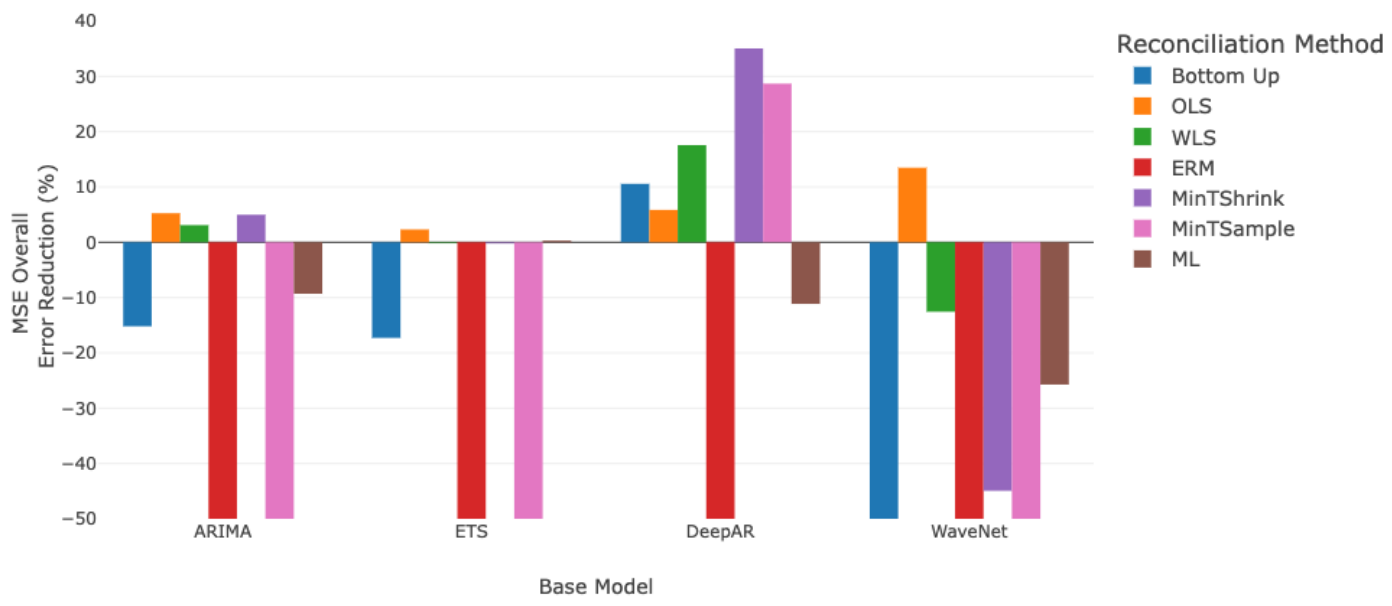
ARIMA	ETS	DeepAR	WaveNet
OLS	ML Reconciliation	ML Reconciliation	ML Reconciliation

Results – Labour (Short Horizon)



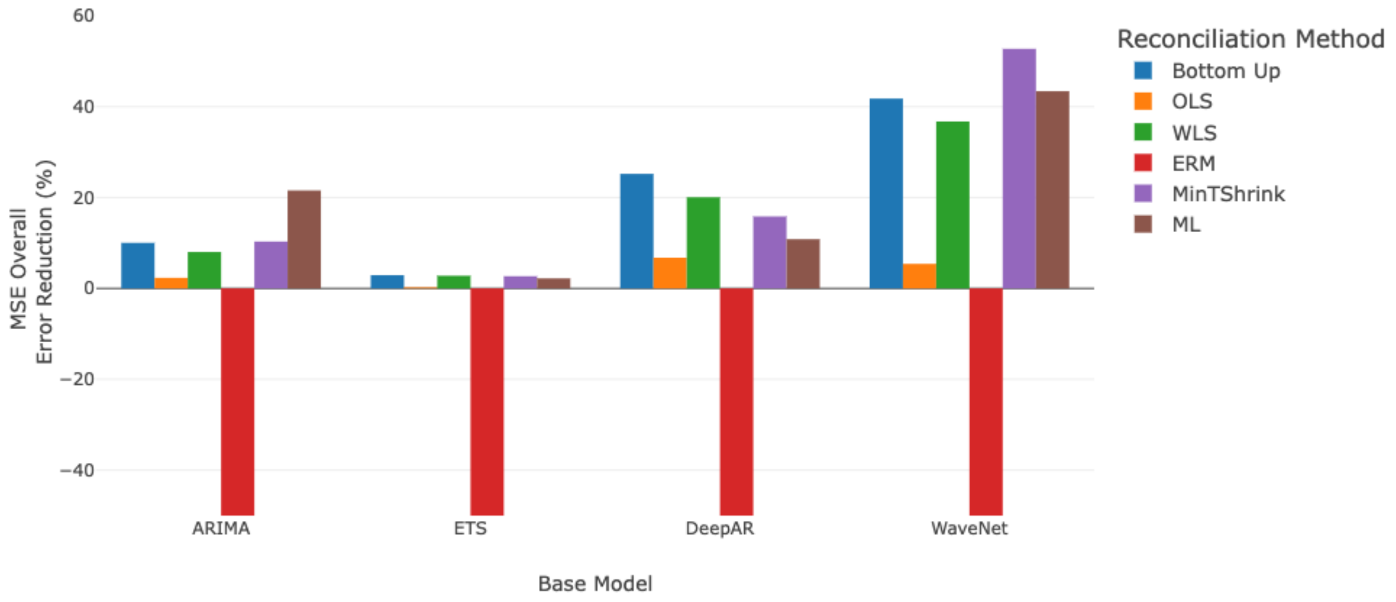
ARIMA	ETS	DeepAR	WaveNet
OLS	ML Reconciliation	MinTSample	ML Reconciliation

Results - Labour (One step Horizon)



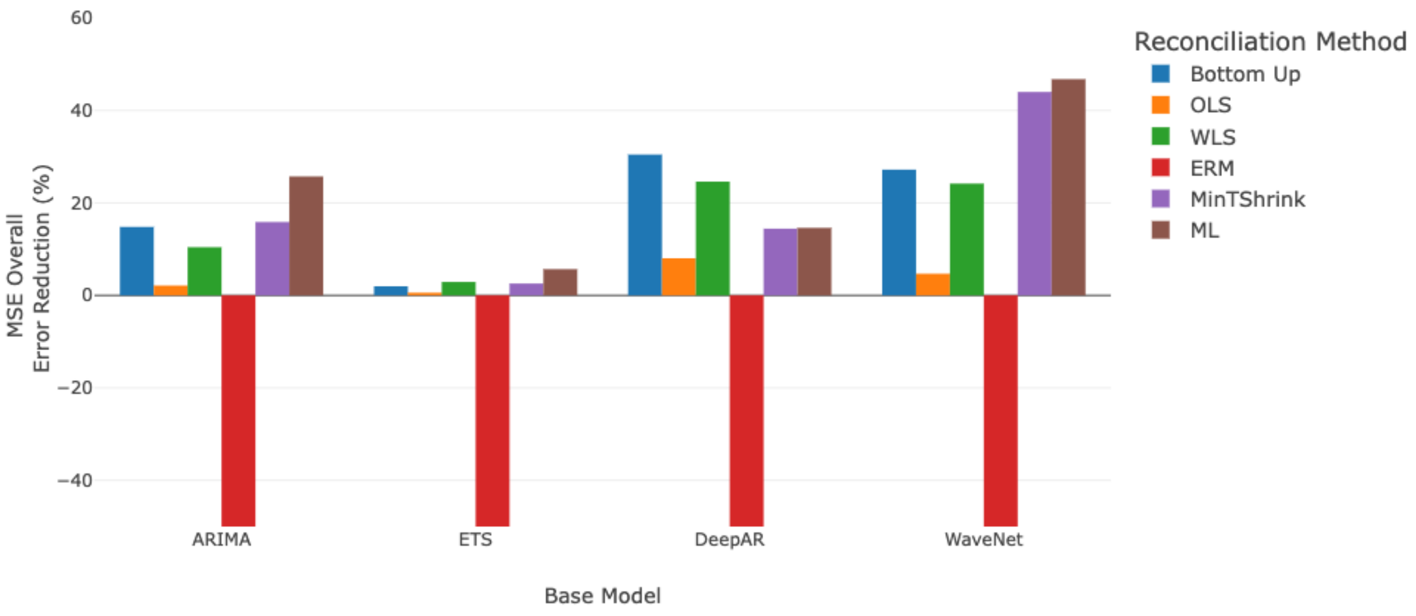
ARIMA	**ETS**	**DeepAR**	**WaveNet**
OLS	OLS	MinTShrink	OLS

Results - Wikipedia



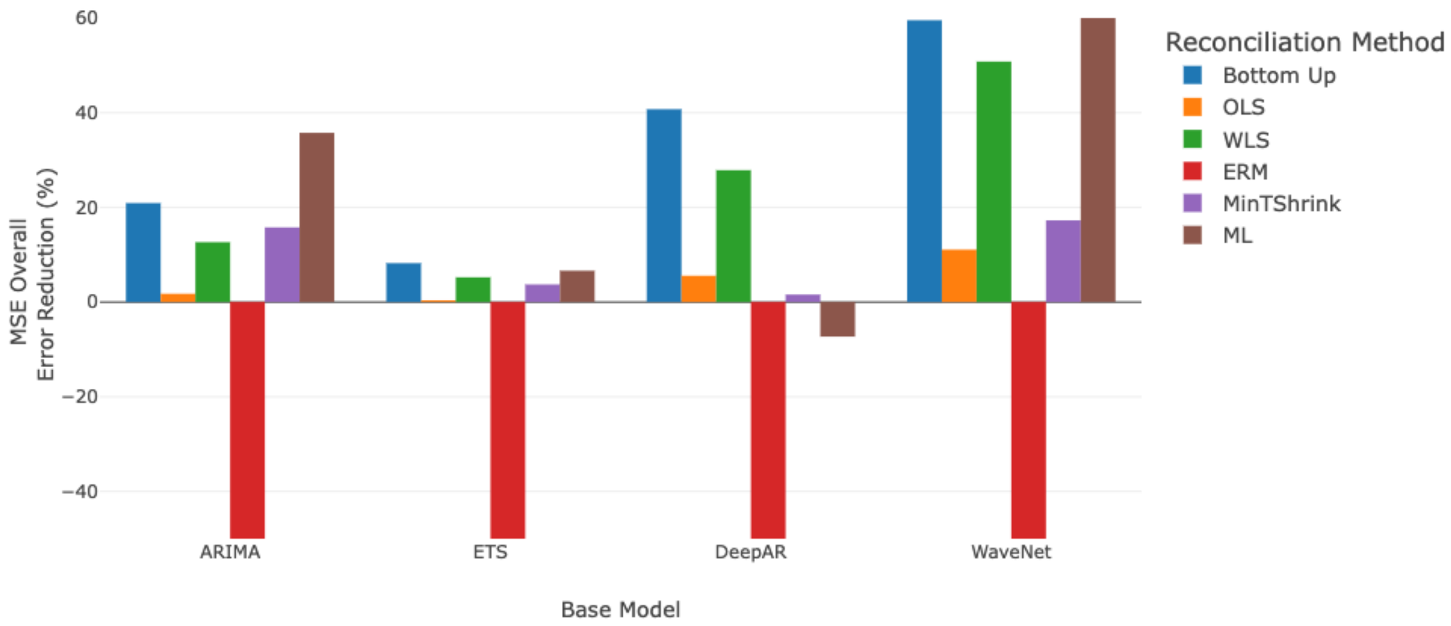
ARIMA	ETS	DeepAR	WaveNet
ML Reconciliation	Bottom Up	Bottom Up	MinTShrink

Results - Wikipedia (Short Horizon)



ARIMA	ETS	DeepAR	WaveNet
ML Reconciliation	ML Reconciliation	Bottom Up	ML Reconciliation

Results - Wikipedia (One step Horizon)



ARIMA	ETS	DeepAR	WaveNet
ML Reconciliation	Bottom Up	Bottom Up	ML Reconciliation