Chapter 4 Matrix Algebra

4.2 Introduction to Matrix concepts

A matrix is a rectangular (or square) array or real numbers arranged in rows and columns.

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} = \begin{pmatrix} 5 & 1 & 5 \\ 4 & 2 & 7 \end{pmatrix}$$

The order (or dimension) of the matrix is a $m \times n$ where m is the number of rows and n is the number of columns. A matrix is square if $m = n$.

Vectors:

Row vector: Example: $\begin{pmatrix} 1 & 2 & 4 \end{pmatrix}$ is a 1×3 row vector.

Column vector: Example: $\begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$ is a 3×1 column vector.

Scalar: Single number. Example: 4 is a scalar

4.3 Matrix operations

$A = \begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 5 \\ -1 & 1 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{pmatrix}$, $D = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$ and $E = \begin{pmatrix} -2 & 4 \end{pmatrix}$.

Matrix addition:

- $A + B = \begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} 2 & 5 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 3 \\ 2 & 5 \end{pmatrix}$

- $B + D = \begin{pmatrix} 2 & 5 \\ -1 & 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \neq$ Not possible to calculate

Matrix subtraction:

- $C - C = \begin{pmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$

A matrix with all elements equal to zero is called a zero matrix.

Transpose of a matrix:

- $E = (-2 \quad 4)$ with $e_{11} = -2$ (first row, first column) and $e_{12} = 4$ (first row, second column).

$E' = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$ with -2 in the first row, first column and 4 in the second row, first column.

- $B = \begin{pmatrix} 2 & 5 \\ -1 & 1 \end{pmatrix}$ with $b_{11} = 2$, $b_{12} = 5$, $b_{21} = -1$ and $b_{22} = 1$. Therefore $B' = \begin{pmatrix} 2 & -1 \\ 5 & 1 \end{pmatrix}$.

- Note that the transpose of a row vector gives a column vector & the transpose of a column vector gives a row vector.

Multiplying a matrix by a scalar:

$2A = 2\begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 2 & -4 \\ 6 & 8 \end{pmatrix}$.

Matrix multiplication:

$AB = \begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 2 & 5 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} (1)(2) + (-2)(-1) = 4 & (1)(5) + (-2)(1) = 3 \\ (3)(2) + (4)(-1) = 2 & (3)(5) + (4)(1) = 19 \end{pmatrix} = \begin{pmatrix} 4 & 3 \\ 2 & 19 \end{pmatrix}$.

$BA = \begin{pmatrix} 2 & 5 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 17 & 16 \\ 2 & 6 \end{pmatrix}$. Note that $AB \neq BA$.

$BC = \begin{pmatrix} 2 & 5 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{pmatrix} \neq$ Not possible to calculate.

Note: Matrix multiplication could also be asked in the following way:

Let $AB = Y = \begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix}$. The value of y_{22} is? In this case you only need to compute the value of one entry. Answer: $y_{22} = 19$

The order of the matrix EAE' is: $\underset{(1\times 2)}{E}\,\underset{(2\times 2)}{A}\,\underset{(2\times 1)}{E'} = 1 \times 1$.

The Identity Matrix:

An Identity matrix has one's on the diagonal and zero's elsewhere.

An Identity matrix is always square.

Let $A = \begin{pmatrix} 1 & 5 \\ -1 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 1 \\ 1 & -2 \\ 2 & 0 \end{pmatrix}$.

$AI = \begin{pmatrix} 1 & 5 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 5 \\ -1 & 3 \end{pmatrix} = A.$

$IA = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 5 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 5 \\ -1 & 3 \end{pmatrix} = A.$

Note: In the previous example the Identity matrix is of order 2.

$IB = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -2 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & -2 \\ 2 & 0 \end{pmatrix} = B.$

Note: In the previous example the Identity matrix is of order 3.

Take note: We DO NOT have to give you the Identity matrix, for example, if we ask you to calculate $A + 4I$ you have to know what the Identity matrix (I) looks like. The answer to $A + 4I$ is:

$$A + 4I = \begin{pmatrix} 1 & 5 \\ -1 & 3 \end{pmatrix} + 4\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 5 \\ -1 & 3 \end{pmatrix} + \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} 5 & 5 \\ -1 & 7 \end{pmatrix}$$

Practical example:

	Model			
	L	**GL**	**GLX**	**GLE**
Series A	500	300	280	90
Series B	400	150	220	190

Let $S = \begin{pmatrix} 500 & 300 & 280 & 90 \\ 400 & 150 & 220 & 190 \end{pmatrix}$, $E_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $E_4 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$. Then $E_2' = (1 \quad 1)$ and $E_4' = (1 \ 1 \ 1 \ 1)$.

Let $P' = (10 \ \ 13 \ \ 18 \ \ 20)$ be the profit for the models. Then $P = \begin{pmatrix} 10 \\ 13 \\ 18 \\ 20 \end{pmatrix}$.

Use matrix multiplication to calculate the following:

1. The total number of cars sold for series A and series B.

$$SE_4 = \begin{pmatrix} 500 & 300 & 280 & 90 \\ 400 & 150 & 220 & 190 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1170 \\ 960 \end{pmatrix}.$$

Test: Series A: 500 + 300 + 280 + 90 = 1170 and Series B: 400 + 150 + 220 + 190 = 960.

2. The total number of cars sold for each model.

$$E_2'S = (1 \quad 1) \begin{pmatrix} 500 & 300 & 280 & 90 \\ 400 & 150 & 220 & 190 \end{pmatrix} = (900 \ \ 450 \ \ 500 \ \ 280).$$

Test: Model L: 500 + 400 = 900.
Model GL: 300 + 150 = 450.
Model GLX: 280 + 220 = 500
Model GLE: 90 + 190 = 280.

3. Calculate the total profit of Series A and Series B.

$$SP = \begin{pmatrix} 500 & 300 & 280 & 90 \\ 400 & 150 & 220 & 190 \end{pmatrix} \begin{pmatrix} 10 \\ 13 \\ 18 \\ 20 \end{pmatrix} = \begin{pmatrix} 15740 \\ 13710 \end{pmatrix}.$$

Test: The profit for Series A is: (500)(10)+(300)(13)+(280)(18)+(90)(20) = 15 740.
The profit for Series B is: (400)(10)+(150)(13)+(220)(18)+(190)(20) = 13 710.

4.5 The determinant of a matrix

- The determinant is indicated by $|A|$.
- The matrix A must be square.
- $|A|$ is a unique scalar value.

The determinant of a 2 × 2 matrix:
For a 2×2 matrix $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ the determinant is calculated using

$$|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}.$$

Example

$A = \begin{pmatrix} 2 & 5 \\ -1 & 1 \end{pmatrix}$. Then $|A| = \begin{vmatrix} 2 & 5 \\ -1 & 1 \end{vmatrix} = (2)(1) - (-1)(5) = 7.$

Property 1

The determinant of a matrix, *A*, has the same value as that of its transpose matrix, *A'*. That is $|A| = |A'|$.

If $A = \begin{pmatrix} 2 & 5 \\ -1 & 1 \end{pmatrix}$ then $|A| = 7$.

Now $A' = \begin{pmatrix} 2 & -1 \\ 5 & 1 \end{pmatrix}$ so that $|A'| = \begin{vmatrix} 2 & -1 \\ 5 & 1 \end{vmatrix} = (2)(1) - (-1)(5) = 7$

Property 2

The interchanging of any two rows (or two columns) will alter the sign, but not the numerical value, of the determinant.

If $A = \begin{pmatrix} 2 & 5 \\ -1 & 1 \end{pmatrix}$ then $|A| = 7$.
Now by interchanging two rows we get $\begin{vmatrix} -1 & 1 \\ 2 & 5 \end{vmatrix} = (-1)(5) - (2)(1) = -7$.

If $A = \begin{pmatrix} 2 & 5 \\ -1 & 1 \end{pmatrix}$ then $|A| = 7$.
Now by interchanging two columns we get $\begin{vmatrix} 5 & 2 \\ 1 & -1 \end{vmatrix} = (5)(-1) - (2)(1) = -7$.

Property 3

The multiplication of any one row (or column) by a scalar *k*, will change the value of the determinant *k*-fold.

If $A = \begin{pmatrix} 2 & 5 \\ -1 & 1 \end{pmatrix}$ then $|A| = 7$.
Now multiplying the first row by $k = 10$ we obtain $\begin{vmatrix} 20 & 50 \\ -1 & 1 \end{vmatrix} = (20)(1) - (50)(-1) = 70$.

Property 4

If each element of a row (or column) is added to (or subtracted from) the corresponding element of another row (or column), the value of the determinant remains the same.

If $A = \begin{pmatrix} 2 & 5 \\ -1 & 1 \end{pmatrix}$ then $|A| = 7$. Now if the first row and the second row are added together, and the sum is placed in the first row we obtain $\begin{vmatrix} 1 & 6 \\ -1 & 1 \end{vmatrix} = (1)(1) - (6)(-1) = 7$.

Property 5

If one row (or column) is a multiple of another row (or column) then the determinant is zero.

$|B| = \begin{vmatrix} 1 & 4 \\ 2 & 8 \end{vmatrix} = (1)(8) - (2)(4) = 0$

Note that the second row is a multiple ($\times$ 2) of the first row.

Property 6

If one row (or column) of a matrix contains only zeros then the value of the determinant is zero.

$|B| = \begin{vmatrix} 1 & 2 \\ 0 & 0 \end{vmatrix} = (1)(0) - (2)(0) = 0$

The determinant of a 3 $\times$ 3 matrix:

$$|B| = \begin{vmatrix} 1 & 0 & 2 \\ 2 & 3 & 0 \\ 3 & 4 & -2 \end{vmatrix}$$

$$= (1)(3)(-2) + (0)(0)(3) + (2)(2)(4) - (3)(3)(2) - (4)(0)(1) — (2)(2)(0)$$

$$= -6 + 0 + 16 - 18 - 0 - 0 = -8$$

- If $|A| = 0$ then the matrix A is called a singular matrix.
- If $|A| \neq 0$ then the matrix A is called a non-singular matrix.

Cramer's rule:

Consider the set of two linear equations:

$$a_{11}x_1 + a_{12}x_2 = b_1$$

$$a_{21}x_1 + a_{22}x_2 = b_2$$

Matrix notation: $AX = B$ with $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$, $X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ and $B = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$.

The linear equations can be solved using Cramer's rule

$x_1 = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{|A|}$ and $x_2 = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{|A|}$. Note: This holds if *A* is non-singular ($|A| \neq 0$).

Example

$$2x_1 - 5x_2 = 31$$

$$8x_1 + 9x_2 = -21$$

Matrix notation: $AX = B$ with $A = \begin{pmatrix} 2 & -5 \\ 8 & 9 \end{pmatrix}$, $X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ and $B = \begin{pmatrix} 31 \\ -21 \end{pmatrix}$.

$|A| = \begin{vmatrix} 2 & -5 \\ 8 & 9 \end{vmatrix} = (2)(9) - (-5)(8) = 58$

$x_1 = \frac{\begin{vmatrix} 31 & -5 \\ -21 & 9 \end{vmatrix}}{|A|} = \frac{174}{58} = 3$ and $x_2 = \frac{\begin{vmatrix} 2 & 31 \\ 8 & -21 \end{vmatrix}}{|A|} = \frac{-290}{58} = -5$

Consider the set of three linear equations:

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

Matrix notation: $AX = B$ with $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$, $X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ and $B = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

The linear equations can be solved using Cramer's rule

$x_1 = \frac{\begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}}{|A|}$ and $x_2 = \frac{\begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}}{|A|}$ and $x_3 = \frac{\begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}}{|A|}$.

Example

$$x_1 - 2x_3 = 1$$
$$4x_1 - 2x_2 + x_3 = 2$$
$$x_1 + 2x_2 - 10x_3 = -1$$

Matrix notation: $AX = B$ with $A = \begin{pmatrix} 1 & 0 & -2 \\ 4 & -2 & 1 \\ 1 & 2 & -10 \end{pmatrix}$, $X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ and $B = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$.

$$|A| = \begin{vmatrix} 1 & 0 & -2 \\ 4 & -2 & 1 \\ 1 & 2 & -10 \end{vmatrix} = -2$$

$$x_1 = \frac{\begin{vmatrix} 1 & 0 & -2 \\ 2 & -2 & 1 \\ -1 & 2 & -10 \end{vmatrix}}{|A|} = \frac{14}{-2} = -7 \quad \text{and}$$

$$x_2 = \frac{\begin{vmatrix} 1 & 1 & -2 \\ 4 & 2 & 1 \\ 1 & -1 & -10 \end{vmatrix}}{|A|} = \frac{34}{-2} = -17 \quad \text{and}$$

$$x_3 = \frac{\begin{vmatrix} 1 & 0 & 1 \\ 4 & -2 & 2 \\ 1 & 2 & -1 \end{vmatrix}}{|A|} = \frac{8}{-2} = -4 \; .$$

The inverse of a matrix:
If there exists a square, non-singular matrix A then there exists a square matrix A^{-1} such that $A^{-1}A = I$ and $AA^{-1} = I$ then A^{-1} is the inverse matrix of A.

Let $A = \begin{pmatrix} 3 & 5 \\ 4 & 7 \end{pmatrix}$ and test if $\begin{pmatrix} 7 & -5 \\ -4 & 3 \end{pmatrix}$ is the inverse of matrix A.

$$AA^{-1} = \begin{pmatrix} 3 & 5 \\ 4 & 7 \end{pmatrix}\begin{pmatrix} 7 & -5 \\ -4 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Note: You will never be asked to calculate the inverse of matrix by hand, but you have to be able to calculate the inverse of a matrix using Microsoft Excel.

The matrix A^{-1} is used to solve a set of linear equations

$$AX = B$$
$$A^{-1}(AX) = A^{-1}(B)$$
$$IX = A^{-1}B$$
$$X = A^{-1}B$$

Example

$$3x_1 + 5x_2 = 1$$
$$4x_1 + 7x_2 = 0$$

Solve x_1 and x_2 using the inverse matrix.

Given: $\begin{pmatrix} 3 & 5 \\ 4 & 7 \end{pmatrix}^{-1} = \begin{pmatrix} 7 & -5 \\ -4 & 3 \end{pmatrix}$

$AX = B$

$\begin{pmatrix} 3 & 5 \\ 4 & 7 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

Therefore,

$X = A^{-1}B$

$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 7 & -5 \\ -4 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 7 \\ -4 \end{pmatrix}$

Example

$$8x_1 - x_2 = 15$$
$$x_2 + 5x_3 = 1$$
$$2x_1 + 3x_3 = 4$$

Solve x_1, x_2 and x_3 using the inverse matrix.

Given: $\begin{pmatrix} 8 & -1 & 0 \\ 0 & 1 & 5 \\ 2 & 0 & 3 \end{pmatrix}^{-1} = \begin{pmatrix} \frac{3}{14} & \frac{3}{14} & \frac{-5}{14} \\ \frac{10}{14} & \frac{24}{14} & \frac{-40}{14} \\ \frac{-2}{14} & \frac{-2}{14} & \frac{8}{14} \end{pmatrix} = \frac{1}{14} \begin{pmatrix} 3 & 3 & -5 \\ 10 & 24 & -40 \\ -2 & -2 & 8 \end{pmatrix}$

$AX = B$

$\begin{pmatrix} 8 & -1 & 0 \\ 0 & 1 & 5 \\ 2 & 0 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 15 \\ 1 \\ 4 \end{pmatrix}$

Therefore,

$X = A^{-1}B$

$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \frac{1}{14} \begin{pmatrix} 3 & 3 & -5 \\ 10 & 24 & -40 \\ -2 & -2 & 8 \end{pmatrix} \begin{pmatrix} 15 \\ 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$

Practical applications:

Example

Consider the sales of 3 shops at a motor show. Flags, T-shirts and caps are sold.

Shop	Flags	T-shirts	Caps
1	8	4	10
2	2	3	14
3	5	0	6

The sales matrix $S = \begin{pmatrix} 8 & 4 & 10 \\ 2 & 3 & 14 \\ 5 & 0 & 6 \end{pmatrix}$.

Suppose the prices of the items are known and are the same for the 3 shops. The prices are R10 for a flag, R8 for a T-shirt and R5 for a cap. Let $P = \begin{pmatrix} 10 \\ 8 \\ 5 \end{pmatrix}$ and $E_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

Use matrix multiplication to calculate the following:

1. The total number of items sold at the 3 shops.

$$SE_3 = \begin{pmatrix} 8 & 4 & 10 \\ 2 & 3 & 14 \\ 5 & 0 & 6 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 22 \\ 19 \\ 11 \end{pmatrix}.$$

The total number of items sold at Shop 1 = 22, at Shop 2 = 19 and at Shop 3 = 11.

2. The total number of flags, T-shirts and caps sold.

$$E_3'S = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 8 & 4 & 10 \\ 2 & 3 & 14 \\ 5 & 0 & 6 \end{pmatrix} = \begin{pmatrix} 15 & 7 & 30 \end{pmatrix}.$$

The total number of flags sold = 15, T-shirts = 7 and caps = 30.

3. The total income for each shop.

$$SP = \begin{pmatrix} 8 & 4 & 10 \\ 2 & 3 & 14 \\ 5 & 0 & 6 \end{pmatrix}\begin{pmatrix} 10 \\ 8 \\ 5 \end{pmatrix} = \begin{pmatrix} 162 \\ 114 \\ 80 \end{pmatrix}.$$

The income for Shop 1 = R162, for Shop 2 = R114 and Shop 3 = R80.

Suppose the income for the 3 shops is known and the prices of the items are the same at the 3 shops.

Shop	Income
1	162
2	114
3	80

To calculate the prices for the items we need to solve for the following linear equations:

$$8x_1 + 4x_2 + 10x_3 = 162$$
$$2x_1 + 3x_2 + 14x_3 = 114$$
$$5x_1 + 0x_2 + 6x_3 = 80$$

or $SX = B$ such that $\begin{pmatrix} 8 & 4 & 10 \\ 2 & 3 & 14 \\ 5 & 0 & 6 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 162 \\ 114 \\ 80 \end{pmatrix}$.

The following is given: $|S| = 226$ and $S^{-1} = \frac{1}{113}\begin{pmatrix} 9 & -12 & 13 \\ 29 & -1 & -46 \\ -7.5 & 10 & 8 \end{pmatrix}$.

Use the information and calculate the following:

1. Test if S^{-1} is the inverse matrix of S.

Test this: If you multiply S^{-1} and S you will get the 3×3 Identity matrix.

2. Calculate the price of a flag by using Cramer's rule.

$$x_1 = \frac{\begin{vmatrix} 162 & 4 & 10 \\ 114 & 3 & 14 \\ 80 & 0 & 6 \end{vmatrix}}{|S|} = \frac{2260}{226} = 10$$

3. Calculate the price of a flag by using the inverse matrix.

$SX = B$

$X = S^{-1}B$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \frac{1}{113}\begin{pmatrix} 9 & -12 & 13 \\ 29 & -1 & -46 \\ -7.5 & 10 & 8 \end{pmatrix}\begin{pmatrix} 162 \\ 114 \\ 80 \end{pmatrix} = \begin{pmatrix} 10 \\ 8 \\ 5 \end{pmatrix}.$$

Excel functions for Matrix Algebra:

Calculating the transpose of a matrix using Excel.

Formula worksheet:

	A	B	C	D	E
1	Matrix A				
2		1	0	4	
3		5	9	2	
4		3	5	6	
5					
6	Transpose	=tranpose(B2:D4)			
7					
8					
9					

Press Ctrl + Shift + Enter

Value worksheet:

	A	B	C	D	E
1	Matrix A				
2		1	0	4	
3		5	9	2	
4		3	5	6	
5					
6	Transpose	1	5	3	
7		0	9	5	
8		4	2	6	
9					

Calculating the determinant of a matrix using Excel.

Formula worksheet:

	A	B	C	D	E
1	Matrix A				
2		1	0	4	
3		5	9	2	
4		3	5	6	
5					
6	Determinant	=MDETERM(B2:D4)			
7					

Press Enter

Value worksheet:

	A	B	C	D	E
1	Matrix A				
2		1	0	4	
3		5	9	2	
4		3	5	6	
5					
6	Determinant	36			
7					

Calculating the inverse of a matrix using Excel:

Formula worksheet:

	A	B	C	D	E
1	Matrix A				
2		1	0	4	
3		5	9	2	
4		3	5	6	
5					
6	Inverse	=MINVERSE(B2:D4)			
7					
8					
9					

Press Ctrl + Shift + Enter

Value worksheet:

	A	B	C	D	E
1	Matrix A				
2		1	0	4	
3		5	9	2	
4		3	5	6	
5					
6	Inverse	1.222222	0.555556	-1	
7		-0.66667	-0.16667	0.5	
8		-0.05556	-0.13889	0.25	
9					

Multiplying two matrices using Excel:

Formula worksheet:

	A	B	C	D	E
1	Matrix A				
2		1	0	4	
3		5	9	2	
4		3	5	6	
5					
6	Matrix B				
7		1	0	-3	
8		0	4	2	
9		-2	1	1	
10					
11	Matrix AB	=MMULT(B2:D4,B7:D9)			
12					
13					
14					

Press Ctrl + Shift + Enter

Value worksheet:

	A	B	C	D	E
1	Matrix A				
2		1	0	4	
3		5	9	2	
4		3	5	6	
5					
6	Matrix B				
7		1	0	-3	
8		0	4	2	
9		-2	1	1	
10					
11	Matrix AB	-7	4	1	
12		1	38	5	
13		-9	26	7	
14					

Typical Excel exam questions:

Consider the following Excel spreadsheet:

Formula sheet

	A	B	C	D
1				
2	4	0	4	0
3	5	6	3	3
4	2	3	6	1
5	1	8	2	1
6				
7		=MDETERM(A2:D5)		
8				

Value sheet

	A	B	C	D
1				
2	4	0	4	0
3	5	6	3	3
4	2	3	6	1
5	1	8	2	1
6				
7		-316		
8				

If $B = \begin{pmatrix} 4 & 4 & 4 & 0 \\ 5 & 9 & 3 & 3 \\ 2 & 9 & 6 & 1 \\ 1 & 10 & 2 & 1 \end{pmatrix}$, the determinant of *B* is equal to:

Answer:
You have to use the properties of determinants in order to answer this question, because you can't calculate the determinant of a 4×4 matrix by hand. Property 4 of determinates state: "If each element of a row (or column) is added to (or subtracted from) the corresponding element of another row (or column), the value of the determinant remains the same".

Note that the matrix in the Excel sheet and the matrix *B* are similar, but we added the 2nd column and 3rd column of the matrix in the Excel sheet together and placed the sum in the 2nd column of the matrix *B*. Using property 4 of determinants, we know that $|B| = -316$.