$\boxed{\text{Salsa}}$

AVP: $a(y,\varphi) = \langle \overset{=0}{f} + Bu, \varphi \rangle, \quad \forall \varphi \in V \qquad (y = y(u))$

Observation operator: C

Target: z_d

Cost function:
- normal: $\tilde{J}(y,u) = \frac{1}{2}\|Cy - z_d\|_Z^2 + \cancel{\frac{\beta}{2}(Nu,u)_U}$ $=0$
- reduced: $J(u) = \frac{1}{2}\|Cy(u) - z_d\|_Z^2 + \cancel{\frac{\beta}{2}(Nu,u)_U}$ $=0$

Problem: $\begin{cases} \tilde{J}(y,u) \to \min \\ \text{s.t.} \\ \text{AVP} \end{cases}$ $\qquad$ solution: $(\hat{y},\hat{u})$

Lagrangian: $\mathcal{L}(y,\underset{\text{Lagr. multiplier}}{p},u) = \underbrace{\frac{1}{2}\|Cy - z_d\|_Z^2}_{\text{cost}} + \cancel{\frac{\beta}{2}(Nu,u)_U} \underbrace{- a(y,p) + \langle p, f + Bu \rangle_*}_{\text{AVP}} \qquad [y \perp\!\!\!\perp u]$

- $\mathcal{L}_p(\hat{y},\hat{p},\hat{u})\varphi = -a(\hat{y},\varphi) + \langle f + B\hat{u}, \varphi \rangle_* = 0$ <u>State Equation</u> (<u>primal problem</u>)
- $\mathcal{L}_y(\hat{y},\hat{p},\hat{u})\psi = (C\hat{y} - z_d, C\psi)_Z - a(\psi,\hat{p}) = 0$ <u>Adjoint Equation</u> (<u>dual problem</u>)
- $\mathcal{L}_u(\hat{y},\hat{p},\hat{u})v = (\beta N\hat{u}, v)_U + \langle \hat{p}, Bv \rangle_*$

$\hookrightarrow \mathcal{L}_u(\hat{y},\hat{p},\hat{u})(w - \hat{u}) \geq 0$ <u>Variational inequality</u>

$\boxed{\text{P-M-S}}$

$a(u,v) = \int_\Omega \sigma_\rho(u) : \varepsilon(v)\,dx, \quad C(v) = \int_{\Gamma_N} f v \, d\Gamma \qquad \sigma_\rho = \rho^p \sigma$

$?\rho: \quad \min_\rho C(u(\rho)): \quad \begin{cases} a(u(\rho),v) = C(v) & \forall v \in U \\ \int_\Omega \rho\,dx \leq d|\Omega| \\ \rho_{\min} \leq \rho \leq 1 \end{cases}$

$(y \; p \; u)$ Salsa

$\mathcal{L}(\overset{\uparrow}{u},\overset{\uparrow}{z},\overset{\uparrow}{\rho}) = \underbrace{C(u)}_{\text{cost}} + \underbrace{a(u,z) - C(z)}_{\text{AVP}} \qquad [u \perp\!\!\!\perp \rho \; (?)]$

- $\mathcal{L}_z(u,z,\rho)\varphi = a(u,\varphi) - C(\varphi)$ <u>S.E.</u> (<u>primal</u>)
- $\mathcal{L}_u(u,z,\rho)\psi = C(\psi) + a(\psi,z)$ <u>A.E.</u> (<u>dual</u>)

$\Rightarrow \begin{cases} a(u,\varphi) = C(\varphi) & \forall \varphi \\ a(z,\psi) = C(\psi) & \forall \psi \end{cases} \Rightarrow z = -u \quad (*)$

- $\mathcal{L}_\rho(u,z,\rho)v \underset{(?)}{=} \int_\Omega \underset{(?)}{p\rho^{p-1}} \sigma(u) : \varepsilon(z)\,dx \overset{(*)}{=} -\int_\Omega p\rho^{p-1}\sigma(u) : \varepsilon(u)\,dx$

$\boxed{\text{Fem 2D}}$

$U = [U_1, U_2]$

$\sigma_{ij} = \sum_{k,l} E_{ijkl}\, \varepsilon_{kl}$ $\quad \varepsilon = 1/2\,(\nabla U + \nabla^T U)$ $\quad E = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1}{2}(1-\nu) \end{bmatrix}$

$$E^H_{ijkl} = \frac{1}{|Y|} \int_Y \sum_{p,q} \left(\sigma_{pq}^{0,ij} - \sigma_{pq}^{*,ij} \right) \left(\varepsilon_{pq}^{0,kl} - \varepsilon_{pq}^{*,kl} \right)$$ AVP ?

$$J(U(\rho)) = \sum_{ijkl} \left(E^H_{ijkl}(\rho) - \underbrace{E^W_{ijkl}}_{\text{target}} \right)^2$$ cost

$\mathcal{L} = ?$ $\quad \mathcal{L}_\rho = \nabla J = ?$