Numerical analysis for Partial Differential Equations

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Topology optimization for lattice materials

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with

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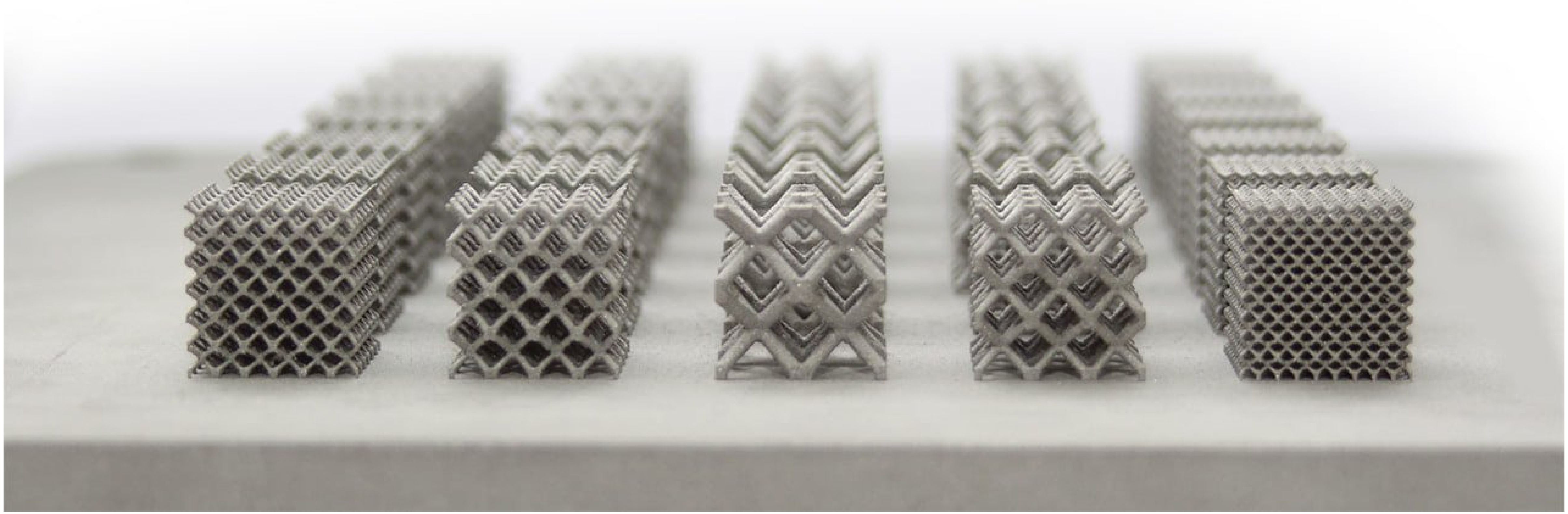
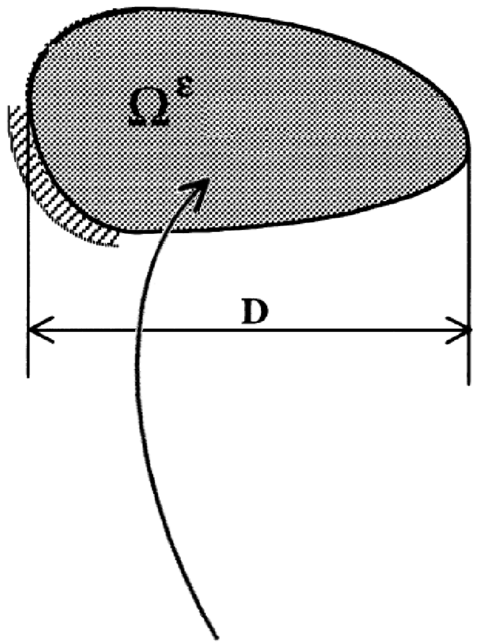
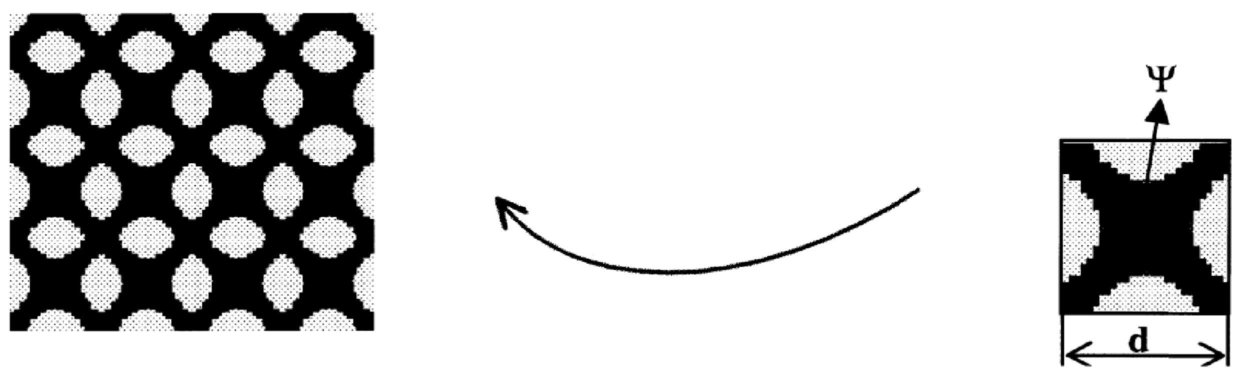
Lattice structures and metamaterials



Homogenization



Direct



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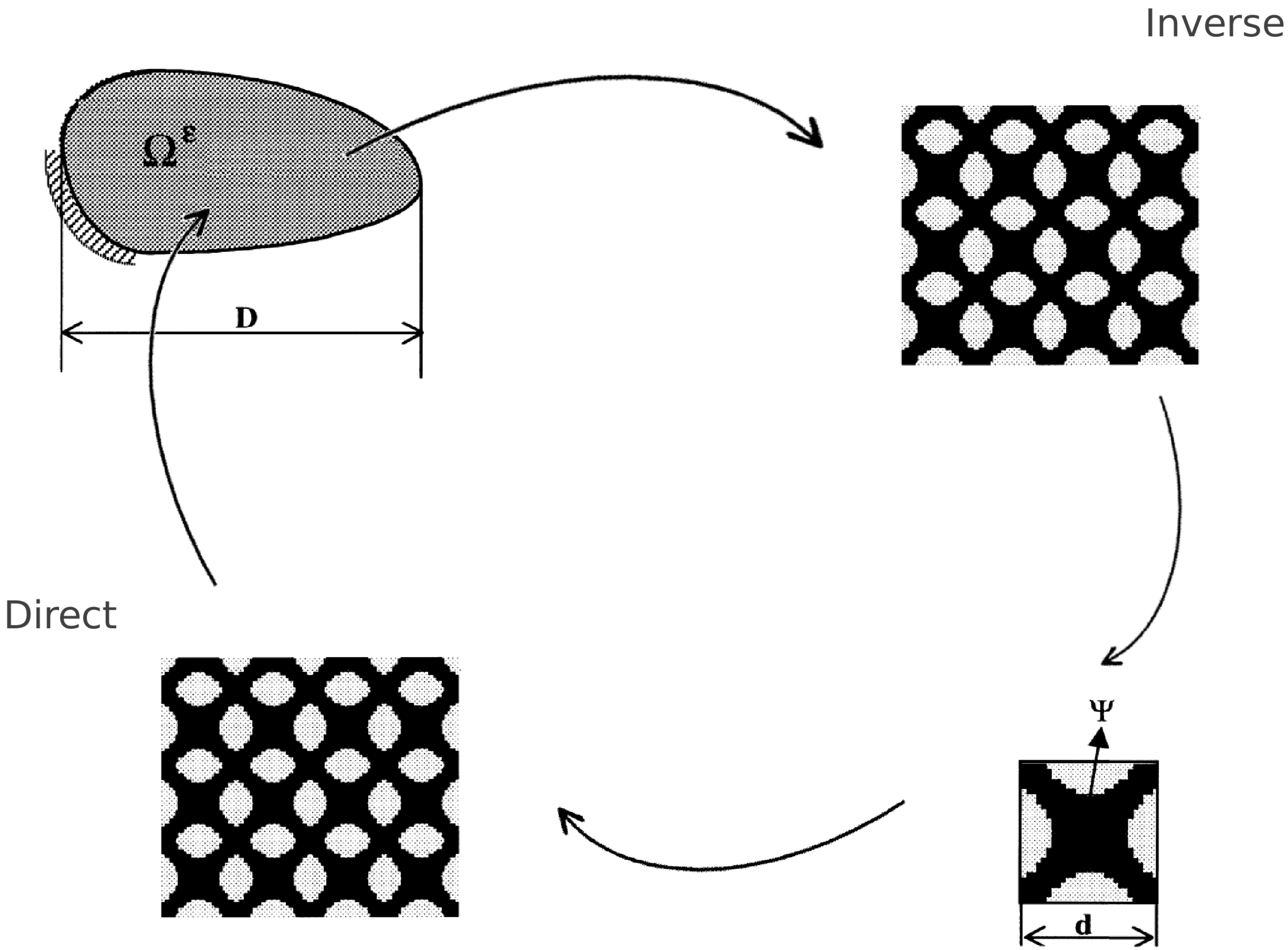

Topology optimization with SIMP

from [MPS]

Compliance

$$\int_{\Gamma_N} \mathbf{f} \cdot \mathbf{u} \, d\gamma, \qquad \mathbf{f} : \Gamma_N \to \mathbb{R}^2$$

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Elasticity problem

$$\begin{cases} -\nabla \cdot \sigma(\mathbf{u}) = \mathbf{0} & \text{in } \Omega \\ \mathbf{u} = \mathbf{0} & \text{on } \Gamma_D \\ \sigma(\mathbf{u})\mathbf{n} = \mathbf{f} & \text{on } \Gamma_N \\ \sigma(\mathbf{u})\mathbf{n} = \mathbf{0} & \text{on } \Gamma_F, \end{cases}$$

$$\sigma(\mathbf{u}) = 2\mu\varepsilon(\mathbf{u}) + \lambda \operatorname{tr}(\varepsilon(\mathbf{u}))I$$

$$\varepsilon(\mathbf{u}) = \left(\nabla\mathbf{u} + \nabla\mathbf{u}^T\right)/2$$

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}, \qquad \mu = \frac{E}{2(1+\nu)}$$

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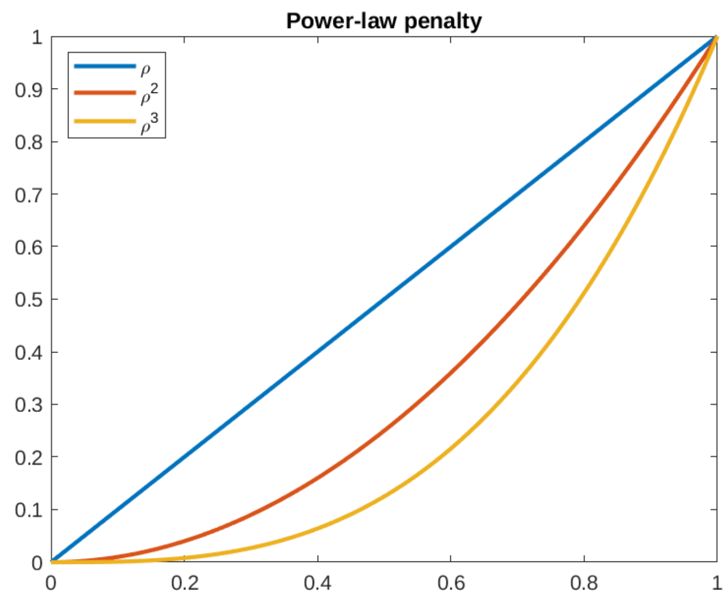

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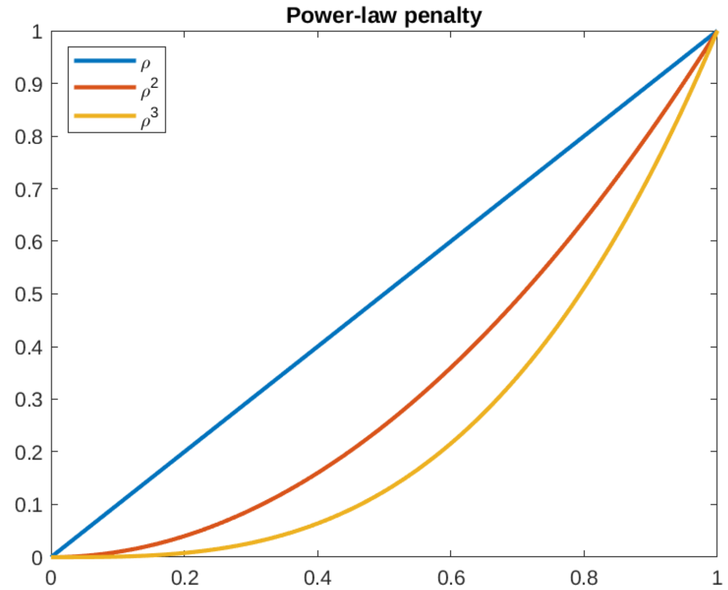

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Abstract Variational Problem

$$\mathbf{u} \in U = \left\{ \mathbf{v} \in \left[H^1(\Omega)\right]^2 : \ \mathbf{v} = \mathbf{0} \text{ on } \Gamma_D \right\}$$

$$a(\mathbf{u}, \mathbf{v}) = \mathcal{C}(\mathbf{v}) \quad \forall \mathbf{v} \in U,$$

$$a(\mathbf{u}, \mathbf{v}) = \int_\Omega \sigma_\rho(\mathbf{u}) : \varepsilon(\mathbf{v}) \, d\mathbf{x}, \quad \mathcal{C}(\mathbf{v}) = \int_{\Gamma_N} \mathbf{f} \cdot \mathbf{v} \, d\gamma,$$

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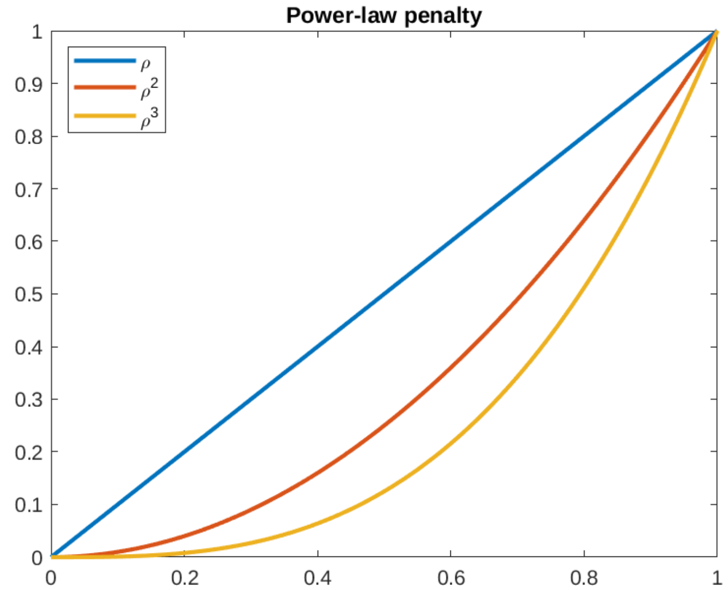

SIMP problem

$$\min_{\rho\in L^\infty(\Omega)} \mathcal{C}(\mathbf{u}(\rho)) : \begin{cases} a(\mathbf{u}(\rho),\mathbf{v}) = \mathcal{C}(\mathbf{v})\ \forall \mathbf{v}\in U \\ \int_\Omega \rho\, d\mathbf{x} \leqslant \alpha|\Omega| \\ \rho_{\min} \leqslant \rho \leqslant 1, \end{cases}$$

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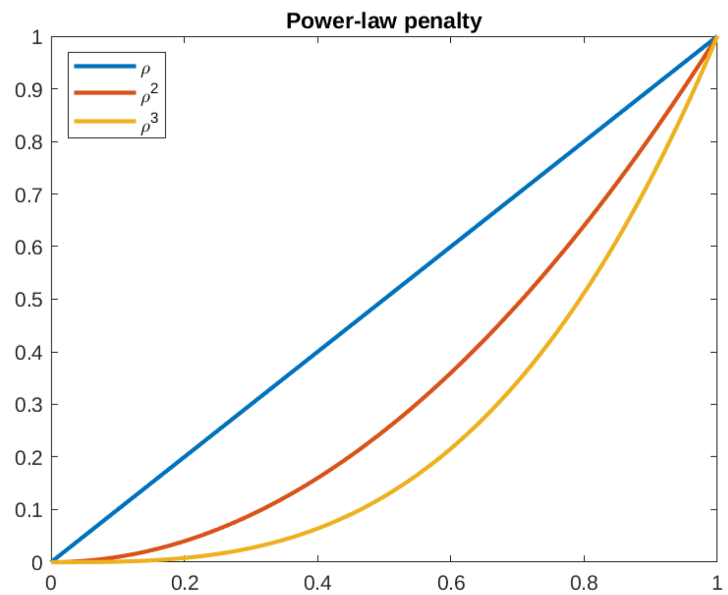

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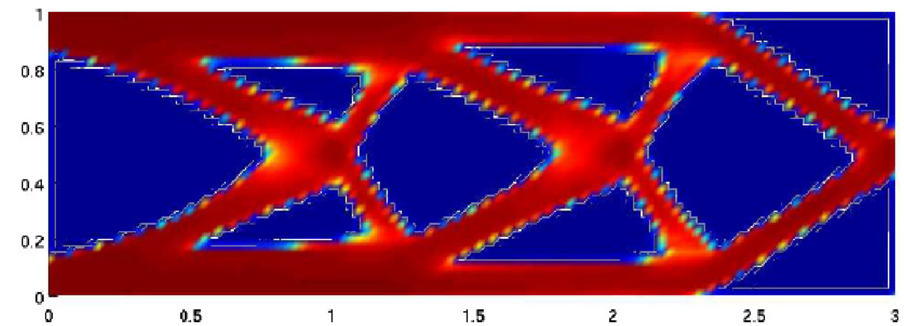
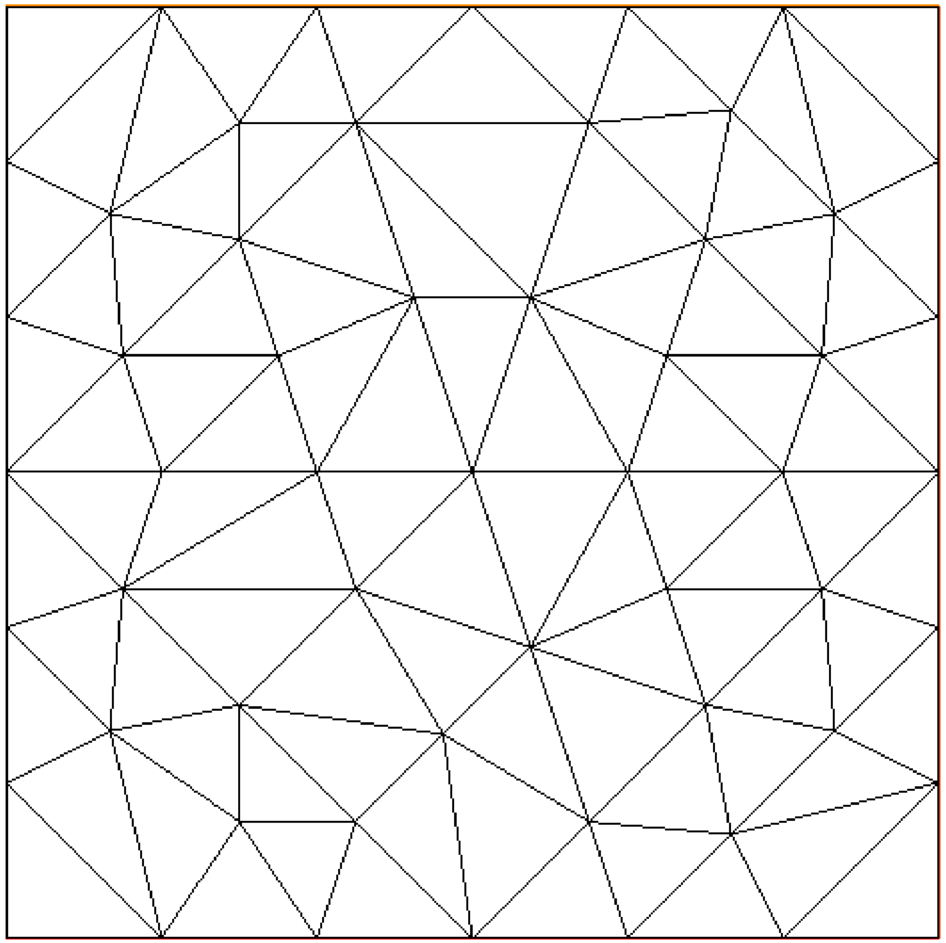
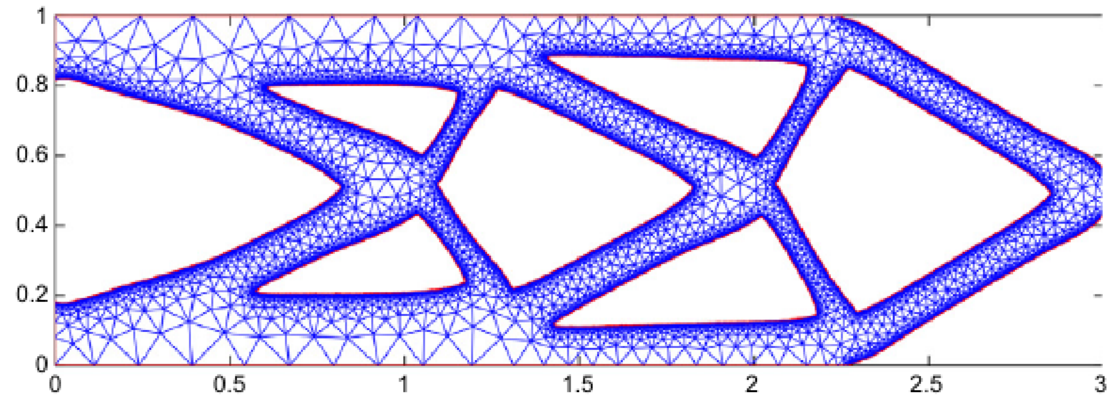
Cantilever example with SIMP algorithm



Mesh adaptivity ...

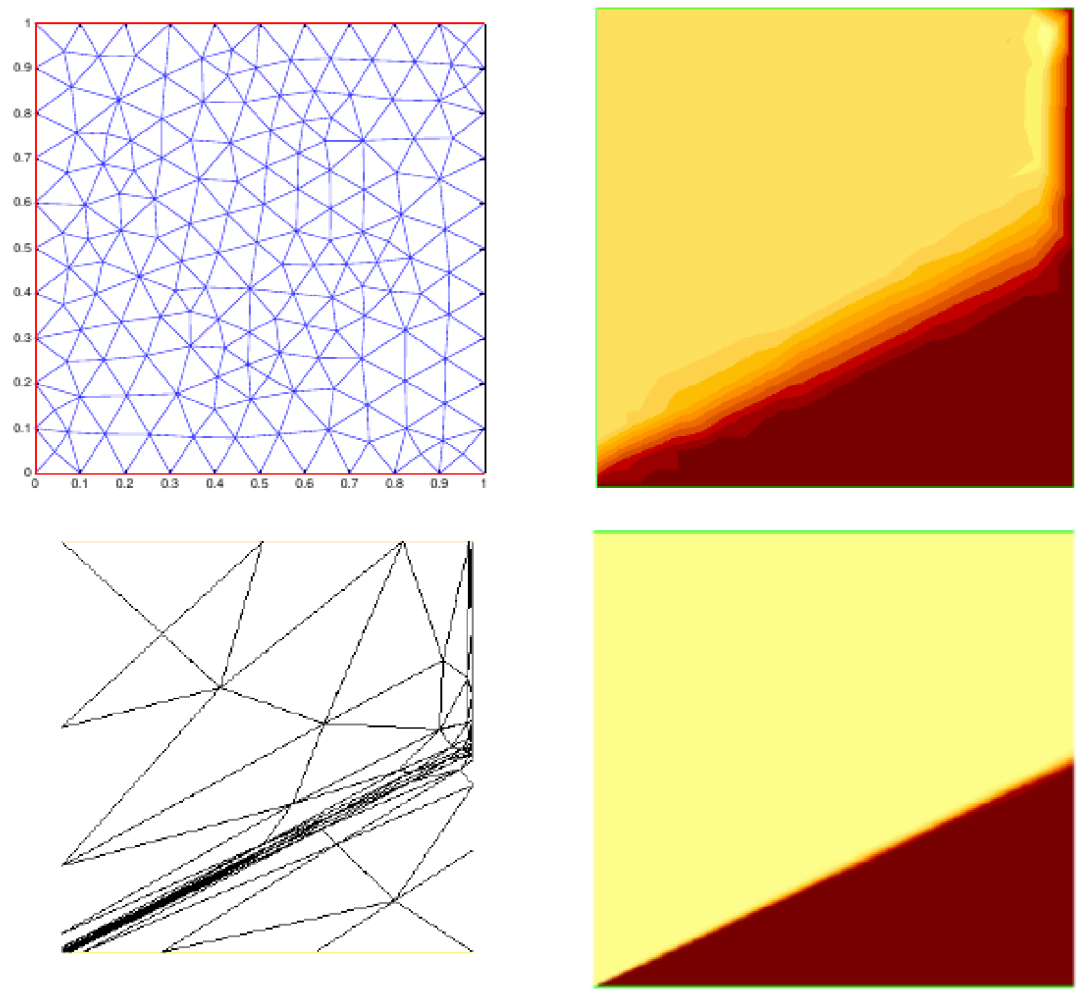


Example of mesh adaptation



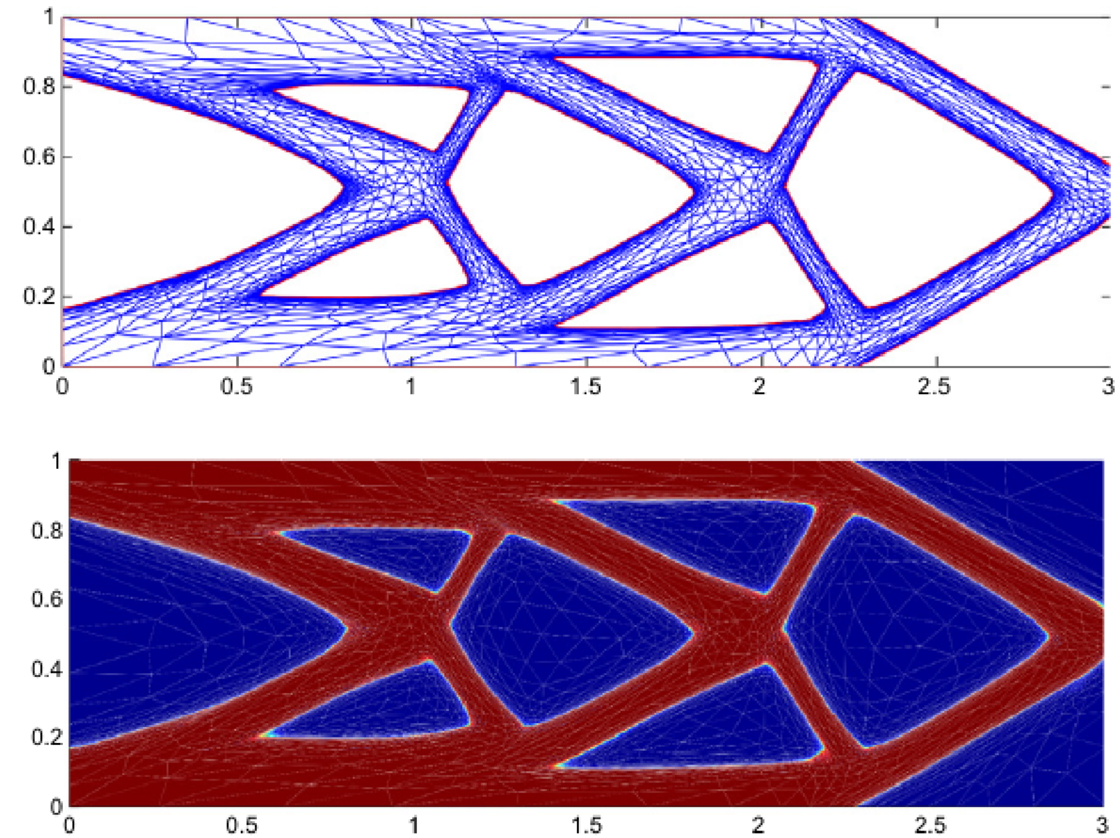
Cantilever example with isotropic mesh adaptation
from [MPS]

... with anisotropic mesh



Isotropic vs anisotropic mesh to capture the boundary
and internal layers of an advection-diffusion problem

from [MPP]

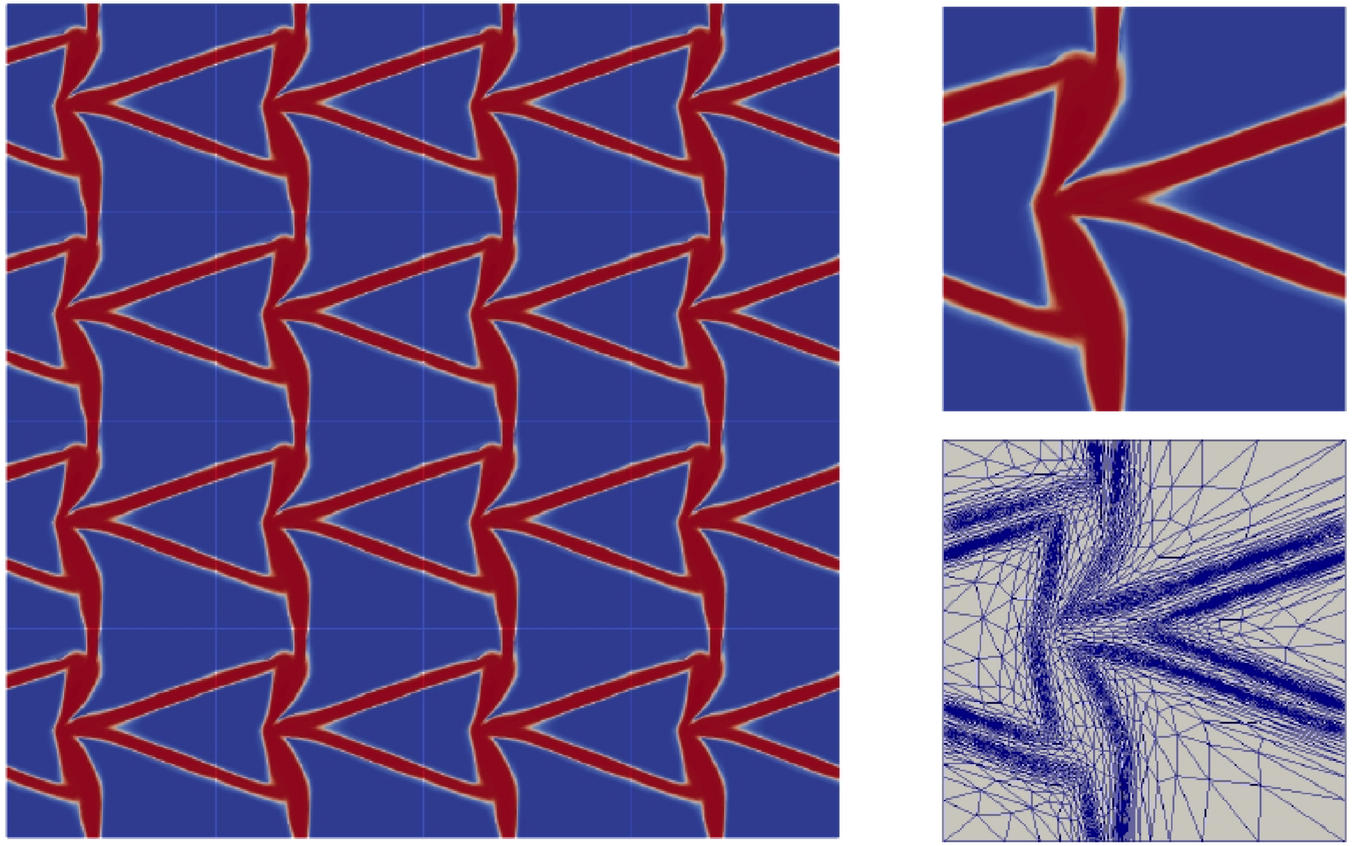


Cantilever example with anisotropic mesh adaptation

from [MPS]

Development roadmap

- Extend the method to the 3D model of lattice materials
- Extend the method from the linear to the non-linear regime in 2D case
- Extend the method from the linear to the non-linear regime in 3D



TO with anisotropic mesh adaptation for an optimized microstructure
from [FMP]

Thank you
for your attention!

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Essential bibliography

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[VDP] A. Vigliotti, V. Deshpande, D. Pasini – Non linear constituive models for lattice materials. J. Mech. Phys. Solids 64 (2014) 44–60

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