equazione (4)

$$\int_Y \rho^P \sum_{k,e} E_{kepq}\, \varepsilon^{*,ij}_{pq}\, \varepsilon_{ke}(r) = \int_Y \rho^P \sum_{k,e} E_{kepq}\, \varepsilon^{0,ij}_{pq}\, \varepsilon_{ke}(r) \quad \forall r \in V$$

$$\Leftrightarrow \int_Y \rho^P \sum_{k,e} \sigma^{*,ij}_{ke}\, \varepsilon_{ke}(r) = \int_Y \rho^P \sum_{k,e} \sigma^{0,ij}_{ke}\, \varepsilon_{ke}(r)$$

prodotto scalare (componente per componente)

corrisponde a microijprimal([u1starij, u2starij], [v1, v2]) = $(\rho = w)$

int2d(Th) (w^P * sigma(u1starij, u2starij)'*epsilon(v1, v2))

- int2d(Th) (w^P * sigma(u10ij, u20ij)'*epsilon(v1, v2));

equazione (5)

$$E^H_{ijke} = \frac{1}{|Y|} \int_Y \rho^P \sum_{p,q} \sum_{r,s} E_{pqrs} \left(\varepsilon^{0,ke}_{pq} - \varepsilon^{*,ke}_{pq}\right)\left(\varepsilon^{0,ij}_{rs} - \varepsilon^{*,ij}_{rs}\right) =$$

$$= \frac{1}{|Y|} \int_Y \rho^P \sum_{p,q} \left(\sigma^{0,ij}_{pq} - \sigma^{*,ij}_{pq}\right)\left(\varepsilon^{0,ke}_{pq} - \varepsilon^{*,ke}_{pq}\right)$$

prodotto scalare (come prima)

corrisponde a righe 109 - 113

Eijke = int2d(Th) (w^P (sigma(u10ij, u20ij)-sigma(u1starij, u2starij))' *

epsilon(u10ke, u20ke) - epsilon(u1starke, u2starke))

Poi [u10ij, u20ij] corrisponde a $\varepsilon^{0,ij}$ / $\sigma^{0,ij}$ ovvero $\varepsilon^{0,ij} = \begin{cases} 1 & \text{per la componente } ij \ (1/2 \text{ se } i \neq j) \\ 0 & \text{altrimenti} \end{cases}$

così [u1starij, u2starij] corrisponde a $\varepsilon^{*,ij}$ / $\sigma^{*,ij}$ (soluzione (4))

$$\mathcal{L} = J(u) + lhs(u) - rhs(z) = \frac{1}{2}\left(\left[C(u) - \bar{r}\right]\right)^2 + lhs(u) - rhs(z) = \mathcal{L}(\rho, u, z)$$

$$= \frac{1}{2}\|(C(u,\rho) - \bar{r})\|^2$$

ρ densità

u spostamento

z moltiplicatore lagrange

$$\int_Y \rho^p (\sigma^{0,11} - \sigma^{*,11})^T (\varepsilon^{0,22} - \varepsilon^{*,22})$$

$$C(u) = E1122(u) = (E1111 = \text{const})$$

$$rhs(u) = \int_Y \rho^p \underbrace{(\sigma^{0,ij})^T}_{\text{fixed}} \varepsilon(z) = \langle f_\rho, z \rangle = \rho^p \langle f, z \rangle$$

$$lhs(u) = \int_Y \rho^p (\sigma^{*,ij})^T \varepsilon(z) = a_\rho(u^{(*)}, z) \quad (\forall i,j)$$

$$= \rho^p a(u,z)$$

Potenze densità di ogni pb (check) (codice)

$\boxed{s}$ → eq (5) sul tensore Eijkl

$\boxed{q}$ → $lhs(u)$

$\boxed{r}$ → $rhs(u)$

?

(*) dipende da u (forse) perché $u^{(*)}$ che corrisponde a σ^*, ε^* solo soluzione dell'eq (4)

↳ lì c'è (1 z test) $\varepsilon(u)$

$$\mathcal{L}(\rho, u, z) = \frac{1}{2}\left[\rho^p (C(u) - \bar{r})\right]^2 - \langle f_\rho, z \rangle + a_\rho(u, z)$$

↳ $\mathcal{L}_z(\hat{\rho}, \hat{u}, \hat{z})\varphi = -\langle f_{\hat{\rho}}, \varphi \rangle + a_\rho(\hat{u}, \varphi) = 0 \quad \forall \varphi \in V$ → STATE EQ. (4) $\forall i,j$

Solution: $\hat{u} = u^*$

(that corresponds to $\varepsilon^{*,ij}$)

corrisponde a microij primal

↳ $\mathcal{L}_u(\hat{\rho}, \hat{u}, \hat{z})\psi = (C(\hat{u}) - \bar{r}) \cdot C(\psi) + a_\rho(\psi, \hat{z}) = 0 \quad \forall \psi \in V$ → ADJOINT PB (PROBLEMA DUALE)

Solution: $\hat{z}$

PROBLEMA DUALE

$$a_\rho(\psi, \hat{z}) = -(C(\hat{u}) - \bar{r}, C(\psi))$$ ⇝ microij dual

$$a_\rho(\psi, \hat{z}) = \int \rho^p \sigma(\psi_1 \text{ stan } ij, \psi_2 \text{ stan } ij)^T \varepsilon(z_1, z_2) \quad (\forall i,j)$$

Solution: z

PROBLEMA DUALE

$a_\rho(\varphi, \hat{z}) = -\left(C(\hat{u}) - \bar{r}, C(\varphi)\right)$ ⇝ micro ij dual

$a_\rho(\varphi, \hat{z}) = \int_Y \rho^{p}\, \sigma(\varphi_1 \text{ star } ij, \varphi_2 \text{ star } ij) \, \varepsilon(z_1, z_2)$ $(\forall\, i, j)$

$\varphi = (\varphi_1 \text{ star } ij, \varphi_2 \text{ star } ij)$ (funzione test)

$\hat{z} = (z_1, z_2)$ soluzione del pb duale

$(C(\hat{u}) - \bar{r}, C(\varphi)) = \int_Y \underbrace{[E1122(\hat{u}) - \bar{r}]}_{\text{corrisponde a CurreC (codice) (penso...)}} \cdot \overbrace{E1122(\varphi)}^{\text{è un valore (costante in Y)}} \, dY =$; costante perché $E1122(\hat{u}) = E1122$ calcolato nel funzionale J

$= \underbrace{|Y|}_{1} \cdot (E1122(\hat{u}) - \bar{r}) \cdot E1122(\varphi) = (E1122(\hat{u}) - \bar{r})\, E1122(\varphi)$

$E1122(\varphi) = \int_Y \rho^{p} (\sigma^{0,11} - \sigma^{*,11})'(\varepsilon^{0,22} - \varepsilon^{*,22})$ (eq (5), dove dipende da φ? Forse equazione (4)? ⊛)

↳ $\mathcal{L}_\rho$:

$\nabla_\rho \left[\frac{1}{2} \| C(u) - \bar{r} \|^2\right] = \nabla_\rho \left[\frac{1}{2} \left(\underbrace{\int_Y \rho^{p} (\sigma^{0,11} - \sigma^{*,11})'(\varepsilon^{0,22} - \varepsilon^{*,22})}_{E1122(\hat{u})} - \bar{r} \right)^2 \right] =$ (~ derivazione per parti)

$= \int_Y p \rho^{p-1} (\sigma^{0,11} - \sigma^{*,11})'(\varepsilon^{0,22} - \varepsilon^{*,22}) \cdot \frac{1}{2} \cdot 2 (E1122(\hat{u}) - \bar{r})$

$\langle \nabla_\rho [\cdots] \big|_{(\hat{\rho}, \hat{u}, \hat{z})}, v \rangle = \int_Y p \hat{\rho}^{p-1} (\sigma^{0,11} - \sigma^{*,11})'(\varepsilon^{0,22} - \varepsilon^{*,22}) \underbrace{(E1122(\hat{u}) - \bar{r})}_{\text{CurreC } (\sim)} v \, dY$

$\mathcal{L}_\rho(\hat{\rho}, \hat{u}, \hat{z})\, v = \int_Y p \rho^{p-1} (C(\hat{u}) - \bar{r}) (\sigma^{0,11} - \sigma^{*,11})'(\varepsilon^{0,22} - \varepsilon^{*,22})\, v - p \rho^{p-1} \langle f, \hat{z} \rangle v + p \rho^{p-1} a(\hat{u}, \hat{z}) v =$

$$= \int_Y p \rho^{p-1} \underbrace{v}_{\text{psi (codice)}} (E1122(\hat{u}) - \bar{r}) (\sigma^{0,11} - \sigma^{*,11})' (\varepsilon^{0,22} - \varepsilon^{*,22}) \Big] \rightarrow \text{ok riga } 205\text{-}210$$

$$\underbrace{- \int_Y p \rho^{p-1} v \left(\sigma^{0,is}(\hat{u})\right)' \varepsilon(\underset{\substack{\uparrow \\ (?)}}{\hat{z}})}_{\text{dovrebbe corrispondere a riga } 216-219 \,+\, 226-229} + \underbrace{\int_Y p \rho^{p-1} v \left(\sigma^{*,is}(\hat{u})\right)' \varepsilon(\underset{\substack{\uparrow \\ (?)}}{\hat{z}})}_{\text{dovrebbe corrispondere a riga } 211-214 \,+\, 221-224}$$

$$\text{grad } J = \mathcal{L}_\rho \Big|_{\hat{\rho}, \hat{u}, \hat{z}}$$