Short presentation (5 min)

(1. Cover)
Hi everybody we are Manfred Nesti and Teresa Babini and our project deals with Topology Optimization for Lattice Materials, in collaboration with professors Perotto and Micheletti and dr. Ferro of MOX and professor Pasini of McGill University.

(2. Lattice structures and metamaterials)
Lattice structures and metamaterials are a class of cellular materials characterized by a regular, periodic microstructure which can be idealised as a network of slender beams or rods. The periodic cell can be designed to make the material have certain properties such as stiffness, strength along some directions, high energy absorbing, lightness, negative Poisson's ratio etc.
These materials are becoming popular thanks to Additive Manufacturing techniques, in particular 3D printing. Different properties of a structure can be obtained by solving a topology optimization problem. The idea is to modify the periodic unit cell that build up the lattice material, to have at macroscale the desired properties.

(3-4. Homogenization)
This is possible thanks to direct and inverse homogenization techniques that links the micro and macro scales.
Homogenization is an asymptotic technique that allows to compute the average (or homogenized) solution: starting from a microscopic description of a problem, it finds a macroscopic effective property of the domain at macroscale. In our work we'll use the inverse technique, where the aim is to find the optimal arrangement of material at the microscale, solving a problem in the base cell, so that the desired effective properties are guaranteed at the macroscale.

(5-6-7-8-9-10. Topology Optimization)
So, we have a Topology Optimization problem that, thanks to inverse homogenization, we can formulate in the same framework at macro and microscale. We start from the compliance of the displacement field u that we want to minimize, we formulate the elasticity problem and we insert the unknown density function rho, that shows where we need to print material. Since, ideally, we want rho assuming only value 0 or 1, we can adopt the standard power law penalty function rho^p which penalizes the intermediate values. The Abstract Variational Problem obtained is a mathematical formulation for Topology Optimization problem known as SIMP (Solid Isotropic Material with Penalization) in which we want to find rho. Unfortunately, the SIMP method has a lot of numerical issues, for example the solution presents staircase patterns and is not smooth. Moreover, because of the non convexity of the problem, we could have multiple minimum, therefore we can not guarantee a priori the uniqueness of the solution, which strongly depends on the selected mesh.

(11. Mesh adaptivity …)
This problem suggests that can be useful to use mesh adaptivity techniques. At every iteration the mesh is changed according to the solution found as you can see in left pictures. The choice on how to change each element is done according to a posteriori error estimator; in our case, we resort to a recovery based error estimator which employs the density as driving quantity.
In this way we find the optimal adapted grid, minimizing the cardinality of the mesh for fixed accuracy and assuring that the error is equidistributed. In right pictures you can see the cantilever example solved by an adaptive isotropic mesh.

(12. … with anisotropic mesh)
We can also use anisotropic meshes, in which the triangles can be very stretched. As you can see on the left, this allows to sharply capture the solution where it has strong gradient, like, as in the left example, in the boundary and internal layers of an advection-diffusion problem. Since the unknown density function has very strong gradient across the material-void interfaces, anisotropic mesh is preferred to the isotropic

one and on the right you can see the same example shown in the previous slide solved by anisotropic mesh adaptation.

(13. Development roadmap)

There are different open challenges over the mechanical properties modelling of lattice structures and over metamaterials design. Pasini's group has worked on different topics, such as the mechanic response of lattice materials in 2D and 3D, in linear and non-linear regime, and such as soft metamaterials properties. Our goal is to extend the method implying inverse homogenization and anisotropic mesh adaptation to the 3D model of lattice materials, then from the linear regime of the elastic behaviour to the non-linear one in 2D and then finally in 3D.