Modelling and Analysis of a Flexible Spine Quadruped

End Term Project

SC 618

Analytical & Geometrical Dynamics

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Contents

Stage 1

Objective

To create a planar biped model (a cross-section of the quadruped model). To the tip of each leg bottom prescribe some cyclic motion. The motion should be out of phase but same frequency for the front and back legs.

Components of the model

1. **Walking Path:** A planar block modelled from a brick solid component on which the quadruped model can walk.
2. **Body:** Modelled from a single brick solid component. Two legs are attached to the front and back of this block.
3. **Legs:** 2 legs, each one modelled from two brick solid components of the same size, attached to each other, first link of the leg is attached to the body and the second link via revolute joints.
4. **Angle inputs:** Inputs to the revolute joints calculated from inverse kinematics model so that the end of second link can trace a circular path.
5. **Rigid Transform:** to give translation transformation between the walking path and the body which is at rest.

Explanation of the model

To compute the angular displacement and angular velocity of the two revolute joints for the end point of the second link to follow a circular motion, **inverse kinematics** was used to find the angles and then **differentiation** to find the velocities.

- Each leg consists of two links connected by two revolute joints.
- The first joint rotates the first link by an angle θ_1.
- The second joint rotates the second link by an angle θ_2.
- The lengths of the links are L_1 and L_2 respectively.

To make the end point of the second link follow a circular trajectory. The position of the end effector in cartesian coordinates (x(t), y(t)) must follow a circle.

The circular path of the end effector is defined parametrically by the following equations:

$$x(t) = x_0 + r\cos(\omega t)$$

$$y(t) = y_0 + r\sin(\omega t)$$

Here, r is the radius of the circle, (x_0, y_0) is the centre of the circle, and ω is the angular velocity determining the speed of the motion along the path.

The position of the end effector is related to the joint angles θ_1 and θ_2 though the forward kinematic equations:

$$x(t) = L_1 \cos(\theta_1(t)) + L_2 \cos(\theta_1(t) + \theta_2(t))$$

$$y(t) = L_1 \sin(\theta_1(t)) + L_2 \sin(\theta_1(t) + \theta_2(t))$$

To ensure the ed effector follows the circular trajectory, solve for the joint angles using inverse kinematics.

From the law of cosines:

$$\cos(\theta_2(t)) = \frac{x(t)^2 + y(t)^2 - L_1^2 - L_2^2}{2L_1L_2}$$

$$\theta_2(t) = \cos^{-1}\left(\frac{x(t)^2 + y(t)^2 - L_1^2 - L_2^2}{2L_1L_2}\right)$$

First computing the intermediate angles α(t), and β(t) to find θ_1.

$$\alpha(t) = \tan^{-1}\left(\frac{y(t)}{x(t)}\right)$$

$$\beta(t) = \tan^{-1}\left(\frac{L_2 \sin(\theta_2(t))}{L_1 + L_2 \cos(\theta_2(t))}\right)$$

$$\theta_1(t) = \alpha(t) - \beta(t)$$

Since the system is discretized in time, the angular velocities can be computed numerically by finite differences:

$$\dot{\theta}_1(t) = \frac{\theta_1(t_i) - \theta_1(t_{i-1})}{t_i - t_{i-1}}$$

$$\dot{\theta}_2(t) = \frac{\theta_2(t_i) - \theta_2(t_{i-1})}{t_i - t_{i-1}}$$

The values of θ_1 and θ_2 are then fed to the revolute joints via ***From workspace*** block of Simulink. The phase difference between the two legs is of 180^0 which is given in the code named ***"phase1.m"*** attached with this report.

Code Snippet :

```
% Defining the parameters

g = -9.80665;        % Acceleration due to gravity (m/s^2)
l = 3;               % Length of the block (m)
b = 1;               % Width of the block (m)
rho = 1000;          % density of the block (kg/m^3)
h = 0.5;             % Height/thickness of the blocks (m)

l_walk = 30;             % Length of walking path (m)
b_walk = 15;             % width of walking path (m)
h_walk = 0.1;            % Height/thickness of walking path (m)

l_thigh = 0.75;           % length of the upper part of leg (m)
b_thigh = 0.05;
t_thigh = b_thigh;        % thickness of the upper part of legs (m)

t_feet = t_thigh*1.1;             % length of square cross section of feet (m)
b_feet = t_feet*1.1;
l_feet = 0.3*t_feet;        % feet thickness, feet makes contact with the ground (m)
r_feet_sphere = l_feet/2;

contact_stiffness = abs(l*b*h*rho*2*g/0.1);     % approx Weight/ deflection (N/m)
contact_damping = contact_stiffness/10;      % Arbitrary Value (can be tuned) (Ns/m)

joint_stiffness = abs(l*b*h*rho*g*l/10);          % moment created by other block / deflection angle (Nm/deg)
joint_damping = joint_stiffness/10;               % Arbitrary Value (can be tuned) (Ns/m)

% center of leg endpoint rotation, measured from body-leg hinge point.
rotation_center = -0.95*l_thigh;
y_centre = 0.75*(l_thigh*2 + h_walk/2 + l_feet/2);
```

Figure 1: The list of parameters used in the model

Simulation plots

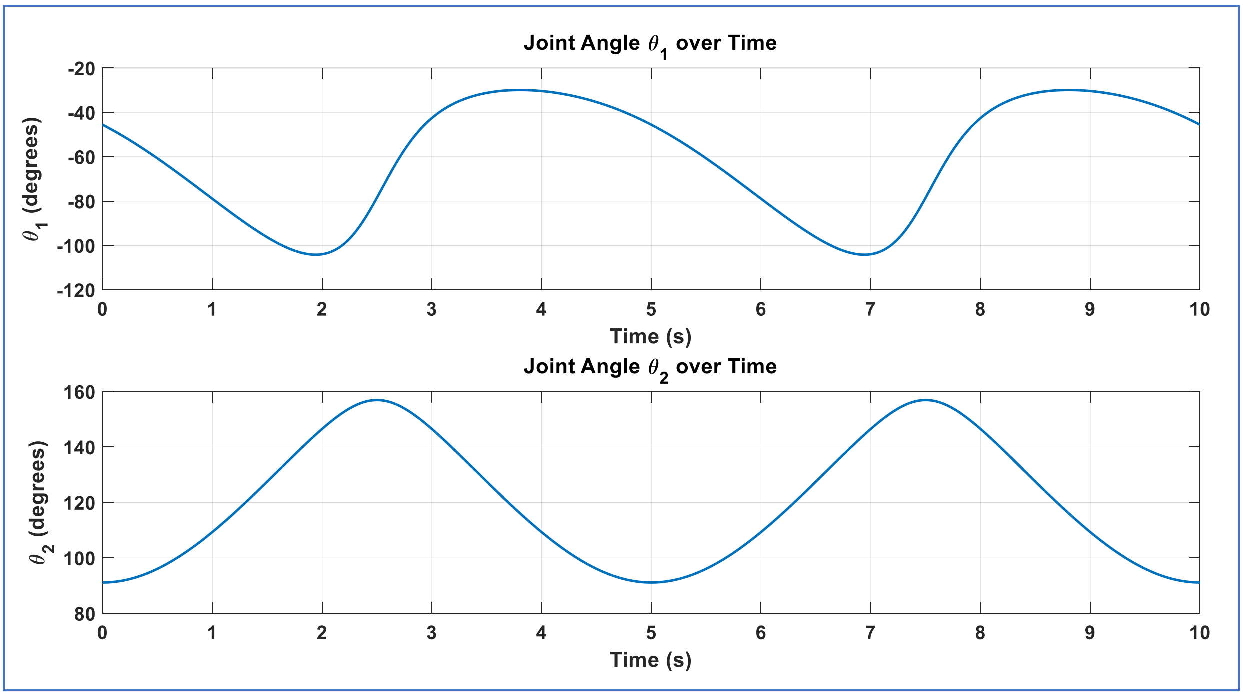

Figure 2: Angle variation over time.

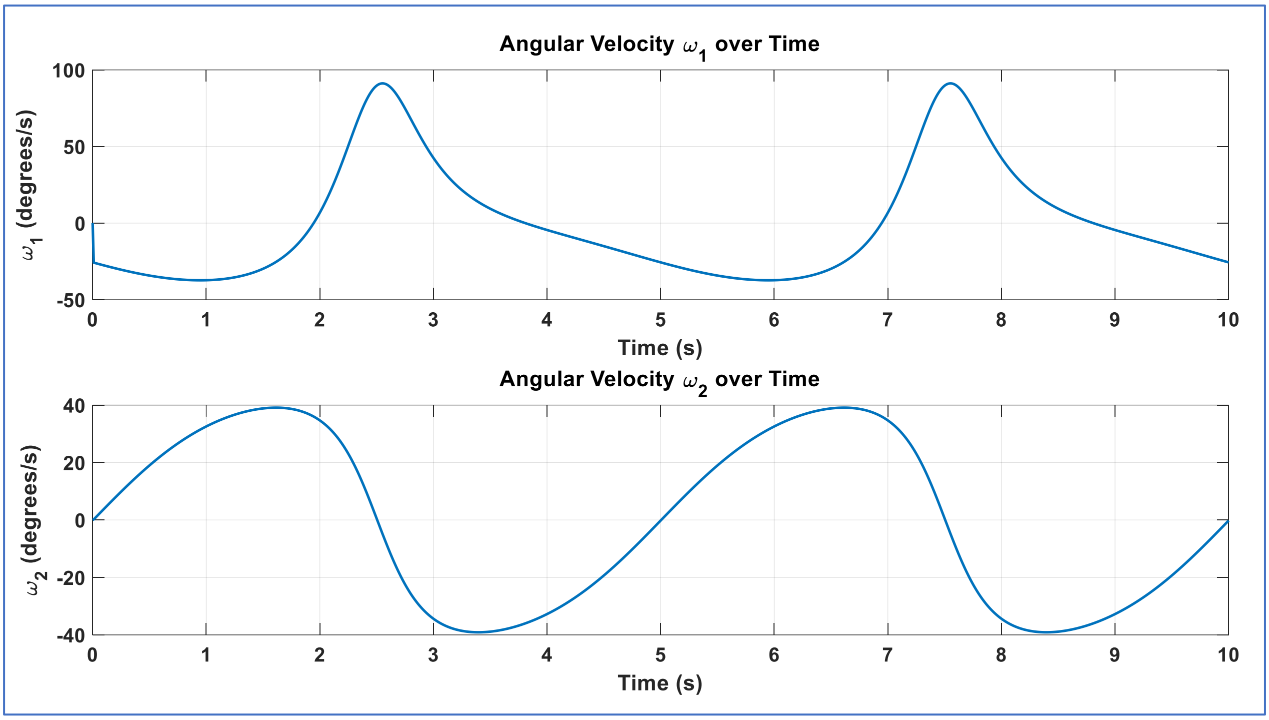

Figure 3: Omega variations over time.

phase_1.mp4

Figure 4: Video for phase 1 simulation

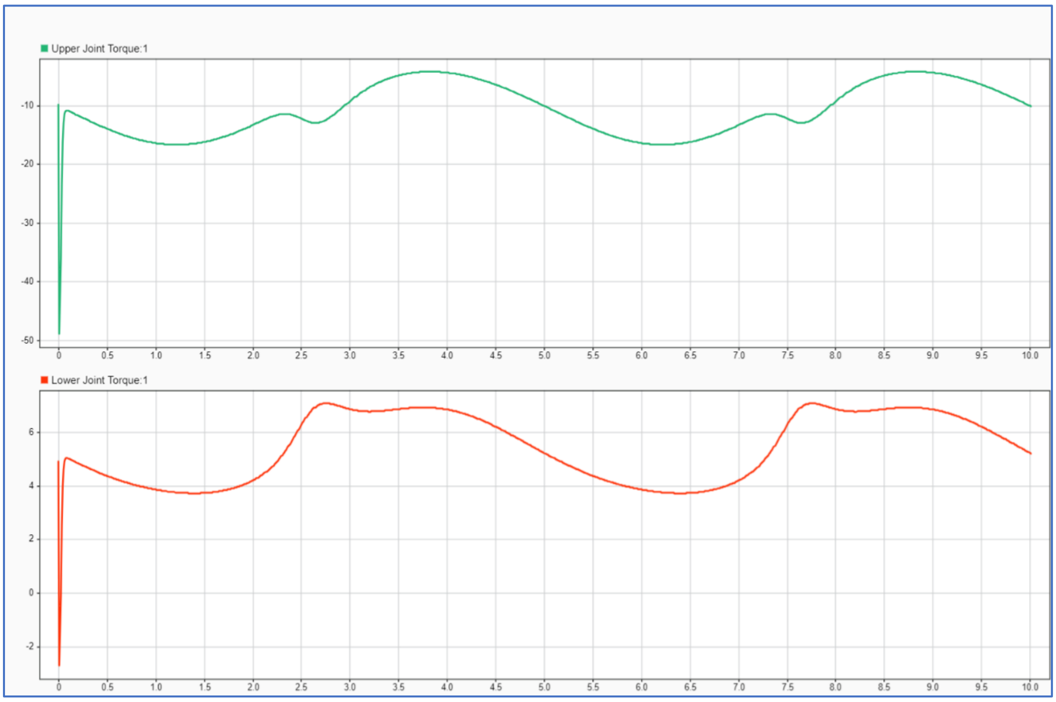

Figure 5: Torque variation (Nm) for the two links

Stage 2

Objective

To replicate the natural discontinuous walking pattern observed in humans, where the back leg pushes upwards and forwards while the other leg moves in the air. Upon collision of the front leg with the ground, it pushes forward as the back leg lifts off. Thus, we need to figure out how to create this realistic motion on our stage 1 model.

Components of the model

1. **Walking Path:** A planar block modelled from a **brick solid** component on which the quadruped model can walk.
2. **Body:** Modelled from 2 **brick solid** components joint together by a real revolute joint. The 2 legs are attached to the front and back edges of these block.
3. **Legs:** 2 legs, each one modelled from two brick solid components of the same size, attached to each other, first link of the leg is attached to the body and the second link via revolute joints.
4. **Angle inputs:** Inputs to the revolute joints calculated from inverse kinematics model so that the end of second link can trace a circular path.
5. **Rigid Transforms:** To give translation transformation between the walking path and the body which is at rest.
6. **2 Feet:** For having contacts with the surface below and giving stability by increasing surface area of contact (wider). Created using 2 **brick solids**.
7. **Spatial Contact Forces:** We define the contact forces such stiffness and damping for the contact to be like real world collision.
8. **Spherical body:** Placed at the 4 corners of the foot, so that contact can be made symmetrically through them with surface, instead of having 1 point of the foot directly to surface. Also, the contact model can distribute forces accurately based on the position and orientation of the brick, simulating real-world interactions more closely.

Explanation of the model

This model simulates a bipedal walking mechanism, utilizing **inverse kinematics** and **spatial contact forces** to replicate a natural, human-like walking pattern. The primary objective is to achieve realistic motion in a preliminary (stage 1) prototype by designing the robot's lower limb movements to mimic the characteristic discontinuous walking gait of humans.

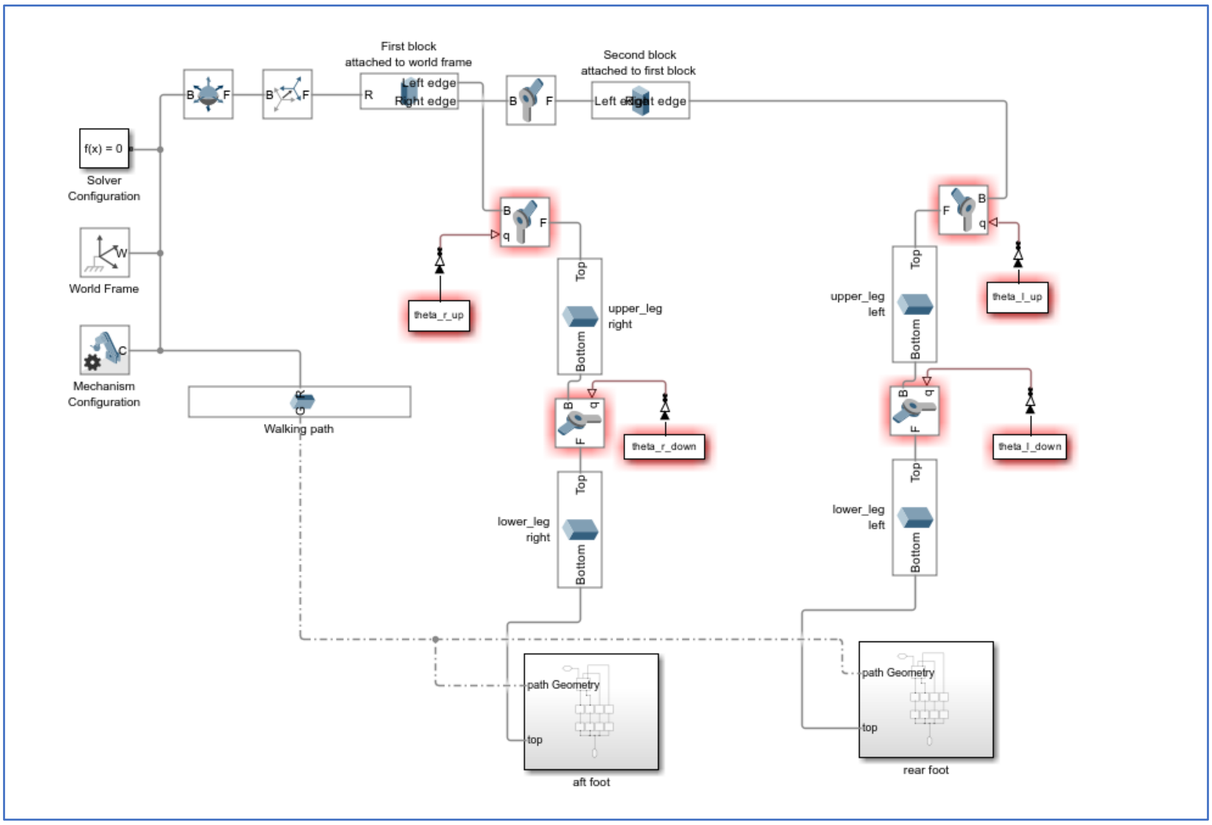

Figure 6: Simulink Model for Phase 2 simulation.

Key Features:

Inverse Kinematics: (Same working as explained in the 1st model)

The variables `theta_r_up`, `theta_l_up`, `theta_l_down`, and `theta_r_down` represent the joints' angles of the robot's legs. These angles are computed using inverse kinematics to ensure the robot follows a desired walking trajectory.

Spatial Contact Forces:

These forces simulate the interactions between the robot's feet and the walking surface.

They are crucial for realistic ground reactions, enabling the robot to adjust its posture dynamically as it transitions between steps.

Contact forces also help replicate the push-and-lift mechanics observed during a natural human gait.

Revolute Joint having spring stiffness and internal damping:

This provides the actual movement/motion of the robot's abdomen. We added the internal mechanics value for the spring stiffness, and damping coefficients so that whenever it bends it can mimic the muscles and abdomen of a robot and can come back to the resting state after the force is removed.

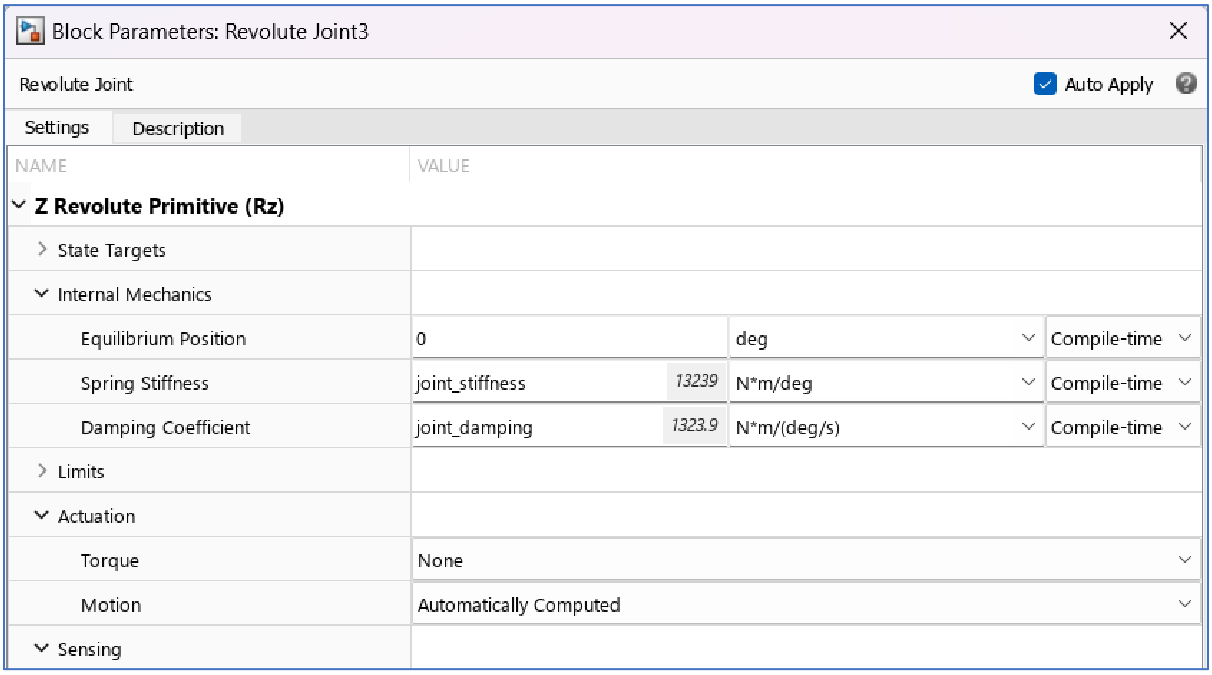

Figure 7: Revolute joint acting as torsional spring

Simulation plots

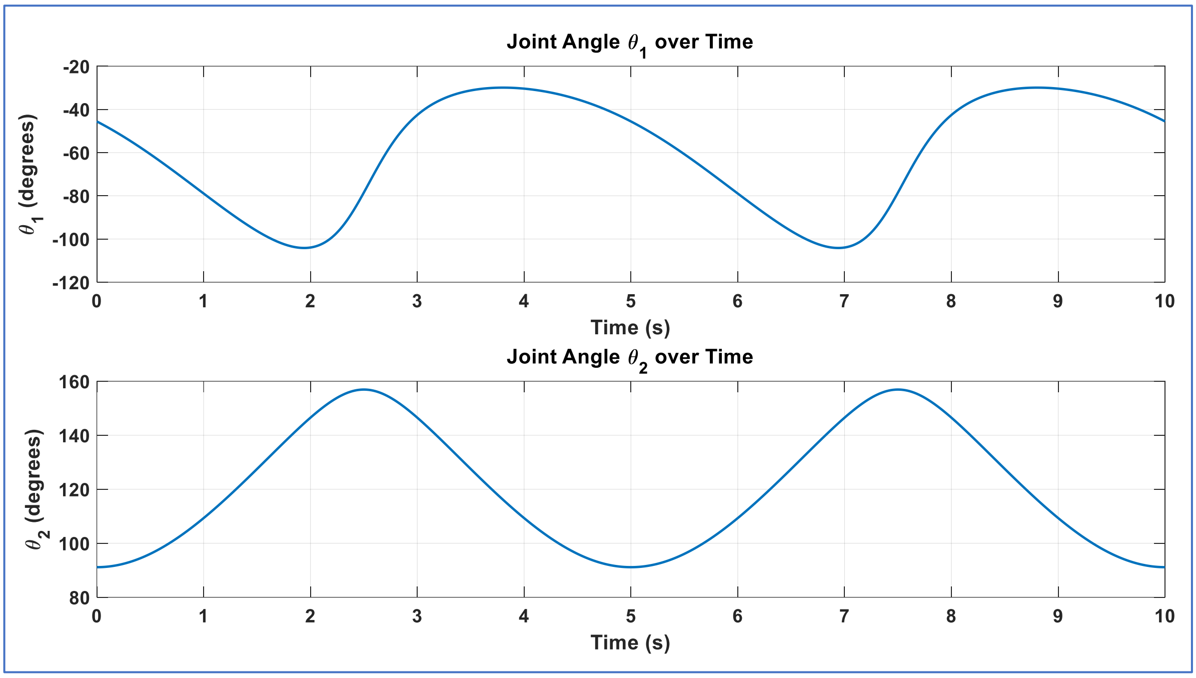

Figure 8: Joint angle variation over time.

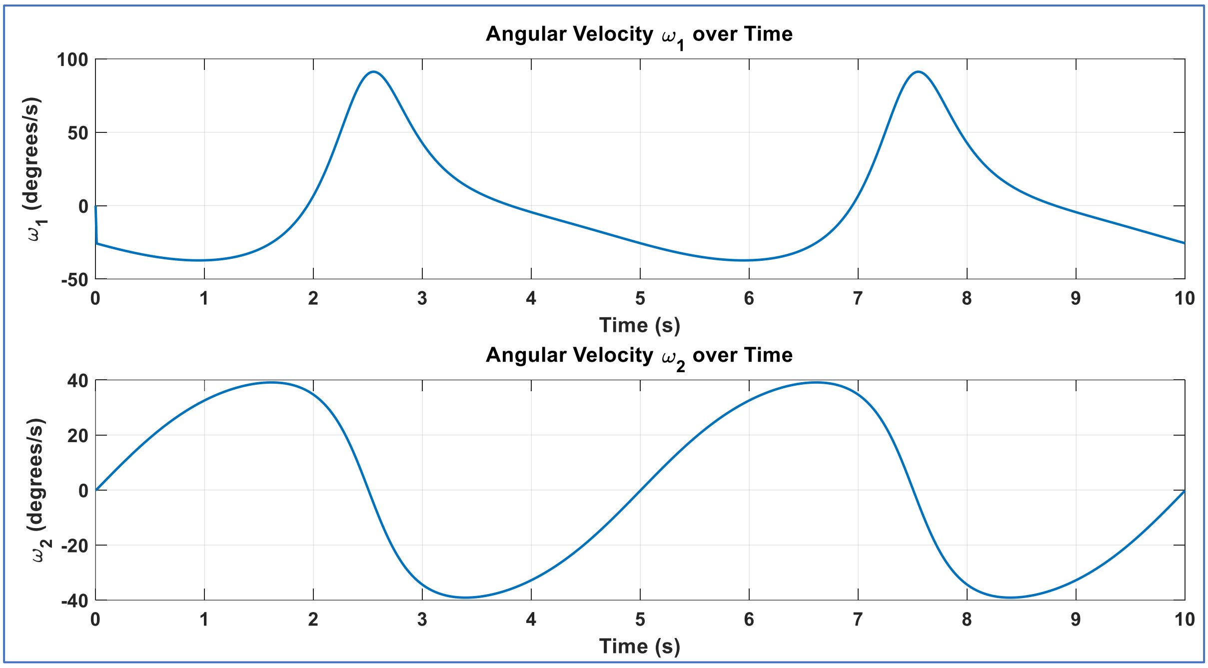

Figure 9: Joint's angular velocity variation over time.

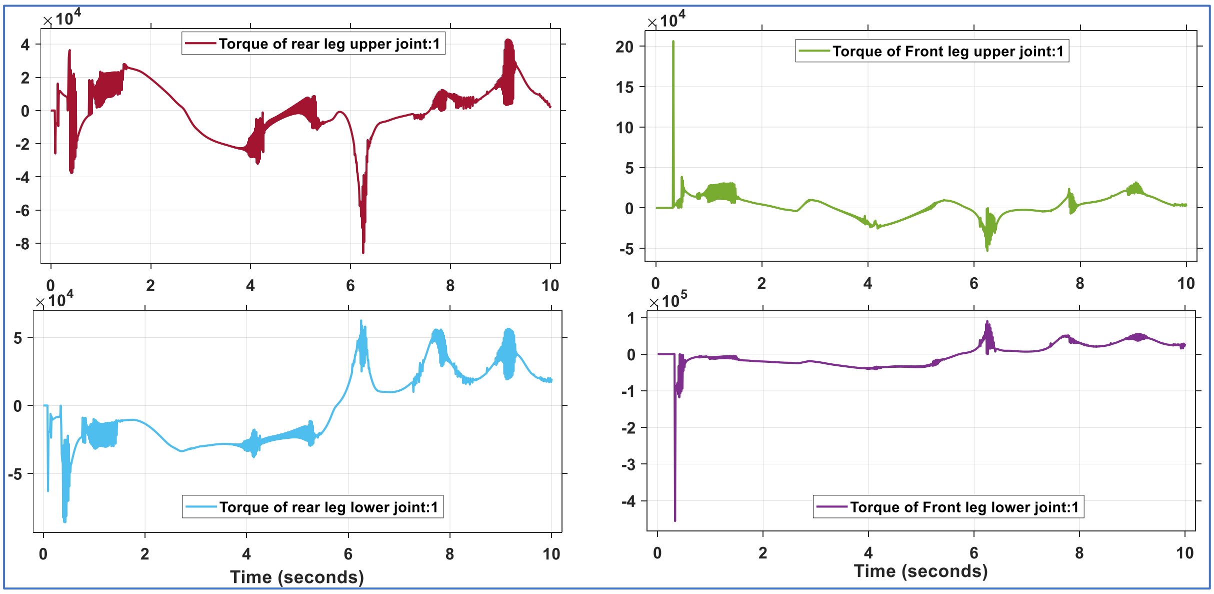

Figure 10: Actuator torque variation at joints of both the legs.

phase_2.mp4

Figure 11: Video of phase 2 simulation.

Stage 3

Objective

To extend the bipedal robot model into a quadrupedal robot, enabling realistic and natural motion. This involves designing and implementing a trajectory planning system to coordinate the movements of all four legs for dynamic stability and efficient locomotion.

Components of the model

1. **Walking Path:** A planar block modelled from a **brick solid** component on which the quadruped model can walk.
2. **Body:** Modelled from 2 **brick solid** components joint together by a real revolute joint. The 4 legs are attached to the front and back edges of these block.
3. **Legs:** 4 legs, each one modelled from **two brick solid** components of the same size, attached to each other, first link of the leg is attached to the body and the second link **via revolute joints**.
4. **Angle inputs:** Inputs to the revolute joints calculated from inverse kinematics model so that the end of second link can trace a circular path.
5. **Rigid Transforms:** To give translation transformation between the walking path and the body which is at rest.
6. **4 Feet:** For having contacts with the surface below and giving stability by increasing surface area of contact (wider). Created using 4 **brick solids**.
7. **Spatial Contact Forces:** We define the contact forces such stiffness and damping for the contact to be like real world collision.
8. **Spherical body:** Placed at the 4 corners of the foot, so that contact can be made symmetrically through them with surface, instead of having 1 point of the foot directly to surface. Also, the contact model can distribute forces accurately based on the position and orientation of the brick, simulating real-world interactions more closely.

Explanation of the model

Just the stage 2 model has been extrapolated by adding the symmetrically opposite legs, thus making the quadrupedal robot. The front two legs are joint to the front block using revolute joints at the front edges of the block, and similarly, the model combines kinematics, dynamics, and contact physics to try to emulate natural quadrupedal locomotion. Also, the walking speed was increased to see the effects of legs rotation on trajectory of the robot.

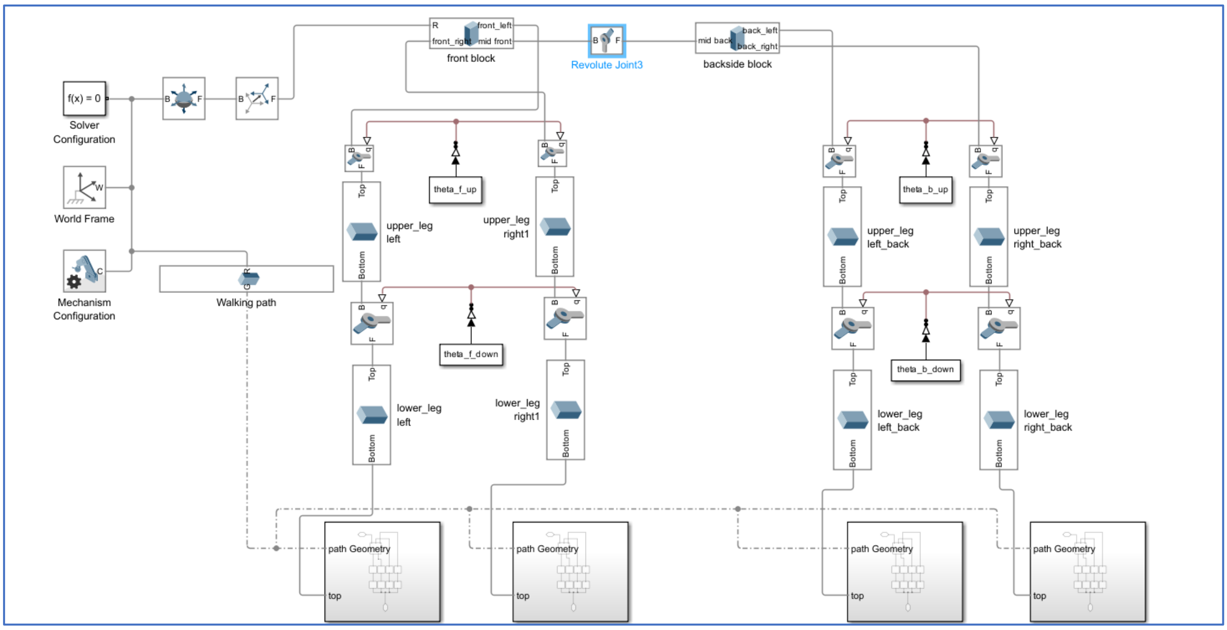

Figure 12: Model for phase 3 simulation.

Simulation plots

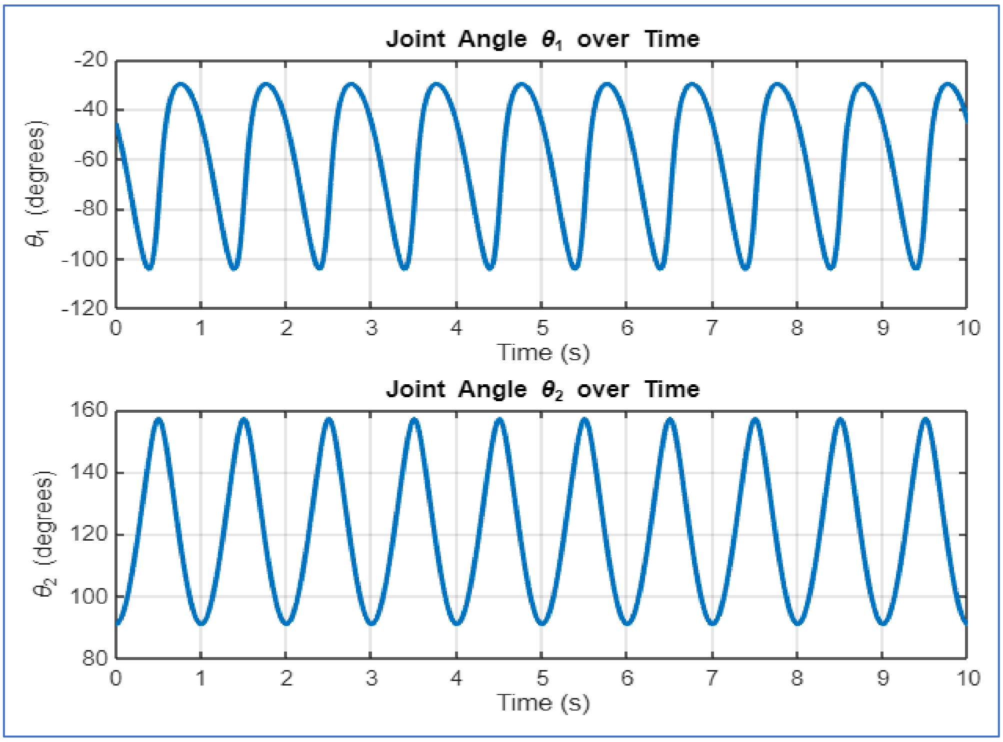

Figure 13: Angle variation over time for different joints of a leg.

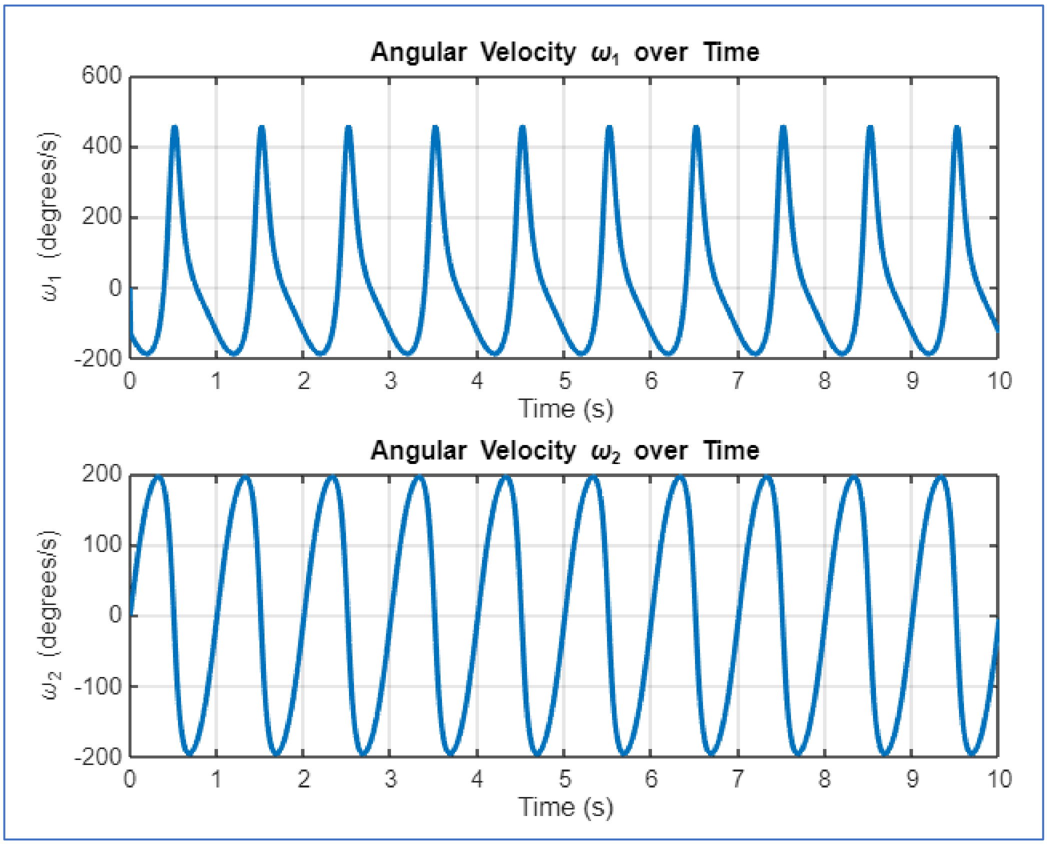

Figure 14: Angular velocity variation over time for different joints of a leg.

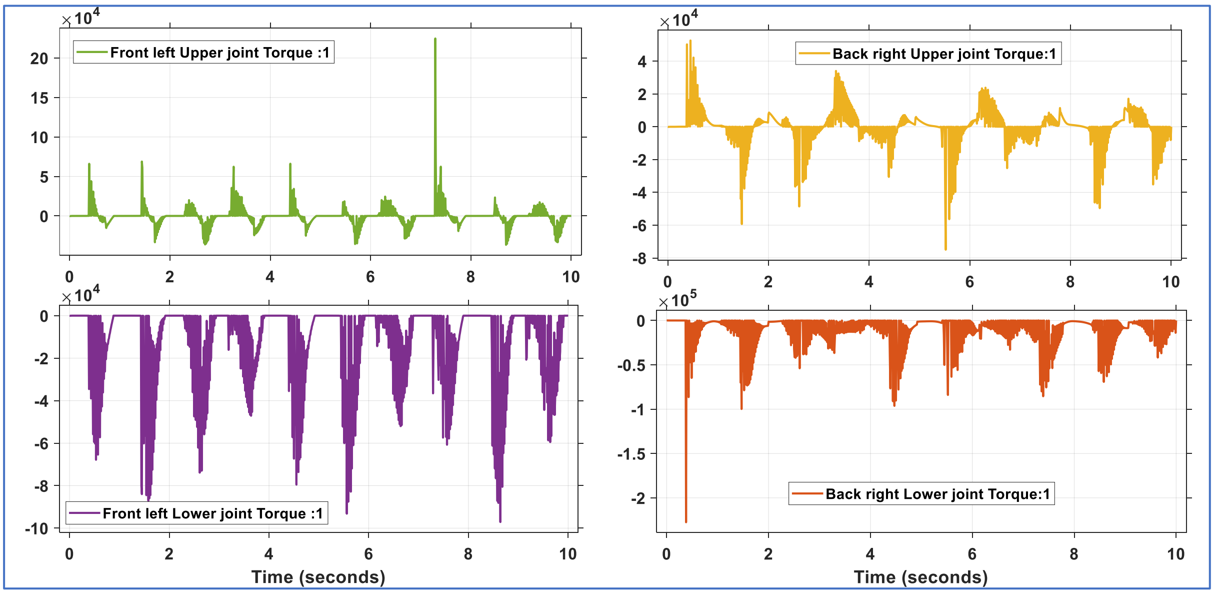

Figure 15: Torque Variation for two of the four legs joints.

phase_3.mp4

Figure 16: Video of phase 3 simulation.

Discussion of results for different cases

Stage 1

Key findings include the ability of the model to trace the desired trajectory, as shown in the simulation plots. The phase difference of 180° between the legs, implemented in Simulink, provided symmetry in movement. The angle and torque variation graphs validated the theoretical predictions, showing smooth motion dynamics over time. However, challenges such as precise tuning of angular velocities and maintaining consistent torque across simulations were noted.

The Stage 1 simulation demonstrated that the basic walking mechanism was feasible. The results serve as a foundation for more complex dynamics, such as the introduction of discontinuous walking patterns in Stage 2.

Stage 2

Stage 2 extended the planar biped model to simulate a more natural, discontinuous human-like walking pattern. This involved incorporating spatial contact forces and designing leg movements that replicate the push-and-lift dynamics of human gait. The model introduced features like spherical contacts at foot corners to enhance stability and revolute joints with spring stiffness and damping to mimic muscle-like behaviour.

Simulation results highlighted realistic interactions between the robot's feet and the walking surface. Angular velocities and torques of joints showed distinct patterns reflecting the discontinuous gait. Also, after sometime, the robot tends to roll and fall off the walking path. This was mitigated by adding more legs to give roll stability.

Despite achieving a more human-like gait, challenges persisted, such as optimizing stiffness coefficients to balance movement fluidity and control. The results of Stage 2 validated the model's adaptability and laid the groundwork for extending the biped design into a quadruped in Stage 3.

Stage 3

It expanded the bipedal model into a quadrupedal robot, focusing on dynamic stability and efficient locomotion. By adding two symmetrically opposite legs, the model integrated kinematics, dynamics, and contact physics to emulate quadrupedal motion. Key components included enhanced spatial contact forces and coordinated trajectory planning across all four legs.

The simulation demonstrated the feasibility of maintaining dynamic stability while coordinating movements. Angular velocity and torque variations across joints indicated balanced motion with well-distributed forces during transitions. The use of spherical contacts ensured realistic interactions with the walking surface, improving force distribution and stability.

The quadruped model faced challenges in synchronization and maintaining stability during high-speed locomotion. Despite these, the results confirmed the success of trajectory planning and coordination, making the model suitable for further developments in realistic robotic motion. This stage marked a significant step toward achieving efficient and adaptable robotic locomotion.

References

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4. MathWorks. (n.d.). *Modeling and simulation of walking robots*. Retrieved November 18, 2024, from https://in.mathworks.com/videos/modeling-and-simulation-of-walking-robots-1576560207573.html