

MECHANICS OF SOLIDS

STRAIN

01

Longitudinal stress = $\frac{\Delta L}{L} = \frac{\text{Change in length}}{\text{Original length}}$

02

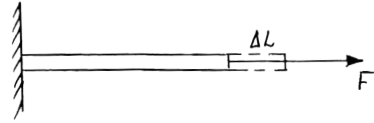
Volumetric strain = $\frac{\Delta V}{V} = \frac{\text{Change in volume}}{\text{Original volume}}$

03

Shearing strain = $\phi = \frac{\Delta x}{L}$

LONGITUDINAL STRESS

Tensile stress causes increase in length



Tensile stress = $\frac{F}{A}$ unit N/m^2

Diagram showing masses m and M hanging from a wire. Tensile stress $T.S = \frac{F}{A} = \frac{Fm}{(M+m)A}$

Diagram showing a wire fixed to a wall with three masses m, 2m, and 3m hanging from it. Tensile stress at different points:

$$T.S_1 = \frac{T_1}{A} = \frac{6mg}{A}$$

$$T.S_2 = \frac{T_2}{A} = \frac{5mg}{A}$$

$$T.S_3 = \frac{T_3}{A} = \frac{3mg}{A}$$

VOLUME STRESS

- Same as pressure
- Causes change in volume

volume stress = $\frac{F}{A}$ = pressure

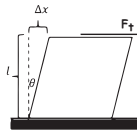
F = normal force/thrust

SHEARING STRESS

- Causes change in shape

shearing stress = $\frac{F_t}{A}$

F_t = tangential force



HOOKE'S LAW

$E = \frac{\text{Stress}}{\text{Strain}}$

(E = modulus of elasticity)

$\tan \theta$ = modulus of elasticity

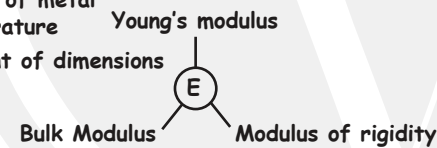
1. For rigid body E = infinity

2. Steel is more elastic than rubber

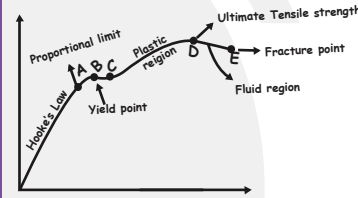
3. Depends on :-

- (a) Nature of metal
- (b) Temperature

4. Independent of dimensions



STRESS STRAIN CURVE



OA - Hooke's law obeyed

A - Proportional limit

AB - Not proportional but body regains its original shape and size when load is removed

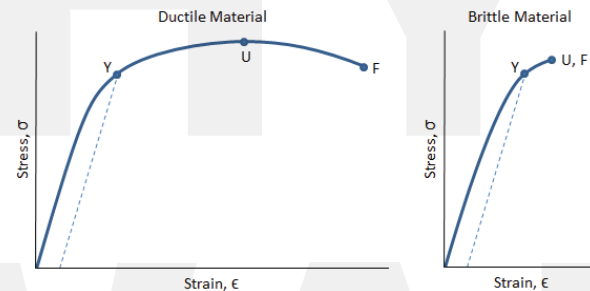
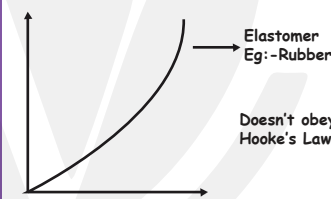
B - Yield point

B to D - Body doesn't regain its original dimension. Beyond B is plastic region.

D - Ultimate stress point

beyond D - added strain is produced even for a small applied force.

E - Fracture occur



Plastic region is large for ductile materials and smaller for brittle materials

BREAKING STRESS

B.F = breaking stress x area, B.F \propto A

Diagram showing two wires of different cross-sectional areas A_1 and A_2 and lengths L_1 and L_2 under a force F.

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \quad (B.S)_1 = (B.S)_2$$

BREAKING OF WIRE UNDER IT'S OWN WEIGHT

$B.S \times A \times T = mg$

$B.S \times A = V \times \rho \times g = A L_{\max} \rho g$

$L_{\max} = \frac{B.S}{\rho g}$



RATIO OF DENSITY OF BODY TO THAT OF LIQUID IN WHICH BODY IS IMMERSSED

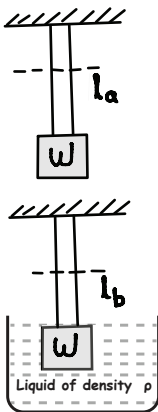
$W = Kl_a$

$W(1 - \frac{\rho}{\sigma}) = Kl_b$

$(1 - \frac{\rho}{\sigma}) = \frac{l_b}{l_a}$

$\frac{\rho}{\sigma} = \frac{l_a - l_b}{l_a}$

$\frac{\sigma}{\rho} = \frac{l_a}{l_a - l_b}$



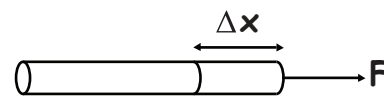
YOUNG'S MODULUS

$Y = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}} = \frac{FL}{A\Delta L}$

Comparing with a spring of force constant K

$Y = \frac{FL}{Ax} \quad F = \frac{AYx}{L} = kx \quad k = \frac{AY}{L}$

ELASTIC POTENTIAL ENERGY



$E.P.E = \frac{1}{2} kx^2 = \frac{F^2}{2k} = \frac{1}{2} Fx$

$= \frac{1}{2} \left(\frac{YA}{L} \right) x^2$

$= \frac{1}{2} \times \frac{\text{Stress}}{\text{Strain}} \times \text{volume} \times \text{strain}^2$

$= \frac{1}{2} \times \text{Stress} \times \text{volume} \times \text{strain}$

INCREASE IN LENGTH DUE TO IT'S OWN WEIGHT

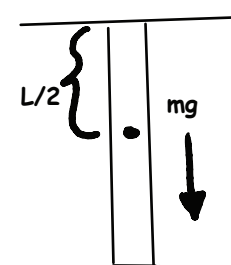
$F = kx$

$Mg = \frac{YA}{L} x$

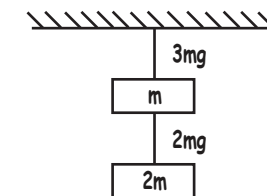
$x = \frac{mgL}{2YA}$

$P.E = \frac{F^2}{2k}$

$= \frac{M^2 g^2}{2 \frac{YA}{L}} = \frac{1}{4} \frac{M^2 g^2 L}{YA}$



RATIO OF EXTENSION



$\frac{x_1}{x_2} = \frac{\frac{F_1}{k_1}}{\frac{F_2}{k_2}} = \frac{\frac{3mg \times L_1}{Y_1 \times A_1}}{\frac{2mg \times L_2}{Y_2 \times A_2}} = \frac{3l}{2yd^2}$

where $l = \frac{L_1}{L_2} \quad y = \frac{Y_1}{Y_2} \quad d = \frac{D_1}{D_2}$

BULK MODULUS

Bulk modulus, $B = \frac{\Delta P}{-\frac{\Delta V}{V}}$

$K = \frac{1}{B}$ = compressibility

$B_{\text{isothermal}} = P \quad B_{\text{adiabatic}} = \gamma P$

MODULUS OF RIGIDITY

$\eta = \frac{F}{A\phi} = \frac{Fl}{Ax}$

