

- Density ( $\rho$ ) =  $\frac{\text{mass (m)}}{\text{volume (v)}}$



- unit =  $\text{kg/m}^3$
- density of water =  $1000 \text{ kg / m}^3 = 1 \text{g/cc}$
- for same mass  
 $\rho_1 V_1 = \rho_2 V_2$   
 if  $\rho_1 > \rho_2$   
 $V_1 < V_2$

### Mixing of liquid

Calculation of resultant/final density

1) Volumes are equal

$$d = \frac{d_1 + d_2}{2}$$

2) Masses are equal

$$\text{For 2- liquid} \Rightarrow d = \frac{2d_1 d_2}{d_1 + d_2}$$

$$\text{For 3- liquid} \Rightarrow d = \frac{3d_1 d_2 d_3}{d_1 d_2 + d_2 d_3 + d_1 d_3}$$

3) Masses and volumes are different

$$d = \frac{\text{Total mass}}{\text{Total volume}} = \frac{m_1 + m_2}{V_1 + V_2}$$

$$m_1 = \rho_1 V_1 \text{ \& } m_2 = \rho_2 V_2$$

$$V_1 = \frac{m_1}{\rho_1} \text{ \& } V_2 = \frac{m_2}{\rho_2}$$

### Relative density (R.D)/Specific gravity

1) Relative density of a body

$$(R.D)_s = \frac{d_s}{d_w} = \frac{w_a}{w_a - w_w}$$

2) Relative density of liquid

$$(R.D)_L = \frac{d_L}{d_w} = \frac{w_a - w_L}{w_a - w_w}$$

3) Relative density of a solid to that of liquid

$$\frac{(R.D)_s}{(R.D)_L} = \frac{w_a}{w_a - w_L}$$

### Pressure

Normal force or thrust exerted by liquid at rest per unit area

$$P = \frac{F}{A}$$

Pressure depth relation

$$P = h \rho g$$

Hydrostatic paradox

Whatever the shape or width the pressure at any particular depth is same



- Gauge pressure =  $P - P_{atm} = h \rho g$
- $P_{atm} = 1.01325 \times 10^5 \text{ Pa}$

### Inclined barometer

if  $\theta$  = angle with horizontal

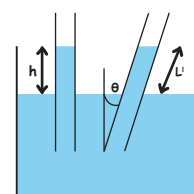
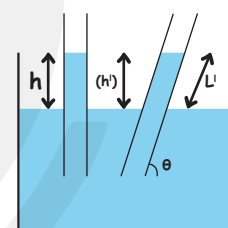
$$\sin \theta = \frac{h}{L'}$$

$$L' = \frac{h}{\sin \theta}$$

if  $\theta$  = angle with vertical

$$\cos \theta = \frac{h}{L'}$$

$$L' = \frac{h}{\cos \theta}$$

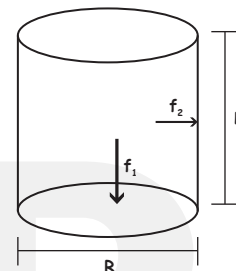


### Bubble rising up at constant temperature

radius 'r' becomes 'R' when bubble rises in liquid from bottom to the surface

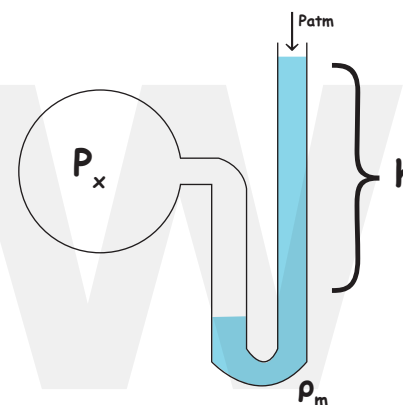
$$\rho g h = P_{atm} [n^3 - 1] \quad R = n r$$

Conditions for equal forces on wall and bottom in a cylinder



If  $L = R$   
then,  $F_1 = F_2$

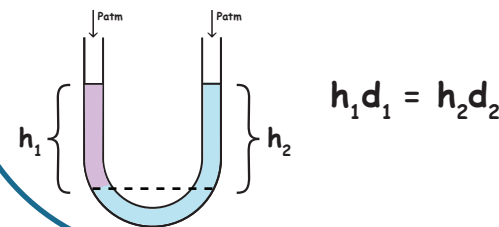
### Manometer



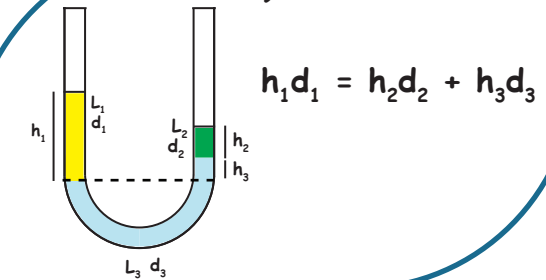
$$P_x = P_o + h \rho_m g$$

$$h \rho_m g = P_x - P_o$$

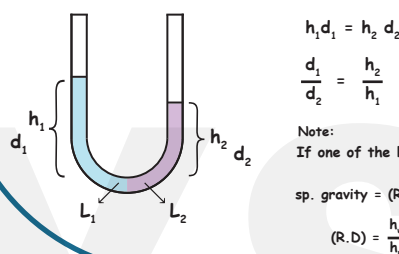
### 1) U-Tube manometer



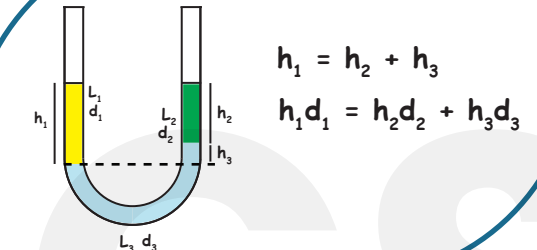
4)



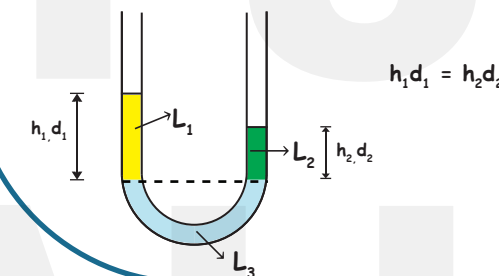
### 2) U-Tube type



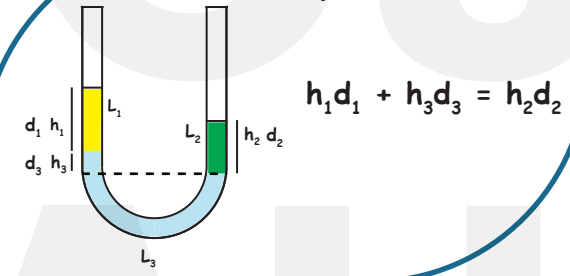
5)



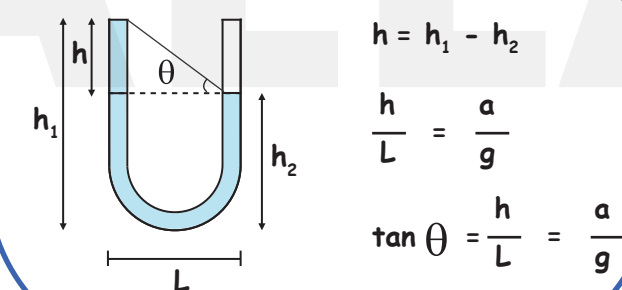
### 3) The third liquid is in level with other



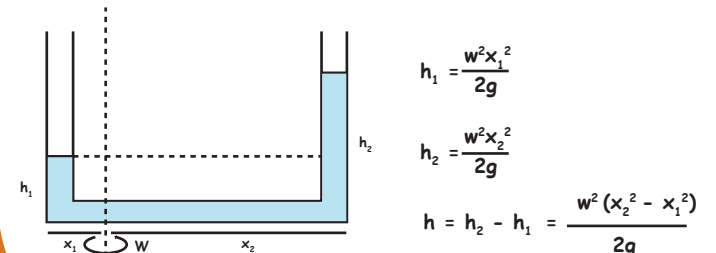
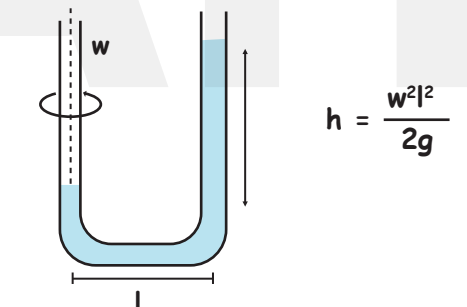
6)



### U - tube accelerating



### Special case: U - tube rotating



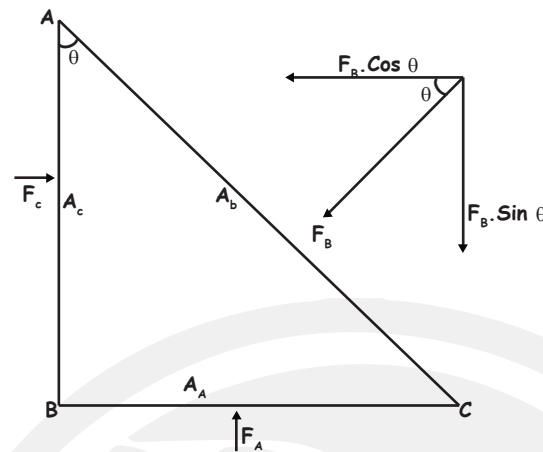
# FLUID MECHANICS 01



PHYSICS  
WALLAH

## Pascals Law

If gravity effect is neglected, the pressure at every point of liquid in static equilibrium is same



## Application

Hydraulic Lift  
As  $A \gg a$  therefore



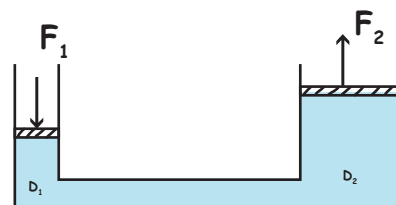
$$\frac{F}{A} = \frac{f}{a}$$

If the cylinders are connected

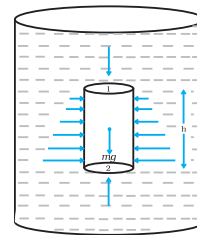
$$\frac{F_1}{\pi R_1^2} = \frac{F_2}{\pi R_2^2}$$

$$\frac{F_1}{R_1^2} = \frac{F_2}{R_2^2}$$

$$\frac{F_1}{D_1^2} = \frac{F_2}{D_2^2}$$

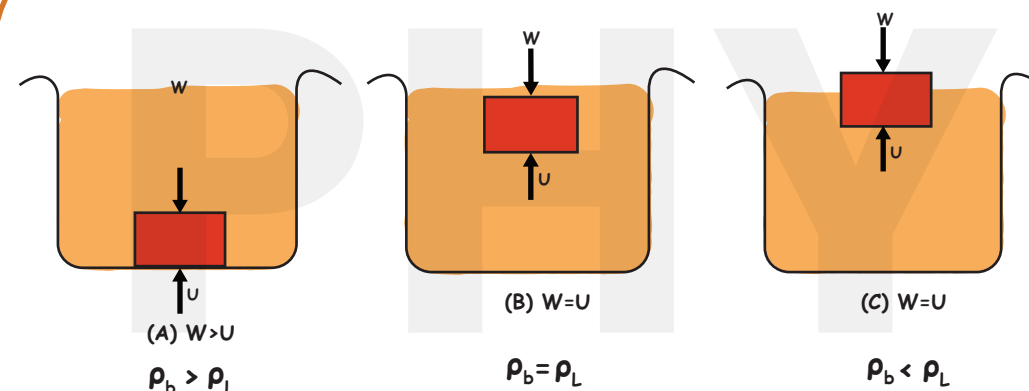


## Archimedes principle



upthrust = weight of the liquid displaced =  $V\rho g$   
Apparent weight = Actual weight - upthrust  
 $W_{app} = W_{air} - U$   
 $= W_{air} \left[ 1 - \frac{\sigma}{\rho} \right]$   
net force acting upward =  $V \times \rho_L \times g$

## Law of floatation



$W \Rightarrow$  Weight  
 $U \Rightarrow$  Upthrust

## Unit of Coefficient of viscosity

- 1) The CGS Unit of  $\eta$  is dyne s cm<sup>-2</sup> and is called poise.
  - 2) The SI unit of  $\eta$  is Nsm<sup>-2</sup> or decapoise or poiseuille
- 1 poiseuille = 10 poise

## Poiseuille's formula

$$Q = \frac{\pi P r^4}{8 \eta l}$$

## Stoke's law

$$F = 6 \eta \pi r v$$

$F_{net} =$  Apparent weight - viscous force

## Terminal velocity

$$V_t = \frac{2 r^2}{9 \eta} (\rho - \sigma) g$$

- 1) If  $\rho > \sigma$ , the body will attain terminal velocity in the downward direction.
- 2) If  $\rho < \sigma$  the terminal velocity will be negative and the body will move in the upward direction.
- 3)  $\rho = \sigma$ , the body remain suspended in the fluid.

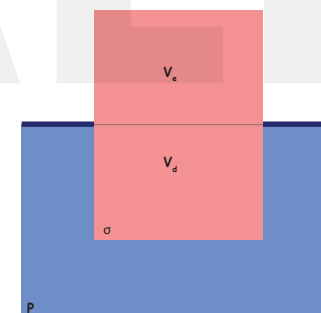
## Fractional submerged volume

$$\frac{\text{Displaced volume}(V_d)}{\text{Total volume}(V)} = \frac{\sigma}{\rho} \text{ (submerged fraction)}$$

$$\frac{\text{Exposed volume}(V_e)}{\text{Total volume}(V)} = 1 - \frac{\sigma}{\rho} \text{ (Exposed fraction)}$$

$$\text{Relative density of a solid} = \frac{\text{weight of solid in air}}{\text{Loss of weight in water}} = \frac{W_a - W_w}{W_a - W_w} = \frac{\rho_b}{\rho_w}$$

$$\text{Relative density of a liquid} = \frac{\text{Loss of weight in liquid}}{\text{Loss of weight in water}} = \frac{W_a - W_L}{W_a - W_w} = \frac{\rho_L}{\rho_w}$$



## Critical velocity

Reynold number

$$R_e = \frac{\rho v D}{\eta}$$

Significances of Reynold number.

- If  $R_e$  lies between 0 and 2000 the flow is stream lined or laminar.
- If  $R_e > 3000$ , the liquid is turbulent.
- If  $R_e$  lies between 2000 & 3000 the flow of liquid is unstable. It may change from laminar to turbulent and vice versa.

## Velocity gradient

$$\text{Velocity gradient} = \frac{dv}{dx}$$

$$F \propto A \frac{dv}{dx} \Rightarrow F = -\eta A \frac{dv}{dx}$$

$$\eta = \frac{F}{A \frac{dv}{dx}} \Rightarrow \text{coefficient of viscosity} \Rightarrow \eta = \frac{F/A}{\frac{dv}{dx}} = \frac{F/A}{\frac{d}{dt}(x/l)}$$

$$\Rightarrow \eta = \frac{\text{shearing stress}}{\text{strain rate}}$$

## Equation of continuity

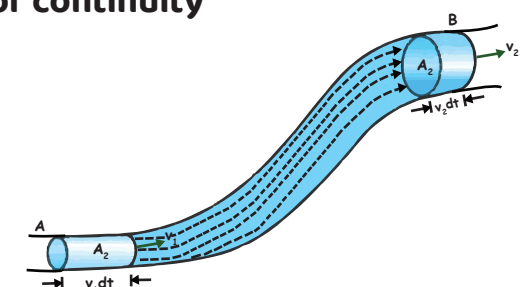
$$V_1 A_1 \Delta t \rho_1 = V_2 A_2 \Delta t \rho_2$$

since the liquid is incompressible  $\rho_1 = \rho_2$

$$V_1 A_1 = V_2 A_2$$

$Av =$  constant.

$$Av = \frac{dv}{dt} = Q \Rightarrow \text{Volume rate of flow}$$



# FLUID MECHANICS 02

## Energy of fluid in a study flow

$$\text{kinetic Energy} = \frac{1}{2} mv^2$$

$$\text{kinetic energy per unit mass} = \frac{1}{2} v^2$$

$$\text{kinetic energy per unit volume} = \frac{1}{2} \rho v^2$$

$$\text{Potential Energy} = mgh$$

$$\text{Potential energy per unit mass} = gh$$

$$\text{Potential energy per unit volume} = \rho gh$$

$$\text{Pressure energy} = PV$$

$$\text{Pressure energy per unit mass} = \frac{P}{\rho}$$

$$\text{Pressure energy per unit volume} = P$$

### BERNOULLI'S PRINCIPLE

$$P_1 V_1 - P_2 V_2 = \frac{1}{2} m (v_2^2 - v_1^2) + mg (h_2 - h_1)$$

$$(P_1 - P_2) V = \frac{1}{2} m (v_2^2 - v_1^2) + mg (h_2 - h_1)$$

$$(P_1 - P_2) V = \frac{1}{2} \frac{m}{V} (v_2^2 - v_1^2) + \frac{mg}{V} (h_2 - h_1)$$

$$\Rightarrow P + \frac{1}{2} \rho v^2 + \rho gh = \text{constant}$$

$$\frac{P}{\rho g} + \frac{v^2}{2g} + h = \text{constant}$$

$$\frac{P}{\rho g} = \text{pressure head}$$

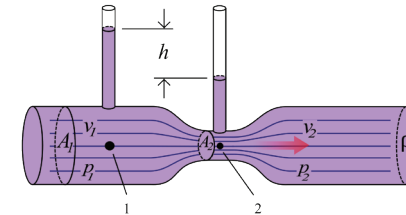
$$\frac{v^2}{2g} = \text{velocity head}$$

$$h = \text{Gravitational head}$$

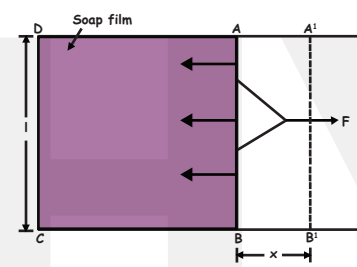
### VENTURIMETER

Device to measure the flow of speed of incompressible fluid

$$v_1 = \sqrt{\frac{2hg}{(A_1^2/A_2^2) - 1}}$$



### SURFACE ENERGY



$$\text{Work done } W = F \times x$$

$$\text{But } F = 2TL$$

$$\Rightarrow W = 2TL \times x$$

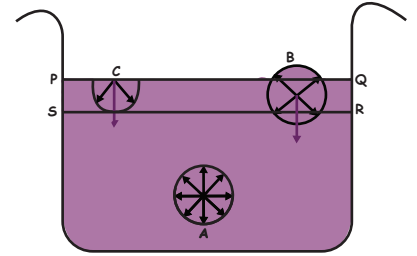
$$\begin{aligned} \text{Energy of the additional surface} &= W = 2TLx \\ &= T(2Lx) = T\Delta A \end{aligned}$$

### SURFACE TENSION

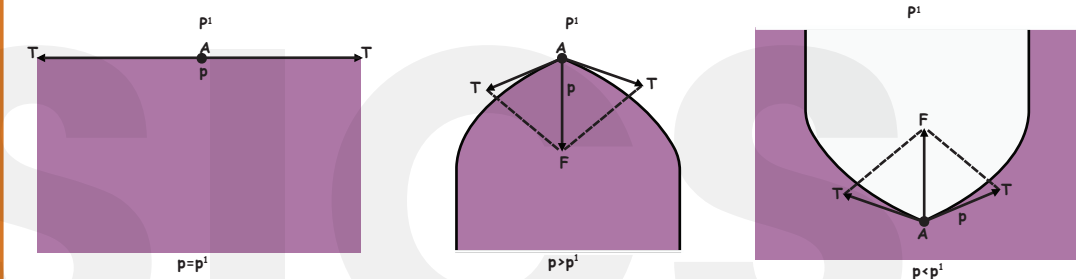
$$\text{Surface tension } T = \frac{\text{Force}}{\text{Length}} = \frac{F}{l}$$

$$\text{Unit in SI system} = \frac{\text{N}}{\text{m}}$$

$$\text{Unit in CGS system} = \text{dyne} / \text{cm}$$



### PRESSURE DIFFERENCE ACROSS A CURVED LIQUID SURFACE

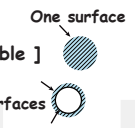


Pressure on concave side > pressure on convex side

$$P_{\text{concave}} - P_{\text{convex}} = \frac{2T}{R}$$

$$P_{\text{inside}} - P_{\text{outside}} = \frac{2T}{R} \quad [\text{Liquid drop or air bubble}]$$

$$P_{\text{inside}} - P_{\text{outside}} = \frac{4T}{R} \quad [\text{Soap bubble}]$$



### APPLICATIONS OF BERNOULLI'S PRINCIPLE

#### Torricelli's Law of Efflux

$$v = \sqrt{\frac{2(P - P_0)}{\rho} + 2gh}$$

If tank is open,  $P = P_0$

$$\text{Then } v = \sqrt{2gh}$$

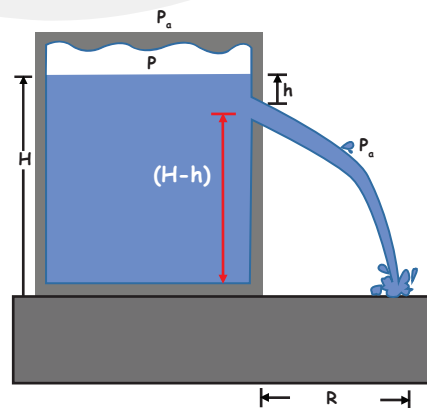
$$\text{Time of fall, } t = \sqrt{\frac{2(H-h)}{g}}$$

$$\begin{aligned} \text{Range } R &= v_x \times t \\ &= \sqrt{2gh} \times \sqrt{2(H-h)/g} \end{aligned}$$

$$\Rightarrow R = 2\sqrt{h(H-h)}$$

$v_x$  = Horizontal component of velocity

R is max. when  $h = \frac{H}{2}$



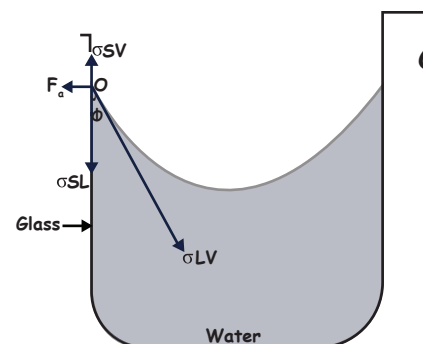
#### Excess pressure inside a liquid drop

$$P_i - P_o = \frac{2T}{R}$$

#### Excess pressure inside a soap bubble

$$P_i - P_o = \frac{4T}{R}$$

### Shape of liquid meniscus



Consider the equilibrium of at line of contact

$$F_a = \sigma_{LV} \sin \theta$$

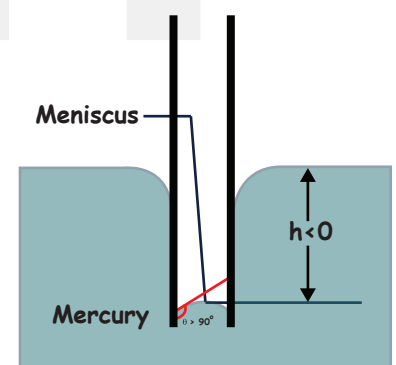
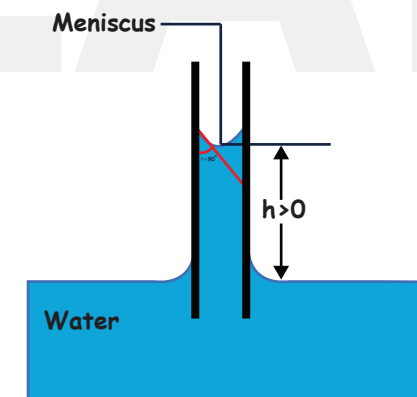
$$\sigma_{SV} = \sigma_{SL} + \sigma_{LV} \cos \theta$$

$$\cos \theta = \frac{\sigma_{SV} - \sigma_{SL}}{\sigma_{LV}}$$

$\theta$  = Angle of contact.

## FLUID MECHANICS 03

### Capillarity



Ascent formula:

$$h = \frac{2T}{R\rho g} \quad h > 0 (\theta < 90^\circ)$$

$$h = \frac{2T \cos \theta}{r\rho g} \quad h < 0 (\theta > 90^\circ)$$