

Point Estimate: The value of any statistics that estimates the value of a parameter is called Point Estimate.

\bar{x} (u) \rightarrow population
Sample mean

$\bar{x} \rightarrow u$
Statistics \downarrow Parameter

Parameter = Population mean

Sample mean = Statistics or Point Estimate

$$\text{Point Estimate} \pm \left[\begin{array}{l} \text{margin of} \\ \text{error} \end{array} \right] = \text{Parameter (pop. mean)}$$

Lower $\hat{x} - C\%I$: Point Estimate - margin of error
higher $\hat{x} + C\%I$: Point Estimate + margin of error

$$\text{margin of error} = Z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \rightarrow \text{standard error}$$

$\hat{x} = \text{Population } S \pm I$

- Q. On the Quant test of CAT Exam, a sample of 25 test takers has a mean of 520 with a population standard deviation of 80. Construct a 95% C.I about the mean?

$$n = 25$$

Sample

$$\bar{x} = \$20$$

Sample mean

$$\sigma = 100$$

Std population

Standard deviation

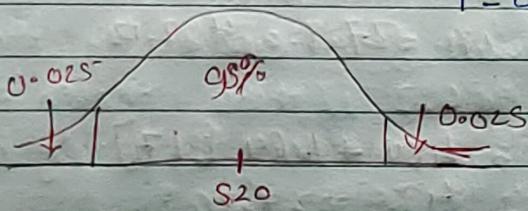
$$C.I = 95\%$$

Confidence Interval

$$\alpha = 5\% = 0.05$$

Significance value

$$1 - 0.025 = 0.975$$



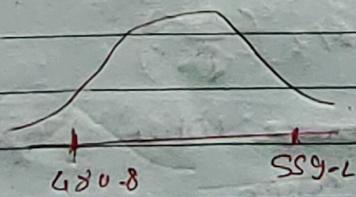
Lower C.I = Point Estimate - margin of Error

$$= \$20 - Z_{0.25} \frac{\sigma}{\sqrt{n}}$$

$$= \$20 - Z_{0.25} \frac{100}{\sqrt{25}} = \$20 - 1.96 \times 20$$

$$= \$20 - 1.96 \times 20 = \$480.8$$

Higher C.I = ~~\$20~~ + 1.96 × 20 = \$59.2



C.I range between 480.8 to 59.2

$$\bar{x} = 480$$

$$S = 85$$

$$n = 25 \quad C.I 90\%$$

$$\text{Lower CI} = 480 - Z_{0.10/2} \left[\frac{85}{\sqrt{5}} \right]$$

$$= 480 - Z_{0.05} \left[\frac{85}{\sqrt{5}} \right]$$

$$= 480 - 1.64 [17]$$

$$= 480 - 27.8 = 452.12$$

$$\text{Higher} = 480 + 1.64 [17]$$

$$= 480 + 27.8$$

$$= 507.8$$

C.I Range between $[452.12 \leftrightarrow 507.8]$

→ on the quant test of Cat Gram, cl Sample
cg 25 test taken mean of 520, with a sample
standard deviation of 80, construct 95% CI
about the mean

$$\Rightarrow \bar{x} = 520 \quad S = 80 \quad C.I = 95\% \quad S: V = 0.05 \\ z \bar{x} + t_{\alpha/2} \left(\frac{S}{\sqrt{n}} \right) \quad n = 25$$

We use t-test ~~because~~ when population std deviation is not given.

t test

$$\text{Degree of freedom} = n - 1 = 24$$

$$2S - 1 = 24$$

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$$\text{Lower CI} = 520 - t_{0.05/2} \left(\frac{80}{\sqrt{16}} \right) \quad t_{0.025}$$

$$= 520 - 2.064 \times 16$$

$$= 486.976$$

$$\text{Higher CI} = 553.024$$

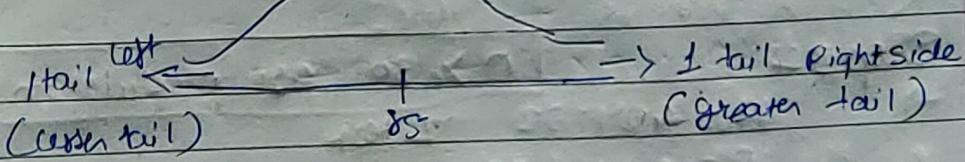
→ 1 tail and 2 tail test

Q ⇒ Colleges in Town A has 85% placement rate.
 A new college was recently opened and it was found that a sample of 180 students had a placement rate 85% with a standard deviation of 4%. Does this college has a different placement rate with 95% C.I?

⇒ greater than 85%.

⇒ lesser than 85%

Two tail



Hypothesis Testing Problem

→ A factory has a machine that fills 40 ml of baby medicines in a bottle. An employee believes the average amount of baby medicine is not 80ml. Using 40 sample, he measures the average amount dispersed by the machines to be 78ml with a standard deviation of 2.5.

- (a) State Null & Alternative Hypothesis
- (b) At 95% CI is there enough evidence to support machine is working properly or not.

Step 1

→ Null Hypothesis $H_0 = 80$

Alt Hypothesis $H_1 = \mu \neq 80$

$$\mu = 80 \text{ ml}, n = 40, \bar{x} = 78, S = 2.5$$

Step 2

$$CI = 75.095 \quad S.O.V (\alpha) = 1 - 0.95 = 0.05$$

Step 3 :

$$n = 40$$

$$S = 2.5$$

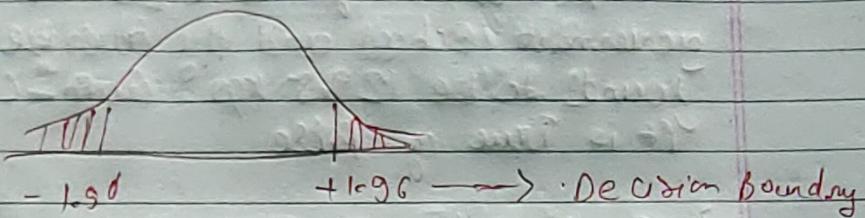
$n > 30$ or population sd

Z test

$n \leq 30$ and Sample sd t test

Z test

Let perform Experiment



Calculate test statistics (Z test)

$$Z = \bar{x} - \mu$$

$$\left[\frac{S}{\sqrt{n}} \right] \rightarrow \text{standard error}$$

$$Z = \frac{\bar{x} - \mu}{\frac{S}{\sqrt{n}}} = \frac{78 - 80}{\frac{2.5}{\sqrt{40}}} = -5.05$$

→ Conclusion

Decision Rule → If $Z = -5.05$ is less than -1.96 or greater $+1.96$; Reject the Null Hypothesis with 95% C.I

Reject the Null Hypothesis [there is some fault in the machine]

→ A complain was registered, the boys in government school are underfed. Average weight of the boys of age 10 is 32 kgs with $S_d = 9 \text{ kgs}$. A sample of 25 boys were selected from the government School and the average weight was found to be 29.5 kgs? with $C\% 98\%$ check if it is True or False

→ Condition for the Z test

- ① We know the population S_d
- ② We do not know the population S_d but our sample is large $n \geq 30$.

→ Condition for T test

- ① We do not know the population S_d
- ② Our sample size is small $n < 30$
- ③ Sample S_d is given

Step 1

$$\begin{aligned} H_0 : \mu &= 32 \\ H_1 : \mu &\neq 32 \end{aligned}$$

$$\rightarrow C\% 1 = 0.95 \quad \alpha = 1 - 0.98 = 0.02$$

→ Z test



$$Z\text{-score} = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{29.5 - 32}{9 / \sqrt{25}} = -1.39$$

Conclusion $\rightarrow -1.39 > -1.96$ Accept the Null hypothesis 95% CI

We fail to reject the Null Hypothesis
The Boys are fed well.

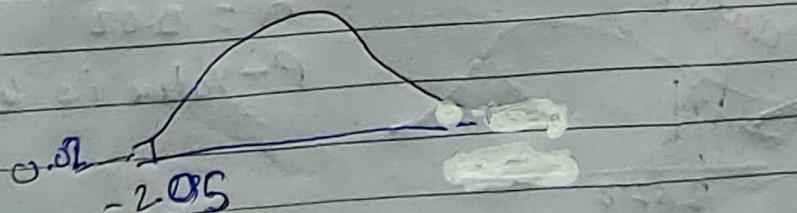
\rightarrow A factory manufactures cars with a warranty of 5 years or more on the engine and transmission. An engineer believes that the engine or transmission will ~~not~~ malfunction in less than 5 years. He tests a sample of 25 cars and finds the average time to be 4.8 years with a standard deviation of ~~0.50~~ 0.50

① State the null & alternate hypothesis.

② At a 2% Significance level, is there enough evidence to support the idea that the warranty should be reduced?

$$\text{Step 1: } H_0: \mu \geq 5 \\ H_1: \mu < 5$$

$$\alpha = 0.02 \quad C.I = 0.98$$



Z score table

Confidence Interval	Significance value
90%	0.10
95%	0.05
98%	0.02
99%	0.01

$$Z \text{ score} = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{4.8 - 3}{0.50 / \sqrt{40}}$$

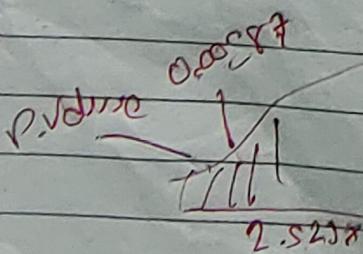
$$= \frac{-0.2}{0.0790}$$

$$= -2.52$$

Conclusion: $-(-2.52) > -2.205$, Reject the null hypothesis 98% CI

With the P-value

Out of all 100% of Z time is



$$\alpha = 0.02$$

P-value is less than

→ A company manufactures bike batteries on average life span of 2 years or more years. An engineer believes this value to be less. Using 10 samples, he measures the average life span to be 1.8 years with a standard deviation of 0.15?

a) State the Null and Alternate hypothesis?

② At a 99% C.I., is there enough evidence to discard the H_0 ?

Step ①

$$H_0: \mu \geq 2$$

$$H_1: \mu < 2$$

② Step ②

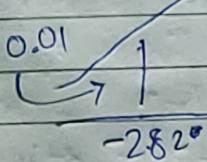
$$C.I = 0.99$$

$$\alpha = 0.01$$

$n < 30$. \rightarrow sample standard deviation.

$$C.I = -0.09 \quad \alpha = 0.01$$

Step 3



Degrees of freedom: $n-1$
 $= 10-1=9$

Calculate t-test statistics

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = 1$$

$$= \frac{1.8 - 2}{0.15 / \sqrt{10}} = -4.216$$

Conclusion $-4.216 \leq -2.22$ (Reject the Null Hypothesis)

The average life of the battery is less than 2 years.

→ Chi Square Test

- ① Chi square test claims about population prop.
- ② It is a non parametric test that is performed on categorical data.

↓
Original Data
Nominal Data

- In the 2000 U.S Census the age of individuals in a small town found to be the following

≤ 18	18 - 35	> 35
20%	30%	50%

In 2010, ages of $n = 500$ individuals were sampled. Below are the results.

≤ 18	18 - 35	> 35
121	286	91

Using $\alpha = 0.05$, would you conclude the population distribution of ages has changed the last 10 years

Now →

	≤ 18	18 - 35	≥ 35
Expected	20%	30%	50%

n = 500

	≤ 18	18 - 35	≥ 35
Observed	121	288	91
Expected	100	150	250

Step 1: Null Hypothesis H_0 : The data meets the expected distribution
 H_1 : The data does not meet the expected distribution.

Step 2) $\alpha = 0.05$ $C.I = 95\%$

Step 3 Degree of Freedom
 $df = C - 1 = 3 - 1 = 2$
 ↳ No. of Categories

Step 4: Decision Boundary = 5.991

Step 5: Chi Square Test Statistics

$$\chi^2 = \sum \frac{(f_o - f_e)^2}{f_e} = \frac{(121 - 100)^2}{100} + \frac{(288 - 150)^2}{150} + \frac{(91 - 250)^2}{250}$$

$$\chi^2 = 252.494$$

Conclub

262.YS99 Reject No

Distribution has been changed.