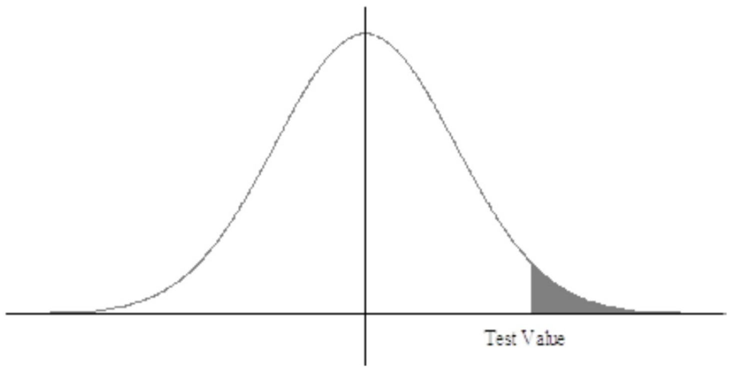
Hypothesis Testing

The ***P*-value** (or probability value) is the probability of getting a sample statistic (such as the mean) or a more extreme sample statistic in the direction of the alternative hypothesis when the null hypothesis is true.



Hypothesis Testing

- In this section, the traditional method for solving hypothesis-testing problems compares ***z*-values**:
  - critical value
  - test value
- The *P*-value method for solving hypothesis-testing problems compares **areas**:
  - alpha
  - *P*-value

Procedure Table

Solving Hypothesis-Testing Problems (*P*-Value Method)

Step 1 State the hypotheses and identify the claim. ✓

Step 2 Compute the test value. ✓

Step 3 Find the *P*-value. ✓

Step 4 Make the decision.

Step 5 Summarize the results.

Example 8-6: Cost of College Tuition

A researcher wishes to test the claim that the average cost of tuition and fees at a four-year public college is greater than \$5700. She selects a random sample of 36 four-year public colleges and finds the mean to be \$5950. The population standard deviation is \$659. Is there evidence to support the claim at a 0.05? Use the *P*-value method.

Step 1: State the hypotheses and identify the claim.

H_0: μ = \$5700 and H_1: μ > \$5700 (claim)

Example 8-6: Cost of College Tuition

A researcher wishes to test the claim that the average cost of tuition and fees at a four-year public college is greater than \$5700. She selects a random sample of 36 four-year public colleges and finds the mean to be \$5950. The population standard deviation is \$659. Is there evidence to support the claim at a 0.05? Use the *P*-value method.

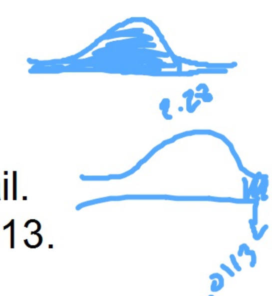
Step 2: Compute the test value.

$$z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \frac{5950 - 5700}{659/\sqrt{36}} = 2.28$$

Example 8-6: Cost of College Tuition

A researcher wishes to test the claim that the average cost of tuition and fees at a four-year public college is greater than \$5700. She selects a random sample of 36 four-year public colleges and finds the mean to be \$5950. The population standard deviation is \$659. Is there evidence to support the claim at a 0.05? Use the *P*-value method.

Step 3: Find the *P*-value.
Using Table E, find the area for z = 2.28.
The area is 0.9887.
Subtract from 1.0000 to find the area of the tail.
Hence, the *P*-value is 1.0000 – 0.9887 = 0.0113.



Example 8-6: Cost of College Tuition

Step 4: Make the decision.

Since the *P*-value is less than 0.05, the decision is to reject the null hypothesis.

Step 5: Summarize the results.

There is enough evidence to support the claim that the tuition and fees at four-year public colleges are greater than $5700.

Note: If α = 0.01, the null hypothesis would not be rejected.

Example 8-7: Wind Speed

A researcher claims that the average wind speed in a certain city is 8 miles per hour. A sample of 32 days has an average wind speed of 8.2 miles per hour. The standard deviation of the population is 0.6 mile per hour. At α = 0.05, is there enough evidence to reject the claim? Use the *P*-value method.

Step 1: State the hypotheses and identify the claim.

H_0: $\mu = 8$ (claim) and H_1: $\mu \neq 8$

Step 2: Compute the test value.

$$z = \frac{\overline{X} - \mu}{\sigma / \sqrt{n}} = \frac{8.2 - 8}{0.6/\sqrt{32}} = 1.89$$

Example 8-7: Wind Speed

A researcher claims that the average wind speed in a certain city is 8 miles per hour. A sample of 32 days has an average wind speed of 8.2 miles per hour. The standard deviation of the population is 0.6 mile per hour. At $\alpha = 0.05$, is there enough evidence to reject the claim? Use the *P*-value method.

Step 3: Find the *P*-value.

The area for $z = 1.89$ is 0.9706.

Subtract: 1.0000 – 0.9706 = 0.0294.

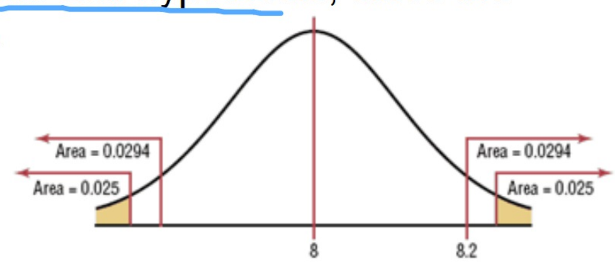
Since this is a two-tailed test, the area of 0.0294 must be doubled to get the *P*-value.

The *P*-value is 2(0.0294) = 0.0588.

Example 8-7: Wind Speed

Step 4: Make the decision.

The decision is to not reject the null hypothesis, since the *P*-value is greater than 0.05.



Step 5: Summarize the results.

There is not enough evidence to reject the claim that the average wind speed is 8 miles per hour.

Guidelines for *P*-Values With No α

- If *P*-value ≤ 0.01, reject the null hypothesis. The difference is highly significant.
- If *P*-value > 0.01 but *P*-value ≤ 0.05, reject the null hypothesis. The difference is significant.
- If *P*-value > 0.05 but *P*-value ≤ 0.10, consider the consequences of type I error before rejecting the null hypothesis.
- If *P*-value > 0.10, do not reject the null hypothesis. The difference is not significant.

Significance

- The researcher should distinguish between **statistical significance** and **practical significance**.
- When the null hypothesis is rejected at a specific significance level, it can be concluded that the difference is probably not due to chance and thus is statistically significant. However, the results may not have any practical significance.
- It is up to the researcher to use common sense when interpreting the results of a statistical test.