



**MAE 548- Probabilistic Methods for Engineering Design
and Analysis**

Prof. Dr. Yongming Liu

**Project 2- Reliability Analysis of Static Structural
and Crash Simulation of Vehicle Architecture.**

Mangirish Sanjeev Kulkarni

ASU ID: 1223229852

Reliability Analysis of Static Structural and Crash Simulation of Vehicle Architecture.

Mangirish Sanjeev Kulkarni, ASU ID- 1223229852

Kaustubh Uttam Nalawade, ASU ID-122645996

Prathamesh Parag Naik, ASU ID-1223474239

Abstract: The objective of this paper is to perform a Reliability Analysis with the primary focus on exit (output) parameters of Static Structural and Crash analysis simulation (i.e., Yield Stress and Impact Force) of a Roll-cage vehicle integrated system. For these two types of simulations i.e., Yield Stress Concentration and Frontal Crash is performed on a Finite Element (FE) based model with input loading conditions. This was performed to observe the Material Behavior in Plastic loading for High strength steel (4130 Chrome Moly) and Advance High Strength Steel (Docol r8 Steel). Uncertainty propagation and Quantification was performed to find goodness of fit of underlying distribution of a generated random variable using KS and Chi-Square testing. Reliability Analysis and Probability of failure of output variable was calculated using FORM and Monte Carlo simulations. Sensitivity analysis was performed and Results were benchmarked.

Keywords: Crash Analysis, Reliability Analysis, Finite-Element Method, Monti-Carlo Method, Design Failure Mode and Effect Analysis, KS and Chi square Testing, FORM.

I. INTRODUCTION

The purpose of static structural analysis and crash analysis of a FE based model is to see how the vehicle will behave under different loading conditions; which in our case is frontal dynamic impact and stress concentration. By following the process of Design Failure mode and Effects Analysis (DFMEA), we evaluated the probability of product failure at different time steps with respect to its input variables. The data was randomly generated using obtained upper and lower limits from both Ansys analysis and SAE papers for material properties. Later we identified the effect of impact, the severity of failure was detected and analyzed to decrease the product cost at manufacturing stage and to overall improve the safety of vehicle. This are the standard norms used in today's automotive industry as the entire process is cost effective, concise and time efficient. This can only be possible by recent development in high-speed computer simulation technologies by use of packages such as Ansys, LS Dyna and HyperMesh and faster mathematical solving approaches such as MATLAB, which will be used below for further simulation.

This crash analysis simulation is used to case study the Material Behavior in initial Plastic loading phase for High strength steel (4130 Chrome Moly) and Advance High Strength Steel (Docol r8 Steel). In recent years Advance high strength steel has replaced good old high strength steel in vehicle architecture i.e., body-in-white, Due to its incredibly advantageous combination of low yield, high-tensile strength, easy cold working, and weldability. To determine this advantage, we used both deterministic and probabilistic approach to observe Yield stress concentration and damage of both materials at plastic failure. For this the distribution of input material parameters (i.e., Normal and lognormal distribution) was validated using statistical test such as KS and chi-square test. The PDF and CDF was plotted for the following distribution and Limit state equation was formulated according to system resistance and load. This was used to perform Form and bruit force Monti Carlo simulation to further verify the failure of system and the final results were benchmarked and validated. The end result will increase the vehicle quality to satisfy the primary goal of safety of passengers in real world scenarios. In the future scope, this can be

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Group 4-

Mangirish Kulkarni, MS in Mechanical Engineering, Arizona state University, Tempe, US. Email: mkulka17@asu.edu.

Kaustubh Nalawade, MS in Mechanical Engineering, Arizona state University, Tempe, US. Email: kunalawa@asu.edu.

Prathamesh Naik, MS in Mechanical Engineering, Arizona state University, Tempe, US. Email: pnaik8@asu.edu.

further analyzed to find crashworthiness of current body frame and design optimization methods can be used to further investigate ways to improve the design.

II. EASE OF USE

Previously traditional approach used multiple design modification and iterations, impact of multiple prototypes and crash validation. This process is extremely expensive, time consuming and lead to inaccurate results of data. Thus, the failure to predict severity has been the key concern of driver and passengers, as it effects the rider's safety. To tackle this limitation which is proven over years of data, The government and automobile manufacturers have imposed various rules to limit the failure of system and to ease concern of passengers.

So instead of relying on experimental validations, the overall safety of model is supplemented with FE simulation and probability estimation in order to evaluate the design. The simulations are controlled in every boundary condition, thus various impact analysis such as side, frontal, rear etc. are carried out effectively. Due to this the product cycle of crash simulation is reduced to half and the end result is a safer vehicle for both the passengers and driver. This results in minimum risk of injury in case of crash, and lower probability of failure.

III. LITERATURE REVIEW

In frontal impact, the frame and front sheet metal absorb most of the crash energy by plastic deformation. The three structural modules are bolted together to form the vehicle structure. The vehicle body is attached to the frame by shock absorbing body mounts, designed to isolate from high frequency vibrations. These vehicles combine the body, frame, and front sheet metal into a single unit constructed from stamped sheet metal and assembled by spot welding or other fastening methods. The construction of the unit body structure, also known as unit-frame-and-body or frame-less body, is claimed to enhance whole vehicle rigidity and provide for weight reduction. [1]

There are three categories of tests: component tests, sled tests, and full-scale barrier impacts. The complexity of the test and associated variables increase from component to full-scale tests. The typical full-scale barrier test involves collision of a guided vehicle, propelled into a barrier at a predetermined initial velocity and angle. Typically, a barrier test uses a complete vehicle. To evaluate individual substructures, a sled test can be equally effective, especially in evaluation of the restraint systems. Safety engineers run this barrier test to ensure vehicle structural integrity. [1]

The tools used to simulate the vehicle impact and predict its response in both processes are extreme in two areas. FE analysis requires a complete and detailed description of the components' geometry and associated material properties. This information is readily available after "freezing" the design status, and solutions for potential problems are limited to minor modifications. By contrast, LMS analysis uses simple simulation models that synthesize the design and determine the functions of the structural parts through an experimental procedure. Furthermore, the quality of the LMS model data and the simulation results are highly dependent on the experience of the crash engineer. [2]

K-S testing is a non-parametric and distribution-free test: It makes no assumption about the distribution of data. The KS test can be used to compare a sample with a reference probability distribution, or to compare two samples. A chi-square test is a statistical test used to compare observed results with expected results. The purpose of this test is to determine if a difference between observed data and expected data is due to chance, or if it is due to a relationship between the variables you are studying. Therefore, a chi-square test is an excellent choice to help us better understand and interpret the relationship between our two categorical variables. These both tests are used to validate input parameters for suitable distributions to be used in uncertainty quantification analysis. [3]

In the reliability analysis of complex engineering systems, the limit state function (LSF) is generally implicit and very often, highly nonlinear. Furthermore, each function evaluation is usually

computationally expensive. This is the case of virtual crash simulation. It is therefore extremely important to choose a reliability analysis method that minimizes the number of function evaluations needed for an estimation of the failure probability P_f , which is of an acceptable accuracy. [3]

IV. PROBLEM STATEMENT

Complete validation of the FE based static structural and crash analysis of model is tedious and extremely difficult because of the complexity of the crash responses and the limited number of measurements in the tests. This also adds to the material behavior of model with respect to different loading conditions. Thus, correct simulation of the crash responses requires modeling responses of both the FE based model and Probabilistic approach to validate the probability of failure.

V. PROPOSED MODEL

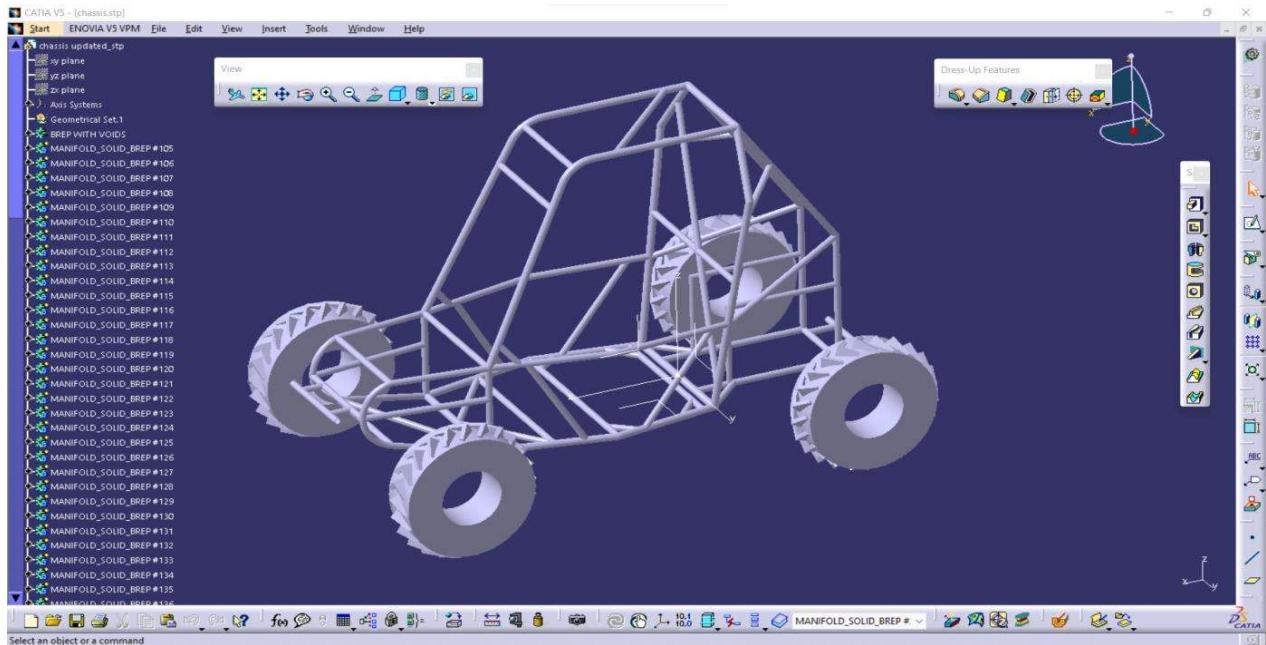


Fig 1- Full Roll-cage Architecture
(BAJA SAE India-Team Gear Falcons)

The following closed model consists of assembly of different parts such as Lower Floor, Upper Roll-cage and Wheels. First geometry cleanup was performed on following model where free edges, overlapping surfaces and contact optimization was performed. A closed geometry setup was performed for 14 individual components using Catia V5 as shown in figure and BIW assembly is prepared for further analysis.

A. Full Model for Roll-cage for Static Structural Simulation-

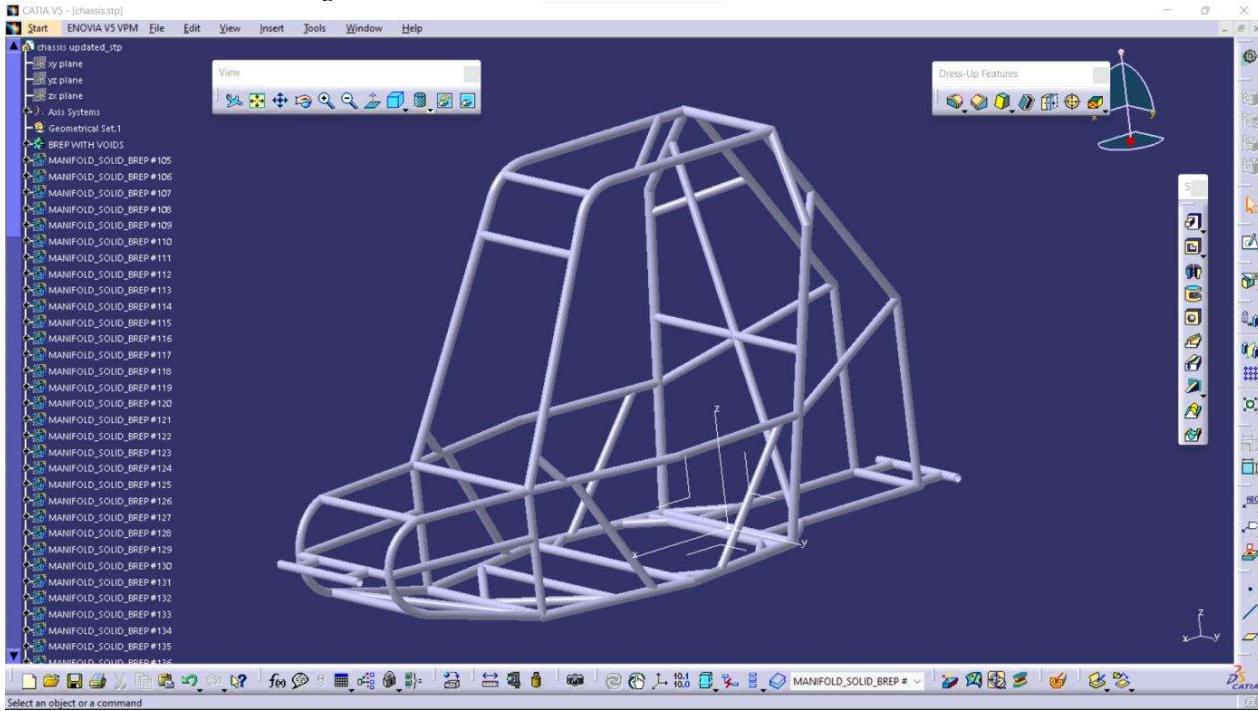


Fig 1.1- Full Vehicle Roll-Cage Architecture

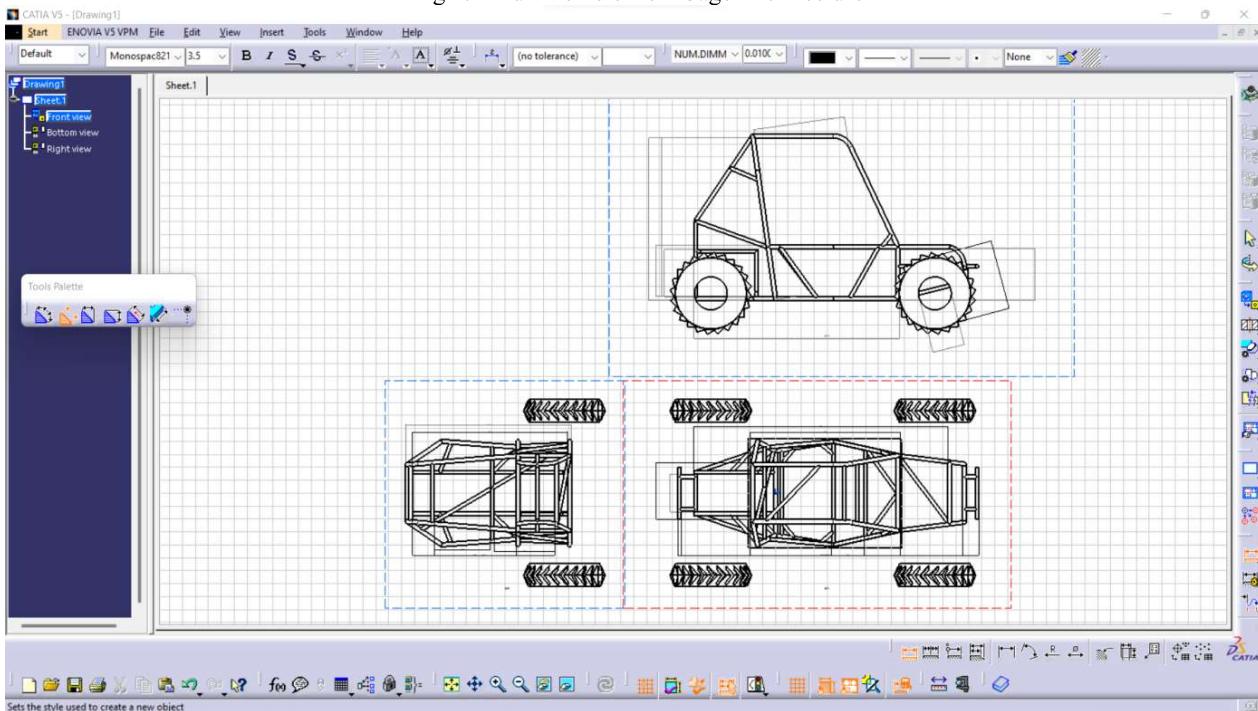


Fig 1.2- Dimensional Draft of Full Vehicle Roll-cage

The Yield stress failure of both materials will be performed based on its upper and lower limits which is obtained from SAE material papers. This will determine the yield stress failure for both HSS and AHSS in plastic behavior which will be used for further analysis.

B. Proposed Model for Explicit Dynamic Simulation-

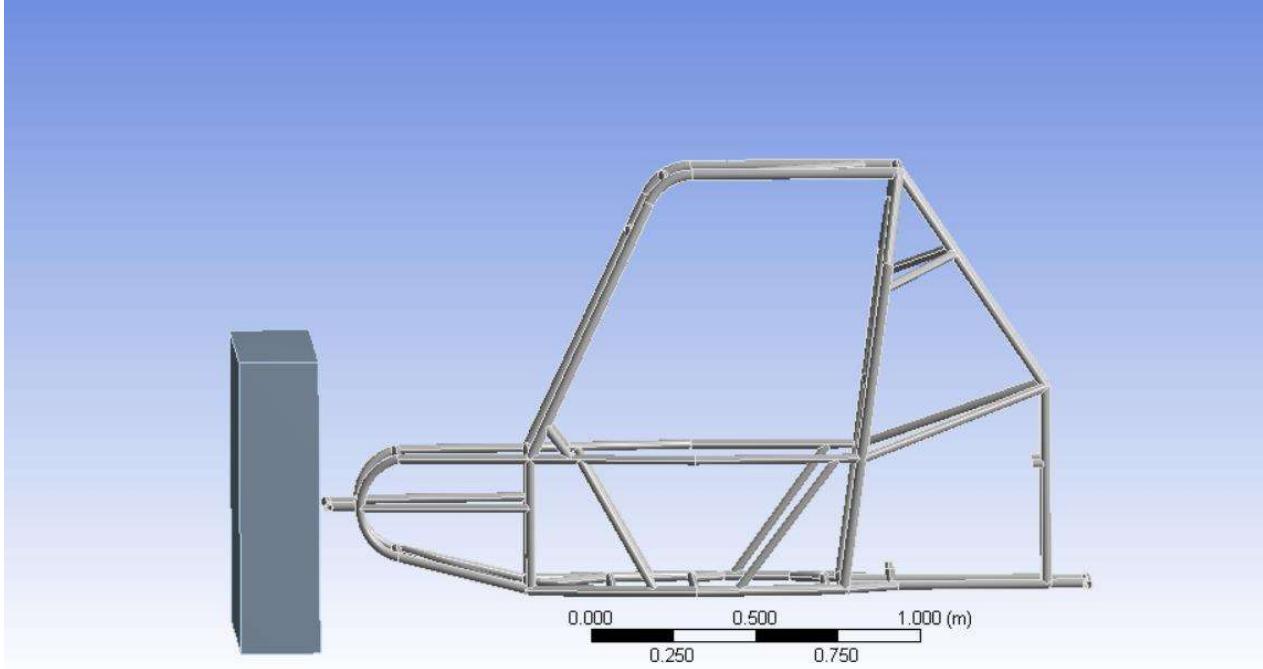


Fig 1.3- Side view of Explicit Dynamic Analysis

Figuratively the first point of impact during the entire simulation will be front tube of roll cage. Total surface deformation and damage will be analyzed during crash simulation. The damage sustained by the A-pillar and Hinge pillar will be key components to the probability of failure of model. This will depend on contacts where the impact loads are going to be applied with respect to speed of the vehicle. As the linear momentum of impact will be seen on the lower floor, the plastic deformation will occur on the front and the rear end will be impacted by resulting vibrational, von mises and thermal stress. Here lower C and B pillar will be considered constant as they will increase complex mathematical calculations.

VI. HIGH STRENGTH STEEL VS ADVANCE HIGH STRENGTH STEEL

High Strength Steel a type of steel that has a yield strength that ranges between 30-80 ksi (210-750 MPa) and a tensile strength between 40-100 ksi (270 to 700 MPa). Steels with yield levels higher than 80 ksi (590 MPa) are considered Advanced High Strength Steels, and when the tensile levels exceed 113 ksi (780 MPa), they are referred to as Ultra High Strength Steels.

The principal difference between conventional High Strength Steel (HSS) (such as 4130 Chrome Moly) and Advanced High Strength Steel (AHSS) (such as Docol® Tube R8) is their microstructure. Conventional HSS are single-phase ferritic steels with a potential for some pearlite in C-Mn steels. AHSS are primarily steels with a microstructure containing a phase other than ferrite, pearlite, or cementite – for example, martensite, bainite, austenite, and/or retained austenite in quantities sufficient to produce unique mechanical properties. Some types of AHSS have a higher strain hardening capacity resulting in a strength-ductility balance superior to conventional steels. Other types have ultra-high yield and tensile strengths and show a bake-hardening behavior.

Advanced High-Strength Steels are complex, sophisticated materials, with carefully selected chemical compositions and multiphase microstructures resulting from precisely controlled heating and cooling processes. Various strengthening mechanisms are employed to achieve a range of strength, ductility, toughness, and fatigue properties. These steels are not the mild steels of yesterday; instead, they are uniquely engineered to meet the challenges.

The AHSS family includes Dual Phase (DP), Complex-Phase (CP), Ferritic-Bainitic (FB), Martensitic (MS or MART), Transformation-Induced Plasticity (TRIP), Hot-Formed (HF), and Twinning-Induced

Plasticity (TWIP). These 1st and 2nd Generation AHSS grades are uniquely qualified to meet the functional performance demands of individual parts. For example, DP and TRIP steels are excellent in the crash zones of the car for their high energy absorption.

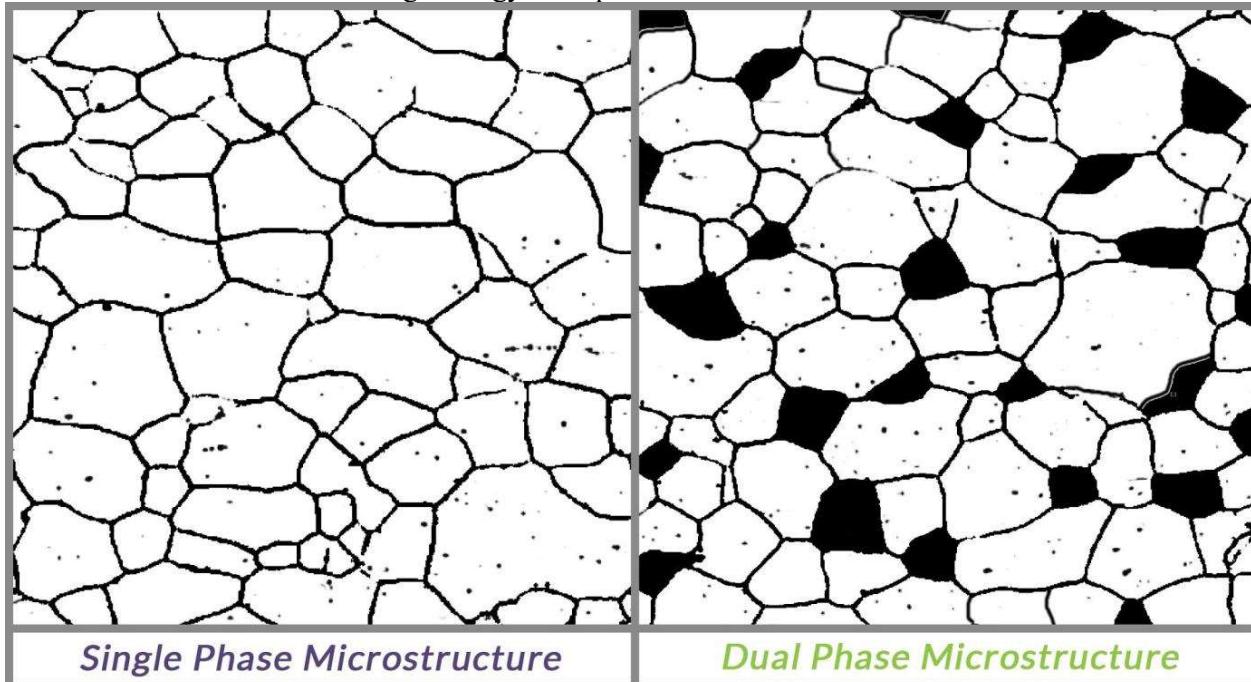


Fig 2- Difference Between microstructure of HSS and AHSS

DP steels, like Docol Tube R8, offer an incredibly advantageous combination of low yield, high-tensile strength, easy cold working, and weldability due to their ferrite-martensite imbued lattice microstructure. The Ferrite phase of DP steels gives it its excellent formability while the hard Martensite islands give it a very high initial work hardening rate.

VII. MATERIAL PROPERTIES

Table- I: Material- SAE 4130 Carbon Chrome Molly.

Property	Value
Density	7850 kg m⁻³
Coefficient of Thermal Expansion	1.2e-005 C ⁻¹
Specific Heat	434 J kg ⁻¹ C ⁻¹
Thermal Conductivity	60.5 W m ⁻¹ C ⁻¹
Resistivity	1.7e-007 ohm m
Yield Strength MPa	435-979
Tensile Strength Pa	670-1404
Elongation in 50 mm	18.1-25.5
Reference Temperature C	22
Strength Coefficient Pa	9.2e+008
Ductility Coefficient	0.213
Hardness Pa	197-372
Cyclic Strain Hardening Exponent	0.2
Young's Modulus Pa	2.e+011
Poisson's Ratio	0.3
Bulk Modulus Pa	1.6667e+011
Shear Modulus Pa	7.6923e+010
Relative Permeability	10000

Table- 2: Material- Docol Tube R8.

Property	Value
Density	8653 kg m⁻³
Coefficient of Thermal Expansion	1.2e-005 C ⁻¹

Specific Heat	434 J kg^-1 C^-1
Thermal Conductivity	60.5 W m^-1 C^-1
Resistivity	1.7e-007 ohm m
Yield Strength MPa	1100-1700
Tensile Strength Pa	670-1404
Elongation in 50 mm	18.1-25.5
Reference Temperature C	22
Strength Coefficient Pa	9.2e+008
Ductility Coefficient	0.213
Hardness Pa	197-372
Cyclic Strain Hardening Exponent	0.2
Young's Modulus Pa	2.e+011
Poisson's Ratio	0.3
Bulk Modulus Pa	1.6667e+011
Shear Modulus Pa	7.6923e+010
Relative Permeability	10000

VIII. MESHED FILES

Meshing is the process in which the continuous geometric space of an object is broken down into thousands or more of shapes to properly define the physical shape of the object. The more detailed a mesh is, the more accurate the 3D CAD model will be, allowing for high fidelity simulations. Here both models were meshed for the further simulations as displayed below:

1) Static Structural Analysis-

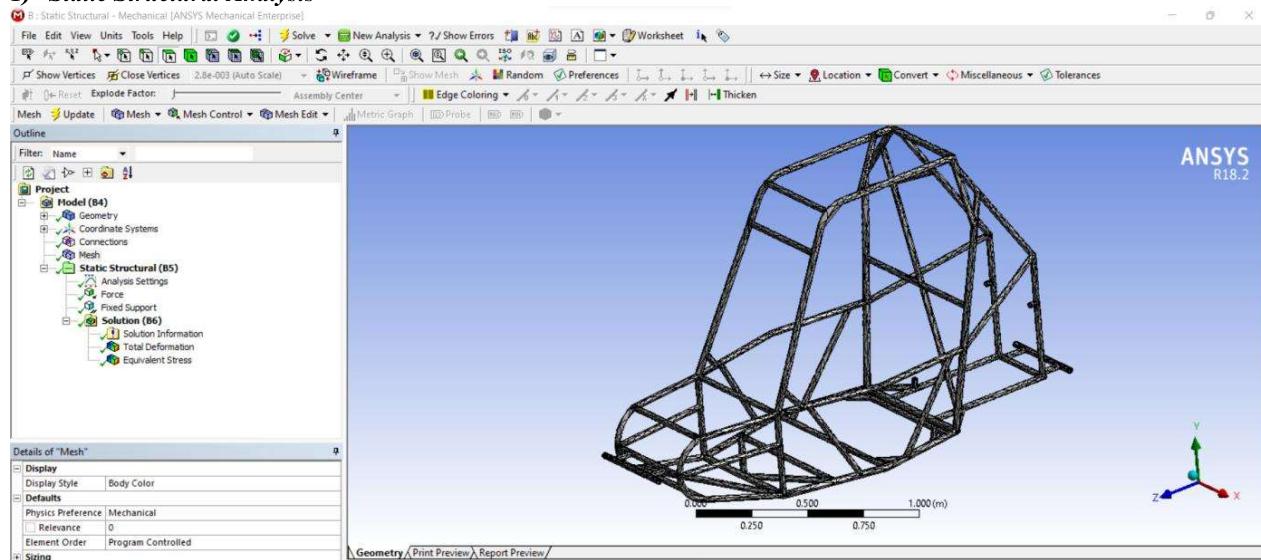


Fig 3.1- Meshed model to perform Static Structural analysis.

No of Nodes- 33363.

No of Elements- 16409.

Mesh was validated checking Jacobian and Aspect ratio.

2) Explicit Dynamic Analysis-

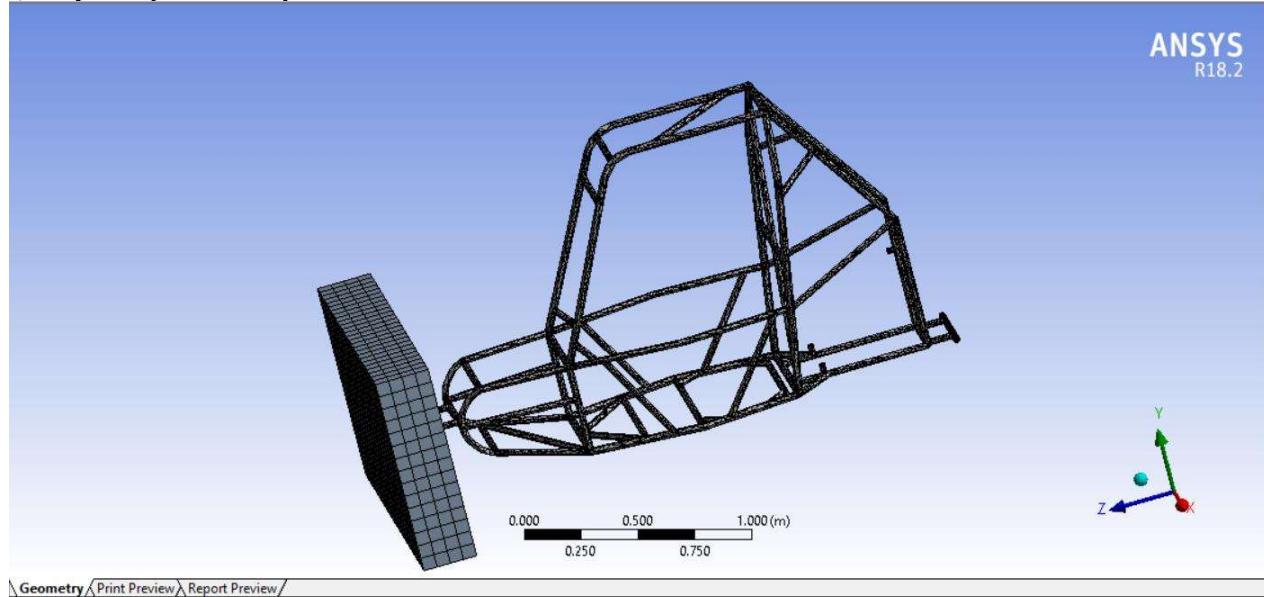


Fig 3.2- Meshed model to perform Explicit Dynamic analysis.

No of Nodes- 13734.

No of Elements- 39764.

Mesh was validated checking Jacobian and Aspect ratio.

IX. FAE RESULTS AND VARIABLE LIMIT VALIDATION

The Finite Element Analysis (FEA) is the simulation of any given physical phenomenon using the numerical technique called Finite Element Method (FEM). Here static structural analysis and Explicit dynamic analysis is used to perform validation on lower and upper limit its of input and output parameters obtained from SAE papers for both materials 4130 chrome molly and Docol R8 steel. Hand calculations are performed for Yield stress and Impact force which is based for uncertainty propagation.

I) Static Structural Analysis-

A static structural analysis calculates the effect of steady (or static) loading conditions on a structure, while ignoring inertia and damping effects, such as those caused by time varying loads.

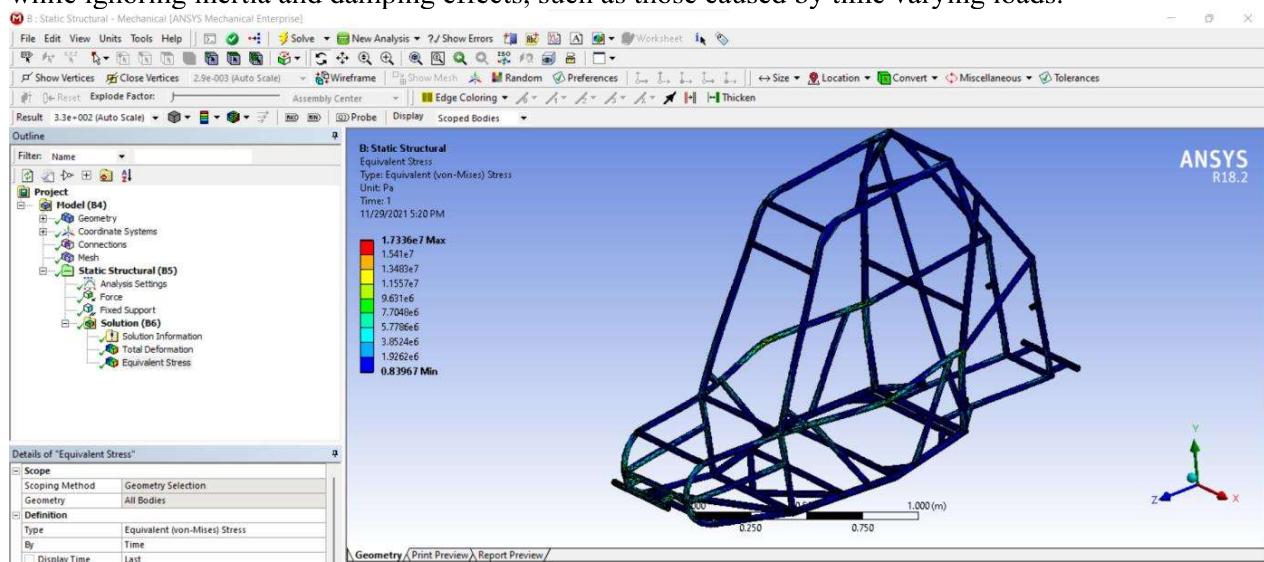


Fig 4.1- Static structural analysis for Yield strength of Docol R8 Steel

Here the yield stress is calculated for Docol R8 steel at plastic conditions. As we can visualize the yield stress concentration is less due to its high tensile strength and high hardness. This system is less plastically deformed compared to 4130 Chrome Molly as represented below.

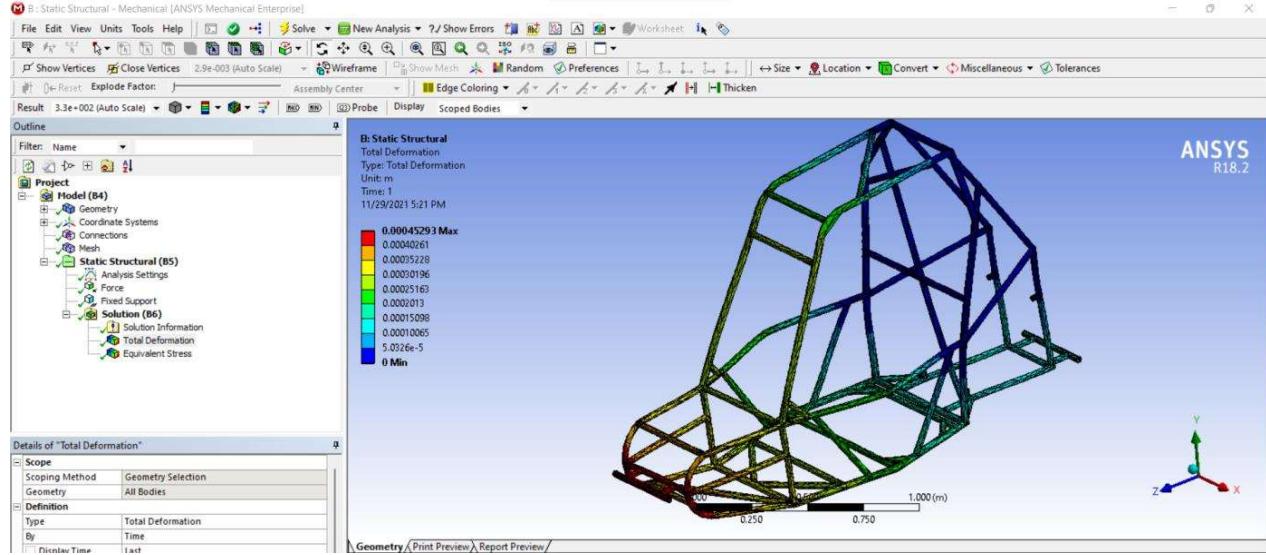


Fig 4.2- Static structural analysis for Yield strength of 4130 Carbon Chrome Moly Steel

Here we can visualize the plastic deformation of material is higher than Docol R8 at similar loading conditions. This further satisfies that probability of failure due to point stress concentration is higher in 4130 Carbon Chrome steel and is represented in table provided below for 20 timesteps. This is further justified by performing uncertainty quantification and propagation and finding the probability of failure and reliability of system.

Table- 3: Max Yield stress for both Materials.

Time	Docol R8 Steel	4130 Chrome Moly Steel
1.	0.83967	1.7336e+007

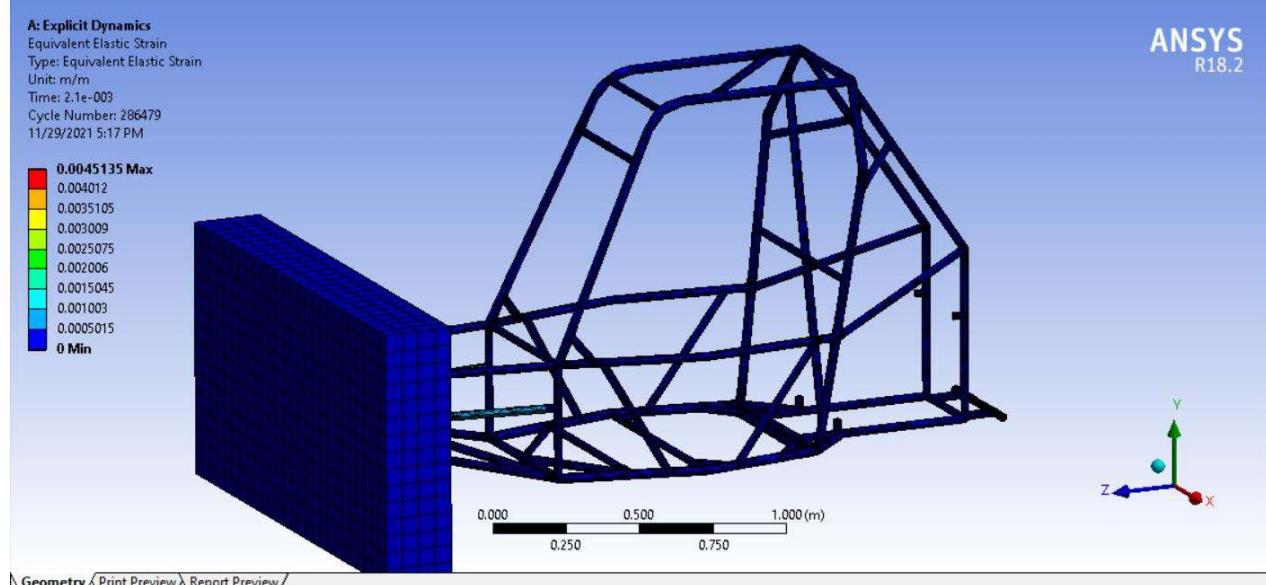


Fig 4.3- Static structural analysis for Yield strain of System

The stress level where the material starts to strain plastically is termed the yield stress, σ_y . When a material is stressed by an amount that is less than the materials yield stress it will only undergo elastic (reversible) strain, and no permanent deformation of the material will occur. The level of stress that corresponds to the yield point is referred to as the yield strength of the material. The yield strength, σ_y , is a material constant. It is often difficult to determine the exact point where yielding begins, so in practice a

proof stress is used. This is where the stress to give a certain amount of strain (for example 0.2%) is used to define the yield strength (and in this case would be called the 0.2% proof stress) which we have considered in our limit state equation.

2) Explicit Dynamic Analysis-

Explicit means that the solution is performed with direct iterative calculations and Implicit algorithms mean that the solution needs to be computed by adding another formulation(s) since some terms cannot be calculated explicitly at the searched step: this is an indirect calculation.

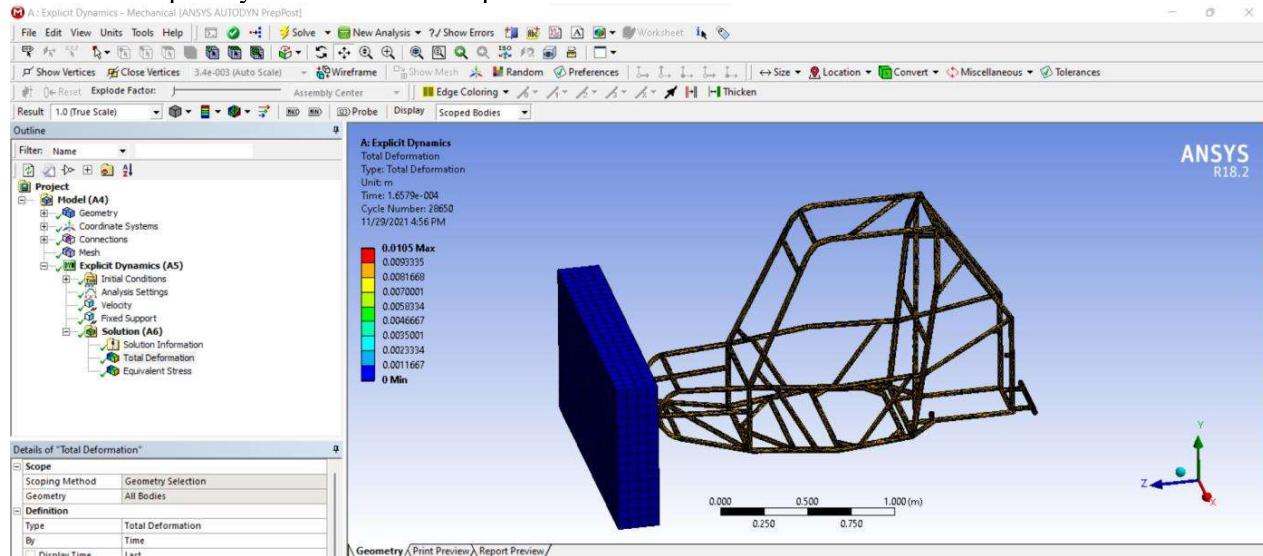


Fig 5.2- Explicit Dynamic analysis for Impact Force of 4130 Chrome Moly Steel

As we can clearly plastic deformation as the impact force is applied. This is due to low hardness and yield strength of material. We can visualize with the FE run obtained below from Docol Steel and there is clearly massive gap in results, this further confirms that Failure will be higher in this system than other system.

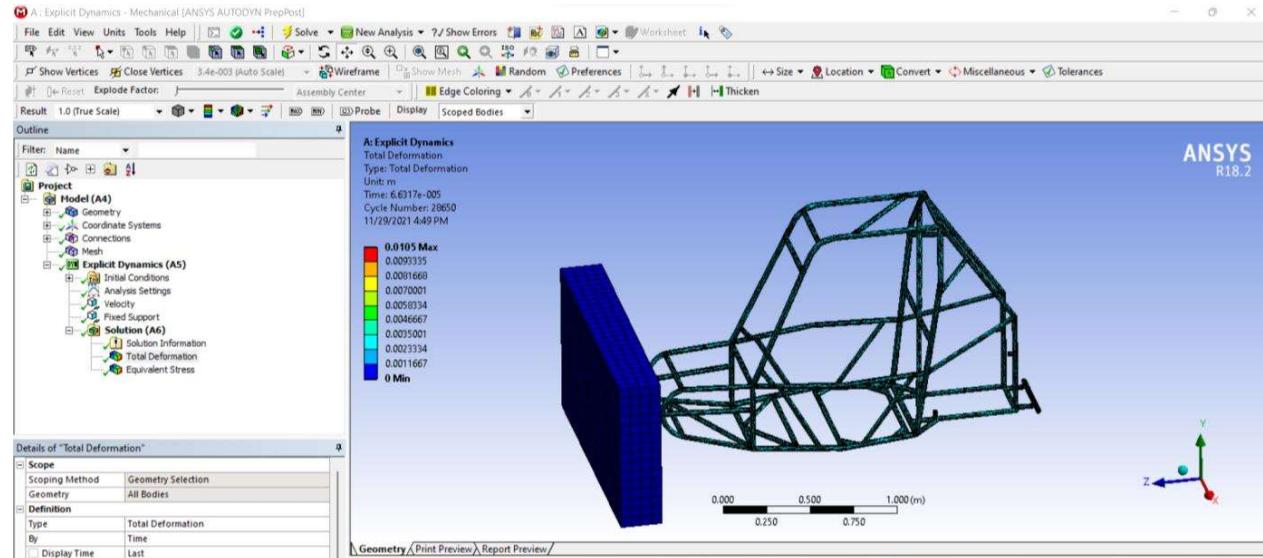


Fig 5.2- Explicit analysis for Impact Force of Docol R8 Steel

Here as we have observed the Impact force/damage lower compared to 4130 Chrome Moly steel. This can be further validated from output table displayed below. The limits used for input parameters will be further used to solve FORM and Monte Carlo method to find probabilistic approach to determine failure of system and check reliability.

Table- 4: Max Impact Force for both Materials.

Time	Maximum- 4130 Chrome Molly	Maximum- Docol r8 Steel
1.1755e-038	0.	0.
1.0506e-005	0.7237 e-003	5.253e-004
2.1003e-005	1.4502e-003	1.0502e-003
3.15e-005	1.678e-003	1.575e-003
4.2005e-005	2.4044e-003	2.1002e-003
5.2502e-005	2.9452e-003	2.6251e-003
6.3007e-005	3.45e-003	3.1503e-003
7.3504e-005	3.834e-003	3.6752e-003
8.4001e-005	4.530e-003	4.2e-003
9.4505e-005	4.9813e-003	4.7253e-003
1.05e-004	5.5591e-003	5.2501e-003
1.1551e-004	5.9354e-003	5.7754e-003
1.26e-004	6.62e-003	6.3002e-003
1.365e-004	6.9351e-003	6.8251e-003
1.4701e-004	7.631e-003	7.3503e-003
1.575e-004	7.98232e-003	7.8752e-003
1.68e-004	8.7632e-003	8.4e-003
1.785e-004	9.3226e-003	8.9252e-003
1.89e-004	9.87501e-003	9.4501e-003
1.9951e-004	10.258e-003	9.9753e-003

Further random variable data samples are generated and both KS and chi-square test are performed to validate goodness to fit of all parameters. Then we will move on to the probabilistic approach to determine if the obtained FEA results and SAE data matches with the probabilistic approach.

X. STATICAL TEST

Even after the plotting distribution of a random variable, a judgment needs to be made as to whether the relationship between the random variable and its CDF is close to linear. A perfect linear relationship is very rarely obtained. More definitive and less cumbersome statistical tests of goodness-of-fit can be conducted to establish the underlying distribution. Two commonly used statistical tests for this purpose are the chi-square and the Kolmogorov-Smirnov (K-S) tests. The chi-square test is based on the error between the observed and assumed PDF of the distribution, and the K-S test is based on the error between the observed and assumed CDF of the distribution.

I) Chi-square test:

In the chi-square goodness-of-fit test, the range of the n observed data is divided into m intervals, and the number of times the random variable is observed in the interval is counted. Observed frequencies of m intervals of the random variable are then compared with the corresponding theoretical frequencies of an assumed distribution. It can be shown that the quantity

$$\sum_{i=1}^m \frac{(n_i - e_i)^2}{e_i}$$

The number of degrees of freedom is a parameter of the chi-square distribution. A significance level alfa is selected. Significance levels between 1% and 10% are common. A significance level of 5% implies that for 5 out of a total of 100 different samples, the assumed theoretical distribution cannot be an acceptable model.

mean value of F

mu_F_1b = 227;

standard deviation of F

sigma_F_1b = 22.7;

generating 100 lognormal random variables of F

```
x_F_1b = lognrnd(mu_F_1b, sigma_F_1b, 1, 100);
```

arranging in increasing order

```
x_F_1b = sort(x_F_1b);
```

Chi square test

```
[h,p] = chi2gof(x_F_1b)
```

$h = 0$

$p = 0.3775$

The returned value $h = 1$ indicates that chi2gof rejects the null hypothesis at the default 5% significance level.

2) K-S Testing:

The K-S test compares the observed cumulative frequency and the CDF of an assumed theoretical distribution. The first step is to arrange the data in increasing order. Then the maximum difference between the two cumulative distribution functions of the ordered data can be estimated as

$$D_n = \max |F_X(x_i) - S_n(x_i)|$$

Mathematically, D_n is a random variable and its distribution depends on the sample size n . The CDF of D_n can be related to the significance level α as

$$P(D_n \leq D_n^\alpha) = 1 - \alpha$$

and the D_n, α values at various significance levels α can be obtained from a standard mathematical table as shown in Appendix 4. Then, according to the K-S test, if the maximum difference D_n is less than or equal to the tabulated value D_n, α , the assumed distribution is acceptable at the significance level α . The advantage of the K-S test over the chi-square test is that it is not necessary to divide the data into intervals; thus the error or judgment associated with the number or size of the interval is avoided.

K-S test

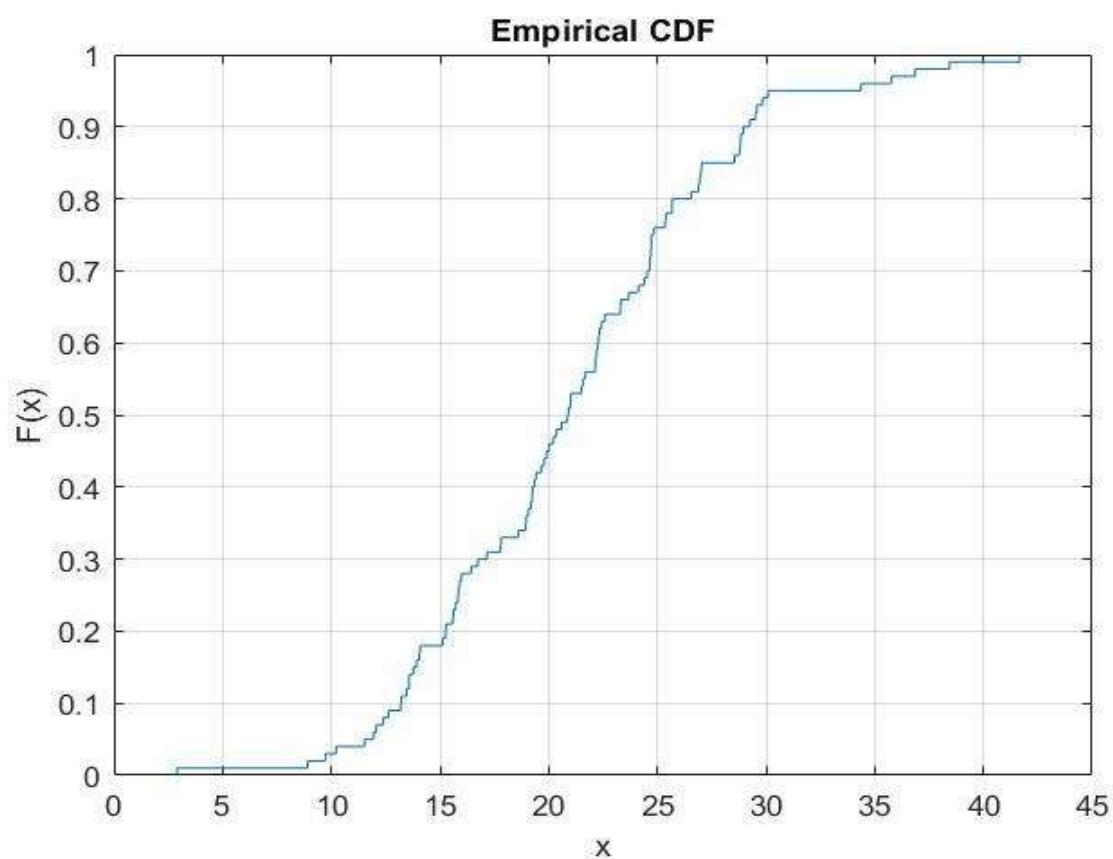
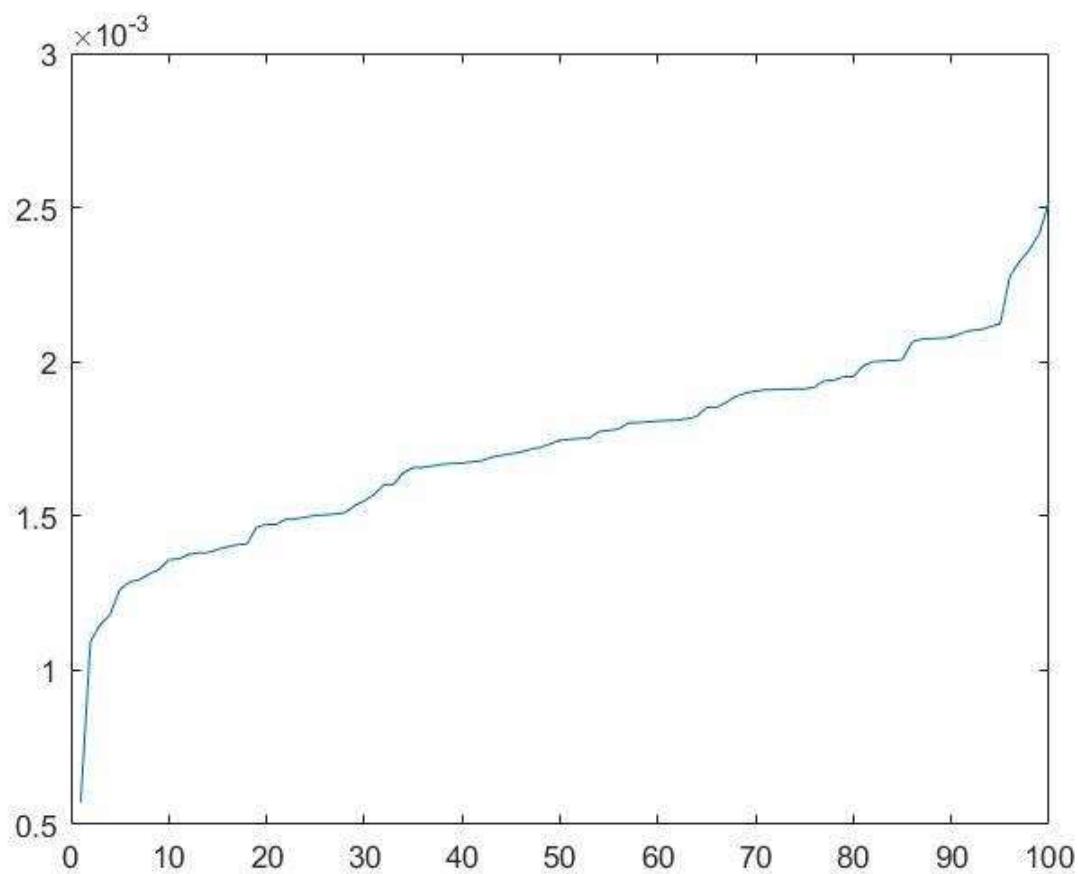
```
[h,p] = kstest(x_F_1b)
```

$h = \text{logical}$

1

$p = 2.4525e-89$

The returned value of $h = 1$ indicates that k-s test rejects the null hypothesis at the default 5% significance level.



XI. QUANTIFICATION OF UNCERTAINTIES IN RANDOM VARIABLE

Reliability estimation is the quantification of uncertainties in all the RVs present in the formulation. The quantification of uncertainty in an RV requires the collection of data on it from as many sources as possible. In most cases, a PDF can be generated by fitting a polynomial whose parameters can be estimated from the samples used to generate it. In most engineering applications, two-parameter PDFs are routinely used. It can be shown that these parameters can be estimated from the first two moments, the mean and variance, of the samples. A normal distribution will be valid when an RV is valid from minus infinity to plus infinity. A lognormal distribution is used when an RV is valid from zero to plus infinity.

1) Normal or Gaussian distribution:

One of the most commonly used distributions in engineering problems is the normal or Gaussian distribution. The PDF of the distribution can be expressed as

$$f_X(x) = \frac{1}{\sigma_X \sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x - \mu_X}{\sigma_X}\right)^2\right], \quad -\infty < x < +\infty$$

where the mean $\mu(X)$ and the standard deviation $\sigma(X)$ are the two parameters of the distribution, estimated from the available data. The corresponding CDF can be expressed as

$$F_X(x) = \int_{-\infty}^x \frac{1}{\sigma_X \sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x - \mu_X}{\sigma_X}\right)^2\right] dx.$$

2) Lognormal Distribution:

In many engineering problems, a random variable cannot have negative values due to the physical aspects of the problem. In this situation, modeling the variable as lognormal (i.e., considering the natural logarithm of the variable X) is more appropriate, automatically eliminating the possibility of negative values. If a random variable has a lognormal distribution, then its natural logarithm has a normal distribution. This is the meaning of the term lognormal. The PDF of a lognormal variable is given by

$$f_X(x) = \frac{1}{\sqrt{2\pi}\zeta_X x} \exp\left[-\frac{1}{2}\left(\frac{\ln x - \lambda_X}{\zeta_X}\right)^2\right], \quad 0 \leq x < \infty$$

where $\lambda(X)$ and $\zeta(X)$ are the two parameters of the lognormal distribution. The two parameters of the lognormal distribution can be calculated from the information on the two parameters of the normal distribution, the mean and standard deviation of the sample population. It can be shown that

$$\begin{aligned} \lambda_X &= E(\ln x) = \ln \mu_X - \frac{1}{2} \zeta_X^2 \\ \zeta_X^2 &= \text{Var}(\ln X) = \ln \left[1 + \left(\frac{\sigma_X}{\mu_X} \right)^2 \right] = \ln(1 + \delta_X^2). \end{aligned}$$

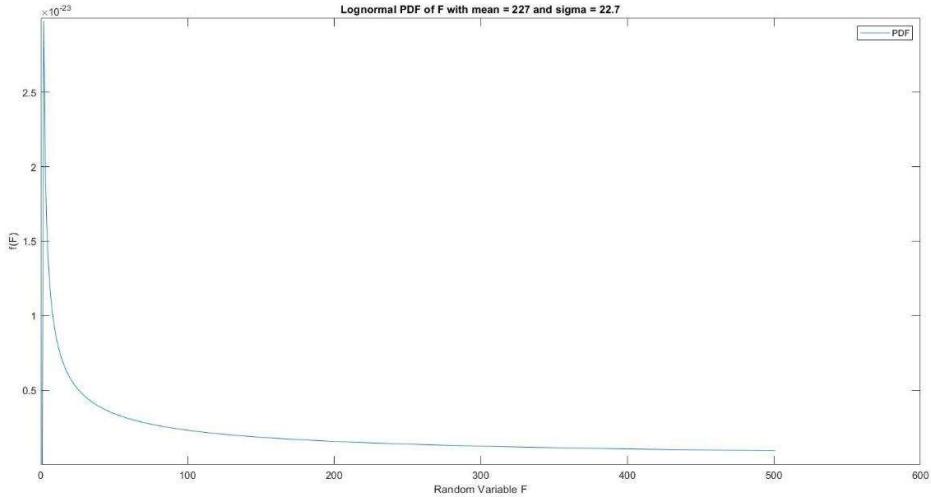
The PDF and CDF of lognormal and normal distribution have been plotted as following:

lognormal pdf of F

```
pdf_F_1b = lognpdf(x_F_1b,mu_F_1b,sigma_F_1b);
```

plot pdf

```
plot(pdf_F_1b)
```

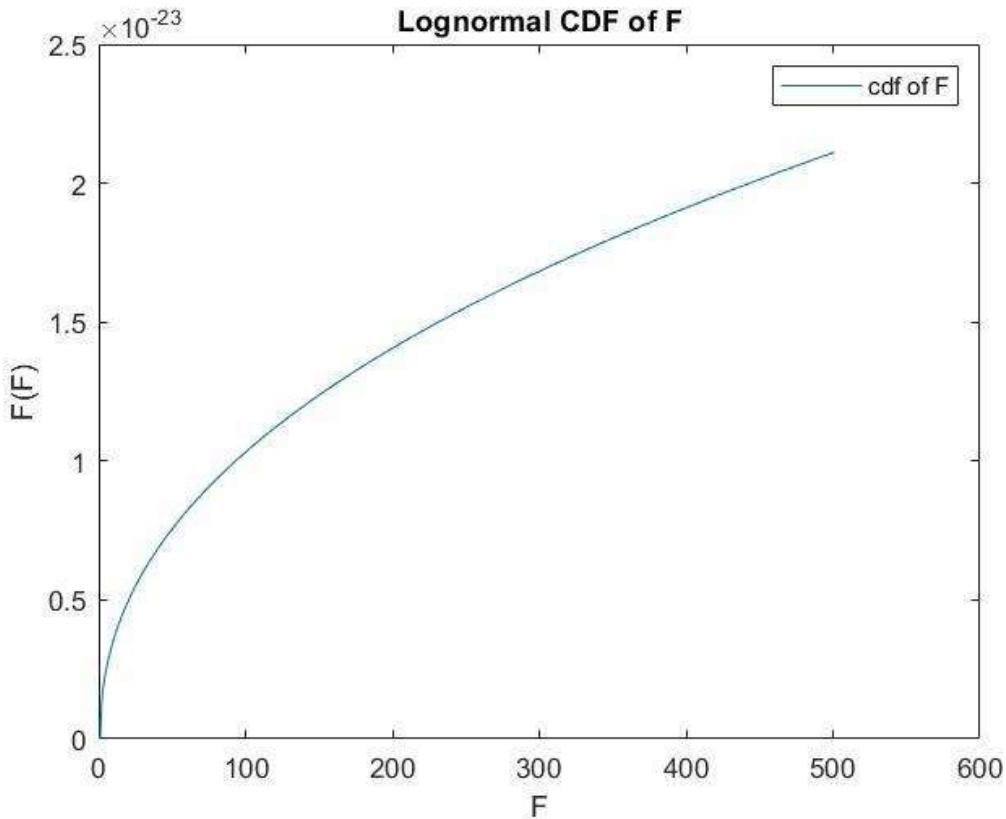


cdf of F

```
cdf_F_1b = logncdf(x_F_1b,mu_F_1b,sigma_F_1b);
```

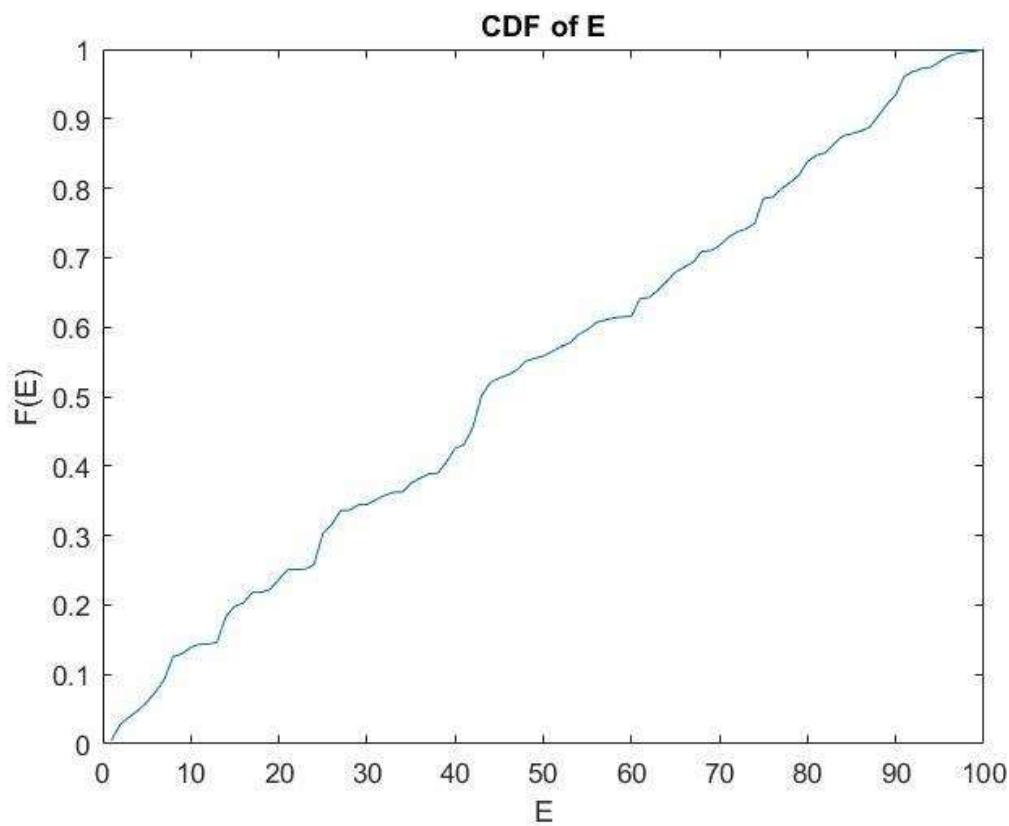
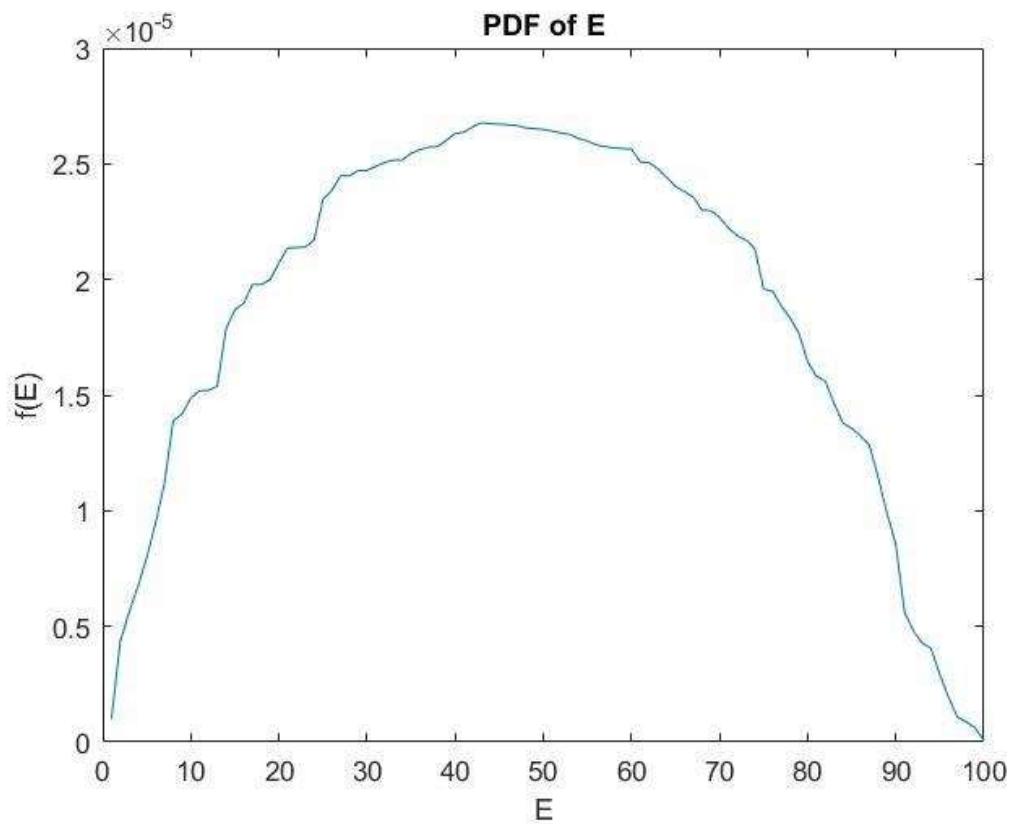
plot cdf

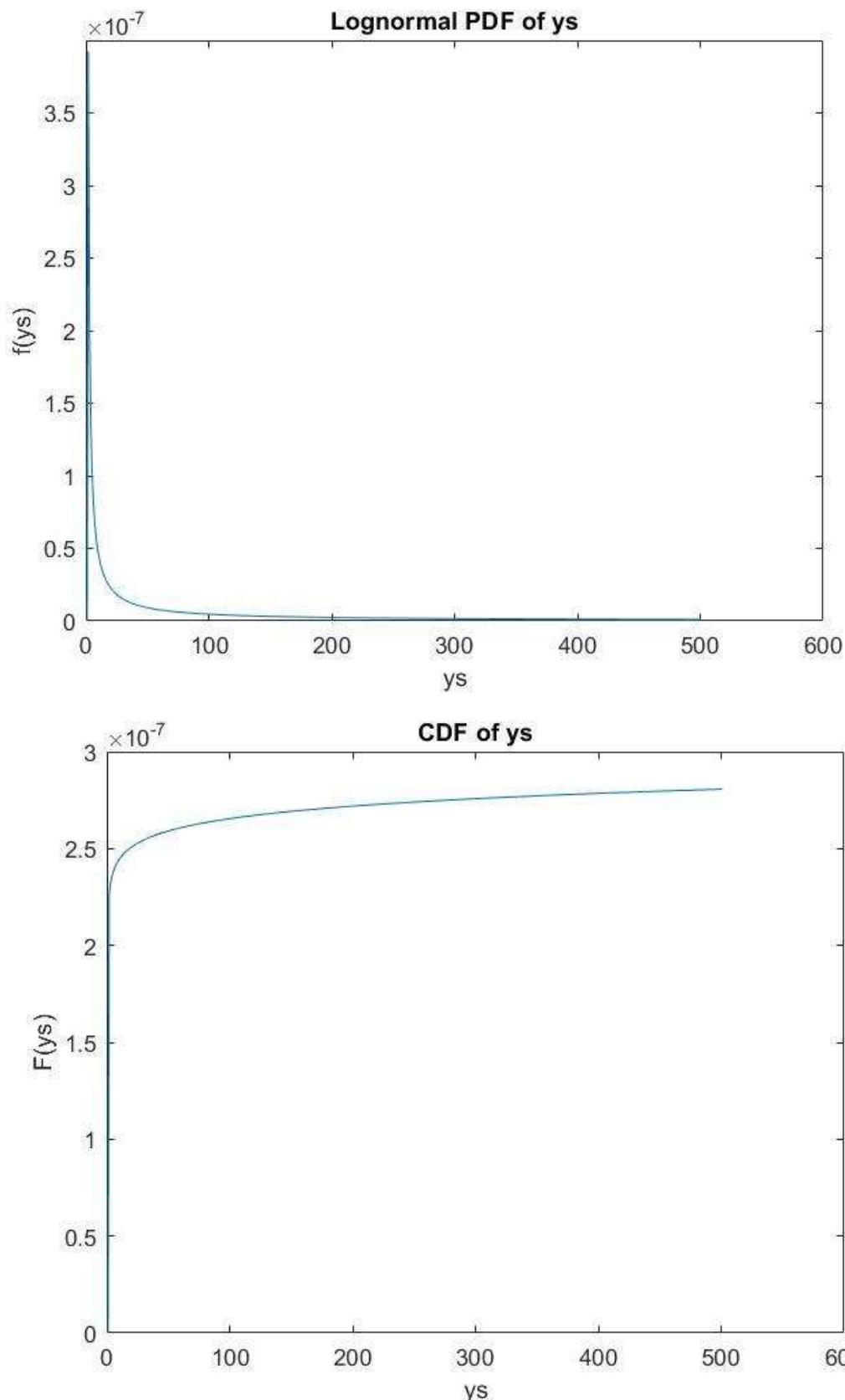
```
plot(cdf_F_1b)
```



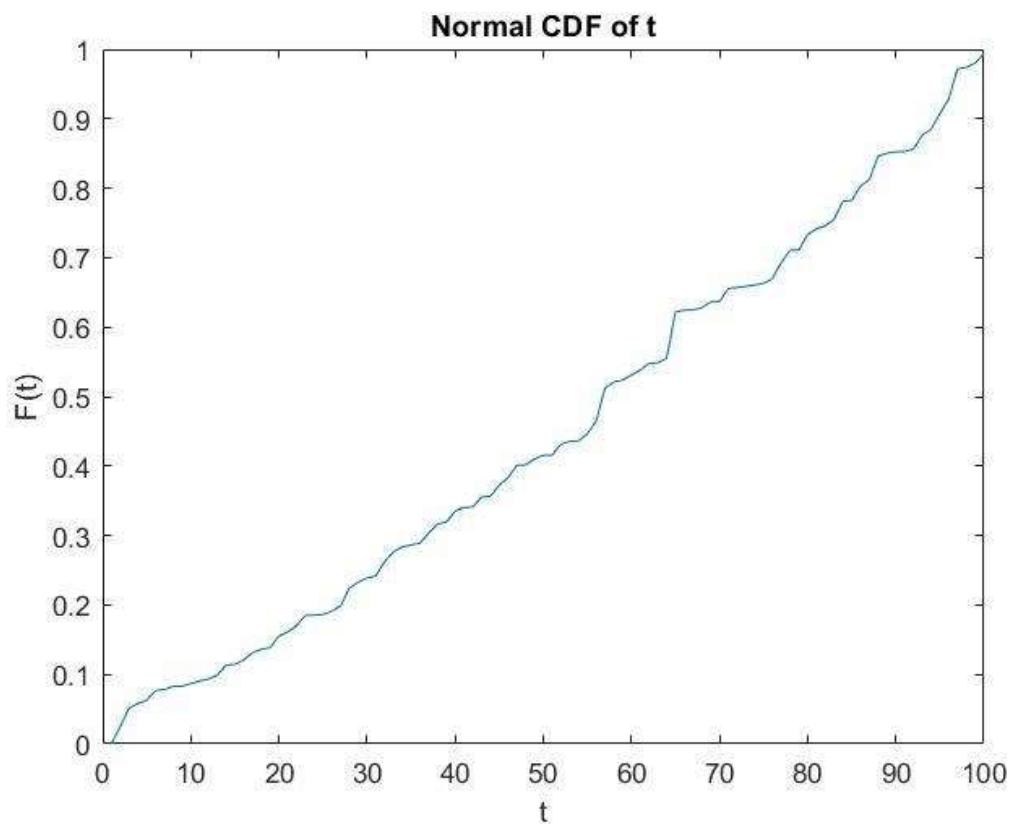
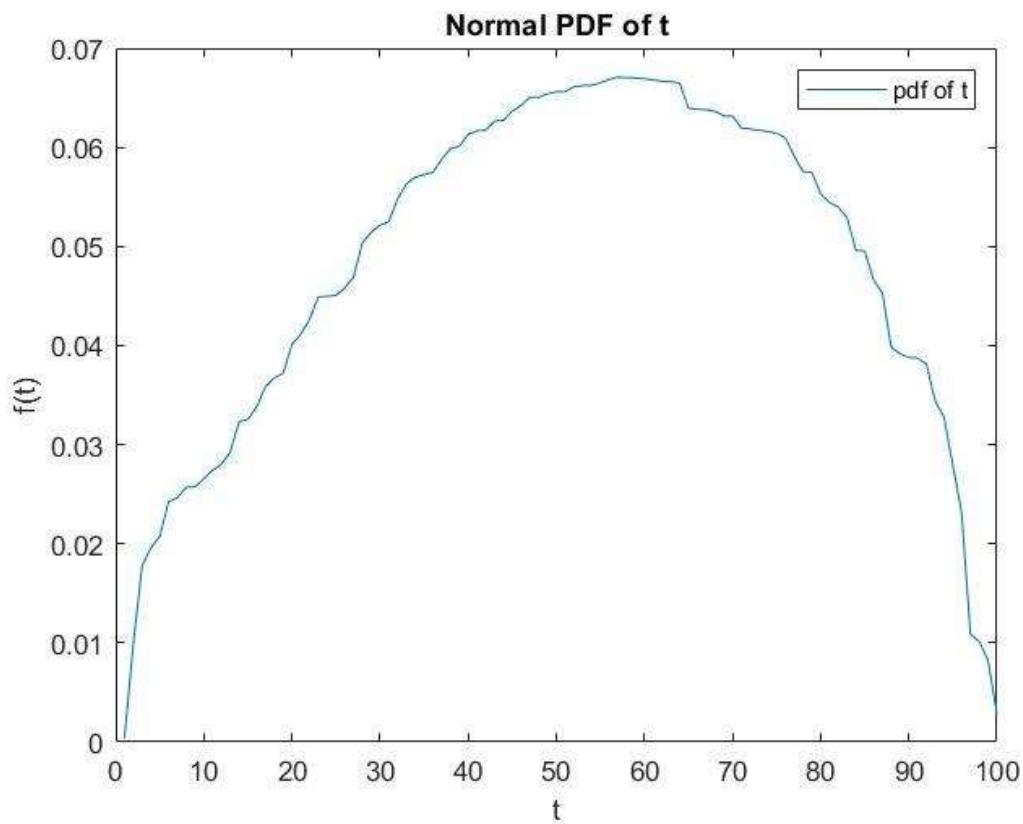
similarly, we have plot the pdf and cdf of all the variables.

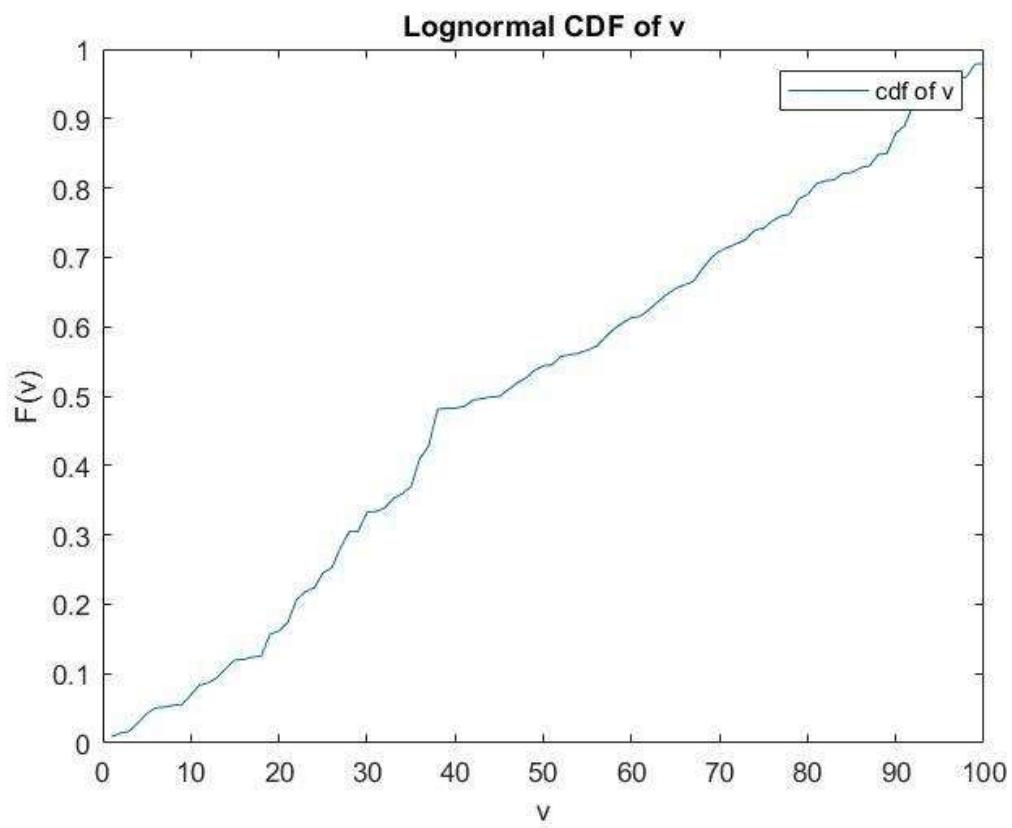
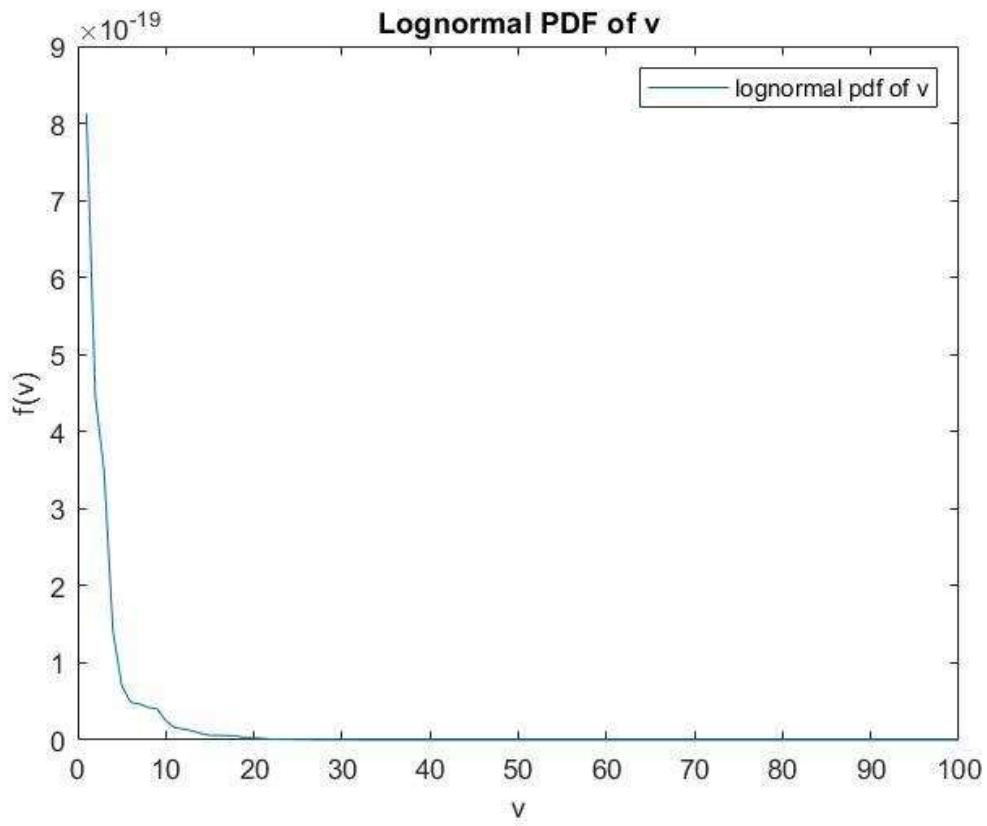
PDF and CDF for variables of Static structural Analysis are as follows:





PDF and CDF for variables of Explicit Dynamic Analysis are as follows:





XII. LIMIT STATE EQUATIONS

The limit state function for estimating the failure probability may be defined as:

$$g(X) = R - L$$

where R is the platform capacity in terms of maximum lateral load that the platform can withstand before system failure or collapse.

L is the total environmental load, which consists of wave and current load, W_v and wind load, W_l , i.e.,
 $g(X) = R - W_v - W_l$

The limit state function $g(X)$ defines a failure criterion that is a function of all random variables X. Failure occurs when the load L exceeds the capacity R or when $g(X) < 0$.

A) Static Structural Analysis-

To manage problems related to yield stress, engineers and scientists rely on a variety of formulas dealing with the mechanical behavior of materials. Ultimate stress, whether it is tension, compression, shearing or bending, is the highest amount of stress a material can withstand. Yield stress is the stress value at which plastic deformation occurs. An accurate value for yield stress can be difficult to pinpoint.

Young's Modulus- It is the slope of the elastic portion of the stress-strain curve for the material being analyzed. Engineers develop stress-strain curves by performing repeated tests on material samples and compiling the data. Calculating Young's Modulus (E) is as simple as reading a stress and strain value from a graph and dividing the stress by the strain.

Stress Equation- Stress (σ) is related to strain (ϵ) through the equation:

$$\sigma = E \times \epsilon$$

The 0.2 Percent Offset Rule- The most common engineering approximation for yield stress is the 0.2 percent offset rule. To apply this rule, assume that yield strain is 0.2 percent, and multiply by Young's Modulus for your material:

$$\sigma = 0.002 \times E$$

Sometimes it is called this the "offset yield stress."

$$G(\sigma) = \sigma - 0.002 \times E$$

B) Explicit Dynamic Analysis-

Force of impact is the total force exerted on an object during a collision. To derive the impact force equation, you can consider the law of conservation of energy. At the beginning, a moving object possesses kinetic energy that reduces to zero after the collision (object stops). To fulfill the conservation law, the change of kinetic energy must be compensated by the work done by the impact force. We express it with the below impact force equation:

$$F = m * v^2 / (2 * d),$$

Where,

F is the average impact force,

m is the mass of an object,

v is the initial speed of an object,

d is the distance traveled during collision.

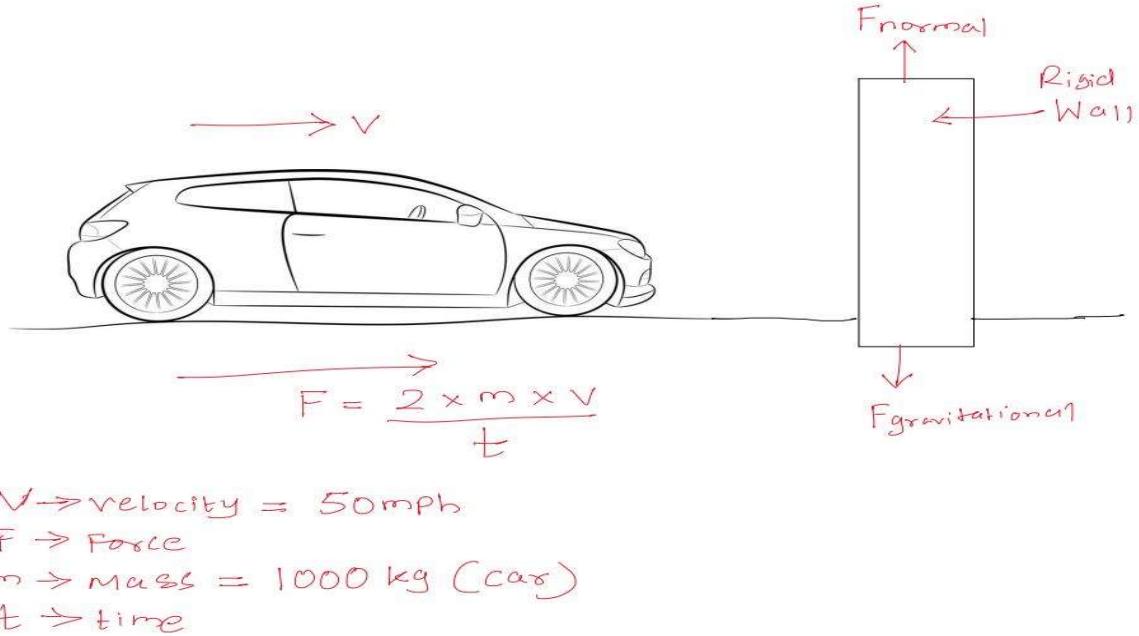
Here extending the distance moved during the collision reduces the average impact force. It should be easier to understand if we rewrite the above impact force formula in the alternative version using the time of collision t instead of the distance d:

$$F = v * m / t$$

This is a special case of Damage/Total deformation of Body.

$$G(t) = F - v * m / t$$

c) Full System impact equation-



XIII. FIRST-ORDER RELIABILITY METHOD(FORM)

The first-order reliability method (FORM) is commonly utilized in structural reliability estimate applications. The approach entails the Taylor expansion of the failure function, i.e., the linearization of the limit state equation. FORM-based designs are often accomplished using commercial software tools, and we employed EXCEL for this purpose. FORM is also known as the Mean Value First Order Second Moment Method (MVFOSM).

I) Results for FORM on Static Structural Analysis-

FORM 1- 4130 Carbon Moly Steel-

Input variables for limit state equation- $g = (\sigma - 0.002 * E)$

Initial value	Type	u	sigma	lamda	kesi
σ	Lognormal	757.5	150.9166	6.610561026	0.038925022
E	Normal	200060	14899.4		

Output Parameters-

Beta Difference- -0.113584244

Beta - 0.036466312

Probability of Failure- 0.48545527

FORM 1- Docot R8 Steel-

Input variables for limit state equation- $g = (\sigma - 0.002 * E)$

Initial value	Type	u	sigma	lamda	kesi
σ	Lognormal	1100	110	6.998090293	0.009950331
E	Normal	210060	21006		

Output Parameters-

Beta Difference- -1.012209572

Beta - 0.139196749

Probability of Failure- 0.444647339

As we can see the **Probability of Failure Is Low** in Docol R8 Steel than 4130 Carbon Moly Steel. Thus, this further verifies the FEA analysis we performed in the initial stage.

2) Results for FORM on Explicit Dynamic Analysis-

FORM 2- 4130 Carbon Moly Steel-

Input variables for limit state equation- $g = (F-v*25*t^(-1))$

Note-Here mass is calculated and kept constant as its standard deviation value was close to zero.

Initial value	Type	u	sigma	lamda	kesi
F	Lognormal	227	22.7	5.419974852	0.009950331
v	Lognormal	50.09	5.896575	3.906940006	0.013762763
t	Normal	20.4	5.944185		

Output Parameters-

Beta Difference- -0.315857366

Beta – 5.141318479

Probability of Failure- 1.36409E-07

FORM 2- Docol R8 Steel-

Input variables for limit state equation- $g = (F-v*23*t^(-1))$

Note-Here mass is calculated and kept constant as its standard deviation value was close to zero.

Initial value	Type	u	sigma	lamda	kesi
F	Lognormal	227	22.7	5.419974852	0.009950331
v	Lognormal	50.09	5.896575	3.906940006	0.013762763
t	Normal	20.4	5.944185		

Output Parameters-

Beta Difference- 5.489966895

Beta – 5.489966895

Probability of Failure- 2.01005E-08

As we can see the **Probability of Failure Is Low** in Docol R8 Steel than 4130 Carbon Moly Steel. Thus, this further verifies the FEA analysis we performed in the initial stage.

Note- All FORM methods were performed in Excel sheet and are attached at the end of report.

XIV. MONTE CARLO METHOD

The technique was first used by scientists working on the atom bomb; it was named for Monte Carlo, the Monaco resort town renowned for its casinos. Since its introduction in World War II, Monte Carlo simulation has been used to model a variety of physical and conceptual systems.

Monte Carlo simulation is a computer-assisted mathematical approach for incorporating risk into quantitative analysis and decision-making. Professionals in a variety of sectors, including finance, project management, energy, manufacturing, engineering, research and development, insurance, oil and gas, transportation, and the environment, employ the approach.

For each given course of action, Monte Carlo simulation provides the decision-maker with a variety of probable outcomes as well as the probability that they will occur. It depicts the extreme outcomes—the implications of going for broke and the most cautious decision—as well as all conceivable outcomes for middle-of-the-road selections.

For our project we have used Monte Carlo using MATLAB.

1) Results for Monte Carlo on Static Structural Analysis-

Number	Material	Probability of Failure
1	4130 Carbon Moly Steel	0.48346211
2	Docol R8 Steel	0.44324519

2) Results for Monte Carlo on Explicit Dynamic Analysis-

Number	Material	Probability of Failure
1	4130 Carbon Moly Steel	1.36247E-07
2	Docol R8 Steel	2.00875E-08

As we can see the **Probability of Failure Is Low** in Docol R8 Steel than 4130 Carbon Moly Steel. This also matches with the FORM results that we performed above. Thus, this further validates the FEA analysis we performed in the initial stage.

Note- All Monte Carlo methods were performed in MATLAB and code is attached at the end of report.

3) Results for Monte Carlo on Full System-

Name	Probability of Failure
Full System	0.0462

Note- All Monte Carlo methods were performed in MATLAB and code is attached at the end of report.

XV. SENSITIVITY

The change in model output values resulting from even minute changes in model input values is given by sensitivity analysis. It is the investigation of the importance of imprecision or uncertainty in model inputs in a modelling process. It provides a general assessment of model precision when used to assess system performance for alternative scenarios, as well as detailed information addressing the relative significance of errors in various parameters.

1) Results for Sensitivity on Form of Static Structural Analysis-

FORM 1- 4130 Carbon Moly Steel-

Initial Value	Value
S	0.022
E	-0.002

FORM 1- Docol R8 Steel-

Initial Value	Value
S	0.026
E	-0.032

The above values make it clear that the uncertainty in sigma is highly connected with the uncertainties in young's modulus of the system. This means that if the young's modulus is uniform, then the reliability of the sigma increases enormously.

2) Results for Sensitivity on Form of Explicit Dynamic Analysis-

FORM 2- 4130 Carbon Moly Steel-

The sensitivity analysis was conducted in the FORM test and gave the following values: -

Initial Value	Values
F	0.0033
V	0.0212
t	0.0084

FORM 2- Docol R8 Steel-

Initial Value	Values
F	0.0034
V	0.17681
t	0.00573

The above values make it clear that the uncertainty in velocity is highly connected with the uncertainties in time of the system. This means that if the time is uniform, then the reliability of the velocity and impact force increases enormously.

XVI. CONCLUSION

This study involved validation of probabilistic method to existing FEA Model. Case study was conducted on both materials i.e., Docol R8 steel and 4130 chrome Molly and benchmarked results proved that Docol R8 was superior. This was validated by using Reliability (i.e., FORM and Monte Carlo Methods) and FEA analysis (Static Structural and Explicit Dynamic Analysis) were Reliability Index and Probability of failure were compared. Sensitivity Analysis was also done for resultant FORM method.

Further improvements are possible in this study, where design optimization can be performed on system to improve Vehicle architecture and further improve safety of passengers. Crashworthiness can also be performed to improve FOS of system.

XVII. ATTACHED ALGORITHMS

Form1-A

Cholesky matrix		
	σ	E
σ	1	-0.1093622
E	0	0.994002
Cholesky Inverse		
	σ	E
σ	1	0.1100221
E	0	1.0060342
Correlation Matrix	σ	E
σ	1	-0.01093622
E	-0.1093622	1
limit state g= (σ -0.002*E)		
Initial value	Type	u
σ	Lognormal	757.5
E	Normal	200060
	#1	#2
Lamda σ	6.610561026	6.610561026
Kesi σ	0.038925022	0.038925022
σ	757.5	739.7655397
E	200060	0.122619873
g	357.38	734.5966146
Eqv. Norm Mean σ	742.7571477	742.8930675
Eqv. Norm Std.Dev. Σ	29.48570452	28.79539025
Eqv. Norm Mean E	757.5	757.5
Eqv. Norm Std.Dev. E	14899.4	14899.4
σ' -correlated	0.5	-0.108612098
E'-correlated	13.37654536	-0.050832744
σ'' -uncorrelated	1.971715612	-0.114204823
E''-uncorrelated	1.971715612	-0.114204823
dg/d σ	1	2
dg/dE	-0.002	-0.002

dg/dσ"	29.48570452	57.59078051			
dg/dE"	-32.84468831	-35.91832125			
DG	44.13819572	67.87358692			
DG2	1948.180321	4606.823801			
σ" new	-0.100238496	-0.030941688			
E" new	0.111657571	0.019297767			
beta	0.150050555	0.036466312			
Beta diff		-0.113584244			
σ'-new-correlated	-0.101459607	-0.031152733			
E'-new-correlated	0.122619873	0.022681619			
σ-new-correlated	739.7655397	741.9960124			
E-new-correlated	2584.462543	1095.442507			
g-new	734.5966146	739.8051274			
Probability of failure	0.48545527				

Monte1-A

```

A_S=rand(1000000,1);
A_E=rand(1000000,1);
lambda_S=6.610561026;
kesi_S=0.038925022;
U_E=200060;
sigma_E=14899.4;
K=1;
Nf=0;
S=zeros(1000000,1);
E=zeros(1000000,1);
g=zeros(1000000,1);
while K<=1000000
    S(K,1)=exp(lambda_S+kesi_S*norminv(A_S(K,1)));
    E(K,1)=U_E+sigma_E-norminv(A_E(K,1));
    g(K,1)=((S(K,1))-(0.002*E(K,1)));
    if g(K,1)<0
        Nf=Nf+1;
    end
    K=K+1;
end
Probability_Of_Failure=Nf/1000000

```

Probability_of_Failure = 0.48346211

FORM1-B

Cholesky matrix					
	σ	E			
σ	1	-0.1093622			
E	0	0.994002			
Cholesky Inverse					
	σ	E			
σ	1	0.1100221			
E	0	1.0060342			
Correlation Matrix	σ	E			
σ	1	-0.01093622			
E	-0.1093622	1			
limit state g= (σ -0.002*E)					
Initial value	Type	u	sigma	lamda	kesi
σ	Lognormal	1100	110	6.998090293	0.009950331
E	Normal	210060	21006		
	#1	#2			
Lamda σ	6.998090293	6.998090293			
Kesi σ	0.009950331	0.009950331			
σ	1100	1091.282073			
E	210060	1.146848092			
g	679.88	1040.900691			
Eqv. Norm Mean σ	1094.527318	1094.536053			
Eqv. Norm Std.Dev. Σ	10.94536394	10.85861769			
Eqv. Norm Mean E	1100	1100			
Eqv. Norm Std.Dev. E	21006	21006			
σ' -correlated	0.5	-0.299667937			
E'-correlated	9.947634009	-0.052311394			
σ'' -uncorrelated	1.594459584	-0.305423347			
E''-uncorrelated	1.594459584	-0.305423347			
dg/d σ	1	2			
dg/dE	-0.002	-0.002			
dg/d σ''	10.94536394	21.71723537			

dg/dE"	-42.9570211	-44.13505666			
DG	44.3295235	49.18883551			
DG2	1965.106654	2419.541539			
σ'' new	-0.284292729	0.061456396			
E'' new	1.115757214	-0.124895342			
beta	1.151406321	0.139196749			
Beta diff		-1.012209572			
σ' -new-correlated	-0.296494896	0.062822279			
E'-new-correlated	1.146848092	-0.131616348			
σ -new-correlated	1091.282073	1095.218216			
E-new-correlated	25190.69102	-1664.733014			
g-new	1040.900691	1098.547682			
Probability of failure	0.444647339				

Monti-1-B

```

A_S=rand(1000000,1);
A_E=rand(1000000,1);
lambda_S=6.998090293;
kesi_S=0.009950331;
U_E=210060;
sigma_E=21006;
K=1;
Nf=0;
S=zeros(1000000,1);
E=zeros(1000000,1);
g=zeros(1000000,1);
while K<=1000000
    S(K,1)=exp(lambda_S+kesi_S*norminv(A_S(K,1)));
    E(K,1)=U_E+sigma_E-norminv(A_E(K,1));
    g(K,1)=((S(K,1)))-(0.002*E(K,1));
    if g(K,1)<0
        Nf=Nf+1;
    end
    K=K+1;
end
Probability_Of_Failure=Nf/1000000

```

>> FormB1

Probability_Of_Failure = 0.44324519

Form2-A

Cholesky matrix				
	s1	v1	t1	
s1	1	0.08495065	0.09396867	
v1	0	0.99638516	0.08783962	
t1	0	0	0.99169254	
Cholesky Inverse				
	s1	v1	t1	
s1	1	-0.08525885	-0.08720401	
v1	0	1.00362795	-0.08889681	
t1	0	0	1.00837705	
Correlation Matrix	s1	v1	t1	
s1	1	0.08495065	0.09396867	
v1	0.084951	1	0.0955048	
t1	0.093969	0.0955048	1	
limit state g= (F-v*25*t^-1)				
Initial value	Type	u	sigma	lamda
F	Lognormal	227	22.7	5.419974852
v	Lognormal	50.09	5.896575	3.906940006
t	Normal	20.4	5.944185	
	#1	#2	#3	
Lamda F	5.419974852	5.419974852	5.419974852	
Kesi F	0.099751345	0.099751345	0.099751345	
Lamda v	3.906940006	3.906940006	3.906940006	
Kesi v	0.117314801	0.117314801	0.117314801	
F	227	127.9524654	194.1218326	
v	50.09	53.4962214	46.58401174	
t	20.4	-0.023924631	-11.76897794	
g	165.6151961	56028.73287	293.0769245	
Eqv. Norm Mean F	225.8706374	200.6699014	223.529133	
Eqv. Norm Std.Dev. F	22.64355534	12.76343053	19.36391392	
Eqv. Norm Mean v	49.74531161	49.60859657	49.64377166	
Eqv. Norm Std.Dev. V	5.876298384	6.275898569	5.464994068	
Eqv. Norm Mean t	20.4	20.4	20.4	
Eqv. Norm Std.Dev. T	5.944185	5.944185	5.944185	
F'-correlated	0.049875673	-5.697326894	-1.518665105	
v'-correlated	0.058657401	0.6194531	-0.559883485	
t'-correlated	0	-3.435950367	-5.411839965	

F"-uncorrelated	0.04487461	-5.450512102	-0.998995936	
v"-uncorrelated	0.04487461	-5.450512102	-0.998995936	
t"-uncorrelated	0.04487461	-5.450512102	-0.998995936	
dg/dF	1	2	3	
dg/dv	-1.225490196	1044.948203	2.124228639	
dg/dt	3.009059016	2336536.801	8.408129607	
dg/dF"	22.64355534	25.52686107	58.09174176	
dg/dv"	-5.2517296	6536.451374	16.50186383	
dg/dt"	19.23303416	13774004.75	56.04280018	
DG	30.16986684	13774006.3	82.38784754	
DG2	910.2208654	1.89723E+14	6787.757422	
F" new	-4.079121585	-1.01136E-05	-3.625148056	
v" new	0.946072436	-0.002589701	-1.029779755	
t" new	-3.464733502	-5.45717523	-3.497286223	
beta	5.434948408	5.457175844	5.141318479	
Beta diff		0.022227436	-0.315857366	
F'-new-correlated	-4.324328515	-0.513033609	-4.041263851	
v'-new-correlated	0.638311662	-0.481936539	-1.333257559	
t'-new-correlated	-3.435950367	-5.411839965	-3.468232658	
F-new-correlated	127.9524654	194.1218326	145.2744476	
v-new-correlated	53.4962214	46.58401174	42.35752701	
t-new-correlated	-0.023924631	-11.76897794	-0.215816541	
g-new	56028.73287	293.0769245	5051.933465	
Probability of failure	1.36409E-07			

Monte 2-A

```

A_S=rand(1000000,1);
A_E=rand(1000000,1);
lambda_S=6.610561026;
kesi_S=0.038925022;
U_E=200060;
sigma_E=14899.4;
K=1;
Nf=0;
S=zeros(1000000,1);
E=zeros(1000000,1);
g=zeros(1000000,1);
while K<=1000000
    S(K,1)=exp(lambda_S+kesi_S*norminv(A_S(K,1)));
    E(K,1)=U_E+sigma_E-norminv(A_E(K,1));
    g(K,1)=((S(K,1)))-(0.002*E(K,1));
    if g(K,1)<0
        Nf=Nf+1;
    end
end
    
```

```

end
K=K+1;
end
Probability_Of_Failure=Nf/1000000

```

Probability_Of_Failure = 0.48346211

FORM2-B

Cholesky matrix					
	s1	v1	t1		
s1		1	0.08495065	0.09396867	
v1		0	0.99638516	0.08783962	
t1		0	0	0.99169254	
Cholesky Inverse					
	s1	v1	t1		
s1		1	-0.08525885	-0.08720401	
v1		0	1.00362795	-0.08889681	
t1		0	0	1.00837705	
Correlation Matrix		s1	v1	t1	
	s1	v1	t1		
s1		1	0.08495065	0.09396867	
v1		0.084951	1	0.0955048	
t1		0.093969	0.0955048	1	
limit state g= (F-v*23*t^-1)					
Initial value	Type	u	sigma	lamda	kesi
F	Lognormal	227	22.7	5.419974852	0.009950331
v	Lognormal	50.09	5.896575	3.906940006	0.013762763
t	Normal	20.4	5.944185		
	#1	#2	#3		
Lamda F	5.419974852	5.419974852	5.419974852		
Kesi F	0.099751345	0.099751345	0.099751345		
Lamda v	3.906940006	3.906940006	3.906940006		
Kesi v	0.117314801	0.117314801	0.117314801		
F	227	125.019916	191.9311732		
v	50.09	53.60855732	45.89730204		
t	20.4	-0.635600348	-14.13938675		
g	170.5259804	2064.913156	273.0826807		
Eqv. Norm Mean F	225.8706374	198.9694213	223.1848649		
Eqv. Norm Std.Dev. F	22.64355534	12.47090479	19.14539269		
Eqv. Norm Mean v	49.74531161	49.60031514	49.59358021		

Eqv. Norm Std.Dev. V	5.876298384	6.289077236	5.384432857		
Eqv. Norm Mean t	20.4	20.4	20.4		
Eqv. Norm Std.Dev. T	5.944185	5.944185	5.944185		
F'-correlated	0.049875673	-5.92976264	-1.632439314		
v'-correlated	0.058657401	0.637333909	-0.68647493		
t'-correlated	0	-3.538853577	-5.81061773		
F"-uncorrelated	0.04487461	-5.675498773	-1.067202085		
v"-uncorrelated	0.04487461	-5.675498773	-1.067202085		
t"-uncorrelated	0.04487461	-5.675498773	-1.067202085		
dg/dF	1	2	3		
dg/dv	-1.225490196	39.33289225	1.768110629		
dg/dt	3.009059016	3317.461382	5.739393724		
dg/dF"	22.64355534	24.94180957	57.43617808		
dg/dv"	-5.2517296	248.5922259	14.36509936		
dg/dt"	19.23303416	19579.85678	40.06605905		
DG	30.16986684	19581.45071	71.48817888		
DG2	910.2208654	383433211.9	5110.55972		
F" new	-4.201287117	-0.007463864	-4.410837164		
v" new	0.974406341	-0.074391496	-1.103174275		
t" new	-3.568498737	-5.859293577	-3.07689105		
beta	5.597719571	5.859770561	5.489966895		
Beta diff		0.26205099	-0.369803666		
F'-new-correlated	-4.453837745	-0.564373494	-4.793683896		
v'-new-correlated	0.657428445	-0.588800704	-1.369459418		
t'-new-correlated	-3.538853577	-5.81061773	-3.051329901		
F-new-correlated	125.019916	191.9311732	131.4079042		
v-new-correlated	53.60855732	45.89730204	42.21981793		
t-new-correlated	-0.635600348	-14.13938675	2.262330574		
g-new	2064.913156	273.0826807	-335.1443584		
Probability of failure	2.01005E-08				

Monti 2-B

```

A_S=rand(1000000,1);
A_E=rand(1000000,1);
lambda_S=6.998090293;
kesi_S=0.009950331;
U_E=210060;
sigma_E=21006;
K=1;
Nf=0;
S=zeros(1000000,1);

```

```

E=zeros(1000000,1);
g=zeros(1000000,1);
while K<=1000000
    S(K,1)=exp(lambda_S+kesi_S*norminv(A_S(K,1)));
    E(K,1)=U_E+sigma_E-norminv(A_E(K,1));
    g(K,1)=((S(K,1))-(0.002*E(K,1)));
    if g(K,1)<0
        Nf=Nf+1;
    end
    K=K+1;
end
Probability_Of_Failure=Nf/1000000

```

>> FormB1

Probability_Of_Failure = 0.44324519

Monte-Full system

```

A_M=rand(1000000,1);
A_v=rand(1000000,1);
A_m=rand(1000000,1);
A_t=rand(1000000,1);
U_M=400;
sigma_M=40;
U_t=20;
sigma_t=2;
lambda_m=6.570197534;
kesi_m=0.099751345;
U_v=50;
sigma_v=5;
K=1;
Nf=0;
M=zeros(1000000,1);
v=zeros(1000000,1);
m=zeros(1000000,1);
t=zeros(1000000,1);
g=zeros(1000000,1);
while K<=1000000
    M(K,1)=U_M+sigma_M-norminv(A_M(K,1));
    v(K,1)=U_v+sigma_v-norminv(A_v(K,1));
    m(K,1)=exp(lambda_m+kesi_m*norminv(A_m(K,1)));
    t(K,1)=U_t+sigma_t-norminv(A_t(K,1));
    g(K,1)=(9.81*(M(K,1)-1))-((2*m(K,1)*v(K,1)*(t(K,1))^-1)));
    if g(K,1)<0
        Nf=Nf+1;
    end
    K=K+1;
end
Probability_Of_Failure=Nf/1000000

```

Project_Crash

Probability_Of_Failure =

0.0462

XVIII. REFERENCES

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THANK YOU.