Vehicle Collision Avoidance System

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***Abstract*— The number of road accidents has increased as the country's road network, motorization, and urbanization have grown. While seatbelts and airbags help to reduce body movement after an accident, the force of the impact can induce abnormal jolting in the neck, upper back, arms, and legs. This work presents an MPC and PID-controlled vehicle collision avoidance system to avoid rear-end collisions with an active control system as a solution to such a problem. When designing the dynamic control system, safety-related aspects (relative speed, relative distance, steering input) were considered. The MPC control system is a potential way for minimizing the likelihood of an impact while maintaining the driver's comfort, according to the MATLAB/Simulink results.**

***Index Terms*— ADAS, CAS, MPC and PID**

I. INTRODUCTION

Most modern automobiles have steadily improved in terms of quality and functionality in recent years. Car manufacturers are concentrating on incorporating various types of ADAS systems into their vehicles. One of these systems is collision avoidance systems. Every year, over 1.3 million individuals are killed in automobile accidents. [1] Speeding, inattentive driving, dangerous vehicles, driving under the influence of alcohol, and a variety of other factors all contribute to road accidents. Different sorts of collision avoidance technologies are being developed to avoid these types of accidents and minimize the death rate. The primary goal of these technologies is to increase safety. Collision avoidance systems include forward collision warning, blind-spot warning, cross-traffic alert, pedestrian detection system, adaptive cruise control, parking assist, and electronic stability control.

The goal of this project is to prevent collisions by designing a system in which vehicles maintain a safe distance from the leading vehicle and continue to follow it. To avoid colliding with the leading car, the vehicle also undertakes lane-changing actions. If the leading car slows or stops, the following vehicle will take the appropriate steps to avoid a collision. MPC (Multi-Point Constraints) and PID (Proportional Integral Derivative) controllers were used to represent the vehicles.

A. Terminology

m	Mass of vehicle
v_x	Longitudinal velocity of vehicle
v_y	Lateral Velocity of Vehicle
d_r	Relative distance
d_s	Distance between vehicles
v_r	Relative speed
v_p	Preceding vehicle velocity
v_e	Ego vehicle velocity
v_i	Initial speed
t_h	Headway time
t_s	Sampling time
a_{ref}	Acceleration reference
a_{com}	Acceleration command given to vehicle
x_{outx}	Position output in x direction
x_{outz}	Position output in z direction
$\boldsymbol{\theta}$	Steering angle
$\boldsymbol{\Psi}$	Yaw angle
v_{ref}	Reference speed
x_{out}	Position output
v_{out}	Output speed
I_z	Polar moment of Inertia
C_f	Cornering stiffness of Front tire
C_r	Cornering stiffness of Rear tire

Table. 01 Terminology

B. Abbreviation

CAS	Collision Avoidance System
ADAS	Advanced Driver Assistance System
MPC	Model Predictive Control
PID	Proportional Integral Derivative Control
ABS	Anti-lock Braking System
DOF	Degree Of Freedom
OV	Output Variables
DMS	Drive Mode Selector

Table. 02 Abbreviation

II. PROPOSED MODEL

The first vehicle in the model was given a constant velocity to represent an automobile moving at a constant pace on a straight road, while the second and third vehicles were altered so that a rear-end accident would be possible, and the controller would have to intervene. The MPC control logic is used in the second vehicle, while the PID controller is used in the third.

The vehicle equipped with an MPC controller concentrates on maintaining a constant speed and keeping a safe distance from the leading vehicle. The other vehicle, which is equipped with a PID controller, is focused on maintaining a safe speed or conducting a lane-change move.

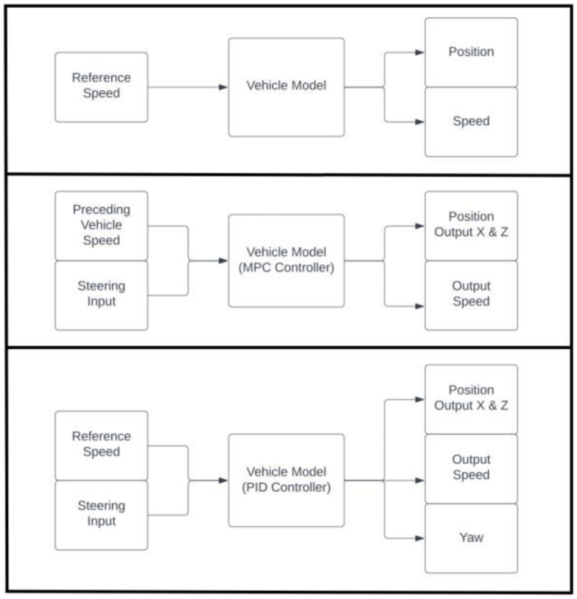

Fig. 01 Proposed Model

III. Vehicle Model

The vehicle dynamics are modeled using a 3-DOF rigid vehicle model (Single Track) imported from a Simulink dynamic block set.

A. *Vehicle Kinematic Model*

Vehicle Motion is on a horizontal plane with 3 degrees of freedom (DOF). Displacement is on a lateral and longitudinal plane and axis of rotation normal to that plane (Yaw Rotation). By controlling these 3 DOF, the trajectory can be described, and path can be controlled. The bicycle model equations can be derived from following kinematic model.[2]

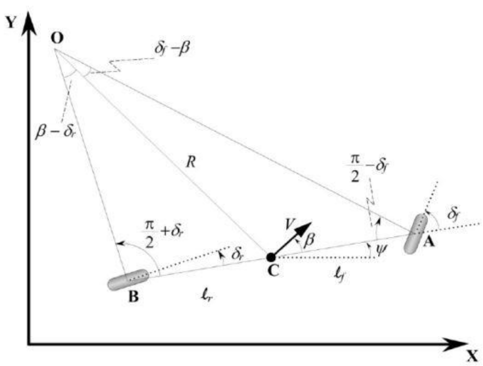

Fig. 02 Vehicle Kinematic Model[2]

As the above Fig. 01 represents 2 front and rear wheels are represented by 1 single central tire at point A and B.

δ_f = steering angle at front wheel,
δ_r = steering angle at rear wheel.

Assumption: Vehicle model assumes front wheel steering angle only, therefore δ_r set to zero.
C – center of gravity,
l_f and l_r – distances from point A and B to Centre of gravity.

Sum of these 2 corresponds to L (wheelbase) i.e.,

$$l_f + l_r. \tag{1}$$

Assumption – Vehicle is in planar motion, therefore 3 co-ordinates required to describe it. (X, Y and Ψ).

V – velocity of vehicle at CG,
β – slip angle with longitudinal axis.

Applying sine rule ΔOCA and ΔOCB,

$$\frac{\sin(\delta_f - \beta)}{l_f} = \frac{\sin\left(\frac{\pi}{2} - \delta_f\right)}{R} \tag{2}$$

$$\frac{\sin(\beta)}{l_r} = \frac{1}{R} \tag{3}$$

Now, multiplying eq (2) by $l_f / \cos(\delta_f)$,

$$\tan(\delta_f)\cos(\beta) - \sin(\beta) = \frac{l_f}{R} \tag{4}$$

Similarly, multiplying eq (3) by l_r,

$$\sin(\beta) = \frac{l_f}{R} \tag{5}$$

Now, adding (4) and (5),

$$\tan(\delta_f)\cos(\beta) = \frac{(l_f + l_r)}{R} \tag{6}$$

.

If radius (R) of vehicle trajectory changes slowly to low velocity, the yaw rate (Ψ') = angular velocity (ω)

$$\omega = \frac{V}{R} \tag{7}$$

Thus, Yaw rate can be described as,

$$\varphi' = \frac{V}{R} \tag{8}$$

Now, substituting R from eq (8) in eq (6), we get,

$$\varphi' = \frac{V cos(\beta)}{l_f + l_r}\tan(\delta_f) \tag{9}$$

Now, the overall equation of kinematic model can be defined as,

$$X' = V\cos(\varphi' + \beta) \tag{10}$$

$$Y' = V sin(\varphi' + \beta) \tag{11}$$

$$\varphi' = \frac{V cos(\beta)}{l_f + l_r}\tan(\delta_f) \tag{12}$$

[2]

B. *Vehicle Dynamic Model*

The limitation of kinematic model is that the vehicle speed and steering angle needs to be low enough to obtain results. During high-speed conditions the tire starts to lose their grip from road(drifting), so dynamics are necessary to be integrated in our system. [2]

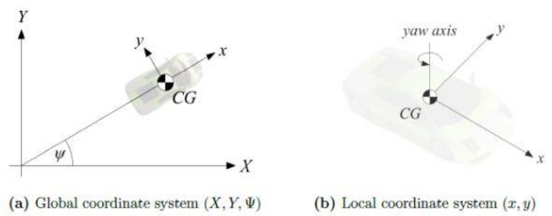

Fig. 03 Model Coordinate System[2]

For our project we have used 3 DOF rigid vehicle model (single track) in Simulink block set. Our test vehicle is steerable from front wheels,

Let,

x and y - Longitudinal and Lateral direction in vehicle frame,
X and Y – Longitudinal and Lateral direction in global frame,
Yaw angle – x, y vehicle frame (ϕ)
Heading angle – X, Y frame,
F_x and F_y – Longitudinal and Lateral tire forces on C.G. of vehicle,
F_z – Normal tire load,
F_l and F_c – Longitudinal and Lateral stiffness force,
ϕ – heading angle
δ – wheel steering
α – slip angle
I_z – Polar moment of inertia

Assumption: No front and rear tire forces in x-axis.

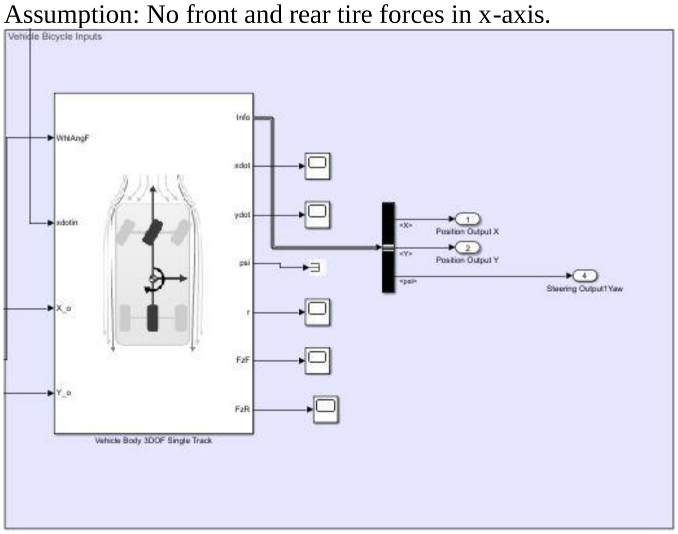

Fig. 04 Simulink Bicycle Model

As shown in figure, only lateral tire forces of the dynamical model individually in co-ordinate frames by the wheels. Here vehicle C.G. will be reference point for calculating Newton's law.

Here longitudinal velocity is constant $V_x = 0$, as a result force $F_{xf} = 0$

$$mv_x' = mv_y\varphi' + F_{xf}\cos(\delta) + F_{xr} - F_{yf}\sin(\delta) \tag{13}$$

$$mv_y' = mv_x\varphi' + F_{xf}\sin(\delta) + F_{yr} + F_{yf}\cos(\delta) \tag{14}$$

$$I_z\varphi' = L_fF_{yf}\cos(\delta) + L_fF_{xf}\sin(\delta) - L_rF_{yr} \tag{15}$$

The vehicle's eq of motion in an absolute inertial frame

$$F_x = F_l\cos(\delta) - F_c\sin(\delta) \tag{18}$$

$$F_y = F_l\sin(\delta) + F_c\cos(\delta) \tag{19}$$

Now, the steering angles of front and rear axles can be described as,

Assuming front wheel drive,

$$J_{wf}\,(\omega_f') = (T_w - T_b - r_wF_{l,f}) \tag{20}$$

$$J_{wr}\,(\omega_r') = (-T_b - r_wF_{l,r}) \tag{21}$$

Where,
I_w = Moment of Inertia of wheels
ω' = angular acceleration of wheel
T_w = wheel torque
T_b = brake torque

Now, the front & rear speeds along x-axis and y-axis are calculated as[2] –

$$V_{x,i} = V_i\cos(\propto_i) \tag{22}$$

$$V_{y,i} = V_i\sin(\propto_i) \tag{23}$$

Where,

α_f and α_r - Front and Rear tire slip angle
V_f and V_r – Front and Rear tire speed

The correlation between tire speeds and host vehicle speed[2] -

$$V_{x,f} = V_{h,f} \tag{24}$$

$$V_{x,r} = V_{h,x} \tag{25}$$

$$V_{y,f} = V_{h,y} + l_f\,\varphi' \tag{26}$$

$$V_{y,r} = V_{h,y} - l_r\,\varphi' \tag{27}$$

The slip angle represents the angle between the wheel velocity and direction of wheel,

Then tire side-slip angles are,

$$\alpha_f = \delta_f - \frac{V_{y,f}}{V_{x,f}} \tag{28}$$

$$\alpha_r = \delta_r - \frac{V_{y,r}}{V_{x,r}} \tag{29}$$

Now, longitudinal forces are assumed to depend on normal force, surface friction and longitudinal slip ratio,

$$F_{x,i} = F_x(\sigma_{i,u}, F_{z,i}) \tag{30}$$

Pacejka model utilized to calculate the friction co-efficient as function of longitudinal slip ratio,

$$\mu(\sigma) = D\sin(C\tan^{-1}(B\sigma - E\,(B\sigma - \tan^{-1}(B\sigma)))) \tag{31}$$

The value of B, C, D and E are different road types,
The longitudinal traction force,
F_x is calculated as

$$F_x = \mu(\sigma) F_z \tag{32}$$

Where, Fz – normal force exerting on wheel,
μ(σ) = Friction Coefficient

The lateral tire forces are crucial for maneuvering range, this depends on normal force, surface friction and slip angle,

$$F_{y,i} = F_y(\alpha, \mu, F_{z,i}) \tag{33}$$

Now, tire force F_y varies proportionally to slip angle α

Cornering stiffness C_f is defined as the ratio between F_y and α. Slip angle commuter in linear region. Therefore, lateral tire forces are linear functions of slip angles,

$$F_{y,i} = 2\, C_{y,i}\, \alpha_i \tag{34}$$

The cornering stiffness are defined for front and rear tires based on static distribution of weight between axles –

$$C_{y,i} = C_s \frac{F_{z,i,static}}{2} \tag{35}$$

Where C_s is cornering stiffness coefficient.

Now, the front wheel slip angle α_f is difference between steering angle δ of front wheel and orientation angle of tire velocity θ_{vf} w.r.t. longitudinal axis.

$$\alpha_f = \delta - \theta_{vf} \tag{36}$$

Similarly, rear wheel slip angle,

$$\alpha_f = -\theta_{vr} \tag{37}$$

Therefore, the lateral tire forces for front and rear wheels of vehicle are –

$$F_{yf} = 2\, C_{\alpha f}(\delta - \theta_{vf}) \tag{38}$$

$$F_{yr} = 2\, C_{\alpha r}(\delta - \theta_{vr}) \tag{39}$$

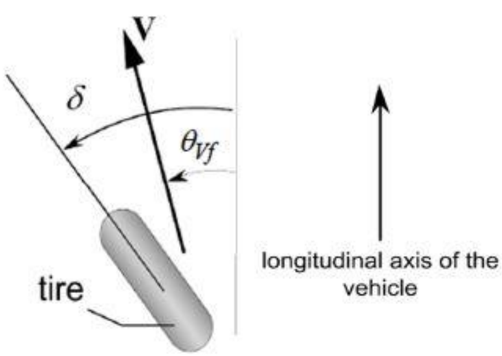

Fig. 05 Tire Slip Angle [2]

Now, to calculate velocity angle of the front wheel θ_{Vf} out and rear wheel θ_{Vr}, we use

$$\tan(\theta_{vf}) = \frac{V_y + l_f \dot{\varphi}}{V_x} \tag{40}$$

$$\tan(\theta_{vr}) = \frac{V_y - l_f \dot{\varphi}}{V_x} \tag{41}$$

Here we assume small angle approximations, therefore equation (40) and (41) can be rewritten as:

$$\theta_{Vf} = \frac{V_y + l_f \dot{\varphi}}{V_x} \tag{42}$$

$$\theta_{Vr} = \frac{V_y - l_f \dot{\varphi}}{V_x} \tag{43}$$

Therefore, relationship of tire longitudinal forces can be represented by

$$F_{yf} = 2C_{\alpha f}(\delta - \frac{V_y + l_f \dot{\varphi}}{V_x}) \tag{44}$$

$$F_{yr} = 2C_{\alpha r}(- \frac{V_y - l_f \dot{\varphi}}{V_x}) \tag{45}$$

Now, the error for controller internal plant with respect to reference trajectory. The error is lateral displacement error e_1 which can be represented by equations as follows

$$\dot{e_1} = V_x e_{2+} V_y \tag{46}$$

$$\dot{e_2} = \varphi - \varphi_{der} \tag{47}$$

The desired yaw angle rate is given by

$$\dot{\varphi}_{der} = V_x \text{k} \tag{48}$$

Where k denotes road curvature (if applicable)
The state-space model for lateral dynamics can be obtained by linearizing the bicycle model.

Therefore $\dot{x} = A_x + B_y$ is represented by –

$$\begin{bmatrix} \dot{y} \\ \ddot{y} \\ \dot{\varphi} \\ \ddot{\varphi} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-2c_{\alpha f} + 2c_{\alpha r}}{mV_x} & 0 & \frac{-V_x - 2L_f c_{\alpha f} - 2L_r c_{\alpha r}}{mV_x} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-2L_f c_{\alpha f} - 2L_r c_{\alpha r}}{\text{Iz}V_x} & 0 & \frac{-2L_f^2 c_{\alpha f} - 2L_r^2 c_{\alpha r}}{\text{Iz}V_x} \end{bmatrix} * \begin{bmatrix} y \\ \dot{y} \\ \varphi \\ \dot{\varphi} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{2c_{\alpha f}}{m} \\ 0 \\ \frac{2L_f c_{\alpha f}}{I_z} \end{bmatrix} \text{d}$$

[2]

Now for longitudinal dynamics, the plant model used for control design is transfer function between desired acceleration and actual vehicle speed

$$\text{G(s)} = \frac{1}{s(T_s + 1)} \tag{49}$$

[3]

Where, T_s is time constant.

C. *Vehicle Parameters (CarSIM)*

<u>CarSIM – E Class Sedan (Vehicle 02)</u>

Mass	1650 kg
Longitudinal distance for CG to front axel	1.4 m
Longitudinal distance for CG to rear axel	1.650 m
Polar Moment of inertia	3234 kgm^2
Front tire corner stiffness	$12*10^3$ n/rad
Rear tire axle corner stiffness	$11*10^3$ n/rad

Table. 03 Vehicle 02 Parameters

<u>CarSIM – D Class Sedan (Vehicle 03)</u>

Mass	1270 kg
Longitudinal distance for CG to front axel	1.015 m
Longitudinal distance for CG to rear axel	1.895 m
Polar Moment of inertia	3536.7 kgm^2
Front tire corner stiffness	$12*10^3$ n/rad
Rear tire axle corner stiffness	$11*10^3$ n/rad

Table. 04 Vehicle 03 Parameters

IV. Modelling of Vehicle on Simulink

The Lead vehicle which is the base line model of this project was modeled on Simulink. To represent the dynamics of acceleration of the vehicle body transfer function mentioned in equation (49) was used. helps to approximate the dynamics of throttle body and vehicle inertia. [3]

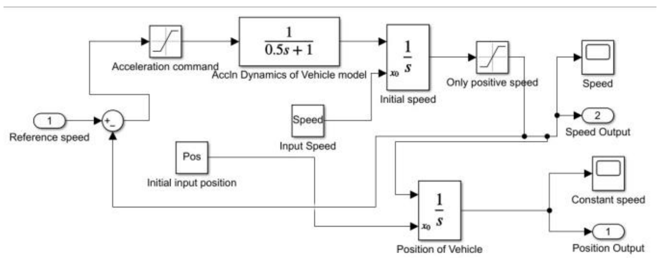

Fig. 06 Lead Vehicle Simulink Model

This model has a transfer function block representing the transfer function G in above equation. To setup the input speed and initial position of the integrator blocks are used. To simulate the output in more realistic way saturation blocks are used as in real life the vehicle has dynamic limitations for acceleration and deceleration.

Lead Vehicle has been modeled on travel in a straight line. Also, it can be controlled by providing the initial position and initial speed inputs. For this project the vehicle one is setup at initial speed of 20 m/s and initial position of 100 m from starting position.

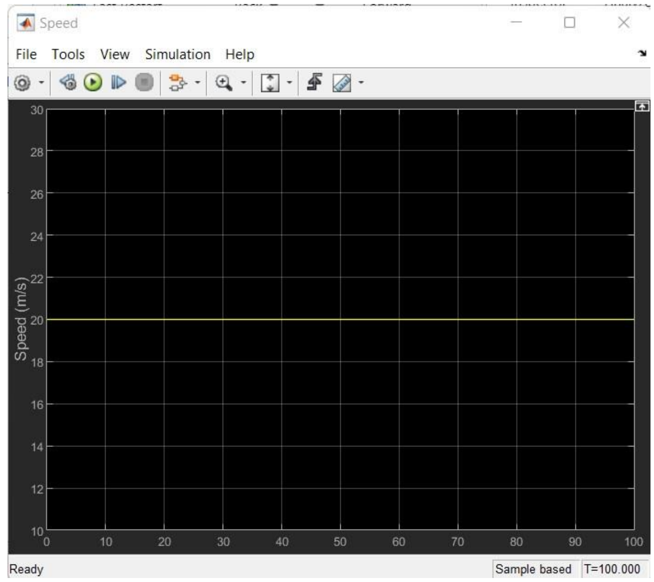

Fig. 07 Plot for lead vehicle speed

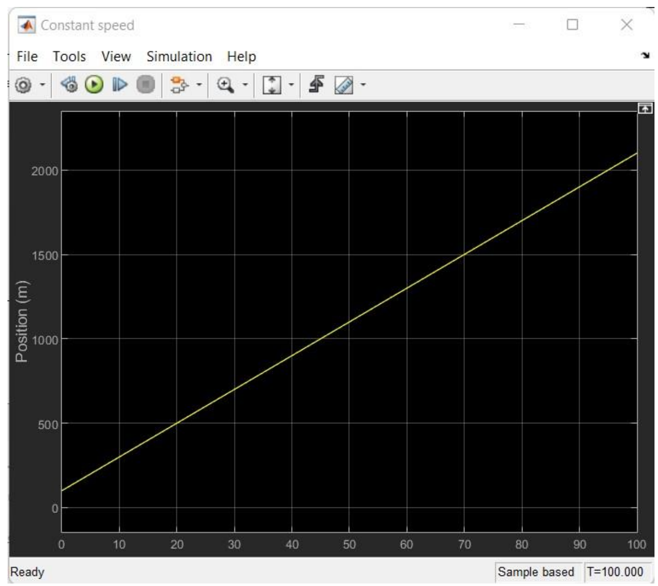

Fig. 08 Plot for lead vehicle position output

The plot in Fig shows how vehicle one travels at constant speed of 20 m/s over the time. The plot in fig shows that the vehicle 01 starts 100 m ahead from the initial position as the home send it over

V. Drive Mode Selector

The vehicle 2 and 3 by following the lead vehicle utilize this feature to select a particular mode driving mode by assessing the surrounding conditions.

A. *Drive Modes*

1. Driving Mode 01: This is a free drive mode in this driving mode the vehicle would continue to drive to at a constant speed in a straight direction.

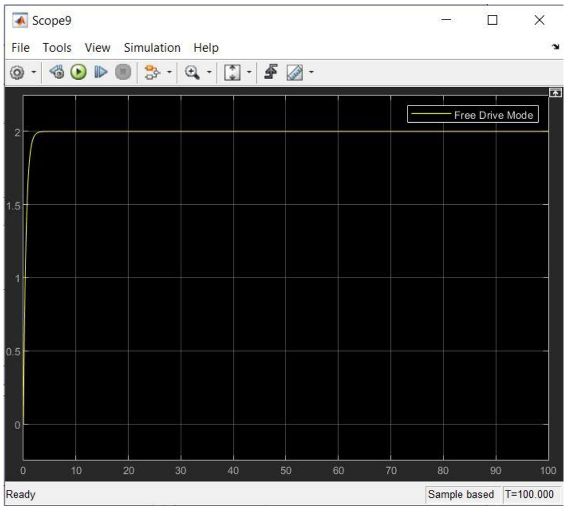

Fig. 09 Acceleration at Free Drive

2. Driving Mode 02: In this mode the vehicle ends up following the lead vehicle. The following vehicles maintain a safe travelling distance from the lead vehicle. The following vehicles 02 and 03 which are modeled using MPC and PID controller respectively. These controllers are responsible for maintaining a safe distance and relative speed between the cars.

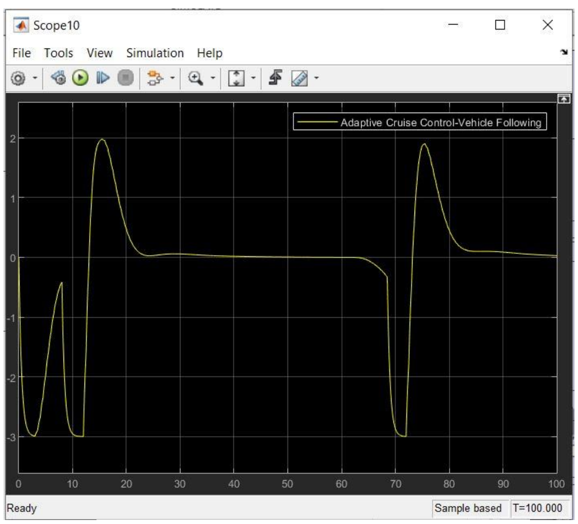

Fig. 10 Vehicle Acceleration with MPC Controller

3. Driving Mode 03: In this mode of the vehicle will end applying brakes on the following the vehicle. This mode would only be selected when the vehicle would sense any unnecessary obstacle, or the lead vehicle would stop suddenly.

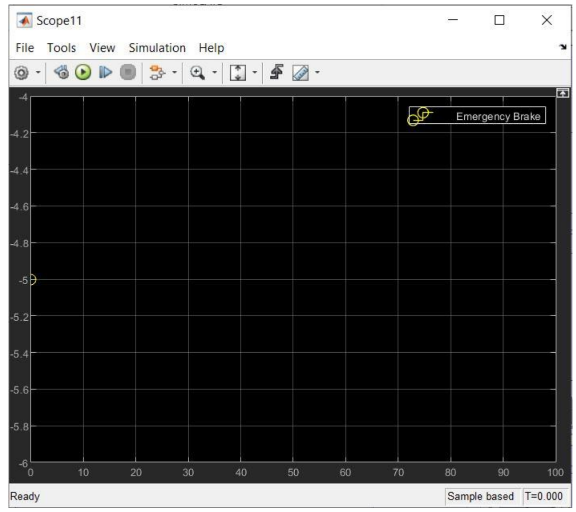

Fig. 11 Mode 03 Emergency Braking

B. *Vehicle Modeled Using MPC Controller*

MPC may control systems with several inputs and outputs that may interact with one another. Using the expected plant outputs, the MPC controller solves a quadratic programming optimization problem. At each time step, a controller solves the optimization problem by attempting to determine the best control signal at sampling time. The MPC controller attempts to find a trajectory that is as near as feasible to the reference trajectory.

MPC controller is modelled after the linear state space model after the linear system stated below:

$$x_1(k+1) = P\, x1(k) + Q_{u1}\, u_1(k) + Q_{d1}\, v_1(k)$$

$$y_1(k) = R\, x_1(k) \qquad [4]$$

P = state matrix

Q_{u1} and Q_{d1} are the input matrices corresponding to inputs u and v respectively.

R is the output matrix.

$$\min_{u_1} J = \sum_{j=1}^{N} \left|\left| y_{p1}\,(k+j|k) - y_{ref}\,(k+j|k) \right|\right|_{A_y} + \sum_{j=0}^{M-1} \left|\left| u1(k+j|k) \right|\right|_{B_u}$$

[4]

$$\textit{subject to} \quad x_1(k+j+1|k) = P x_1(k+j|k) + Q_{u_1} u_1(k+j|k) + Q_d v_1(k+j|k)$$

$$x_1(k|k) = x_1(k)$$

$$y_1(k+j|k) = R x_1(k+j|k)$$

$$|u_1(k+j|k)| \leq u_{1_{limit}} \qquad [4]$$

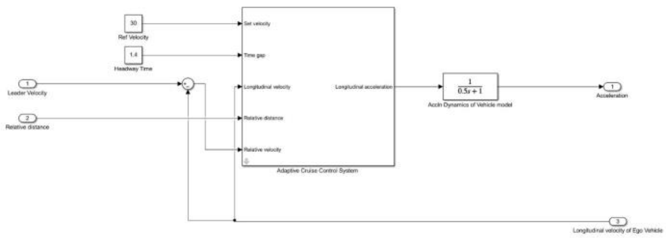

Fig. 12 Simulink Model for MPC Controller.

Where u_1 = Manipulated Variable

A_y and B_u = Weights for output and manipulated variables respectively.

The optimization problem searches for optimal value of input u where it reaches a minimal point.

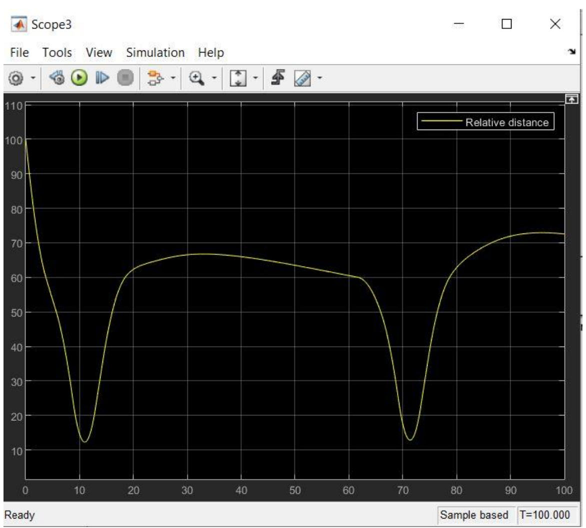

Fig. 13 Relative Distance from Vehicle 01

In the above plot, it can be observed that when the relative distance drops below 25m, vehicle reduces its speed before crossing a threshold value of relative distance.

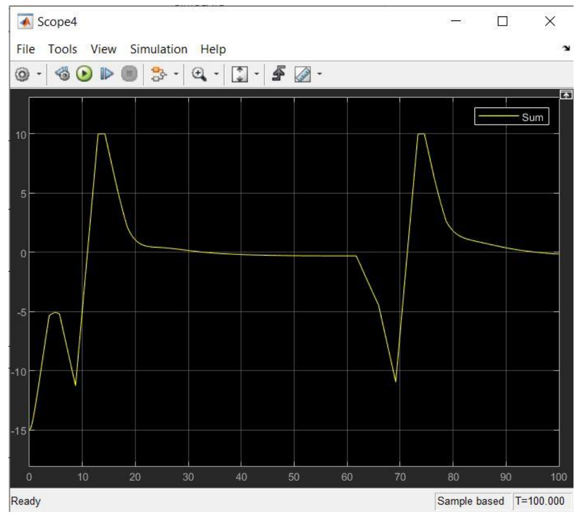

Fig. 14 Relative Velocity Difference between Leader and Longitudinal Velocity

Between 13th – 14th time step the MPC controller is activated, and the relative speed is brought to 0 and maintained for some time.

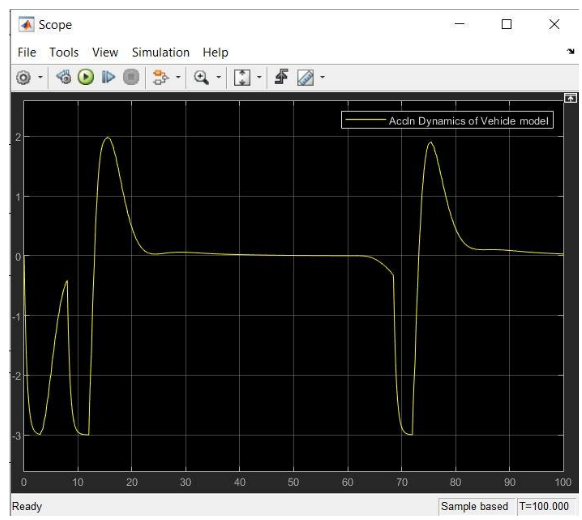

Fig. 15 Output Acceleration Dynamic of Vehicle Model

The output acceleration is a function of relative speed and is affected by it; as relative velocity rises (in the positive y-direction), acceleration rises, and as relative velocity falls, acceleration falls. As a result, the output acceleration plot matches the relative velocity plot, while the MPC controller's response (output) is visible a few time steps later.

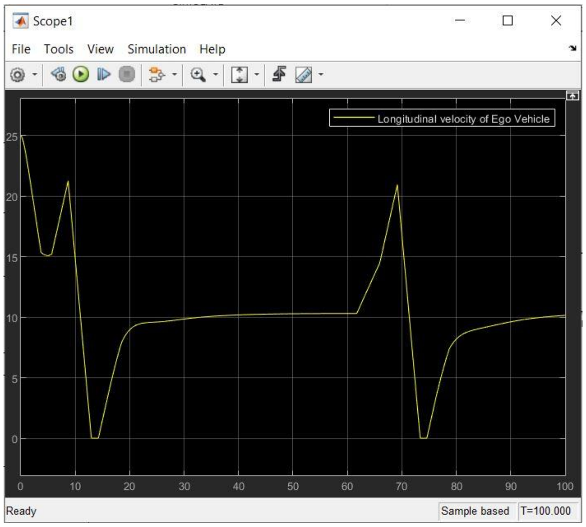

Fig. 16 Output Longitudinal Velocity of Ego Vehicle

In the above plot you can see the longitudinal velocity of ego vehicle changing as the MPC controller steps in to maintain a safe distance between the lead vehicle and it.

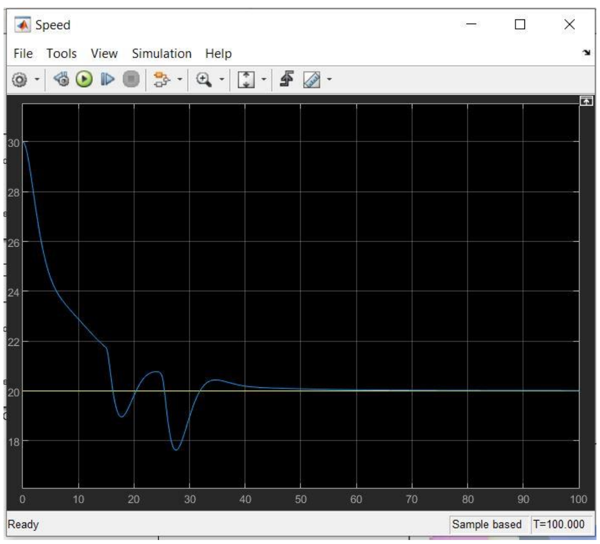

Fig. 17 Stabilized System

The sample time was kept at 0.5 sec, while the prediction horizon was kept at 200 steps. The controller behavior was adjusted so that the ego vehicle cruises at the same speed (i.e. converges) as the lead vehicle without compromising computation time.

C. Vehicle Modeled Using PID Controller

Proportional-Derivative-Integral (PID) Controller. PID is simple and practical compared to MPC control method. Here we use PID to control the distance error and make it to zero. This will allow Vehicle 3 to follow a desired yaw rate and the side slip is reduced as much as possible. This will also help in the lane change maneuver which we will perform on same vehicle.

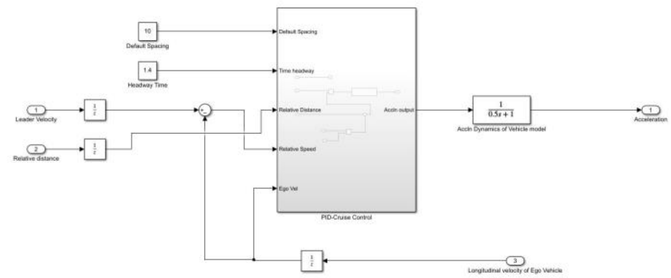

Fig.18 Mode 2: PID Control Dynamics Model

First, the input Relative speed which is difference between the Leader velocity and Longitudinal velocity of ego vehicle is terminated to keep the system simplified. This will help to obtain a system consisting of Single input- Relative Distance and Single output- Acceleration Dynamics of Vehicle.

Thus, the Controller Design to obtain Acceleration output is the difference between Relative distance and Reference distance. The inputs used in PID control are single input-single output, thus they have both pros and cons. As it is easy to control, the distortion analysis needs to be performed to further tune the output parameters.

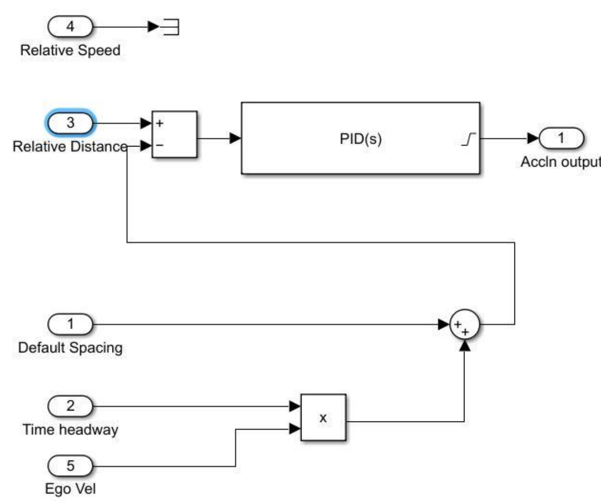

Fig.19 PID Controller.

Now, to further understand how the system behaves, below are represented inputs results for Relative distance and Reference distance-

Relative Distance

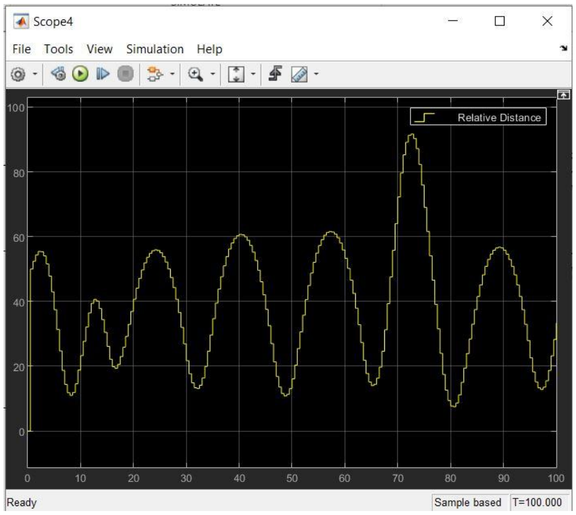

Fig.20 Relative Distance

The Ref. Distance is obtained using following formula

Reference distance = Ego Vehicle Speed + Time Headway + Default Spacing. [6]

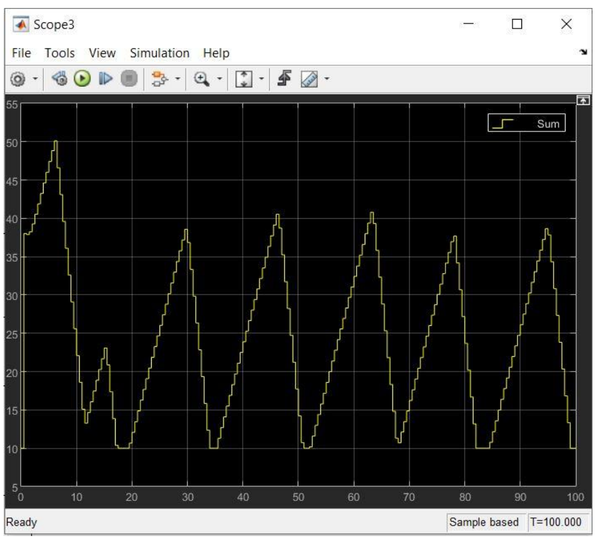

Fig. 21 Reference Distance

Now the Acceleration Output = Relative Distance – Reference Distance. [6]

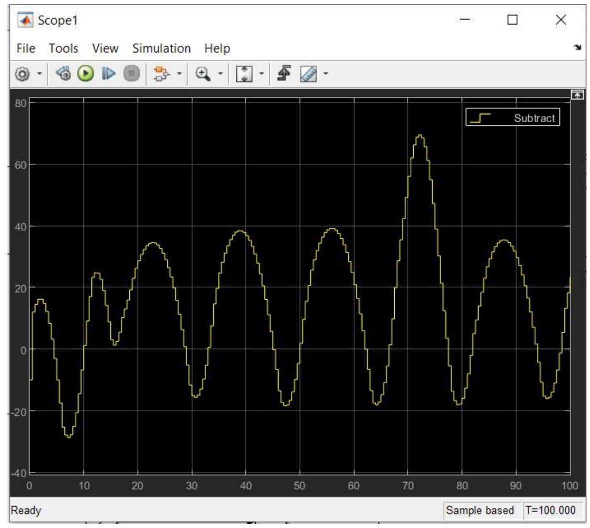

Fig.22 Acceleration Output

PID Tuning is done using PID control law in which proportional gain K_p, Integral gain K_i and Derivative gain K_d is used to form controller transfer block G_c.

$$G_c = K_p + K_i/s + K_d s \quad [5]$$

The output values of K_p, K_d and K_i are tuned using Zieglar-Nichols Method. In this method initially K_i and K_d are set to zero and K_p is increased until loop begins to oscillate. After tuning we obtained satisfactory results using K_p= 2.4, K_i= 2.7, K_d= 0.08 and Filter co-efficient N = 500 which can be represented in below figure.

Fig.23 Tuning PID Controller

After initial tuning using PID, the output signal for Acceleration output is further tuned using Acceleration Dynamics Plant model (G(s) Transfer Function). The resultant acceleration is shown below-

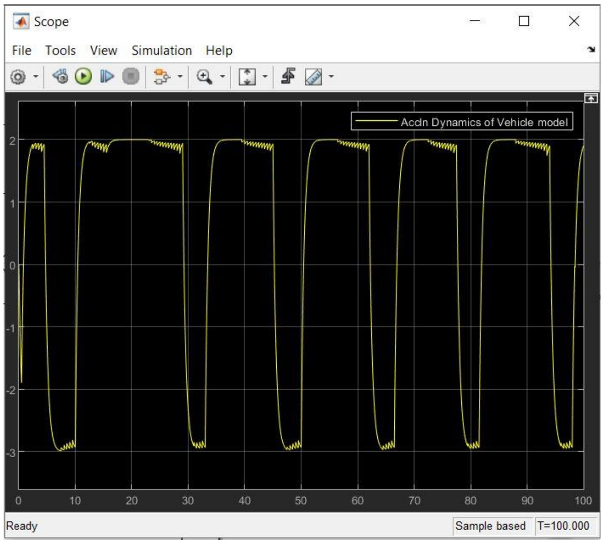

Fig.24 Mode 02- Acceleration Dynamics of Vehicle Model using PID controller.

The limitations of single input and single output can be represented in the above results. As relative speed is not taken into consideration the Vehicle behaves in fixed accelerated and decelerated pattern with respect to relative distance between Vehicle 2(Lead) and Vehicle 3(Ego). Unlike MPC controller where the vehicle converges after maintaining constant speed and fixed relative distance, The PID controller has limited command over the system. This will be further analyzed in the results section.

D. State Flow Model Used for Selecting the Drive Modes:

The state flow model is used for selecting the drive modes. The selection of drive modes is completely dependent on the relative distance and relative speed between the two cars.

- Consider a vehicle travelling on the road in straight direction. This is like the drive mode 01. Now this vehicle which is modeled either using MPC or PID controller, will calculate the relative distance between him and the car in front of it. And if the relative distance is more than 50m the vehicle would shift to the driving mode 02 where it will follow the lead vehicle and maintain a constant relative distance.
- Now if the relative distance goes below 60m the vehicle would shift to drive mode 01.
- Consider the vehicle in driving mode 02 and if in this case the relative distance goes below 25m the vehicle would enter the driving mode 03 were the vehicle would end up applying the brakes.
- And as the relative distance increases and goes above 35m the vehicle would shift back to driving mode 02.
- In another situation, the vehicle would switch from mode 01 to mode 02 as soon as the relative distance fell below 25 meters.

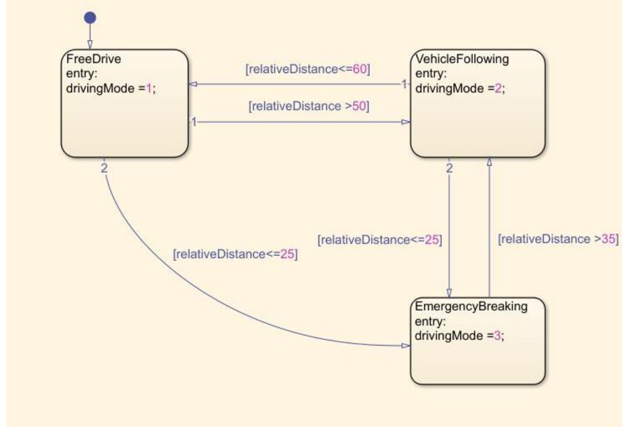

Fig.25 State Flow Model

VI. Lane Changing Maneuver

Minimum lateral jerk trajectory for the lane changing maneuver:

The lateral jerk is considered a function of the vehicle's driving comfort. When changing lanes to prevent a collision, the jerk is an important component to consider. High jerk can be uncomfortable and even dangerous for passengers in vehicles, so it is not recommended. Reduced jerk creates a more comfortable and less dangerous collision avoidance maneuver. As a result, the sixth derivative of the lateral displacement must be zero to provide a smooth lane change movement. [7]

The driving Comfort of the vehicle is given by,

$$\dddot{d}(t) = \frac{d^3 d(t)}{dt^3}$$

Cost Function:

$$H(d(t)) = \frac{1}{2} * \int_{t_i}^{t_f} \dddot{d}(t)^2 \, dt$$

Were,

t_i is the starting time of lane change maneuver.

t_f is the ending time of lane change maneuver.

$$H(d+e\eta) = \frac{1}{2} * \int_{t_i}^{t_f} \left(\dddot{d} + e\dddot{\eta}\right)^2 dt$$

$$\frac{dH(d+e\eta)}{e} = \int_{t_i}^{t_f} \left(\dddot{d} + e\dddot{\eta}\right) \dddot{\eta} \, dt$$

t_i and t_f are the start and end time of lane change trajectory.

After Integrating,

$$\left.\frac{dH(d+e\eta)}{e}\right|_{e=0} = -\int_{t_i}^{t_f} \eta d^{(6)} \, dt = 0 \qquad [7]$$

To have minimum jerk in the trajectory, the sixth derivative of lateral displacement is equal to zero

When $d^{(6)} = 0$,

$$\begin{bmatrix} d(t) \\ \dot{d}(t) \\ \ddot{d}(t) \end{bmatrix} = \begin{bmatrix} a_0 & a_1 & a_2 & a_3 & a_4 & a_5 \\ a_1 & 2a_2 & 3a_3 & 4a_4 & 5a_5 & 0 \\ 2a_2 & 6a_3 & 12a_4 & 20a_5 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ t \\ t^2 \\ t^3 \\ t^4 \\ t^5 \end{bmatrix} \quad [7]$$

These six coefficients define the lateral position of the vehicle during the lane changing maneuver

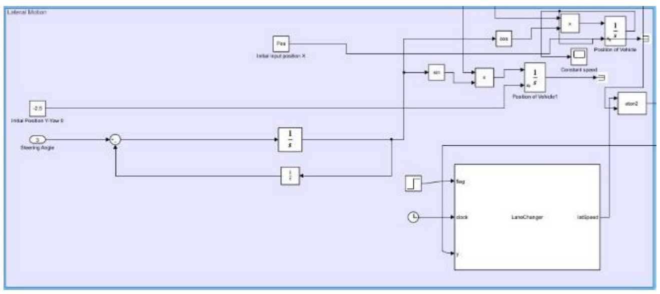

Fig.26 Simulink model for lane changing maneuver.

The output results of Code are represented below which shows the Vehicle competing an initial overtake from lane one to lane 2 and passing the ego vehicle. Similar code is implemented with negative trajectory to implement the car to move to initial lane one after completing the overtaking maneuver.

To implement the minimum lateral jerk to the Simulink model MATLAB code was generated. Now for the lateral displacement, we define d as a function of time with six constants of coefficients for positions one to six. So now for solving six unknowns, six equations are required.
Then taking the derivative of d we get lateral speed. Again, taking the derivative we get lateral acceleration. Now define the initial time value as zero and the final time value is 10. Solving this we would get six equations, we define another matrix by defining the vehicle position, the speed, and acceleration. Finally using the linsolve command we get the six unknown variables for lane changing maneuver. Solving the equation using the six constant and then generating the plots for a lateral position, lateral speed, and lateral acceleration.

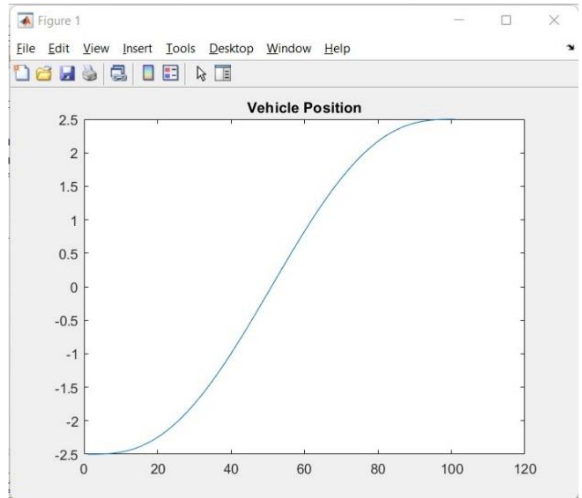

Fig.27 Vehicle position during lane changing maneuver.

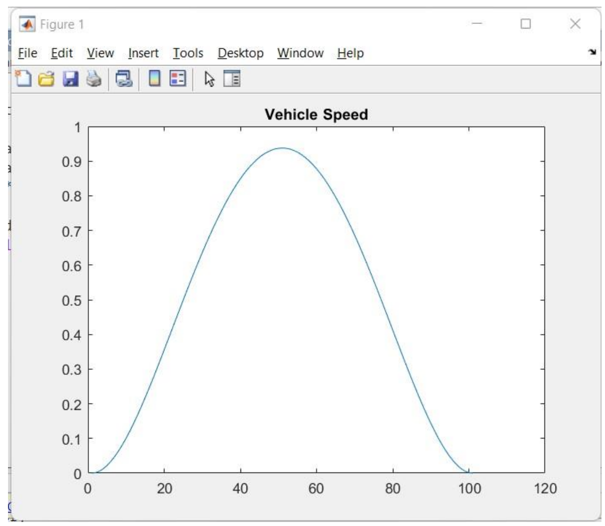

Fig.28 Vehicle Speed during lane changing maneuver.

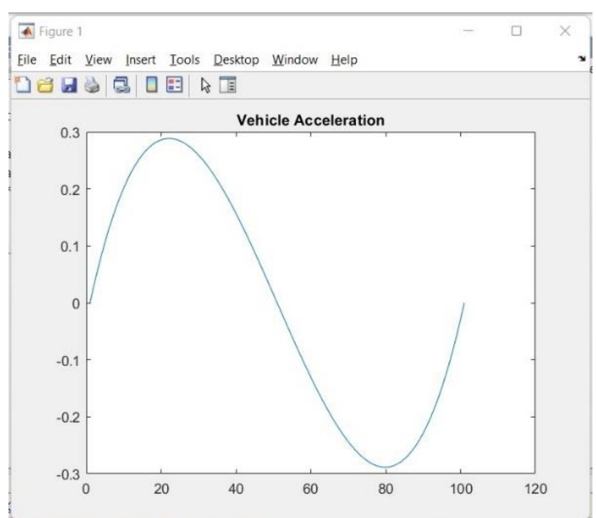

Fig.29 Vehicle Acceleration during lane changing maneuver

In our case, here we consider only speed output to complete the successful lane change. The Mode selector input parameters in Longitudinal block diagram are Relative distance between vehicle 2 and 3, Leader speed and Initial speed. Here Mode 3, i.e., Emergency Breaking is not taken into consideration as there is no car to obstruct (Vehicle in the side lane or incoming vehicle) for our lane changing simulation. Thus Mode 3 can be eliminated. Thus, for successful lane change Mode 1 and Mode 2 can be taken into consideration. In our case, Mode 1 was best scenario for increasing speed and overtaking ego vehicle 2 to shift from lane A to lane B. The speed output is represented below for longitudinal acceleration.

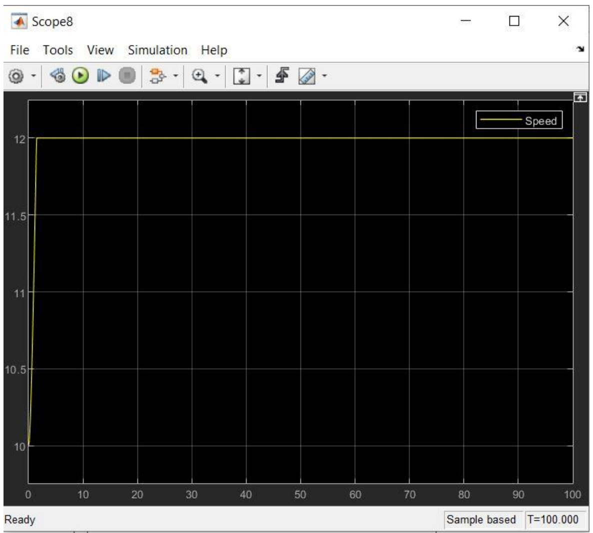

Fig.30 Mode 1 Speed output for Vehicle 3.

This speed output along with lateral speed which is obtained from Lane Changing code will give us input Yaw angle for Bicycle Model. This leads to following result for input Yaw angle which is represented below.

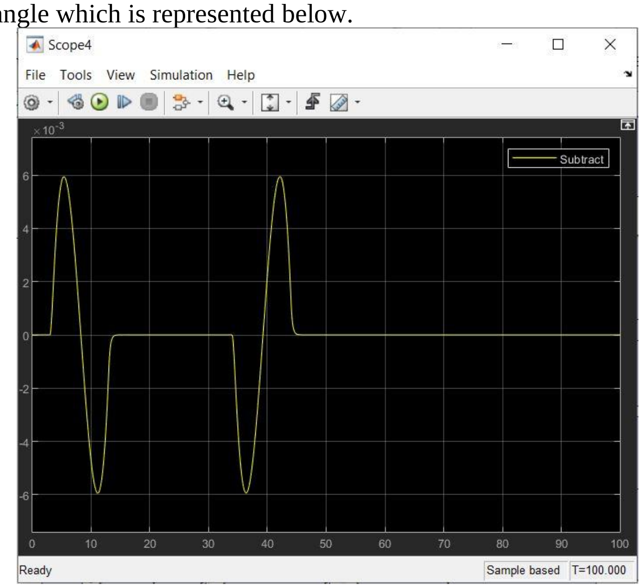

Fig.31 Input Yaw angle

This will give us Results for output position and Yaw angle after successful lane change scenario. A Demux is used to vary states to obtain the required results.

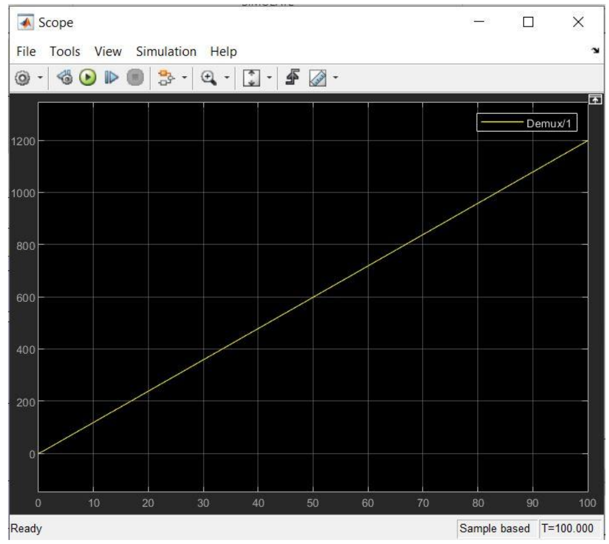

Fig.32 Position Output X Direction

This represents Position of Vehicle in X direction.

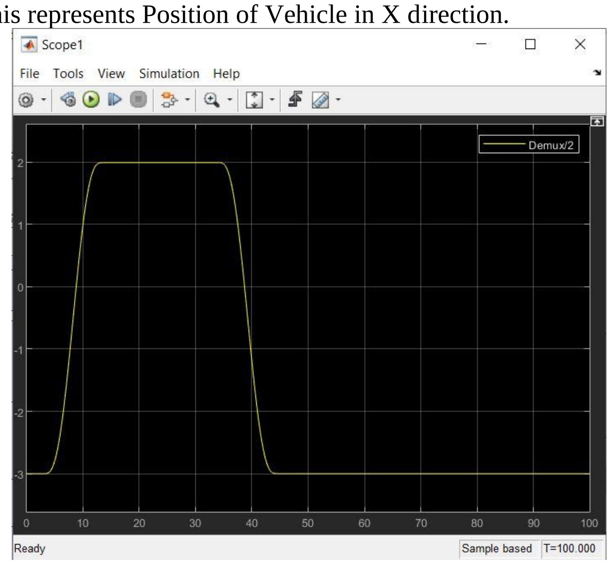

Fig.33 Position Output Y Direction

This represents Position of Vehicle in Y direction.

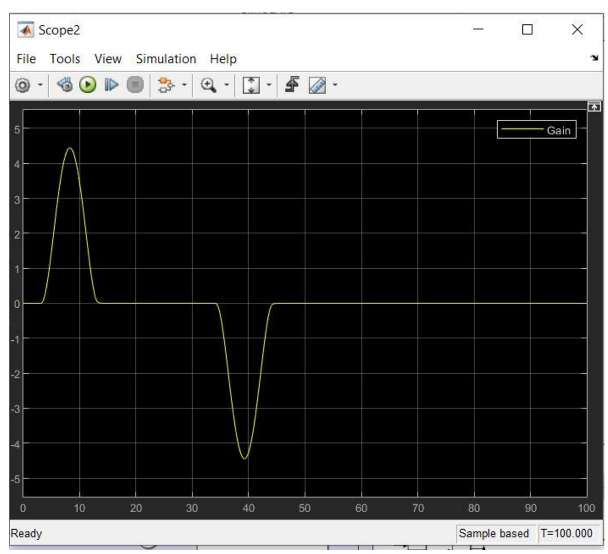

Fig.34 Output Yaw angle.

This Output Steering Angle is verified using the Dashboard Scope that we provided to check the entire lane change maneuver process as represented below.

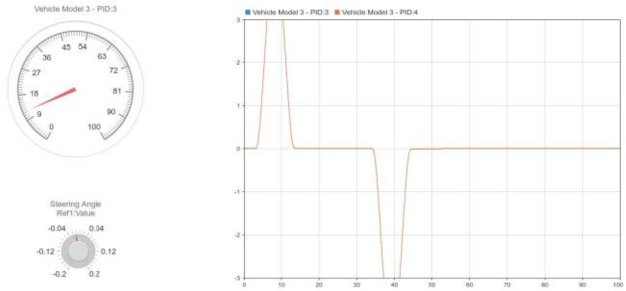

Fig.35 Dashboard Scope which represents Yaw angle and Vehicle

VII. RESULTS

A. Results for All Vehicles Simulation (Without Lane Change Trajectory)

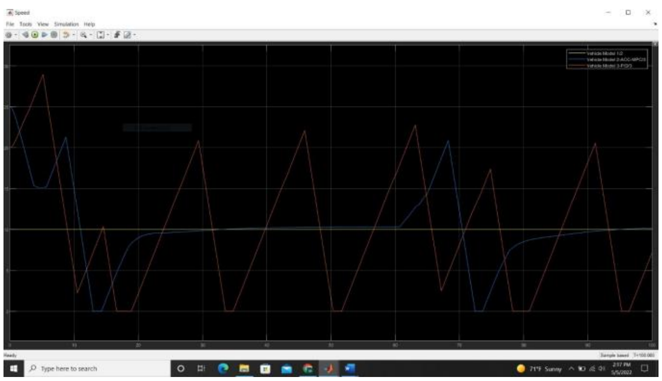

Fig.36 All Vehicles Speed.

As we can already see in above figure the vehicle 2 (MPC controlled) converges after maintaining relative distance that is provided in the State Flow Diagram which accesses the Mode selector in case to avoid collision and maintain smooth control of system. The system detects obstacle i.e., Vehicle A at 0.5 seconds and then applies breaks to gradually decrease its acceleration with respect to relative distance between them. As we ran the simulation keeping in mind of high risk of collision, The vehicle 2 came right to stop at 12 seconds and automatically picked up pace the next second thus maintaining smooth transition even under panic situation. Vehicle 2 then picks up pace with maintaining the relative speed and distance in mind and converges at 30 seconds. Also, Vehicle 2 picking up pace after decelerating is kept avoiding impact from rear vehicle. Thus, driver and passenger comfort are maintained throughout the scenario. In case of Vehicle 3 (PID controlled), as it is a single input- single output system the system although it avoids collision, The downside is its sudden acceleration and deceleration. This creates uncomfortable driving scenarios. In case of Vehicle 3 it detects the collision at 6th second and applies panic emergency breaking. This results to vehicle gradual stop at 17th second, but due to aberrant breaking the system needs time to initiate its longitudinal acceleration. This results in decrease in response of 2-3 seconds as compared to MPC controller as we can see in the above figure which could be deadly in real life scenario. This can be further verified by analyzed by visualizing change in relative distance of both vehicles.

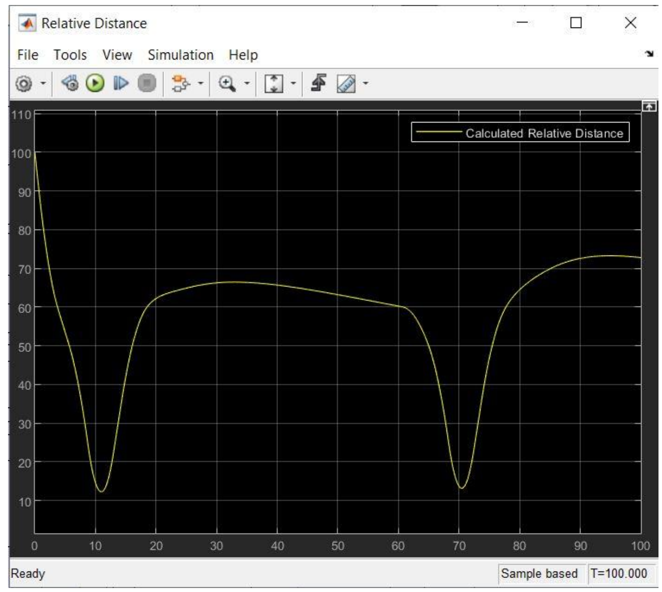

Fig.37 Relative Distance between Vehicle 1 and Vehicle 2(MPC Controlled).

As results shows smooth transition is maintained throughout the scenario.

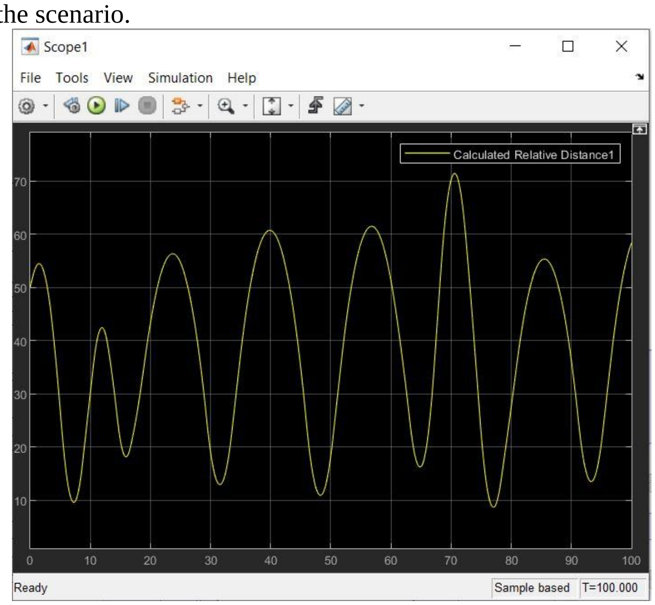

Fig.38 Relative Distance between Vehicle 2 and Vehicle 3(PID Controlled).

Due to sudden acceleration and deceleration the transition is not smooth thus lead to harsh driving scenarios and uncomfortable system for driver and passengers.

Thus, we can conclude that MPC is superior system then PID as it shows better performance when tested in same scenarios.

B. *Results for All Vehicles Simulation (With Lane Change Trajectory initiated by Vehicle 3)*

Fig. All vehicle speeds in Lane changing System. As the result suggests both vehicles 1 and 2 maintain consistent distance and system converges for both at 40 seconds where they maintain relative speed and distance. The lane changing is initiated by Vehicle 3 at relative distance of 10 meters and the speed of vehicle 3 increases gradually to overtake both vehicles 1 and 2. After completing the lane change sequence in 3-4 seconds the vehicle 3 gradually increases speed in order complete the maneuver. The smooth overtaking of both vehicles is completed in 60th second, as after this the vehicle 3 can go back to its initial lane or continue the same lane with Free drive mode activated. The Mode-2 PID controller is utilized during lane changing and as soon as the switch happens the system goes back to Mode 1 Free Drive mode as there are no obstacles in front of the vehicle.
We can further verify the change in relative distance to visualize the system-

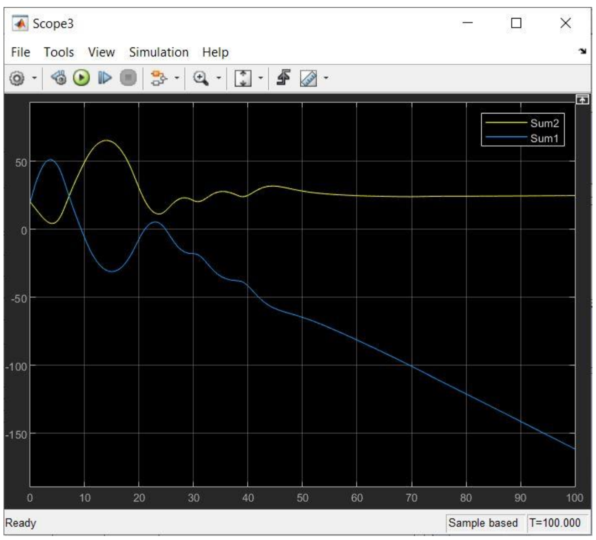

Fig.39 Relative Distance between System A (Vehicles 1 and 2) and System B (Vehicles 2 and 3)

Vehicles 1 and 2 converge at 40 seconds and retain constant relative speed and distance following convergence, as shown in the diagram. During the identical situation, a lane change is initiated at the 5th second, and as soon as the speed differential between vehicles 1 and 2 narrows at the 23rd second, Vehicle 3 overtakes both and contends for the overtaking by the 25th second. Following that, as vehicle 3 overtakes both vehicles and retains its free mode status, the relative distance gradually grows regarding time.

VIII. CONCLUSION

We can verify that Car 2 (E class Sedan) managed by MPC is more stabilized and converges after 40 seconds of sampling time after reviewing the data from the Full system simulation. Because relative speed is not considered, PID is more difficult to converge. To avoid a collision, the emergency brake is activated, and after initial stabilization, the system returns to normal operation. As a result, we apply the Minimum Jerk Trajectory Method to further stabilize the system and improve the vehicle's driving comfort. Even after emergency braking, there is a substantial risk of collision. To ensure that Vehicle 3 (PID controlled) effectively executes the overtaking mechanism and avoids collision, the driving mode selector is turned off.

IX. FUTURE WORK

- The project focuses on a lane changing maneuver for straight future it can be extended for a curve road.
- The project works on collision avoidance system for one way street in future collision avoidance systems can be developed of two-way streets as well.
- In two-way street the vehicle would detect the vehicle coming in other direction and abort the lane changing maneuver
- This project was based obstacle detection and collision avoidance, where the obstacle was another car. In future we can also consider human or any other obstacle detection.
- Try implementing this system on intersection with unpredictable obstacles.

X. REFERENCE

1. "Road traffic injuries," *World Health Organization*. [Online]. Available: https://www.who.int/news-room/fact-sheets/detail/road-traffic-injuries
2. R. Rajamani, Vehicle Dynamics and control. New York: Springer, 2012.
3. "Adaptive Cruise Control System," *MATLAB & Simulink*. [Online]. Available: https://www.mathworks.com/help/mpc/ug/adaptive-cruise-control-using-model-predictive-controller.html. [Accessed: 06-May-2022].
4. "Adaptive MPC controller," *MATLAB & Simulink*. [Online]. Available: https://www.mathworks.com/help/mpc/ug/obstacle-avoidance-using-adaptive-model-predictive-control.html. [Accessed: 06-May-2022].
5. "PID theory explained," *NI*. [Online]. Available: https://www.ni.com/en-us/innovations/white-papers/06/pid-theory-explained.html. [Accessed: 06-May-2022].
6. Zainal, Zainab & Rahiman, Wan & Baharom, Nor Ramdon. (2017). Yaw Rate and Sideslip Control using PID Controller for Double Lane Changing. Journal of Telecommunication, Electronic and Computer Engineering. 9. 99-103.
7. Yang, Qianwei & Shi, Qihang & Saraoğlu, Mustafa & Janschek, Klaus. (2021). A Hybrid Approach using an Adaptive Waypoint Generator for Lane-changing Maneuver on Curved Roads. 10.13140/RG.2.2.30512.58886.
8. "Understanding model predictive control," *MATLAB & Simulink*. [Online]. Available: https://www.mathworks.com/videos/series/understanding-model-predictive-control.html. [Accessed: 06-May-2022].
9. Shuping Chen and Huiyan Chen 2020 *IOP Conf. Ser.: Mater. Sci. Eng.* **892** 012034