CSC 544
Data Visualization

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Lecture 24
FlowVis: Features and Time-Varying Flow

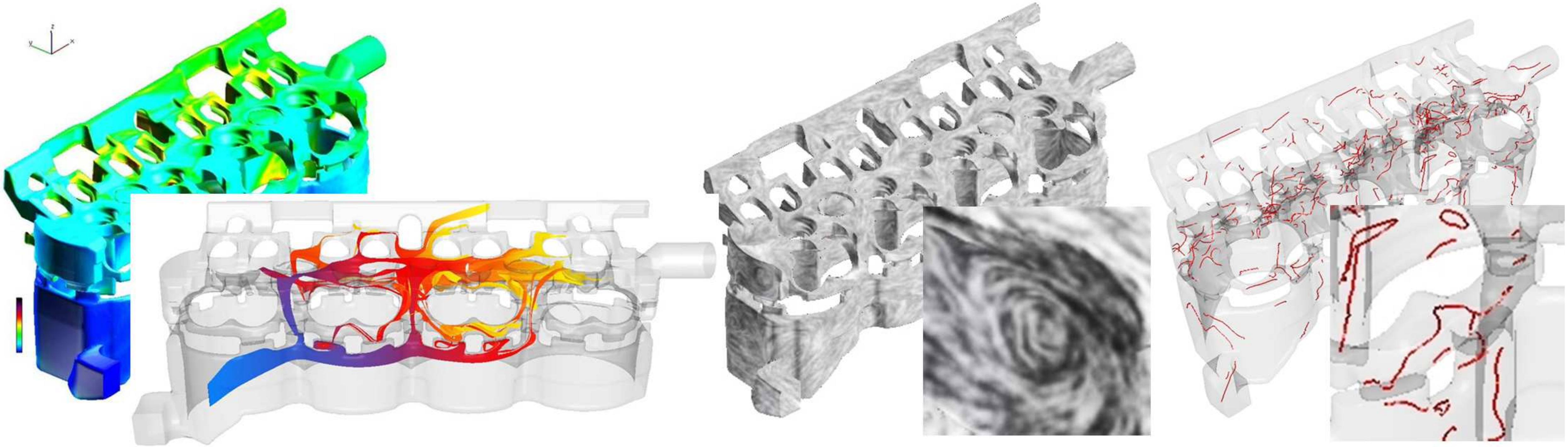
Apr. 12, 2023

Today's Agenda

- Reminders:
  - A06 questions? (due Apr. 24)
  - P03/P04 questions? (due Apr. 26/May 3)
  - Student Course Surveys (SCSs) (due May 3)
    - Currently at 26% class completed (80% threshold unlocks extra credit!)
- Goals for today: Discuss concepts relating to feature-based flow visualization and time-varying flow data

Classification of Visualization Techniques

- **Direct:** overview of vector field, minimal computation, e.g. glyphs, color mapping
- **Texture-based:** covers domain with a convolved texture, e.g., Spot Noise, LIC, ISA, IBFV(S)
- **Geometric:** a discrete object(s) whose geometry reflects flow characteristics, e.g. streamlines
- **Feature-based:** both automatic and interactive feature-based techniques, e.g. flow topology



Feature-Based Methods

- **Feature Extraction**
  - **Features:**
    - Represent interesting structures or objects in the data
    - Extract and visualize them
    - The original (large) field is not needed anymore for visualization
    - Extraction can (usually) be automated
    - Rendering is fast and (usually) interactive
    - Medium to high cognitive load
  - **Typical features:**
    - Topological features
    - Vortices
    - Shock waves
    - Extremal structures

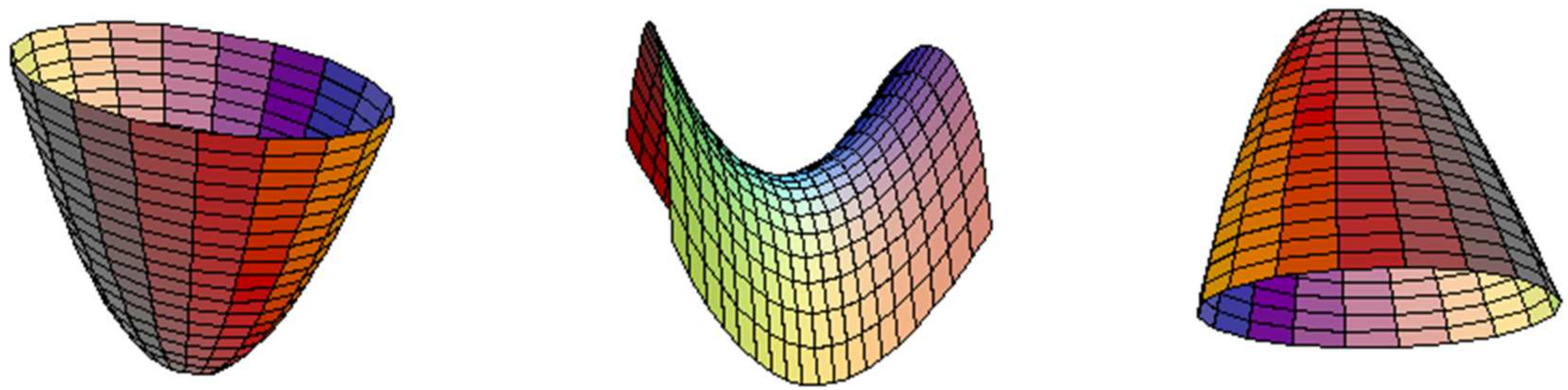
Topological Features of Vector Fields

Recall: What is Topology?

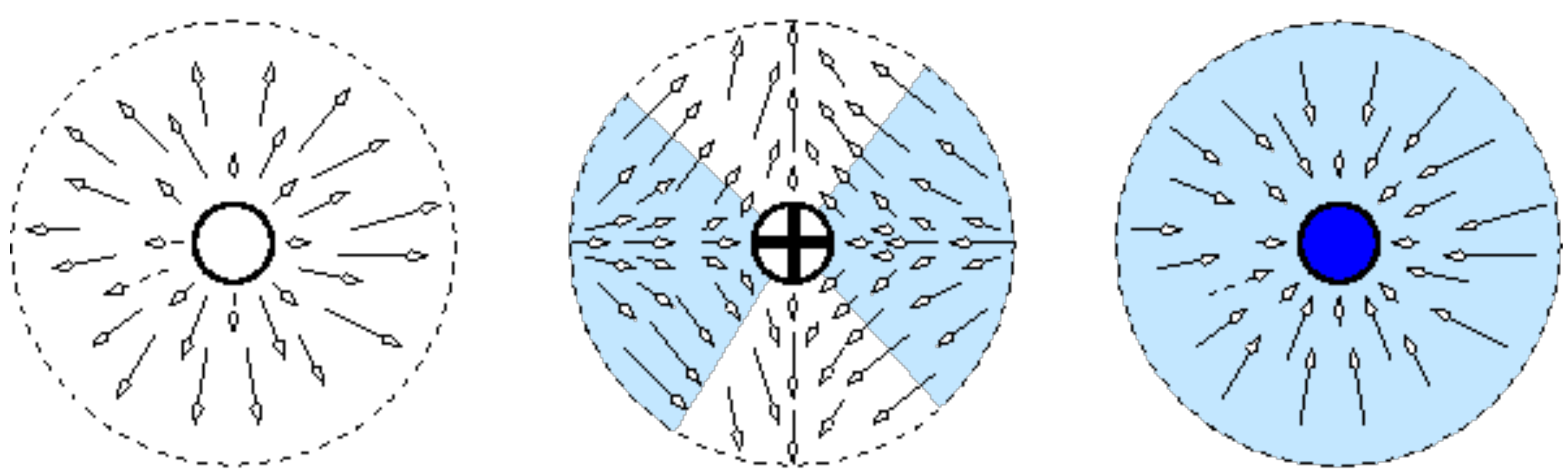
- Field of mathematics which studies properties which are **preserved under continuous transformations**.
  - Stretching, bending = continuous changes.
  - Tearing, gluing = discontinuous changes.
- Also called: “Rubber sheet” geometry.
- Studies the connectedness of a space.

Critical Points

- Critical points in 2D have three possible behaviors

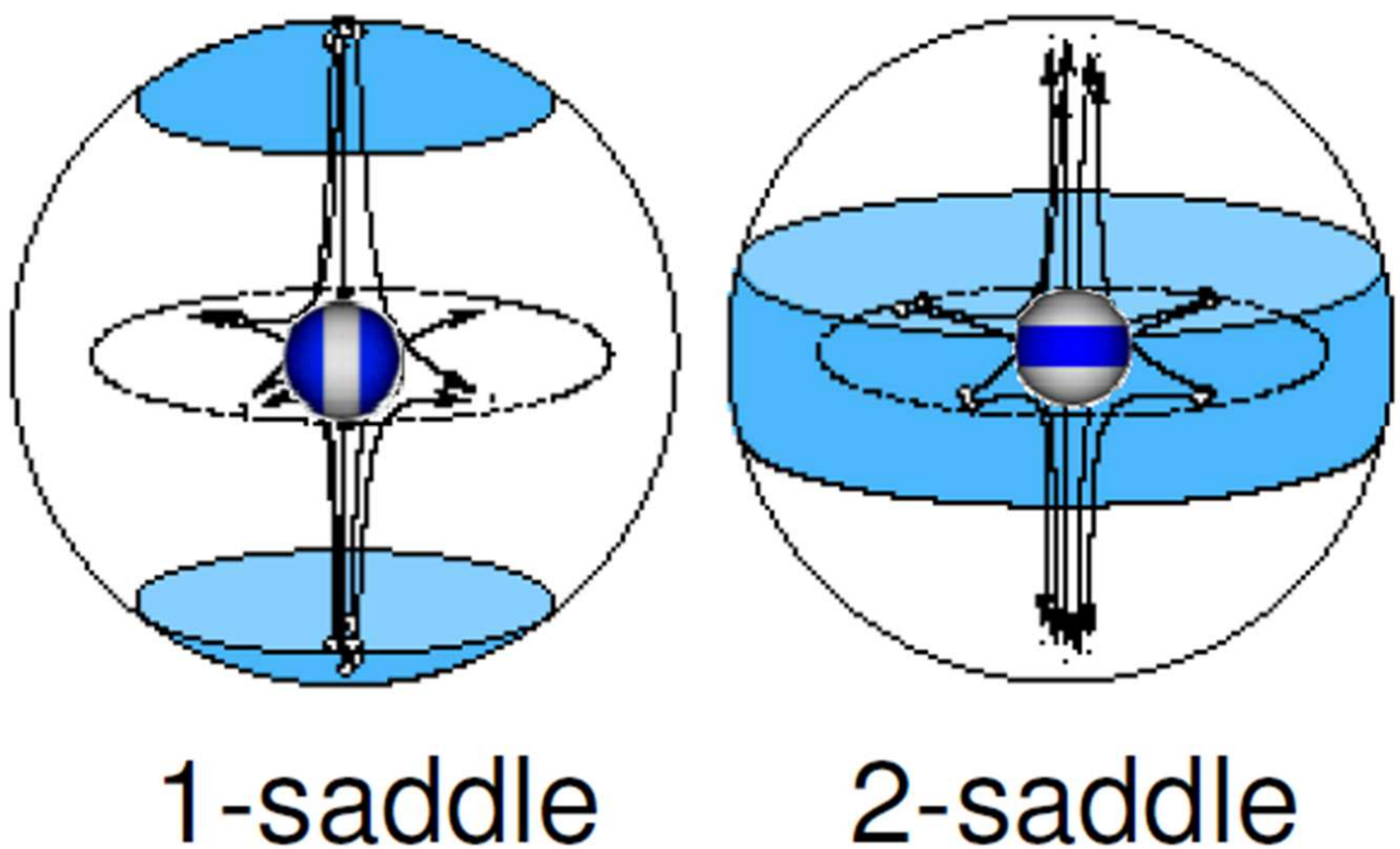


Minimum **Saddle** **Maximum**

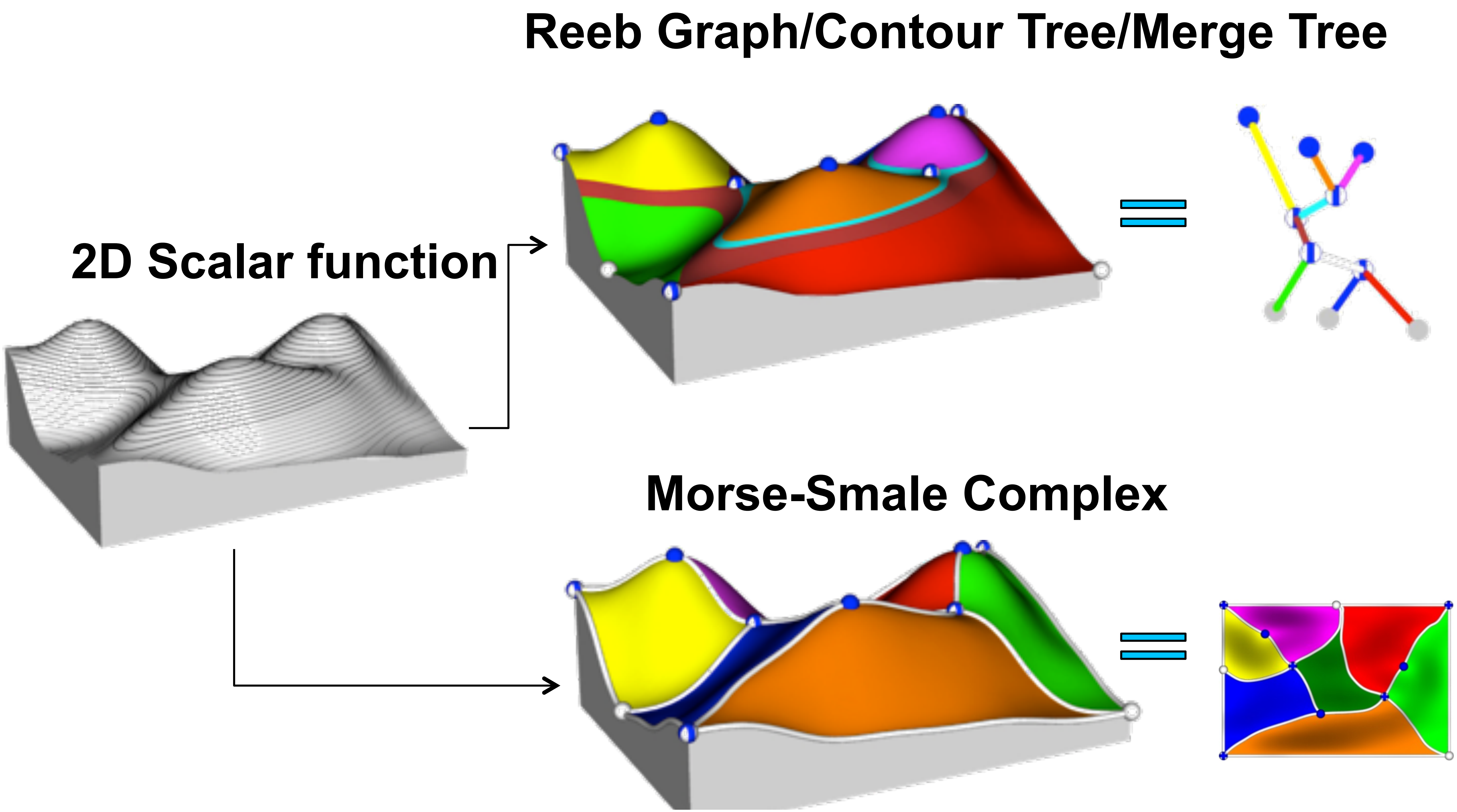


Detection of Critical Points

3D saddles can have two distinct configurations



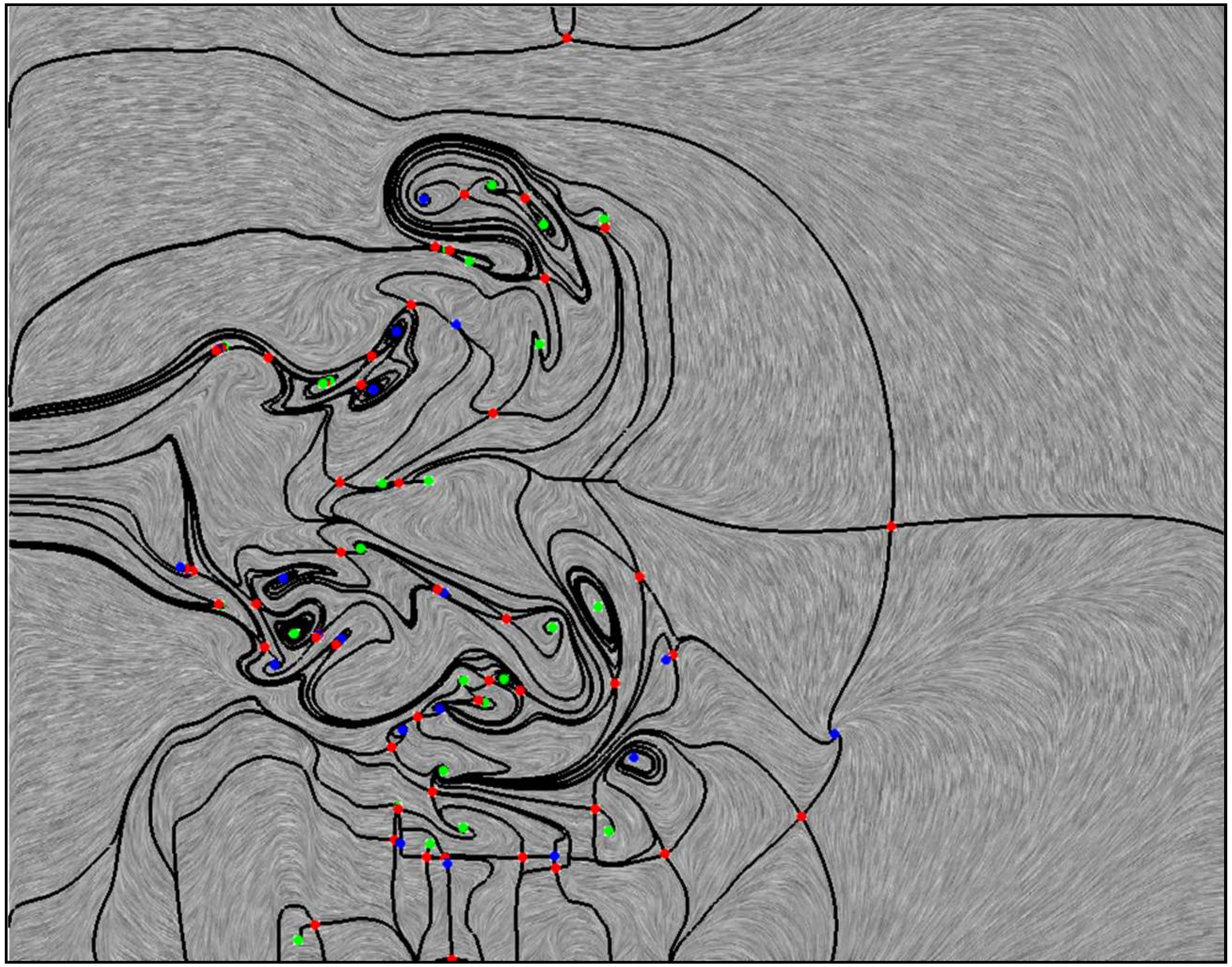
Two Types of Topological Structures For Scalar Data

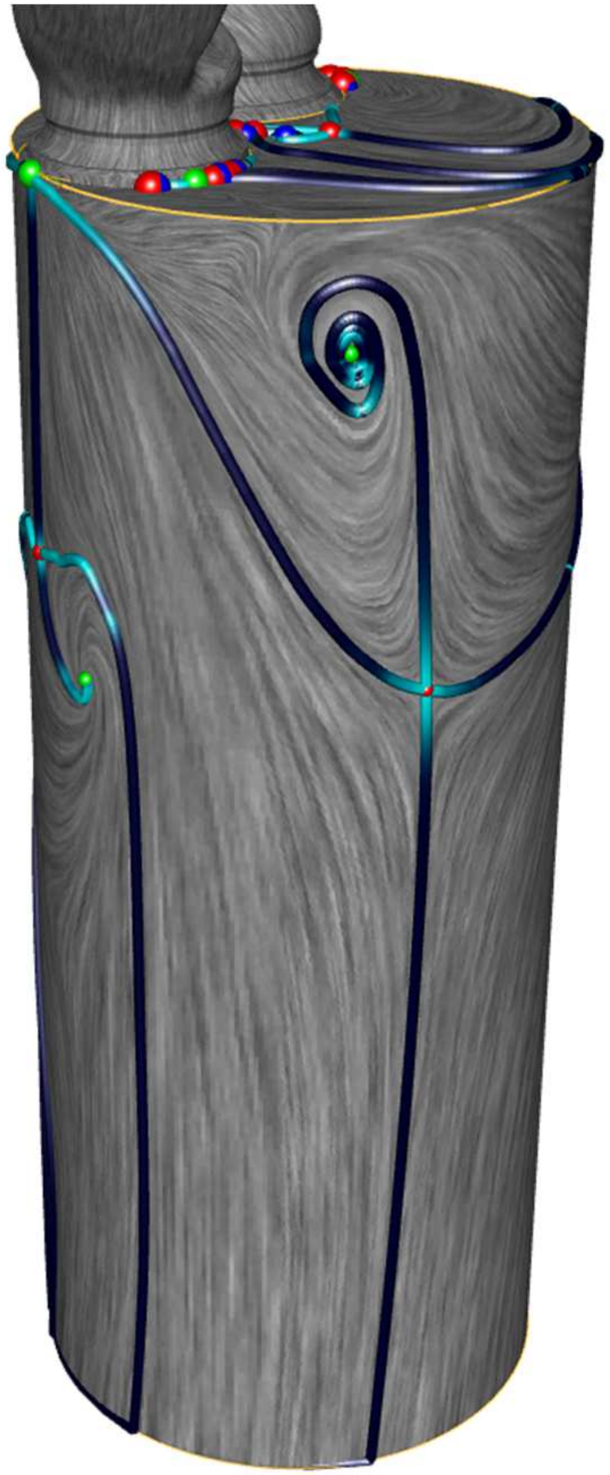


Vector Field Topology

Topological Skeleton and Morse Decomposition

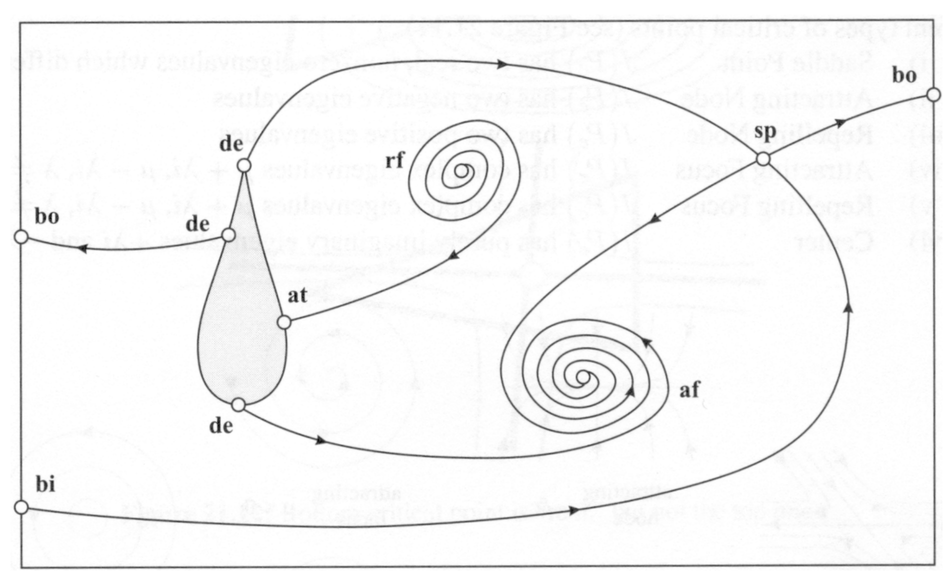
Examples





Vector Field Topology

- Idea:
  Do not draw “all” streamlines, but only the “important” streamlines
- Show only topological skeletons
- Important points in the vector field: critical points
- Critical points:
  – Points where the vector field vanishes: v = 0
  – Points where the vector magnitude goes to zero and the vector direction is undefined
  – Sources, sinks, …
- The critical points are connected to divide the flow into regions with similar properties
- Structure of particle behavior for t → ∞



Critical Points

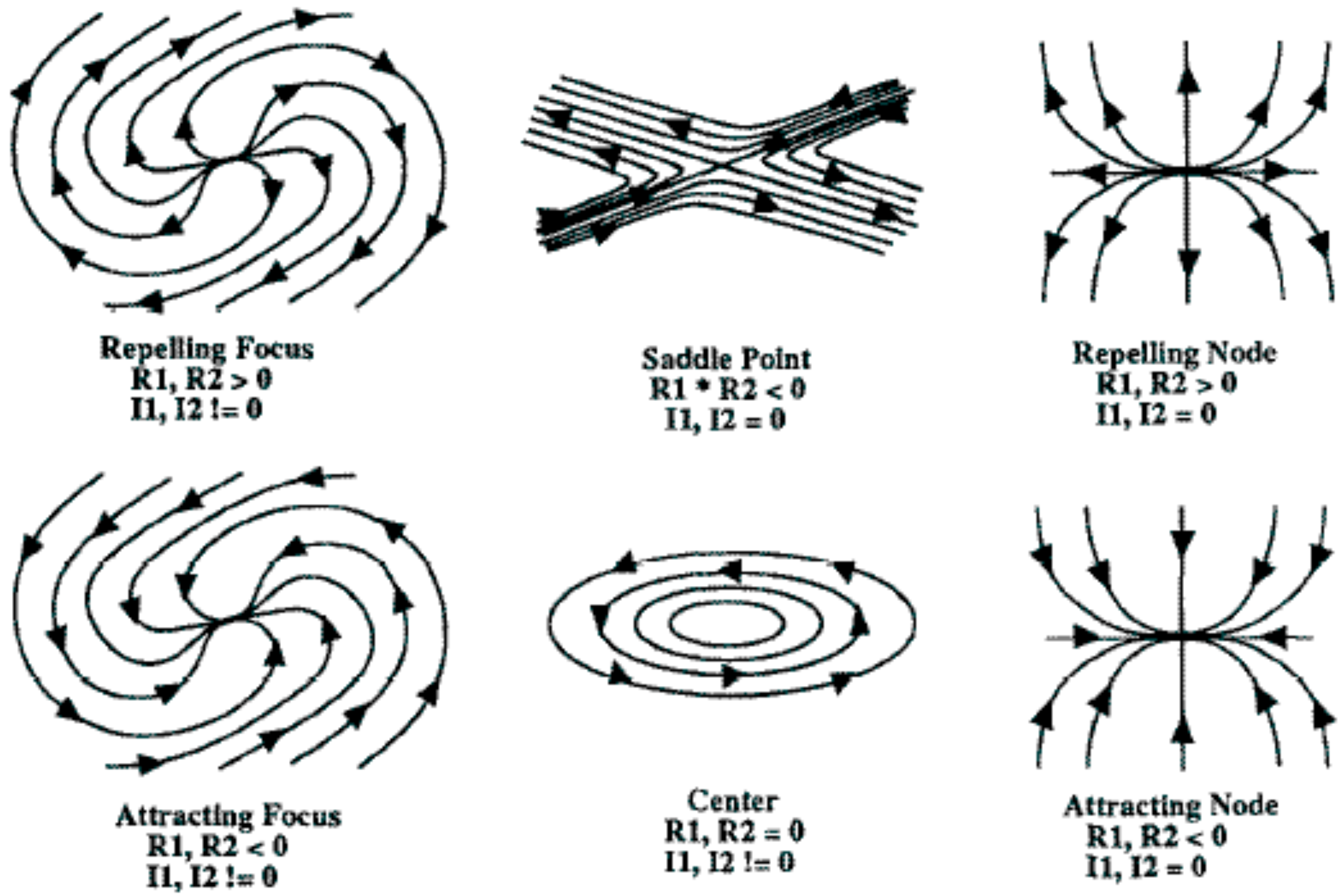


Figure 1: Classification criteria for critical points. R1 and R2 denote the real parts of the eigenvalues of the Jacobian, I1 and I2 the imaginary parts.

Image: Surface representations of 2- and 3-dimensional fluid flow topology, Helman & Hesselink

Sectors & Separatrices

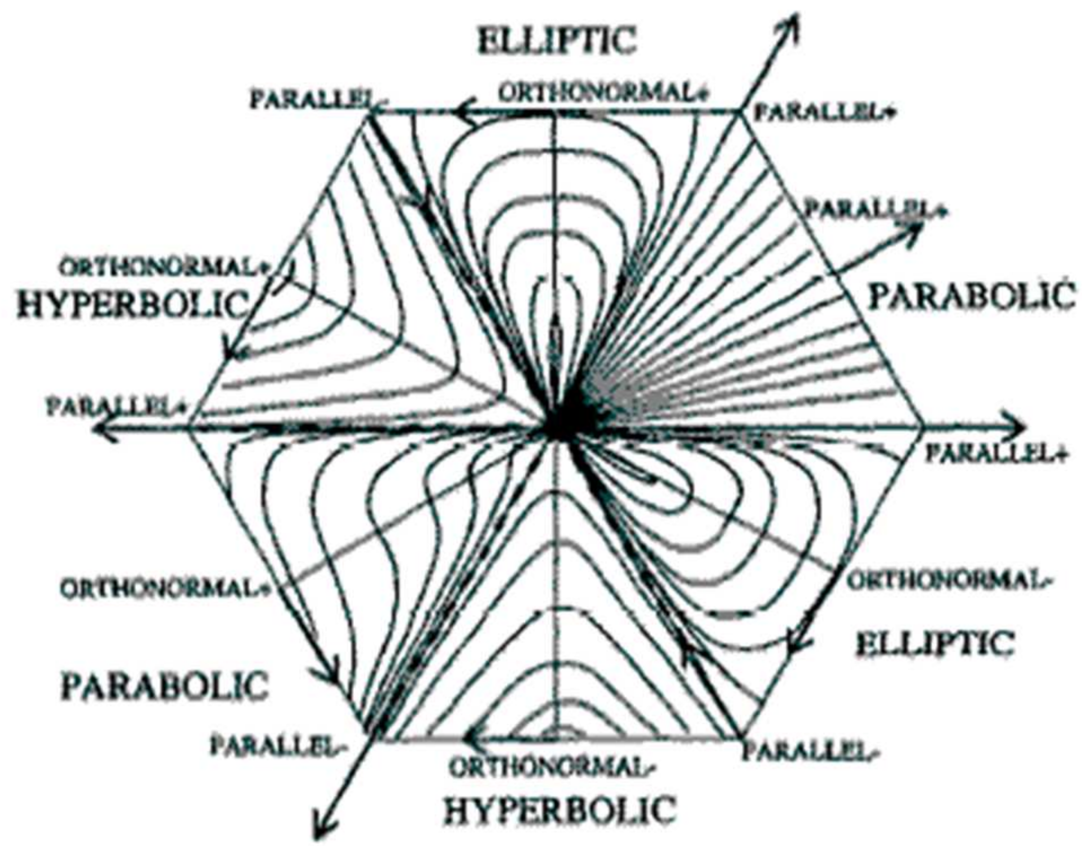


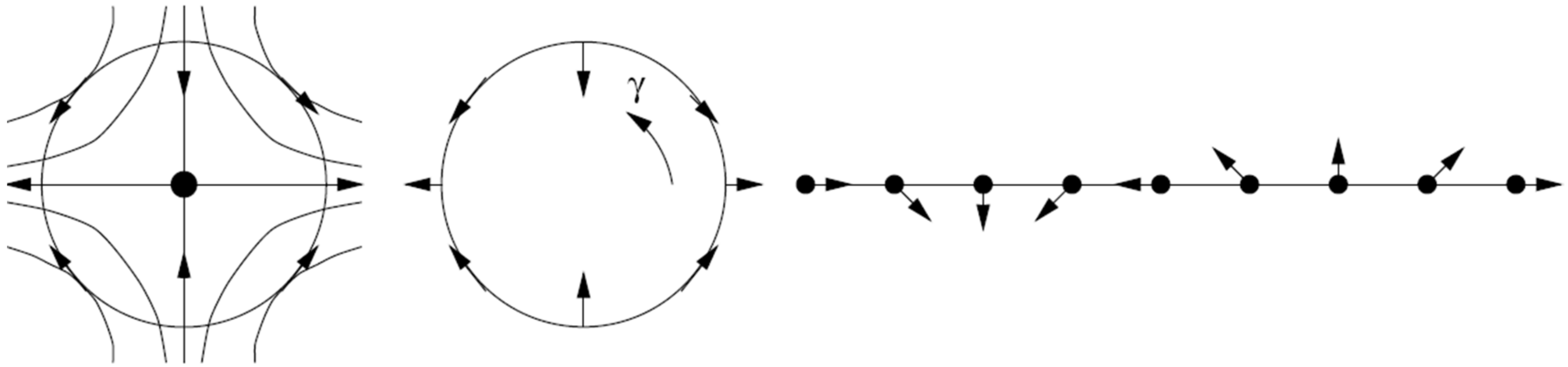
Figure 11: Example of sector type identification

Source: A topology simplification method for 2D vector fields. Xavier Tricoche, Gerik Scheuermann, & Hans Hagen

$$I = 1 + \frac{e - h}{2}$$

Poincaré Index

- **Poincarè index** I(Γ, V) of a simple closed curve Γ in the plane relative to a continuous vector field is the number of the positive field rotations while traveling along Γ in positive direction.

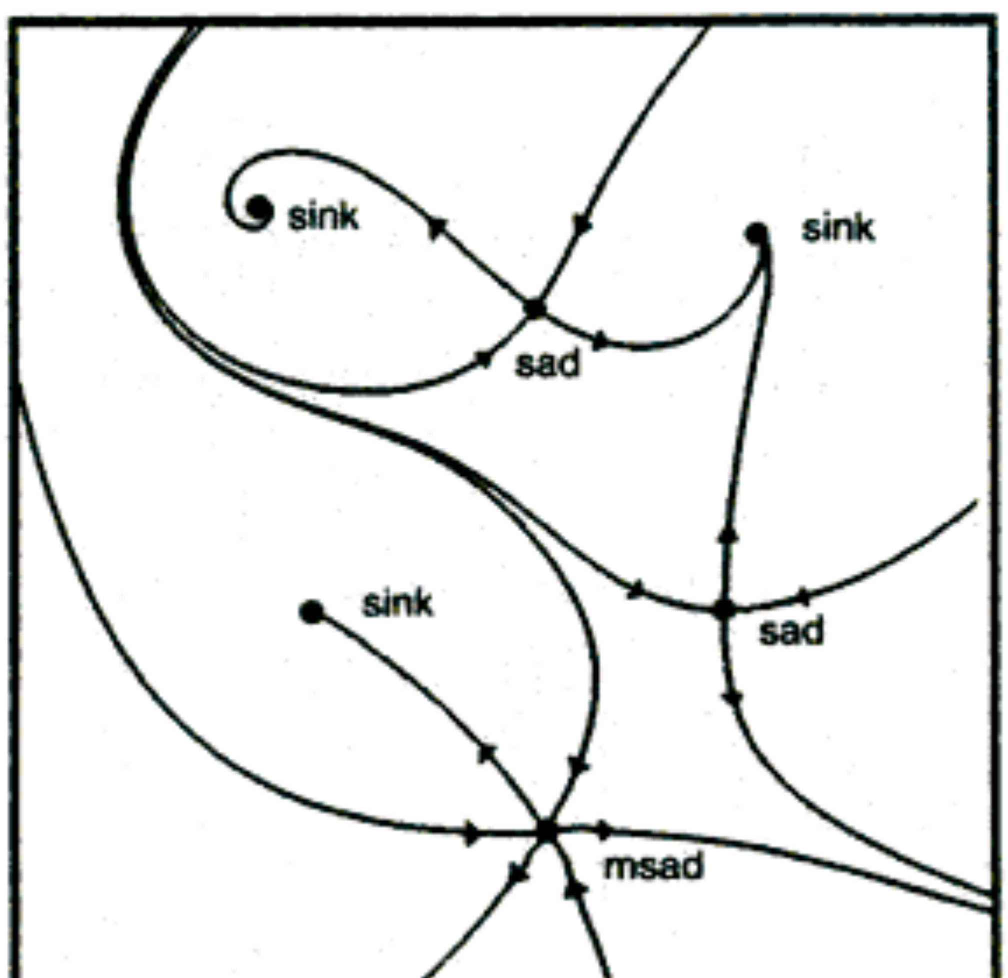


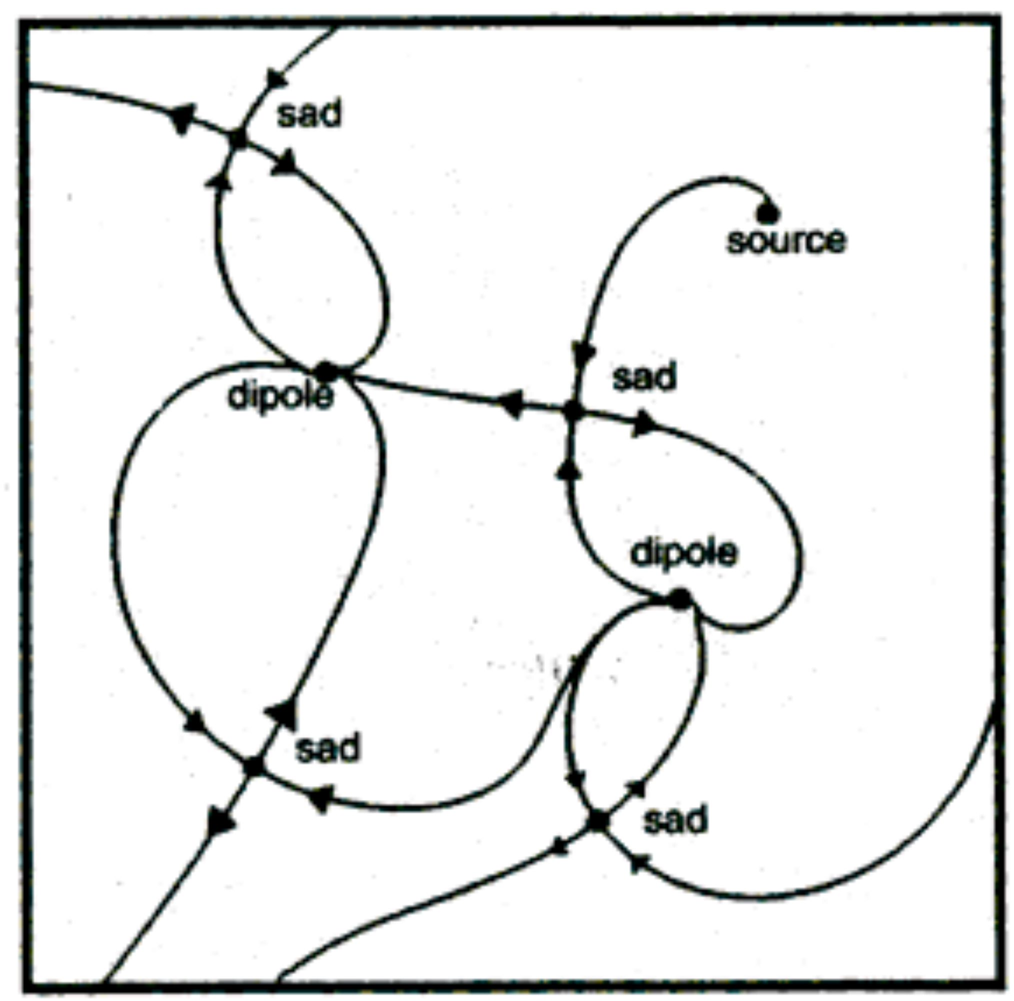
[Tricoche Thesis2002]

- By continuity, always an integer
- The index of a closed curve around multiple fixed points will be the sum of the indices of the fixed points

Topological Skeleton

- Graph connecting the critical points using separatrices





Vector Field Topology

1. Find critical points

- pretty much iso-value algorithm
- but with a twist - since three components are zero
- find iso-values for each component and then only consider cells where all three intersect
- not enough - sub-divide potential cells until a certain bound is reached.

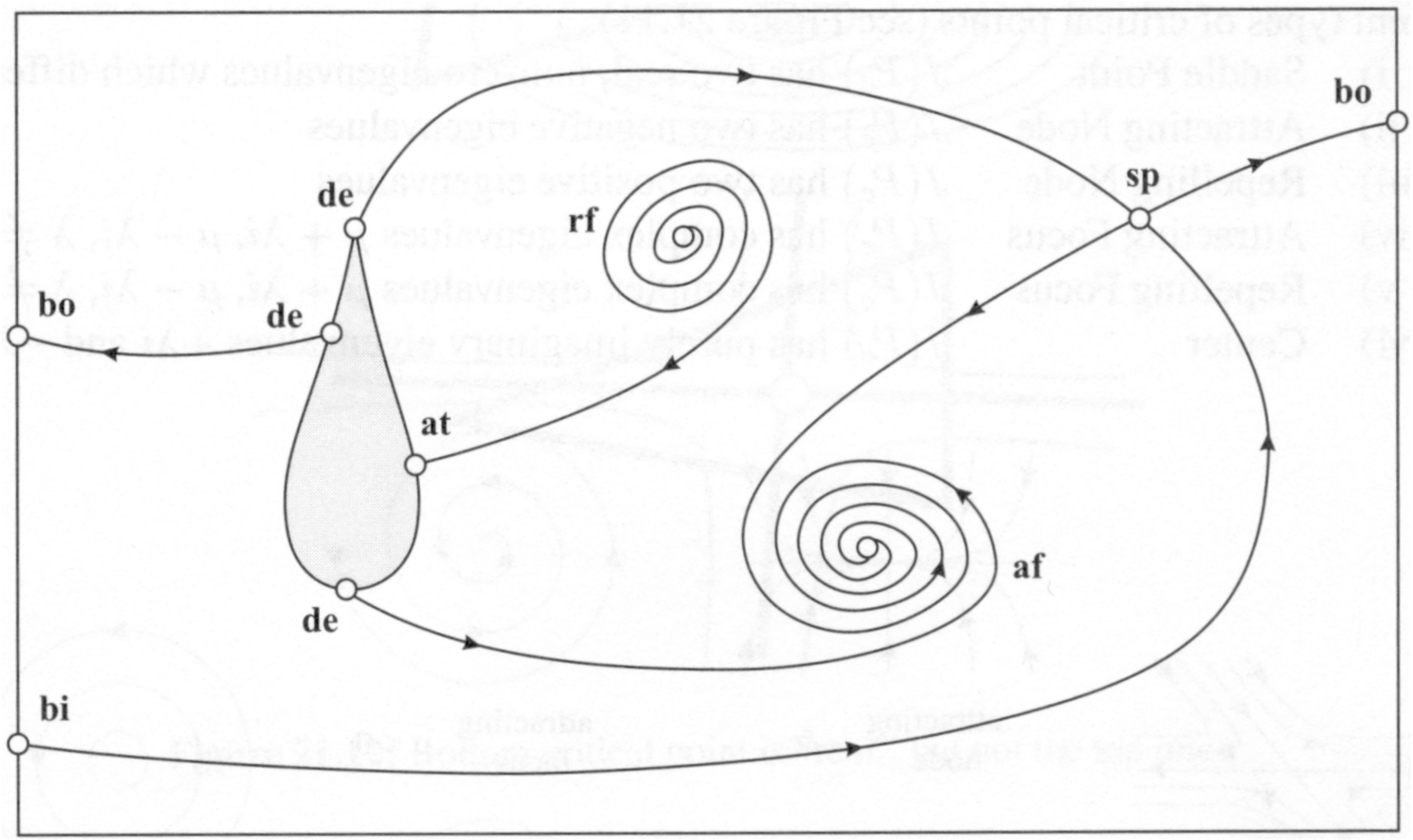
Vector Field Topology

2. classify critical points

- according to what is happening in the neighborhood - attracting or repelling or a combination thereof
- determined by derivative of velocity
- if positive then things move away
- if negative things come closer
- this is 1D

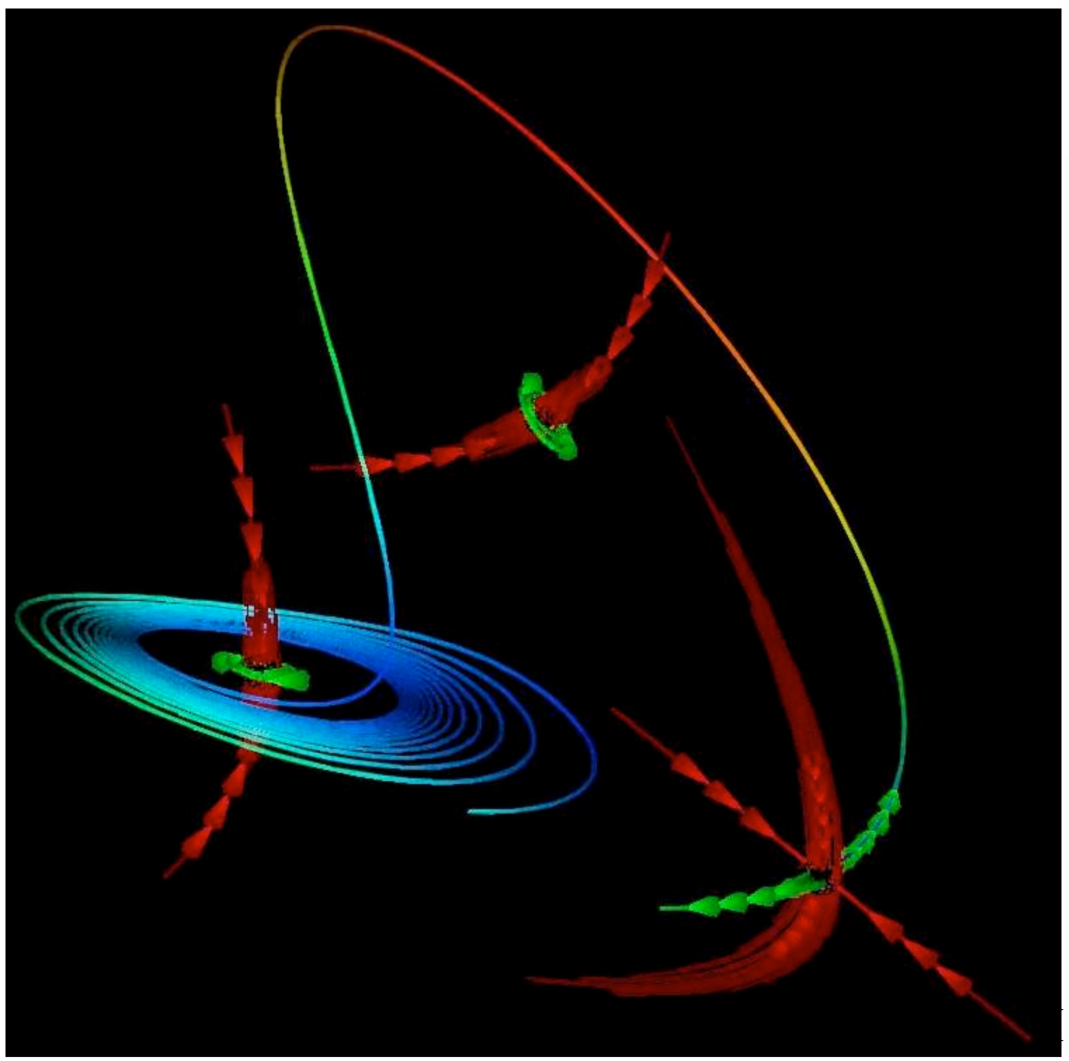
Vector Field Topology

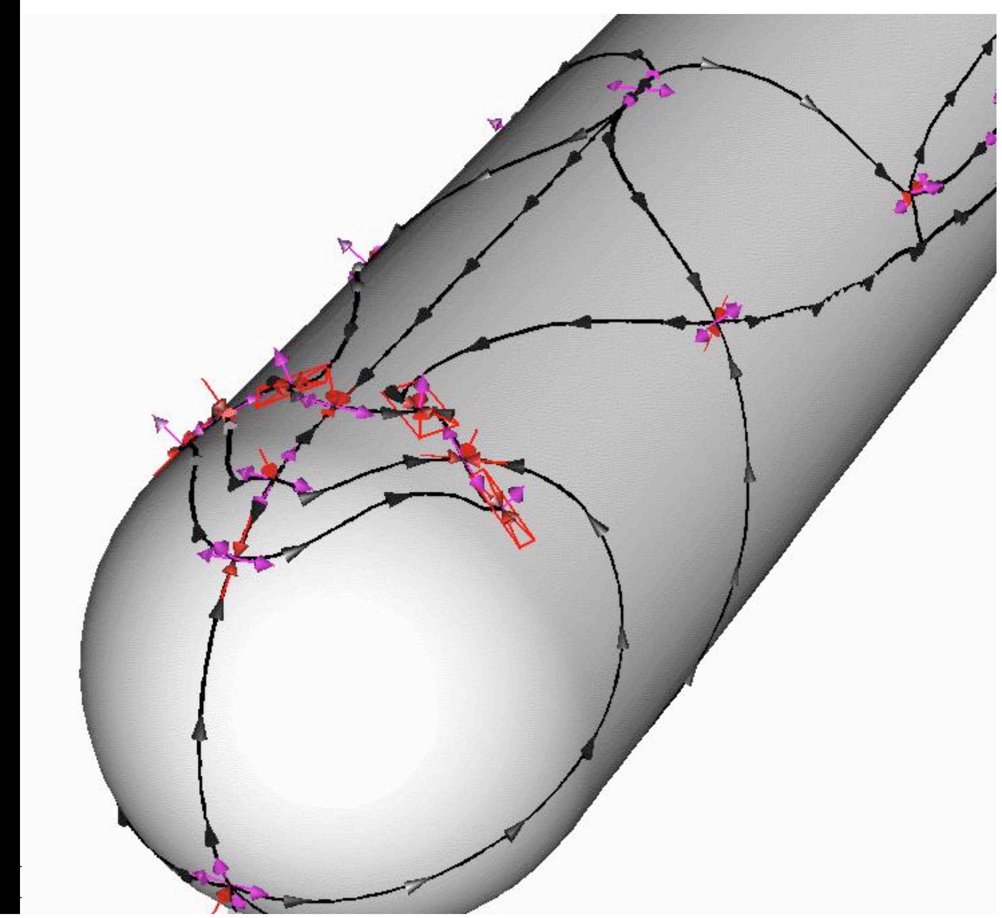
- Example of a topological graph of 2D flow field



Vector Field Topology

- Further examples of topology-guided streamline positioning





Limit Sets

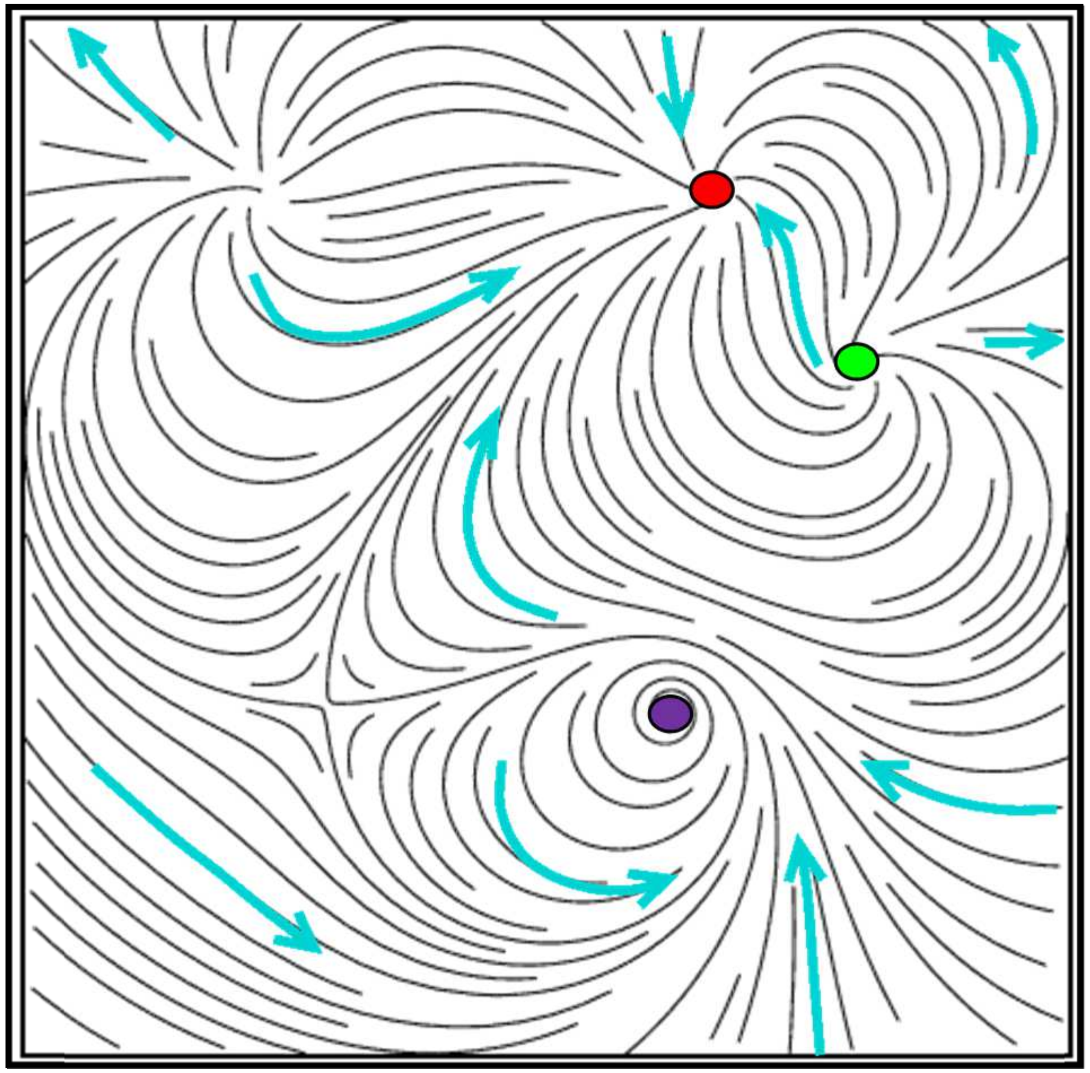
- Limit sets reveal the long-term behaviors of vector fields, correspond to flow recurrence
- The **limit sets** are:

$$\alpha(x)=\cap_{t<0} cl(\varphi((-\infty, t), x))$$

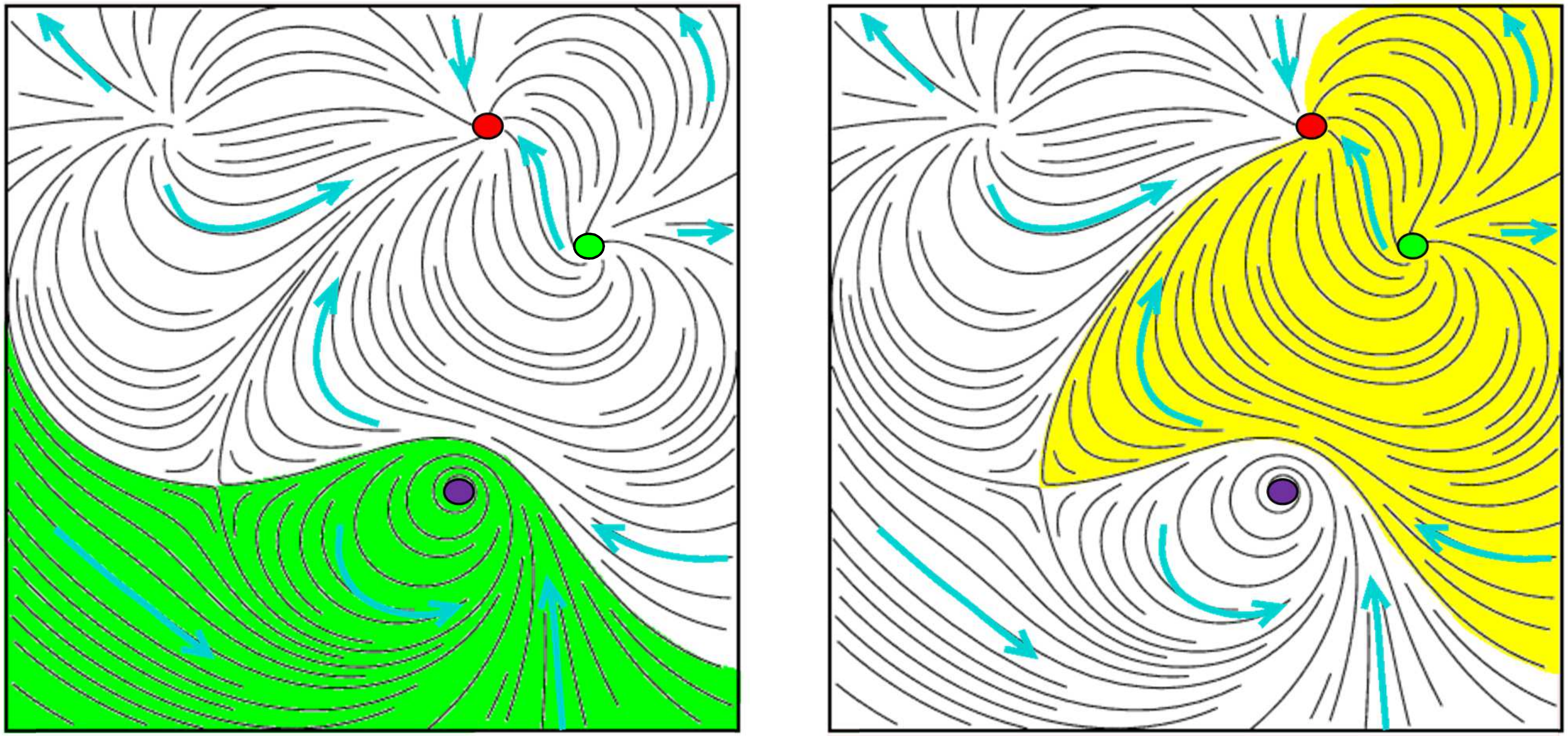
point (or curve) reached after ***forward*** integration by streamline seeded at x

$$\omega(x)=\cap_{t>0} cl(\varphi((t, \infty), x))$$

point (or curve) reached after ***backward*** integration by streamline seeded at x



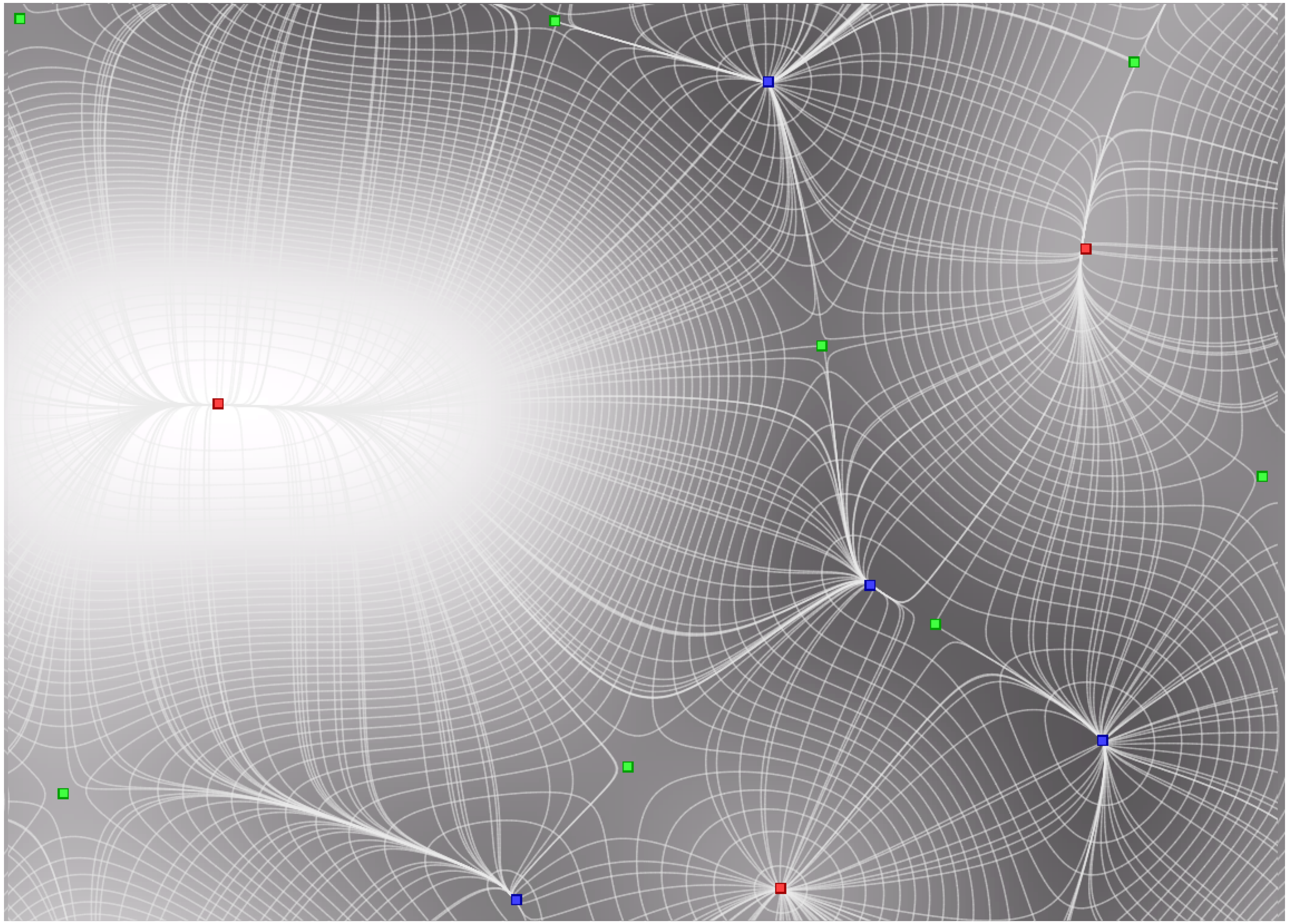
Repellor and Attractor Manifolds



Recall Features of a 2-dimensional Scalar function

"Monotonicity" = pieces of contours grouped by integral lines

$$\frac{\partial}{\partial t}L(t) = \nabla f(L(t))$$



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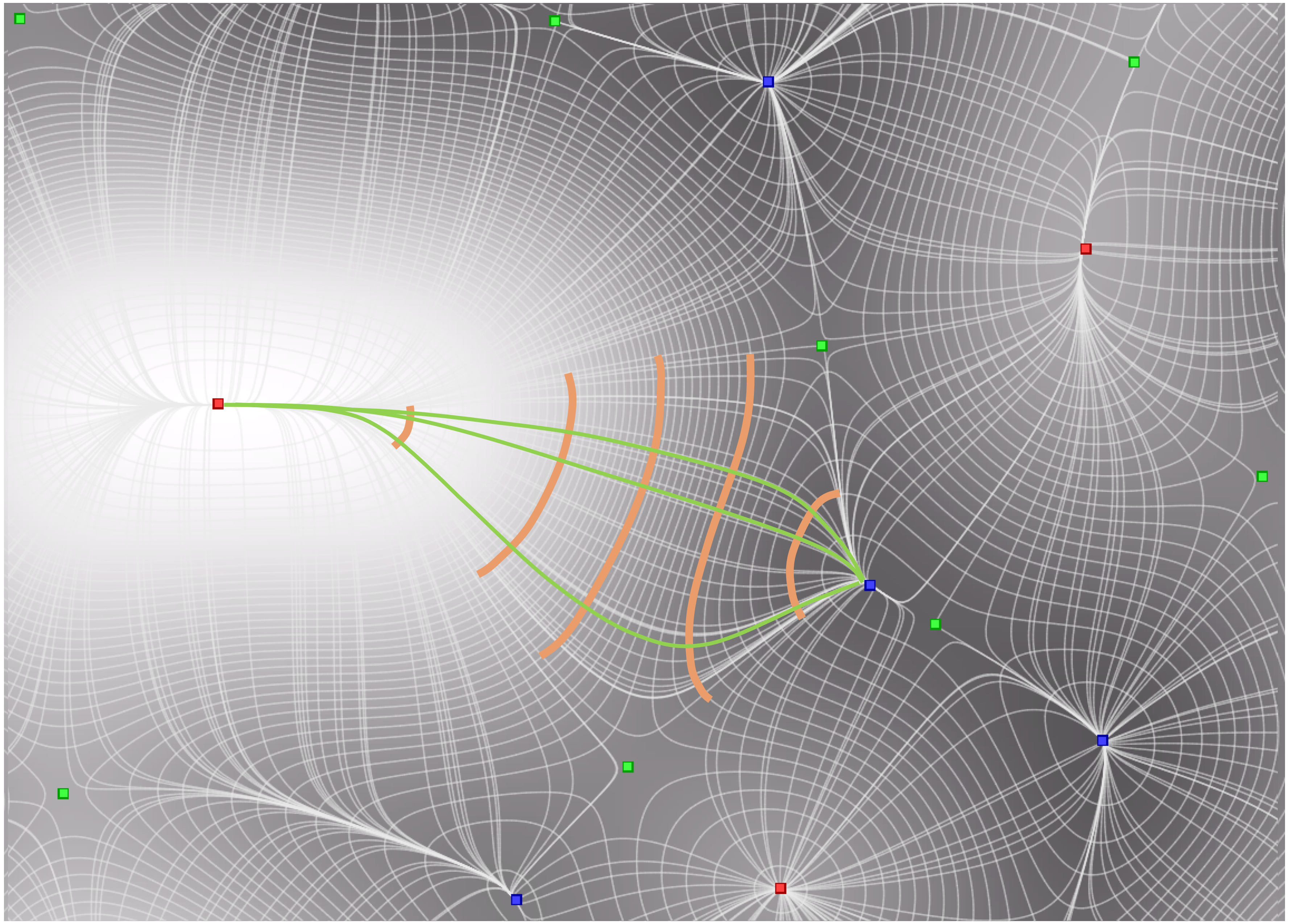
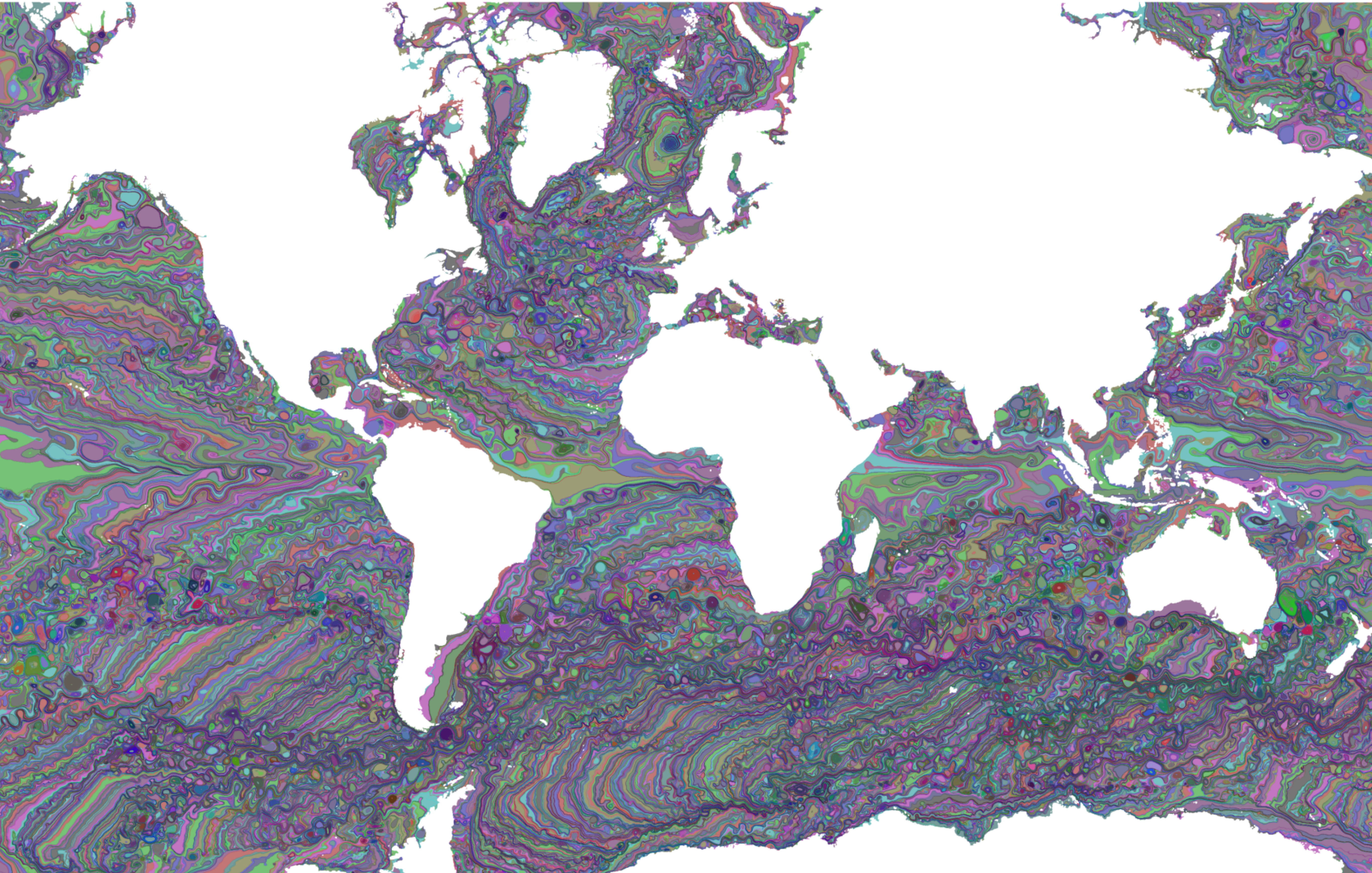


Levine, Jadhav, Bhatia, Pascucci, Bremer, CGF 31(3) 2012

Stable Manifolds of Ocean



Levine, Jadhav, Bhatia, Pascucci, Bremer, CGF 31(3) 2012

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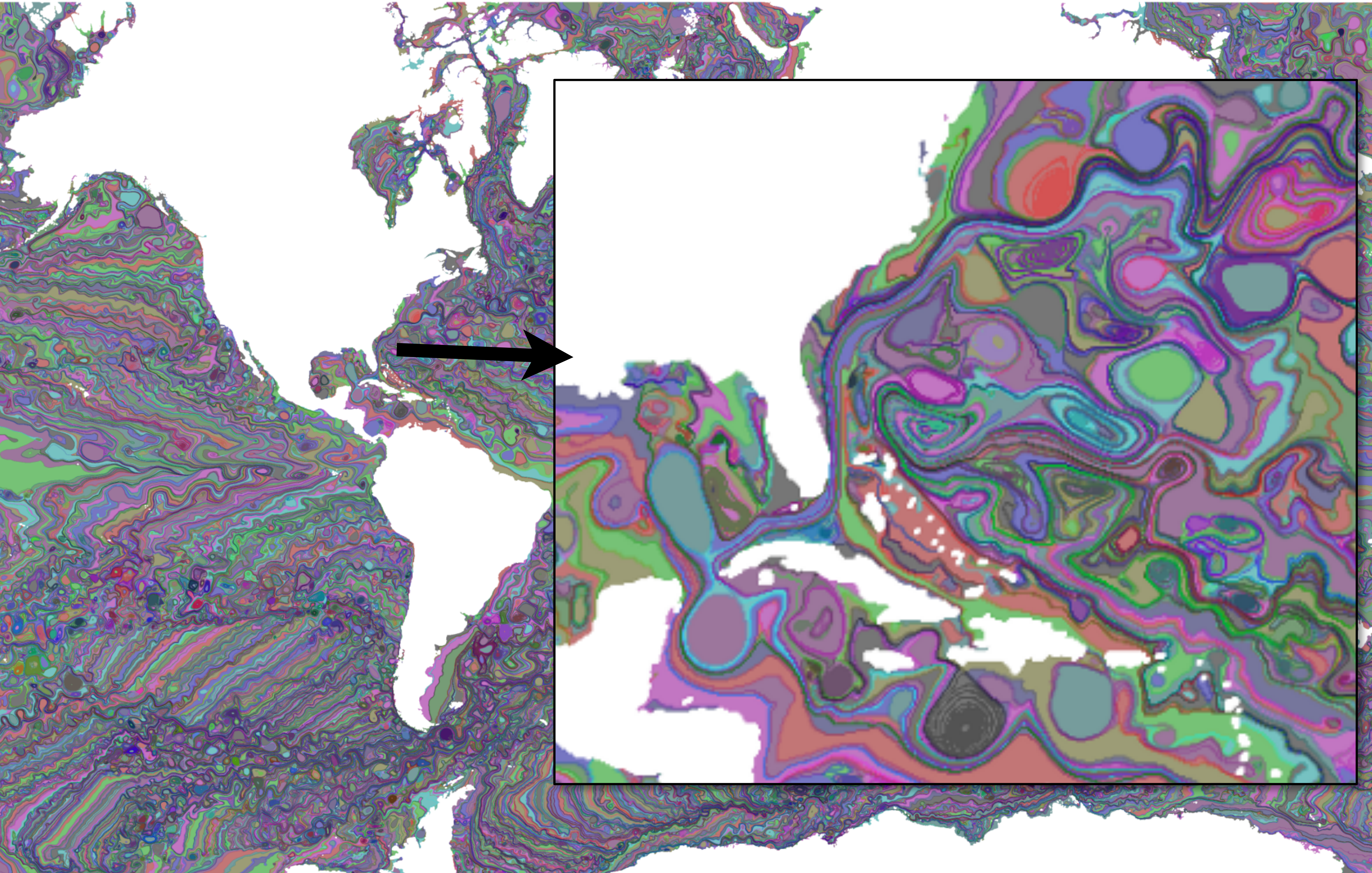

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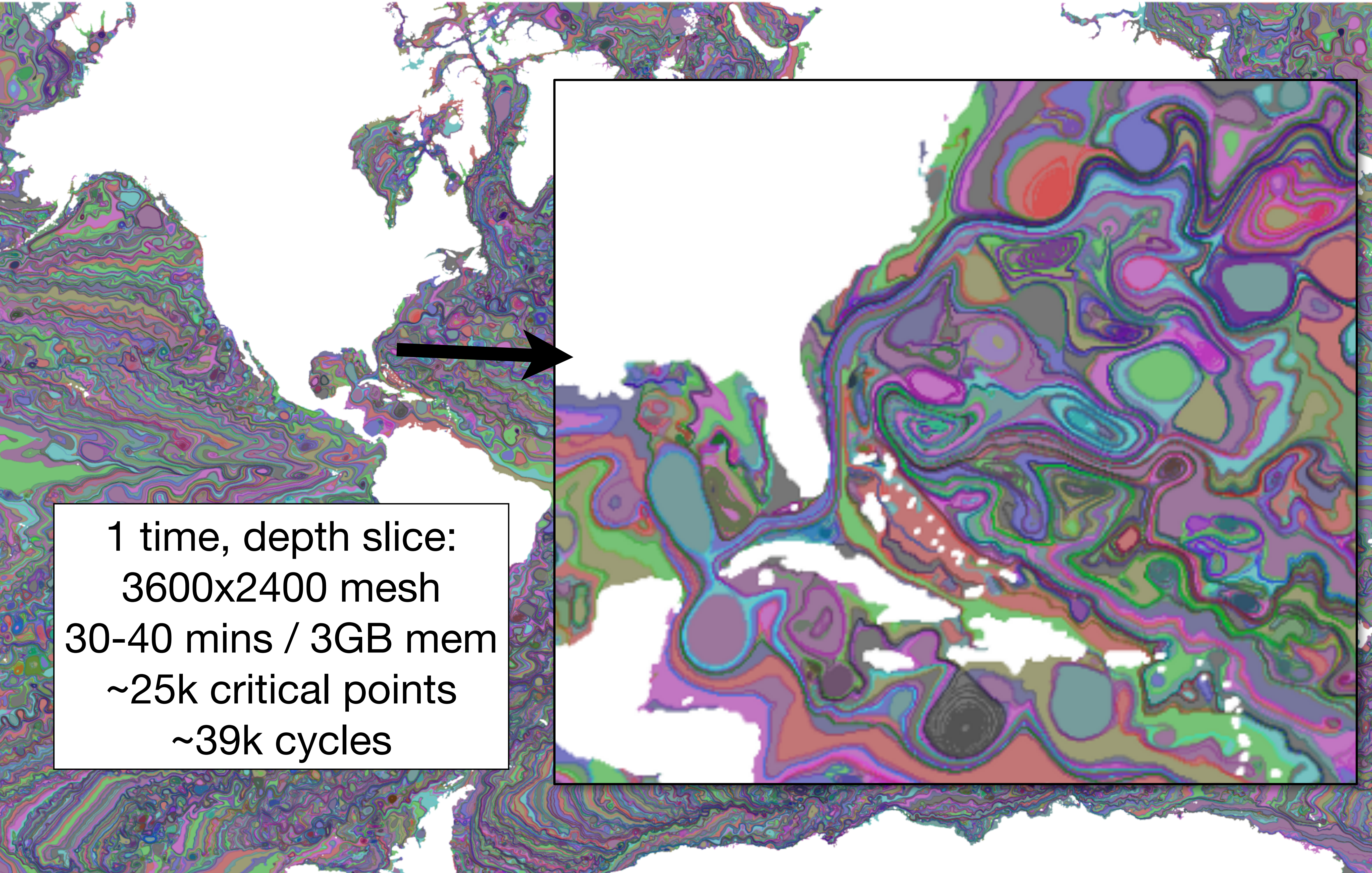
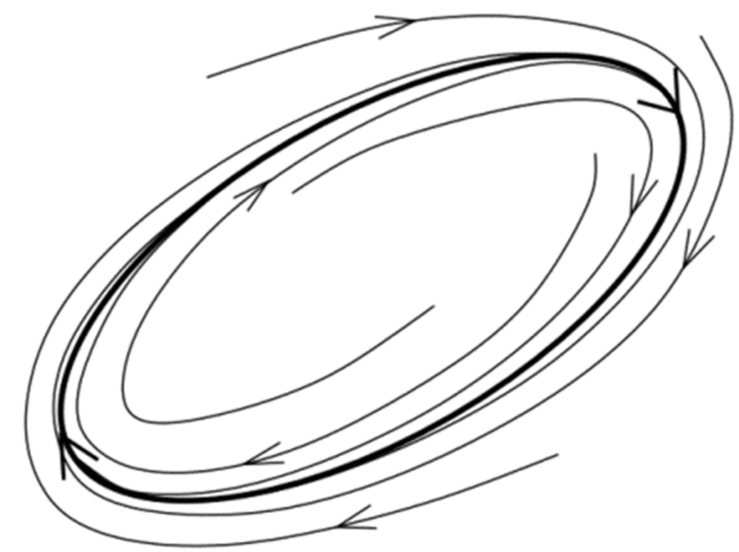
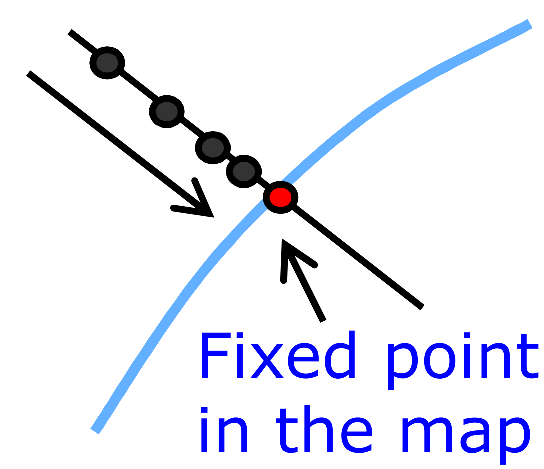


1 time, depth slice:
3600x2400 mesh
30-40 mins / 3GB mem
~25k critical points
~39k cycles

Periodic Orbits

- Curve-type (1D) limit set
- Attracting / repelling behavior
- **Poincaré map**:
  - Defined over cross section
  - Map each position to next intersection with cross section along flow
  - Discrete map
  - Cycle intersects at fixed point
  - Hyperbolic / non-hyperbolic





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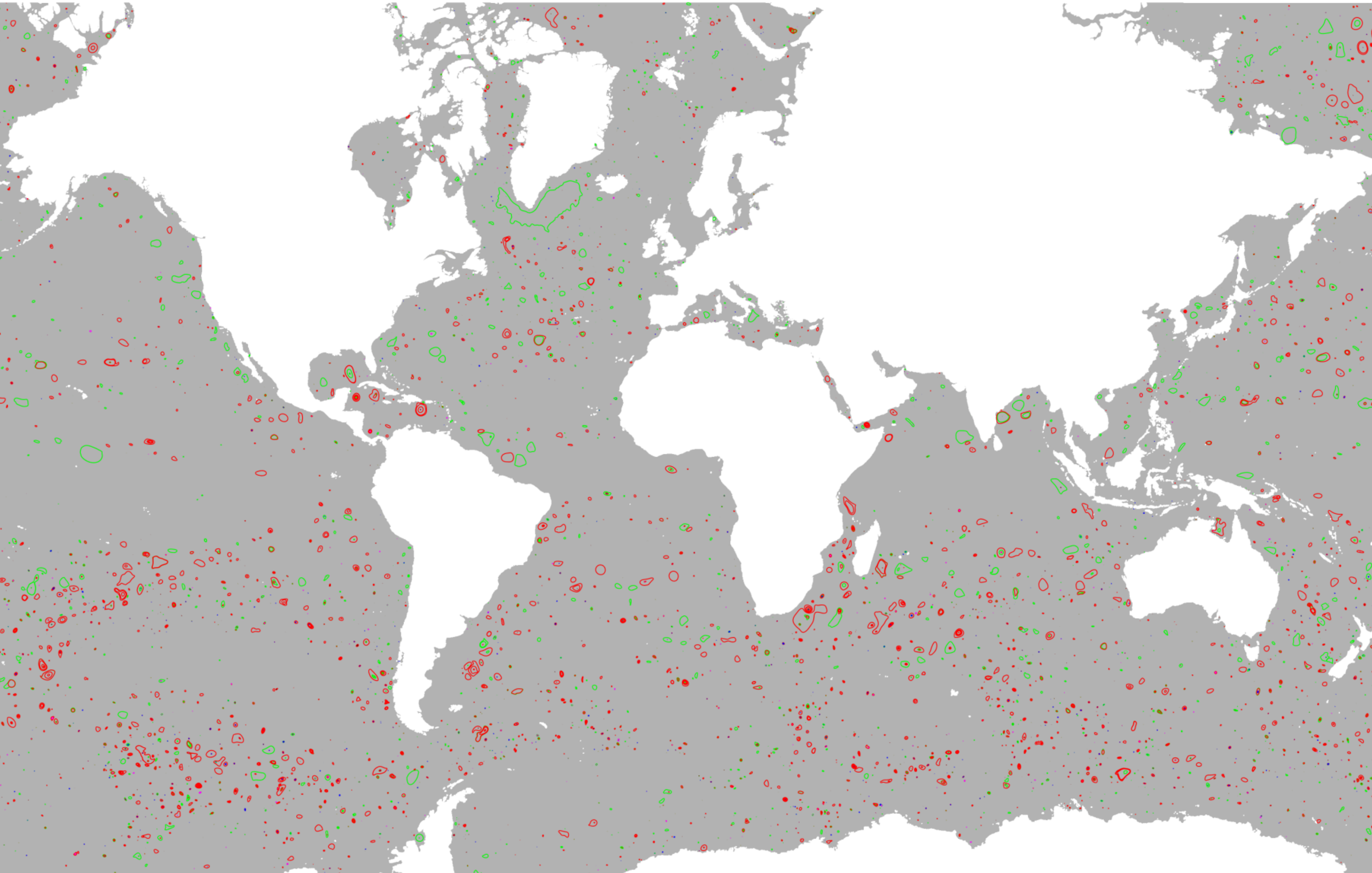
Periodic Orbits in the Ocean



Levine, Jadhav, Bhatia, Pascucci, Bremer, CGF 31(3) 2012

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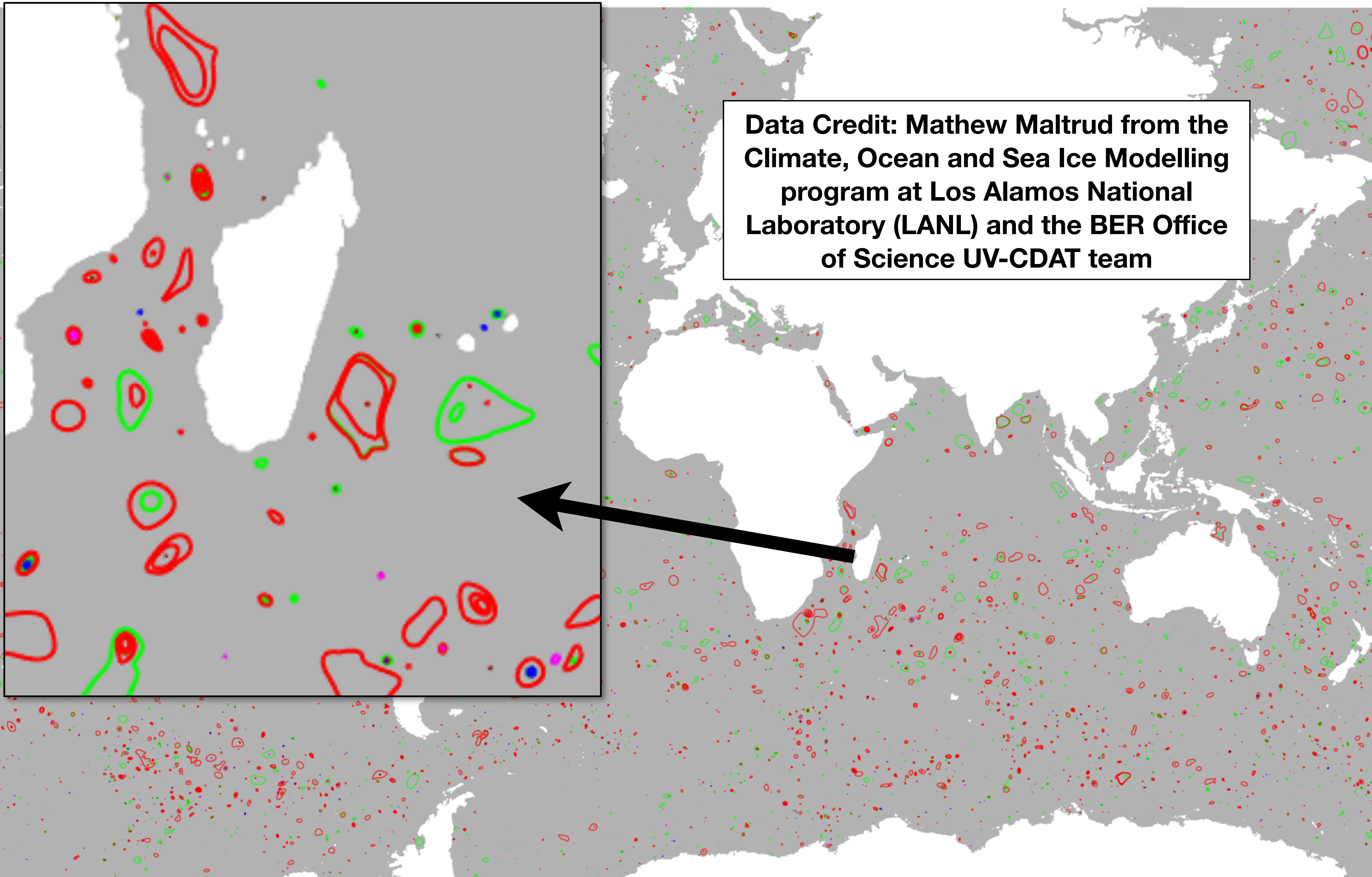
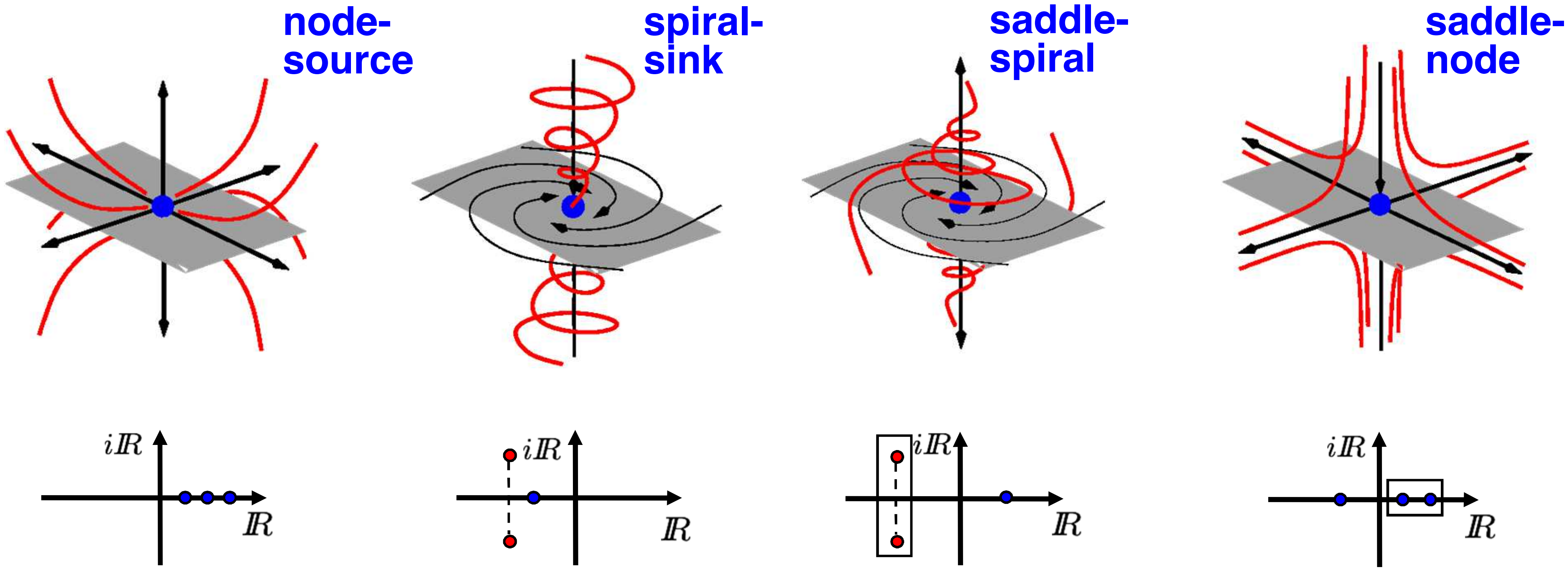


Data Credit: Mathew Maltrud from the Climate, Ocean and Sea Ice Modelling program at Los Alamos National Laboratory (LANL) and the BER Office of Science UV-CDAT team

3D Flow Topology

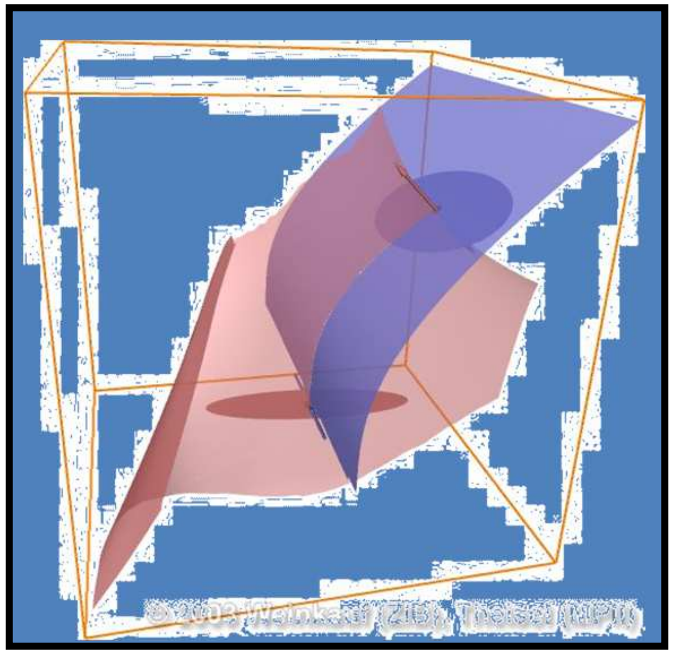
- Fixed points

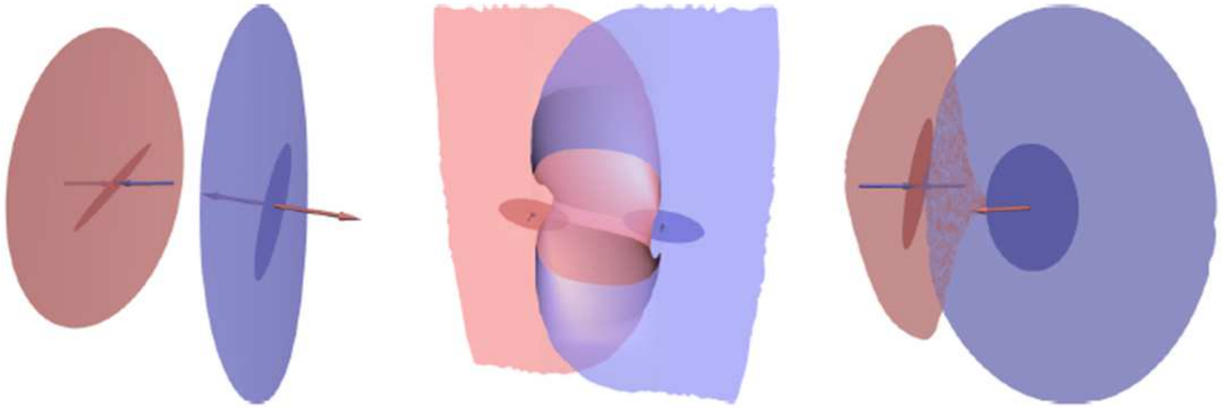


- Can be characterized using 3D Poincaré index
- Both line and surface separatrices exist

Saddle Connectors

- Multiple separating surfaces may lead to occlusion problems
- Idea: reduce visual clutter by replacing stream surfaces with streamlines of interest
- Saddle Connector:
  - Separating surfaces intersection integrated from two saddle points
    of opposite indices (inflow vs. outflow surface)
  - Intersection is a streamline



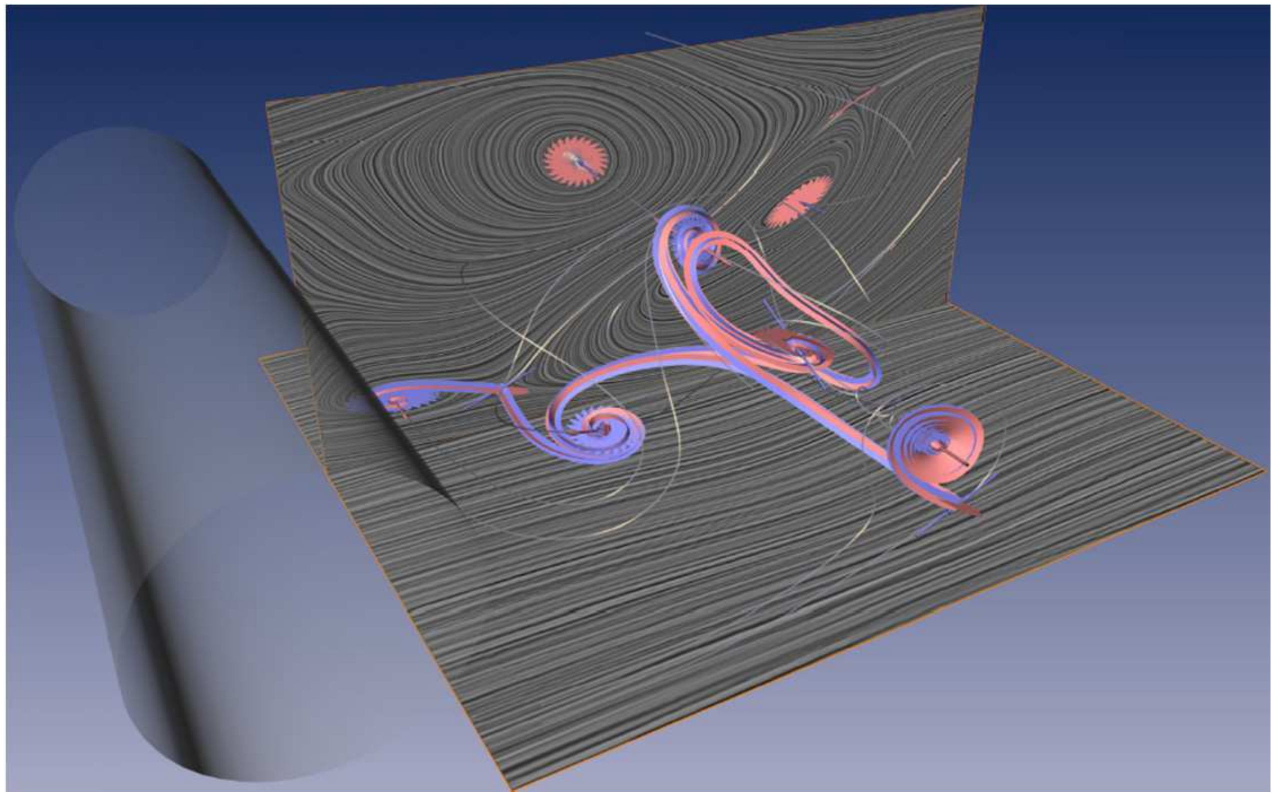


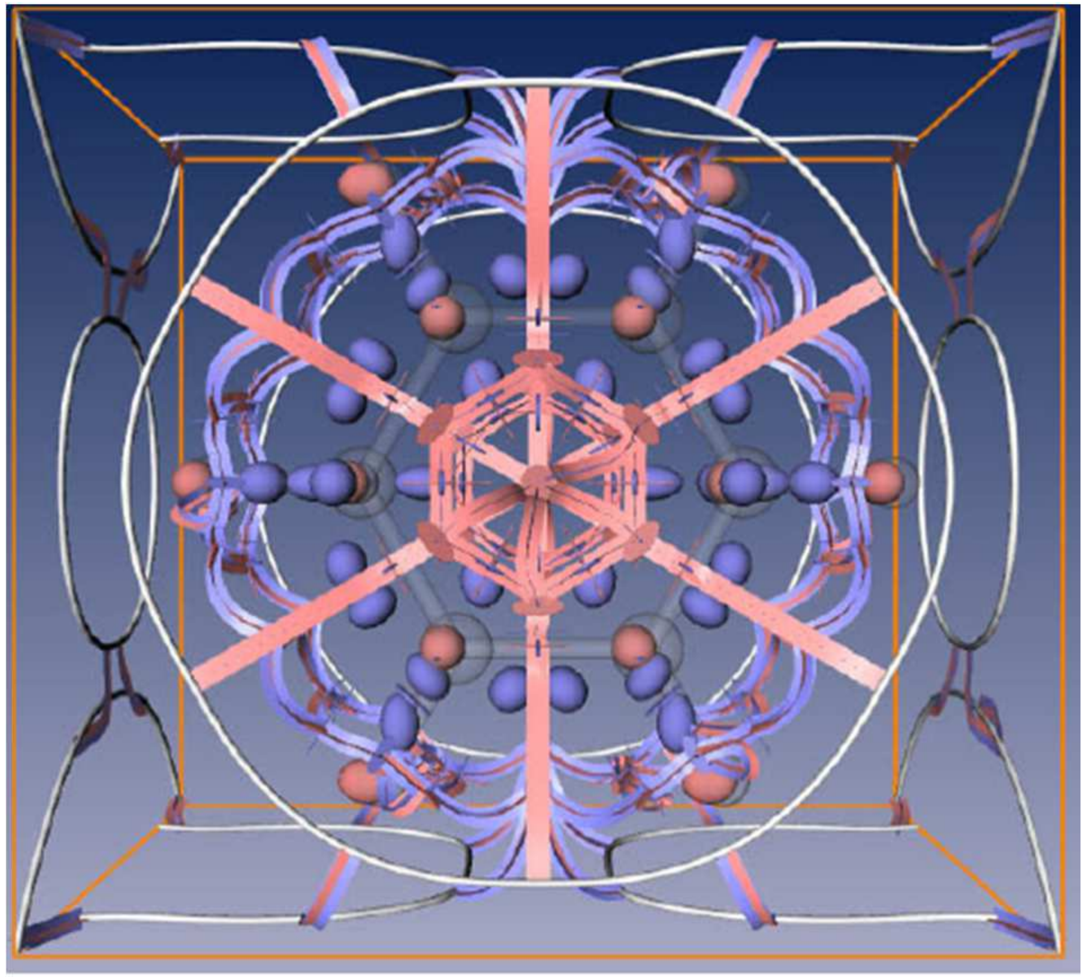
(a) No intersection. (b) Two intersection curves. (c) The separation surfaces collapse.

Source: Theisel et al. Vis 03

Vector field topology

- 3D topology





Saddle connectors:
H. Theisel, T. Weinkauf, H.-C. Hege, H.-P. Seidel. Saddle connectors-an approach to visualizing the topological skeleton of complex 3D vector fields. IEEE Visualization 2003, pp. 225-232

Time-Varying Data

vector field

$$\mathbf{v} : \mathbb{E}^n \rightarrow \mathbb{R}^m$$

vector field

$$\mathbf{v} : \mathbb{E}^n \rightarrow \mathbb{R}^m$$

parameter-independent

steady vector field

vector field

parameter-independent

$$\mathbf{v} : \mathbb{E}^n \rightarrow \mathbb{R}^m$$

steady vector field

one-parameter-dependent

$$\mathbf{v} : \mathbb{E}^{n+1} \rightarrow \mathbb{R}^m$$

$$\mathbf{v}(x, y, t) = \begin{pmatrix} u(x, y, t) \\ v(x, y, t) \end{pmatrix}$$

unsteady vector field

vector field

parameter-independent

$$\mathbf{v} : \mathbb{E}^n \rightarrow \mathbb{R}^m$$

steady vector field

one-parameter-dependent

$$\mathbf{v} : \mathbb{E}^{n+1} \rightarrow \mathbb{R}^m$$

$$\mathbf{v}(x, y, t) = \begin{pmatrix} u(x, y, t) \\ v(x, y, t) \end{pmatrix}$$

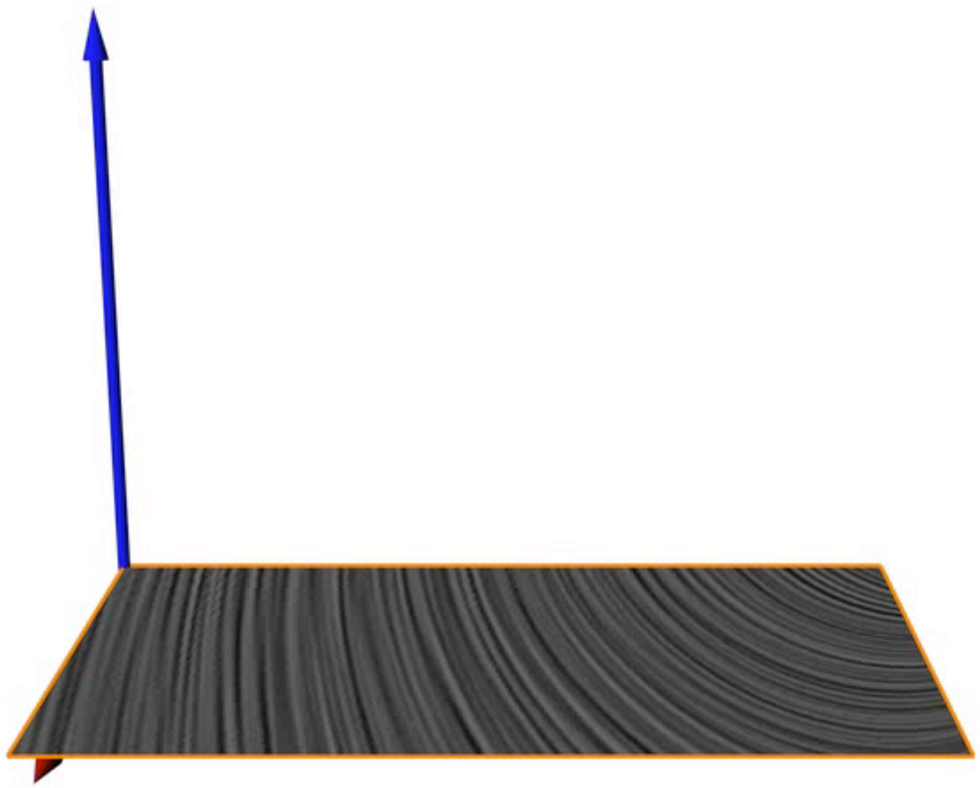
unsteady vector field

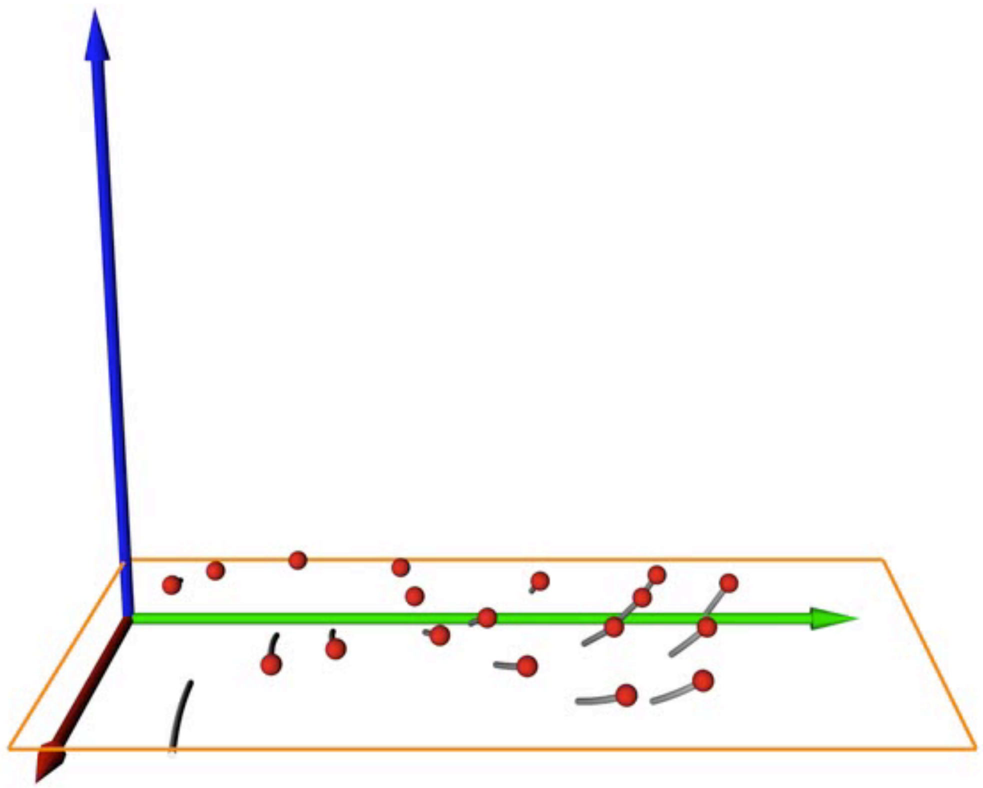
The flow itself changes over time, and leads to more complex phenomena

Geometry-Based Visualization for Unsteady Flow

Characteristic Curves of a Vector Field

- **Streamlines:** curve parallel to the vector field in each point for a fixed time
- **Pathlines:** describes motion of a particles over time through a vector field
- **Streaklines:** trace of dye that is released into the flow at a fixed position
- **Timelines**: describes motion of particles set out on a line over time through a vector field





streamlines

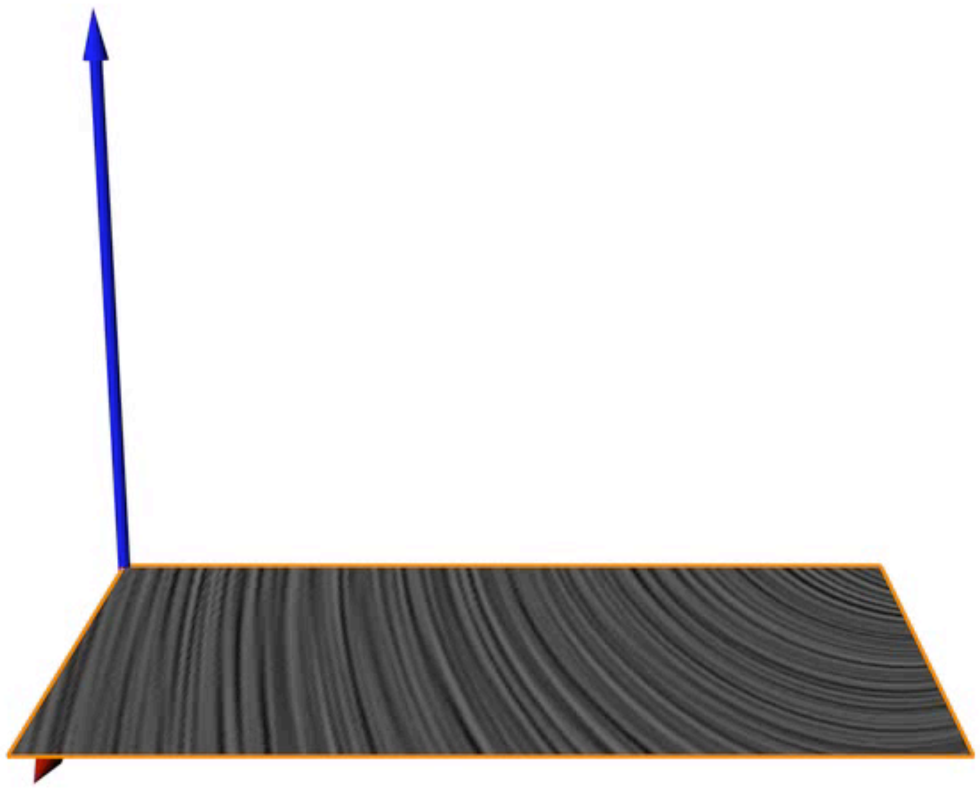
pathlines

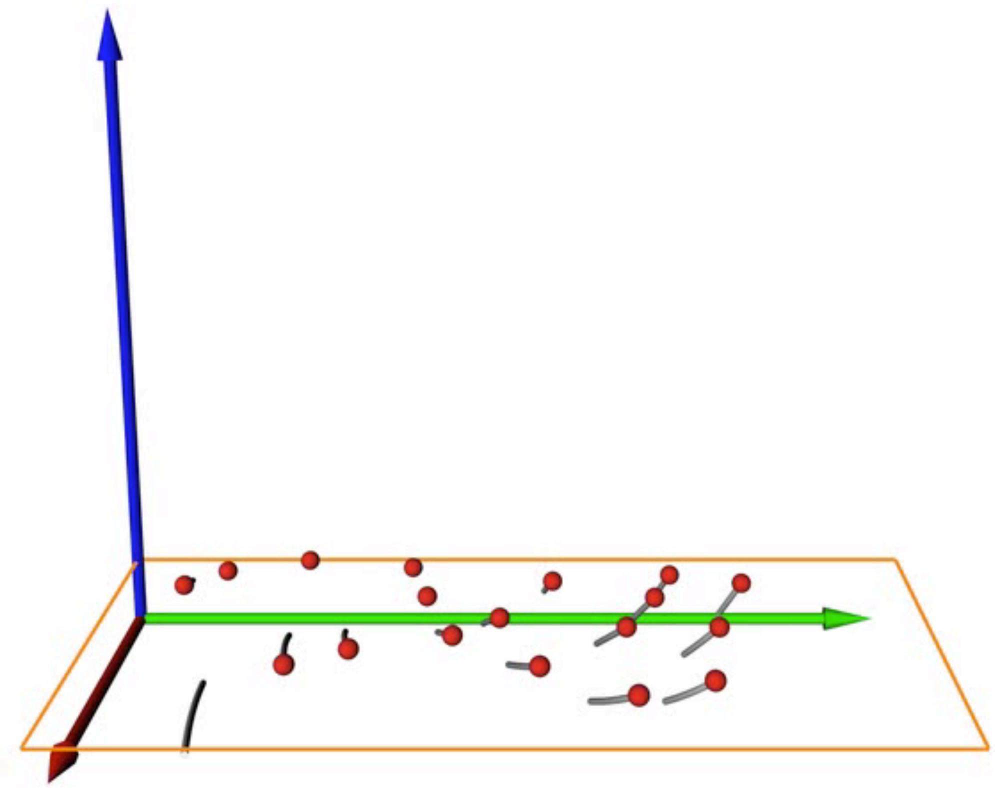
curve parallel to the vector field in each point for a <u>fixed time</u>

describes motion of a massless particle in an <u>steady</u> flow field

curve parallel to the vector field in each point <u>over time</u>

describes motion of a massless particle in an <u>unsteady</u> flow field





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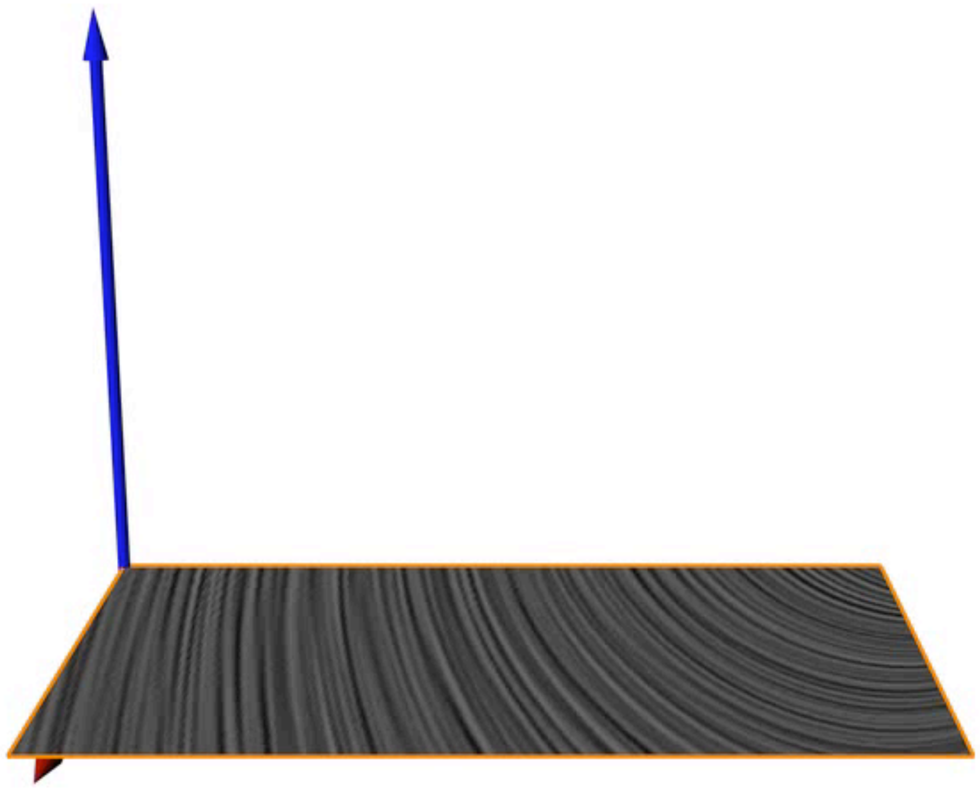

streamlines

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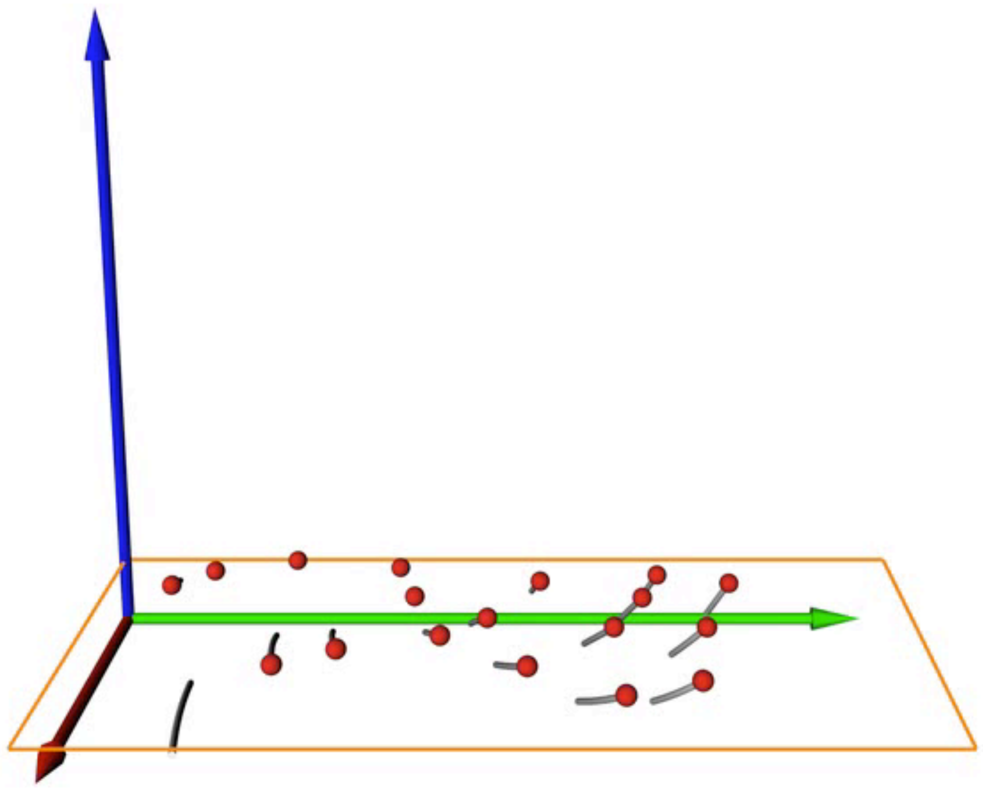

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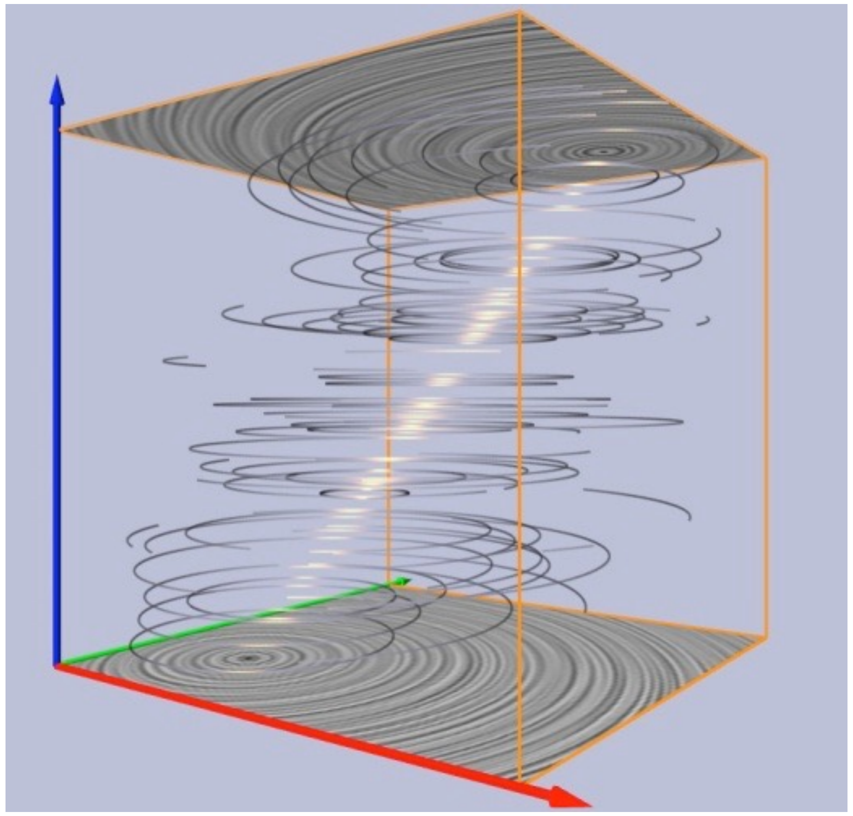

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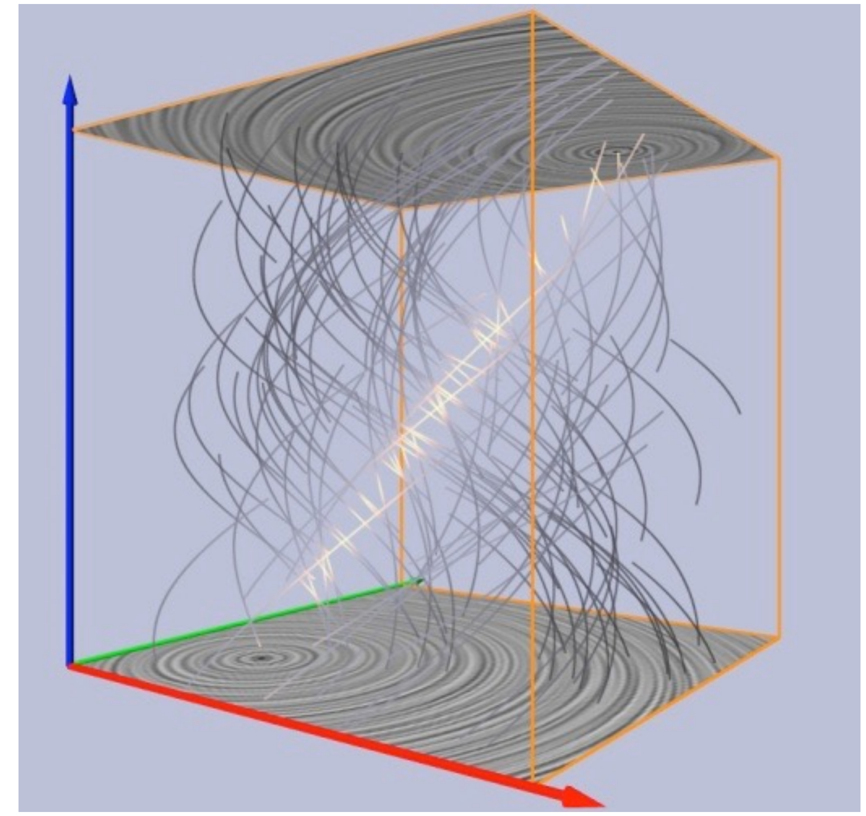

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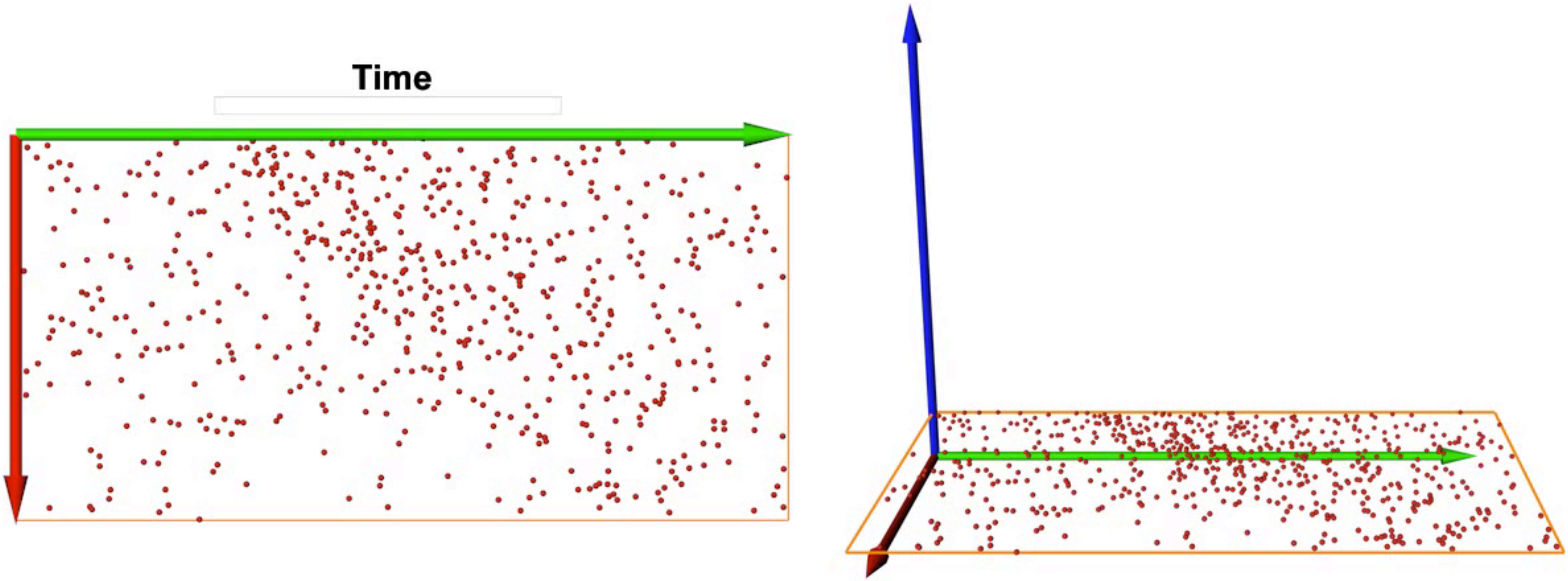
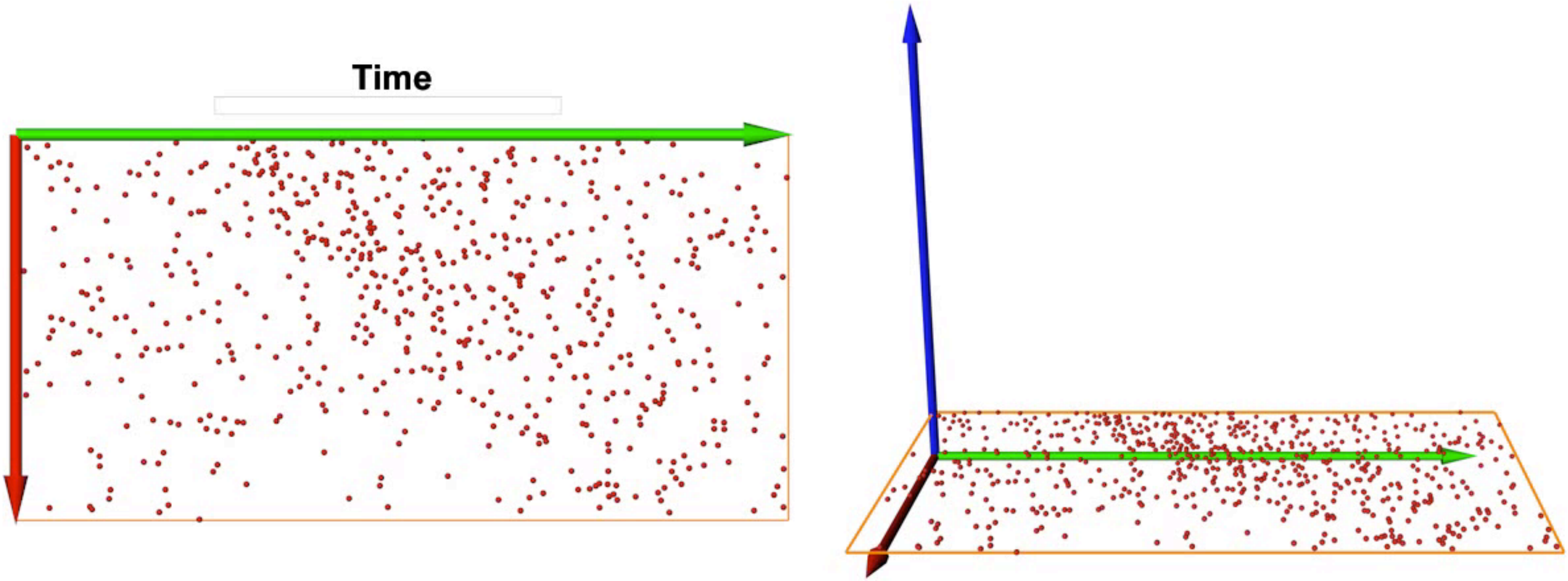
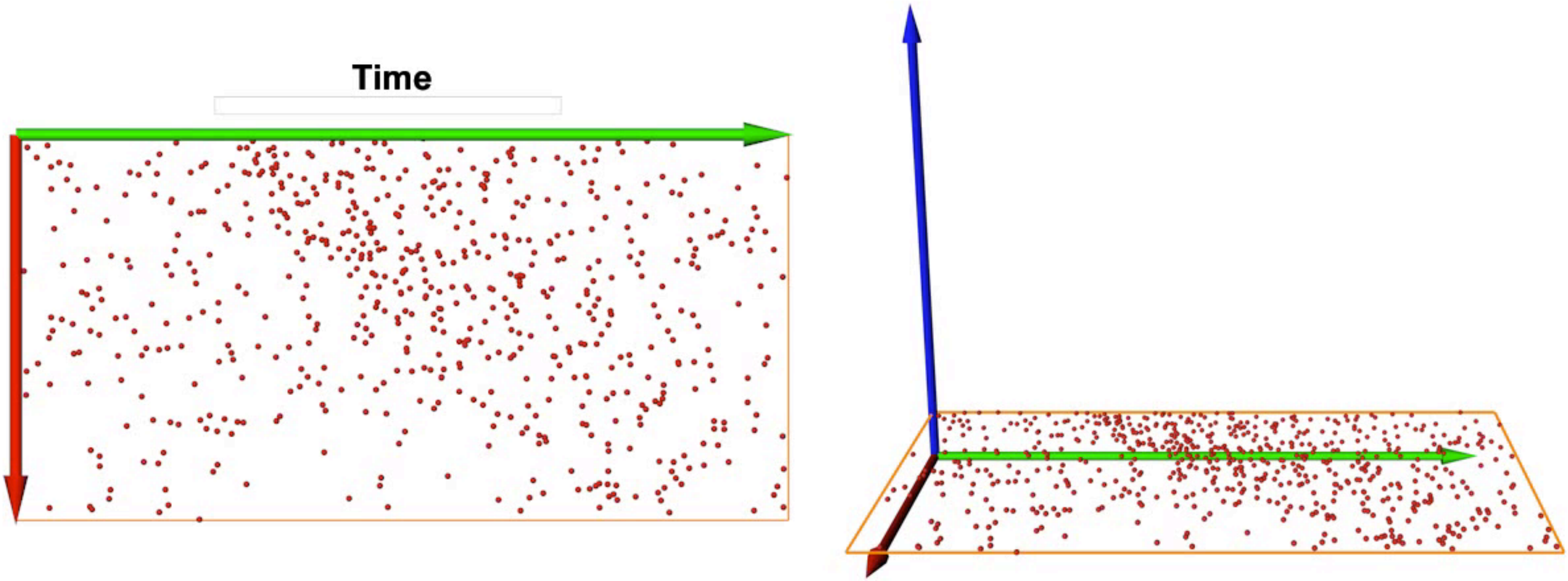
Weinkauf and Theisel, TVCG 2010



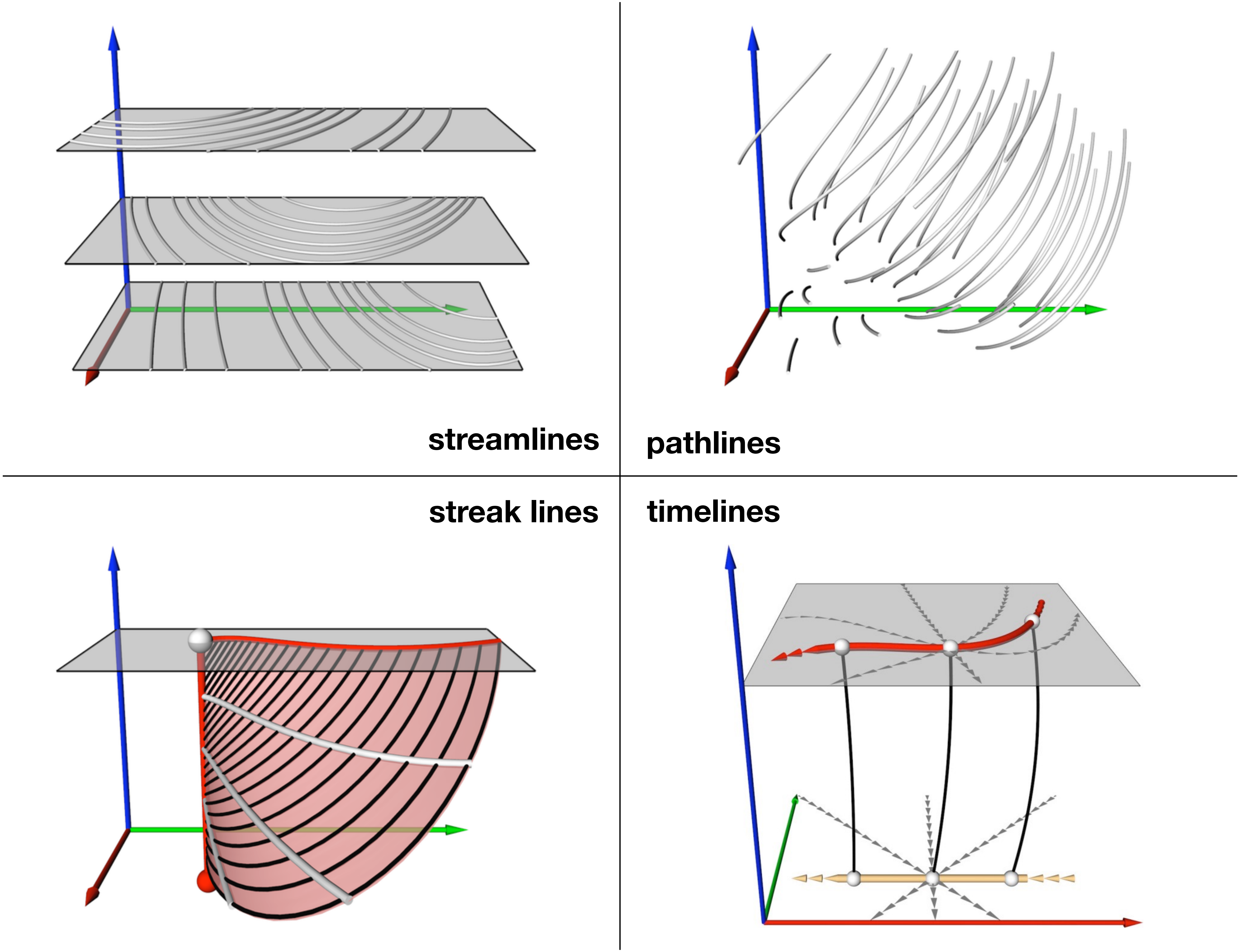
**2D time-dependent vector field
particle visualization**



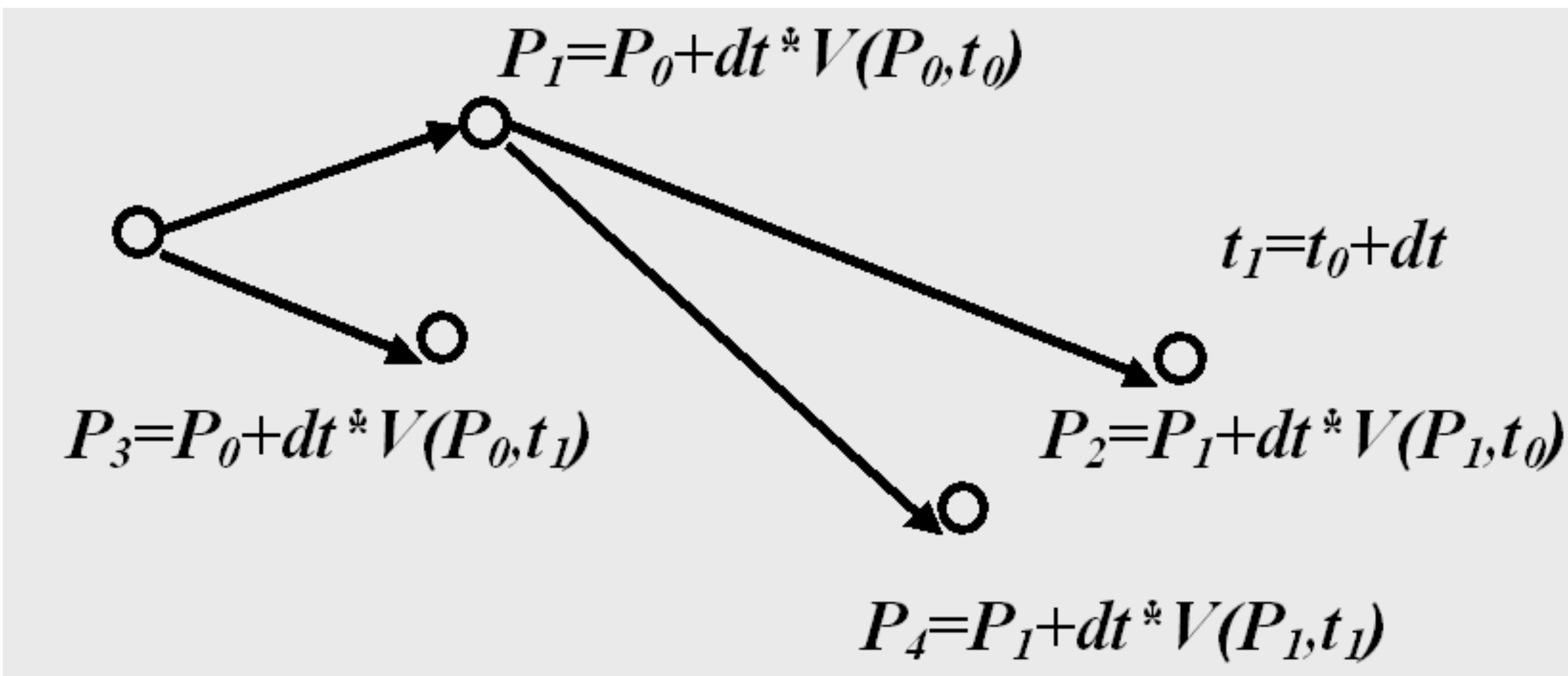
2D time-dependent vector field particle visualization



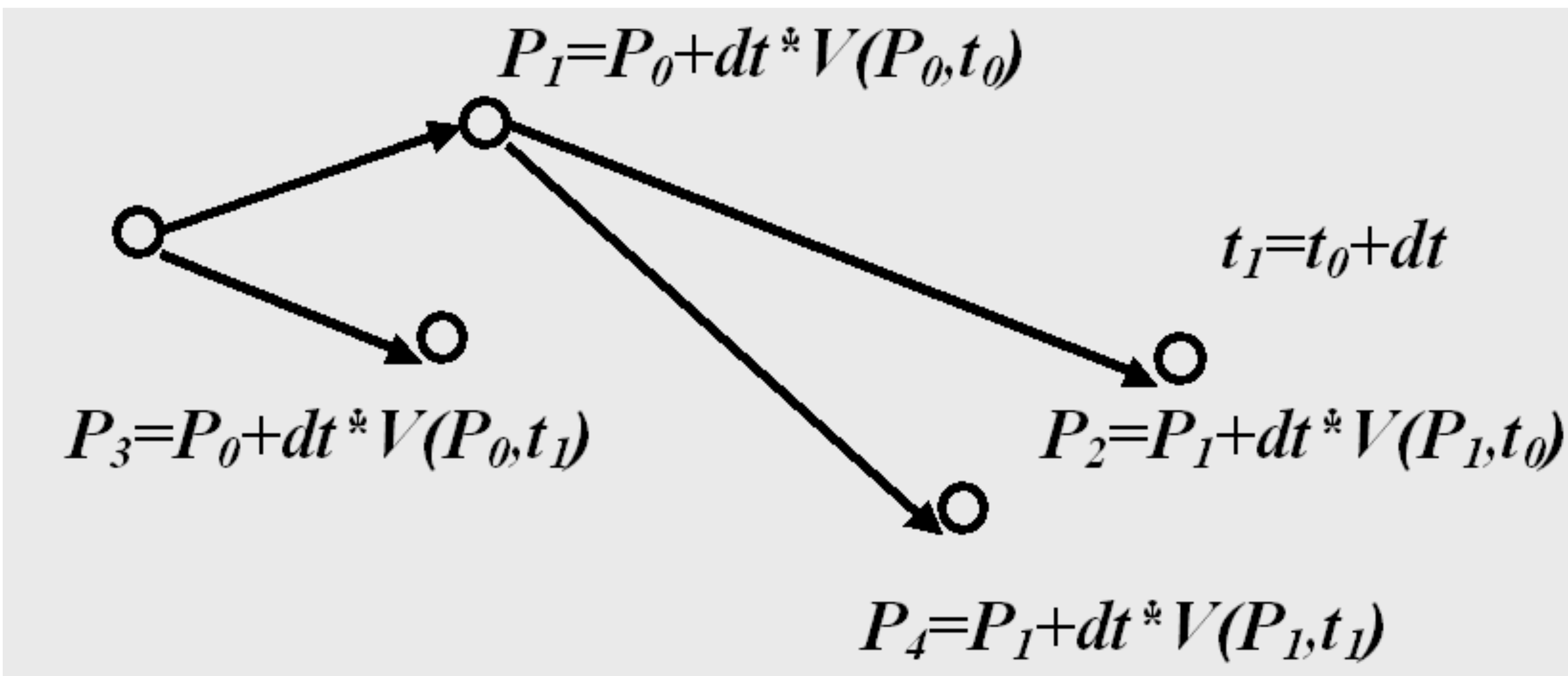
2D time-dependent vector field particle visualization


streamlines
pathlines
streak lines
timelines

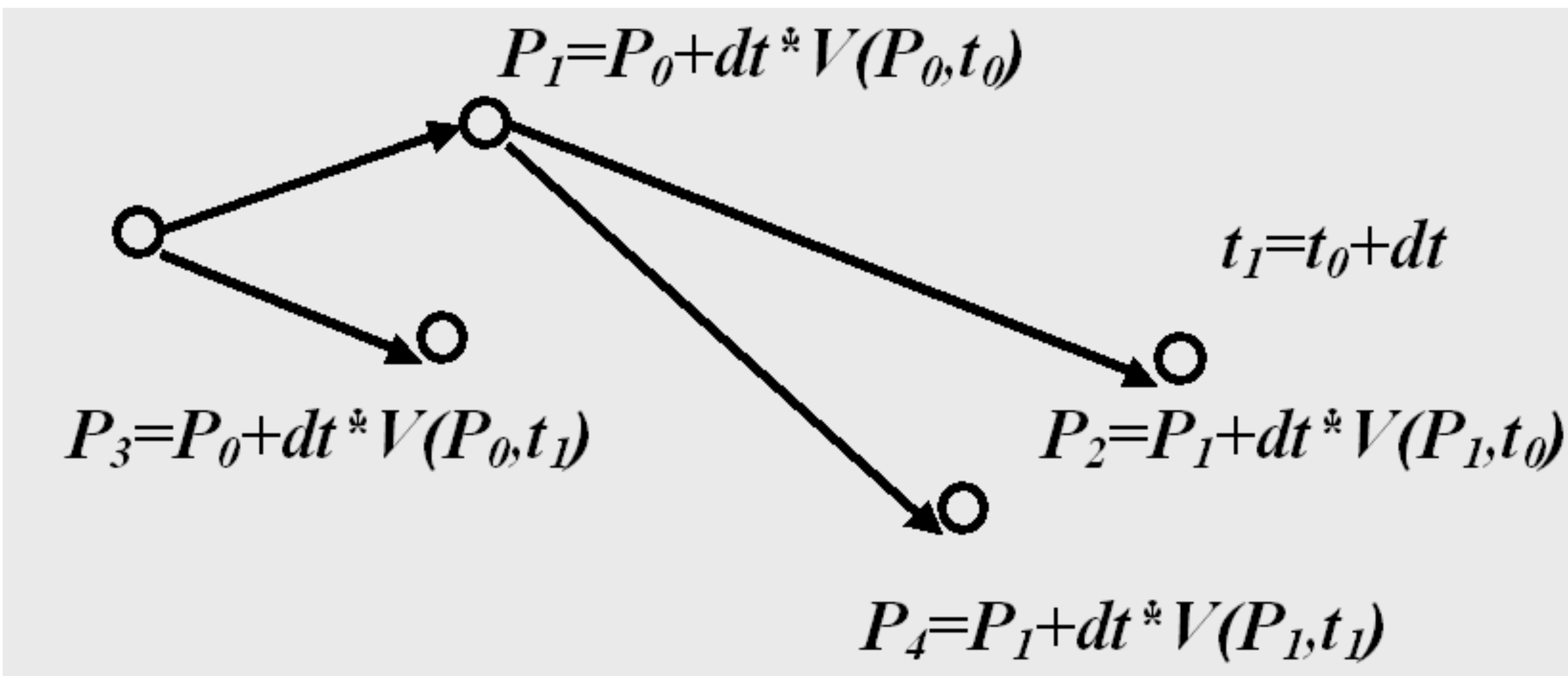
- **Characteristic curves of a vector field:**
  - Streamline:
  - Pathline:
  - Streakline:



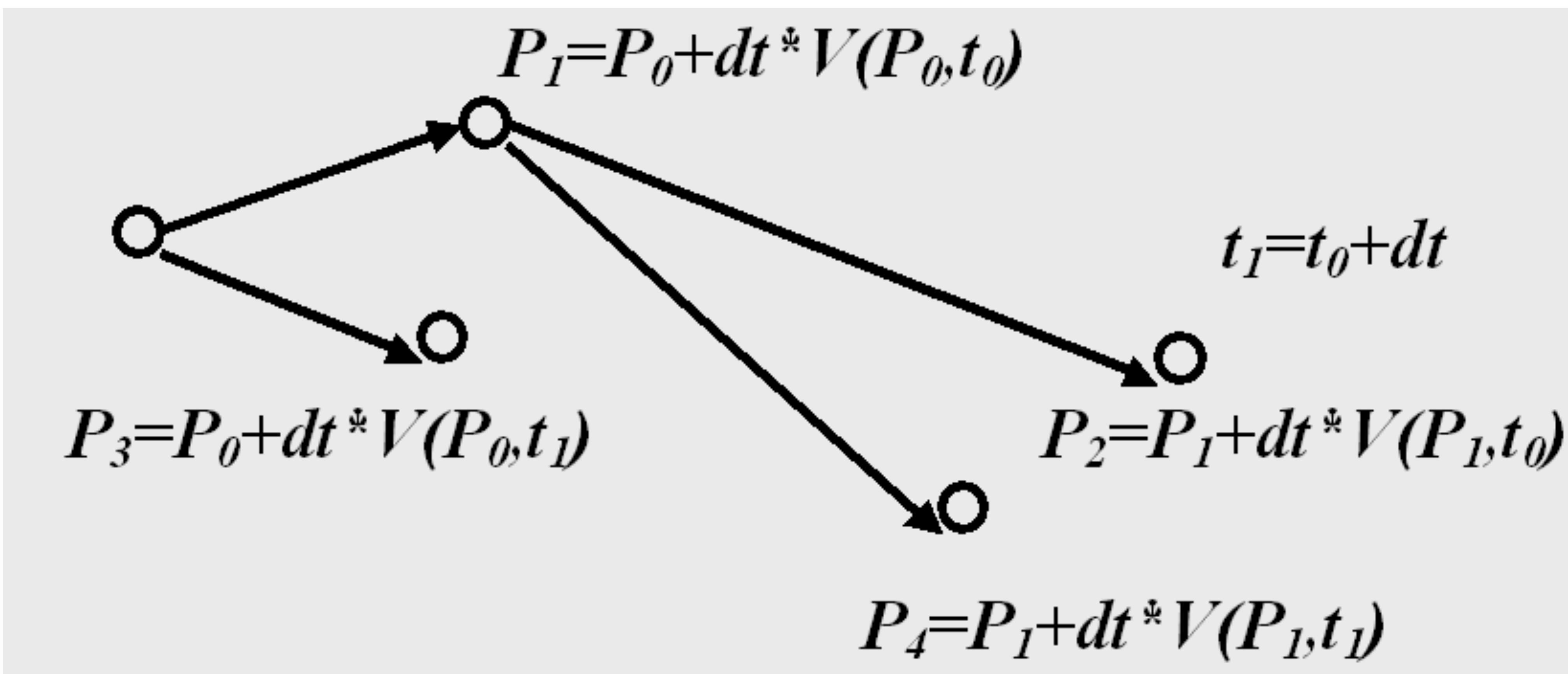
- **Characteristic curves of a vector field:**
  - Streamline: P0, P1, P2
  - Pathline:
  - Streakline:



- **Characteristic curves of a vector field:**
  - Streamline: P0, P1, P2
  - Pathline: P0, P1, P4
  - Streakline:



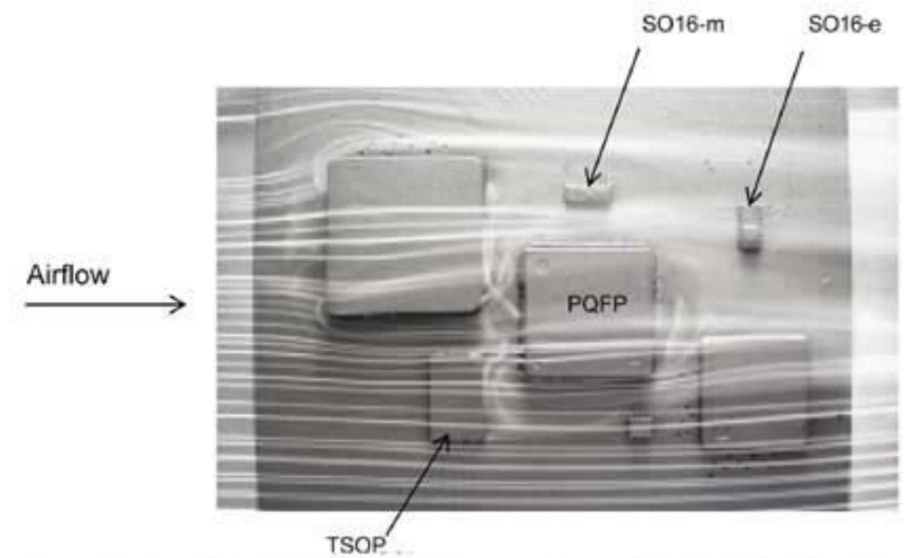
- **Characteristic curves of a vector field:**
  - Streamline: P0, P1, P2
  - Pathline: P0, P1, P4
  - Streakline: P0, P4, and P3



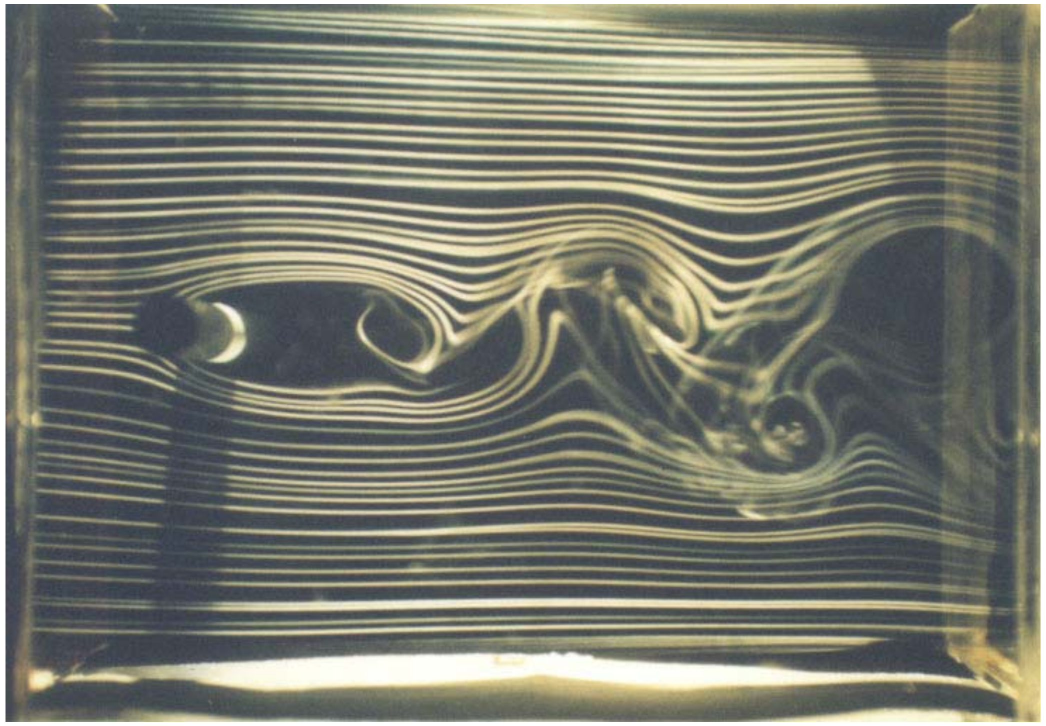
Unsteady Vector Fields

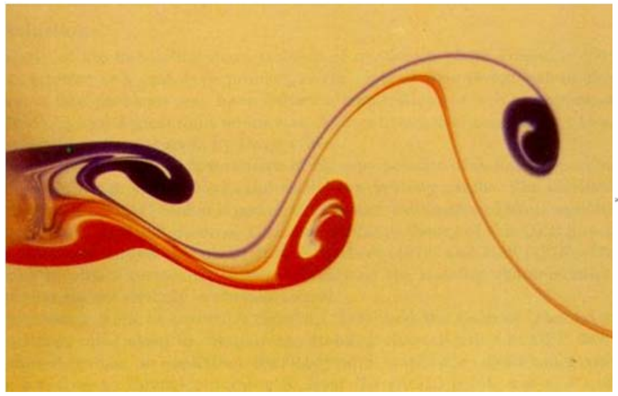
Important feature curves:

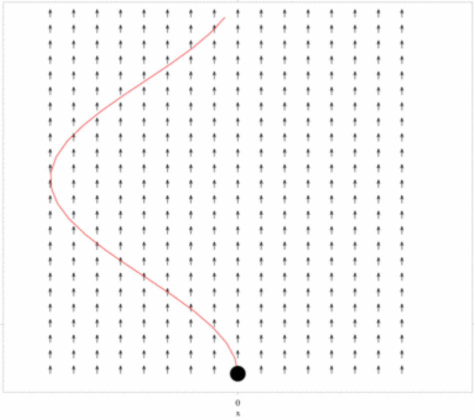
- **Streakline**: a curve traced by the continues release of particles in unsteady flow from the **same position in space** (release infinitely many massless particles)



Note: Smoke wire set flush with the PCB surface, 25 mm upstream of the PCB leading edge.

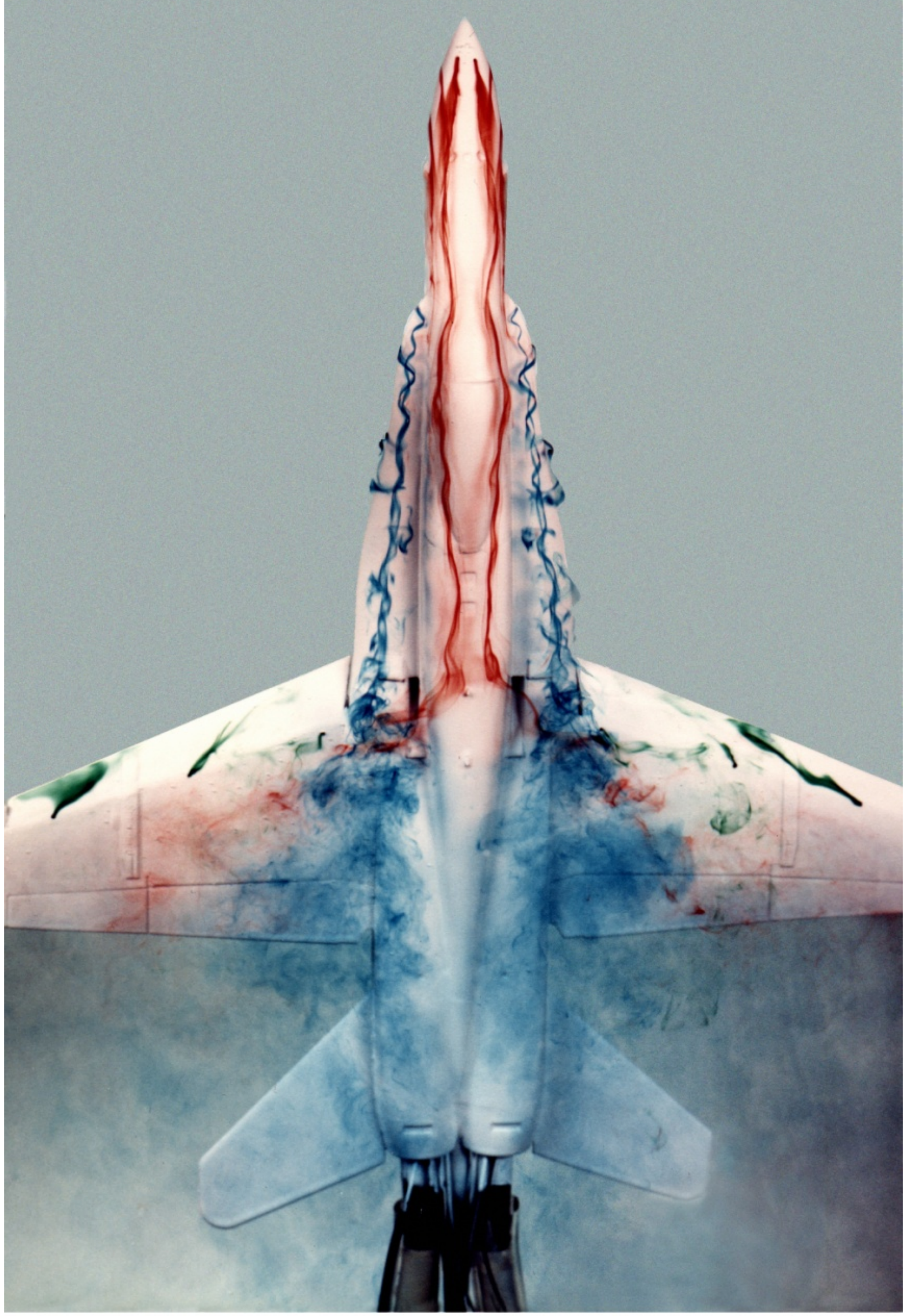






Source: Google images

Streaklines in Experimental Flow Vis



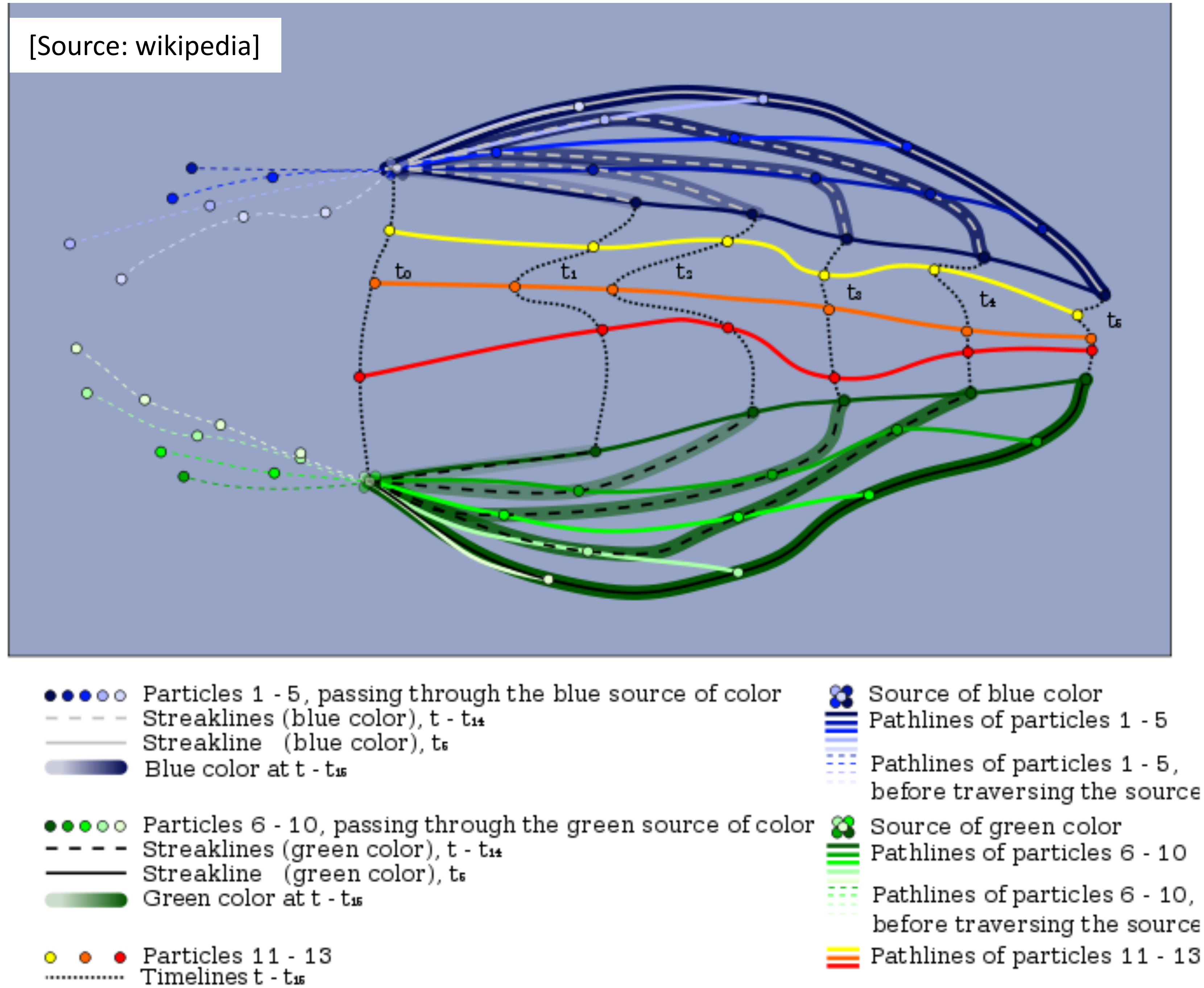
NASA Dryden Flight Research Center Photo Collection
http://www.dfrc.nasa.gov/gallery/photo/index.html
NASA Photo: ECN-33298-03 Date: 1985

1/48-scale model of an F-18 aircraft in Flow Visualization Facility (FVF)

Dryden Flight Research Center ECN 33298-47 Photographed 1985
F-18 water tunnel test in Flow Visualization Facility NASA/Dryden

Streaklines computation

- Not tangent curves to the vector fields
- Union of the current positions of particles released at the same point in space



[Weinkauf et al. 2010]

Computing Streaklines

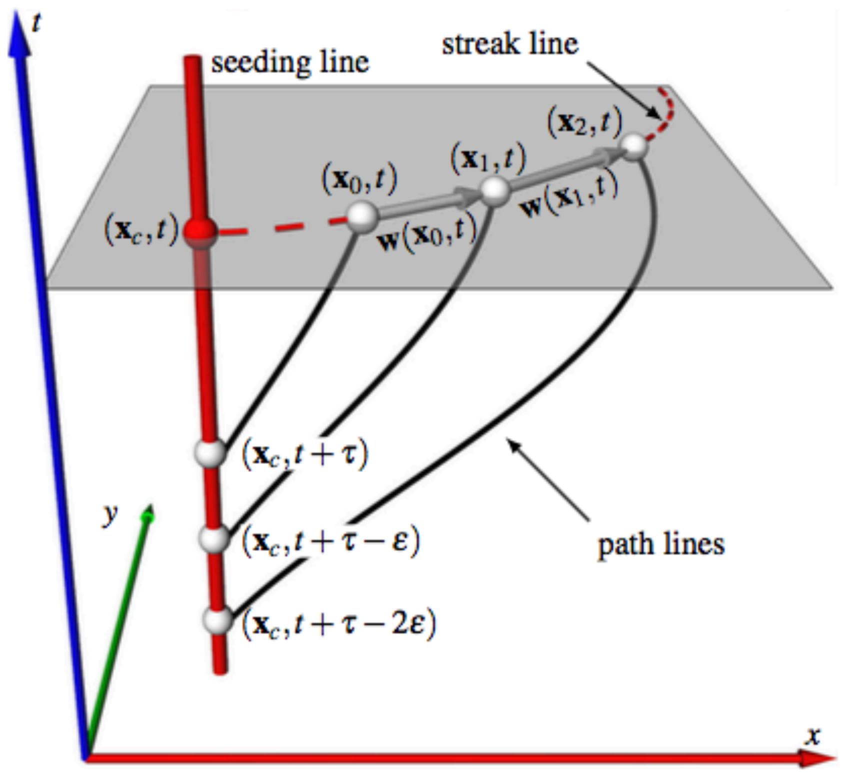


Figure 2. Definition of the vector field $\mathbf{w}(\mathbf{x}, t, \tau)$, which is the main ingredient of the streak line vector field $\bar{\bar{\mathbf{q}}}$.

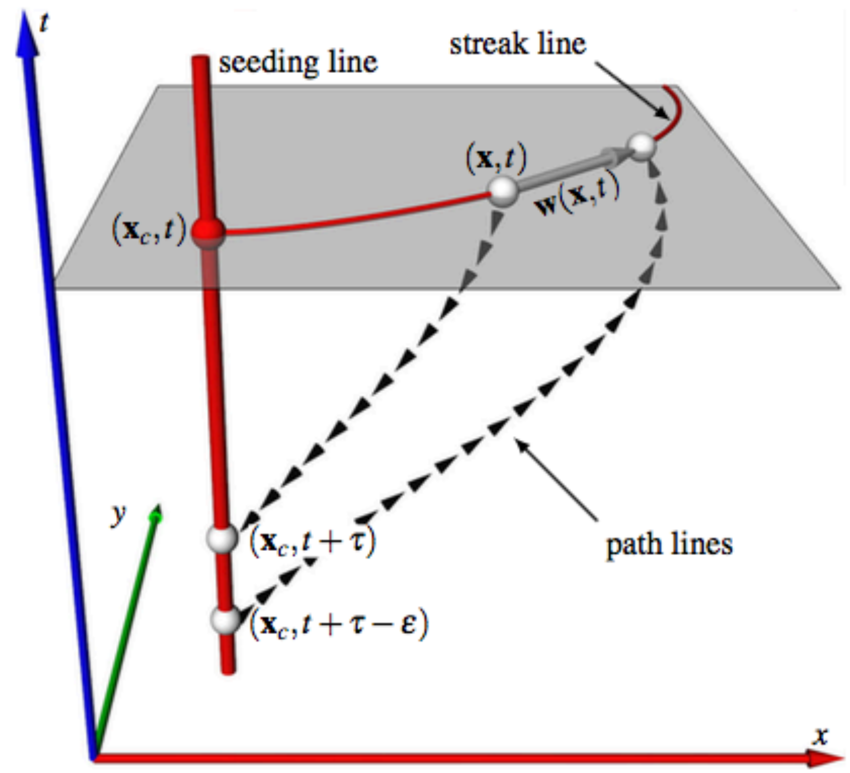


Figure 3. Straightforward, but costly way for computing $\mathbf{w}(\mathbf{x}, t, \tau)$ with two consecutive path line integrations.

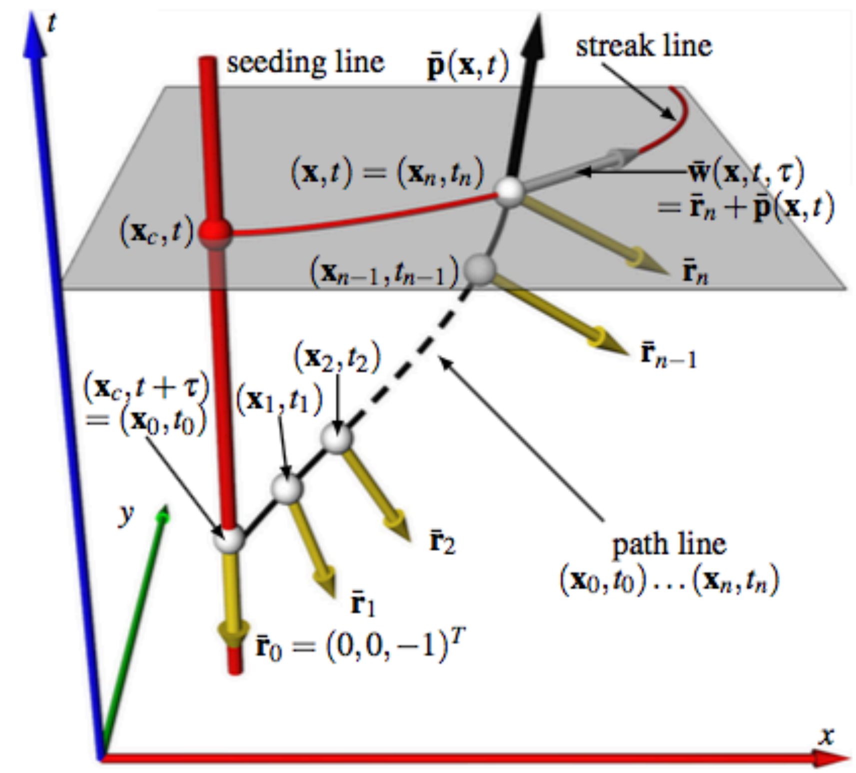
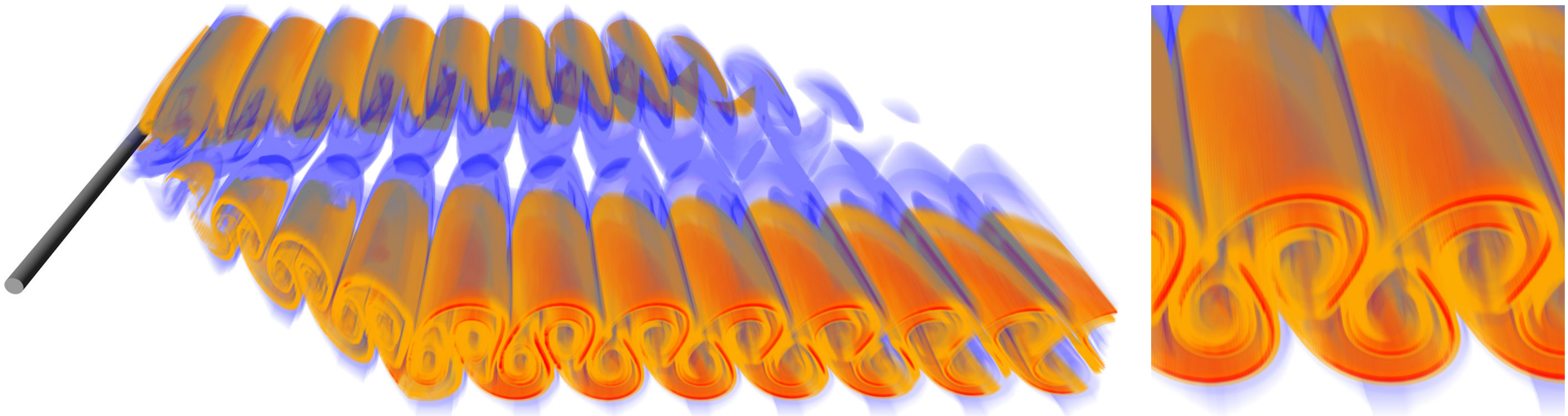


Figure 4. Computing $\mathbf{w}(\mathbf{x}, t, \tau)$ with a single path line integration by keeping track of the direction vector $\mathbf{r}_0$.

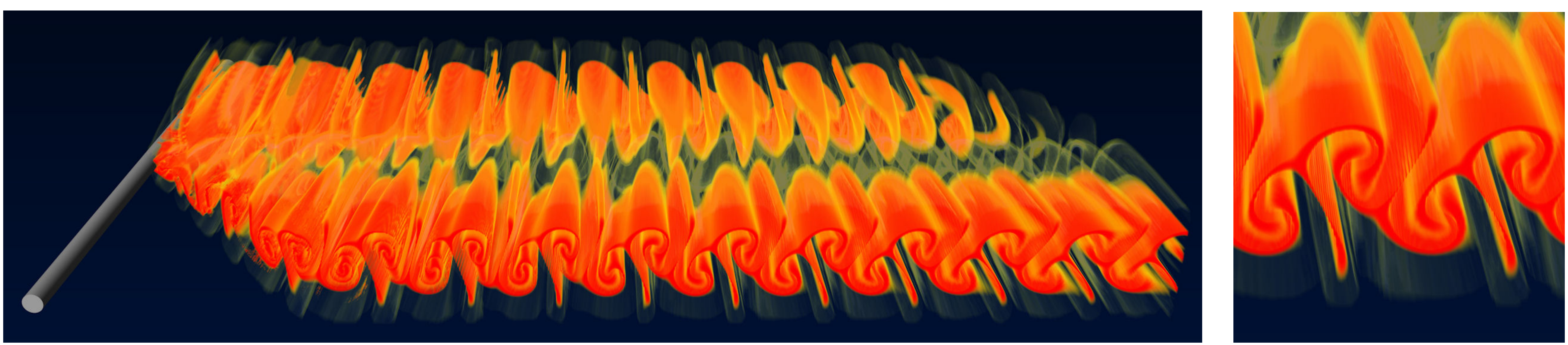
$$\bar{\bar{\mathbf{q}}}(\mathbf{x}, t, \tau) = \begin{pmatrix} \mathbf{w}(\mathbf{x}, t, \tau) \\ 0 \\ -1 \end{pmatrix}$$

Weinkauf and Theisel, TVCG 2010

Computed Streaklines



(a) Velocity magnitude of the streak lines between $\tau = (-3, 3)$.



(b) Curvature of the streak lines between $\tau = (-1.5, 1.5)$.

- **Stream and Path lines:**
  - Through all non-critical points (**x**,t) in space-time there is exactly one stream/path line passing through it.

- **Streak and Time lines:**
  - Many streak/time lines through every point (of the spatial domain)
  - ➔ makes it difficult to describe streak/time lines as tangent curves of some vector field
    - But it is possible. We may discuss it in a later session.

- Stream, Path, and Streak lines ☐ in a steady vector field.

?

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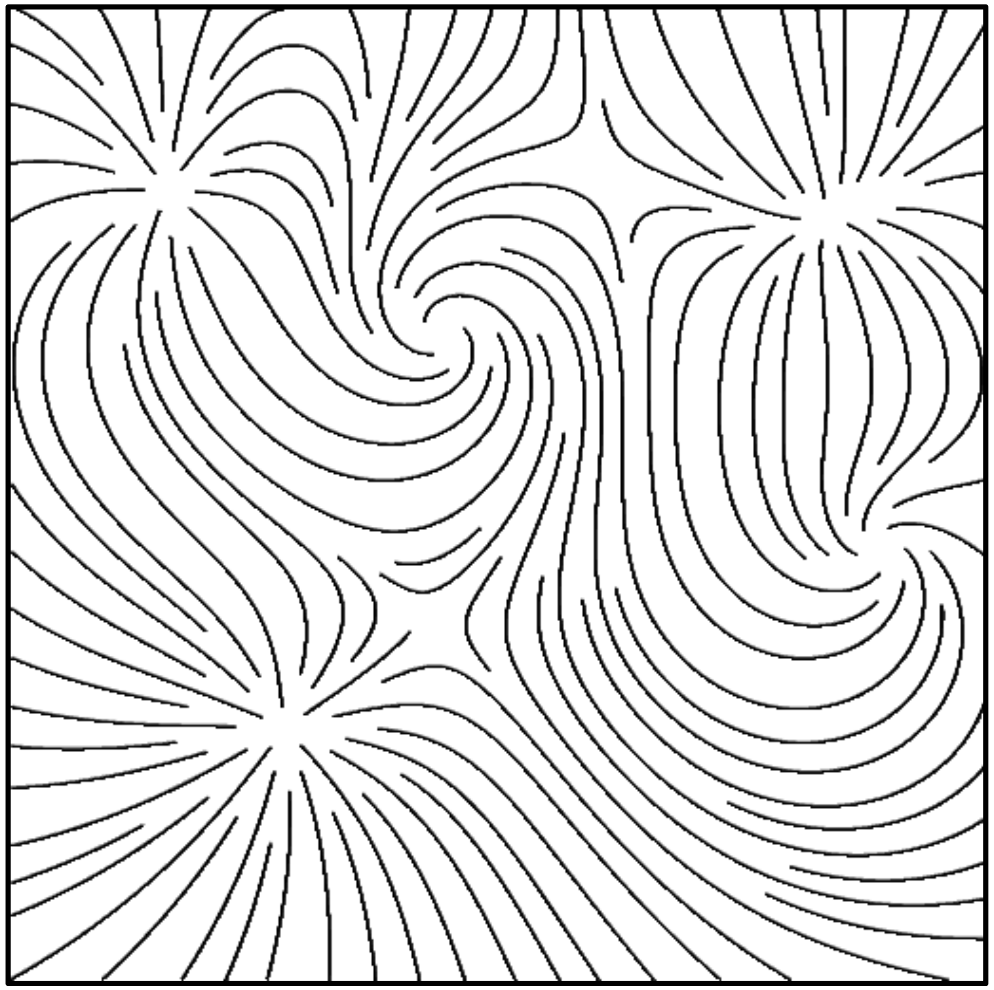
    - But it is possible. We may discuss it in a later session.

- Stream, Path, and Streak lines coincide in a steady vector field.

In **steady** flow, streamlines, pathlines and streaklines are **identical** because the vector field is NOT changing over time**.**

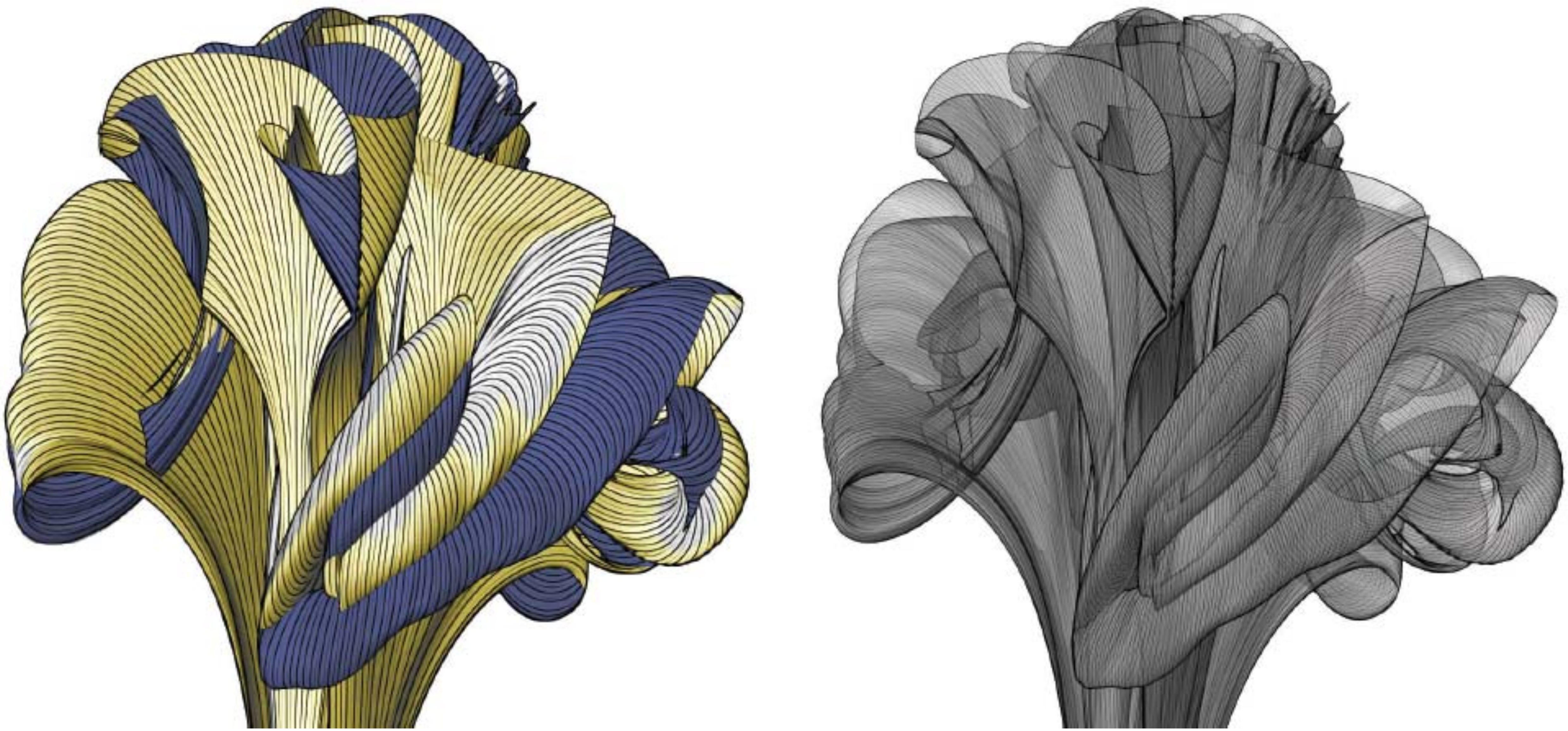


Other Geometry-Based Techniques

Higher-Dimensional Feature Descriptors

Path surface:

The extension of the path lines into unsteady 3 dimensional flows

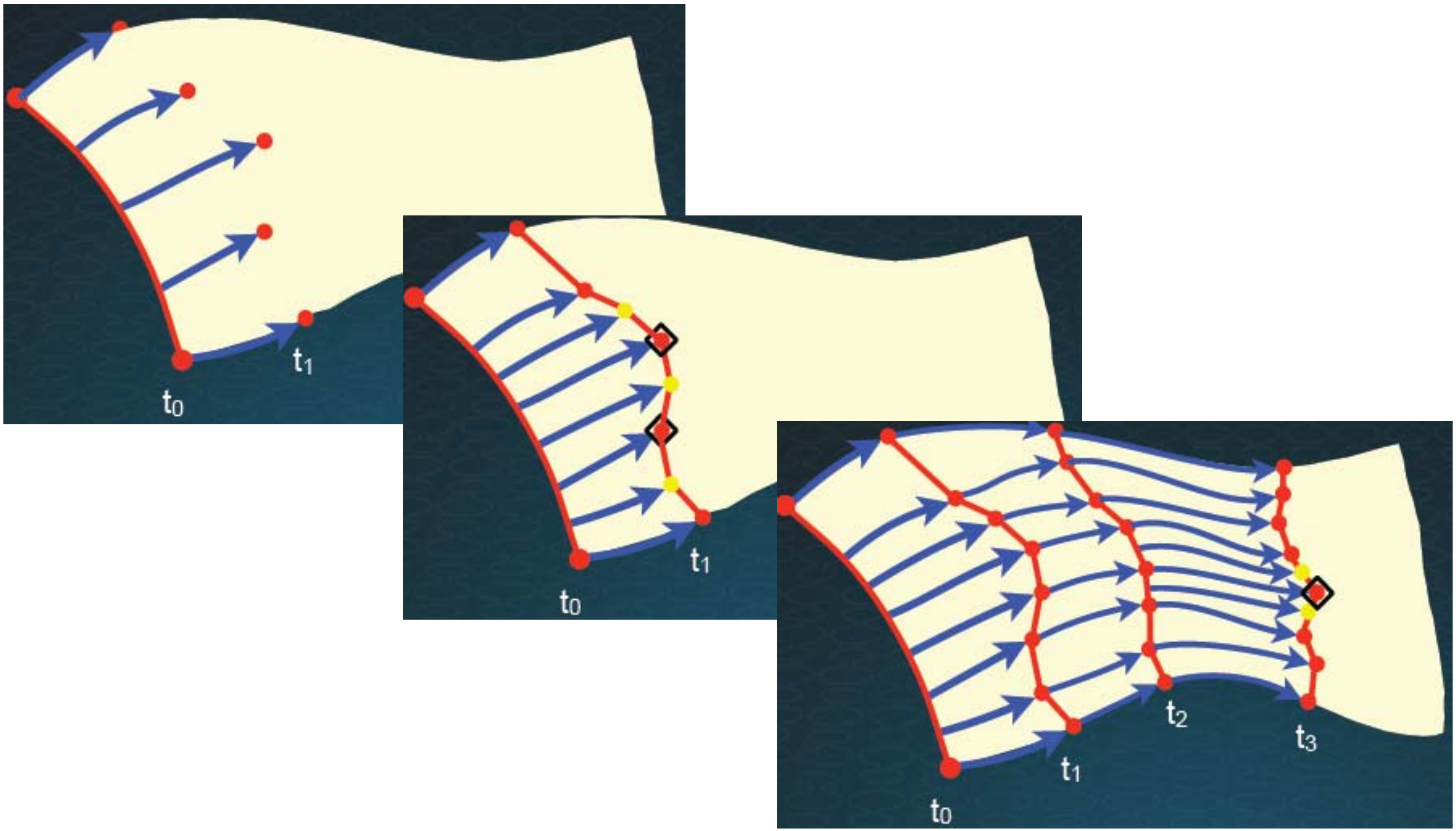


Illustrative path surfaces: texture + transparency [Hemmel et al. 2010]

Higher-Dimensional Feature Descriptors

Path surface:

Its computation can use **timeline** advection



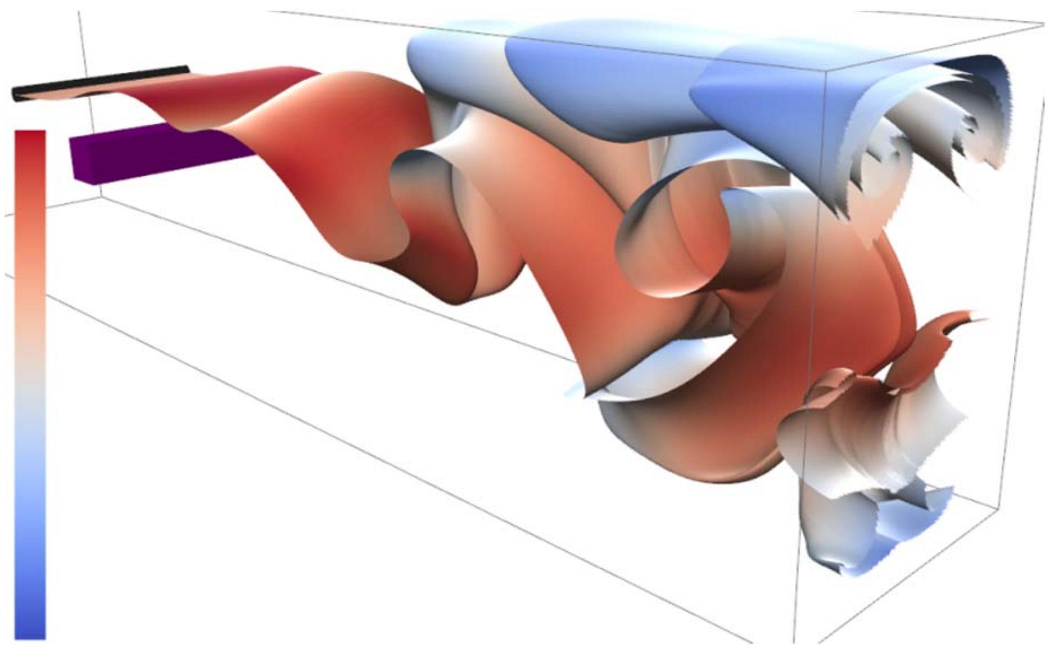
Source: Garth Vis09 Tutorial

Streak Surfaces: Challenges

Streak surfaces are an extension of streak lines (next higher dimension)

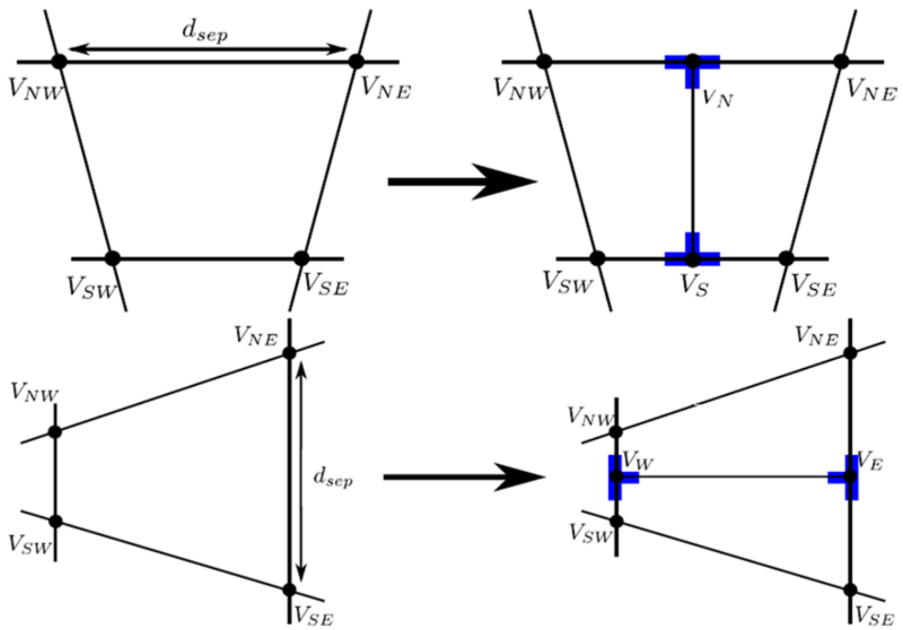
Challenges:

- Computational cost: surface advection is **very** expensive
- Surface completely dynamic: entire surface (all vertices) advect at each time-step
- Mesh quality and maintaining an adequate sampling of the field.
  - Divergence
  - Convergence
  - Shear
- Large size of time-dependent (unsteady) vector field data, out-of-core techniques

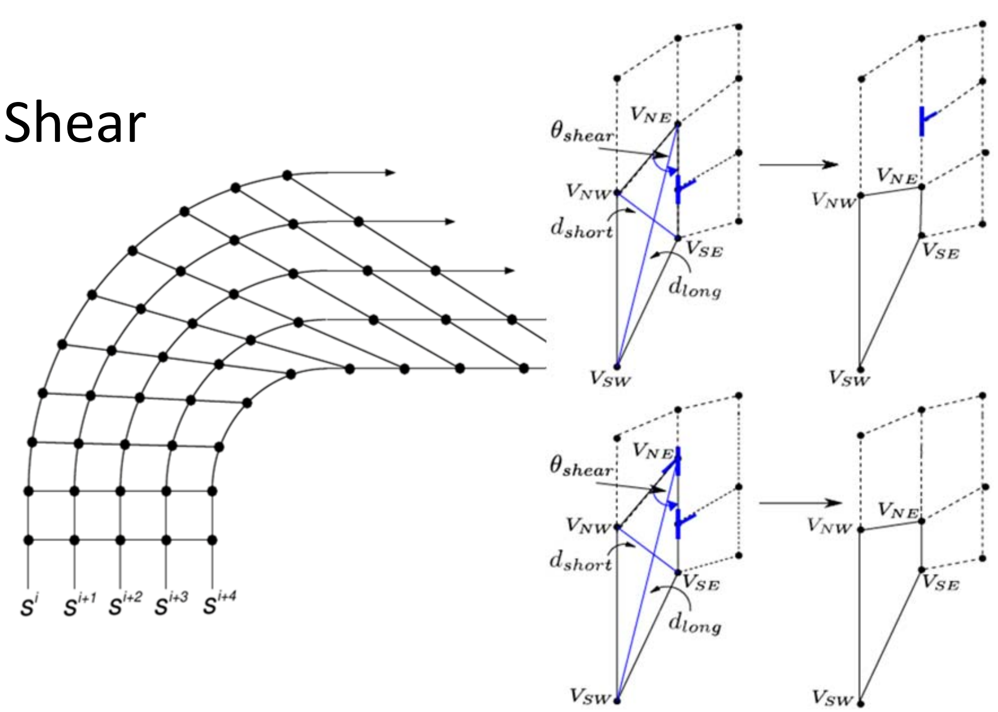


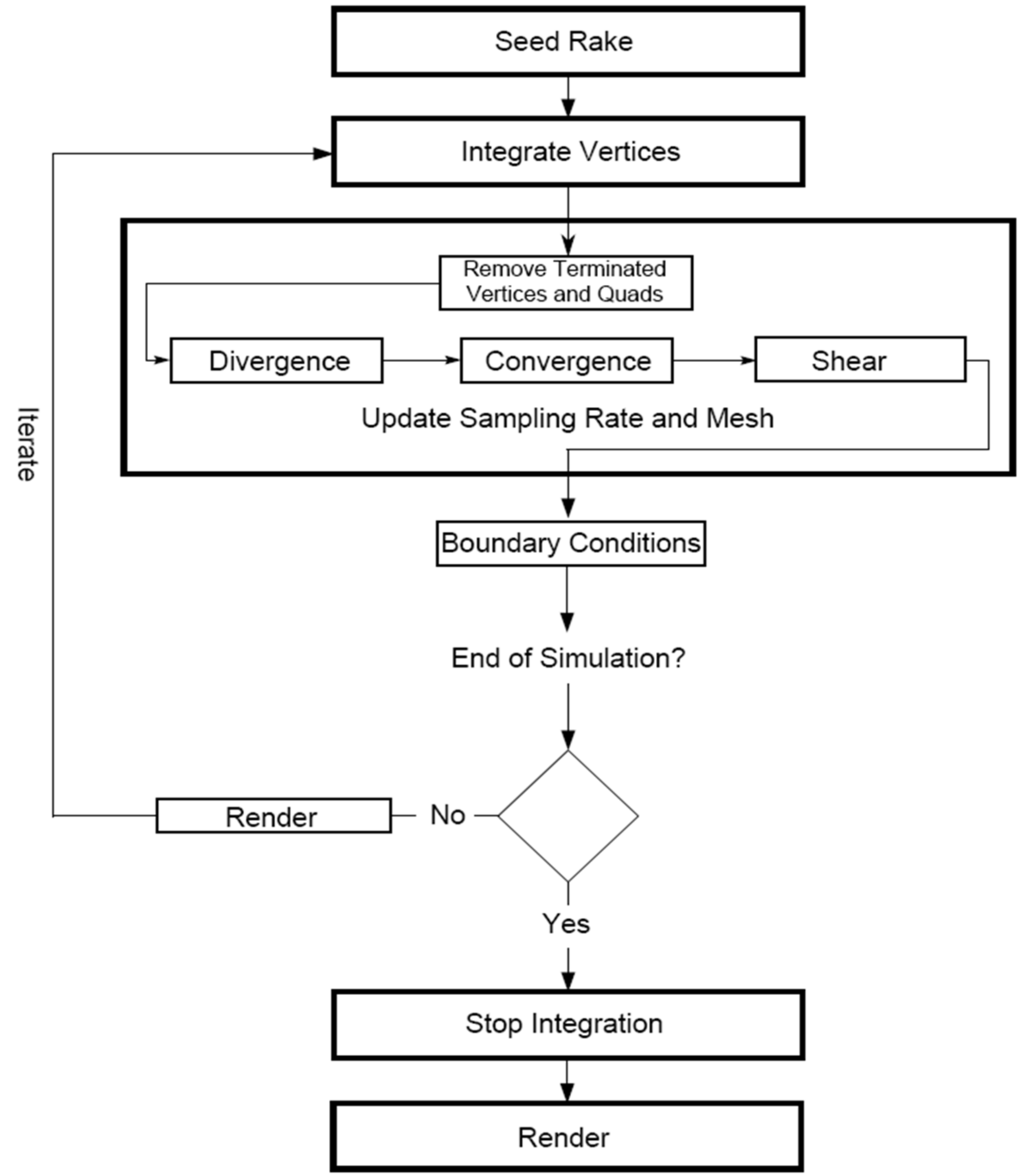
A Streak Surface Computation Pipeline

Divergence: Quad Splitting

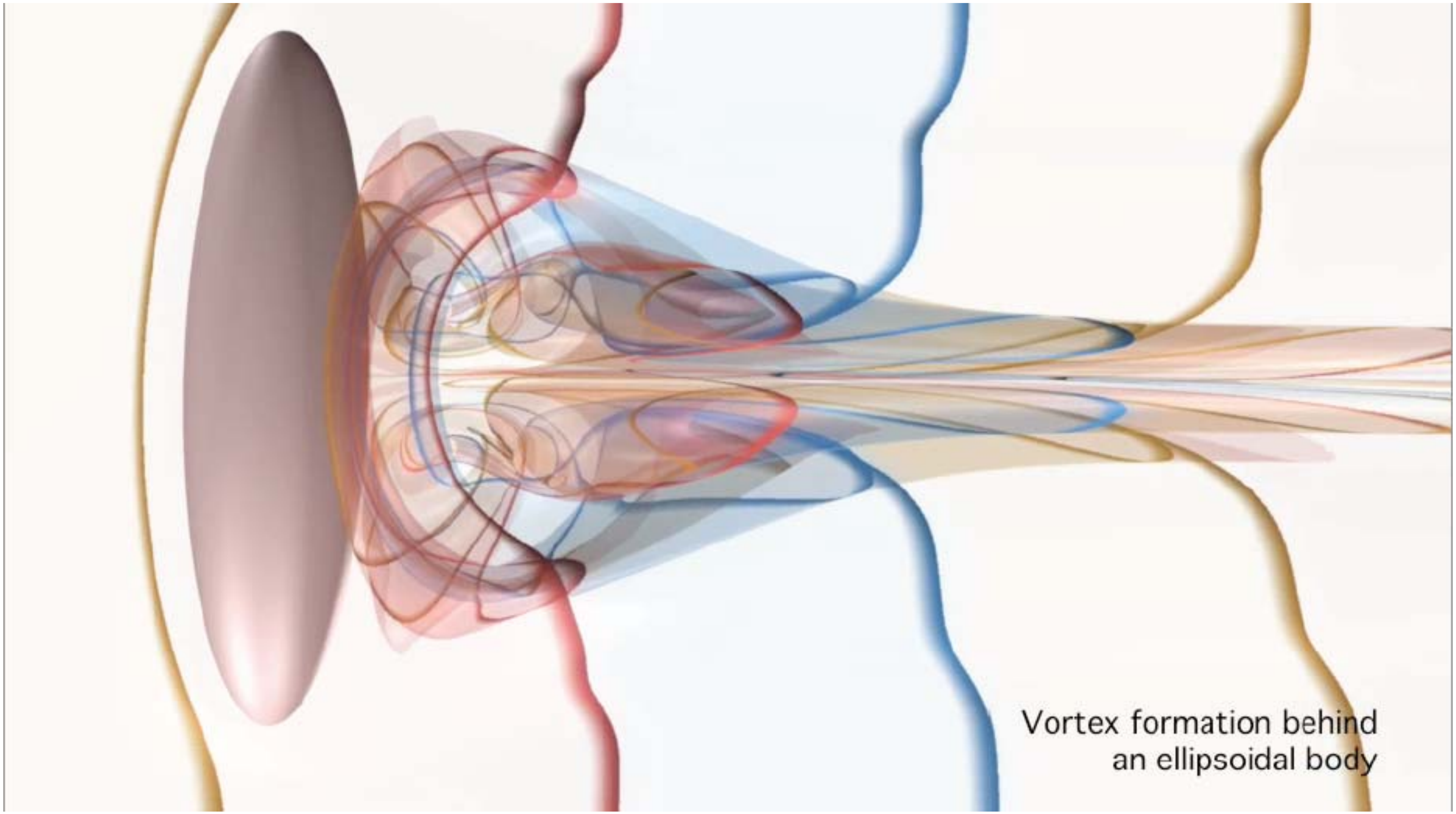


Shear





Time Lines on Streak Surfaces



Source: Garth et al. Vis 2008

Texture-Based Techniques for Unsteady Flow?

Texture-Based Methods

Unsteady flow LIC (UFLIC): forward scattering + collecting

IBFV: texture advection in forward direction + hardware acceleration

Texture-Based Methods

Unsteady flow LIC (UFLIC): forward scattering + collecting

IBFV: texture advection in forward direction + hardware acceleration

Unsteady Vector Field Topology

Topology-based Method

Challenges:

Additional time dimension
Finite range in time dimension

So:

Conventional vector field topology cannot be directly extended to time-dependent setting

Why?

Solution 1: Feature Tracking

- Consider the time-dependent vector field as a stack of time-independent vector fields with slow changes.

- This is actually how we store a time-dependent vector field by recording their values at some sampled time-steps

Solution 1: Feature Tracking

- Parameter dependent topology:
  - “Fixed points” (no more fixed) move, appear, vanish, transform
  - Topological graph connectivity changes

- **Structural stability**: topology is stable w.r.t. small but arbitrary changes of parameter(s) if and only if
  - 1) Number of fixed points and periodic orbits is finite and all are hyperbolic
  - 2) No <u>saddle-saddle</u> connection

Bifurcations

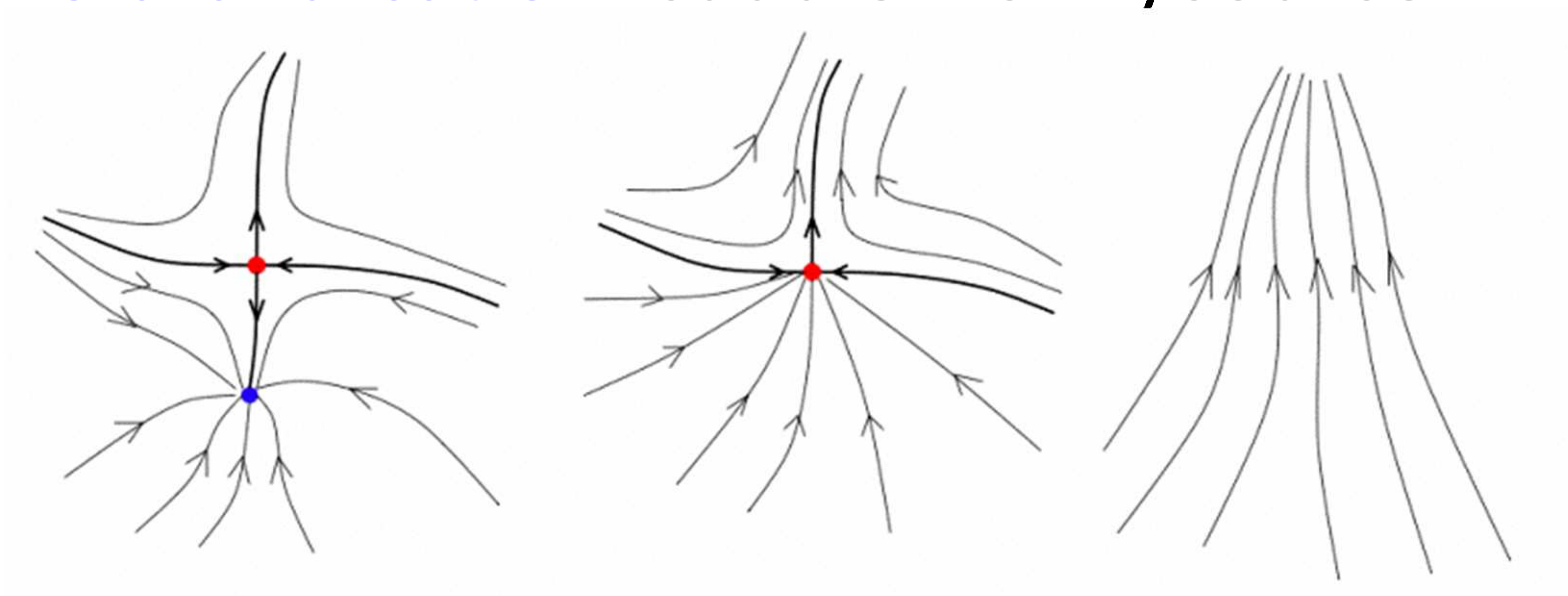
- Transition from one stable structure to another through *unstable state.*

- **Bifurcation value**: parameter value inducing the transition.

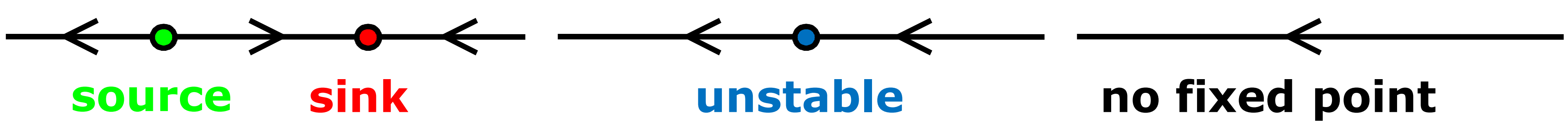
- Local vs. global bifurcations

Local Bifurcations

- Transformation affects local region
- Fold bifurcation: saddle + sink/source

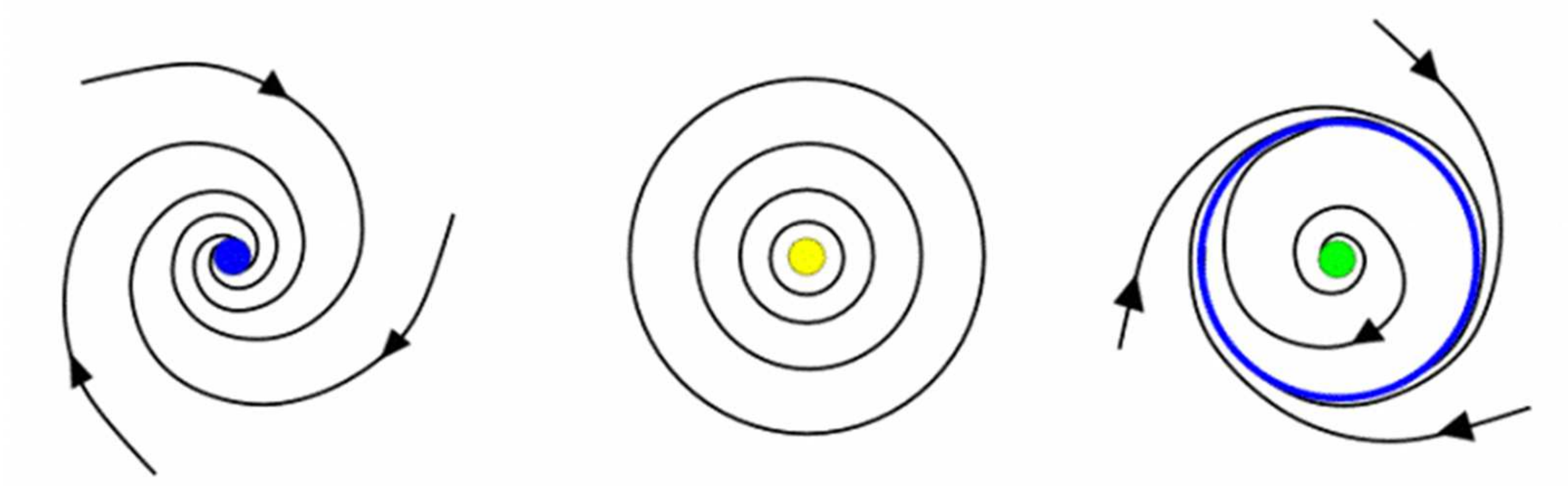


1D equivalent:



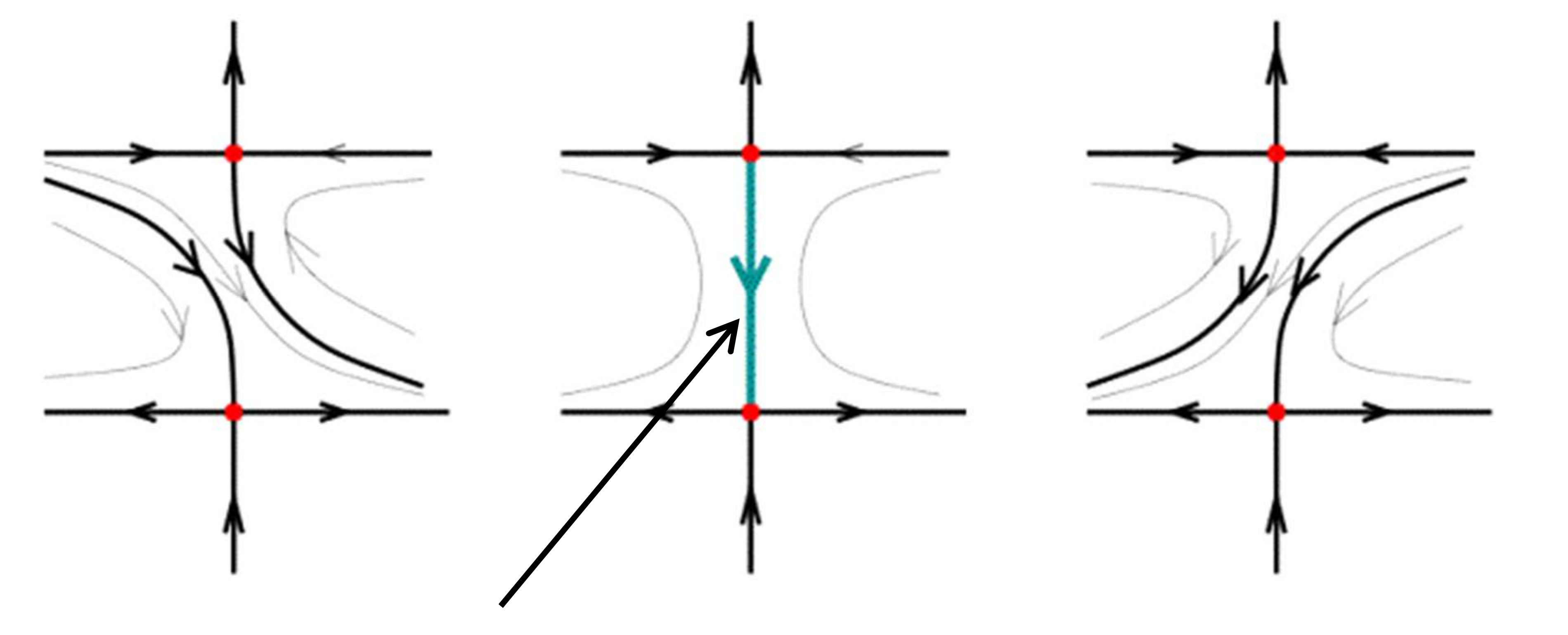
Local Bifurcations

- Transformation affects local region
- Hopf bifurcation: sink/source + closed orbit



Global Bifurcations

- Affects overall topological connectivity
- Basin bifurcation

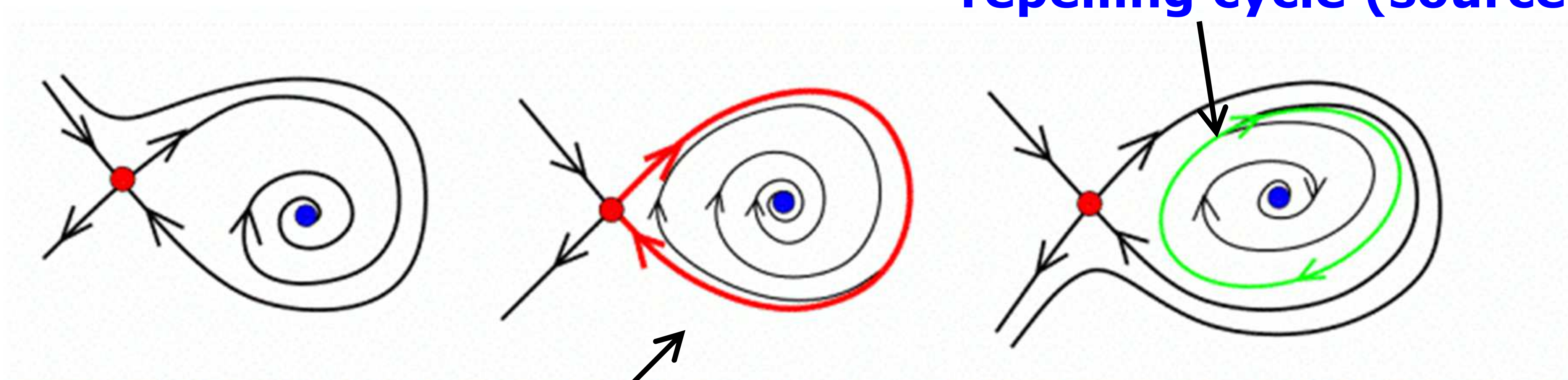


Saddle-saddle connection

Global Bifurcations

- Modifies overall topological connectivity
- Homoclinic bifurcation

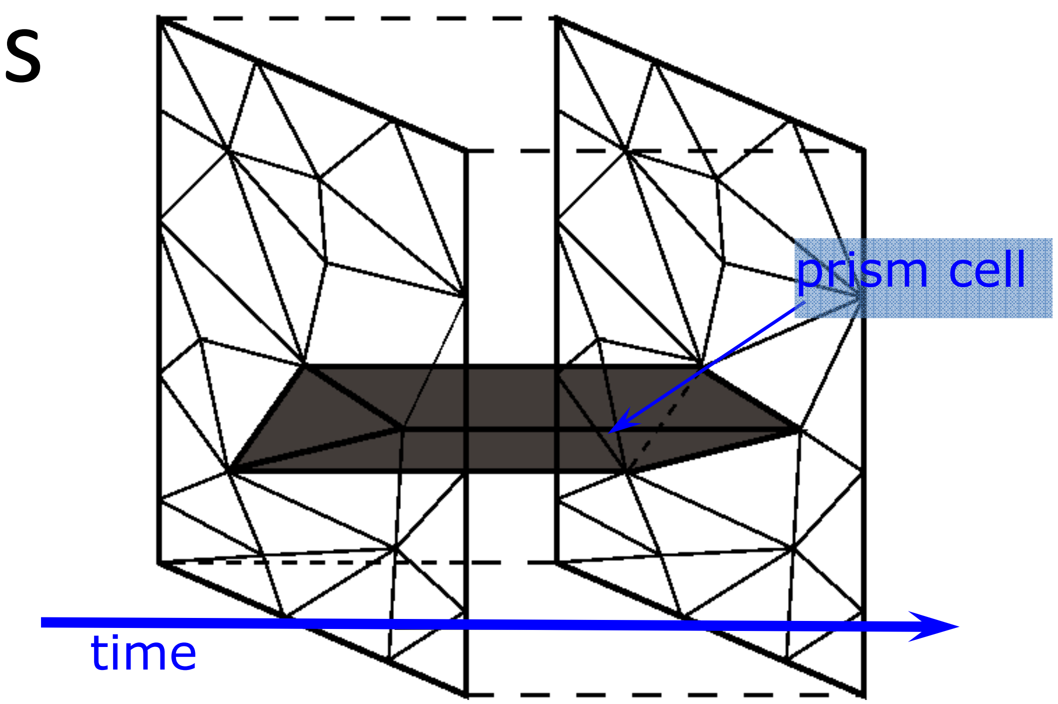
repelling cycle (source)



Saddle-saddle connection

2+1D Topology-A General Pipeline

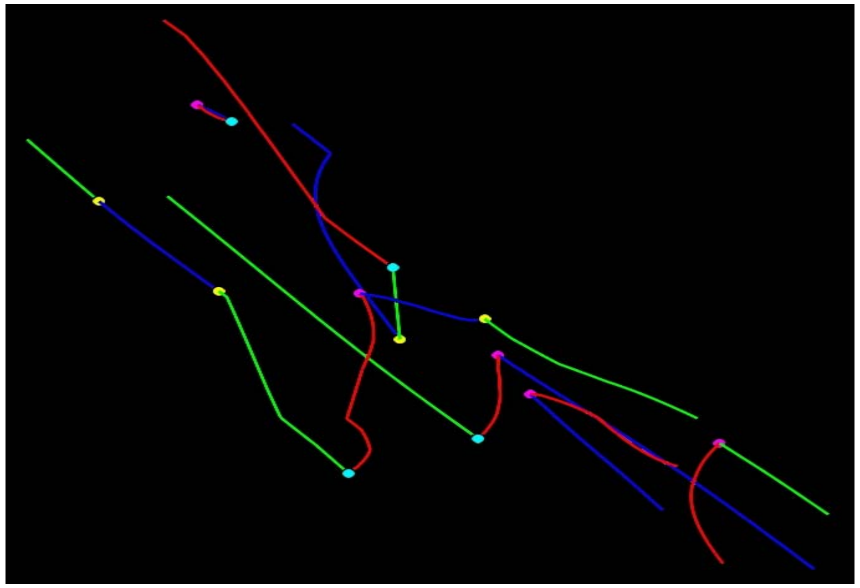
- Time-wise interpolation
- Cell-wise tracking over 2+1D grid
- Detect local bifurcations

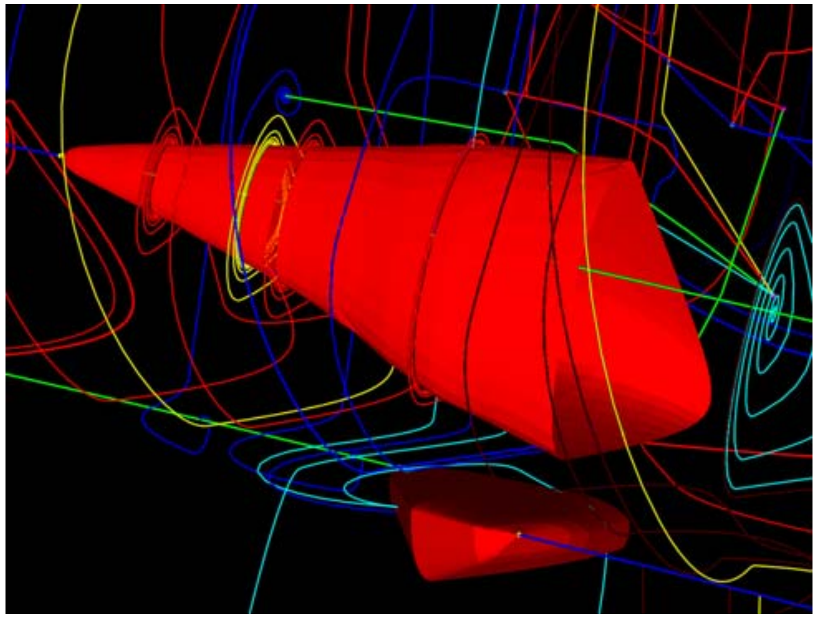


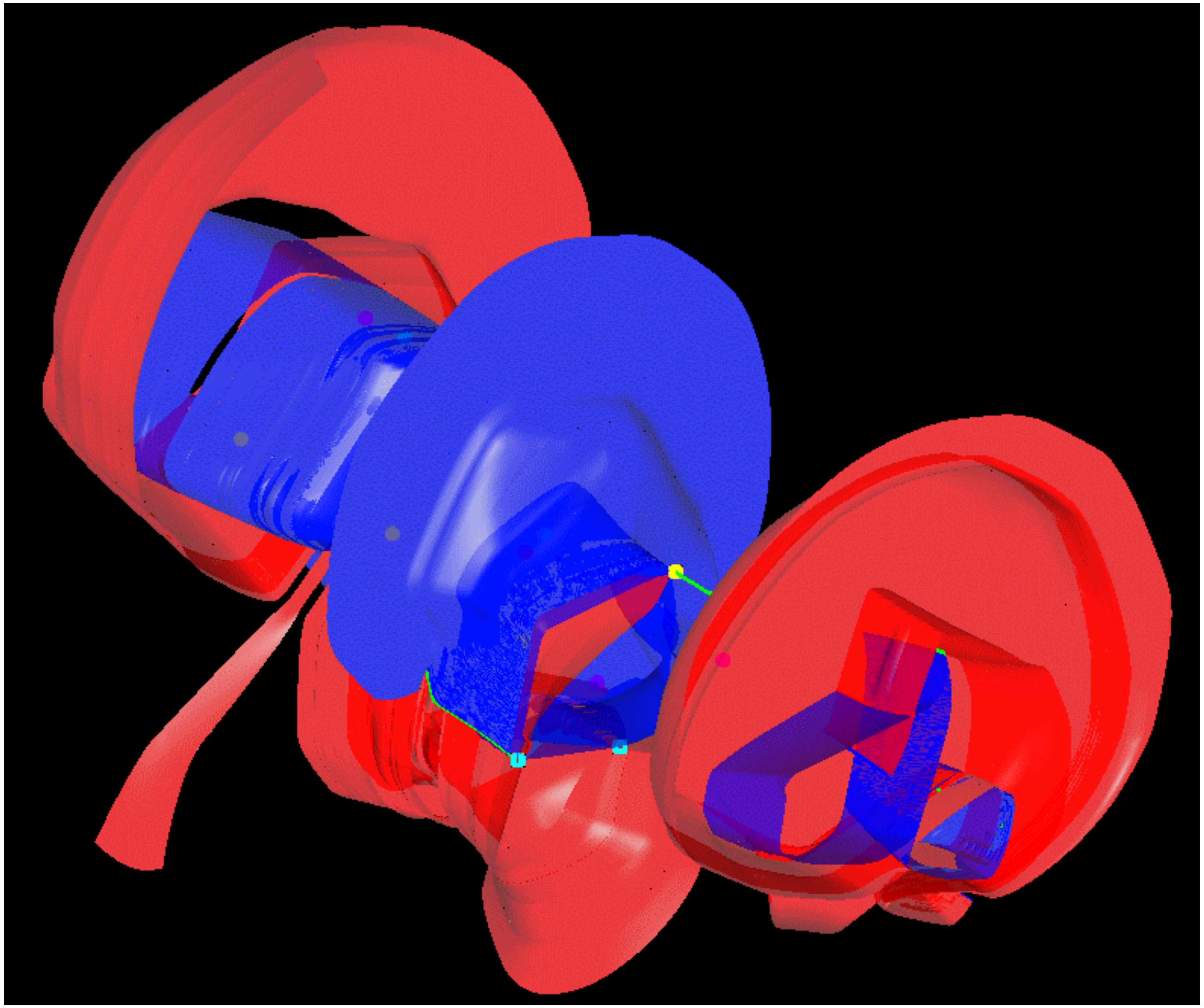
- Track associated separatrices (surfaces)

[Tricoche et al., 2001, 2002]

Some Results







[Tricoche et al., 2001, 2002]

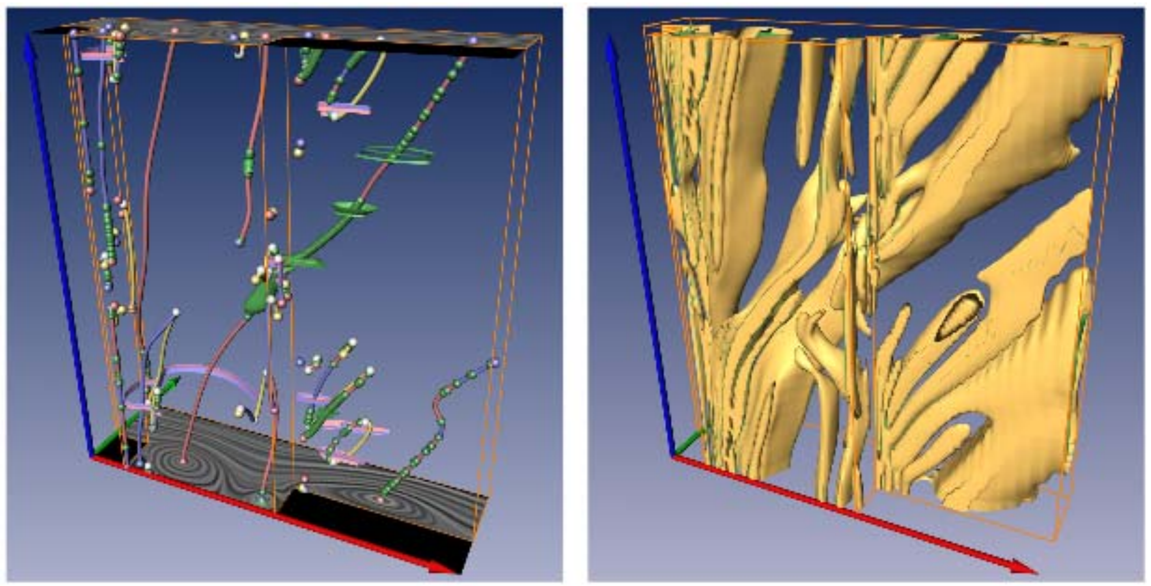
Problem of Topology Tracking

It is still based on the evolution of “streamlines” and their classification!

Under time-dependent setting, what feature curves should be used to describe the true dynamics of particle behaviors?

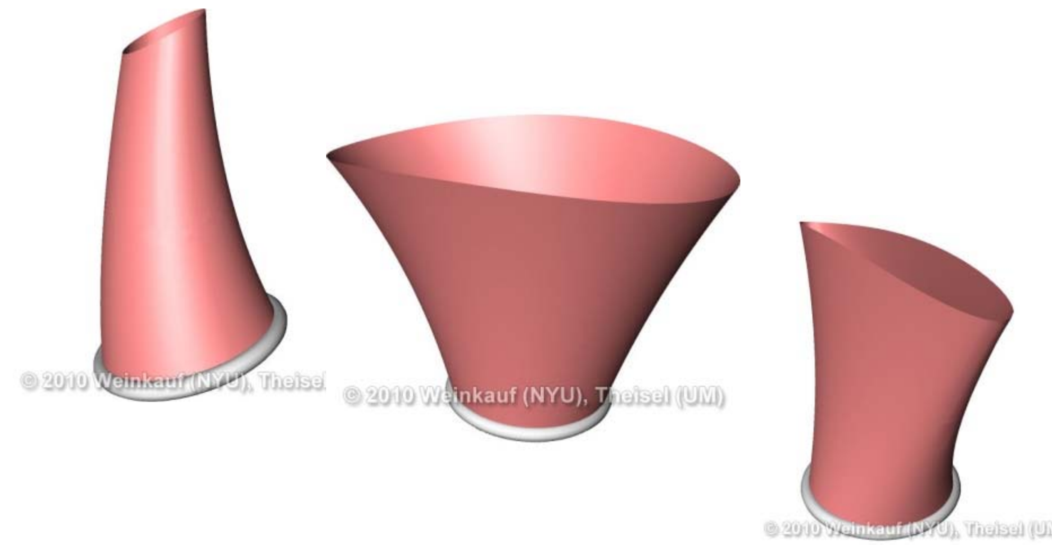
Should be “pathlines”!

Pathline-Oriented Topology

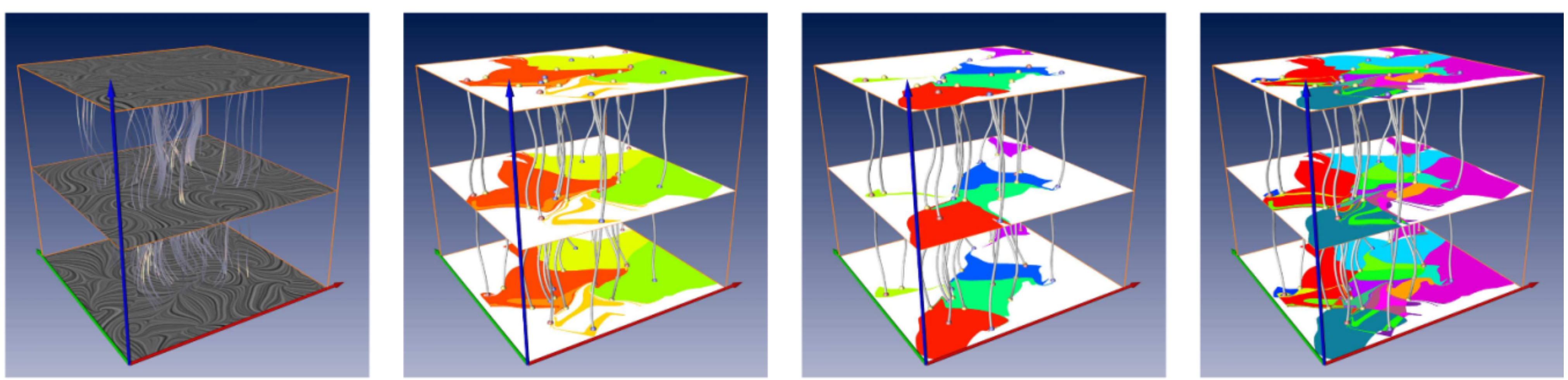


(b) Stream line oriented topology of the first 100 time steps. (c) Path line oriented topology of the first 100 time steps.

[Theisel et al. Vis04, TVCG05]



[Weinkauf et al. 2011]

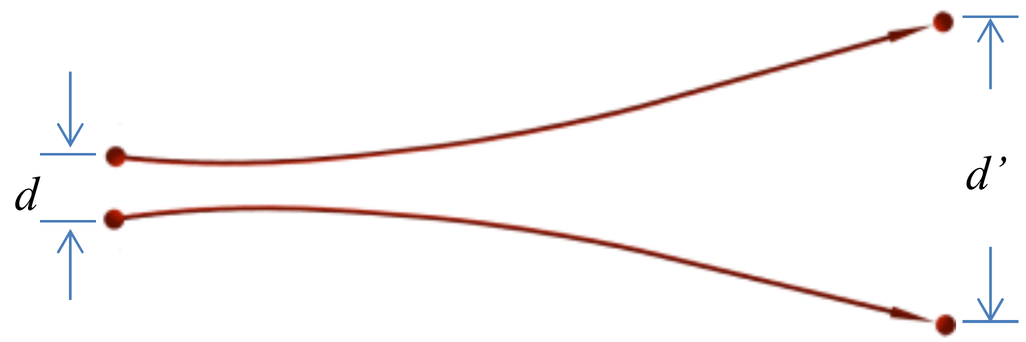


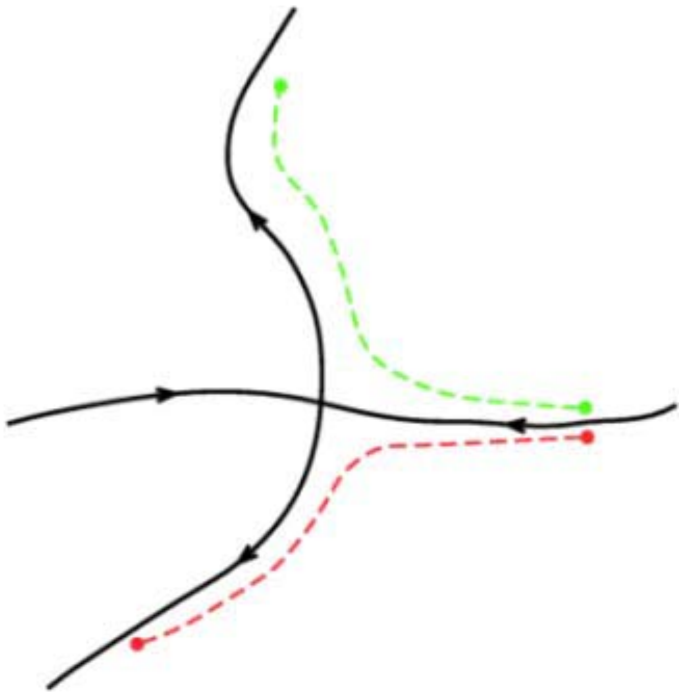
(a) The vector field **p**. (b) Critical path lines and basins for forward integration. (c) Critical path lines and basins for backward integration. (d) Overlayed basins for forward and backward integration.

[Shi et al. EuroVis06]

Finite-Time Lyapunov Exponent

- Some observation
  - Observe particle trajectories
  - Measure the **divergence** between trajectories, i.e. how much flow stretch





[Shadden]

Finite-Time Lyapunov Exponent

- Description
  - Lyapunov exponents describe rate of separation or stretching of two infinitesimally close points over time in a dynamical system
  - FTLE refers to the largest Lyapunov exponent for only a **limited time** and is measured **locally**
  - *Largest exponent is governing the behavior of the system, smaller ones can be neglected*

Defining Separation

- First, we define the **flow map**, ϕ

$$\mathbf{x} \mapsto \phi_{t_0}^{t_0+T}(\mathbf{x})$$

  - This encodes the position a particle travels starting at position x, time t_0 and across a window of time T

- It's spatial derivatives will encode how much local stretch occurs (this is the (right) Cauchy-Green tensor, Δ)

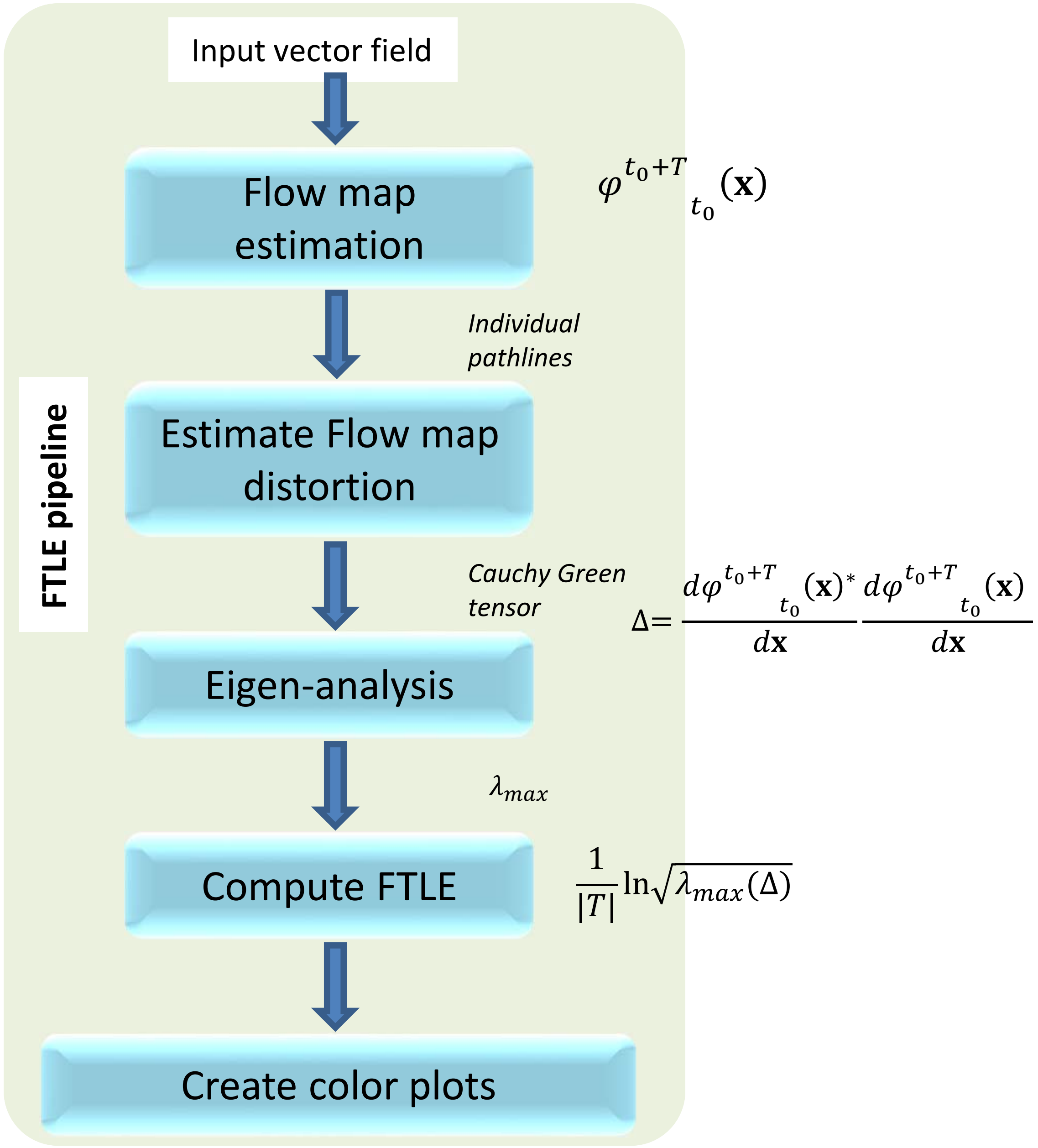
$$\Delta = \frac{d\phi_{t_0}^{t_0+T}(\mathbf{x})}{d\mathbf{x}}^{*} \frac{d\phi_{t_0}^{t_0+T}(\mathbf{x})}{d\mathbf{x}}$$

- This allows us to define the **FTLE**, σ, based on the maximum eigenvalue, $\lambda_{\max}$, of this tensor

$$\sigma_{t_0}^{T}(\mathbf{x}) = \frac{1}{|T|} \ln \sqrt{\lambda_{\max}(\Delta)}\ ,$$

Finite-Time Lyapunov Exponent

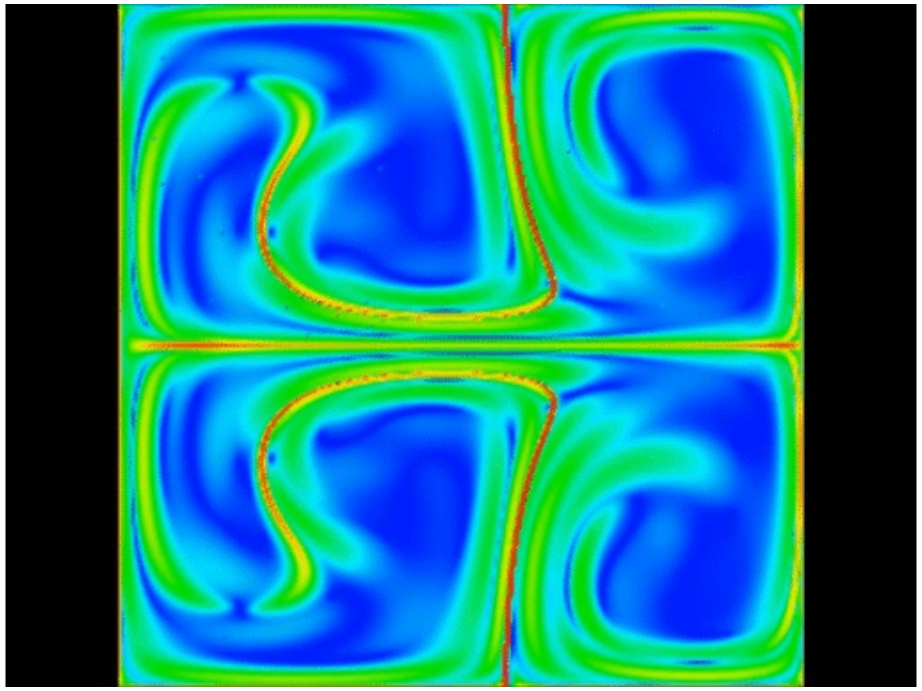
A computation framework



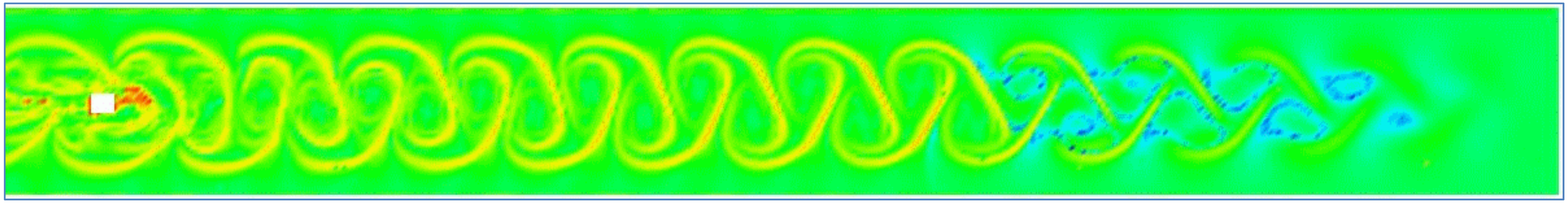
Finite-Time Lyapunov Exponent

- Examples:

unsteady quad-gyre



Flow around a cylinder



Other examples: http://shaddenlab.berkeley.edu/uploads/LCS-tutorial/contents.html

Lec25 Reading

- Uncertainty Visualization (Chapter 22). Lace Padilla, Matthew Kay, and Jessica Hullman. In W. Piegorsch, R. Levine, H. Zhang, and T. Lee (Eds.), Computational Statistics in Data Science (pp. 405-421). Wiley, 2022.

Reminder
Assignment 06

Assigned: Monday, April 10
Due: Monday, April 24, 4:59:59 pm

Reminder
Project Milestones 03/04

Assigned: Wednesday, March 29

03 (Talk) Due: Wednesday, April 26, 4:59:59 pm

04 (Report) Due: Wednesday, May 3, 4:59:59 pm