CSC 544
Data Visualization

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Lecture 22
Flow Data

Apr. 5, 2023

Today's Agenda

- Reminders:
    - A05 questions? (due Apr. 10)
    - P03/P04 questions? (due Apr. 26/May 3)
- Goals for today: Discuss designing modeling data for flow

Vector Fields

What is a Vector Field?

$$s: \mathbb{R}^n \to \mathbb{R}$$

Scalar Field

$$\mathbf{v}: \mathbb{R}^n \to \mathbb{R}^m$$

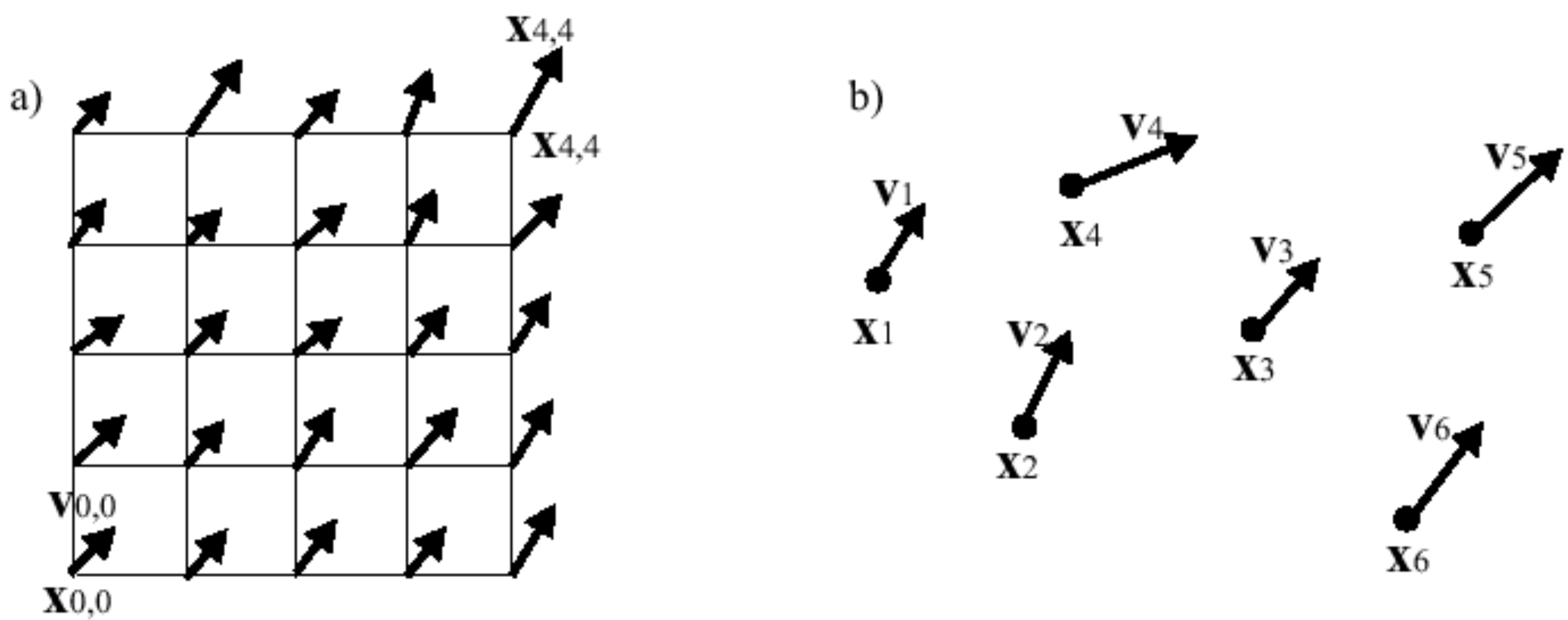
Vector Field

- m will often be equal to n, but not necessarily

- Main application of vector field visualization is flow visualization
  - Motion of fluids (gas, liquids)
  - Geometric boundary conditions
  - Velocity (flow) field $\boldsymbol{v}(\boldsymbol{x},t)$
  - Pressure p
  - Temperature T
  - Vorticity $\nabla\times\boldsymbol{v}$
  - Density ρ
  - Conservation of mass, energy, and momentum
  - Navier-Stokes equations
  - CFD (Computational Fluid Dynamics)

Flow Data

- Vector data on a 2D or 3D grid
- Additional scalar data may be defined per grid point
- Can either be on a regular grid (a) or scattered data points (b)



Cylinder Data Set

IEEE Visualization 2007

Weinkauf, Sahner, Theisel, Hege

Cores of Swirling Particle Motion in Unsteady Flows

Cylinder Data Set

IEEE Visualization 2007

Weinkauf, Sahner, Theisel, Hege

Cores of Swirling Particle Motion in Unsteady Flows

Experimental Flow Visualization

Tuesday, 7 February 2012

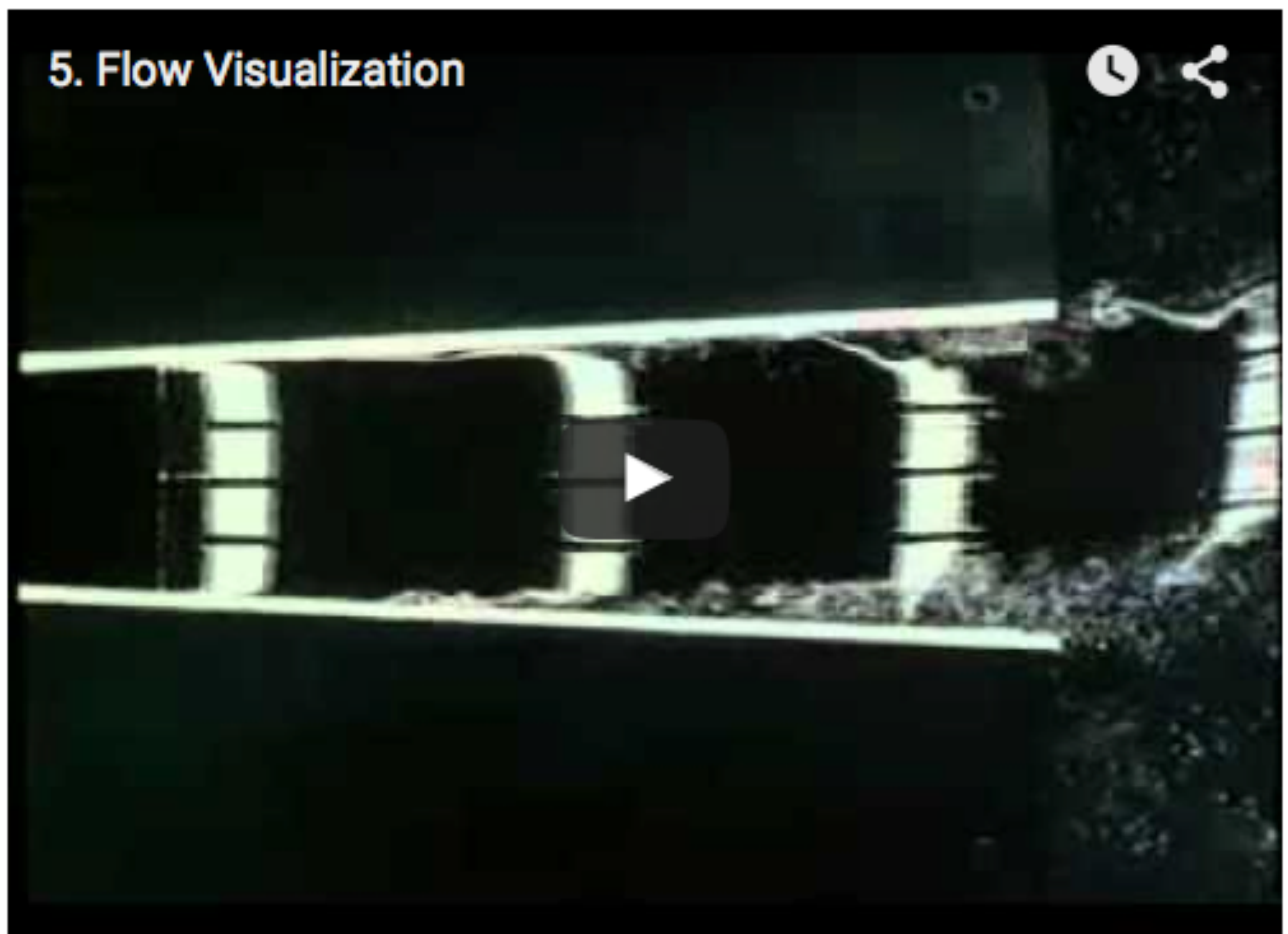
Flow Visualization

National Committee for Fluids Mechanics Films.

With Stephen J. Kline, Stanford University.

Film notes.

Other videos from this series



Milestones in Flight History
Dryden Flight Research Center

L-1011

Airliner Wing Vortice Tests at Langley

Circa 1970s

Milestones in Flight History
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L-1011

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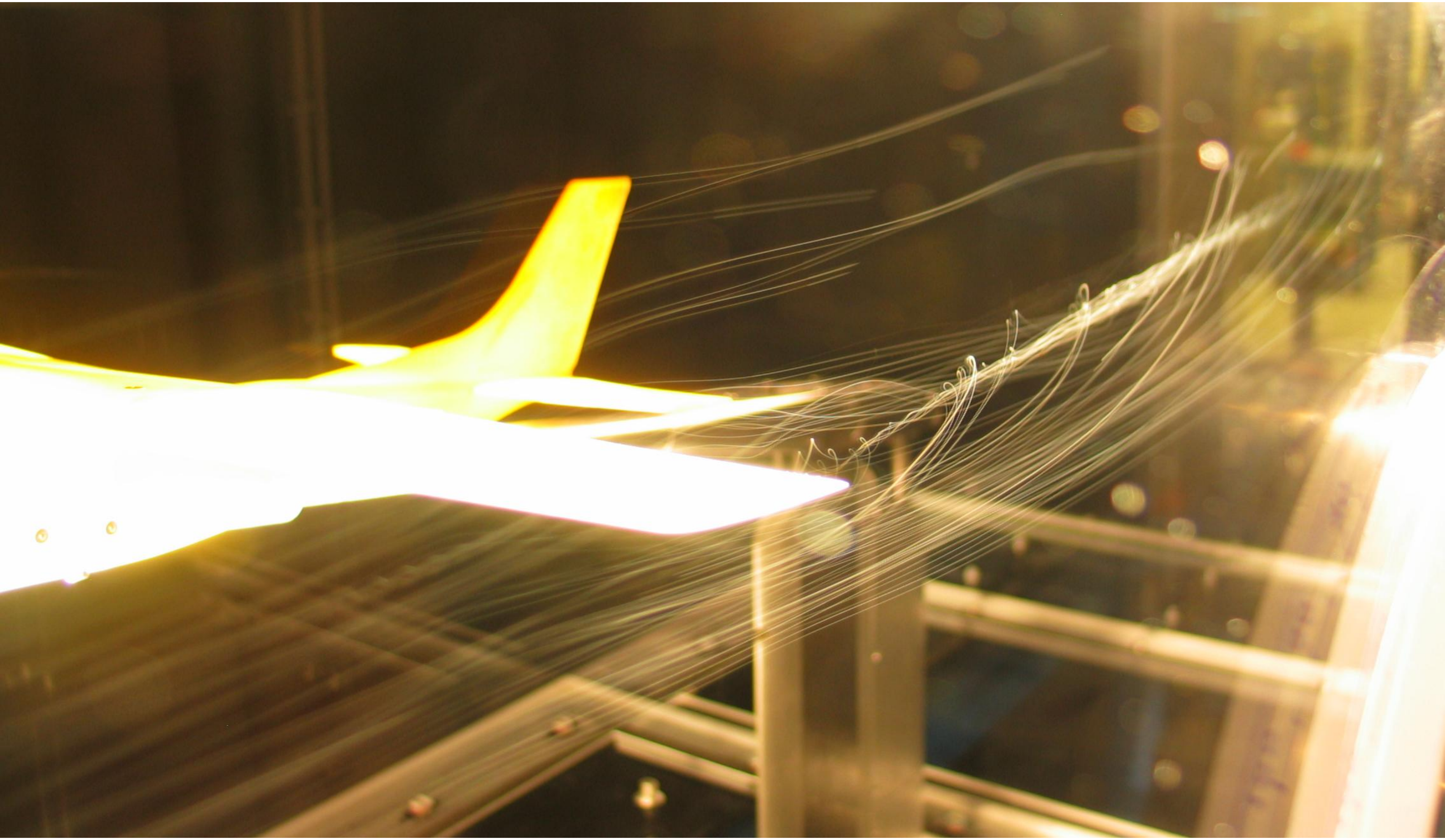
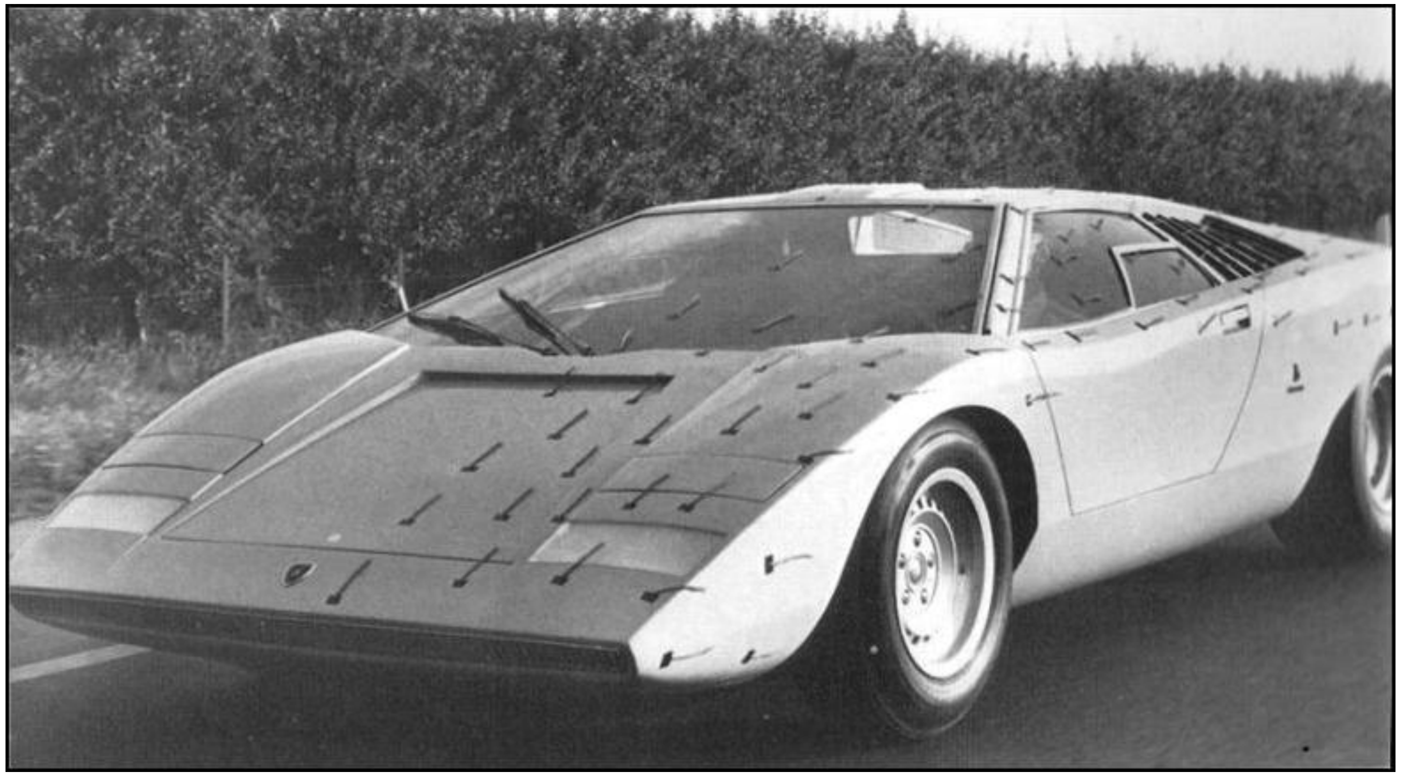
Circa 1970s

Smoke angel
A C-17 Globemaster III from the 14th Airlift Squadron, Charleston Air Force Base, S.C. flies off after releasing flares over the Atlantic Ocean near Charleston, S.C., during a training mission on Tuesday, May 16, 2006. The "smoke angel" is caused by the vortex from the engines.
(U.S. Air Force photo/Tech. Sgt. Russell E. Cooley IV)



A wind tunnel model of a Cessna 182 showing a wingtip vortex.
Tested in the RPI (Rensselaer Polytechnic Institute) Subsonic Wind Tunnel.
By Ben FrantzDale (2007).



Flow Visualization: Problems and Concepts

Wool Tufts

http://autospeed.com/cms/A_108677/article.html

Smoke Nozzles

Smoke Injection

http://autospeed.com/cms/A_108677/article.html

Smoke Nozzles

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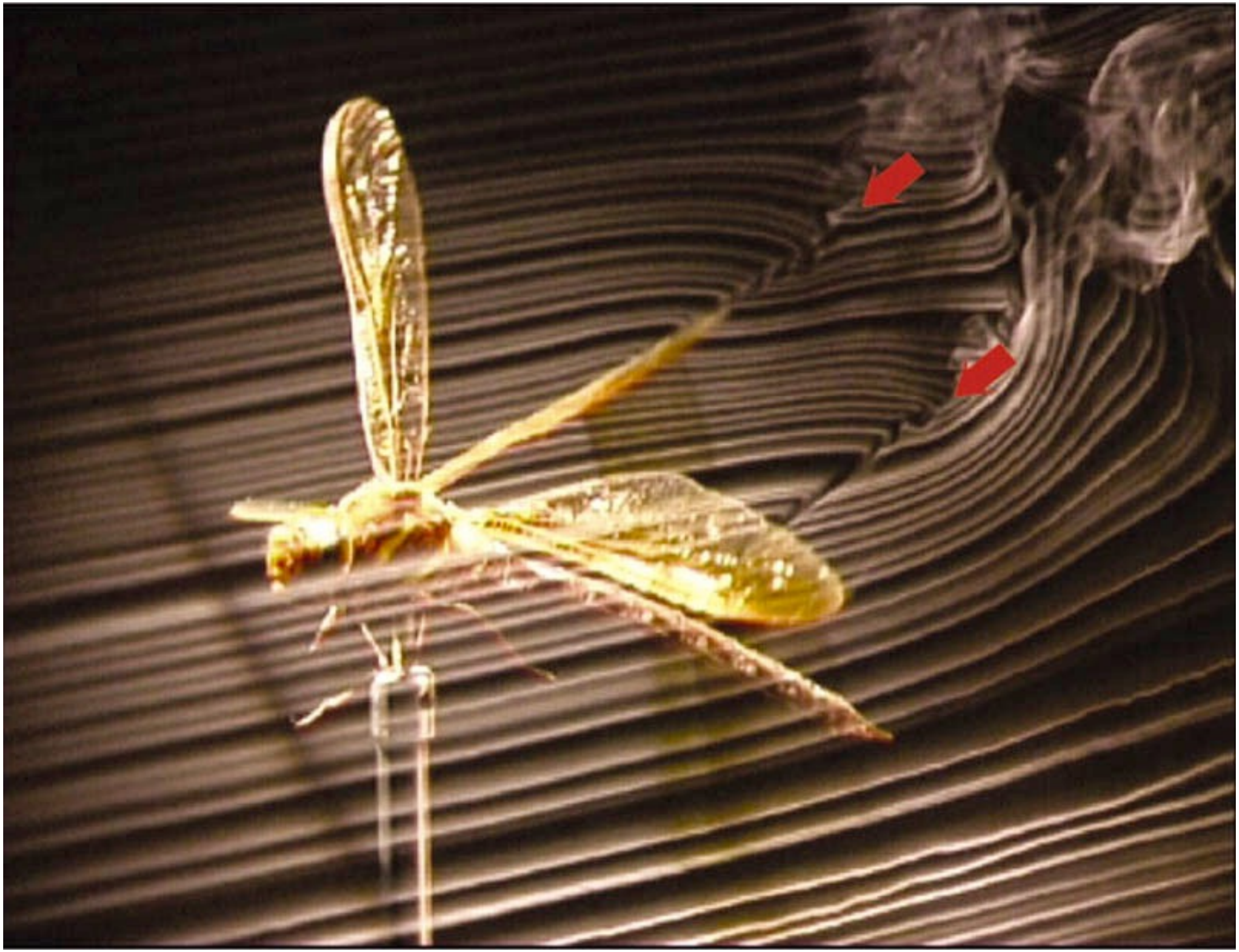
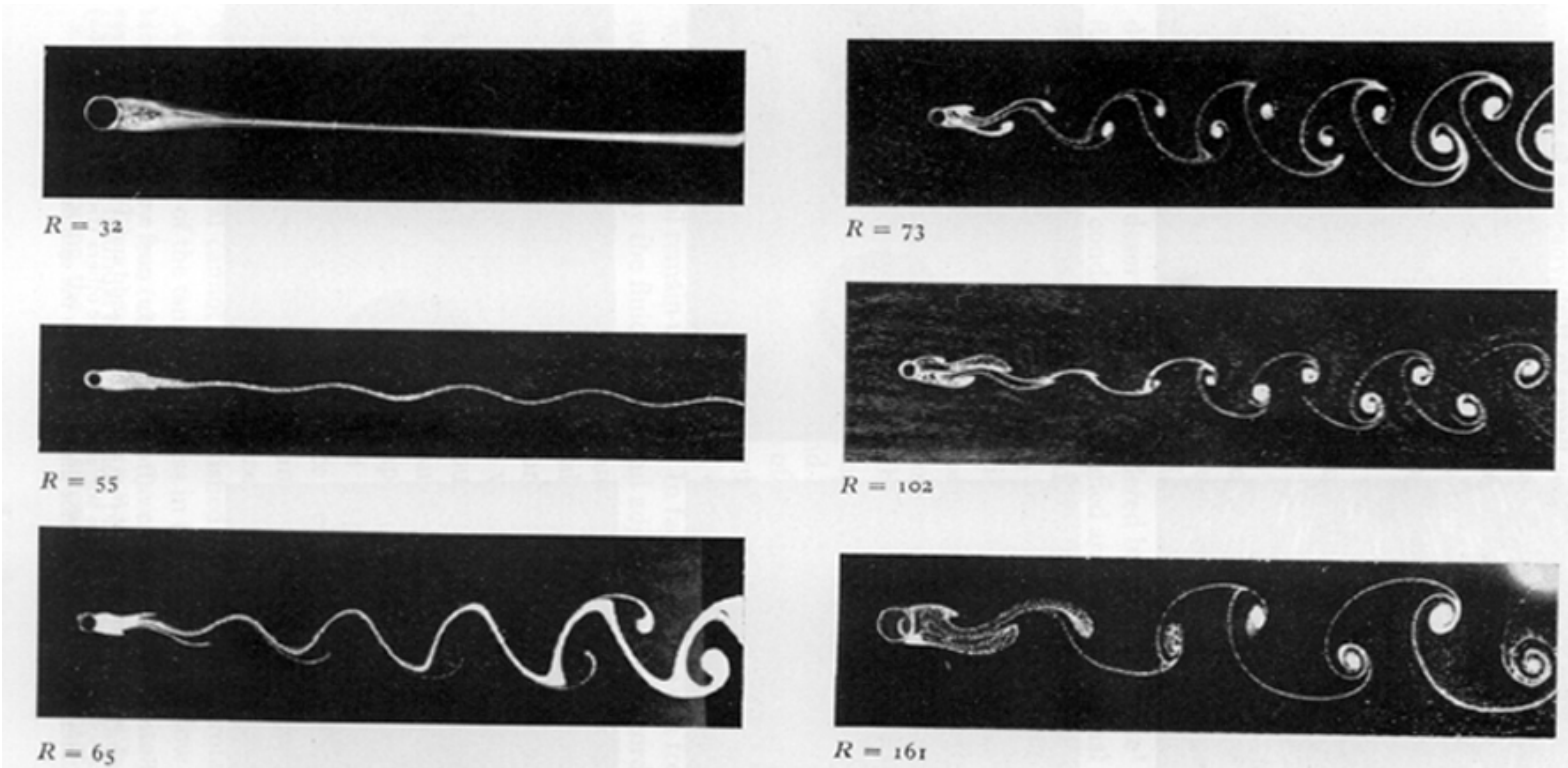


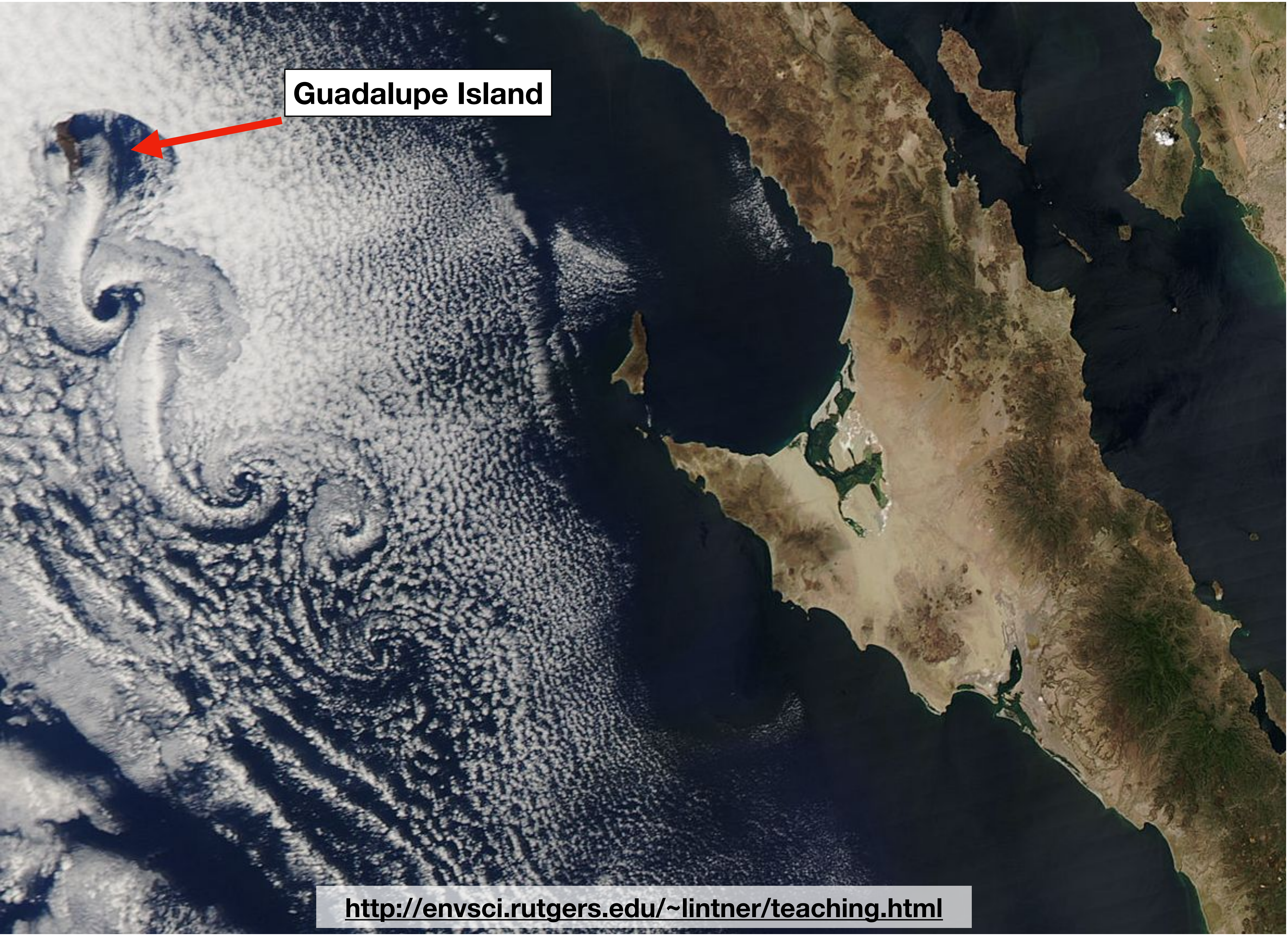
A. L. R. Thomas, G. K. Taylor, R. B. Srygley, R. L. Nudds, and R. J. Bomphrey. Dragonfly flight: free-flight and tethered flow visualizations reveal a diverse array of unsteady lift-generating mechanisms, controlled primarily via angle of attack. J Exp Biol, 207(24):4299–4323, 2004.

http://de.wikipedia.org/wiki/Bild:Airplane_vortex_edit.jpg

Experimental Flow Vis



von Kármán vortex street, depending on Reynolds number


Guadalupe Island
http://envsci.rutgers.edu/~lintner/teaching.html

Experimental Flow Vis

T. Gerhold and P. Krogmann. Investigation of the hypersonic turbulent flow past a blunt fin/wedge configuration. AIAA-93-5026, 1993.

Vector Field Mathematics

scalar field

$$s : \mathbb{E}^n \rightarrow \mathbb{R}$$

vector field

$$\mathbf{v} : \mathbb{E}^n \rightarrow \mathbb{R}^m$$

tensor field

$$\mathbf{T} : \mathbb{E}^n \rightarrow \mathbb{R}^{m \times b}$$

scalar field

$$s : \mathbb{E}^n \rightarrow \mathbb{R}$$

$$s(\mathbf{x})$$

with $\mathbf{x} \in \mathbb{E}^n$

vector field

$$\mathbf{v} : \mathbb{E}^n \rightarrow \mathbb{R}^m$$

$$\mathbf{v}(\mathbf{x}) = \begin{pmatrix} c_1(\mathbf{x}) \\ \vdots \\ c_m(\mathbf{x}) \end{pmatrix}$$

with $\mathbf{x} \in \mathbb{E}^n$

tensor field

$$\mathbf{T} : \mathbb{E}^n \rightarrow \mathbb{R}^{m \times b}$$

$$\mathbf{T}(\mathbf{x}) = \begin{pmatrix} c_{11}(\mathbf{x}) & \ldots & c_{1b}(\mathbf{x}) \\ \vdots & & \vdots \\ c_{m1}(\mathbf{x}) & \ldots & c_{mb}(\mathbf{x}) \end{pmatrix}$$

with $\mathbf{x} \in \mathbb{E}^n$

scalar field

$$s : \mathbb{E}^n \rightarrow \mathbb{R}$$

$$s(\mathbf{x})$$

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$$\mathbf{v}(\mathbf{x}) = \begin{pmatrix} c_1(\mathbf{x}) \\ \vdots \\ c_m(\mathbf{x}) \end{pmatrix}$$

with $\mathbf{x} \in \mathbb{E}^n$

$$\mathbf{v}(x, y) = \begin{pmatrix} u(x, y) \\ v(x, y) \end{pmatrix}$$

2D vector field

tensor field

$$\mathbf{T} : \mathbb{E}^n \rightarrow \mathbb{R}^{m \times b}$$

$$\mathbf{T}(\mathbf{x}) = \begin{pmatrix} c_{11}(\mathbf{x}) & \dots & c_{1b}(\mathbf{x}) \\ \vdots & & \vdots \\ c_{m1}(\mathbf{x}) & \dots & c_{mb}(\mathbf{x}) \end{pmatrix}$$

with $\mathbf{x} \in \mathbb{E}^n$

scalar field

$$s : \mathbb{E}^n \rightarrow \mathbb{R}$$

$$s(\mathbf{x})$$

with $\mathbf{x} \in \mathbb{E}^n$

Could be the gradient of a scalar field.

vector field

$$\mathbf{v} : \mathbb{E}^n \rightarrow \mathbb{R}^m$$

$$\mathbf{v}(\mathbf{x}) = \begin{pmatrix} c_1(\mathbf{x}) \\ \vdots \\ c_m(\mathbf{x}) \end{pmatrix}$$

with $\mathbf{x} \in \mathbb{E}^n$

$$\mathbf{v}(x, y) = \begin{pmatrix} u(x, y) \\ v(x, y) \end{pmatrix}$$

2D vector field

tensor field

$$\mathbf{T} : \mathbb{E}^n \rightarrow \mathbb{R}^{m \times b}$$

$$\mathbf{T}(\mathbf{x}) = \begin{pmatrix} c_{11}(\mathbf{x}) & \dots & c_{1b}(\mathbf{x}) \\ \vdots & & \vdots \\ c_{m1}(\mathbf{x}) & \dots & c_{mb}(\mathbf{x}) \end{pmatrix}$$

with $\mathbf{x} \in \mathbb{E}^n$

vector field

$$\mathbf{v} : \mathbb{E}^n \rightarrow \mathbb{R}^m$$

tensor field

$$\mathbf{T} : \mathbb{E}^n \rightarrow \mathbb{R}^{m \times b}$$

$$\mathbf{v}(x, y) = \begin{pmatrix} u(x, y) \\ v(x, y) \end{pmatrix}$$

2D vector field

$$\nabla \mathbf{v}(x, y) = \mathbf{J}(x, y) = \begin{pmatrix} u_x & u_y \\ v_x & v_y \end{pmatrix}$$

Jacobian of 2D vector field

vector field

$$\mathbf{v} : \mathbb{E}^n \rightarrow \mathbb{R}^m$$

tensor field

$$\mathbf{T} : \mathbb{E}^n \rightarrow \mathbb{R}^{m \times b}$$

Supposed to be continuous and differentiable. This means we can compute its partial differentials and write them into the 2x2 matrix:

$$\mathbf{v}(x, y) = \begin{pmatrix} u(x, y) \\ v(x, y) \end{pmatrix} \quad \nabla \mathbf{v}(x, y) = \mathbf{J}(x, y) = \begin{pmatrix} u_x & u_y \\ v_x & v_y \end{pmatrix}$$

2D vector field

Jacobian of 2D vector field

vector field

$$\mathbf{v} : \mathbb{E}^n \longrightarrow \mathbb{R}^m$$

vector field

parameter-independent

$$\mathbf{v} : \mathbb{E}^n \to \mathbb{R}^m$$

steady vector field

vector field

parameter-independent

$$\mathbf{v} : \mathbb{E}^n \rightarrow \mathbb{R}^m$$

steady vector field

one-parameter-dependent

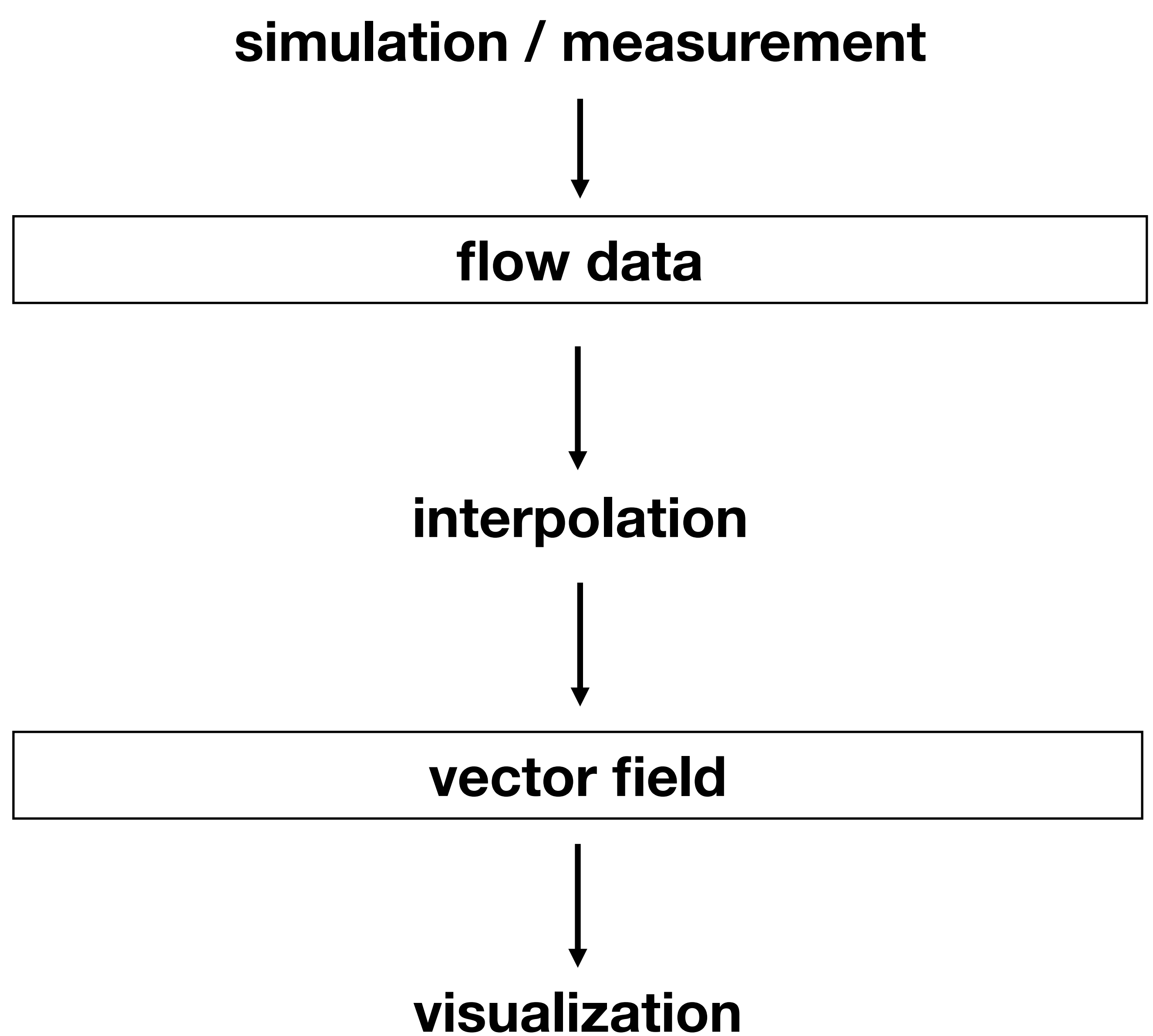
$$\mathbf{v} : \mathbb{E}^{n+1} \rightarrow \mathbb{R}^m$$

$$\mathbf{v}(x, y, t) = \begin{pmatrix} u(x, y, t) \\ v(x, y, t) \end{pmatrix}$$

unsteady vector field

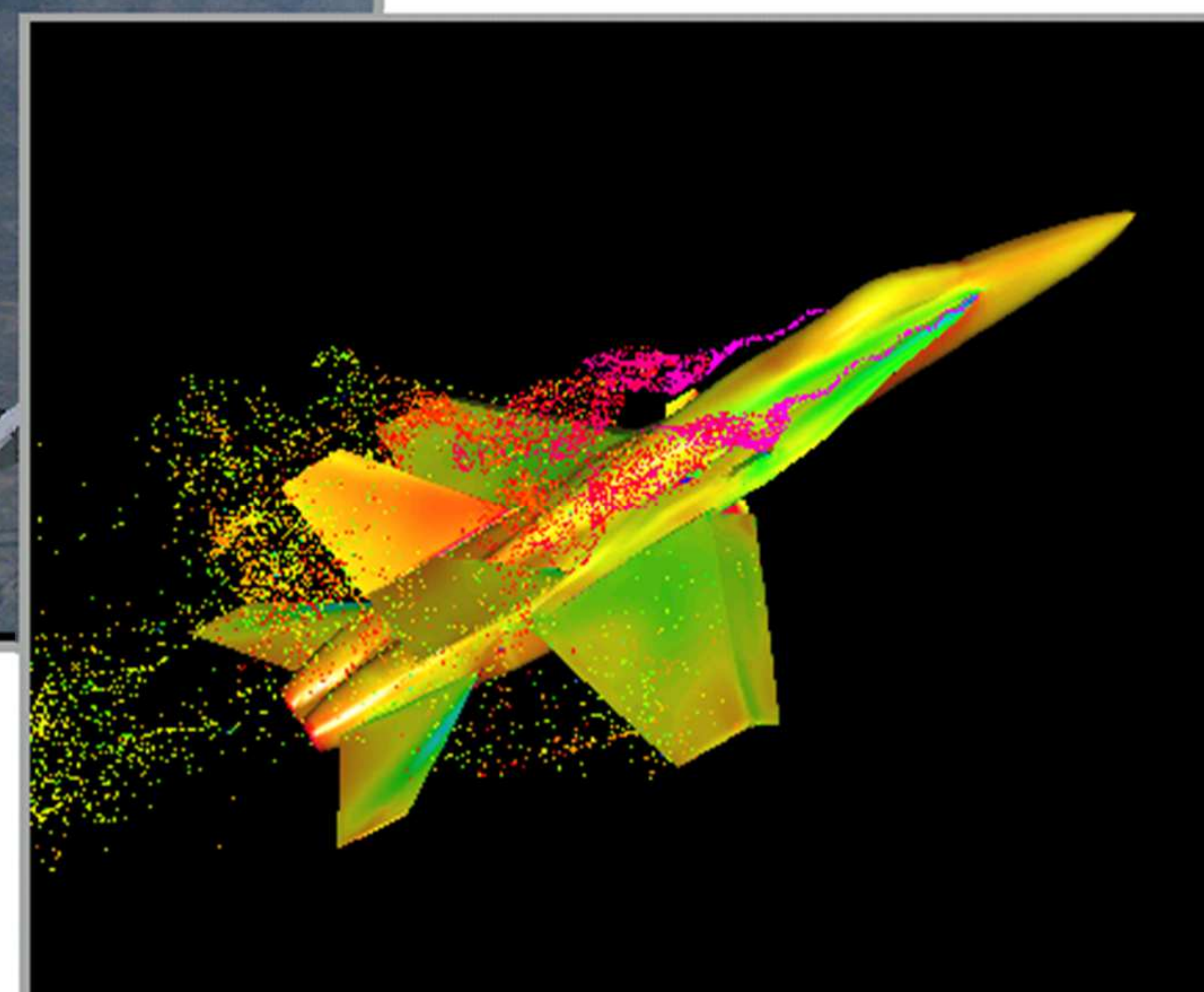
Computing w/ Flow Data

- **Flow data** is converted to a **vector field** by **interpolation** of the vectors inside the cells.

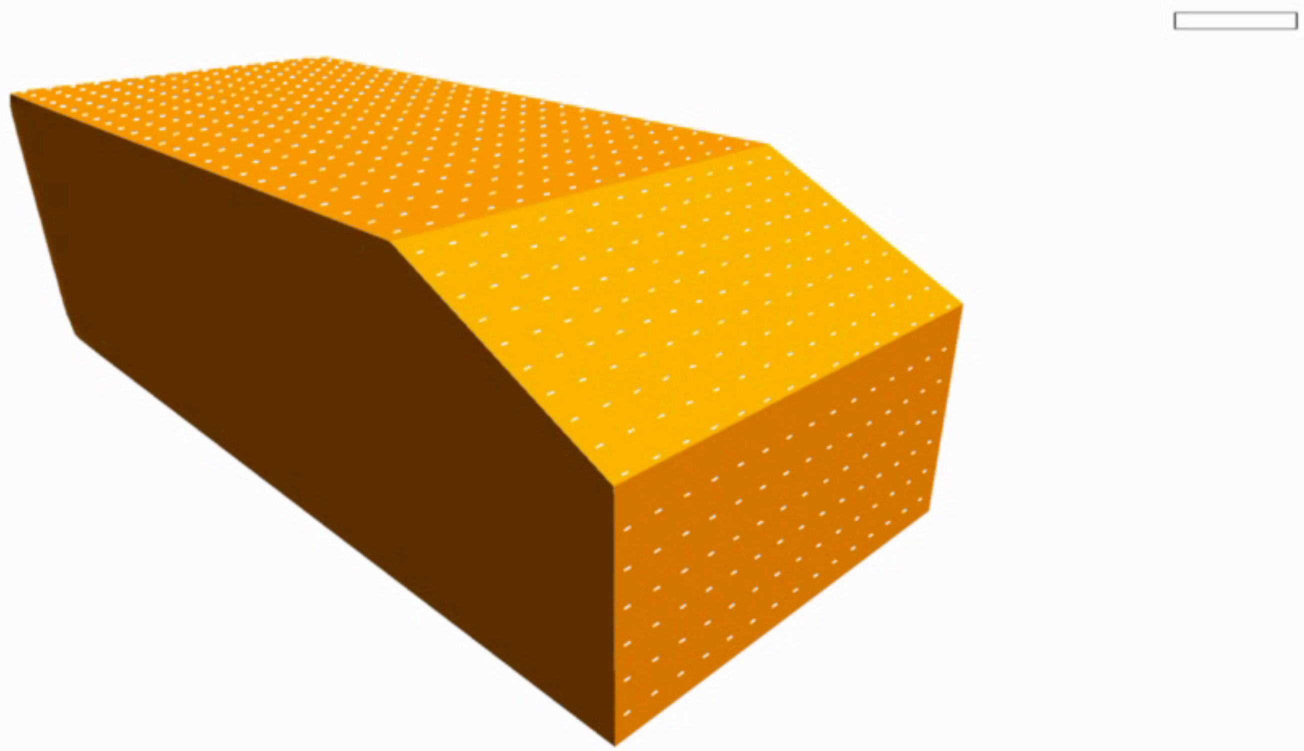


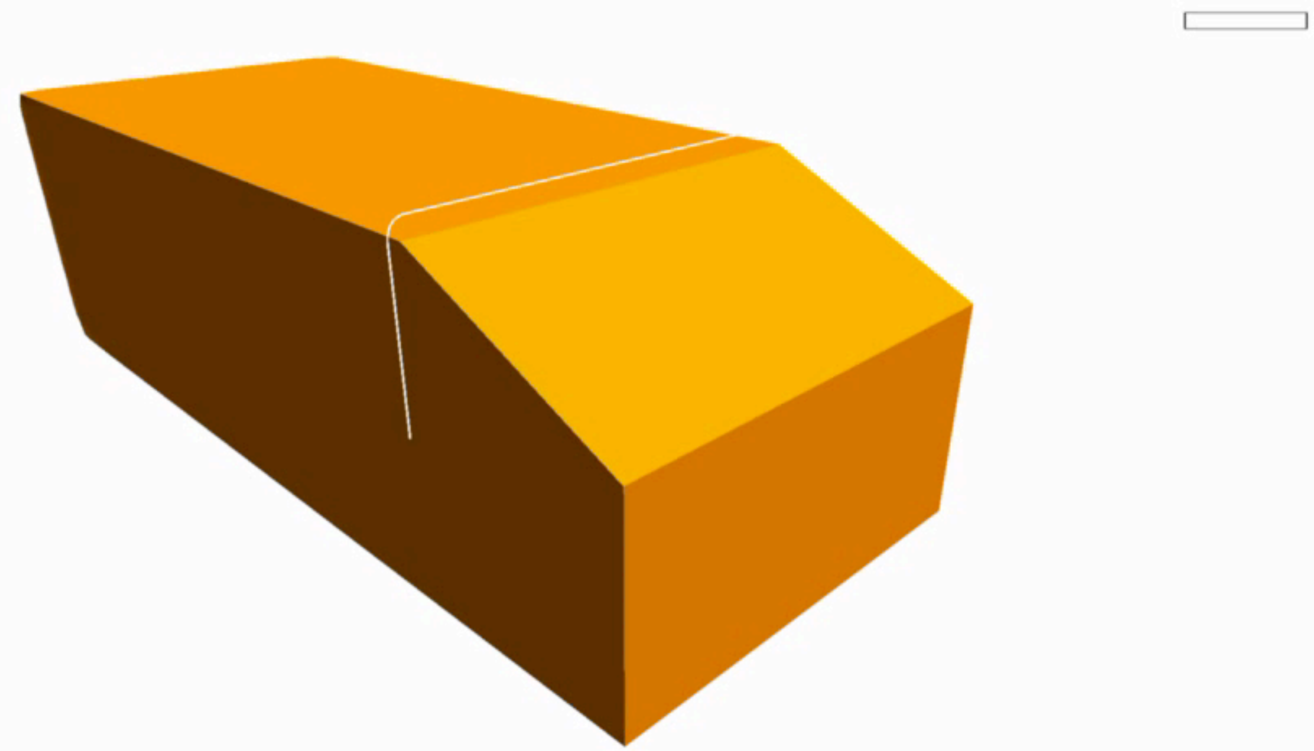
Comparison with Reality

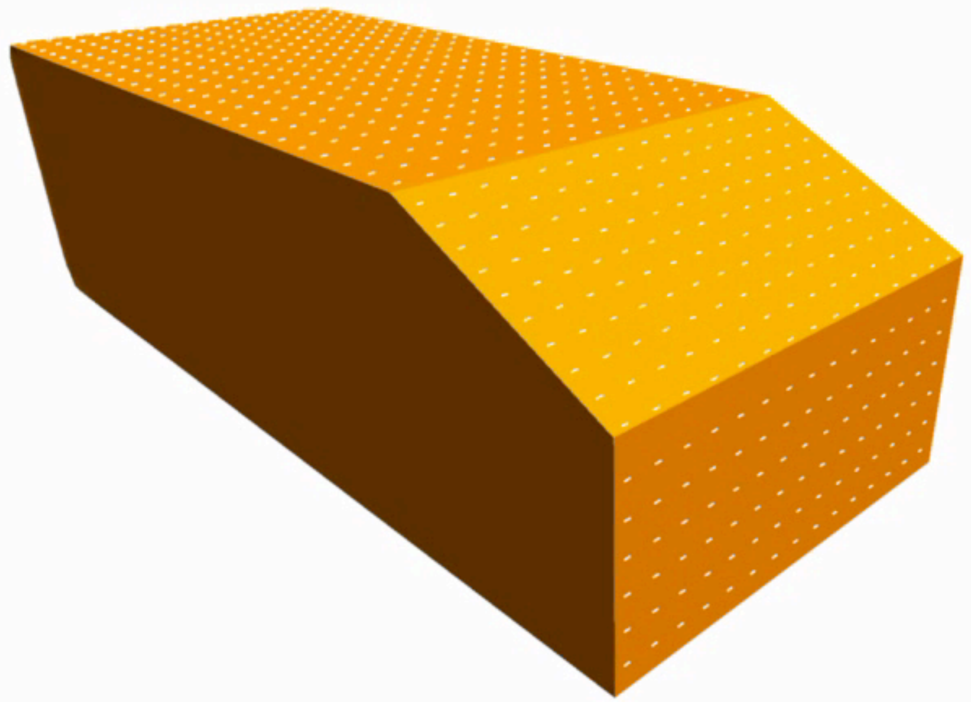
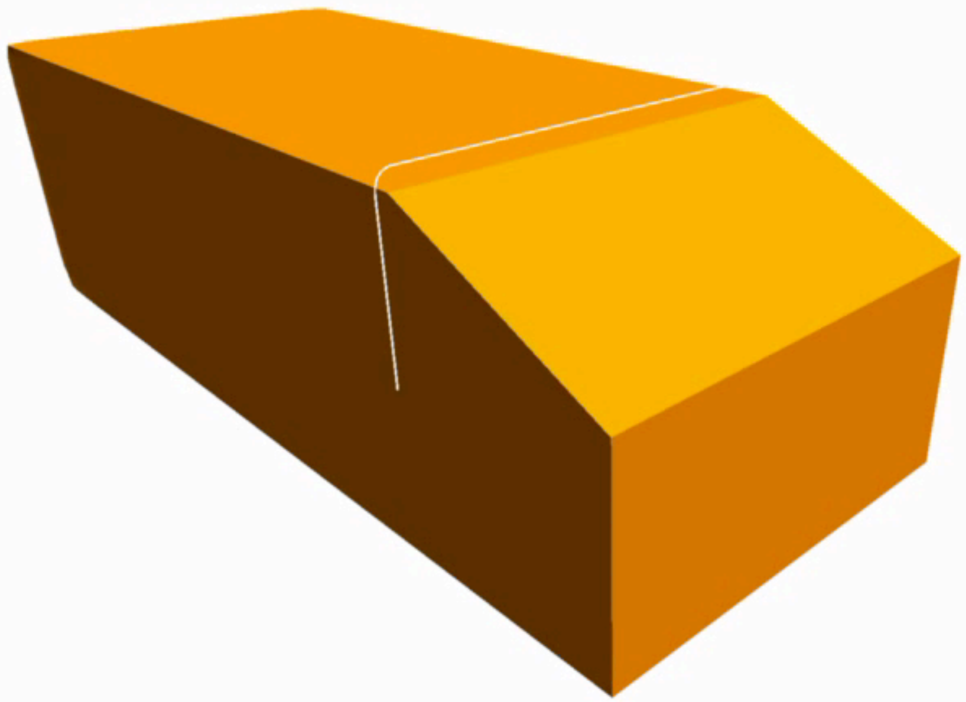
Experiment



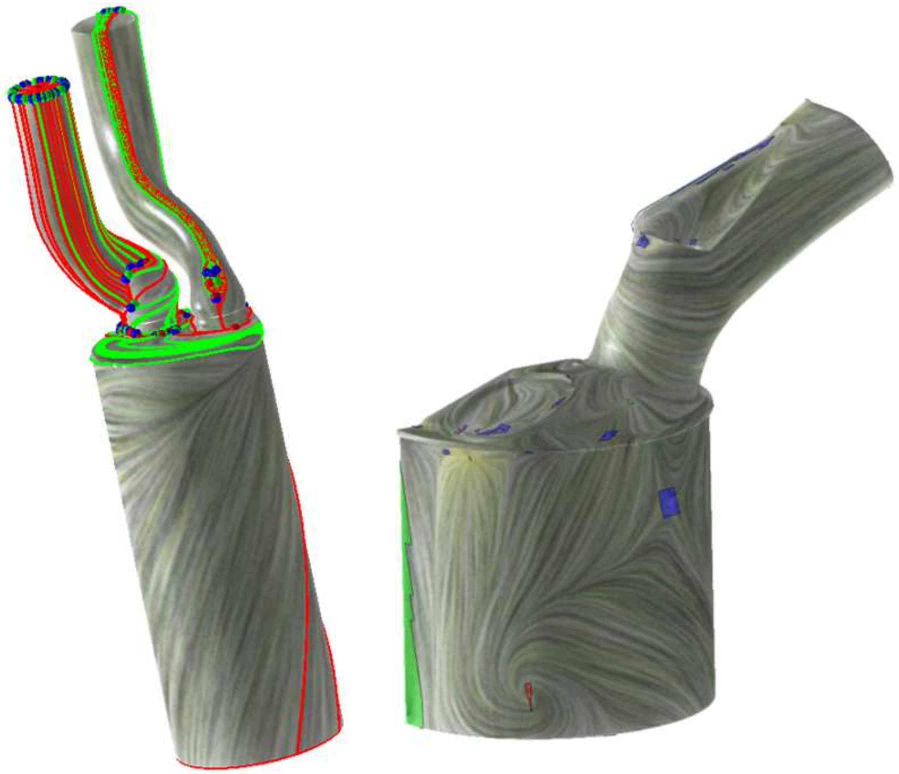
Simulation



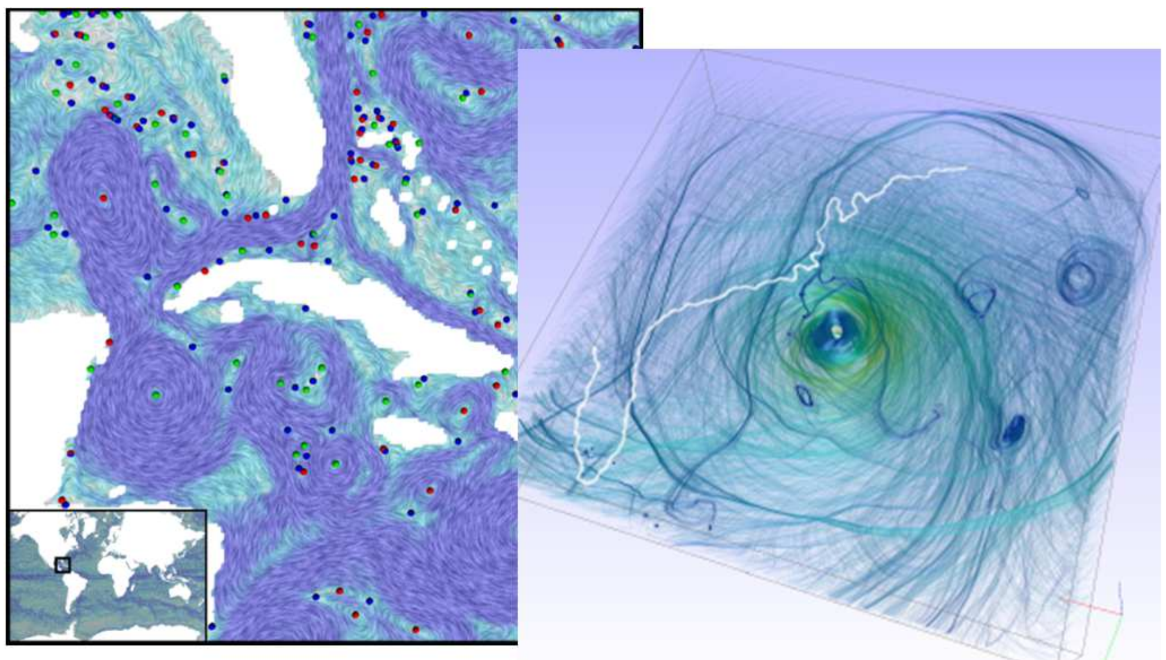


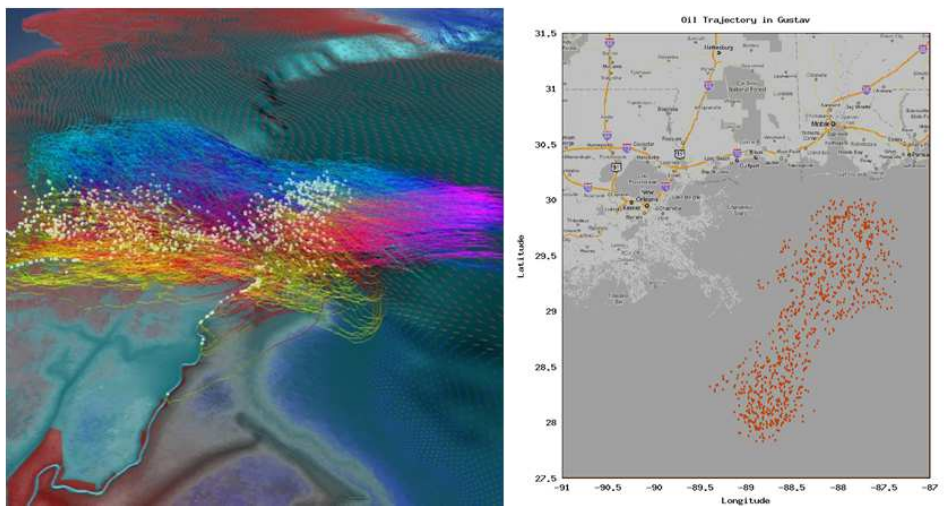
Vector Fields in Engineering and Science



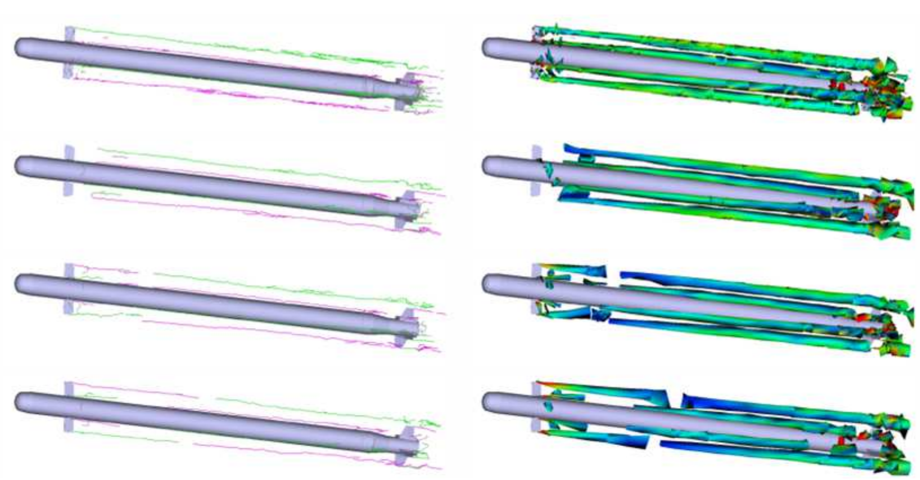
Automotive design
[**Chen** et al. TVCG07,TVCG08]



Weather study [Bhatia and Chen et al. TVCG11]

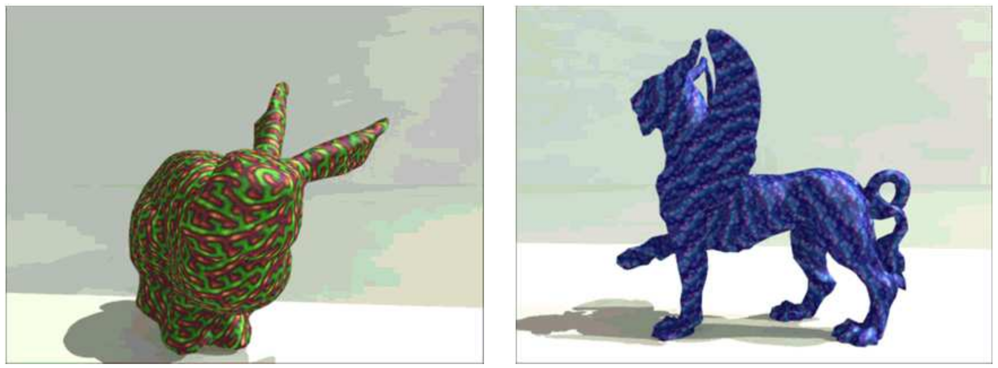


Oil spill trajectories [Tao et al. EMI2010]

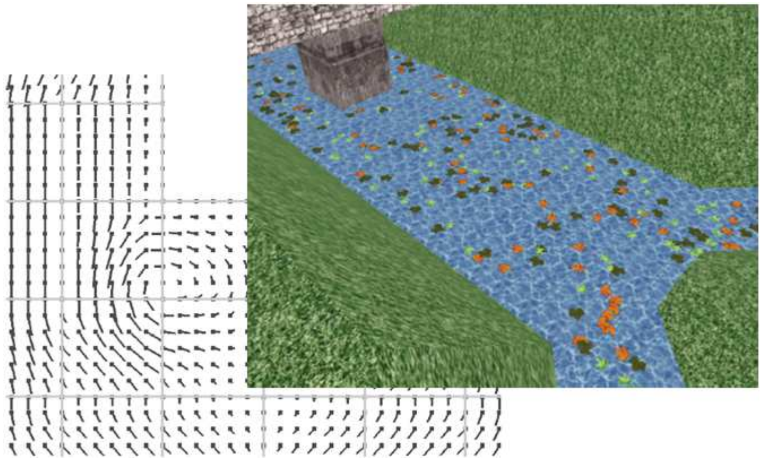


Aerodynamics around missiles [Kelly et al. Vis06]

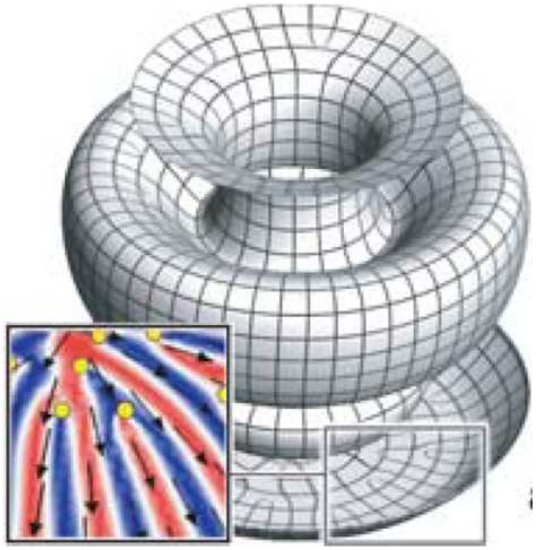
Vector Field Design in Computer Graphics



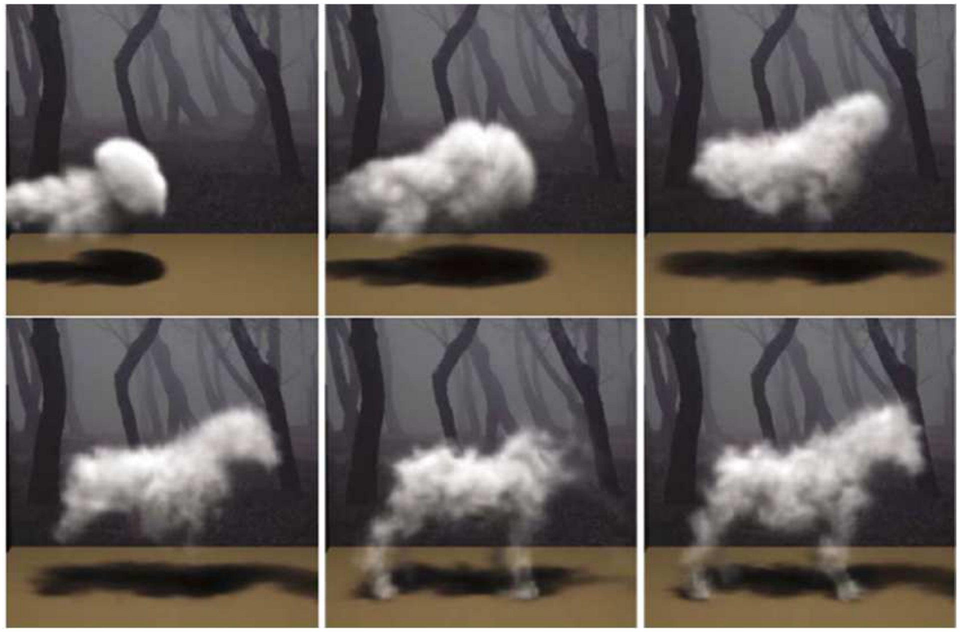
Texture Synthesis [**Chen** et al. TVCG11b]



River simulation [Chenney SCA2004]

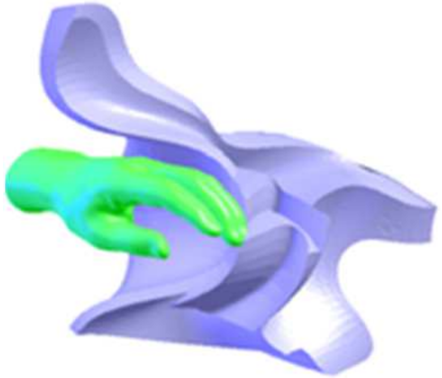


Parameterization
[Ray et al. TOG2006]



Smoke simulation [Shi and Yu TOG2005]

Painterly Rendering [Zhang et al. TOG2006]

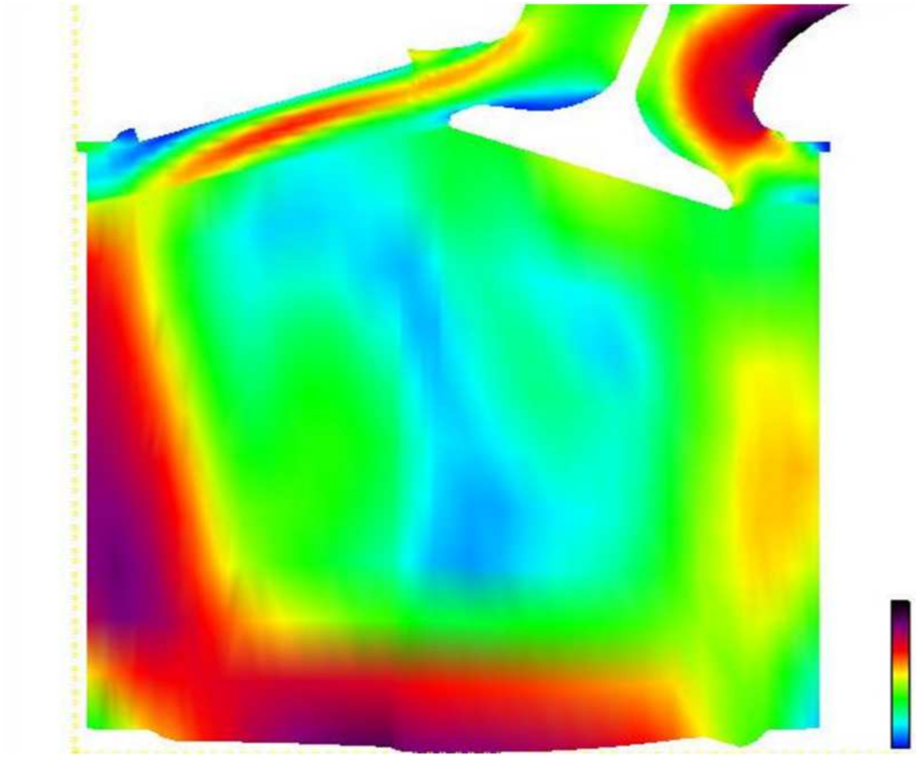


Shape Deformation
[von Funck et al. 2006]

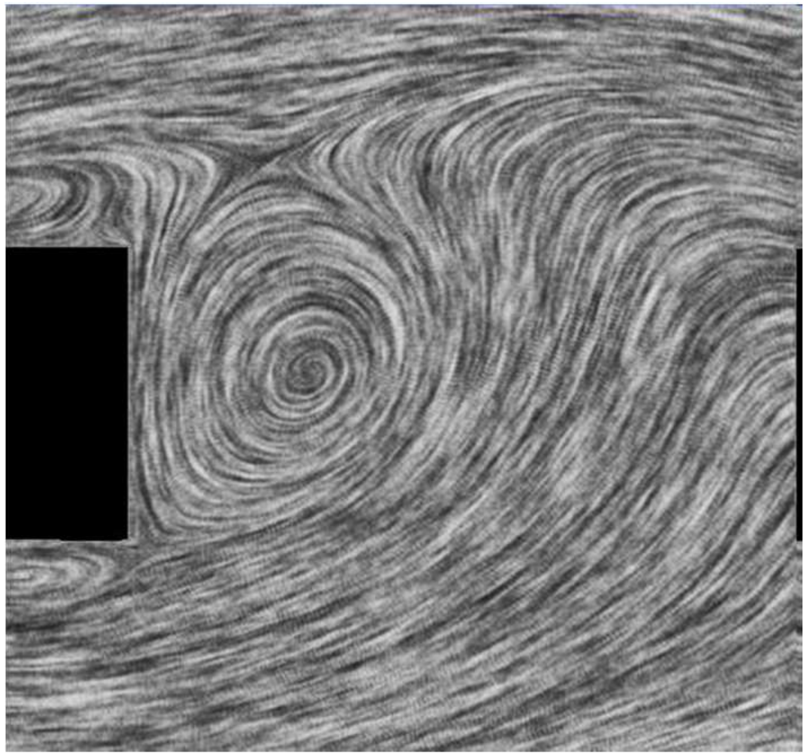
Why is It Challenging?

- to effectively visualize both *magnitude + direction,* often simultaneously
- large data sets
- time-dependent data

magnitude only

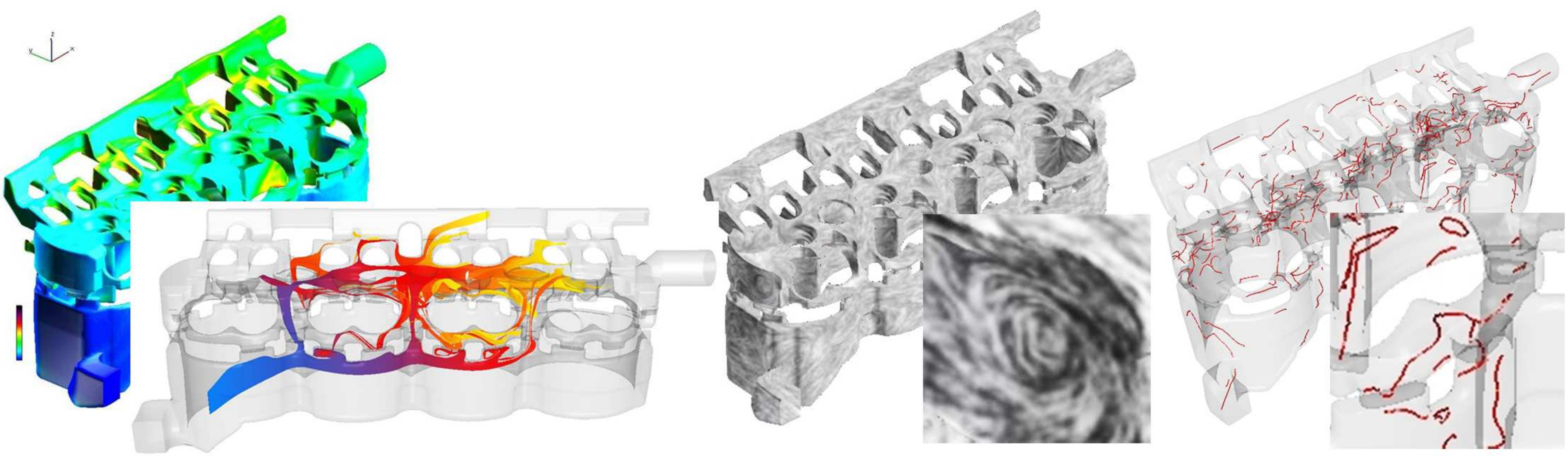


direction only



Classification of Visualization Techniques

- **Direct:** overview of vector field, minimal computation, e.g. glyphs, color mapping
- **Texture-based:** covers domain with a convolved texture, e.g., Spot Noise, LIC, ISA, IBFV(S)
- **Geometric:** a discrete object(s) whose geometry reflects flow characteristics, e.g. streamlines
- **Feature-based:** both automatic and interactive feature-based techniques, e.g. flow topology

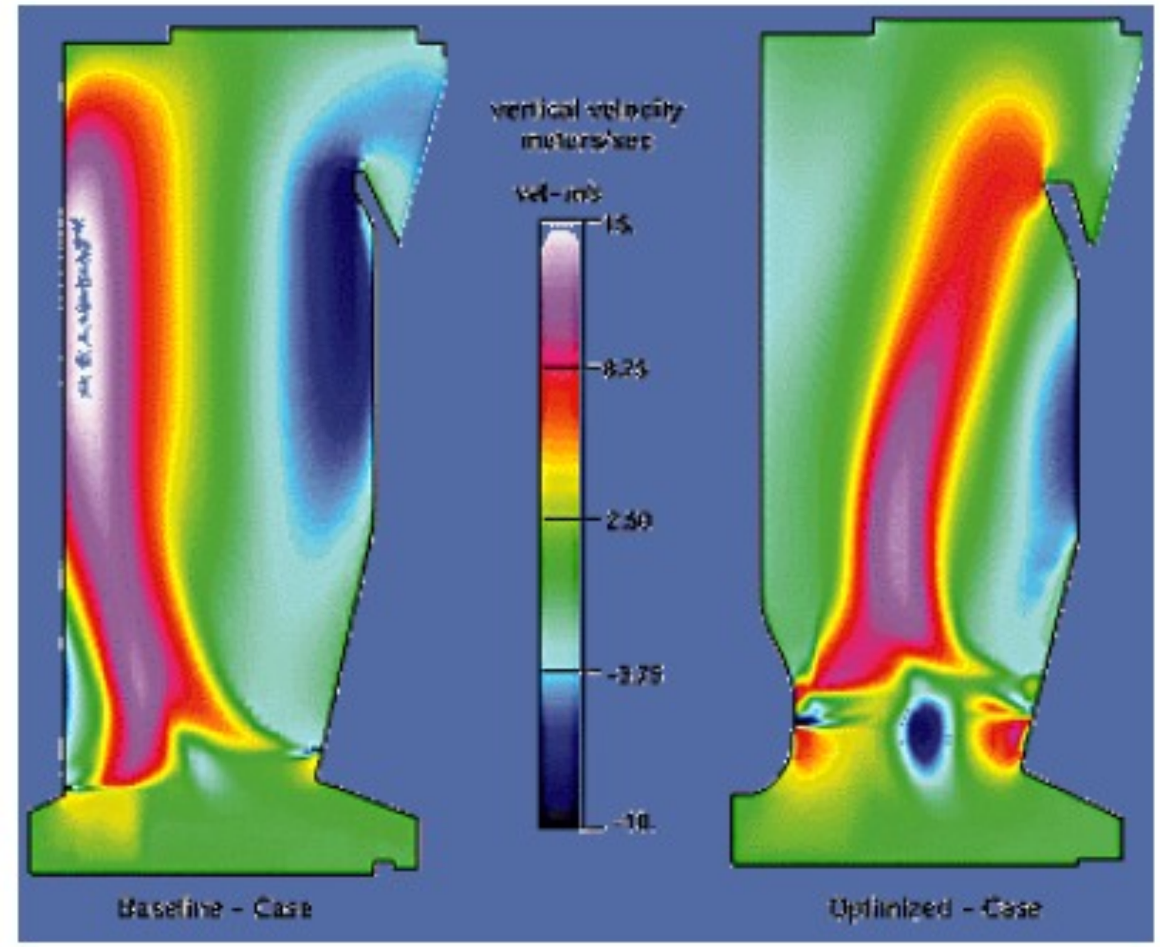


Direct Visualization of Vector Fields

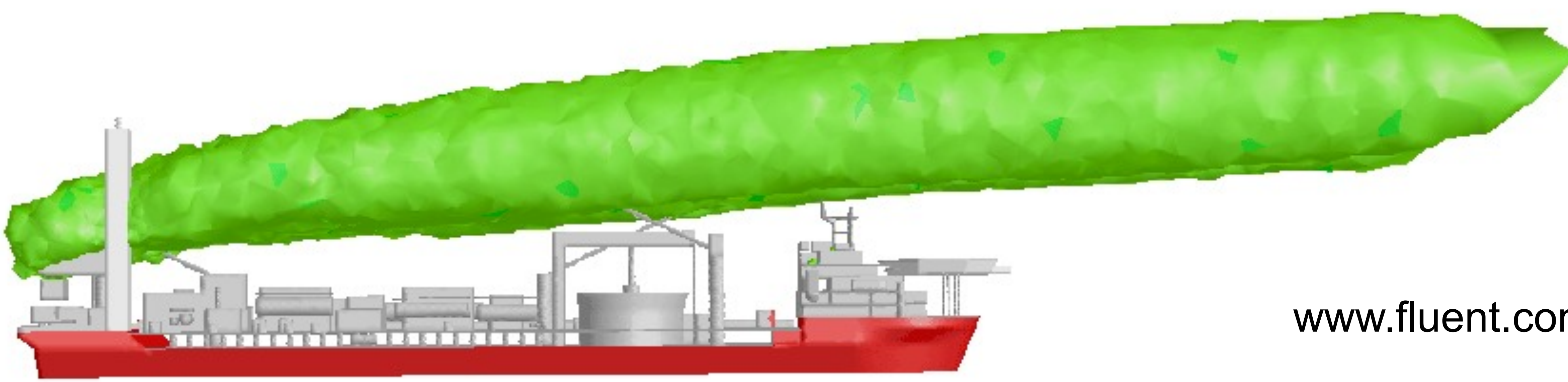
- **Color Coding:**
- Extract scalar field from vector field (i.e. magnitude, curvature, pressure, temperature)
- 2D: direct color coding to visualize them
- same for slices or boundary surfaces in 3D
- 3D ➔ volume visualization (FlowVis ➔ VolVis)

- ➔ loss of information

- **Examples for Color Coding:**



Vertical Velocity Distribution. The baseline condition illustrates the flow pattern expressing the vertical velocity components over the range front-10 to 15 m/s. The baseline case with only a single elevation of overfire air produces a high velocity flow channel attached to the front wall with an associated recirculation down the rear wall of the main combustor section. The optimized case includes a revised overfire air configured to centralize the vertical flow region. The peak vertical velocities and size of the recirculation



www.fluent.com

• Volume illustration for flow visualization [Svakine et al 05]

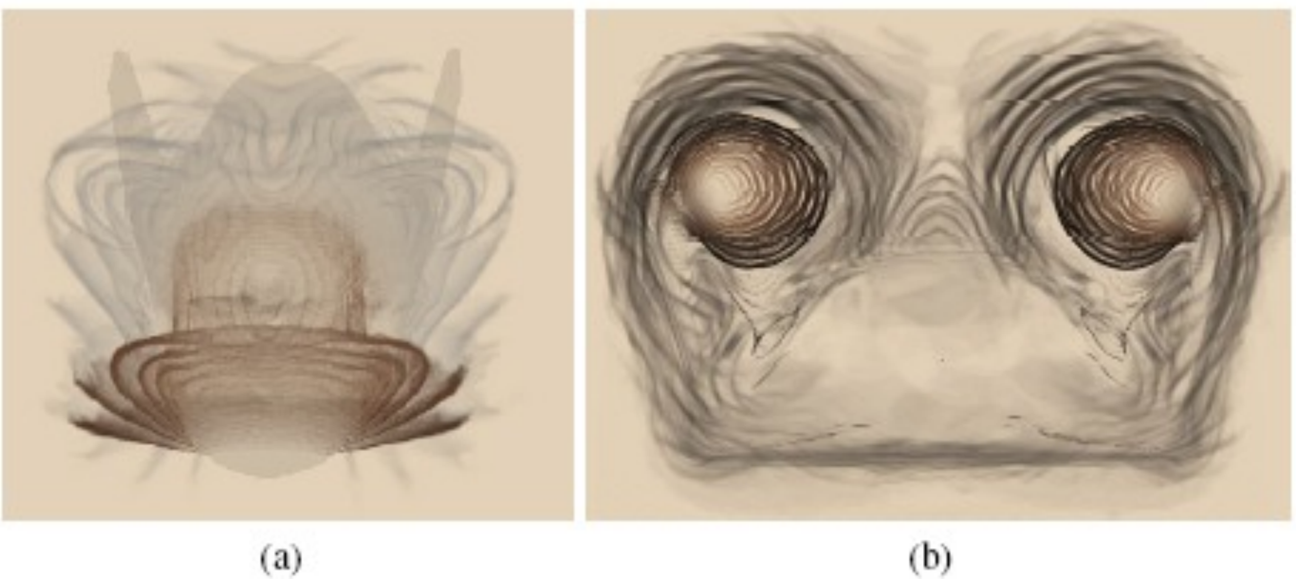


Figure 3: Volume illustrations of flow around the X38 spacecraft. (a) is an illustration of density flow and shock around the bow, while (b) highlights the vortices created above the fins of the spacecraft.

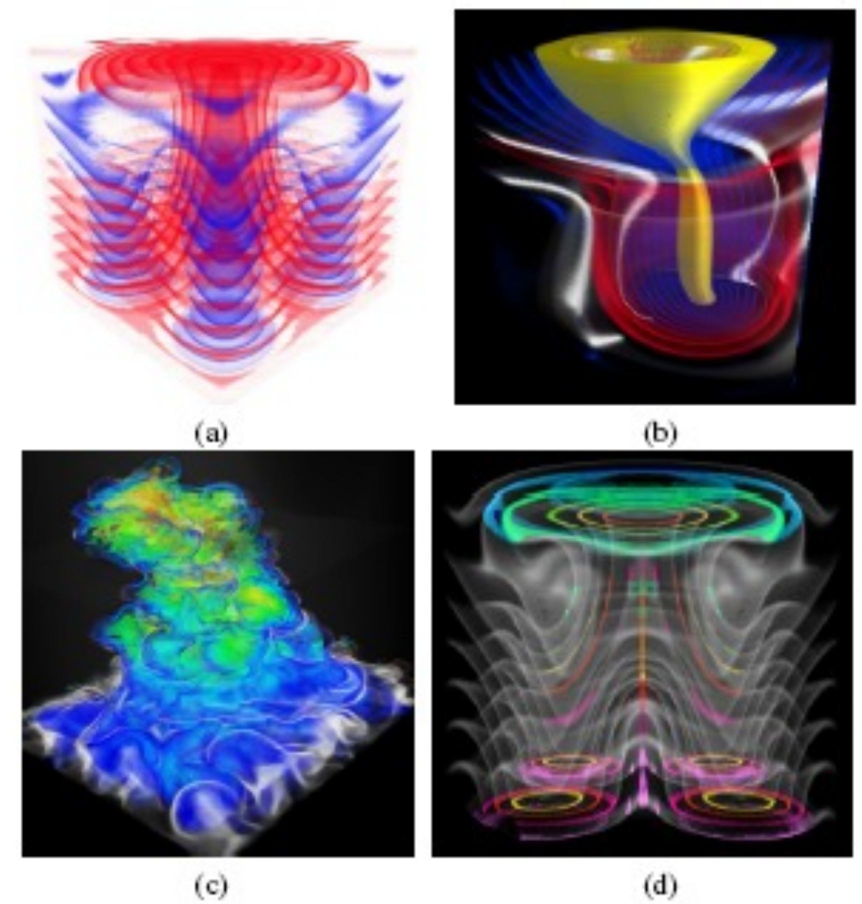
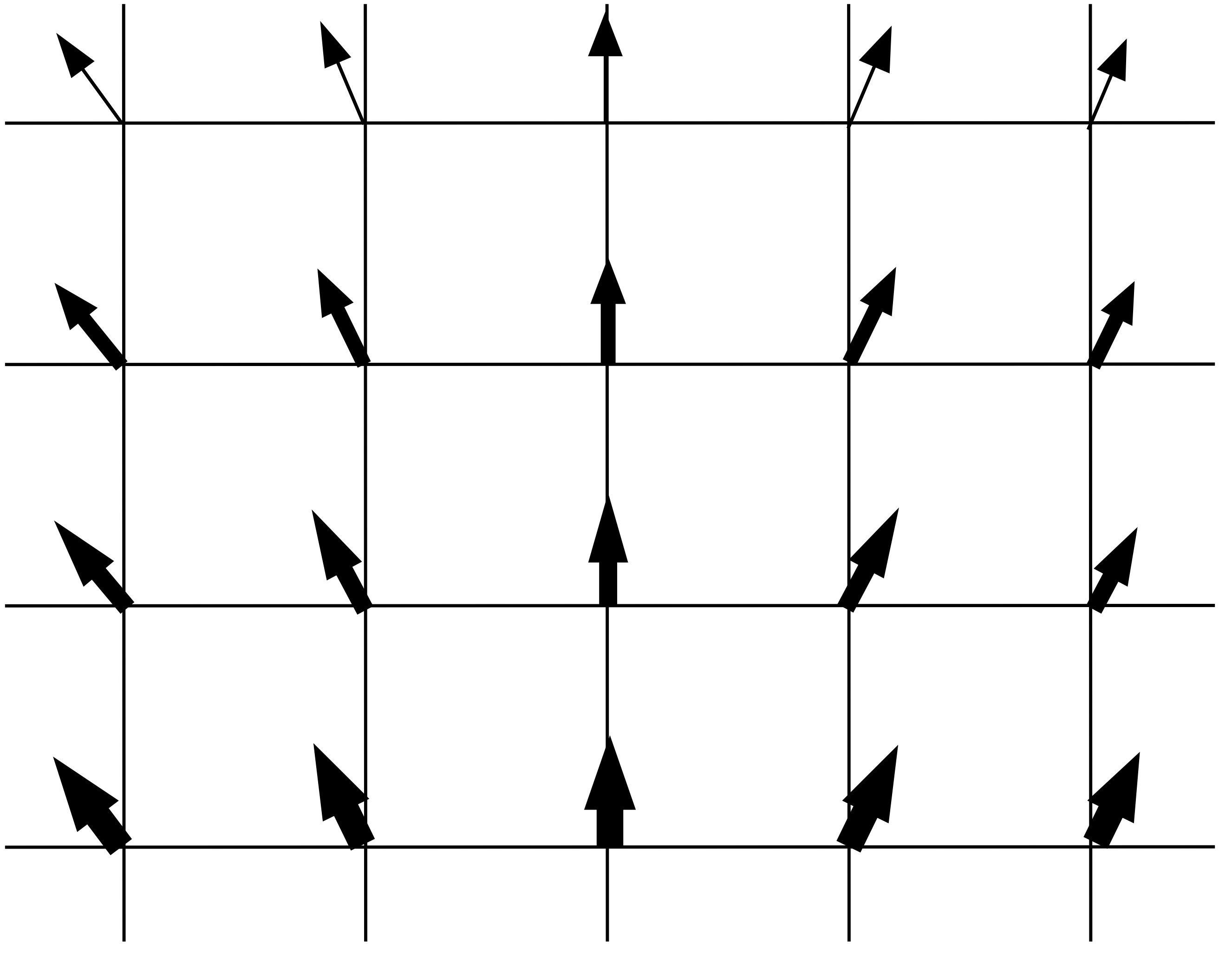


Figure 6: Use of two-dimensional transfer function with the Laplacian operator and other flow quantities. (a) shows heat inflow (red) and outflow (blue). (b) shows all values of the Laplacian of velocity magnitude in the tornado dataset. (c) visualizes the cloud TKE using the Laplacian to highlight boundaries (white) and velocity for silhouetting. (d) highlights emerging flow structures in the convection dataset using banding of the second derivative magnitude of the temperature field.

- **Elementary Methods:**
- Present the complete data set at low level of abstraction
- Mapping is direct, without complex conversions or extractions
- 3 main representations:
  - color coding
  - arrow plots
  - icons
- ➔ very frequently used

- **Arrow plots:**
- also called hedgehog plots
- represent velocity as arrows at regular locations, e.g., place arrows at grid points
- ➔ overloading possible
- arrows: (scaled) unit length or encode magnitude
- well-established for 2D

- Arrows visualize
  - Direction of vector field
  - Orientation
  - Magnitude:
    - Length of arrows
    - Color coding



- [Kirby et al 99]: multiple values of 2d flow data by layering concept related to painting process of artists

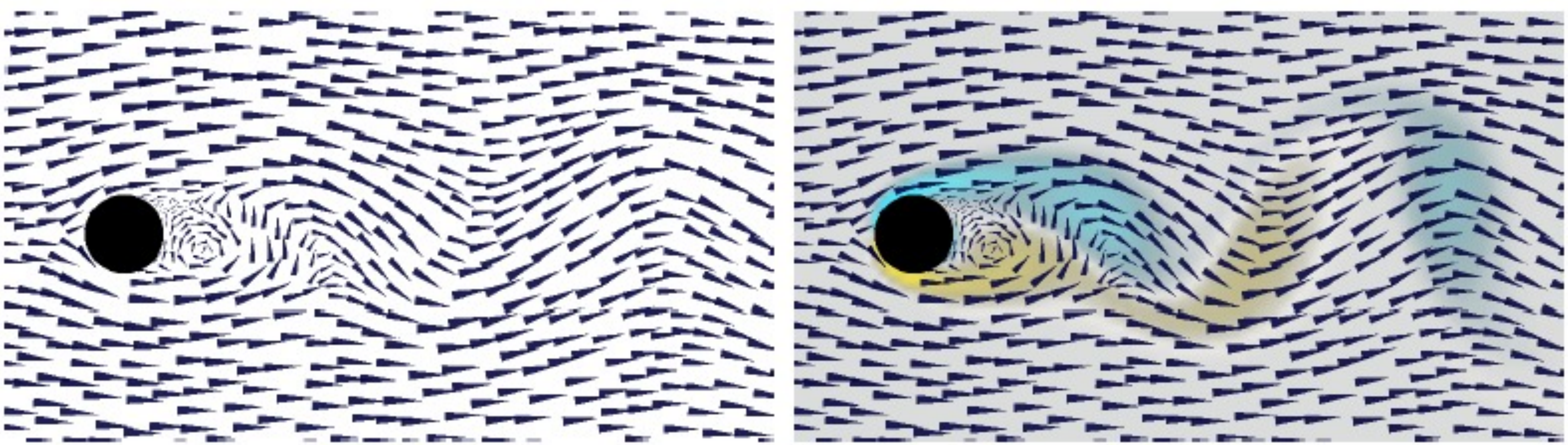
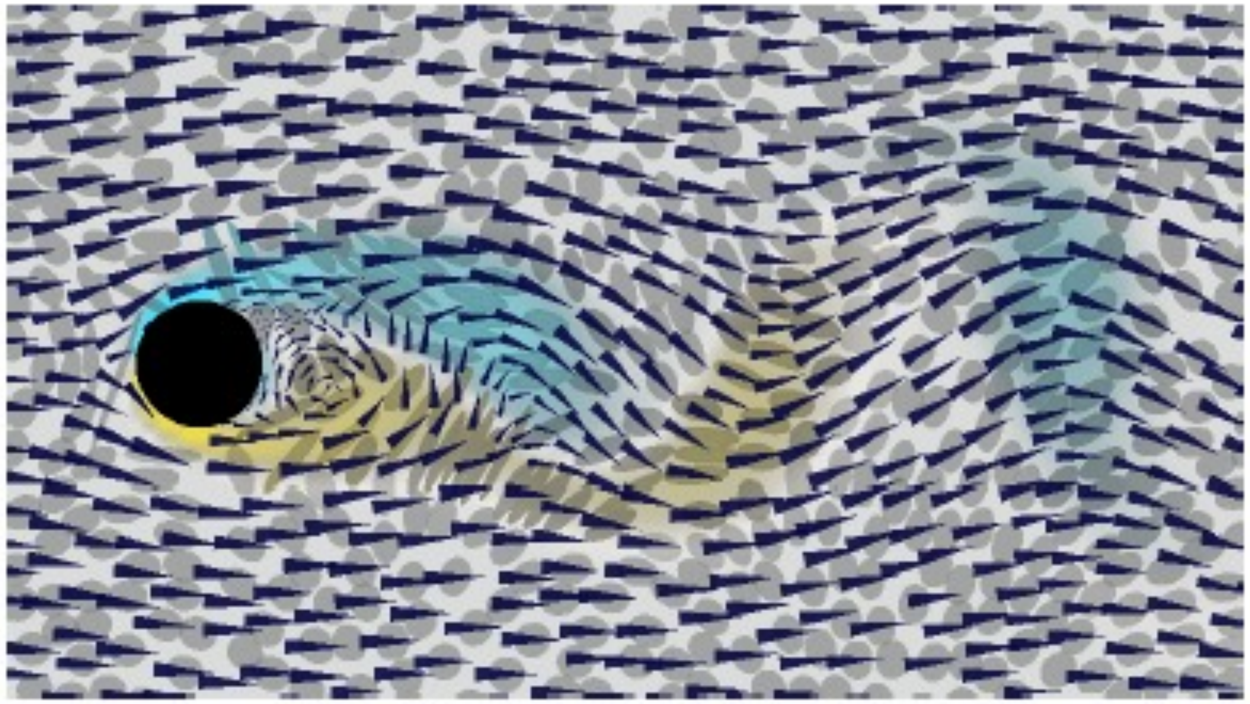
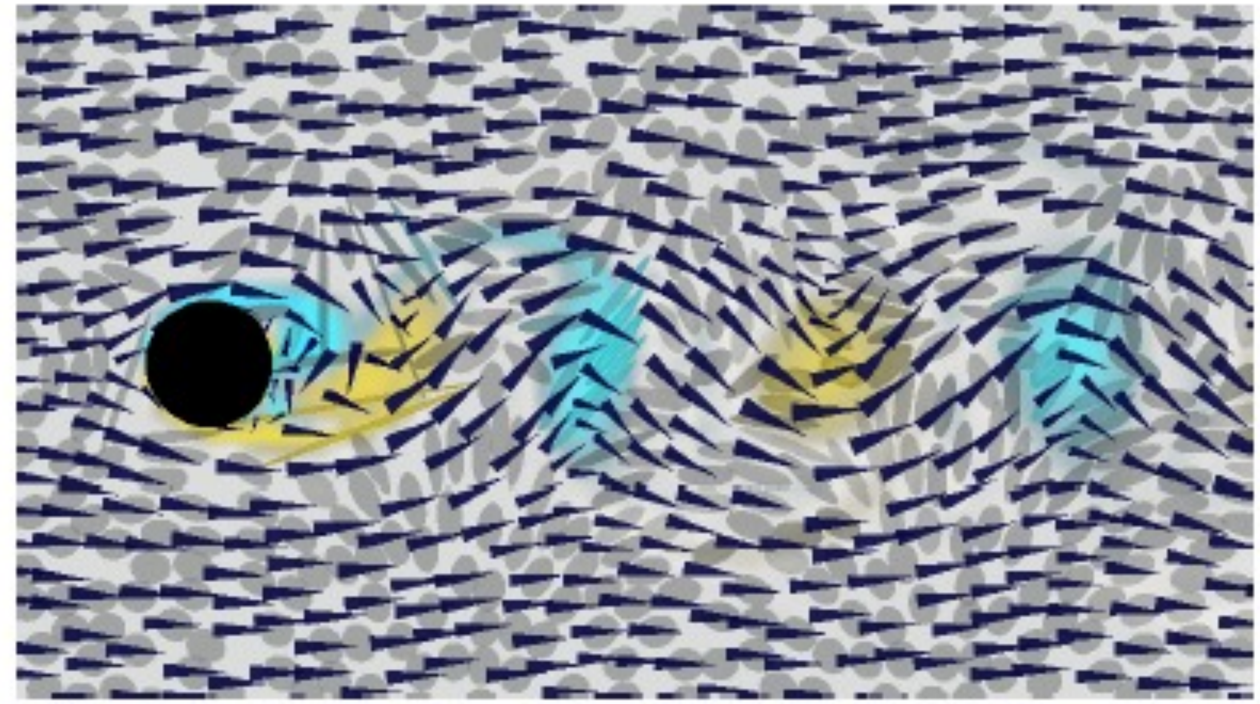


Figure 1: Typical visualization methods for 2D flow past a cylinder at Reynolds number 100. On the left, we show only the velocity field. On the right, we simultaneously show velocity and vorticity. Vorticity represents the rotational component of the flow. Clockwise vorticity is blue, counterclockwise yellow.

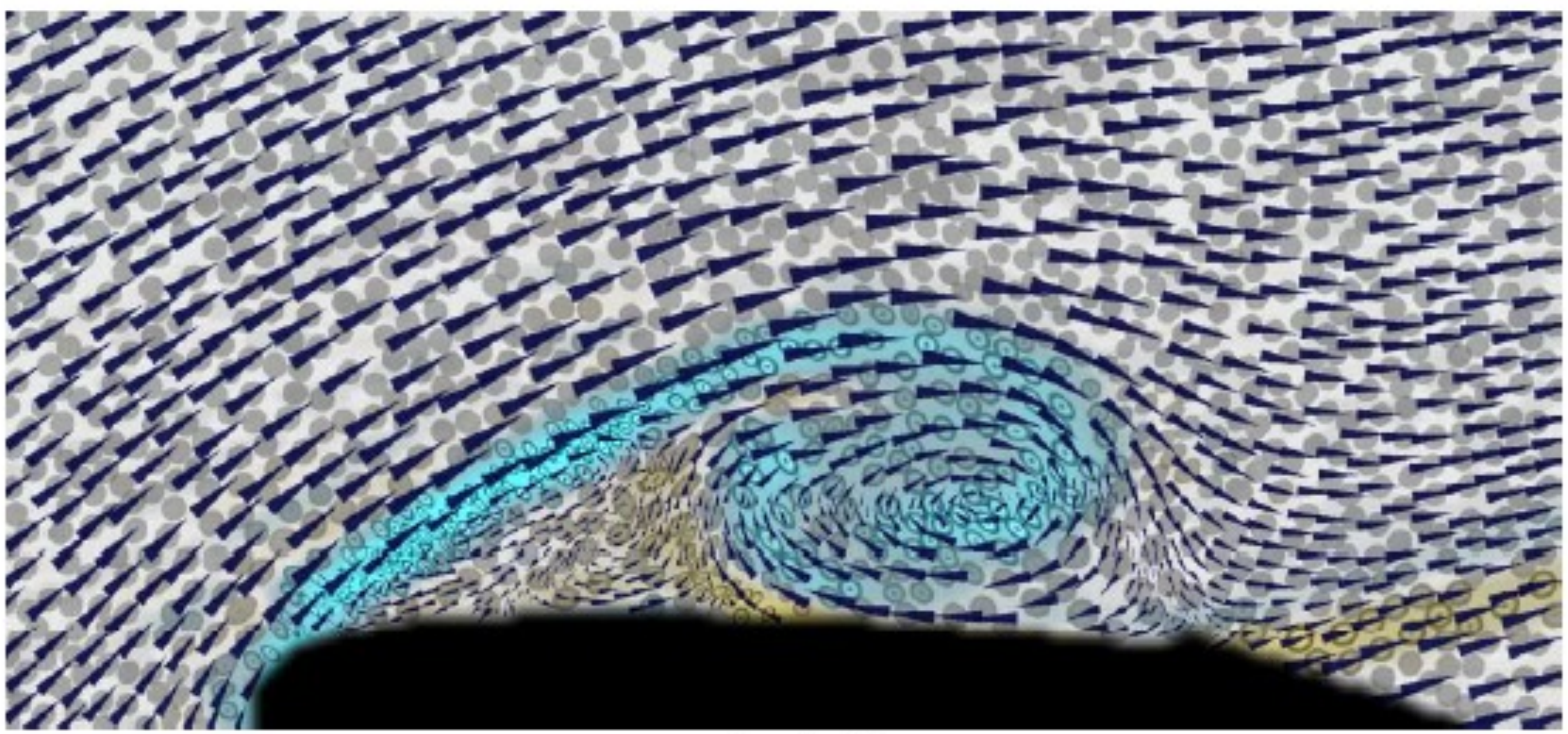


Reynolds number 100



Reynolds number 500

simulated flow



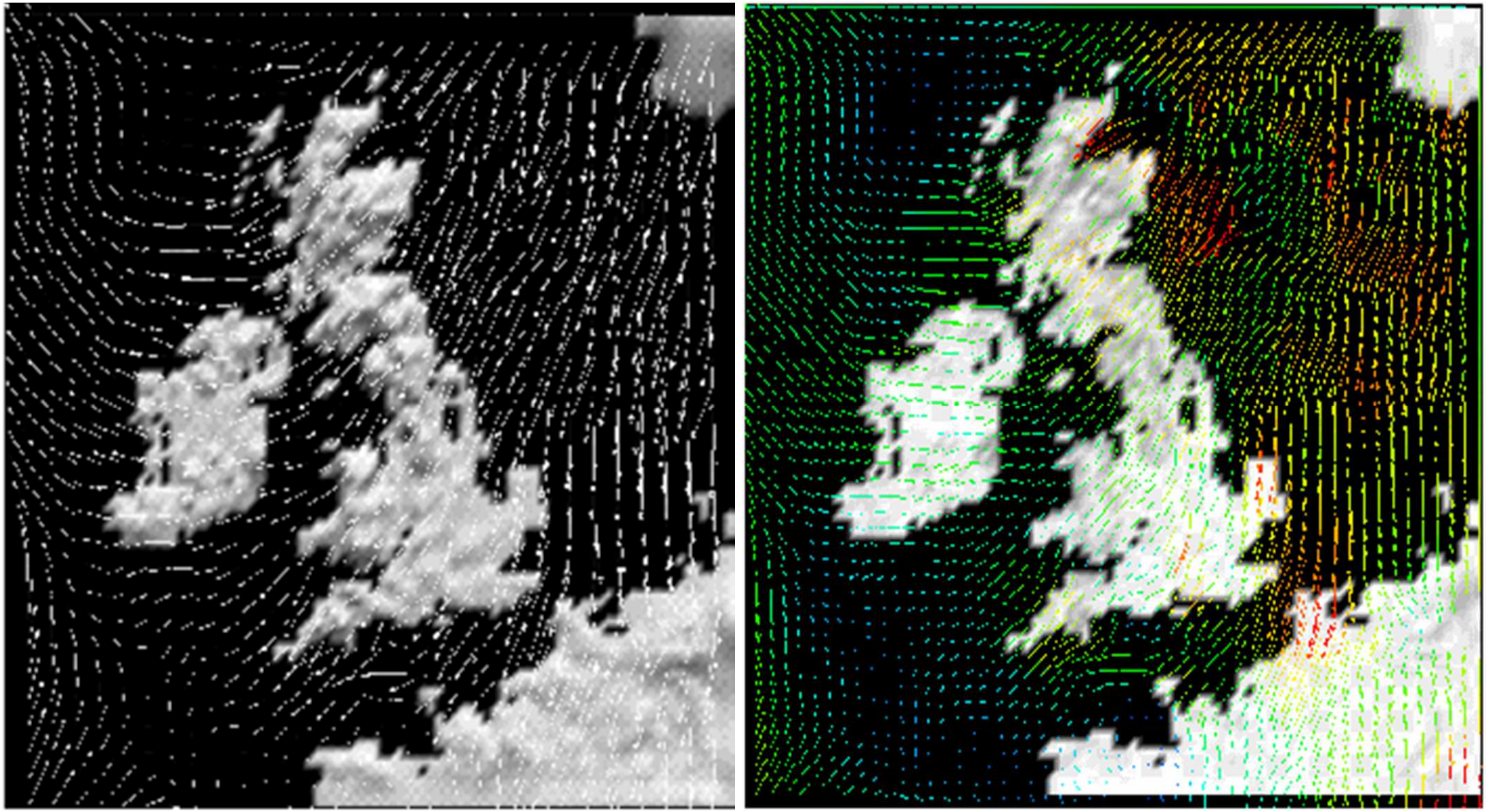
experimental flow

KEY

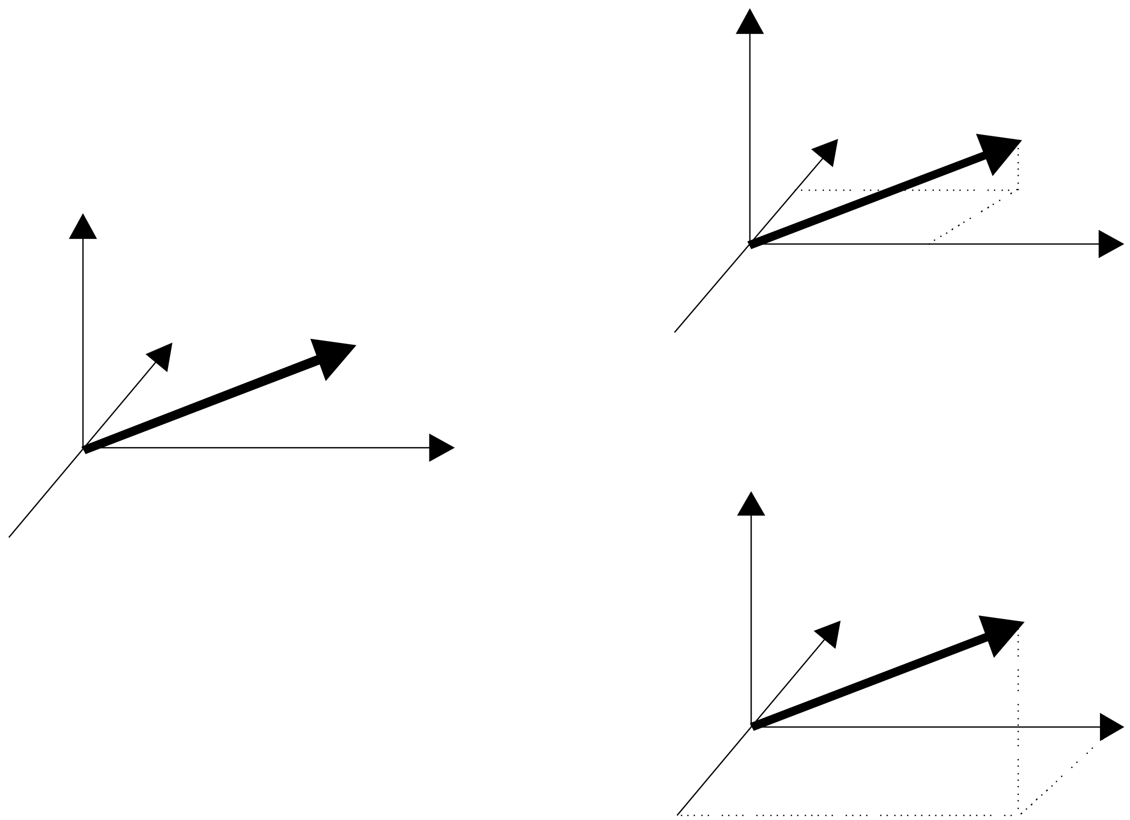
data	**visualization**
velocity	arrow direction
speed	arrow area
vorticity	underpainting/ellipse color (blue=cw, yellow=ccw), and ellipse texture contrast
rate of strain	log(ellipse radii)
divergence	ellipse area
shear	ellipse eccentricity

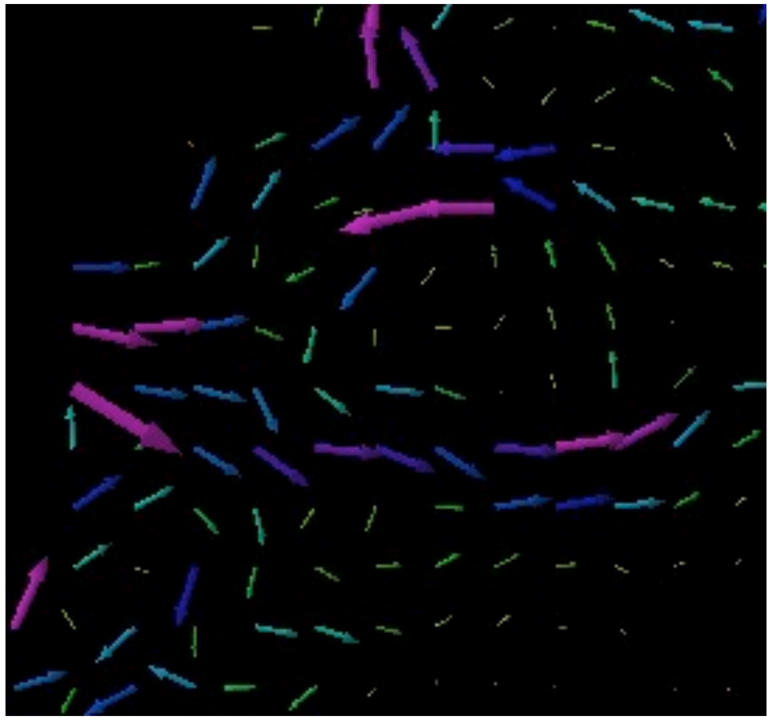
Arrows in 2D

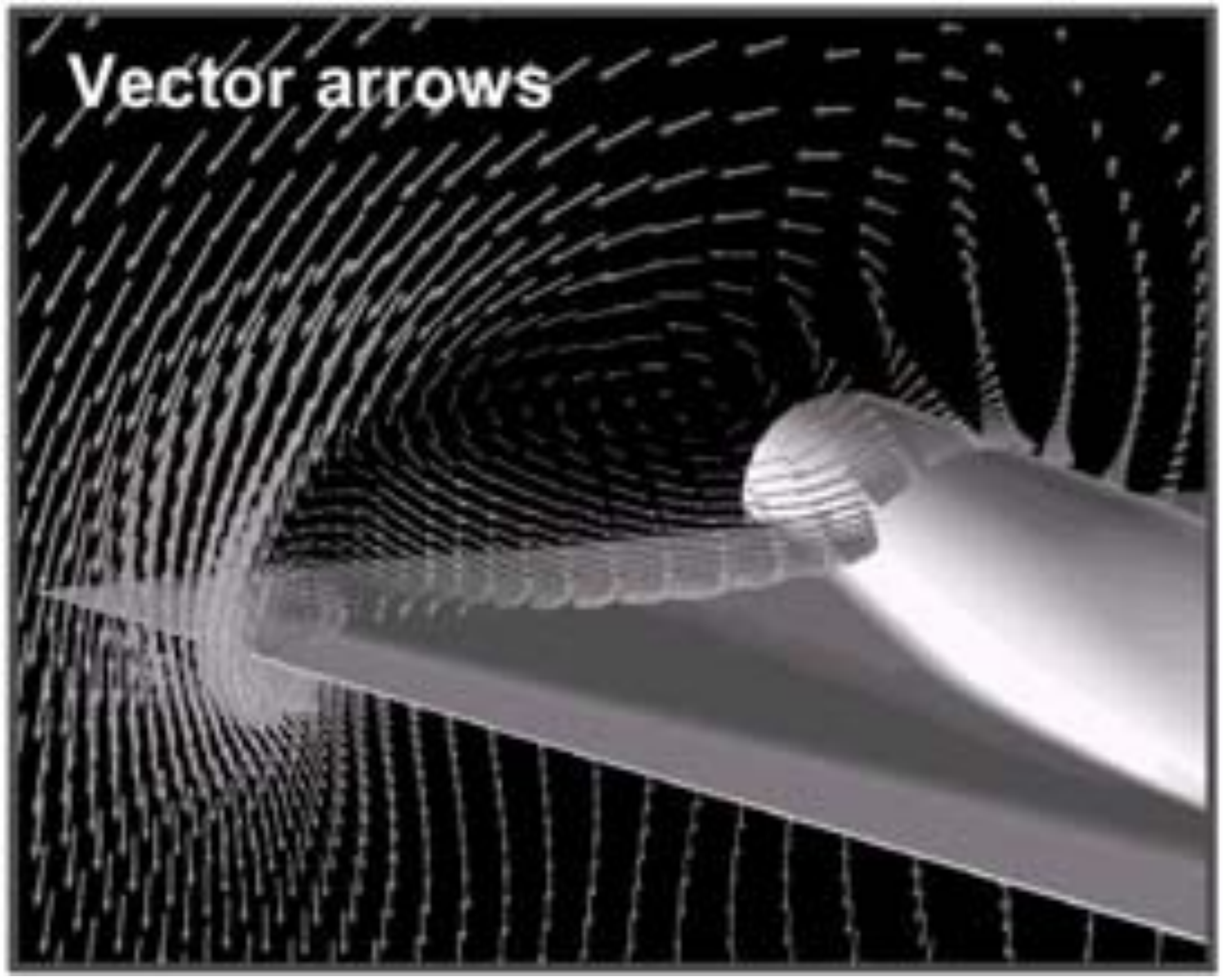
Scaled arrows vs. color-coded arrows

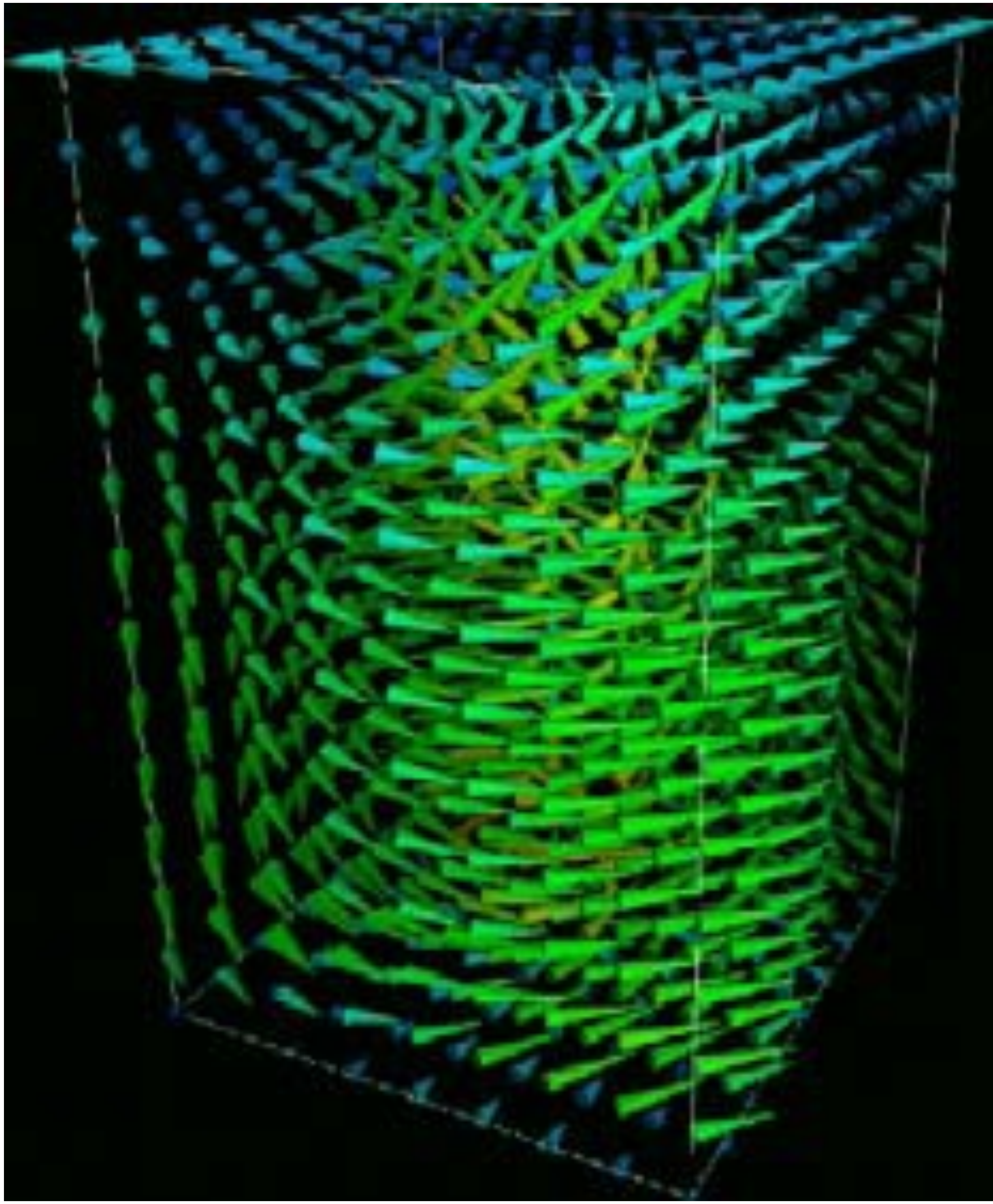


- **3D arrow plots:**
- Occlusion problem ➔ careful seeding
- Ambiguity problem
- Solutions:
  - 3D icons (cylinder + cone)
  - Highlight parts of error plot which points into a user defined direction

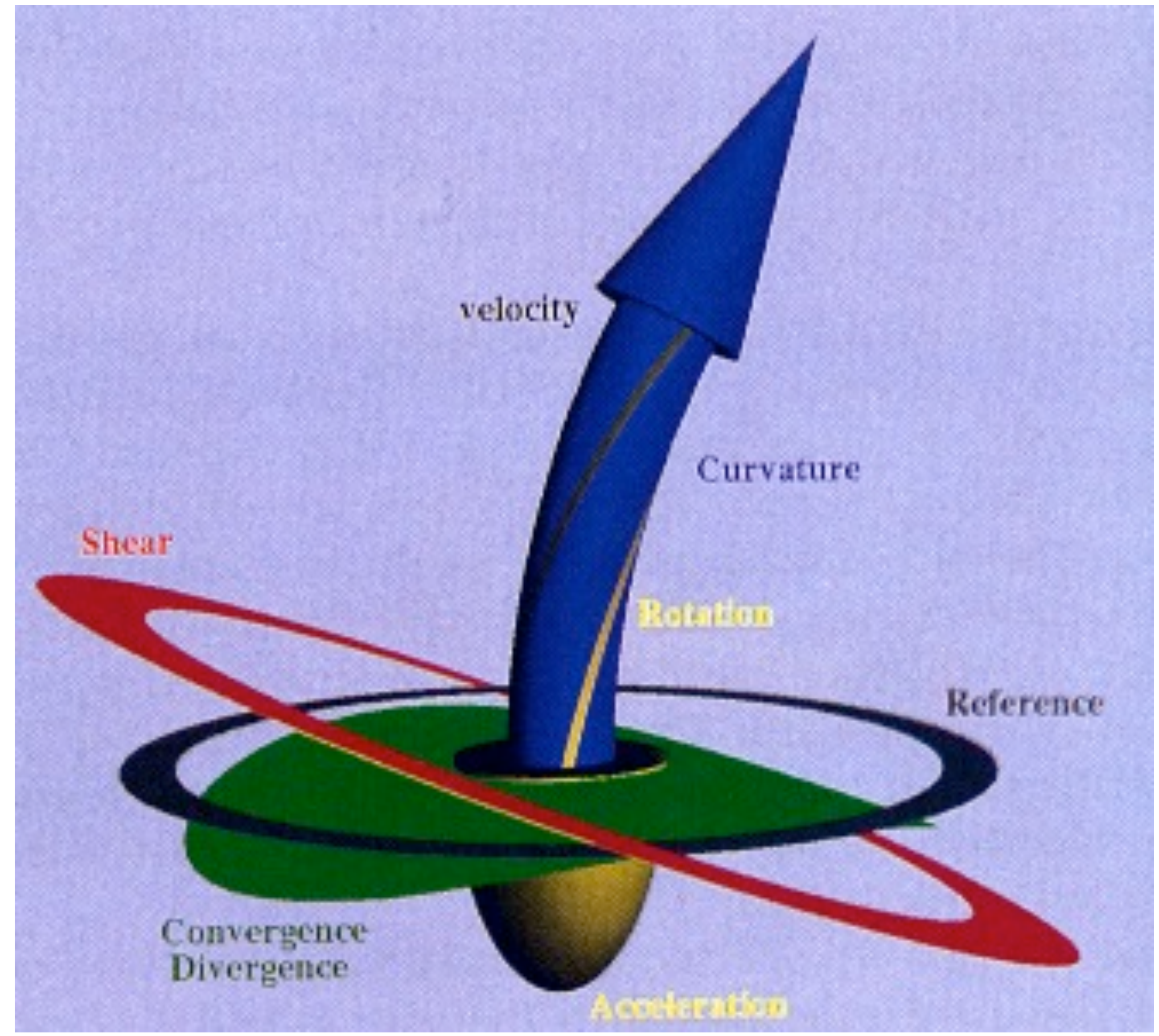





Vector arrows

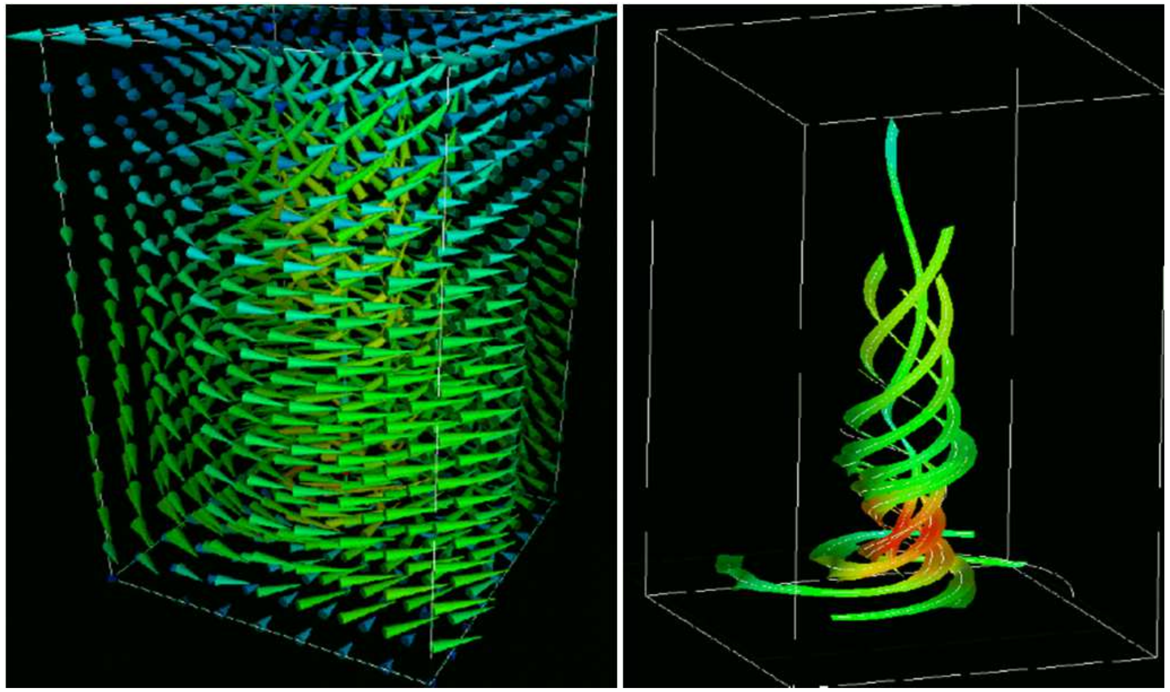


- **Icons:**
- Place icon at selected locations and encode different values of the flow
- Seeding strategy necessary (usually interactive)
- Example: probe for local flow visualization [de Leeuw, van Wijk 93]



Geometry-Based Visualization

Direct vs. Geometric FlowVis



Direct flow visualization:

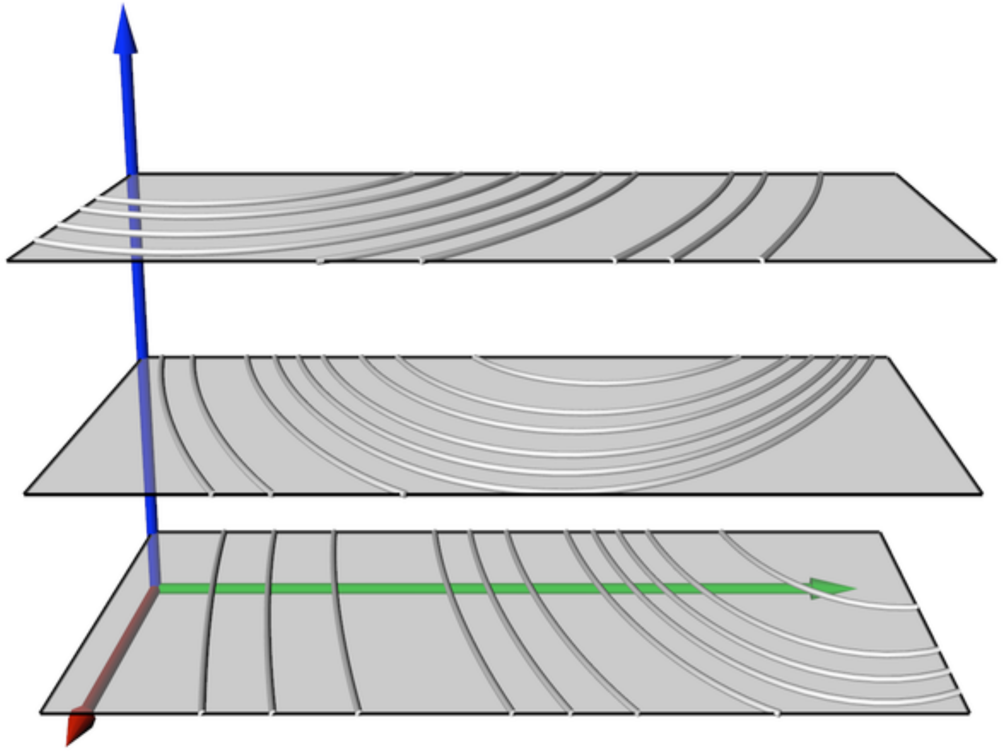
- overview of current state of flow
- visualization with vectors popular
- arrows, icons, glyph techniques

Geometric flow visualization:

- use of intermediate objects,
  e.g., after vector field integration over time
- visualization of development over time
- streamlines, stream surfaces
- analogous to indirect (vs. direct) volume visualization

Characteristic Curves of a Vector Field

- **Streamlines:** curve parallel to the vector field in each point for a fixed time
- **Pathlines:** describes motion of a particles over time through a vector field
- **Streaklines:** trace of dye that is released into the flow at a fixed position
- **Timelines**: describes motion of particles set out on a line over time through a vector field



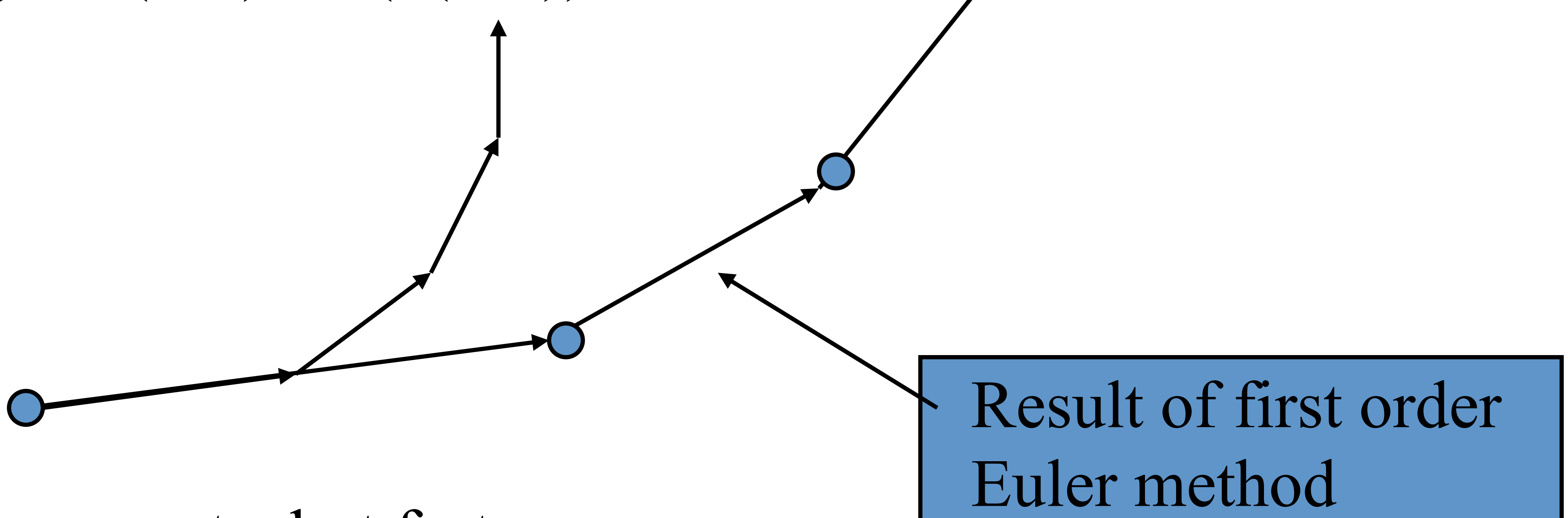
streamlines

Integration Techniques

Numerical Integration

First Order Euler method:

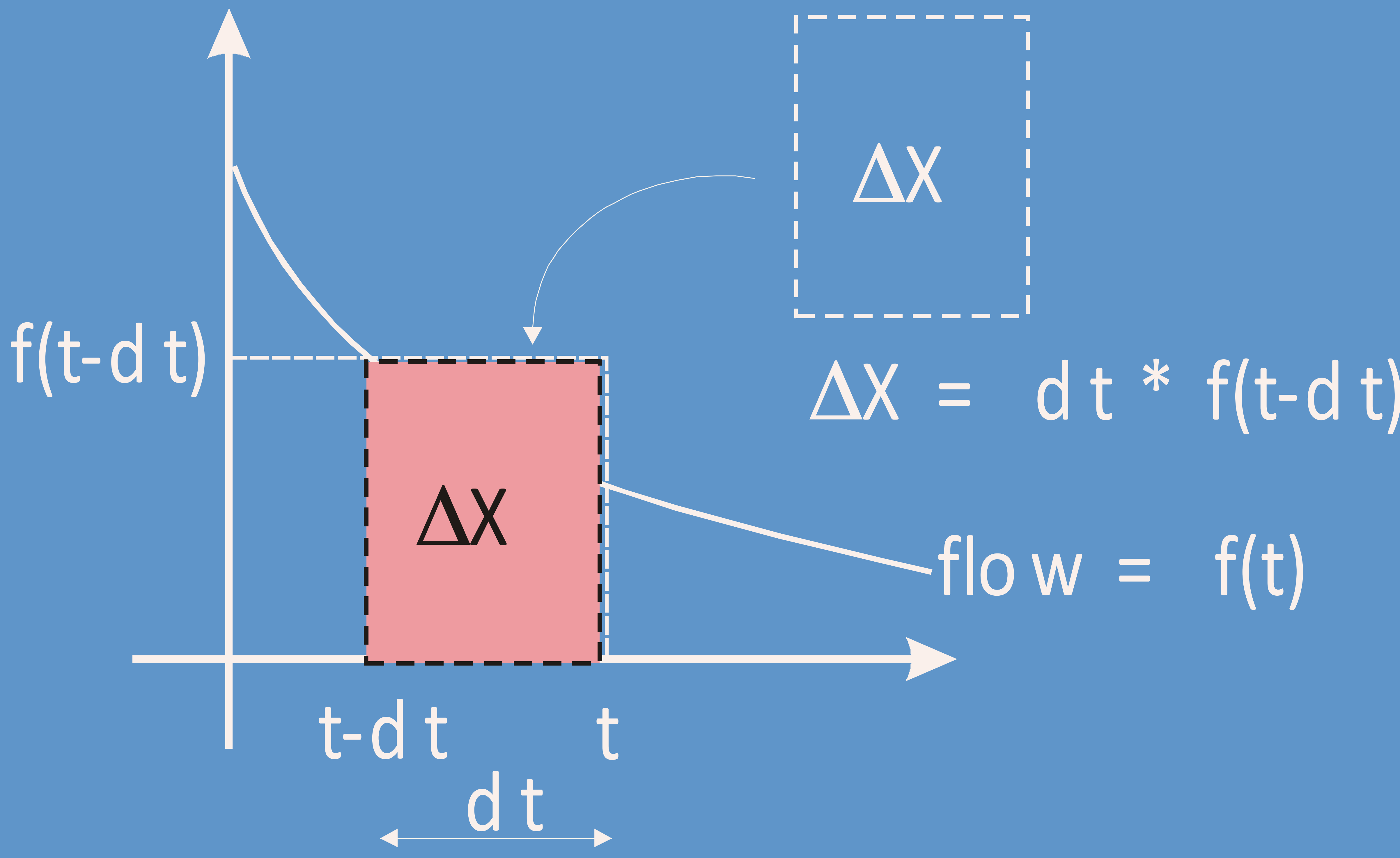
$$x(t) = x(t\text{-}dt) + v(x(t\text{-}dt)) * dt$$



- Not very accurate, but fast
- Other higher order methods are avilable: Runge-Kutta second and fourth order integration methods (more popular due to their accuracy)

Euler's Method

Assume flow = f(t)



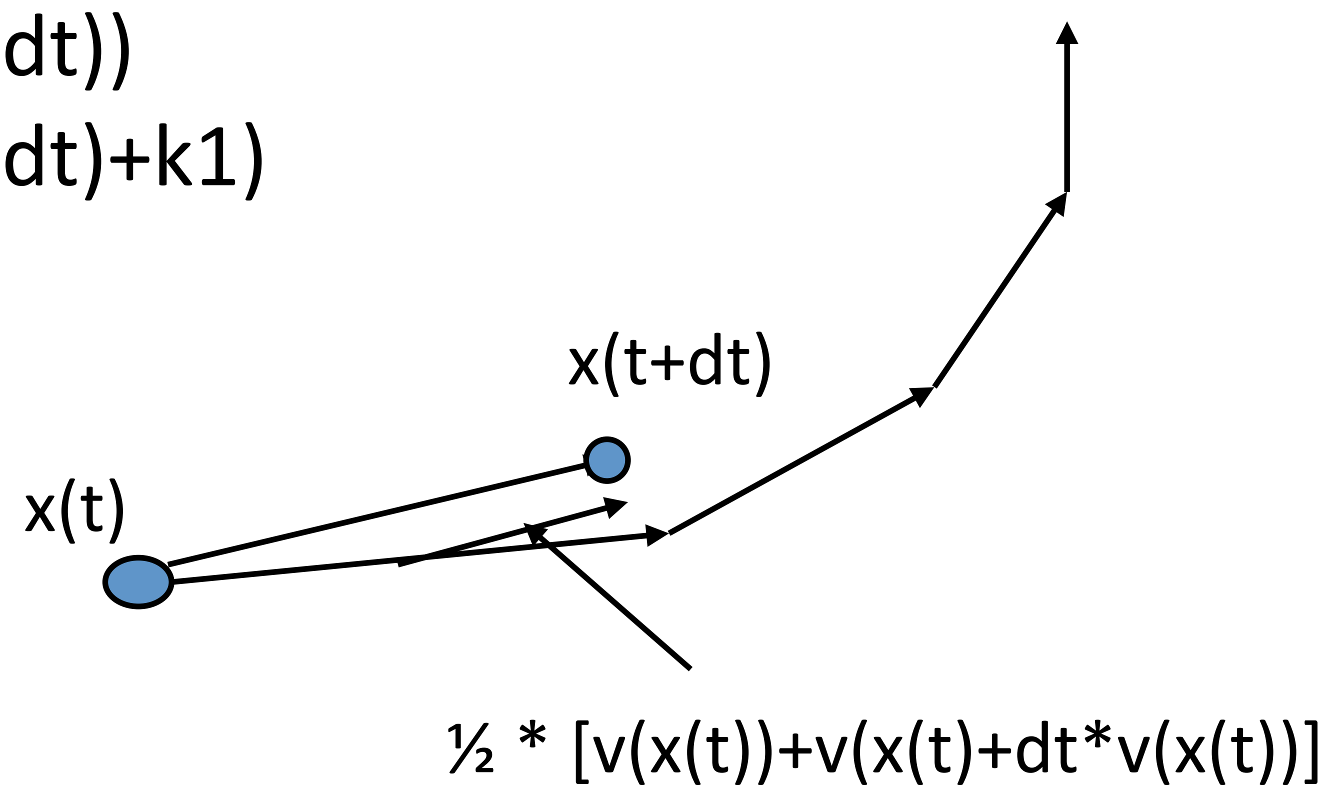
Numerical Integration (2)

Second Runge-Kutta Method

x(t) = x(t-dt) + ½ * (K1 + K2)

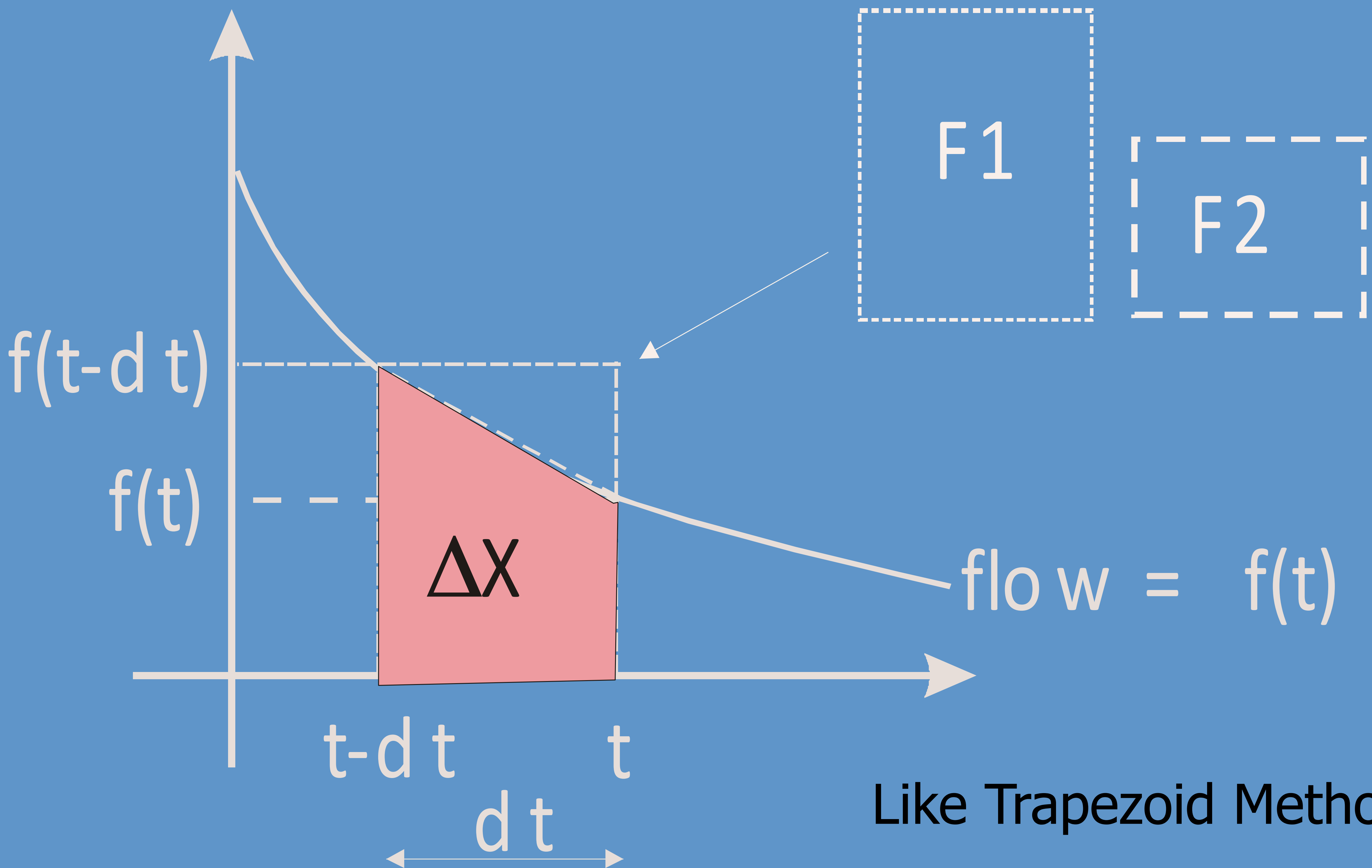
k1 = dt * v(x(t-dt))

k2 = dt * v(x(t-dt)+k1)



Runge-Kutta 2

Assume flow = f(t)



Like Trapezoid Method.

Numerical Integration (3)

Standard Method: Runge-Kutta fourth order

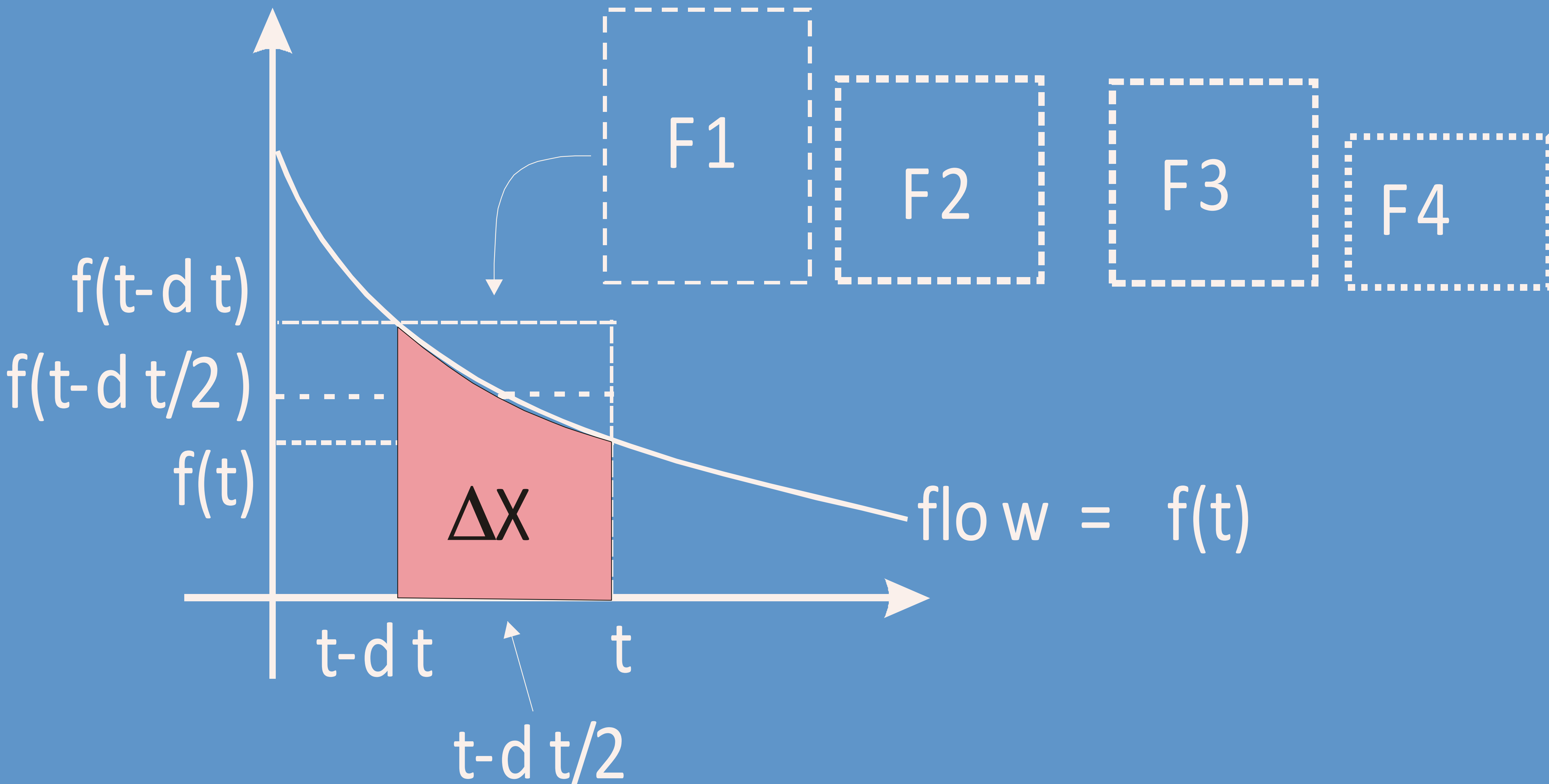
$$x(t) = x(t\text{-}dt) + 1/6\ (k_1 + 2k_2 + 2k_3 + k_4)$$

$k_1 = dt * v(t\text{-}dt)$; $k_2 = dt * v(x(t\text{-}dt) + k1/2)$

$k_3 = dt * v(x(t\text{-}dt) + k2/2)$; $k_4 = dt * v(x(t\text{-}dt) + k3)$

Runge-Kutta 4

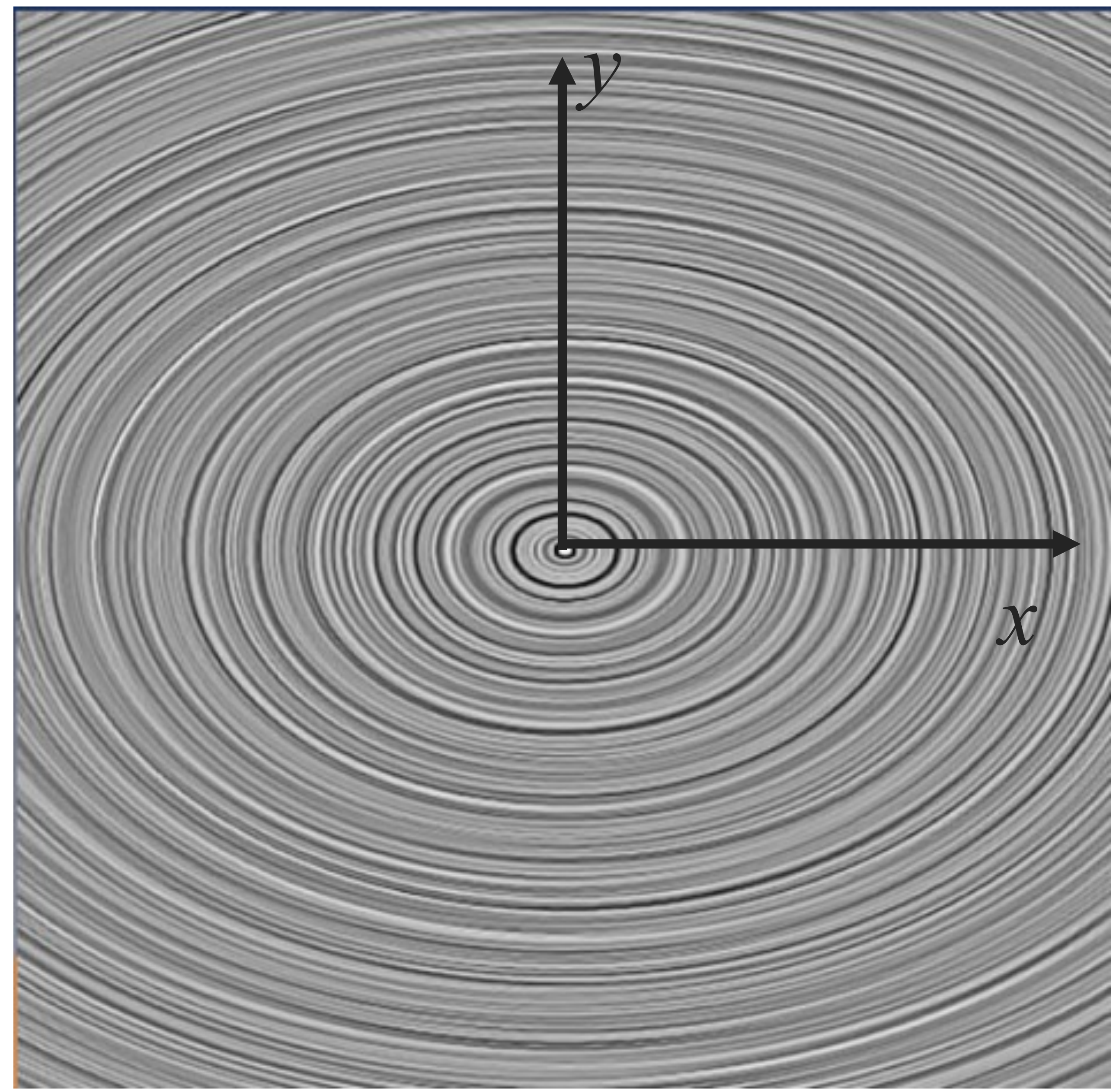
Assume flow = f(t)



- **Numerical integration of stream lines:**

- approximate streamline by polygon $\mathbf{x}_i$

- **Testing example:**
  - $\mathbf{v}(x,y) = (-y, x/2)^T$
  - exact solution: ellipses
  - starting integration from (0,-1)



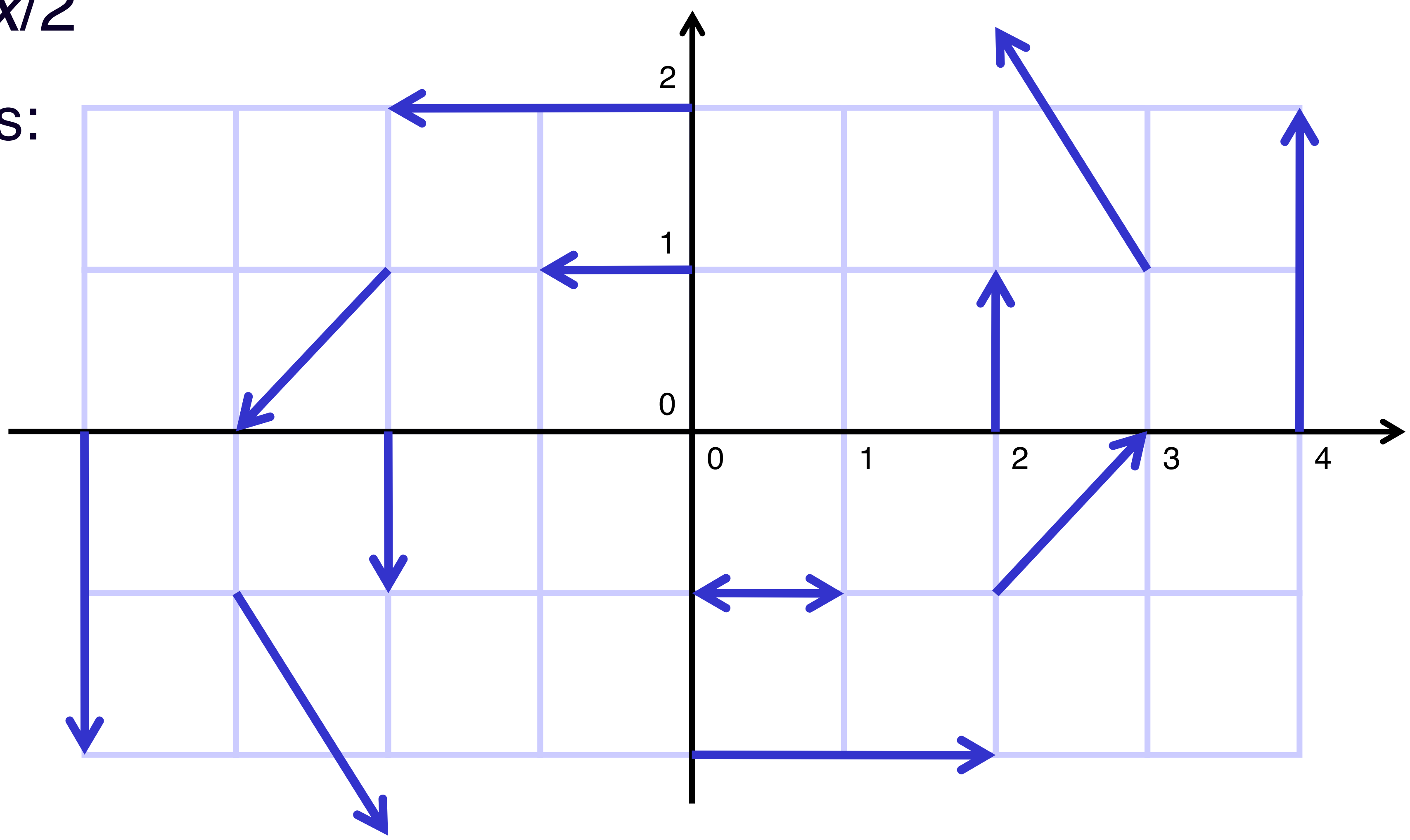
Euler Integration – Example

2D model data:

$v_x = dx/dt = -y$

$v_y = dy/dt = x/2$

Sample arrows:



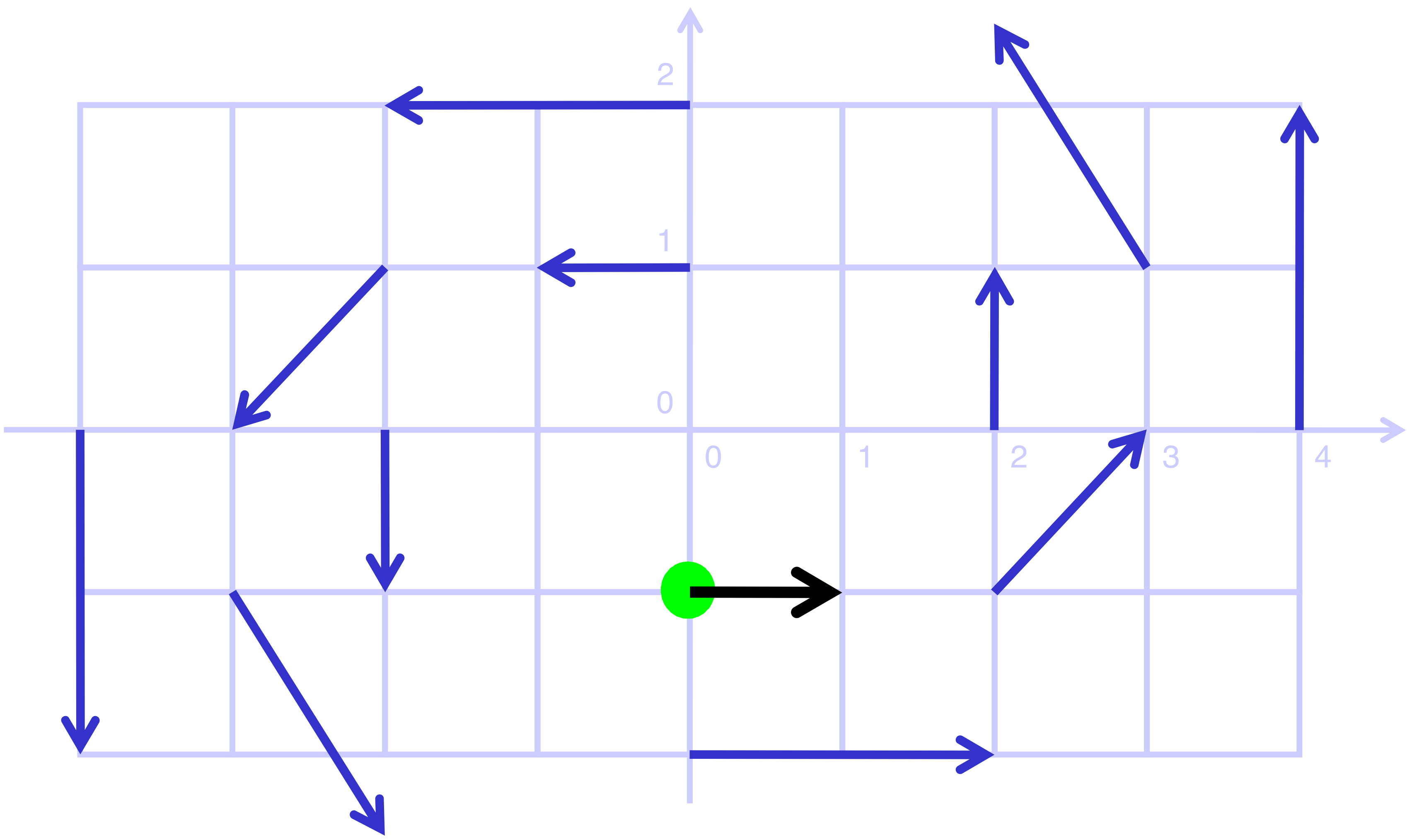
True solution: ellipses.

Euler Integration – Example

- Seed point $\mathbf{s}_0 = (0|-1)^T$;
current flow vector $\mathbf{v}(\mathbf{s}_0) = (1|0)^T$;
$\mathrm{d}t = ½$

$v_x = \mathrm{d}x/\mathrm{d}t = -y$

$v_y = dy/dt = x/2$

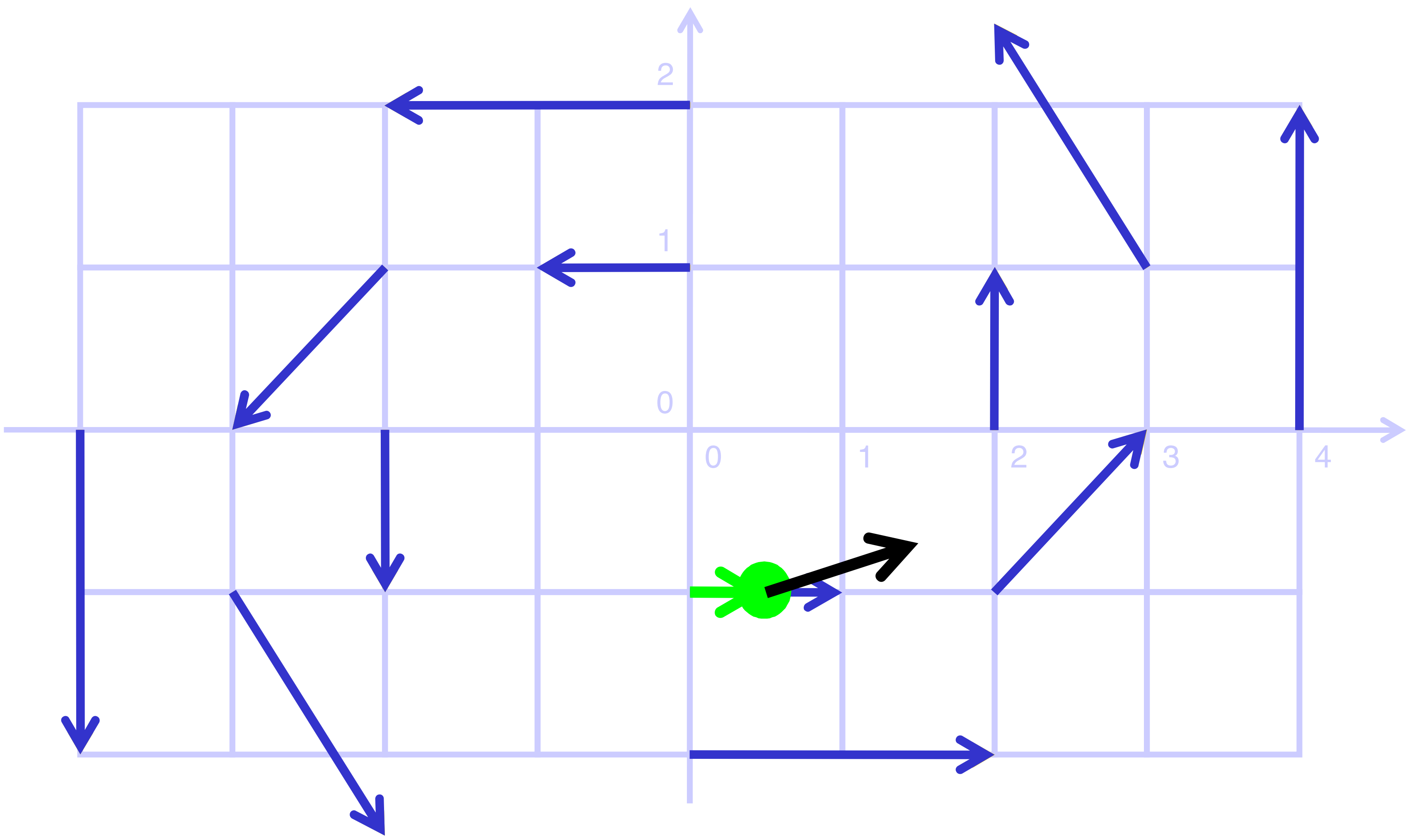


Euler Integration – Example

- New point $\mathbf{s}_1 = \mathbf{s}_0 + \mathbf{v}(\mathbf{s}_0) \cdot \mathrm{d}t = (1/2 | -1)^T$; current flow vector $\mathbf{v}(\mathbf{s}_1) = (1 | 1/4)^T$;

$v_x = \mathrm{d}x/\mathrm{d}t = -y$

$v_y = dy/dt = x/2$

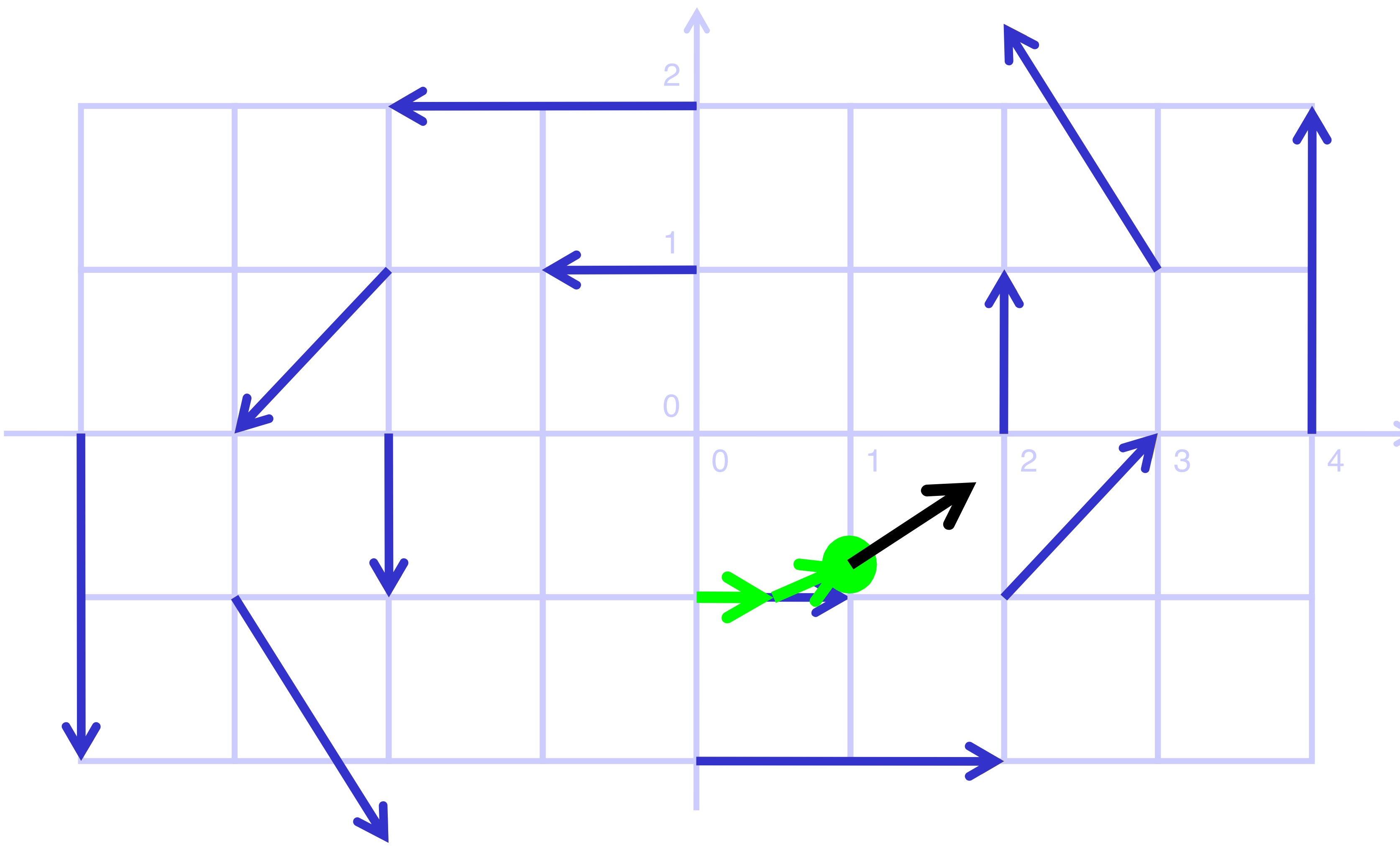


Euler Integration – Example

- New point $\mathbf{s}_2 = \mathbf{s}_1 + \mathbf{v}(\mathbf{s}_1) \cdot \mathrm{d}t = (1 | -7/8)^T$; current flow vector $\mathbf{v}(\mathbf{s}_2) = (7/8 | 1/2)^T$;

$v_x = \mathrm{d}x/\mathrm{d}t = -y$

$v_y = dy/dt = x/2$



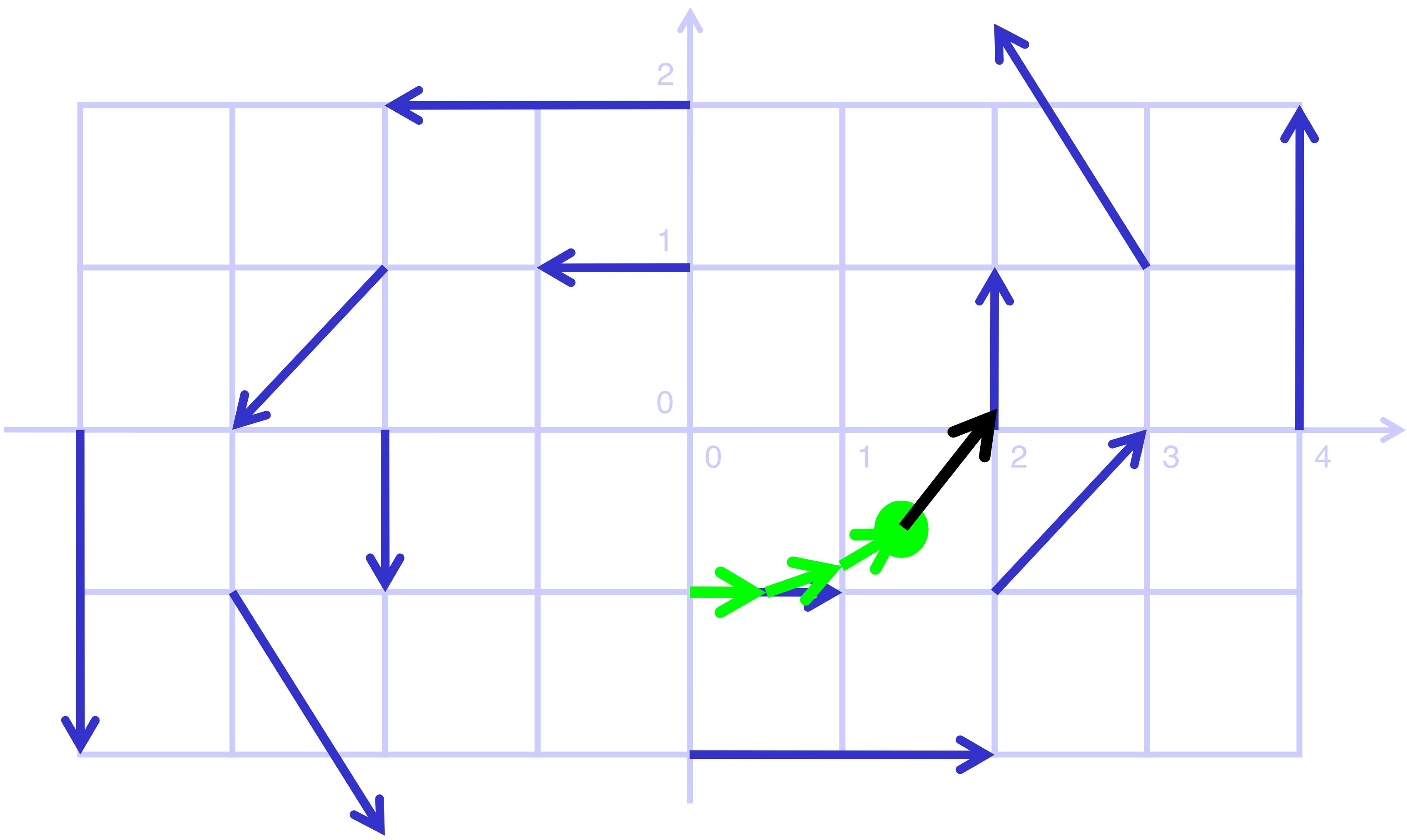
Euler Integration – Example

- $\mathbf{s}_3 \quad = (23/16 \mid -5/8)^T \approx (1.44 \mid -0.63)^T;$

$\mathbf{v}(\mathbf{s}_3) = (5/8 \mid 23/32)^T \approx (0.63 \mid 0.72)^T;$

$v_x = \mathrm{d}x/\mathrm{d}t = -y$

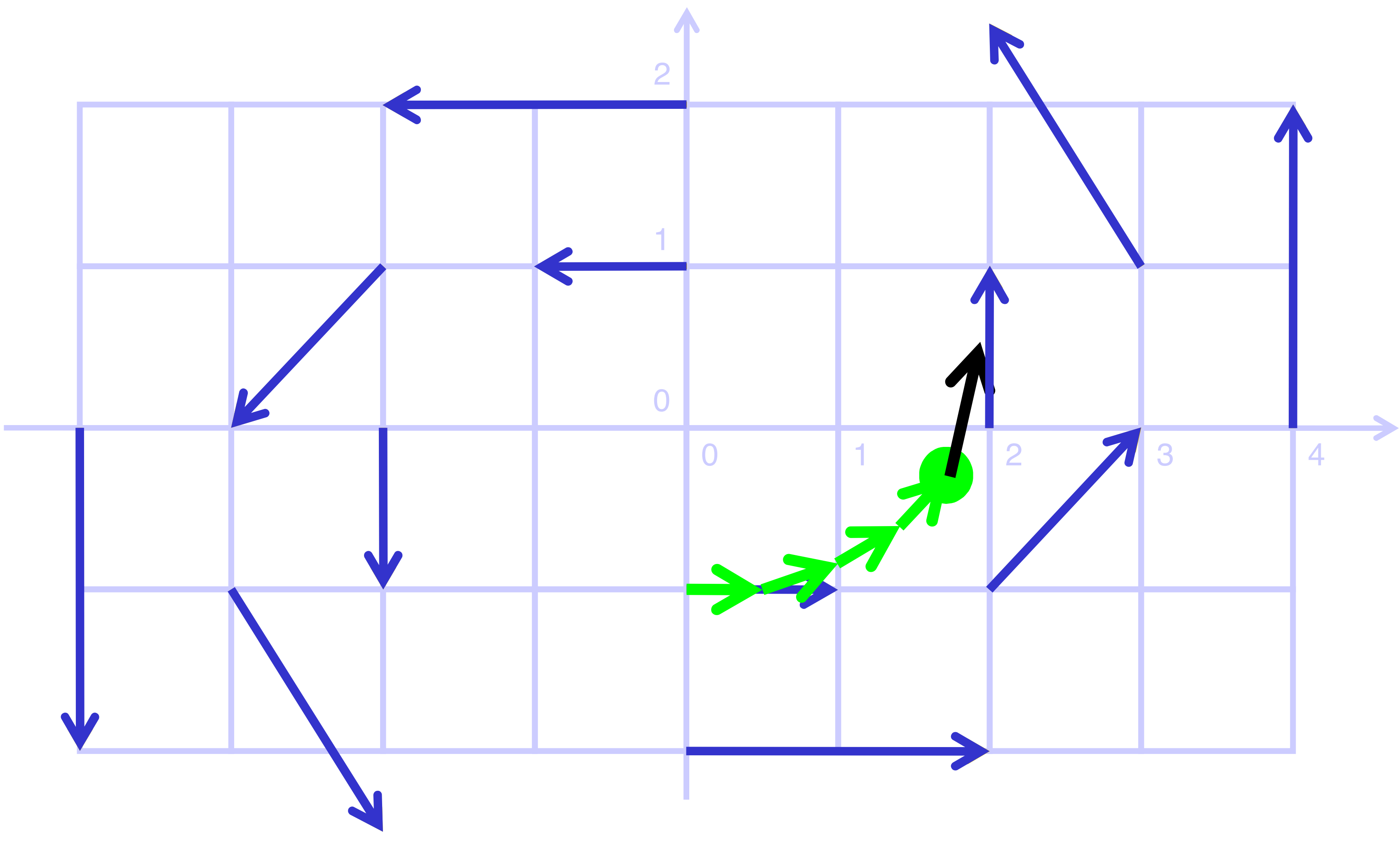
$v_y = dy/dt = x/2$



Euler Integration – Example

- $\mathbf{s}_4 \quad = (7/4 \mid -17/64)^T \approx (1.75 \mid -0.27)^T$;

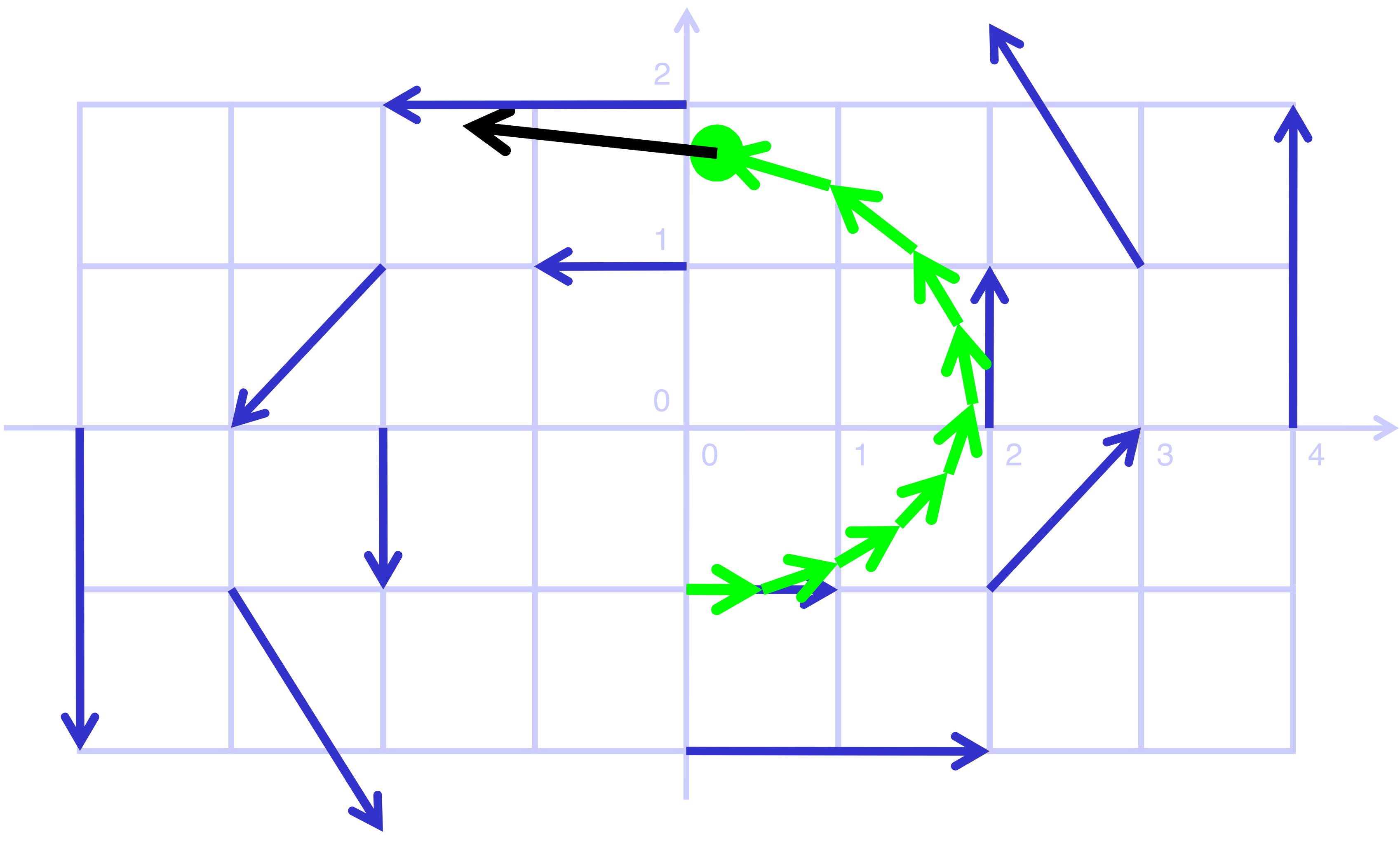
$\mathbf{v}(\mathbf{s}_4) \quad = (17/64 \mid 7/8)^T \approx (0.27 \mid 0.88)^T$;



Euler Integration – Example

- $\mathbf{s}_9 \approx (0.20|1.69)^T$;

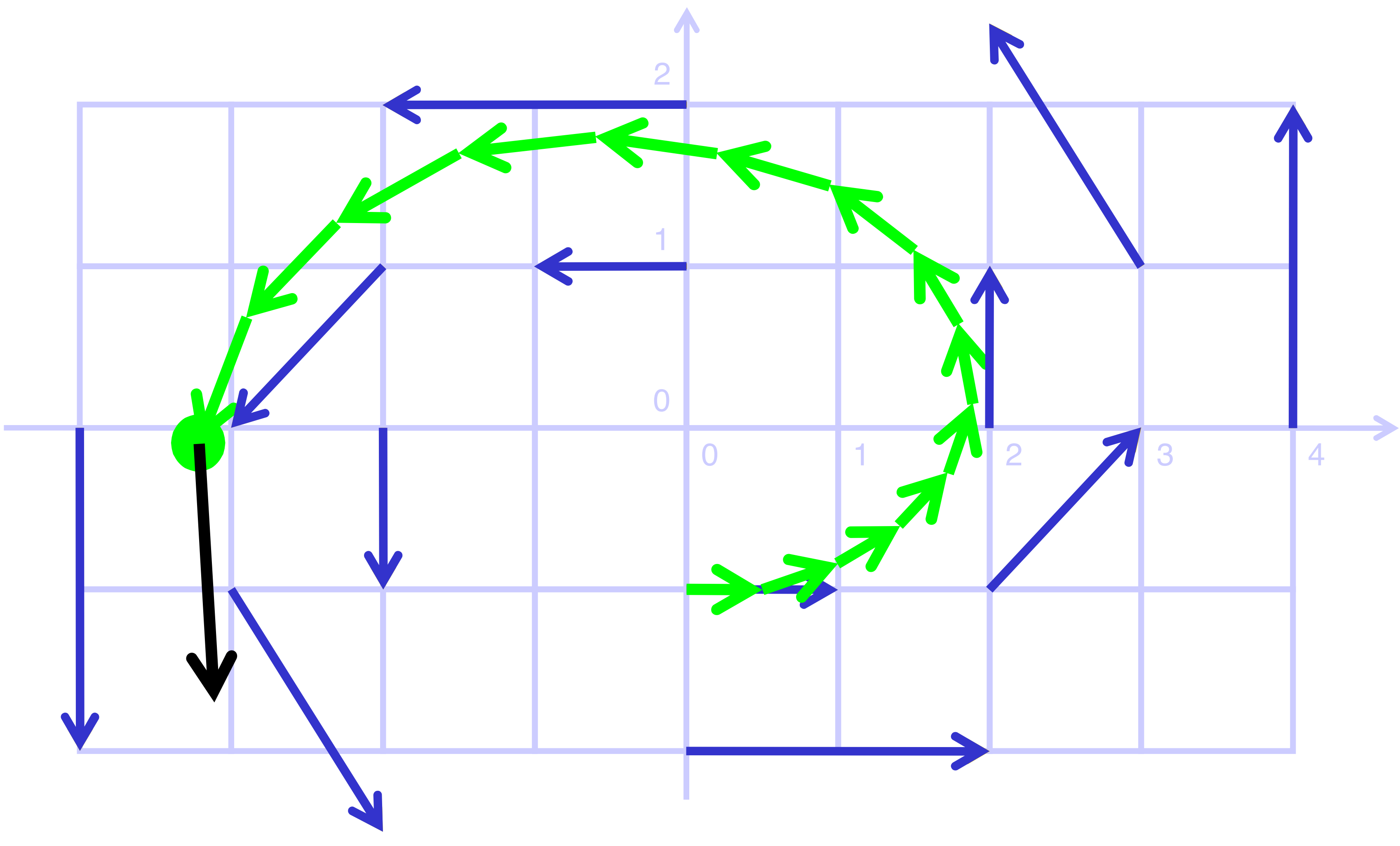
$\mathbf{v}(\mathbf{s}_9) \approx (-1.69|0.10)^T$;



Euler Integration – Example

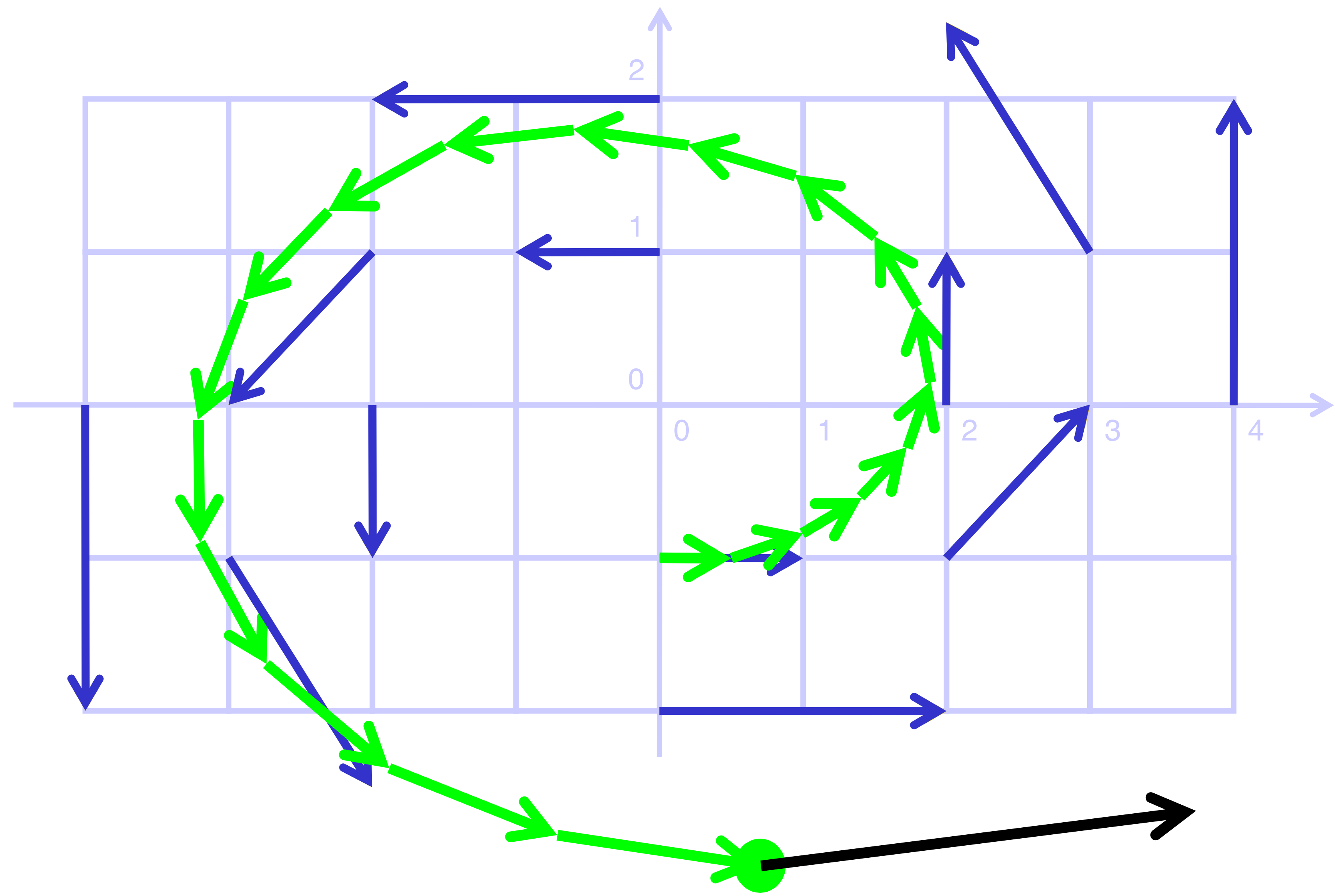
- $\mathbf{s}_{14} \approx (-3.22 \mid -0.10)^T$;

$\mathbf{v}(\mathbf{s}_{14}) \approx (0.10 \mid -1.61)^T$;



Euler Integration – Example

- $\mathbf{s}_{19} \approx (0.75 \mid -3.02)^T$; $\mathbf{v}(\mathbf{s}_{19}) \approx (3.02 \mid 0.37)^T$; clearly: large integration error, d*t* too large, 19 steps



Lec23 Reading

- Comparing 2D Vector Field Visualization Methods: A User Study. David H. Laidlaw, Robert M. Kirby, Cullen D. Jackson, J. Scott Davidson, Timothy S. Miller, Marco da Silva, William H. Warren, Michael J. Tarr. IEEE Trans. Vis. Comput. Graph. 11(1): 59-70 (2005).

Reminder
Assignment 05

Assigned: Monday, March 27

Due: Monday, April 10, 4:59:59 pm

Reminder
Project Milestones 03/04

Assigned: Wednesday, March 29

03 (Talk) Due: Wednesday, April 26, 4:59:59 pm

04 (Report) Due: Wednesday, May 3, 4:59:59 pm