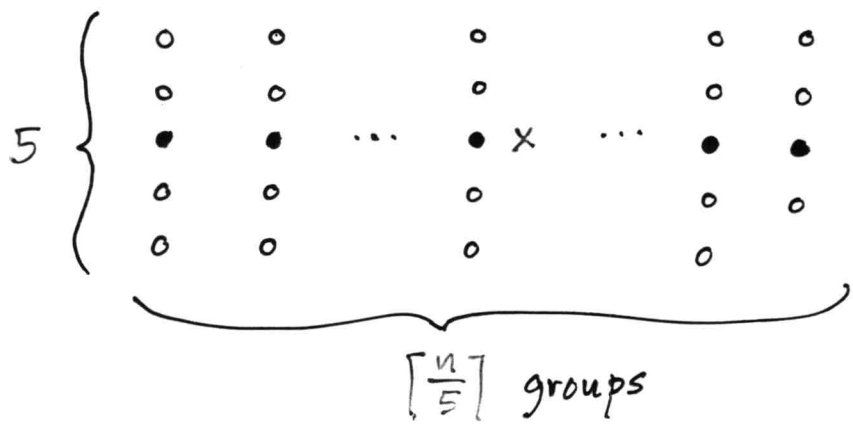
# Finding the $k^{th}$-smallest in $\Theta(n)$ worst-case time

- Like the randomized algorithm, we recursively partition the array.
  But now we guarantee a good split in 5 steps:

(1) Divide the $n$ elements into $\lfloor \frac{n}{5} \rfloor$ groups of 5 elements, and $\leq 1$ group of $<5$ elements:

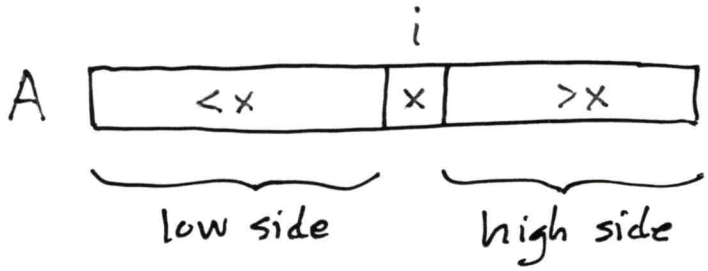



(2) Find the median of each group (by say running insertion sort on the $\leq 5$ elements and taking the middle element).

## Finding the $k^{th}$-smallest, cont'd

(3) Recursively find the median $x$ of the $\lceil \frac{n}{5} \rceil$ medians found in step (2) (by recursively calling the algorithm with $k' = \left\lceil \frac{\lceil \frac{n}{5} \rceil + 1}{2} \right\rceil$).

(4) Partition the input array A around element $x$ from step (3):




(5) Let $i$ be the rank of $x$.

$k = i$ : Return $x$.

$k < i$ : Recursively find $k^{th}$-smallest in the low side of A.

$k > i$ : Recursively find $(k-i)^{th}$-smallest in the high side.