Searching VI (Skiplists)

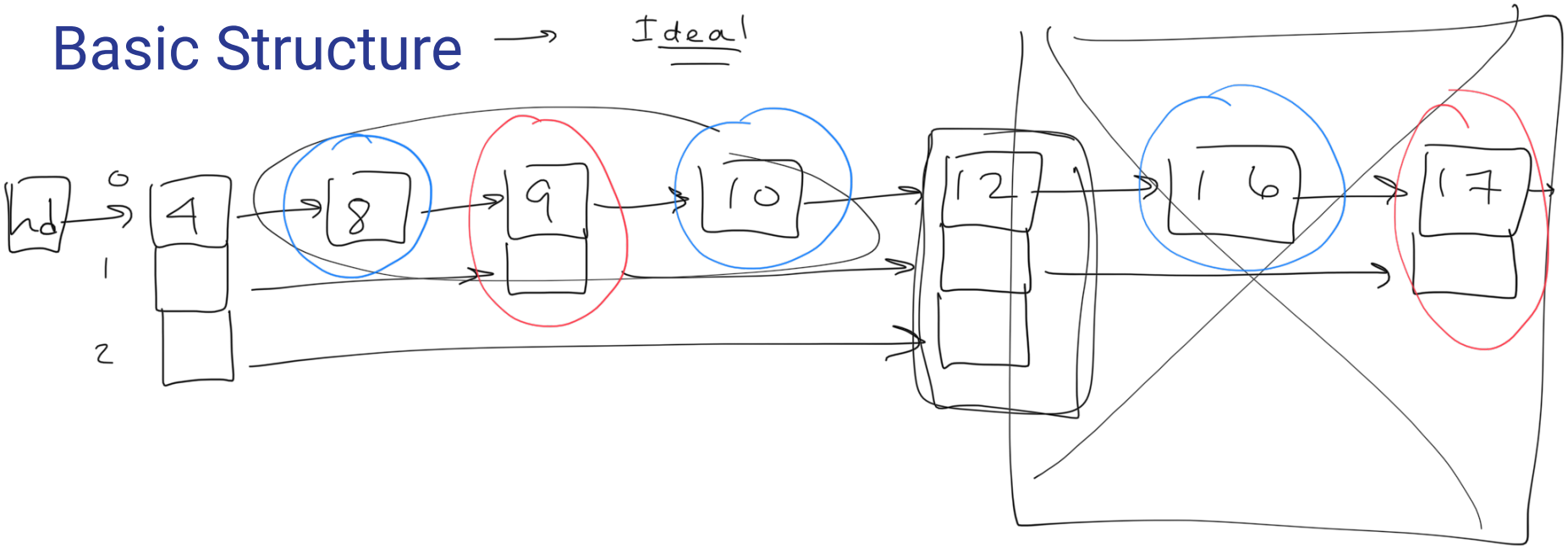
Motivation

BST pros: ordered, time is good for random data, simple

BST cons: worst-case is O(N)

build randomization into structure

Basic Structure

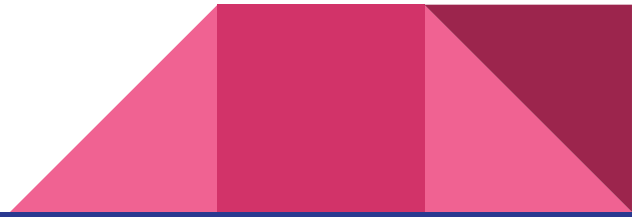


Operations

insert → at what level?

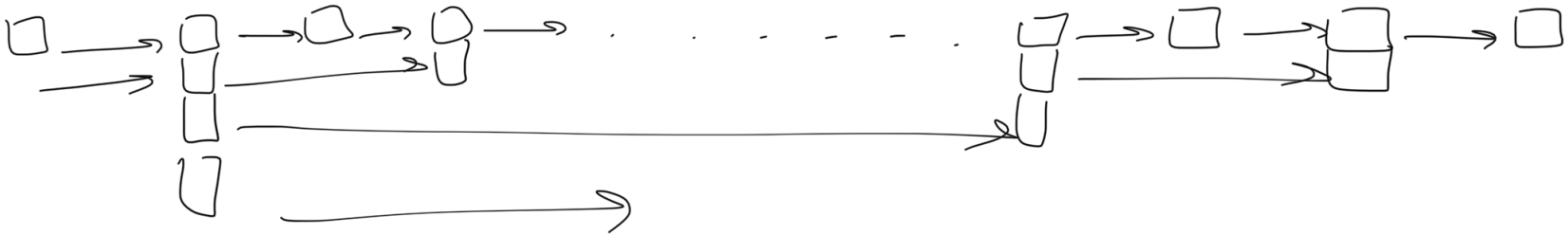
search → start at bottom level

delete



The Skiplist...

- ...does not *guarantee* O(logN) runtime for operations,...
- ...does provide O(logN) performance *with a high probability*.
- ...can provide a balance between good performance and ease of implementation.



N 1024

Hashtables

ordered
- BST
- 2-3
- RB
- Skiplists

Ideally...

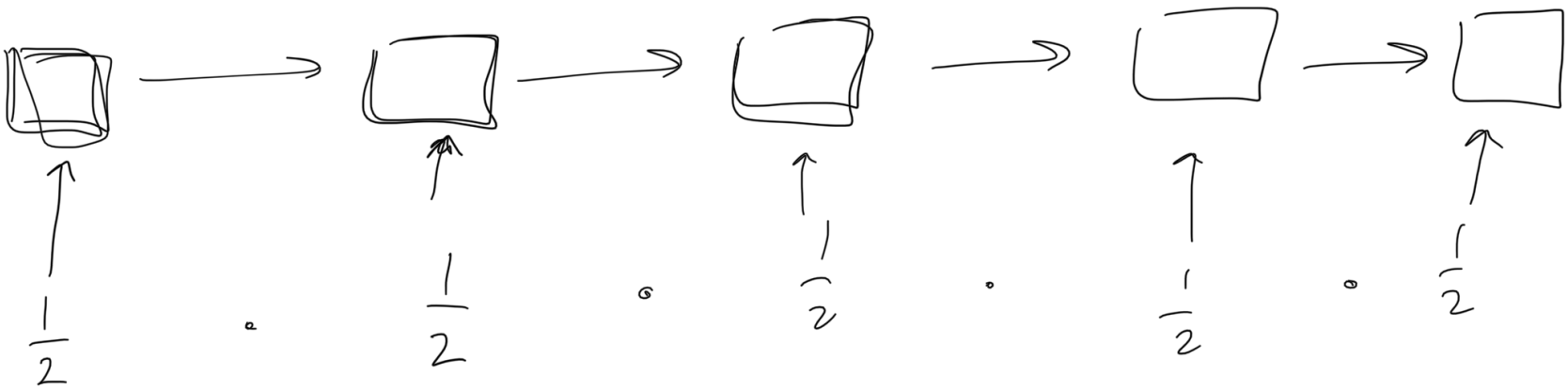
- You would have enough levels so that the ideal search would skip half the values, then a quarter of the remaining values, then an eighth of the remaining values, etc…
- Which would mean that a *search* (or an *insert*) would be O(logN) with high probability.
- Space: If the head node isn't counted,
  - ½ the nodes should have 1 pointer...
  - ¼ should have 2 pointers...
  - ⅛ should have 3 pointers…
  - and so on…
- However, we don't enforce the *ideal* situation.
- Instead, we rely on probability and randomization.

Inserting & removing nodes...

- Instead, a new node is assigned a level at random where there is
  - a 50% chance that it will have 1 pointer (the lowest level),
  - a 25% chance that it will have 2 pointers (the second lowest level)...
  - and so on...
- This is what makes skiplists *probabilistic*.
- https://opendsa-server.cs.vt.edu/ODSA/Books/CS3/html/SkipList.html#id1

Worst case:

level 0 : 50%
level 1 : 25%



$\frac{1}{32}$

P(inserting ~~N~~ in a row at level 0) = $\frac{1}{1024}$ $\left(\frac{1}{2^N}\right)$

Analysis

- Consider the extremes:
  - All the nodes are at a high level, so lots of pointers. In that case, the insert cost could be quite high (even O(N)). (Basically, this would be a linked list with extra pointers).
  - All the nodes are at a low level (e.g. level 0), so they only have 1 pointer each, which is basically just a linked list. In that case, insert and search could be O(N).
- The good news: These extremes are unlikely to happen. For example, there is only 1/1024 chance that 10 nodes in a row will be in level 0.
- Similar to Quicksort, we accept that the randomization will make sure that the worst case is unlikely to happen.
- Expected memory use: O(N)