

CSC 544

Data Visualization

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Lecture 21

Topology

Apr. 3, 2023

Today's Agenda

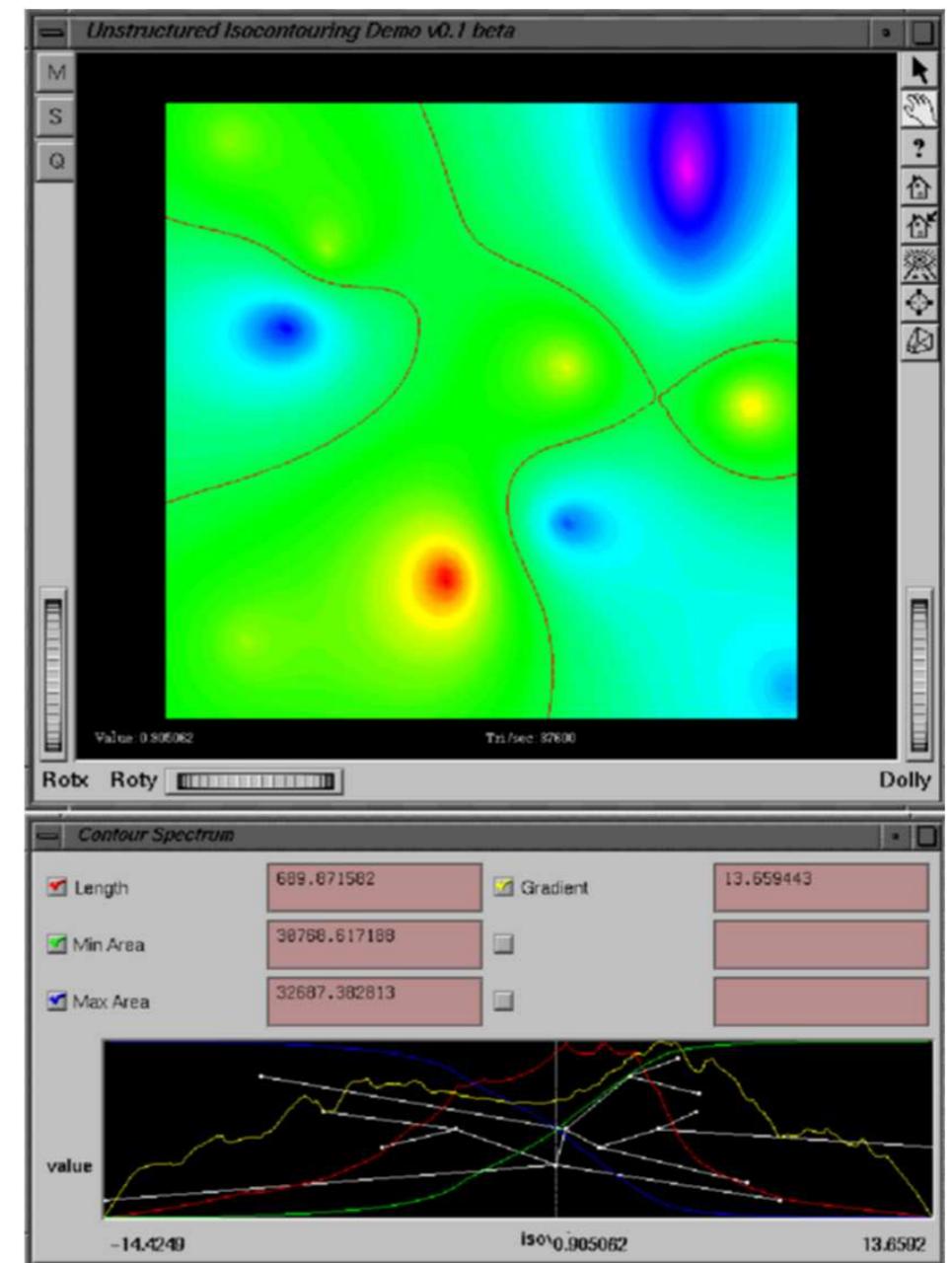
- Reminders:
 - A05 questions? (due Apr. 10)
 - P03/P04 questions? (due Apr. 26/May 3)
- Goals for today: Discuss topological data analysis for scalar field data

Topological Features

Salient Isovalues

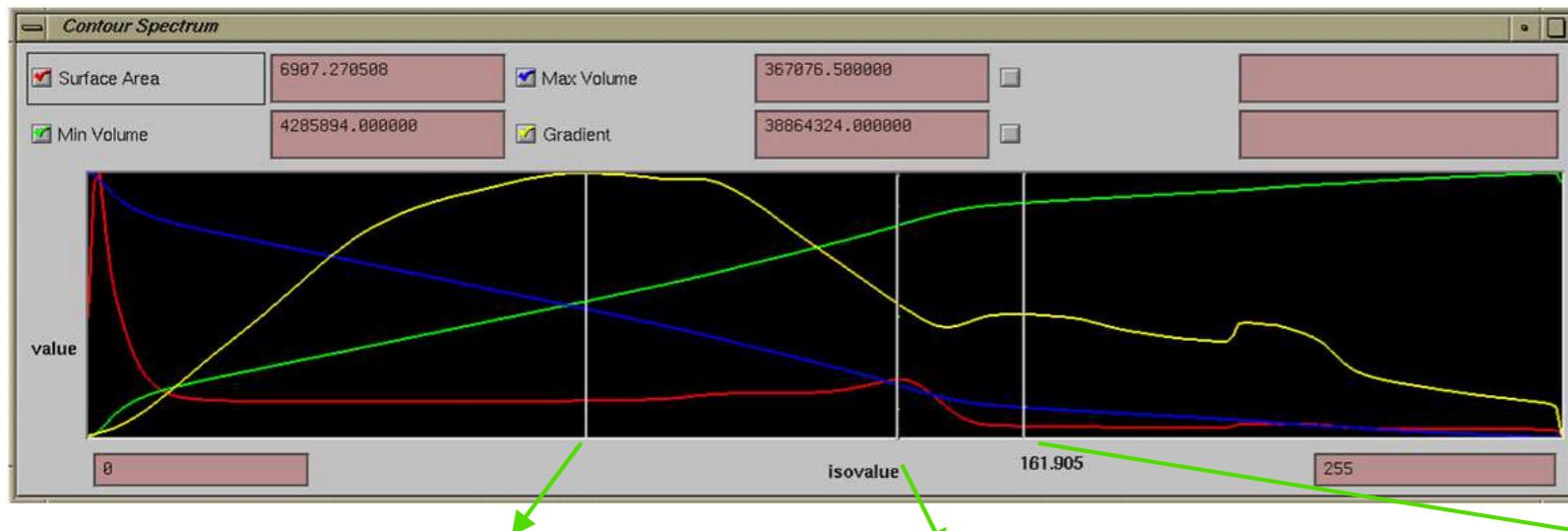
What are the “best” isovalue for extracting the main structures in a volume dataset?

- Contour Spectrum (Bajaj, Pascucci, Schikore: Vis ’97; Transfer Function Bake-Off: Vis ’00)
- Efficient computation of isosurface metrics
 - Area, enclosed volume, gradient surface integral, etc.
 - Efficient connected-component topological analysis
 - **Interface itself concisely summarizes data**

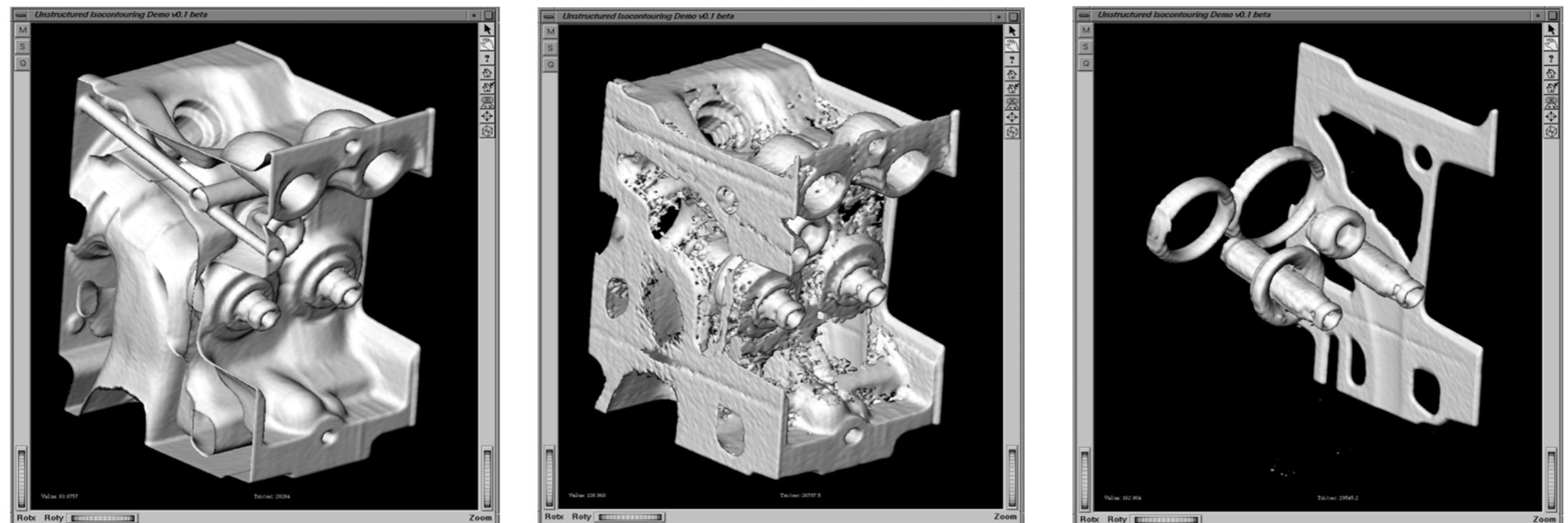


The Contour Spectrum

(colored lines correspond to different isosurface metrics)

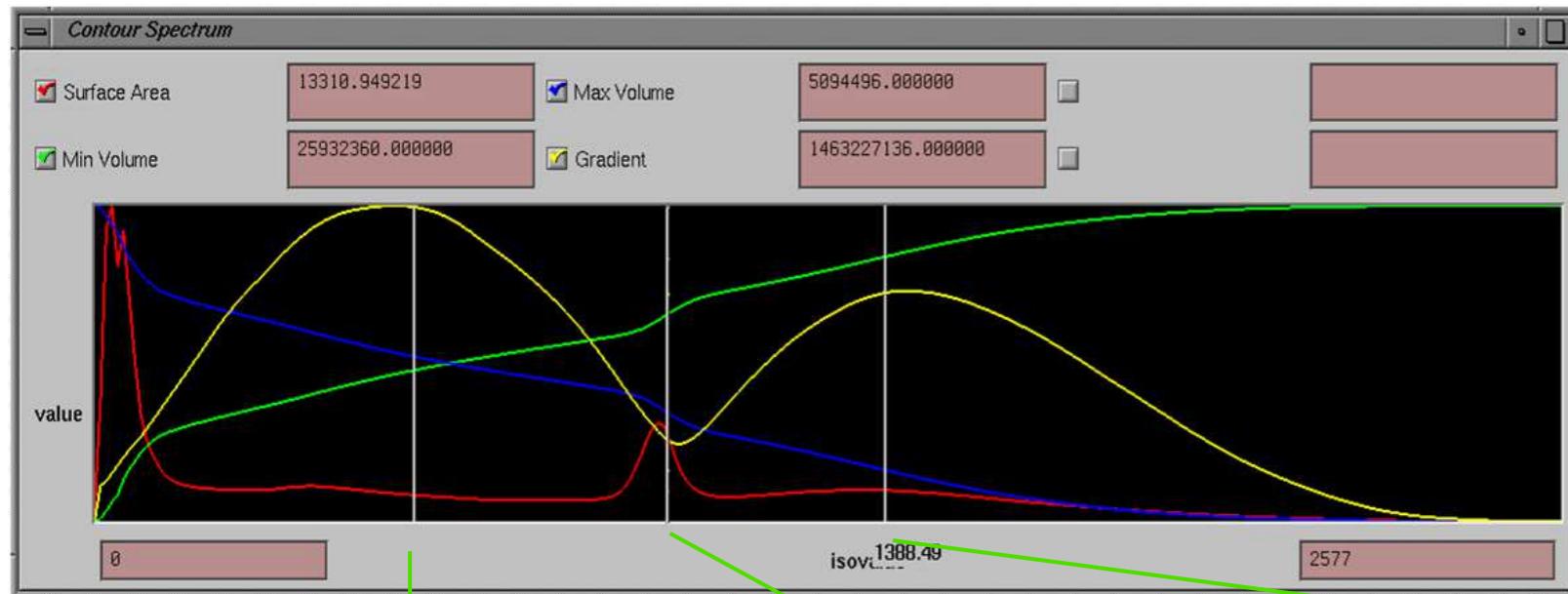


The contour spectrum allows the development of an adaptive ability to separate *interesting* isovales from the others.

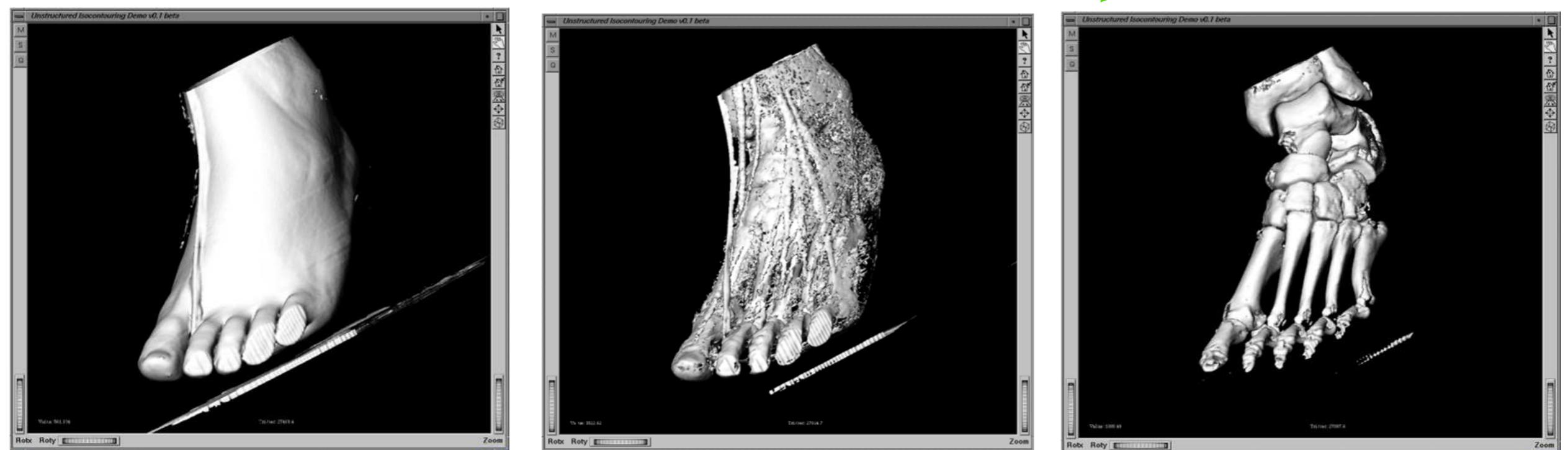


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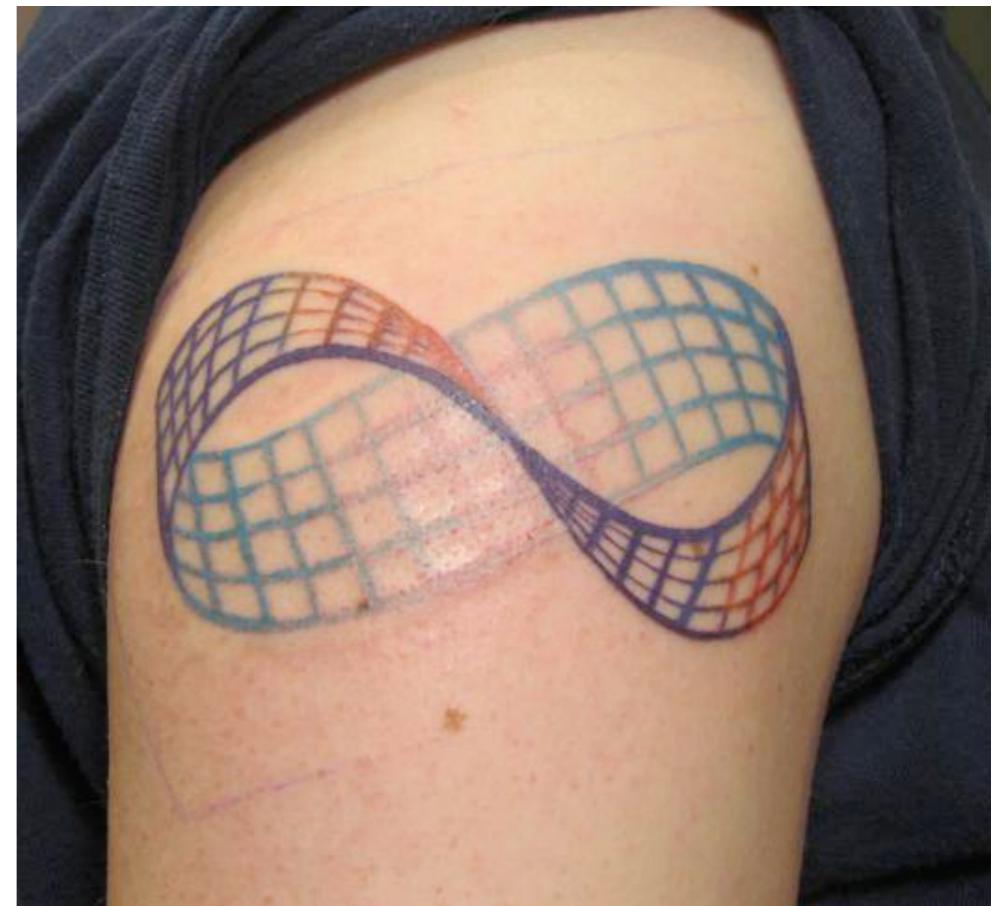
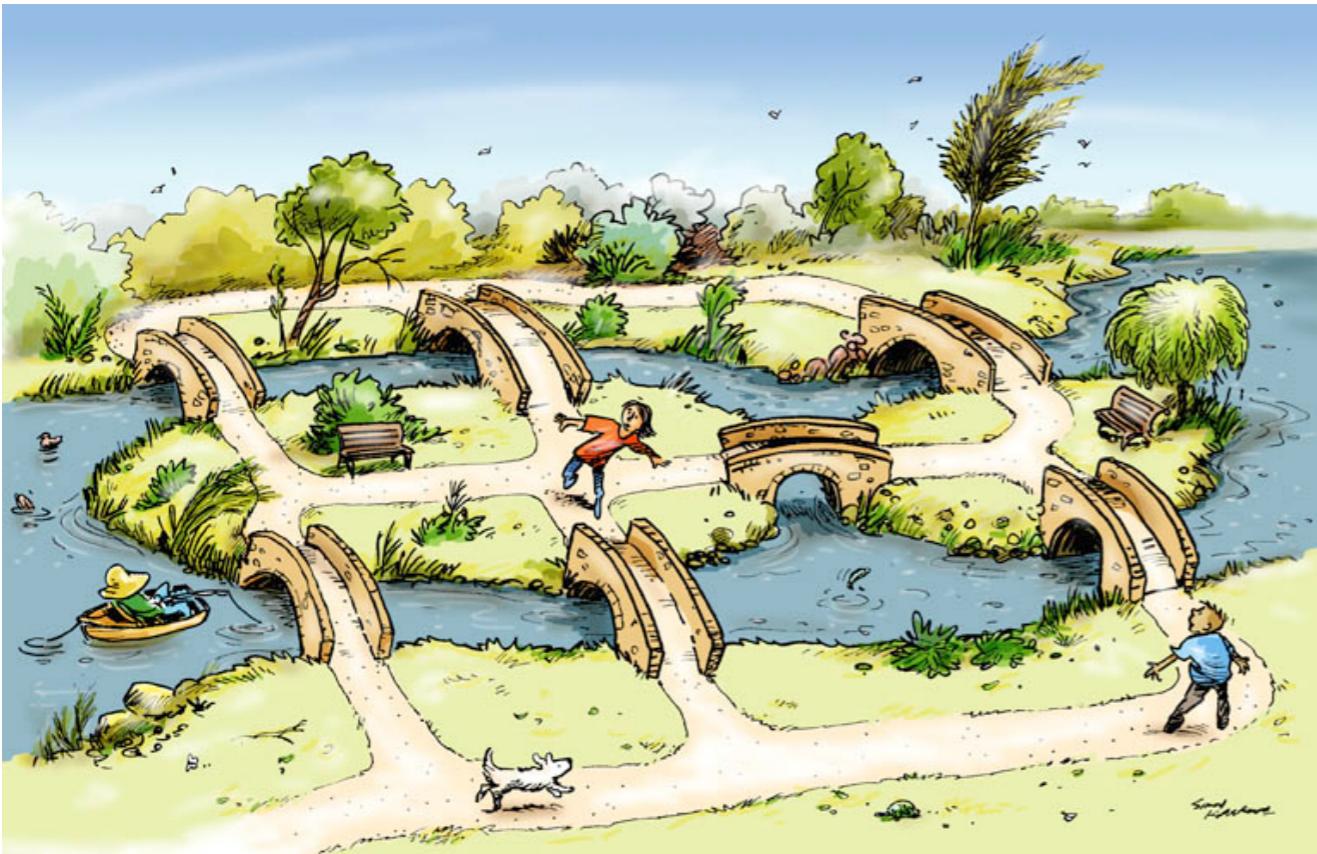


How to Describe All Possible Isosurfaces?

What is Topology?

- Field of mathematics which studies properties which are **preserved under continuous transformations**.
 - Stretching, bending = continuous changes.
 - Tearing, gluing = discontinuous changes.
- Also called: “Rubber sheet” geometry.
- Studies the connectedness of a space.

<http://simonkneebone.files.wordpress.com/2011/11/konigsberg-puzzle.jpg>



<http://talklikeaphysicist.com/wp-content/uploads/2008/09/image-497.jpg>



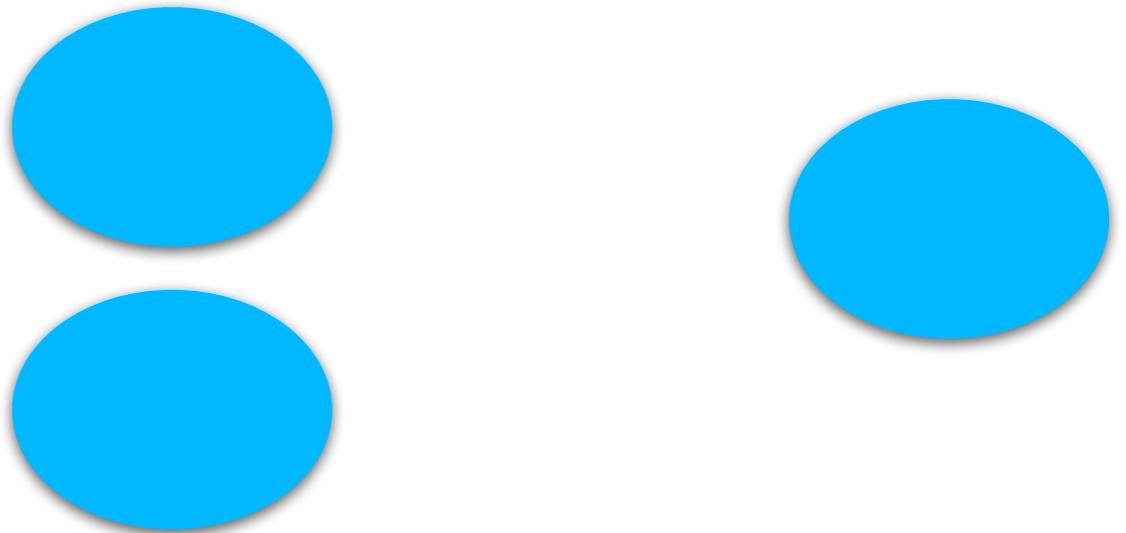
<http://math.arizona.edu/~models/Topology/source/2.html>

What is Topological Data Analysis (TDA)?

- Broadly, TDA refers to a family of techniques that study the ***shape*** of data
 - Helps to understand relationships between how data varies and the domain on which it is defined
 - E.g., connectedness, arrangement, and cycles
- It is a “Swiss army knife” for feature extraction from raw data
 - Robust, Multi-scale

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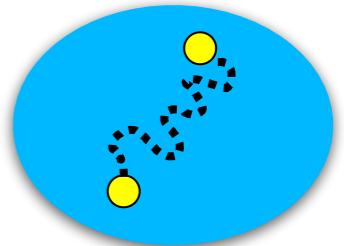
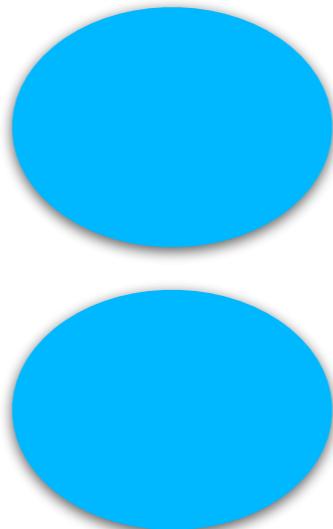
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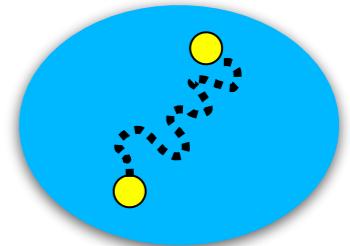
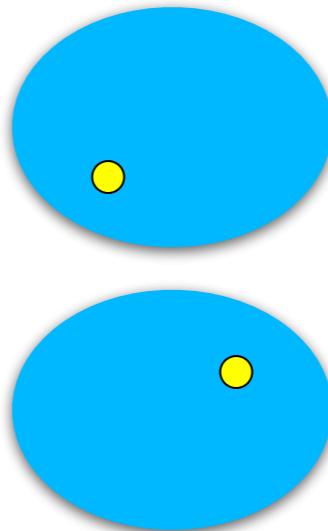
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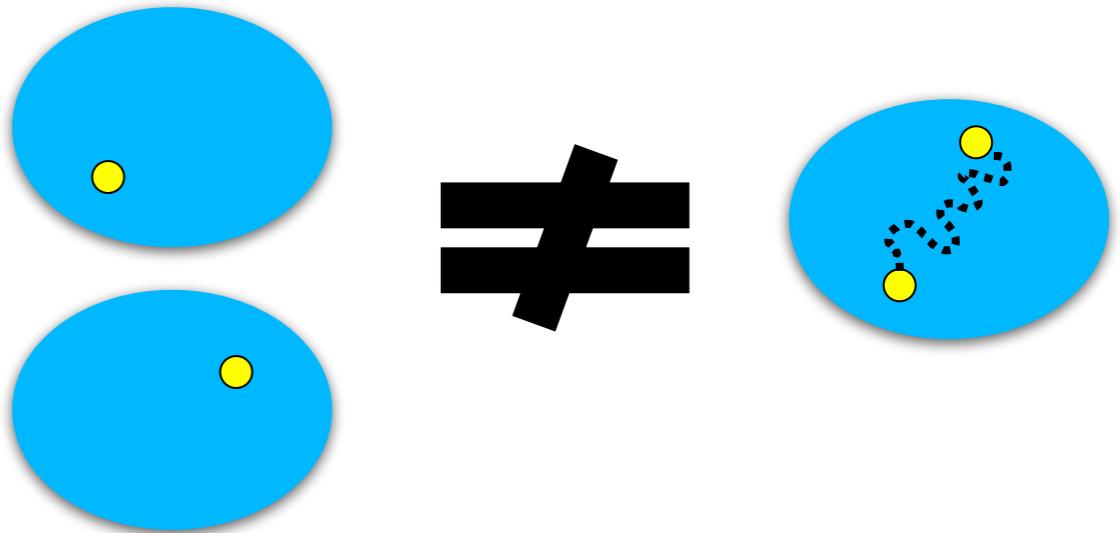
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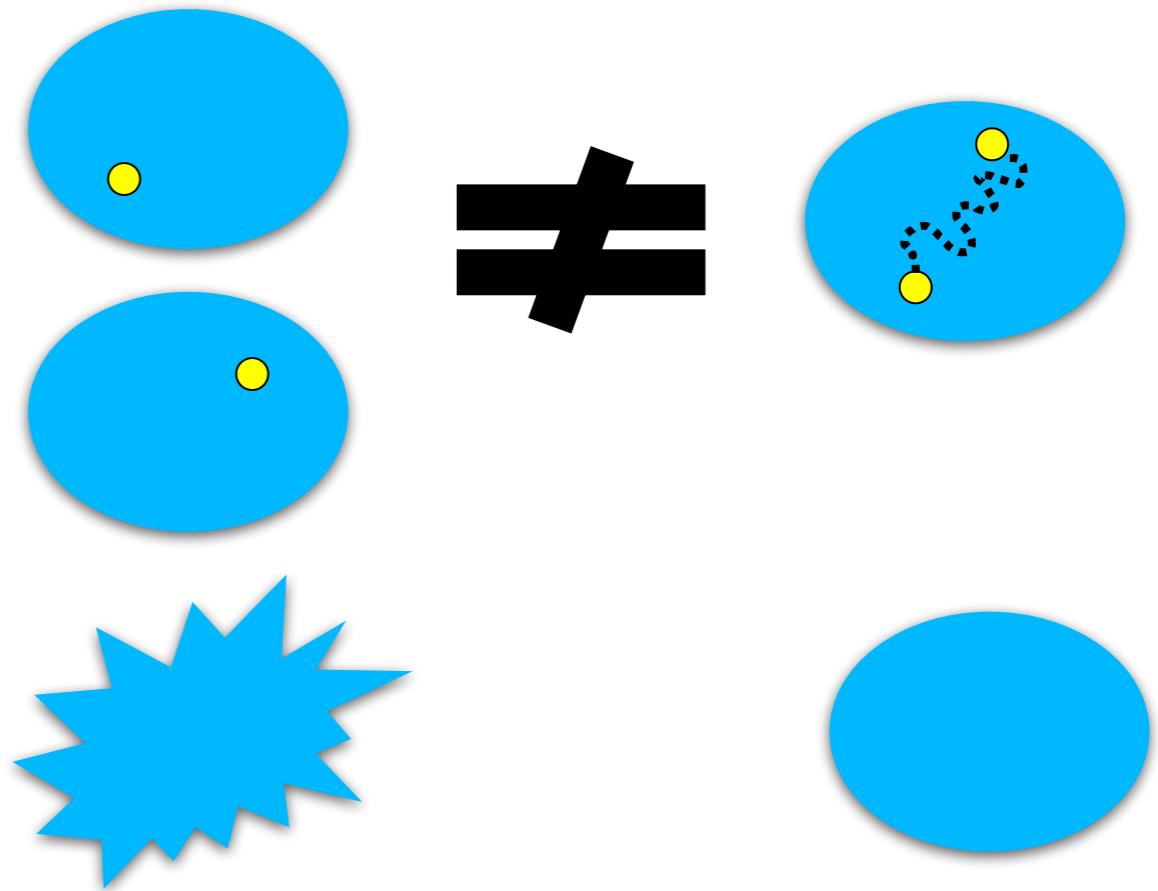
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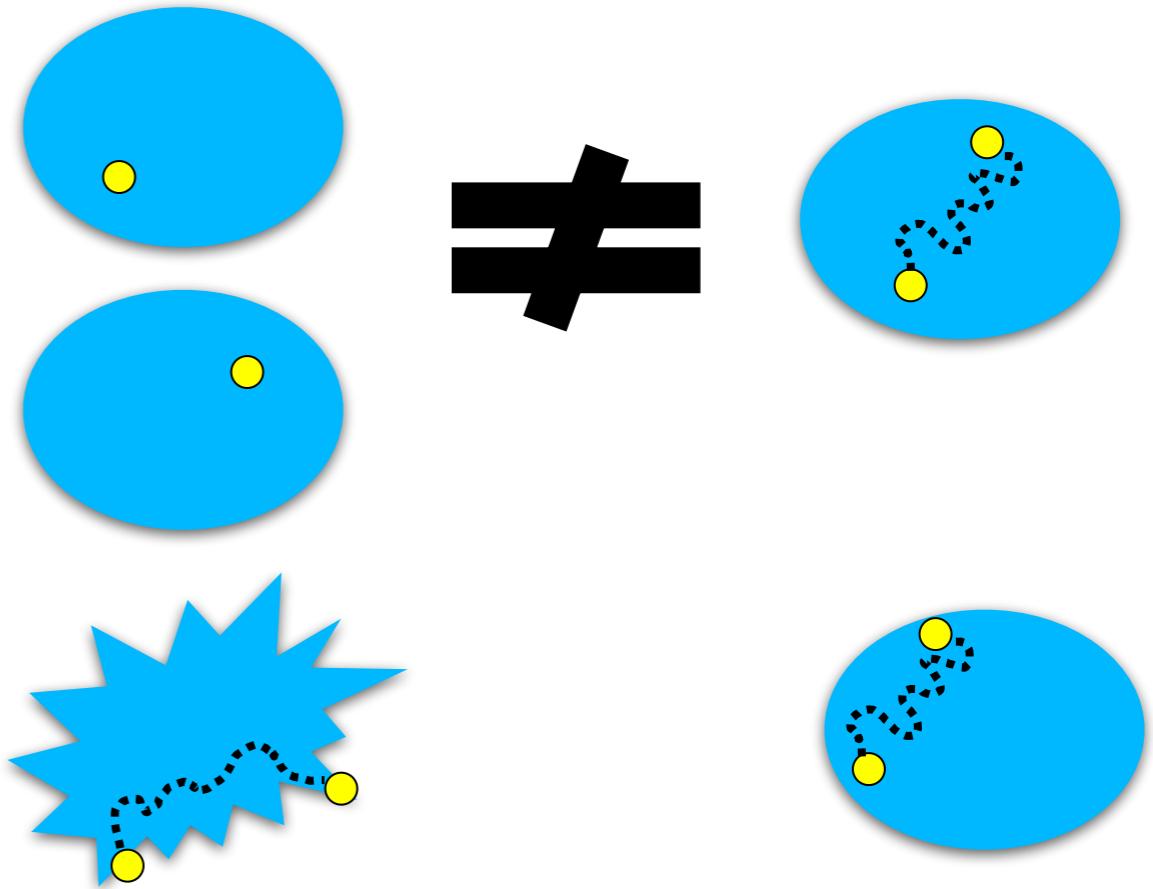
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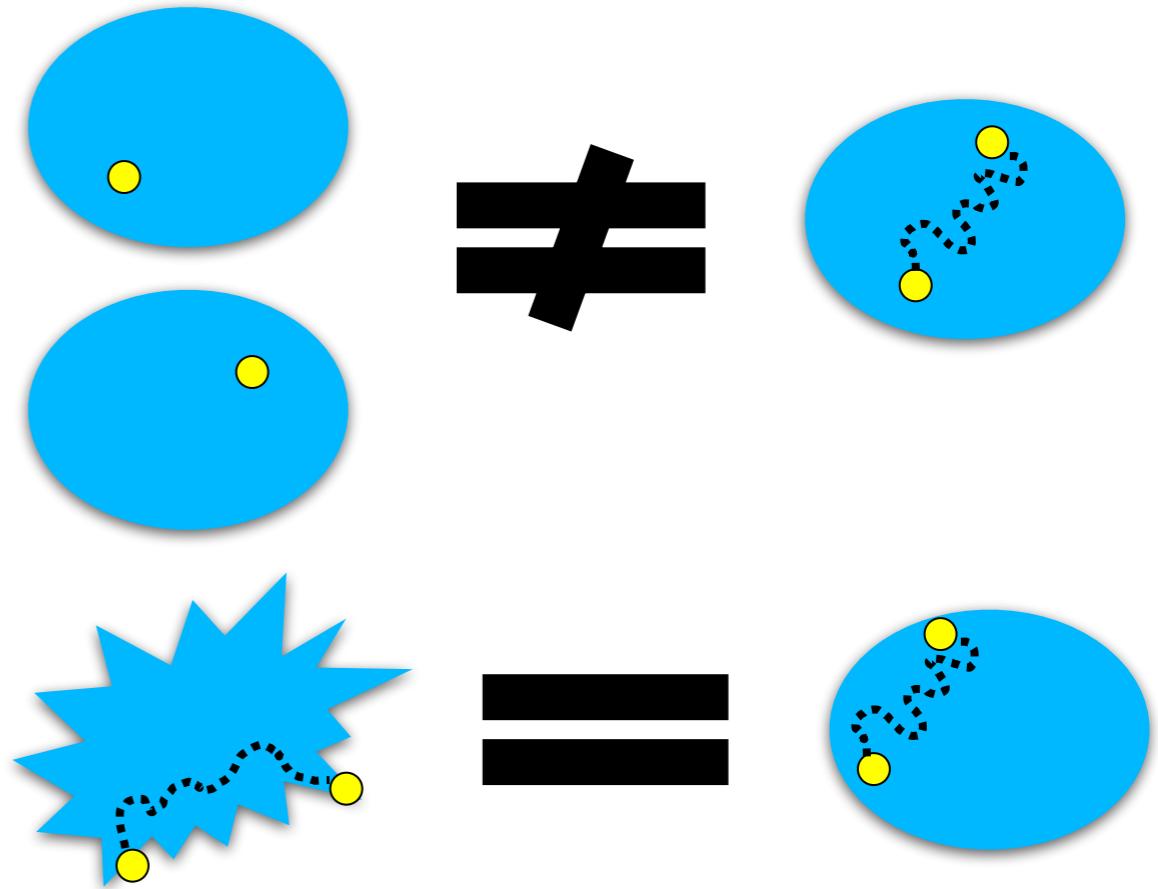
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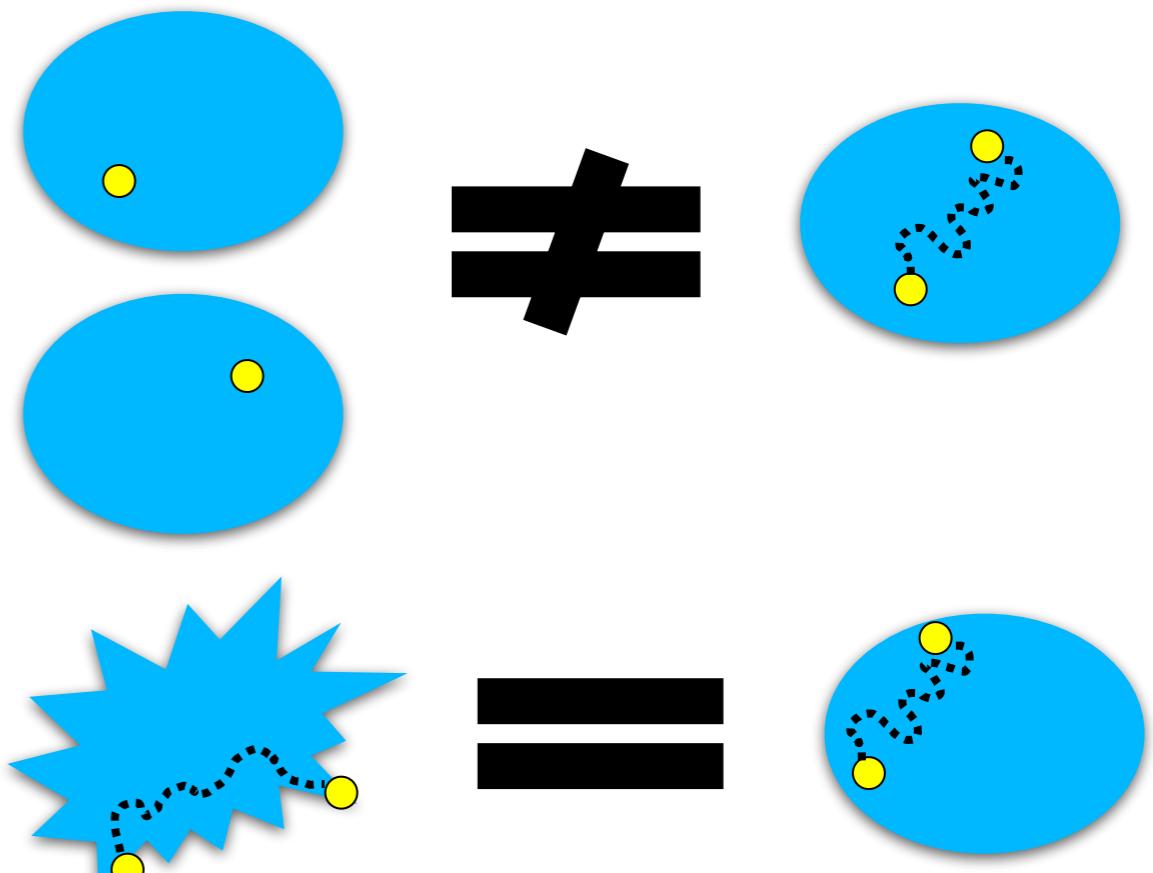


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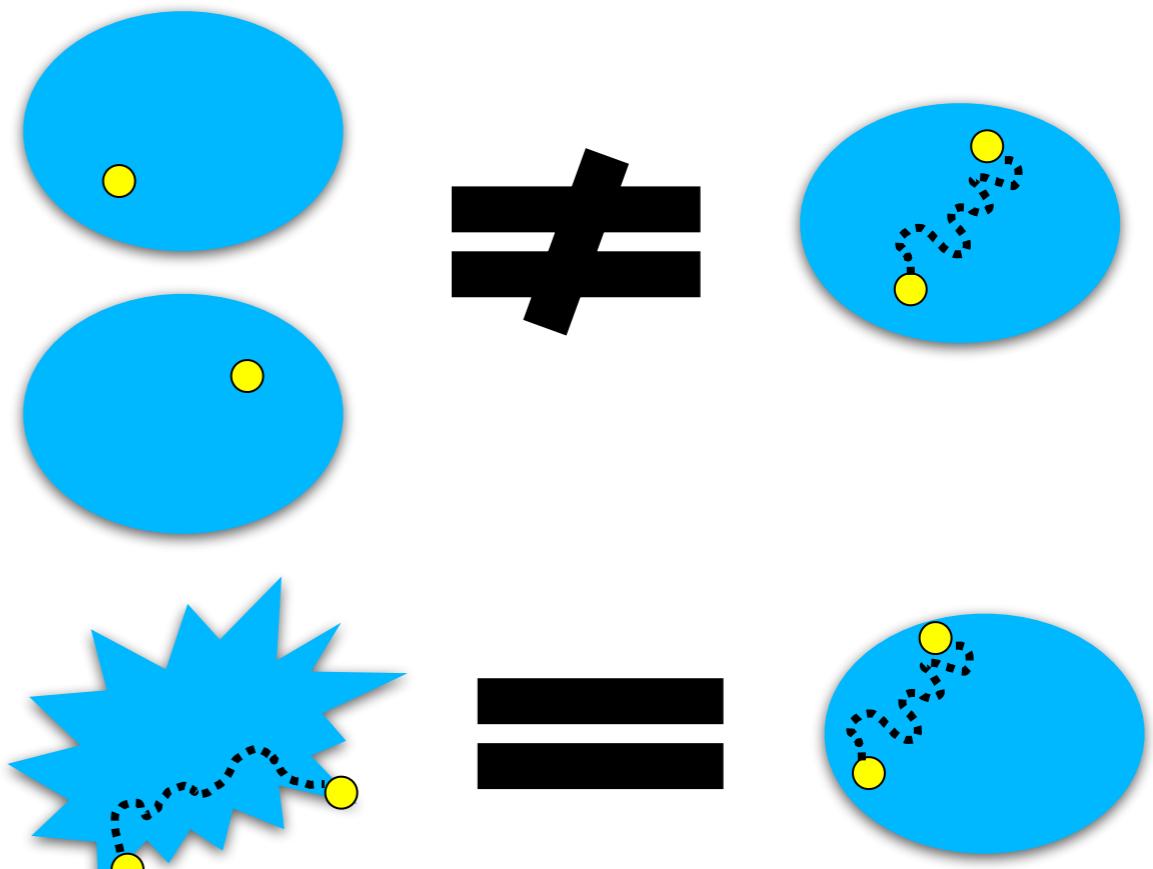


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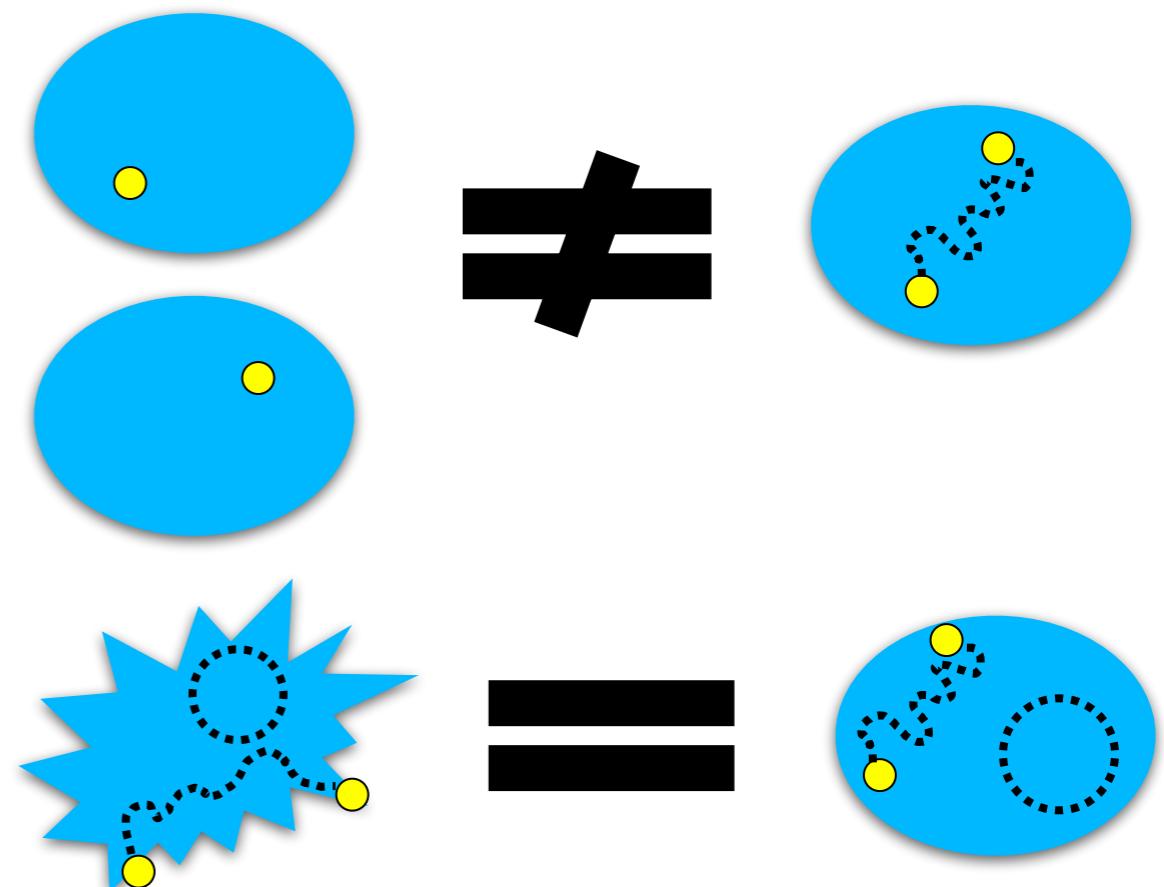
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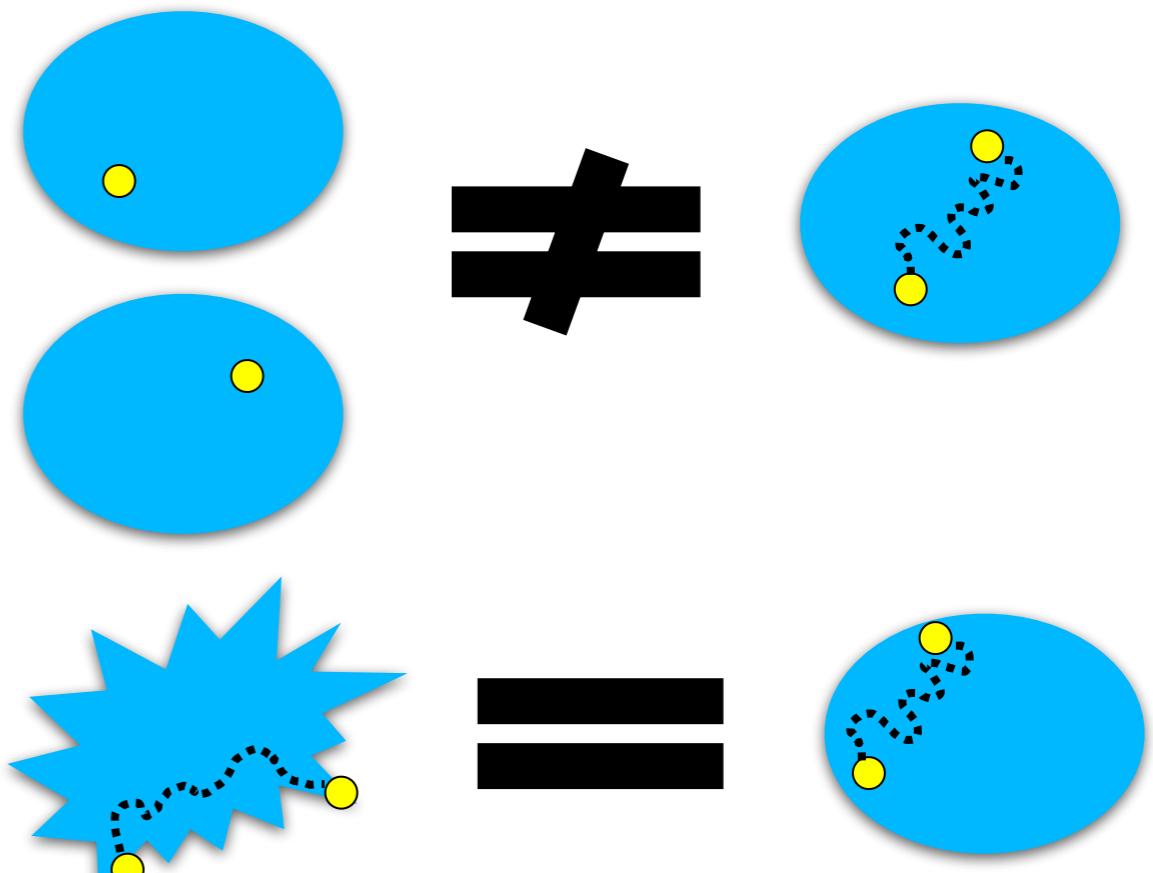
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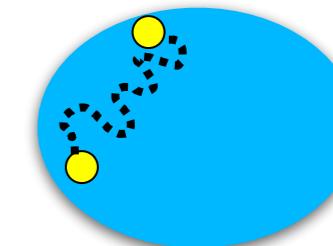
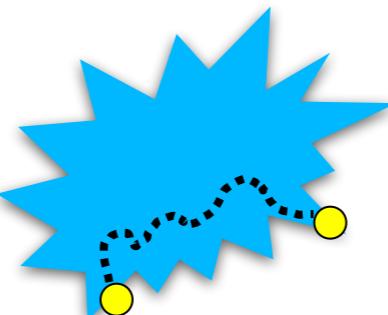
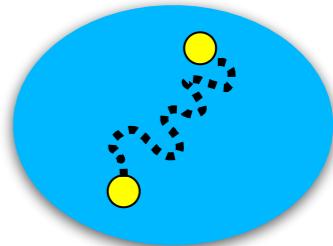
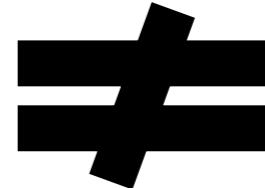
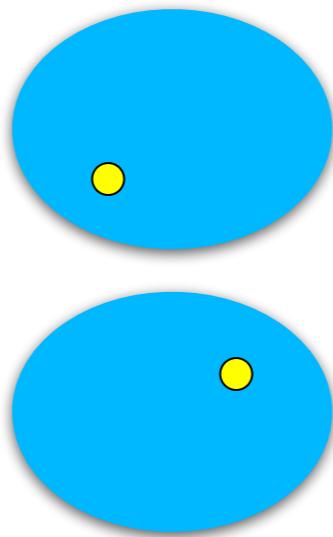


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- Broadly, TDA refers to a family of techniques that study the **shape** of data
 - Helps to understand relationships between how data varies and the features it exhibits



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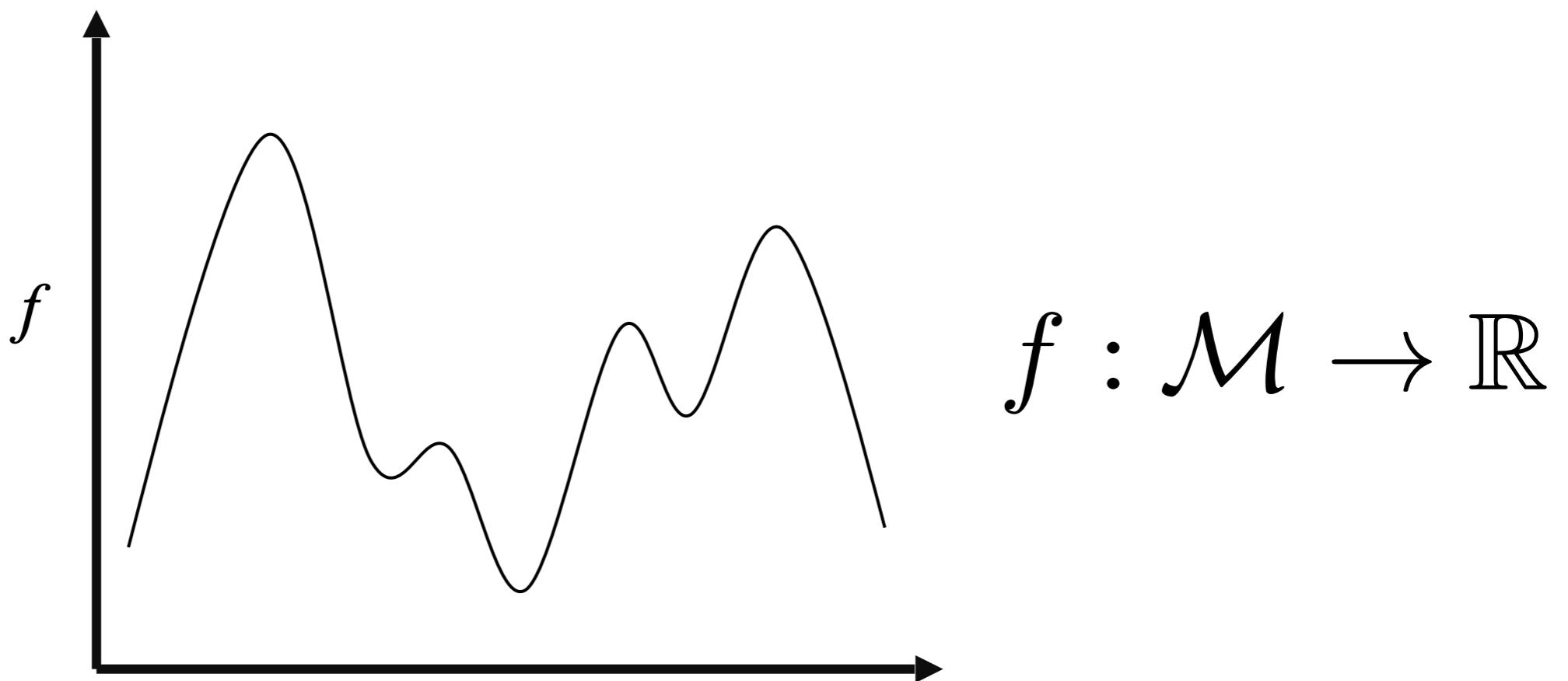


More General Question: What are Features in Field Data?

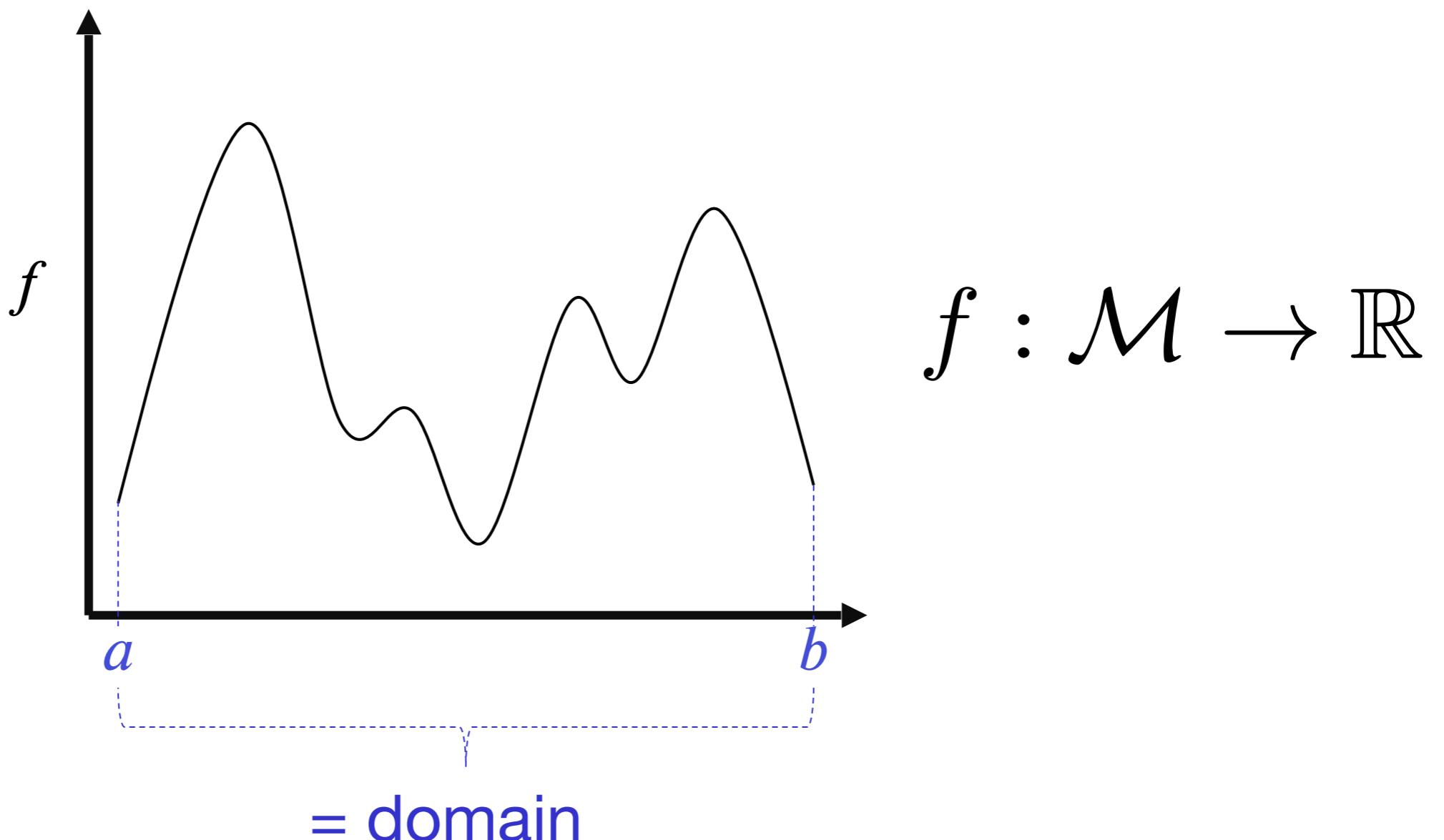
- Spatial Segmentation / Classification
- “Interesting” positions, neighborhoods, curves, surfaces, etc.
- How can this help visualization?
 - Compute features, and then augment a visualize with them to aid the user.

Background Mathematics

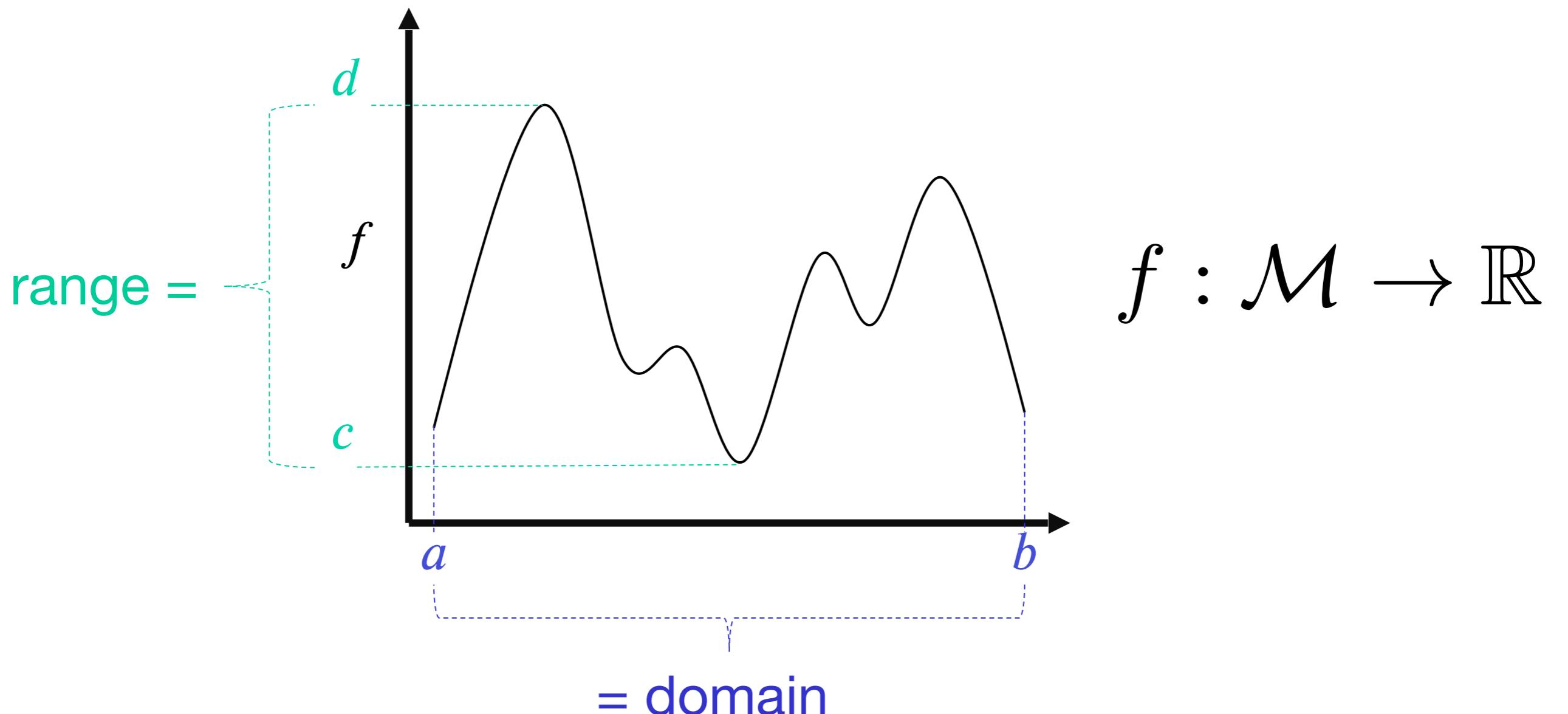
Scalar Field Topology Understood through Filtrations



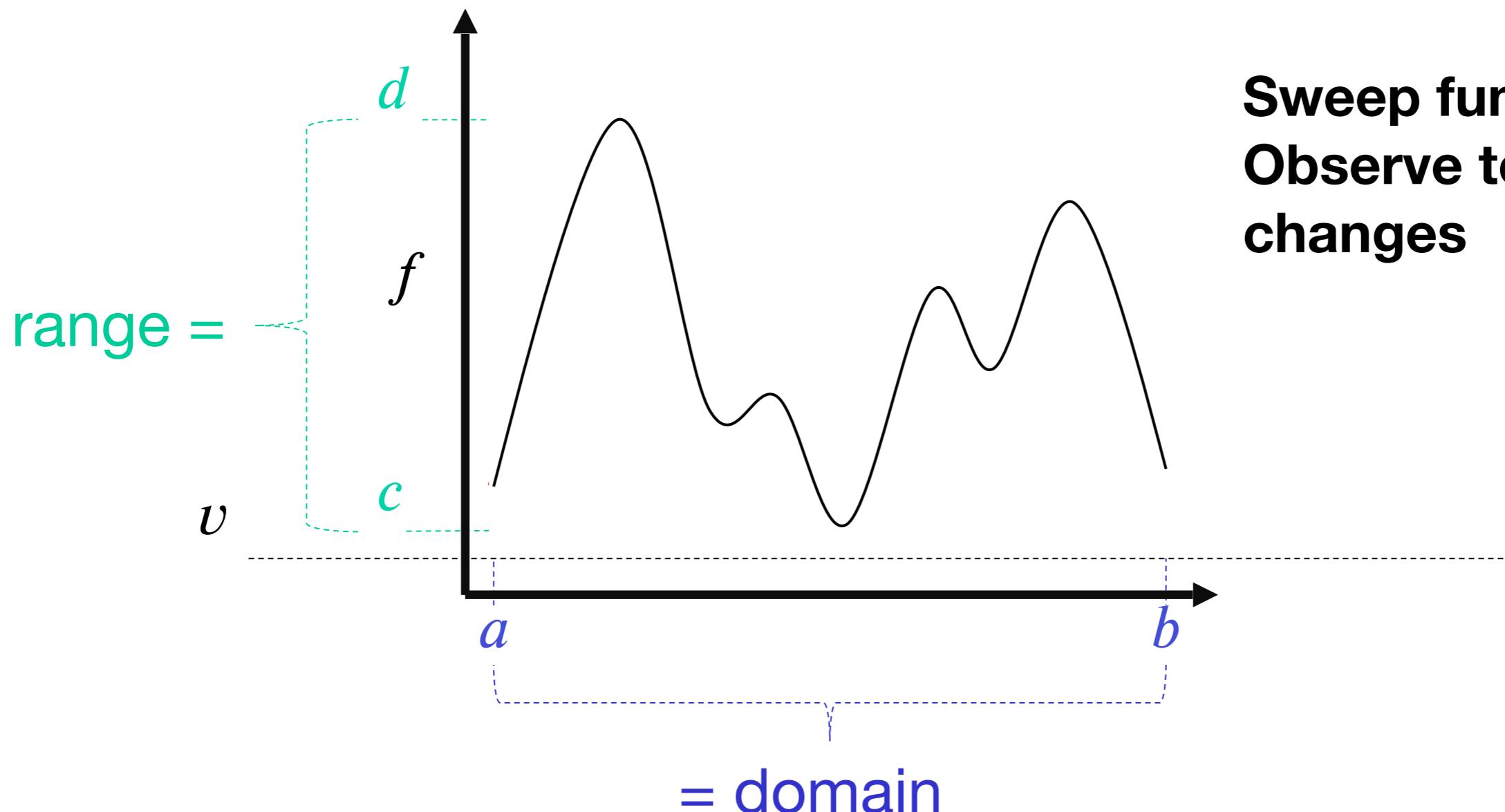
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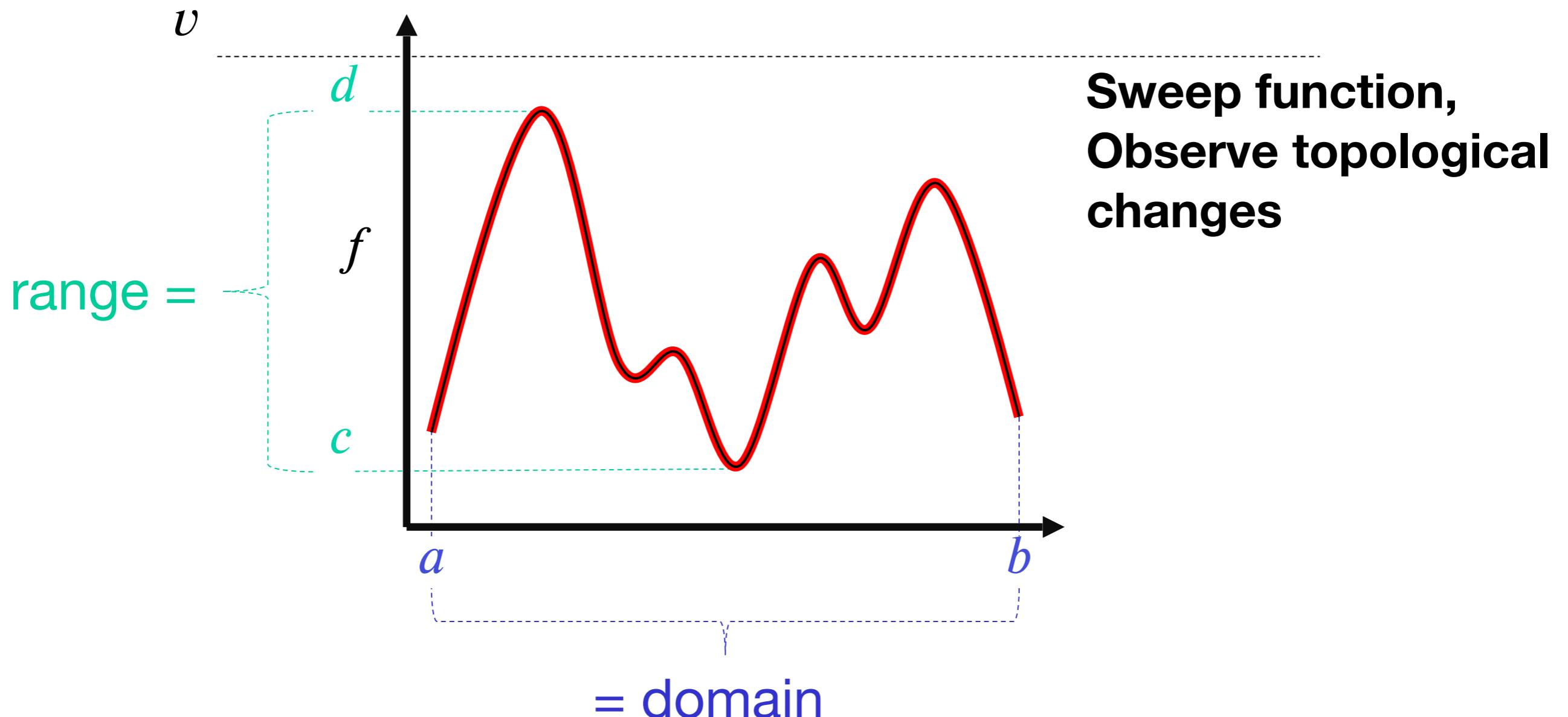
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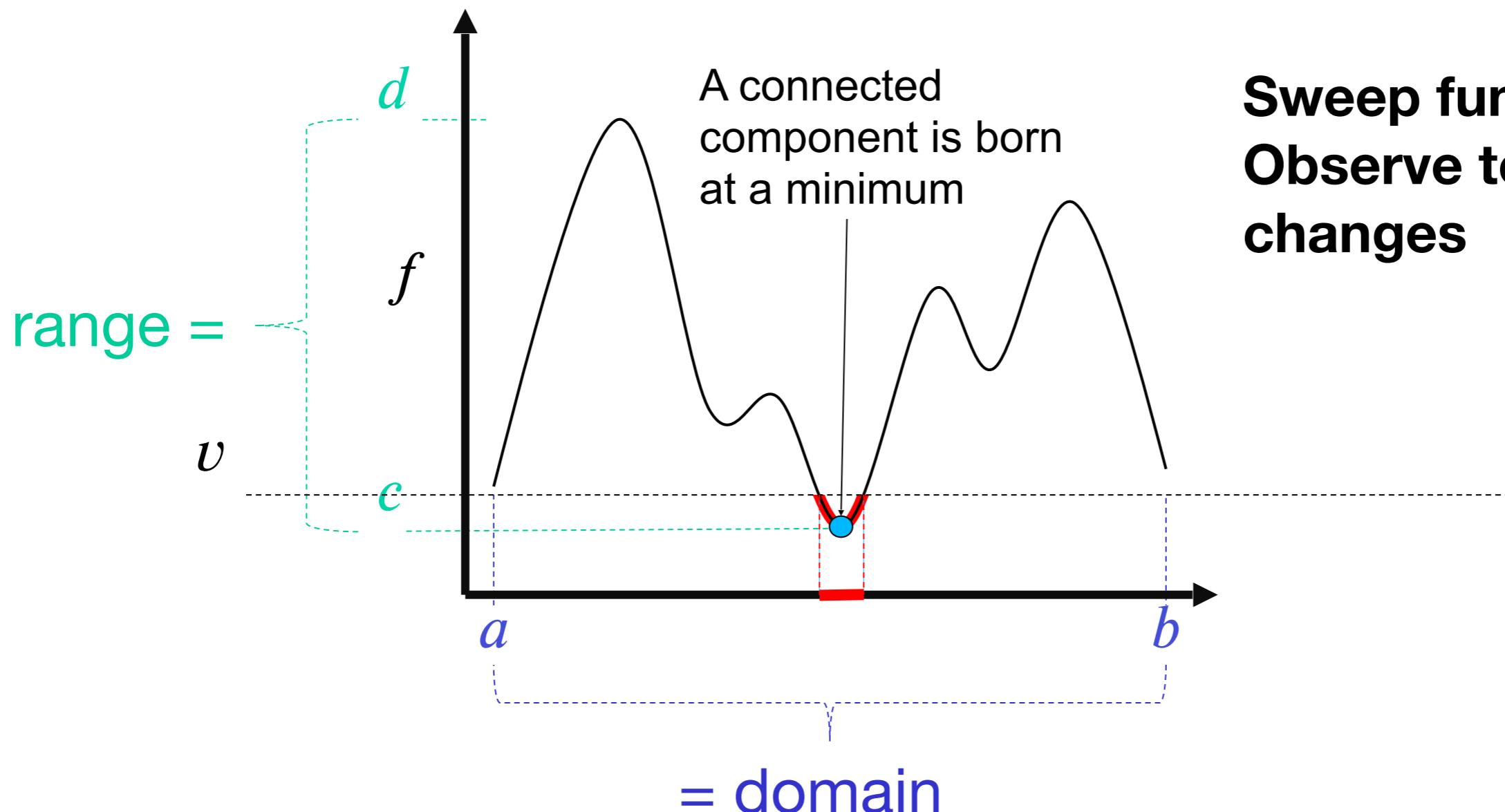
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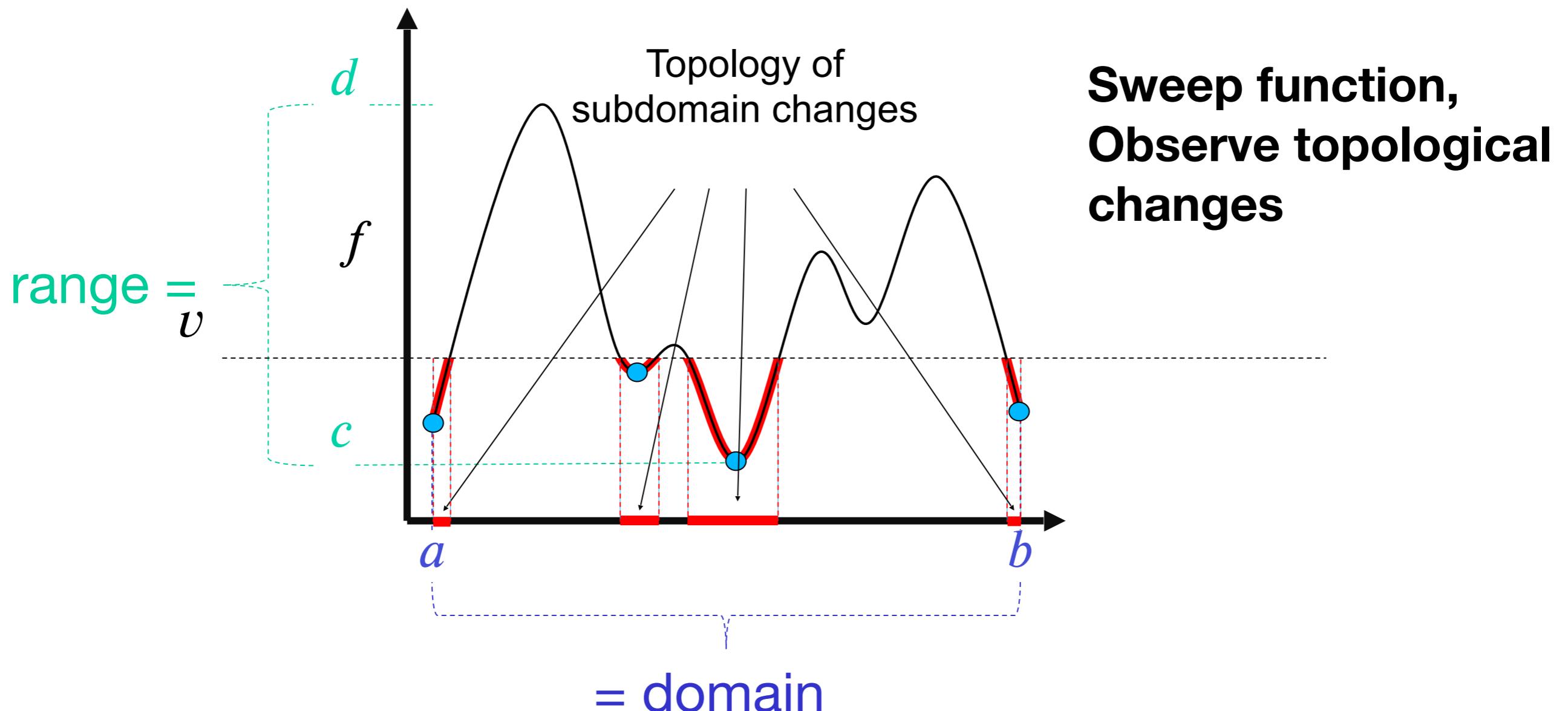
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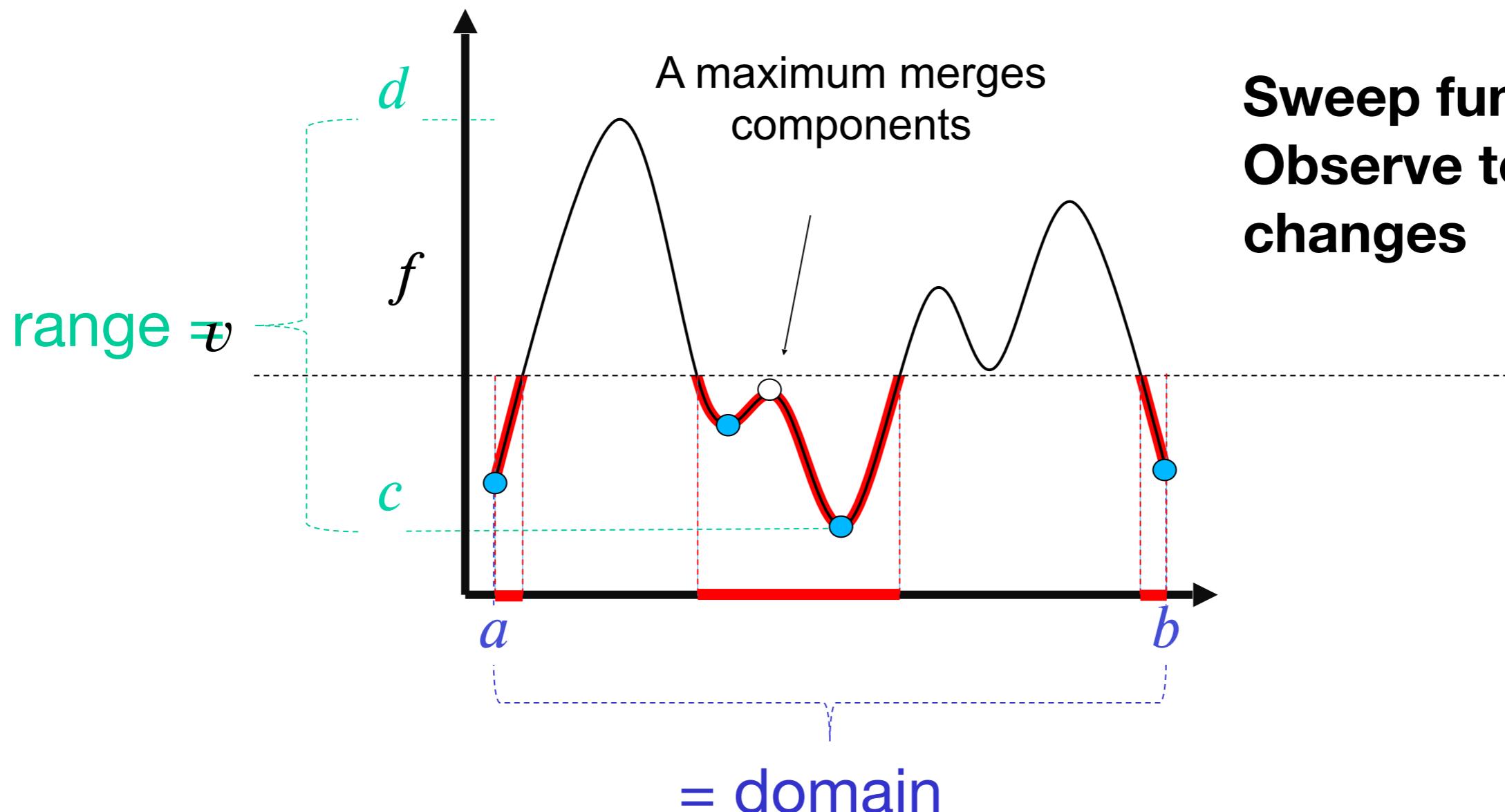
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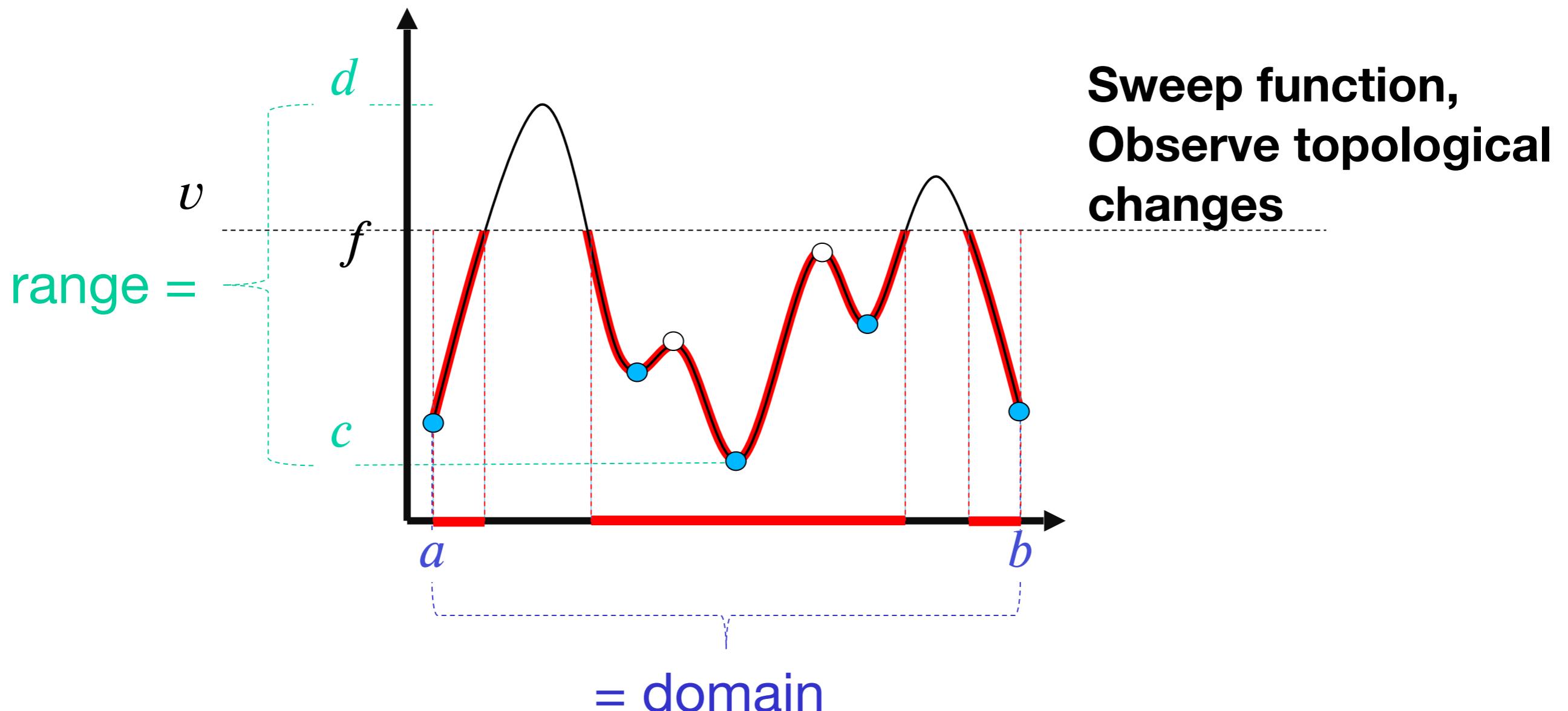


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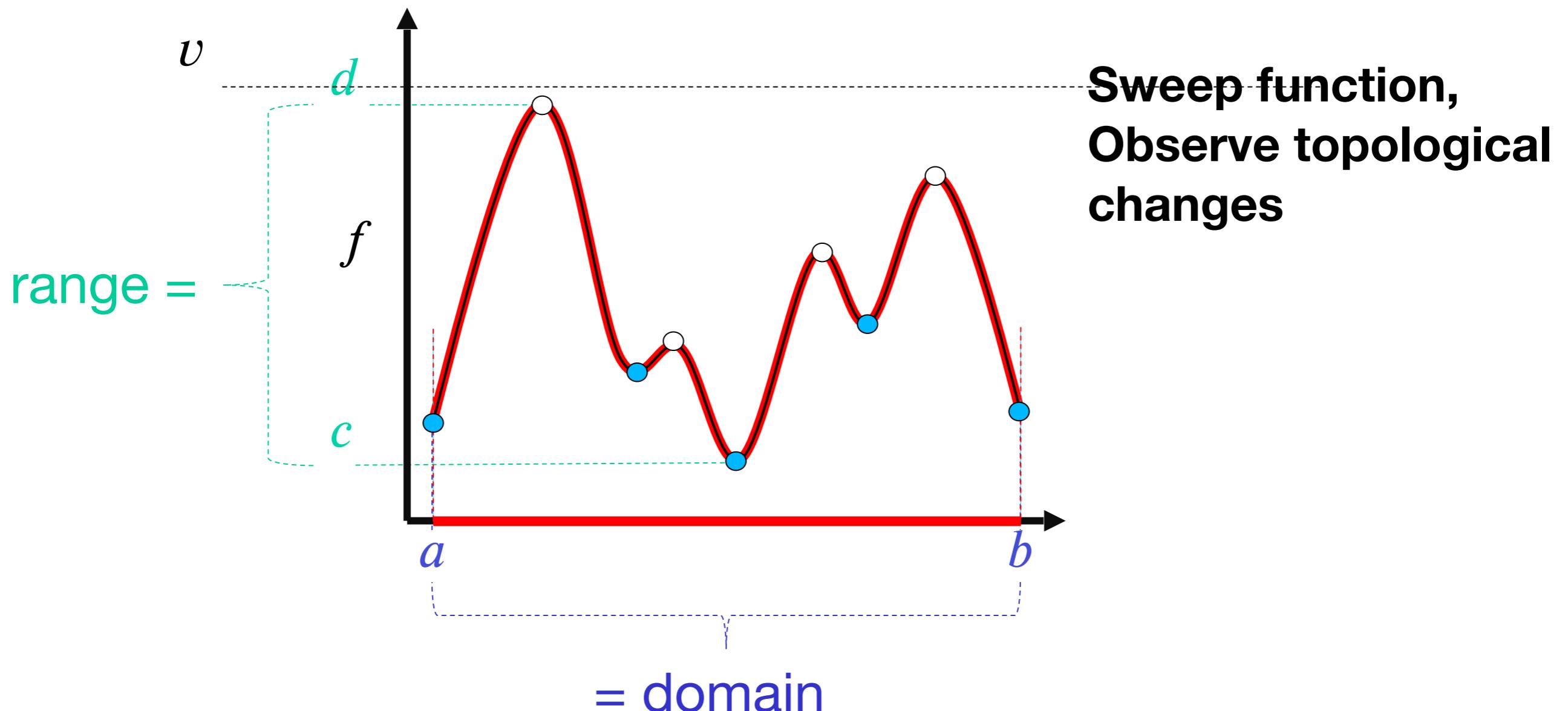


Slide credit Attila Gyulassy

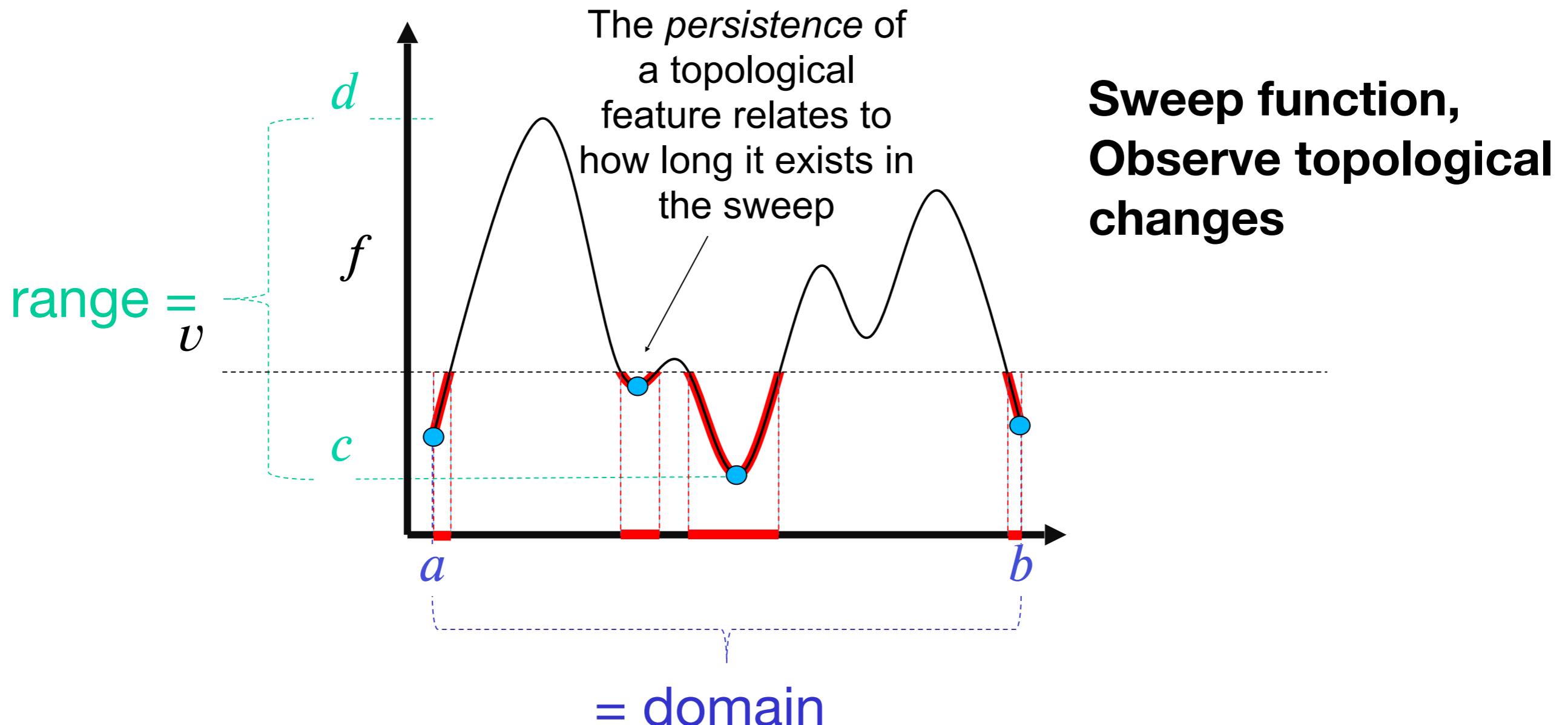
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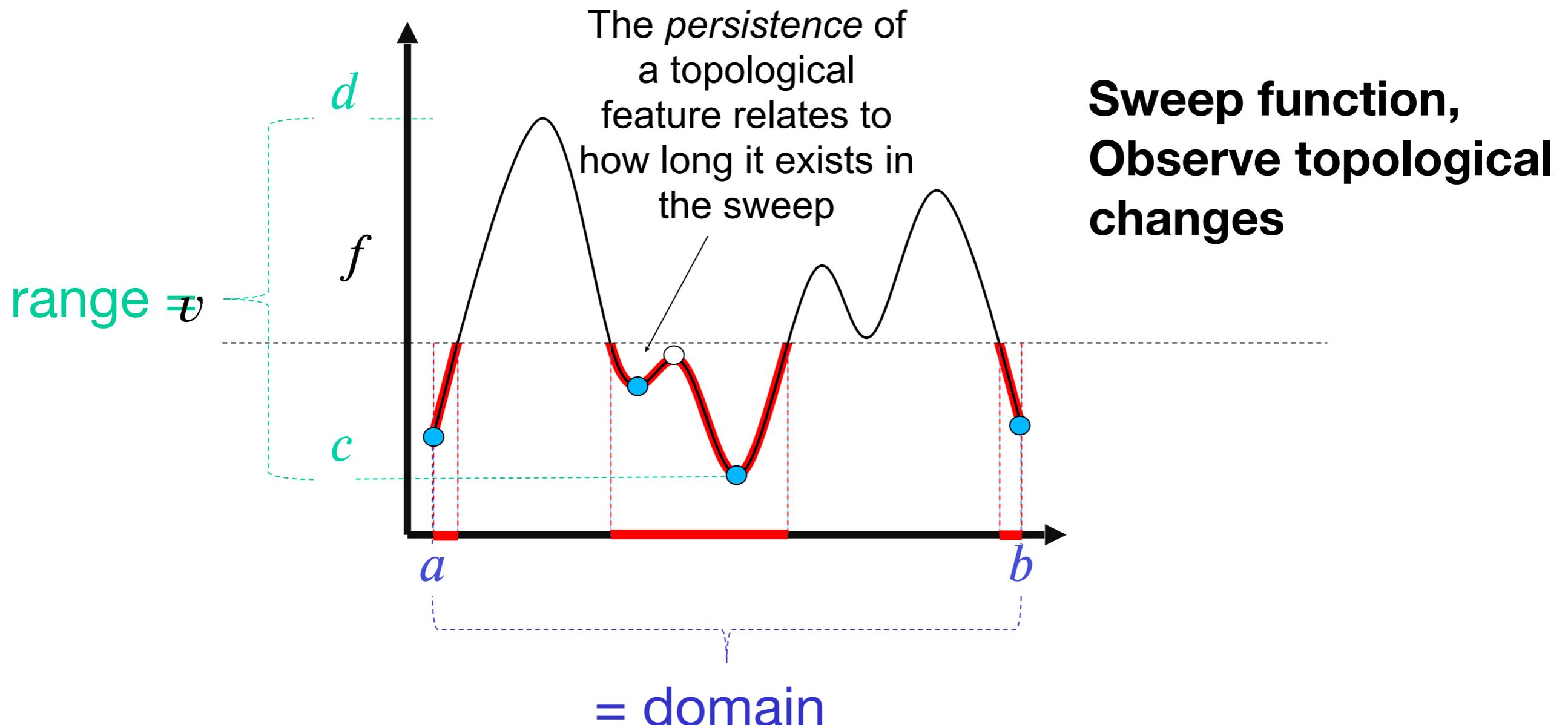
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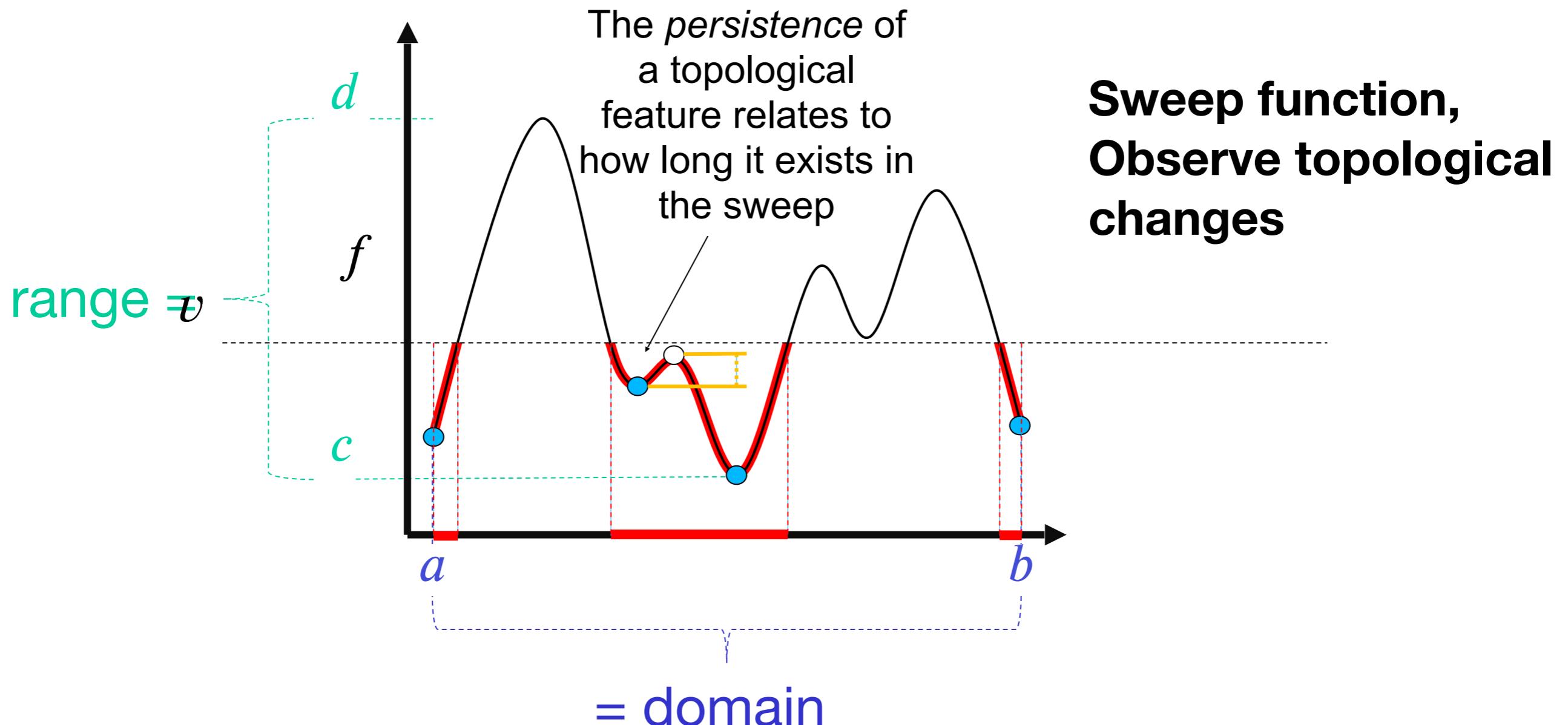
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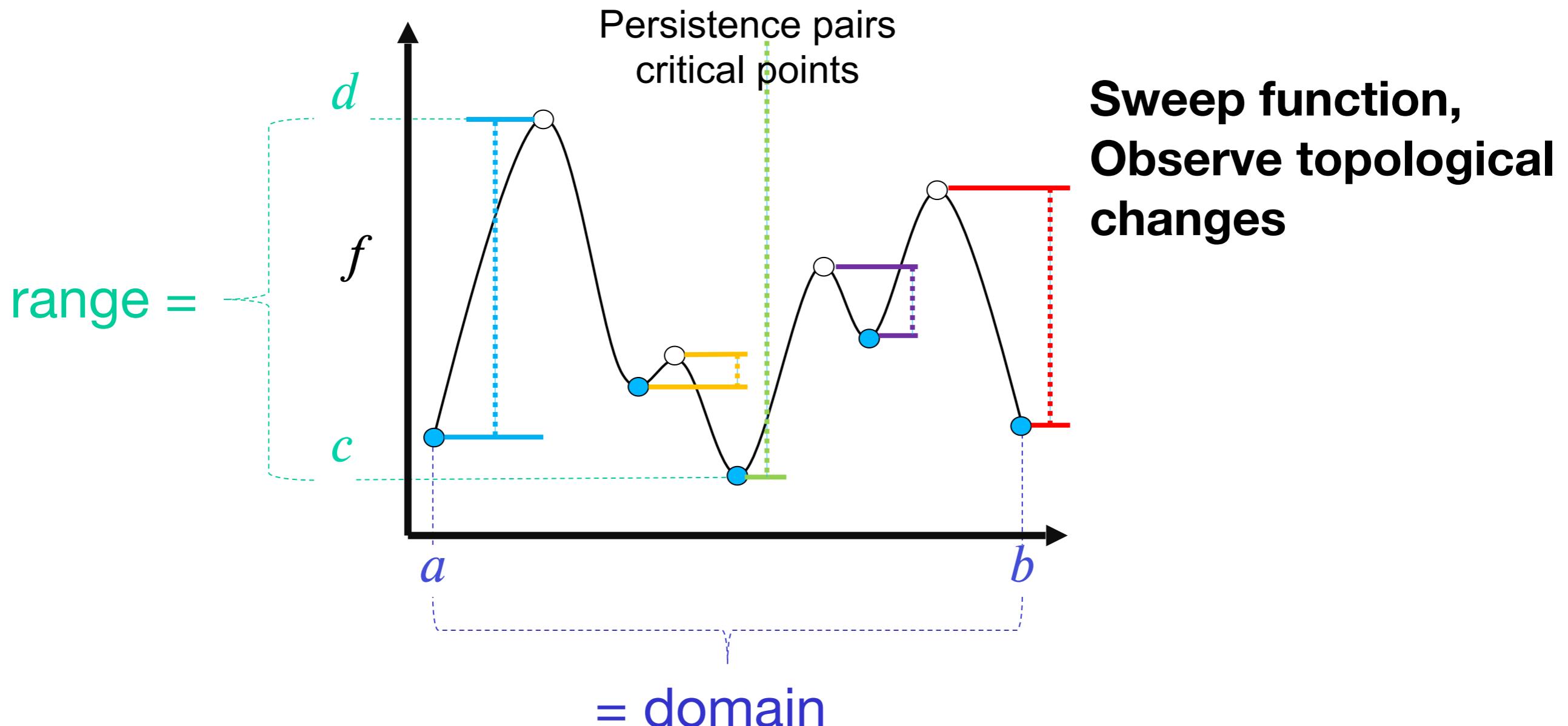
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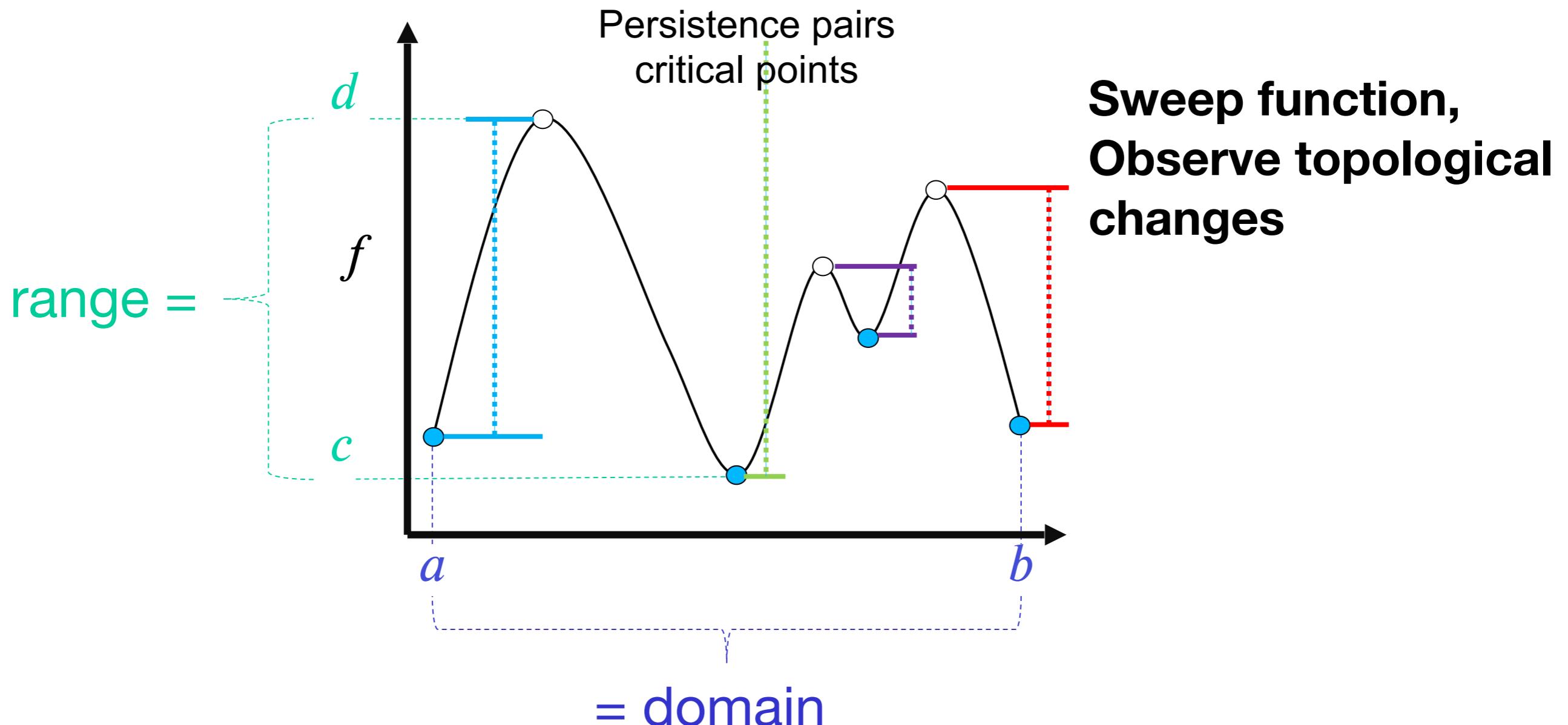
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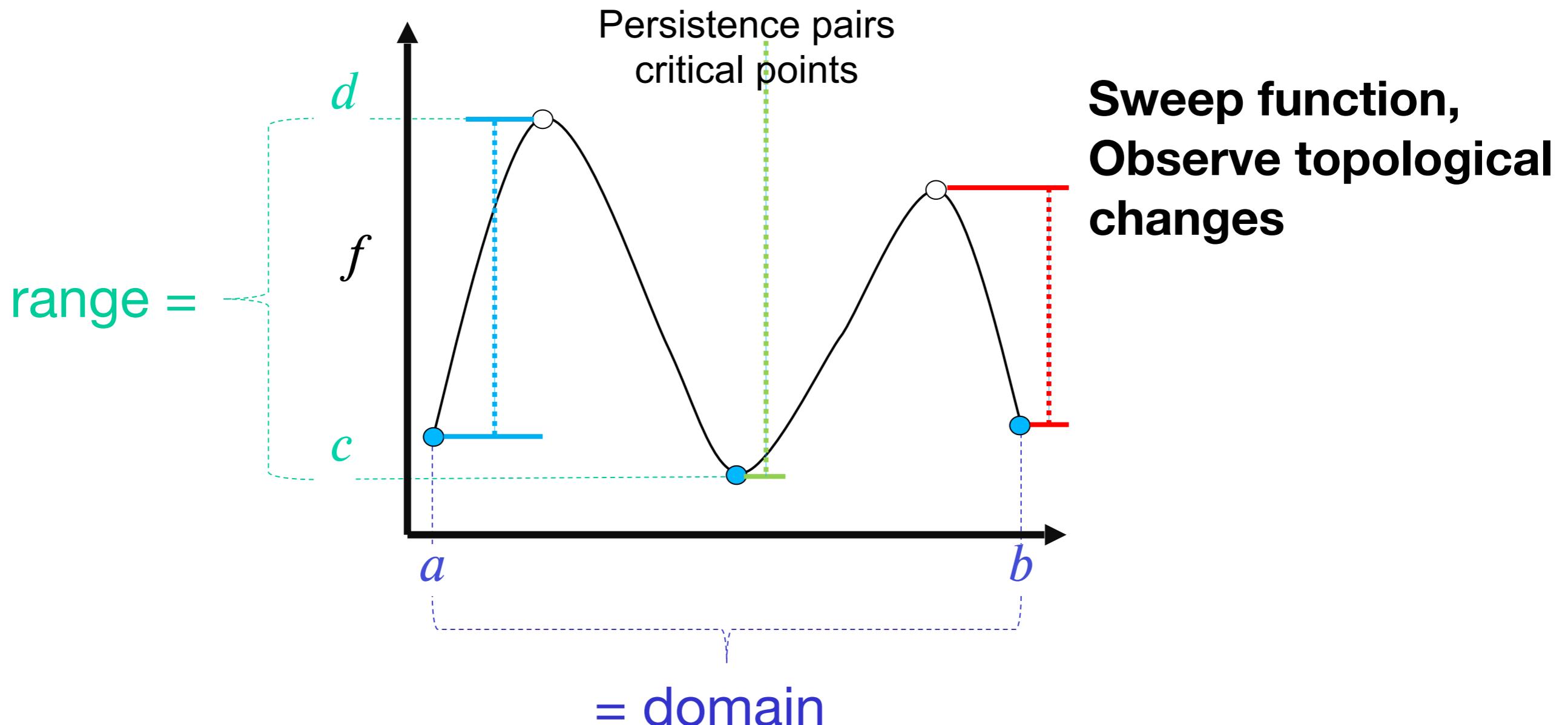
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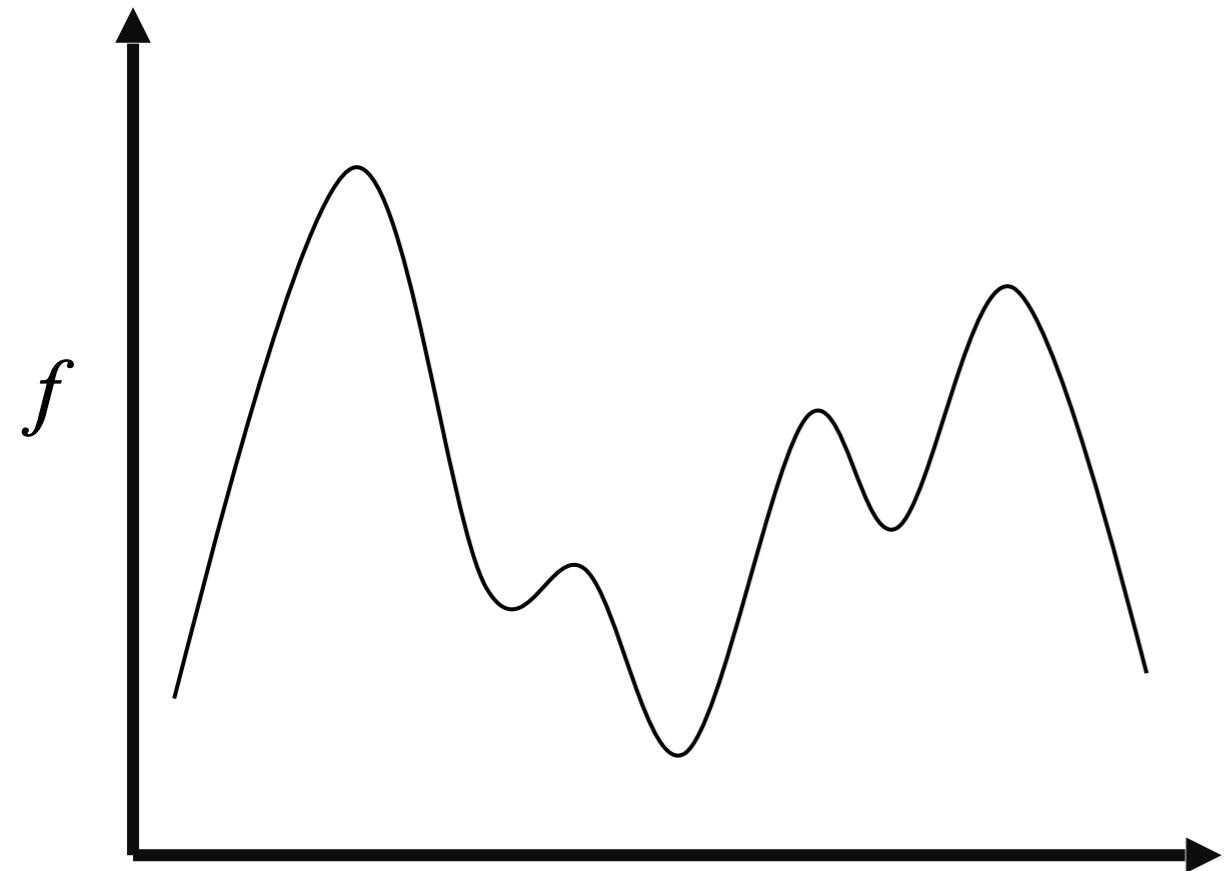
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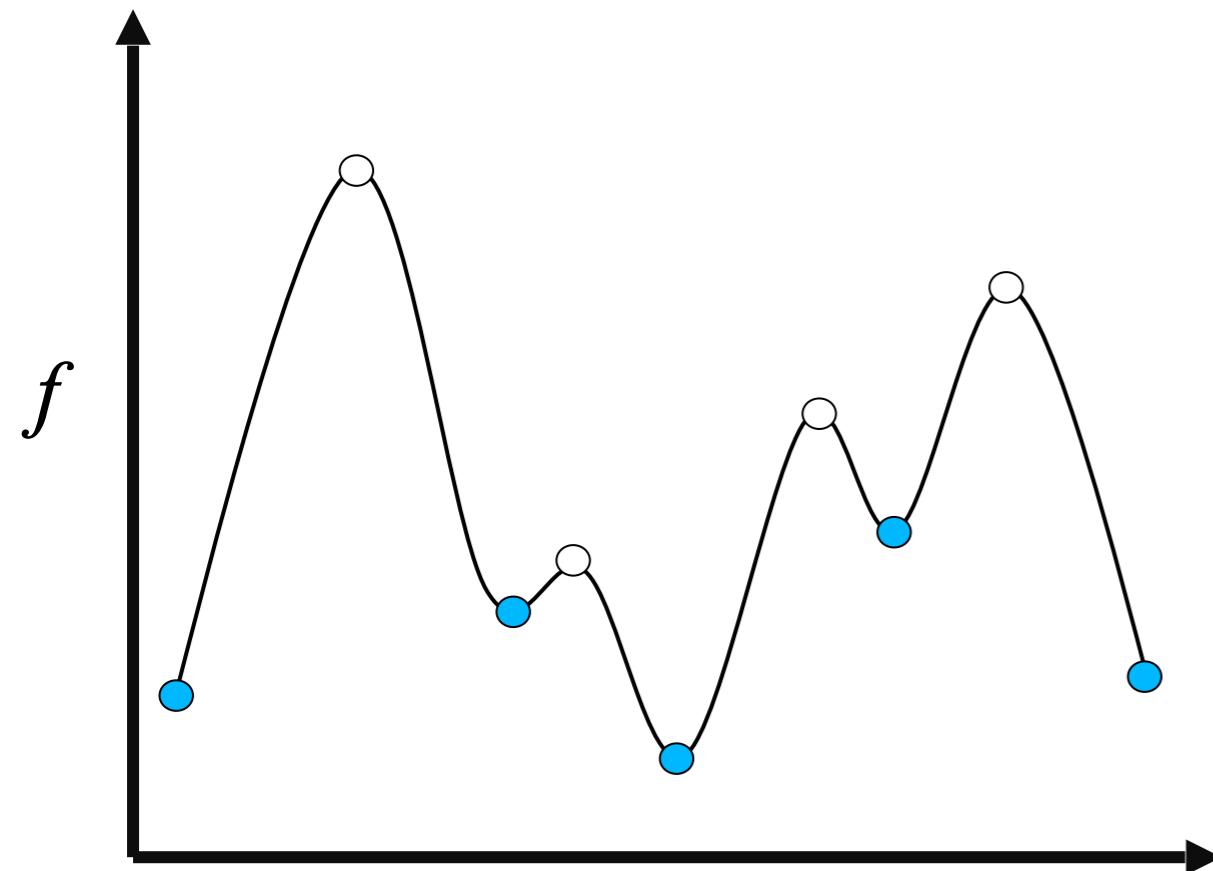
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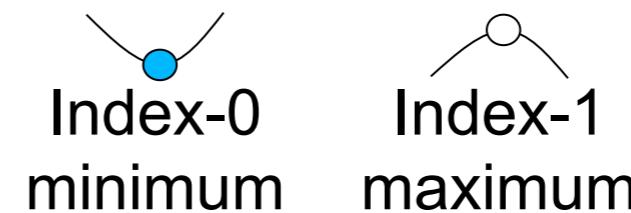
Features of a 1-D Scalar Field



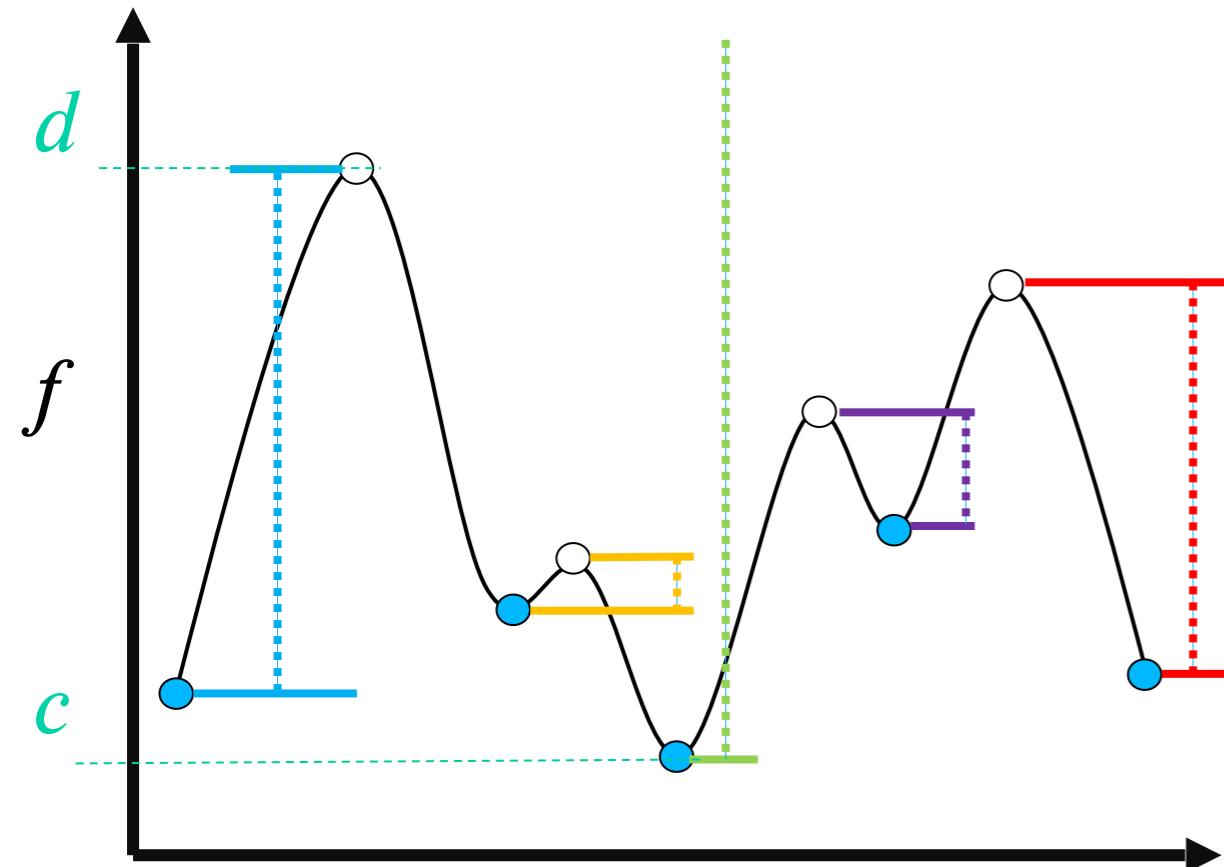
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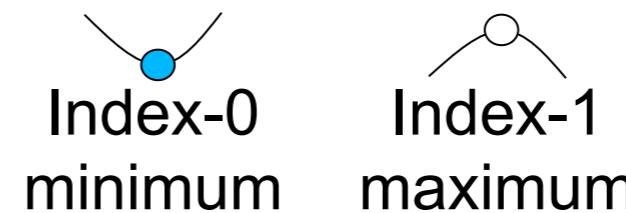
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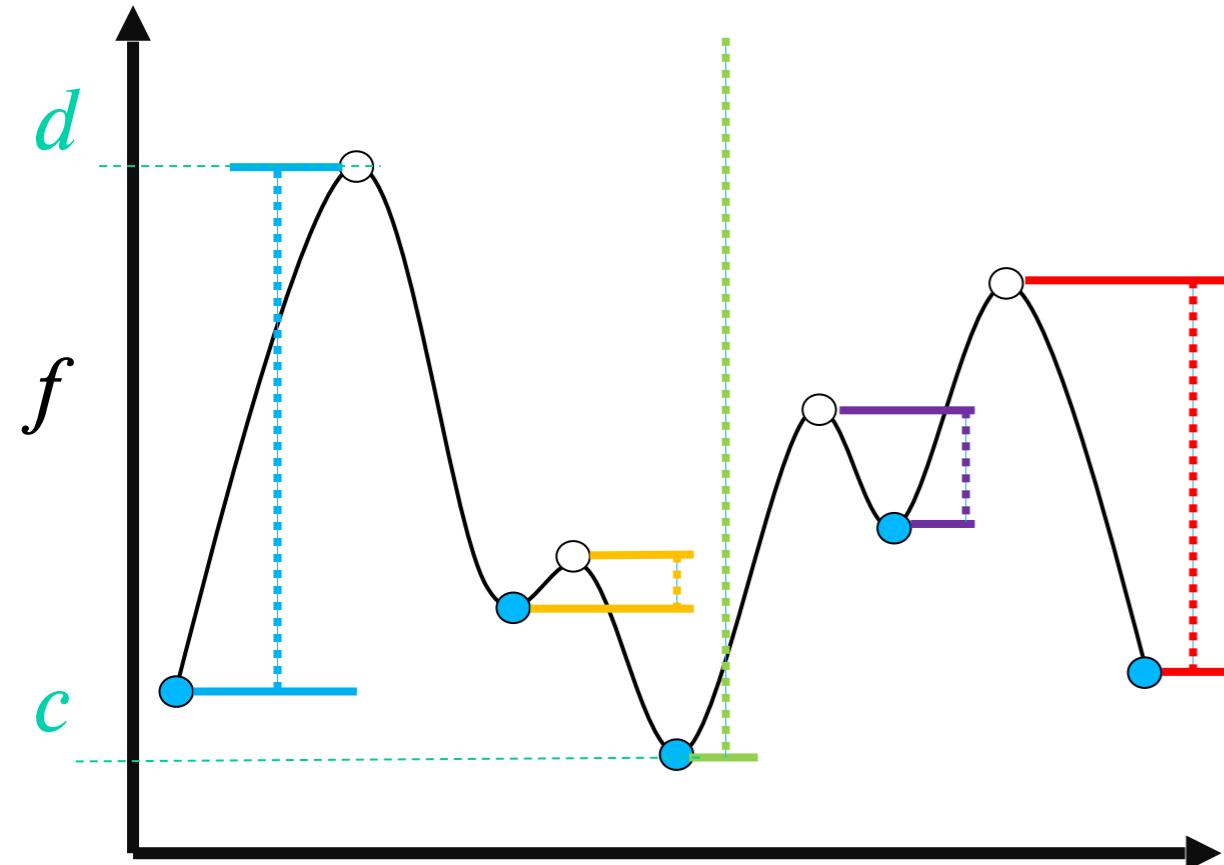
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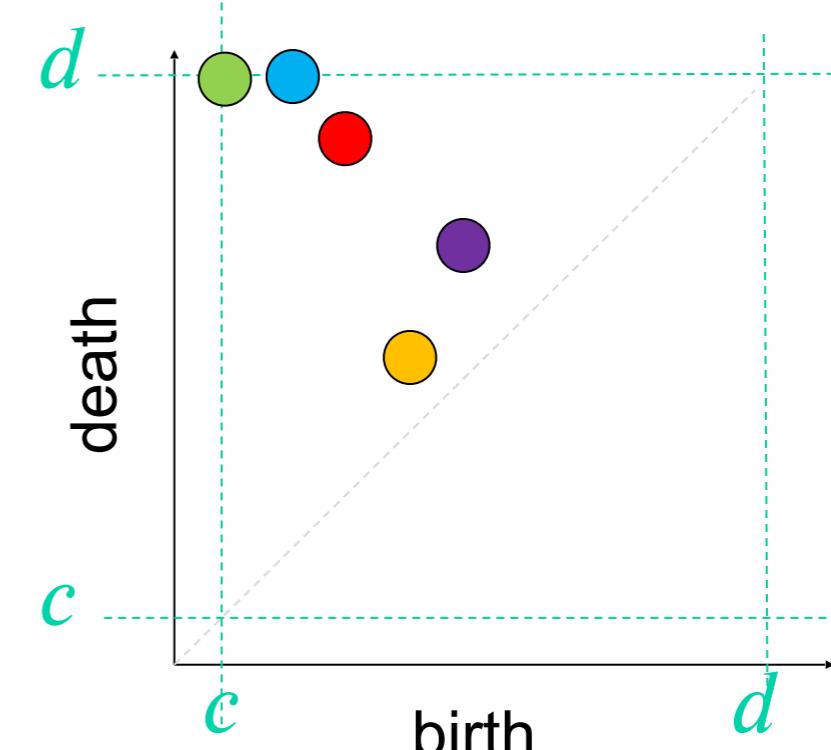
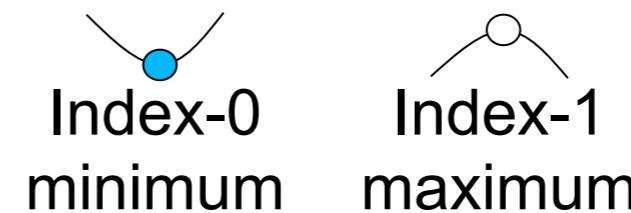
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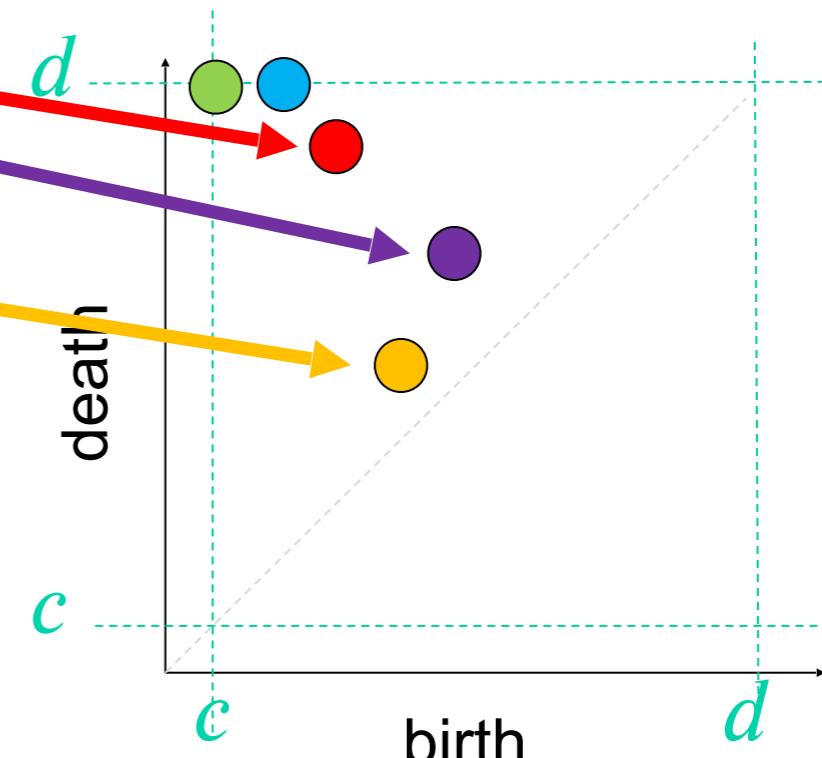
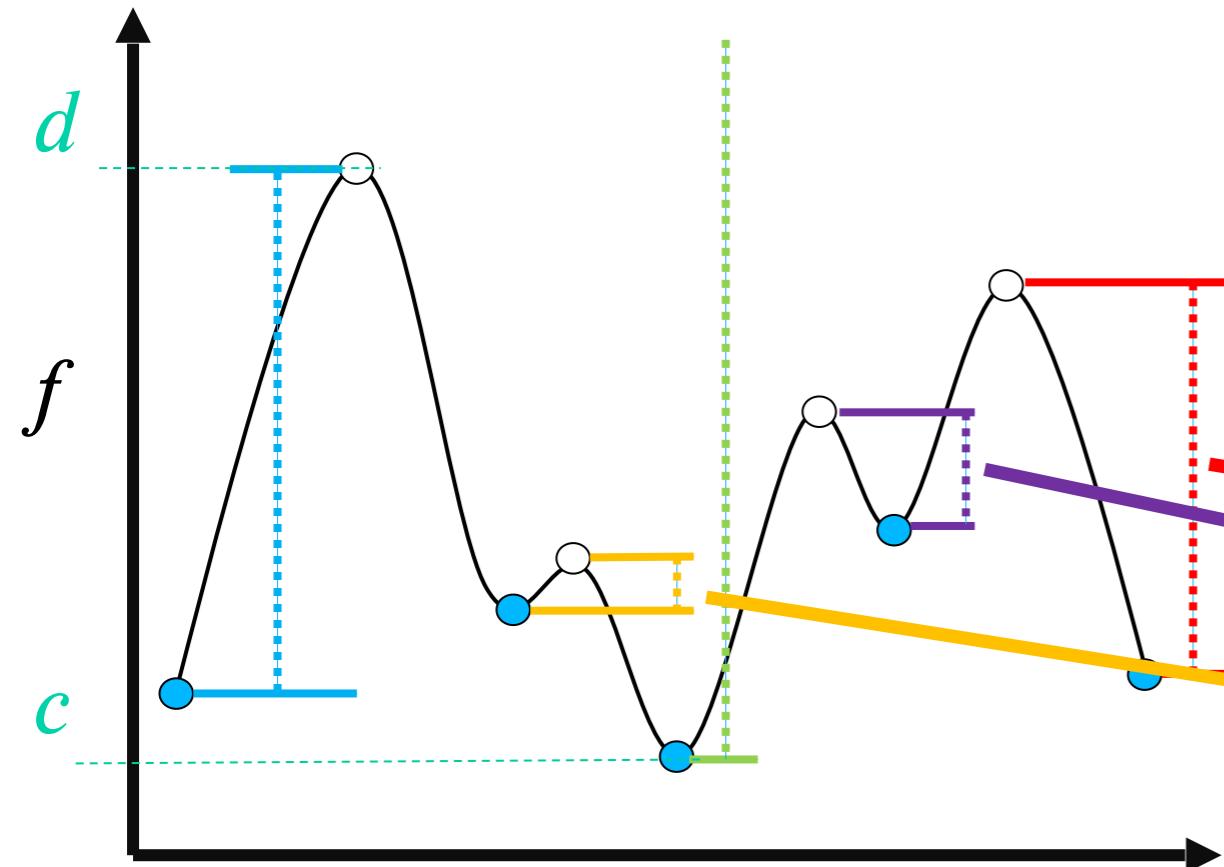
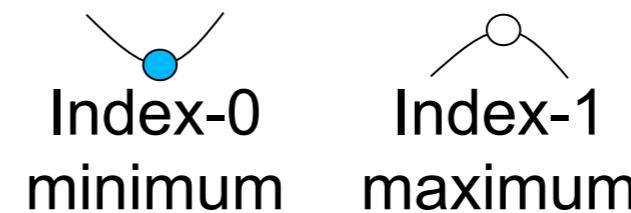
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Persistence diagram

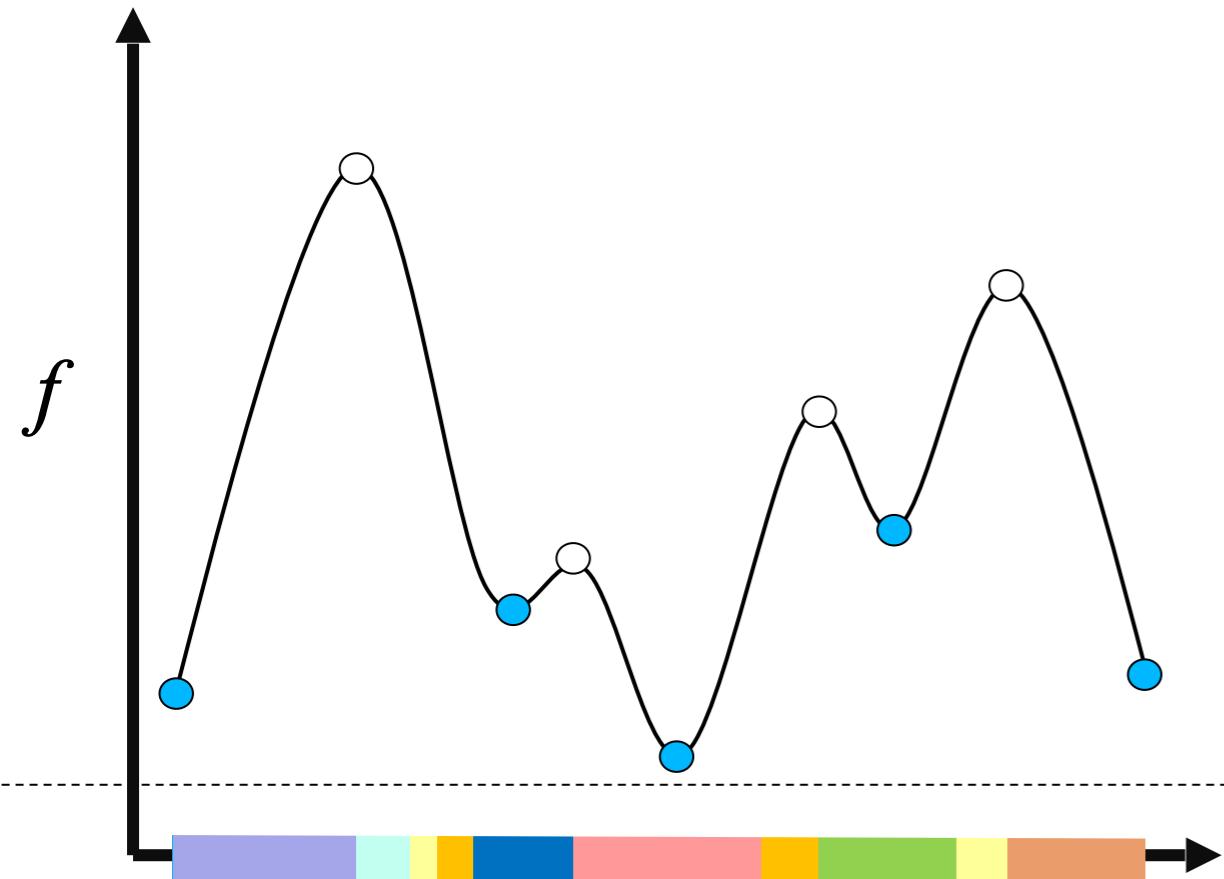
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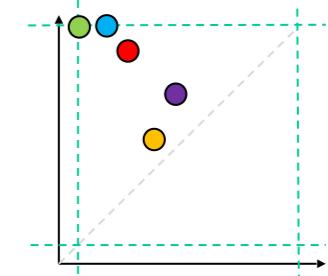
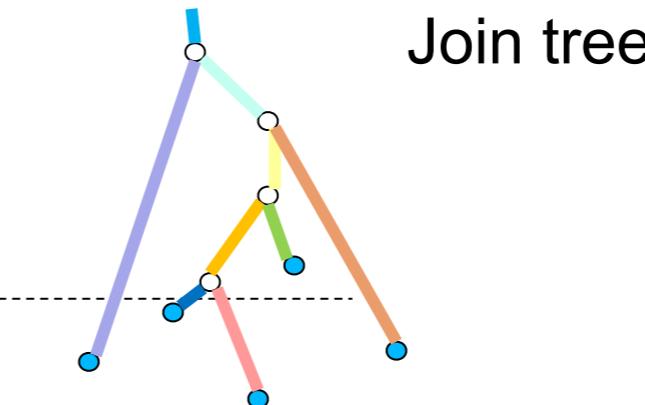


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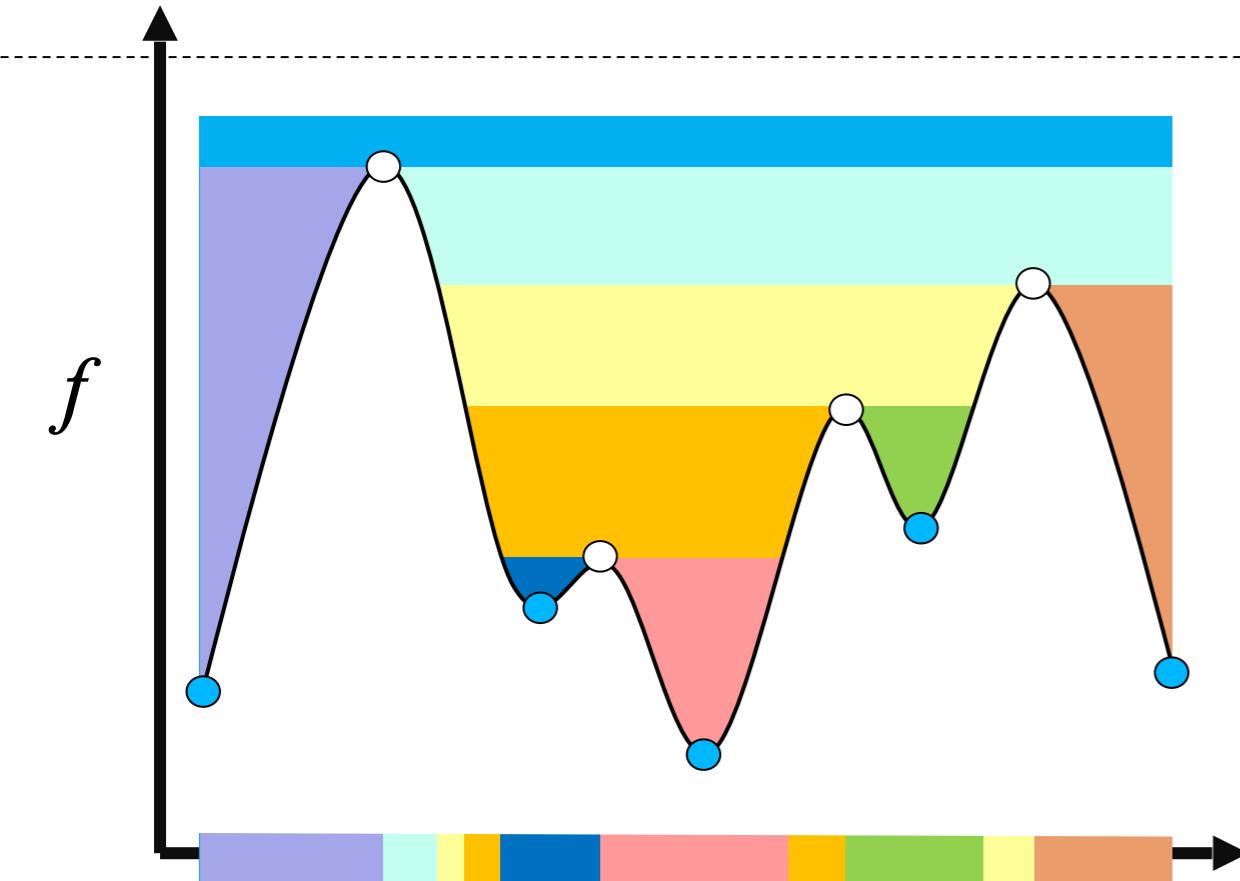
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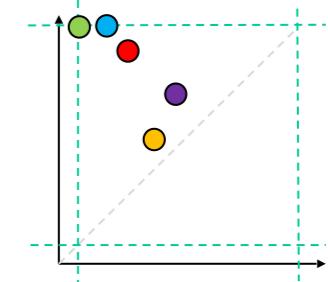
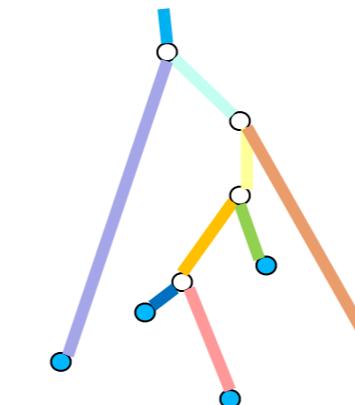
- Critical points where
 - Index-0 minimum
 - Index-1 maximum
- Components from the filtration



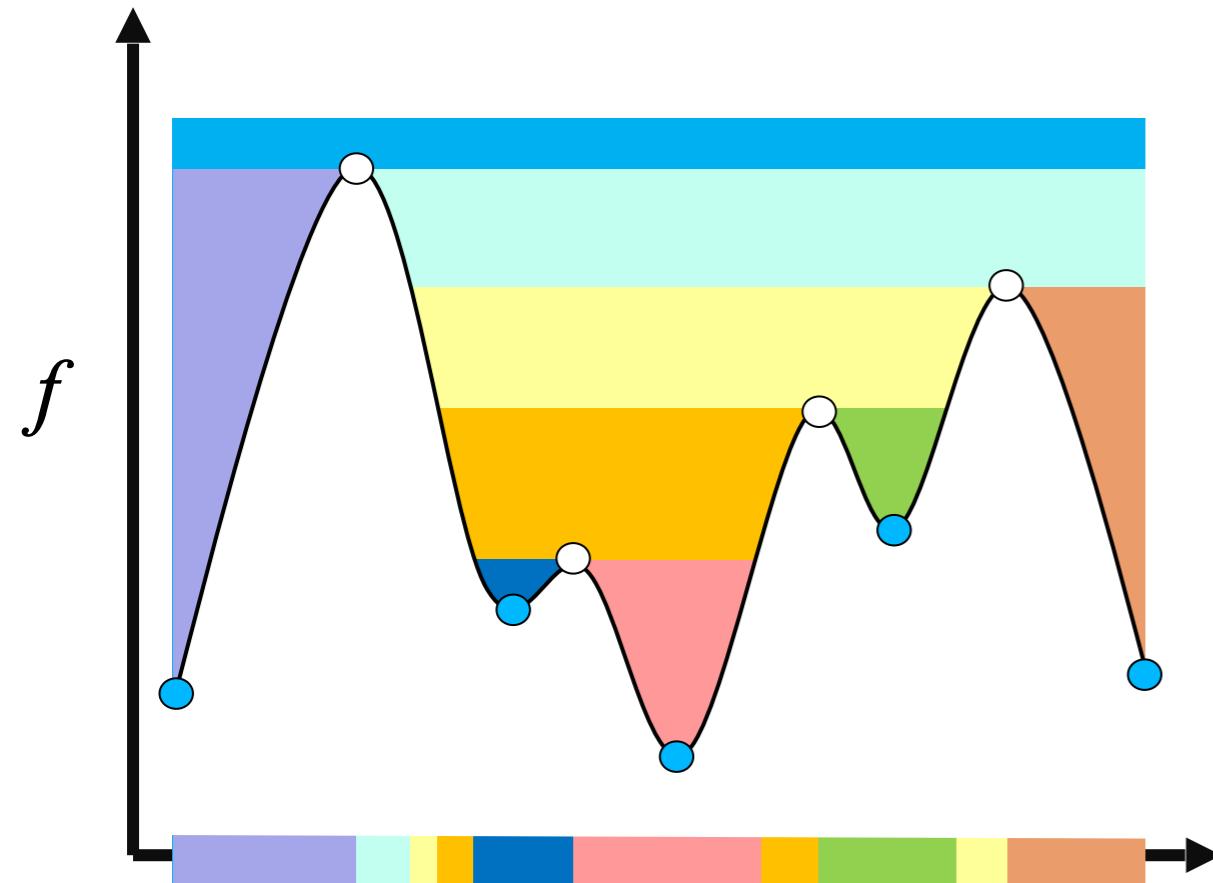
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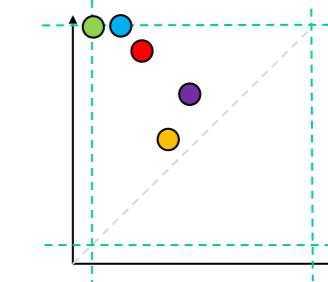
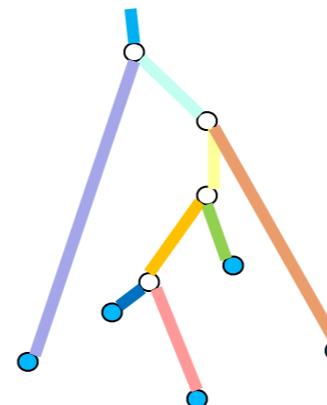
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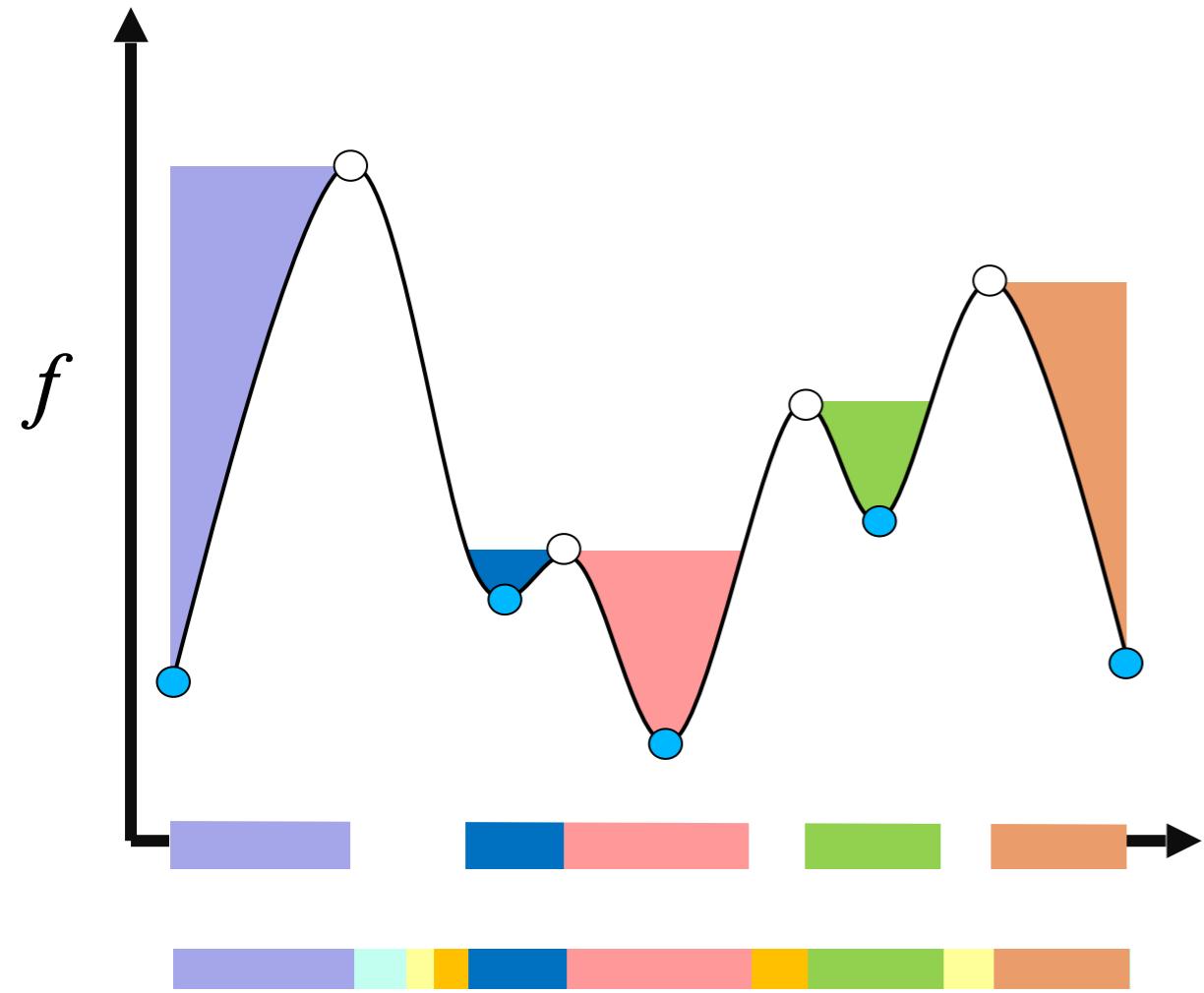
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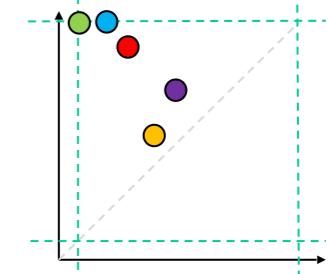
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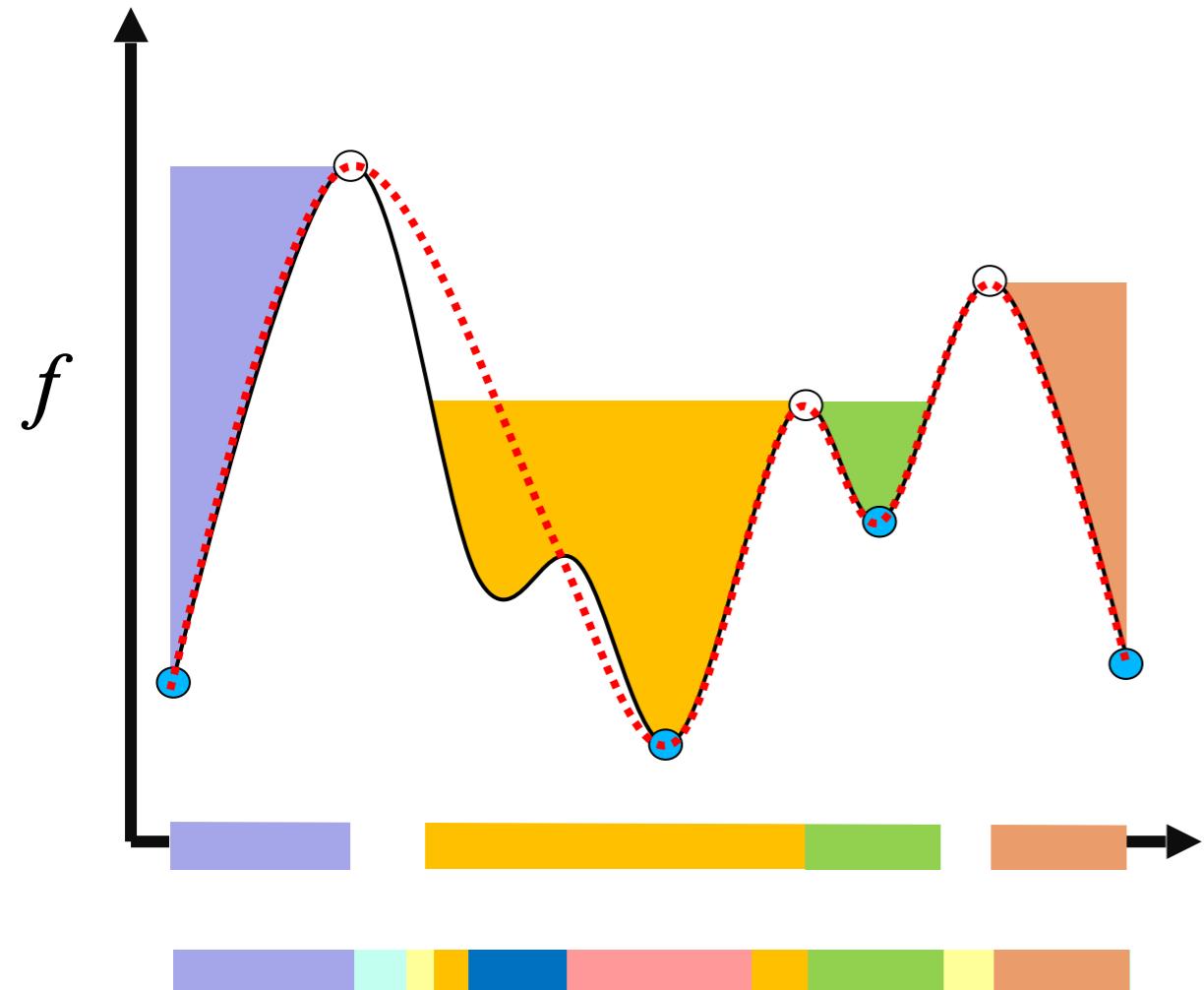
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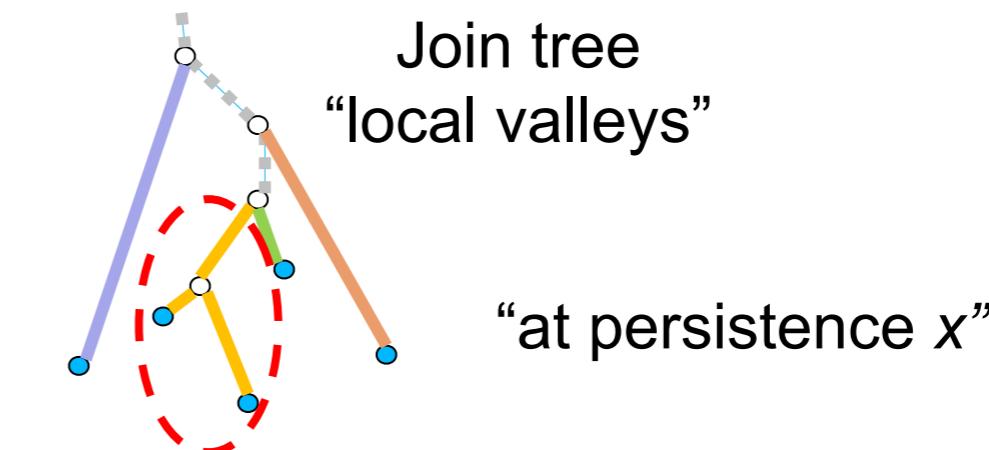
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 - Join tree
“local valleys”



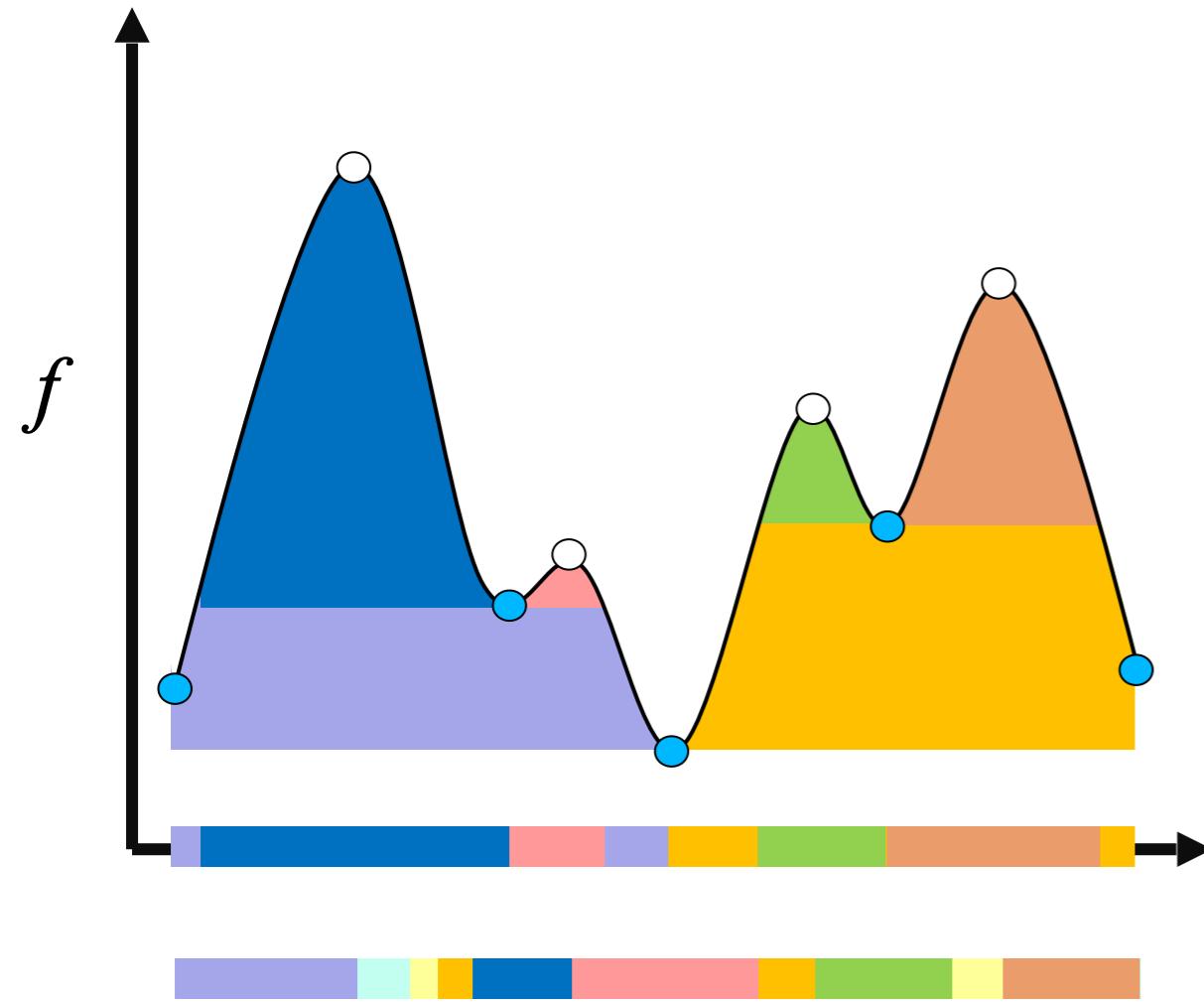
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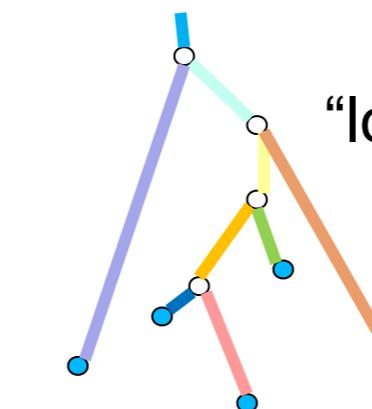
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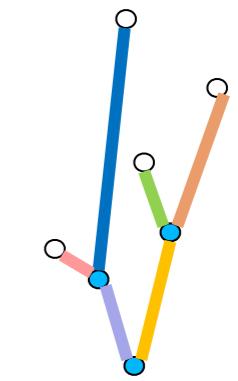


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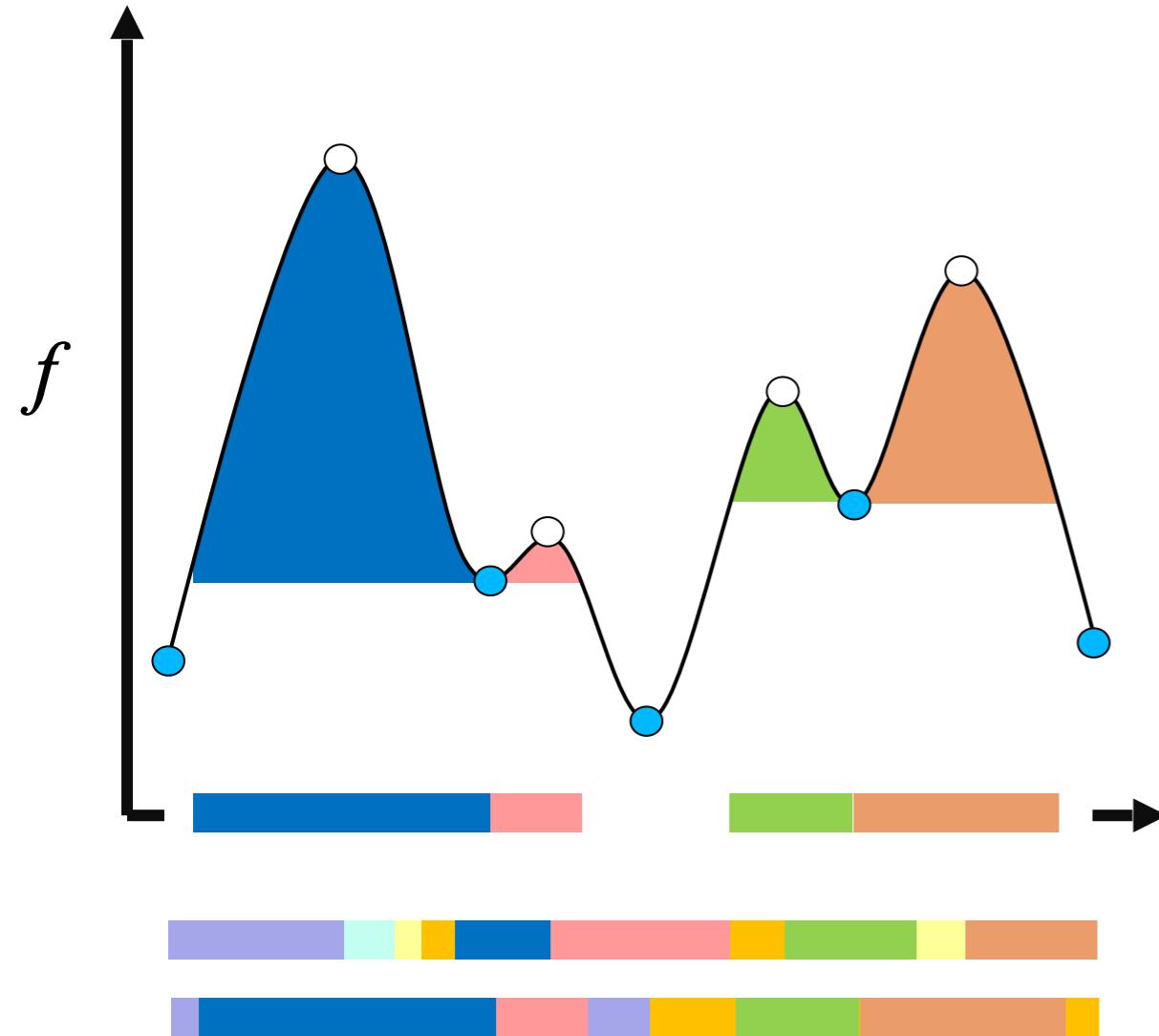


Join tree
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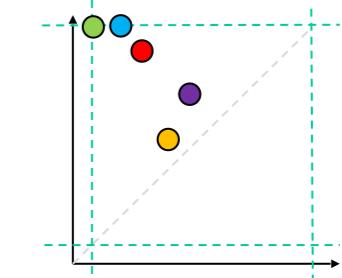
Split tree



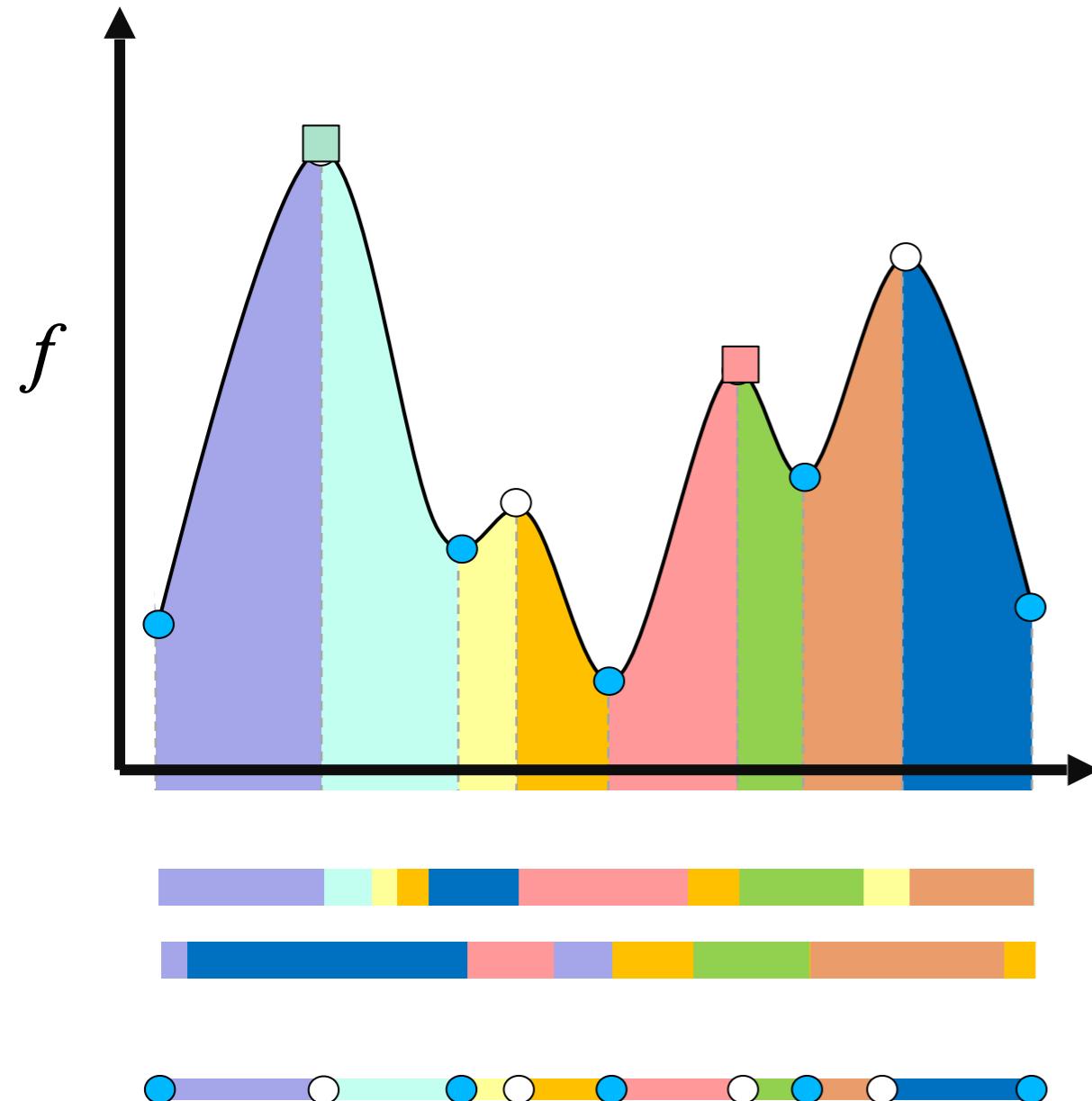
Features of a 1-D Scalar Field



- Critical points where
 - Index-0 minimum
 - Index-1 maximum
- Components from the filtration
 - Join tree “local valleys”
 - Split tree “local peaks”

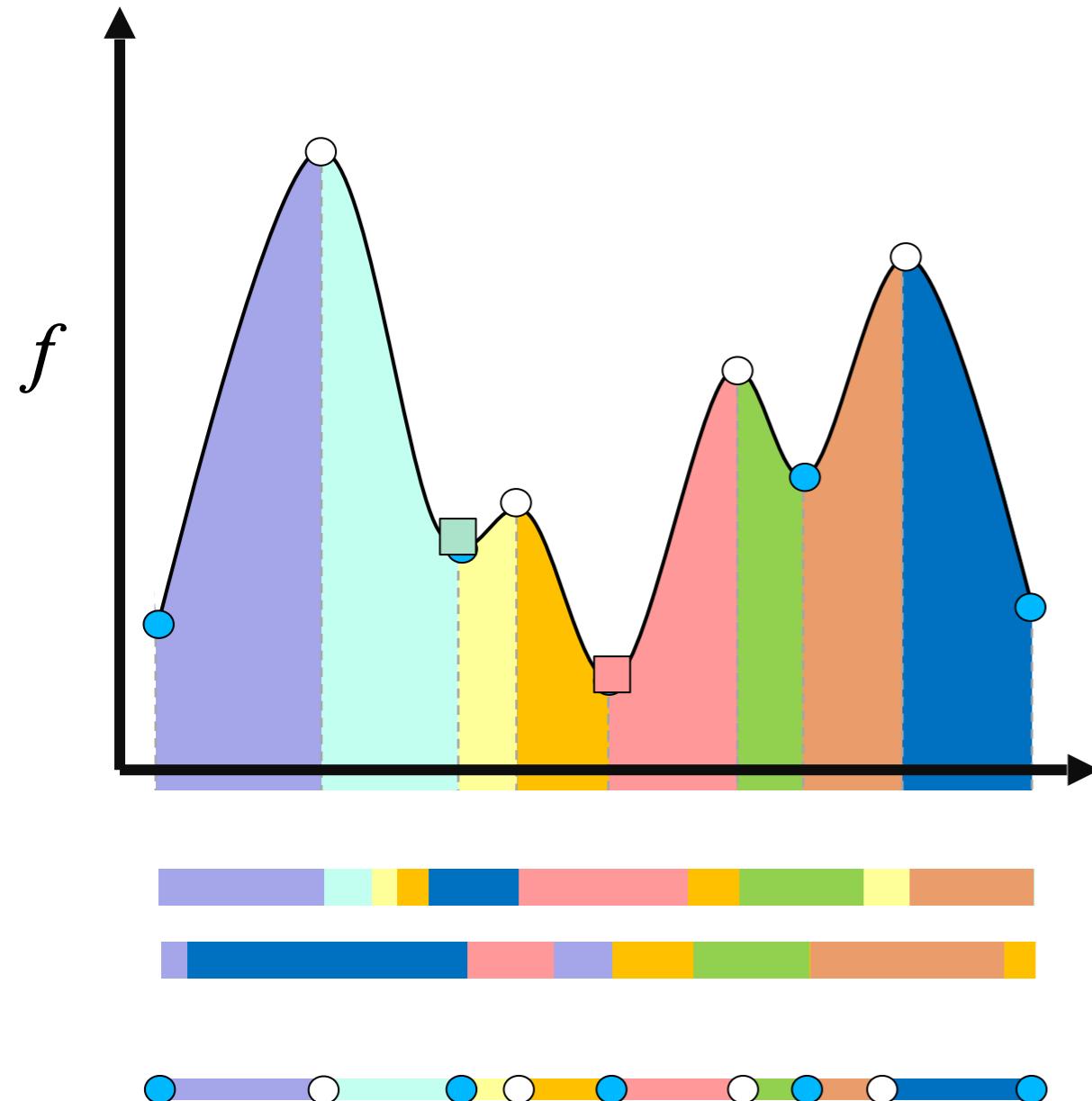


Features of a 1-D Scalar Field



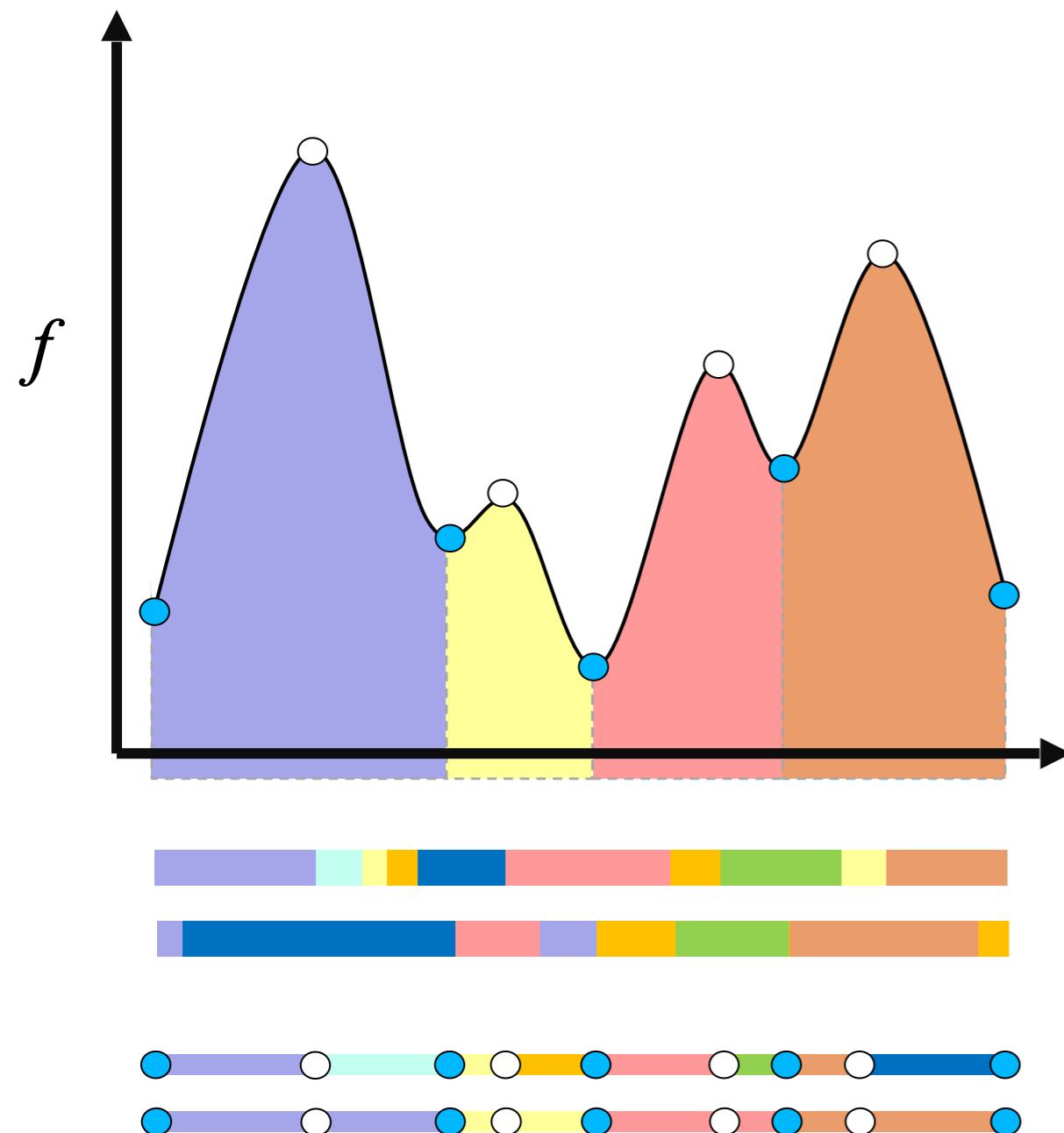
- Critical points where
 - Index-0 minimum
 - Index-1 maximum
- Components from the filtration
 - Join tree “local valleys”
 - Split tree “local peaks”
- Locally monotonic regions
 - Morse-Smale complex

Features of a 1-D Scalar Field



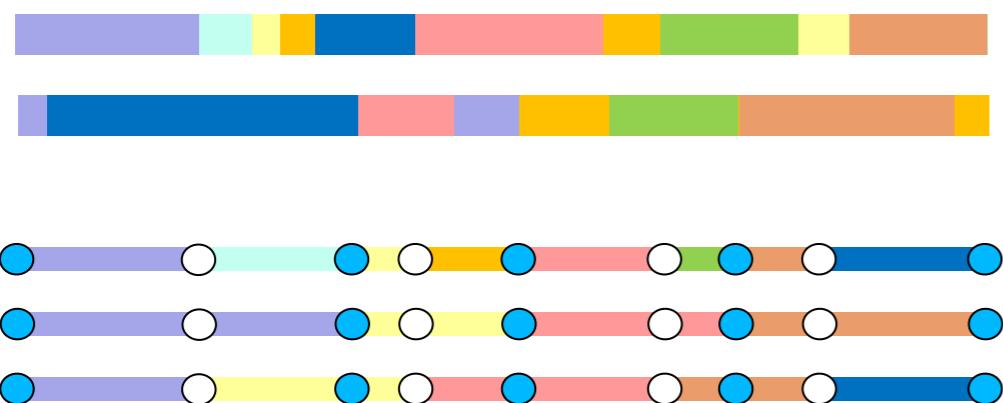
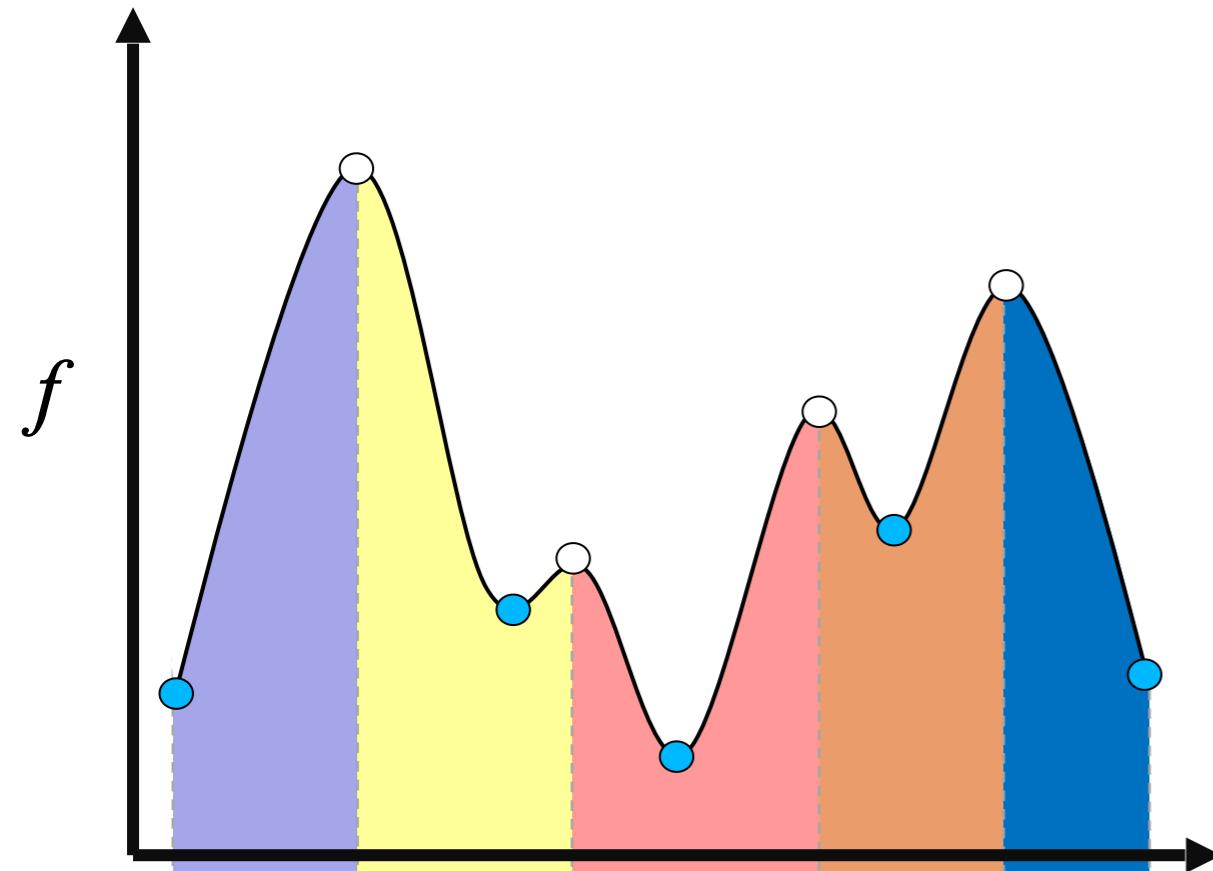
- Critical points where
 - Index-0 minimum
 - Index-1 maximum
- Components from the filtration
 - Join tree “local valleys”
 - Split tree “local peaks”
- Locally monotonic regions
 - Morse-Smale complex

Features of a 1-D Scalar Field



- Critical points where
 - Index-0 minimum
 - Index-1 maximum
- Components from the filtration
 - Join tree “local valleys”
 - Split tree “local peaks”
- Locally monotonic regions
 - Morse-Smale complex
 - Mountains

Features of a 1-D Scalar Field

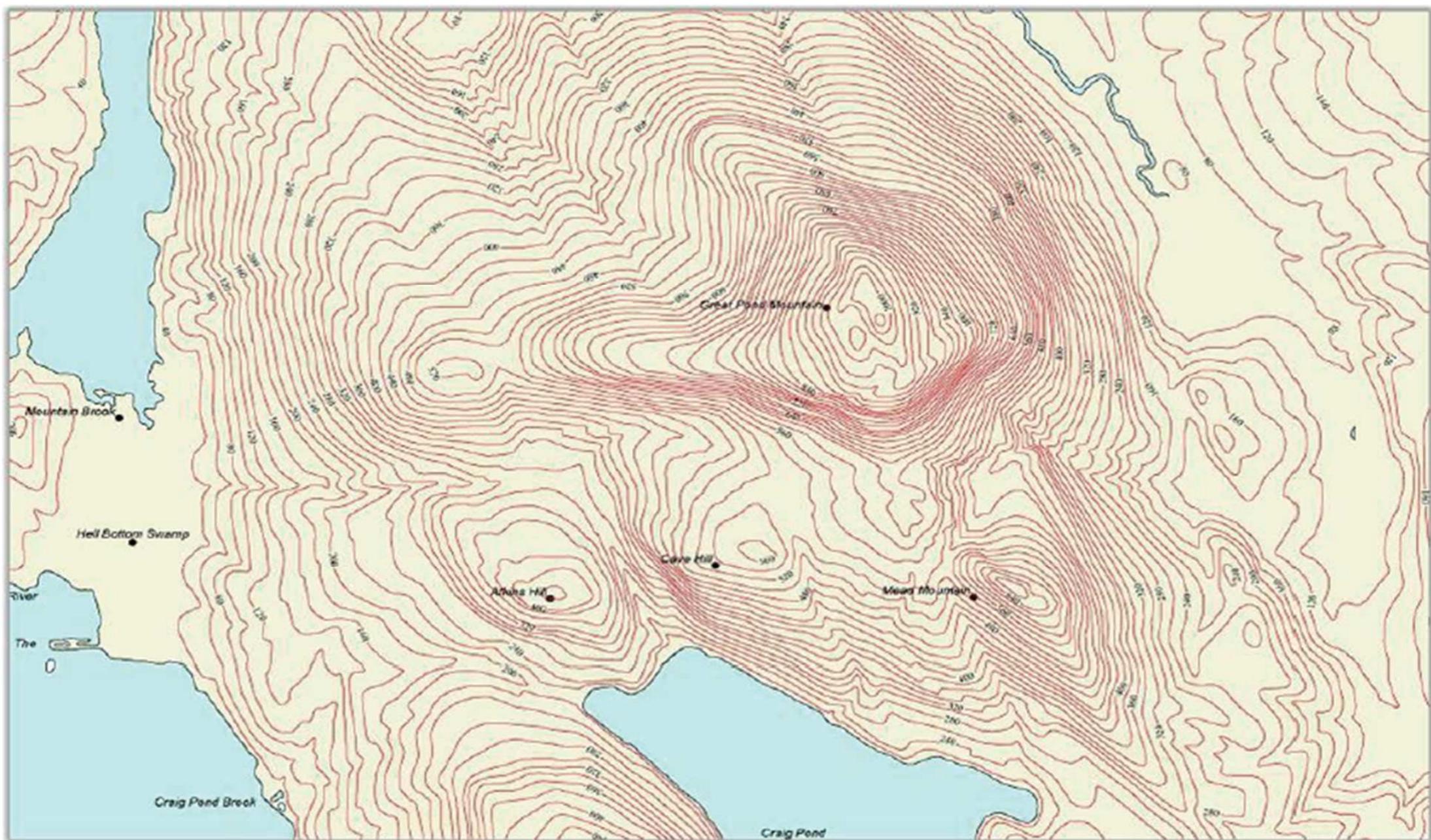


- Critical points where
 - Index-0 minimum
 - Index-1 maximum
- Components from the filtration
 - Join tree “local valleys”
 - Split tree “local peaks”
- Locally monotonic regions
 - Morse-Smale complex
 - Mountains
 - Basins

2D and 3D

How About 2D Case?

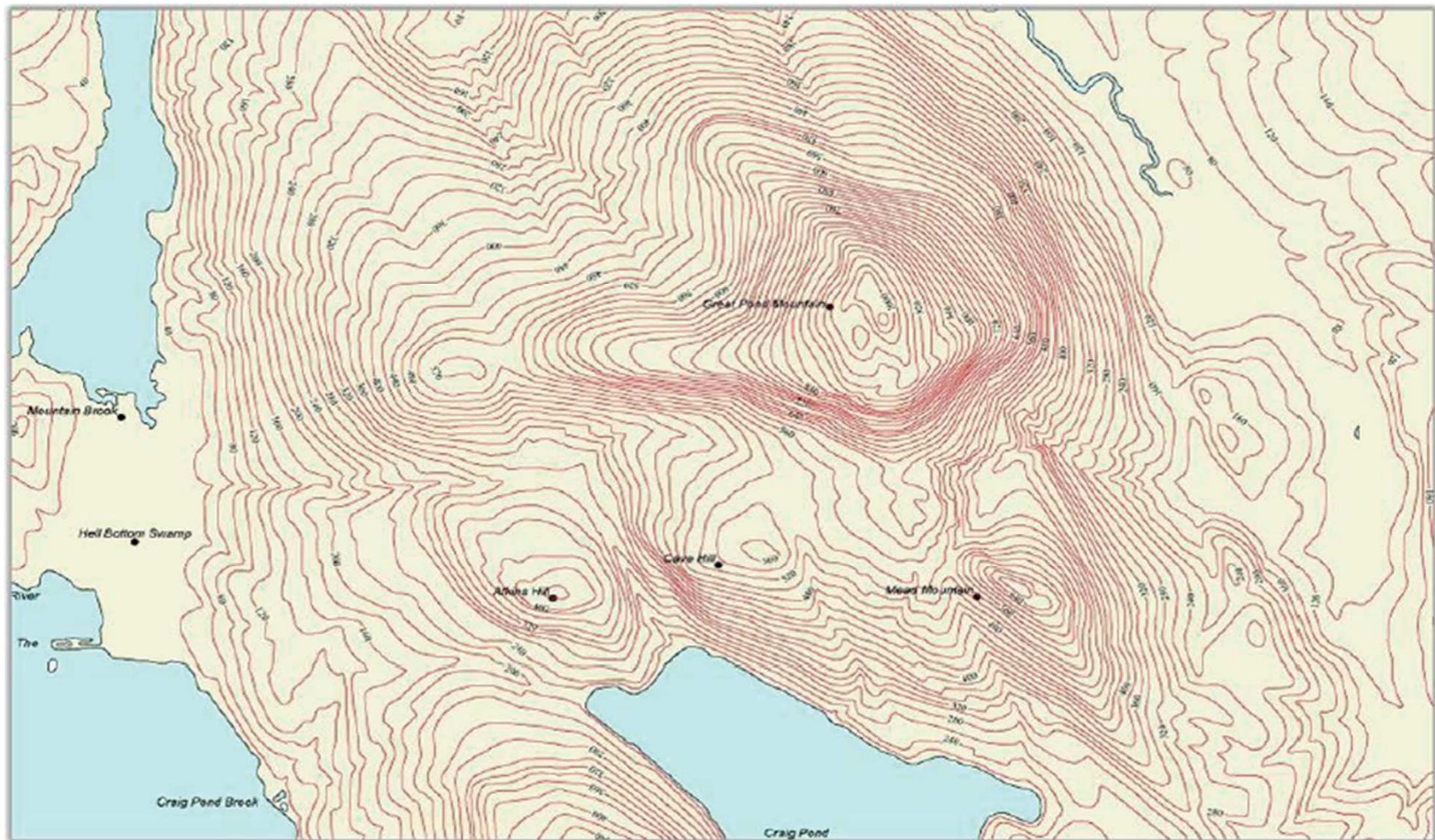
Pre-image of an iso-value: Iso-contours



We Want to Extract Similar Information

Q: Which iso-contours are interesting?

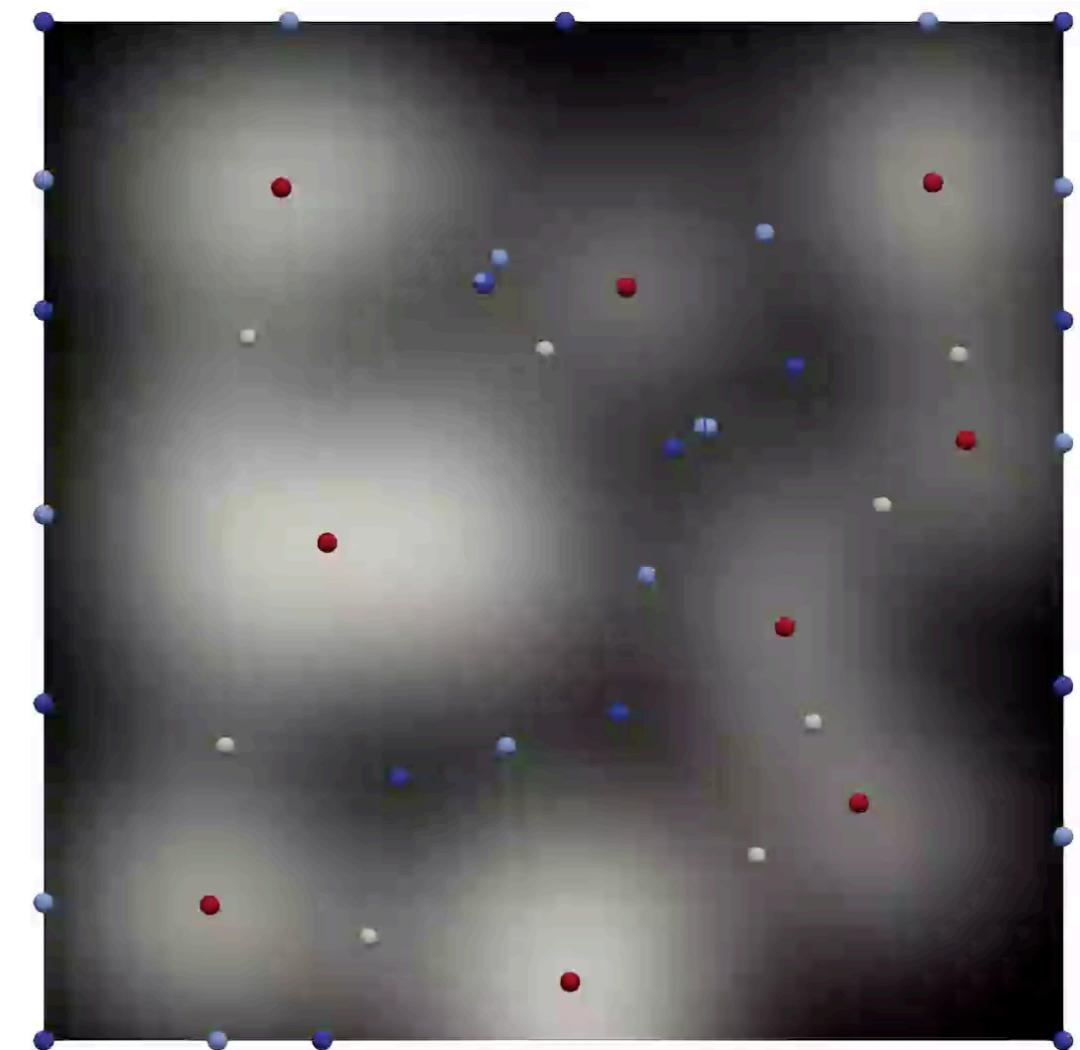
Q: Summarize the evolution of iso-contours?



Features of a 2-dimensional function



Grayscale rendering
Of 2d scalar function

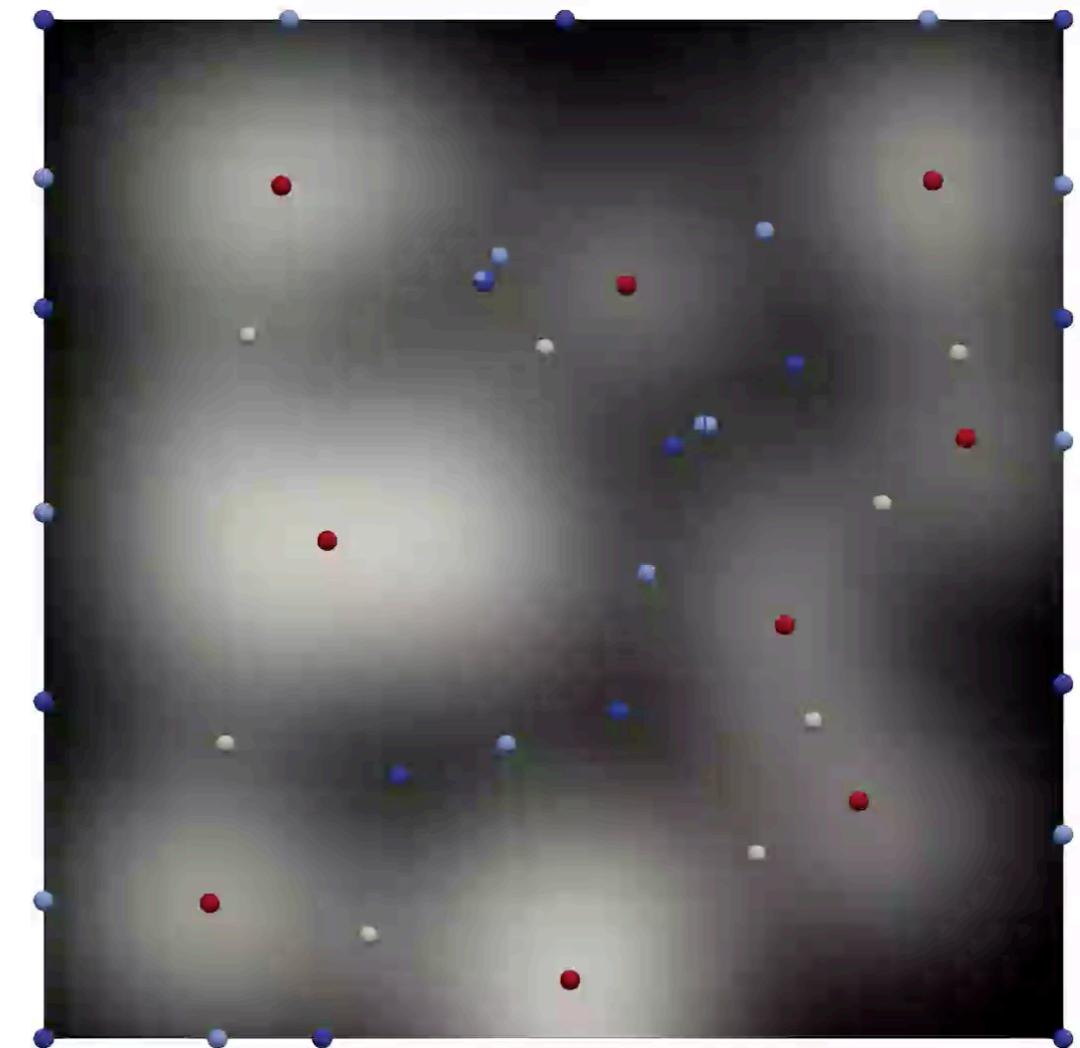


$$f^{-1}((- \infty, v])$$

Features of a 2-dimensional function



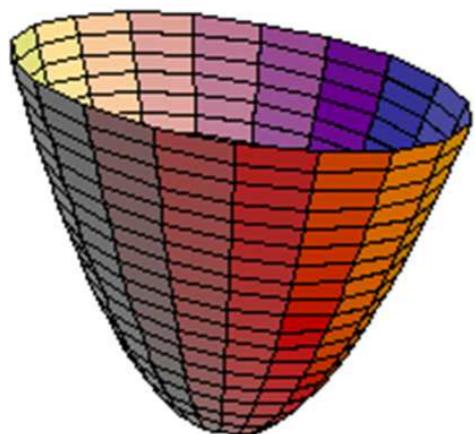
Grayscale rendering
Of 2d scalar function



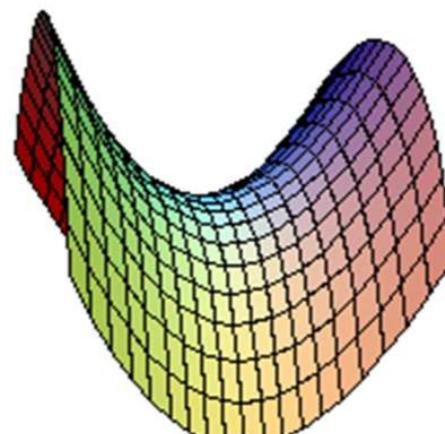
$$f^{-1}((- \infty, v])$$

Critical Points

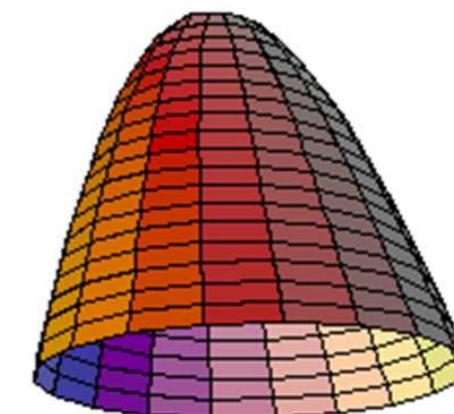
- Critical points in 2D have three possible behaviors



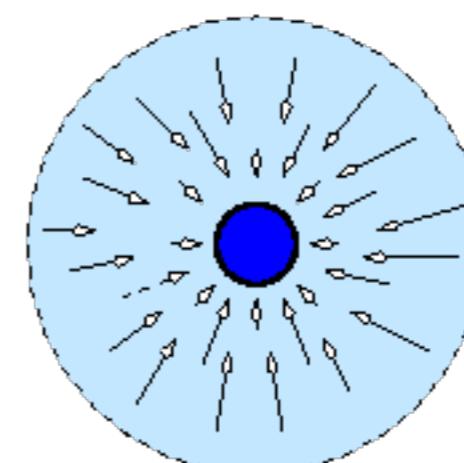
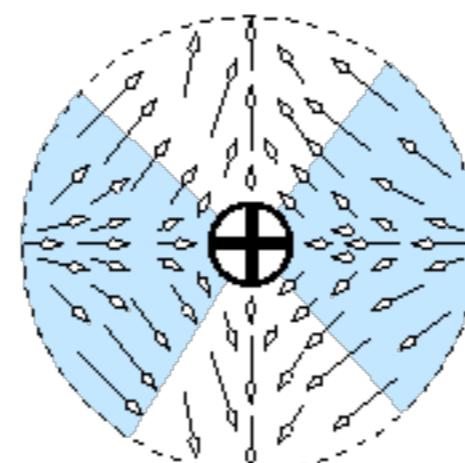
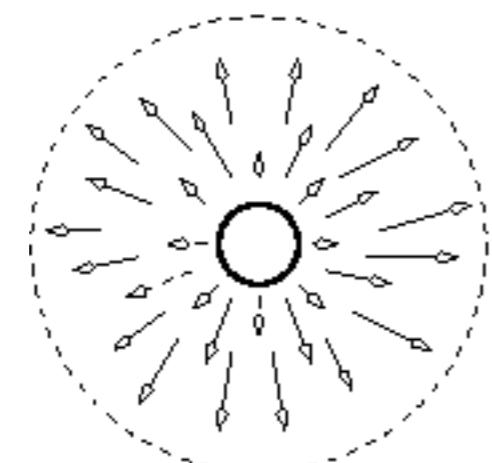
Minimum



Saddle



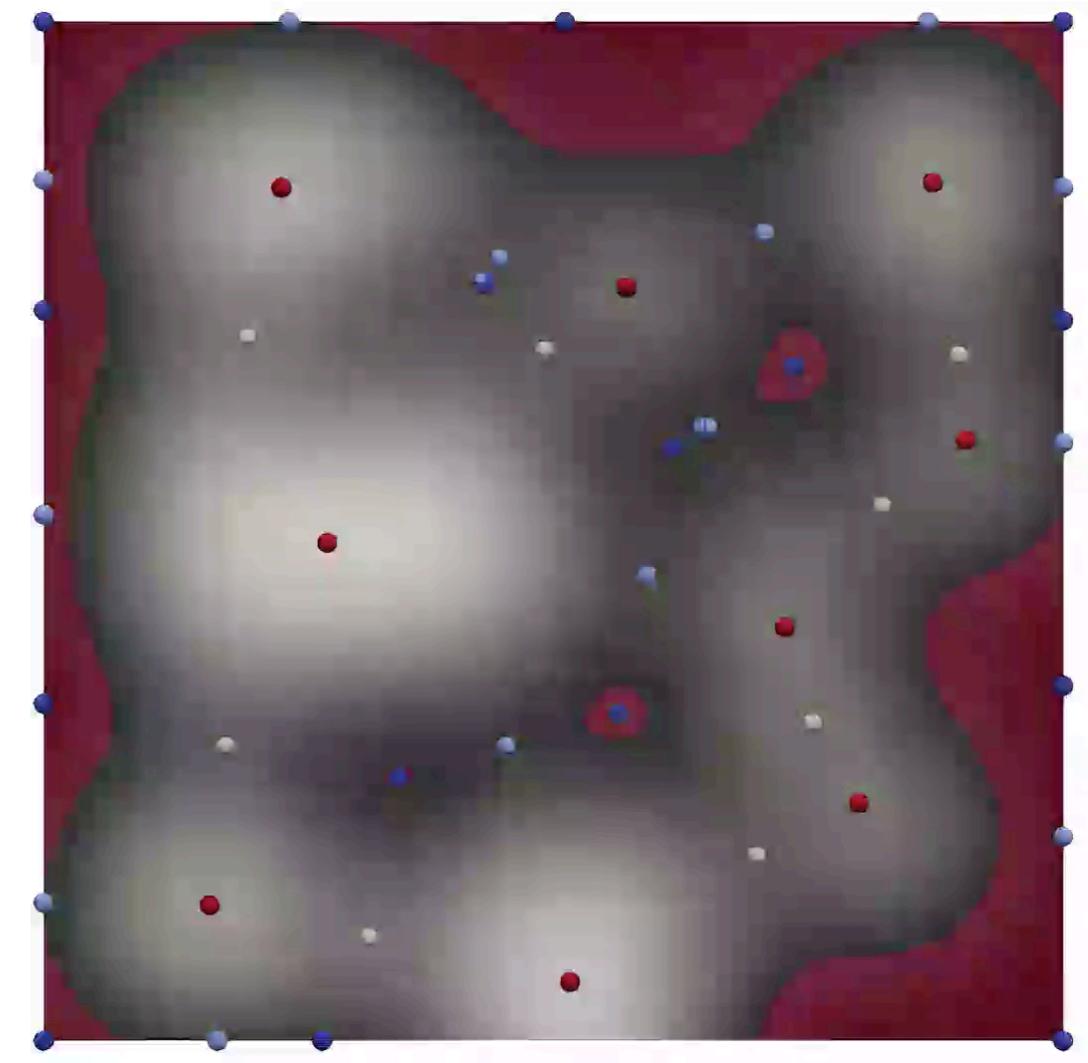
Maximum



Features of a 2-dimensional function



Grayscale rendering
Of 2d scalar function

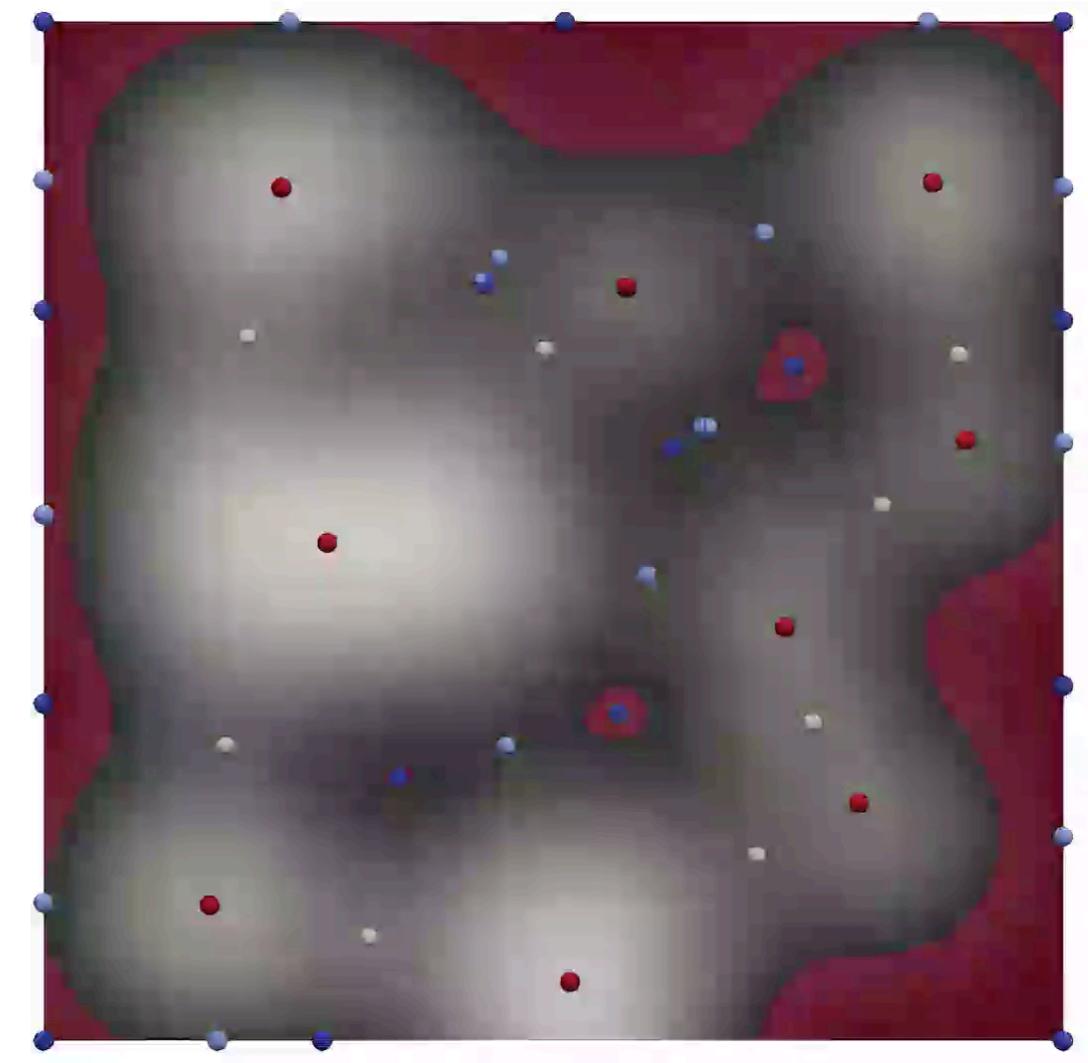


$$f^{-1}((-\infty, v])$$

Features of a 2-dimensional function



Grayscale rendering
Of 2d scalar function

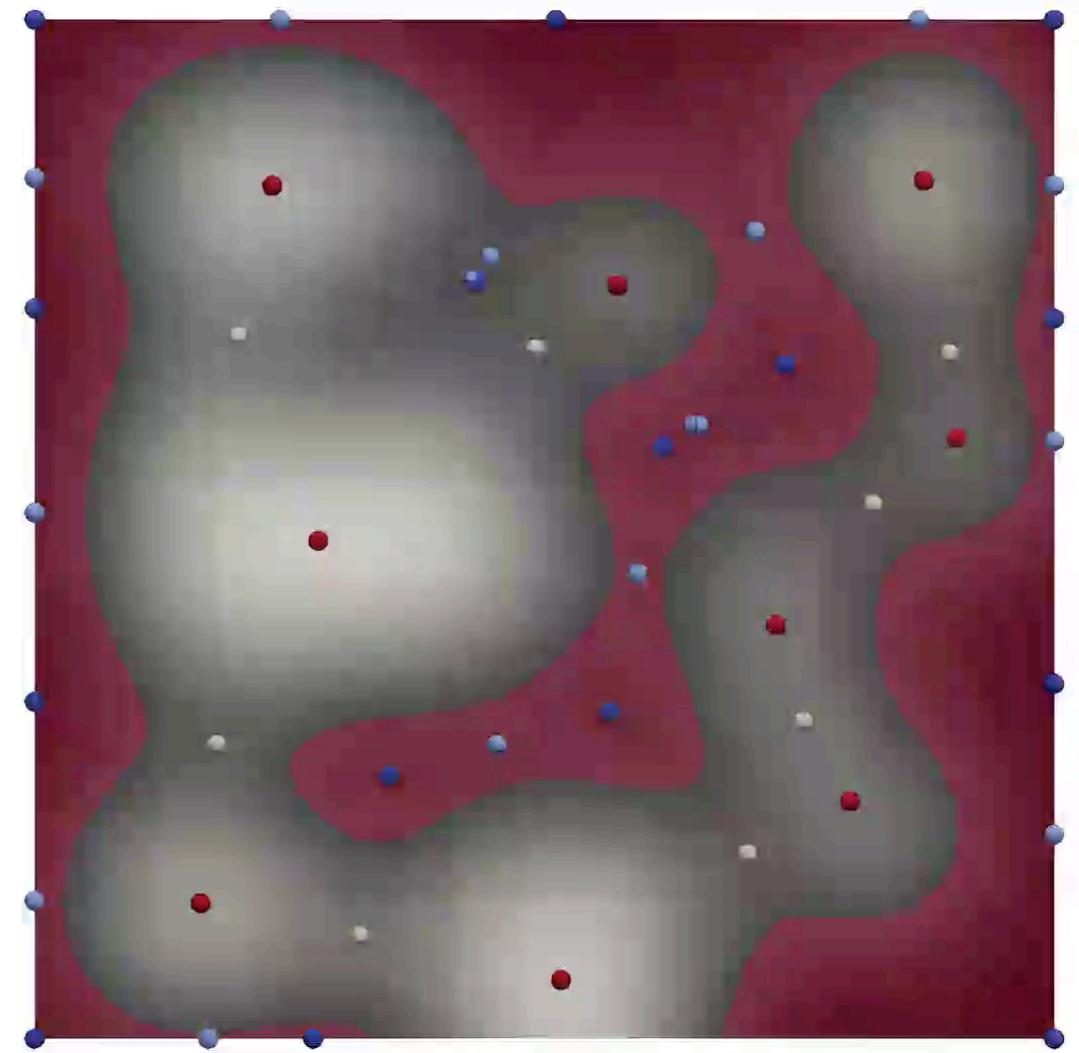


$$f^{-1}((-\infty, v])$$

Features of a 2-dimensional function



Grayscale rendering
Of 2d scalar function

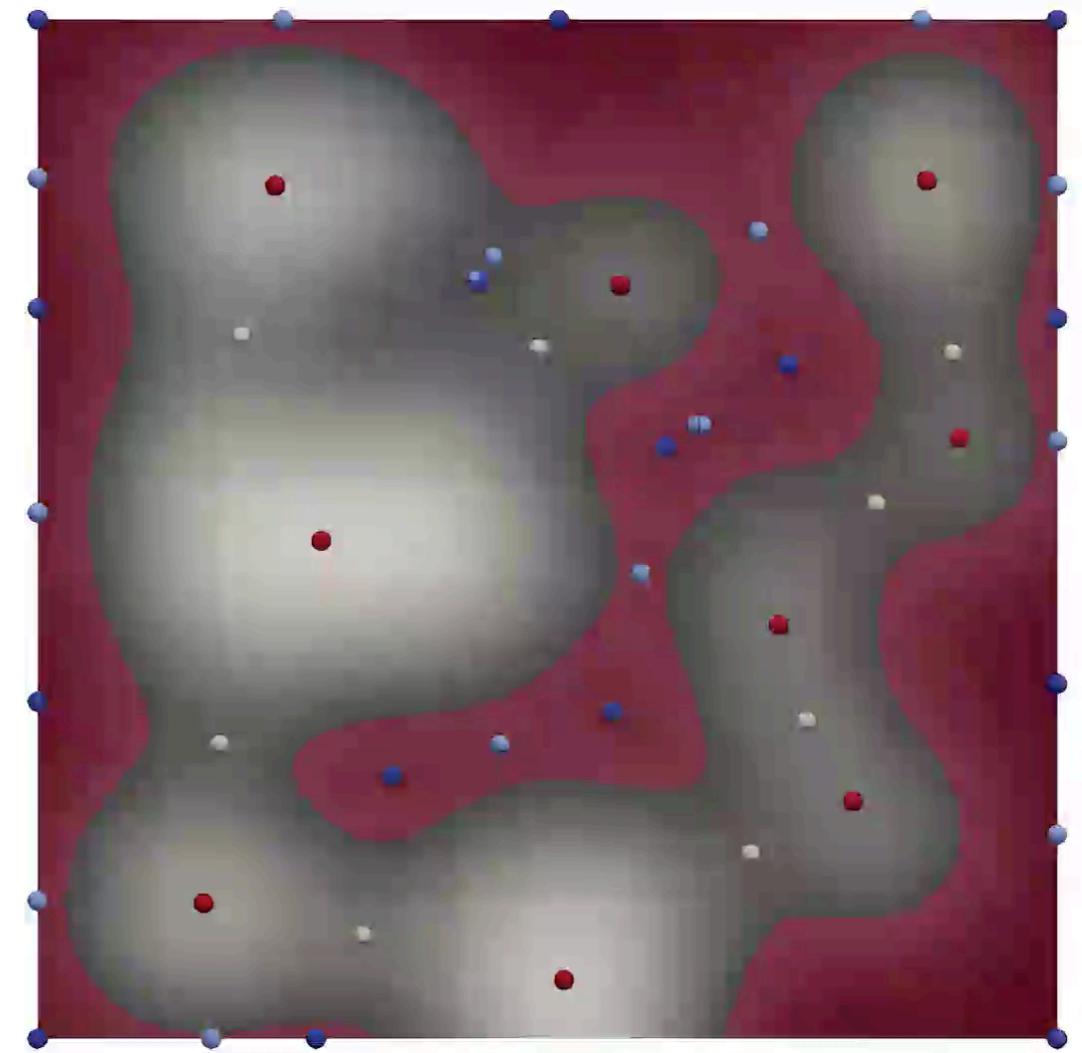


$$f^{-1}((-\infty, v])$$

Features of a 2-dimensional function

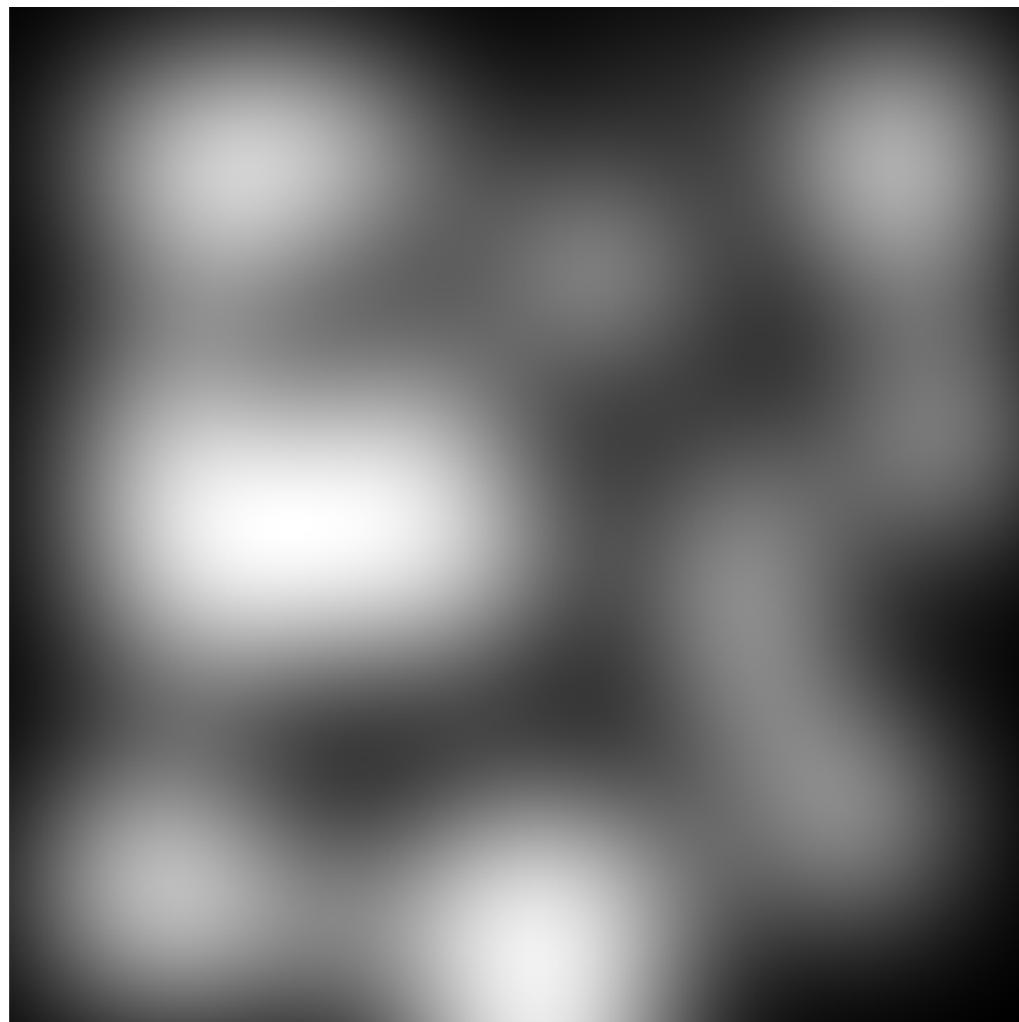


Grayscale rendering
Of 2d scalar function

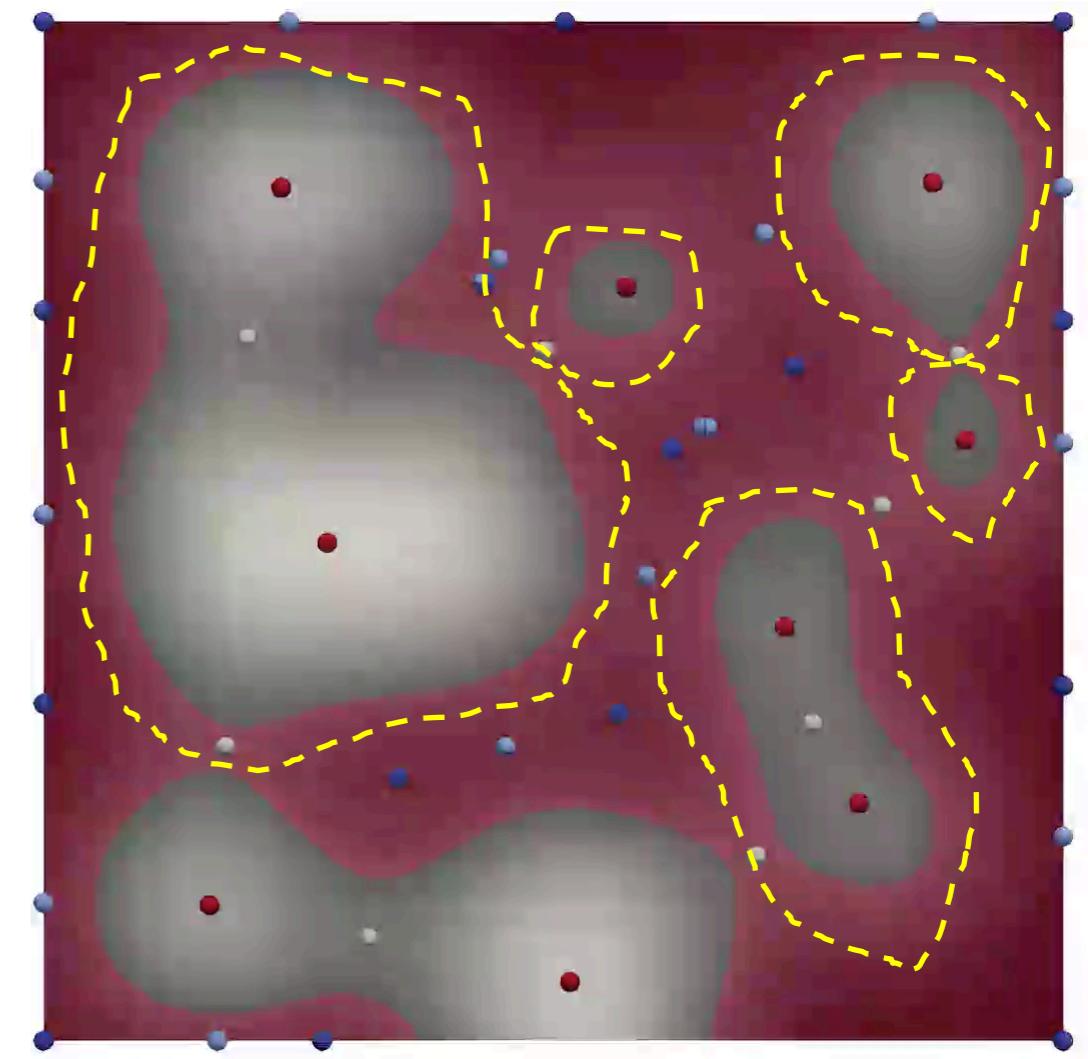


$$f^{-1}((-\infty, v])$$

Features of a 2-dimensional function



Grayscale rendering
Of 2d scalar function

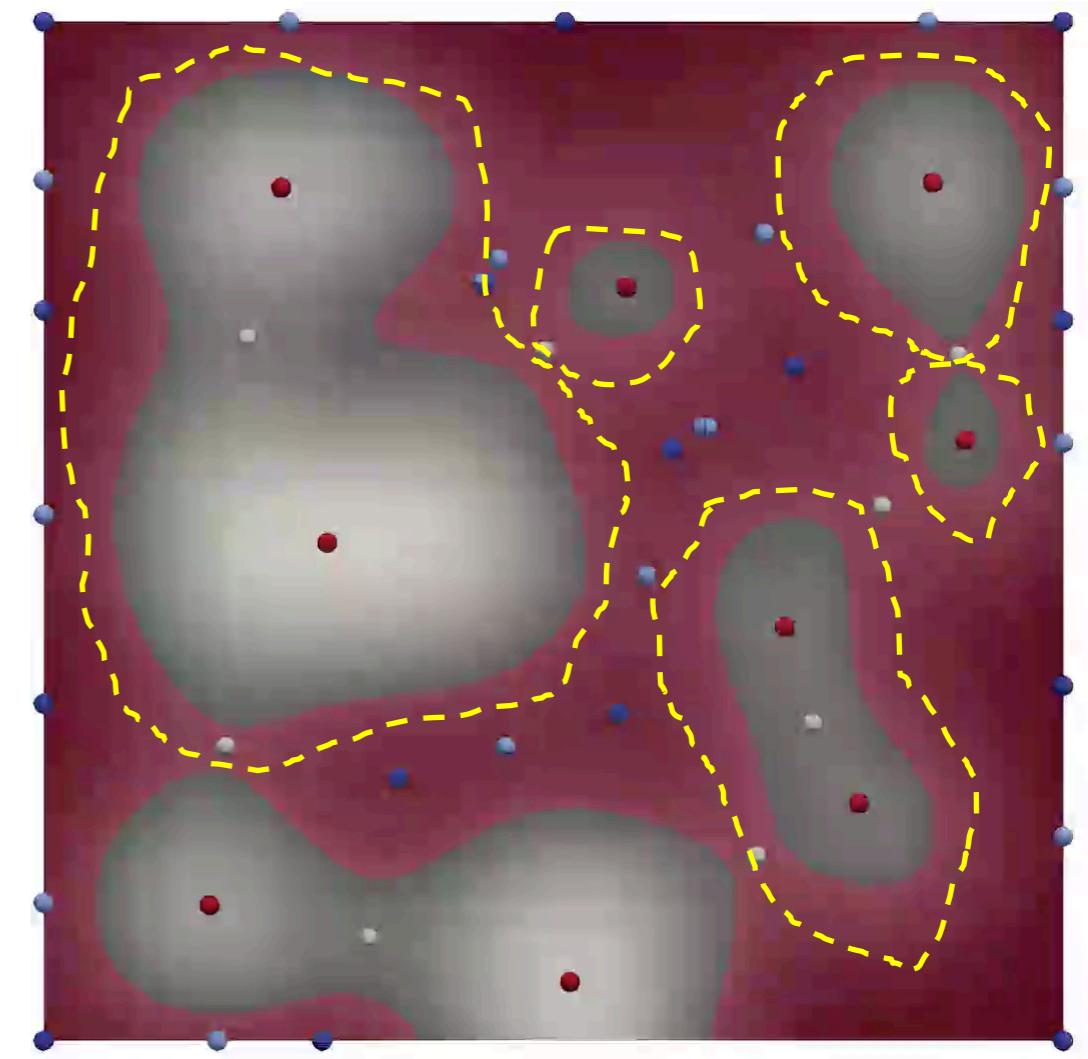


$$f^{-1}((-\infty, v])$$

Features of a 2-dimensional function



Grayscale rendering
Of 2d scalar function

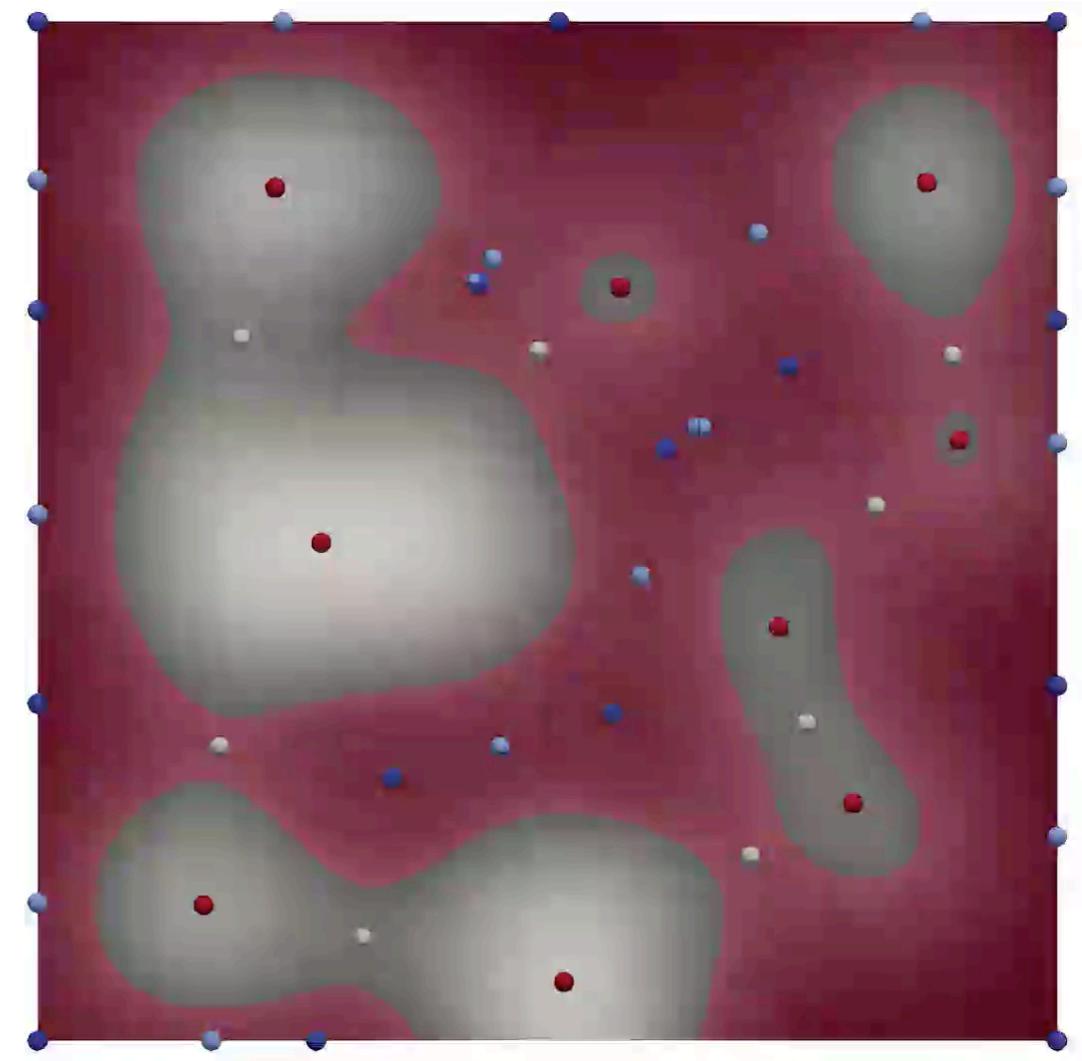


$$f^{-1}((-\infty, v])$$

Features of a 2-dimensional function



Grayscale rendering
Of 2d scalar function

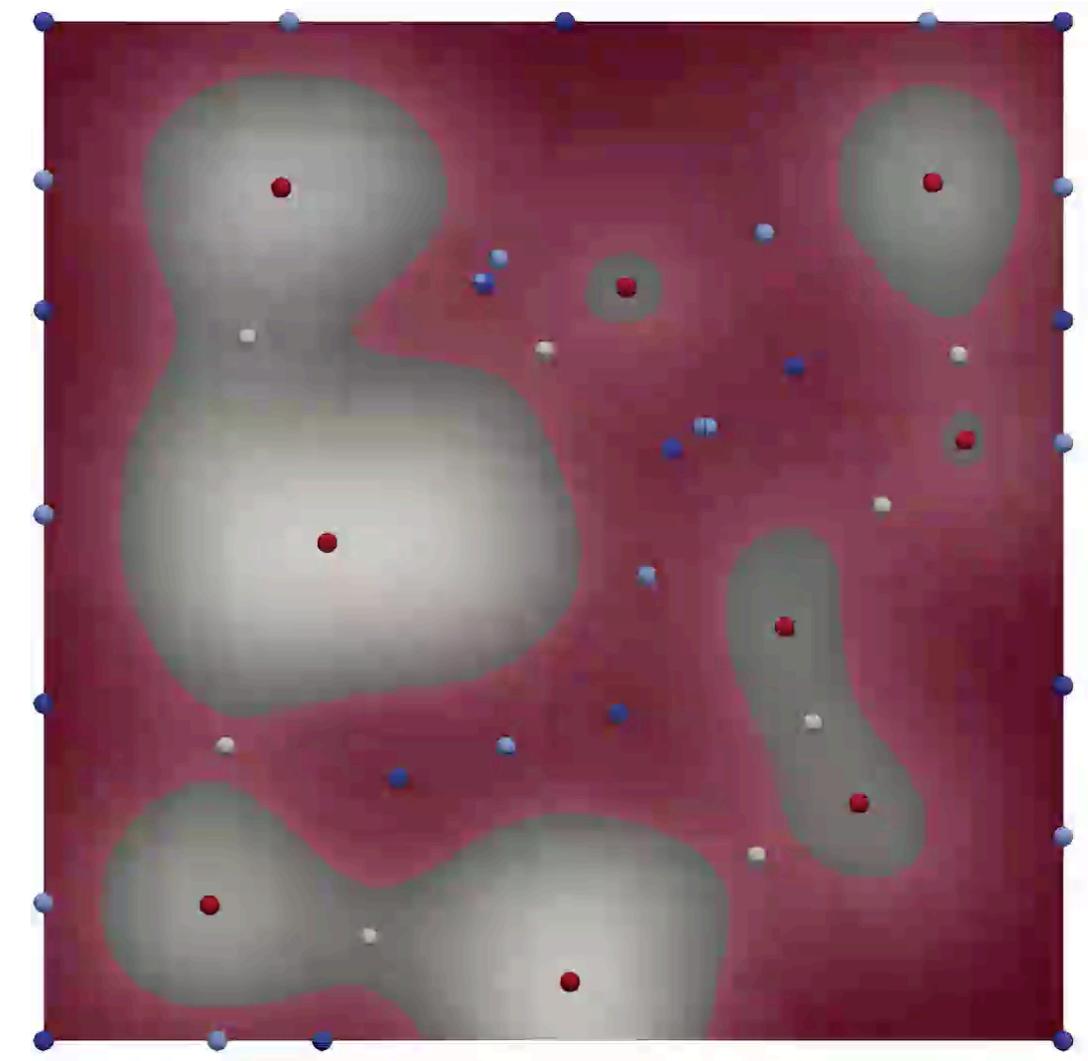


$$f^{-1}((-\infty, v])$$

Features of a 2-dimensional function



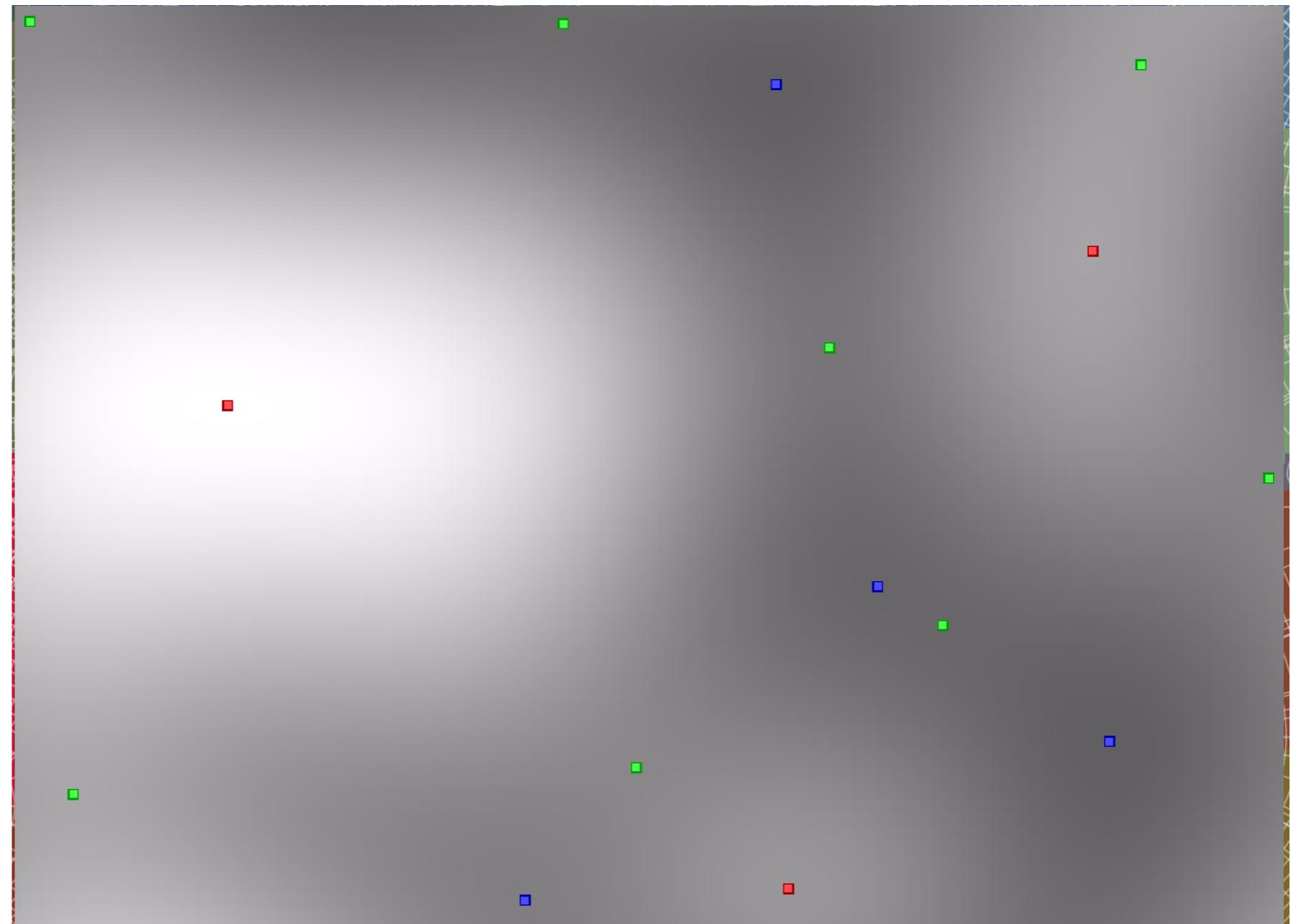
Grayscale rendering
Of 2d scalar function



$$f^{-1}((-\infty, v])$$

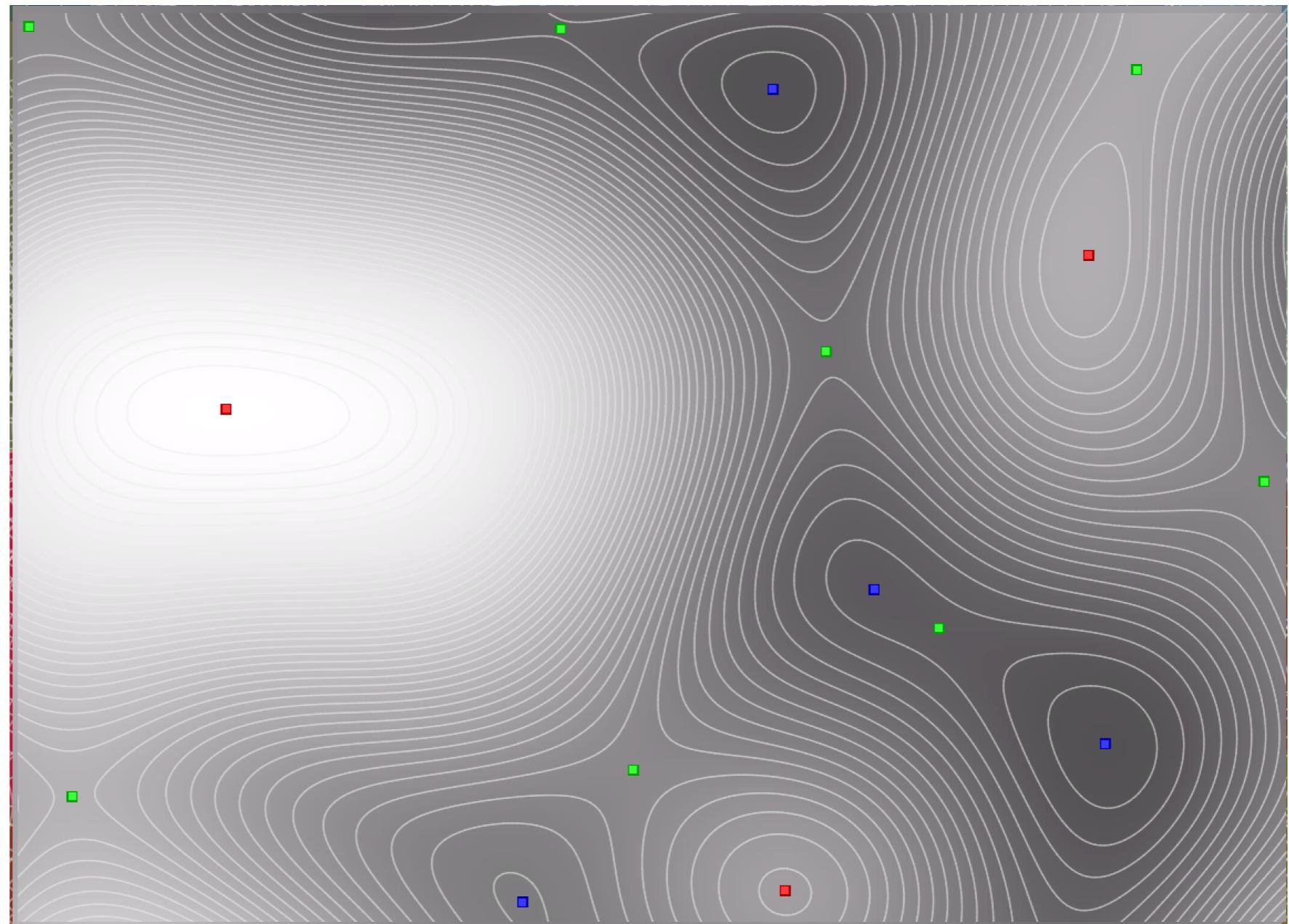
Features of a 2-dimensional function

**“Monotonicity” =
pieces of contours
grouped by integral
lines**



Features of a 2-dimensional function

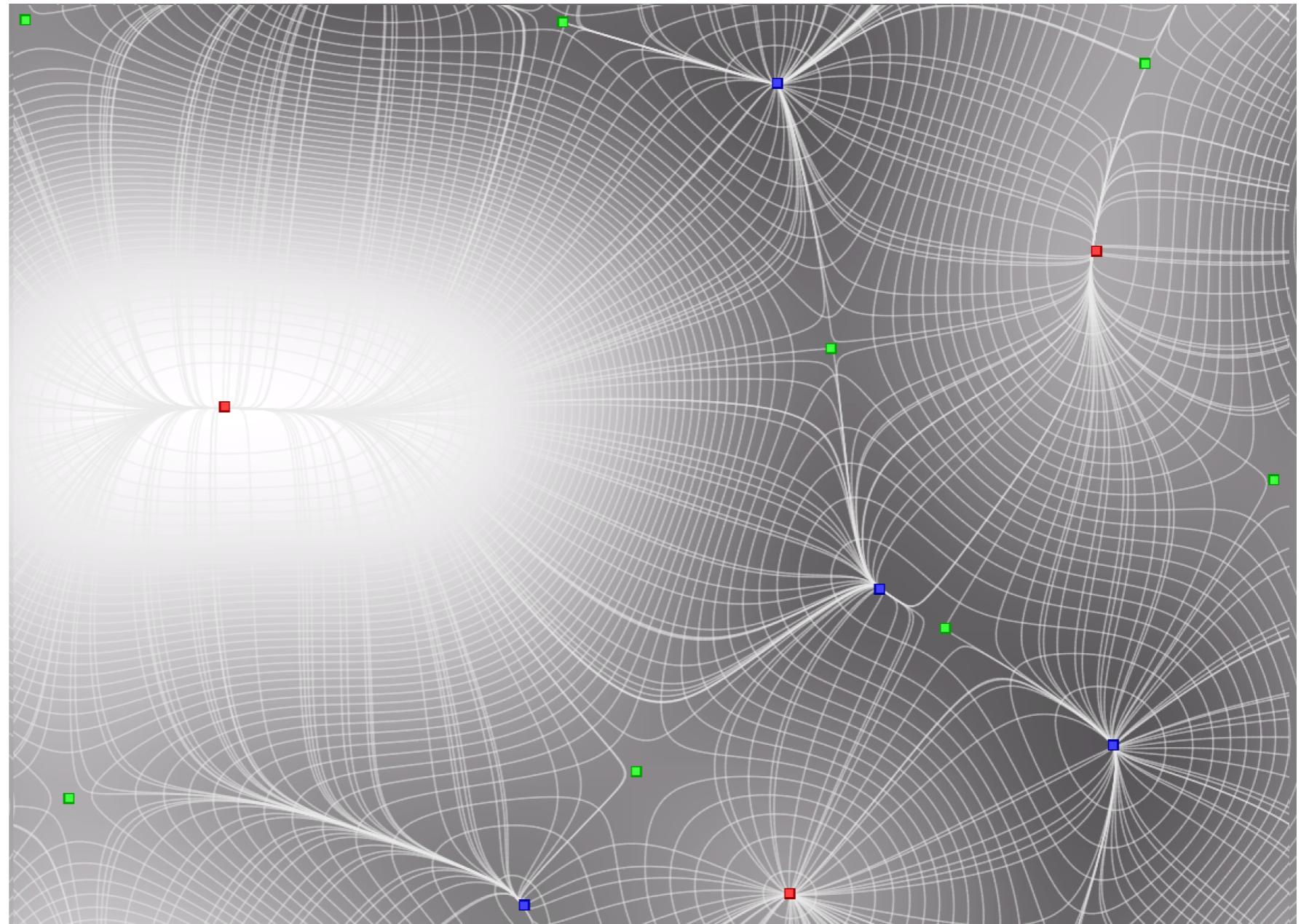
**“Monotonicity” =
pieces of contours
grouped by integral
lines**



Features of a 2-dimensional function

**“Monotonicity” =
pieces of contours
grouped by integral
lines**

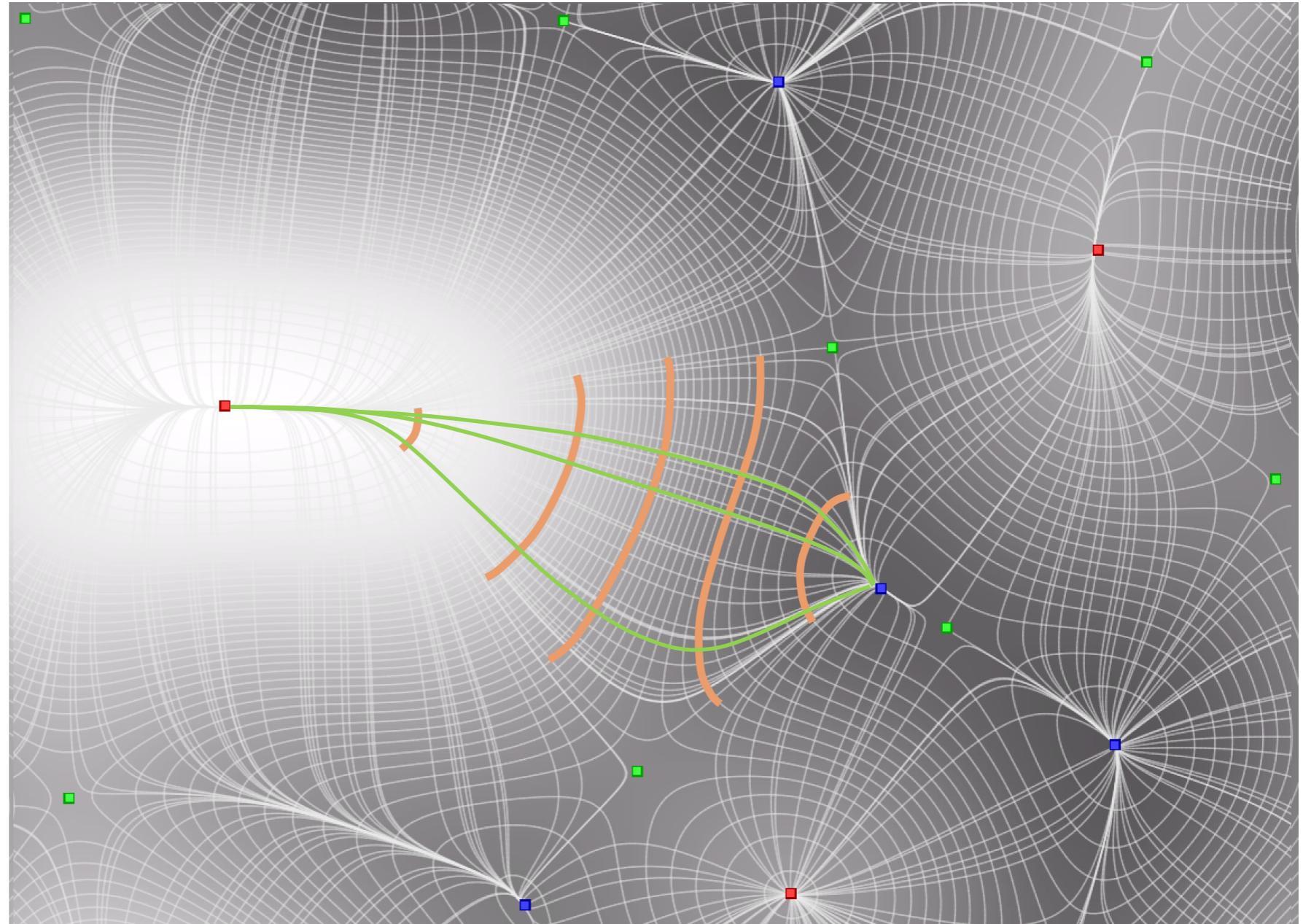
$$\frac{\partial}{\partial t} L(t) = \nabla f(L(t))$$



Features of a 2-dimensional function

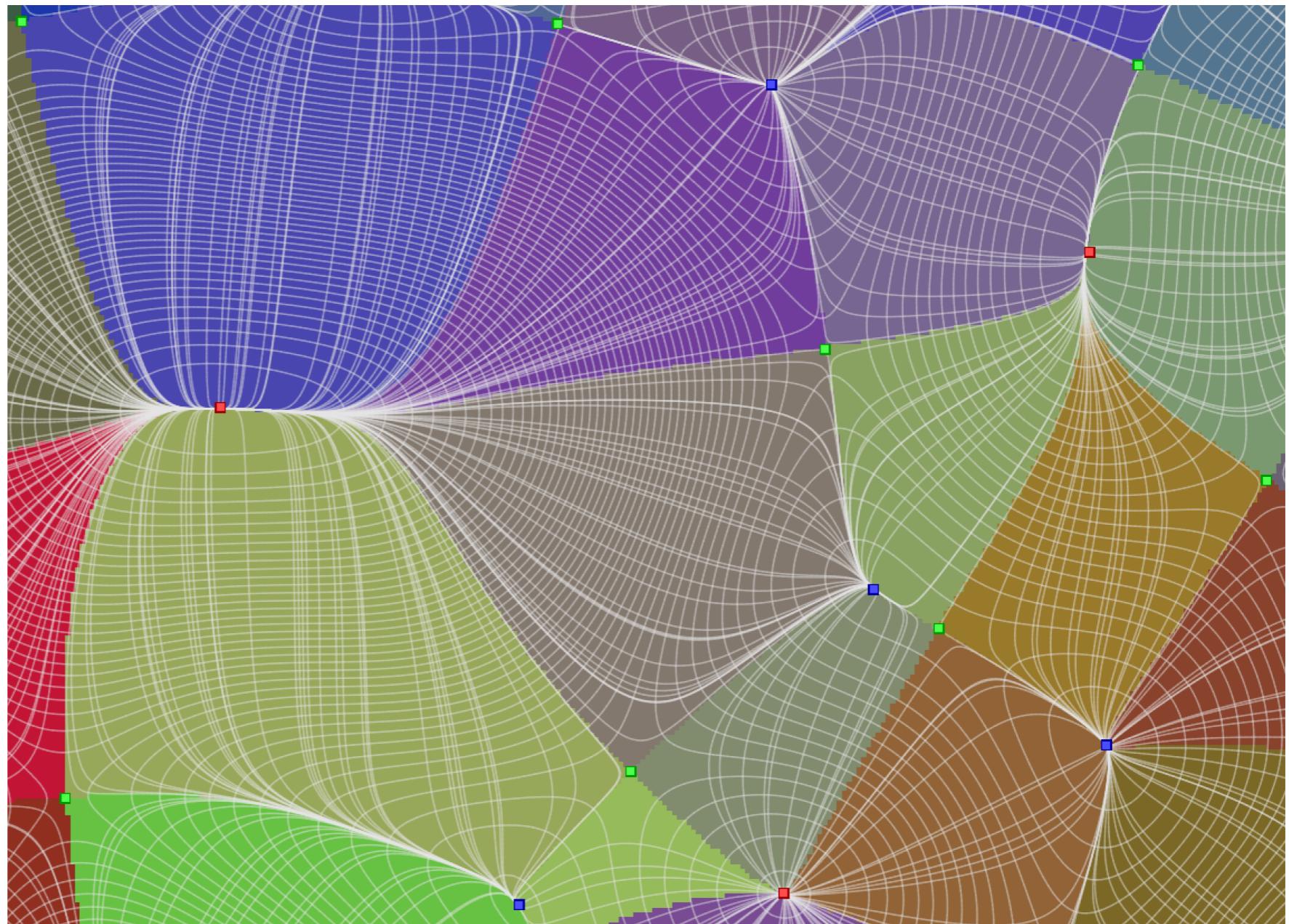
**“Monotonicity” =
pieces of contours
grouped by integral
lines**

$$\frac{\partial}{\partial t} L(t) = \nabla f(L(t))$$



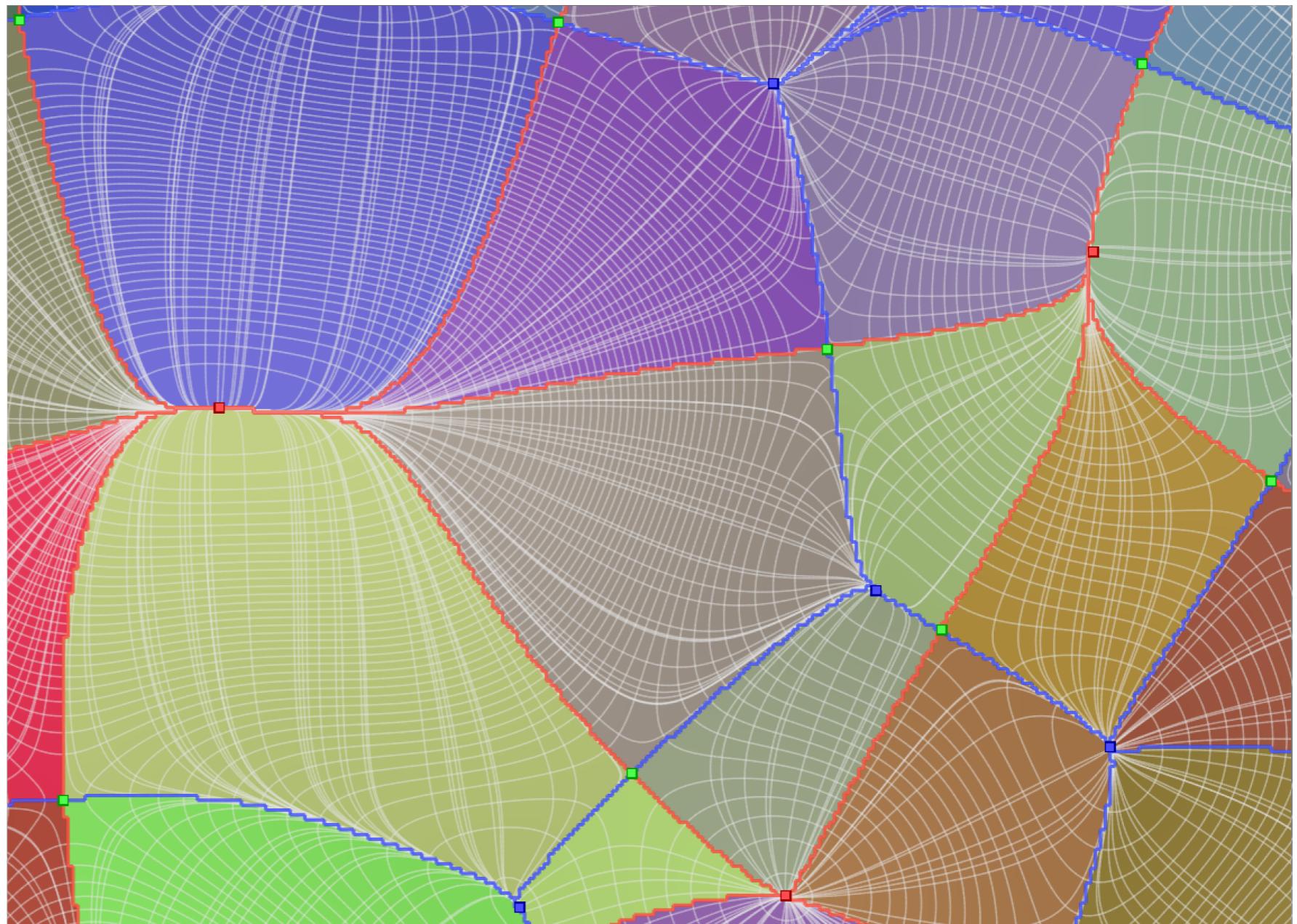
Features of a 2-dimensional function

**“Monotonicity” =
pieces of contours
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lines**



Features of a 2-dimensional function

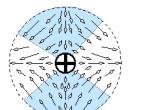
**“Monotonicity” =
pieces of contours
grouped by integral
lines**



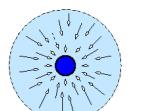
Features of a 2-dimensional function



Minimum

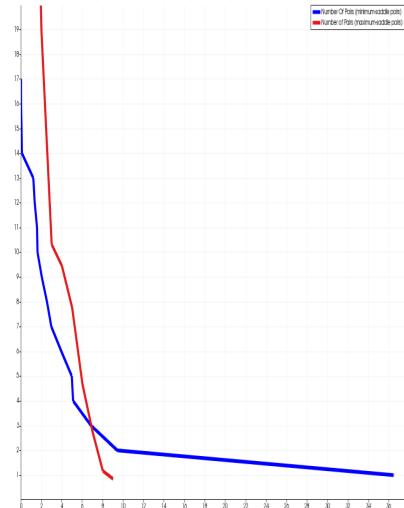


Saddle



Maximum

Features of a 2-dimensional function



**Persistence
curve**



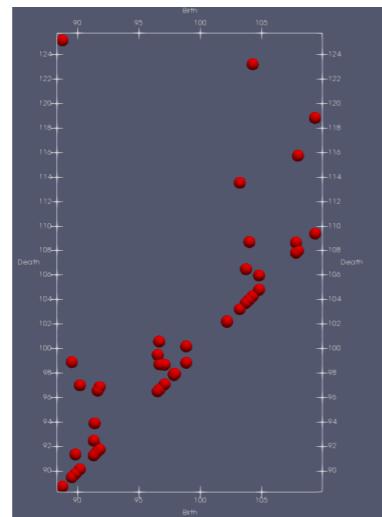
Minimum



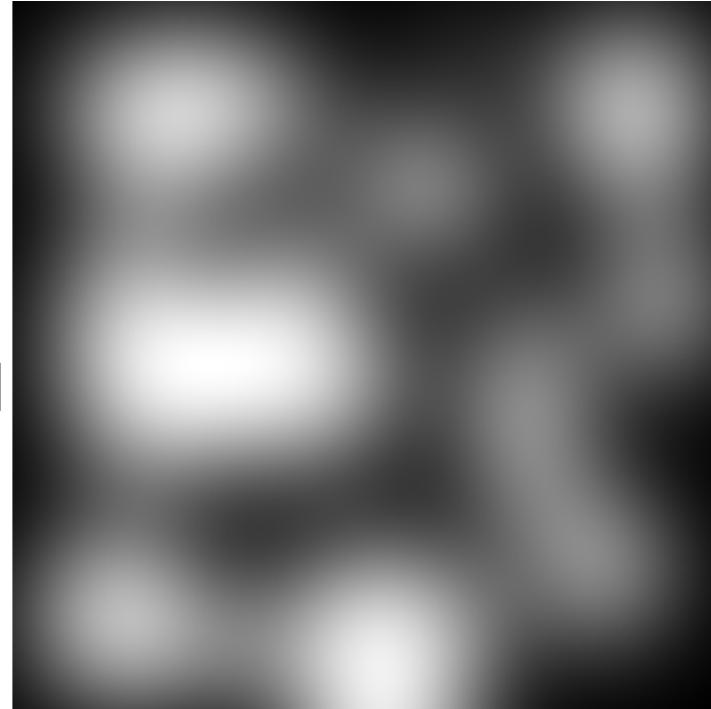
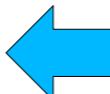
Saddle



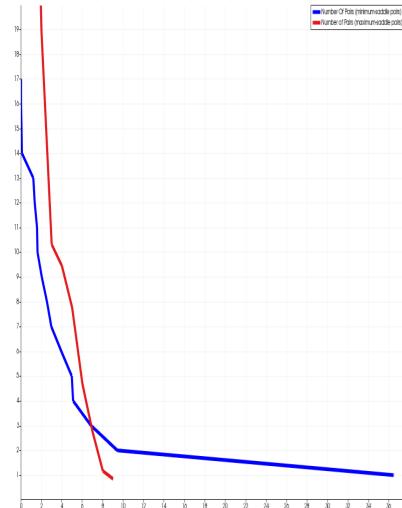
Maximum



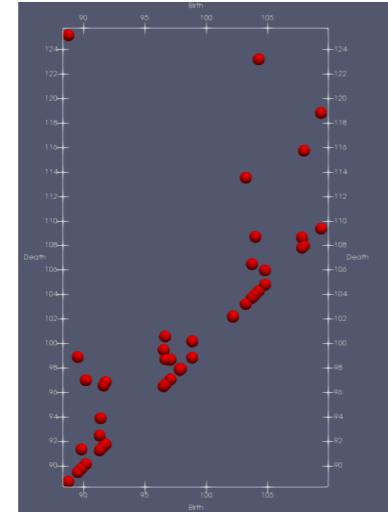
**Persistence
diagram**



Features of a 2-dimensional function



Persistence
curve

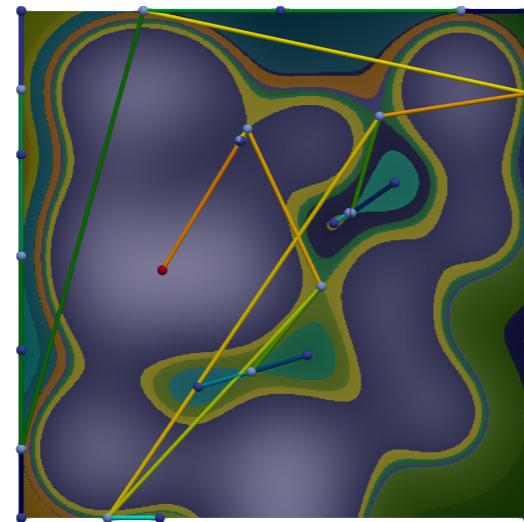


Persistence
diagram

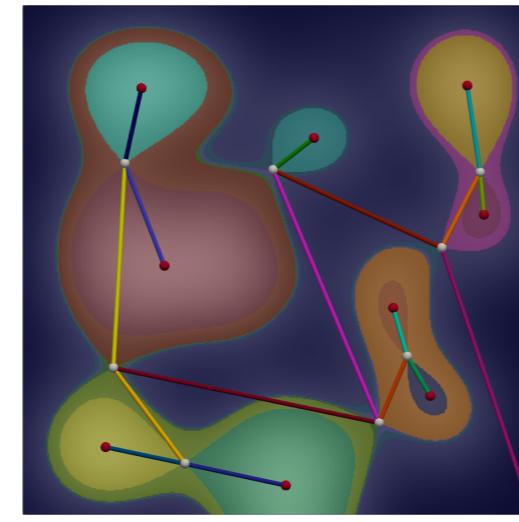


- Minimum**
- Saddle**
- Maximum**

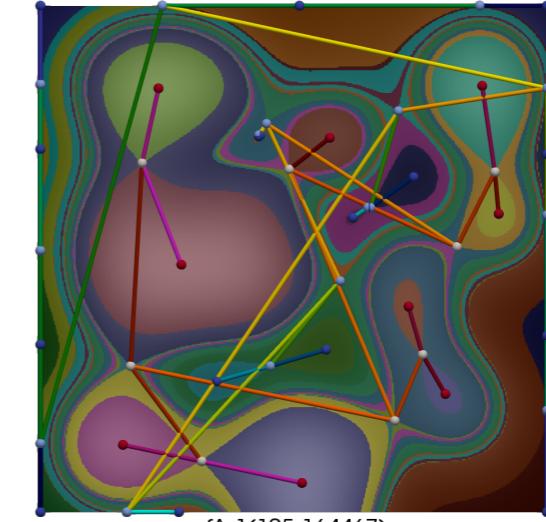
Join tree



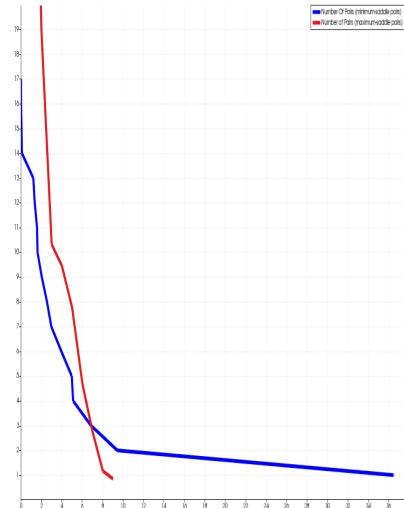
Split tree



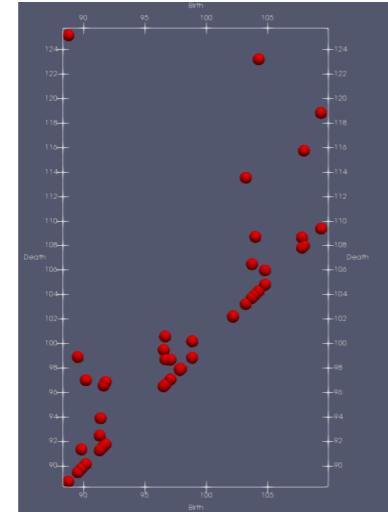
Contour tree



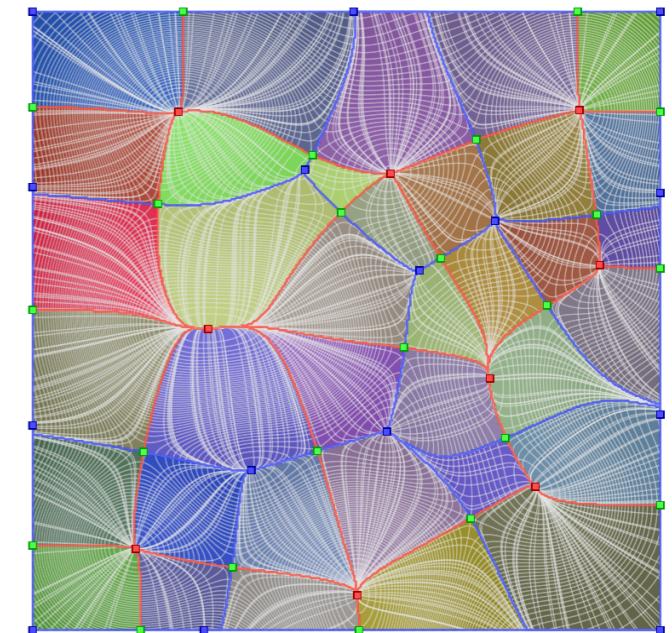
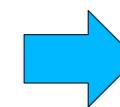
Features of a 2-dimensional function



Persistence
curve



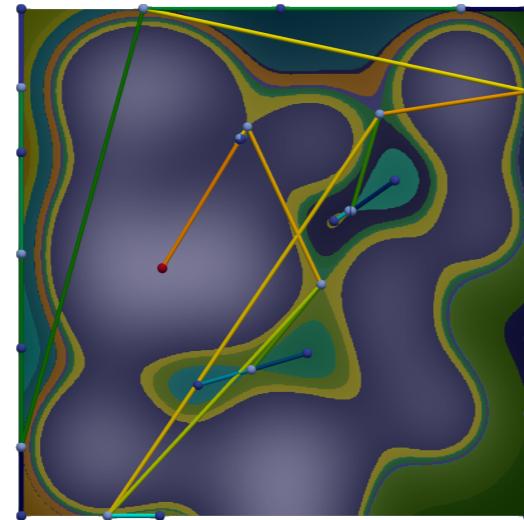
Persistence
diagram



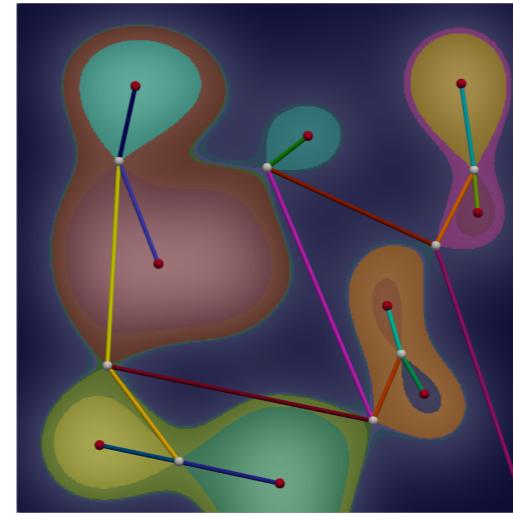
Morse-Smale Complex

- Minimum
- Saddle
- Maximum

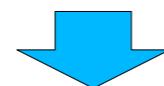
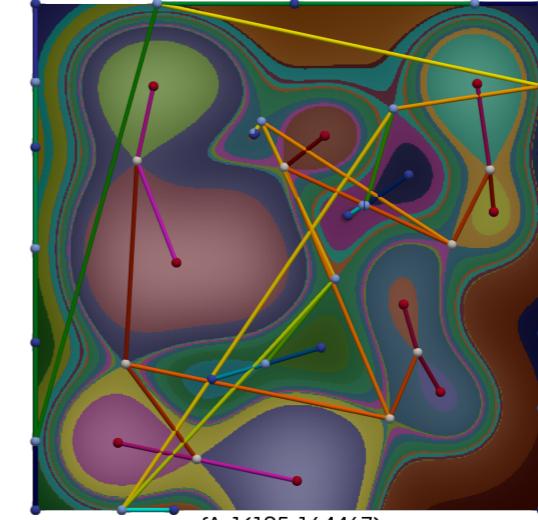
Join tree



Split tree

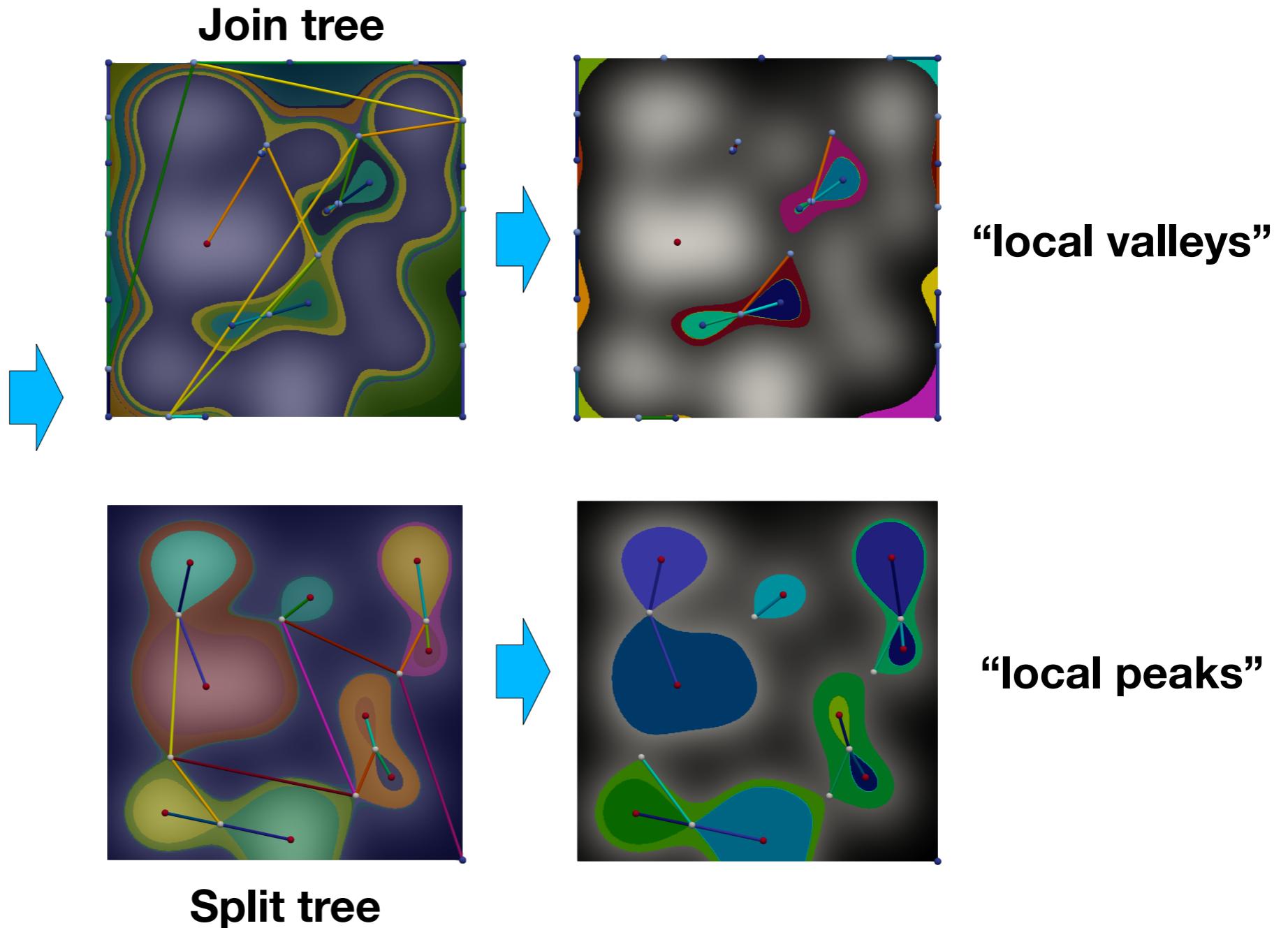


Contour tree



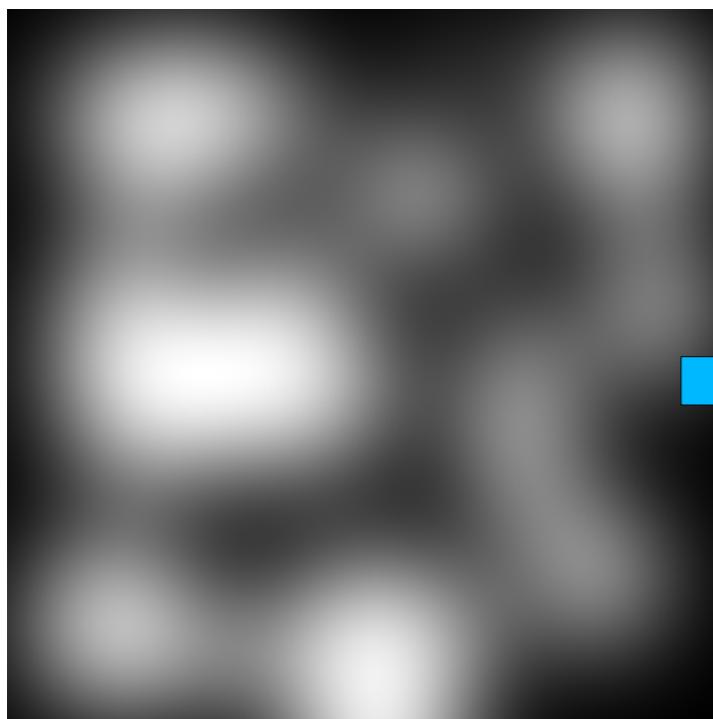
Features of a 2-dimensional function

Contour-based features

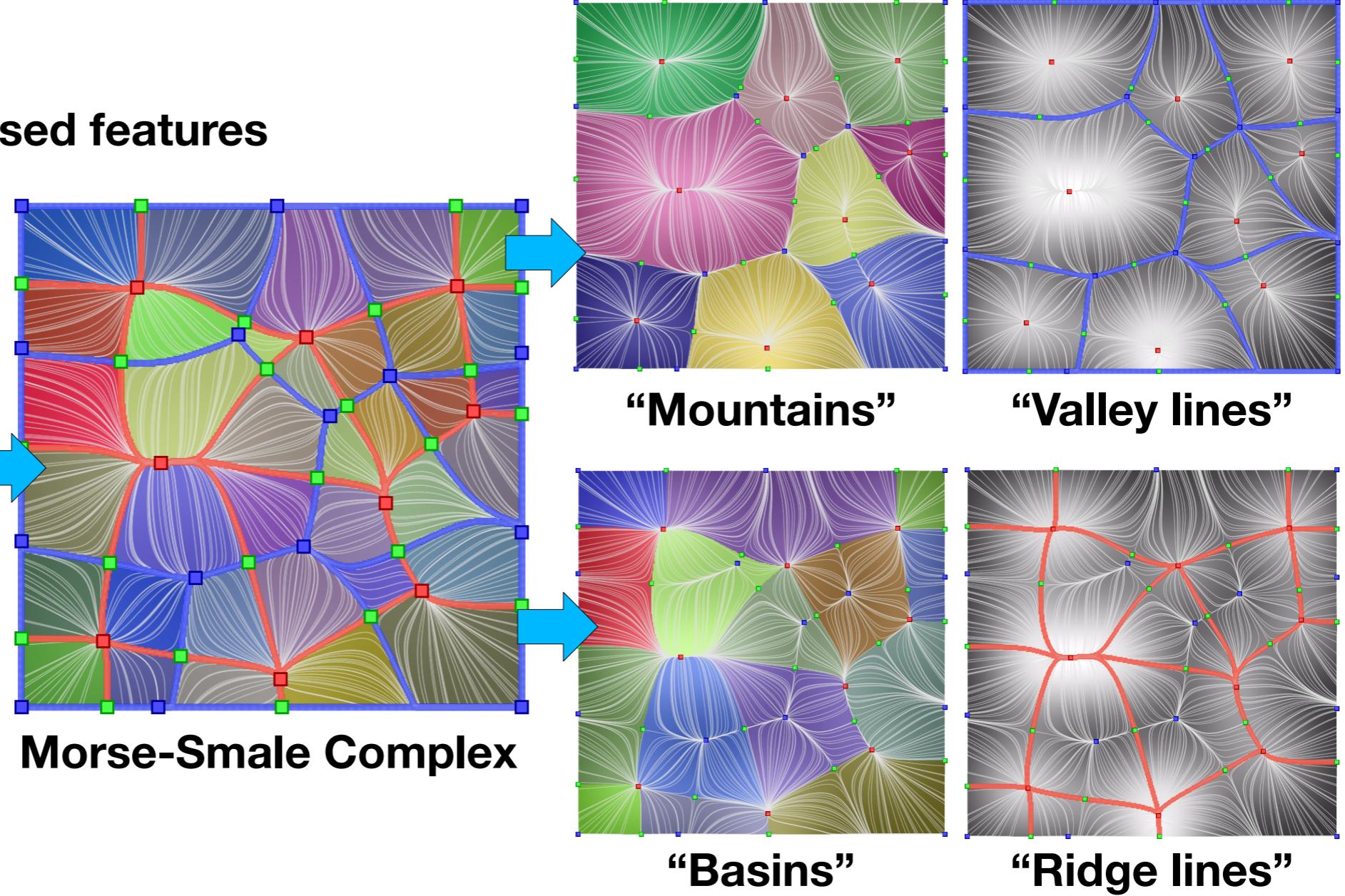


Features of a 2-dimensional function

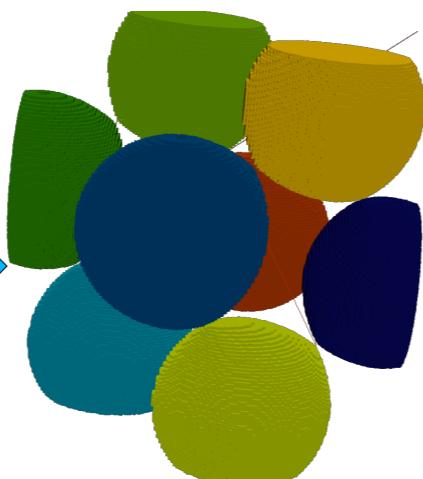
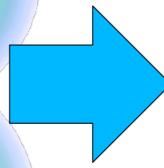
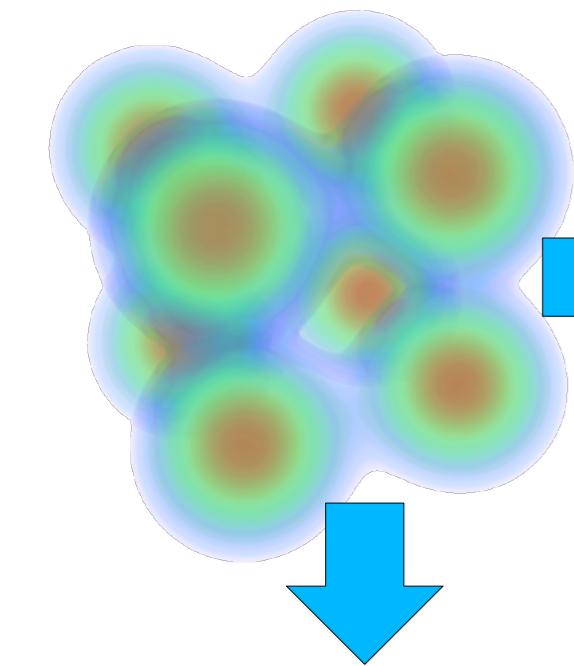
Gradient based features



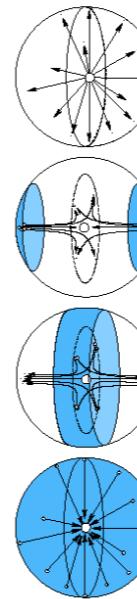
Morse-Smale Complex



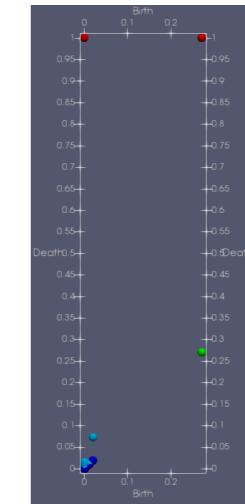
Features of a 3-dimensional function



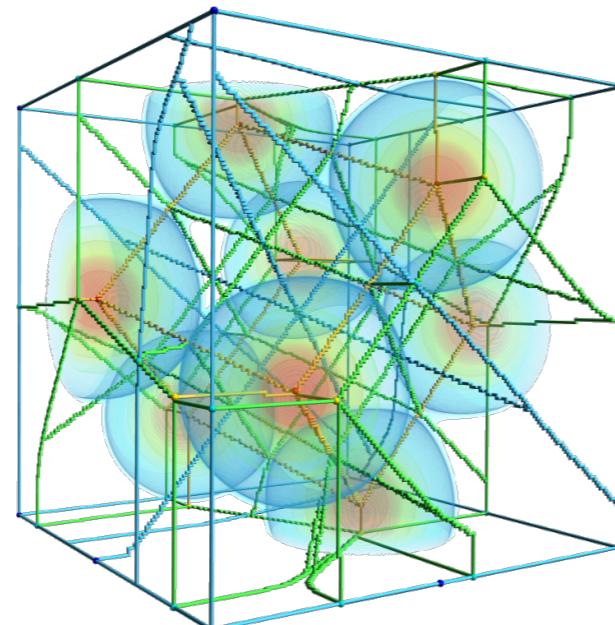
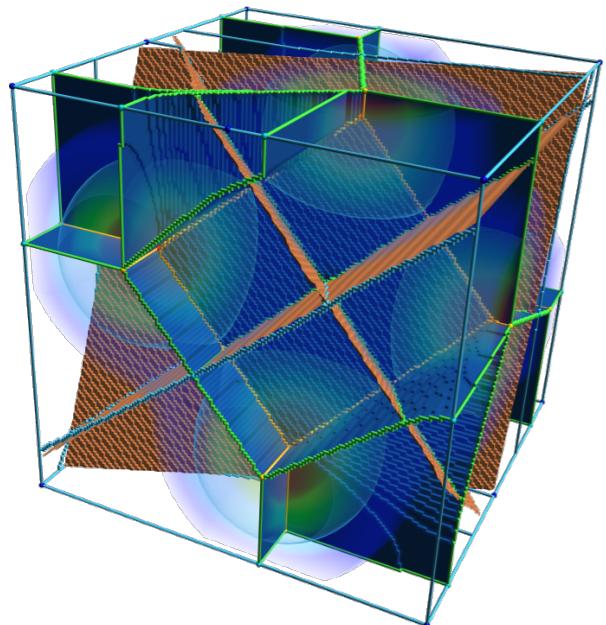
Split tree,
Join tree,
Contour trees
work the
same as 1-
and 2d



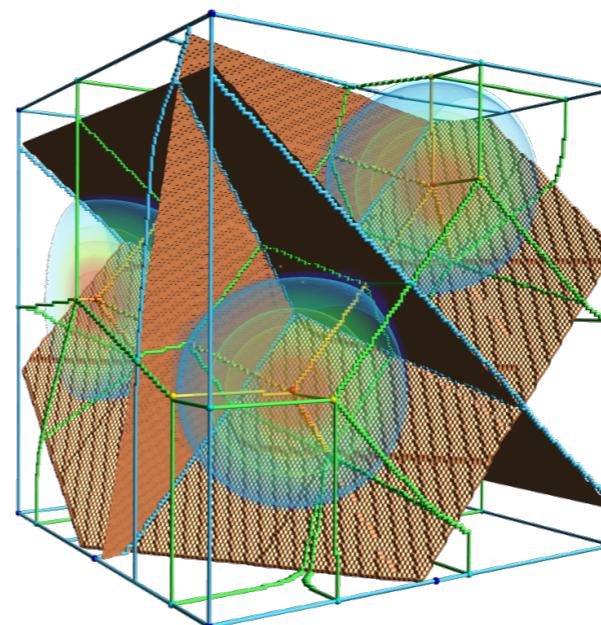
Minimum
1-saddle
2-saddle
Maximum



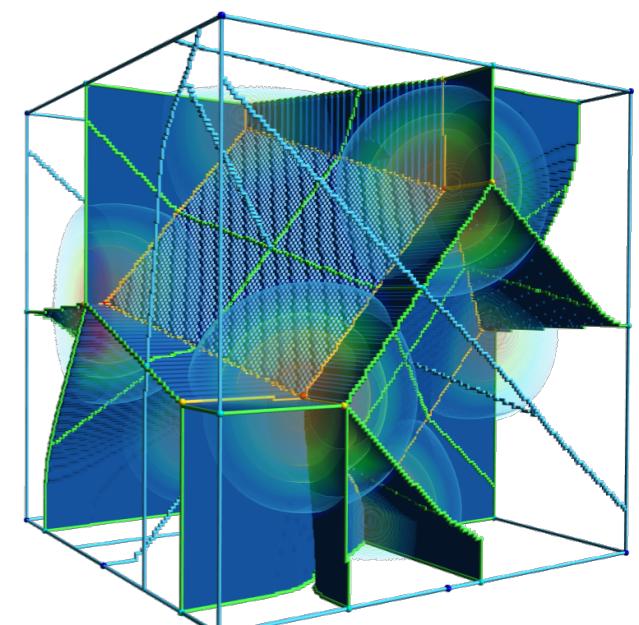
Persistence
diagrams also
record saddle-
saddle pairs



“Ridge lines” “Valley lines”
“Saddle connectors”



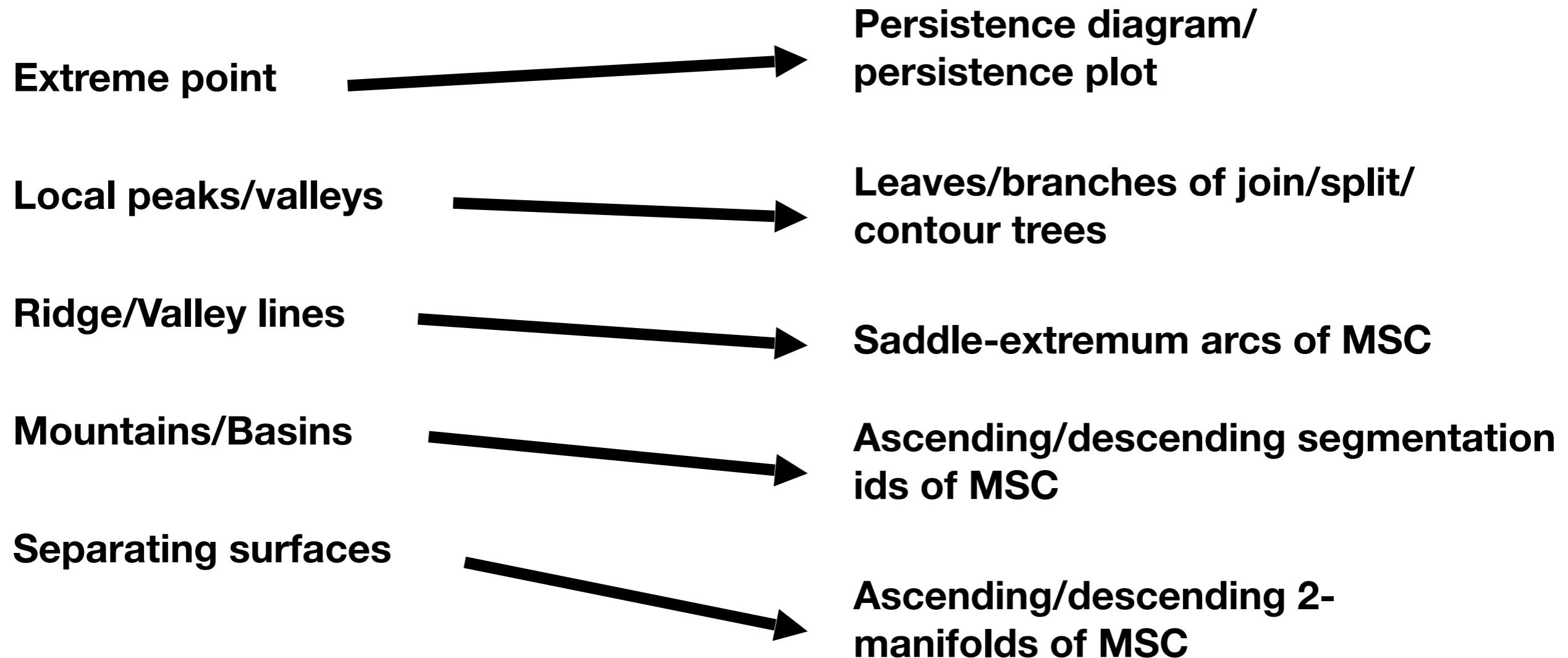
“Ridge surfaces”
Descending 2-manifolds Ascending 2-manifolds



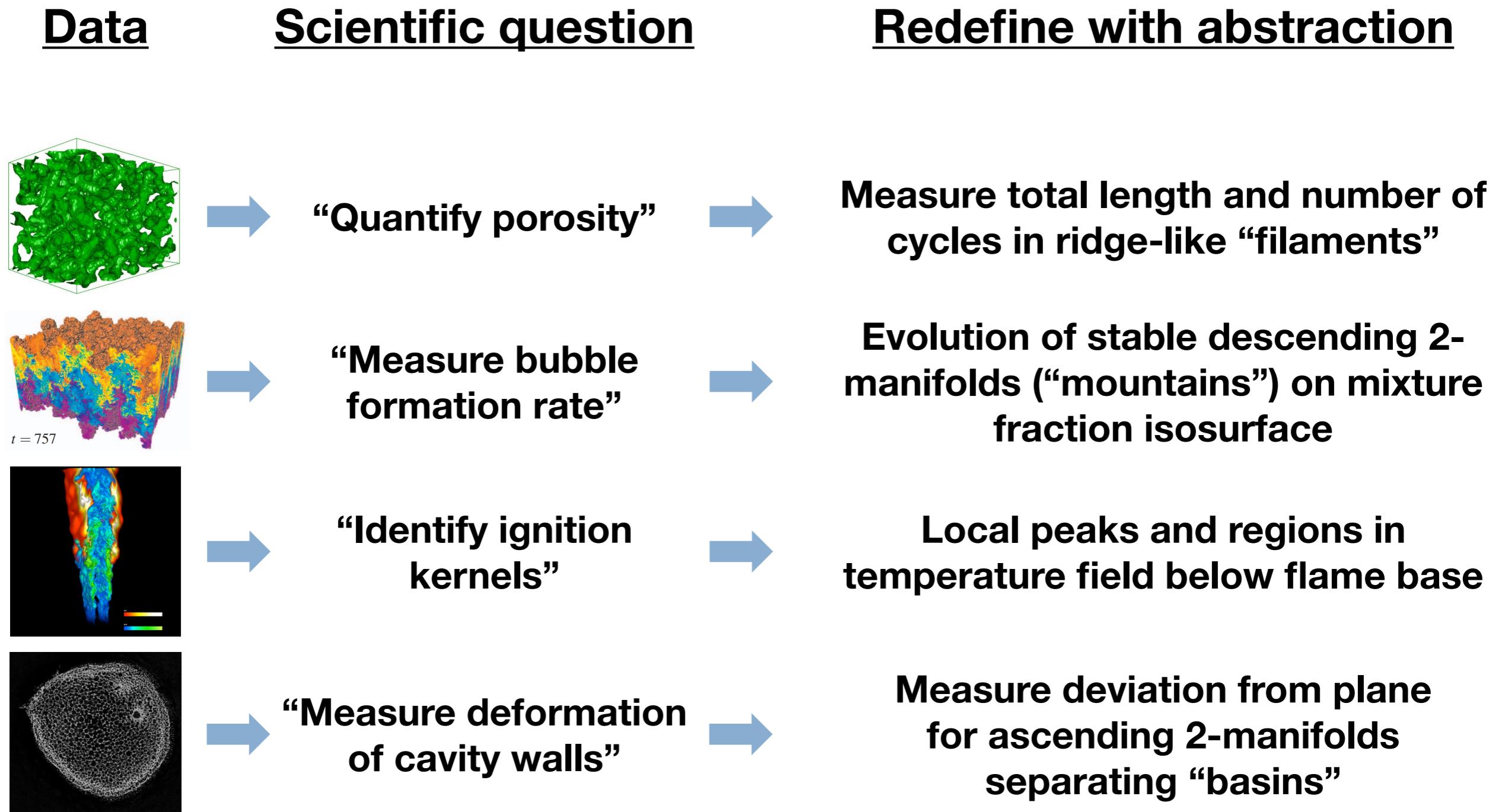
**How do we approach
an analysis problem?**

Answer: in reverse

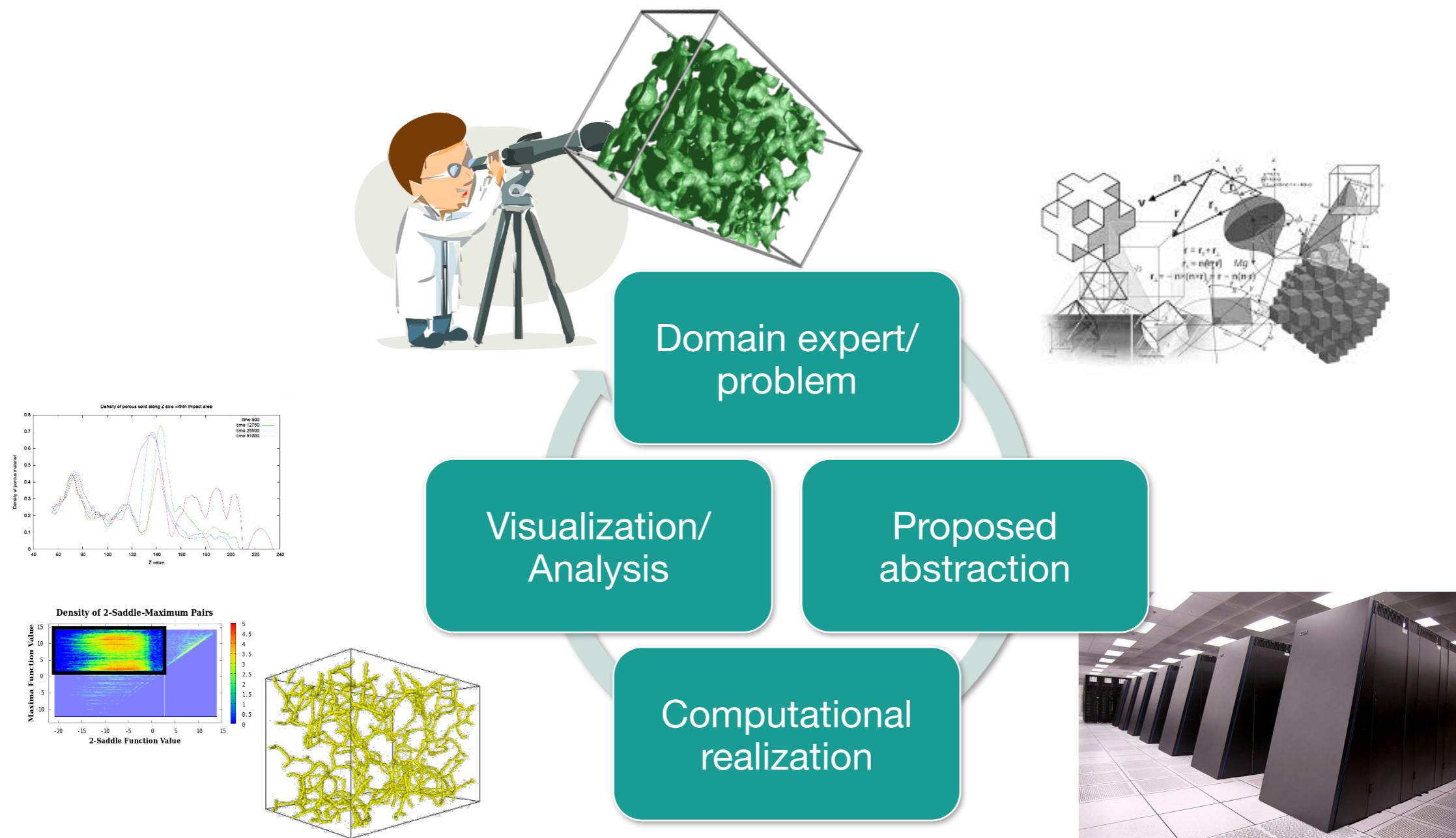
“I need to extract/identify/count/measure....”



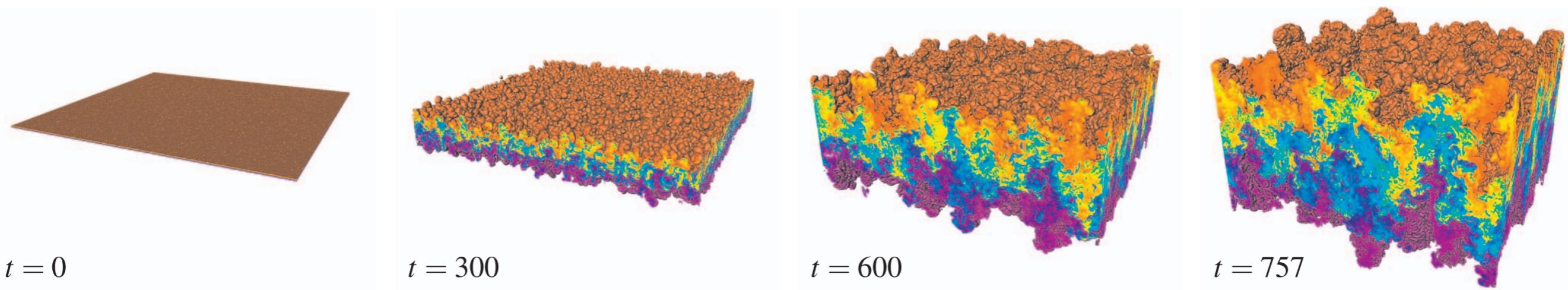
Create a hypothesis for topological features



Continuously re-evaluate hypothesis



Counting bubbles in turbulent mixing



$t = 0$

$t = 300$

$t = 600$

$t = 757$

Pick a value, extract isosurface, analyze height function of it

f

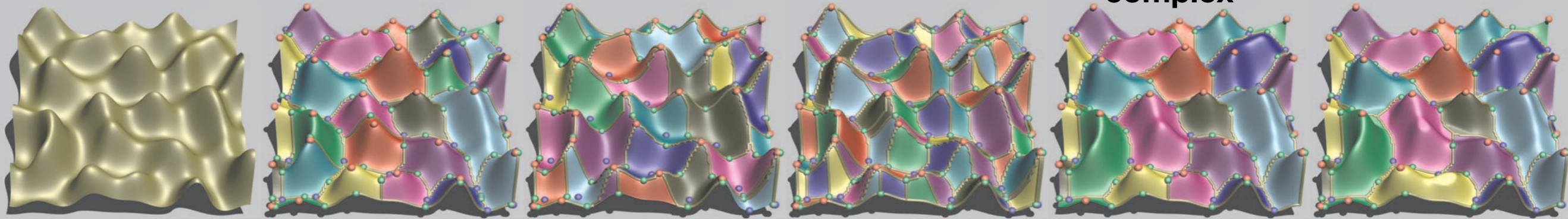
Descending

Ascending

MS-complex

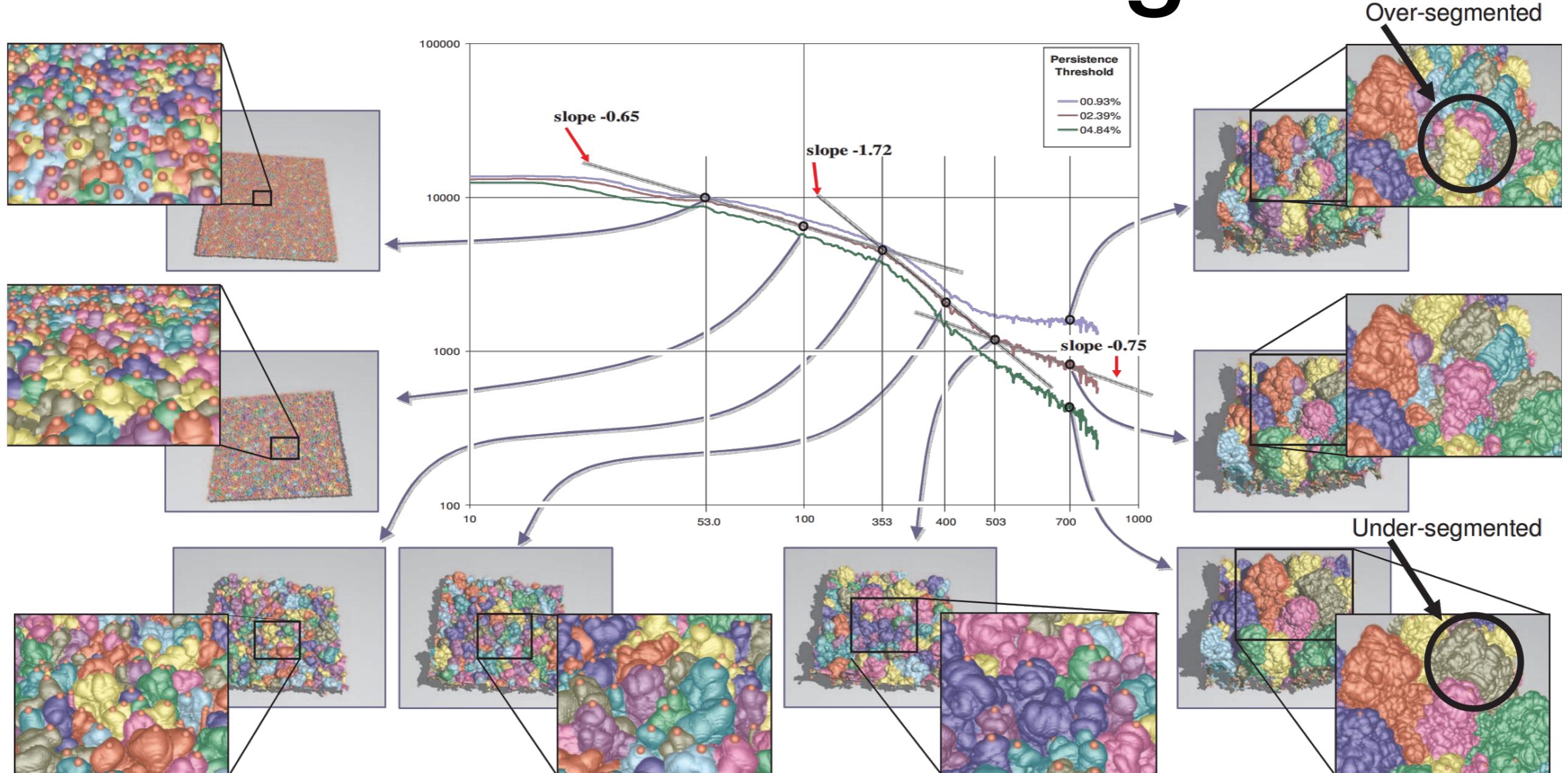
Simplified
complex

Simplified f

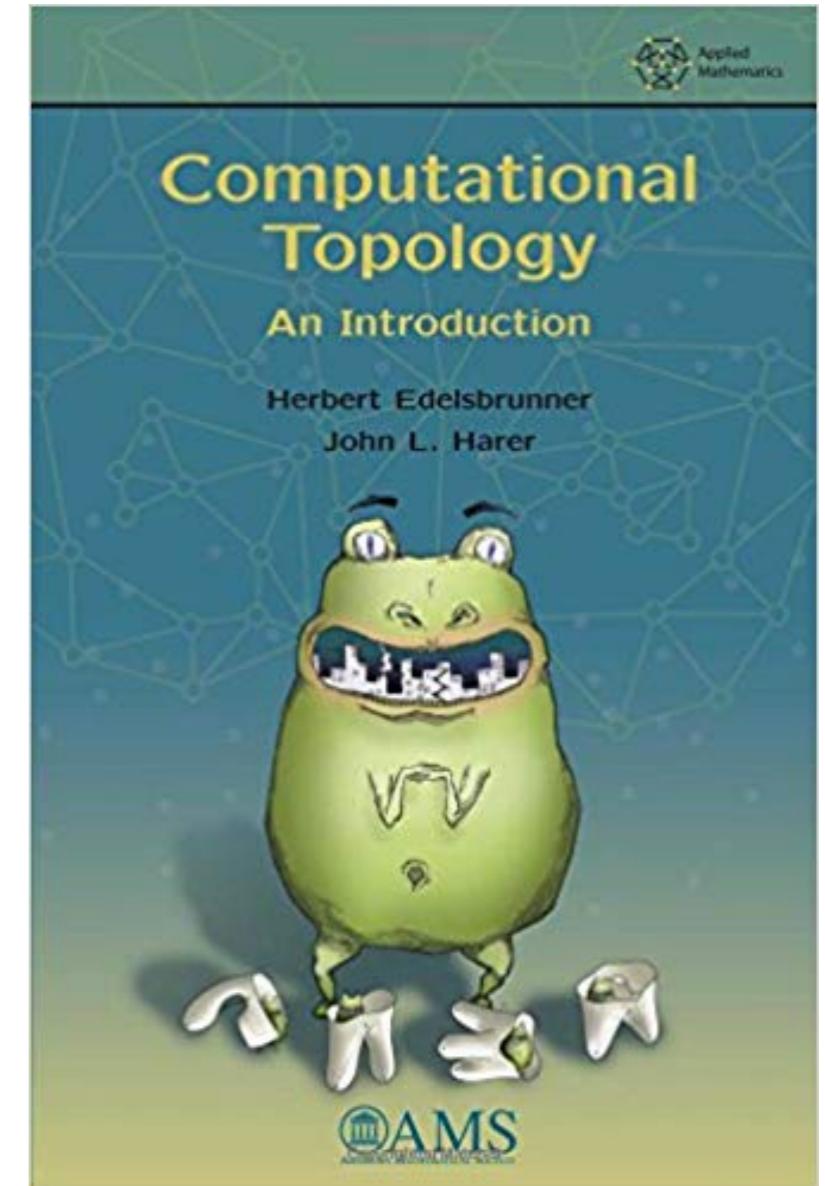
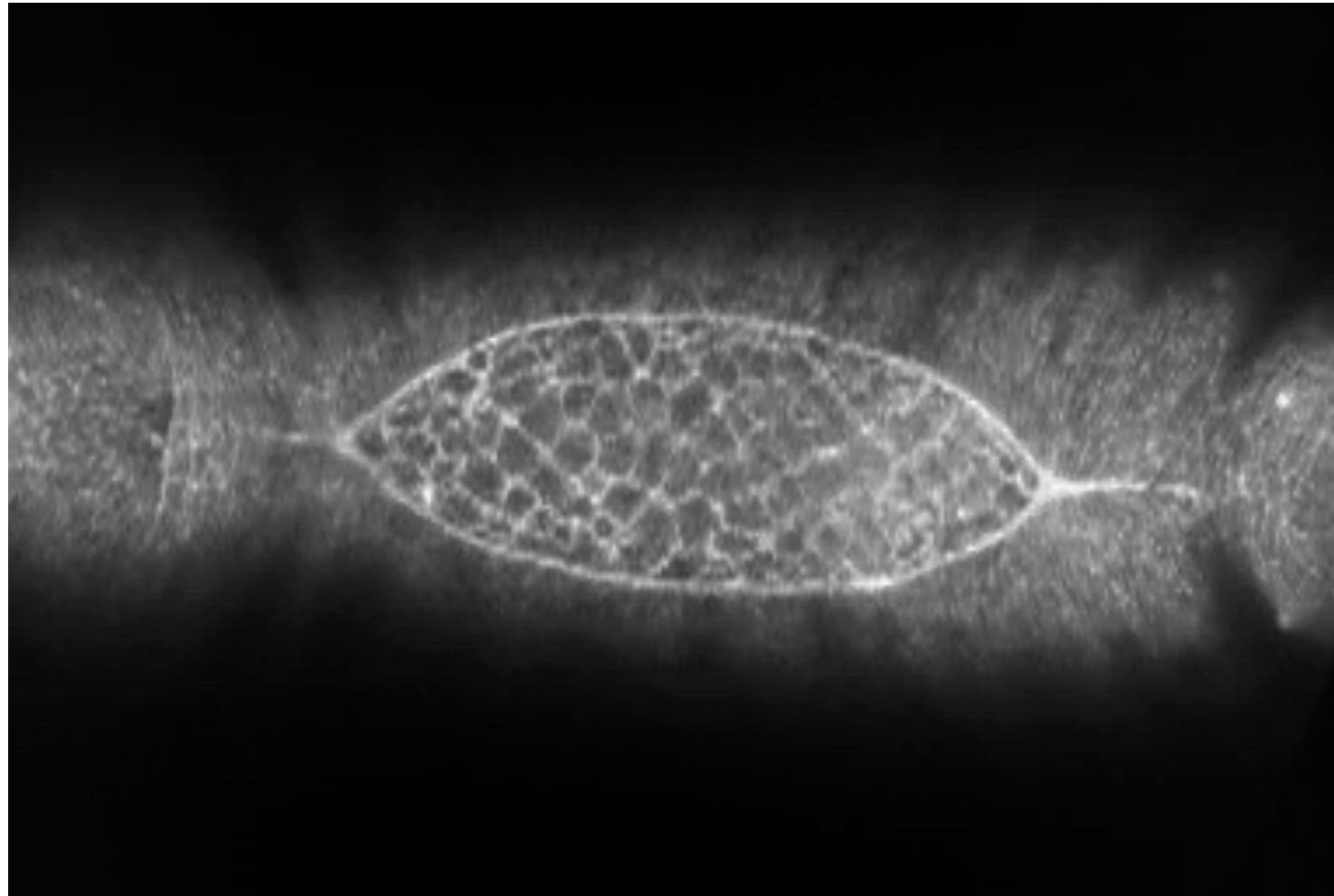


D. Laney et al., "Understanding the Structure of the Turbulent Mixing Layer in Hydrodynamic Instabilities," in IEEE TVCG, 12(5) Sept-Oct. 2006.

Counting bubbles in turbulent mixing



D. Laney et al., "Understanding the Structure of the Turbulent Mixing Layer in Hydrodynamic Instabilities," in IEEE TVCG, 12(5) Sept-Oct. 2006.



Live Demo

Confocal microscopy data from:

Kiehart et al. Multiple Forces Contribute to Cell Sheet Morphogenesis for Dorsal Closure in Drosophila. *The Journal of Cell Biology*, 149(2), 471–490 (2000).

<https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2175161/>

Lec22 Reading

- Munzner, Ch. 8.5
- Streak Lines as Tangent Curves of a Derived Vector Field.
Tino Weinkauf, Holger Theisel. IEEE Trans. Vis. Comput. Graph. 16(6): 1225-1234 (2010).

Reminder

Assignment 05

Assigned: Monday, March 27

Due: Monday, April 10, 4:59:59 pm

Reminder

Project Milestones 03/04

Assigned: Wednesday, March 29

03 (Talk) Due: Wednesday, April 26, 4:59:59 pm

04 (Report) Due: Wednesday, May 3, 4:59:59 pm