Practice Problem Set Solutions

Week 1

PP 1.0

Question 1 (0.5 points)

You should submit regrade requests within how much time of the grade being released?

- ○ whenever
- ○ 1 month
- ○ 72 hours
- ○ 10 minutes

Question 2 (0.5 points)

Homework and practice problems are never accepted late.

- ○ True
- ○ False

Question 3 (0.5 points)

The penalty for turning a project in late is 1 point per minute late.

- True
- False

Question 4 (0.5 points)

It is okay to work on the exams with another person.

- True
- False

PP 1.1

Question 1 (0.5 points)

$$\log_2(2^n) + \log_9\left(\frac{27}{81}\right) = n\log_2 2 + \log_9 27 - \log_9 81$$

$$= n + \frac{\log_3 27}{\log_3 9} - 2$$

$$= n + \frac{3}{2} - 2$$

$$= n - \frac{1}{2}$$

- $n - \frac{1}{2}$
- $2^n - \frac{1}{2}$
- $2^n + \frac{3}{2}$
- None of the above.

Question 2 (0.5 points)

$$16^{\log_4 N+2} + \left(2N^{-2}\right)^3 \left(N^4\right)^7 = 16^2 \cdot (4^2)^{\log_4 N} + 2^3 N^{-6} (N^{28})$$

$$= 256N^2 + 8N^{22}$$

- $16N + 8N^{22}$
- $16N^2 + 4N^{22}$
- $256N^2 + 8N^{22}$
- $256N^2 + 4N^{22}$
- None of the above.

Question 3 (1 point)

Consider the following "proof" that $\log_2\left(4^n\right) \;=\; 2^n \, for\ all\ n \geq 0.$

$$\log_2\left(4^n\right) \;=\; \log_2\left(2^2\right)^n = 2^n$$

$$\log_2\left(2^2\right)^n = \log_2 2^{2n} = 2n\log_2 2$$
$$= 2n$$

Which of the following is true?

- ◯ The proof is valid.
- ◯ The proof is invalid because $\log_2\left(4^n\right) \;\neq\; \log_2\left(2^2\right)^n$.
- ◯ The proof is invalid because $\log_2\left(2^2\right)^n \neq 2^n$.
- ◯ None of the above.

PP 1.2

Question 1 (0.5 points)

$2 + 4 + 6 + 8 + \ldots + N =$ $2\left(1+2+3+4+\ldots+N/2\right) = 2\sum_{k=1}^{N/2} k = \cancel{2}\left[\frac{\frac{N}{2}\left(N/2+1\right)}{\cancel{2}}\right]$

$= \frac{N^2}{4} + \frac{N}{2}$

- $\frac{N^2}{2} + N$
- $2N^2 + 2N$
- $N^2 + N$
- X $\frac{N^2}{4} + \frac{N}{2}$

Question 2 (0.5 points)

$2 + 4 + 8 + 16 + \ldots + N =$ $2\left(1+2+4+8+\ldots+N/2\right) = 2\sum_{k=0}^{\log_2 N/2} 2^k$

- $2N - 1$
- $2^{N/2+1} - 1$
- $2^{N+1} - 1$
- $\cancel{}$ $2N - 2$

$$= 2\left[\frac{2^{\log_2 N/2 + 1} - 1}{1}\right]$$

$$= 2\left(2\left(N/2\right) - 1\right)$$

$$= 2N - 2$$

Question 3 (0.5 points)

$\sum_{k=1}^{N} \sum_{j=1}^{N} \sum_{i=1}^{N} kj =$

- $\frac{N^3(N+1)^2}{4}$
- $\frac{N(N+1)}{2}$
- $\frac{N^3(N+1)^3}{8}$
- $\frac{N^2(N+1)^2}{2}$

$$\sum_{k=1}^{N} \sum_{j=1}^{N} kjN = \sum_{k=1}^{N} kN \left(\frac{N(N+1)}{2} \right)$$

$$= \frac{N^2(N+1)}{2} \sum_{k=1}^{N} k$$

$$= \frac{N^2(N+1)}{2} \cdot \frac{N(N+1)}{2}$$

$$= \frac{N^3(N+1)^2}{4}$$

Question 4 (0.5 points)

4 + 7 + 10 + 13 + ... + N =

3(1)+1 3(2)+1 3(3)+1 3(4)+1

$3x + 1 = N$

$x = \frac{N-1}{3}$

- $\sum_{k=1}^{N} 3^k$
- $\sum_{k=1}^{\frac{N-1}{3}} (3k + 1)$
- $\sum_{k=1}^{\log_3 N} \left(3^k + 1\right)$
- $\sum_{k=1}^{N} (3k + 1)$

PP 1.3

Question 1 (0.5 points)

Based on the table below, which of the following statements is true about f(n) and g(n)?

n	f(n)	g(n)
0	1 <	10
1	2 <	15
2	5 <	20
4	17 <	30
8	65 >	50
16	257 >	90

- ○ f(n) grows faster than g(n)
- ○ g(n) grows faster than f(n)
- ○ f(n) and g(n) grow at the same rate

Question 2 (0.5 points)

constant

Given that $\frac{f(n)}{g(n)} = \frac{2}{3}$, which of the following statements is true about f(n) and g(n)?

- ( ) f(n) and g(n) grow at the same rate
- ( ) f(n) grows faster than g(n)
- ( ) g(n) grows faster than f(n)

Question 3 (0.5 points)

Given that $\frac{f(n)}{g(n)} = \frac{2 \log n}{3}$, which of the following statements is true about f(n) and g(n)?

- f(n) grows faster than g(n)
- f(n) and g(n) grow at the same rate
- g(n) grows faster than f(n)

Question 4 (0.5 points)

Given the table below, which of the following statements is true about f(n) and g(n)?

slope = 3 → f(n) is cubic

slope = 2 → f(n) is quadratic

logn	log(f(n))	log(g(n))
1	0 +3	1 +2
2	3 +3	3 +2
3	6 +3	5 +2
4	9 +3	7 +2
5	12 +3	9 +2
6	15	11

- ○ f(n) grows faster than g(n)
- ○ f(n) and g(n) grow at the same rate
- ○ g(n) grows faster than f(n)

PP 1.4

Question 1 (1 point)

$\theta(n^3)$ $\theta(n^3)$

Given that $f(n) = 2n^3 + 4n$ and $g(n) = \frac{n^2 \log(4^n)}{2}$, select all the statements that are true.

$g(n) = \frac{n^2 \log(4^n)}{2} = \frac{n^3 \cancel{\log 4}}{\cancel{2}} = n^3$

- [x] $f(n)$ is $\theta(g(n))$
- [x] $g(n)$ is $O(f(n))$
- [x] $f(n)$ is $O(g(n))$
- [x] $f(n)$ is $\Omega(g(n))$

Question 2 (1 point)

$\Theta(n^4)$ $\Theta(n^2 \log n)$

Given that $f(n) = 2n^4 + 4n^2$ and $g(n) = \frac{n^2 \log(n^4)}{2}$, select all the statements that are true.

$= \frac{4n^2 \log n}{2}$

- [x] $g(n)$ is $O(f(n))$
- [ ] $f(n)$ is $O(g(n))$
- [x] $f(n)$ is $\Omega(g(n))$
- [ ] $f(n)$ is $\theta(g(n))$

PP 1.5

Question 1 (0.25 points)

Given that $f(n) = 4n^3 + 4n^2 + 8n + 1$, which values of c and n_0 are sufficient for showing that $f(n)$ $is\ O(n^3)$?

- ✓ $c = 32\ and\ n_0 = 1$
- $c = 8\ and\ n_0 = 2$
- $c = 8\ and\ n_0 = 1$
- $c = 4\ and\ n_0 = 2$

$4n^3 + 4n^2 + 8n + 1$

$\leq 8n^3 + 8n^3 + 8n^3 + 8n^3$

$\leq 32n^3$

Question 2 (0.25 points)

Given that $f(n) = 4n^3 \log n + 4n^2 \log n + 8n \log n + \log n + 1$, which values of c and n_0 are sufficient for showing that $f(n)$ *is* $O(n^3 \log n)$?

- $c = 40$ *and* $n_0 = 1$
- $c = 8$ *and* $n_0 = 1$
- $c = 8$ *and* $n_0 = 2$
- $c = 40$ *and* $n_0 = 2$

Question 3 (0.25 points)

$$7n + 3n^2 + \underbrace{n\log\left(2^{n^2}\right)}_{n^3 \log 2 \,=\, n^3} \text{ is } O\left(n^2\right).$$

- ○ True
- ○ False

Question 4 (0.25 points)

$$4n\log^2 n \; + \; 2^n + 3n^3 \text{ is } \Omega\left(n^3\right).$$

- ○ True
- ○ False

Question 5 (0.25 points)

$$\underbrace{16n\log(16)}_{16(4)n} + \underbrace{16^{\log_2 n}}_{(2^4)^{\log_2 n} = n^4} \; is\ \theta(n).$$

- [ ] True
- [x] False

Question 6 (0.25 points)

$$n\log\left(n^2\right) + n^2\log n \; is\ O(n\log n).$$

- [ ] True
- [x] False

Question 7 (0.25 points)

$$4n\log\left(\frac{n}{4}\right) + 5n \text{ is } \Omega\left(n\log n\right).$$

- [x] True
- [ ] False

Question 8 (0.25 points)

$$7(2^n) \text{ is } \Omega(1).$$

- [x] True
- [ ] False

PP 1.6

Question 1 (1 point)

Given that $f(n)$ *is* $O(g(n))$, $g(n)$ *is* $O(h(n))$, *and* $h(n)$ *is* $\Omega(i(n))$, select all the statements below that *must be true.*

- [x] $h(n)$ *is* $\Omega(f(n))$.
- [ ] $f(n)$ *is* $O(i(n))$.
- [x] $f(n) + g(n) + h(n) + i(n)$ *is* $O(h(n))$.
- [x] $f(n) + g(n) + h(n) + i(n)$ *is* $\Omega(h(n))$.

h i
g h i
h i
f g h i
i

PP 1.7

Question 1 (0.25 points)

Given a proposition P(n), if you prove P(1) and P(k) --> P(k+1), then you have proven P(n) for

$P(1) \to P2$

$P(2) \to P(3)$

$\vdots$

- ◯ all even integers
- ◯ all odd integers
- ◯ no integers--you have not proven P(n)
- ◯ all positive integers
- ~~◯~~ all integers

Question 2 (0.25 points)

Given a proposition P(n), if you prove P(1) and P(k) --> P(k+2), then you have proven P(n) for

$P(1) \to P(3)$
$P(3) \to P(5)$
$\vdots$

- ◯ all even integers
- ◯ no integers--you have not proven P(n)
- ◯ all odd integers
- ◯ all positive integers
- ◯ all integers

Question 3 (0.25 points)

Given a proposition P(n), if you prove P(1) and P(floor(k/2)) --> P(k), then you have proven P(n) for

$P(1) \to P(2), P(3)$

$P(2) \to P(4), P(5)$

$P(3) \to P(6), P(7)$

$\vdots$

- ( ) all positive integers
- ( ) all powers of 2
- ( ) no integers--you have not proven P(n)
- ( ) all even integers
- ( ) all odd integers

Question 4 (0.25 points)

Given a proposition P(n), if you prove P(1), P(2), and P(floor(k/3)) --> P(k), then you have proven P(n) for

$P(1) \to P(3), P(4), P(5)$

$P(2) \to P(6), P(7), P(8)$

$P(3) \to P(9), P(10), P(11)$

$\vdots$

- ◯ no integers--you have not proven P(n)
- ✓ all positive integers
- ◯ all odd integers
- ◯ all powers of 3
- ◯ all even integers

PP 1.8

Question 1 (0.5 points)

Consider the proof below that $n! > n^3$ for all $n \geq 6$.

1 Basis Step

2 $$6! = 720 > 216 = 6^3$$

3 Inductive Step

4 Assume as the IH that $k! > k^3$ for some $k \geq 6$.

5 $(k+1)! = (k+1)k!$

6 $> (k+1)^3(k^3)$ by the IH

7 $> (k+1)^3$

The IH can only be applied to k!. It should not affect (k+1).

In which line is there an error?

- ◯ Line 2
- ◯ Line 4
- ◯ Line 5
- ◯ Line 6
- ◯ None of the above.

Question 2 (0.5 points)

Consider the proof below that $a^{n-1} = 1$ for all $n \geq 1$.

1 Basis Step

2 $a^{1-1} = a^0 = 1$

3 Inductive Step

4 Assume as the IH that $a^{k-1} = 1$ for some $k \geq 1$.

5 $a^k = a^{k-1} * a^1$

6 $= 1 * 1$ by the IH → uses $a^1 = 1$, which has not been proven.

7 $= 1$

In which line is there an error?

- Line 2
- Line 4
- Line 5
- Line 6
- None of the above.

Question 3 (1 point)

Consider the proof below.

Let T(N) = T(N-1) + N and T(1) = 1.

Conjecture: T(N) is $O\left(N^2\right)$.

Proof.

1 Basis Step.

2 $T\left(1\right) \; = \; 1 \; \leq 1^2$

3 Inductive Step.

4 Assume as the IH that $T\left(k\right) \; \leq k^2$ for some $k \geq 1$.

5 $T(k+1) \; = \; T(k) \; + \; k \; + \; 1$

6 $\leq k^2 + k + 1$ by the IH

7 $\leq k^2 + 2k + 1$

8 $= \; (k+1)^2$

Select all the statements that are true about the proof.

- [x] The basis step is correct.
- [x] The IH is correct.
- [x] Line 5 is correct.
- [x] The application of the IH is correct.
- [x] Line 7 is correct.
- [x] Line 8 is correct.
- [x] The proof correctly proves the conjecture.