CSC 545 **Final Exam** **Spring 2022**

Name _____

Instructions This exam consists of a *single* problem worth 100 points. From Problems (1)–(3), choose *one*. (If you do more than one problem, only one will be graded.)

In general, when asked to design an algorithm, you should:

(a) describe your algorithm using prose and pictures,
(b) argue that your algorithm is correct, and
(c) analyze its running time.

Pseudocode is not required.

During this take-home exam, the only resources you are allowed to use are: your personal *course notes*, the *course handouts*, the *textbook*, and the videos of *recorded lectures* from this course. Individual work is required. Use of *any* resource in preparing your answers *must be cited*.

You must submit your answers on `Gradescope` (at `www.gradescope.com`) for the course CSC 545 by 11:59pm MST on Wednesday, May 11. Upload a single PDF file containing your submission, and be sure to mark which pages of your submission correspond to which problem. If you write your answers by hand on paper, use an application to scan your pages such as `CamScanner` (a free smartphone app).

I will be available online through `Zoom` during 3:30–4:30pm MST to answer questions concerning the exam at:

`https://arizona.zoom.us/j/5128057331`

Please only enter the waiting room if you have a question about the exam.

Good luck!

(1) **(Greedy algorithms)** (100 points) Given a list $x_1, x_2, \ldots, x_n$ of distinct real numbers, and another list $y_1, y_2, \ldots, y_n$ of distinct real numbers, reorder the x_i into a new list x_i', and reorder the y_i into a new list y_i', so as to

$$\text{minimize} \max_{1 \le i \le n} \left| x_i' - y_i' \right|.$$

Design an efficient *greedy algorithm* that finds an optimal solution, analyze its running time, and prove that it is correct.

(Note: For your correctness proof, be sure to prove a greedy augmentation lemma.)

(2) **(Amortized algorithms)** (100 points) Suppose we want to support a balanced search tree data structure that does *automatic backup* according to the following scheme. After executing each search tree operation, we check whether at least n operations on the tree have elapsed since the last backup was performed, where n is the current size of the tree. If this many operations have elapsed, a backup is performed by writing a copy of the entire tree to a file.

Show how to implement automatic backup of balanced search trees so that *all* tree operations, including those that cause backups to occur, run in $O(\log n)$ *amortized time*.

(Hint: Use the accounting method in your analysis.)

(3) **(Graph algorithms)** (100 points) Given an undirected graph $G = (V, E)$, a *cloak* for G is an edge subset $C \subseteq E$ such that every vertex in V is touched by some edge in C. The *minimum cloak problem* is, given graph G, to find a cloak C of minimum cardinality $|C|$.

Design an efficient *graph algorithm* that, given G, finds a cloak for G of minimum cardinality. You may assume that every vertex in graph G is touched by some edge, so that a cloak always exists.

(Note: Be sure to prove that your algorithm finds an optimal cloak.)