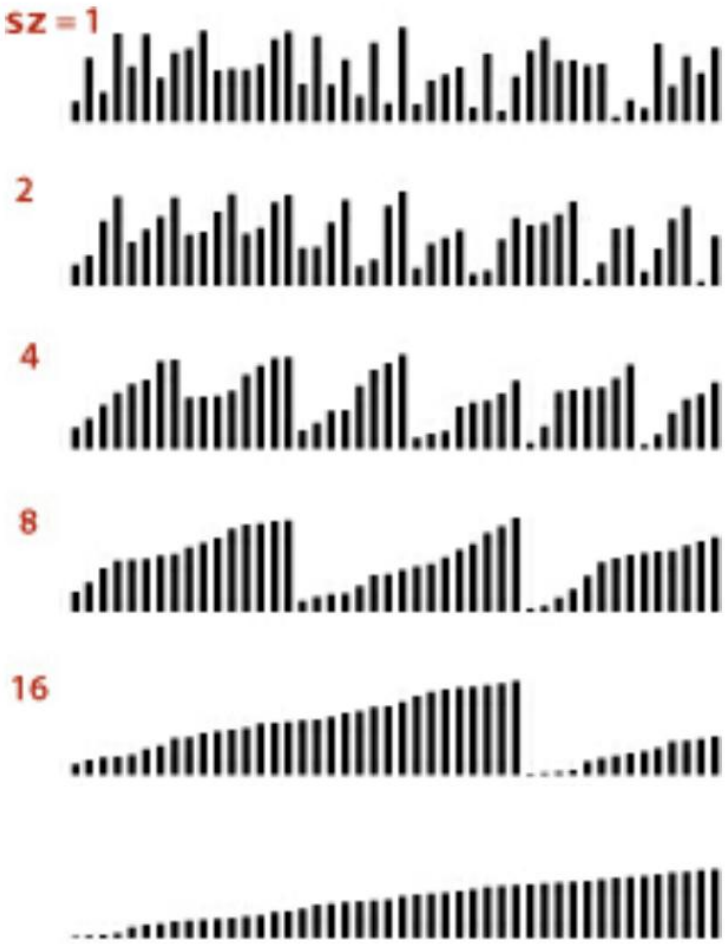
Sorting IV

- Divide-and-Conquer Paradigm
- MergeSort Overview
- Analysis

Divide-and-Conquer

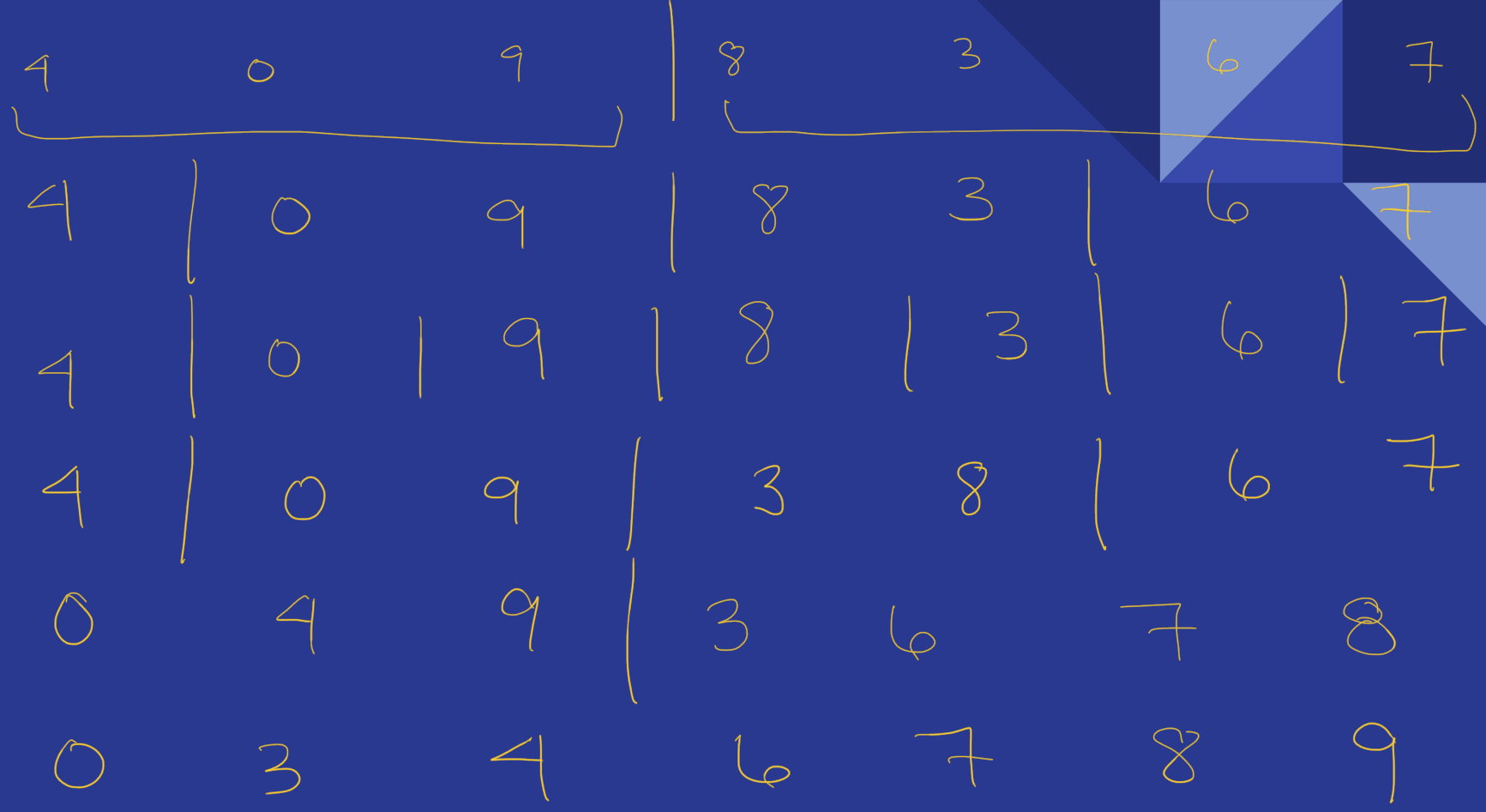
Solve a problem by

- dividing it into smaller problems
- solving the smaller problems
- combining them back together



Visual trace of bottom-up mergesort

Picture Sedgewick and Wayne


4 0 9 | 8 3 6 7
4 | 0 9 | 8 3 | 6 7
4 | 0 | 9 | 8 | 3 | 6 | 7
4 | 0 9 | 3 8 | 6 7
0 4 9 | 3 6 7 8
0 3 4 6 7 8 9

MergeSort Overview

split in half

↓

base case (1 element)

↓

combines with merging

sorting →

$T(N) = 2T(N/2) + N$

$T(1) = 1$

Algorithm *mergeSort*(*S*, *C*)

Input sequence *S* with *n* elements, comparator *C*

Output sequence *S* sorted according to *C*

if *S.size*() > 1

O(1) $(S_1, S_2) \leftarrow partition(S, n/2)$

$mergeSort(S_1, C)$

$mergeSort(S_2, C)$

O(N) → $S \leftarrow merge(S_1, S_2)$

Picture from Tamassia and Goodrich

MergeSort Runtime

```
Algorithm mergeSort(S, C)
    Input sequence S with n
        elements, comparator C
    Output sequence S sorted
        according to C
    if S.size() > 1
        (S1, S2) ← partition(S, n/2)
        mergeSort(S1, C)
        mergeSort(S2, C)
        S ← merge(S1, S2)
```

Picture from Tamassia and Goodrich

```
Algorithm mergeSort(Array A of size N):
        temp = an array of size N
        mergeSortAux(A, temp, 0, N-1)
end mergeSort

procedure mergeSortAux(Array A, Array temp, int i, int j)
        if (j-i<1):
                return
        m = (j+i)/2
        mergeSortAux(A, temp, i, m)
        mergeSortAux(A, temp, m+1, j)
        merge(A, temp, i, m, m+1, j)
end mergeSortAux
```

base case

split

```
Algorithm mergeSort(Array A of size N):
      temp = an array of size N
      mergeSortAux(A, temp, 0, N-1)
end mergeSort

procedure mergeSortAux(Array A, Array temp, int i, int j)
      if (j-i<1):
            return
      m = (j+i)/2
      mergeSortAux(A, temp, i, m)
      mergeSortAux(A, temp, m+1, j)
      merge(A, temp, i, m, m+1, j)
end mergeSortAux
```

```
procedure merge(Array A, Array temp, int a, int b, int x, int y)
      k = a
      c = a
      while(a <= b || x <= y)
            if (a > b):
                  temp[c] = A[x]
                  c++
                  x++
            else if (x > y):
                  temp[c] = A[a]
                  c++
                  a++
            else if (A[a] <= A[x]):
                  temp[c] = A[a]
                  c++
                  a++
            else
                  temp[c] = A[x]
                  c++
                  x++
      end while
      for i from k to y:
            A[i] = temp[i]
end merge
```

Analysis: Recurrence Relation

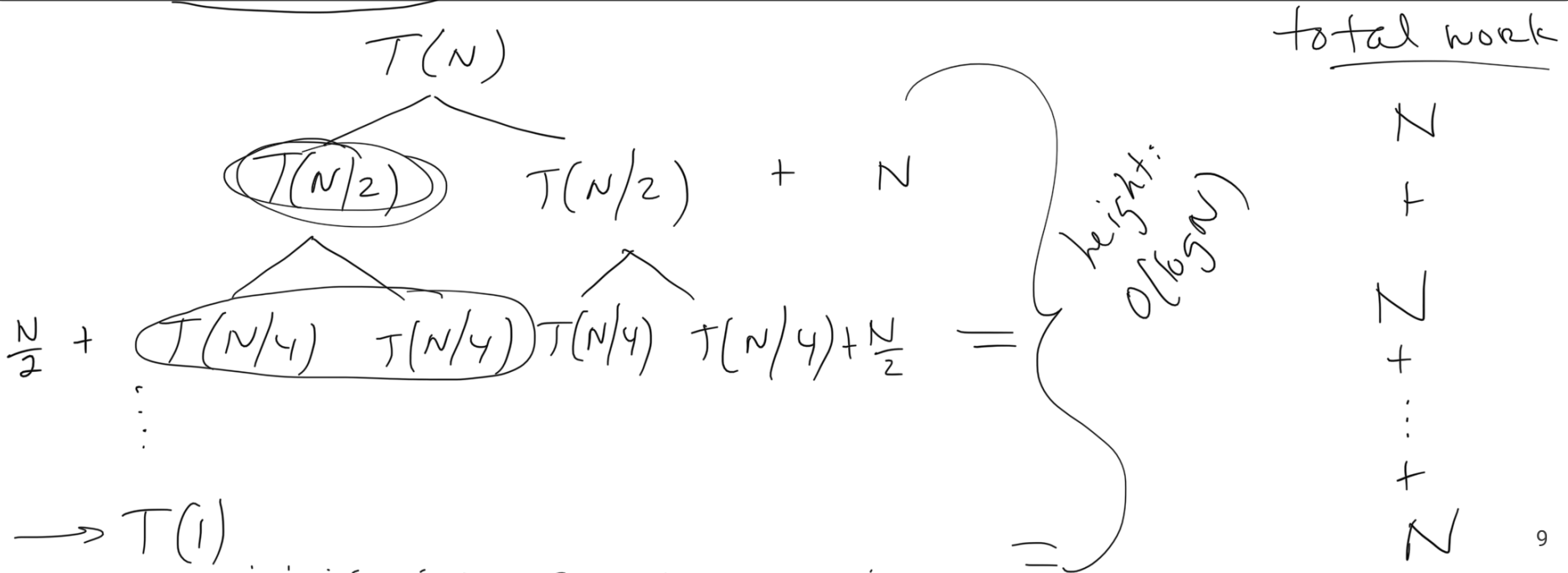
$a = 2$
$b = 2$
$d = 1$

$2 = 2^1$

$O(N \log N)$

Recurrence Relation:
$T(N) = 2T(N/2) + N$

Base Case:
$T(1) = 1$



Analysis: Tree Representation (N elements)

- How much work is done at each level?

N

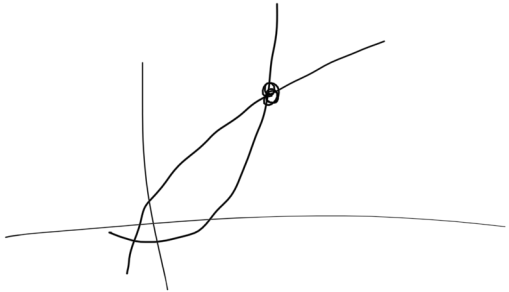
- How many levels are there?

$\log N$

- What's the total work required?

$O(N \log N)$

Mergesort Summary



- Guarantees the proven best worst case runtime for comparison-based sorting methods: O(NlogN)
- Uses a Divide & Conquer approach where subproblems are smaller sorted arrays that are then combined through merging. Note that the actual sorting is done through the merging because the base case (1 element) is already sorted.
- Not typically done in place. Space usage takes into account the call stack (which is O(logN)) and the extra copy of the array (which is O(N)), so the space usage is O(N).
- Sometimes, Mergesort is combined with Insertion Sort since the latter works well on tiny arrays. For example, you might use Mergesort but instead of dividing and combining all the way to 1-element arrays, you stop at an array of size 16 or less and sort those with Insertion Sort before combining them back together. An example of this kind of hybrid sorting is Timsort.