Cryptocurrencies without Proof of Work

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Abstract

We study decentralized cryptocurrency protocols in which the participants do not deplete physical scarce resources. Such protocols commonly rely on *Proof of Stake*, i.e., on mechanisms that extend voting power to the stakeholders of the system. We offer analysis of existing protocols that have a substantial amount of popularity. We then present our novel pure *Proof of Stake* protocols, and argue that they help in mitigating problems that the existing protocols suffer from.

1 Introduction

The decentralized nature of Bitcoin [14] means that anyone can become a "miner" at any point in time, and thus participate in the security maintenance of the Bitcoin system and be compensated for this work. The miners continuously perform *Proof of Work* (PoW) computations, meaning that they attempt to solve difficult computational tasks. The purpose of the PoW element in the Bitcoin system is to reach consensus regarding the ledger history, thereby synchronizing the transactions and making the users secure against double-spending attacks.

The miners who carry out PoW computations can be viewed as entities who vote on blocks of transactions that the users recently broadcasted to the network, so that the decision-making power of each miner is in proportion to the amount of computational power that she has. Thus, an individual miner who has p fraction of the total mining power can create each new block with probability $\approx p$, though "selfish mining" [8] and other factors [2, 4] can influence p.

Under the assumption that the majority of the PoW mining power follows the Bitcoin protocol, the users can become increasingly confident that the payment transactions that they receive will not be reversed [9, 14, 18].

By means of the PoW mechanism, each miner depletes physical scarce resources in the form of electricity and mining equipment erosion, and thereby earns cryptographic scarce resources in the form of coins that can be spent within the Bitcoin system.

Hence the following question is of interest: can a *decentralized* cryptocurrency system be as secure as Bitcoin even if the entities who maintain its security do not deplete physical scarce resources?

Cryptocurrency protocols that attempt to avoid wasting physical scarce resources commonly rely on *Proof of Stake*, i.e., on mechanisms that give the decision-making power regarding the continuation of the ledger history to entities who possess coins within the system. The rationale behind *Proof of Stake* is that entities who hold stake in the system are well-suited to maintain its security, since their stake will diminish in value when the security of the system erodes. Therefore, in an analogous manner to Bitcoin, an individual stakeholder who possesses p fraction of the total amount of coins in circulation becomes eligible to create the next extension of the ledger with probability $\approx p$.

We use the terminology "pure" *Proof of Stake* to refer to a cryptocurrency system that relies on *Proof of Stake* and does not make any use of PoW. To the best of our knowledge, the idea of *Proof of Stake* in the context of cryptocurrencies was first introduced in [23], though that discussion focused on non-pure *Proof of Stake* variants.

PoW based cryptocurrencies become insecure when a significant enough portion of the total mining power colludes in an attack. Likewise, the security of pure *Proof of Stake* cryptocurrencies deteriorates when enough stakeholders wish to collude in an attack. If the majority of the stake wishes to participate in attacks on a pure *Proof of Stake* system, it can be argued that there is no longer enough interest that this system should continue to exist, hence assuming that the majority of the stake will not participate in an (overt) attack is sensible. The same does not necessarily hold in a PoW based system, i.e., the majority of the mining power might be under the control of an external adversary during some time period, while the majority of the participants in this system still wish for it to remain sound. See [4] and Section 3 for additional considerations.

1.1 Related work

In [4], authors of this work presented a hybrid protocol that relies both on PoW and *Proof of Stake*, where the objective is to combine to advantageous properties of the PoW element and the *Proof of Stake* element into a system that is superior to relying on only one of these two elements.

An alternative to PoW that is based on data storage is presented in [1, 7], though in such *Proof of Space* systems the participants may still opt to make use of PoW due to an inherent time-space tradeoff. While no concrete construction for a cryptocurrency system that is based on *Proof of Space* has been given yet, recent progress towards this goal is worth consideration [16]. In any case, *Proof of Space* based protocols still require depletion of physical scarce resources to some significant degree, unlike the focus of this work.

1.2 Organization of the Paper

The main contributions of this work are organized as follows. In Section 2.1 we analyse an existing pure *Proof of Stake* system. In Section 2.2 we present a novel pure *Proof of Stake* protocol, analyse its properties, and argue that it is more secure than the existing system. In Section 2.3 we present another variant of our pure *Proof of Stake* protocol, and explore the tradeoffs that it offers in terms of security, fairness, and efficiency. In Section 3 we explain the greater importance of checkpointing the ledger history of pure *Proof of Stake* systems compared to PoW based systems, and specify our proposal for handling this issue. In Section 4 we examine how to conduct the initial issuance of coins in a pure *Proof of Stake* system.

2 Pure *Proof of Stake*

There are two apparent hurdles with decentralized pure *Proof of Stake* systems: fair initial distribution of the money supply to the interested parties, and network fragility if the nodes are rational rather than altruistic. PoW offers an elegant solution to the first hurdle, by converting physical scarce resources into coins in the system. We provide here an analysis of the second hurdle in an existing pure *Proof of Stake* system, and also describe our novel "CoA" and "Dense-CoA" pure *Proof of Stake* systems that seek to mitigate this problem. Let us note this second hurdle is less severe in PoW systems, though bribe attacks on Bitcoin have indeed been considered, for example in [21].

2.1 The PPCoin system

PPCoin is a pure *Proof of Stake* system, in the sense that PoW is used only for distributing the initial money supply[1]. Stakeholders in the PPCoin network can create the next block according to the following type of condition:

$$\texttt{hash}(\texttt{prev\_blocks\_data}, \texttt{time\_in\_seconds}, txout_A) \leq d_0 \cdot \texttt{coins}(txout_A) \cdot \texttt{timeweight}(txout_A) \quad (*)$$

[1] See http://peercoin.net/assets/paper/peercoin-paper.pdf. It should be noted that the PPCoin protocol keeps being tweaked, which can easily be done because it is controlled by a centralized entity via signed checkpoints, as opposed to being controlled by a decentralized network. For security, the users of the PPCoin system currently rely on PoW blocks and on the checkpoints of the centralized entity.

Where `time_in_seconds` should correspond to the current time (with some leniency bounds), thus restricting hash attempts to 1 per second and preventing PoW use at creating the next block, because nodes will regard a new block as invalid unless the difference between its time and their local time is within the bounds. The notation $\texttt{coins}(txout_A)$ refers to the amount of coins of some unspent transaction output $txout_A$, hence if stakeholder A has the private key sk_A that controls $txout_A$ then she can create a valid block by signing it with sk_A and attaching the evidence for condition (*). This means that a stakeholder who controls an output of e.g. 50 coins is 10 times more likely to create a block than a stakeholder who controls an output of 5 coins. See Section 2.1.3 regarding $\texttt{timeweight}(txout_A)$, and Section 2.1.4 regarding `prev_blocks_data`. The constant d_0 is readjusted according to a protocol rule that dictates that blocks should be created in intervals of 10 minutes on average, i.e. if fewer stakeholders are online during a certain time period then d_0 gets increased. The winning blockchain is the one with the largest cumulative stake, i.e., the blockchain with the most blocks such that stake blocks are weighted according to their d_0 difficulties, and PoW blocks have a negligible weight.

Although the PPCoin cryptocurrency had a market cap of over $100 million in 2014, the PPCoin protocol has the following problems:

2.1.1 Rational forks

On every second we have that $\Pr[\{\text{some block is solved}\}] \approx \frac{1}{600}$, therefore multiple blocks will be solved simultaneously every ≈ 360000 seconds ≈ 4 days. Rational stakeholders can increase their expected reward by maintaining and trying to solve blocks on the multiple forked chains that were transmitted to them, which would lead to a divergent network. An individual stakeholder can either tie her hands behind her back by ignoring all the forked chains except for one, or opt to gain more rewards by keeping all the forked chains, which may render her entire stake worthless in case the network becomes divergent. The strategy of tying your hands behind your back is not a Nash equilibrium: if all the stakeholders follow this strategy then it is better for an individual stakeholder to deviate and maintain all the forked chains, as her influence on the overall convergence of the network is minor. Network propagation lag implies an even greater frequency of forks, as a stakeholder will get competing blocks sent to her even if those blocks were honestly solved a few seconds apart from one another. Worse still, when a rational stakeholder who currently tries to extend the block B_i receives B_{i+1} from her peers, she may opt to increase her expected reward by attempting to extend both the chain $\ldots, B_i, B_{i+1}$ and the chain $\ldots, B_i$ simultaneously. Rational stakeholders may thus prefer to reject blocks whose timestamp is later than another block that they currently try to extend, though an attempt to extend both $\ldots, B_i, B_{i+1}$ and $\ldots, B_i$ can still be possible if the rule that the stakeholders deploy does not retrace to an earlier chain that is received late due to propagation lag.

2.1.2 Bribe attacks on PPCoin

An attacker can double-spend quite easily. After the merchant waits for e.g. 6 block confirmations and sends the goods, the attacker can publicly announce her intent to create a fork that reverses the last 6 blocks, and offer bribes to stakeholders who would sign blocks her competing branch that starts 6 blocks earlier. The attacker may offer a larger bribe to stakeholders who sign only her branch, and may commit to giving bribes even after her competing branch wins, to encourage more stakeholders to participate in the attack. Notice that the stakeholders who collude with the attacker will not lose anything in case the attack fails. As long as the value of the goods is greater than the total value of the bribes, this attack will be profitable. Let us note that a bribe attack in a pure PoW network has to surmount far greater obstacles: miners who join the attack would deplete their resources while working on a fork with a 6 blocks deficit, and it is a nontrivial task to assess the success probability by measuring how many other miners participate in the attack. See also [4, Section 5.3].

2.1.3 Attacks associated with accumulation of timeweight

In PPCoin protocol $v0.2$, $\texttt{timeweight}(txout_A)$ is proportional to the time elapsed since the transaction whose output is $txout_A$ was included into a block, and is not capped. The meaning of this is that the probability

to generate a block immediately after a stake was used is very low, but it grows as time passes. This avoids the exponential distribution between payouts by boosting the chances of stakeholders who possess a small amount of coins to generate a block in a reasonable time frame, while large stakeholders will still be eligible to create blocks without much wait. Since there is no good way to have stake pools, the `timeweight` component is particularly useful.

However, this also means that an attacker can boost her chances to generate consecutive blocks by waiting. For example, if an attacker controls 10% of all the coins, and she waits until the average `timeweight` of the transaction outputs she controls is 5 times higher than the average `timeweight` of transaction outputs that belong to the other stakeholders who participate, she will be able to produce a chain of multiple consecutive blocks, as the probability that the next block is produced by her is close to 50% under these conditions.

The way by which `timeweight` is computed was changed in PPCoin $v0.3$, and in this protocol version `timeweight` stops growing after 90 days. This means that under the condition that the average `timeweight` is close to the cap $60 \cdot 60 \cdot 60$, an attacker will get no advantage by waiting. This condition is met when total number of participating transaction outputs is higher than $2 \cdot 90 \cdot 60 \cdot 60$, and in that case most unspent transaction outputs will wait for more than 90 days until they generate a stake.

Hence, this attack is now considerably less effective, but the beneficial properties that `timeweight` was designed to achieve are diminished.

2.1.4 Opportunistic attacks in relation to the need to disallow PoW

A stakeholder who holds a significant fraction of all the coins is able to generate a significant fraction of the blocks, as the probability to generate a block is proportional to the amount of coins that a stakeholder holds. Therefore, from time to time a stakeholder will be able to generate chains of consecutive blocks.

We can analyze this event by using a simplified model where stakeholders who owns $\frac{1}{M}$ of all coins can generate a block with probability $\frac{1}{M}$, and the probability to generate k sequential blocks is $(\frac{1}{M})^k$. This approximation is accurate under the assumption that the stakeholder holds a number of unspent transaction outputs significantly larger than k, so that `timeweight` will have no impact. We can estimate the average number of blocks between groups of k sequential blocks generated by one stakeholder as a mean of exponential distribution, which would be equal to $1/(1/M)^k = M^k$.

If merchants waits for k confirmations before sending their goods, the stakeholder has a chance to attack the merchant when she is able to generate k sequential blocks, thus the mean number of blocks between such attacks is M^k. For example, a stakeholder who holds $\frac{1}{4}$ of all coins participating in stake mining will be able to carry out a 6-block reorganization each $4^6 = 4096$ blocks, i.e., approximately once per month if one block is generated every 10 minutes.

An attacker who is able to create k sequential blocks would prefer to know about it as early as possible, so that she has enough time to send the payment transaction (that she intends to reverse) to the merchant. If the possible stakeholders' identities who may create the next blocks are derived from a low entropy process that only takes into account the identities who created the previous blocks, then the attacker can "look into the future" by carrying out brute-force computations to assess the probabilities that she will be able to create the k consecutive blocks at certain points in time. In order to gain a measure of unpredictability, PPCoin re-calculates once every 6 hours a "stake modifier" value that depends on the transactions that the previous blocks included, i.e., this stake modifier is part of the `prev_blocks_data` in condition (*). Therefore, a stakeholder who obtains an opportunity to generate k blocks in a row can know about this approximately 6 hours in advance, so she has plenty of time to mount an attack. If the protocol required the stake modifier to be re-calculated at a much shorter time interval, this would open the door for a stakeholder to do PoW attempts at deriving herself as being able to create future blocks more frequently.

2.2 The CoA pure *Proof of Stake* system

The Chains of Activity (CoA) system that we hereby present is a pure *Proof of Stake* protocol that aims to overcome the problem of rational forks (c.f. Section 2.1.1) by dictating that only a single stakeholder identity

may create the next block, and solidifying the random choices for these identities in the earlier ledger history via an interleaving mechanism.

The CoA protocol is based in part on the core element of PoA [4], i.e., on a lottery among the online stakeholders via the *follow-the-satoshi* procedure. This procedure takes as input an index of a satoshi (smallest unit of the cryptocurrency) between zero and the total number of satoshis in circulation, fetches the block of ledger data in which this satoshi was minted, and tracks the transactions that moved this satoshi to subsequent addresses until finding the stakeholder who can currently spend this satoshi (c.f. [4, Section 3 and Appendix A]).

The CoA protocol is parameterized by a maximal coins amount 2^κ, a subchain length $w \geq 1$, a chain length $\ell = \kappa \cdot w$, a function $\mathbf{comb} : \{0,1\}^\ell \to \{0,1\}^\kappa$, a minimal block interval time G_0, a minimal stake amount C_0, an award amount $0 \leq C_1 < C_0$, and a double-spending safety bound T_0.

The blocks creation process of CoA assembles a blockchain that is comprised of groups of ℓ consecutive blocks:

$$\overbrace{\square\square\cdots\square}^{\ell},\overbrace{\square\square\cdots\square}^{\ell},\overbrace{\square\square\cdots\square}^{\ell},\ldots$$

The rules of the CoA protocol are specified as follows:

The CoA Protocol

1. Each block is generated by a single stakeholder, whose identity is fixed and publicly known (as will be explained in the next steps). This stakeholder collects transactions that are broadcasted over the CoA network as she sees fit, and then creates a block B_i that consists of these transactions, the hash of the previous block, the current timestamp, the index i, and a signature of these pieces of data as computed with her private key.

2. Every newly created block B_i is associated with a supposedly uniformly distributed bit b_i that is derived in a deterministic fashion, for example by taking the first bit of $hash(B_i)$.

3. The time gap between B_i and B_j must be at least $|j - i - 1| \cdot G_0$. This means that if for example the next four blocks $B_i, B_{i+1}, B_{i+2}, B_{i+3}$ were supposed to be generated by the four stakeholders $A_i, A_{i+1}, A_{i+2}, A_{i+3}$ but A_{i+1} and A_{i+2} were inactive, then the difference between the timestamp of B_{i+3} and B_i must be at least $2G_0$. Nodes in the network will consider a newly created block to be invalid if its timestamp is too far into the future relative to their local time.

4. After a group of ℓ valid blocks $B_{i_1}, B_{i_2}, \ldots, B_{i_\ell}$ is created, the network nodes will form a κ-bit seed $S^{B_{i_\ell}} = \mathbf{comb}(b_{i_1}, \ldots, b_{i_\ell})$. The function **comb** can simply concatenate its inputs (if $w = 1$), and several other alternatives are explored in Section 2.2.1.

5. The seed $S^{B_{i_\ell}}$ is then used in an interleaved fashion to derive the identities of the *after* next ℓ stakeholders, via *follow-the-satoshi*. That is, if the next ℓ valid blocks are $B_{i_\ell+j_1}, B_{i_\ell+j_2}, \ldots, B_{i_\ell+j_\ell}$, then the nodes who follow the protocol will derive the identity of the stakeholder who should create the block $B_{i_\ell+j_\ell+x}$ by invoking *follow-the-satoshi* with $hash(i_\ell + j_\ell + x, S^{B_{i_\ell}})$ as input.

6. If the derived satoshi is part of an unspent output of $c < C_0$ coins, the stakeholder must also attach an auxiliary signature that proves that she controls another output of at least $C_0 - c$ coins, or else she will not be able to create a valid block. Neither the derived output nor this auxiliary output may be spent in the first T_0 blocks that extend the newly created block. In case the stakeholder A_i who should create the i^{th} block signs two different blocks B_i, B'_i, any stakeholder A_j among the next T_0 derived stakeholders can include it as evidence in the block that she creates, in order to confiscate at least C_0 coins that A_i possessed. The stakeholder A_j is awarded with C_1 of the confiscated coins, and the rest of the confiscated coins are destroyed.

7. If the network nodes see multiple competing blockchains, they consider the blockchain that consists of the largest number of blocks to be the winning blockchain.

Notice that $\ell < \kappa$ is insufficient for selecting identities that are uniformly distributed over the entire range of eligible stakeholders. If ℓ is very large (in the extreme $\ell = \infty$ i.e. practically equivalent to selecting the identities of the stakeholders via a round-robin), then an attacker may try to gain possession of future consecutive satoshis in order to mount a double-spending attack (c.f. Section 2.2.3). For example, the parameters $\kappa = 51$ (for ≈ 21 million coins of 10^8 satoshis each), $w = 9$ with **comb** as the iterated majority function (see Section 2.2.1 and Figure 1), $\ell = 459$, $G_0 = 5$ minutes, and $T_0 = 5000$ can make sense. It may also make sense to readjust some of the CoA protocol parameters dynamically (c.f. [19]).

The aforementioned concern regarding $\ell = \infty$ can be restated as follows: to have a cryptocurrency protocol that is resistant to double-spending attacks, with no need for PoW and with potentially malicious stakeholders who distrust each other, the only assumptions needed are (1) that the stakeholder who possesses the i^{th} coin will be online to sign a block when her round (of e.g. 5 minutes) is due, and (2) that the punishment for a dishonest stakeholder who exposes herself by signing two versions of her block in two competing chains is severe enough to prevent the stakeholder from doing that. Alas, assumption (1) is unrealistic, and assumption (2) is nontrivial as it depends on how much stake and anonymity the dishonest stakeholder loses relative to the value of the payment that is being reversed via double-spending. Additionally, any decentralized cryptocurrency system could be forked by some majority group to follow different protocol rules, hence it is crucial to design a protocol such that in equilibrium the protocol benefits the participants of the cryptocurrency.

2.2.1 Using low-influence functions to improve chain selection

We compare different ways in which the stakeholders involved in the current chain - denoted $A_1, \ldots, A_\ell$, can choose the stakeholders who will generate the next (interleaved) chain. The basic framework is the following.

- At round i, the stakeholder A_i will generate and publish a bit b_i that is supposed to be uniformly distributed.
- The ℓ stakeholders of the next chain - denoted $A'_1, \ldots, A'_\ell$ - will be chosen as follows. A'_i will be determined by applying *follow-the-satoshi* on $hash(i_0 + i, \mathbf{comb}(b_1, \ldots, b_\ell))$, where i_0 is the index of the stakeholder the precedes A'_1. Here $\mathbf{comb} : \{0,1\}^\ell \to \{0,1\}^\kappa$ will be some publicly known function, such that 2^κ is at least as large as the number of coins in the system.

Denote $s \triangleq \mathbf{comb}(b_1, \ldots, b_\ell)$, i.e., s is the "seed" with which we choose the next set of stakeholders. The main issue is how to choose the function **comb** so that a colluding subset of $\{A_1, \ldots, A_\ell\}$ will have a low probability of unfairly influencing the choice of s, and therefore have low probability of influencing the choice of $\{A'_1, \ldots, A'_\ell\}$. This is quite similar to the problem of collective sampling considered by Russell and Zuckerman [20] which arises in the context of well-studied questions in cryptography and distributed computing such as leader election and collective coin-flipping (c.f. [3]). In fact, we could have used the function **comb** from their 'collective sampling protocol', but do not do so for two reasons.

1. Their protocol requires a rather involved construction of a 'hitting set for combinatorial rectangles' that might be hard to implement in practice.
2. Their construction is not tailored to work well for the ranges of parameters we will consider - when the number of values of the seed - 2^κ - is 'almost exponential but not exponential' in the number of participants - i.e., $2^\kappa = 2^{\gamma \cdot \ell}$ for constant $\gamma < 1$.

We proceed to give a few choices for the function **comb**. To give a simple illustration of the advantages of different choices, we only analyze the probability that the last stakeholder in the chain, A_ℓ, can choose herself again as the last stakeholder, A'_ℓ, assuming he has a p-fraction of the coins in the system. Denote this probability by μ. We also make the simplifying assumptions that the previous players have indeed chosen their bits b_i randomly, and that the function *hash* gives random values on new inputs.

Simple concatenation: We let $\mathbf{comb}(b_1, b_2, \ldots, b_\ell) \triangleq b_1 \circ b_2 \circ \cdots \circ b_\ell$. The probability μ that A_ℓ can choose herself as A'_ℓ is the probability that either $hash(\ell, \mathbf{comb}(b_1, \ldots, b_{\ell-1}, 0))$ or $hash(\ell, \mathbf{comb}(b_1, \ldots, b_{\ell-1}, 1)$ map to a coin of A_ℓ under *follow-the-satoshi*. Using the simplifying assumption that these are random independent values we have $\mu = 1 - (1-p)^2 = 2p - p^2 \approx 2p$.

Combining majority with concatenation: Assume now that $\ell = \kappa \cdot w$ for positive integer w. We now split the ℓ stakeholders into groups of size w: $A_1, \ldots, A_w, A_{w+1}, \ldots, A_{2w}, \ldots, A_{(\kappa-1)\cdot w+1}, \ldots, A_{\kappa \cdot w} = A_\ell$. Each group will determine a bit of the seed using the majority function. That is, the i^{th} bit of the seed, denoted s_i, will be the majority of the bits $b_{(i-1)\cdot w+1}, \ldots, b_{iw}$. And $s = \mathbf{comb}(b_1, \ldots, b_\ell) \triangleq s_1 \circ s_2 \cdots \circ s_\kappa$. Note first that when the bits b_i are all chosen randomly, s is random - as the majority of random inputs is a random bit. Now, we analyze again the probability μ that A_ℓ can choose herself as A'_ℓ. It can be shown, using Stirling's approximation, that with probability roughly[2] $1 - \sqrt{2/\pi w}$ the last bit of the seed, s_κ, will already be determined by the bits of the previous stakeholders. This is because when w players vote choose a bit randomly, the probability that *exactly half* of the bits came out one tends to $\binom{w}{w/2}/2^w \approx \frac{2^{w+1/2}}{\sqrt{\pi w}}/2^w = \sqrt{2/\pi w}$. In this event, the probability that $A_\ell = A'_\ell$ is p, "as it should be". When this event does not happen, and A_ℓ can determine s_κ, as before he can get to probability $\approx 2p$. In total we have $\mu \approx (1-\sqrt{2/\pi w}) \cdot p + (\sqrt{2/\pi w}) \cdot 2p$. Taking a large enough w, this is much closer to the "correct" probability p than in the previous choice of **comb**.

Improving further by using low-influence functions: There were two reasons why the majority function was useful above. First, on a random set of inputs, the majority function returns a random bit. In other words, it is a *balanced* function. Second, when all but one of the w inputs are chosen randomly, the probability that the function is not yet determined is low - $\theta(1/\sqrt{w})$. Are there balanced functions where this probability is even lower? If so, we could use them to get an even better construction of the function **comb**. It turns out that this is a very well-known question about the "influence" of boolean functions initially raised in [3], and studied in the seminal paper of Kahn, Kalai and Linial [11] where it was shown that the probability above will always be at least $\Omega(\log w/w)$. Ben-Or and Linial, in fact gave an example of a function where this probability is that low: The TRIBES-function where we divide the w players into "tribes" of size approximately $\log w$, take the AND of the bits in each group, and then take the OR of all resulting bits.

Protection against larger coalitions: We now address the scenario of a coalition $C \subseteq \{A_1, \ldots, A_\ell\}$ of $|C| = c > 1$ players (i.e., stakeholders) trying to influence the choice of the output of **comb**. For simplicity, we make the following assumptions.

- As before, the $\ell - c$ honest players outside of the coalition choose their input bit randomly.
- We make the worst-case assumption that the c players in the coalition see all the input bits of the honest players before choosing theirs.
- We measure how "successful" a choice for the function **comb** is by the *statistical distance* of the resulting seed to the uniform distribution: For constant $\varepsilon > 0$, a distribution P on $\{0,1\}^\kappa$ is *ε-close to uniform* if for any set $T \subseteq \{0,1\}^\kappa$, the probability that $P \in T$ is at most $|T|/2^\kappa + \varepsilon$. In words, the probability of any event grows at most by an additive factor of ε compared to it's 'correct' probability.

It turns out that under these assumptions, the problem we are trying to solve is exactly that of constructing *extractors for non-oblivious bit-fixing sources*, considered by Kamp and Zuckerman [12].

Let us use the terminology that a function $\mathbf{comb} : \{0,1\}^\ell \to \{0,1\}^\kappa$ is an *ε-extractor* if for any choice of the coalition C of size c, and any strategy of C to choose their input bits after seeing the bits of the honest players, $\mathbf{comb}(b_1, \ldots, b_\ell)$ produces an output that is ε-close to unifrom.

[2] More precisely, as w goes to infinity this is the limit of the probability of the event.

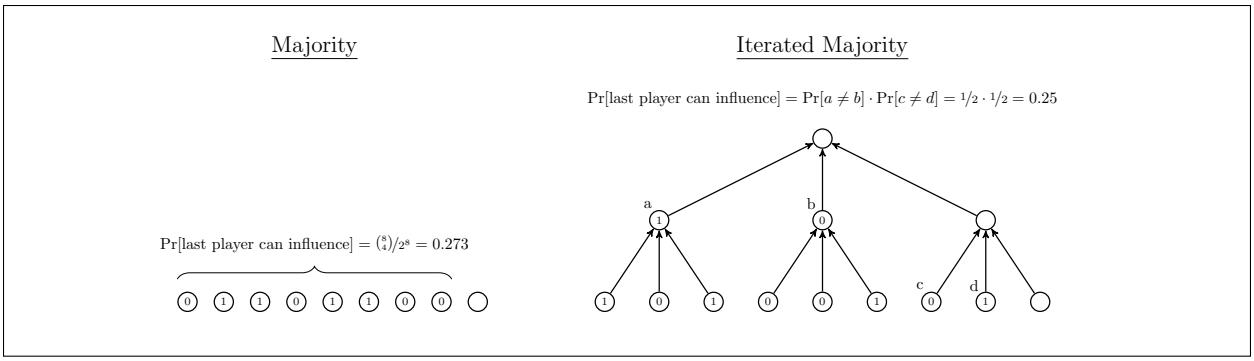


Figure 1: Majority versus Iterated Majority.

[12] give the following construction of an ε-extractor - that is in fact the same one we described earlier when replacing the majority function with the *iterated majority* (defined in [3] and illustrated in Figure 1) function.

$KZ(b_1, \ldots, b_\ell)$**:**

- Choose $w = 3 \cdot (c/\varepsilon)^{1/\alpha}$, where $\alpha = \log_3 2$. Set $\ell = w \cdot \kappa$.
- Similarly to before, output κ bits by taking the *iterated majority* of consecutive groups of w inputs.

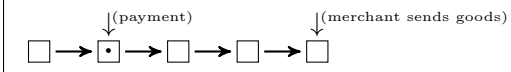
[12], using the analysis of [3] of the iterated majority function, were able to show that the function KZ is an ε-extractor. Upon fixing ε as the desired statistical error, κ as the desired output length, and ℓ as the total number of players in a chain, KZ can handle a coalition of size $c \leq \varepsilon \cdot (1/3 \cdot \ell/\kappa)^\alpha$.

[12] show, on the other hand, that any such ε-extractor can handle coalitions of size at most $c \leq \varepsilon \cdot 10 \cdot \ell/(\kappa - 1)$.

Since $\alpha > 1/2$, it follows that this choice of **comb** is less than quadratically worse than the optimal choice. Notice that this assumes that stakeholders who are not honest are non-oblivious, i.e., that they see the choices of the honest stakeholders before they play. This conservative assumption makes a certain sense in our context, as it easier for stakeholders who play in the last locations to try to collude in order to influence the seed.

2.2.2 Rational collusions

Stakeholders may wish to collude and skip the last several blocks as if they did not exist, i.e., to extend the blockchain from an earlier block, in order to gain the fees that went to previous stakeholders. This can be mitigated by including in each transaction the index of the latest block that the user who made this transaction is aware of. Thus, if for example block B_i contains a transaction tx_i, and block B_{i+1} contains a transaction tx_{i+1} which specifies that block i exists, then the colluding parties cannot reverse these two blocks in order to obtain the fees of both tx_i and tx_{i+1}, because the block B_i must exist in the chain that contains tx_{i+1}. The user could even specify in her transaction the index of the block that is currently being created, but this implies that the user would need to send another transaction in the case that the current stakeholder is offline. The colluding stakeholders diminish the overall value of their stake when they participate in such attacks, hence this strategy is not necessarily rational. It is also possible to reward stakeholders via monetary inflation and have the transaction fees destroyed to provide a counterbalance, though bribe attacks may then become more likely (see Section 2.2.3).



To demonstrate how a successful double-spending attack on CoA looks like, in this example an alternative history of 5 blocks needs to be created, by extending the previous block with a chain that includes a conflicting transaction:

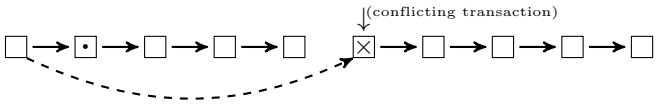


Figure 2: Illustration of a double-spending attack in the CoA system.

2.2.3 Bribe attacks on CoA

Suppose that the number of blocks that merchants consider to be secure against double-spending attacks is d, i.e., a merchant will send the goods after she sees that the payment transaction that she received in block B_{i_1} has been extended by $B_{i_2}, B_{i_3}, \ldots, B_{i_d}$ extra blocks. An attacker can now offer bribes to the next $i_d + 1, i_d + 2, \ldots, i_d + d + 1$ stakeholders, so that they would extend the blockchain starting from the block that preceded B_{i_1} and exclude that payment transaction. The attacker will need to bribe more than $d + 1$ stakeholders if some of them refuse the bribe. Since rational stakeholders will not participate in the attack without an incentive, the cost of the attack is at least $\varepsilon(d + 1)$ where ε is the average bribe amount that is given to each stakeholder.

Observe that $\Pr[\{\text{successful attack}\}] < 1$ since some of the stakeholders might be altruistic, some of the rational stakeholders may think that it would be unprofitable to participate in such attacks, and the attacker's funds are not unlimited. Hence, a rational stakeholder will choose to accept the bribe by weighing whether $(\varepsilon + F') \cdot \Pr[\{\text{successful attack}\}] > F \cdot (1 - \Pr[\{\text{successful attack}\}])$, where F and F' are the fee amounts that this stakeholder will collect on the honest chain and the attacker's chain, respectively. The implication is that the attacker may need to spend substantially more than $\varepsilon(d + 1)$ coins for the attack to succeed.

This stands in stark contrast to Section 2.1.2, as the short-term dominant strategy of the PPCoin stakeholders is to participate in the attack, while the CoA stakeholders will forfeit their reward F in case the attack fails.

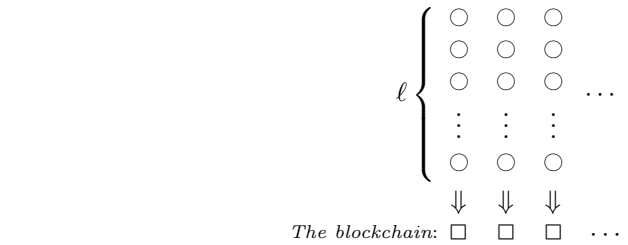
Note that the attacker cannot simply bribe the stakeholders who generated the blocks $B_{i_1}, B_{i_2}, B_{i_3}, \ldots, B_{i_d}$ to create an alternative history of length d in a risk-free manner, as their coins will be confiscated if they double-sign.

In Figure 2 we illustrate the nature of a double-spending bribe attack.

2.3 The Dense-CoA pure *Proof of Stake* variant

The Dense-CoA protocol is an alternative variant of CoA in which the identities of stakeholders who should create the next blocks are not known far in advance, with the objective of making collusions and bribe attacks more difficult. Another plus point of Dense-CoA is that it makes it more difficult for rational stakeholders to obtain disproportional rewards. The disadvantages of the Dense-CoA protocol are susceptibility to DoS attacks by large stakeholders, and greater communication and space complexities.

In Dense-CoA, each block is created by a group of ℓ stakeholders, rather than by a single stakeholder:



Let $h : \{0,1\}^n \to \{0,1\}^n$ be a one-way permutation. Let us assume for a moment that the block B_{i-1} is associated with a seed $S^{B_{i-1}}$ that was formed by the ℓ stakeholders who created B_{i-1}. Now, the identity of the stakeholder A_ℓ who determines which transactions to include in a block B_i is derived by invoking *follow-the-satoshi* with $hash(i, \ell, S^{B_{i-1}})$ as input, and the identities of the rest of the stakeholders $A_1, A_2, \ldots, A_{\ell-1}$ who must participate in the creation of B_i are derived by invoking *follow-the-satoshi* with $hash(i, j, S^{B_{i-1}})$ for $j \in \{1, 2, \ldots, \ell-1\}$. These ℓ stakeholders engage in a two-round protocol to create the block B_i:

- In round 1, for every $j \in \{1, 2, \ldots, \ell\}$, the stakeholder A_j picks a random secret $R_j \in \{0,1\}^n$, and broadcasts $h(R_j)$ to the network.
- In round 2, for every $j \in \{1, 2, \ldots, \ell-1\}$, the stakeholder A_j signs the message $M \triangleq h(R_1) \circ h(R_2) \circ \cdots \circ h(R_\ell)$, and broadcasts her signature $\mathsf{sign}_{sk_j}(M)$ and her preimage R_j to the network.

We require Dense-CoA to use a signature scheme with multisignature [5, 10, 13, 15] support, therefore A_ℓ can aggregate the signatures $\{\mathsf{sign}_{sk_j}(M)\}_{j=1}^{\ell}$ into a single signature $\hat{\mathsf{s}}(M)$. Note that the size of $\hat{\mathsf{s}}(M)$ depends only on the security parameter of the signature scheme (and not on ℓ), and the verification time is faster than verifying ℓ ordinary (ECDSA) signatures.

Hence, the stakeholder A_ℓ signs and broadcasts a block B_i that consists of the (Merkle root of the) transactions that she wishes to include, the hash of the previous block B_{i-1}, the current timestamp, the index i, the ℓ preimages $R_1, R_2, \ldots, R_\ell$, and $\hat{\mathsf{s}}(M)$. To verify that the block B_i is valid, the network nodes invoke h to compute the images $h(R_1), h(R_2), \ldots, h(R_\ell)$, then concatenate these images to form M, and then check that $\hat{\mathsf{s}}(M)$ is a valid signature of M with respect to the public keys $pk_1, pk_2, \ldots, pk_\ell$ that control the winning satoshis of the stakeholders $A_1, A_2, \ldots, A_\ell$.

The seed S^{B_i} is defined as $hash(R_1 \circ R_2 \circ \cdots \circ R_\ell)$. Notice that S^{B_i} is computationally indistinguishable from random even if only a single stakeholder A_j picked a random R_j, under the assumption that n is sufficiently large.

If some of the ℓ stakeholders are offline or otherwise withhold their signatures, then after G_0 time the nodes who follow the protocol will set $t = 1$ and derive alternative ℓ identities from the previous block B_{i-1}, by invoking *follow-the-satoshi* with inputs $hash(i, t\ell + j, S^{B_{i-1}})$ for $j \in \{1, 2, \ldots, \ell\}$. The starting index $t\ell + j$ should be specified in the new block B_i so that the verification of blocks will be simpler, and the gap between the timestamps of B_{i-1} and B_i must be at least tG_0. As with CoA, the honest nodes consider the blockchain with the largest amount of valid blocks to be the winning blockchain, and disregard blocks with a timestamp that is too far into the future relative to their local clock.

The parameters C_0, C_1, T_0 of the CoA protocol (c.f. Section 2.2) are used in the same way in the Dense-CoA protocol.

The parameter ℓ should be big enough in order to resist large stakeholders from controlling consecutive seeds $\{S^{B_i}, S^{B_{i+1}}, \ldots\}$ and re-deriving themselves. For example, to force a stakeholder who holds 5% or 10% of the total stake into making $\approx 2^{100}$ *hash* invocations on average until re-deriving herself as all of the ℓ identities of the next block, we need $\ell = 23$ or $\ell = 30$, respectively. However, if we set $G_0 = 5$ minutes and $\ell = 23$, a malicious stakeholder with e.g. 10% of the total stake will have $1 - (90/100)^{23} \approx 91\%$ probability to be one of the derived stakeholders $A_1, A_2, \ldots, A_\ell$ and then refuse to participate in creating the next block, hence it will take $5 \cdot (91\%)^{-1} \approx 5 \cdot 11 = 55$ minutes on average to create each next valid block while this

attack is taking place (actually less than 55 minutes because chains that extend blocks prior to the last block can also become the longest valid chain).

Overall, the main difference between the Dense-CoA and CoA protocols is that Dense-CoA offers improved security over CoA in terms of double-spending attacks, but weaker security against DoS attacks by large stakeholders who wish to harm the cryptocurrency. Also, Dense-CoA prevents a rational stakeholder from influencing the seed in an attempt to earn more rewards than her fair share, unless she colludes with all the other $\ell - 1$ stakeholders who create the next block. The Dense-CoA protocol is less efficient than CoA due to the preimages $R_1, R_2, \ldots, R_\ell$ that need to be stored in each valid block, and the two-round protocol that requires a greater amount of network communication to create each block.

3 Solidification of the ledger history

Any decentralized cryptocurrency system in which extending the ledger history requires no effort entails the danger of costless simulation [17], meaning that an alternative history that starts from an earlier point of the ledger can be prepared without depleting physical resources and hence without a cost. This is a problem because a rational adversary who has little or no stake in the system may try to attack by replacing an arbitrarily long suffix of the current ledger history with an alternative continuation that benefits her. Further, a malicious adversary who does not operate out of self-interest is also more likely to attempt this kind of an attack, as she would not incur a monetary loss for executing the attack.

In the case of pure *Proof of Stake* systems, this danger can manifest itself in the following form. Consider participants who held coins in the system a long time ago and have since traded those coins in exchange for other goods, so they are no longer stakeholders of this system. These participants can now collude to extend the ledger from the point at which they had control over the system, and it may indeed be rational for them to mount this attack because it is costless and would have no detrimental outcome from their standpoint, as they have no stake in the current system.

More specifically, let us examine how this attack looks like in the CoA or Dense-CoA systems. Even a single stakeholder with few coins can fork the blockchain and create an alternative branch with large enough time gaps as she re-derives herself to create subsequent blocks, but according to the timestamp rules for valid blocks, the other participants will reject this alternative branch (even though it contains more blocks) because the timestamps will be too far ahead in the future relative to their local time. Therefore, if the average participation level among current stakeholders is $p\%$, and the stakeholders who collude to carry out this attack have had control at the earlier history over $q\%$ of the coins, then $q > p$ implies that the attack will succeed. Because $p\% = 1$ is highly unlikely, and collusion among participants who held $q\% > p\%$ stake at an earlier point is costless and rational, this attack vector appears to be quite dangerous.

To mitigate this attack, we propose periodic checkpointing as a rigid protocol rule that extends the CoA and Dense-CoA protocols, as follows:

- Denote by $T_0 = 2T_1$ the double-spending safety bound of Section 2.2.
- The blocks at gaps of T_1 are designated as *checkpoint* blocks: the genesis block is a checkpoint block, and any block that extends a checkpoint block by exactly T_1 additional blocks is a candidate checkpoint block.
- When a node that follows the protocol receives for the first time a candidate checkpoint block B_j that extends the candidate checkpoint block B_i such that $j = i + T_1$ (or $j > i + T_1$ if some stakeholders were inactive), she solidifies B_i and disallows any changes to the history from the genesis block until B_i, though B_j can still be discarded as a result of a competing fork.

Since the double-spending safety bound is T_0, a stakeholder who creates a block can spend the coins only after an intermediate checkpoint block is already solidified, so the costless simulation threat is mitigated (if C_0 is substantial). See Figure 3 for an illustration.

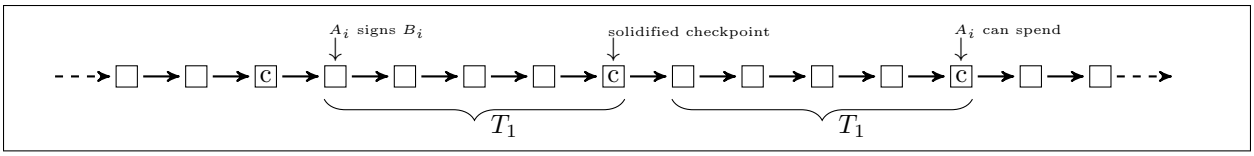


Figure 3: Checkpoint solidification prior to the $T_0 = T_1 + T_1$ bound.

However, this checkpointing mechanism presents two problems:

1. New nodes who enter the decentralized network for the first time cannot tell whether the checkpoint blocks that they receive are trustworthy.

2. Due to propagation lag, adversarial stakeholders can collude by preparing an alternative branch of length $T_1 + 1$, and broadcast the competing forks at the same time, thus creating an irreversible split among the network nodes.

The first problem needs to be handled by utilizing a "Web of Trust" type of mechanism that is external to the cryptocurrency system. This means that participants who are unaware of the current state of the system should rely on reputable sources to fetch the blockchain data up to the latest checkpoint.

The second problem should also be resolved manually, meaning that participants who become aware of a network split can decide to instruct their node to switch to the other faction, e.g. if they see that they are in the minority. Note, however, that the second problem becomes increasingly unlikely for larger T_1 values. The exemplary parameters that we proposed in section 2.2 imply that a fork of $T_1 + 1$ blocks represents more than one week of ledger history.

Let us mention that PoW based systems such as Bitcoin also benefit from having checkpoints, as it helps to bootstrap new nodes who enter the network in a more efficient manner. The Bitcoin checkpoints also thwart DoS attacks that would extend the genesis block with low-difficulty large blocks, although "headers-first" client behavior[3] can be sufficient for mitigating DoS, as the client will quickly discard an alternative branch that is being transmitted unless its cumulative PoW outcompetes the current best branch. Both CoA and Bitcoin can utilize SNARK[4] to represent the ledger state (a.k.a. the set of unspent coins) that is reached by a checkpoint, as sending a SNARK to new nodes is more efficient than sending the entire ledger history. While CoA checkpoints rely on an element of trust, Bitcoin can take advantage of SNARK checkpoints that are trust-free in the sense that a new node will have exactly the same level of confidence in the ledger state as the existing nodes, because the SNARK will prove that the protocol rules have been followed from the genesis block until the current state.

4 Issuance of the money supply

With a cryptocurrency that does not incorporate a PoW component, one straightforward way to distribute the money to the interested parties is by having an IPO of some sort. However, this implies that initially the money supply is controlled by a central party, which means that it is significantly less likely that this cryptocurrency can be decentralized.

We propose to use PoW only for the initial issuance of the coins that should then circulate in the cryptocurrency system, in a way that pegs the value of the newly minted coins to the cost of producing these coins (c.f. [22]). This can be done by dictating that the cost of producing a coin in terms of electricity and erosion of the equipment will be approximately fixed throughout the issuance process, and hence:

- If the value of each coin is more than the cost of producing a coin, then more mining equipment will be

[3] See http://sourceforge.net/p/bitcoin/mailman/message/32921390/
[4] See e.g. http://www.scipr-lab.org/

brought online to produce larger amounts of coins at the fixed cost, and then larger amounts of newly minted coins will come into existence - which implies that the value of each coin decreases.

- If the value of each coin is less than the cost of producing a coin, then some of the mining equipment that participates in the minting process will quit, and then smaller amounts of coins will come into existence - which implies that the value of each coin increases.

In order to have a fixed cost of production, we simply need to remove the difficulty readjustment mechanism that Bitcoin based systems use. This means that there will no longer be a predicable gap of x minutes on average (e.g. $x = 10$ with Bitcoin) between the PoW blocks that are being created, and instead the gap between PoW blocks will directly correspond to the amount of mining power that participates in the block creation process. This way, if an individual miner could create a block once every y minutes on average in accord with the fixed PoW difficulty, then she will still be able to create blocks once every y minutes even if additional mining power joins the process, though her blocks will represent a smaller portion of the total amount of blocks that are being created. To avoid the possibility that blocks get generated extremely fast in the case that the cryptocurrency continues to have a high market value even when many newly minted coins come into existence, the protocol can still specify a difficulty readjustment rule that imposes a minimal average gap of e.g. 1 minute between PoW blocks. The minimal gap is desirable as otherwise there can be many orphaned blocks, which amplifies the phenomenon where miners who are well-connected to high concentrations of the mining power receive more rewards than what their proportion relative to the total mining power should imply.

Bitcoin's difficulty readjustment mechanism is different than the above mechanism because the value of a coin is ultimately determined by long-term fundamentals that are derived from the adoption level of the cryptocurrency among merchants, rather than the amount of miners who wish to mint new coins during a certain time period. Hence, given the current market value of a coin, the cost of minting new coins in the Bitcoin system is determined according to how many miners currently participate in the production process, i.e., a greater participation level implies that an individual miner will be rewarded with a smaller proportion of the fixed amount of coins that is being produced every 10 minutes of average. When the difficulty readjustment mechanism is not deployed, the cost of production for each individual miner is not affected by the overall mining power that participates in the production process.

For the CoA system to deploy such a PoW issuance scheme, the overall protocol should specify that the CoA network nodes are allowed to create transactions that spend newly minted coins from the PoW blockchain if and only if the minted coins are buried behind a sufficiently large amount n of PoW blocks. In Bitcoin $n = 100$, but here $n > 100$ may be prudent, in case the fixed PoW difficulty becomes relatively easier for future mining hardware. If the minimal average gap rule is in place, then a sensible value for n can be specified with no uncertainty issues. For a cryptocurrency without infinite monetary inflation, the CoA network nodes can stop listening for new PoW blocks after the last PoW block is created (for example the last spendable PoW block can be solidified by making the PoW blocks that follow it unspendable), as well as completely discard the PoW blockchain after all the minted coins have been spent.

Let us note that an alternative approach for the initial issuance of the coins is "one-way pegging" a.k.a. "proof of burn" (see e.g. [6]), by which coins of an earlier cryptocurrency are destroyed in exchange for newly minted coins in the system that is being launched. Under the assumption that the earlier cryptocurrency is decentralized, this issuance method is indeed decentralized as well. However, this issuance method does not imply that the value of minted coins in the new system is pegged to the cost that it took to produce these coins.

5 Conclusion

Even if we take it as a premise that PoW based systems such as Bitcoin are sustainable over the long term, it is still a nontrivial task to design sustainable decentralized cryptocurrency protocols that do not rely on depletion of physical scarce resources for their security maintenance. Our analysis argues that the security of existing such protocols is lacking. We offer novel constructions of pure *Proof of Stake* protocols that

avoid depletion of scarce physical resources, and argue that our protocols offer better security than existing protocols. However, we do not claim that the security of the protocols that we present is strictly better than that of Bitcoin, and depending on short-term human behavior the security of our protocols may possibly be worse relative to Bitcoin.

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