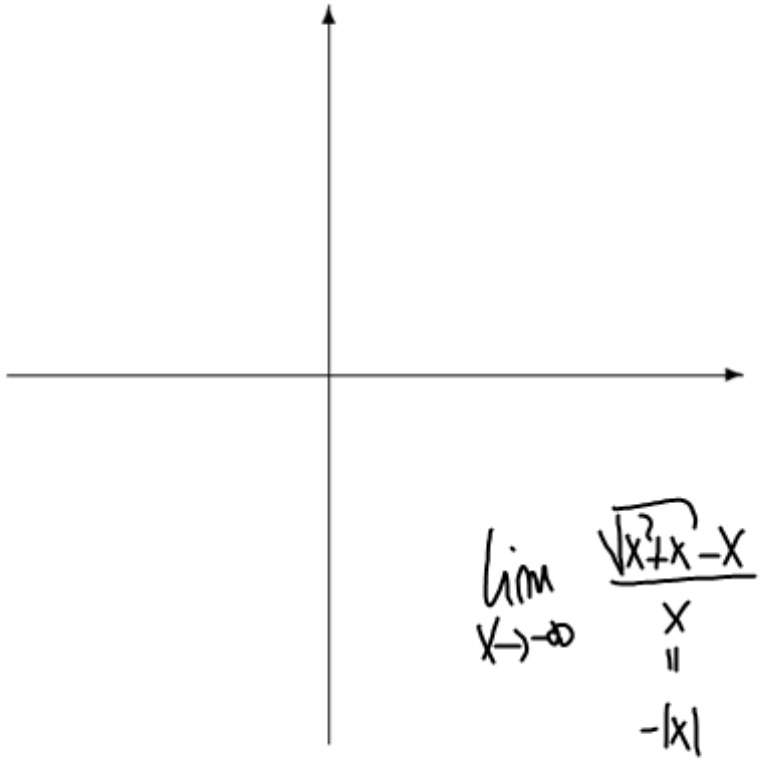
Esempio I

$$f(x) = \sqrt{x^2 + x} - x$$



$$D = (-\infty, -1] \cup [0, +\infty)$$

$$\lim_{x \to +\infty} f(x) = 1/2$$

$$\lim_{x \to -\infty} f(x) = +\infty \qquad y = -2x - 1/2$$

$$\lim_{x \to +\infty} \left(\sqrt{x^2+x} - x\right) = \lim_{x \to +\infty} \frac{x^2 + x - x^2}{\sqrt{x^2+x} + x} = \frac{1}{2}$$

$$\lim_{x \to -\infty} \frac{\sqrt{x^2+x} - x}{x} = \lim_{x \to -\infty} -\sqrt{\frac{x^2+x}{x^2}} - 1 = -2 = m$$

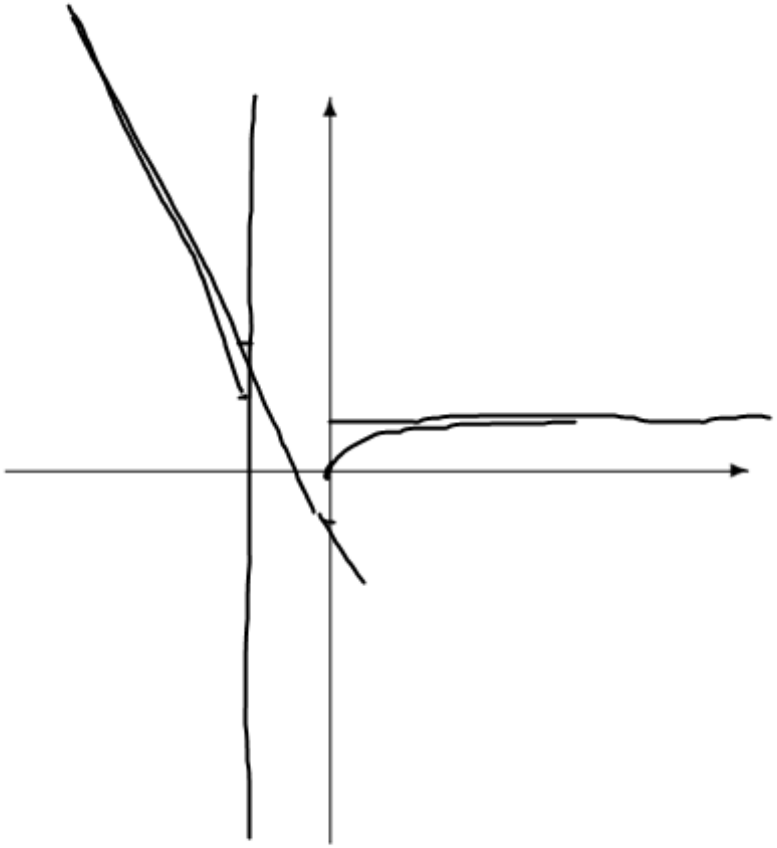
$x = -|x|$

$$\lim_{x \to -\infty} \left[\sqrt{x^2+x} - x - (-2)x\right] = \lim_{x \to -\infty} \frac{x^2 + x - x^2}{\sqrt{x^2+x} - x} = -\frac{1}{2}$$

($-x - (-2)x = +x$)

Esempio I

$$f(x) = \sqrt{x^2 + x} - x$$



$$D = (-\infty, -1] \cup [0, +\infty)$$

$$\lim_{x \to +\infty} f(x) = 1/2$$

$$\lim_{x \to -\infty} f(x) = +\infty \quad y = -2x - \tfrac{1}{2}$$

$$f'(x) = \frac{2x+1}{2\sqrt{x^2+x}} - 1$$

$f'(0) =$

$$\lim_{x \to 0^+} f'(x) = +\infty$$

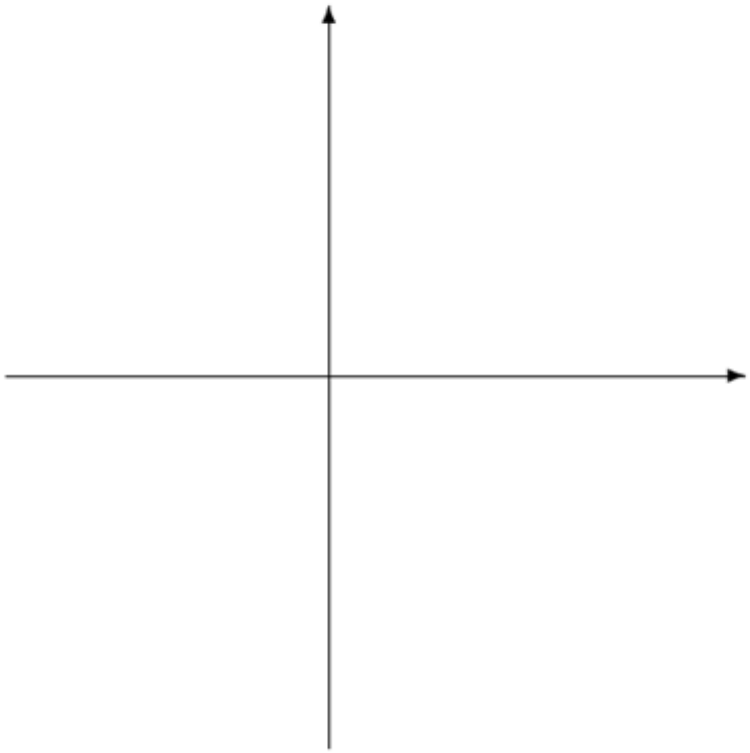
$$f''(x) = -\frac{1}{4(x^2+x)^{3/2}}$$

$f'(-1) =$

$$\lim_{x \to 1^-} f'(x) = -\infty$$

Esempio II

$$f(x) = \frac{x^2 + 3}{x - 1}$$



$$D = (-\infty, 1) \cup (1, +\infty)$$

$$\lim_{x \to \pm\infty} f(x) = \pm\infty \qquad y = x + 1$$

$$\frac{f(x)}{x} = \frac{x^2+3}{x^2-x} \longrightarrow 1 \qquad x \to \pm\infty$$

$$\left(f(x) - x\right) = \frac{x^2+3-x^2+x}{x-1} = \frac{x+3}{x-1} \longrightarrow 1$$

Esempio II

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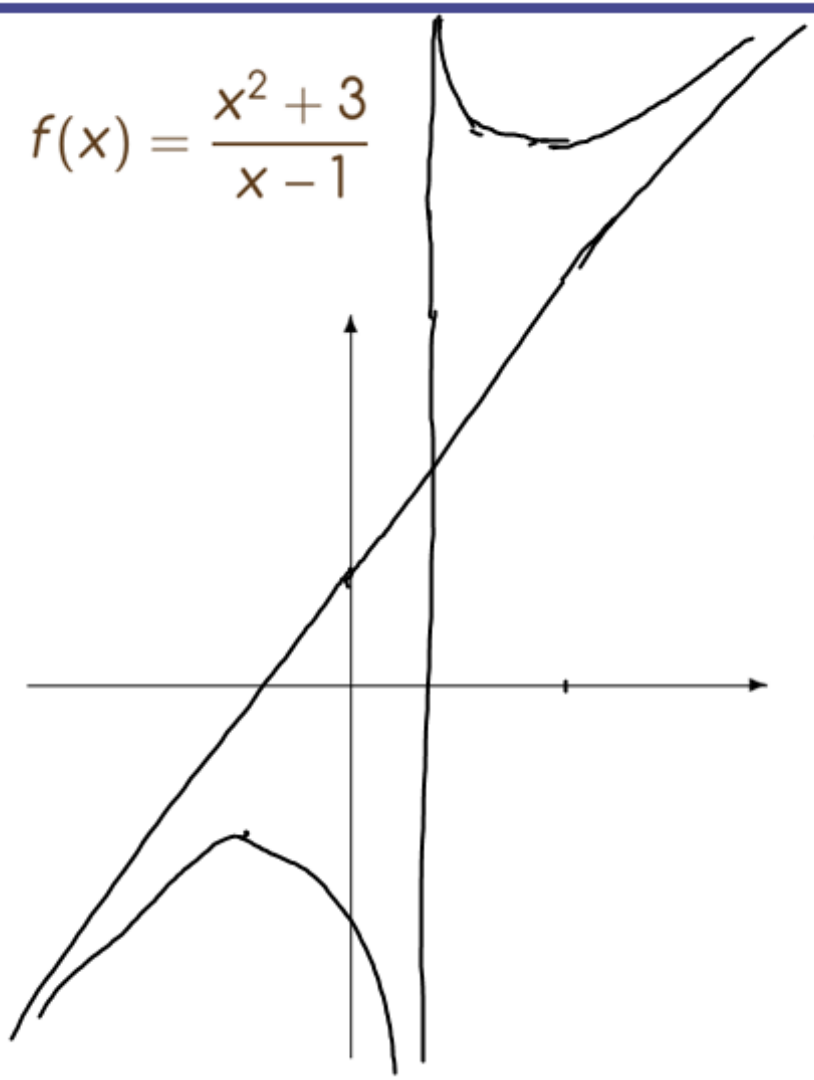
$$f(x) = \frac{x^2+3}{x-1}$$



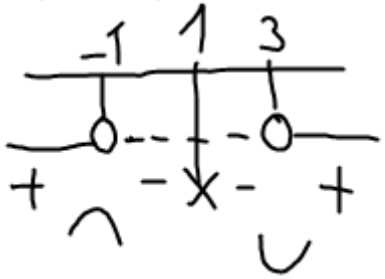
$$D = (-\infty, 1) \cup (1, +\infty)$$

$$\lim_{x \to \pm\infty} f(x) = \pm\infty \qquad y = x + 1$$

$$\lim_{x \to 1^{\pm}} f(x) = \pm\infty$$

$$f'(x) = \frac{x^2 - 2x - 3}{(x-1)^2}$$

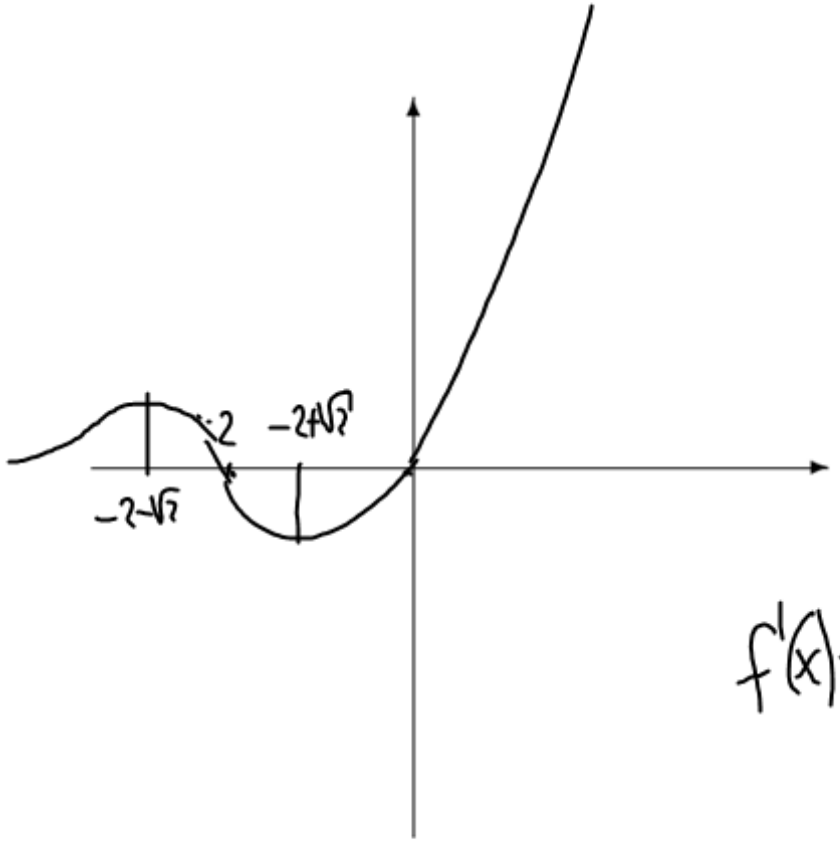
$$(x+1)(x-3) \geq 0$$



$$f''(x) = \frac{8}{(x-1)^3}$$

Esempio III

$$f(x) = (x^2 + 2x)e^x$$



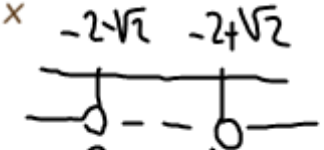
$$D = \mathbb{R}$$

$$\lim_{x \to +\infty} f(x) = +\infty$$

$$\lim_{x \to -\infty} f(x) = 0$$

$$f'(x) = (x^2 + 4x + 2)e^x$$

$$f''(x) = (x^2 + 6x + 6)e^x$$

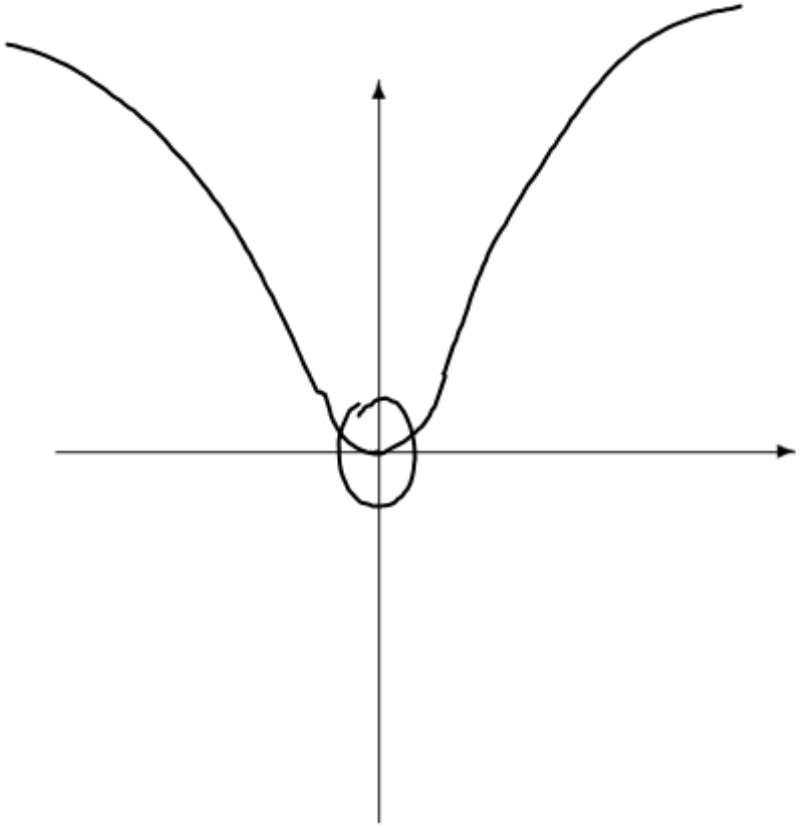


$$f'(x) = (2x+2)e^x + (x^2+2x)e^x = (x^2+4x+2)e^x$$

$$x_{1,2} = -2 \mp \sqrt{4-2} \quad \begin{matrix} -2-\sqrt{2} \\ -2+\sqrt{2} \end{matrix}$$

Esempio IV

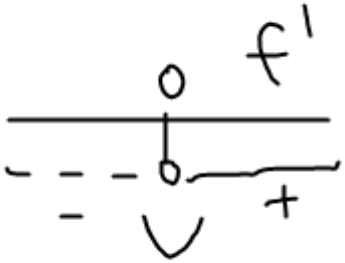
$$f(x) = \ln(1+x^2)$$



$$D = \mathbb{R}$$

$$\lim_{x\to\pm\infty} f(x) = +\infty$$

$$f'(x) = \frac{2x}{1+x^2}$$



$$f''(x) = 2\frac{1-x^2}{(1+x^2)^2}$$