UNITELMA SAPIENZA

SAPIENZA
UNIVERSITÀ DI ROMA
DIPARTIMENTO DI INFORMATICA

Calcolo Differenziale

Eugenio Montefusco

20. Studi di funzione

Uno schema

- insieme di definizione

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- comportamento della funzione "vicino" alla frontiera

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- comportamento della funzione "vicino" alla frontiera
- individuazione di asintoti
- ricerca dei punti stazionari e del loro valore
- studio della concavità e della convessità

Asintoti obliqui e orizzontali

Se una funzione ha un asintoto, allora

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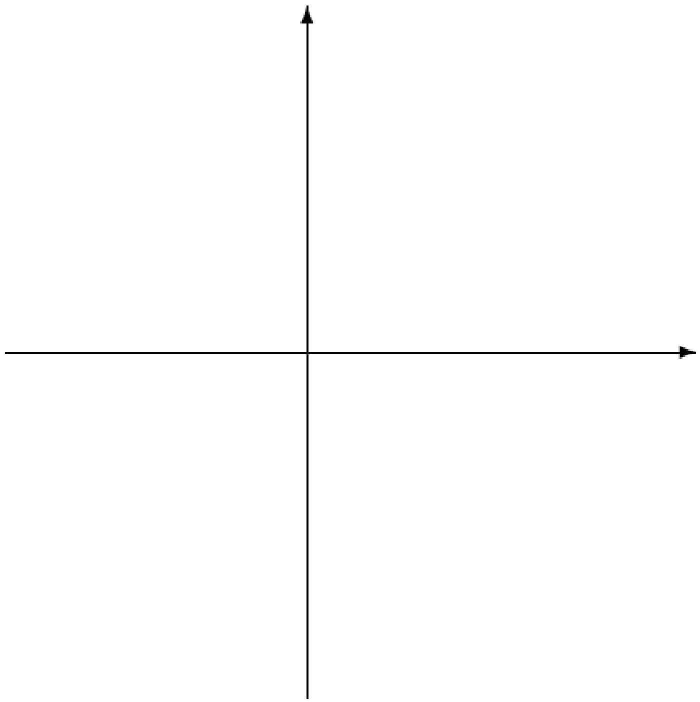
quindi

$$m = \lim_{x \longrightarrow \pm\infty} \frac{f(x)}{x}$$

$$q = \lim_{x \longrightarrow \pm\infty} (f(x) - mx)$$

Esempio I

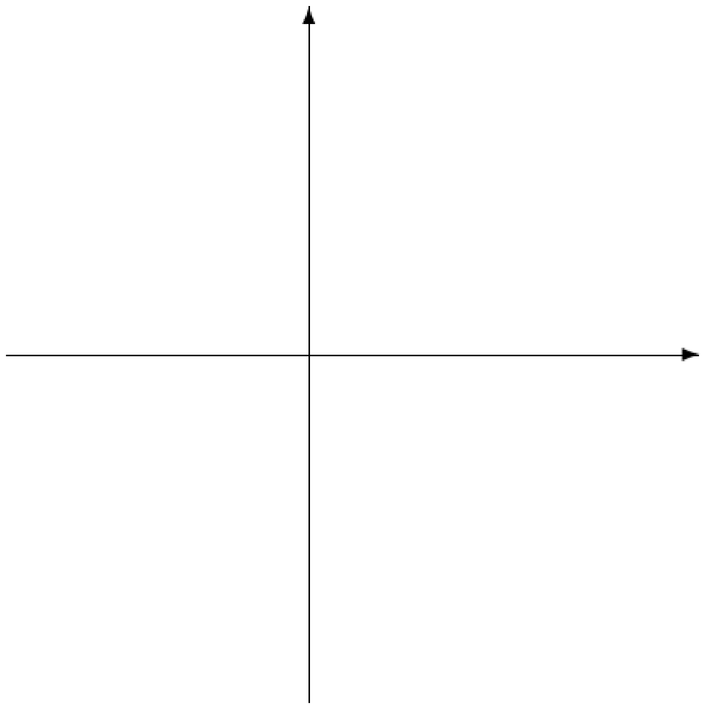
$$f(x) = \sqrt{x^2 + x} - x$$



Esempio I

$$f(x) = \sqrt{x^2 + x} - x$$

$$D = (-\infty, -1] \cup [0, +\infty)$$



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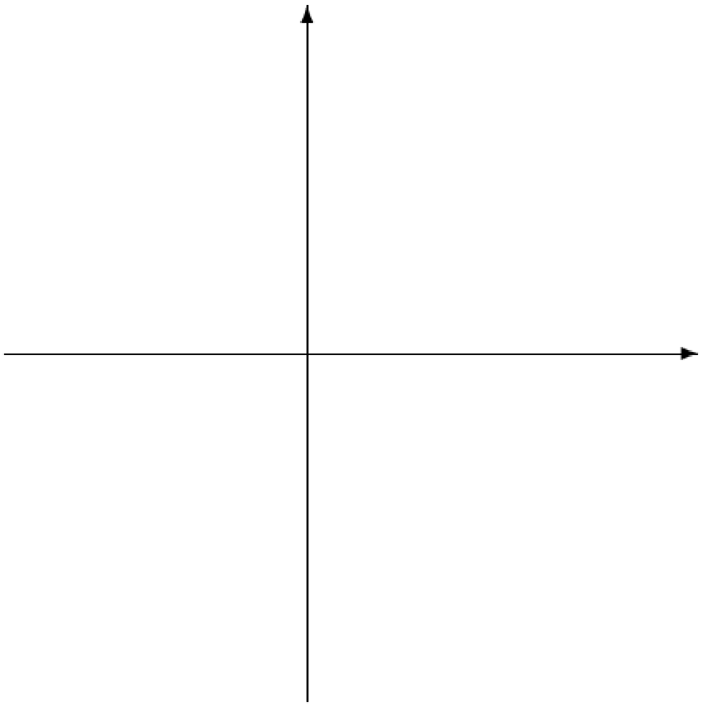

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$$\lim_{x \to +\infty} f(x) = 1/2$$

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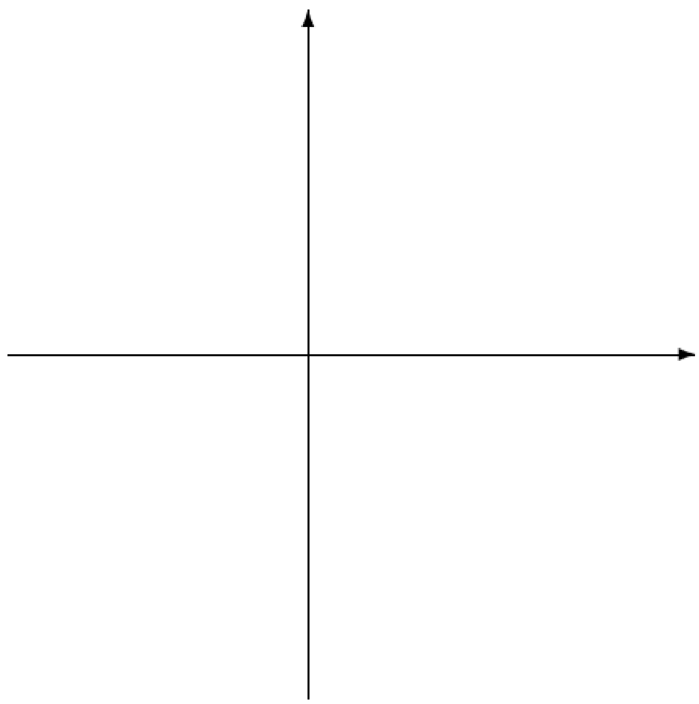

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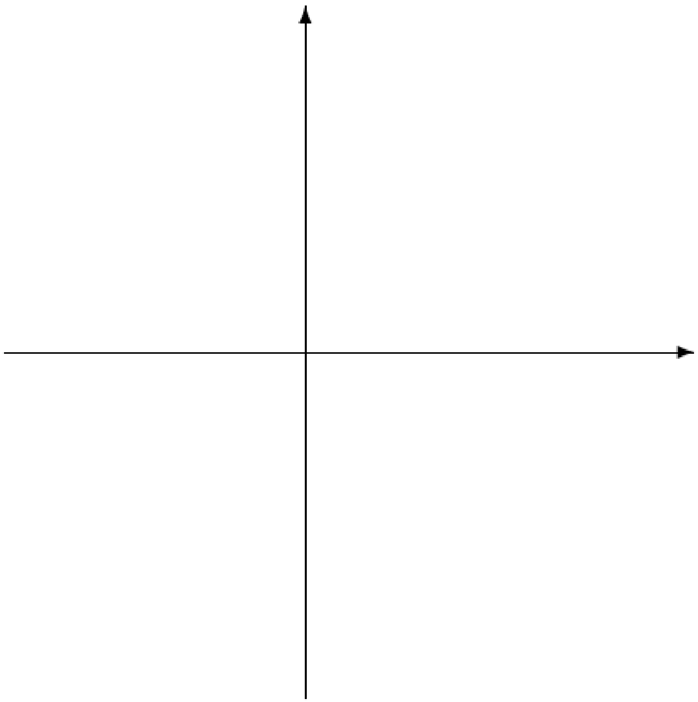

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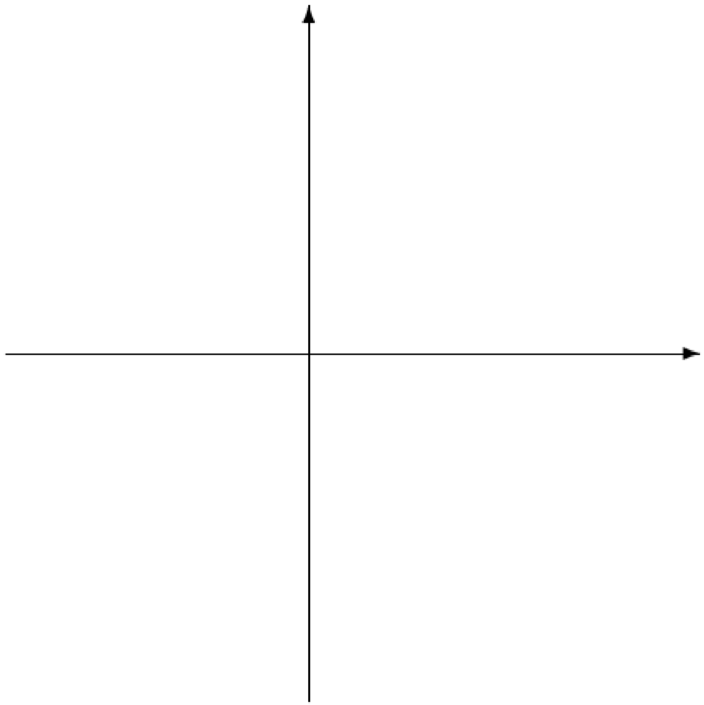

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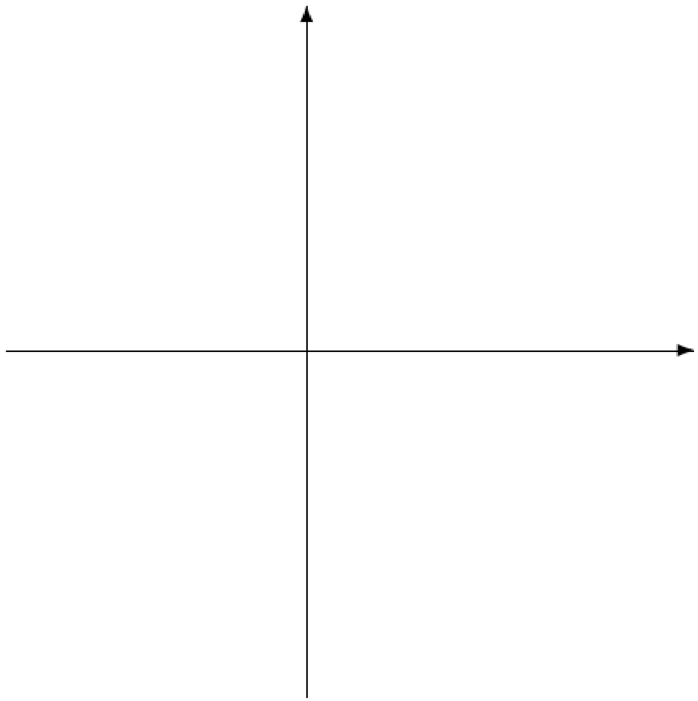
$$f''(x) = -\frac{1}{4(x^2 + x)^{3/2}}$$

Esempio II

$$f(x) = \frac{x^2 + 3}{x - 1}$$



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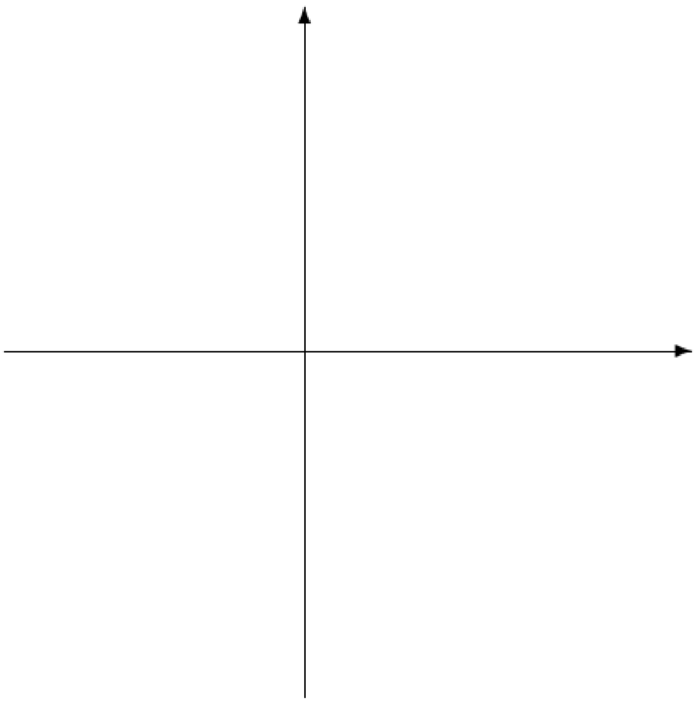

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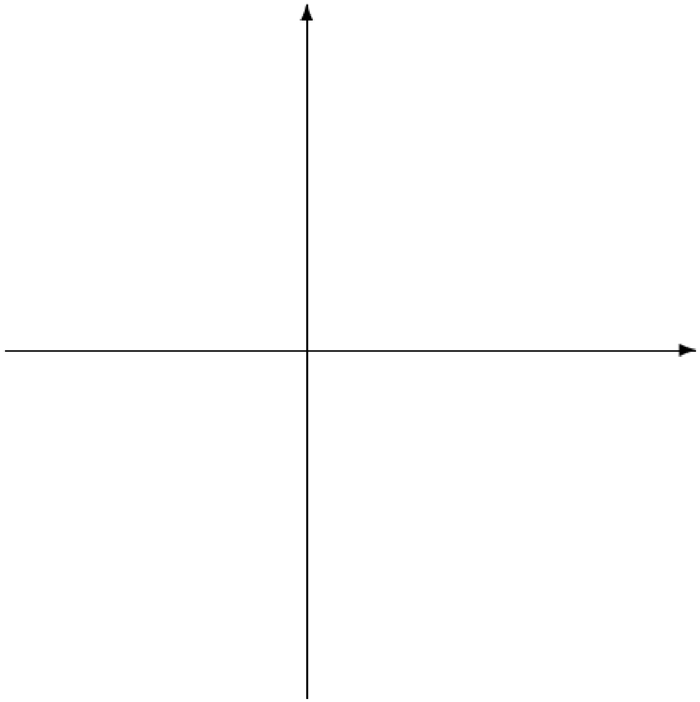

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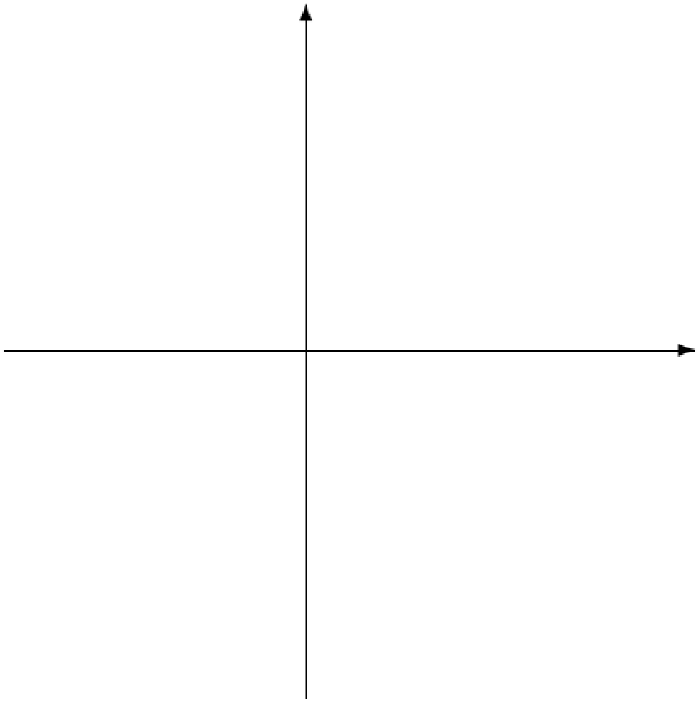

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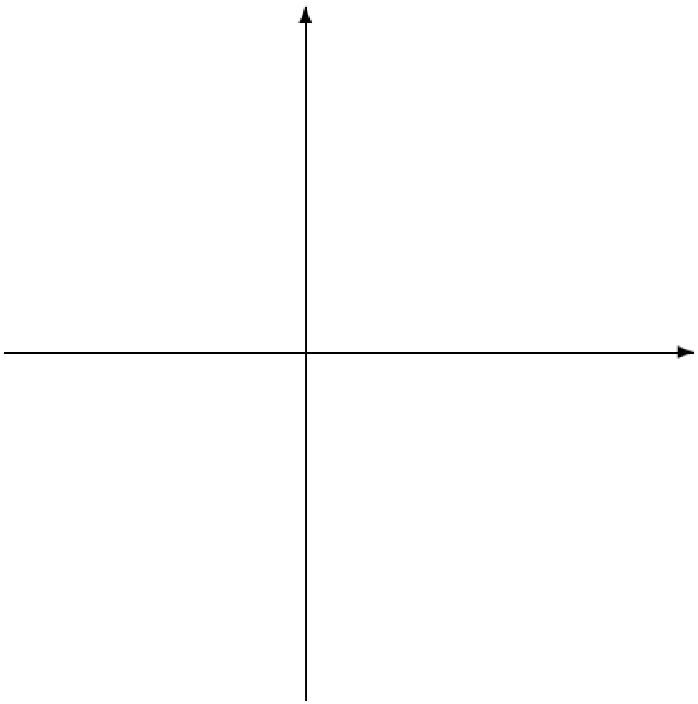

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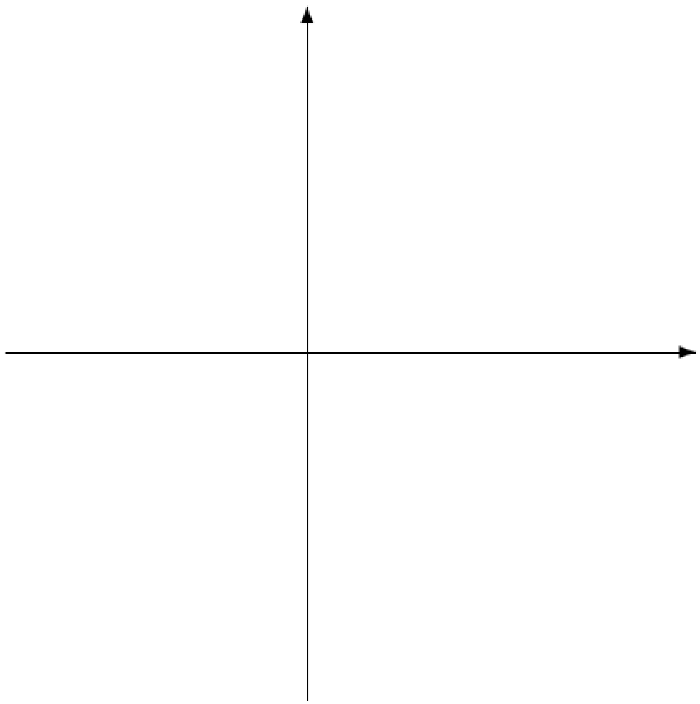

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$$\lim_{x \to \pm\infty} f(x) = \pm\infty$$

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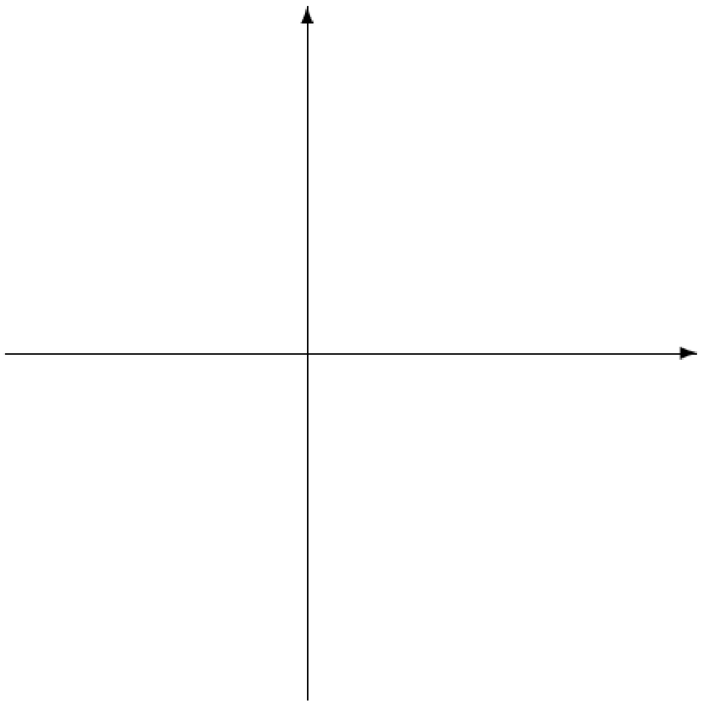
$$f'(x) = \frac{x^2-2x-3}{(x-1)^2}$$

$$f''(x) = \frac{8}{(x-1)^3}$$

Esempio III

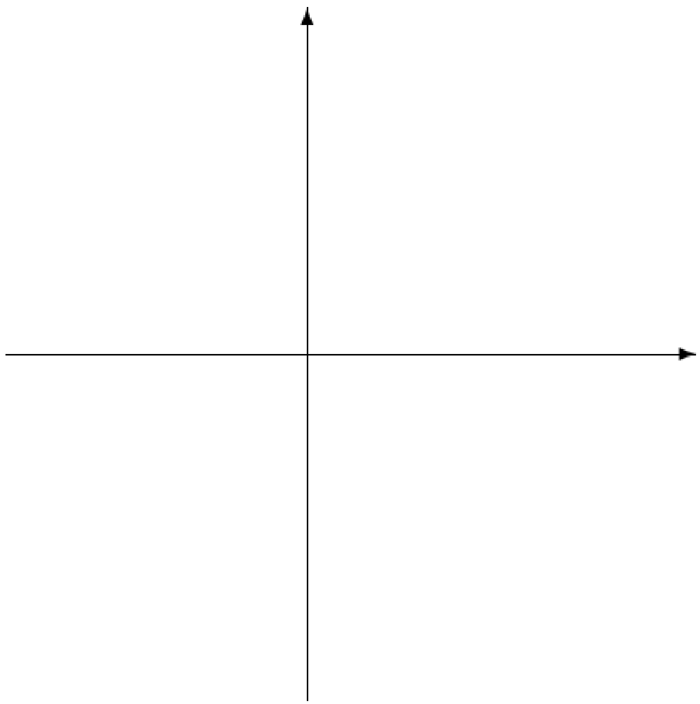
$$f(x) = (x^2 + 2x)e^x$$



Esempio III

$$f(x) = (x^2 + 2x)e^x$$

$$D = \mathbb{R}$$



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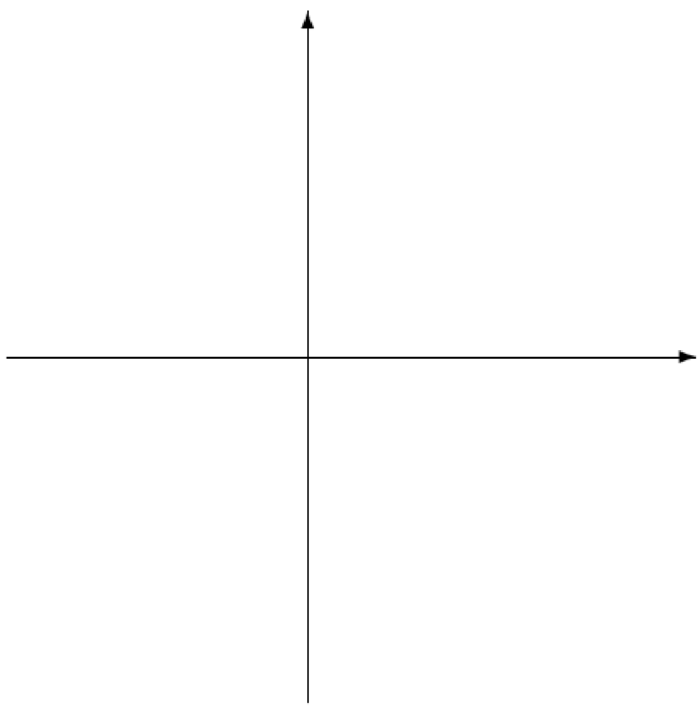

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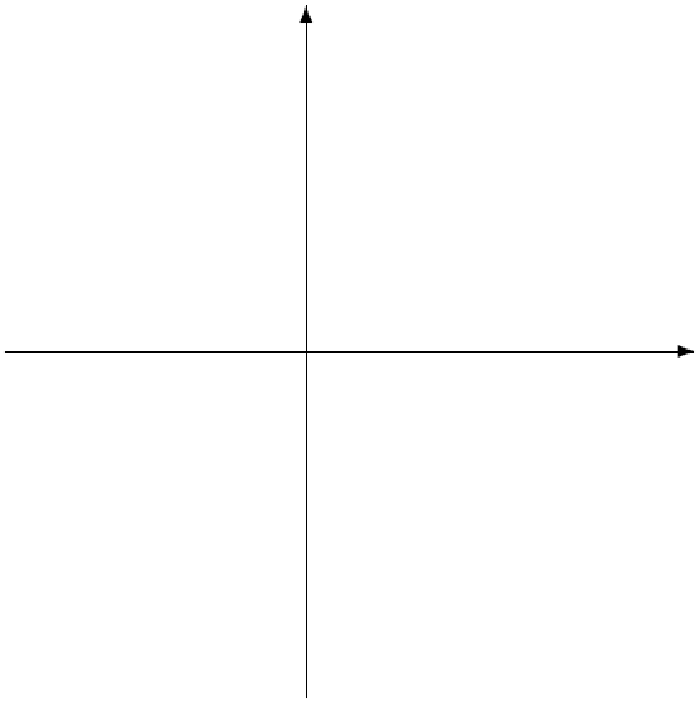

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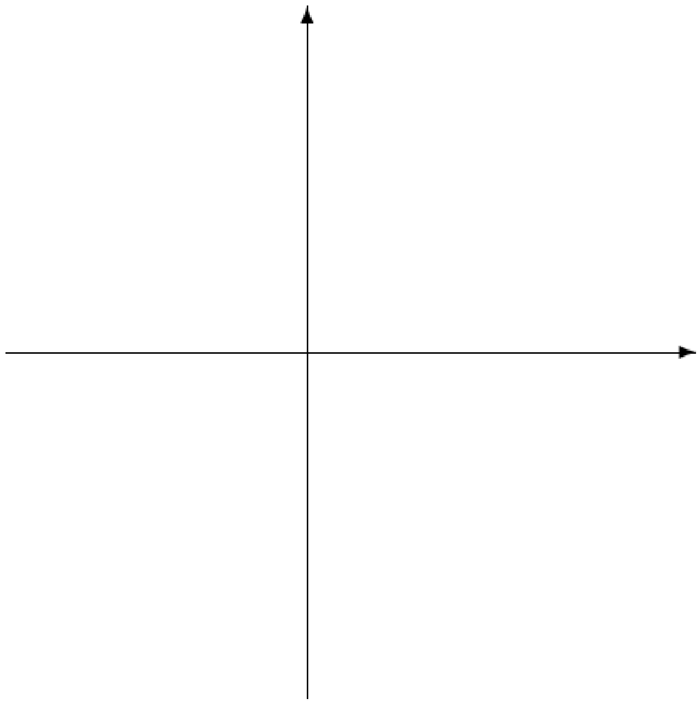

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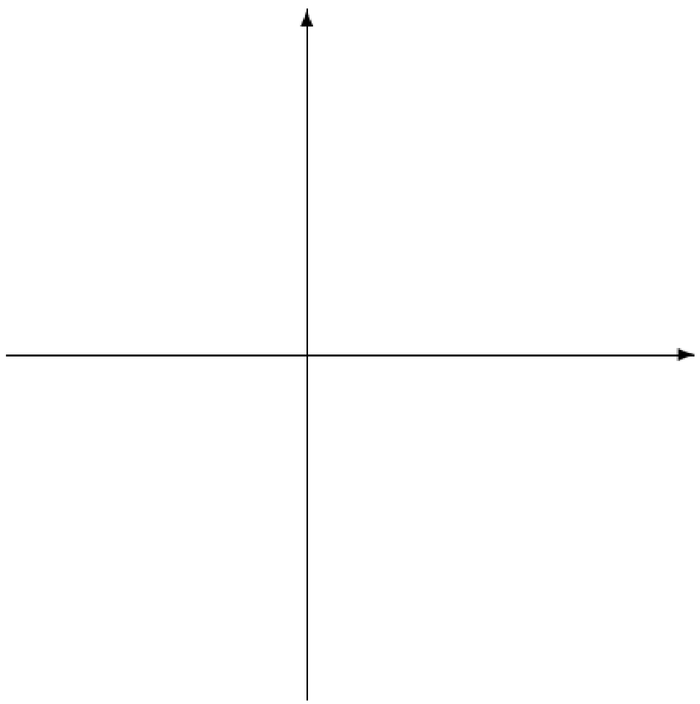

$D = \mathbb{R}$

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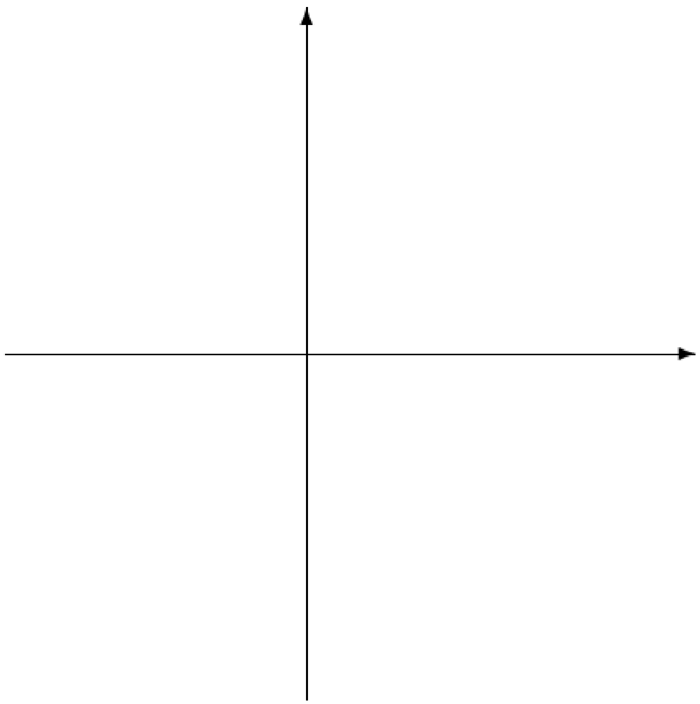
$$f'(x) = (x^2 + 4x + 4)e^x$$

$$f''(x) = (x^2 + 6x + 8)e^x$$

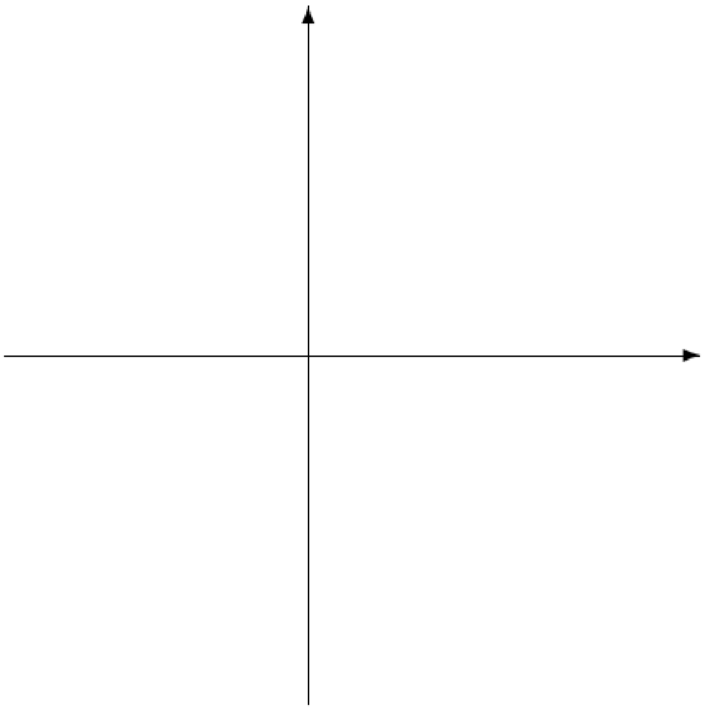
Esempio IV

$$f(x) = \ln(1 + x^2)$$



Esempio IV

$$f(x) = \ln(1 + x^2)$$



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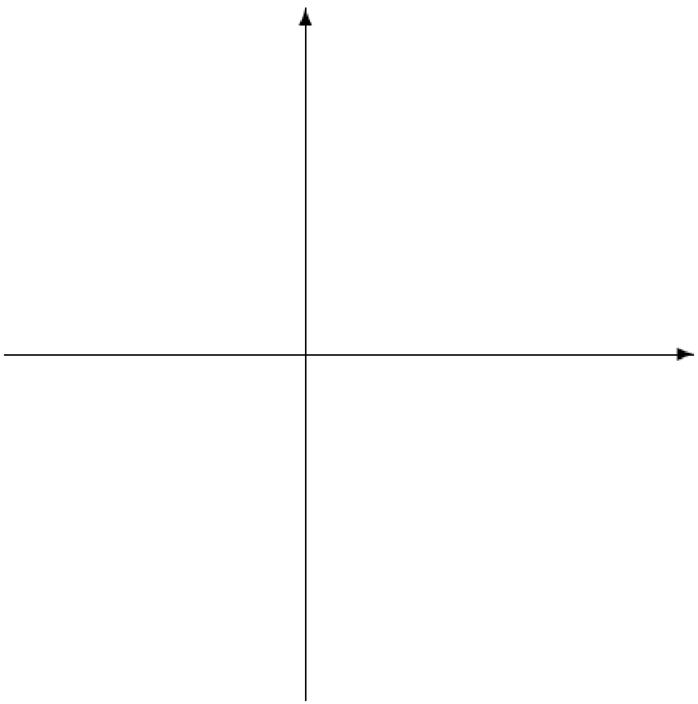

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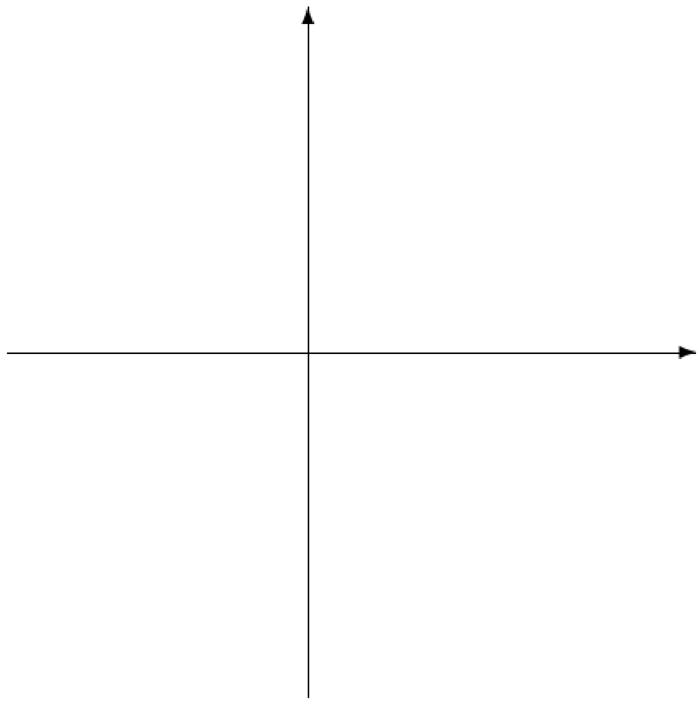

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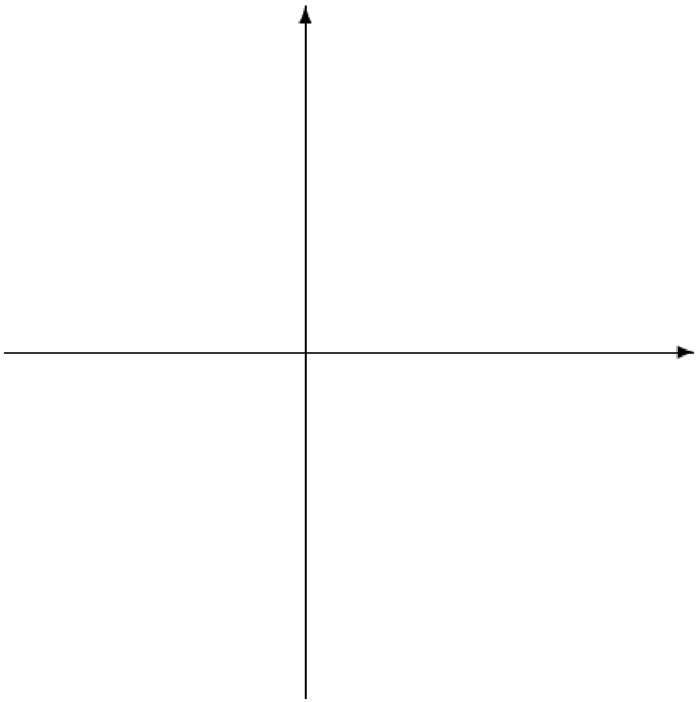

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$$f'(x) = \frac{2x}{1+x^2}$$

$$f''(x) = 2\frac{1-x^2}{(1+x^2)^2}$$