



Metodi matematici per l'Informatica

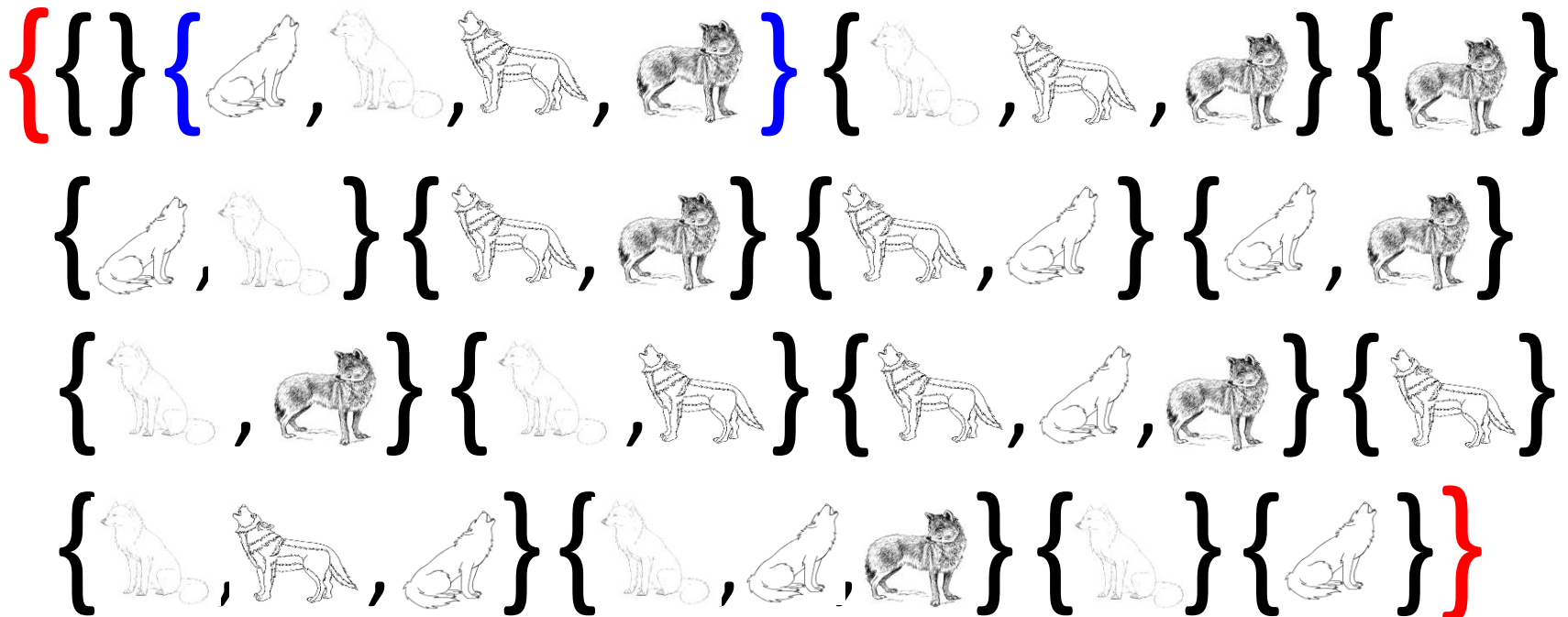
Modulo 2.2 - Operatori su insiemi

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Potenza



L'*insieme delle parti* di A, o insieme potenza, è l'insieme dei sottoinsiemi di A



Potenza



$$A = \left\{ \text{dog}_1, \text{dog}_2, \text{dog}_3, \text{dog}_4 \right\} \quad 4$$

$$\mathcal{P}(A) = \left\{ \{\}, \{\text{dog}_1\}, \dots, \{\text{dog}_1, \text{dog}_2, \text{dog}_3, \text{dog}_4\} \right\} \quad 16$$

$$\{\} \quad 0$$

$$\{\{\}\} \quad 1$$

$$\{\star\} \quad 1$$

$$\{\{\}, \{\star\}\} \quad 2$$

$$\{\star, \odot\} \quad 2$$

$$\{\{\}, \{\star\}, \{\odot\}, \{\star, \odot\}\} \quad 4$$

Potenza

$$| A | = n$$

$$| \mathcal{P}(A) | = 2^n$$

$\mathcal{P}(A)$ si indica spesso con 2^A

Coppie ordinate

$$(a, b) \neq (b, a) \quad (\text{...in genere!})$$

$$\{a, b\} = \{b, a\}$$

$$(a, b) \stackrel{?}{=} \{a, \{b\}\}$$

$$(\{\star\}, \text{☀}) = \{\{\star\}, \{\text{☀}\}\} = (\{\text{☀}\}, \star)$$



Coppie ordinate

$$(a, b) \neq (b, a) \quad (\text{...in genere!})$$

$$\{a, b\} = \{b, a\}$$

$$(a, b) = \{a, \{a, b\}\}$$

$$(\star, \odot) = \{\star, \{\star, \odot\}\} \neq \{\odot, \{\odot, \star\}\} = (\odot, \star)$$

Prodotto cartesiano

$$a \in A, b \in B$$

$$(a, b) = \{a, \{a, b\}\} \subseteq A \cup \mathcal{P}(A \cup B)$$

$$(a, b) \in \mathcal{P}(A \cup \mathcal{P}(A \cup B)) = K$$

$$A \times B = \{x \in K : x = (a, b), a \in A \text{ e } b \in B\}$$

Prodotto cartesiano

$$A = \{\heartsuit, \diamondsuit\} \quad B = \{\clubsuit, \spadesuit\}$$

$$A \times B = \{(\heartsuit, \clubsuit), (\heartsuit, \spadesuit), (\diamondsuit, \clubsuit), (\diamondsuit, \spadesuit)\}$$

$$A \times B = \{x \in K : x = (a, b), a \in A \text{ e } b \in B\}$$

Prodotto cartesiano

$$A \times \emptyset = \emptyset \times A = \emptyset$$

$$A \times (B \cup C) = (A \times B) \cup (A \times C)$$

$$A \times (B \cap C) = (A \times B) \cap (A \times C)$$

$$(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$$



$$(A \times B) \cup (C \times D) = (A \cup C) \times (B \cup D)$$

Prodotto cartesiano

Se $(x, y) \in (A \times B) \cup (C \times D)$ allora

$(x, y) \in (A \times B)$ oppure $(x, y) \in (C \times D)$

se $(x, y) \in (A \times B)$ allora $x \in A$ e $y \in B$

e dunque $x \in (A \cup C)$ e $y \in (B \cup D)$

e dunque $(x, y) \in (A \cup C) \times (B \cup D)$

$$(A \times B) \cup (C \times D) \subseteq (A \cup C) \times (B \cup D)$$

Prodotto cartesiano

Se $(x, y) \in (A \times B) \cup (C \times D)$ allora

$(x, y) \in (A \times B)$ oppure $(x, y) \in (C \times D)$

se $(x, y) \in (C \times D)$ allora $x \in C$ e $y \in D$

e dunque $x \in (A \cup C)$ e $y \in (B \cup D)$

e dunque $(x, y) \in (A \cup C) \times (B \cup D)$

$$(A \times B) \cup (C \times D) \overset{\checkmark}{\subseteq} (A \cup C) \times (B \cup D)$$

Prodotto cartesiano

$$A = \{\heartsuit, \diamondsuit\} \quad C = \{\clubsuit, \spadesuit\} \quad B = \{\text{torre}, \text{cavallo}\} \quad D = \{\text{pedone}, \text{re}\}$$

$$\heartsuit \in A \cup C \quad \text{re} \in B \cup D$$

$$(\heartsuit, \text{re}) \in (A \cup C) \times (B \cup D)$$

$$(\heartsuit, \text{re}) \notin (A \times B) \cup (C \times D)$$

$$(A \times B) \cup (C \times D) \neq (A \cup C) \times (B \cup D)$$

Fenomeni accidentali...

$$3 \in 7$$

$$\{a, b\} \in (a, b)$$

$$2 = (0, 0) \quad \text{😄}$$

$$2 = \{\{\}, \{\{\}\}\} = \{0, \{0, \textcolor{red}{0}\}\} = (0, 0)$$

... surreali.

