

Logical Identities

- $\neg(p \rightarrow (q \rightarrow p))$
Step 1: $p \wedge \neg(q \rightarrow p)$ (Since $\neg(p \rightarrow q) \equiv p \wedge \neg q$)
Step 2: $p \wedge q \wedge \neg p$ (Since $\neg(p \rightarrow q) \equiv p \wedge \neg q$)
Step 3: $p \wedge \neg p$ (Since $p \wedge \neg p$ will always equal false, q has no influence on the identity)
- $\neg((p \wedge q) \rightarrow (q \vee p))$
Step 1: $(p \wedge q) \wedge \neg(q \vee p)$ (Since $\neg(p \rightarrow q) \equiv p \wedge \neg q$)
Step 2: $p \wedge q \wedge \neg q \wedge \neg p$ (De Morgan's law. This expression will always evaluate to false since it has p and $\neg p$ and q and $\neg q$)

Logical Consequence

- Jimmy is smart, smart people are rich, therefore, Jimmy is rich.
This inference is valid since it states that all smart people are rich, therefore Jimmy must be rich because he is smart.
- Islands are surrounded by water, Puerto Rico is surrounded by water, therefore, Puerto Rico is an island.
This inference is not valid. It was stated that all islands are surrounded by water, but it did not say whether all lands surrounded by water are islands or not. So just knowing that Puerto Rico is surrounded by water is not enough to conclude that Puerto Rico is indeed an island based on the claims made.

Translating English Sentences into Formulas

- Not every student in CSE015 plays a musical instrument Translation: $\forall x \neg(S(x) \rightarrow M(x))$
- A person is either a CSE015 student or plays a musical instrument, but not both. Translation: $\forall x (S(x) \vee M(x)) \wedge \neg(S(x) \wedge M(x))$
- There exists at least one student in CSE015 who does not play musical instrument.
Translation: $\exists x (S(x) \wedge \neg M(x))$

Logical equivalence

$$\forall x(A(x) \wedge B(x)) \equiv \forall x(A(x) \rightarrow B(x))$$

$A(x)$	$B(x)$	$A(x) \wedge B(x)$	$A(x) \rightarrow B(x)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

As we can see by the truth table, this logical equivalence is not valid.