Nested Quantifiers

1. $\exists$x $\forall$y A(x,y) Value: True
Reasoning: when x = 0, x times any y will equal 0.
2. $\exists$x $\exists$y B(x,y) Value: True
Reasoning: there is multiple values for x and y that result in x+y = 0. For example, x = -1 and y = 1.
3. $\forall$x $\exists$y A(x,y) Value: True
Reasoning: when y = 0, y times any x will equal 0.
4. $\exists$x $\forall$y (A(x,y) $\wedge$ B(x,y)) Value: False
Reasoning: only 1 x value makes x*y=0 always true, x = 0. And if you plug in x = 0 to B(x,y) , you can see that there is only 1 y value that will result in x + y = 0, that is y = 0. But for the rest on the y values, the result will not be 0. Therefore, there does not exist a value x for all y that would satisfy both A(x,y) and B(x,y).
5. $\exists$x $\exists$y (A(x,y) $\wedge$ $\neg$ B(x,y)) Value: True
Reasoning: For example, take x = 0 and y = 5. If you plug them in A(x,y), you get 0*5 = 0. However, if you plug them in B(x,y), you get 0 + 5 = 0, which is not true. So there exists at least one value for x and for y that results in A(x,y) being true and $\neg$ B(x,y) being false.