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CSE-015: Discrete Mathematics
Homework 4 / Functions

1. A function is considered to be R to R if f is a function from the real numbers to the real numbers. In other words, if the input of the function is a real number, the function will output a real number.
a) $f(x) = \pm\sqrt{x^2+1}$ This function is R to R because the value inside the square root will always be positive (if the input is real) since x (the input) is squared.
b) $f(x) = 1/x$ This function is NOT R to R because if the input is zero, which is a real number, the output is 1/0, which is undefined.
Undefined is neither real or imaginary.
c) $f(x) = x - x^2$ This function is R to R since when you plug any real number, you just square it (which also results in a real number) and subtract it from itself. So the result will always be real if the inout is real.

2. Finding Domain and Range for functions:
a) The function that assigns to each positive integer the largest perfect square not exceeding this integer. Domain: set of positive integers (Z+). Range: set of perfect square values.
b) The function that assigns to each bit string the number of ones in the string minus the number of zeros in the string. Domain: set of bit strings (of any length). Range: all integers (Z).
c) The function that assigns to each bit string twice the number of zeros in that string. Domain: all bit strings. Range: all even integers (zero included).
d) The function that assigns the number of bits left over when a bit string is split into bytes (which are blocks of 8 bits). Domain: set of bit strings (of any length). Range: integers in the interval from 0 to 8 (zero included, eight excluded).
e) This part answered itself so there is no need to repeat it.

3. Determine whether the function $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$
a) $f(m, n) = m^2 - n^2$ this function is not onto because there is no integer values for m and n(in Z) which will result in $f = 2$. In order for a function to be onto, there should be m and n integer value for any f=c (where c is an integer in Z). In this case, f can only equal to 1 or an integer greater than 2.
(since our condition is $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$).
b) $f(m, n) = m + n + 1$ this function is onto because for any integer c there will be an integer m and n such that $f(m, n) = c$.
c) $f(m, n) = |m| - |n|$ this function is onto because for any integer c there will be an integer m and n such that $f(m, n) = c$.
d) $f(m, n) = m^2 - 4$ this function is not onto because there is no integer values for m and n(in Z) which will result $f = 1$. In this case, f can only equal to -4,-3,0,5... As you can see, there is no 1.